# EXPERIMENT ON CRACK PROPAGATION IN A VISCOELASTIC STRIP

N69-38798
CR-106101

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GALCIT SM 69-8

SEPTEMBER 1969



This work was supported by the
National Aeronautics and Space Administration
Research Grant No. NGL-05-002-005
GALCIT 120

and

NASA International University Fellowships
In Space Science
1968-1969

Firestone Flight Sciences Laboratory
Graduate Aeronautical Laboratories
California Institute of Technology
Pasadena, California

## INTRODUCTION AND SUMMARY

Crack propagation in viscoelastic material lies over two fields of solid mechanics, fracture mechanics and viscoelasticity. The classical principle of fracture mechanics "Griffith Theory" gives the condition for crack propagation and the application of visco-elasticity gives time-dependence of crack growth. This research treats crack propagation in a viscoelastic strip with an initial crack at the center.

The first part of this report is an abstract from the research of W. G. Knauss [1] and H. K. Mueller [2]. This part includes the energy equation, stress analysis and failure criteria for elastic material. These theories are expanded to viscoelastic material by using the Correspondence Principle.

The second part includes illustrations of this experiment.

The comparison between theory and experiment is in the third part of this report.

The failure criterion of a linearly viscoelastic material is discussed by W. G. Knauss and the crack propagation with constant velocity is discussed by H. K. Mueller. The purpose of this experiment is investigation of the possibility of determining crack velocity for short crack lengths where crack velocity is not constant upon the foundation of theories mentioned above.

## 1. THEORETICAL FOUNDATION

Let us consider a tip of a moving crack which is blunt. It may be reasonable to assume existence of a small region where the material is no longer continuous but forms a gathering of filaments.

A simple model of a crack tip is shown in Figure 1. Suppose the filamentized region is so small that this region has no effect on the entire stress strain field and these filaments give surface traction.

Let the length of this region be $\Delta a$ and its width infinitesimally small. The amount of surface traction is equal to that of a continuously elastic medium which has no separated region $\Delta a$.

The crack travels a distance of $\Delta a$ in the time $\Delta t$. A point P at position $\xi$ on x-axis moves vertically to the point P' at the distance $U_y$. Figure 2 shows this process. The solid line indicates original position and the dotted line indicates the crack position after crack motion and this transition of the crack position creates a new open surface $\Delta a$.

The energy, $\Delta E_c$, to create new surface $\Delta a$ is equal to the work done by the vertical movement of new surface against the surface traction $\sigma_y$,

$$-\Delta E_c = z \int_{\xi=a}^{a+\Delta a} \frac{1}{2} \sigma_y(\xi)\, U_y(\xi-\Delta a)\, d\xi \tag{1}$$

The surface traction at position $U_y$ is zero and at center line is $\sigma_y$, the relation between surface traction and vertical displacement is supposed to be linear.

Linear elastic material in plane stress situation shows the following stress $\sigma_y$ and displacement $U_y^e$ in the vicinity of a crack tip.

$$\sigma_y(x,0) = \frac{K_n}{\sqrt{\frac{x-a}{b}}}\sigma_o \qquad \text{for } x \geq a \tag{2}$$

$$\sigma_y(x,0) = 0 \qquad \text{for } x < a$$

$$U_y^e(x,0) = 4K_n \frac{\sigma_o}{E}\sqrt{b(a-x)} \qquad \text{for } x \leq a \tag{3}$$

$$U_y^e(x,0) = 0 \qquad \text{for } x > a$$

where a: half length of the crack

b: width of the crack

$K_n$: stress intensity factor. Fig. 3

$\sigma_o$: stress at the infinite distance.

By substituting equation (2) and (3) into (1), one can get

$$\int_{x=a}^{a+\Delta a} \sigma_y(x)\, U_y^e(x-\Delta a)\, dx$$

$$= 4\frac{\sigma_o^2 K_n^2}{E} b \int_{x=a}^{a+\Delta a} \sqrt{\frac{(a+\Delta a)-x}{x-a}}\, dx. \tag{4}$$

Let us change variables $(x-a)/\Delta a = \cos^2\theta$,

$$\int_{x=a}^{a+\Delta a} \sqrt{\frac{(a+\Delta a)-x}{x-a}} = -\int_{\frac{\pi}{2}}^{o} \sqrt{\frac{1}{\cos^2\theta} - 1}\ \Delta a \sin\theta \cos\theta d\theta$$

$$= -\frac{\pi \Delta a}{2}.$$

Thus,

$$\Delta E_c = 2\pi \frac{\sigma_o^2}{E} K_n^2 b \Delta a \tag{5}$$

Equation (5) leads the critical stress for crack propagation to the following expression

$$2S \leq \frac{\Delta E_c}{\Delta a} = \frac{\sigma_o}{E} K_n^2 b\pi$$

$$\sigma_{o\,crit.} = \sqrt{\frac{S E}{\pi b K_n^2}} \tag{6}$$

S is the surface energy which is necessary for creating a new unit surface.

Equation (6) corresponds to the Griffith criterion for an infinitely large plate with crack under uniaxial tension when b tends to be infinitely large. That is,

$$\sigma_{o\,crit.} = \sqrt{\frac{2SE}{\Delta a}}.$$

The displacement $U_y$ for viscoelastic materials is given by equation (7) by virtue of the Correspondence Principle [3].

$$U_y(x,o,t) = D_{cr}(t)\ E\ U_y^e\ (x,o) \tag{7}$$

where $D_{cr}$ is the creep function.

The crack half length $a$ grows as time goes on, so one can say $U_y^e$ is a function of time. Now one can get displacement $U_y$ in the case of moving crack.

$$U_y(x,o,t) = D_{cr}(t)\ U_y^e(x,o) + \int_o^t \frac{\partial}{\partial a}(U_y^e) \cdot \frac{\partial a(\tau)}{\partial \tau}\ D_{cr}(t-\tau)\, d\tau \tag{8}$$

Supposing the crack length is zero at $t=0$ and

$$\frac{\partial U_y^e}{\partial a} = 2\frac{\sigma_o}{E}\sqrt{\frac{b}{a-x}}\ K_n(a/b)\ ,$$

equation (8) becomes

$$U_y(x,o,t) = 2\sigma_o \int_o^t \sqrt{\frac{b}{a(\tau)-x}}\ K_n\left(\frac{a(\tau)}{b}\right) \frac{\partial a(\tau)}{\partial \tau} D_{cr}(t-\tau)\, d\tau$$

$$\text{for } 1-\left|\frac{x}{a(t)}\right| << 1 \tag{9}$$

When acceleration of the moving crack tip is small, one can assume

$$\frac{\partial a(t)}{\partial t} = V$$

where $V$ is the crack velocity.

The variable change $\xi = Vt$ leads to the following form:

$$U_y(x,o,v) = 2\sigma_o \int_x^{a_t} \sqrt{\frac{b}{\xi-x}}\ K_n(\xi/b)\ D_{cr}\left(\frac{a_t-\xi}{v}\right) d\xi$$

$$\text{for } 1-\left|\frac{x}{a_t}\right| << 1 \tag{10}$$

Substitute equation (2) and equation (10) into equation (1); then one gets

$$-\Delta E_c = \int_{x=a}^{a+\Delta a} \left[ \sigma_o \frac{K_n}{\sqrt{\frac{x-a}{b}}} \int_{x-\Delta a}^{a(t)} 2\sigma_o \sqrt{\frac{b}{\xi-(x-\Delta a)}}\, K_n\left(\frac{\xi}{b}\right) D_{cr}\left(\frac{a_t - \xi}{V}\right) d\xi \right] dx \tag{11}$$

By using spectrum form of the creep function

$$D_{cr}(t) = \frac{1}{E_r} - \int_o^{\infty} L(\tau)\, \exp(-t/\tau)\, \frac{d\tau}{\tau}$$

and

$$\dot{E}_c = \frac{\Delta E_c}{\Delta a} V \; ,$$

equation (11) is reduced to the following form:

$$\dot{E}_c = 2\pi K_n^2 \sigma_o^2 b\, V\, D_{cr}\left(\frac{\Delta a}{V}\right) \tag{12}$$

where $K_n$ is regarded as a constant for small intervals between a and $a+\Delta a$.

Taking the surface energy S and effect of temperature T ($^oK$) into consideration, equation (11) changes into

$$D_{cr}\left(\frac{\Delta a}{V a_T}\right) = \frac{1}{\pi} \frac{1}{b\sigma_o^2} \frac{S}{K_n^2(a/b)} \frac{T}{273} . \tag{13}$$

Equation (13) governs crack velocity for small $\Delta a$ and small acceleration of crack velocity.

One can get the fracture criterion for viscoelastic materials by setting limit value of $D_{cr}$ as $1/E_r$; $E_r$ is the rubbery modulus.

$$\sigma_o \text{ crit.} = \sqrt{\frac{S E_r T}{\pi b K_n^2 (a/b) 273}} \tag{14}$$

## 2. EXPERIMENT

### 2.1 Apparatus and Specimen

A scheme of the method of this experiment is shown in Figure 4. A viscoelastic strip of width 2b is stretched between upper and lower edge with no initial strain as shown by the solid line. A small initial crack is located at the center. This initial crack opens and propagates to right and left direction by edge displacement which gives strain $\epsilon_o$ as shown by the dotted line.

Pictures of a propagating crack are taken by a motor-driven 35 mm camera which is controlled by a timing device.

#### Apparatus

Figure 5 shows a scheme of a mechanism which gives a certain strain to the specimen and an overall view of the apparatus. The lower edge is fixed to the base and the upper edge is supported by two screws. An induction motor drives the screws through a chain and gives a certain amount of edge displacement to the upper edge, edge displacement is regulated by a limit switch. The moving velocity of the upper edge is 0.05 in/sec., which is equivalent to the strain rate 3.74%/ sec for the width of specimen; 1.375 inches.

#### Specimen

| | |
|---|---|
| Chemical Property | Solithane 113 (Polyester Elastomer) manufactured by the Thiokol Chemical Company. This material is a synthetic resin of two components, Urethane Resin and Catalyst. Mechanical |

| | |
|---|---|
| | properties depend on the mixing ratio of the two components. Mixing ratio of 50%-50% in volume is employed in the experiment. |
| Mechanical Characteristics: | Rubbery Young's modulus is about 440 psi at room temperature. Relaxation modulus is shown in Figure 6. |
| | Viscoelastic properties do not have any influence on the loading process because the relaxation time is much smaller than the initial time [4] of crack propagation. |

Geometric specification of the specimen is shown in Figure 7.

## 2.2 Condition of Experiment

Temperature of environment $23^{\circ}C$ ($73^{\circ}F$)

| Experiment No. | Strain Applied | Interval Time of Each Picture |
|---|---|---|
| 601 | 6.5% | 10 sec. |
| 701 | 7.5% | 8 sec. |
| 805-805 | 8.5% | 2.5 sec. |
| 901 | 9.5% | 2.5 sec. |

### 2.3 Results

The velocity of crack propagation is obtained by graphical differentiation of time-crack position plots. Figure 8 shows velocities versus crack tip position for each experiment. Six cases out of 8 experiments show maximum velocity between $a/b \simeq 0.5$ and $a/b \simeq 1.0$. The crack velocity reaches a certain value for $a/b > 1.5 \sim 2.0$. The crack velocity is very sensitive to change of strain.

## 3. COMPARISON BETWEEN THEORY AND EXPERIMENT

### Theoretical Calculation

Equation (13) can be changed into

$$V = \frac{\Delta a}{a_T} \Big/ D_{cr}^{-1}\left[\frac{1}{\pi} \cdot \frac{1}{b\,\sigma_o^2} \cdot \frac{S}{K_n^2(a/b)} \cdot \frac{T}{273}\right] \quad . \tag{15}$$

$D_{cr}^{-1}$ denotes the inverse function of the creep function $D_{cr}$, that is

$$D_{cr}(A) = B$$

$$D_{cr}^{-1}(B) = A.$$

One can simplify equation (15) into a function of the stress intensity factor $K_n$ and the strain $\epsilon_o$ when other values and creep function are specified.

$$V = \mathfrak{F}\,(\epsilon_o K_n) \tag{16}$$

Velocity is calculated and ascertained experimentally by H. K. Mueller [2] in the case of steady state crack propagation in which the crack length is large enough for considering stress intensity factor $K_n$ as a constant value.

Figure 9 shows the crack velocity at the steady state crack propagation for specified experiment in the previous section. This curve is calculated under assumption of

| | |
|---|---|
| Characteristic length | $\Delta a = 10^{-8}$ inches |
| Surface energy | $S = 3.21 \times 10^{-2}$ lbs/in. |
| Rubbery relaxation modulus | $E_r \simeq 443$ psi |
| Creep function | Figure 10 |

Now one can make an attempt to determine crack velocity by introducing equivalent strain $\epsilon_o^*$.

$$\epsilon_o^* \simeq \epsilon_o \frac{K_n}{0.346} \tag{17}$$

The number 0.346 is the value of the stress intensity factor $K_n$ for $a/b > 1.5$ where $K_n$ is considered as a constant value. $K_n$ is shown in Figure 4. Velocity can be read from Figure 9 by using $\epsilon_o^*$ which is obtained through equation (17).

Experimental values have a certain scattering range around the theoretical curve, so it may be reasonable to set upper and lower limits such that most of the experimental values drop between these two limits in Figure 9.

The essential difficulty is this experiment is that the crack velocity is very sensitive against the condition of the experiment. The leading factors which cause experimental errors are:

(1) Error of setting strain

(2) Error of estimated temperature of specimen

(3) Error of measured crack velocity through graphical differentiation.

One can reduce these errors by improvement method and apparatus of experiment.

It will be desirable to use statistical judgement for these experiments in which measured values scatter widely like this experiment.

## COMPARISON BETWEEN THEORY AND EXPERIMENT

Figure 11 shows measured velocity and calculated velocity versus crack tip position for each strain. Measured velocities from this experiment lie within the upper and lower limit in the case of $\epsilon_o = 6.5\%$ and $8.5\%$. The experiment of $\epsilon_o = 7.5\%$ shows exceptionally high crack velocity. It is unclear to say whether velocity is reasonable or not in the case of $\epsilon_o = 9.5\%$. These experimental values are plotted again in Figure 9. One can see that most of the measured values of this experiment correspond to that of steady state crack propagation and also one can say that unusual value in the case of 7.5% can be negligible because steady velocity at $a/b > 1.5$ is outside of the limit.

## CONCLUSION

The equation (13) is essentially adequate to the steady state (constant crack velocity) crack propagation in a viscoelastic strip.

This experiment intends to escalate applicable area of the equation (13) such that one can use the same equation for a smaller range of crack length where stress intensity factor $K_n$ is not a constant but a function of crack length/strip width ratio (a/b).

There are some assumptions for making use of the equation (13) for $a/b>1.5$. One is that the acceleration of crack propagation is small and another is that the characteristic length $\Delta a$ is small. This assumption makes an unsteady problem. This experiment suggests a significant possibility to make use of the equation (13) for smaller crack lengths.

# REFERENCES


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2. Mueller, H. K., "Stable Crack Propagation in a Viscoelastic Strip", NASA CR-1279, March 1969.

3. Fung, Y. C., Foundations of Solid Mechanics, Prentice Hall, 1965.

4. Flügge, W., Viscoelasticity, Braisdel, 1967.

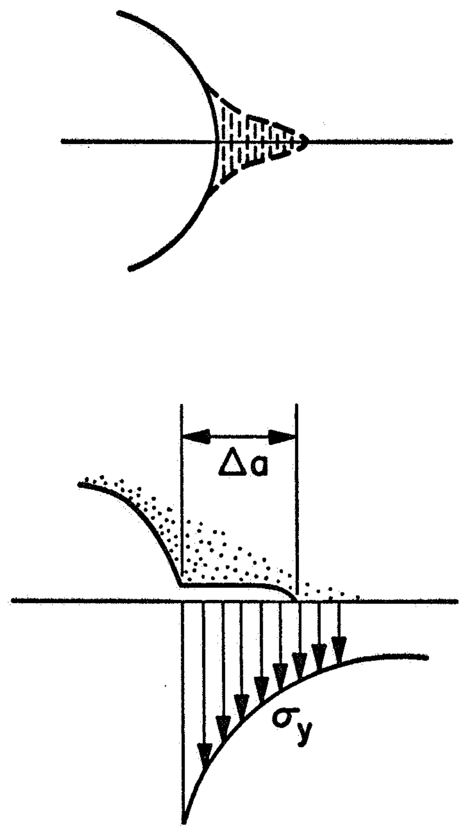



FIG. 1
MODEL OF CRACK TIP

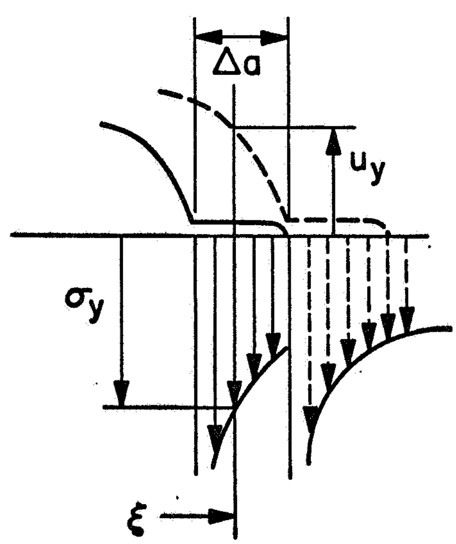



FIG. 2
OPENING CRACK

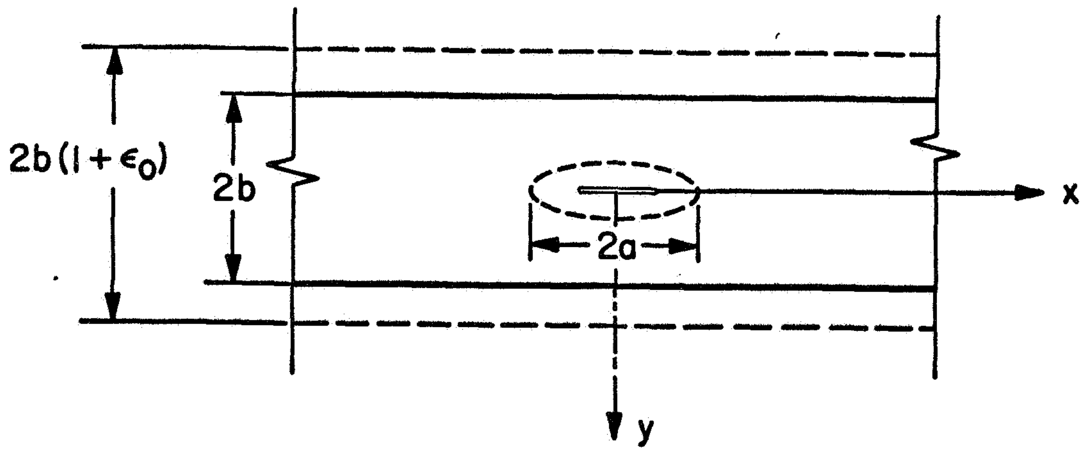



FIG. 3 STRIP GEOMETRY

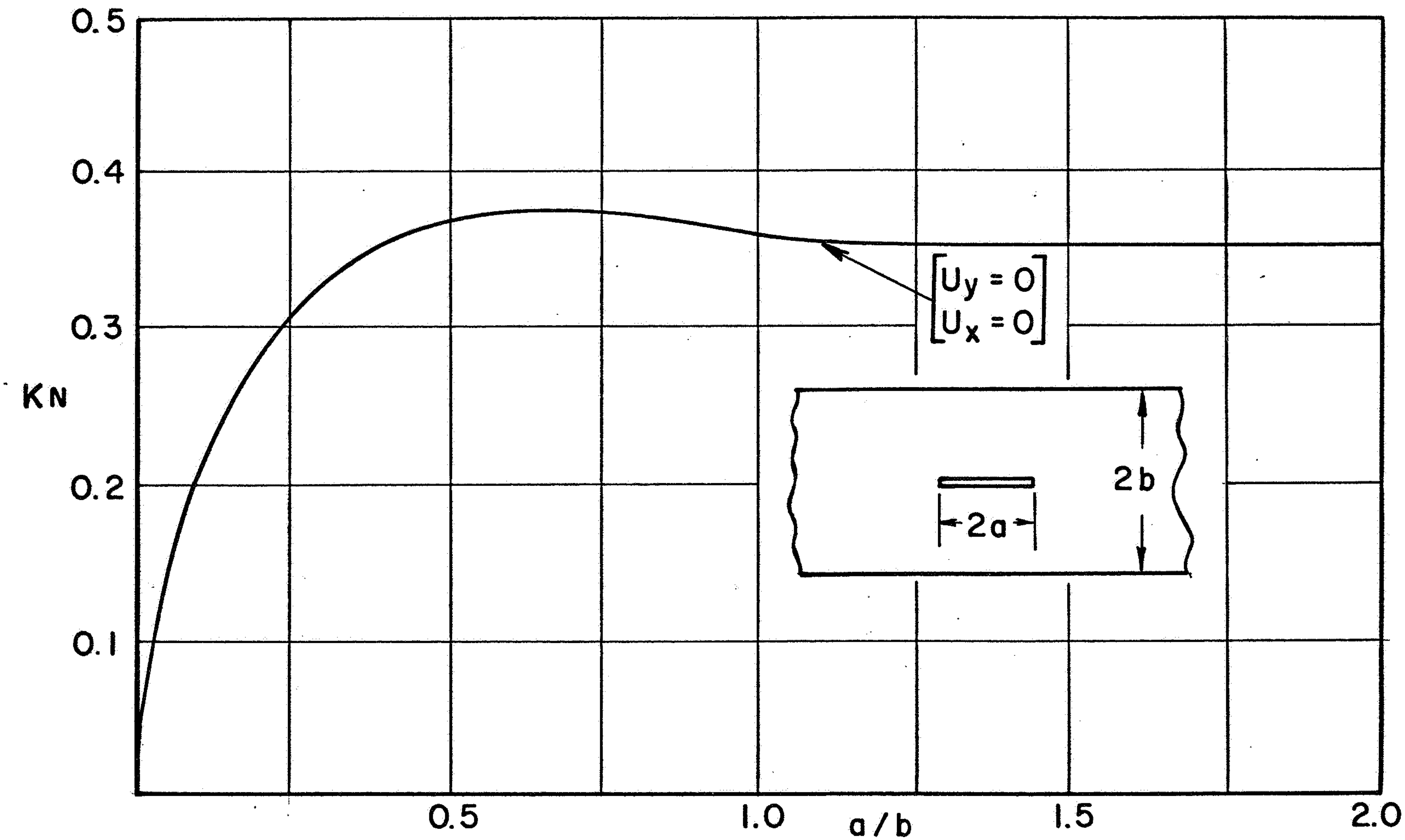
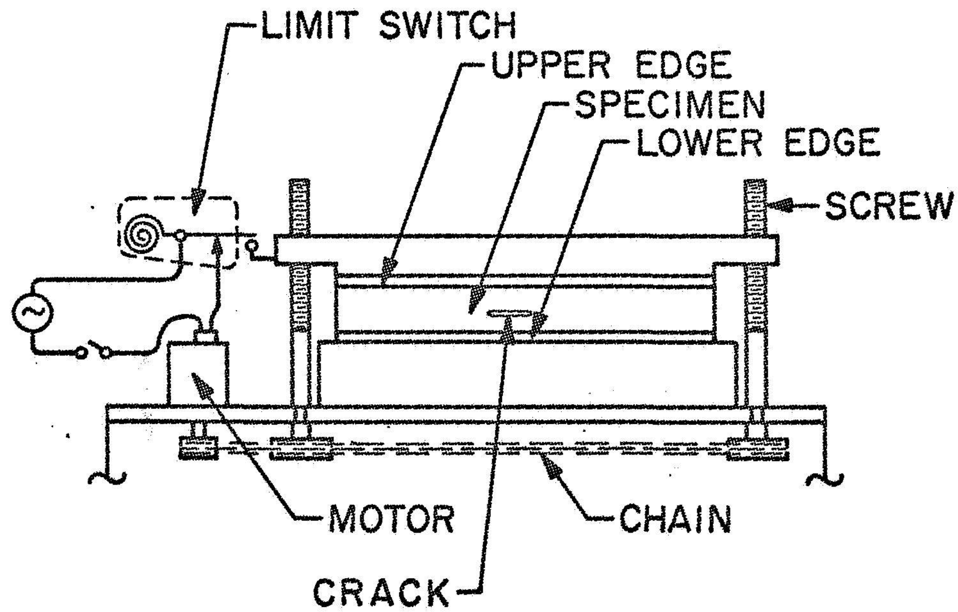



FIG. 4 STRESS INTENSITY FACTOR KN VERSUS CRACK TIP POSITION a/b



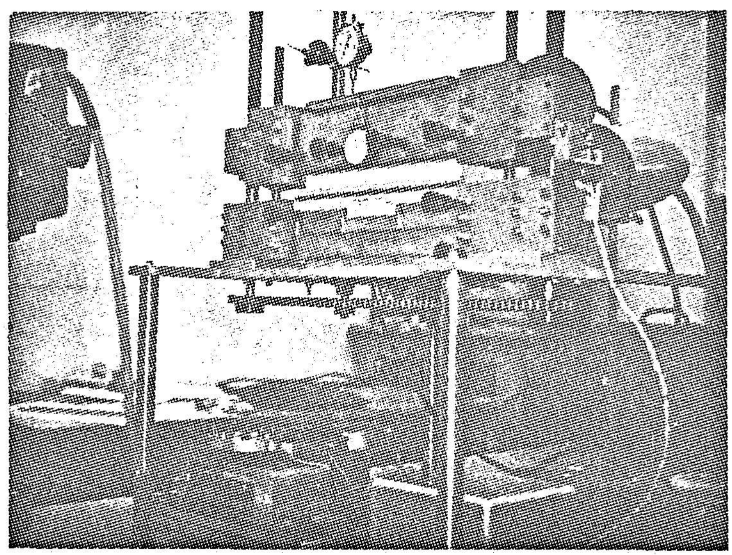



FIG. 5 APPARATUS

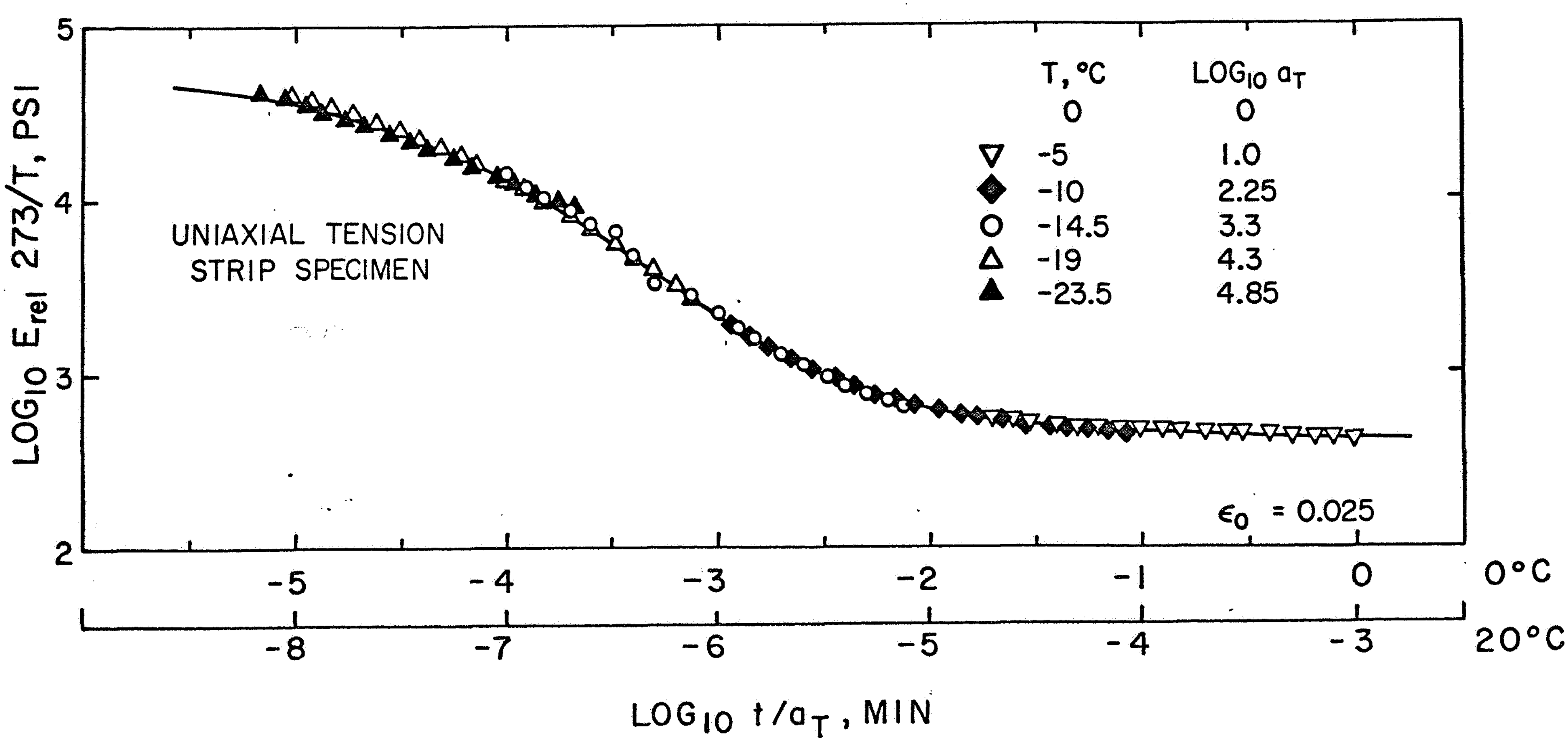



FIG. 6 RELAXATION MODULUS OF SOLITHANE 50/50

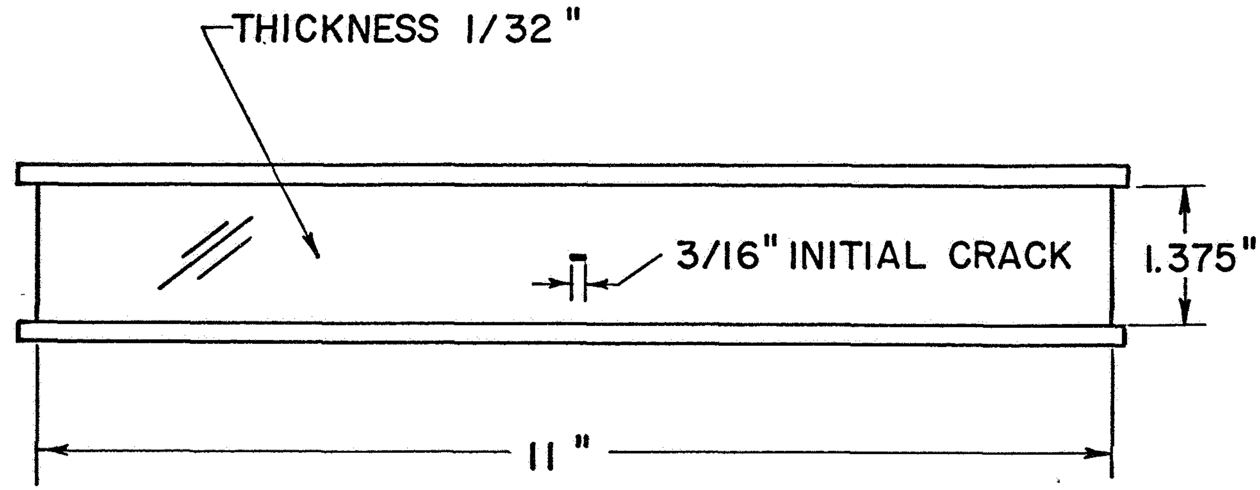



FIG. 7 SPECIMEN

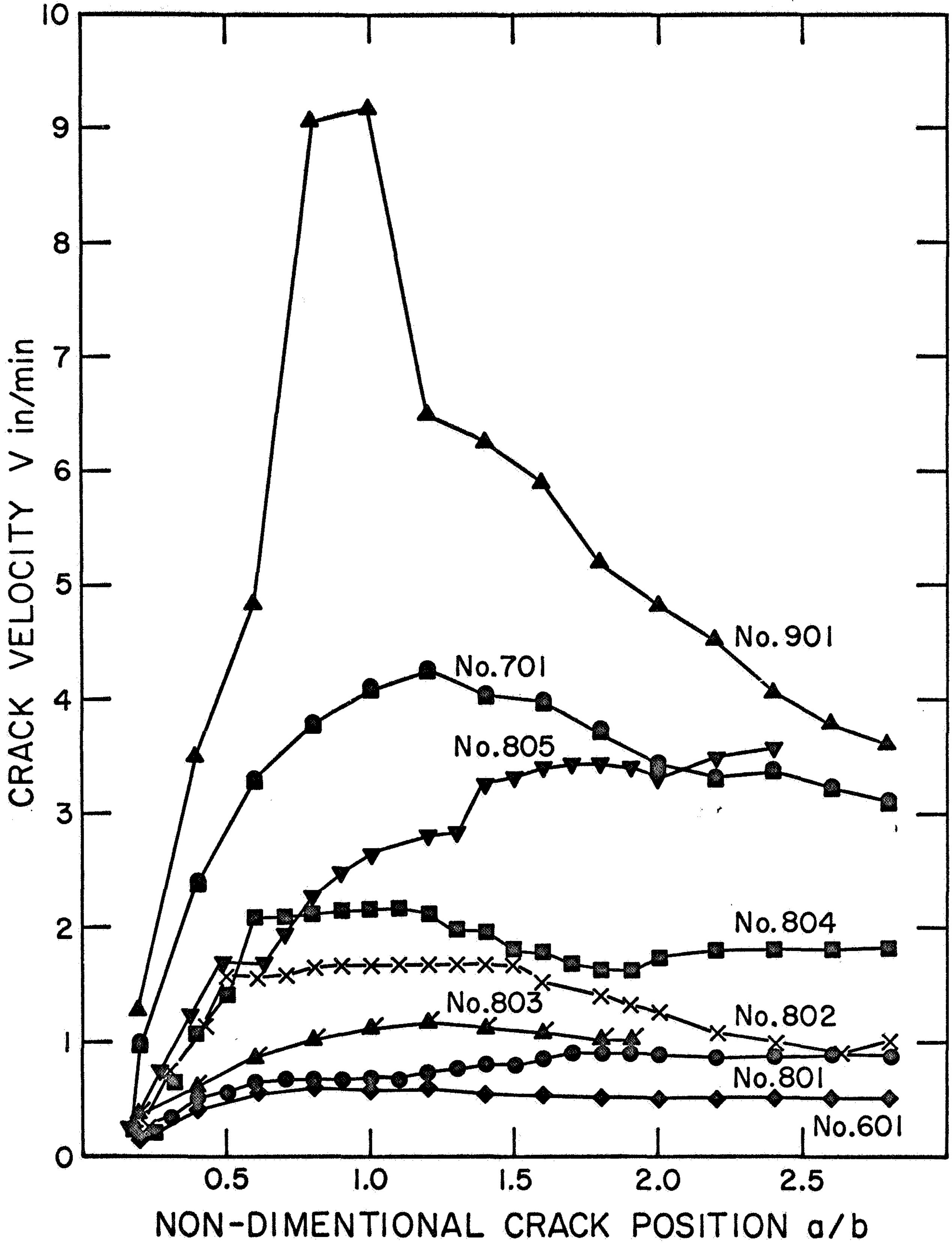



FIG. 8 RESULT OF EXPERIMENT

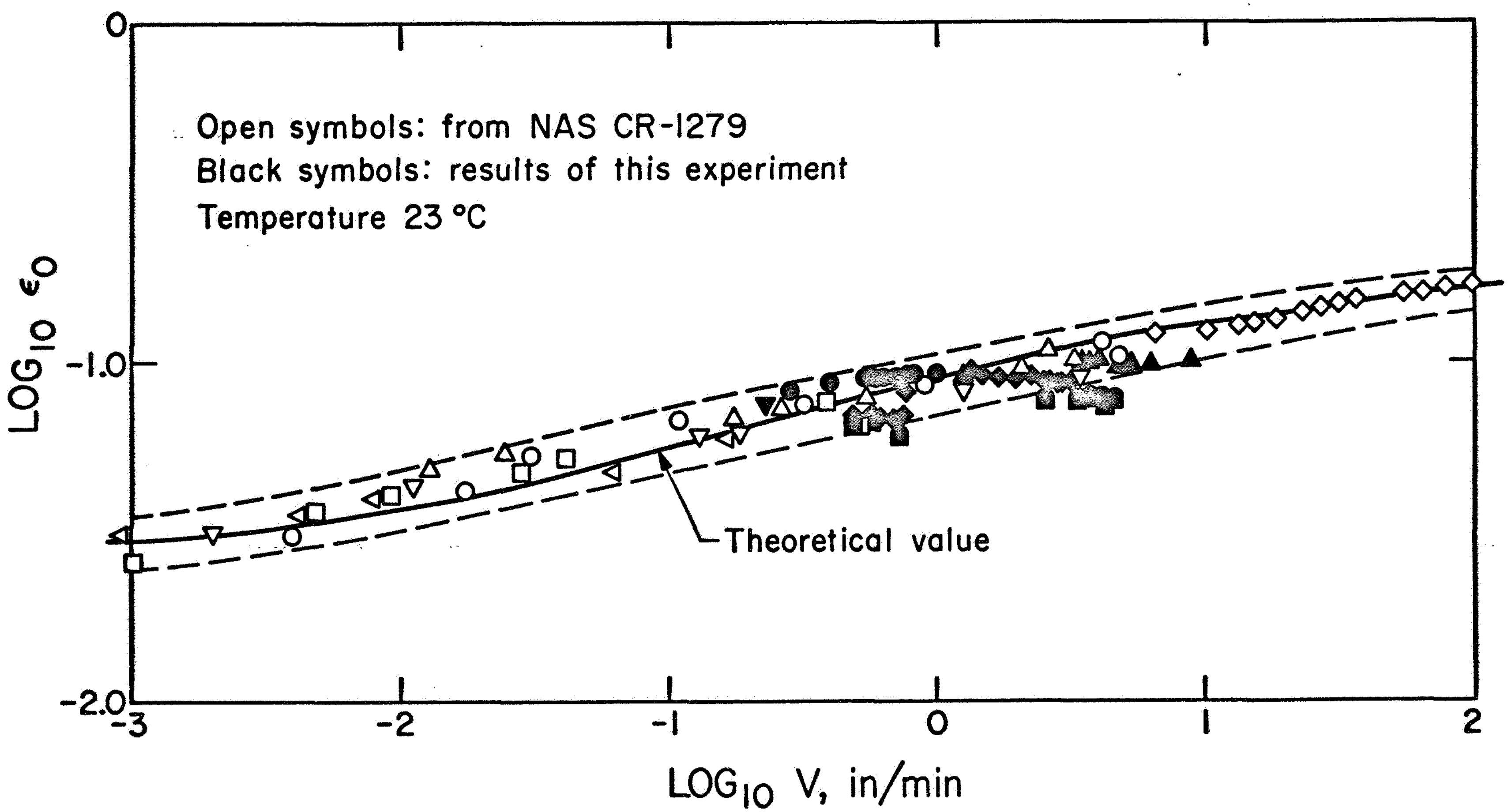



FIG. 9 CRACK VELOCITY VERSUS STRAIN IN STEADY STATE

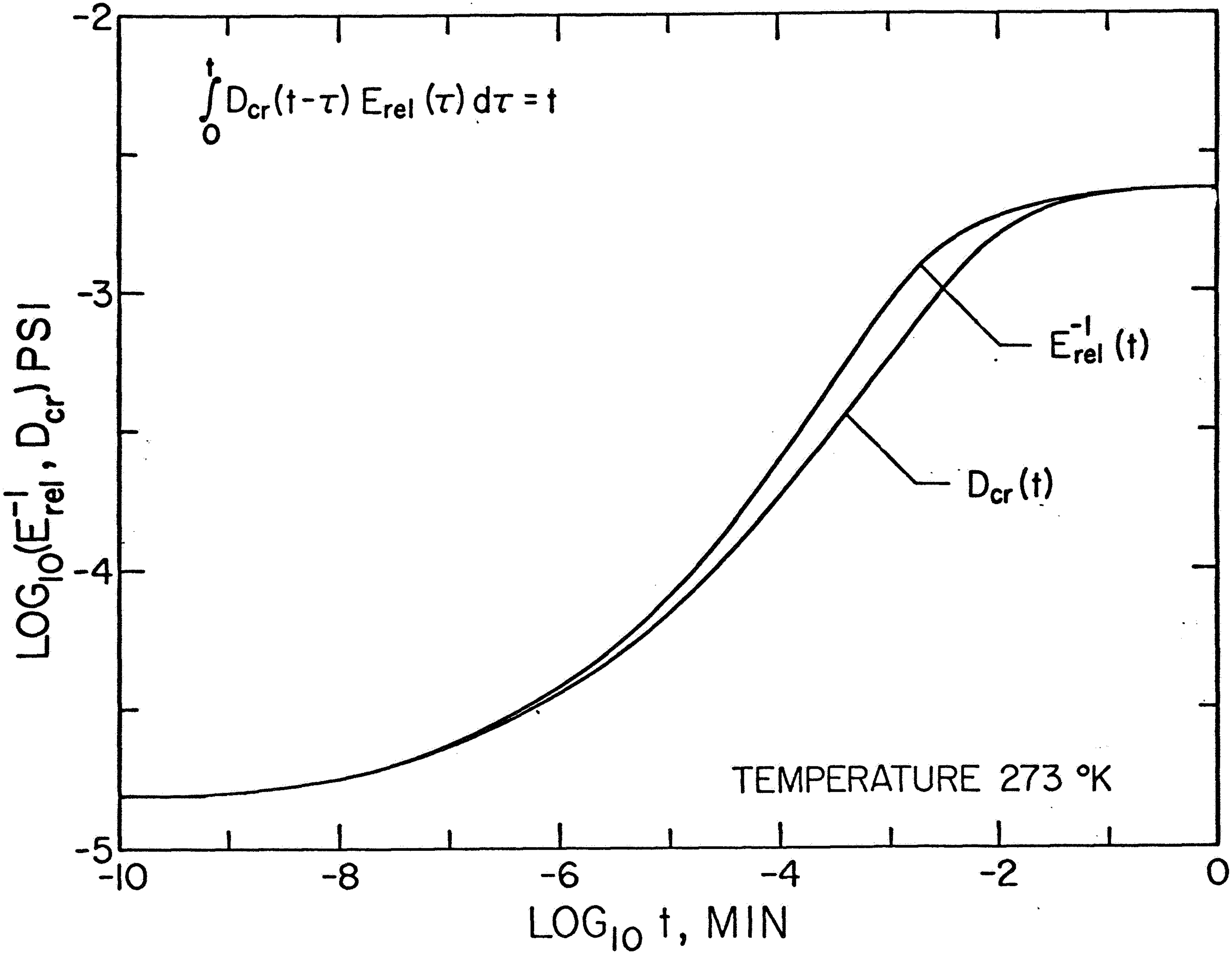



FIG. 10 THE RELAXATION FUNCTION, CREEP FUNCTION
SOLITHANE 50/50

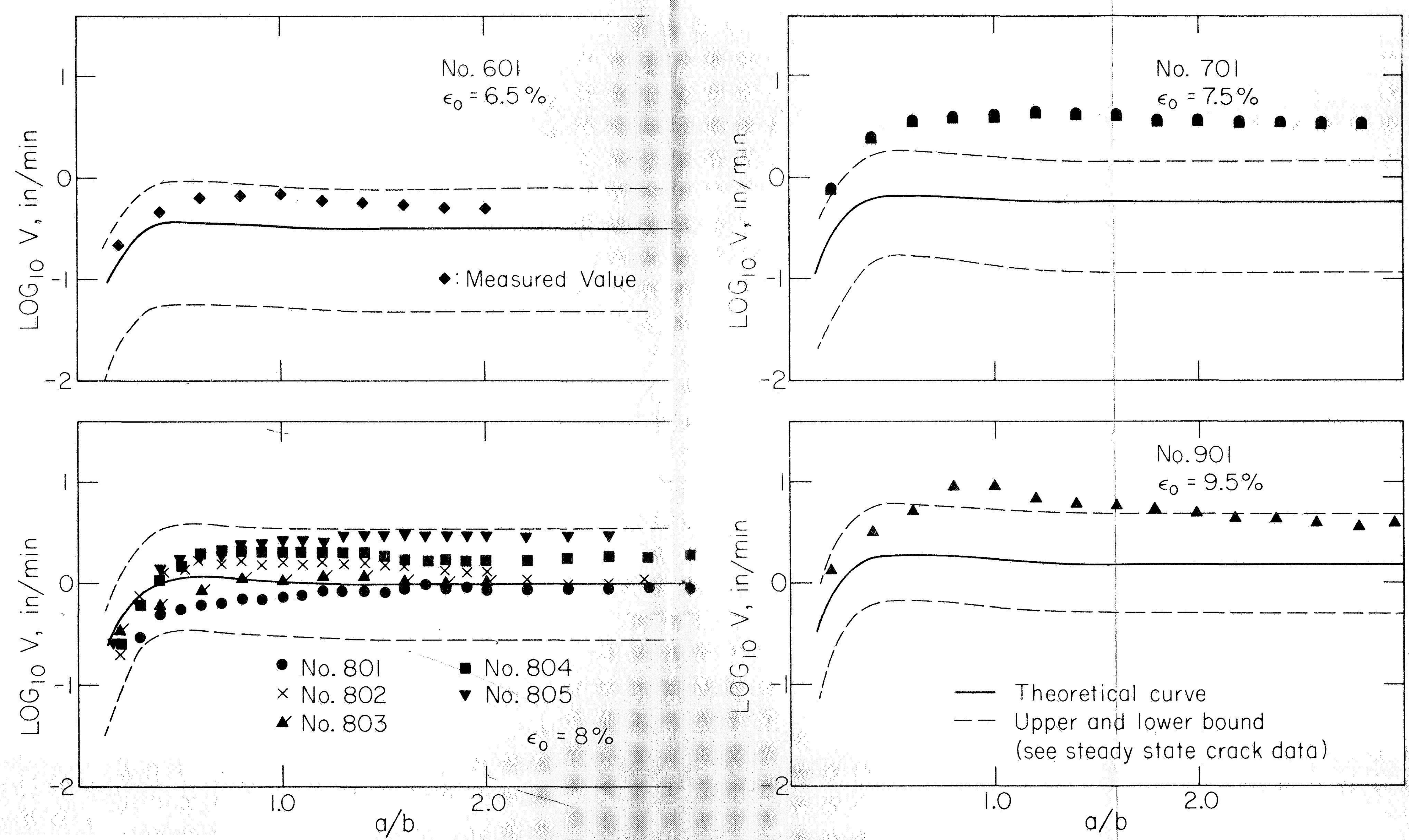



FIG. 11 MEASURED AND CALCULA    OCITY VERSUS CRACK TIP POSITION