NASA TECHNICAL REPORT

NASA TR R-320

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NASA TR R-320

COMMUNICATION EFFICIENCY OF QUANTUM SYSTEMS

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • SEPTEMBER 1969

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TR R-320		
4. Title and Subtitle: Communication Efficiency of Quantum Systems		5. Report Date: September 1969
		6. Performing Organization Code
7. Author(s): Sherman Karp		8. Performing Organization Report No.: C-78
9. Performing Organization Name and Address: Electronics Research Center, Cambridge, Mass., 02139		10. Work Unit No.: 125-22-01-02
		11. Contract or Grant No.
		13. Type of Report and Period Covered: Technical Report
12. Sponsoring Agency Name and Address: National Aeronautics and Space Administration, Washington D.C. 20546		14. Sponsoring Agency Code
15. Supplementary Notes		

16. Abstract

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17. Key Words	18. Distribution Statement
	Unclassified - Unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	19	$3.00

*For sale by the Clearinghouse for Federal Scientific and Technical Information
Springfield, Virginia 22151

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SUMMARY

This report presents a method for evaluating the theoretical performance of quantum communication systems based on the communication efficiency parameter, $\beta = P/N_oR$. For Gaussian systems, β represents the minimum average amount of energy required to decipher a bit of information, with zero error, in the presence of white noise of spectral density N_o. For certain quantum systems, the Gaussian result is directly applicable with N_o replaced by $h\nu$. In general, β is recognized to be the minimum number of average (photon) events required to decipher a bit of information with zero error. To make comparisons with Gaussian systems, the parameter κ is introduced and $h\nu/\kappa$ becomes the effective Gaussian spectral density of the quantum system. Several systems are considered in addition to the capacity bound for the narrow-band quantum channel introduced by Gordon. Unlike the Gaussian channel, there is no lower bound on β as the bandwidth-to-data-rate ratio, α, is increased. Thus β can be continually decreased at the expense of average data rate. For large values of α, PPM direct detection asymptotically approaches the lower bound for β.

INTRODUCTION

As more and more attention is given to optical communication systems, it becomes necessary to develop effective guides so that they can be compared with each other and to other communication systems. At present, the only comparison that has been used is the post-detection signal-to-noise ratio (refs. 1,2). However, as has been pointed out (ref. 3), this can be misleading and a more objective criterion is needed. For communication in Gaussian noise, an accepted criterion is the β-efficiency (ref. 4) or the minimum energy required to decipher a bit of information with zero error in the presence of white Gaussian noise, the spectral density of which is N_o. Thus:

$$\beta = \frac{P}{N_oR} \tag{1}$$

with P the average power and R the data rate. This is generally a function of the ratio of the available bandwidth B to the data rate. In this report, it will be shown that this criterion is also applicable to optical systems and can yield some interesting results. Unity quantum efficiency will be assumed throughout the report.

BACKGROUND

As is well known, the capacity of an additive white Gaussian channel can be written as

$$R = B \log_2 \left(1+\frac{P}{N_o B}\right) \tag{2}$$

If this is rewritten in terms of β and $\alpha = B/R$, it takes the form:

$$\beta = \alpha(2^{1/\alpha}-1).$$

As $\alpha \to \infty$, β is monotone, decreasing and has a lower bound, β_{min}, given by

$$\beta_{min} = \log_e 2. \tag{3}$$

Hence, for any given noise spectral density N_o, there is a minimum amount of energy required to decipher a bit of information with zero error. Thus from a theoretical point of view, it is best to use all the available bandwidth to send any information rate R.

For a narrow-band photon channel, Gordon (refs. 5,6) has calculated the maximum source entropy, which has been verified (refs. 7,8) as being a tight bound to the capacity of a noiseless narrow-band optical channel. This expression takes the form:

$$R = B\left[\log\left(1+\frac{P}{h\nu B}\right)+ \frac{P}{h\nu B} \log\left(1+\frac{h\nu B}{P}\right)\right]. \tag{4}$$

An important comparison can be made between Eq. (4) and the capacity of the additive Gaussian noise channel in the limit as $P/h\nu B$ becomes large. Then, to first order:

$$\begin{aligned} R &\cong B\left[\log\left(1+\frac{P}{h\nu B}\right)+ 1\right] \\ &\cong B\left[\log\left(1+\frac{P}{h\nu B}\right)\right] \end{aligned} \tag{5}$$

and it is recognized that even with no additive noise the photon channel looks approximately like an additive Gaussian noise channel with spectral density equal to $h\nu$. The associated signal-to-noise ratio, $P/h\nu B$, is referred to as the *photon-limited*, signal-to-noise ratio. It is also common to associate an equivalent Gaussian temperature T_g to this expression by the relation:

$$T_g = \frac{h\nu}{k} = 4.8 \times 10^{-11}\nu.$$

It must be remembered, however, that these latter remarks *only* apply to the approximation in Eq. (5).

The β-efficiency of the narrow-band channel can be computed in the following way. With the result taken from the Gaussian channel, β can be associated with $P/h\nu R$. Thus Eq. (4) can be written as

$$\frac{1}{\alpha} = \log\left(1+\frac{\beta}{\alpha}\right) + \frac{\beta}{\alpha}\log\left(1+\frac{\alpha}{\beta}\right). \tag{6}$$

The β-efficiency of this channel is plotted in Figure 1, together with that of the additive Gaussian channel. Notice that for the narrow-band photon channel, β is a monotonically decreasing function with no lower bound.

OPTICAL HETERODYNE DETECTION

It was first suggested by Oliver (refs. 9,10), and has since been substantiated experimentally, that one can obtain the channel characteristics of Eq. (5) by using optical heterodyne detection. In this technique a local oscillator is aligned coherent with received field over the surface of a quantum detector. The net result is that, because of the non-linear behavior of the detector, a signal appears at the difference frequency of the two fields. By making the local oscillator field very large it becomes the major contributor to the noise, and by central limit theorem arguments this noise is Gaussian. The resulting signal-to-noise ratio can then be shown to be:

$$\left(\frac{S}{N}\right) = \frac{P}{h\nu B} \quad \text{and} \quad \beta = \frac{P}{h\nu R}\ .$$

with $\beta_{min}^{het} = \log_e 2$. It can also be shown that even in the presence of additive thermal background noise, the net result of this procedure will remain substantially the same (ref. 11).

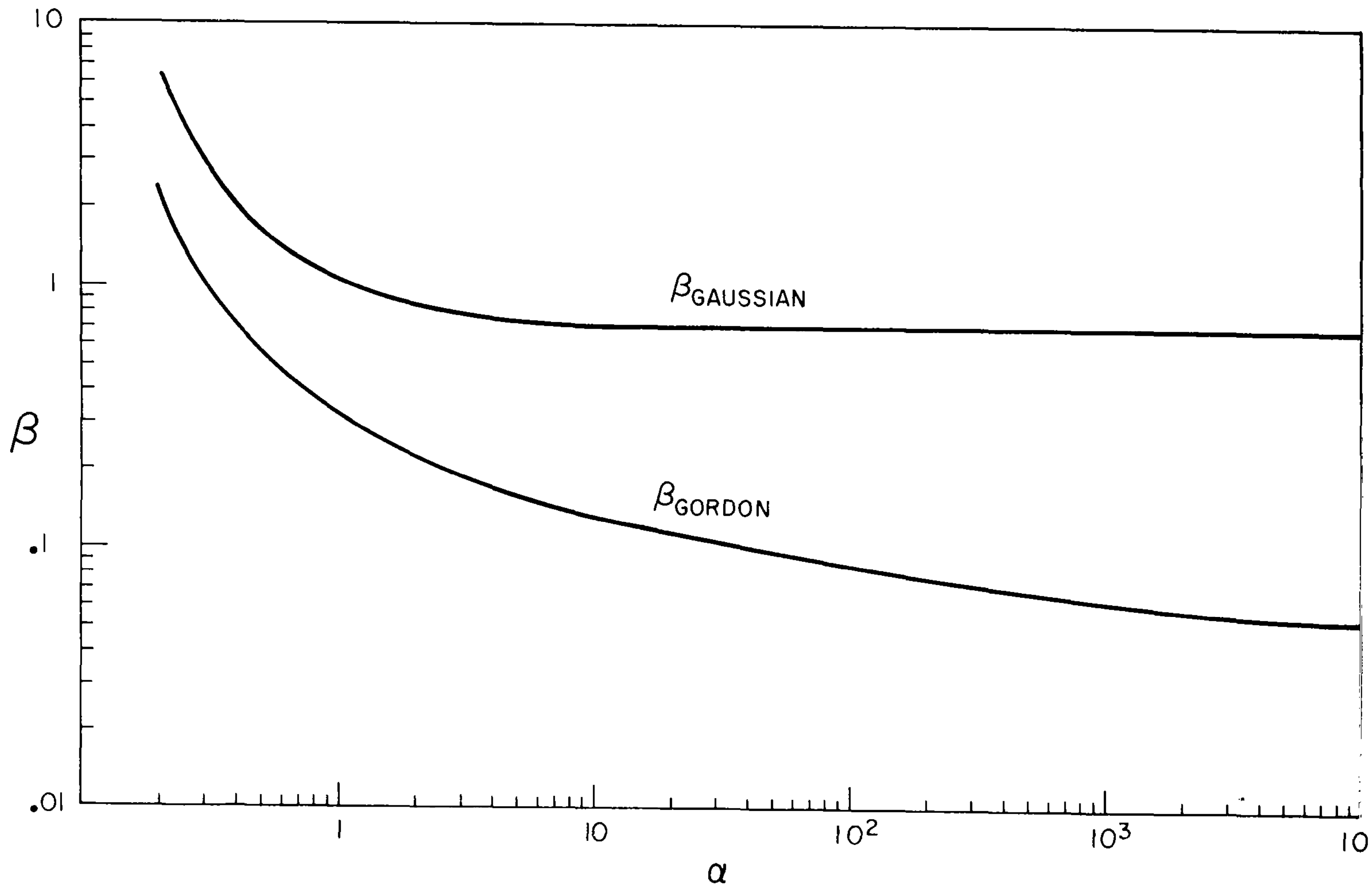

Figure 1.- Theoretical β-efficiency of the Gaussian and narrow-band photon channels as a function of α (bandwidth-to-data-rate ratio)

For the narrow-band optical channel, β lends itself to an interesting interpretation. For this case, one can write P as the number of photons arriving per second, $\bar{n}$, multiplied by the energy per photon, $h\nu$. Thus:

$$\beta = \frac{P}{h\nu R} = \frac{\bar{n}h\nu}{h\nu R} = \frac{\bar{n}}{R} ,$$

or the number of photons per bit. Thus we see that the β efficiency of an optical channel can be interpreted as the minimum number of photons required to decipher a bit of information with zero error The Gordon bound indicates that this number is monotonically decreasing with α.

OPTICAL HOMODYNE DETECTION

A unique feature of the heterodyne system is the performance at zero difference frequency when the signal local oscillator fields are homodyned. It can be shown that the signal-to-noise ratio becomes $2P/h\nu B$, or, that the noise spectral density is $h\nu/2$. This does not happen with RF systems and can be considered to be a quantum effect. To see this, again, consider Eq. (5):

$$R \cong B\left[\log\left(1+\frac{P}{h\nu B}\right)+ 1\right]$$

for high signal-to-noise ratio. The first term on the right side of Eq. (5) is treated as the classical portion while the second term is the quantum portion. This can be approximated by

$$R \cong B\left[\log\left(\frac{P}{h\nu B}\right)+ 1\right]= B \log\left(\frac{eP}{h\nu B}\right)$$

or an effective noise spectral density of $h\nu/e$. Thus the capacity of the heterodyne system does not approach the Gordon bound, but is 4.34 dB away. By contrast the capacity of the homodyne system is:

$$R \cong B \log\left[1+\frac{2P}{h\nu B}\right]\cong B \log\left(\frac{2P}{h\nu B}\right)$$

with $\beta_{min}^{hom} = 1/2\beta_{min}^{het} = 1/2 \log_e 2$. This system recovers 3 of the 4.34 dB. One can view this gain as a coherent addition of the positive and negative signal frequencies, but an incoherent addition of the quantum noise.* The Gaussian temperature associated

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*Private communication with R. Kennedy and E. Hoversten, Dept. of Elec. Engng., M.I.T.

with homodyning is:

$$T_{hom} = \frac{h\nu}{2k} = \frac{\beta^{hom} T_g}{\beta^{het}} = \frac{T_g}{2} = 2.4 \times 10^{-11}\nu .$$

DIRECT DETECTION - WIDEBAND OPTICAL FILTER

At optical frequencies one is afforded the additional latitude of direct or incoherent detection without the use of diffraction limited receiving structures. This is due primarily to the availability of quantum detectors and the validity of geometric optics. In direct detection the received field is collected and focused onto a quantum detector, the response of which is proportional to the instantaneous power collected. The resulting current flow can be accurately modeled as a conditional shot noise process (ref. 12) with the rate parameter proportional to the instantaneous intensity integrated over the active area of the detector.

Initial analyses of direct detection systems were based on the signal-to-noise ratio. This quantity can be shown to be less than or equal to the quantum-limited signal-to-noise ratio. Unlike the heterodyne system, however, it is very sensitive to any form of additive noise. Thus, from a cursory examination one would only consider direct detection systems where heterodyne detection could not be used. More recently, however, Reiffer and Sherman (ref. 13) and then Abend (ref. 14) recognized that the performance of a direct detection system was also sensitive to signal design. Later it was shown that, based on the same Poisson model, the optimum M-ary system was discrete pulse position modulation (ref. 3). Karp and Gagliardi (ref. 15) designed a system based on these results which minimized β for each given value of α. The efficiency of this system is plotted in Figure 2 for zero background noise, together with the Gordon bound and both the heterodyne and homodyne systems. Notice that for $\alpha > 5$ the PPM system is more efficient than heterodyning, and for $\alpha > 20$, it is more efficient than homodyning. In fact, in the limit as $\alpha \to \infty$, the PPM system approaches the Gordon bound. For small alphabets, $\alpha < 5$, the PPM system is, however, outperformed by the optimum binary quantum receiver (ref. 16) as shown by the hatched lines in Figure 2 (for M=2 on-off modulation is superior to PPM and comes very close to the optimum quantum receiver). This latter system is the only one, so far, which has been designed strictly by quantum mechanical considerations. In fact, the physical structure of this receiver is, as yet, undetermined. It is believed that this system reduces to a biorthogonal homodyne system, gaining an additional 3 dB in performance, when signal design is optimized.*

*Private communication with R. Kennedy and E. Hoversten, Dept. of Elec. Engng., M.I.T.

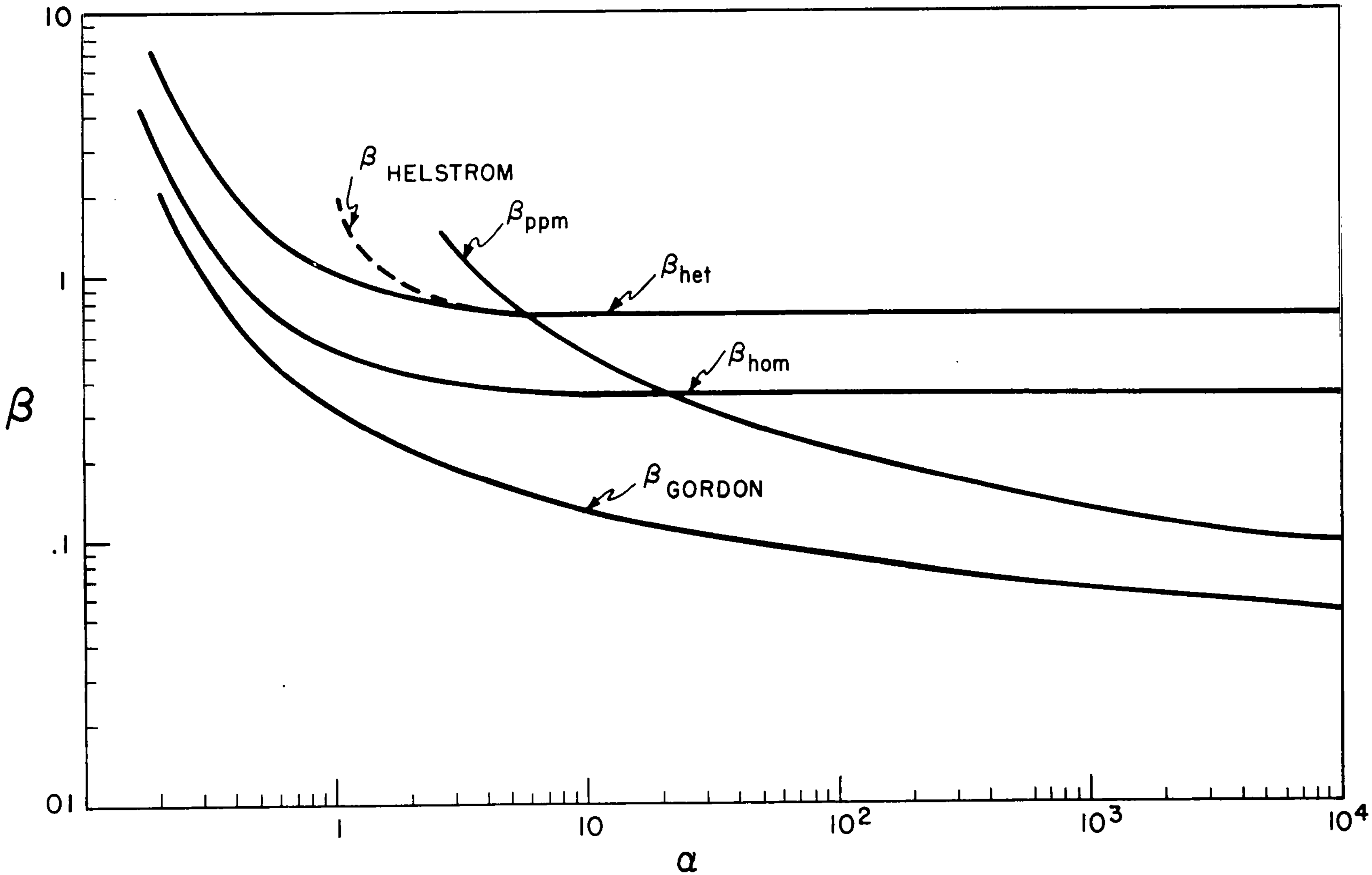

Figure 2.- Theoretical β-efficiency of some noiseless systems as a function of α

When background noise is considered the channel performance of the PPM system naturally degrades. In Figure 3, this performance is plotted for the PPM system with the background noise, K_n a parameter. K_n, which is proportional to the background noise, is a measure of the average number of noise events present per pulse position. Notice that for each value of K_n, performance follows the zero noise system until a break point is reached. At this point, efficiency flattens out. The smaller the value of K_n, the lower the value of β at which the break point is reached. It is clear that efforts to reduce K_n will result in increased performance. On the other hand, K_n is a measure of the quality of the components used (i.e., width of the optical filter, quality of the collector for a given diameter, speed of the electronic processing for low duty cycle); hence a balance must be reached between cost and performance. Notice, for example, that for $\alpha = 10$ there is no gain in reducing K_n below 0.01.

DIRECT-DETECTION, NARROW-BAND OPTICAL FILTER

It has been shown by Liu (ref. 17) that a system, dual in performance to PPM, is frequency-position modulation or frequency-shift keying. This brings up the question as to the range of validity of the Karp-Gagliardi system. They have assumed Poisson statistics, and hence have inherently assumed that the optical filter has a larger bandwidth than the overall system bandwidth. If the bandwidth of the optical filter is reduced to minimize noise by matching the system bandwidth, the statistics change to a Laguerre density in Bose-Einstein noise (ref. 18). With these latter statistics, the efficiency of the PPM narrow-band system (and the dual system) was recalculated and is plotted in Figure 4, together with the result for the Poisson case.

For $K_n = 0$, both systems are identical, while for a given value of K_n the β-efficiency of the wideband system is lower than that of the narrow-band system. Of course one cannot compare the two systems for the same value of K_n since they correspond to two different levels of prefiltered noise intensity. From physical intuition one always expects the performance to improve as the background noise decreases. Thus, the following observations can be made:

1. All other factors being constant, one always improves system performance by decreasing the filter bandwidth to match the system bandwidth (up to the point where the $K_n = 0$ curve is approached).
2. All other factors being constant, one always improves system performance by decreasing the duty cycle to match the filter bandwidth (up to the point where the $K_n = 0$ curve is reached).

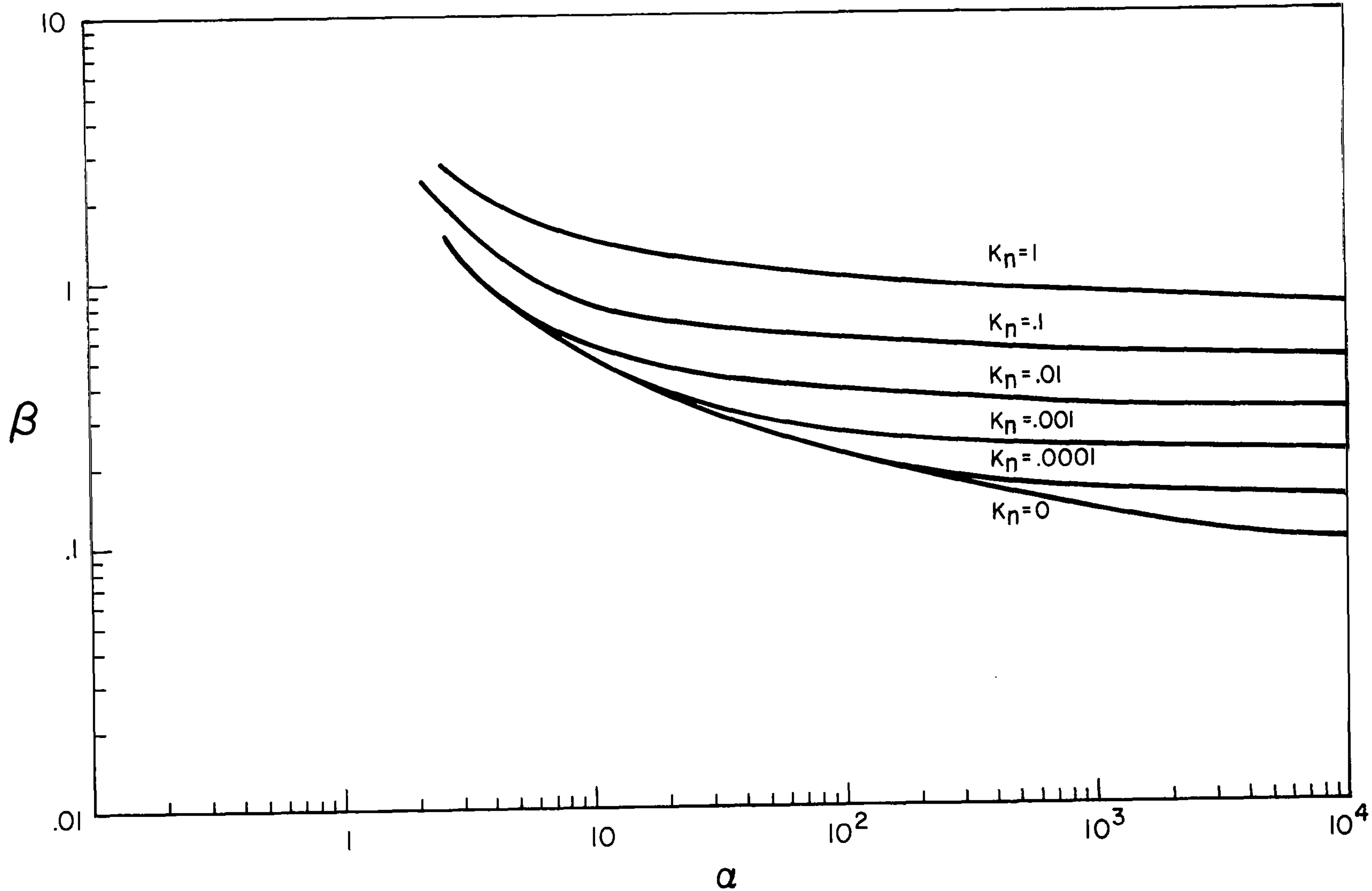

Figure 3.- Theoretical β-efficiency of wideband PPM systems with additive noise as a function of α

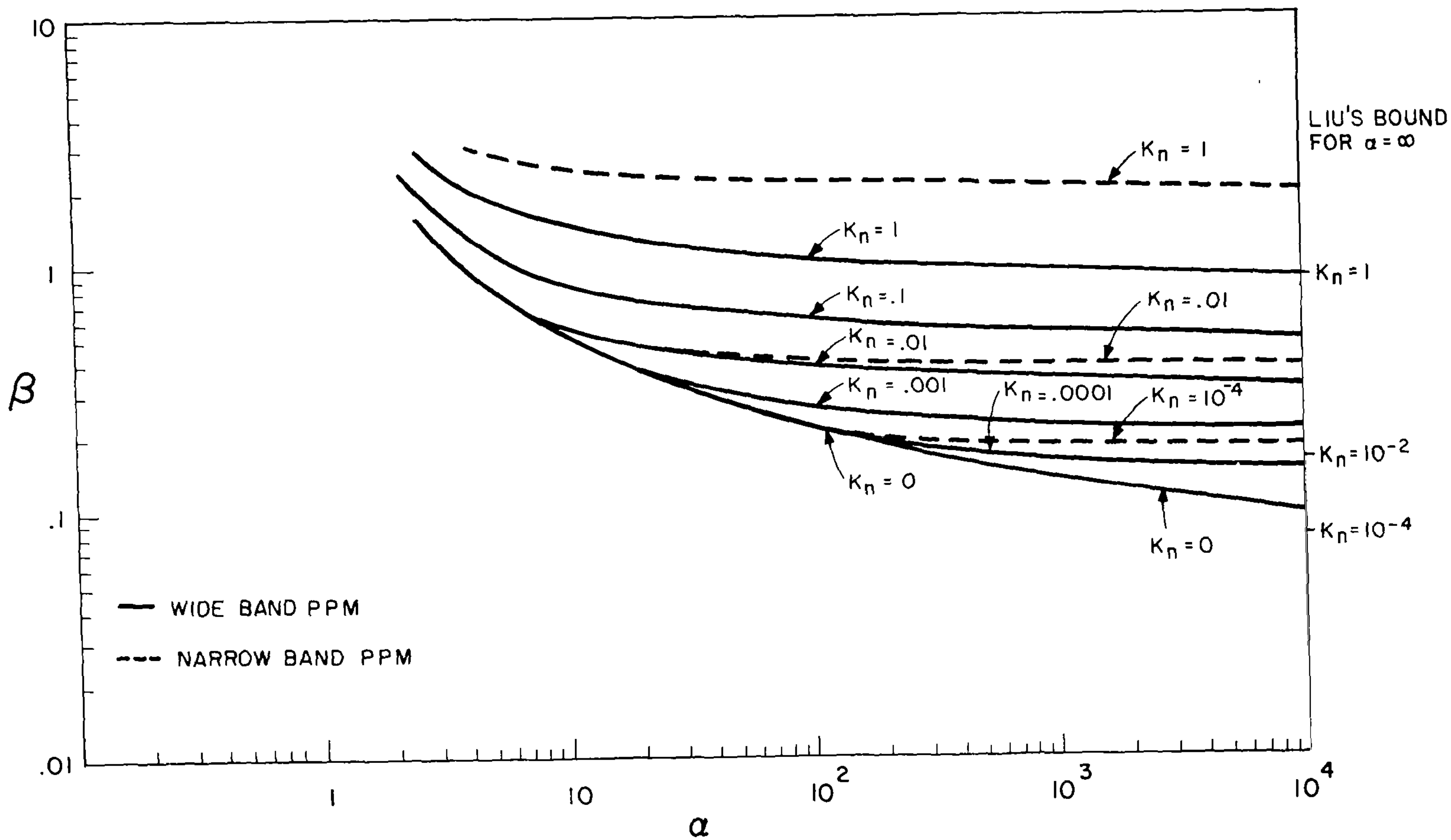

Figure 4.- Theoretical β-efficiency of PPM systems as a function of α

3. If one has a choice of either decreasing K_n by (1) or (2) <u>or</u> decreasing K_n by a statistics preserving means (such as decreasing the field of view of the collector), the latter is preferable.

COMMUNICATION EFFICIENCY WITH ADDITIVE GAUSSIAN NOISE

In private correspondence, it has been suggested by Gordon that the capacity equation in reference 5, Eq. (5), is valid for the quantum channel in the presence of additive Gaussian noise. This equation is:

$$R = B\left\{\log\left[1 + \frac{P}{h\nu B+N}\right] + \left(\frac{P+N}{h\nu B}\right)\log\left[1 + \frac{h\nu B}{P+N}\right] - \frac{N}{h\nu B}\log\left[1 + \frac{h\nu B}{N}\right]\right\} \tag{7}$$

where N is the average noise power. It follows immediately that $N/h\nu B = K_n$ in the notation used here. Therefore in terms of α, β, and K_n we can write this as

$$\frac{1}{\alpha} = \log\left[1 + \frac{\beta/\alpha}{1+K_n}\right] + \left(\frac{\beta}{\alpha} + K_n\right)\log\left[1 + \frac{1}{\beta/\alpha+K_n}\right] - K_n\log\left(1+\frac{1}{K_n}\right) \tag{8}$$

Introducing the variable $\beta/\alpha = \eta$, multiplying both sides of Eq. (8) by β and rearranging, yields

$$\beta = \frac{\eta}{\log\left(1 + \frac{\eta}{1+K_n}\right) + (\eta+K_n)\log\left(1 + \frac{1}{\eta+K_n}\right) - K_n\log\left(1 + \frac{1}{K_n}\right)}$$

$$\alpha = \beta/\eta. \tag{9}$$

The limit of large α corresponds to $\eta \to 0$. It can be shown after a little manipulation that

$$\lim_{\eta\to 0}\beta = \frac{1}{\log\left[1 + \frac{1}{K_n}\right]}, \tag{10}$$

which is the identical result as that obtained by Liu (ref. 17). Since the capacity of the Poisson channel can be shown to approach that of the quantum channel for P, N small, we can conclude that the result in Eq. (10) is also valid for the wideband PPM channel. Equation (9) is plotted for several values of K_n in Figure 5 together with the results for the wideband PPM channel.

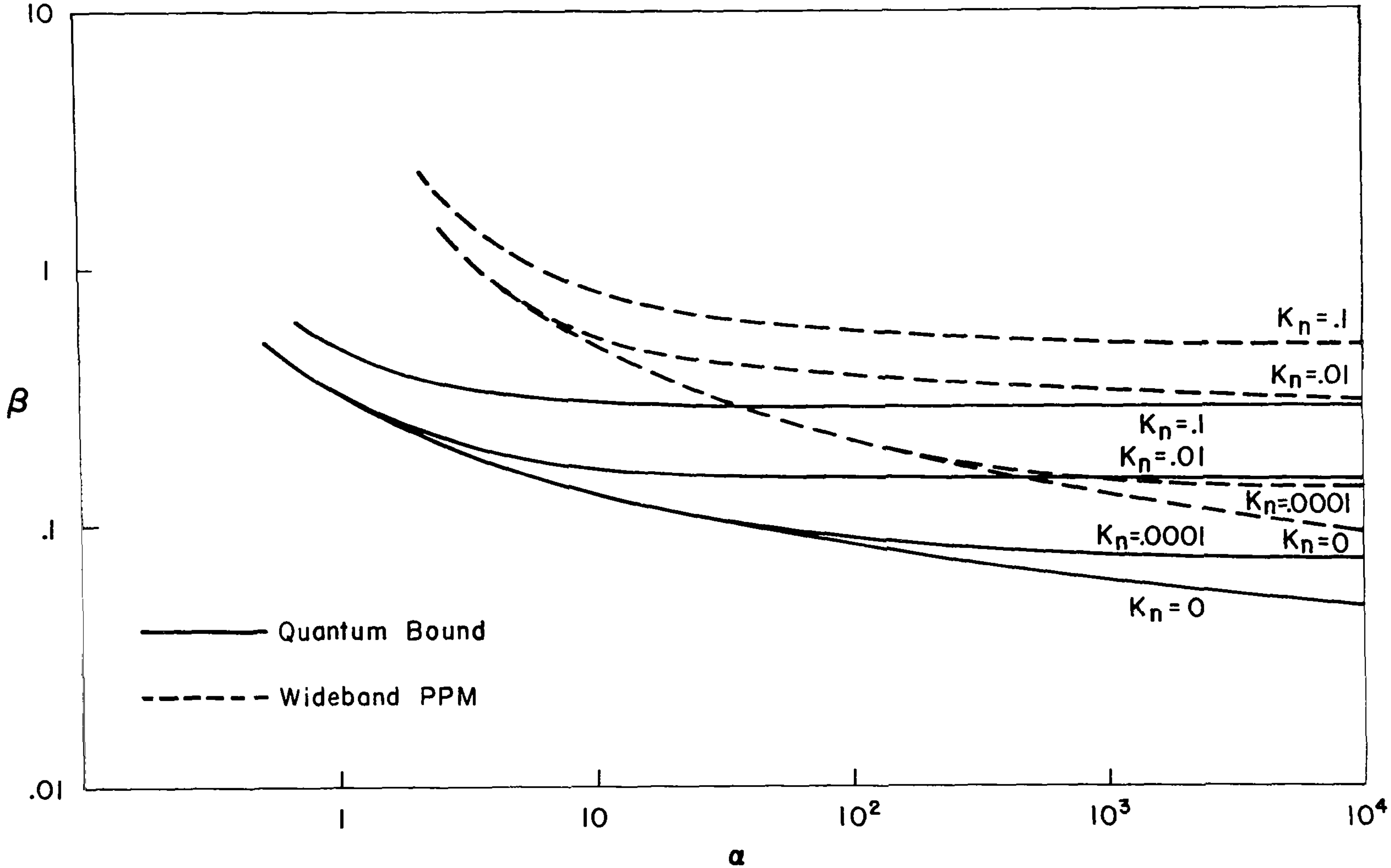

Figure 5.- Theoretical β-efficiency of wideband PPM systems with additive noise and the quantum bound with additive noise α

Another interesting result also suggested by Gordon can be obtained for large K_n. For this case it can be shown that

$$\lim_{K_n \text{ large}} \beta = (1+K_n) \log 2.$$

Suppose K_n arose from a blackbody at a temperature T_n. Then

$$K_n = \frac{1}{e^{\frac{h\nu}{KT_n}} - 1} . \tag{11}$$

For this to be large implies that $h\nu/KT_n << 1$ or $T_N >> h\nu/K$. But this is merely the classical Gaussian limit. To see this we write

$$\lim_{K_n \text{ large}} \beta = \frac{P}{h\nu R} = \left(1 + \frac{1}{e^{\frac{h\nu}{KT_n}} - 1}\right) \log 2$$

$$\cong \frac{KT_n}{h\nu} \log 2$$

or

$$\frac{P}{KT_n R} = \log 2,$$

the classical result discussed earlier or,

$$R = \frac{P}{KT_n \log 2} \text{ bits/sec.}$$

Similarly, for β large, K_n small and the assumption in Eq. (11),

$$\beta = \frac{1}{\left(\log 1 + \frac{1}{K_n}\right)}$$

$$\cong \frac{1}{\log e^{\frac{h\nu}{KT_n}}}$$

$$= \frac{KT_n}{h\nu} \log 2\ e$$

or again,

$$\frac{P}{KT_n R} = \log 2\ e.$$

EFFECTIVE GAUSSIAN TEMPERATURE

As mentioned earlier, it is common to associate a noise spectral density $h\nu$ with a quantum system. On the other hand, it has also been shown that to associate an irreducible temperature $T = h\nu/k$ with a quantum system is fallacious. Still one desires a figure of merit whereby various systems can be compared. It is proposed here that an *effective Gaussian temperature* can be established whereby quantum systems, in general, can be compared with Gaussian systems. If one defines

$$\kappa = \frac{\beta_{het}}{\beta}$$

where β is the communication efficiency of the system to be compared, then the *effective Gaussian temperature* of this system is, T_g, where

$$T_g = \frac{h\nu}{\kappa}\ .$$

The values of κ are plotted in Figure 6 for some of the noiseless systems considered previously, in Figure 7, for the wideband PPM systems, and in Figure 8 for the narrow-band PPM system and in Figure 9 for the maximum value predicted by Eq. (9). For the homodyne system, κ was equal to 2. Two examples follow.

<u>Example I</u> - Wideband PPM

$\alpha = 100$

$K_n < 0.002$

$\kappa = 3.3$

T_g at 0.53μ $(6\text{x}10^{14}) \approx 8000^{\circ}K$

at 10.6μ $(3\text{x}10^{13}\ \text{Hz}) \approx 400^{\circ}K$

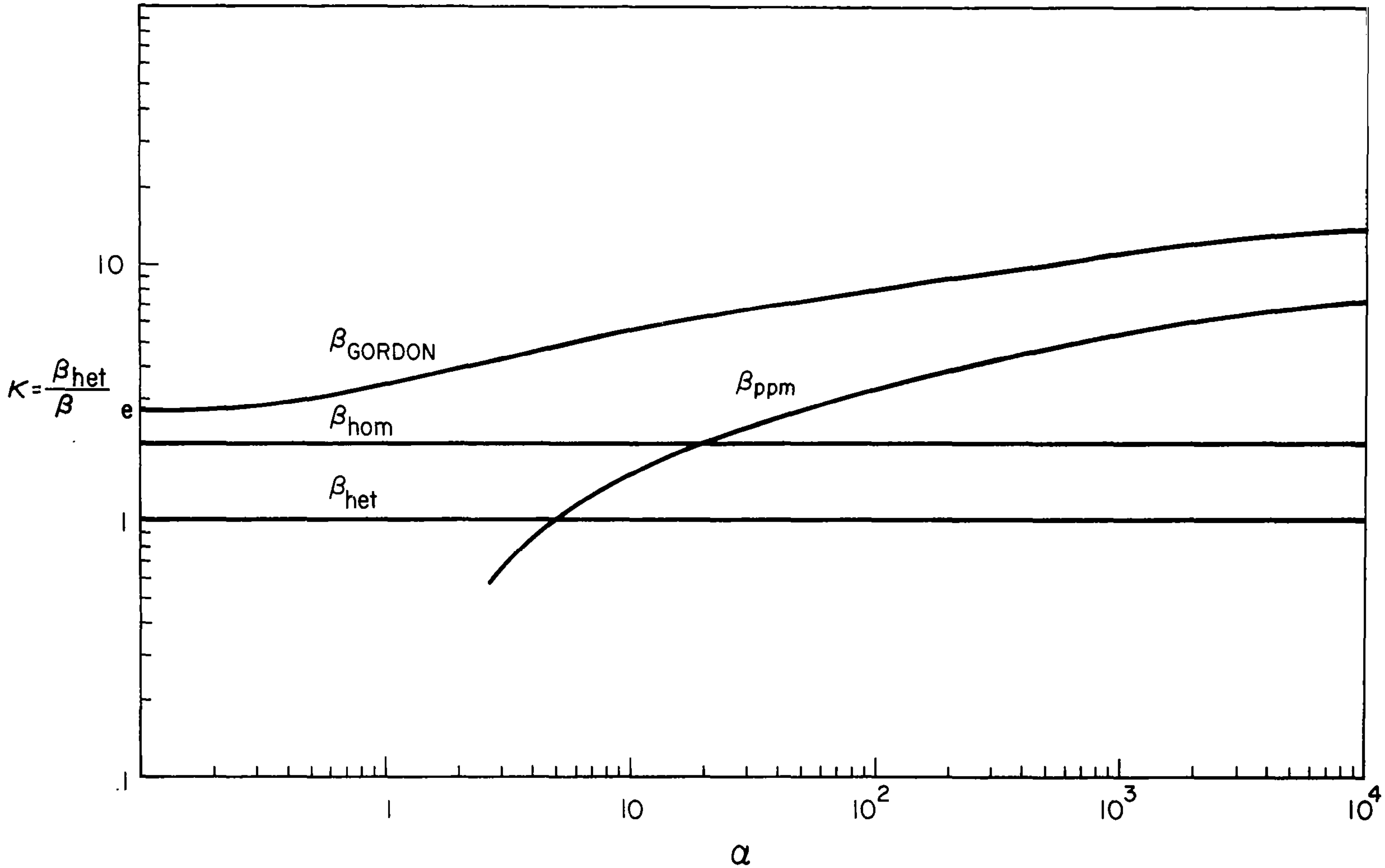

Figure 6.- Theoretical values of $\kappa = \frac{\beta_{het}}{\beta}$ for some noiseless systems as a function of α

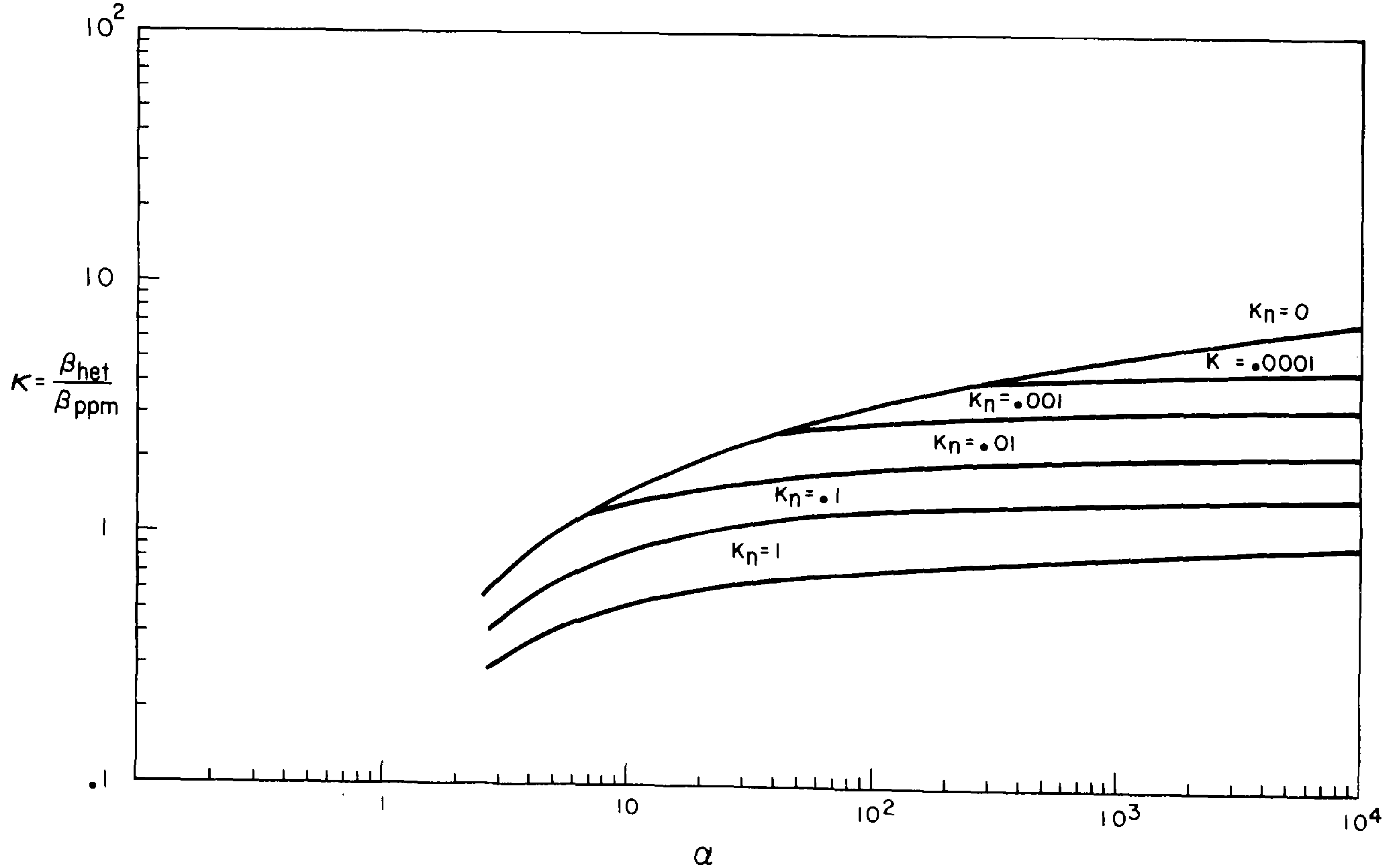

Figure 7.- Theoretical values of $\kappa = \frac{\beta_{het}}{\beta_{PPM}}$ for the wideband PPM system as a function of α

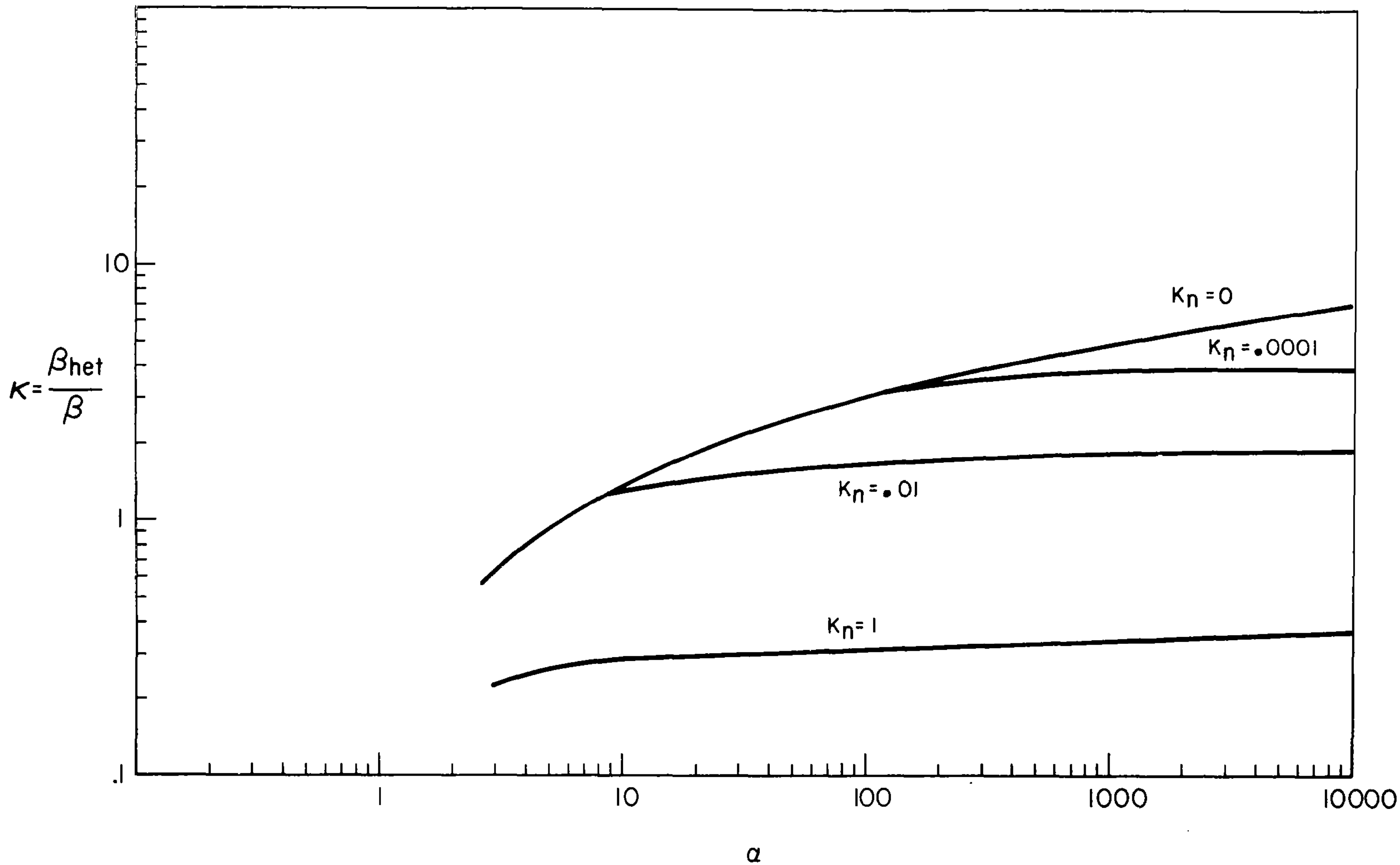

Figure 8.- Theoretical values of $\kappa = \frac{\beta_{het}}{\beta_{PPM}}$ for the narrow-band PPM system as a function of α

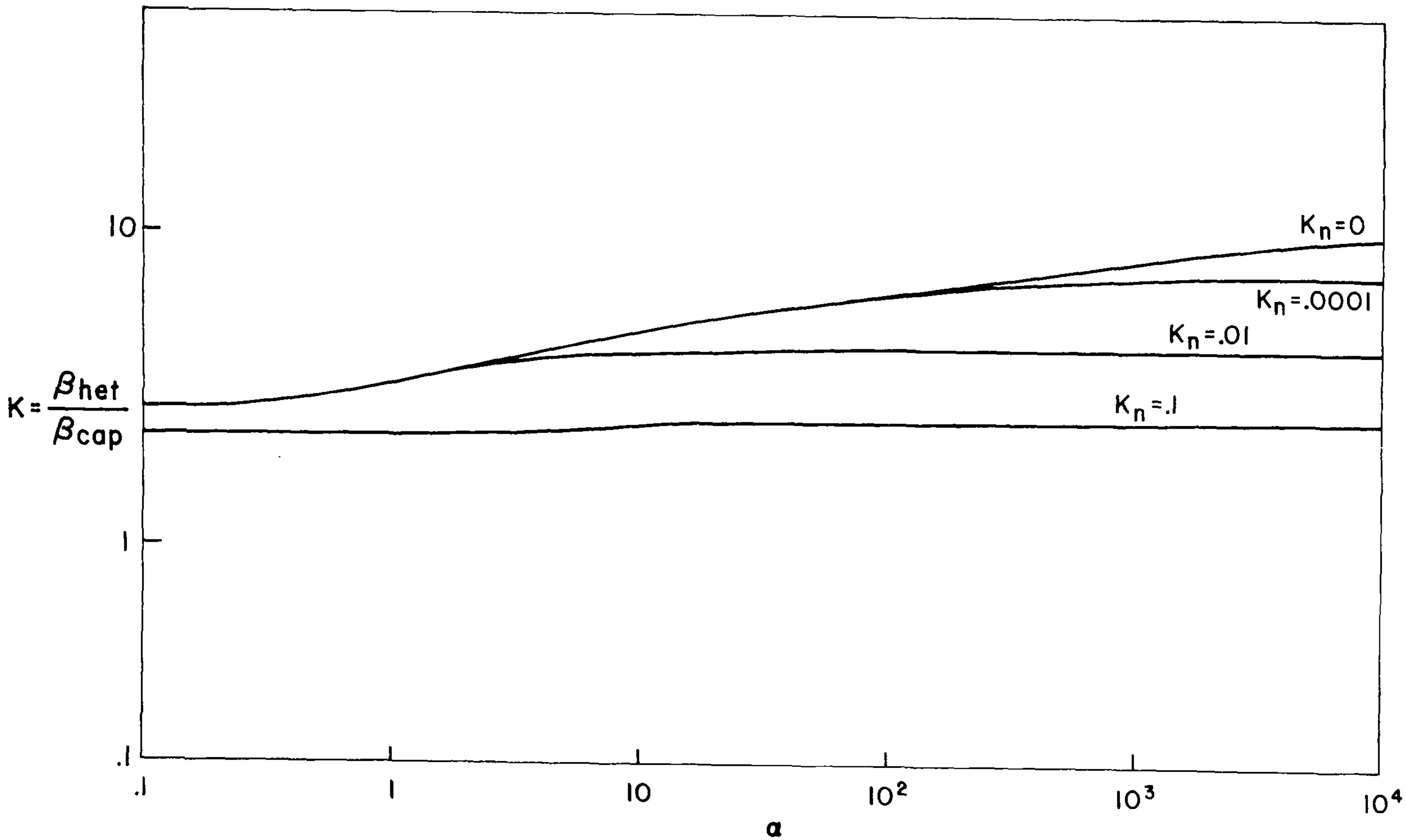

Figure 9.- Theoretical values of $\kappa = \frac{\beta_{het}}{\beta_{capacity}}$ for the capacity of the quantum channel with additive Gaussian noise versus α

Example II - Gordon Bound

$$\alpha = 100$$

$$\kappa = 8$$

$$T_g \text{ at } 0.53\mu \simeq 3400^{\circ}K$$

$$\text{at } 10.6\mu \simeq 170^{\circ}K$$

Another common figure of merit used is (S/N)B, where (S/N) is the signal-to-noise ratio in a bandwidth B. For the heterodyne system, which appears to behave as an additive Gaussian noise channel, this figure of merit has the same meaning. For direct detection systems with analog signalling, one may also assume that this figure of merit has substantially the same meaning, particularly for high values of (S/N) where central limit theorem arguments would yield Gaussian statistics. For digital direct detection systems the comparable figure of merit would become κ(S/N)B where (S/N) is merely $P/h\nu B$, the quantum-limited signal-to-noise ratio. This is true because κ has been scaled to this value.

Although no proof has been given, the previous arguments suggest that a digital direct detection system is, in fact, more efficient than an analog direct detection system. This would not be too surprising since a digital system should be more compatible with a discrete photon channel. On the other hand, the results from the quantum detection theory indicate that there is something better than direct detection (although how much is not yet known).

AREAS FOR FUTURE STUDY

From the data presented here, it seems clear that a broader view than that allowed from experience with Gaussian channels must be taken, and although the PPM systems do appear quite efficient, there still are large gaps between obtainable performance and theoretical performance, particularly for values of α close to 1.

In addition, efficient PPM systems require large alphabets with M-ary codes. If the alphabet size is reduced, the efficiency of the system deteriorates. The results of Helstrom (ref. 16) indicate that a rigorous quantum mechanical treatment can yield superior performance. Thus it may be possible to obtain comparable performance with a smaller alphabet size and lower values of α.

From a broader perspective, no one has yet been able to demonstrate that this performance can be maintained when traversing deleterious channels. For the atmosphere it appears that "aperture averaging" (ref. 19), while not the optimum form of diversity combining (ref. 20), can eliminate most of the adverse effects when used with direct detection systems. On the other hand, very little is known about optical communications through rain, fog, clouds, haze, etc.*

CONCLUSIONS

In this report quantum communication systems have been treated in the more universally accepted context of communication efficiency. As a result it has been shown that some of the accepted criteria such as "fundamental quantum noise temperature" and "two photons per bit" can indeed be misleading. Some of the fundamental bounds have been presented in this context, and have shown where existing systems fit and where large gaps appear between existing systems and the bounds. No attempt was made to treat anything but the free space channel (no channel disturbances), although additive noise was considered. Indications were made as to where the problem areas lie and where future work could be usefully directed.

*Joint NASA/MIT Workshop on Optical Communications, Williams College, Williamstown, Mass., August 1968

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