X-322-69-399
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NASA TM X-63699

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SEPTEMBER 1969

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ABSTRACT

The problems associated with the measurement of transmittance of light through relatively thick plates of optical material have been reviewed. Measurements have been made to determine the spectral transmittance of a quartz plate of thickness one inch using a quartz iodine lamp scanned spectrally by a Perkin-Elmer monochromator, with and without the plate interposed between the source and the slit. The relevant theoretical expressions have been derived and the agreement between theory and experiment has been established. The transmittance of the plate is presented in the form of graphs and tables.

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INTRODUCTION

Transmittance of light through relatively thick and large samples of optical materials presents several interesting problems of experimental technique and optical theory. A small plate of size less than 2 inch square and thickness about a tenth of an inch presents no special difficulty, since it can be readily mounted in a standard instrument like a Beckman DK2 spectrophotometer and transmittance data of sufficient accuracy can be obtained. A large size, thick plate of quartz of the type used for windows of vacuum chambers cannot be handled by conventional techniques.

Two sets of conflicting results were recently reported by experimenters (private communication). John Arvesen of Ames Research Center, Moffett Field, Cal., obtained for the Dynasil window of NASA 711 Galileo aircraft transmittance values which are higher than those theoretically possible throughout the wavelength range where quartz is fully transparent and energy losses are caused only by surface reflection. The window was one inch thick. The differences vary from 0.9% at 4500 Å to 0.7% at 15,000 Å. Arvesen had employed two independent techniques, one using a Leiss double prism monochromator and the other using a Cary 14 spectroradiometer. Values lower by 2 to 3 percent than those theoretically possible were reported on a thick sample of suprasil quartz by G. C. Hunter of Perkin-Elmer Corporation. The instrument was a Spectracord spectrophotometer. The sample was 40 mm thick. These measurements were made in connection with a major space experiment, namely, C. O. Alley's retroreflector on the Apollo 11 mission. These large and systematic errors occur in a wavelength range where the losses due to reflection are a function of the refractive index. Since the refractive index of quartz is known with an accuracy of a few parts in a million[1], the transmittance can be computed quite accurately. The theoretical values can be verified on thin plates of quartz. The discrepancies illustrate the special problems associated with thick plates.

EXPERIMENTAL PROCEDURE

The present investigation was undertaken for determining the spectral transmittance of a quartz plate of grade Dynasil 1000. It is a circular plate of diameter 13-1/2" and thickness 1". It is intended for the window of a vacuum chamber for research purposes, and hence an accurate knowledge of its transmittance was considered necessary. The measurements were made with a

Perkin-Elmer Monochromator, Model 112, fitted with a lithium fluoride prism. The source of light was a 1000 W quartz iodine lamp which was mounted about 30 cm away from a diffusing screen. The diffusing screen, which was a ground glass surface coated with aluminum, was located in front of the entrance slit of the monochromator, the normal to the screen being at 45° to the incident beam as also to the optic axis of the monochromator. A divergent beam of light illumines the diffusing screen. A diaphragm mounted in front of the entrance slit limits the view of the monochromator to an area of about two inch square in the center of the diffusing screen.

The procedure was to scan the spectrum of the quartz iodine lamp in the wavelength range 0.258 μ to 3.75 μ. The Dynasil 1000 plate was next interposed between the source and the diffusing screen and the spectrum was again scanned. Both these scans were repeated once again. The spectral range accessible to the monochromator extends from 0.235 μ to about 5 μ. The UV range 0.235 μ to 0.258 μ was not scanned since the energy from the 1000 W lamp as diffused by the screen is too small for reliable measurement. The IR range beyond 3.75 μ was also omitted because the quartz plate is totally opaque in this range.

This method of using a divergent beam was preferred to the alternate method of using a collimated beam, since a perfectly collimated beam is practically impossible to obtain, and a slight degree of decollimation can introduce a significant systematic error. The correction for the lack of collimation can be computed if the significant parameters are known. The corrected value can be compared with the theoretical value in the wavelength ranges of zero absorption.

THEORETICAL CONSIDERATIONS

A brief review of the relevant aspects of optical theory will be helpful.

In Figure 1, let Q_1Q_2 be a plate of transparent material. The two surfaces Q_1 and Q_2 are parallel to each other. Let AB be a beam of monochromatic light incident on the upper surface Q_1. It is partially reflected along BA_1 and partially refracted along BC. The refracted beam again undergoes partial reflection and refraction at C. The refracted beam CD is the first and major component of the transmitted light. The reflected beam CB_1 undergoes multiple reflections within the plate and contributes the other components C_1D_1, C_2D_2, C_3D_3, etc. The energy in the transmitted beam is the sum total of the energy in each of these component beams, which are parallel to each other and to the incident beam AB. As the angle of incidence decreases, the transmitted beams become closer to each other, and in the limiting case of normal incidence, they are coincident.

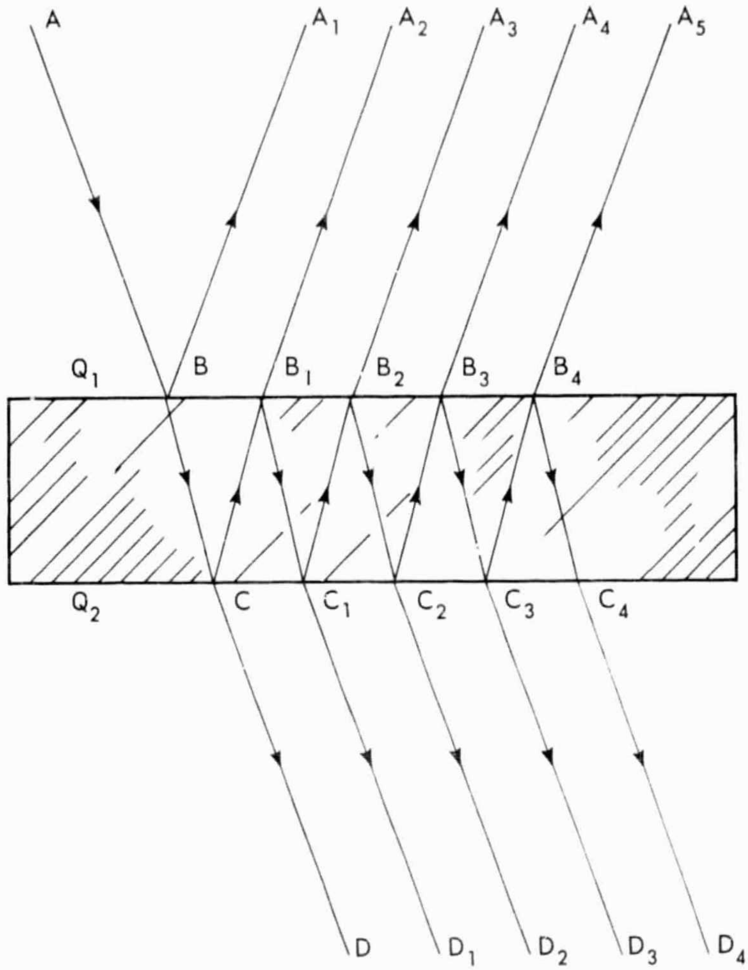

Figure 1. Multiple Reflections Within a Transparent Plate of Parallel Sides

Here we are concerned only with the case of normal incidence. The reflectance (or reflection coefficient) at normal incidence is given by the Fresnel formula[2]

$$r = \left(\frac{n - 1}{n + 1} \right)^2 , \tag{1}$$

where n is the refractive index.

If P is the energy in the incident beam AB, assuming that the ray is normal to the surface Q_1 and not inclined as shown in Figure 1, the energy in the beams BA_1, BC, CB_1 and CD are respectively Pr, $P(1 - r)$, $P(1 - r)r$ and $P(1 - r)^2$. It is also assumed that no energy is absorbed in the quartz while traversing from B to C.

It can be readily seen that the energy values in the other emergent beams C_1D_1, C_2D_2, C_3D_3, etc., are respectively $P(1 - r)^2 r^2$, $P(1 - r)^2 r^4$, $P(1 - r)^2 r^6$, etc.

The total energy in all the components of the transmitted beam is $P(1 - r)^2 (1 + r^2 + r^4 + r^6 + \ldots)$, which can be simplified by the application of the binomial theorem as $P(1 - r)^2 (1 - r^2)^{-1}$.

The transmittance T_r (the subscript r denotes that losses are due to reflection only) is the ratio of the transmitted energy to the incident energy.

$$T_r = \frac{(1 - r)^2}{1 - r^2} . \qquad (2)$$

Substituting for r from Equation (1), we obtain

$$T_r = \frac{2n}{n^2 + 1} . \qquad (3)$$

This expression holds true in the wavelength ranges where the material of the plate does not absorb any energy. In these ranges the transmittance does not change with thickness. For Dynasil 1000 this wavelength range extends from 0.3 to 1.2 μ and from 1.55 to 1.8 μ.

Outside these ranges quartz is an absorber. Consider the plate as made up of many parallel layers of infinitesimally small thickness dx. If P is the energy incident on one of the layers and dP the energy absorbed in it,

$$\frac{dP}{P} = - a\, dx , \qquad (4)$$

where a is a constant of the material and is dependent on wavelength. Distance x is measured along the direction of the beam. Let x_1 and x_2 be the values of x at the upper and lower surface, and P_1 and P_2, the values of the energy in the beam just below the upper surface and just above the lower surface. By integrating Equation (4) and applying the limits, it can be readily shown that T_a, the transmittance within the material where losses are due to absorption, is given by

$$T_a = \frac{P_2}{P_1} = e^{-a(x_2 - x_1)}. \tag{5}$$

T_a is dependent on the thickness of the material $(x_2 - x_1)$, and decreases exponentially as the thickness increases. By doubling the thickness the transmittance is reduced to the square of the former value.

In the wavelength ranges where absorption losses have to be considered in addition to the reflection losses, an expression for transmittance T can be derived by following a procedure similar to that for deriving Equation (2). For the plate shown in Figure 1, let T_a be denoted by q. The energy in the refracted beam BC (normal incidence is assumed) is Pr at the surface Q_1 and Prq at the surface Q_2. The values of energy in the components CD, C_1D_1, C_2D_2, etc. of the transmitted light are $P(1 - r)^2 q$, $P(1 - r)^2 q^3 r^2$, $P(1 - r)^2 q^5 r^4$, etc. The transmittance of the plate is

$$T = \frac{(1 - r)^2 q}{1 - q^2 r^2}. \tag{6}$$

The application of Equations (3) and (6) to our case will be discussed later.

The experimental determination of the transmittance T is by finding the ratio of the signals due to a monochromatic beam of light with and without the plate interposed between the source of light and the diffusing screen which is viewed by the monochromator. The same area on the diffusing screen is viewed by the monochromator in both cases, but that area subtends a larger solid angle at the source when the plate is interposed than when there is no plate. The correction factor can be calculated with the aid of Figure 2, where for the sake of clarity the beam of light is shown considerably more divergent than in the actual case. Let S be the source and AA' the illumined area on the screen. When the plate is introduced the apparent position of the source moves from S to S'.

$$SS' = S_1C = S_1B - CB = t\left(1 - \frac{\tan r}{\tan i}\right),$$

where t is the thickness of the plate, i and r are respectively the angles of incidence and refraction. For very small values of i, sin i = tan i = i, so

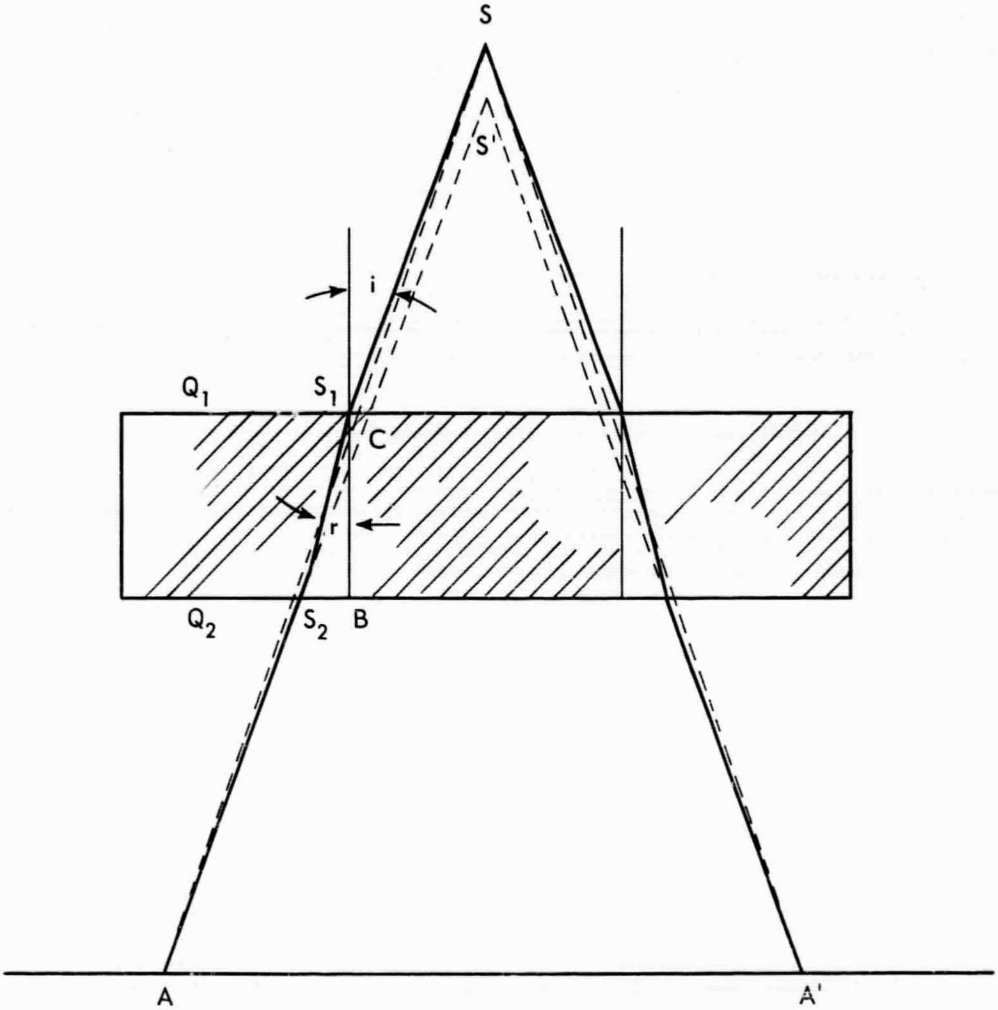

Figure 2. Transmittance of Oblique Rays Through a Parallel Plate

that $SS' = t(1 - 1/n)$. In our case i is about 5° for the extreme rays so that $\tan i / \tan r$ is about $0.997/n$. Let R be the distance of the source from the screen, dR the apparent change SS' in the distance, I the radiance of the source (energy emitted per unit solid angle), P and P' the irradiance (energy incident per unit area) on the diffuse screen in the absence of the plate and with the plate interposed respectively.

$$P = \frac{I}{R^2}, \tag{7}$$

and

$$P' = \frac{I\,T}{(R - dR)^2} \tag{8}$$

Dividing Equation (8) by Equation (7) and transposing the terms,

$$T = \frac{P'}{P}\left(1 - \frac{dR}{R}\right)^2, \text{ where } dR = t\left(1 - \frac{\tan r}{\tan i}\right). \tag{9}$$

Let $\left(1 - \frac{dR}{R}\right)^2 = f$. Since the signals S and S' without and with the plate are proportional to P and P',

$$T = f\,\frac{S'}{S}. \tag{10}$$

The correction factor f can be computed from known values of n and t. Computed values of f corrected to 3 significant figures are given in Table 1 for $R = 30.3$ cm, $t = 2.54$ cm and $i = 5°$. In the wavelength ranges where $T = T_r$, the computed values can be checked for accuracy from the theoretical value of T_r as given by Equation (3).

DATA ANALYSIS AND RESULTS

As stated earlier, four scans of the spectrum of the quartz iodine lamp were made, two with and two without the Dynasil 1000 plate. The signals were read off from the charts at 110 discrete wavelengths. The wavelengths were chosen at rather close intervals in the spectral ranges where due to absorption in the quartz, the transmittance changes rapidly with wavelength, as also in the range below 0.31 μ where the high noise to signal ratio introduces larger uncertainties in S'/S. In the spectral range where quartz is not an absorber, relatively wider intervals were chosen. The wavelength markers superposed on the spectrum charts at increments of ten drum counts enabled signals to be read at precisely the same wavelengths on all four charts. An average value of S'/S based on the four scans was determined for each of the 110 wavelengths, and the transmittance T as given by Equation (10) was computed. In the spectral range where Equation (3) is applicable, it was observed that the theoretical value agreed with the experimental value within the limits of experimental error

Table 1

Correction Factor $f = \left(1 - \frac{dR}{R}\right)^2$

Wavelength in Microns; See Equation (10)

Wavelength Range	f
< 0.2686	0.944
0.2686 - 0.3143	0.945
0.3143 - 0.3854	0.946
0.3854 - 0.6098	0.947
0.6098 - 1.314	0.948
1.314 - 2.223	0.949
2.223 - 2.877	0.950
2.877 - 3.422	0.951
> 3.422	0.952

(± 1/2 percent). In the range 0.258 to 0.29 μ the experimental errors are considerably larger, but here also the average of the observed transmittance values agrees with the theoretical value.

No measurements were made in the wavelength range below 0.258 μ. For this range as also for the range up to 0.29 μ where our experimental values are not sufficiently reliable, an alternate procedure was adopted to determine the transmittance of the plate. A transmittance curve for a plate of half inch thickness of Dynasil 1000 had been supplied by the Dynasil Corporation. The transmittance of a plate of 1" thickness of the same material can be derived from this curve by the following procedure.

In Equation (6) let q denote the absorptance due to half inch of path length in quartz and T the transmittance of the half-inch plate. From Equation (6) we obtain

$$T r^2 q^2 + (1 - r)^2 q - T = 0, \tag{11}$$

which is a quadratic equation in q. T is known experimentally and r is computed from Equation (1). Hence q for a 1/2-inch plate can be computed. Since q for a 1" plate is the square of that for a 1/2" plate, the transmittance of the 1" plate is

$$T = \frac{(1 - r)^2 q^2}{1 - q^4 r^2} . \qquad (12)$$

In the wavelength range 0.15 to 0.30 μ at intervals of 0.01 μ, the values of q were computed by solving Equation (11) and the transmittance values of the 1" plate were calculated by evaluating the expression on the right hand side of Equation (12). A recently acquired IME 86S electronic desk calculator with its associated magnetic core memory programming unit proved to be highly useful for these computations which on an ordinary desk calculator would have been very tedious and time-consuming.

It should be observed that spectral transmittance is defined for an infinitesimally small wavelength band, and the measurements were made over a wider band. In the thermocouple range the slit width was 2 mm and the corresponding wavelength resolution, as defined by the half-width observed at the exit slit when an infinitesimally narrow spectral line is incident at the entrance slit, is relatively large. It varies from a maximum of 0.11 μ at 1.4 μ to 0.078 μ at 2.2 μ, 0.061 μ at 2.7 μ and 0.046 μ at 3.8 μ. The effect on the profile of an absorption line is to increase its half-width and to decrease correspondingly the depth, keeping the area the same. Another slight source of error is due to the oblique rays of the divergent beam travelling a slightly longer path in the plate than the central rays. Thus the observed value of T_a of Equation (5) for a given wavelength is slightly lower than for a collimated beam at the same wavelength. No attempt was made to correct for these since a non-collimated beam and finite spectral resolution correspond more closely to the conditions under which the plate will be used.

The results are presented in Table 2 and in Figures 3 and 4. The values for wavelength longer than 1.9 μ and for the vicinity of the 1.4 μ absorption band are based directly on our measurements as computed from Equation (10). For the UV range below 0.30 μ, the values computed from Equation (12) were used. For the remaining portions of the curve two sets of values which confirm each other were used, one theoretical computed from Equation (3) and the other experimental based on our measurements as computed from Equation (10). The measurements were made at wavelengths which could be precisely identified by the markers on the spectrum charts, and they are not the same as those listed in Table 2. The values given in Table 2 for transmittance were obtained by

Table 2

Transmittance of Quartz Plate, 1" Thick, Dynasil 1000
(λ wavelength in microns, T transmittance)

λ	T	λ	T	λ	T	λ	T
.15	0	.90	.934	1.9	.926	2.95	.249
.16	0	1.0	.935	2.0	.913	3.0	.380
.17	.006	1.1	.935	2.05	.905	3.05	.475
.18	.205	1.2	.935	2.1	.884	3.1	.526
.19	.510	1.25	.934	2.15	.787	3.15	.556
.20	.618	1.27	.934	2.2	.576	3.2	.590
.21	.727	1.30	.926	2.25	.417	3.25	.615
.22	.789	1.32	.914	2.3	.583	3.29	.631
.23	.843	1.34	.902	2.35	.761	3.30	.629
.24	.881	1.36	.892	2.375	.772	3.35	.611
.25	.899	1.38	.883	2.4	.766	3.40	.565
.26	.919	1.40	.876	2.45	.693	3.45	.477
.27	.920	1.42	.886	2.5	.565	3.50	.348
.28	.923	1.44	.897	2.55	.403	3.55	.201
.29	.925	1.46	.906	2.6	.202	3.6	.133
.30	.926	1.48	.914	2.65	.091	3.65	.062
.40	.930	1.50	.922	2.7	.022	3.7	.028
.50	.932	1.55	.929	2.75	.000	3.75	.002
.60	.933	1.6	.936	2.8	.004	3.8	.000
.70	.934	1.7	.937	2.85	.021	3.85	.000
.80	.934	1.8	.938	2.9	.097		

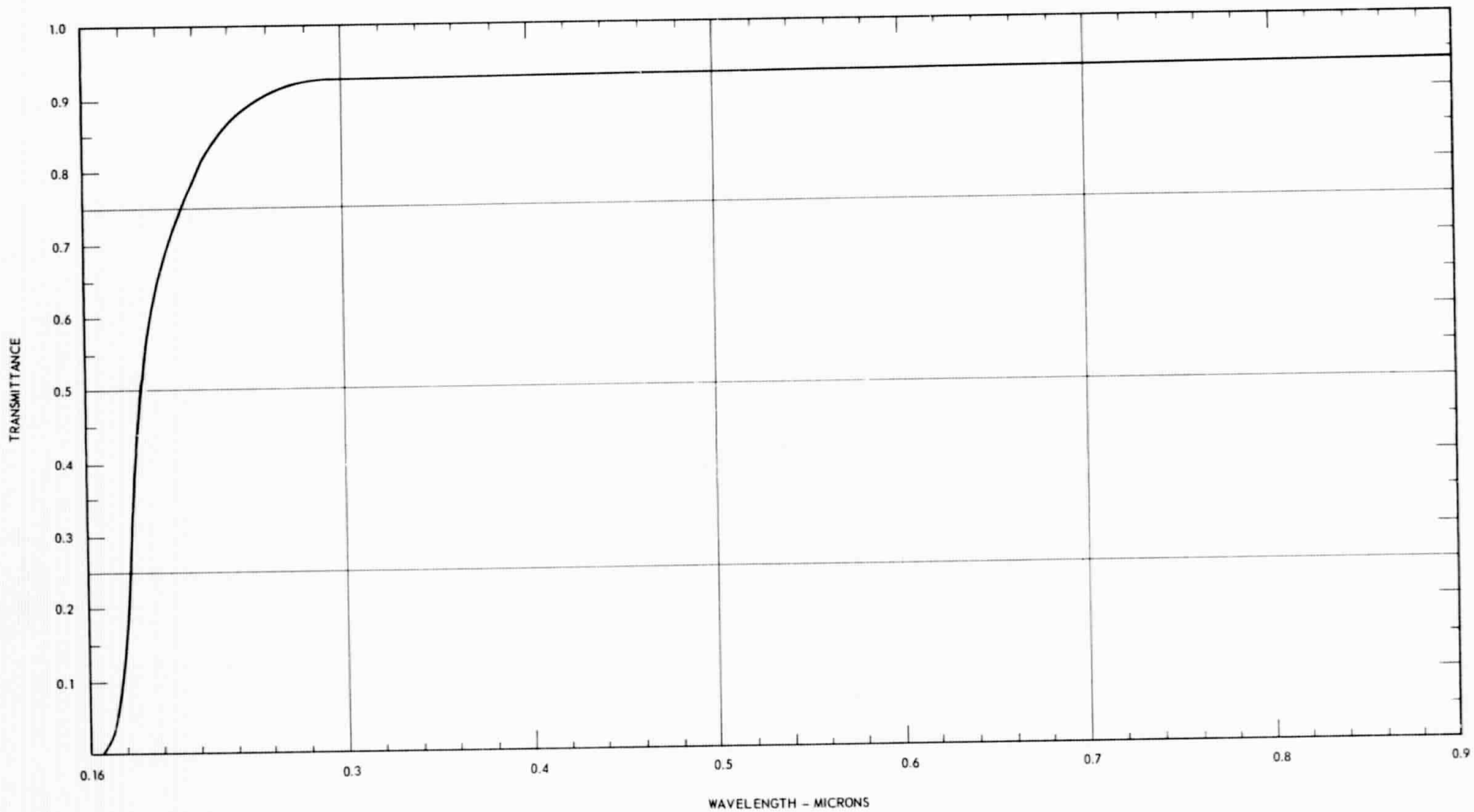

Figure 3. Transmittance of Quartz, Dynasil 1000, 1 Inch Thick, Range 0.16 to 0.9 μ

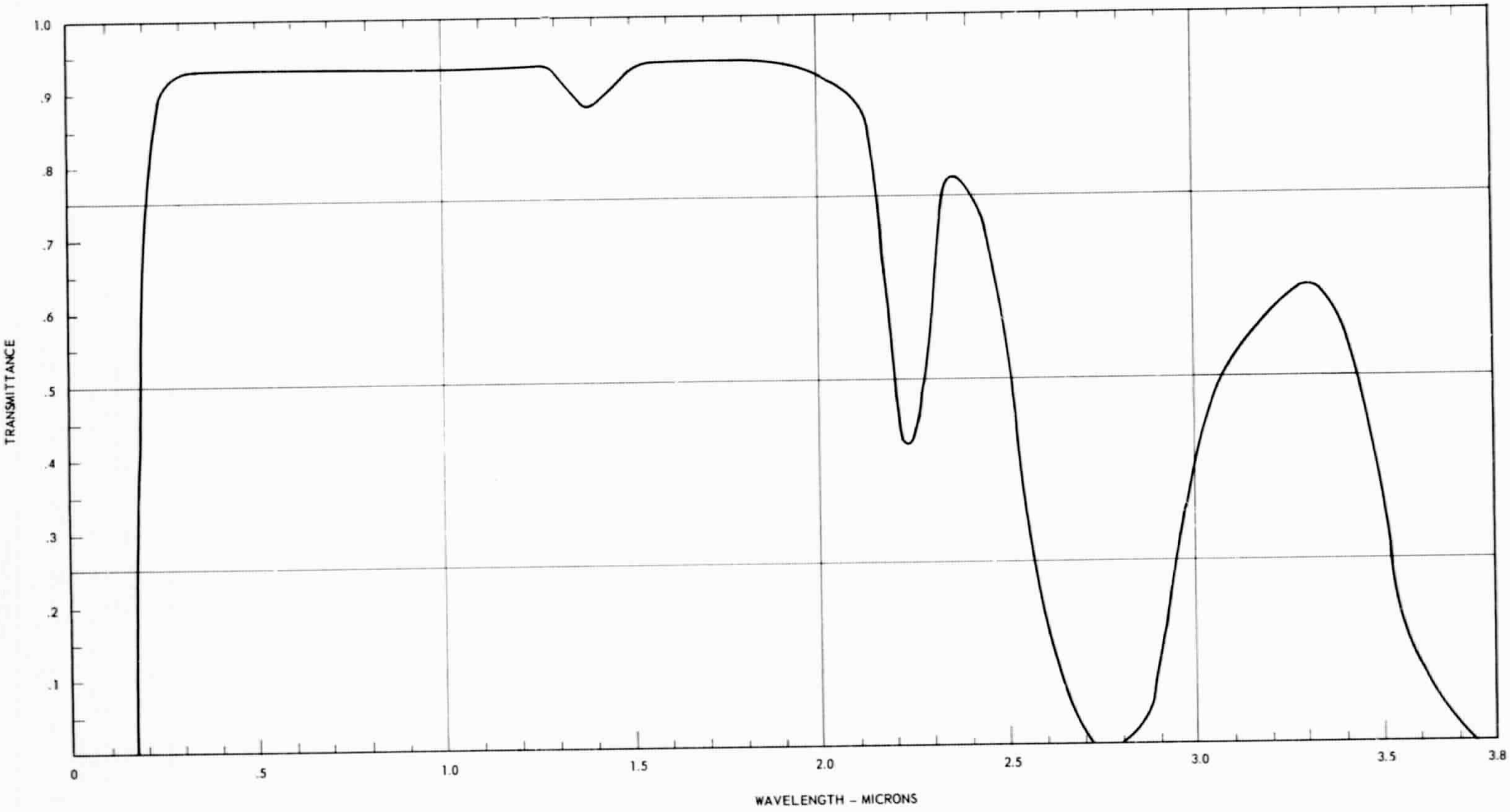

Figure 4. Transmittance of Quartz, Dynasil 1000, 1 Inch Thick

linear interpolation from the measured values. Figure 3 gives the transmittance curve in the range 0.16 to 0.9 μ on an expanded wavelength scale. Figure 4 covers the entire wavelength range over which the plate has non-zero transmittance.

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