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PRESSURE EFFECT OF A SOUND WAVE ON A CRYOGENIC FLUID

by Fritz Kramer

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PRESSURE EFFECT OF A SOUND WAVE ON A CRYOGENIC FLUID

SUMMARY

It has been observed that a sound wave incident upon the surface of a liquid in an open container can produce a positive pressure differential between the liquid and the ambient atmosphere. This pressure is noticeable only when the liquid is close to its boiling point. The proposal has been made to utilize such a pressure differential for keeping a cryogenic propellant settled at the bottom of its tank during gravity-free flight in space. The report shows that this pressure can be explained as an effect of cavitation of the liquid during the negative half-cycle of the acoustic pressure wave.

INTRODUCTION

When a sound wave encounters a discontinuity in its path of propagation such as the boundary between two dissimilar materials, only a part of the sound wave will be transmitted into the second medium, while the remaining part will be reflected back into the first medium. This reflection of the sound energy causes the reflecting surface to experience a pressure which is commonly known as the sound pressure. This sound pressure is a periodic function of time according to the periodic character of the sound wave; however, its overall, integrated effect on the surface upon which it acts is zero, since the effect of the positive half-wave is fully compensated by the effect of the subsequent negative half-cycle of the pressure wave. It has been demonstrated, however, that a positive pressure differential can be observed between a liquid in an open container and the atmosphere outside the container, if the liquid is near its boiling point and its surface is subjected to an acoustic sound field. A typical liquid close to its boiling point may be represented by a cryogen under ambient atmospheric pressure.

This phenomenon of a positive pressure component has been termed "acoustical pumping," and it has been postulated that this positive pressure could be utilized to cause residuals of cryogenic propellants to agglomerate at a container wall opposite a sound source when the container system is in a gravity-free environment.

This report shows that the phenomenon of "acoustical pumping" can be explained by cavitation of the liquid during the negative pressure half-cycle of the incident sound wave.

EFFECT OF SOUND WAVE INCIDENT UPON A FLUID SURFACE

The apparatus in which the positive pressure component has been observed is shown schematically in Figure 1. Before an attempt is made to explain the

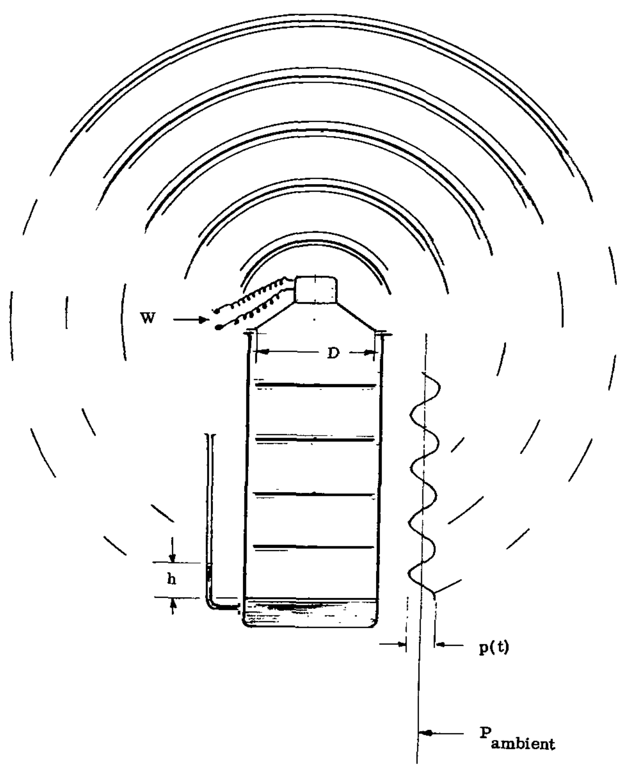


FIGURE 1. SCHEMATIC OF DEMONSTRATION APPARATUS

observed pressure differential, it is necessary to reiterate the process which takes place when a sound wave encounters a discontinuity, such as the interface between the air and the liquid in the container. It is known that only a part of the sound wave normally incident upon the liquid surface will be transmitted into the liquid. The remaining part of the sound wave will be reflected back into the air or vapor on top of the liquid. The intensities of the reflected and the transmitted wave are known from the physical properties of the two media; the relevant equations are given in Appendix A. These equations have been published in the available literature on acoustics [1, 2], and are given in the Appendix to make this report self contained. These equations are valid for the idealized conditions that (1) the second medium extends to infinity or is terminated in such a manner as to resemble acoustically a medium of semi-infinite extension; and (2) that the surface of the second medium at the interface is optically flat and smooth and remains smooth and unaffected by the presence of the sound wave.

In the demonstration apparatus, however, the liquid was only about 5 cm (2 in.) deep as compared to the container diameter of about 30 cm (12 in.), or to the wavelength of from 30 to 60 cm (12 to 24 in.) for the employed sound frequencies. Also, the surface of the liquid did not remain optically smooth, but became rippled and somewhat agitated under the influence of the sound vibration. It is evident that the conditions underlying the theory were not fully realizable in the demonstration apparatus. The reflection and transmission actually taking place in the apparatus must, therefore, to some degree be different from the theoretically predicted values. However, for the purpose of this report, the theoretical relations as given by the equations in Appendix A are accepted also for conditions as they prevailed in the demonstration apparatus. As a consequence, the incident wave should be almost totally reflected from the liquid surface, and the pressure experienced by the surface of the liquid is expected to be twice the pressure in the incident wave.

If a liquid is subjected to a pressure, it will transmit this pressure as an omnidirectional stress through its entire volume. This is basically the interpretation of equation (A-9) when viewed from the field of fluid mechanics. It is also known that a liquid has an unlimited capacity for supporting compressive stresses (positive gage or absolute pressure), but that it is limited in its capability to transmit a tensile stress (or negative gage pressure). This limit is established by the vapor pressure of the liquid. If the tensile stress in the liquid tends to be lower than the vapor pressure, the liquid will tear or separate, and form cavities. These cavities are filled with saturated vapor of the liquid. This formation of vapor-filled cavities is known as cavitation.

The vapor pressure is a function of temperature. The relation $p_V = f(T)$ is shown for water in Figure 2. It shows that at 372.25°K (99.1°C) water has a vapor pressure of 1 kg/cm^2 or one atmosphere absolute. At

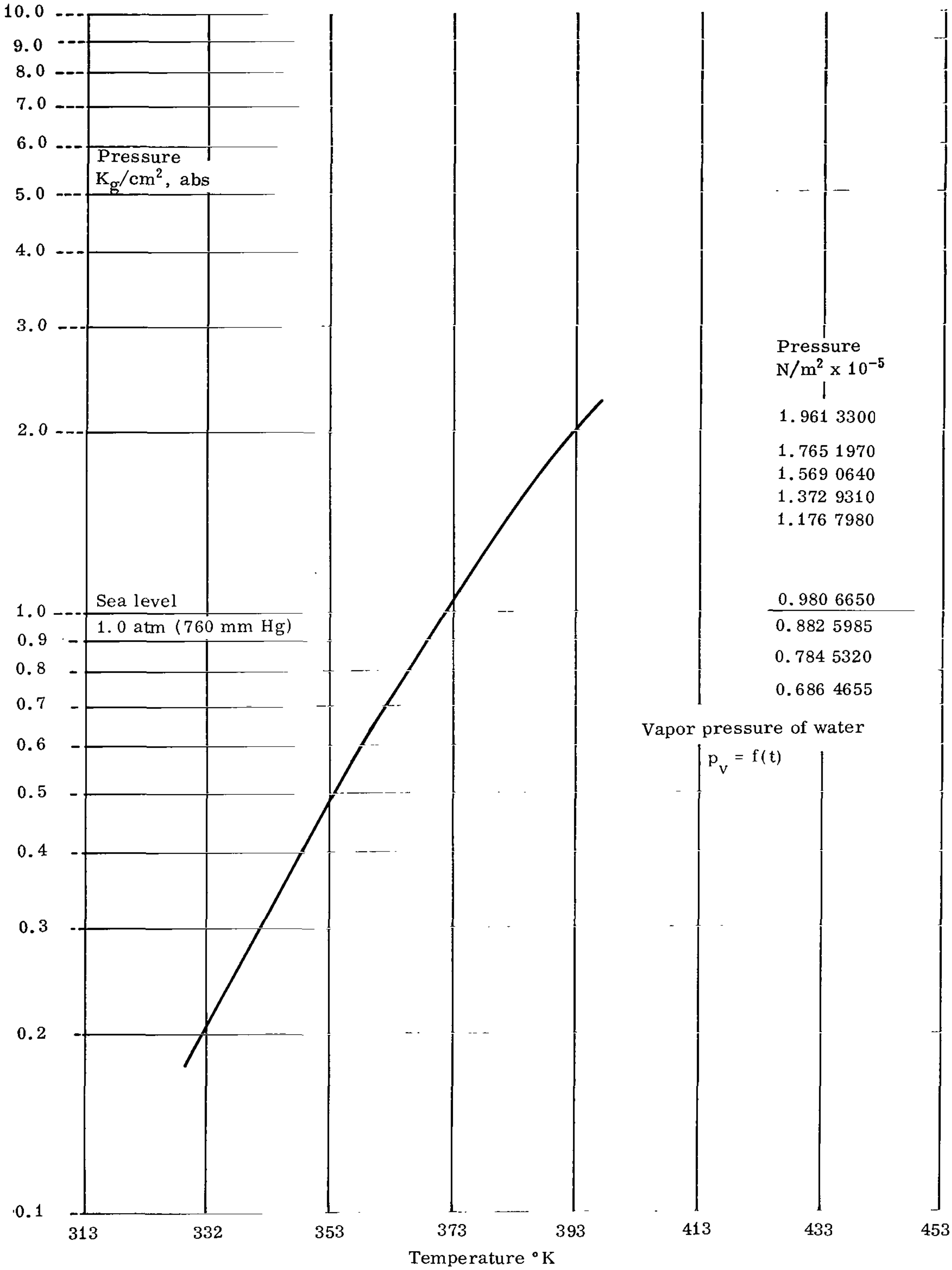


FIGURE 2. VAPOR PRESSURE OF WATER AS A FUNCTION OF TEMPERATURE

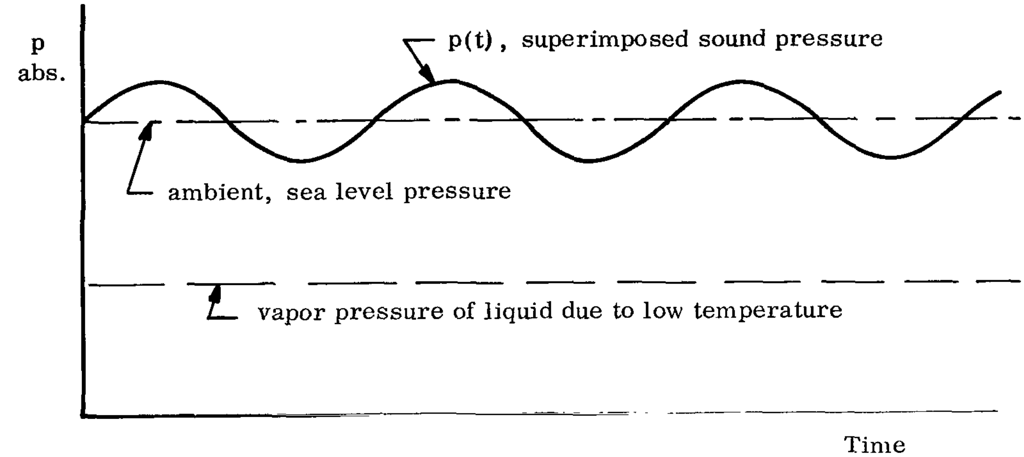
354.15°K (81°C) the vapor pressure is only 0.5 kg/cm^2 or one-half atmosphere. Therefore, water at 372.25°K (99.1°C) cannot support any tensile stress (or negative gage pressure below one atmosphere ambient) but it can sustain a tensile stress or negative gage pressure of -0.5 kg/cm^2 if its temperature is lowered to 354.15°K (81°C). If the attempt were made to lower this pressure to a more negative value below -0.5 kg/cm^2, the liquid would again cavitate, that is, form cavities filled with saturated vapor at 354.15°K (81°C); the pressure in the liquid would not change in spite of the added vapor volume.

If a periodically varying pressure is superimposed to the ambient pressure which prevails on the surface of a liquid, a total absolute pressure on the liquid's surface results as a function of time as shown in Figure 3(a). If the temperature of the liquid is low, the vapor pressure pertaining to this low temperature will also be well below the absolute pressure acting on the surface.

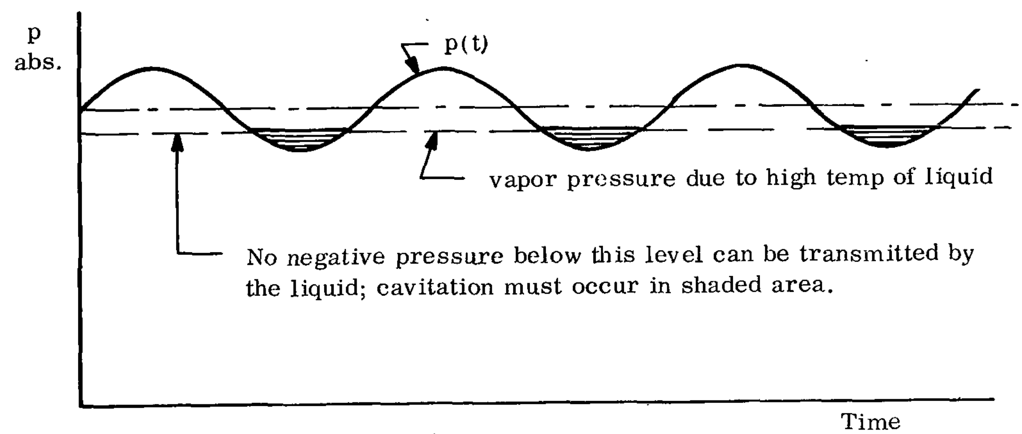
If the temperature of the liquid is now raised, the vapor pressure of the liquid will also rise until it falls within the negative half-wave of the superimposed pressure fluctuation. This is illustrated in Figure 3(b). Since the liquid cannot sustain a negative pressure below its vapor pressure, cavitation must now occur during that part of the negative pressure cycle which falls below the value of the vapor pressure. This is indicated by the shaded part in Figure 3(b). It is also to be noted that the negative pressure amplitude below the vapor pressure within the shaded part of the negative half-wave is not transmitted into the liquid; therefore, a positive pressure component above ambient results from the integral $\int p \cdot dt$ taken over any number of complete cycles.

In the extreme case illustrated in Figure 3(c), the temperature of the liquid has been raised to the boiling point. In this case, only the positive pressure can be transmitted through the liquid; the negative pressure below ambient causes only cavitation or a release of additional vapor in excess of the boil-off rate caused by external heating. In this case, the positive differential pressure between the liquid in the container and the ambient pressure outside the container will attain its maximum value.

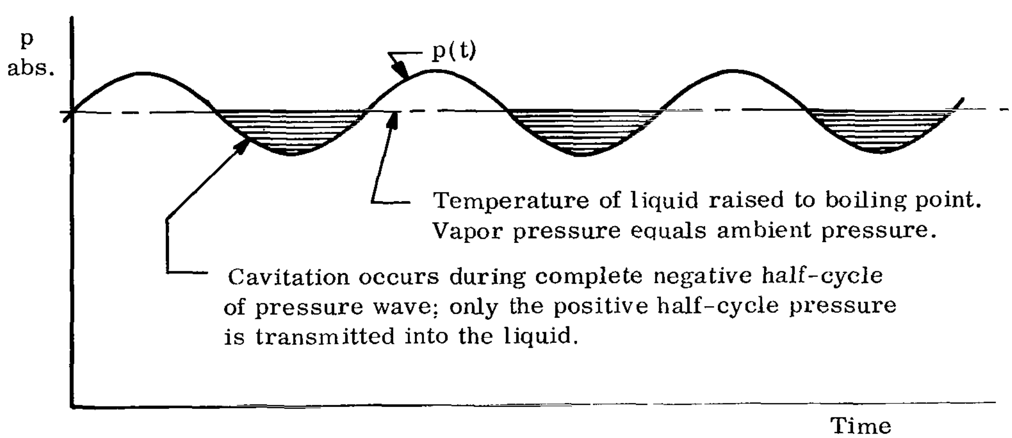
Cavitation or vapor formation in the examples cited above takes place only at the surface of the liquid. This is caused by the gravitational field, which causes the pressure in the liquid to increase toward the bottom of the container. Any vapor release caused by the reduction of the pressure on the liquid's surface must, therefore, take place at the surface where the pressure in the liquid is closest to the vapor pressure.



(a) Cold Liquid



(b) Hot Liquid



(c) Boiling Liquid

FIGURE 3. SOUND PRESSURE WAVE SUPERIMPOSED TO ATMOSPHERIC PRESSURE IN RELATION TO VAPOR PRESSURE OF COLD LIQUID, HOT LIQUID, AND BOILING LIQUID

ANALYSIS OF EFFECT OF CAVITATION ON PERIODIC PRESSURE VARIATION

The periodic pressure variation of a perfect sound wave can be expressed mathematically by a trigonometric function as a pure sine wave. This is not possible for the pressure oscillation acting on the surface of the liquid if the negative half-wave is only partly transmitted and effective because of the cavitating liquid. However, the clipped-off pressure wave shown in Figure 3(a) and (b) can be represented by a Fourier series expansion, which also allows analysis of the particular characteristics and side effects of such a disturbed pressure fluctuation. This has been done in Appendix B for the pressure cycle shown in Figure 4.

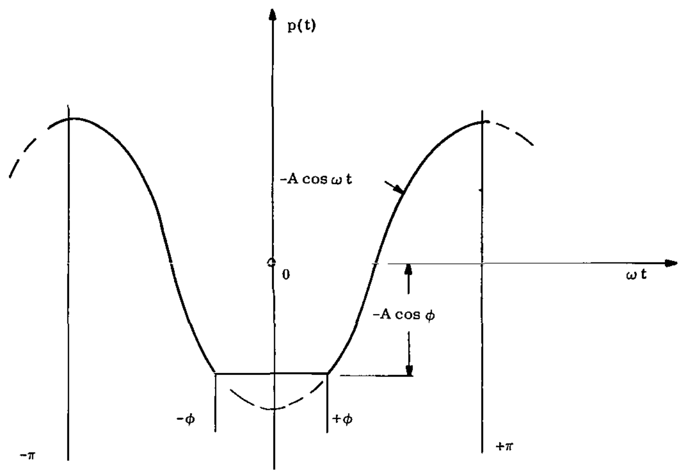


FIGURE 4. PERIODIC PRESSURE FUNCTION LIMITED BY CAVITATION

The analysis shows that this particular wave shape produces a constant, positive pressure component, the magnitude of which depends only on the reduction of the negative pressure peak by the vapor pressure. It also shows that an infinitely large number of higher harmonics is now introduced. These higher harmonics may produce special effects in the vibration pattern of the liquid and the container, and may actually affect the positive pressure component beyond that magnitude which is given by the constant term. Additional cavitation may be caused by the higher frequency harmonics because of their high acceleration levels. These effects, however, are disregarded here.

NUMERICAL ANALYSIS OF ACOUSTICAL PRESSURE CAUSED BY CAVITATION

A loudspeaker may be used as a source of acoustical energy in the test arrangement illustrated in Figure 1. The electrical energy, W, supplied to the terminals of the speaker is not converted fully into acoustical energy because of some losses inherent in the energy conversion. These losses are

1. Copper losses (electrical energy converted into heat in the speaker coil)

2. Hysteresis and induction losses (caused by the rapidly changing magnetic field)

3. Mechanical losses (caused by the deformation of the speaker membrane and its internal material damping)

4. Losses from the non-isentropic boundary layer attached to the membrane surface.

There might be additional losses. The actual efficiency of the sound source used in a test arrangement will have to be determined by calibration.

The sound intensity generated by the loudspeaker is then

$$I_s = \eta_c \frac{W}{\frac{D^2 \pi}{4}} \qquad \mathrm{W/m^2} \qquad (1)$$

where D = diameter of speaker membrane (m)
W = electrical energy fed into the speaker (W)
η_c = conversion efficiency (-) .

The sound energy of a plane, free progressive wave is

$$I_W = \frac{p_0^2}{\rho_0 \cdot c} \qquad \mathrm{W/m^2} \qquad (2)$$

where p_0 = rms value of the sound pressure (N/m^2)

$\rho_0 \cdot c$ = characteristic impedance $(N\text{-sec}/m^2)$.

The sound energy generated by the speaker will be radiated primarily into the interior of the container, its magnitude depending on the directivity characteristics of the speaker. Part of the sound energy remains on the outside of the system as indicated in Figure 1. The ratio of these two energies may be denoted by ϵ. The sound energy inside the container is then given by

$$I_{in} = \epsilon \cdot \eta_c \frac{W}{\frac{D^2 \pi}{4}} \tag{3}$$

and the sound intensity outside the container is

$$I_{out} = (1 - \epsilon) \cdot \eta_c \frac{W}{\frac{D^2 \pi}{4}} \quad . \tag{4}$$

Without sound attenuation, the rms value of the sound pressure inside the cylindrical container on the surface of the liquid is obtained from equations (2) and (3) as

$$p_0 = \left[\frac{4\epsilon \cdot \eta_c}{\pi} (\rho_0 c) \frac{W}{D^2} \right]^{1/2} \quad . \tag{5}$$

Introducing the numerical value of the impedance for air at 295.15° K (22° C) and 751 mm Hg pressure as $\rho_0 \cdot c = 407\ N\text{-sec}/m^2$ results in

$$\frac{p_0}{\epsilon \cdot \eta_c \cdot W^{1/2}} = 22.7 \frac{1}{D} \quad . \tag{6}$$

The diameter, D, initially taken as the diameter of the speaker membrane, is now to be taken as the inside diameter of the container; however, the two diameters should not differ widely from one another.

It is shown in Appendix A that because of the almost complete reflection of the incident sound wave from the surface of the liquid, the instantaneous sound pressure p(t) is twice the pressure of the incident wave. However, the pressure amplitude of the incident wave is $\sqrt{2}$ times the rms pressure p_0. Thus, the amplitude of the pressure acting on the surface of the liquid is

$$A = 2 \cdot \sqrt{2} \cdot p_0 = 2.825\ p_0 \tag{7}$$

Combining equations (6) and (7) finally results in

$$\frac{A}{\epsilon \cdot \eta_c \cdot W^{1/2}} = 64 \frac{1}{D} \quad . \tag{8}$$

Equation (8) states that 1 W of electrical energy supplied to a loudspeaker of efficiency η_c and a diameter of 1 m would produce a pressure amplitude of 64 N/m^2. Since a barometric pressure of 751 mm Hg equals 10^5 N/m^2, the pressure amplitude expected at the surface of the liquid, expressed in mm Hg, is

$$A_{Hg} = (\epsilon \cdot \eta_c) \cdot 0.482 \frac{W^{1/2}}{D} \quad \text{mm Hg.} \tag{9}$$

Expressed in head of water, the pressure amplitude is

$$A_W = (\epsilon \cdot \eta_c) \cdot 6.55 \frac{W^{1/2}}{D} \quad \text{mm water.} \tag{10}$$

This value of A, if introduced into equation (B-10), yields the positive pressure component which can be expected as the "acoustical pumping" pressure for a given electrical input into the speaker

$$p+ = \frac{0.482}{\pi} \left(\frac{\eta_c \cdot \epsilon \cdot W^{1/2}}{D} \right) \left[\sin\phi - \phi \cos\phi \right] \quad \text{mm Hg.} \tag{11}$$

The value of cos ϕ is found from Figure 4 as

$$\cos\phi = \frac{p_a - p_v}{A} \quad ; \quad 0 < (p_a - p_v) < A \quad . \tag{12}$$

Using equation (8), the final value is

$$\cos\phi = \frac{D}{64 \cdot \epsilon \cdot \eta_c \cdot W^{1/2}} \quad (p_a - p_v) \quad . \tag{13}$$

CONCLUSIONS AND RECOMMENDATIONS

The explanation that the observed positive pressure in the liquid may be caused by cavitation seems to be supported by the analysis of the pressure wave with clipped-off negative pressure amplitude. It would be of great interest to verify the numerical values of pressure obtained from 1 W of electrical energy by performing some well controlled but simple tests. The frequency of the imposed sound wave should be in the audible range. The pressure observed in such a test could then be considered an indication of the validity of the theory developed in this report.

It would also be of great interest to determine the effect of the sound frequency of much shorter wavelengths. According to the results shown in Appendix B, the frequency should have no influence on the magnitude of the pressure. However, it is known that ultrahigh frequency sound has some directional characteristics which may cause a deviation from the results shown in Appendix B. The extension of the sound frequency into the ultrasonic range would, therefore, be very desirable.

Generally, the omnidirectional pressure in the liquid, as generated by the sound wave, will not be sufficient in itself to keep the liquid settled at a predetermined location in the tank. The effect of the acoustical pressure on the liquid is the same as a commonly performed tank pressurization from an external pressure source. It can not cause the liquid to stay agglomorated at a definite place in the tank under a gravity-free environment. To accomplish this requires a pressure gradient within the fluid, similar to the static pressure increase with depth in the liquid when subjected to gravity, or similar to the pressure distribution within a rotating fluid due to centrifugal forces. Such a pressure gradient cannot be created by the acoustic pressure investigated in this report. However, sound waves with ultrahigh frequencies may produce a pseudo pressure gradient due to their directional characteristics. For this reason, tests in the ultrasonic frequency range appear to be very desirable.

George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Huntsville, Alabama, April 25, 1967
103-19-05-00-62

APPENDIX A

REFLECTION OF A PLANE WAVE FROM A PLANE BOUNDARY BETWEEN TWO MEDIA

The plane boundary between media I and II establishes the discontinuity at x = 0 in Figure A-1. If p indicates pressure, u the particle velocity, and R the characteristic impedance, a simplified analysis [1, 2] can be performed using the wave equation for sinusoidal waves:

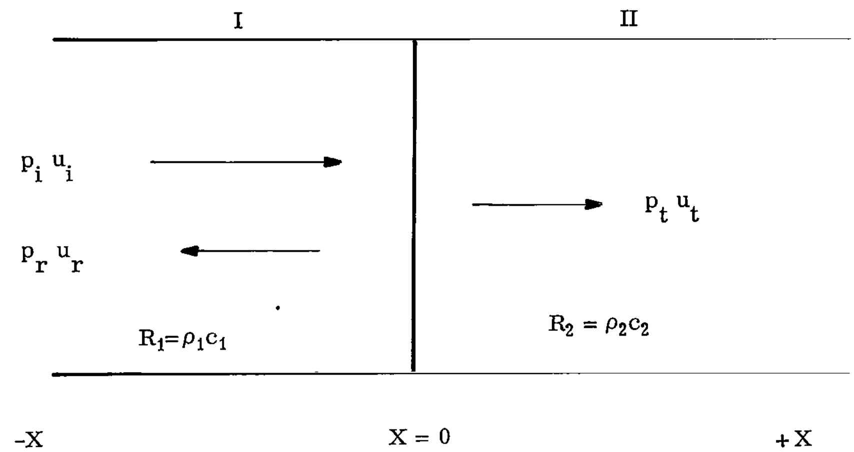


FIGURE A-1. REFLECTION OF PLANE WAVE AT A PLANE BOUNDARY

$$p_i = A_1 \cdot \sin(\omega t - k_1 x) = u_i \cdot R_1 \tag{A-1}$$

$$p_r = B_1 \sin(\omega t - k_1 x) = - u_r \cdot R_1 \tag{A-2}$$

$$p_t = A_2 \sin(\omega t - k_2 x) = u_t \cdot R_2 \ , \tag{A-3}$$

where

$$\frac{p}{u} = \rho \cdot c = R \ , \tag{A-4}$$

A_1, B_1, and A_2 are the pressure amplitudes,

k_1 and k_2 are the wave numbers defined as $\frac{2\pi}{\lambda}$,

and x is the positive direction of the incident wave.

The subscripts i, r, and t refer to the incident, reflected, and transmitted sound wave, respectively.

At the boundary, the pressure and the velocity must be continuous, thus

$$p_t = p_i + p_r \tag{A-5}$$

$$u_t = u_i + u_r \tag{A-6}$$

If equations (A-4) and (A-5) are introduced into equations (A-1), (A-2), and (A-3), the following relations are obtained:

$$A_2 = A_1 + B_1 \tag{A-7}$$

$$R_1 \cdot A_2 = R_2 (A_1 - B_1) \tag{A-8}$$

$$\frac{p_t}{p_i} = \frac{A_2}{A_1} = \frac{2 R_2}{R_1 + R_2} \tag{A-9}$$

$$\frac{p_r}{p_i} = \frac{B_1}{A_1} = \frac{R_2 - R_1}{R_1 + R_2} \quad . \tag{A-10}$$

For the particle velocities follows

$$\frac{u_t}{u_i} = \frac{A_2 R_1}{A_1 R_2} = \frac{2 R_1}{R_1 + R_2} \tag{A-11}$$

$$\frac{u_r}{u_i} = - \frac{B_1}{A_1} = \frac{R_1 - R_2}{R_1 + R_2} \quad . \tag{A-12}$$

The ratios given in equations (A- 9) through (A-12) are the amplitude ratios of particle pressure and particle velocity, respectively.

The transition of sound energy from one medium into the other is defined as the ratio of the intensity of the transmitted wave to that of the incident wave. Intensity is defined as

$$I = \frac{p^2}{\rho \cdot c} = u^2 \cdot \rho \cdot c \quad , \tag{A-13}$$

from which follows

$$\alpha_t = \frac{I_t}{I_i} = \frac{p_t^2 \, R_1}{R_2 \, p_i^2} = \frac{4 \, R_1 \, R_2}{(R_1 + R_2)^2} \tag{A-14}$$

and the ratio of the reflected intensities is obtained as

$$\alpha_r = \frac{I_r}{I_i} = \frac{p_r^2 \cdot R_1}{R_1 \, p_i^2} = \frac{(R_2 - R_1)^2}{(R_1 + R_2)^2} \quad . \tag{A-15}$$

Equations (A-9) through (A-15) show that if R_1 and R_2 are equal, there is no sound reflection; the entire sound energy is transmitted into the second medium.

If both materials are very dissimilar, such that $R_2 \gg R_1$ (as is the case if medium I is air or saturated steam, and medium II is water), then the numerator in equation (A-12) will be negative. This means that the particle velocity changes in phase on reflection; that is, the reflected wave is opposite in phase to the incident wave. The phase of the pressure, however, remains unchanged. If R_2 approaches infinity, as in the case of an absolutely rigid surface, the phase of the particle velocity of the reflected wave is opposite to that of the incident wave by π, and its amplitude is equal to the amplitude of the incident wave; the resultant particle velocity at the boundary is therefore zero. However, since the phase of the pressure does not change, it follows that the pressure at the boundary is doubled. Thus, the reflection results in a standing wave which has a node at the wall, and a pressure amplitude at the wall of twice the amplitude of the incident wave.

APPENDIX B

ANALYSIS OF PRESSURE OSCILLATION LIMITED BY CAVITATION

A periodic function such as shown in Figure 4 can be expressed by a Fourier series expansion of the form

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{n=\infty} \left(a_n \cdot \cos(nx) + b_n \sin(nx) \right) , \tag{B-1}$$

where

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x)\, dx \tag{B-2}$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) \cos nx\, dx \quad , \tag{B-3}$$

and

$$b_n = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) \sin nx\, dx \quad . \tag{B-4}$$

The undisturbed pressure wave is of the form $-A \cdot \cos \omega t$. Cavitation may take place between the values of $\omega t_1 = -\phi$ and $\omega t_2 = +\phi$ at a vapor pressure p_v which limits the negative pressure half-wave to a magnitude of $-A \cdot \cos \phi$. The Fourier coefficients are found as follows:

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(\omega t)\, d(\omega t)$$

$$= \frac{1}{\pi} \int_{-\pi}^{-\phi} - A \cos \omega t\, d(\omega t) + \frac{1}{\pi} \int_{-\phi}^{+\phi} - A \cos \phi\, d(\omega t)$$

$$+ \frac{1}{\pi} \int_{\phi}^{\pi} - A \cos \omega t\, d(\omega t)$$

$$= -\frac{A}{\pi}\left[\sin\omega t \Big|_{-\pi}^{-\phi} + \cos\phi\,\omega t \Big|_{-\phi}^{+\phi} + \sin\omega t \Big|_{\phi}^{\pi}\right];$$

or finally

$$a_0 = -\frac{2A}{\pi}\left[\phi\cos\phi - \sin\phi\right] \quad . \tag{B-5}$$

Likewise,

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(\omega t)\cdot\cos n\,\omega t\cdot d(\omega t) = \frac{1}{\pi}\int_{-\pi}^{-\phi} -A\cos\omega t\cdot\cos n\,\omega t\cdot d(\omega t)$$

$$+\frac{1}{\pi}\int_{-\phi}^{+\phi} -A\cos\phi\cos n\,\omega t\, d(\omega t) + \int_{+\phi}^{\pi} -A\cdot\cos\omega t\cdot\cos n\,\omega t\cdot d(\omega t),$$

which yields

$$a_n = \frac{A}{\pi}\left[\frac{\sin(n-1)\,\phi}{n\cdot(n-1)} - \frac{\sin(n+1)\,\phi}{n\cdot(n+1)}\right] . \tag{B-6}$$

The integrals leading to equation (B-6) are not applicable to the case of $n = 1$; the value of a_1 (for $n = 1$) has to be determined from the original Fourier equation for a_n by introducing $n = 1$ before the integration is performed. This yields

$$a_1 = \frac{1}{\pi}\int_{-\pi}^{-\phi} -A\cos^2\omega t\cdot d(\omega t) + \frac{1}{\pi}\int_{-\phi}^{+\phi} -A\cos\phi\cdot\cos\omega t\cdot d(\omega t)$$

$$+\frac{1}{\pi}\int_{+\phi}^{\pi} -A\cos^2\omega t\cdot d(\omega t) \quad ,$$

or

$$a_1 = -\frac{A}{\pi}\left[\pi - \phi + \cos\phi\cdot\sin\phi\right] \quad . \tag{B-7}$$

Combining equations (B-5), (B-6), and (B-7), the Fourier series expansion for the pressure function shown in Figure 4 reads

$$p(t) = -\frac{A}{\pi}\left[(\phi \cos\phi - \sin\phi) + (\pi - \phi + \cos\phi \sin\phi)\cdot\cos\omega t + \right.$$

$$\left. + \sum_{n=2}^{n=\infty}\left(\frac{\sin(n-1)\phi}{n\quad(n-1)} - \frac{\sin(n+1)\phi}{n\,(n+1)}\right)\cos n\cdot\omega t\right]. \qquad (B\text{-}8)$$

Equation (B-8) shows that for $\phi = 0$ (no cavitation), the pressure is given by

$$p(t)_{\phi=0} = -A\cdot\cos\omega t \quad , \qquad (B\text{-}9)$$

which is the original, undisturbed sound pressure oscillation.

In case of cavitation, $\phi \neq 0$, there is a positive, constant pressure component independent of the sound frequency, but dependent only on the maximum pressure amplitude A and the cavitation angle ϕ. This constant pressure is given by

$$p+ = \frac{A}{\pi}(\phi\cos\phi - \sin\phi) \quad . \qquad (B\text{-}10)$$

The magnitude of this pressure component, p+/A is given in Table B-I and is illustrated in Figure B-1.

TABLE B-I. MAGNITUDE OF POSITIVE PRESSURE

$$p+/A = \frac{1}{\pi}(\sin\phi - \phi\cos\phi)$$

ϕ	p+/A
0	0.00 000
$\pi/12$	0.00 623
$2\pi/12$	0.04 707
$3\pi/12$	0.15 203
$4\pi/12$	0.34 253
$5\pi/12$	0.62 701
$6\pi/12$	1.00 000

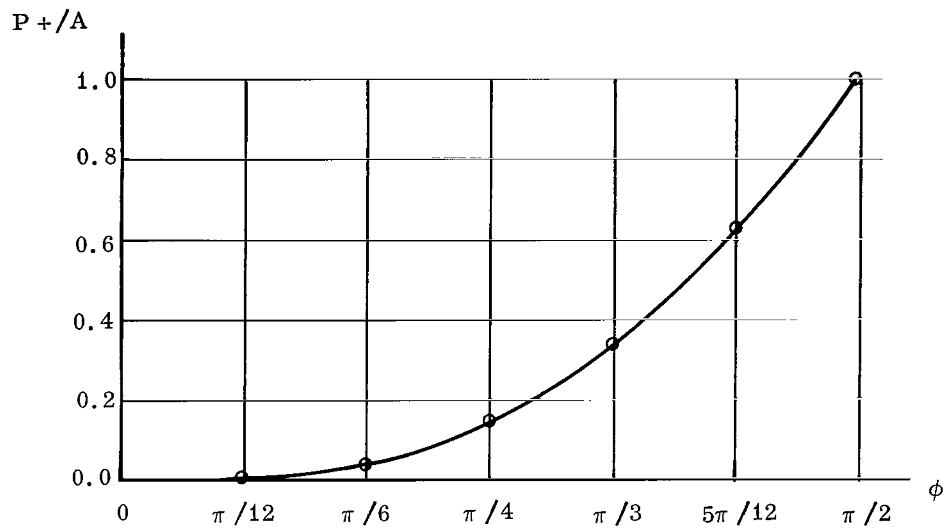


FIGURE B-1. MAGNITUDE OF POSITIVE PRESSURE COMPONENT

$$p+/A = \frac{1}{\pi} (\sin \phi - \phi \cos \phi)$$

In case of cavitation, higher harmonics of the pressure wave are also created with pressure amplitudes as given by the term

$$p_n = \sum_{n=2}^{n=\infty} \frac{A}{\pi} \left[\frac{\sin (n-1) \phi}{n (n-1)} - \frac{\sin (n+1) \phi}{n \cdot (n+1)} \right] \cos n \cdot \omega t \quad .$$

The magnitude of the pressure amplitudes of the first five higher harmonics ($n = 2, 3, 4, 5$) are given in Table B-II for the values of $\phi = \pi/6, \pi/3, \pi/2$, where the case of $\phi = \pi/2$ corresponds to the case of the boiling liquid.

TABLE B-2. PRESSURE AMPLITUDES OF HIGHER HARMONICS

ϕ \ n	2	3	4	5
$\pi/6$	- .02 6526	- .02 2972	- .01 8568	- .01 3783
$\pi/3$	- .13 7833	- .06 8917	- .01 3783	+ .01 3783
$\pi/2$	- .21 2207	0. 0	- .04 2441	0. 0

Figure B-2 shows how closely the true wave shape of the three cases for $\phi = \pi/6$, $\pi/3$, and $\pi/2$ is represented by equation (B-8), when only four higher harmonics (n = 5) are used.

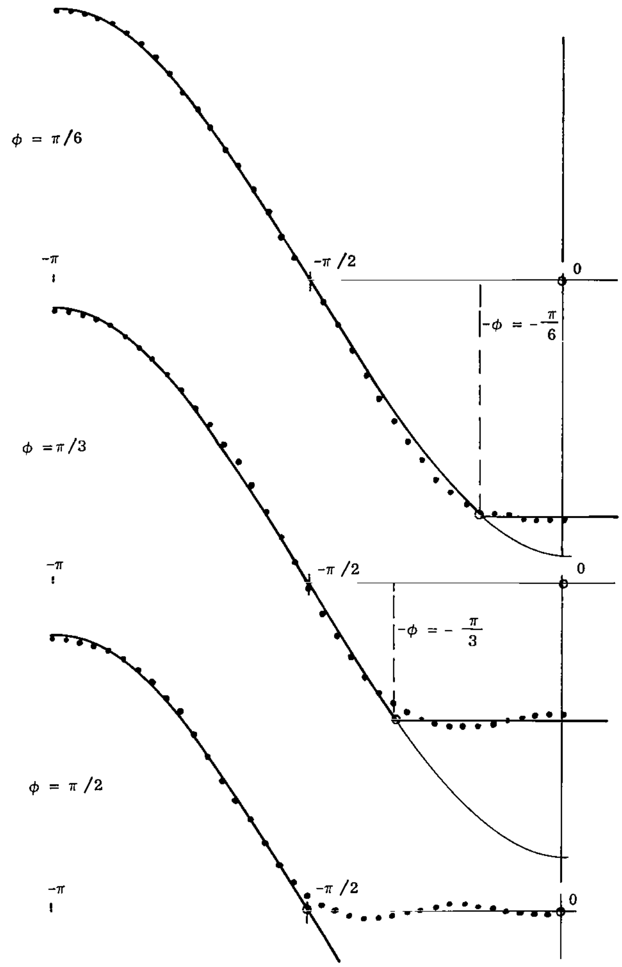


FIGURE B-2. COMPARISON OF EXACT WAVE SHAPE WITH APPROXIMATION BY FOURIER SERIES EXPANSION (for n = 5)

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