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AIRBREATHING PROPULSION SYSTEM TESTING USING SWEEP FREQUENCY TECHNIQUES

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16. Abstract

The use of sweep frequency input in lieu of constant frequency input for determining system characteristics is discussed. Methods are furnished to determine the magnitude of errors that result from using a sweep frequency into a time delay, a first order system, and a second order system with various averaging techniques. The ability of the analysis method to reject noise is indicated for signal-to-noise ratios as low as one. Experience using the system for jet engine - inlet tests indicates a time saving factor of 10 to 1 with results which are as accurate and more complete than constant frequency testing. On-line data processing can be accomplished by components usually found in portable analog computers.

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SUMMARY

The use of sweep frequency input in lieu of a number of constant frequency inputs for determining system characteristics is discussed. Methods are furnished to determine the magnitude of errors that result from tests which use the sweep frequency instead of the constant frequency procedure. Error plots are furnished for a system having the dynamic characteristics of a time delay, a first-order lag, or a second-order system. Also discussed are the errors caused by the averaging techniques. The ability of the analysis method to reject noise is indicated for signal-to-noise ratios as low as one. Experience using the system for jet engine - inlet tests indicates a time saving factor of 10 to 1 with results which are as accurate and more complete than constant frequency testing. On-line data processing can be accomplished by components usually found in portable analog computers.

INTRODUCTION

The determination of the frequency response characteristics of a system is a fundamental step in the process of a dynamic analysis of a system. Often the sheer complexity or lack of information about the system prohibits analytical evaluation of its characteristics. Even when an analysis is made, testing is desirable to assure that some element was not overlooked or oversimplified. Generally, the method used to obtain the frequency characteristics of a system is sinusoidal testing at discrete frequencies.

Discrete frequency testing has limitations in that it is usually time consuming, and selection of the test frequencies, a priori, often results in a nonoptimum distribution of data points for determining the frequency response characteristics. Both of these limitations can, in theory, be avoided by using transient testing. Methods to determine frequency characteristics from transient data are discussed in reference 1 to 4. These methods are more complicated than those of the discrete frequency testing, and they also have a limitation in that, for each analysis frequency, there must be adequate power content for a reliable result.

This report deals with a sweep frequency testing method used to obtain the dynamic characteristics of a jet engine when coupled with a supersonic inlet. As these tests were conducted in a large wind tunnel, the following objectives were considered to be important: (1) rapid acquisition of the data, to minimize costly supersonic wind tunnel operation, (2) on-line data reduction of one or more of the test variables, and (3) preliminary processing of all data channels prior to the next day's testing. All of these objectives were achieved by the testing and analysis methods discussed herein. Based on recent test experience, test time savings of 10 to 1 were accomplished with no loss in accuracy and with greatly increased frequency response resolution.

The testing technique used was to excite the system with a frequency sweep input. The use of frequency sweep is not new. It has been used for years in structural dynamic testing (refs. 5 to 7). The application of this technique to jet engine testing is new, however, and the purpose of this report is to document the technique and present typical results obtained in recent engine - inlet dynamic studies.

This report is similar to reference 7, which is oriented toward lightly damped structures (second order systems) subjected to linear sweep rates. Jet engine dynamic characteristics usually consist of combinations of delay times and first and second order systems. This discussion includes all of these classes of dynamics. Since the logarithmic sweep rates are more practical when considering total testing time, this report includes both logarithmic and constant sweep rates.

DESCRIPTION OF APPARATUS

The type of equipment used for the sweep frequency testing consisted of an oscillator whose frequency can be controlled by an input voltage. Time variation of this voltage thus can determine the sweep rate of the oscillator. The output of the oscillator becomes the input for a wide-band phase shift network (ref. 8) which furnishes as output a sine and a cosine signal. Some variation in phase and amplitude of these signals occurs because of equipment limitations. One of these output signals was then used to drive a high response servo to introduce a disturbance into the jet engine - inlet system. For the test data shown later in this report, the signals from the system transducers and the two oscillator output signals were recorded on magnetic tape. Processing of this tape is essentially identical to the on-line data analysis system to be described in detail later. The only difference between the recorded and on-line data analyses is that for the recorded data analysis all the signals necessary for the processing have been stored on the tape.

The on-line processing of the data channel signal shown in figure 1 was done by electronic analog computing equipment. The components necessary consist of multipliers, averaging networks, and an X-Y plotter. Batch processing of a number of channels can be done by duplicating the single channel system. Analog equipment limitations sets the maximum number of channels which can be analyzed simultaneously.

Final processing is more complicated than indicated in figure 1 in that the final plots are magnitude and phase against frequency, in addition to the polar plots. Corrections are included for variations in amplitude and phase between the sine and cosine sweep frequency signals and for transducer calibrations. While these corrections can be done on analog computer equipment, it may be more expeditious to utilize a digital computer for this work. Thus, for batch processing, the signals after averaging were digitized, and a simple digital computer program made the necessary corrections before making the final plots. Details of the mathematics of the corrections are outlined in the PROCEDURE section.

The averaging indicated in figure 1 may be done in one of two ways with the end object being to eliminate all sinusoidal components resulting from the multiplications and to retain only the direct current or information component. The modern analog computer with electronically controlled integrators, track/store units, and electronic comparators can be made to do period integration and thereby average out sinusoidal components. Usually the test cell analog computer does not have the components necessary for period integration, but a simple, low-pass filter will suffice if it has a 40-decibel decrease at about twice the lowest analysis frequency. For jet engine tests the noise in the system output signals makes filter-type averaging preferable.

Figure 2 shows a cutaway of the supersonic propulsion system which was tested using the sweep frequency technique. The turbojet engine was a General Electric Company Model J-85 GE-13. The inlet was an axisymmetric mixed compression type designed for Mach 2.5 operation with 60 percent of the total supersonic area contraction occurring internally. For these tests, the engine's main fuel control was bypassed and fuel flow was supplied from a high response electrohydraulic servovalve commanded from external electronic inputs, such as the sweep generator described previously. A description of the high response fuel valve may be found in reference 9. Part of the inlet's shock position control consisted of six overboard bypass doors which were also electronically controlled. The bypass door system consisted of sliding plate valves driven by hydraulic actuators controlled by electro-hydraulic servovalves. The servovalves and pressure transducers used have a flat frequency response out to about 120 hertz for the magnitude of disturbances used in the test program.

The engine-inlet test data used in this report are the result of oscillating engine fuel flow to obtain excursions in compressor discharge pressure and oscillating the bypass doors to obtain excursions in throat exit static pressure in the inlet. Logarithmic sweep rates of 1 decade per minute between 0.5 and 140 hertz were used for all test data. The averaging function utilized in the analysis of these data was a first order lag having a 1-second time constant for the on-line data. A second order lag was used for the averaging function during the final processing of the data. The second order lag had a natural frequency of 1.414 radians per second and a damping factor of 0.707.

PROCEDURE

The analysis of sweep frequency data is rigorous only if it is considered to be a transient. The dynamic characteristics can be obtained from the data by use of Fourier transforms or correlation function analysis techniques. Unfortunately, these methods require considerable manipulation of the data and adequate power content and thus are rather time consuming.

An alternate method is to utilize a sweep rate which is sufficiently slow so that the data can be analyzed as though they were the steady-state sinusoidal response. This quasi-steady-state assumption greatly simplifies the analysis procedure, but it does introduce errors in the answers which are functions of sweep rate and the system characteristics. A detailed discussion of the errors is delayed to a later section. If the errors are temporarily assumed to be zero, the mathematics of the multiplication and averaging indicated in figure 1 is equivalent to solving for the Fourier coefficients of the data signal at a discrete frequency. The results of the multiplication are as follows:

$$a = \left[A \sin \theta(t)\right]\left[AC \sin(\theta(t) + \psi)\right] \tag{1}$$

$$b = \left[B \cos(\theta(t) + \varphi)\right]\left[AC \sin(\theta(t) + \psi)\right] \tag{2}$$

or

$$a = \frac{A^2C}{2}\left\{\left[1 - \cos 2\theta(t)\right]\cos \psi + \left[\sin 2\theta(t)\right]\sin \psi\right\} \tag{3}$$

$$b = \frac{ABC}{2}\left\{\sin(\psi - \varphi) + \left[\cos 2\theta(t)\right]\sin(\psi + \varphi) + \left[\sin 2\theta(t)\right]\cos(\varphi + \psi)\right\} \tag{4}$$

The filters (averages) attenuate frequency components which oscillate at twice the analysis frequency ($2\dot{\theta}$). This condition thus eliminates terms in equations (3) and (4) containing $\sin 2\theta$ and $\cos 2\theta$ and leaves the information signals

$$\bar{a} = \frac{A^2C}{2} \cos \psi \tag{5}$$

$$\bar{b} = \frac{ABC}{2} \sin(\psi - \varphi) \tag{6}$$

Implied in equations (1) and (2) is the absence of any direct current component in the data signal. If one does exist, the filter must be able to attenuate frequency components at $\dot{\theta}$ and thereby eliminate the $\sin \theta$ and $\cos \theta$ terms which would occur in equations (3) and (4). Normalization and corrections for oscillator errors ($B \neq A$, $\varphi \neq 0$) can be accomplished by replacing the system signal in figure 1 by each of the oscillator output signals. The results after averaging are

$$\overline{\left[A \sin \theta(t)\right]^2} = \frac{A^2}{2} \tag{7}$$

$$\overline{\left[B \cos(\theta(t) + \varphi)\right]^2} = \frac{B^2}{2} \tag{8}$$

$$\overline{\left[A \sin \theta(t)\right]\left[B \cos(\theta(t) + \varphi)\right]} = \frac{-AB}{2} \sin \varphi \tag{9}$$

Division of equation (5) by equation (7) results in the desired real component:

$$\mathcal{Re} = C \cos \psi \tag{10}$$

The desired imaginary component

$$\mathcal{Im} = C \sin \psi \tag{11}$$

can be obtained from equation (6) by the following method:

$$\frac{\bar{b}}{\frac{AB}{2}} = C(\sin \psi \cos \theta - \cos \psi \sin \theta)$$

After rearranging and substituting into equation (11),

$$\mathcal{I}m = \frac{\dfrac{\bar{b}}{AB/2} + C \cos\psi \sin\varphi}{\cos\varphi} = \frac{\dfrac{\bar{b}}{AB/2} + C \cos\psi \sin\varphi}{\left(1 - \sin^2\varphi\right)^{1/2}}$$

where $AB/2$ may be obtained from equations (7) and (8), $C \cos\psi$ is equation (10), and $\sin\varphi$ may be obtained from equations (7), (8), and (9).

For on-line data, corrections for phase errors and normalization are not necessary. The X-Y plots obtained directly from the magnitude developed in equations (5) and (6) are sufficient.

ERRORS DUE TO SWEEP FREQUENCY TESTING

The process of selecting a sweep rate is one of deciding the magnitude of error in the results which is acceptable. In general, the magnitude of the error is proportional to the sweep frequency rate. For noise free signals and the analysis method discussed in this report there are two sources of error:

(1) The effect of a sweep frequency on the system response

(2) The method used to provide the averaging

Signals with noise influence the selection of the averaging method, for the averaging method also serves as the noise filter.

System Response Errors

The system characteristics usually encountered in a jet engine are a combination of first and second order systems and time delays. The following discussion on errors is concerned with only a time delay, a first order system, or a second order system and not a combination of these terms. This is not a serious drawback, for often only one term dominates, or they can be considered as additive.

The transfer function obtained for a time delay system when driven by a constant rate sweep can be obtained in closed form. This may be done by assuming that the system in figure 1 consists of a time delay of value σ and of unity gain ($C = 1$). If $A = B$ and $\varphi = 0$, or when the source of error are removed,

$$\text{System output} = A \sin\theta(t - \sigma)$$

Using an infinite series expansion around $\sigma = 0$ (only the first three terms exist because the sweep rate is constant) one obtains

$$\text{System output} = A \sin\left[\theta(t) - \sigma\omega(t) + \frac{\sigma^2}{2}\dot{\omega}(t)\right]$$

Then,

$$\left.\begin{aligned} \bar{a} &= \frac{A^2}{2}\cos\left[-\sigma\omega(t) + \frac{\sigma^2}{2}\dot{\omega}\right] \\ \bar{b} &= \frac{A^2}{2}\sin\left[-\sigma\omega(t) + \frac{\sigma^2}{2}\dot{\omega}\right] \end{aligned}\right\} \tag{12}$$

Thus, the answer is correct in amplitude ratio and in error in phase by $(\sigma^2/2)\dot{\omega}$. One has the alternate option of interpretation that the information signals are correct in phase but at an earlier frequency, shifted by $(\sigma/2)\dot{\omega}$.

The transfer function characteristics of a first order system when driven by a constant rate sweep frequency signal are indicated in figure 3. Both increasing and decreasing directions for the sweep are included. The effect is to shift the characteristics in the direction of the sweep. These curves can be made nondimensional by multiplication of the sweep rate by the square of the system time constant. Over the range tested this term was found to correlate the data. The logarithmic sweep rate results are very similar if the correlation between the constant and the logarithmic sweep rates is such that the rates are identical at the frequency where $\tau\omega = 1$. Inspection of figure 3 indicates that sweeping frequency does not destroy the asymptotic property of a first order system and that the greatest error will occur in the neighborhood of $\tau\omega = 1$. Thus, if sufficient frequency range were tested, a true value of the system time constant can readily be obtained from the asymptotes. Also, for values of $\tau\omega > 3.0$, there is negligible error due to sweeping frequency. Crossplots at constant $\tau\omega = 1$ and at an amplitude of 0.707 (-3dB) are shown in figure 4 for estimating errors as a function of the nondimensional sweep rate parameter $\dot{\omega}\tau^2$ and the type of sweep being used. The data required for these plots were obtained by using an analog simulation.

A resonant second order system has a shift in the resonant frequency and a decrease in the amplitude of resonance when subjected to a sweep frequency input. Unlike the first order systems, the amplitude of resonance is always less than would occur at the resonant frequency regardless of the direction of the sweep. The frequency shift due to sweep is always in the direction of the sweep. These effects can be represented on a nondimensional plot if one uses a sweep parameter $\dot{\omega}/2\zeta^2\omega_n^2$ for the abscissa and $(Y_p - Y_s)/Y_p$ and $|\omega_p - \omega_s|/2\zeta\omega_n$ for the ordinates of the resulting error in amplitude and the frequency shift, respectively. Subscripts s and p are used to identify the sweep and steady-state value of the resonant peak, respectively.

Plots showing these characteristics of a resonant second order system are given in figure 5. The data used for the plots were generated by an analog simulation. These plots are somewhat different from those of reference 7, but agree with reference 6, as they indicate a distinct difference for the direction of the sweep. Some points from the referenced reports are indicated for completeness. These plots also show the error for either logarithmic or constant sweep rates and, as was done for the first order system, the correlation between the types of sweep is such that at the system natural frequency ω_n the rates are identical.

Averaging Errors

In addition to system errors, there are errors introduced by the method of averaging. If it is assumed that the system output signal is noise free, the error introduced by the period integration method is small. The primary error, except at very low frequencies, is the tracking error of the track/store devices. These units are required to store the period integration voltage. For the equipment used, a 100-volt computer, this error is less than 0.5 volt at 100 hertz, and less than 2.0 volts at 500 hertz. Other much smaller errors might be introduced because of the switching response time of the computer components, drift in the integrators and track/store devices, and the fact that the period integration output displayed is one cycle behind system output.

If the system output signal has noise on it, which most do, exponential averaging by use of a first or second order lag filter is desirable and, from a computer component viewpoint, much simpler to implement than the period integration method. One does not get noise filtering and simplicity without introducing an error for frequency sweep test data. In some cases, the magnitude of the filter error can be the largest single source of error in the entire system. The filter's ability to average out all of the sinusoidal component resulting from the multiplication (eqs. (3) and (4)) depends on the bandwidth and the slope of the cutoff. A method of determining the error introduced by a first order lag filter when the system is a lightly damped second order system is given in reference 7. Unfortunately, this method is not readily adaptable to the wide range of dynamics encountered in jet engines. The method presented herein does not depend on the system, and, thus, it handles the dynamics encountered in a jet engine.

The basis for the method is that the desired output of the filter is the slowly time varying component (information signal) of the multiplication product of equation (3) or (4).

The frequency content of the information signal is proportional to the sweep rate and to the characteristic of the system under test. However, for slow sweep rates the frequency content approaches zero and, in fact, is zero for a zero sweep rate. Figure 6 can perhaps better emphasize this point for it indicates the real and imaginary components, the information signals, of a system consisting of two second order transfer functions. If the sweep rate was such that it took 120 seconds to sweep logarithmically from 1 to 100 hertz, the dominant frequency content of information signals is less than 0.1 radian per second. For the same frequency range, doubling the sweep rate doubles the frequency content range, and changing the system modifies the shape to more or less humps and dips, thereby affecting the frequency content. A procedure for estimating the frequency content in the information signal is given in appendix B. The important point is that the frequency content of the information signal is near zero frequency (0 to 0.1 rad/sec), whereas the frequencies to be filtered out are at 1 hertz or greater. Therefore, at the low relative frequencies of the information signal the filters can be approximated as time delays.

The validity of this approximation can best be understood by considering the characteristics of a time delay and the first and second order filters under the conditions that the frequency of interest is less than 10 percent of the characteristic frequencies of the filters. These characteristics are listed in table I, and the one to one correspondence

TABLE I. - COMPARISON OF TIME DELAY AND

FIRST AND SECOND ORDER FILTERS

	Time delay	First order filter	Second order filter
Characteristic frequency	$1/\sigma$	$1/\tau$	ω_n
Amplitude ratio	1	1	1
Phase angle, θ	$\sigma\omega$	$\tau\omega$	$2\zeta\omega/\omega_n$
$d\theta/d\omega$	σ	τ	$2\zeta/\omega_n$

can readily be determined. Thus, the mangitude of the time delay is directly proportional to the filter parameters. Knowing the time delay and the sweep rate allows direct calculation of the frequency associated with the filter output from the instantaneous sweep frequency.

The corrected frequency ω_c associated with the output of the filters can be calculated by the following formulas for a first order filter:

$$\omega_c = \omega - \tau\dot{\omega} \tag{13}$$

For a second order filter,

$$\omega_c = \omega - \frac{2\zeta_f \dot{\omega}}{\omega_{nf}} \tag{14}$$

For a constant sweep rate the correction is a constant frequency; for a logarithmic sweep rate the correction is a constant percentage of the instantaneous frequency. The validity of this can be established for a logarithmic sweep from figure 6. In the figure the point are the constant frequency values, and the solid line curve is the sweep frequency result. One may observe that the sweep results have the same values except that they are shifted by a constant percentage of the instantaneous frequency in the direction of the sweep.

NOISE REJECTION

Before discussing the noise rejection capability of the filters used in the analysis, a short discussion of the filter's pass band characteristics is helpful. Presuming some unwanted 60-hertz pickup is occurring on the data channel, the multiplication of the analysis signal and the data channel results in the desired signal plus sine waves having frequencies which are the sum and the difference of the analysis frequency and 60-hertz pickup. Thus, for analysis frequencies far away from 60 hertz, the filter will eliminate the unwanted signals. However, as the sweep frequency passes through 60 hertz the difference frequency becomes low enough to pass through the filter.

The pass band characteristics for the 1 second time constant first order filter and $\omega_{nf} = 1.414$ and $\zeta_f = 0.707$ second order filter mentioned herein are indicated in figure 7(a). The abscissa of this figure is the difference frequency between the instantaneous analysis frequency and the unwanted frequencies. The ordinate is the amplitude ratio of the unwanted signal to that which occurs if the frequency of the unwanted signal and the analysis signal are equal.

Averaging by use of period integration is not recommended for signals with noise. While it gives complete rejection for noise free signals that are multiples of the analysis frequency, the pass bandwidth is proportional to the analysis frequency plus additional side band lobes similar to the function $\sin x/x$ and thus makes a very poor noise filter.

The problem of selecting a suitable bandwidth for the filter can be illustrated in figure 7(b). The pass bandwidth must be wide enough to get the desired information signal through and narrow enough to attenuate the unwanted noise components and the analysis frequency components. For example, the cause of the ripple in the 1 to 2 hertz range in figure 6 is insufficient rejection of the analysis frequency components.

With the exception of a constant sweep rate into a time delay, there is no simplified method of determining the frequency content of the information signals. However, the frequency content can be estimated by using the information signals' power-frequency spectrum (see appendix B). These estimates should be increased by a factor of 5 to 10 to assure accurate correction for the filter's tracking lag, as discussed in the previous section and indicated in figure 7(b).

To demonstrate the noise rejection capability of the analysis method, a system consisting of a pair of first order Padé approximations to a time delay was tested on the analog computer. To the output of this system was added various rms levels of noise whose frequency band was the same as the testing range, namely, 1 to 100 hertz. For this example, the averaging filters used were second order having the characteristics of figure 7(a), and the sweep rate was 1 decade per minute.

The polar plot output of this system is a circular path and the effect of the noise will cause the deviation from the circle. Figure 8 shows the results of a 10, 25, 50, and 100 percent rms noise being added to the output signal. In all cases the basic characteristic of the system is still exhibited.

The noise rejection capability of the analysis method is the same as could be obtained by notch filters. Namely, the reduction is the ratio of the rms value of the noise within the pass band of the filter to that of the total noise bandwidth. Since the pass band requirement for the information signal is proportional to the sweep rate, one can obtain better noise rejection simply by using slower sweep rates and allowing a smaller filter pass band.

Another kind of noise often encountered in test data is the pickup of a pure tone such as 60 hertz. While the discussion of the filter's ability to reject this unwanted noise could be included here, it will be deferred to the next section so that the experimental test data can be discussed as a unit.

APPLICATION AND DISCUSSION

By using the curves and equations developed in the preceding section, one is now able to evaluate the magnitude of error caused by using a sweep frequency signal for obtaining system frequency characteristics. Two examples are used to illustrate the procedure. The first example is the analog computer results which are shown in figure 6. Following this are some data obtained during a jet engine - inlet test program.

Figure 6 indicates the real and imaginary components of a system consisting of two tandem transfer functions. The defining parameters are as follows:

$$\omega_{n1} = 62.8 \qquad \omega_{n2} = 200$$

$$\zeta_1 = 0.4 \qquad \zeta_2 = 0.2$$

The sweep rate used was 1 decade per minute, and the filter used was second order having the pass band characteristics shown in figure 7(a).

As the data are for a logarithmic sweep, it is necessary to calculate the sweep rate at each resonant frequency of the system before the system error can be determined from figure 5. This can be done by the formula

$$\omega = 10^{\frac{Rt}{60}}$$

$$\dot{\omega} = \omega\left(\frac{R \ln 10}{60}\right) \tag{15}$$

where R is the sweep rate in decades per minute. Substitution of the test and system parameters into equation (15) yields

$$\dot{\omega}_{n1} = 2.41$$

$$\dot{\omega}_{n2} = 7.68$$

and applying these into the sweep rate parameter (for second order systems) gives

$$\frac{\dot{\omega}_{n1}}{2\zeta_1^2\omega_{n1}^2} = 0.0019$$

$$\frac{\dot{\omega}_{n2}}{2\zeta_2^2\omega_{n2}^2} = 0.0024$$

When the plot of error is entered against sweep rate parameter (fig. 5), it is obvious that there is no sweep rate effect for the system. The effect of the filter is the remaining source for error. Combining equation (14), the correction for the filter tracking lag, with equation (15) and inserting the filter parameters ($\omega_{nf} = 1.414$ and $\zeta_f = 0.707$) give

$$\omega_c = \left(1 - \frac{2 \times 0.707 \times 2.3}{1.414 \times 60}\right)\omega$$

$$= 0.962\ \omega$$

which is the magnitude of the frequency shift of the sweep results from the true values. From figure 6 a frequency shift of 3.7 to 3.9 percent is obtained. Thus, for this case the filters were the largest source of error and even this error can be readily corrected.

Three polar plots of on-line data for an engine-inlet test are shown in figure 9. Figure 9(a) is for the inlet throat exit static pressure response when the bypass doors are oscillated and figures 9(b) and (c) are compressor discharge pressure and fuel spray nozzle pressure responses when the fuel flow is oscillated. A final plot of compressor discharge pressure response to fuel flow requires the conversion of figure 9(c) from pressure into flow and complex division of this result into corresponding frequency points of figure 9(b). Also, the correction for errors outlined in the PROCEDURE section would be made.

Figure 9(a) presents results from a sweep frequency test and a discrete frequency test. Test conditions for the sweep and discrete frequency data are not identical, but are very close. As may be observed, good agreement was obtained for amplitude and fair agreement for frequency. The frequencies associated with the discrete frequency points should differ from the sweep frequencies, the blip marks, by the filter tracking lag (0.96). Close observations show variations greater than this amount for some points. The major cause of this is the inaccuracy of the blip marking circuit for the on-line data. Because of the limitation of test cell equipment, a maximum error of 4 percent can occur in blip marking.

The sweep frequency data were obtained using a 1 decade per minute sweep rate and thus required approximately 2 minutes to obtain. The discrete frequency data required over 25 minutes to obtain. Thus, a time saving of 10 to 1 was obtained. This magnitude of the time saving factor is not a special case, and it applies to all the dynamic testing done in the engine-inlet test program.

The fuel nozzle spray pressure signal can be used to illustrate that good results can be obtained by the processing method even though a large percentage of unwanted 60-hertz pickup exists on the information signal. Figure 10 shows the unprocessed data which were used to obtain the results in figure 9(c). Beat frequency phenomenon at 30, 60, and 120 hertz and to a lesser extent at 20 hertz is quite pronounced. These beats, while making a visual estimate of amplitude unreliable, can be used to estimate the 60-hertz pickup amplitude to be four lines peak to peak throughout the data.

From figure 9(c) the maximum amplitude occurs near 20 hertz, and the amplitude of the information signal varies by 7 to 1 throughout the frequency range tested. From the unprocessed data (fig. 10), the signal to noise ratio at 20 hertz can be determined to be about 5. Thus, for frequencies in the 70- to 100-hertz range the signal to noise ratio is 5/7 assuming the noise amplitude is constant throughout.

The signature of a constant frequency noise added to a sweep frequency is the beat phenomenon for the unprocessed data. After processing, the signature is a buildup of small loops followed by a large "S" shape loop which occurs when the sweep and noise frequencies are within the filters pass band. The signature can be seen in figure 9(c) as the sweep frequency progresses from 52 toward 60 hertz. This effect is only seen at the noise frequency, even though the unprocessed data show beats at multiples and fractions of the noise frequency. The magnitude of this effect depends on the signal to noise ratio, and the frequency range over which it is apparent depends on the averaging filters' bandwidth. For this case, the signal to noise ratio was approximately 1 and the filter used was a simple first order lag with a 1-second time constant.

CONCLUDING REMARKS

The use of sweep frequency testing for determining the dynamics of a jet engine is feasible and economical. A time saving factor of 10 to 1 is possible with no loss in accuracy and with greatly increased frequency resolution.

Straightforward procedures are furnished for determining the magnitude of the error resulting from using a sweep frequency as an input. A proper selection of the sweep frequency rate can result in an error magnitude which is constant and can be analytically predicted. Noise rejection is good even for a signal to noise ratio of 1 and logarithmic sweeping 1 to 100 hertz in 2 minutes.

The analysis method described herein can be mechanized on a table top size analog computer and used for on-line data reduction. Final plots can be obtained by the addition of corrections for variations in the input signals amplitude and phase.

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720-03.

APPENDIX A

SYMBOLS

A	amplitude of sine wave from oscillator
a	defined by eq. (1)
B	amplitude of cosine from oscillator
b	defined by eq. (2)
C	system gain
$\mathcal{Im}$	imaginary part of transfer function
N	nondimensional frequency index
P	period, time
R	sweep rate, decades/minute
$\mathcal{Re}$	real part of transfer function
t	time
Y	amplitude ratio of system or transfer function
ζ	damping ratio
θ	angle, rad
σ	time delay, sec
τ	time constant of first order system, sec
φ	phase shift angle between sine and cosine, outputs of the sweep generator, rad
ψ	phase shift angle between sine and system, rad
ω	frequency, rad/sec

Subscripts:

b	content in information signal
c	corrected
f	filter
h	highest value during test
p	resonant peak of second order system for constant frequency
n	natural frequency
nf	natural frequency of filter
n1	natural frequency of system 1
n2	natural frequency of system 2
s	resonant peak of second order system for sweep frequency
1, 2	time slice identification

Superscripts:

$\overline{}$	average
$\dot{}$	time derivative, 1/sec

APPENDIX B

FREQUENCY CONTENT IN THE INFORMATION SIGNALS

Time Delay

The information signals of a time delay when driven by a constant sweep rate (given in eq. (12)) are

$$\mathcal{Re} = \frac{A^2}{2}\cos\left[\sigma\omega(t) + \frac{\sigma^2}{2}\dot{\omega}\right]$$

$$\mathcal{Im} = \frac{A^2}{2}\sin\left[\sigma\omega(t) + \frac{\sigma^2}{2}\dot{\omega}\right]$$

When the phase error, due to the sweep rate, $(\sigma^2/2)\dot{\omega}$ is neglected, the information signals are sinusoidal and a function of $\sigma\omega(t)$. The frequency content of the signals depends on the length of time P required for $\sigma\omega(t)$ to change by 2π. Thus, the frequency content is

$$\left.\begin{aligned} \sigma\omega_1 - \sigma\omega_2 &= 2\pi \\ \sigma(\dot{\omega}P) &= 2\pi \\ \sigma\dot{\omega} &= \frac{2\pi}{P} = \omega_b \end{aligned}\right\} \tag{B1}$$

For logarithmic sweep rates the instantaneous sweep rate of the uppermost frequency ω_h sets the bandwidth. This can be determined by using equation (15) to determine the sweep rate:

$$\dot{\omega} = \omega_h\left(\frac{R \ln 10}{60}\right) \tag{B2}$$

Substituting the sweep rate (eq. (B2)) into equation (B1) results in the frequency content being

$$\omega_b = \frac{\sigma\omega_h R \ln 10}{60} = \sigma\omega_h\left(\frac{\dot{\omega}}{\omega}\right)$$

First Order and Second Order Systems

The information signal frequency content of first and second order systems cannot be obtained by straightforward methods. An approximate method, which consisted of three steps, was used: (1) determining the information signal amplitude as a function of frequency in the range of approximately 1/10 to 10 times the characteristic frequency of the system, (2) converting frequency to time by use of the frequency-time relation of the sweep rate, and (3) determining the frequency content of the information signal amplitude-time relation by Fourier transforms.

The frequency spectrum developed by the Fourier transforms can be used to reproduce the information signal if all the frequencies are used. In general, the power content in the upper frequencies is very small, and these frequencies can be truncated with a small loss in accuracy and a large reduction in the frequency content. Thus, for the first and second order systems the percentage of total power against required frequency content is presented.

In order to generalize the figures, a nondimensional frequency index N is used for first order systems and the product $N\zeta$ is used for second order systems. The magnitude of the frequency index is proportional to frequency range required for a selected power content. The curve of frequency index against power for a first order system is given in figure 11(a) and for a second order system in figure 11(b).

The equations necessary to determine the information frequency bandwidth ω_b from the frequency index are as follows:

First order constant sweep rate:

$$\omega_b = \dot{\omega}\tau N$$

First order logarithmic sweep rate:

$$\omega_b = \left(\frac{\dot{\omega}}{\omega}\right)N = \left(\frac{R \ln 10}{60}\right)N$$

Second order constant sweep rate:

$$\omega_b = \left(\frac{\dot{\omega}}{\zeta\omega_n}\right)N\zeta$$

Second order logarithmic sweep rate:

$$\omega_b = \left(\frac{\dot{\omega}}{\zeta\omega}\right)N\zeta = \left(\frac{R \ln 10}{60}\right)N$$

It is interesting to note that if the logarithmic and constant sweep rates are equal at the system characteristic frequencies (ω_n and $1/\tau$), either type of rate results in the same frequency bandwidth. Also, the frequency bandwidth of the information signal is inversely proportional to the system characteristic frequency for constant rate sweeps and is independent of them for logarithmic rate sweeps. If the system errors $\left(\dot{\omega}\tau^2 \text{ and } \dot{\omega}/2\zeta^2\omega_n^2\right)$ are held constant, then the frequency bandwidth of the information signals is proportional to the system characteristic frequency as indicated in the following equations:

$$\frac{\omega_b}{\frac{1}{\tau}} = N(\dot{\omega}\tau^2)$$

$$\frac{\omega_b}{2\zeta\omega_b} = N\zeta\left(\frac{\dot{\omega}}{2\zeta^2\omega_n^2}\right)$$

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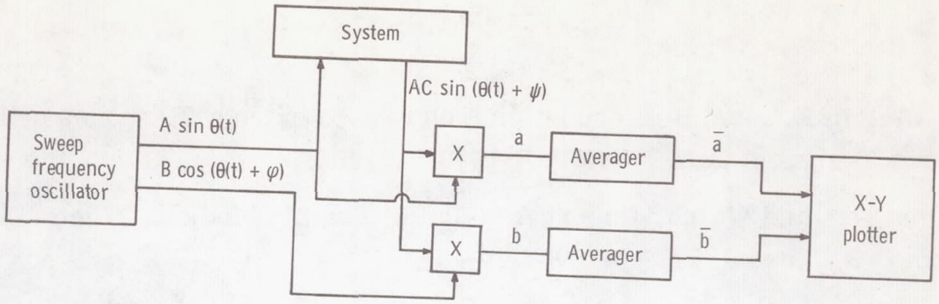


Figure 1. - On-line analysis of sweep frequency data.

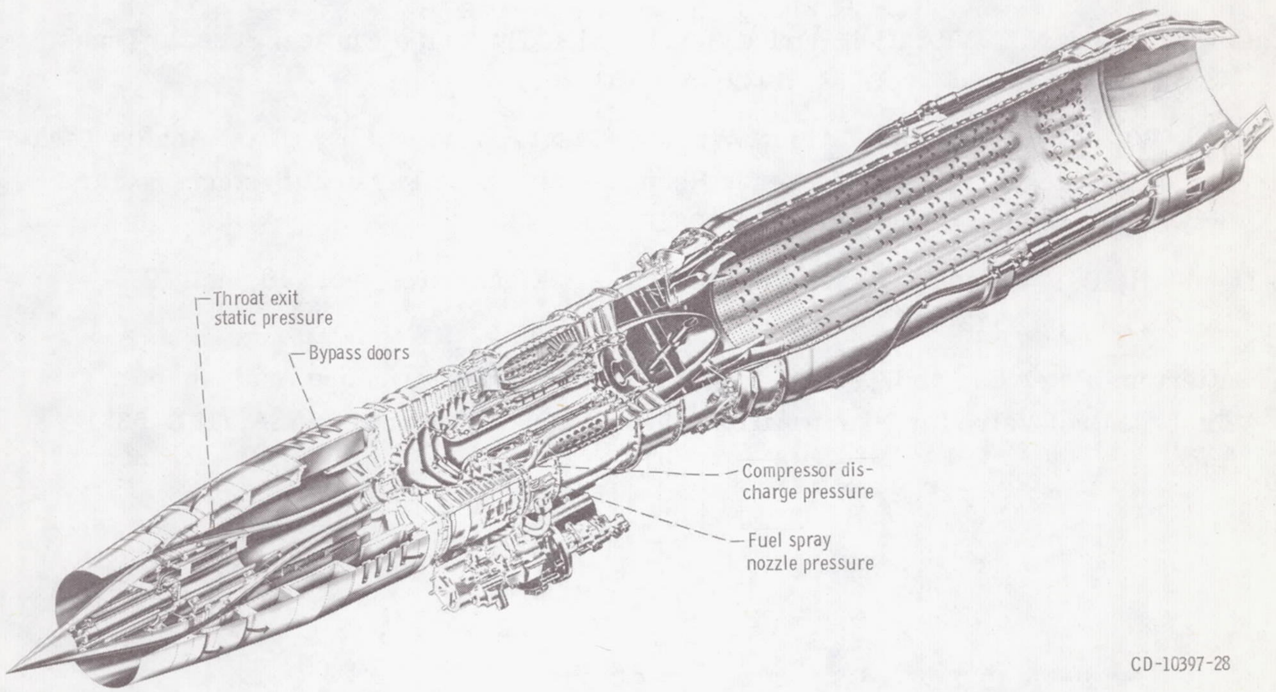


Figure 2. - Cutaway view of inlet engine used for supersonic wind tunnel tests.

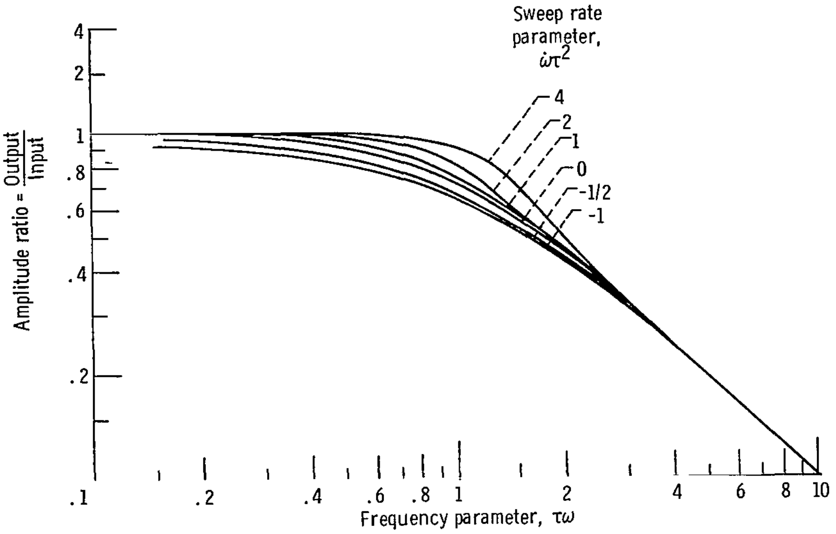
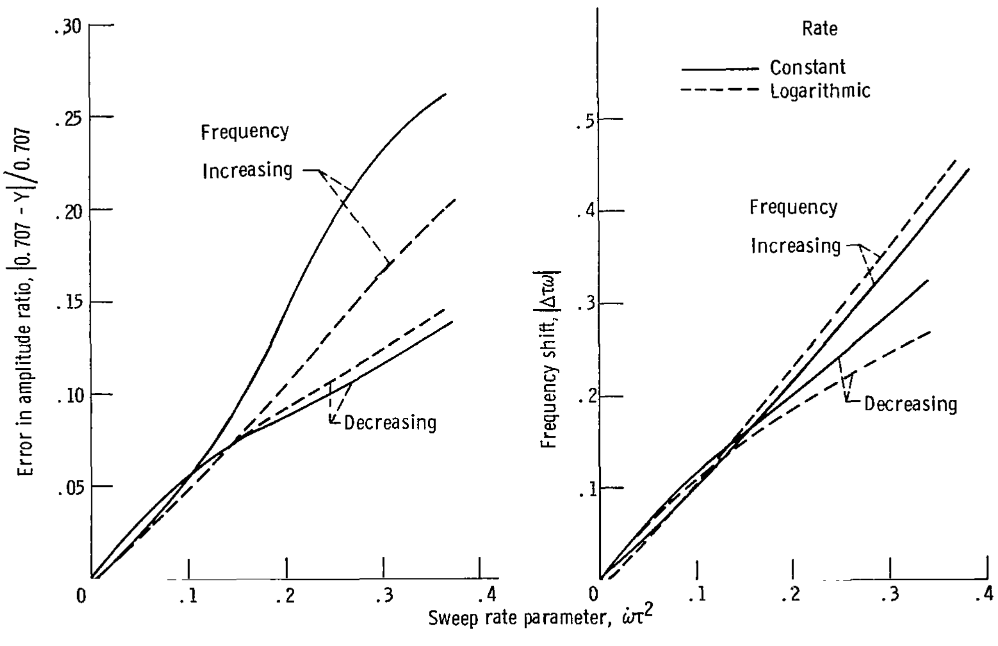


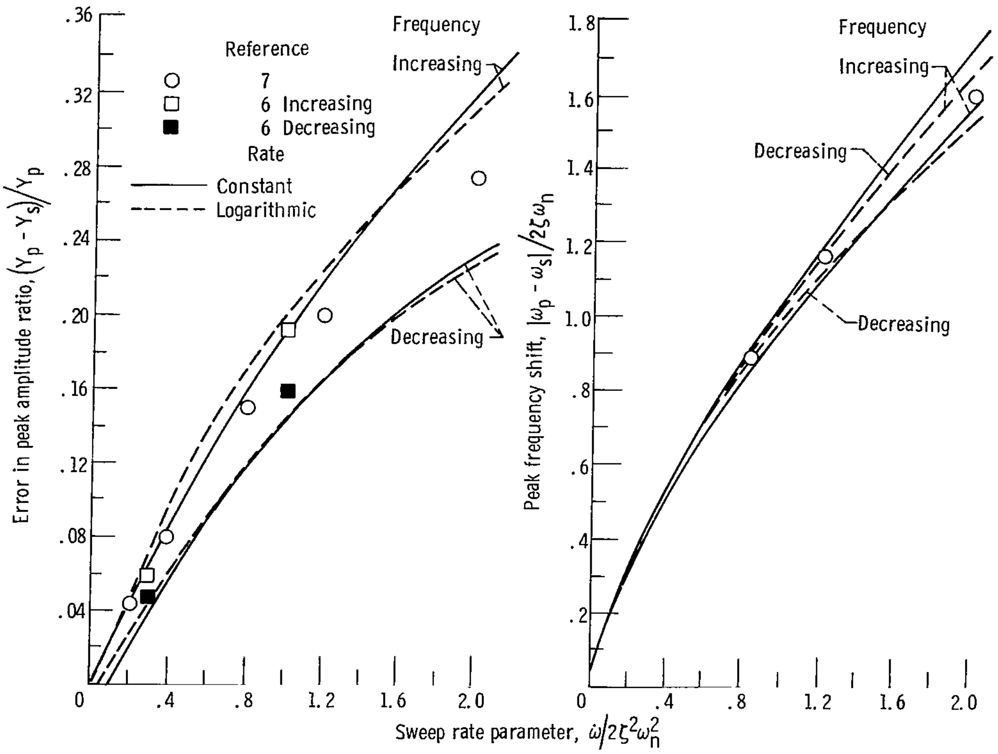
Figure 3. - First order system response for sweep frequency inputs.



(a) Error in amplitude ratio at $\tau\omega = 1$.

(b) Frequency shift at $Y = 0.707$.

Figure 4. - Effect of sweep rate on first order system.



(a) Peak amplitude ratio error.

(b) Frequency shift at peak amplitude.

Figure 5. - Effect of sweep rate on second order system resonance.

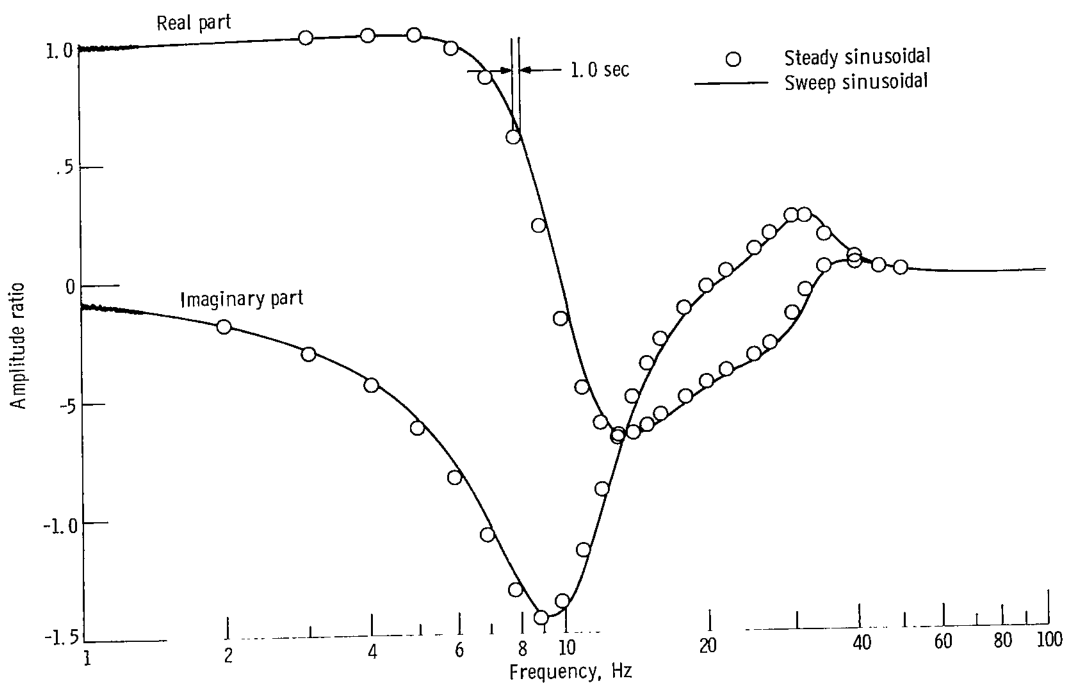
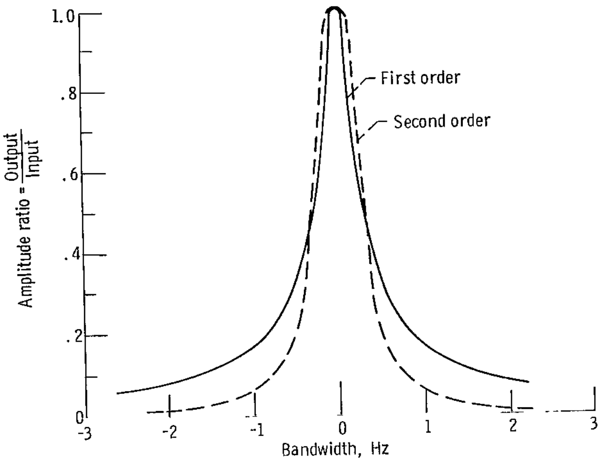
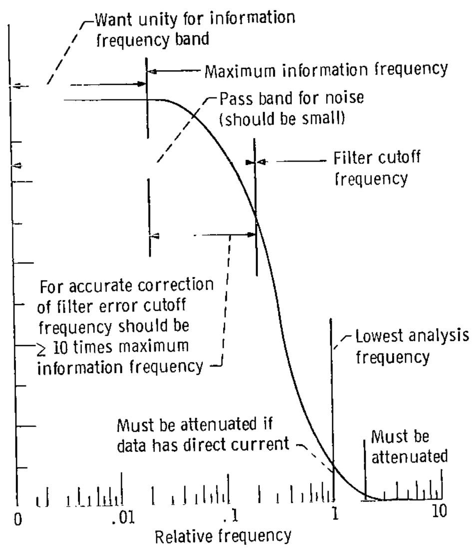


Figure 6. - Comparison of sweep and steady sinusoidal data.

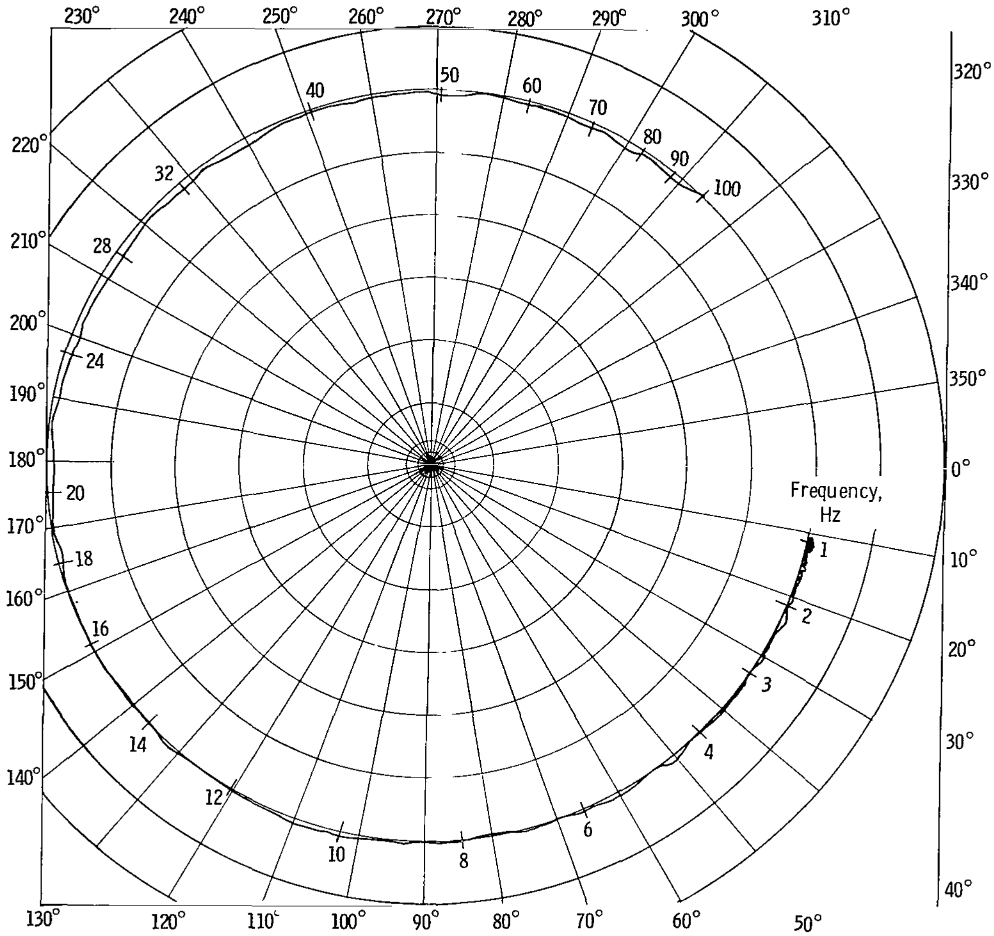


(a) First and second order filter characteristics.



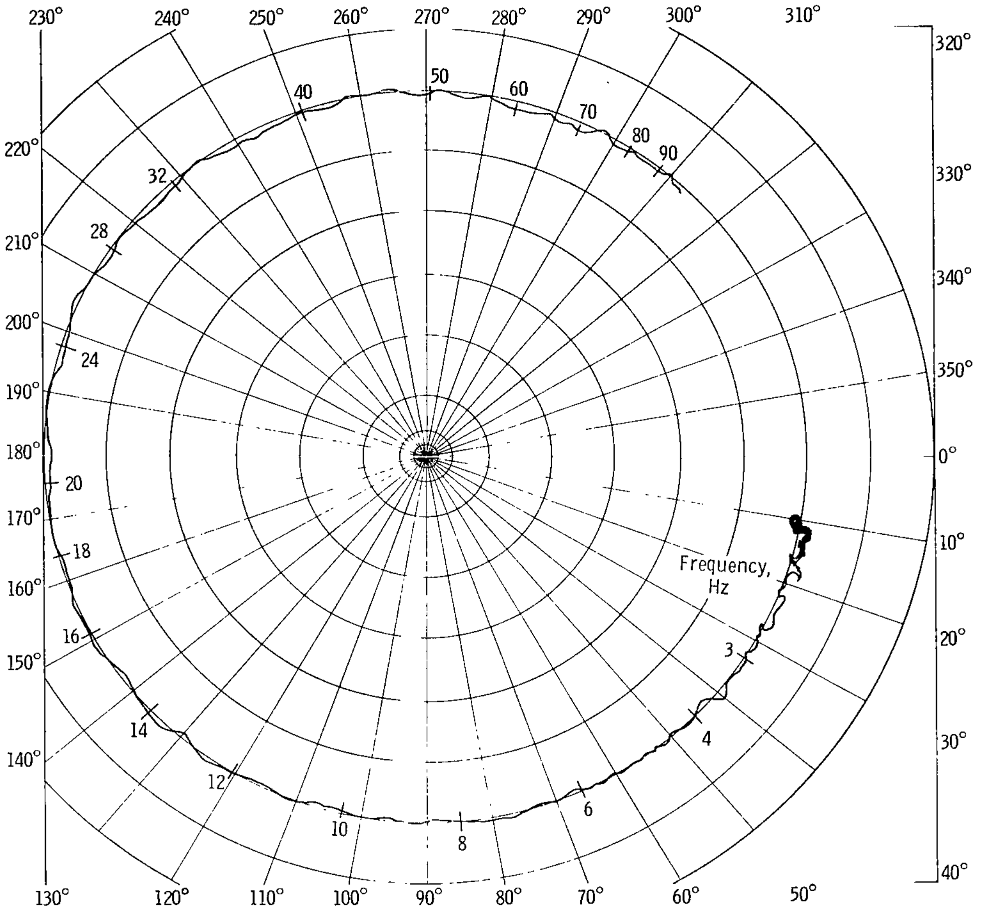
(b) Filter requirements.

Figure 7. - Band pass filter considerations.



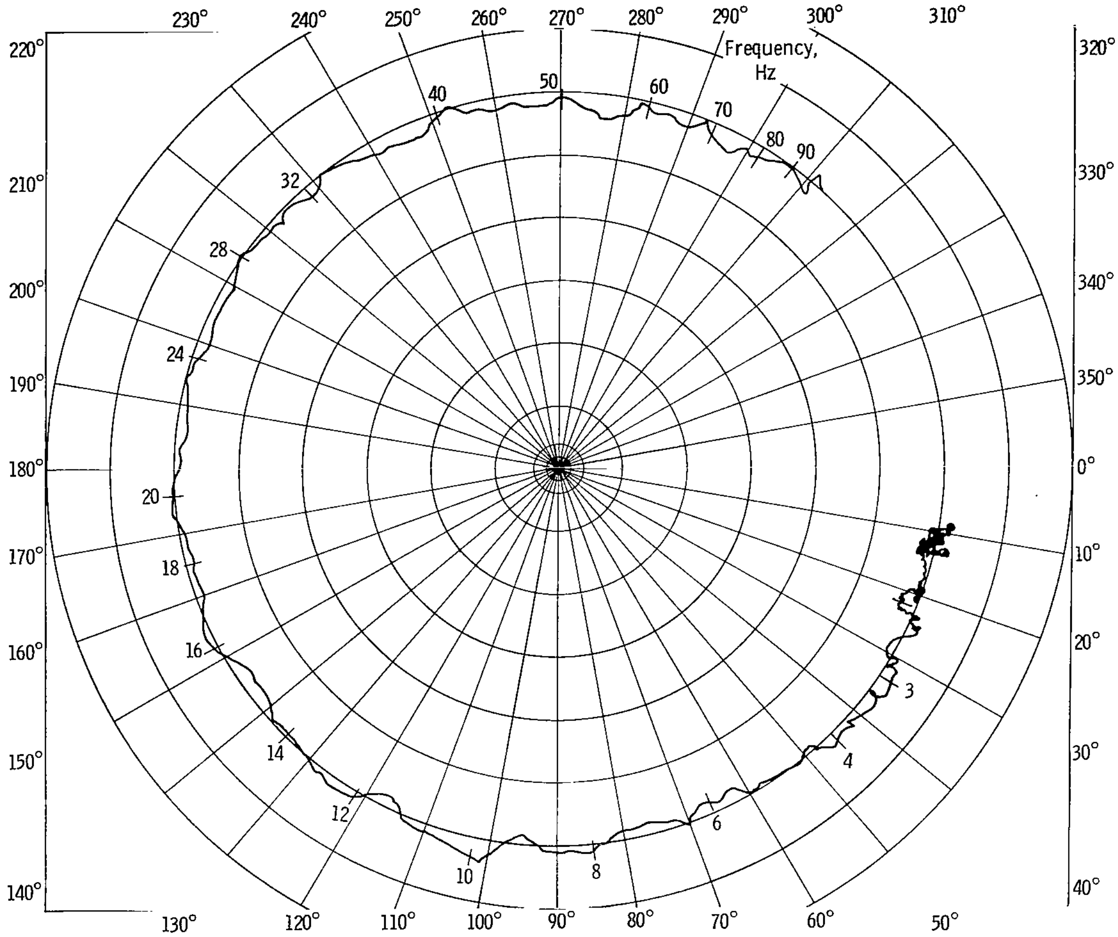
(a) 10-Percent rms noise.

Figure 8. - Noise rejection capability of analysis method.



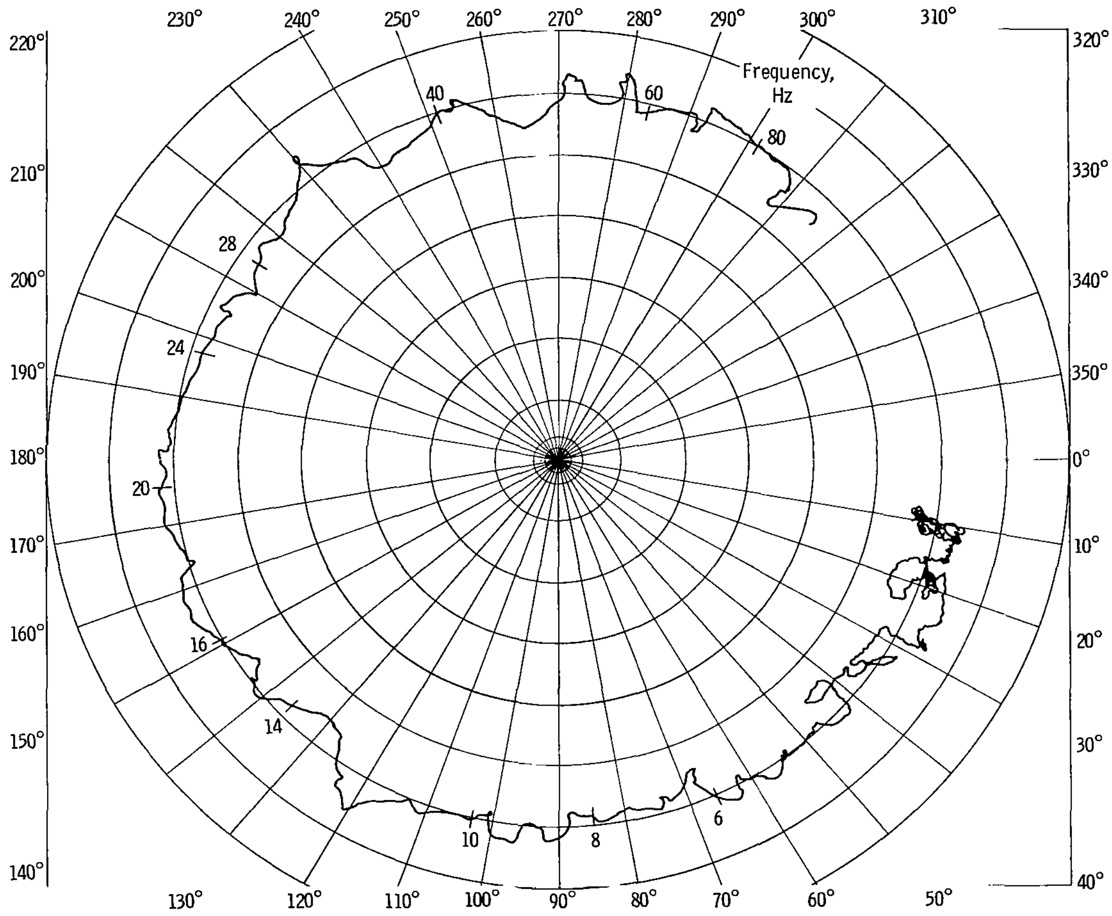
(b) 25-Percent rms noise.

Figure 8. - Continued.



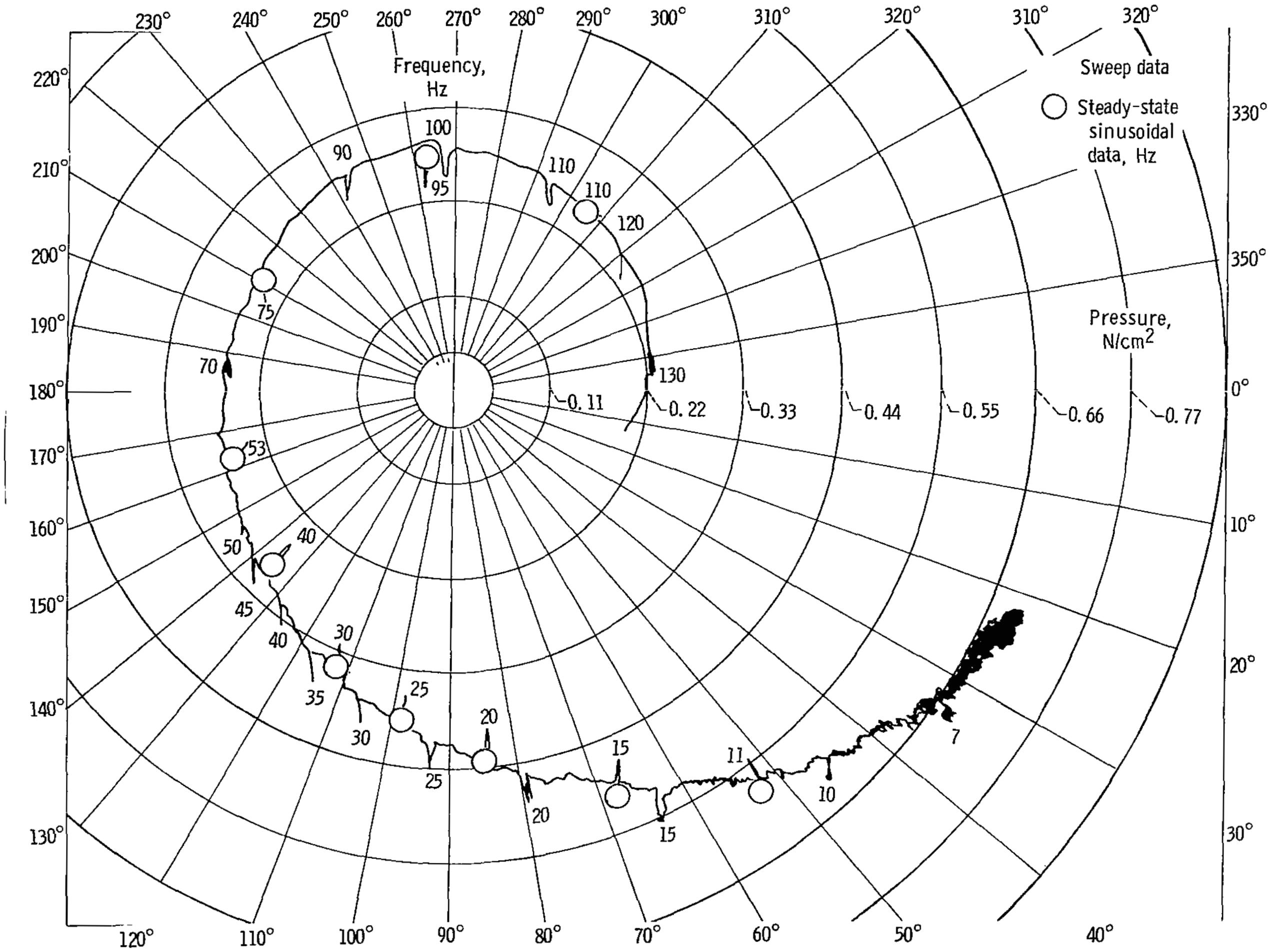
(c) 50-Percent rms noise.

Figure 8. - Continued.



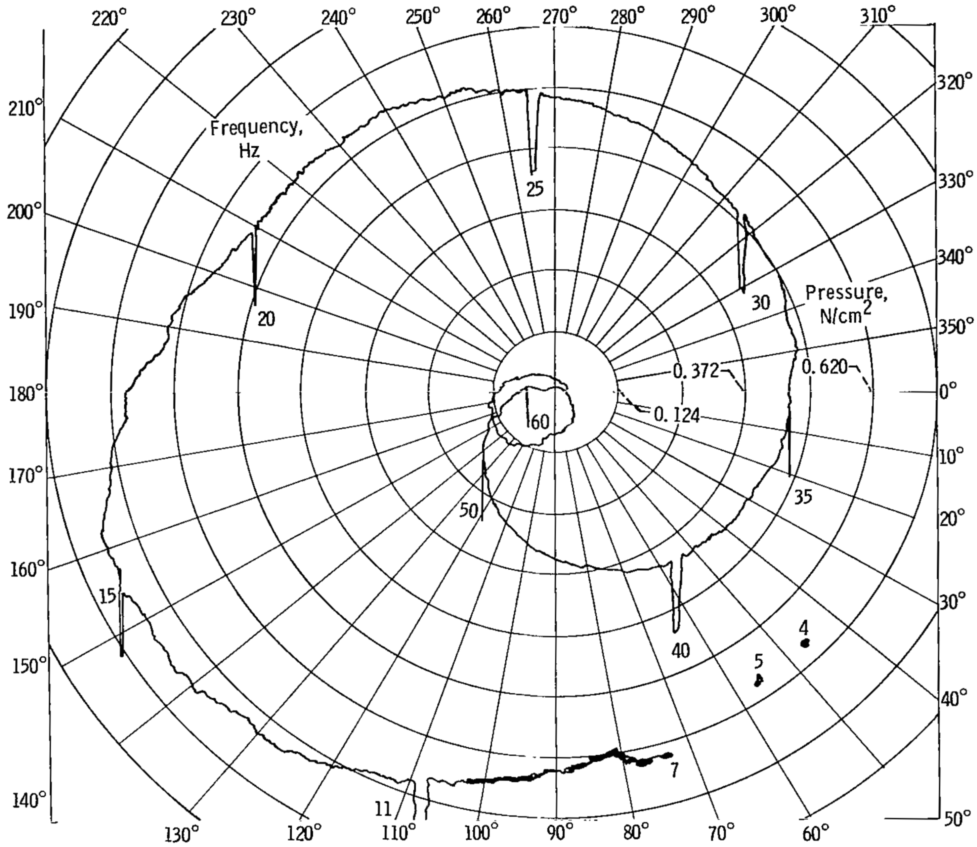
(d) 100-Percent rms noise.

Figure 8. - Concluded.



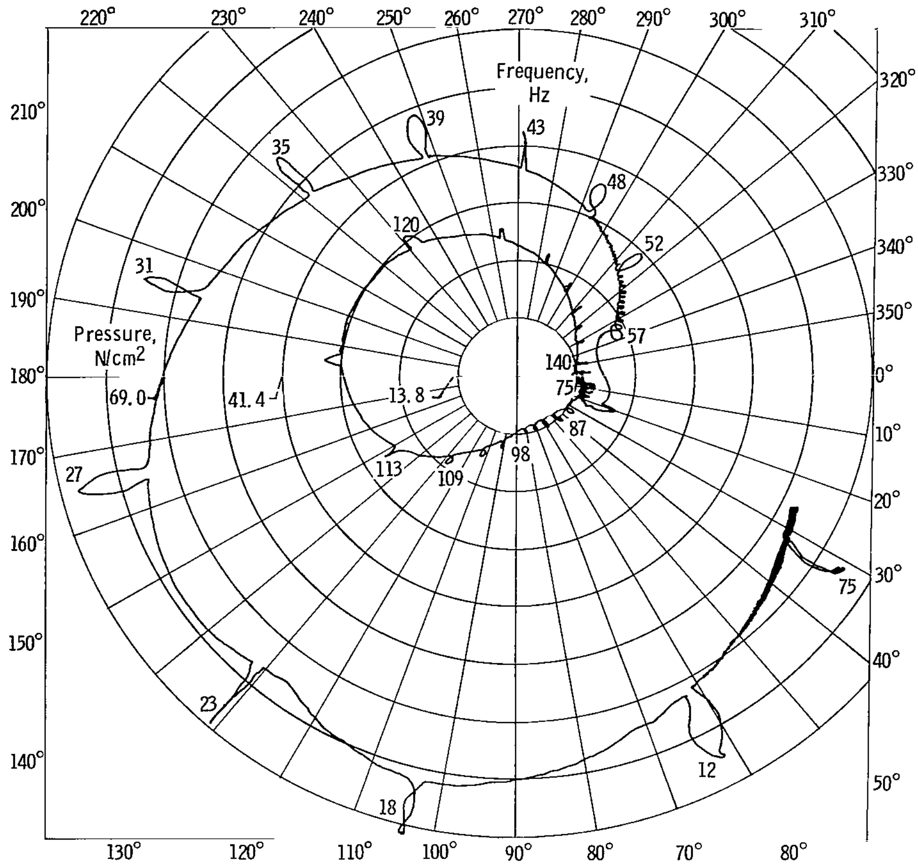
(a) Inlet throat exit static pressure response to bypass door command.

Figure 9. - Sweep frequency test results from engine-inlet test program.



(b) Compressor discharge pressure response to fuel flow command.

Figure 9. - Continued.



(c) Fuel spray nozzle pressure response to fuel command.

Figure 9. - Concluded.

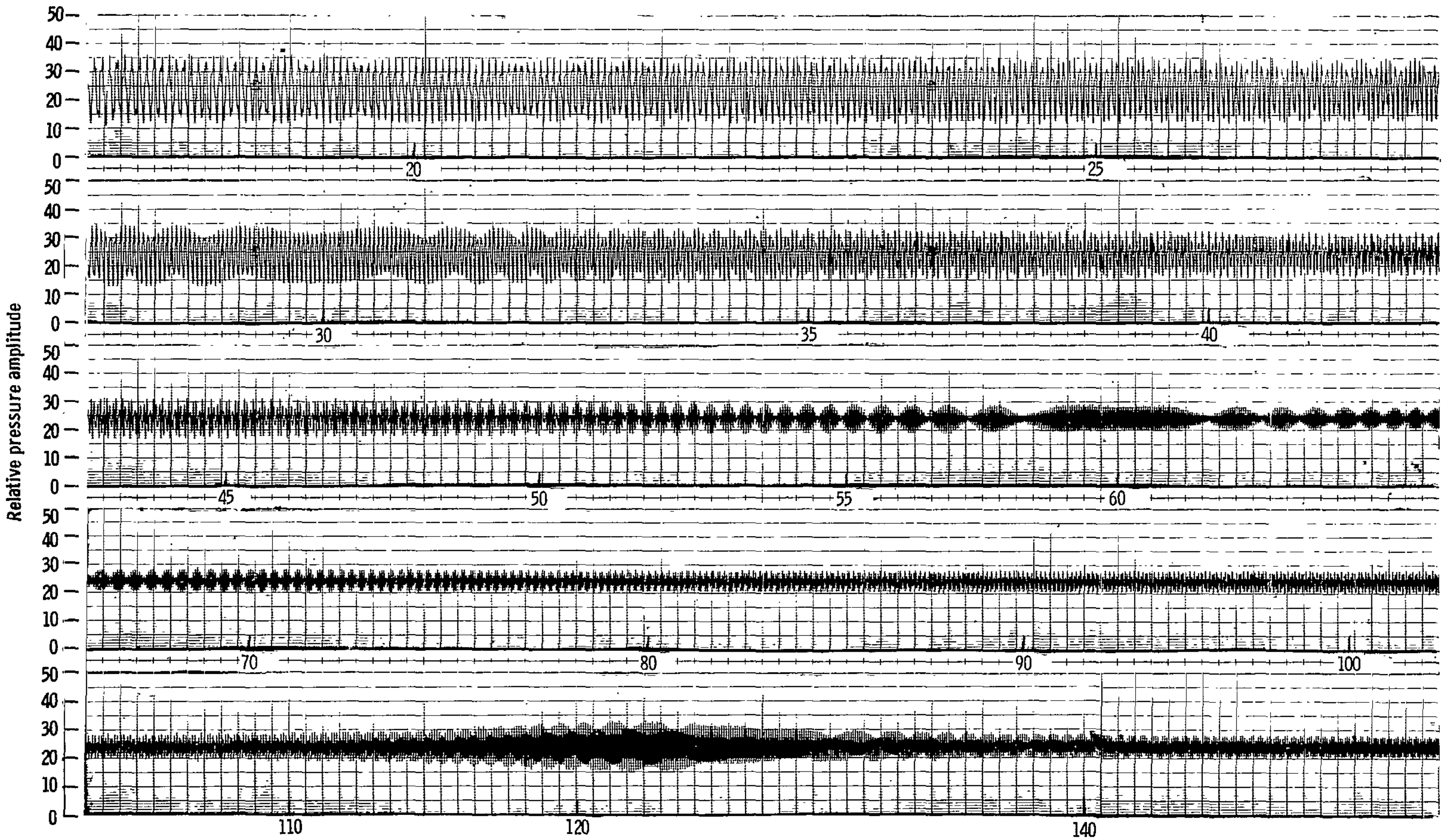
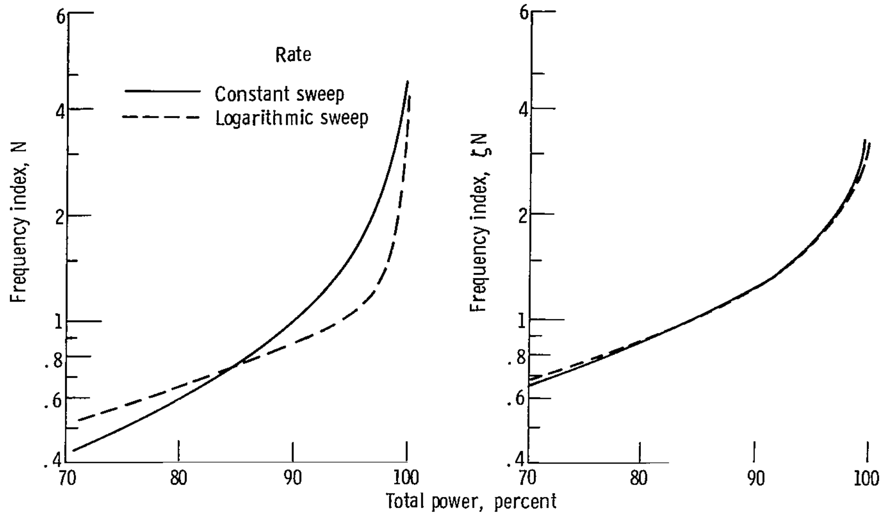


Figure 10. - Raw sweep frequency data for spray nozzle pressure response to fuel command.



(a) First order system.

(b) Second order system.

Figure 11. - Power content in information signals.

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