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PULSE-RESPONSE CURVES OF CONVENTIONAL LOW-PASS FILTERS

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16. Abstract

A catalog of 100 different pulse responses of conventional low-pass filters is presented. The pulse responses were obtained by a computer simulation of a general digital filter whose characteristics could be varied to correspond very closely with the various analog filter functions. The standard Butterworth, the Chebyshev (with various ripples), and the Bessel filter functions are considered. Filter orders from 1 to 5 are included. For each filter type and order, responses corresponding to four different products of bandwidth and pulse width are considered.

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PULSE-RESPONSE CURVES OF CONVENTIONAL LOW-PASS FILTERS

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SUMMARY

A catalog of 100 different pulse responses of conventional low-pass filters is presented. The pulse responses were obtained by a computer simulation of a general digital filter whose characteristics could be varied to correspond very closely with the various analog filter functions.

The standard Butterworth, the Chebyshev (with various ripples), and the Bessel filter functions are considered. Filter orders from 1 to 5 are included. For each filter type and order, responses corresponding to four different products of bandwidth and pulse width are considered.

INTRODUCTION

The response of a low-pass filter to a square-pulse excitation is a topic of practical interest in the design of systems employing pulse-type waveforms. Practical pulse waveforms employed in radar systems and in coded communications systems may often be approximated by the theoretical square-pulse function in many applications. Whenever an assumed ideal square pulse is employed in a system, the effects of a given filter on the waveform must very often be considered since filtering is required in order to separate the desired signal from undesirable signals and noise. For any finite bandwidth, the output of the filter will always be a distorted version of the input pulse. The severity and nature of the distortion depend on two factors: (1) the nature of the filter characteristic and (2) the bandwidth (or cutoff frequency) of the filter.

The proper selection of filter types and cutoff frequencies is frequently required in order to optimize the design objectives in systems employing pulse waveforms. Usually this process necessitates the computation of the pulse responses of several different proposed designs in order to select the form most suited for the particular application. Although the computation of the pulse response is, in theory, a straightforward application of fundamental linear-system techniques, the actual details of the calculations may be extremely unwieldy for reasonably complex filters.

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In this report, a catalog of pulse responses for several different types of low-pass filters is presented. For each particular filter characteristic, curves for different products of pulse width and bandwidth are presented. The curves were obtained by digital-computer computation utilizing a digital-filter simulation. The curves obtained should be useful in the design of systems employing waveforms with approximately square pulses in conjunction with band-limiting filters.

GENERAL DISCUSSION

A comprehensive treatment of the pulse response of the ideal low-pass filter as a function of the cutoff frequency and pulse width was provided by Goldman (ref. 1). Further results, which included the effects of various distortion functions, were provided by Murakami and Corrington (ref. 2). A more recent treatment of some of the previous results was given by Schwartz (ref. 3).

The results of the mentioned investigations and of similar investigations have been employed extensively in analysis of communication systems. However, the major limitation in these studies is the fact that ideal filter characteristics and ideal mathematical distortion functions were assumed. Since the ideal filter is not physically realizable, these results are only approximate. In many applications, the error introduced by assuming an ideal filter may be very critical.

Most filter designs today are patterned after the so-called "modern" filter synthesis approach, in which the transfer function and s-plane concepts are utilized. Notable among such filters are (a) the Butterworth, or maximally flat amplitude response, (b) the Chebyshev, or equiripple amplitude response, and (c) the Bessel linear phase response. Henderson and Kautz (ref. 4) presented a catalog of the step and impulse responses of practical low-pass and high-pass filters belonging to the preceding classifications. Although these curves have been used extensively for calculations of step and impulse responses, they are not ideally suited to the determination of pulse responses.

The present study was made because of a need for a set of pulse responses for the conventional filters currently receiving widespread usage. Although several different computational approaches could be employed in obtaining these curves, the method actually chosen was that of programing a digital-filter simulation of the corresponding analog filter characteristic. The bilinear transformation was employed in the approximation procedure as described in the appendix.

The results of the digital-computer simulation were plotted directly on a computer software system. As a means of checking for gross error, all the results were quantitatively verified in the laboratory. This task was accomplished by synthesizing the filter types with variable components and observing the pulse responses on an oscilloscope.

A systematic approach was employed in which the filter characteristics could be readily changed by changing the dial settings. A standard pulse generator employing a low-duty cycle was used to excite the filters. Although not a precise test, this procedure was expected to serve as an extremely useful complement to the computer simulation and to assist in uncovering possible mistakes in the basic program. No measurable discrepancies were observed between the experimental responses and the computer simulation results.

DISCUSSION OF DATA

The results of this study are presented as 100 different pulse responses in figures 1 to 100. A few comments regarding assumptions and curve parameters are in order. The input square pulse in all cases is normalized to have unit amplitude and unit width. The horizontal scale for the response curves simply reads TIME. The width of the normalized input pulse corresponds to 1 on the TIME scale.

Each filter response curve is characterized by a certain bandwidth B (or equivalent cutoff frequency). The bandwidth is defined for different filter types as follows:

(a) For Butterworth and Bessel filters, the bandwidth B is defined as the frequency at which the response is 3 dB below the dc level. Both Butterworth and Bessel filters have monotonically decreasing amplitude responses, and there is only one frequency possessing the 3-dB constraint.

(b) For Chebyshev filters, the bandwidth B is defined as the highest frequency at which the amplitude response is bounded by the prescribed passband ripple. This definition permitted extensive usage of the transfer-function-coefficient tables provided by Weinberg (ref. 5).

(c) The first-order low-pass filter can be thought of as the limiting case for all the filter types. In this case, the bandwidth B for this filter is the 3-dB frequency in all cases.

The relative bandwidths for the filter response curves are given as the dimensionless product BT. (The bandwidth B is in hertz, or cycles per second, and the pulse width T is in seconds.) Since the pulse width is held constant at a normalized unit value, the various BT products can be interpreted as different values of the bandwidths.

As an aid in determining the contents and figure number of different response curves, a summary of the various filter types and characteristics is provided in table I. The decibel specification of a Chebyshev filter refers to the prescribed passband ripple for that case. The order of a given filter is the number of low-pass, s-plane poles in the transfer function.

CONCLUDING REMARKS

The different pulse responses of conventional low-pass filters have been presented in the form of response curves which were obtained by a computer simulation. The filter functions considered are the standard Butterworth, the Bessel, and the Chebyshev for filter orders from 1 to 5 and for four different products of bandwidth and pulse width.

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National Aeronautics and Space Administration,
Langley Station, Hampton, Va., August 29, 1969.

APPENDIX A

DIGITAL-FILTER SIMULATION

Bilinear Transformation

The pulse responses presented in this report were obtained from a digital-filter simulation employing the bilinear transformation (ref. 6). The bilinear transformation has received a considerable amount of attention recently in the design of digital filters. Since all the filter types considered are low pass in nature, with all zeros of transmission located at infinity, the Laplace transform prototype p-plane transfer function of order k may be represented in the form

$$G(p) = \frac{a_0}{a_0 + a_1 p + a_2 p^2 + \ldots + a_k p^k} \tag{A1}$$

The bilinear transformation is given by

$$p = C \frac{1 - z^{-1}}{1 + z^{-1}} \tag{A2}$$

where C is a mapping constant, and z is the z-transform variable of sampled-data theory.

Substitution of equation (A2) into equation (A1) yields the resulting z-plane transfer function $G(z)$. In general, $G(z)$ may be expressed in the form

$$G(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \ldots + b_k z^{-k}}{c_0 + c_1 z^{-1} + c_2 z^{-2} + \ldots + c_k z^{-k}} \tag{A3}$$

The various constants in equation (A3) may be related to the constants in equations (A1) and (A2) by expansion. A summary of these relationships is given in the following section.

Let $x(n)$ represent the sampled input (pulse function) to the digital filter, and let $y(n)$ represent the sampled output (pulse response). The digital computer realization of equation (A3) is then given by the difference equation

$$\begin{aligned} y(n) = {} & \frac{1}{c_0}\left[b_0 x(n) + b_1 x(n - 1) + b_2 x(n - 2) + \ldots + b_k x(n - k)\right] \\ & - \frac{1}{c_0}\left[c_1 y(n - 1) + c_2 y(n - 2) + \ldots + c_k y(n - k)\right] \end{aligned} \tag{A4}$$

The mapping constant C is chosen by specifying the correspondence between the p-plane and the s-plane. A fundamental relationship of z-transform theory is

$$z = e^{st_s} \tag{A5}$$

where s is the Laplace transform variable representing the actual system response and t_s is the time between samples. Let $s = j\omega$ represent the actual imaginary axis, and let $p = j\lambda$ represent the prototype imaginary axis. It can then be verified by substituting equation (A5) into equation (A2) that

$$\lambda = C \tan \frac{\omega t_s}{2} \tag{A6}$$

Let $\lambda = \lambda_r$ be the prototype reference angular frequency in radians/sec, and let $\omega = \omega_r$ be the actual reference angular frequency. Assume that the width of the input pulse is T, and assume that there are N sample points employed in the interval $0 < t \leqq T$. Then,

$$T = Nt_s \tag{A7}$$

Assuming that the number of sample points is chosen sufficiently large to yield a good digital-to-analog filter correspondence, the constant C may be determined to a close approximation to be

$$C = \frac{N\lambda_r}{\pi f_r T} \tag{A8}$$

In all cases considered, the quantity f_r was interpreted as the cutoff frequency, or the bandwidth B, of the pertinent filter characteristic as explained earlier. With this change in notation

$$C = \frac{N\lambda_r}{\pi BT} \tag{A9}$$

where BT may be interpreted as the product of bandwidth and pulse width.

The computer program was written so that the number of sample points N could be varied. It was found that excellent results were obtained in all cases with $N = 200$. This choice results in a Nyquist, or folding, frequency equal to 100/T. Of course, it is quite possible that other values of N would have worked as well since no attempt was made to determine an optimum value.

Digital Filter Coefficients

The digital filter coefficients in the transfer functions G(p) and G(z) are as follows:

For $k = 1$,

$$b_0 = a_0$$

$$b_1 = a_0$$

$$c_0 = a_0 + a_1 C$$

$$c_1 = a_0 - a_1 C$$

For $k = 2$,

$$b_0 = a_0$$

$$b_1 = 2a_0$$

$$b_2 = a_0$$

$$c_0 = a_0 + a_1 c + a_2 c^2$$

$$c_1 = 2a_0 - 2a_2 c^2$$

$$c_2 = a_0 - a_1 C + a_2 C^2$$

For $k = 3$,

$$b_0 = a_0$$

$$b_1 = 3a_0$$

$$b_2 = 3a_0$$

$$b_3 = a_0$$

$$c_0 = a_0 + a_1 C + a_2 C^2 + a_3 C^3$$

$$c_1 = 3a_0 + a_1C - a_2C^2 - 3a_3C^3$$

$$c_2 = 3a_0 - a_1C - a_2C^2 + 3a_3C^3$$

$$c_3 = a_0 - a_1C + a_2C^2 - a_3C^3$$

For $k = 4$,

$$b_0 = a_0$$

$$b_1 = 4a_0$$

$$b_2 = 6a_0$$

$$b_3 = 4a_0$$

$$b_4 = a_0$$

$$c_0 = a_0 + a_1C + a_2C^2 + a_3C^3 + a_4C^4$$

$$c_1 = 4a_0 + 2a_1C - 2a_3C^3 - 4a_4C^4$$

$$c_2 = 6a_0 - 2a_2C^2 + 6a_4C^4$$

$$c_3 = 4a_0 - 2a_1C + 2a_3C^3 - 4a_4C^4$$

$$c_4 = a_0 - a_1C + a_2C^2 - a_3C^3 + a_4C^4$$

For $k = 5$,

$$b_0 = a_0$$

$$b_1 = 5a_0$$

$$b_2 = 10a_0$$

$$b_3 = 10a_0$$

$$b_4 = 5a_0$$

$$b_5 = a_0$$

$$c_0 = a_0 + a_1C + a_2C^2 + a_3C^3 + a_4C^4 + a_5C^5$$

$$c_1 = 5a_0 + 3a_1C + a_2C^2 - a_3C^3 - 3a_4C^4 - 5a_5C^5$$

$$c_2 = 10a_0 + 2a_1C - 2a_2C^2 - 2a_3C^3 + 2a_4C^4 + 10a_5C^5$$

$$c_3 = 10a_0 - 2a_1C - 2a_2C^2 + 2a_3C^3 + 2a_4C^4 - 10a_5C^5$$

$$c_4 = 5a_0 - 3a_1C + a_2C^2 + a_3C^3 - 3a_4C^4 + 5a_5C^5$$

$$c_5 = a_0 - a_1C + a_2C^2 - a_3C^3 + a_4C^4 - a_5C^5$$

Analog Filter Coefficients

Detailed consideration of filter characteristics are provided in such synthesis texts as Weinberg (ref. 5) and Van Valkenburg (ref. 7). The analog filter coefficients are given in the following table:

Filter	a_0	a_1	a_2	a_3
		$k = 1$		
All	1	1		
		$k = 2$		
Butterworth	1.0000000	1.4142136	1.0000000	
Chebyshev (0.5 dB)	1.5162026	1.4256245	1.0000000	
Chebyshev (1 dB)	1.1025103	1.0977343	1.0000000	
Chebyshev (2 dB)	.6367681	.8038164	1.0000000	
Chebyshev (3 dB)	.7079478	.6448996	1.0000000	
Bessel	3.0000000	3.0000000	1.0000000	
		$k = 3$		
Butterworth	1.0000000	2.0000000	2.0000000	1.0000000
Chebyshev (0.5 dB)	.7156938	1.5348954	1.2529130	1.0000000
Chebyshev (1 dB)	.4913067	1.2384092	.9883412	1.0000000
Chebyshev (2 dB)	.3268901	1.0221903	.7378216	1.0000000
Chebyshev (3 dB)	.2505943	.9283480	.5972404	1.0000000
Bessel	15.0000000	15.0000000	6.0000000	1.0000000

Filter	a_0	a_1	a_2	a_3	a_4	a_5
			k = 4			
Butterworth	1.0000000	2.6131259	3.4142136	2.6131259	1.0000000	
Chebyshev (0.5 dB)	.3790506	1.0254553	1.7168662	1.1973856	1.0000000	
Chebyshev (1 dB)	.2756276	.7426194	1.4539248	.9528114	1.0000000	
Chebyshev (2 dB)	.2057651	.5167981	1.2564819	.7162150	1.0000000	
Chebyshev (3 dB)	.1769869	.4047679	1.1691176	.5815799	1.0000000	
Bessel	105.0000000	105.0000000	45.0000000	10.0000000	1.0000000	
			k = 5			
Butterworth	1.0000000	3.2360680	5.2360680	5.2360680	3.2360680	1.0000000
Chebyshev (0.5 dB)	.1789234	.7525181	1.3095747	1.9373675	1.1724909	1.0000000
Chebyshev (1 dB)	.1228267	.5805342	.9743961	1.6888160	.9368201	1.0000000
Chebyshev (2 dB)	.0817225	.4593491	.6934770	1.4995433	.7064606	1.0000000
Chebyshev (3 dB)	.0626391	.4079421	.5488626	1.4149847	.5744296	1.0000000
Bessel	945.0000000	945.0000000	420.0000000	105.0000000	15.0000000	1.0000000

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TABLE I.- PRESENTATION OF RESULTS

Filter type	Order	BT, Hz-sec	Figure
First-order	1	0.5 1 2 5	1 2 3 4
Butterworth	2	0.5 1 2 5	5 6 7 8
Chebyshev (0.5 dB)	2	0.5 1 2 5	9 10 11 12
Chebyshev (1 dB)	2	0.5 1 2 5	13 14 15 16
Chebyshev (2 dB)	2	0.5 1 2 5	17 18 19 20
Chebyshev (3 dB)	2	0.5 1 2 5	21 22 23 24
Bessel	2	0.5 1 2 5	25 26 27 28
Butterworth	3	0.5 1 2 5	29 30 31 32
Chebyshev (0.5 dB)	3	0.5 1 2 5	33 34 35 36
Chebyshev (1 dB)	3	0.5 1 2 5	37 38 39 40

TABLE I.- PRESENTATION OF RESULTS – Continued

Filter type	Order	BT, Hz-sec	Figure
Chebyshev (2 dB)	3	0.5	41
		1	42
		2	43
		5	44
Chebyshev (3 dB)	3	0.5	45
		1	46
		2	47
		5	48
Bessel	3	0.5	49
		1	50
		2	51
		5	52
Butterworth	4	0.5	53
		1	54
		2	55
		5	56
Chebyshev (0.5 dB)	4	0.5	57
		1	58
		2	59
		5	60
Chebyshev (1 dB)	4	0.5	61
		1	62
		2	63
		5	64
Chebyshev (2 dB)	4	0.5	65
		1	66
		2	67
		5	68
Chebyshev (3 dB)	4	0.5	69
		1	70
		2	71
		5	72
Bessel	4	0.5	73
		1	74
		2	75
		5	76
Butterworth	5	0.5	77
		1	78
		2	79
		5	80

TABLE I.- PRESENTATION OF RESULTS – Concluded

Filter type	Order	BT, Hz-sec	Figure
Chebyshev (0.5 dB)	5	0.5	81
		1	82
		2	83
		5	84
Chebyshev (1 dB)	5	0.5	85
		1	86
		2	87
		5	88
Chebyshev (2 dB)	5	0.5	89
		1	90
		2	91
		5	92
Chebyshev (3 dB)	5	0.5	93
		1	94
		2	95
		5	96
Bessel	5	0.5	97
		1	98
		2	99
		5	100

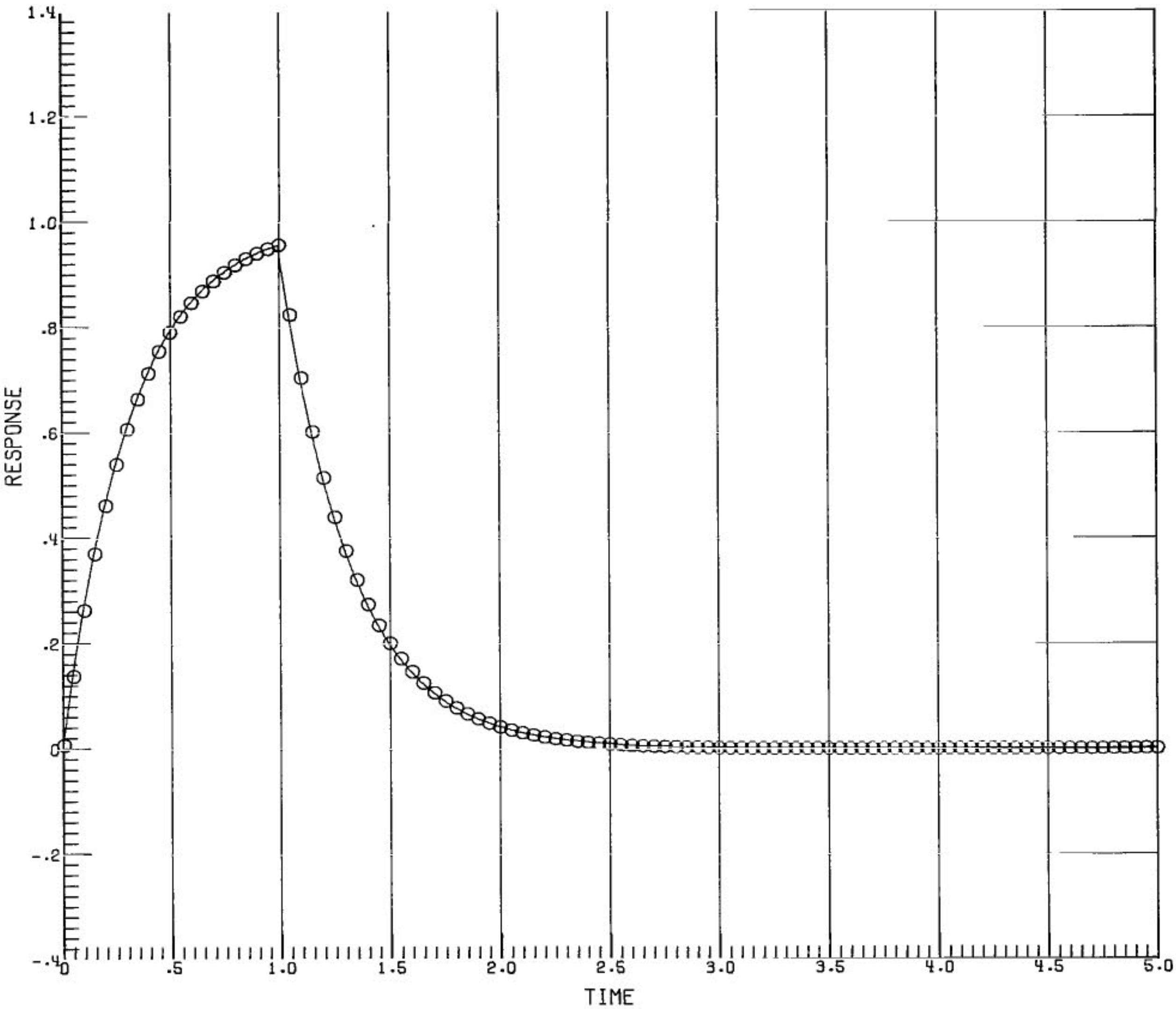

Figure 1.- Response of first-order filter with BT = 0.5.

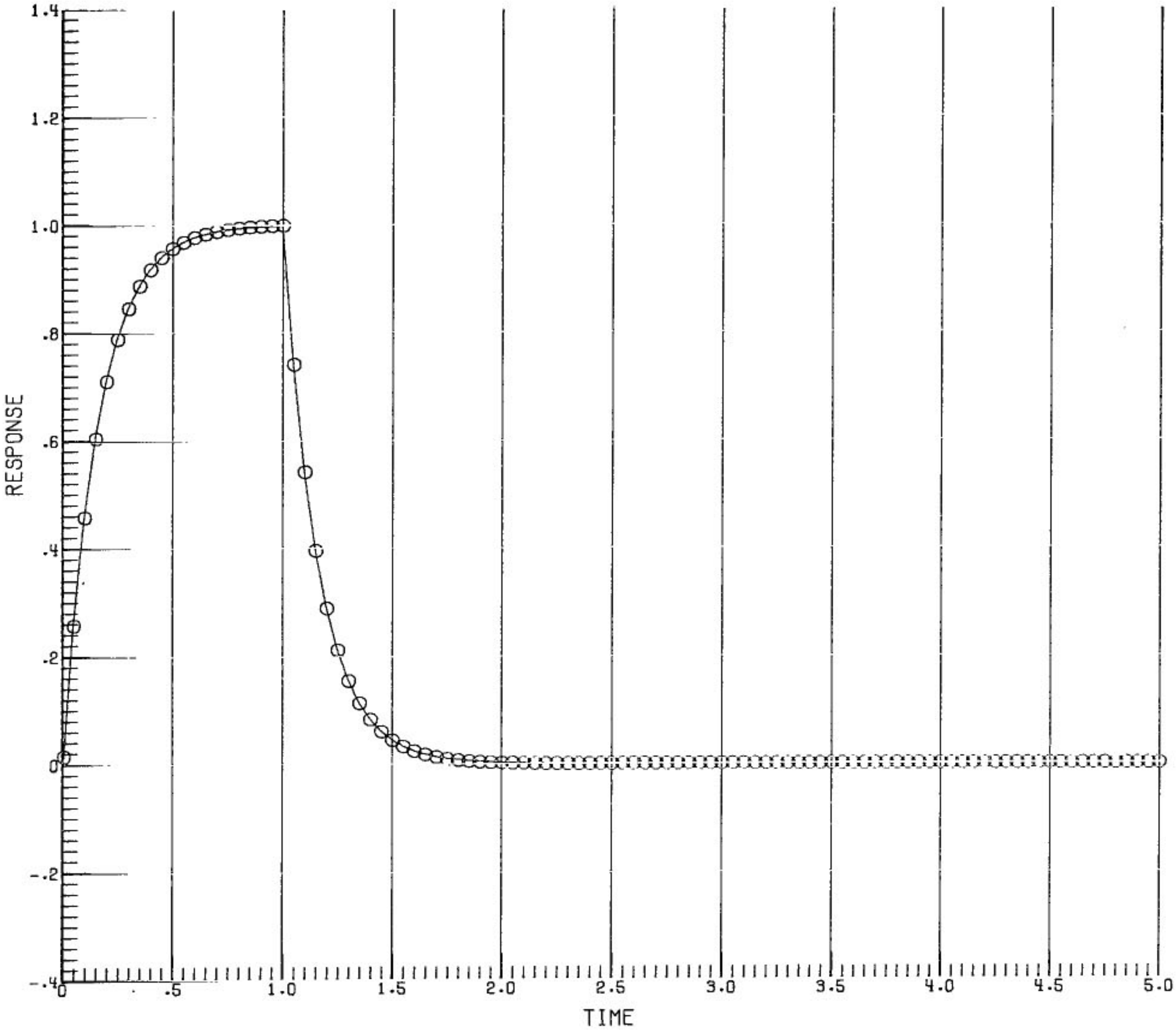

Figure 2.- Response of first-order filter with BT = 1.

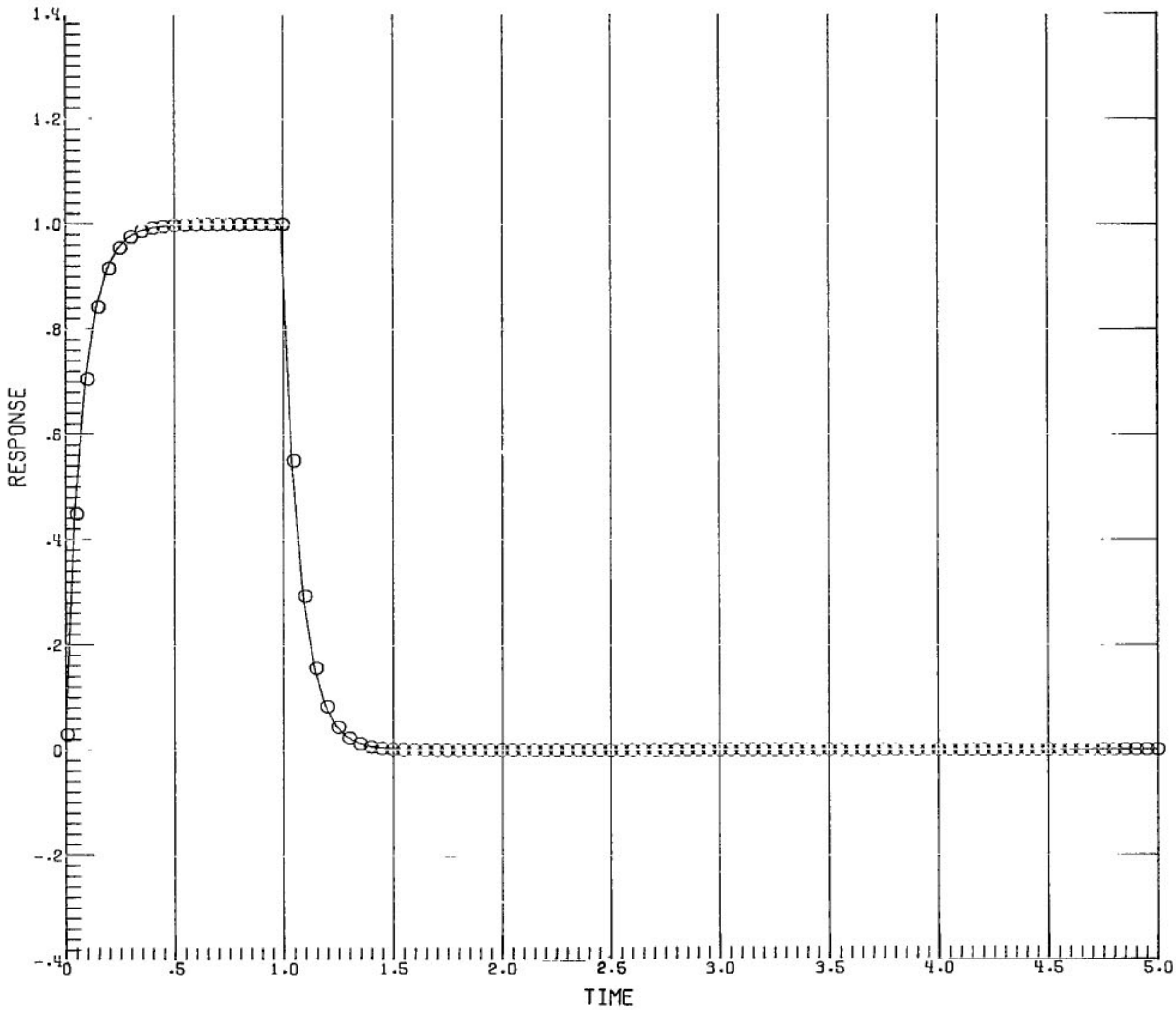

Figure 3.- Response of first-order filter with BT = 2.

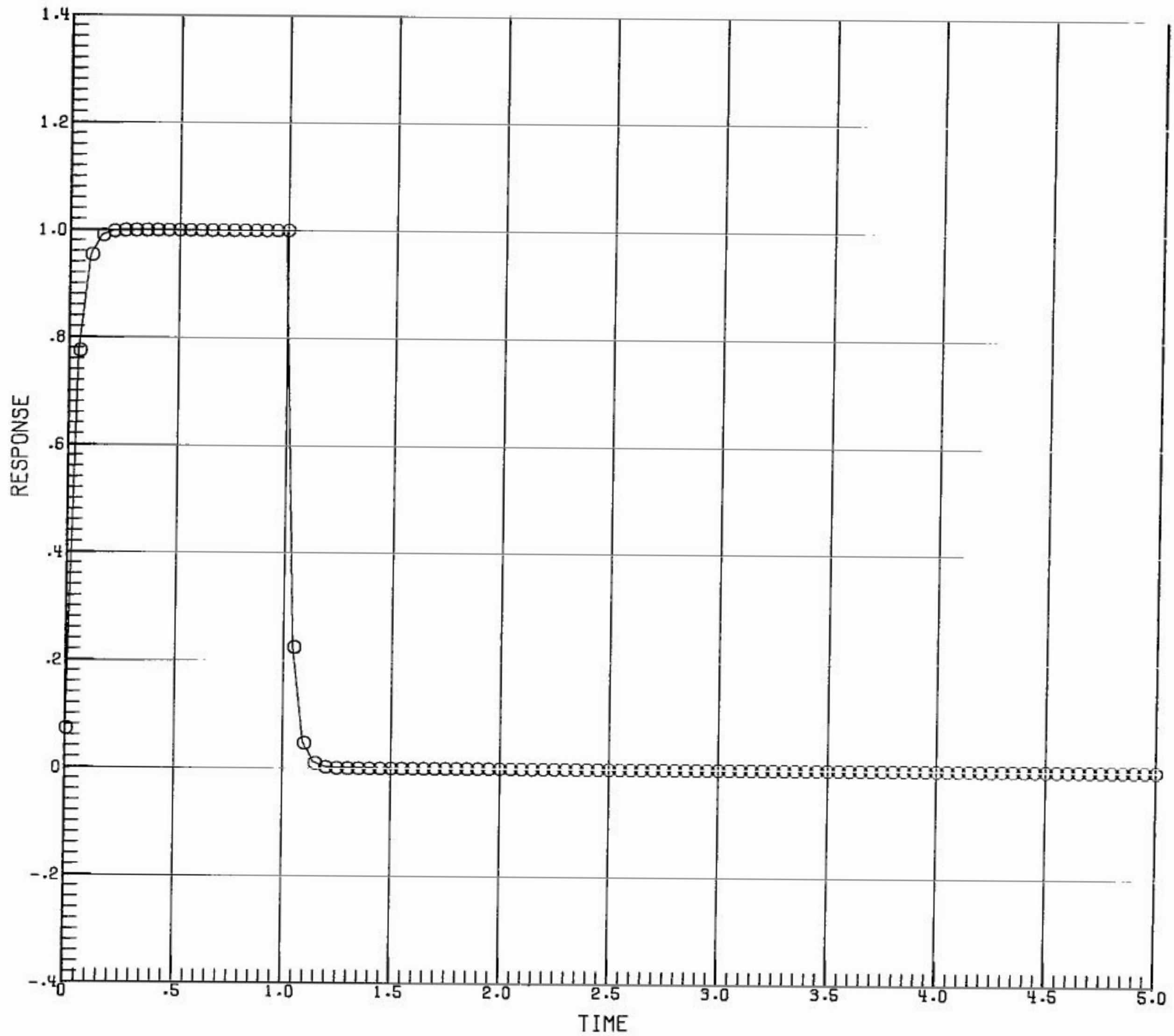

Figure 4.- Response of first-order filter with BT = 5.

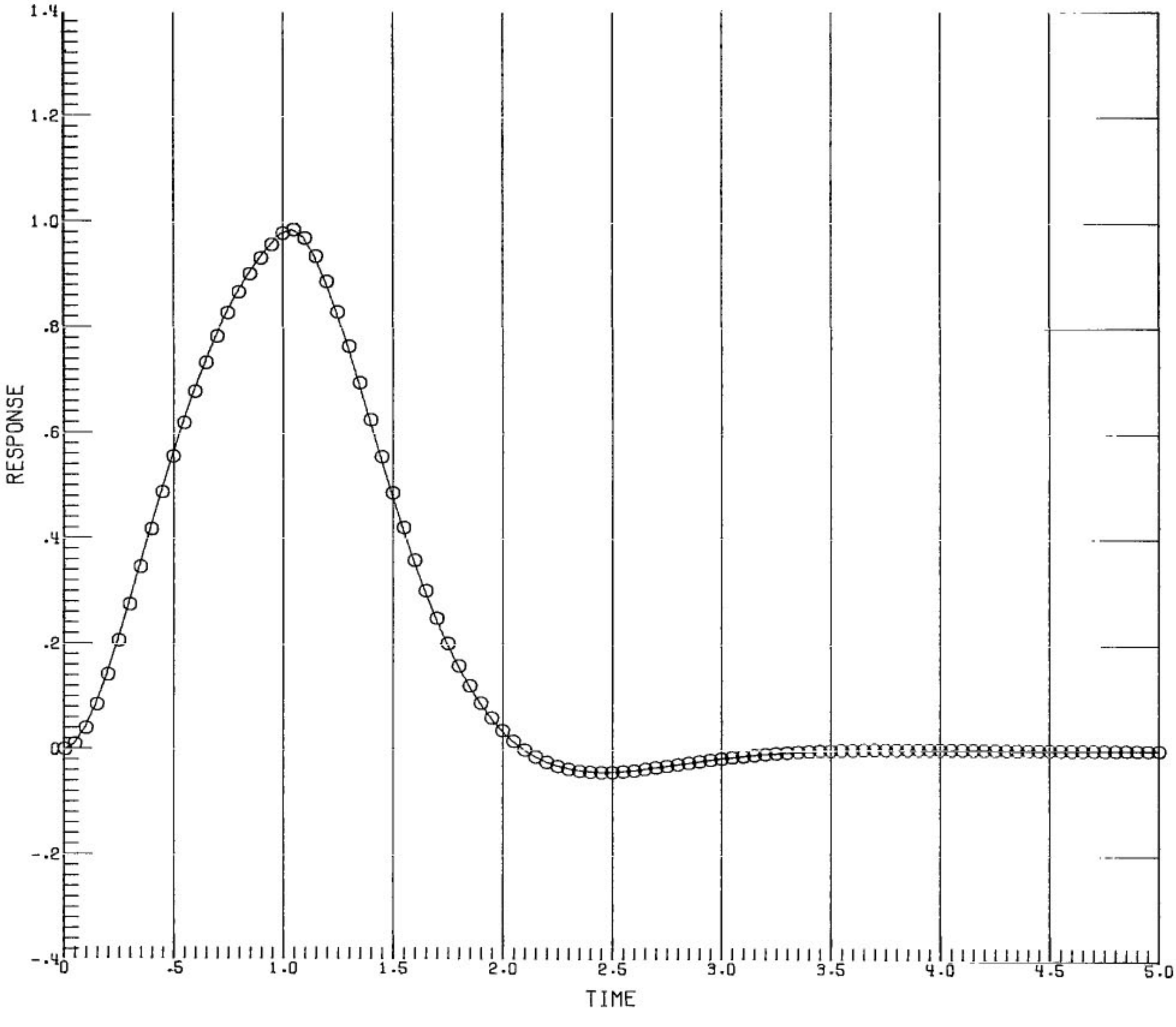

Figure 5.- Response of second-order Butterworth filter with BT = 0.5.

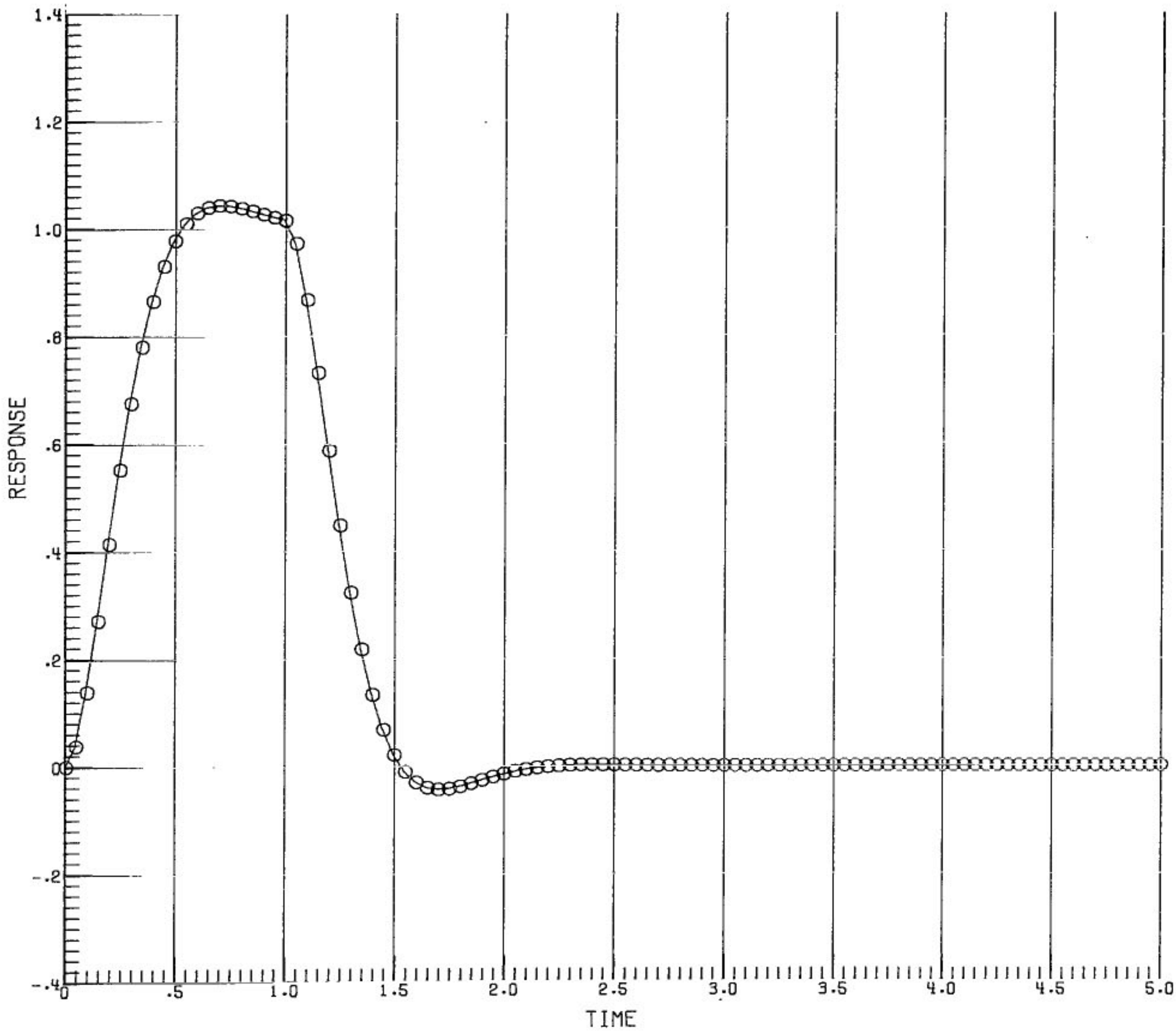

Figure 6.- Response of second-order Butterworth filter with BT = 1.

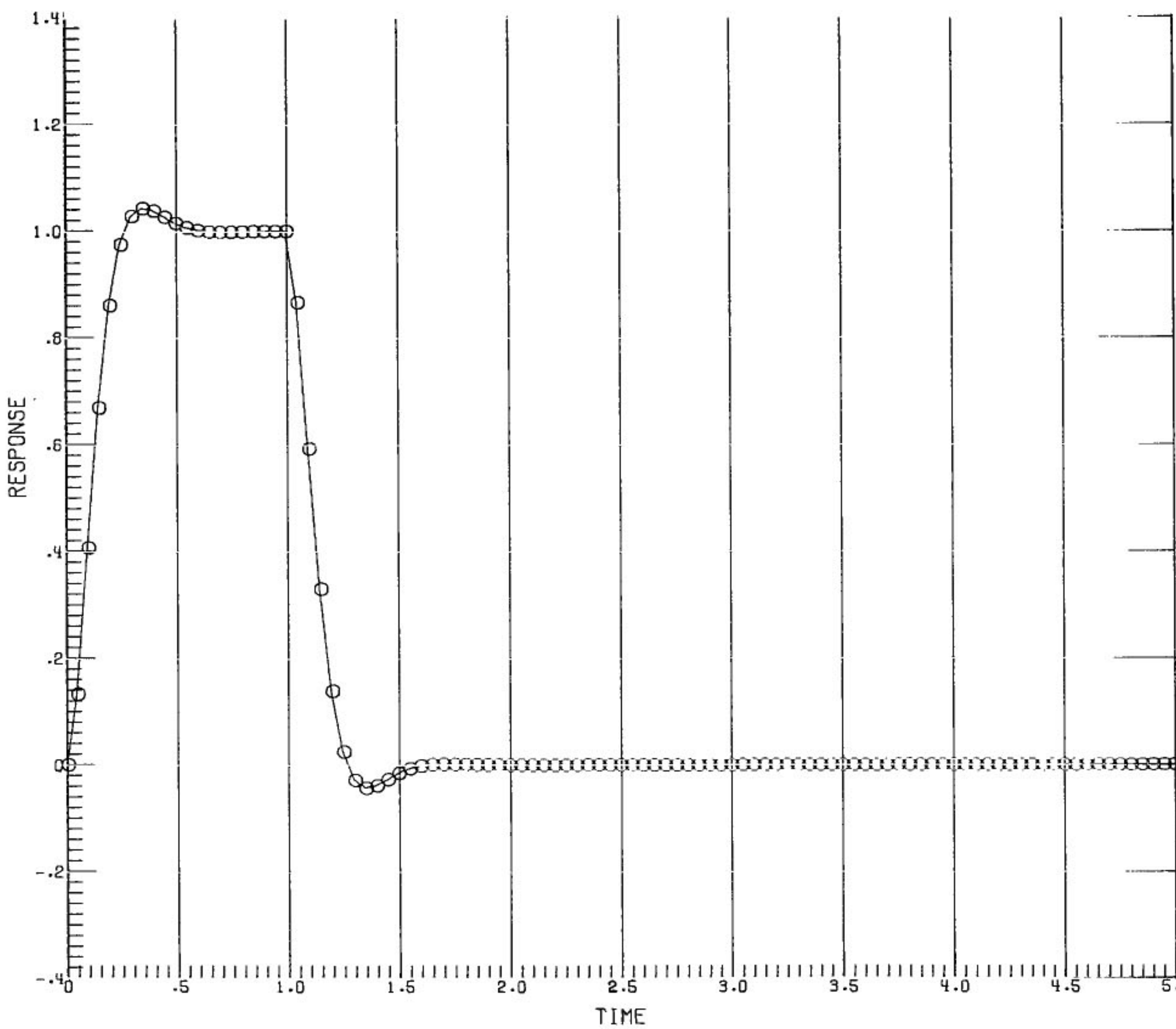

Figure 7.- Response of second-order Butterworth filter with BT = 2.

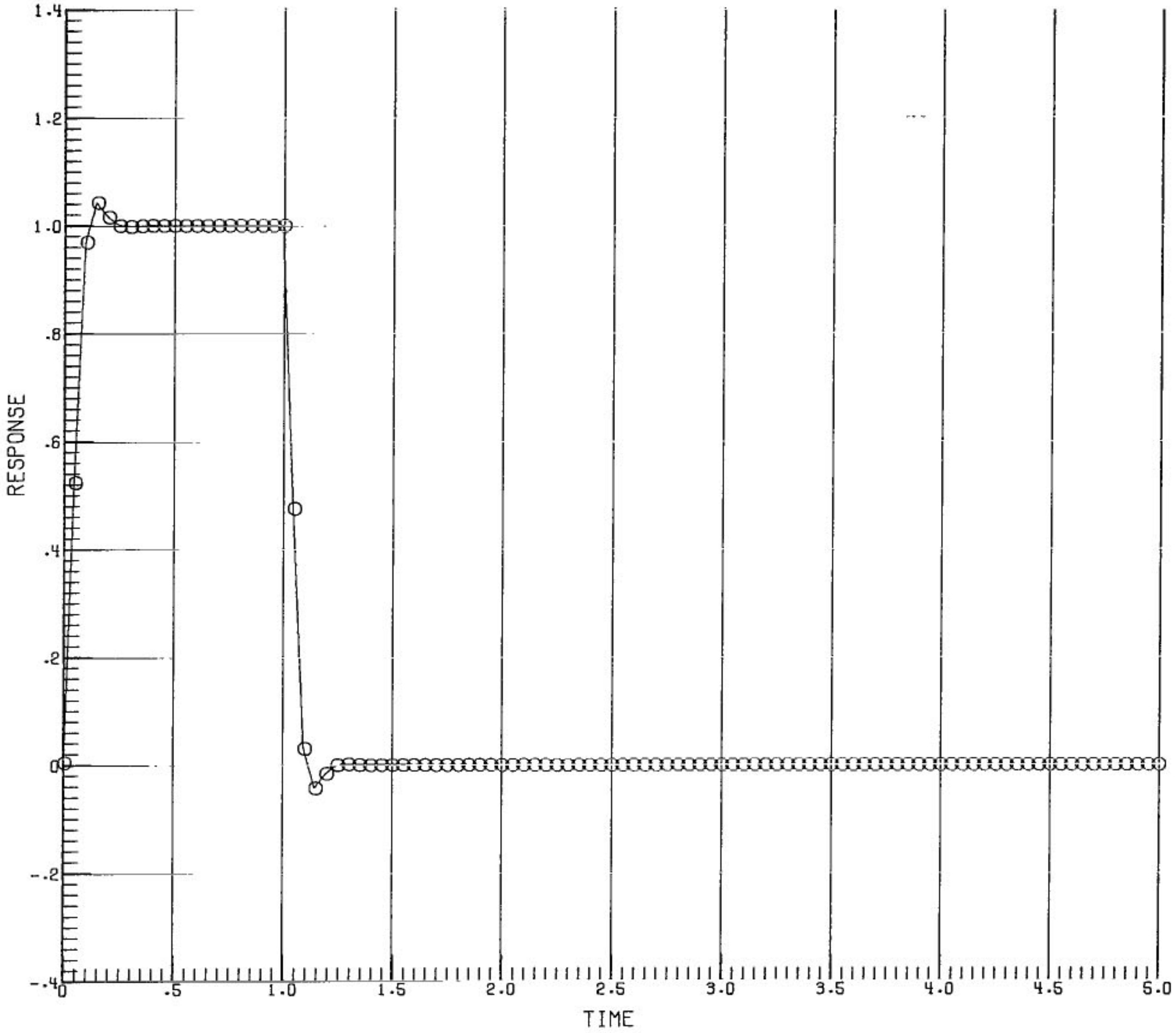

Figure 8.- Response of second-order Butterworth filter with BT = 5.

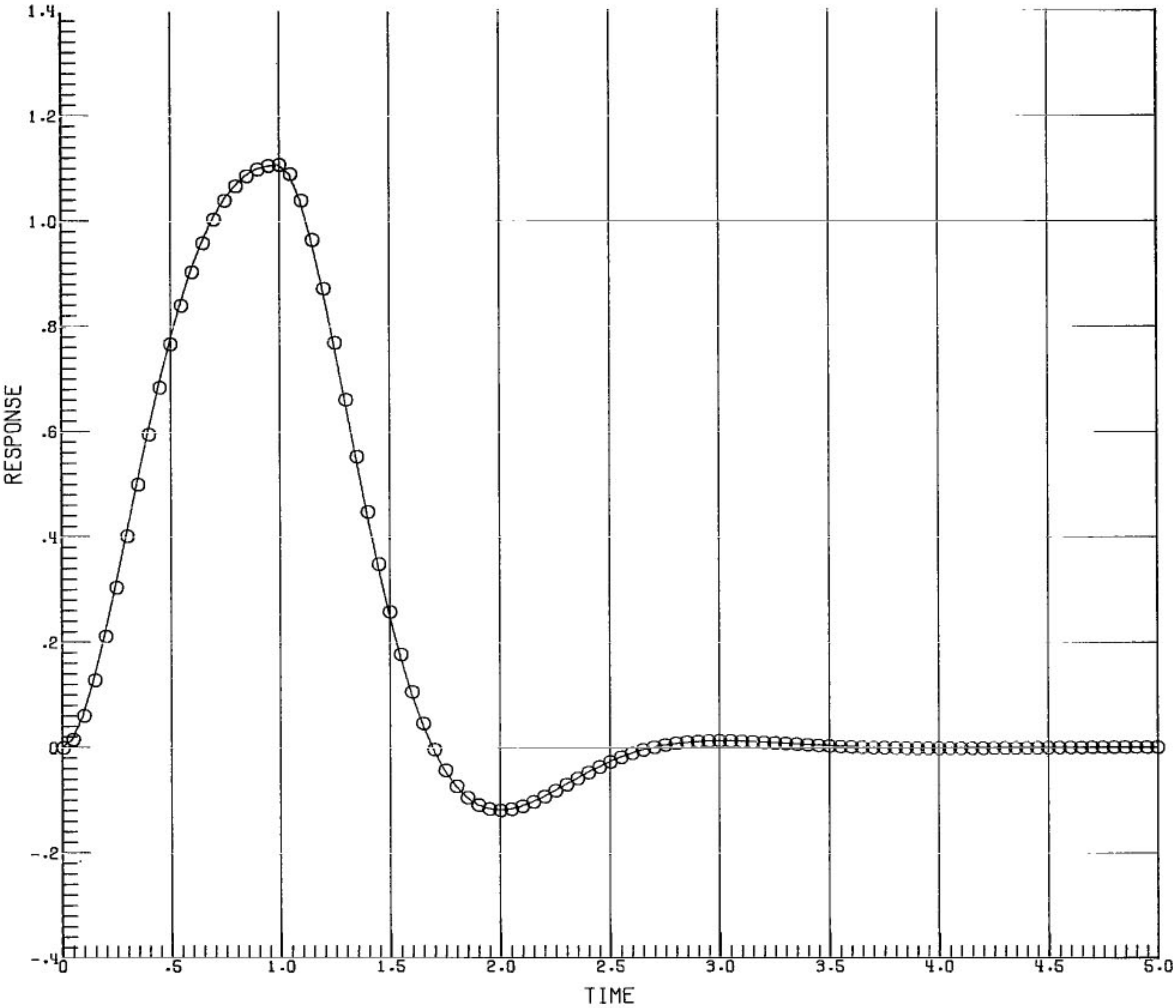

Figure 9.- Response of second-order Chebyshev (0.5-dB ripple) filter with BT = 0.5.

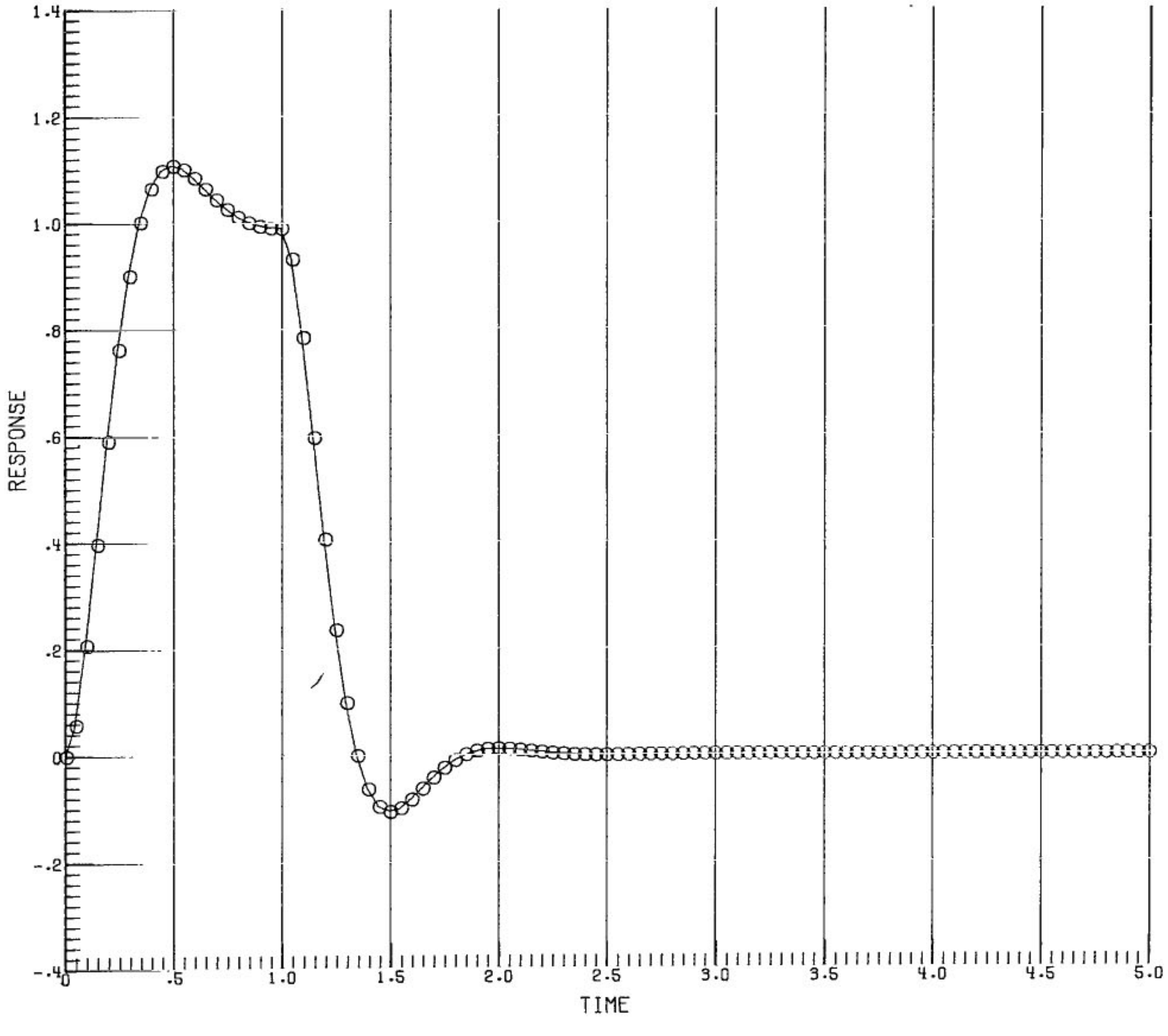

Figure 10.- Response of second-order Chebyshev (0.5-dB ripple) filter with BT = 1.

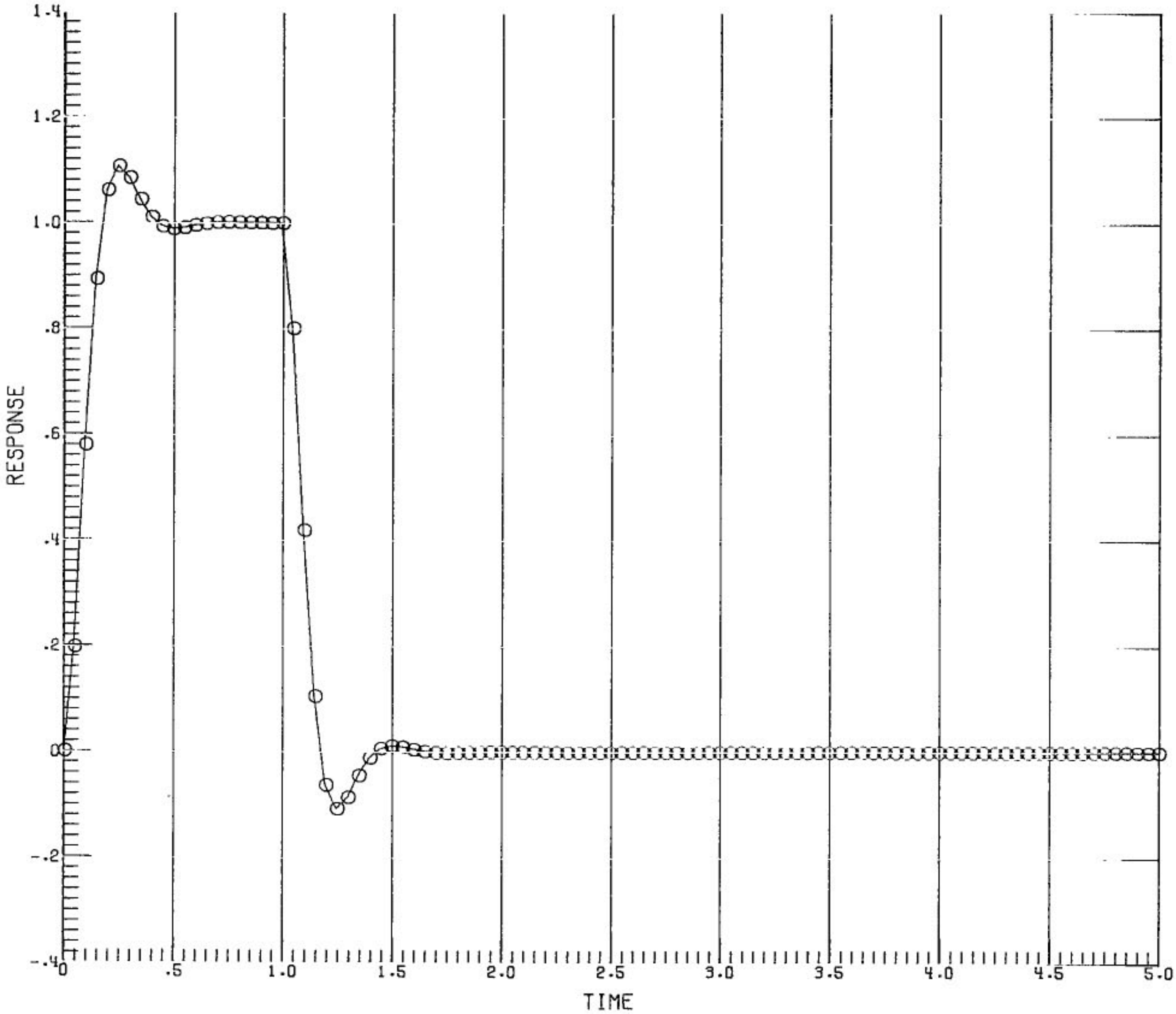

Figure 11.- Response of second-order Chebyshev (0.5-dB ripple) filter with BT = 2.

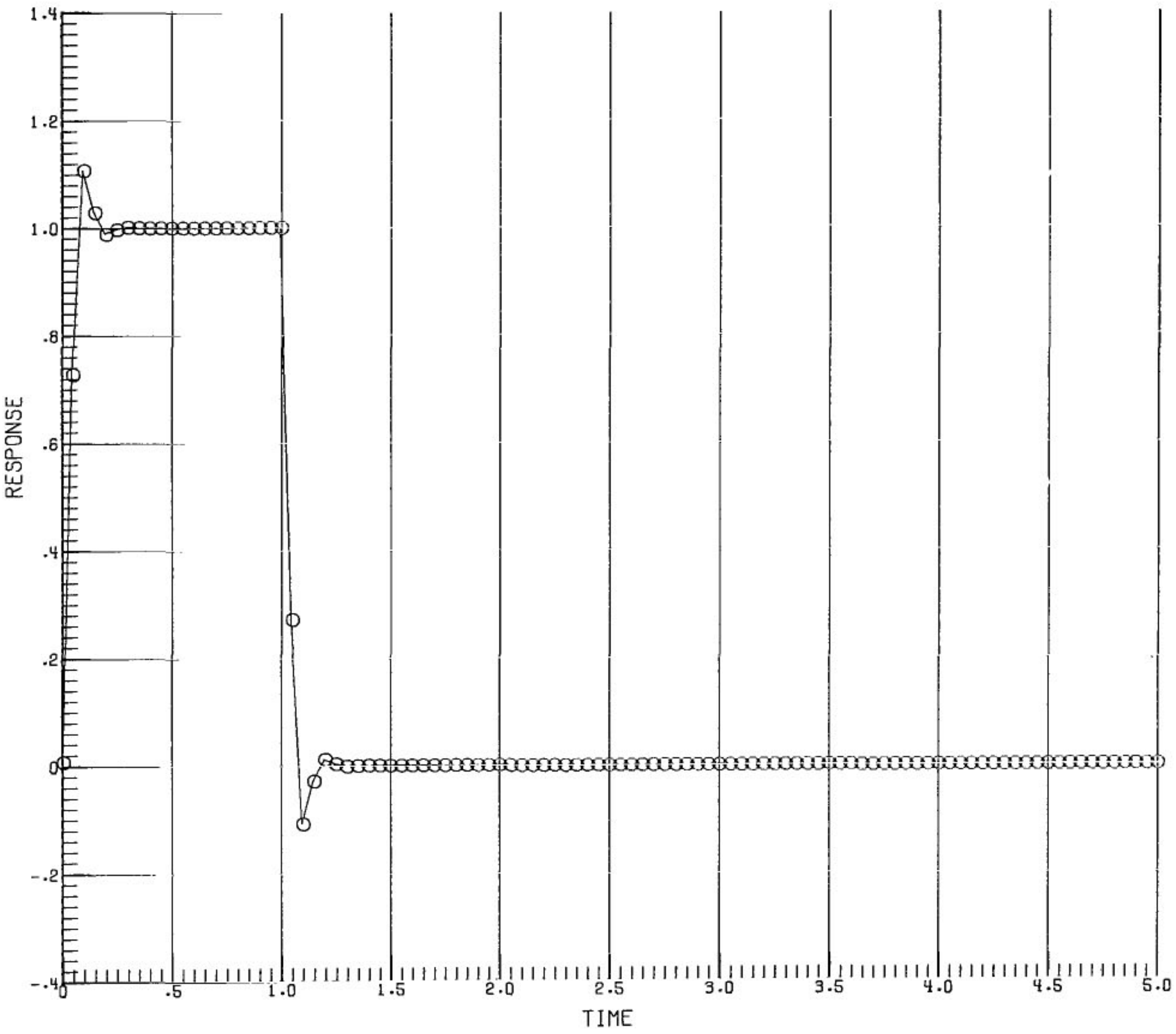

Figure 12.- Response of second-order Chebyshev (0.5-dB ripple) filter with BT = 5.

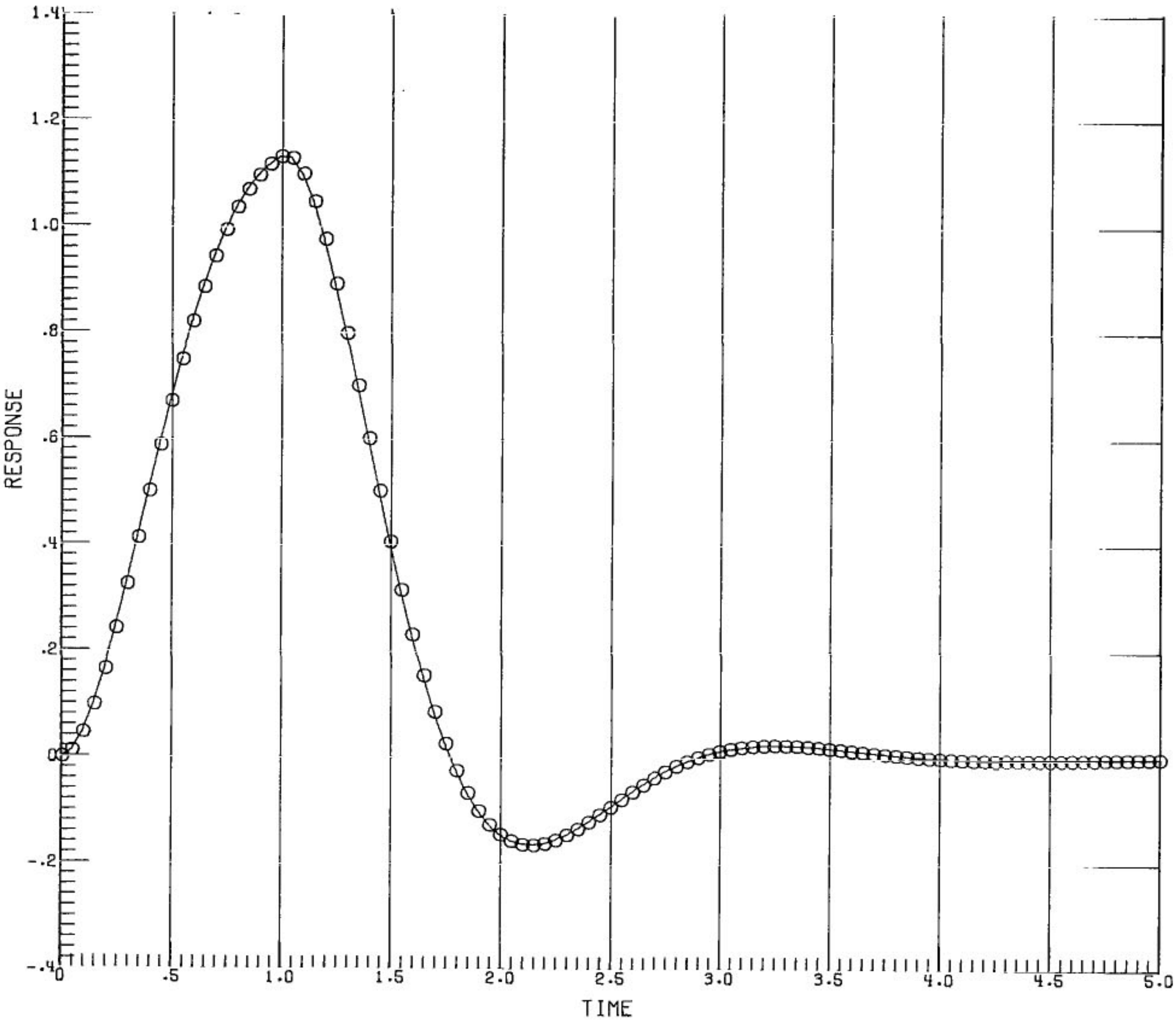

Figure 13.- Response of second-order Chebyshev (1-dB ripple) filter with BT = 0.5.

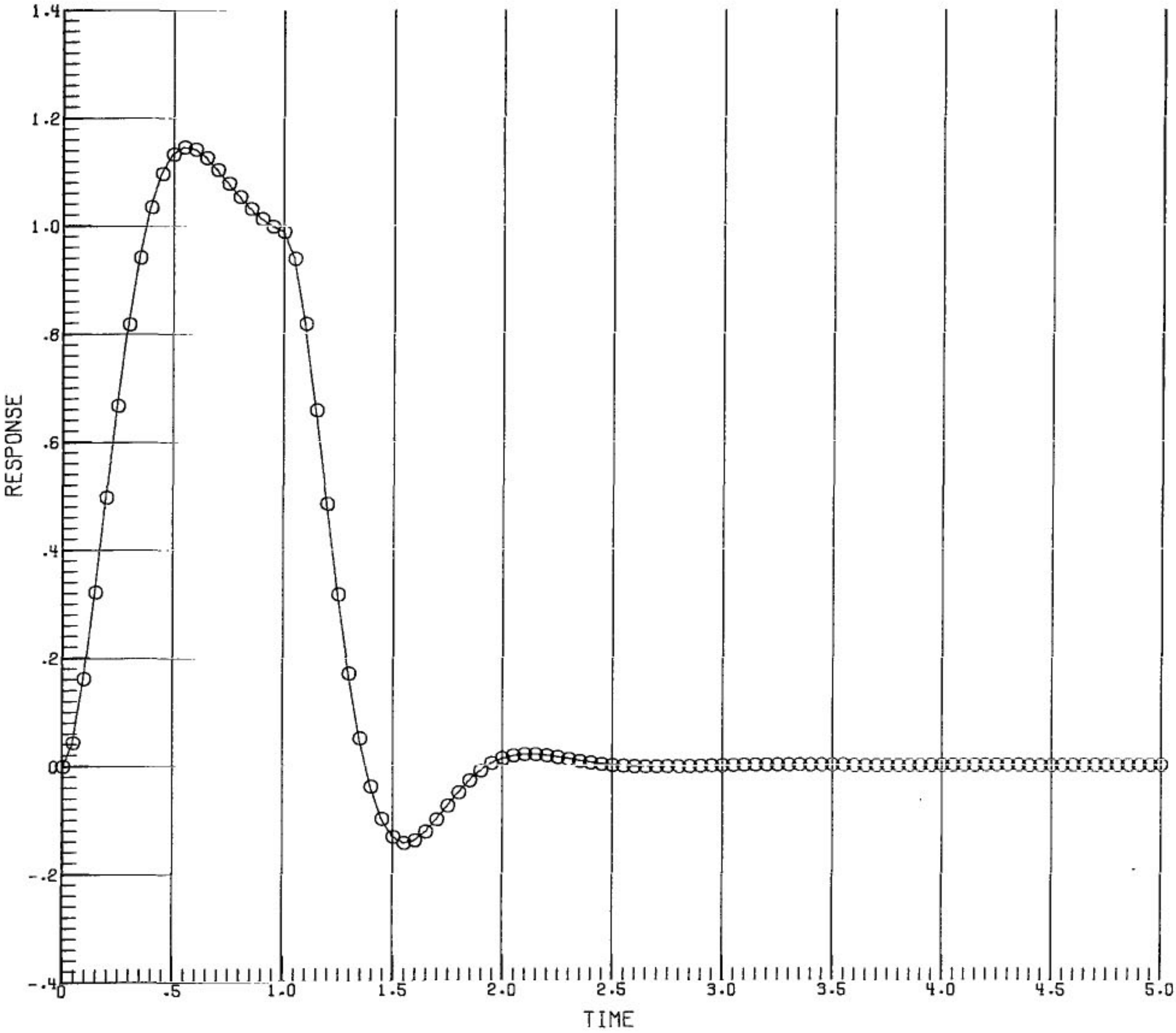

Figure 14.- Response of second-order Chebyshev (1-dB ripple) filter with BT = 1.

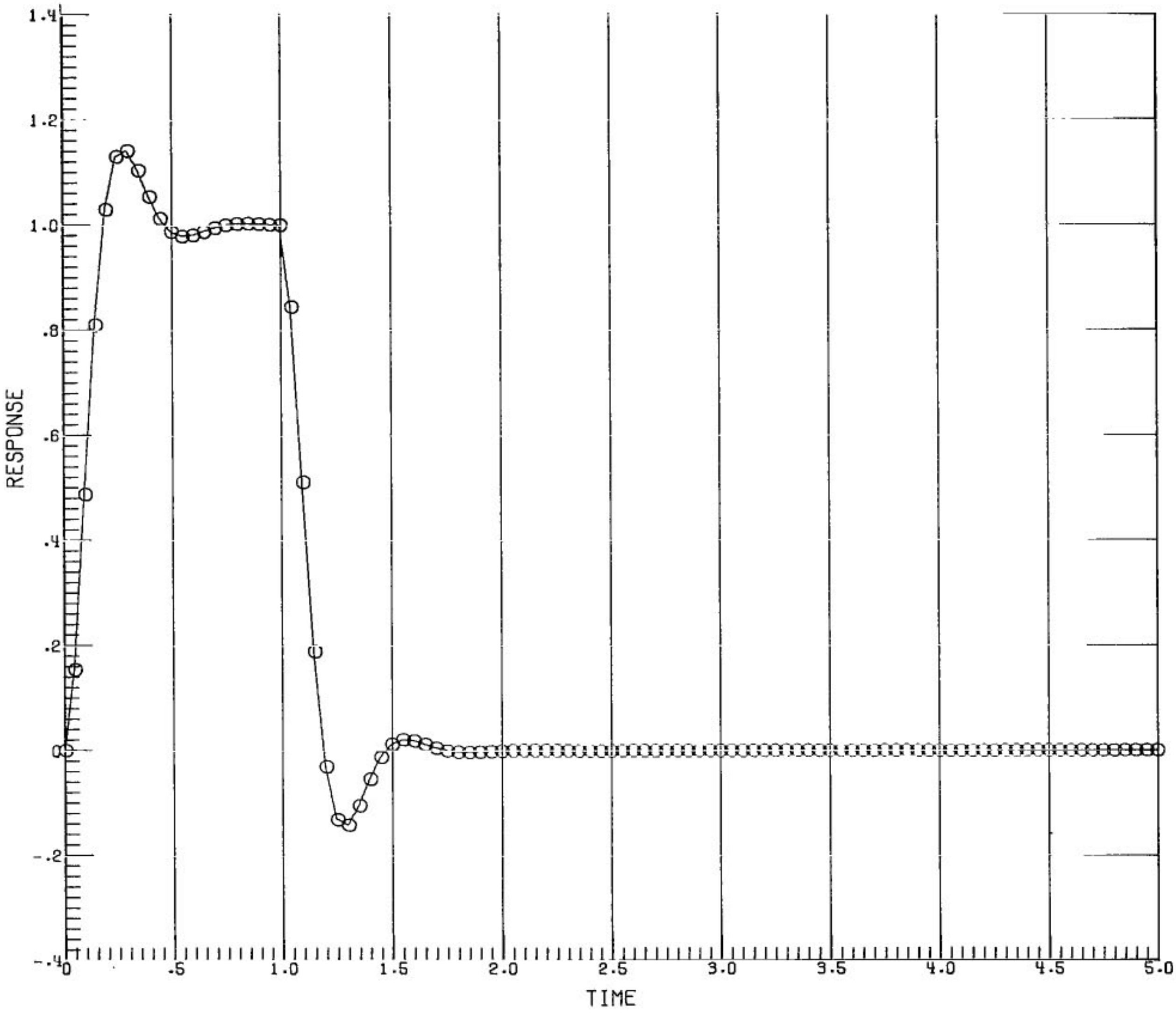

Figure 15.- Response of second-order Chebyshev (1-dB ripple) filter with BT = 2.

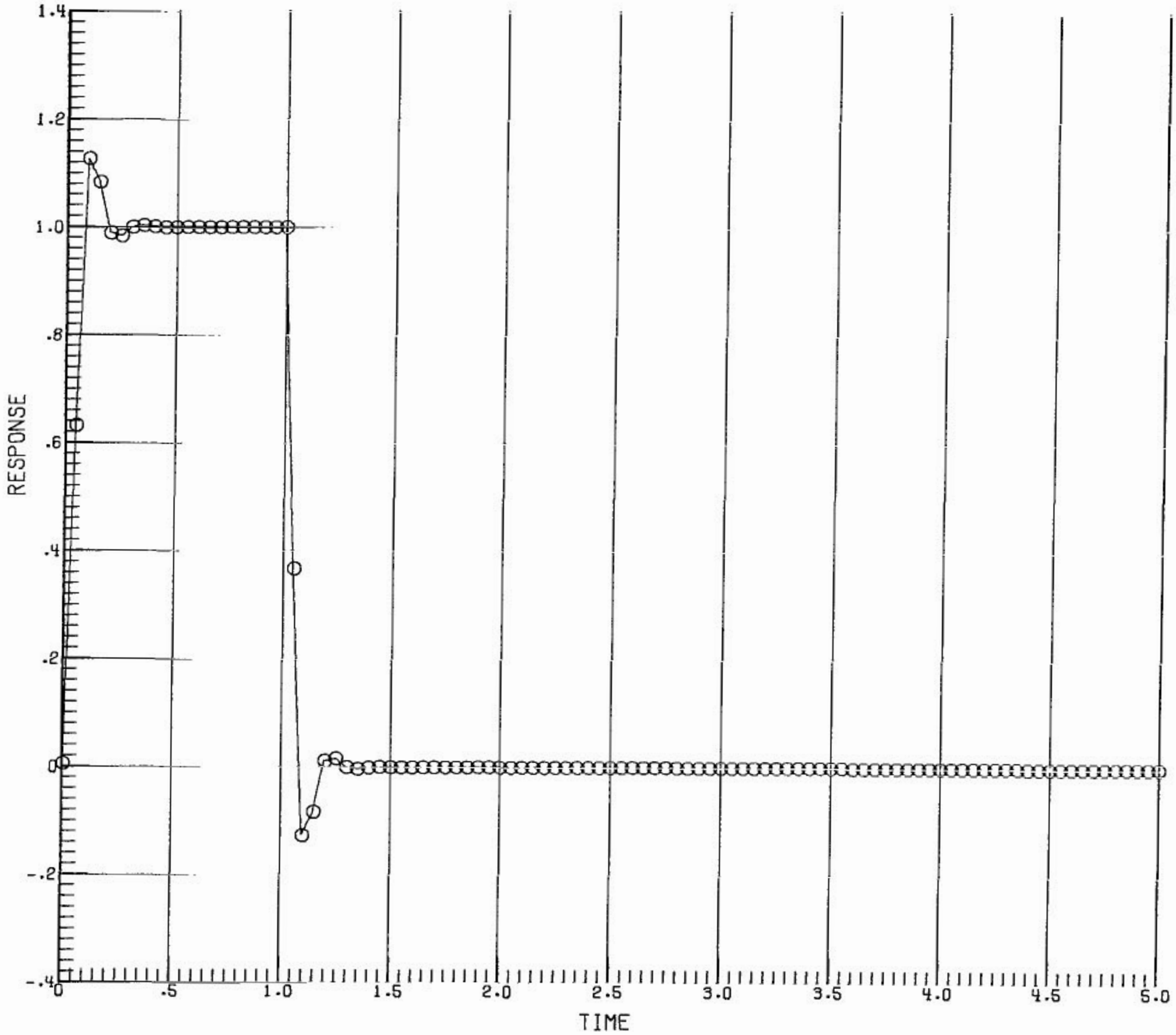

Figure 16.- Response of second-order Chebyshev (1-dB ripple) filter with BT = 5.

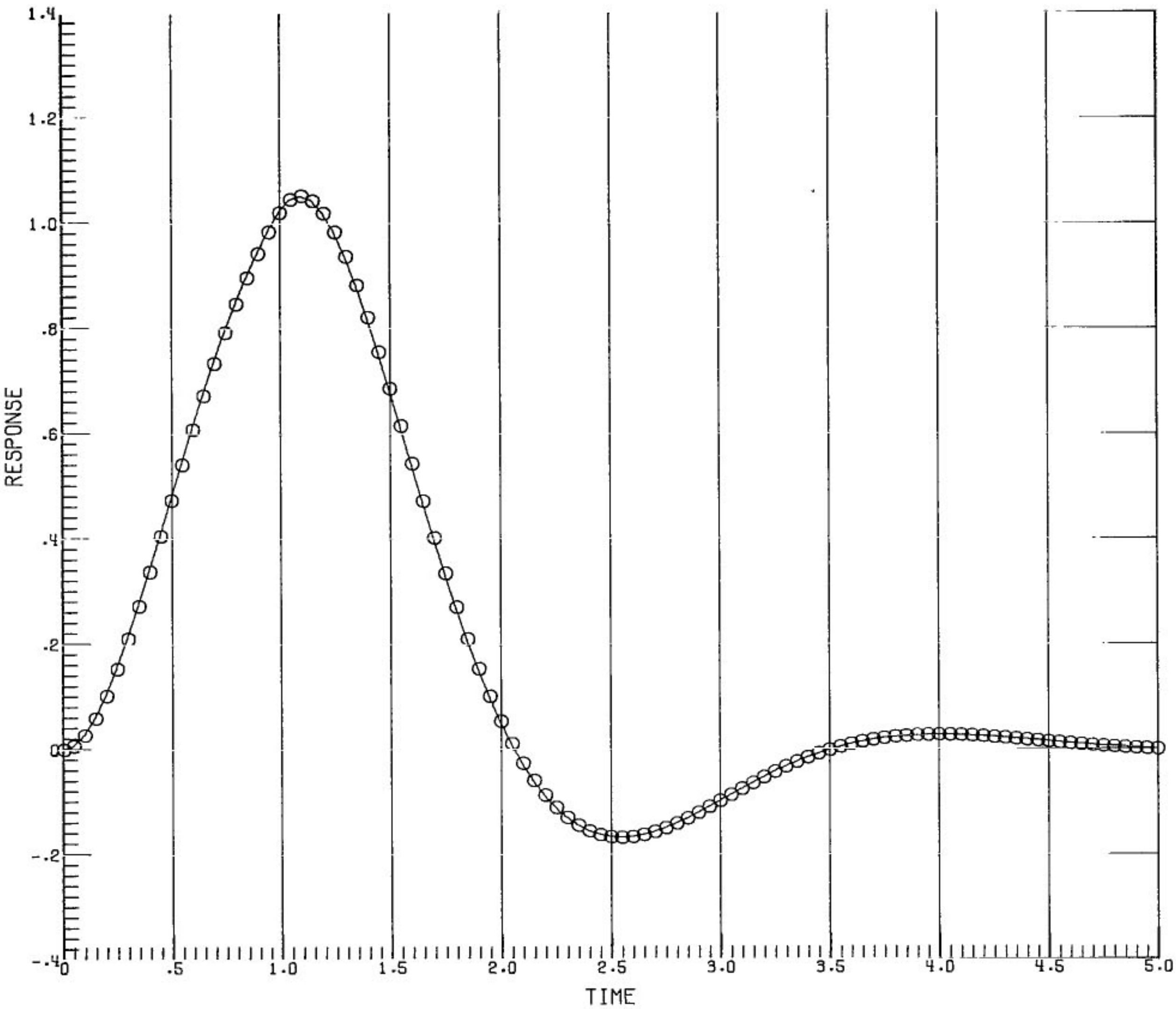

Figure 17.- Response of second-order Chebyshev (2-dB ripple) filter with BT = 0.5.

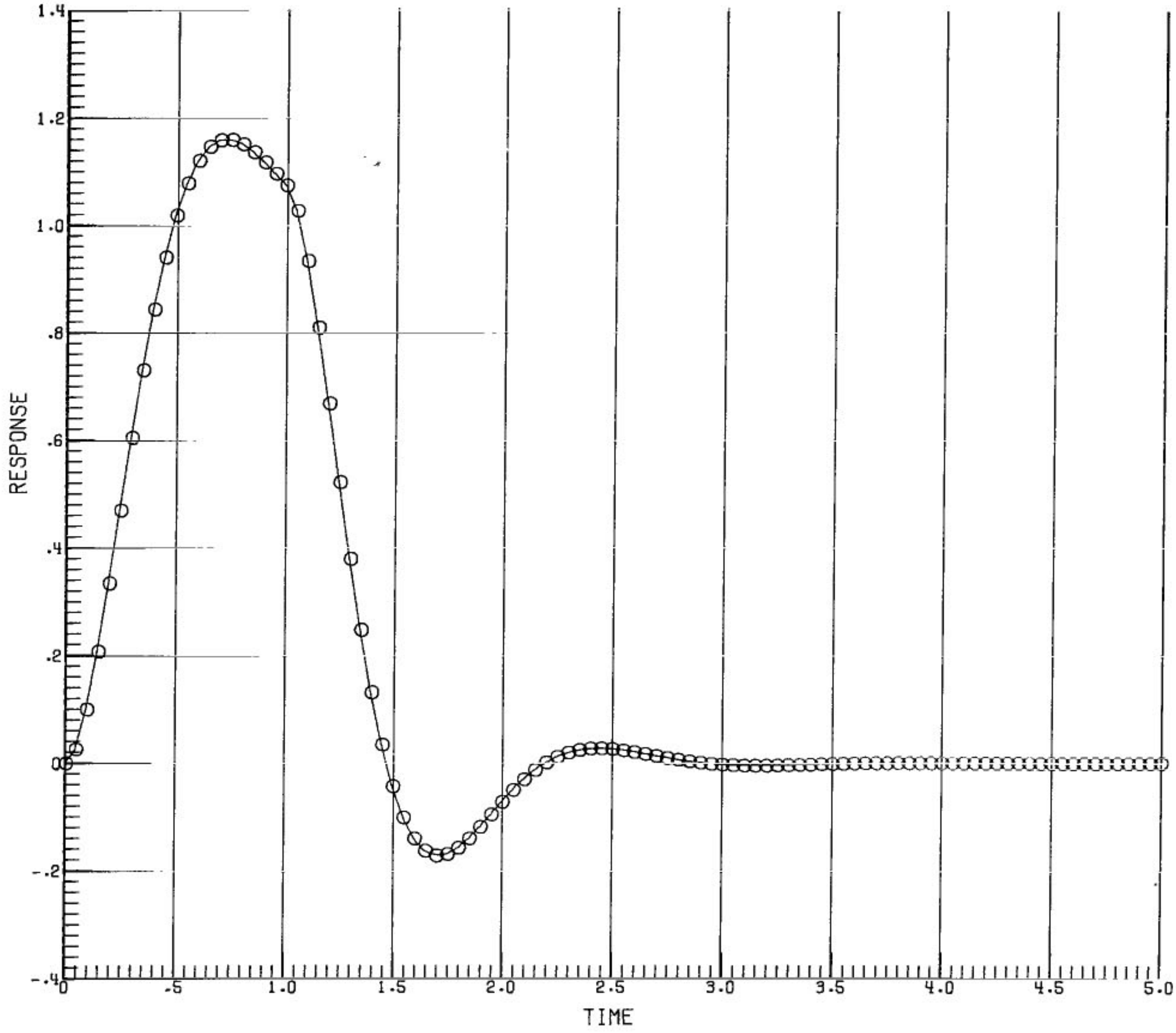

Figure 18.- Response of second-order Chebyshev (2-dB ripple) filter with BT = 1.

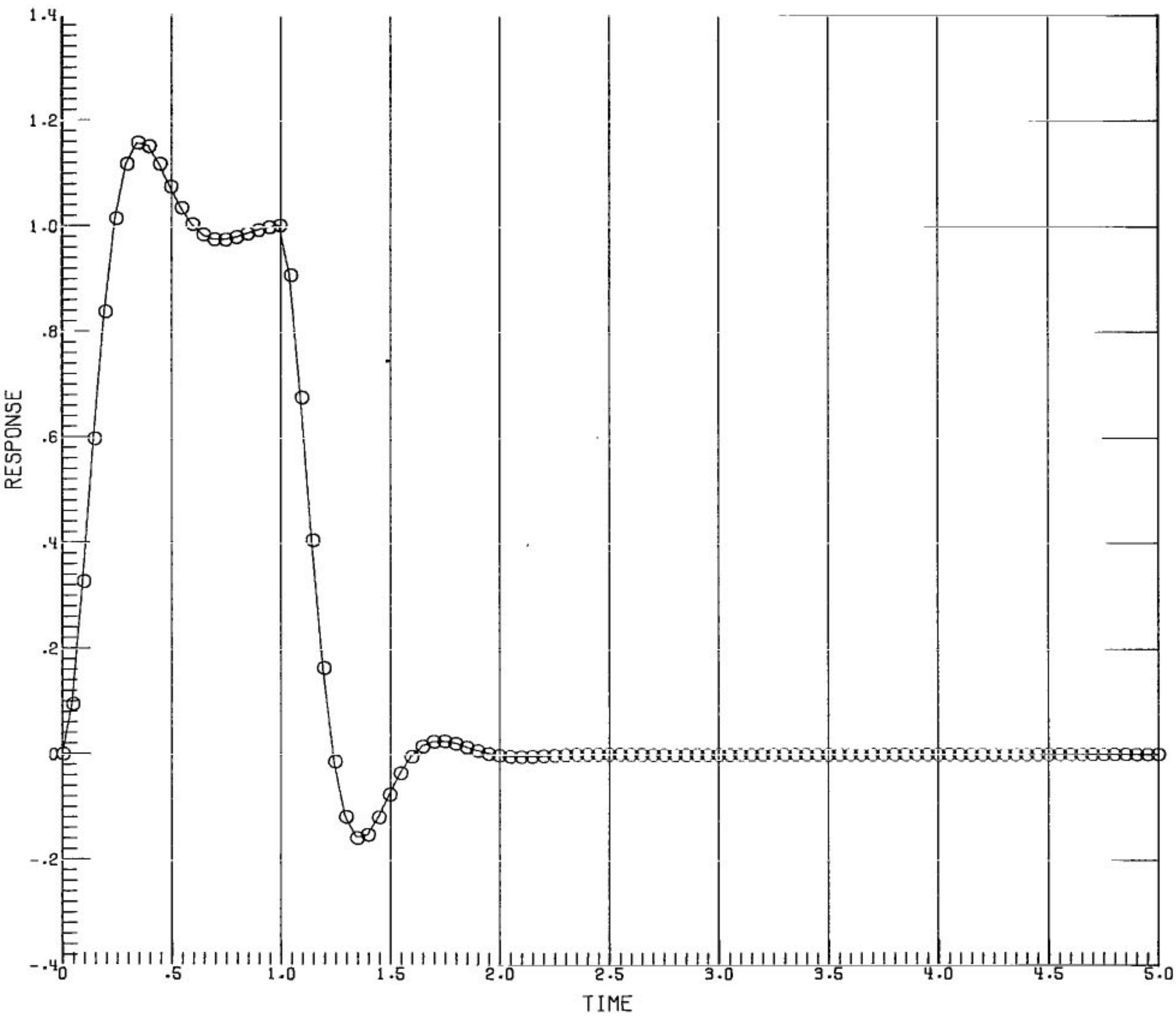

Figure 19.- Response of second-order Chebyshev (2-dB ripple) filter with BT = 2.

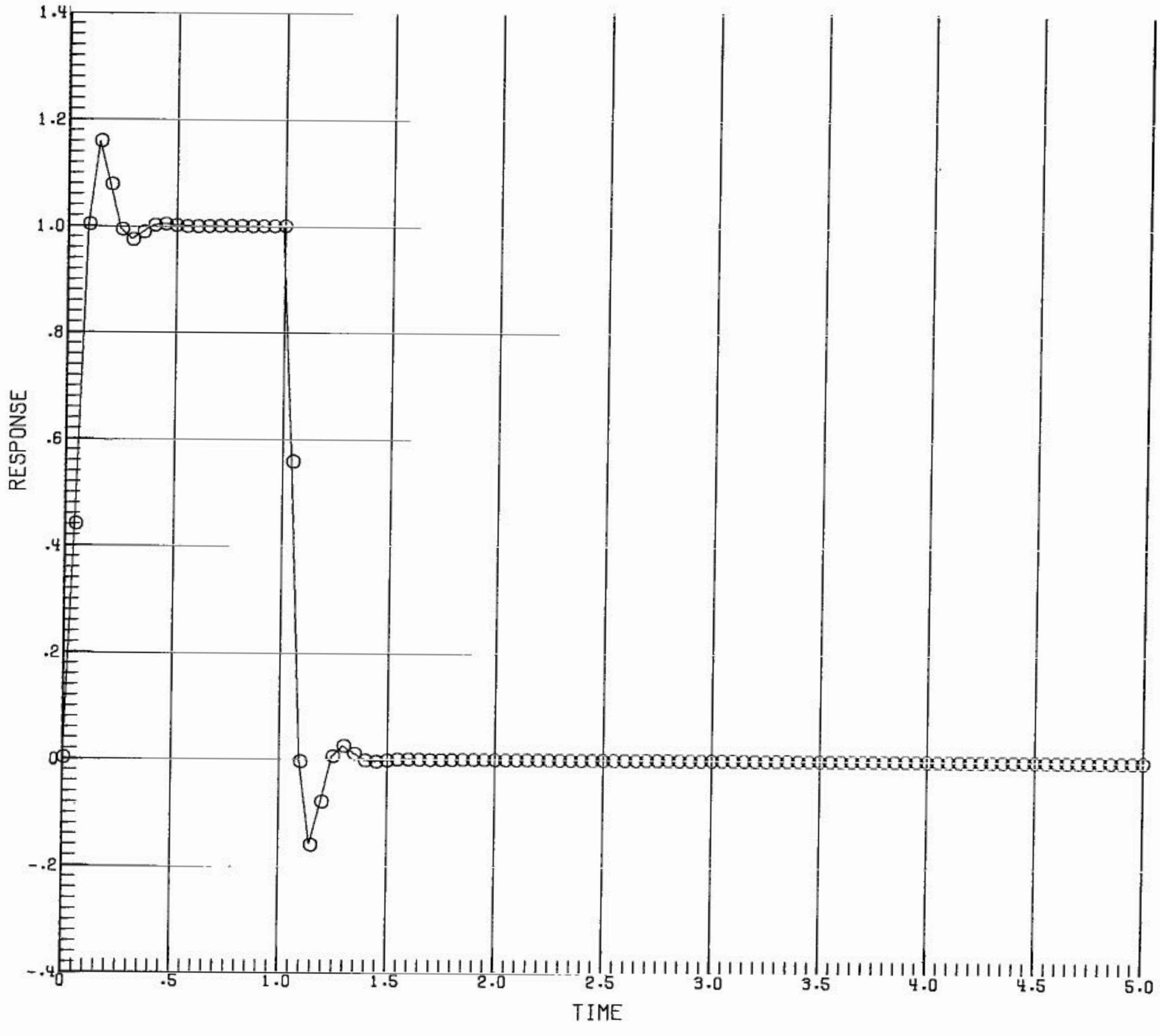

Figure 20.- Response of second-order Chebyshev (2-dB ripple) filter with BT = 5.

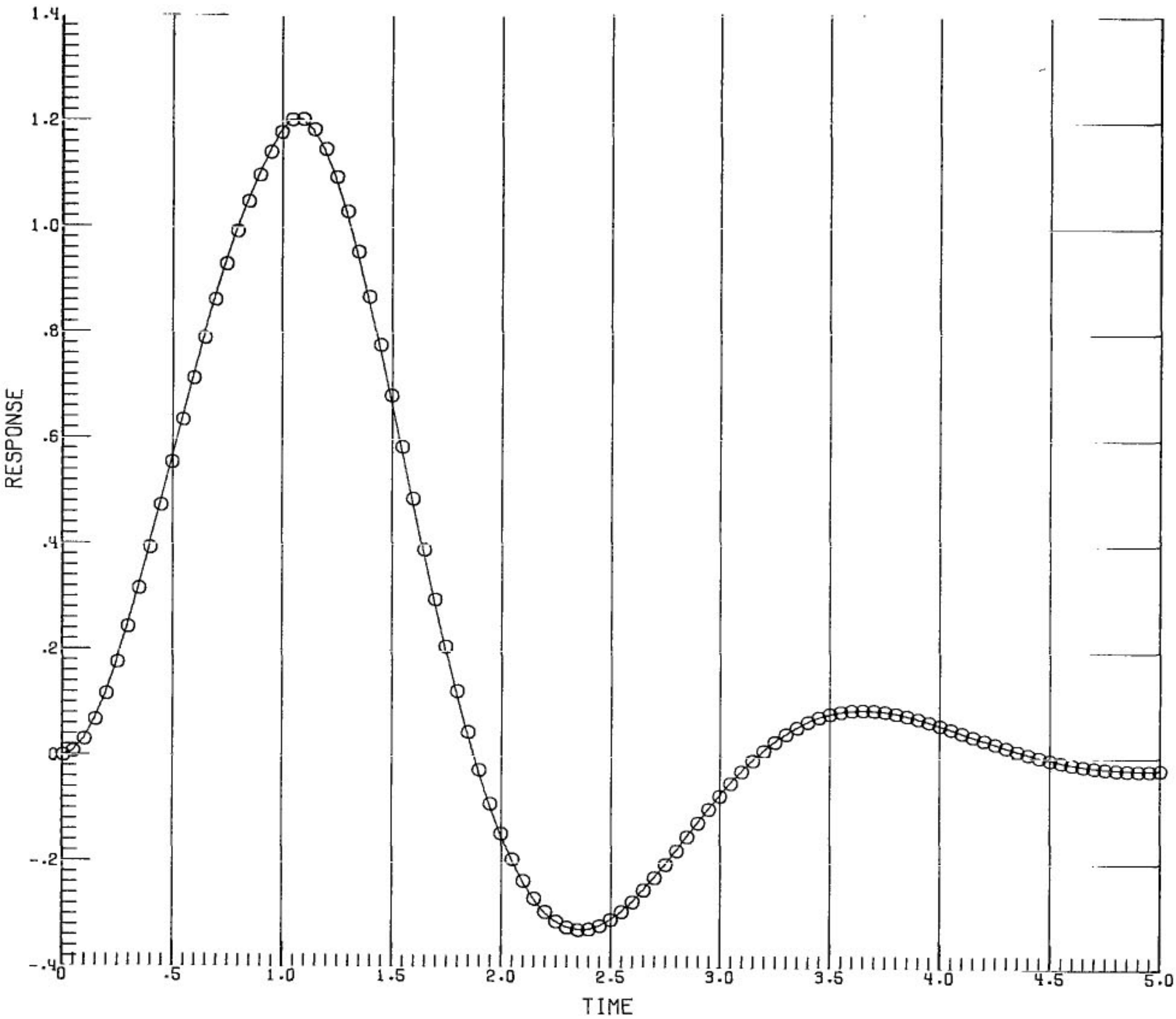

Figure 21.- Response of second-order Chebyshev (3-dB ripple) filter with BT = 0.5.

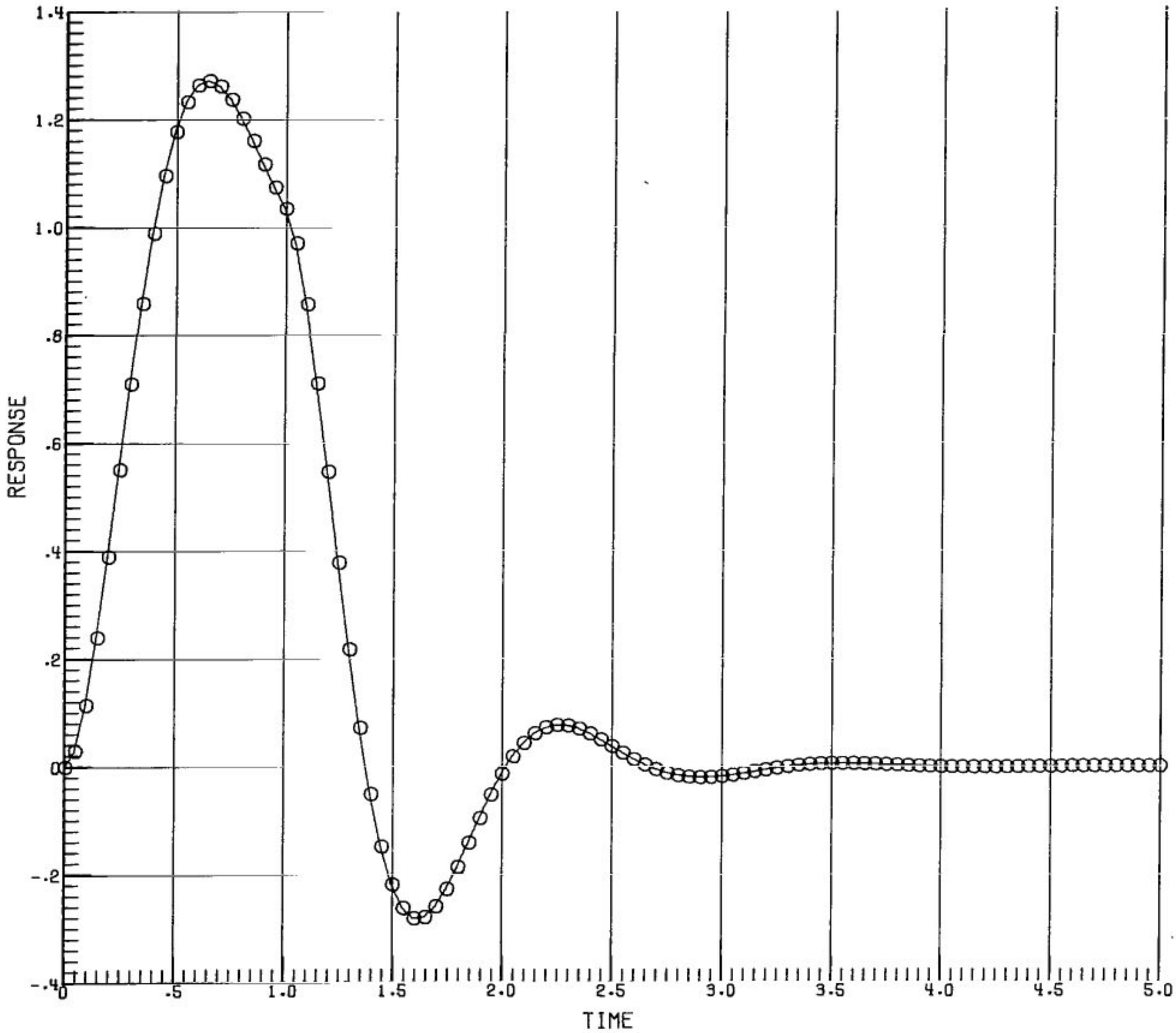

Figure 22.- Response of second-order Chebyshev (3-dB ripple) filter with BT = 1.

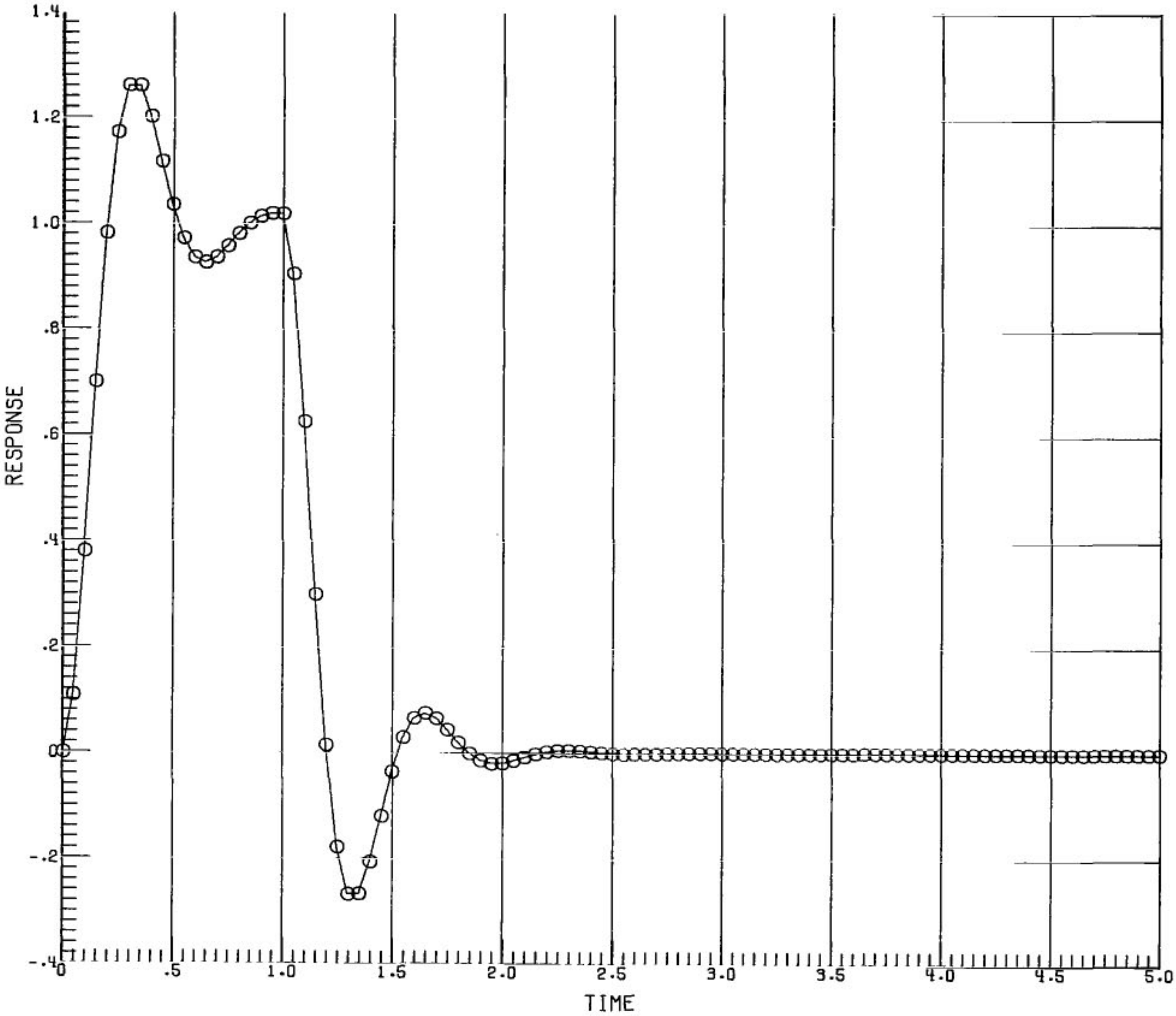

Figure 23.- Response of second-order Chebyshev (3-dB ripple) filter with BT = 2.

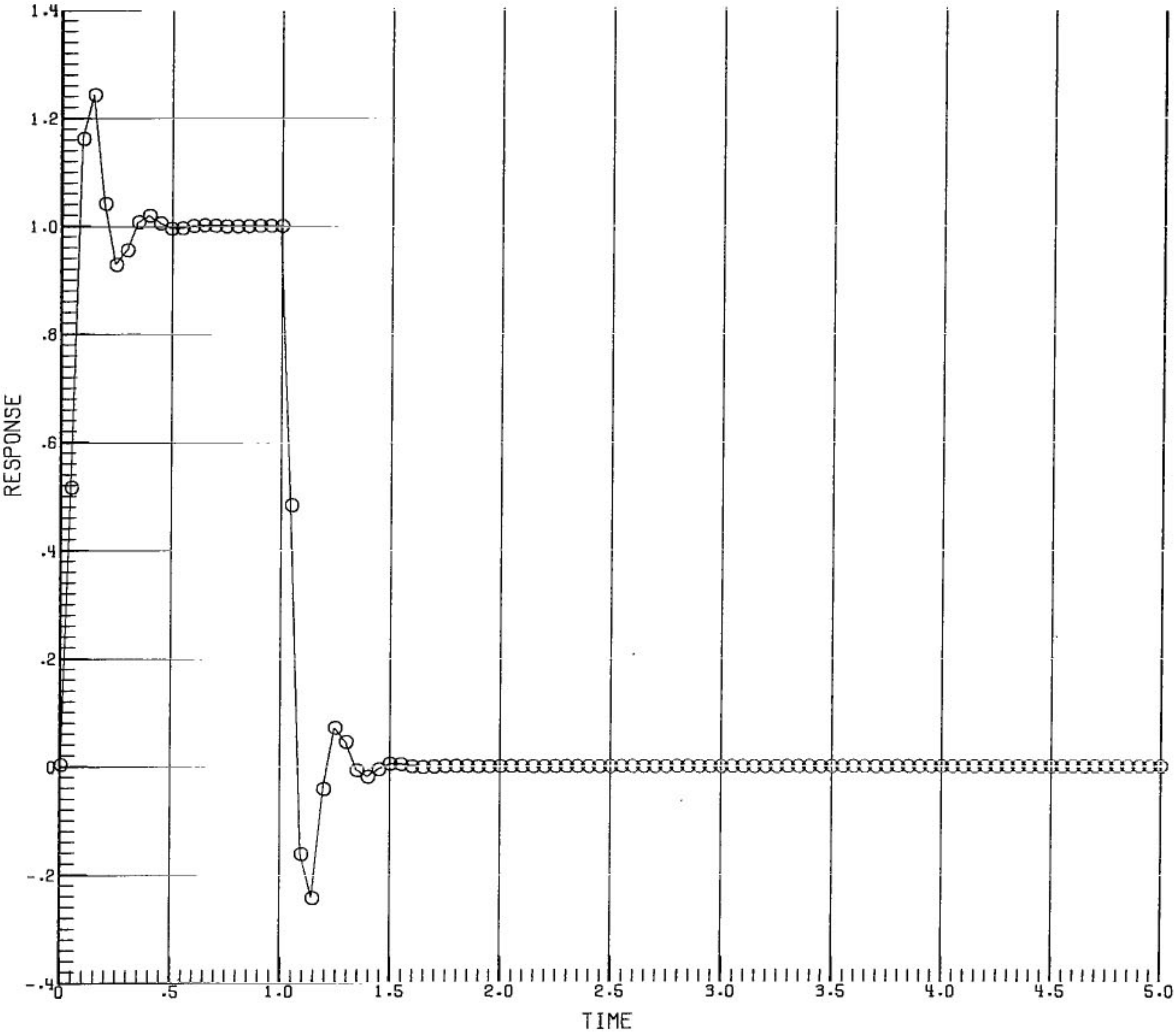

Figure 24.- Response of second-order Chebyshev (3-dB ripple) filter with BT = 5.

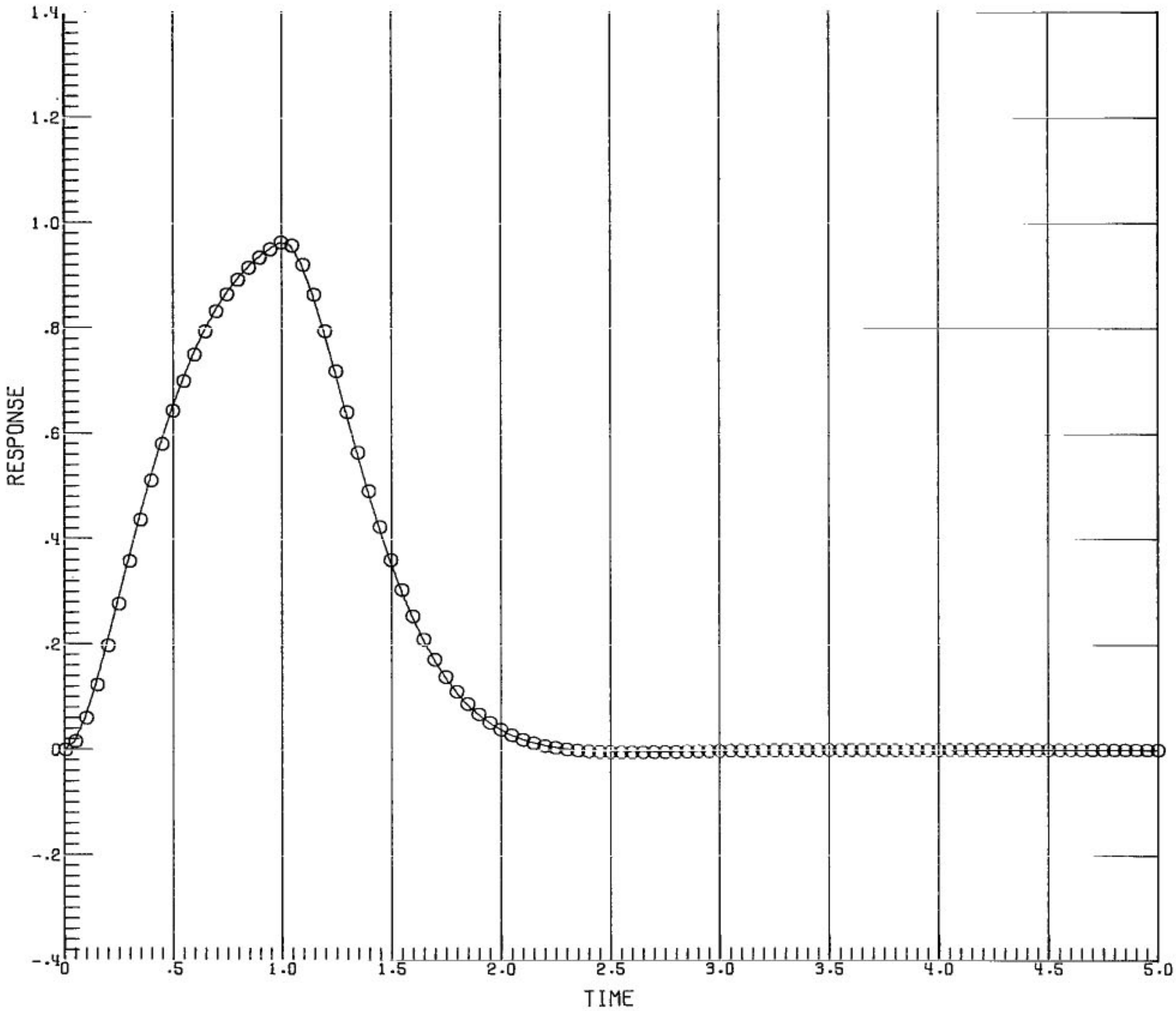

Figure 25.- Response of second-order Bessel filter with BT = 0.5.

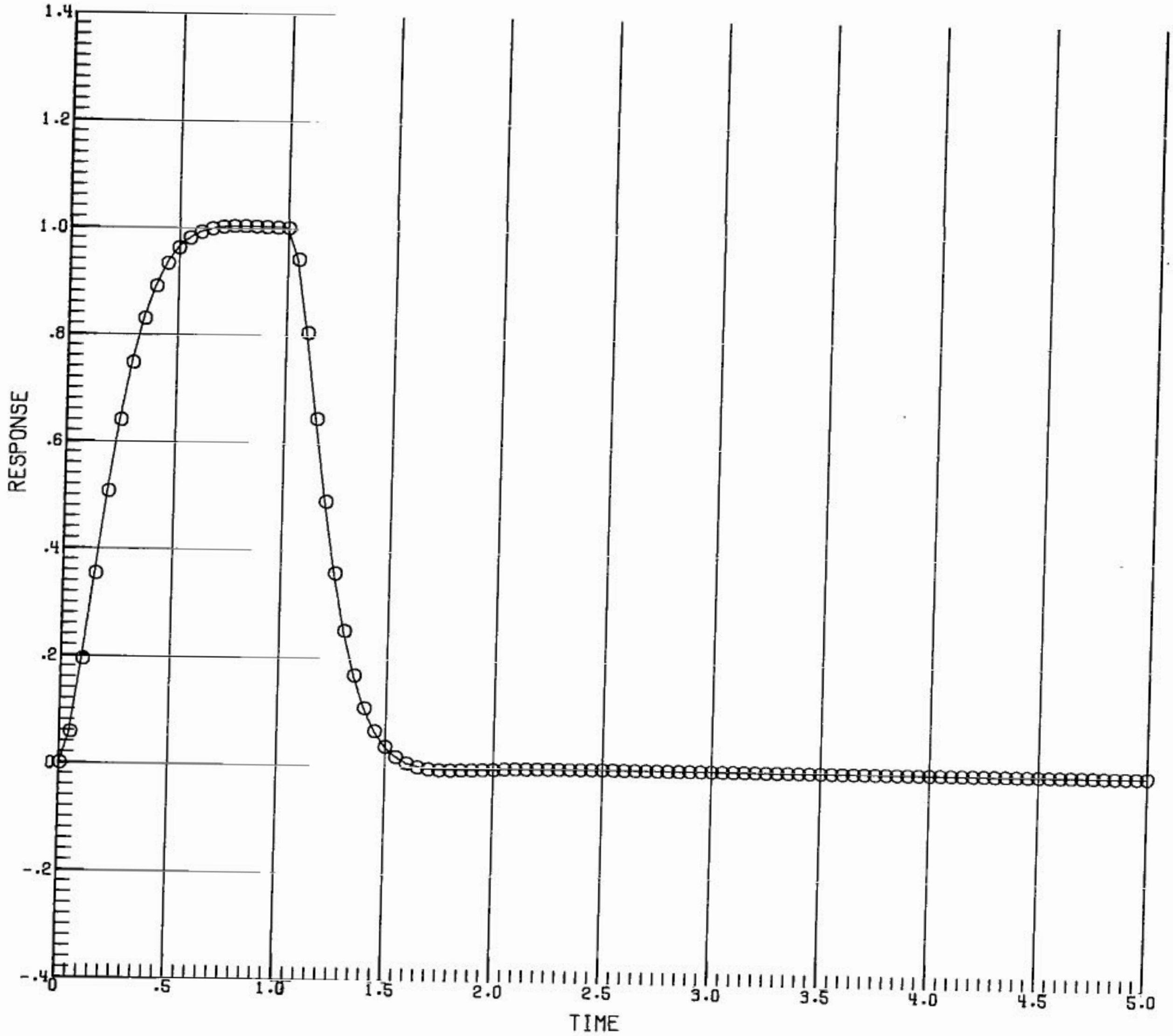

Figure 26.- Response of second-order Bessel filter with $BT = 1$.

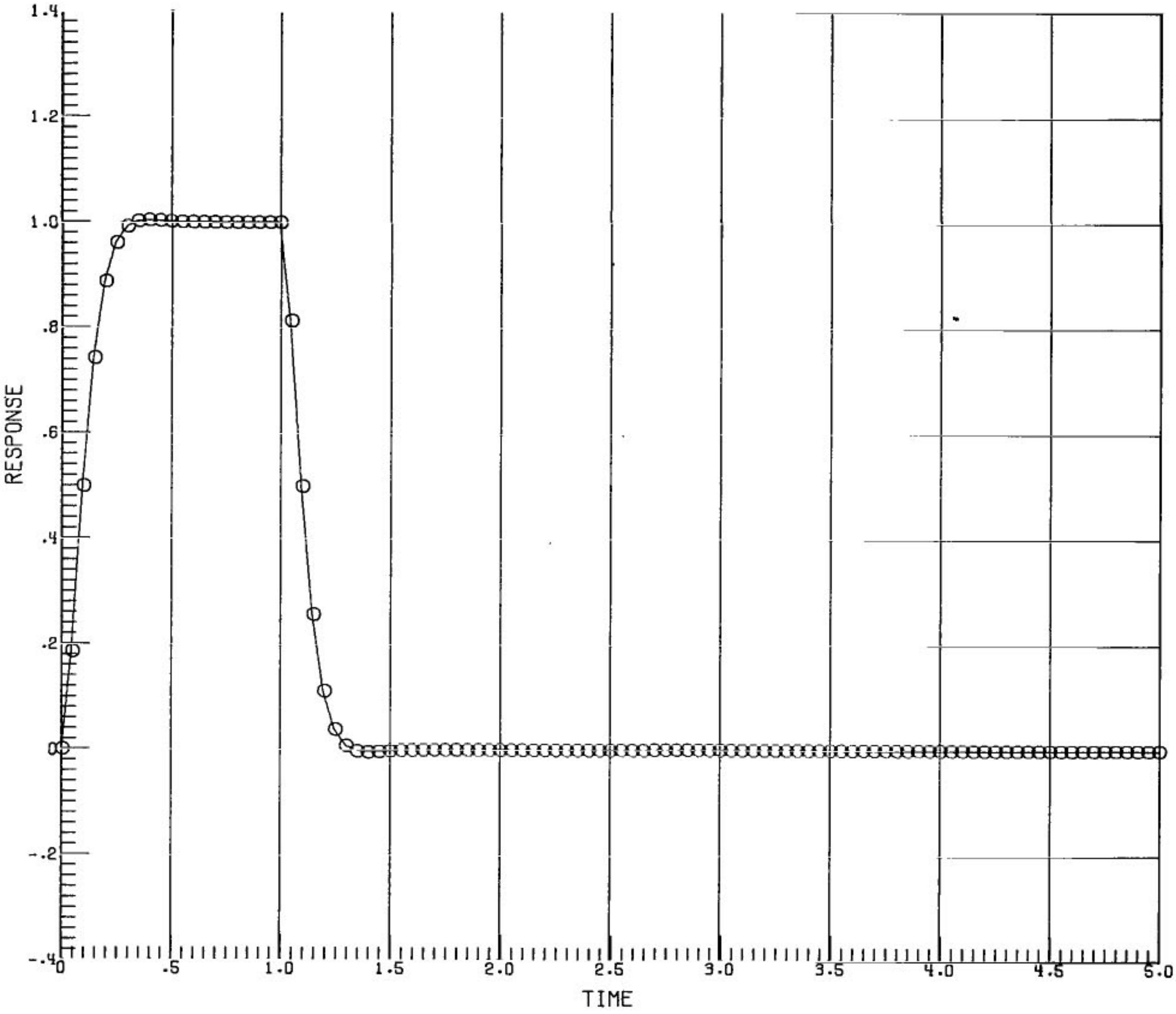

Figure 27.- Response of second-order Bessel filter with BT = 2.

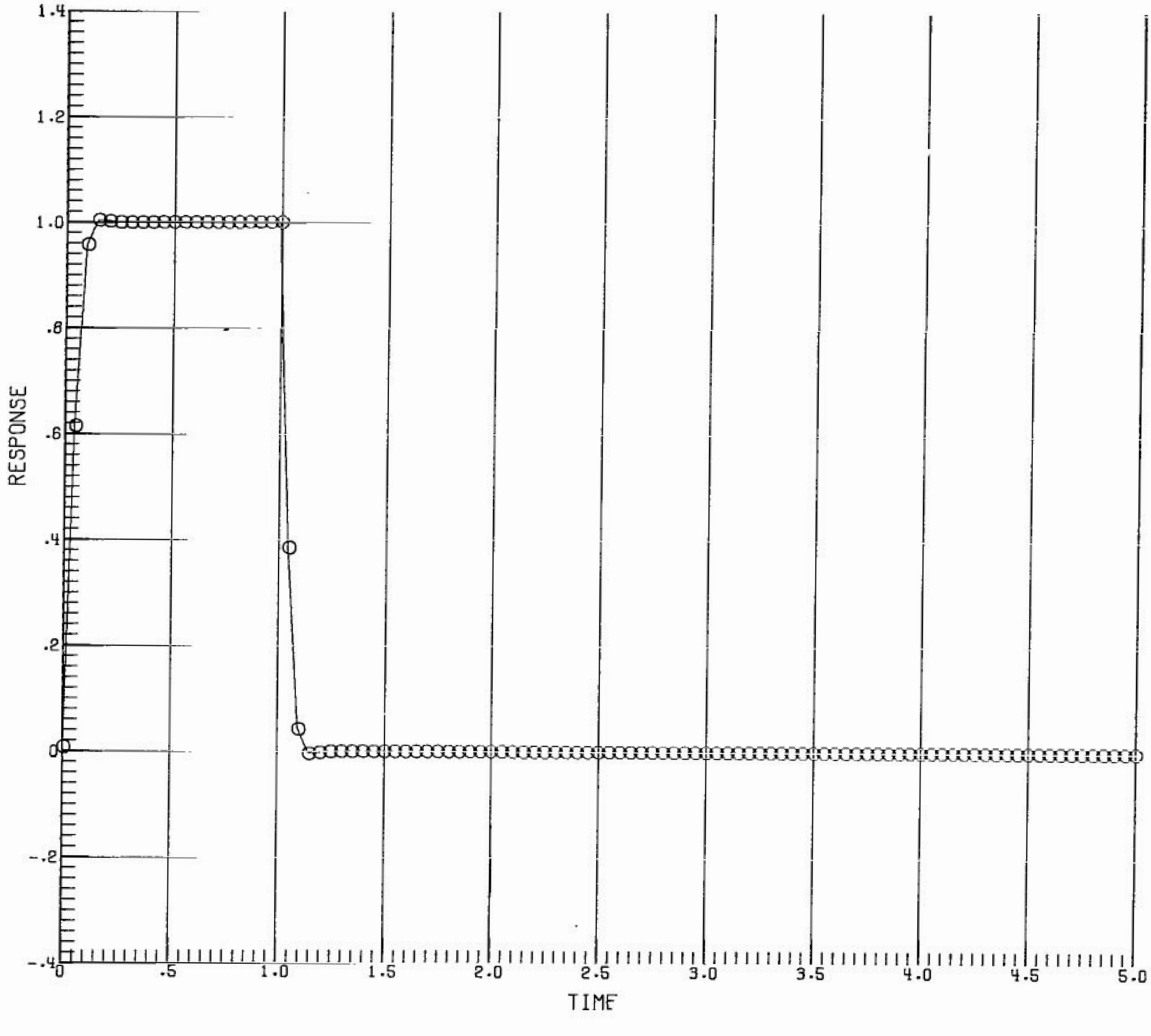

Figure 28.- Response of second-order Bessel filter with BT = 5.

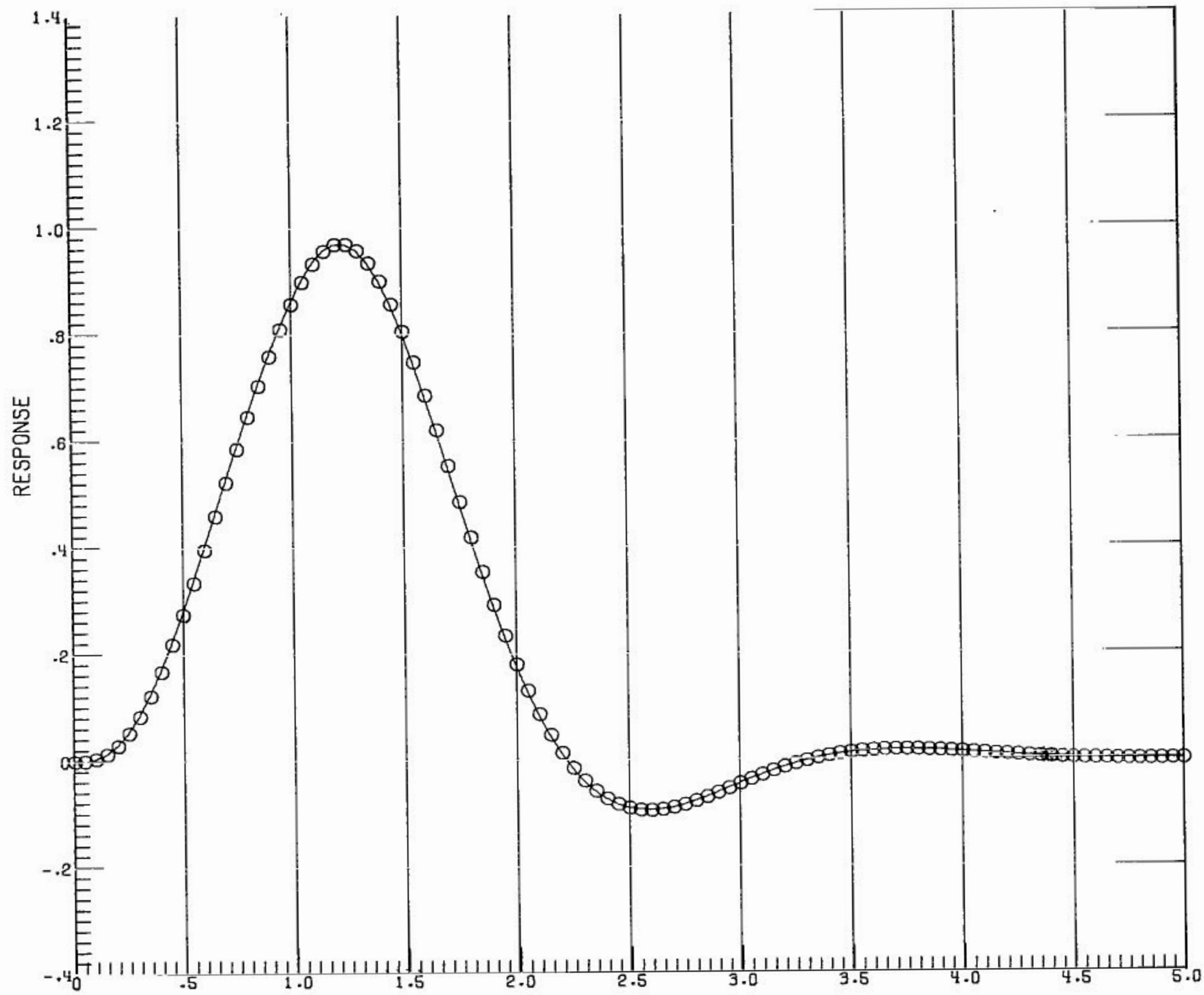

Figure 29.- Response of third-order Butterworth filter with BT = 0.5.

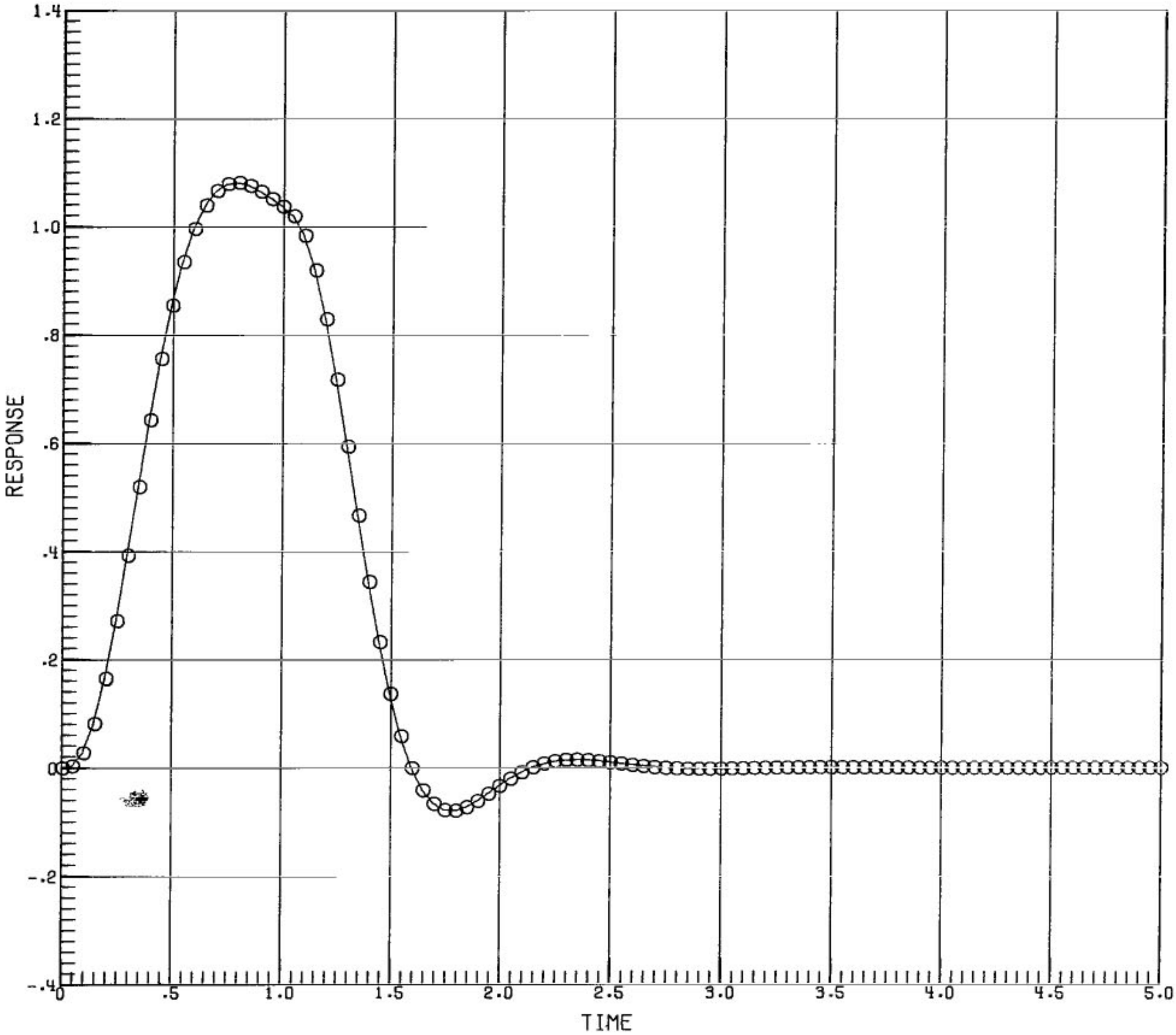

Figure 30.- Response of third-order Butterworth filter with BT = 1.

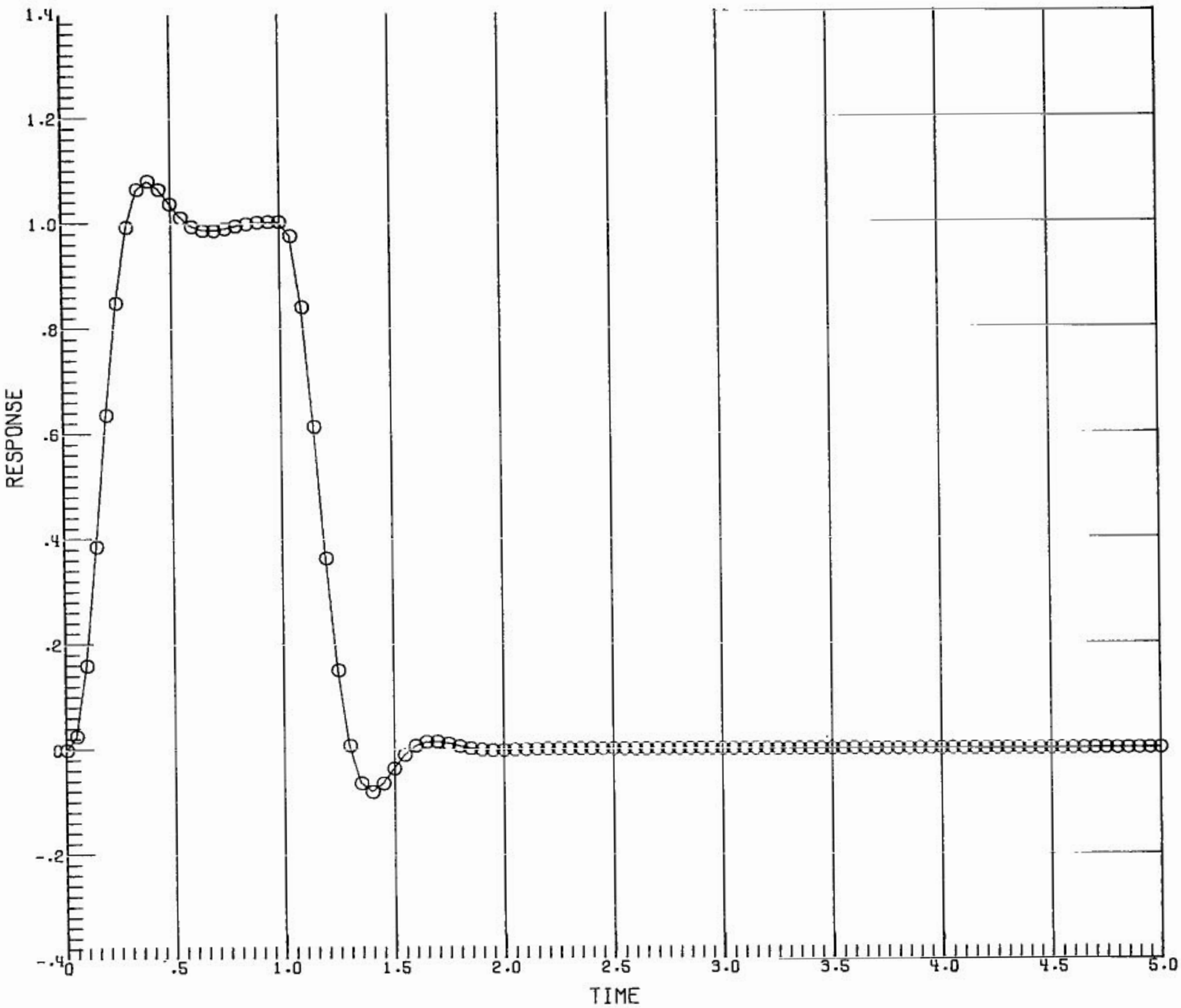

Figure 31.- Response of third-order Butterworth filter with BT = 2.

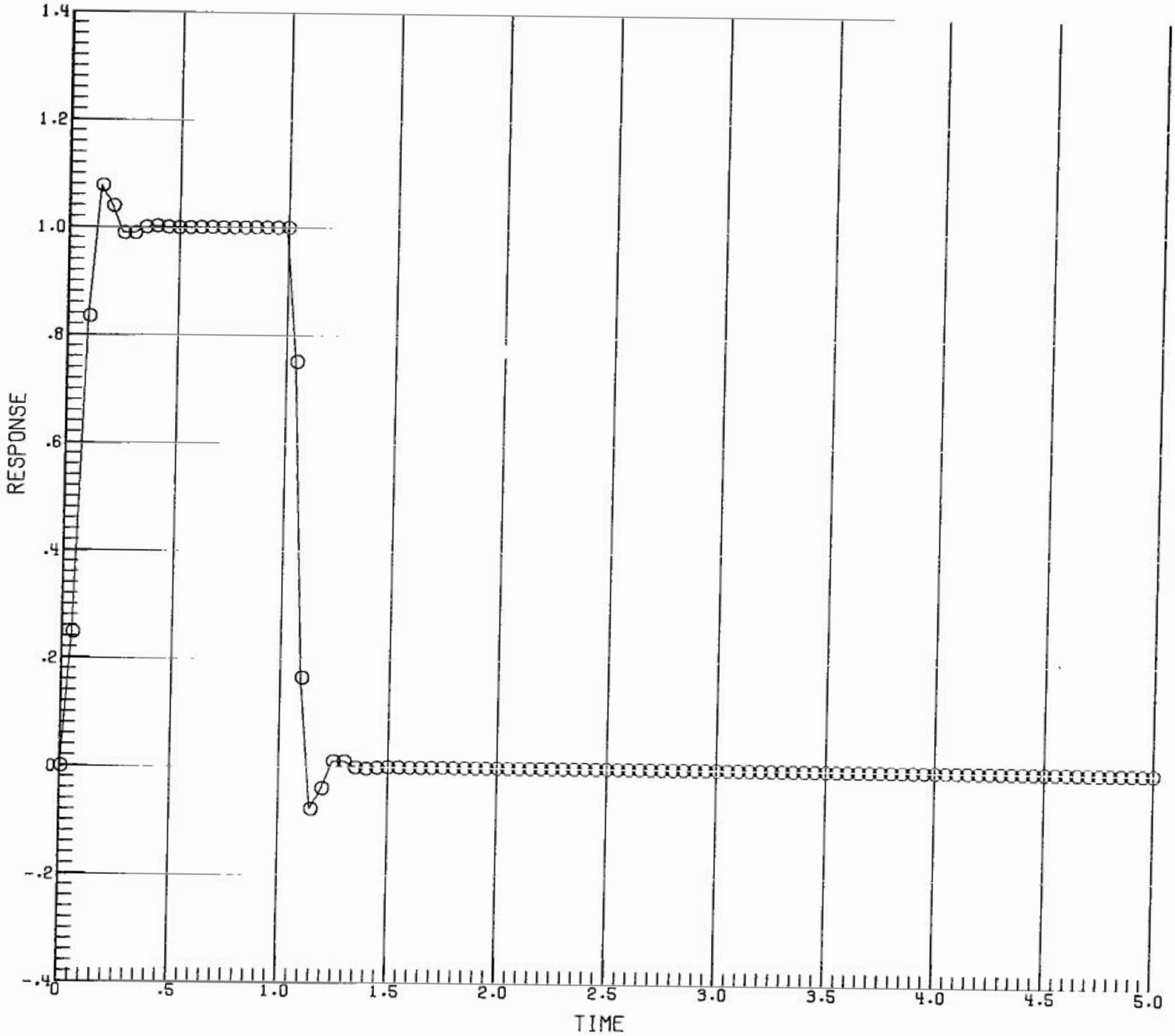

Figure 32.- Response of third-order Butterworth filter with BT = 5.

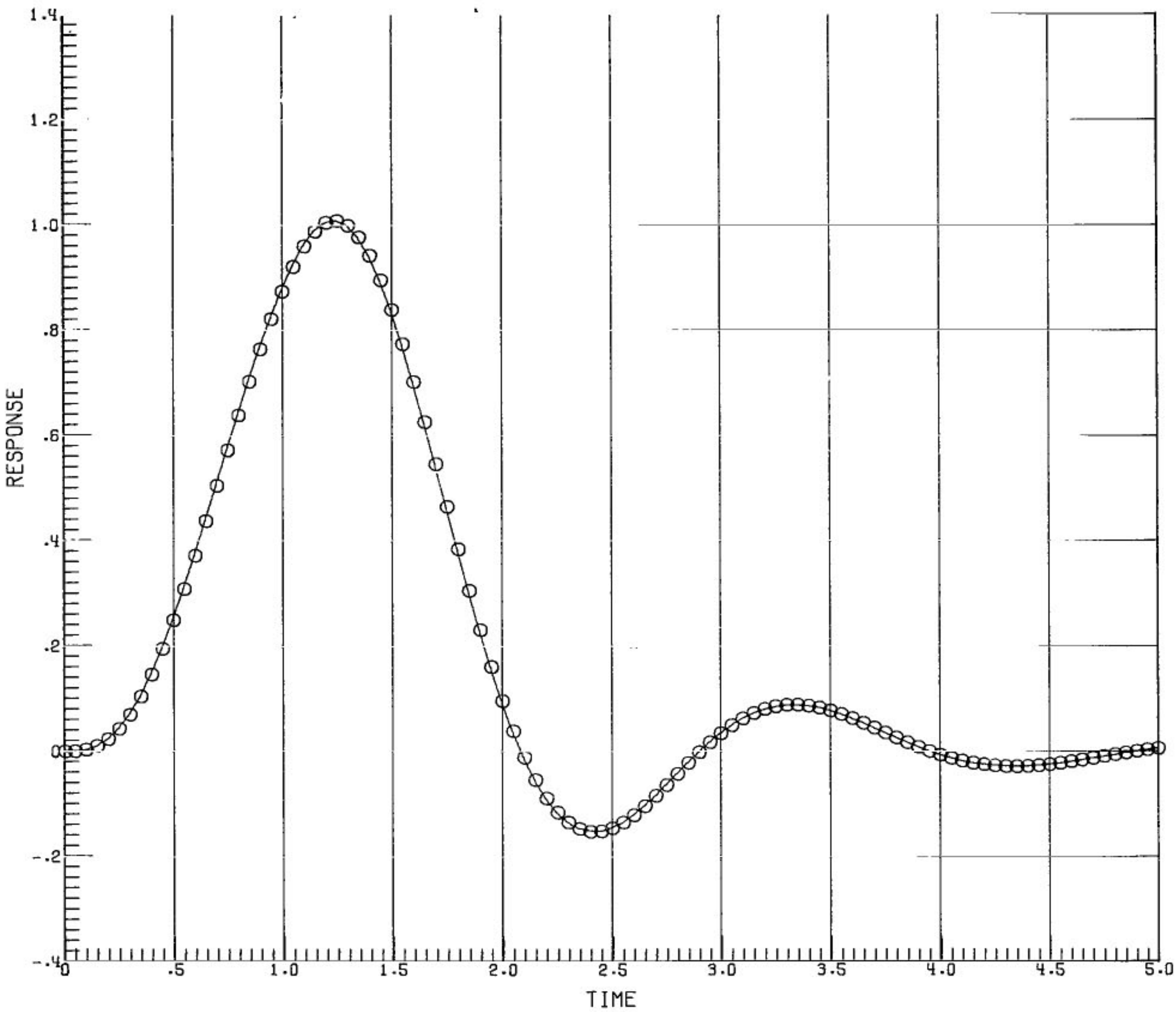

Figure 33.- Response of third-order Chebyshev (0.5-dB ripple) filter with BT = 0.5.

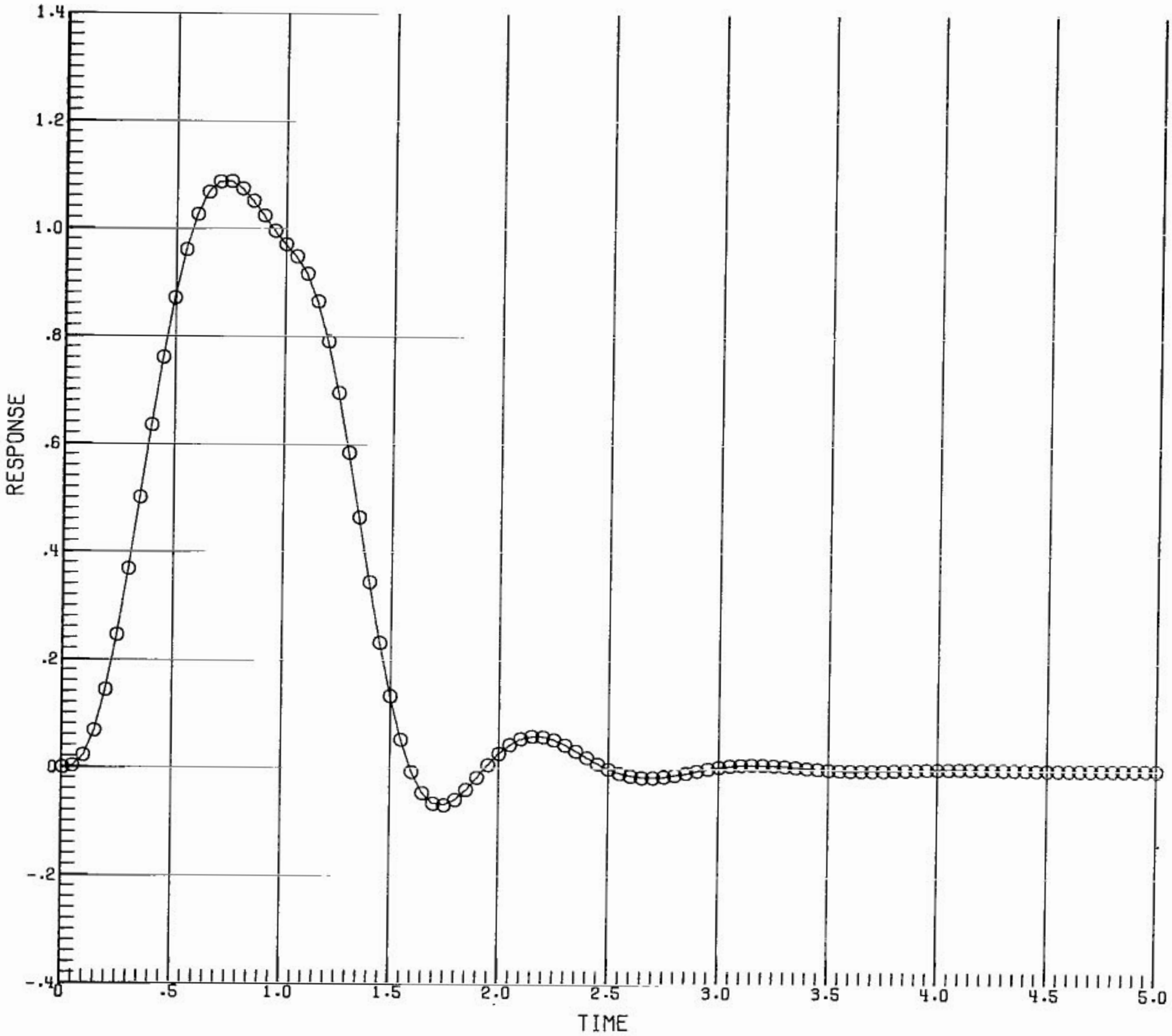

Figure 34.- Response of third-order Chebyshev (0.5-dB ripple) filter with BT = 1.

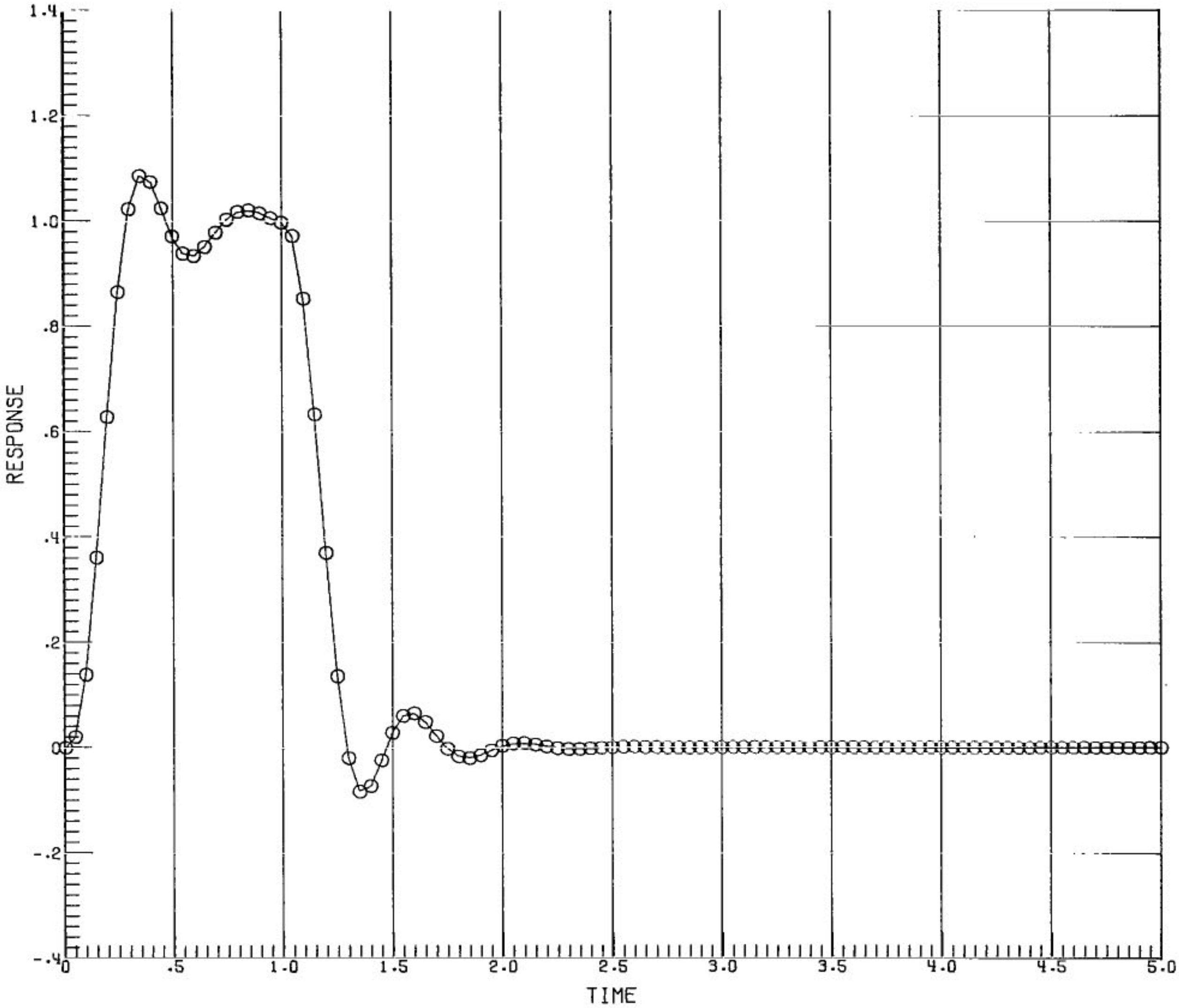

Figure 35.- Response of third-order Chebyshev (0.5-dB ripple) filter with BT = 2.

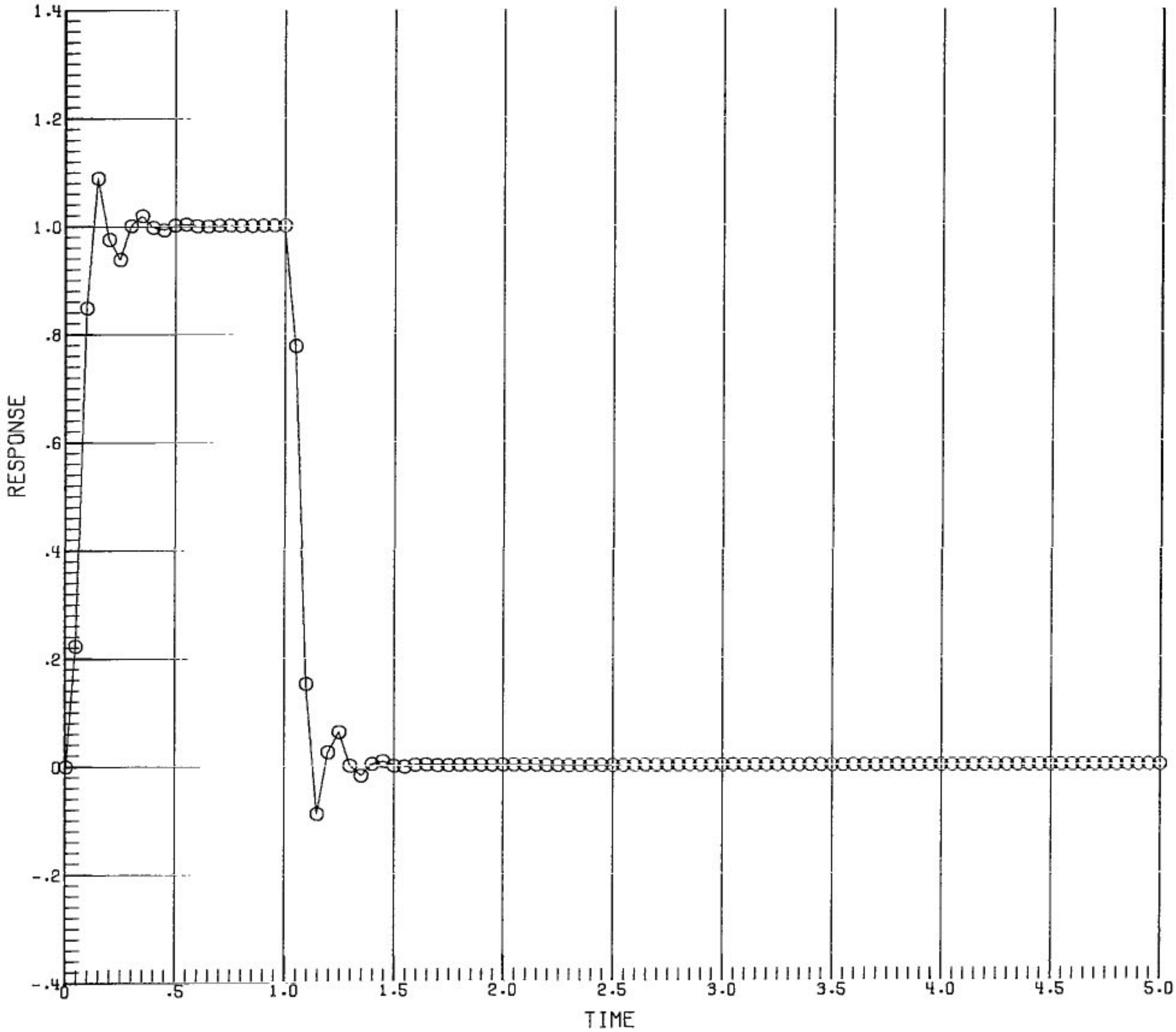

Figure 36.- Response of third-order Chebyshev (0.5-dB ripple) filter with BT = 5.

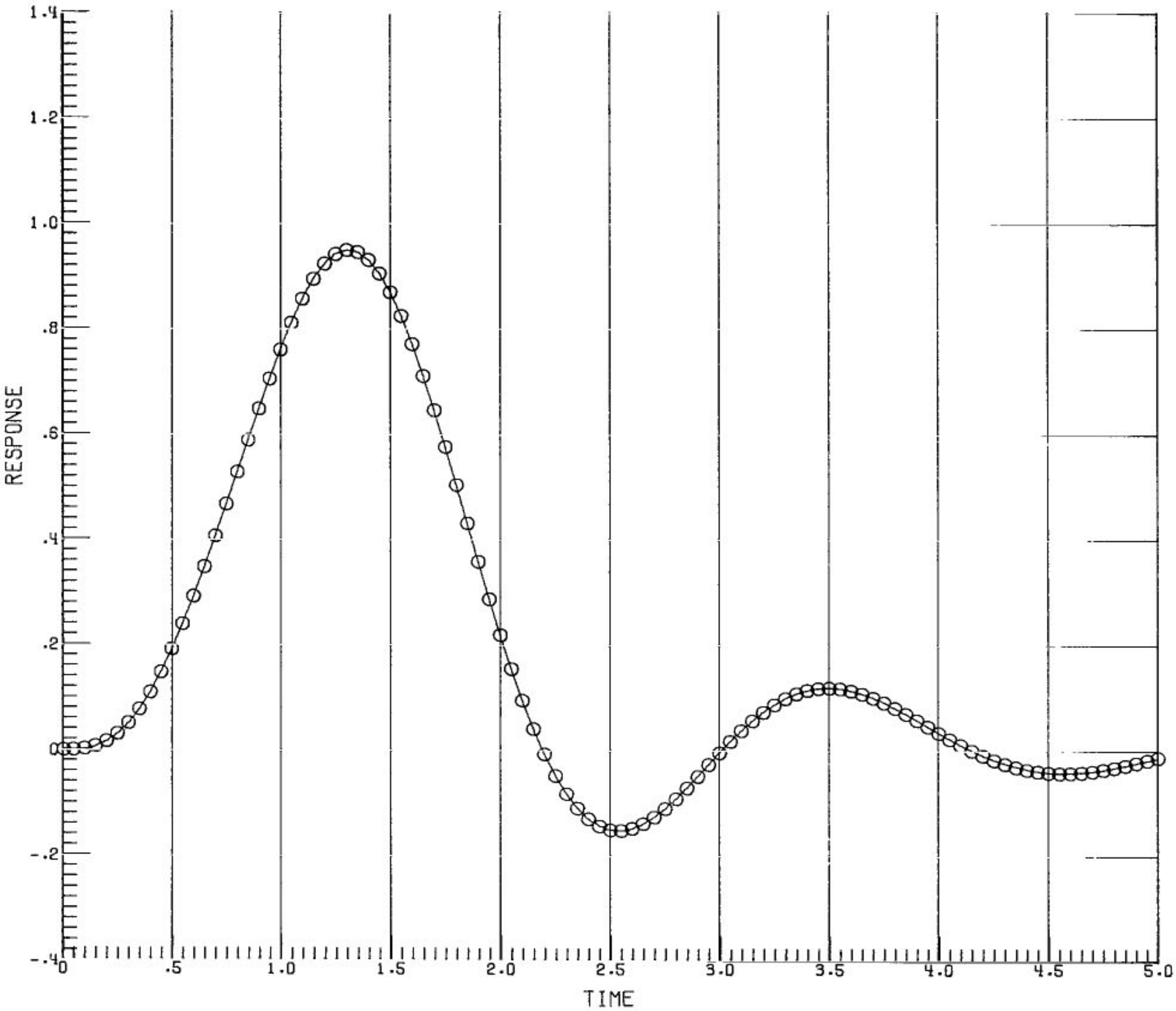

Figure 37.- Response of third-order Chebyshev (1-dB ripple) filter with BT = 0.5.

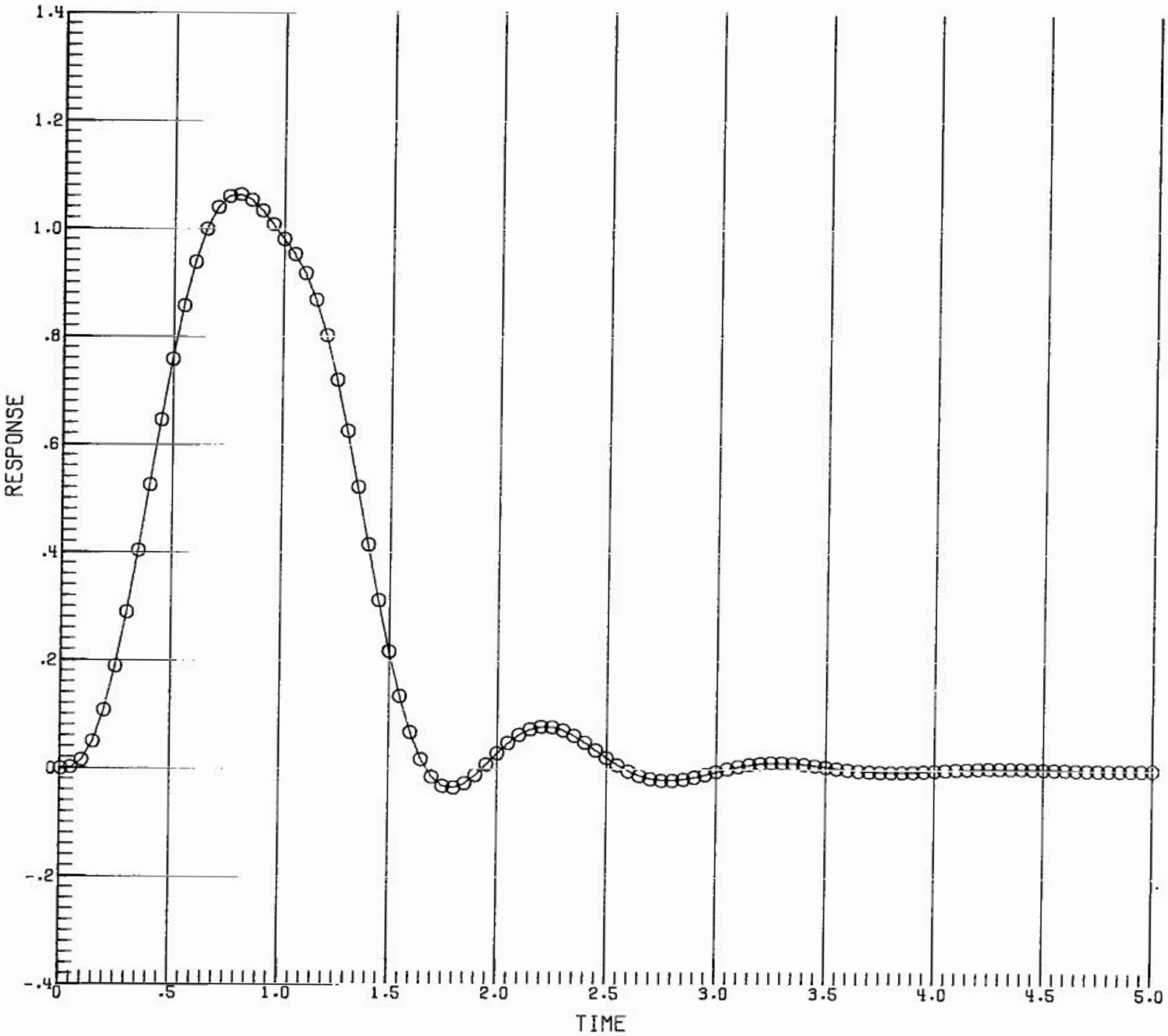

Figure 38.- Response of third-order Chebyshev (1-dB ripple) filter with BT = 1.

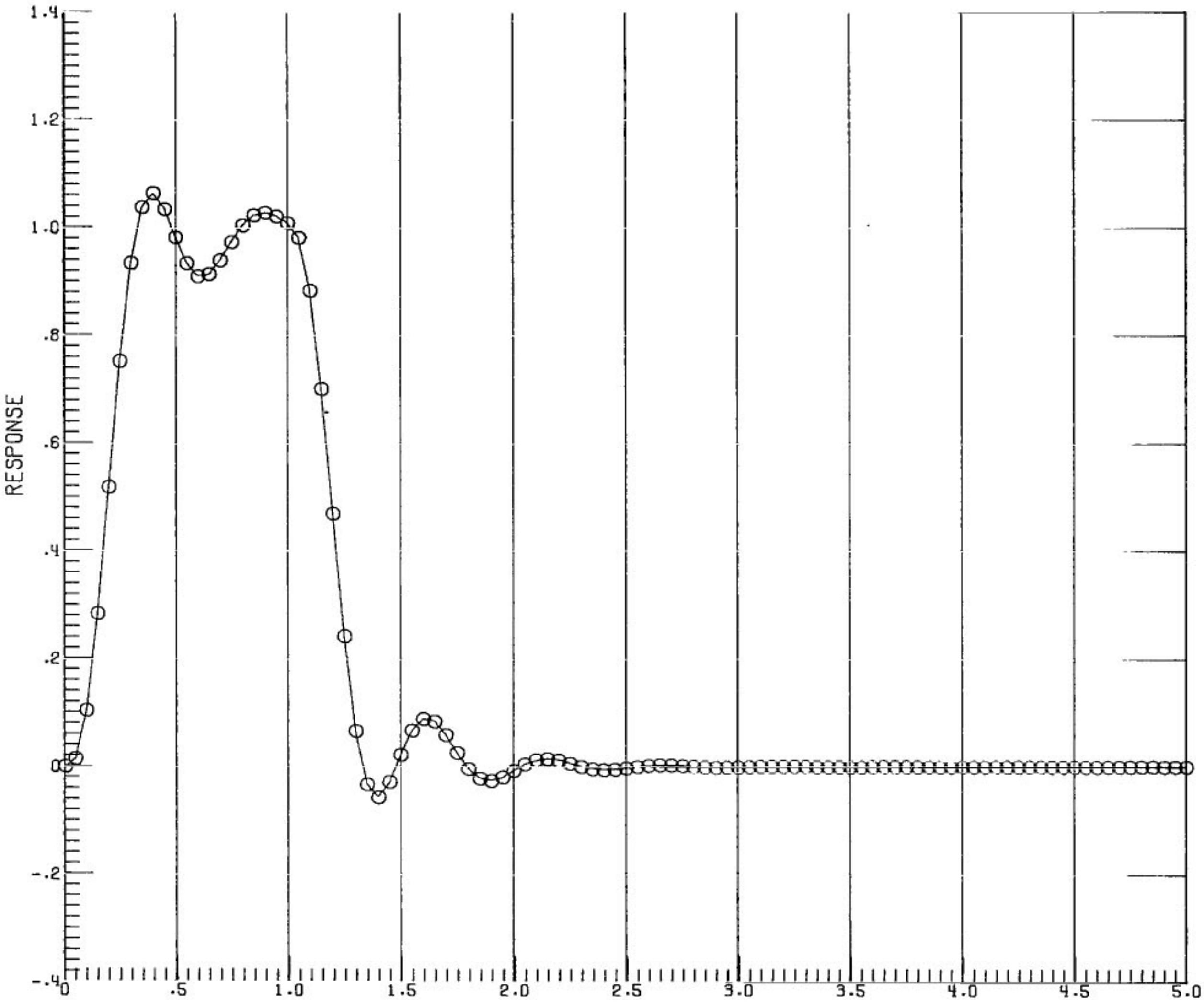

Figure 39.- Response of third-order Chebyshev (1-dB ripple) filter with BT = 2.

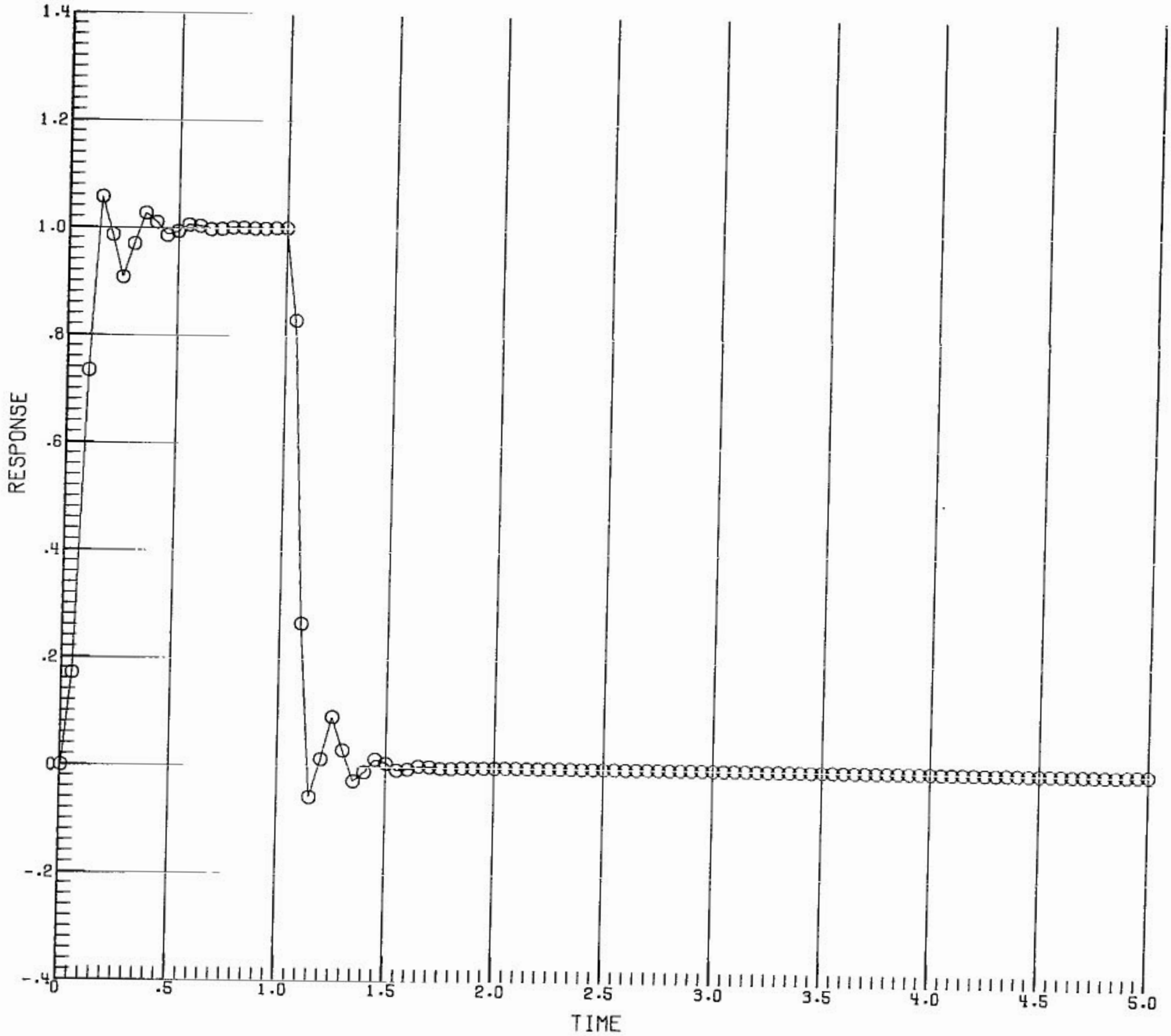

Figure 40.- Response of third-order Chebyshev (1-dB ripple) filter with BT = 5.

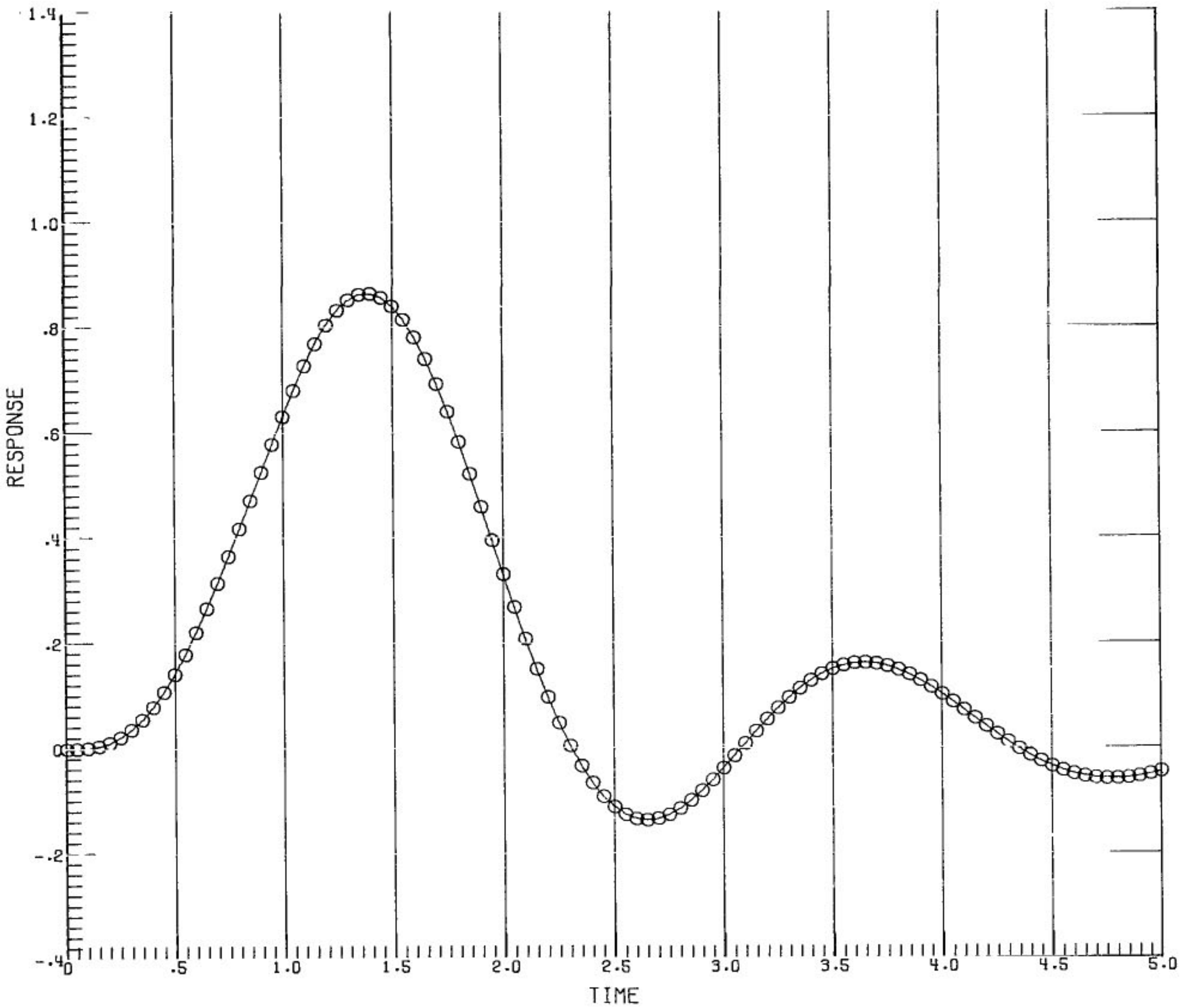

Figure 41.- Response of third-order Chebyshev (2-dB ripple) filter with BT = 0.5.

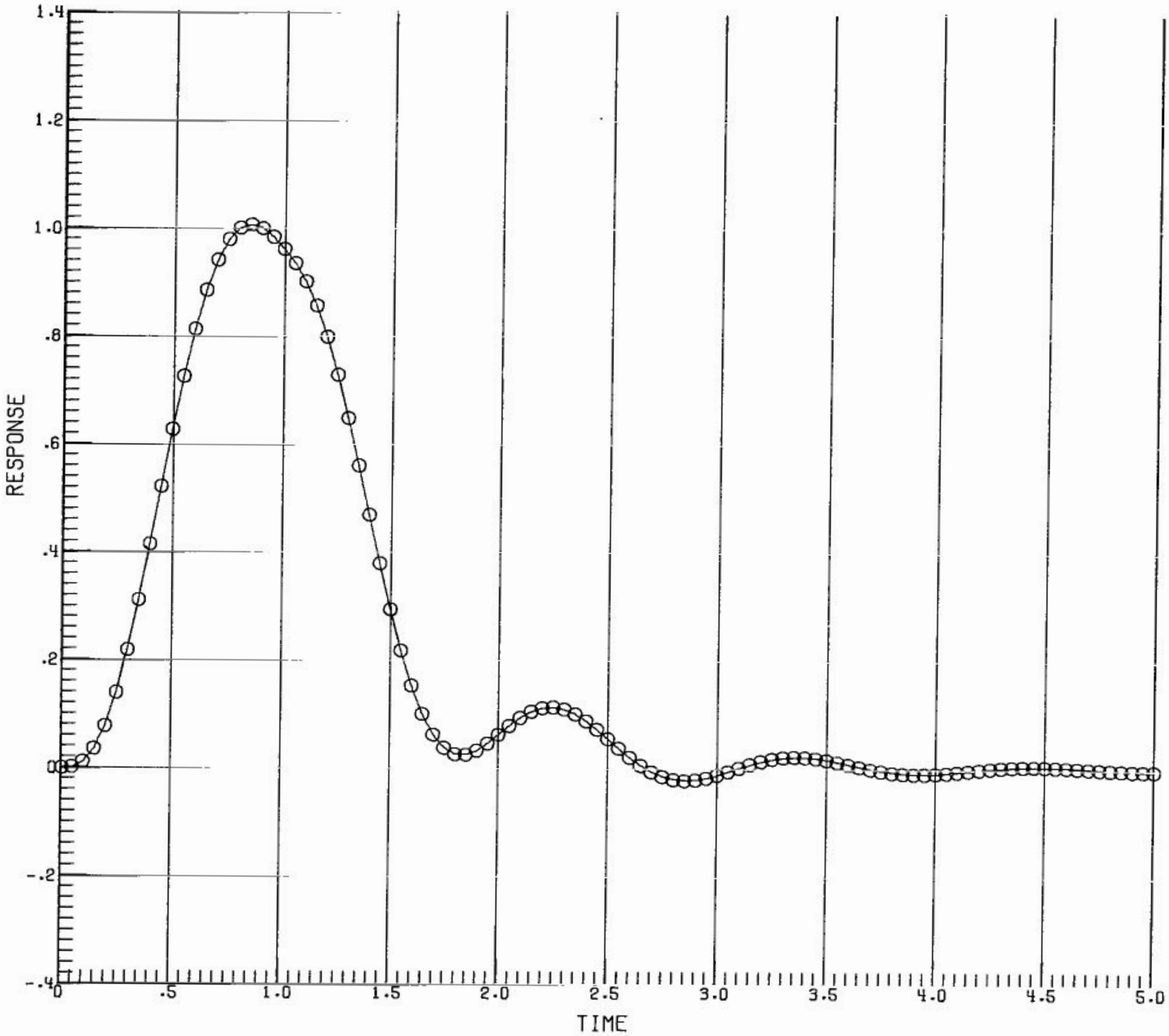

Figure 42.- Response of third-order Chebyshev (2-dB ripple) filter with BT = 1.

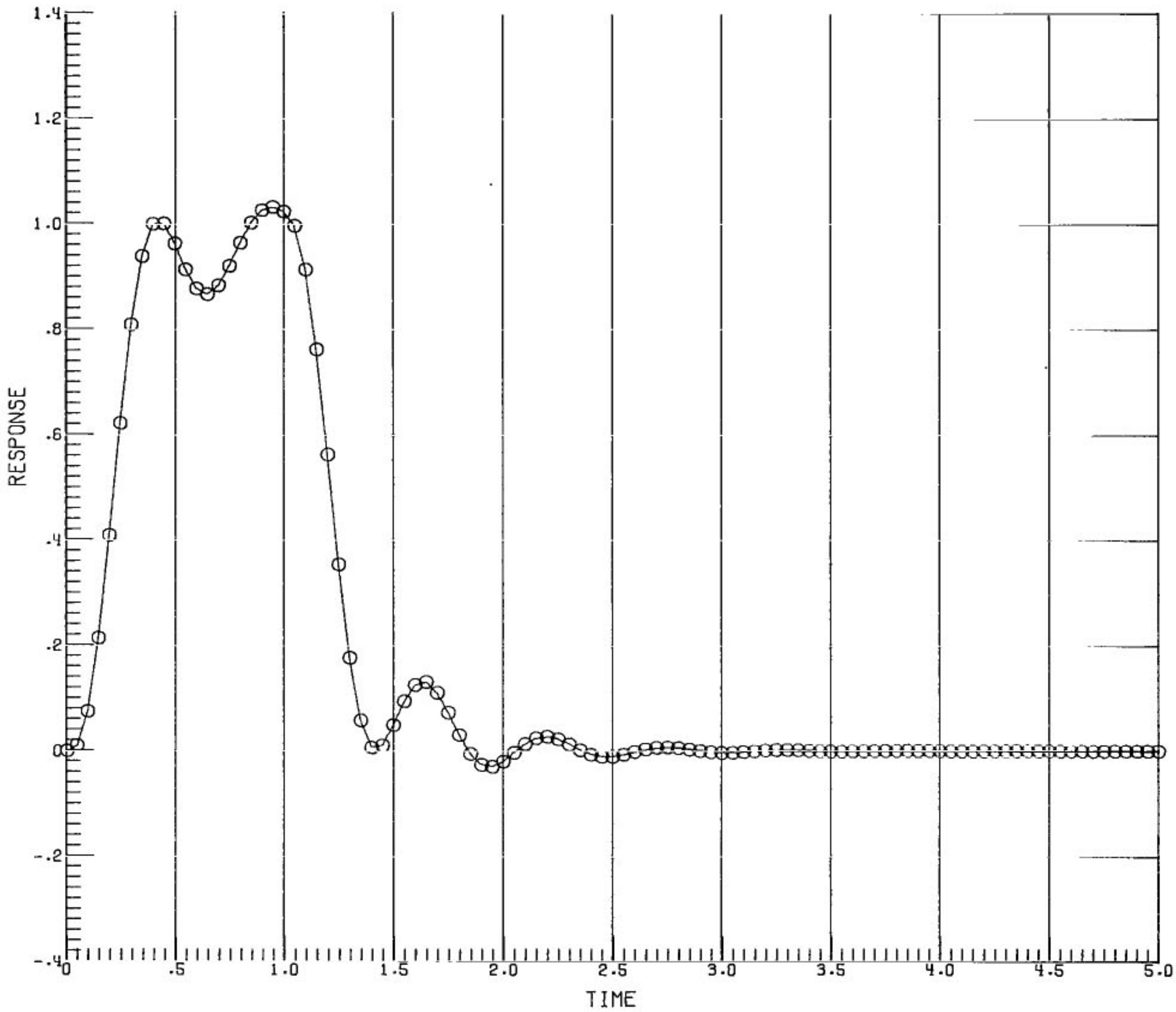

Figure 43.- Response of third-order Chebyshev (2-dB ripple) filter with BT = 2.

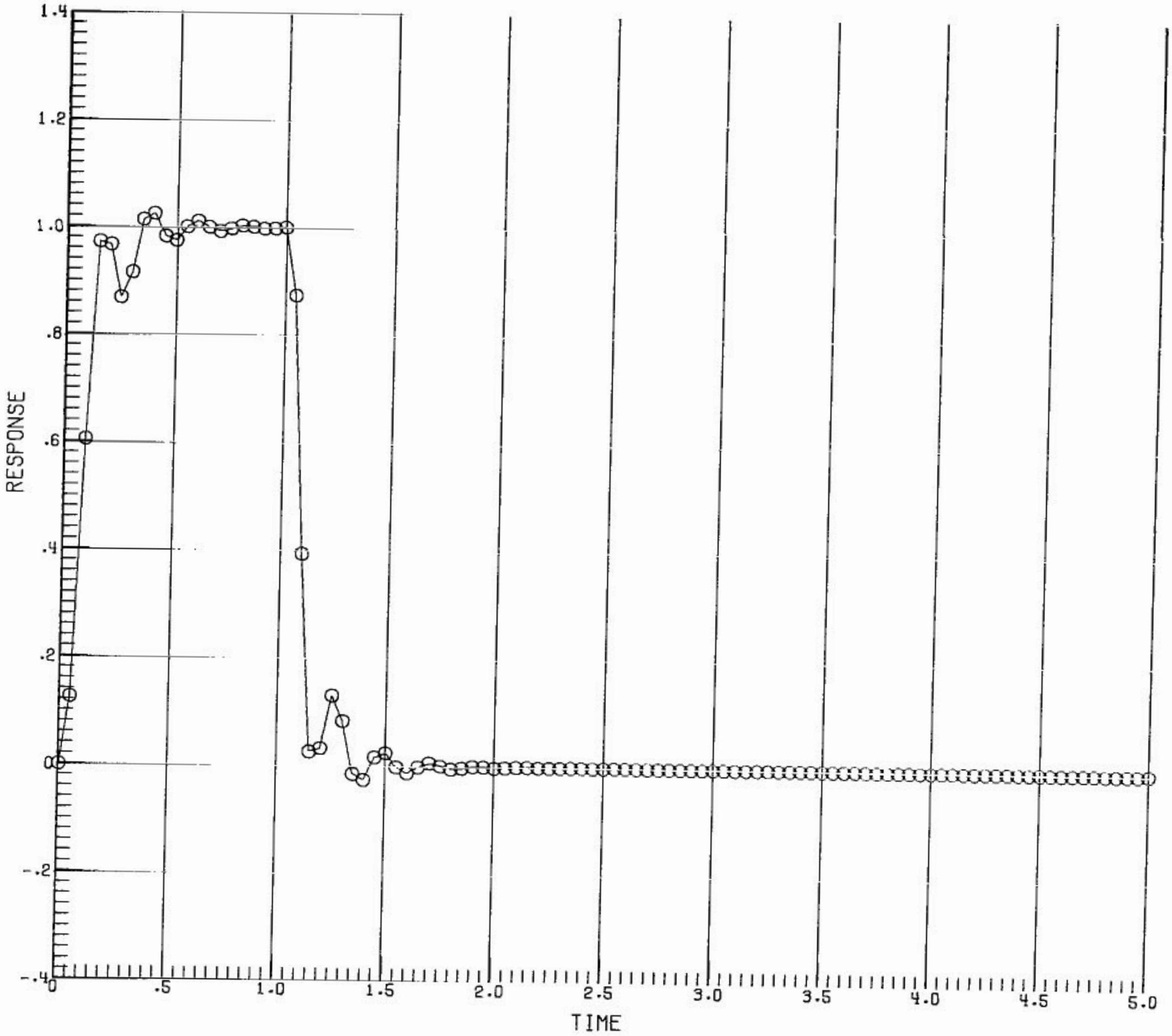

Figure 44.- Response of third-order Chebyshev (2-dB ripple) filter with BT = 5.

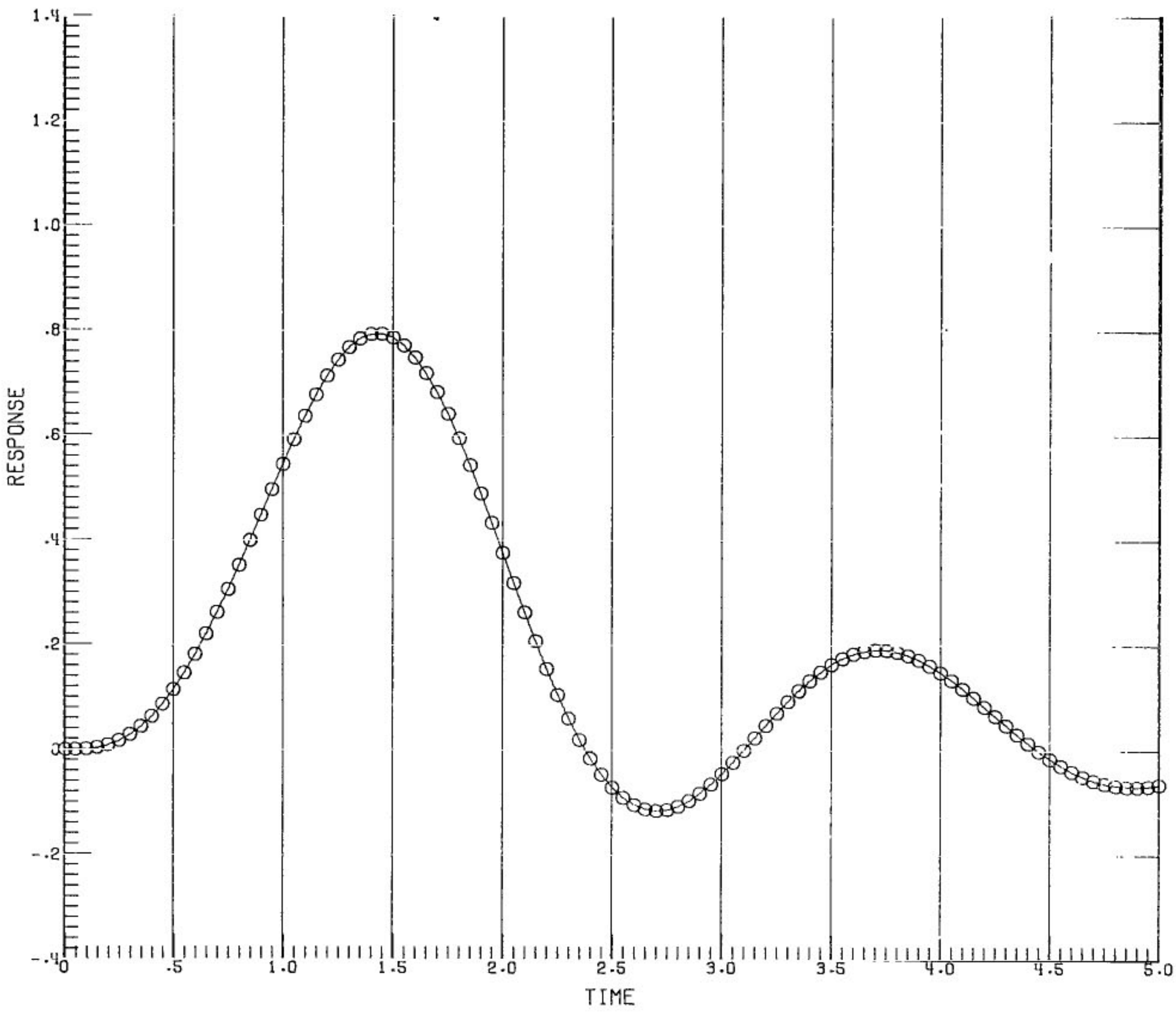

Figure 45.- Response of third-order Chebyshev (3-dB ripple) filter with BT = 0.5.

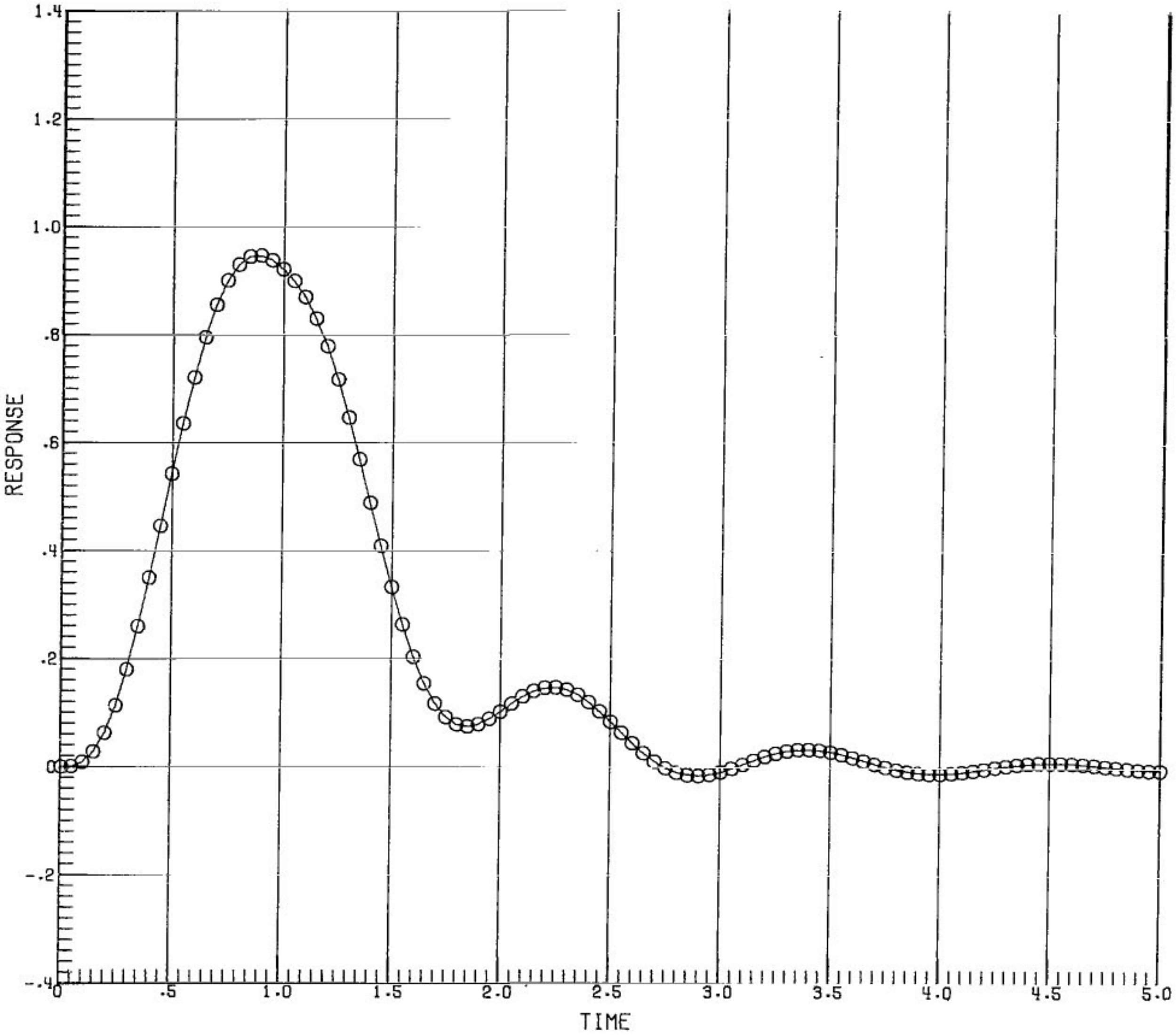

Figure 46.- Response of third-order Chebyshev (3-dB ripple) filter with BT = 1.

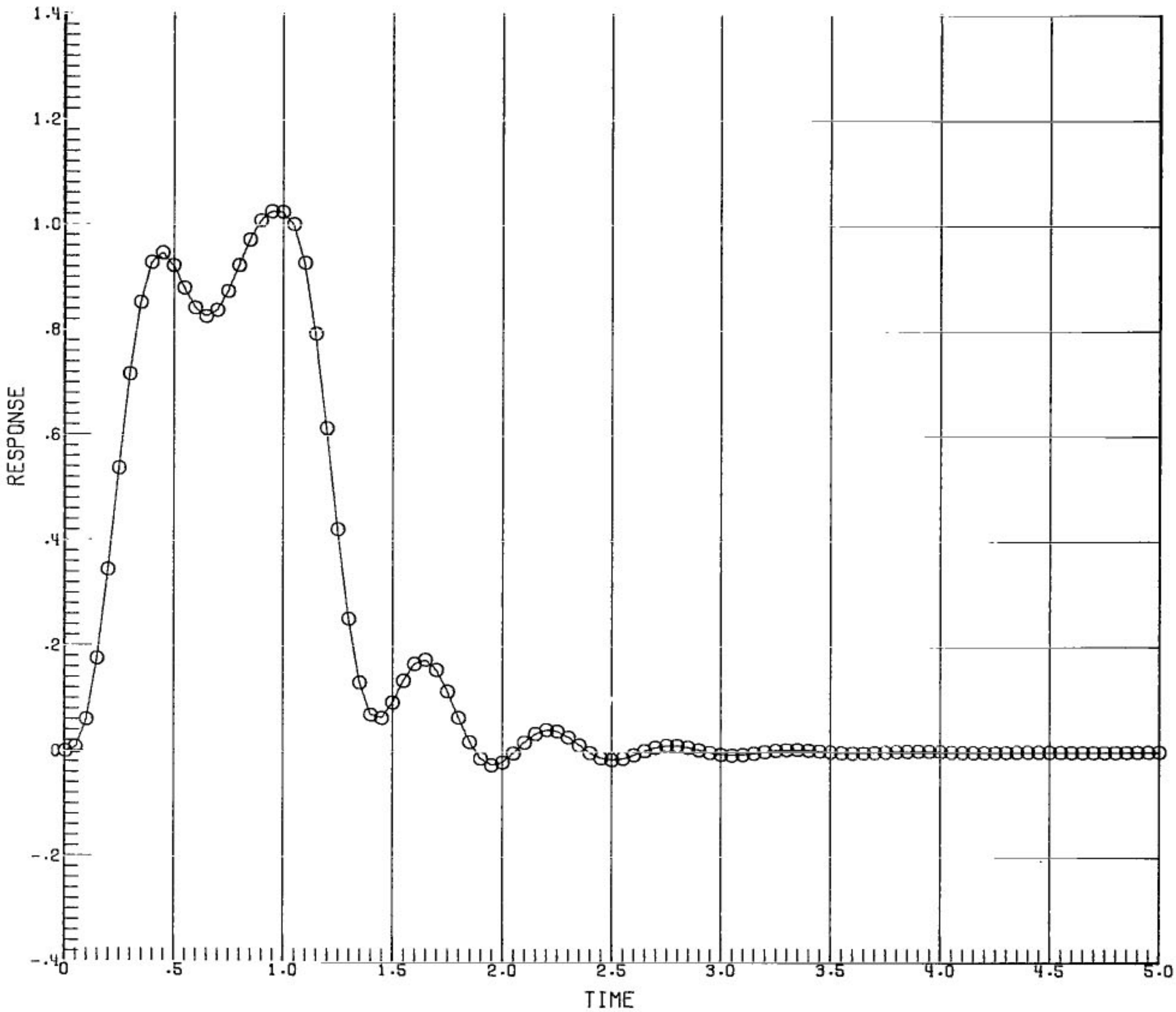

Figure 47.- Response of third-order Chebyshev (3-dB ripple) filter with BT = 2.

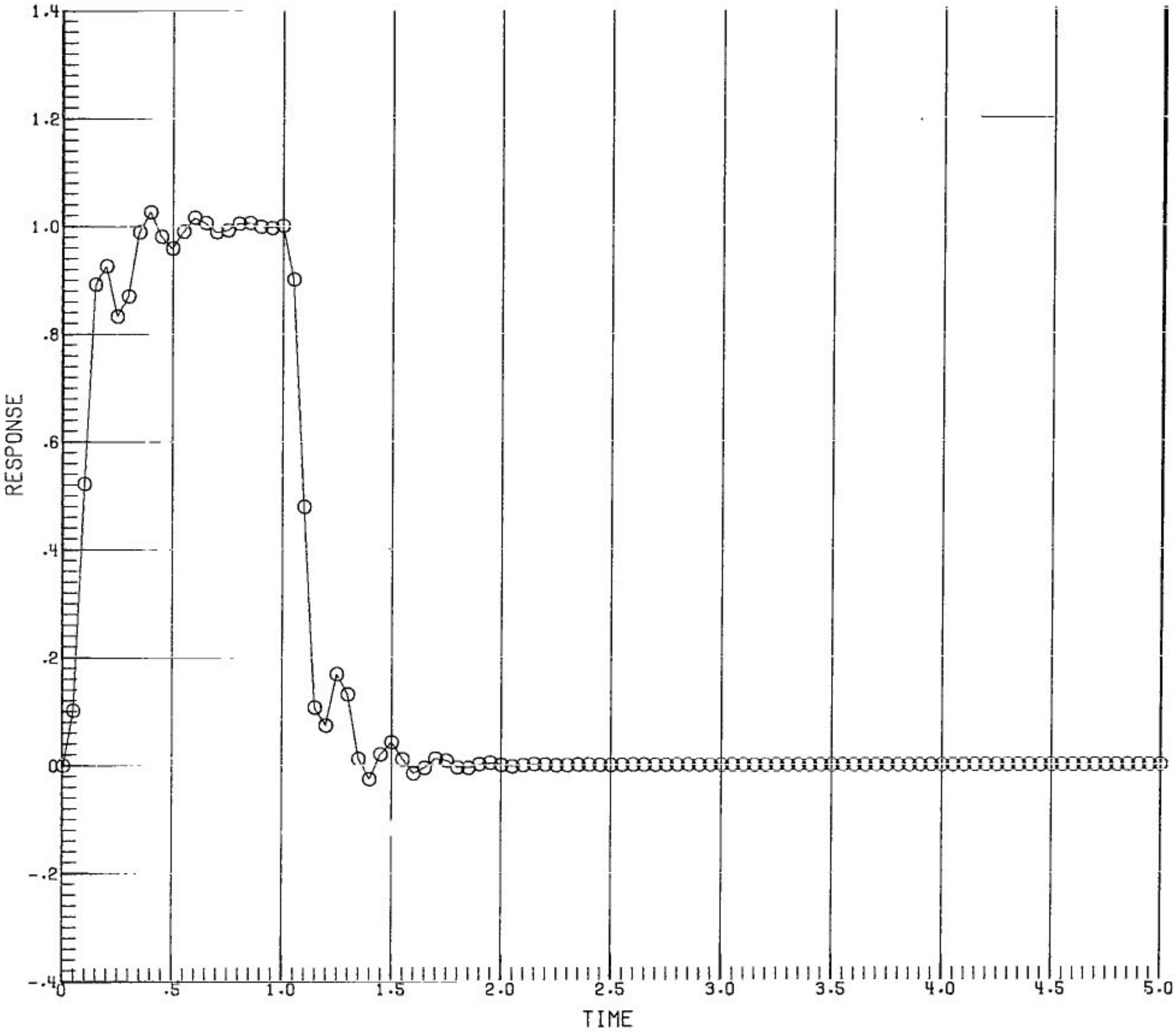

Figure 48.- Response of third-order Chebyshev (3-dB ripple) filter with BT = 5.

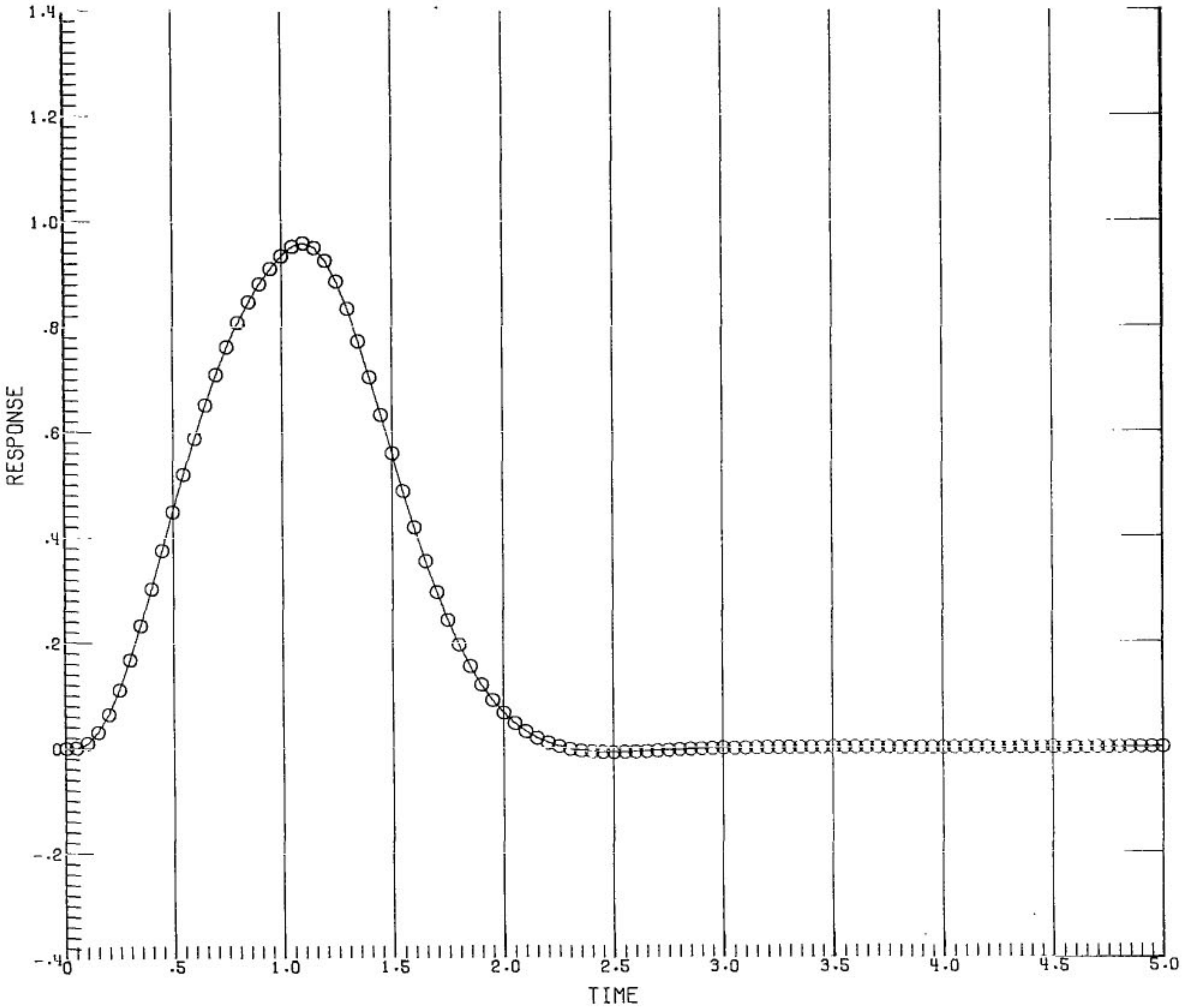

Figure 49.- Response of third-order Bessel filter with BT = 0.5.

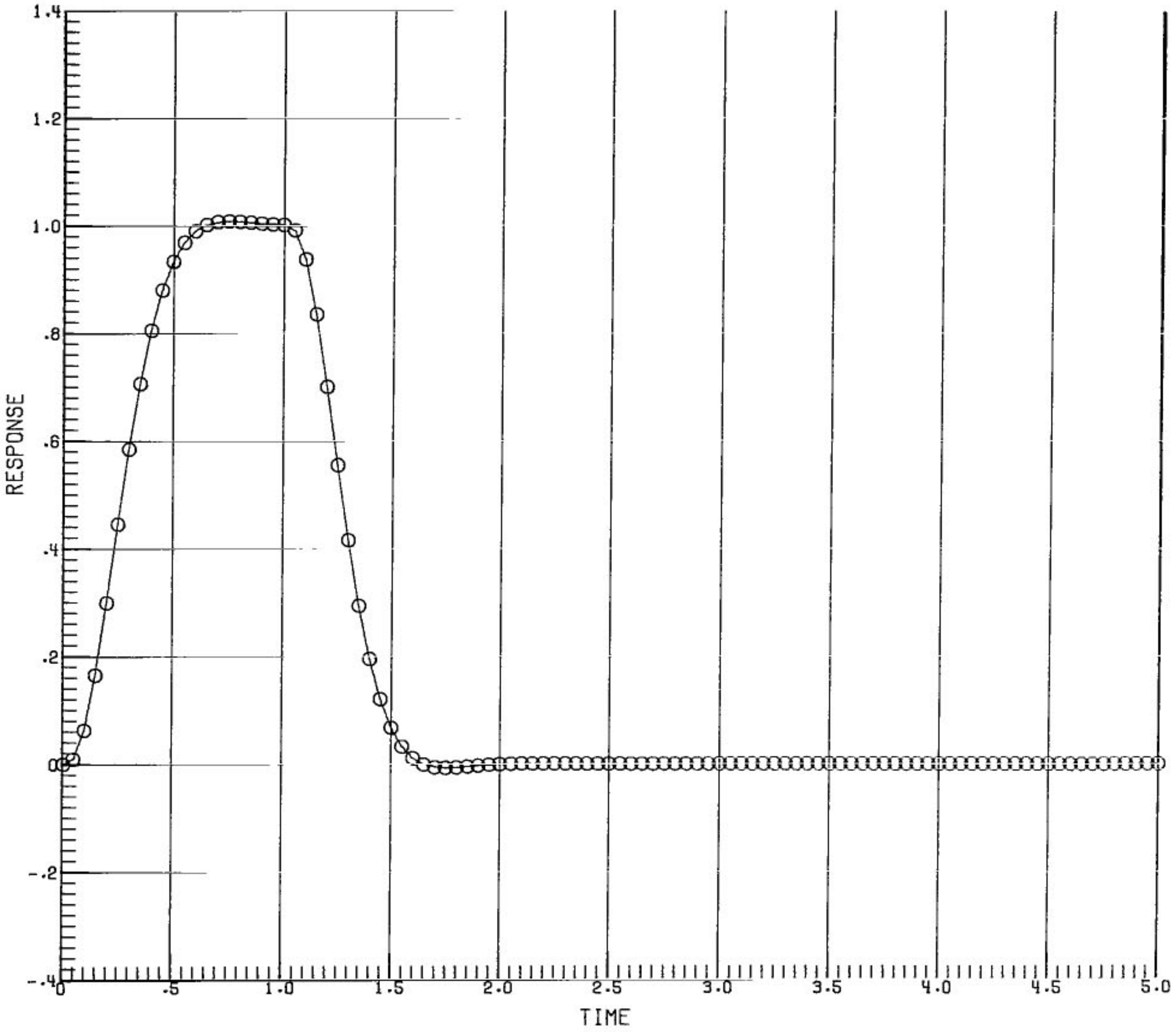

Figure 50.- Response of third-order Bessel filter with BT = 1.

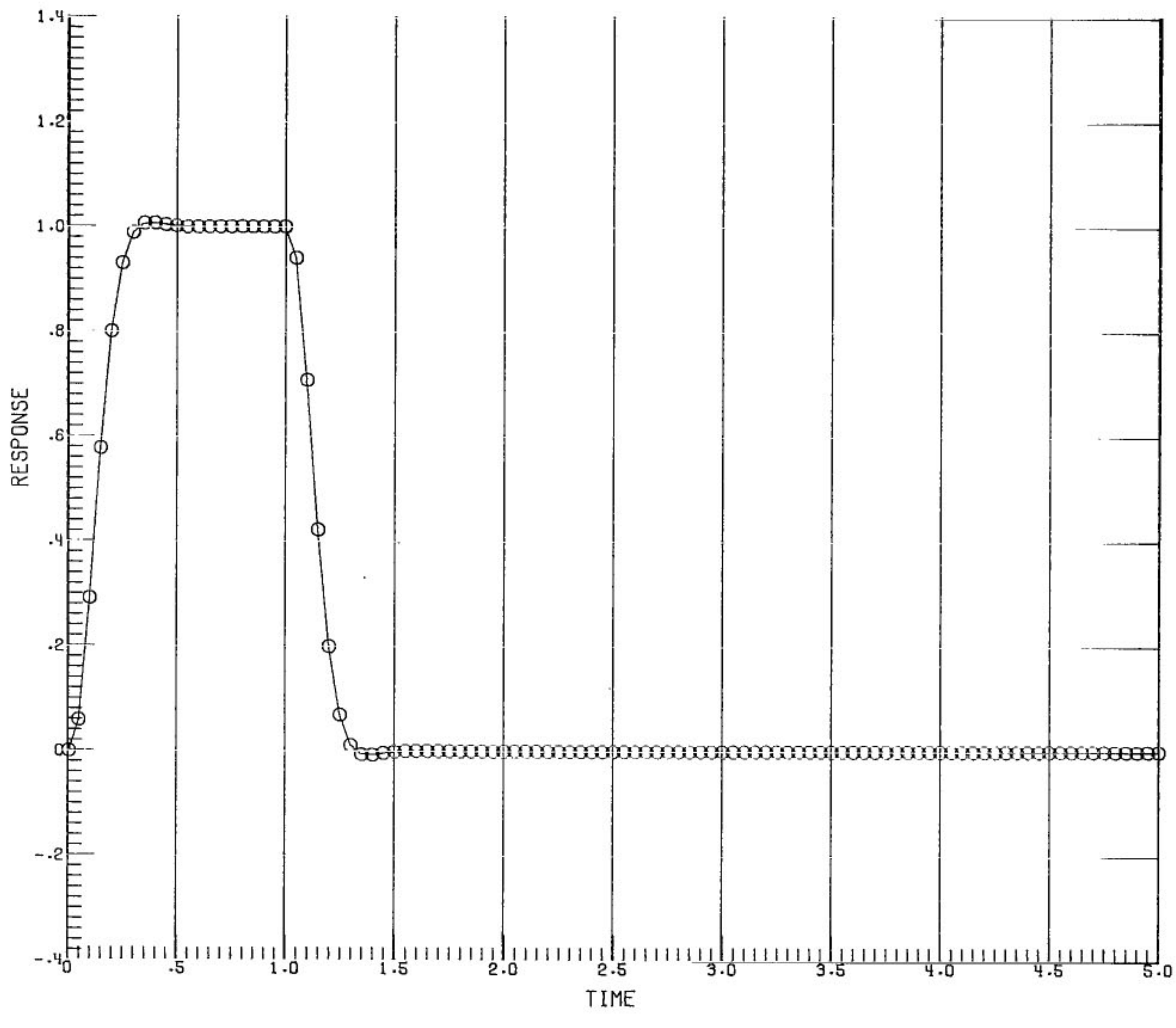

Figure 51.- Response of third-order Bessel filter with BT = 2.

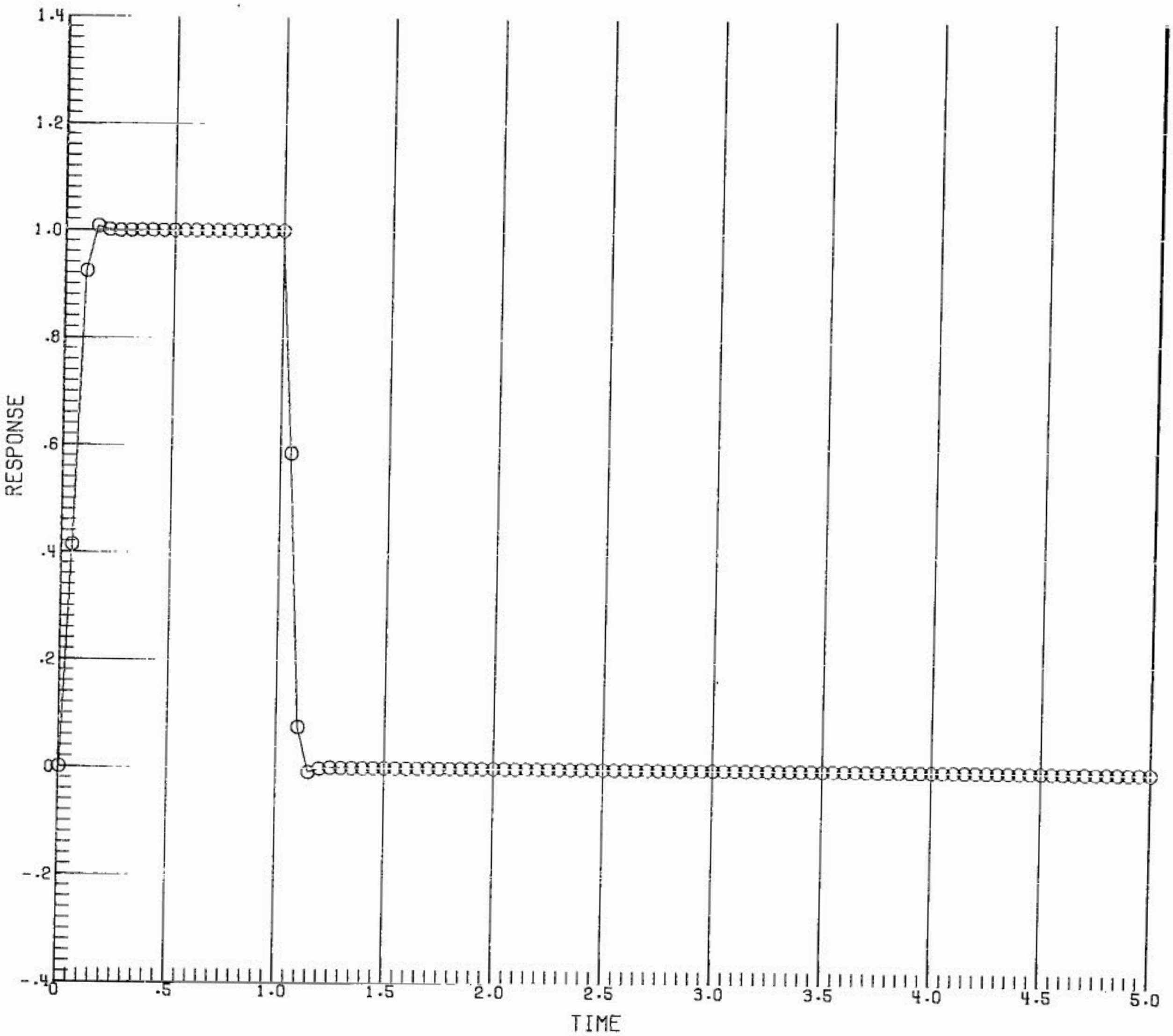

Figure 52.- Response of third-order Bessel filter with BT = 5.

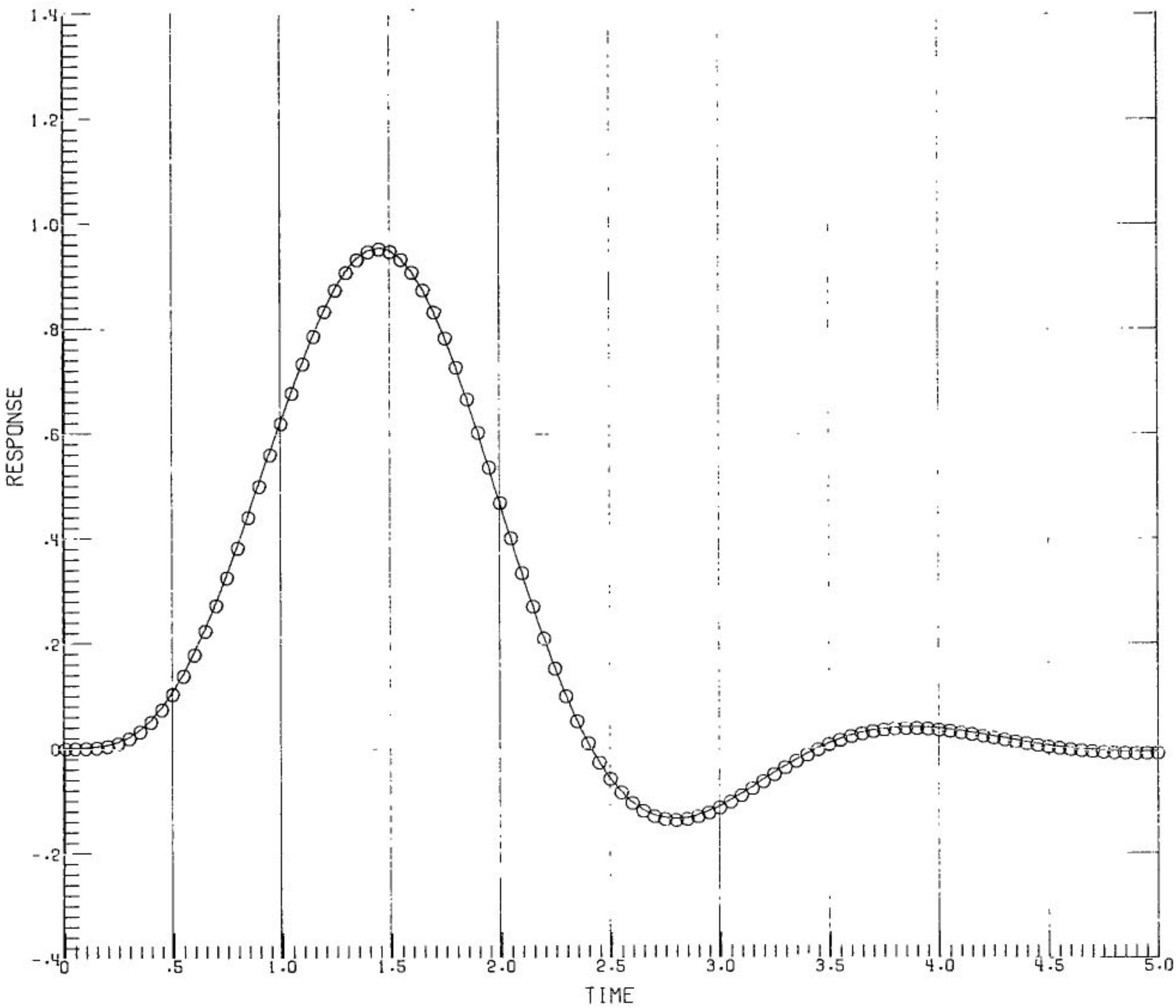

Figure 53.- Response of fourth-order Butterworth filter with BT = 0.5.

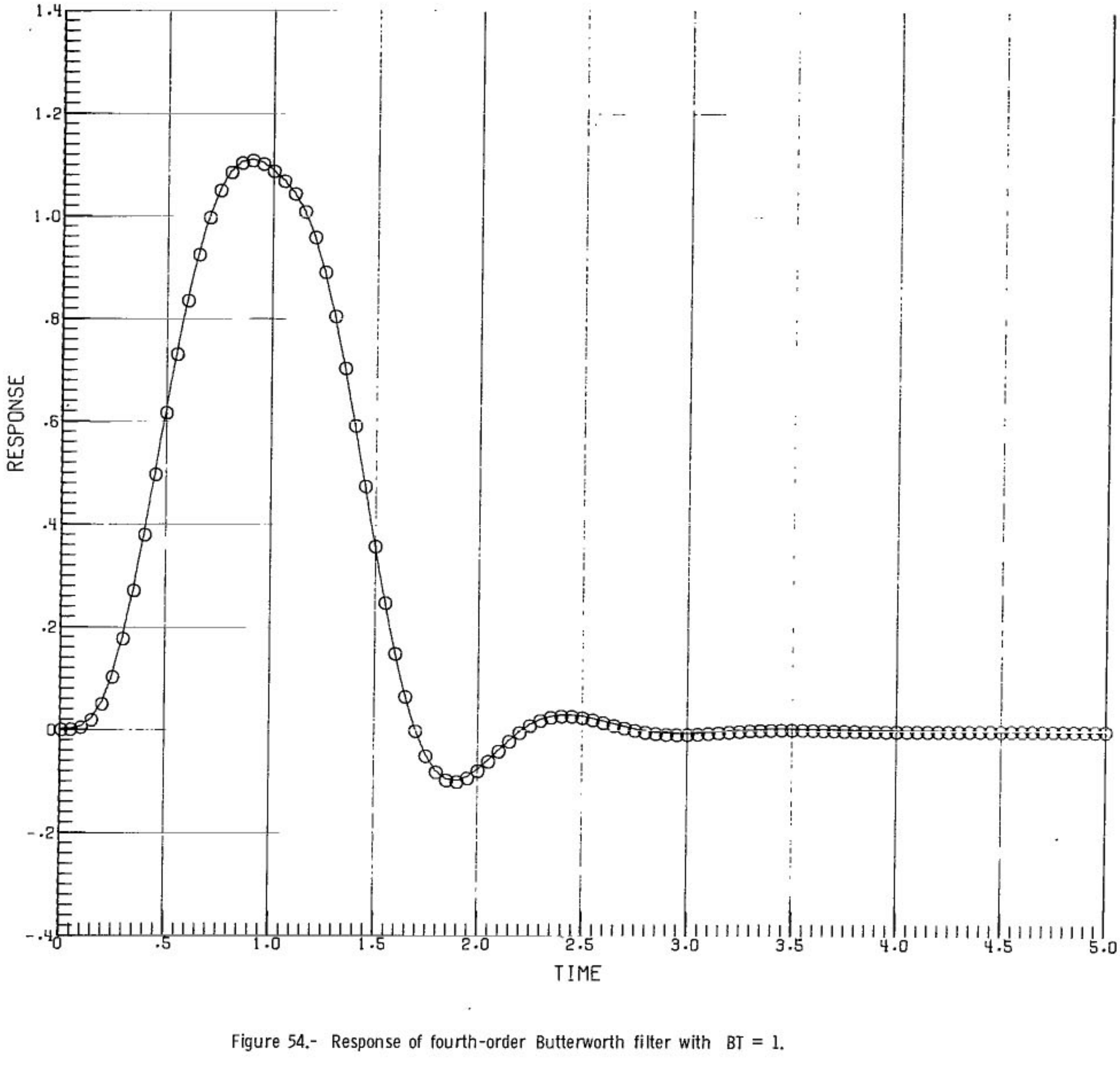

Figure 54.- Response of fourth-order Butterworth filter with BT = 1.

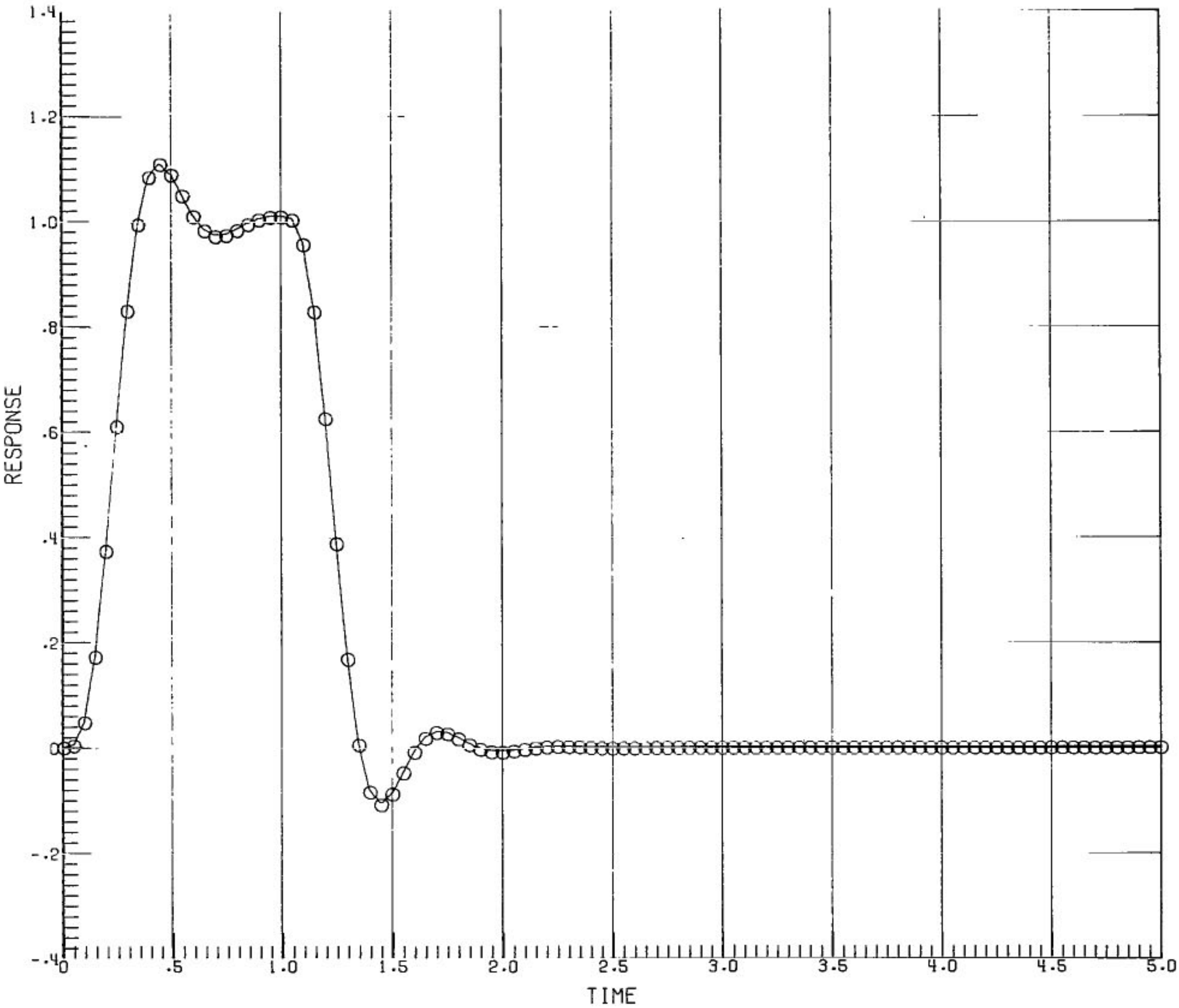

Figure 55.- Response of fourth-order Butterworth filter with BT = 2.

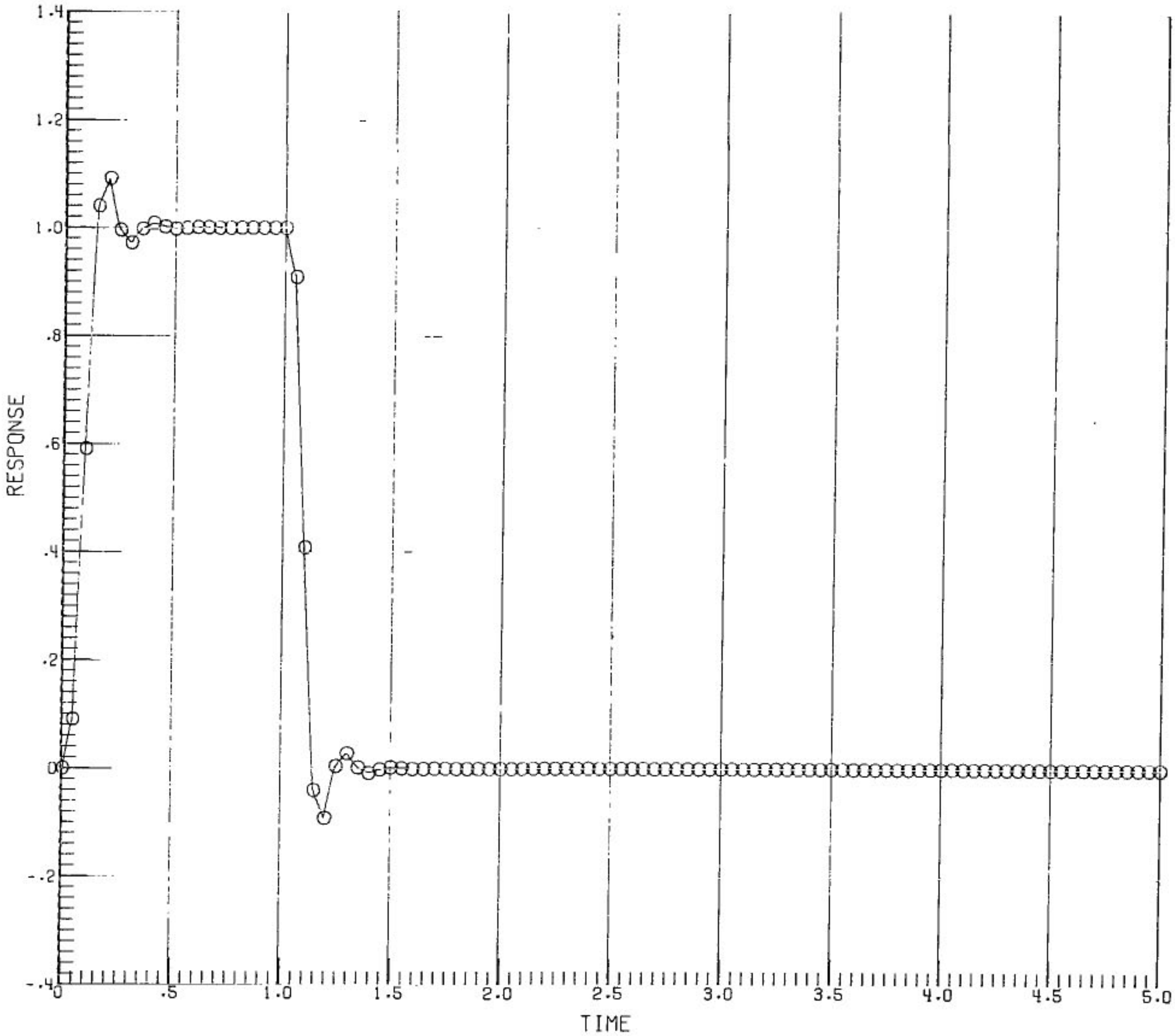

Figure 56.- Response of fourth-order Butterworth filter with BT = 5.

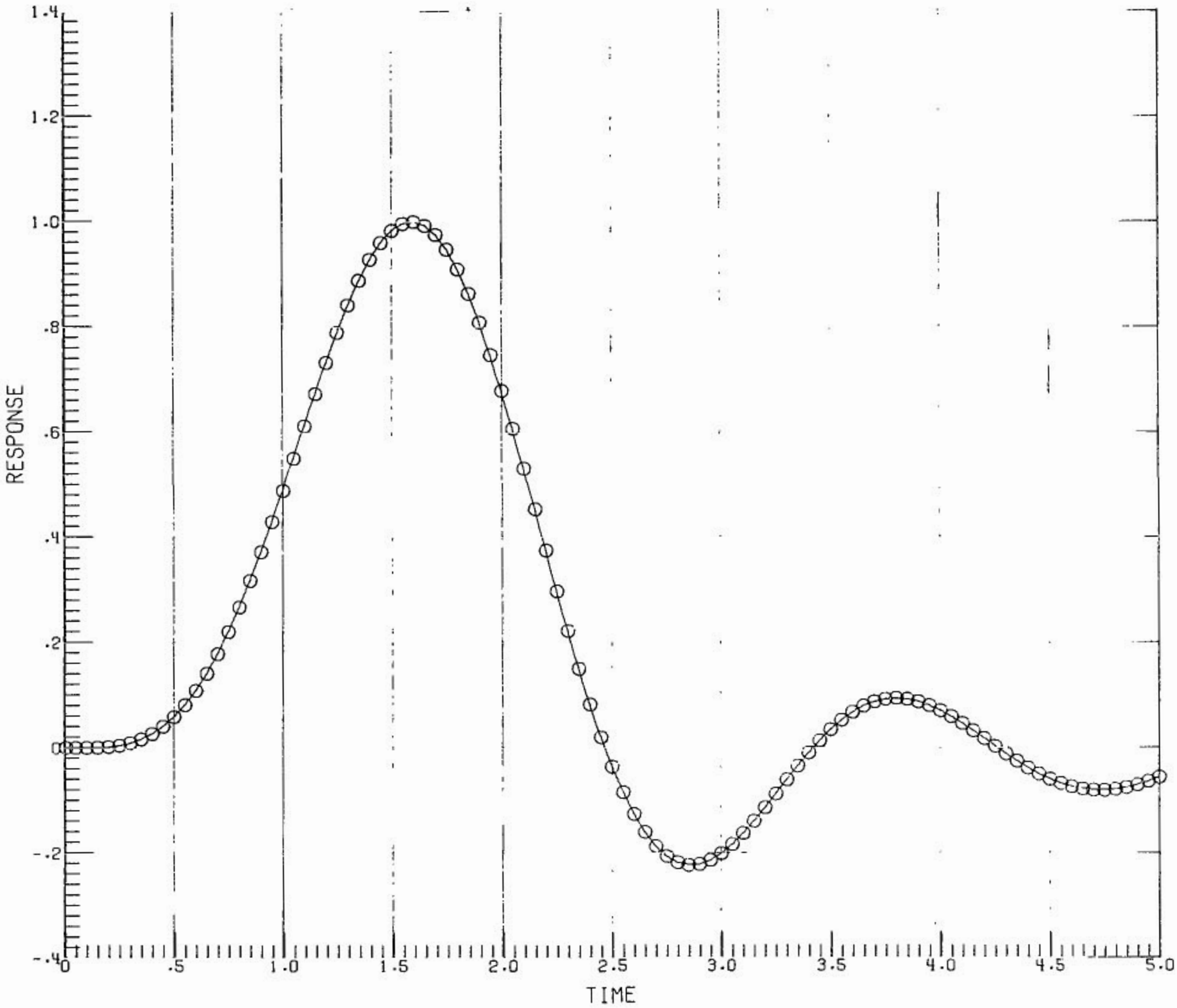

Figure 57.- Response of fourth-order Chebyshev (0.5-dB ripple) filter with BT = 0.5.

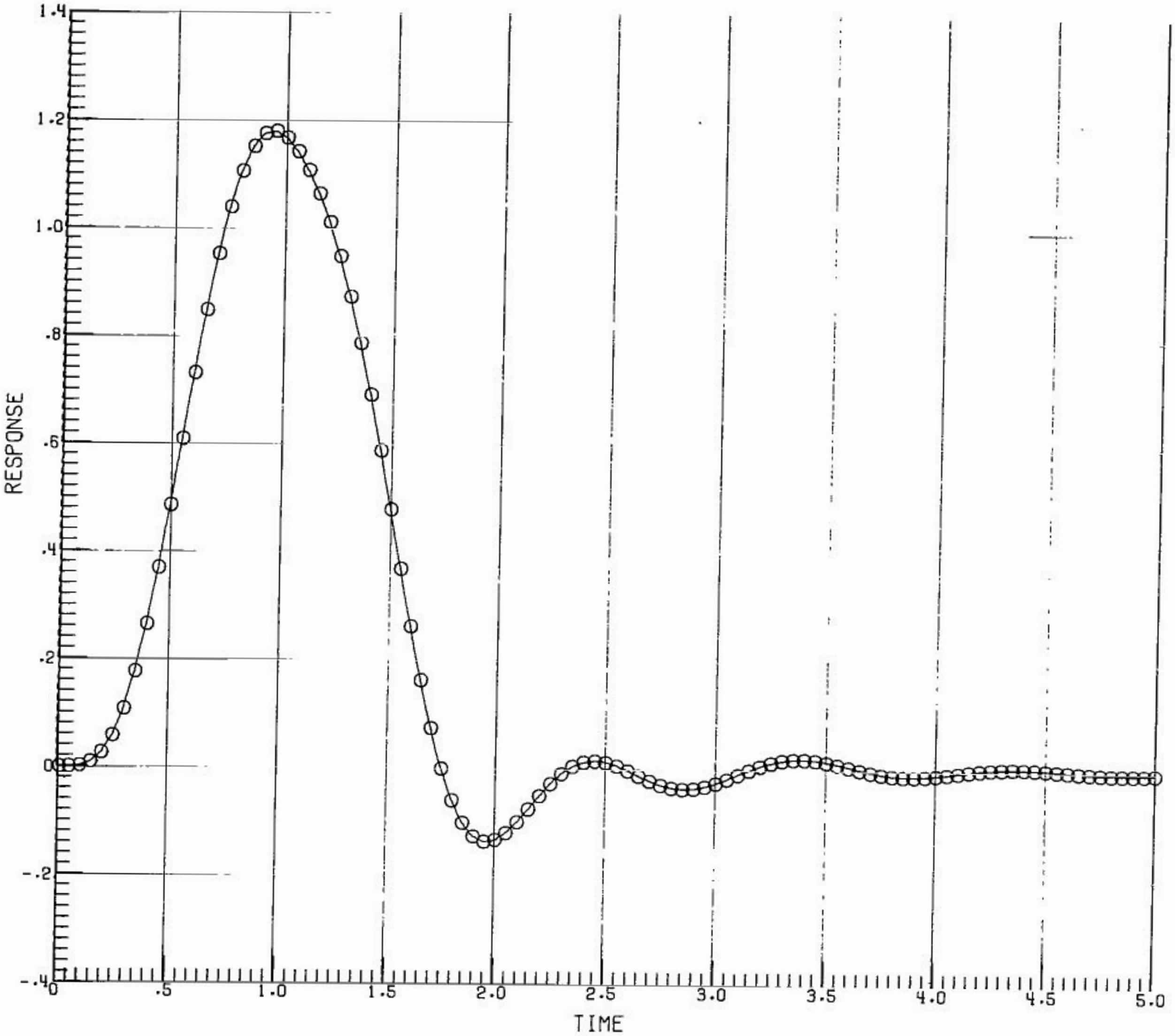

Figure 58.- Response of fourth-order Chebyshev (0.5-dB ripple) filter with BT = 1.

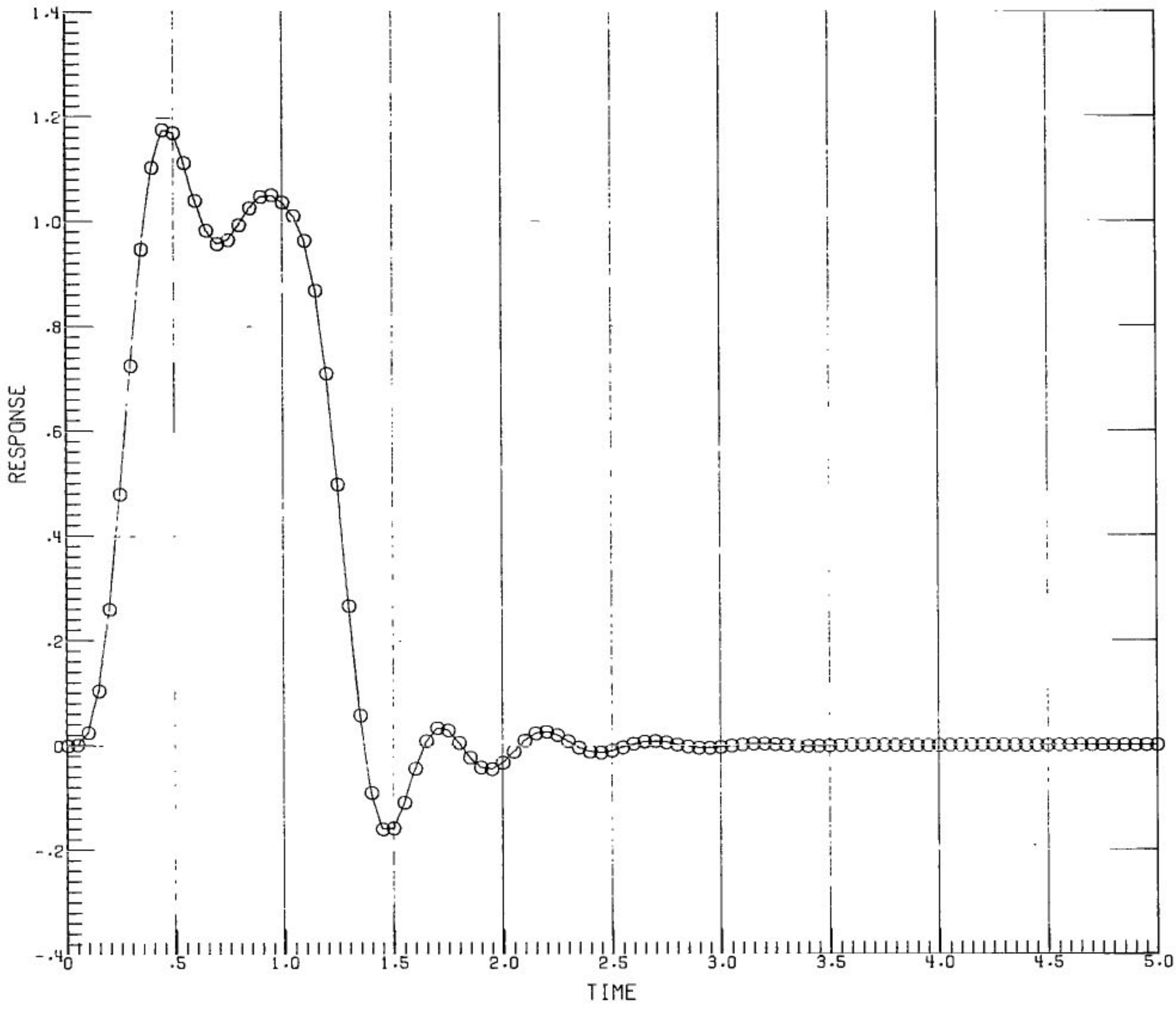

Figure 59.- Response of fourth-order Chebyshev (0.5-dB ripple) filter with BT = 2.

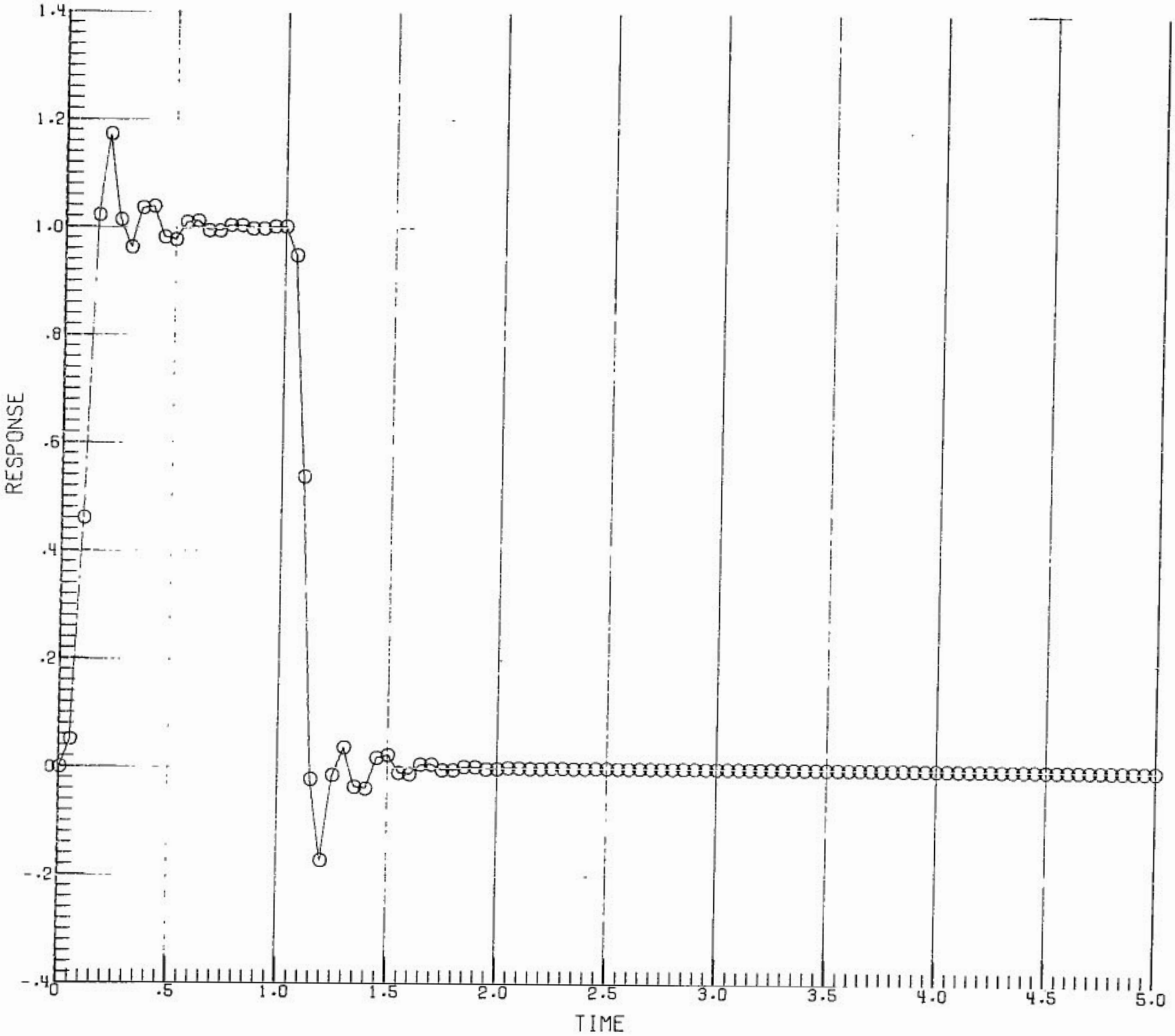

Figure 60.- Response of fourth-order Chebyshev (0.5-dB ripple) filter with BT = 5.

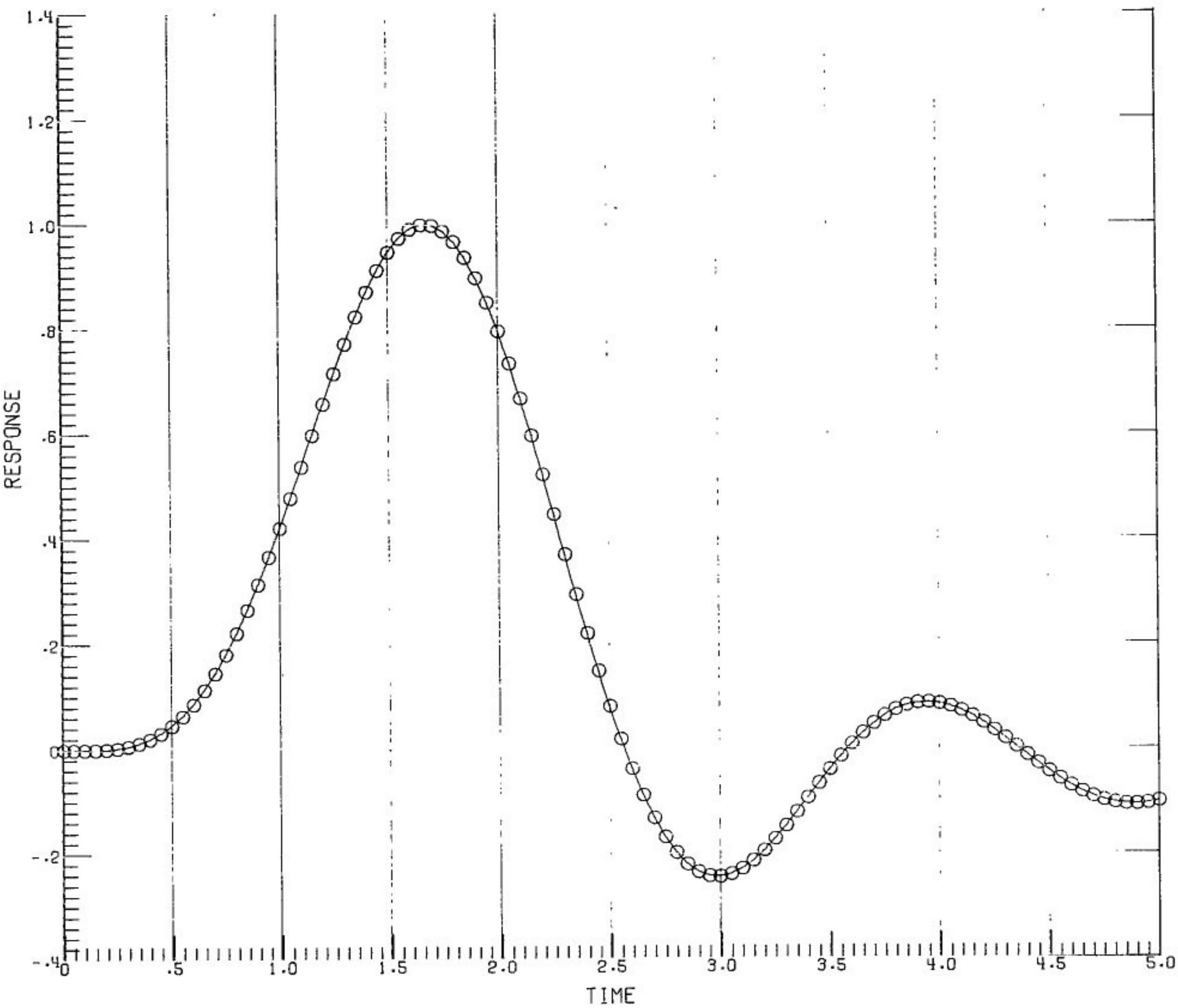

Figure 61.- Response of fourth-order Chebyshev (1-dB ripple) filter with BT = 0.5.

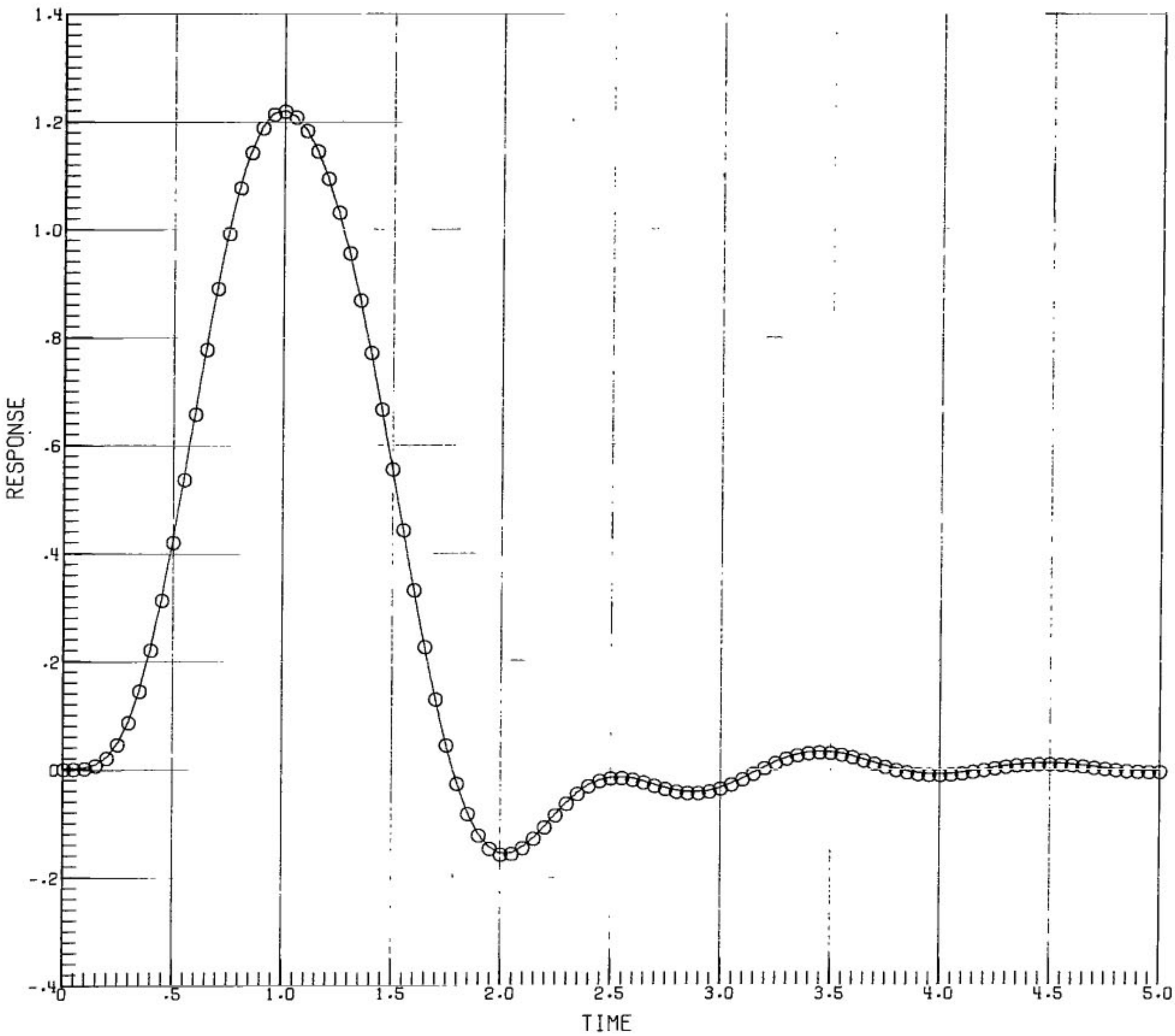

Figure 62.- Response of fourth-order Chebyshev (1-dB ripple) filter with BT = 1.

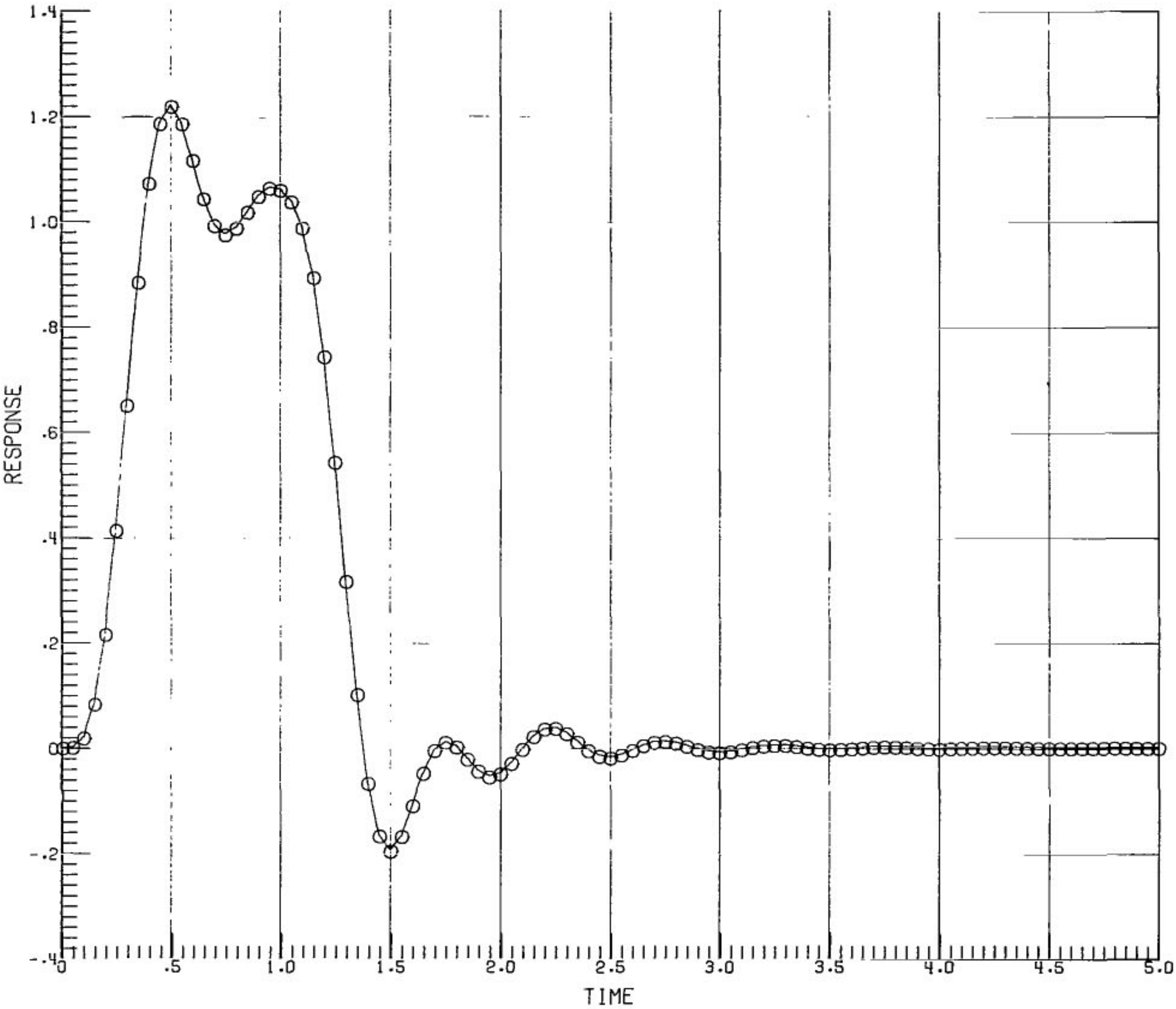

Figure 63.- Response of fourth-order Chebyshev (1-dB ripple) filter with BT = 2.

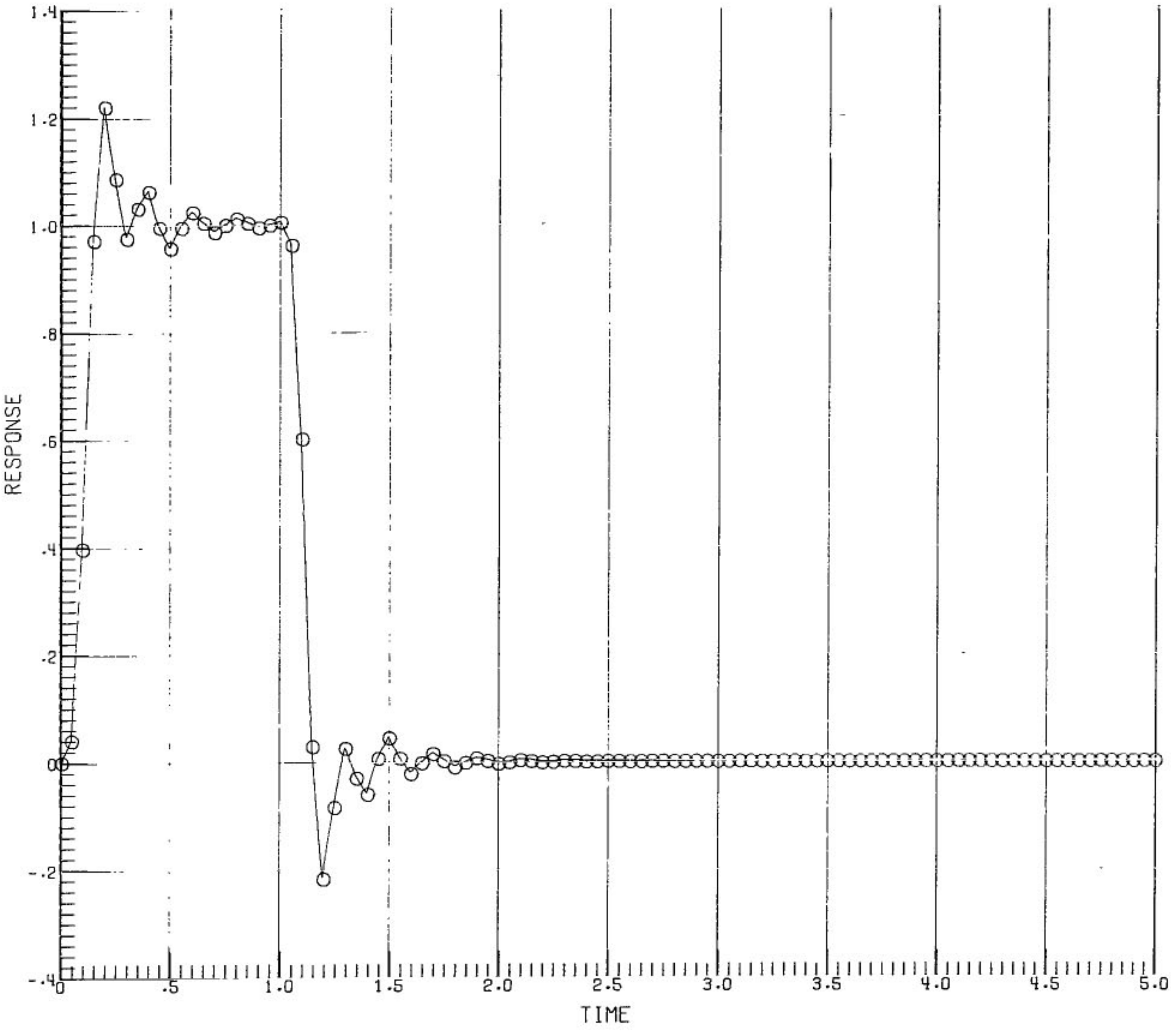

Figure 64.- Reponse of fourth-order Chebyshev (1-dB ripple) filter with BT = 5.

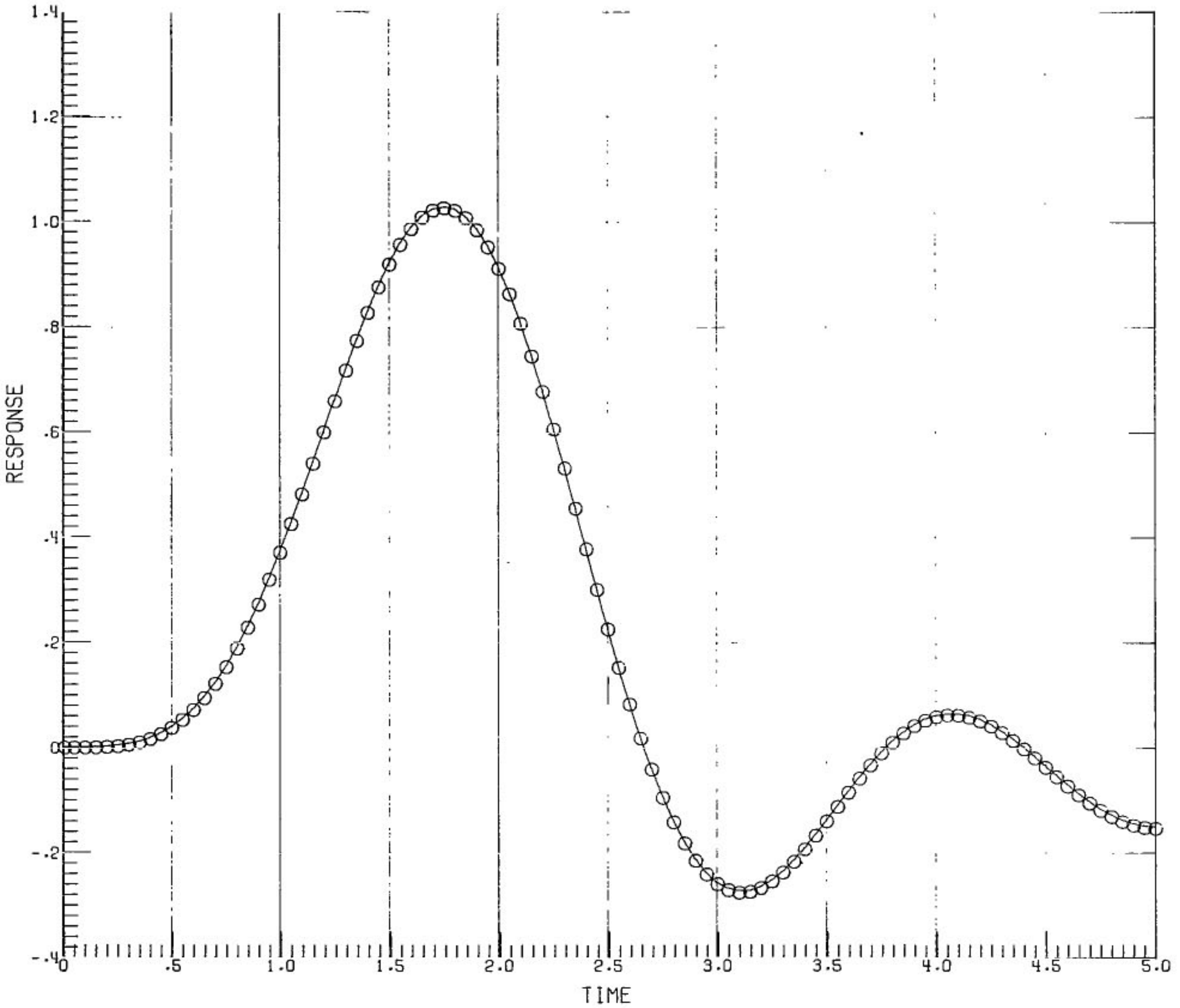

Figure 65.- Response of fourth-order Chebyshev (2-dB ripple) filter with BT = 0.5.

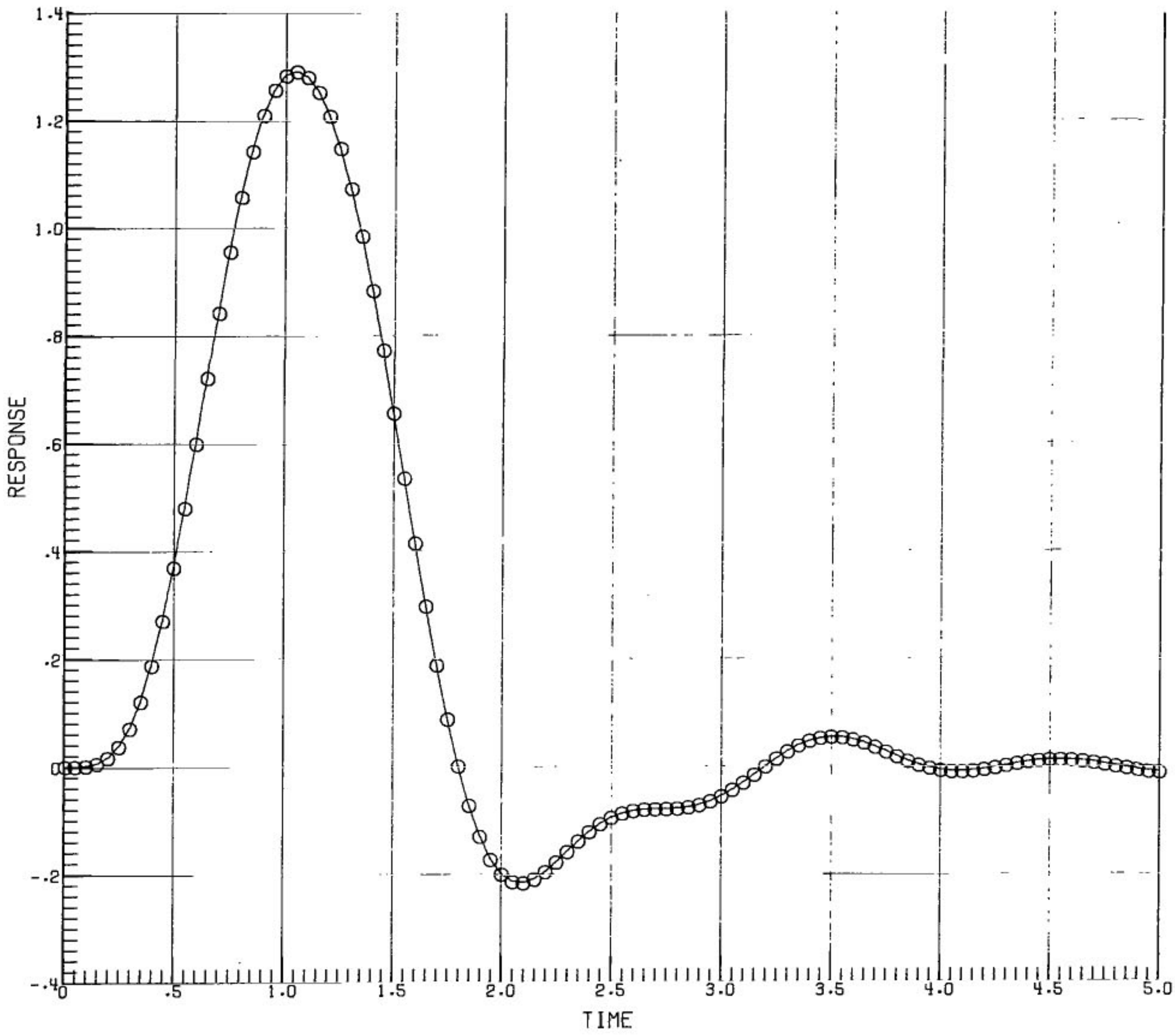

Figure 66.- Response of fourth-order Chebyshev (2-dB ripple) filter with BT = 1.

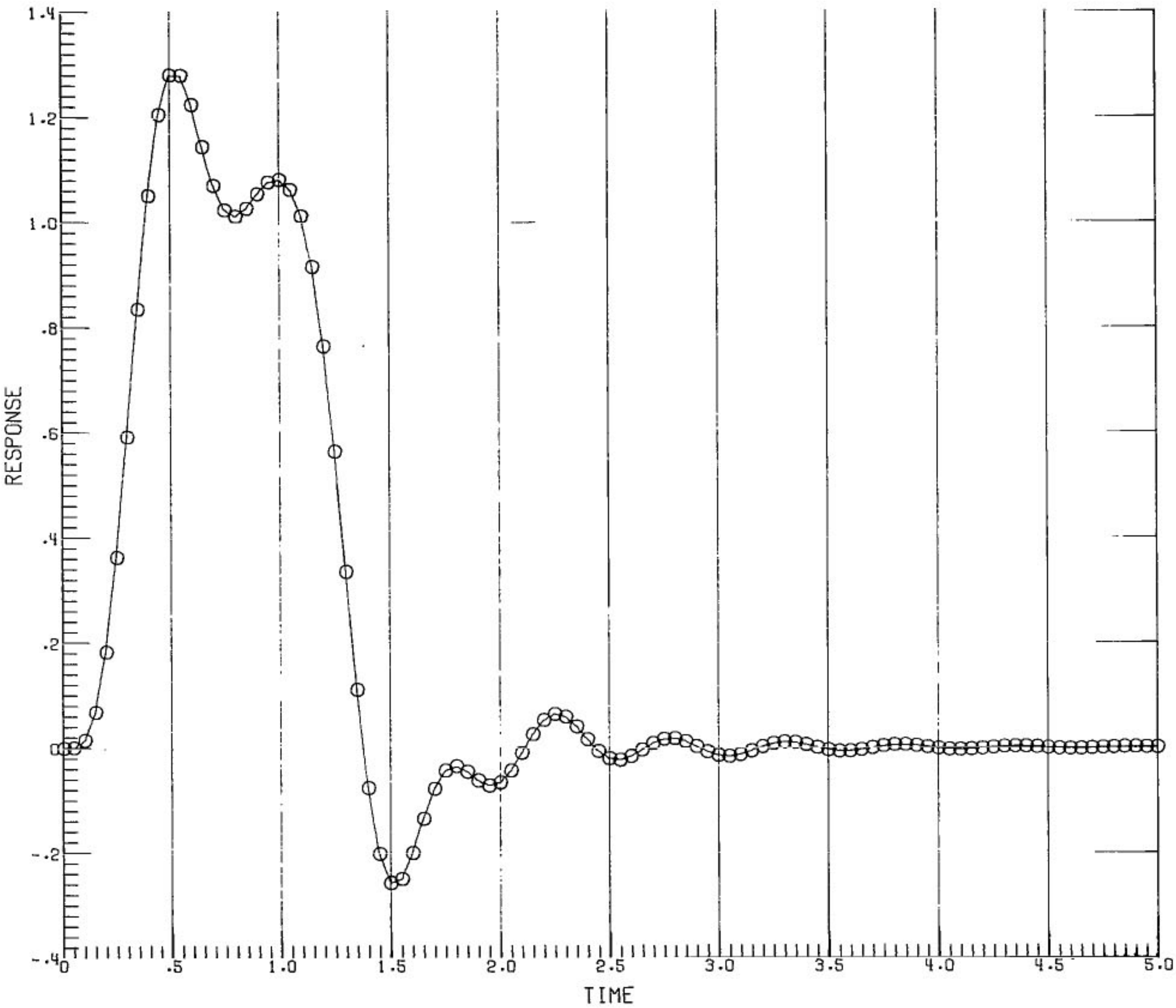

Figure 67.- Response of fourth-order Chebyshev (2-dB ripple) filter with BT = 2.

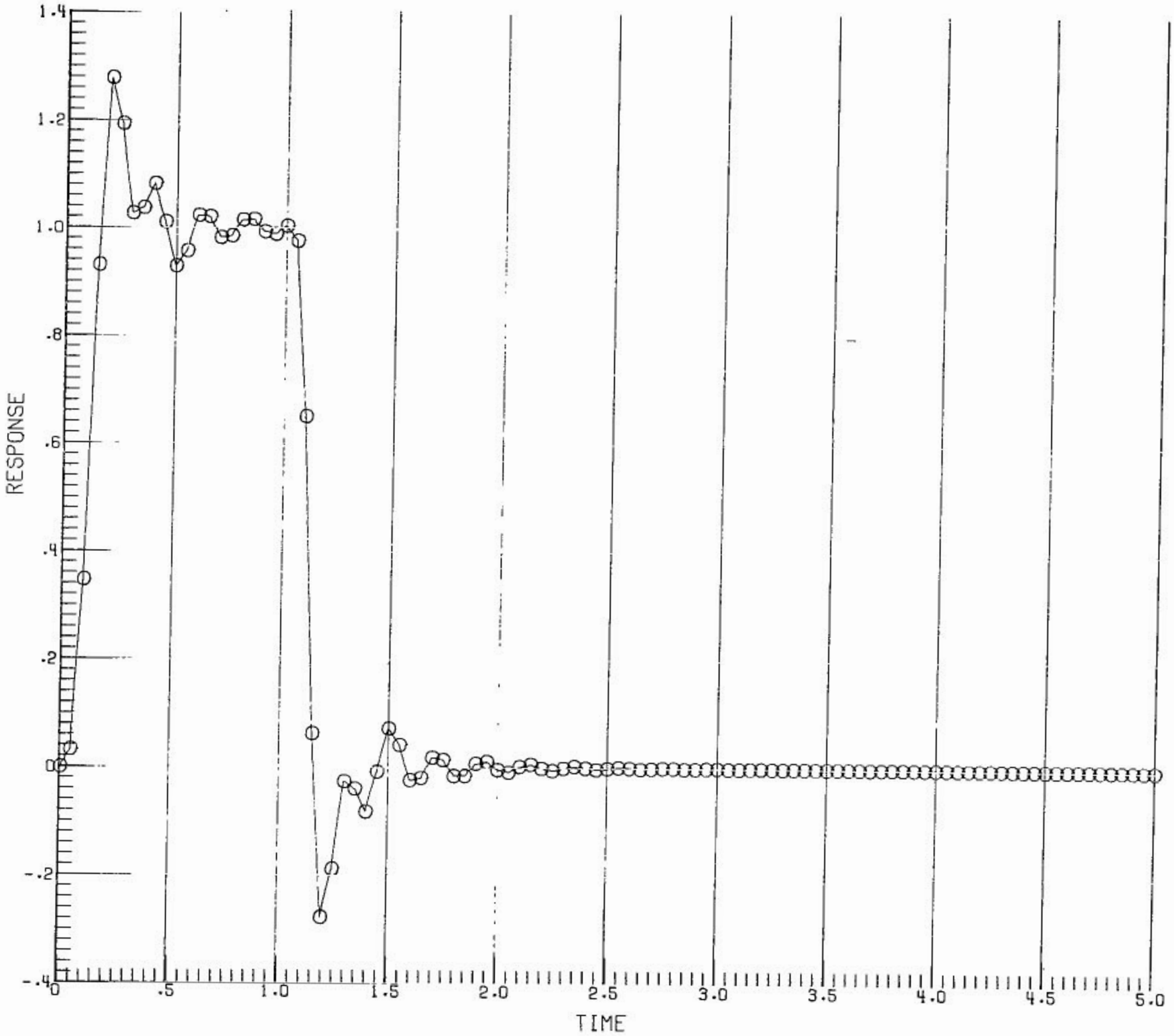

Figure 68.- Response of fourth-order Chebyshev (2-dB ripple) filter with BT = 5.

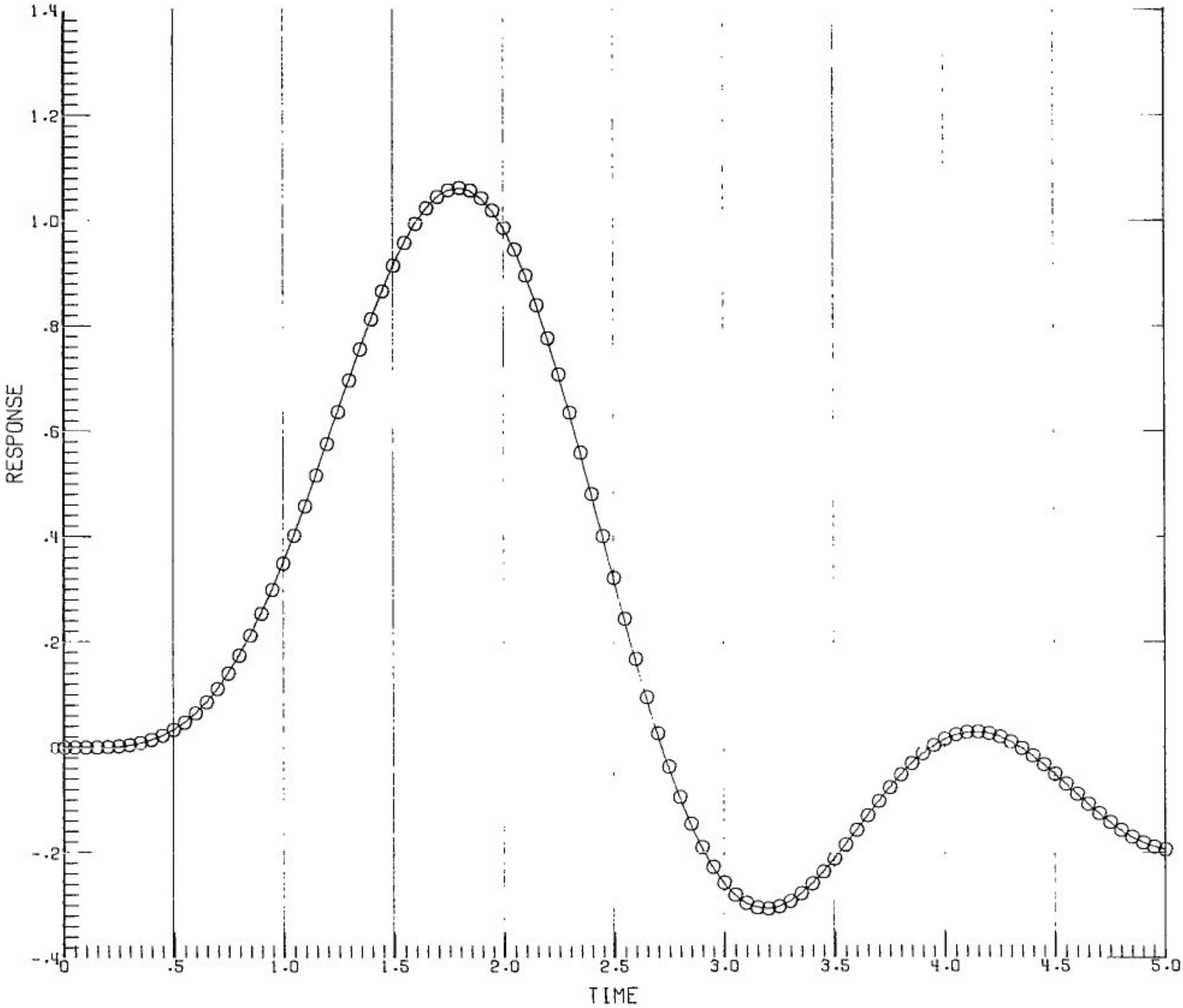

Figure 69.- Response of fourth-order Chebyshev (3-dB ripple) filter with BT = 0.5.

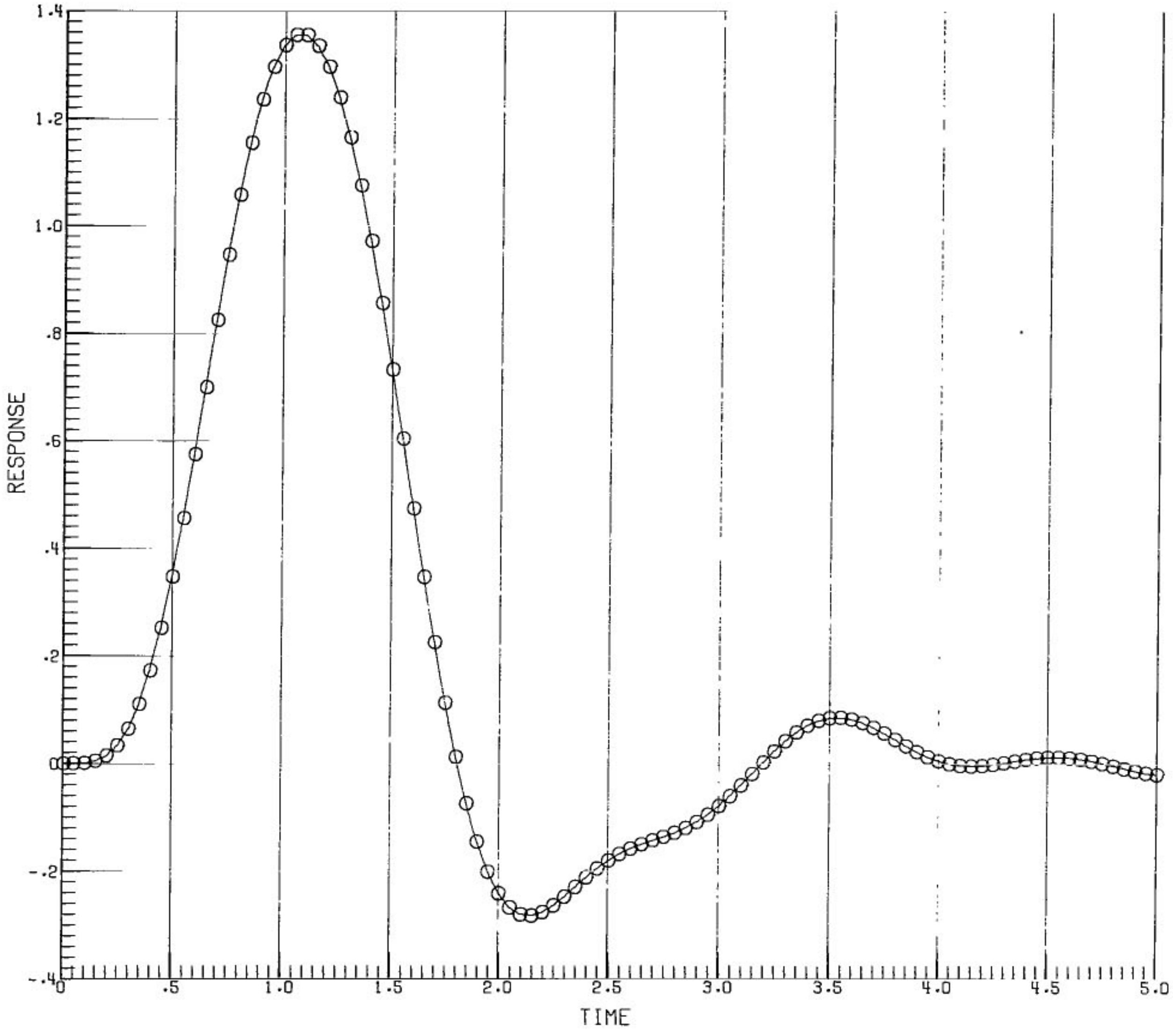

Figure 70.- Response of fourth-order Chebyshev (3-dB ripple) filter with $BT = 1$.

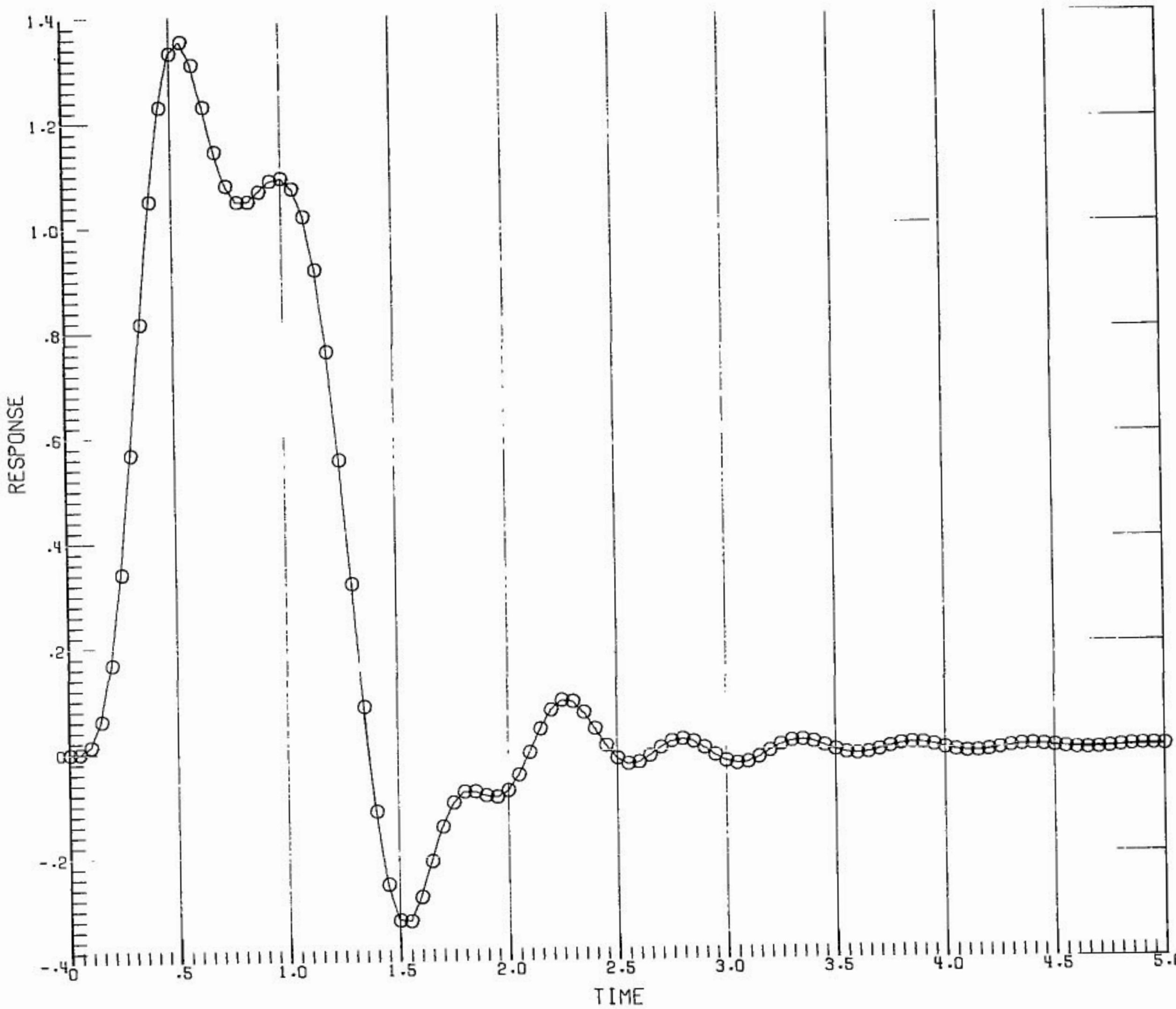

Figure 71.- Response of fourth-order Chebyshev (3-dB ripple) filter with BT = 2.

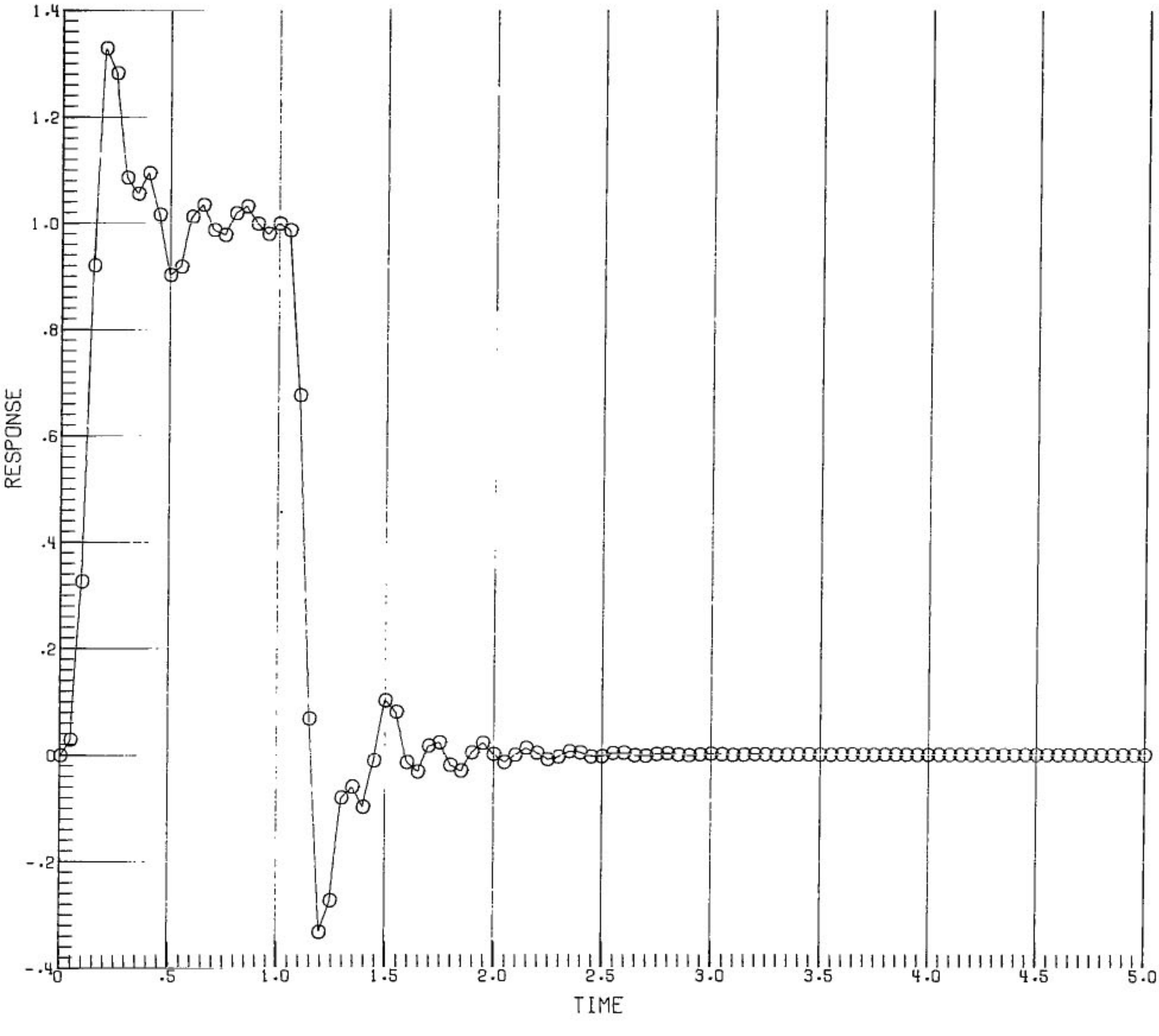

Figure 72.- Response of fourth-order Chebyshev (3-dB ripple) filter with BT = 5.

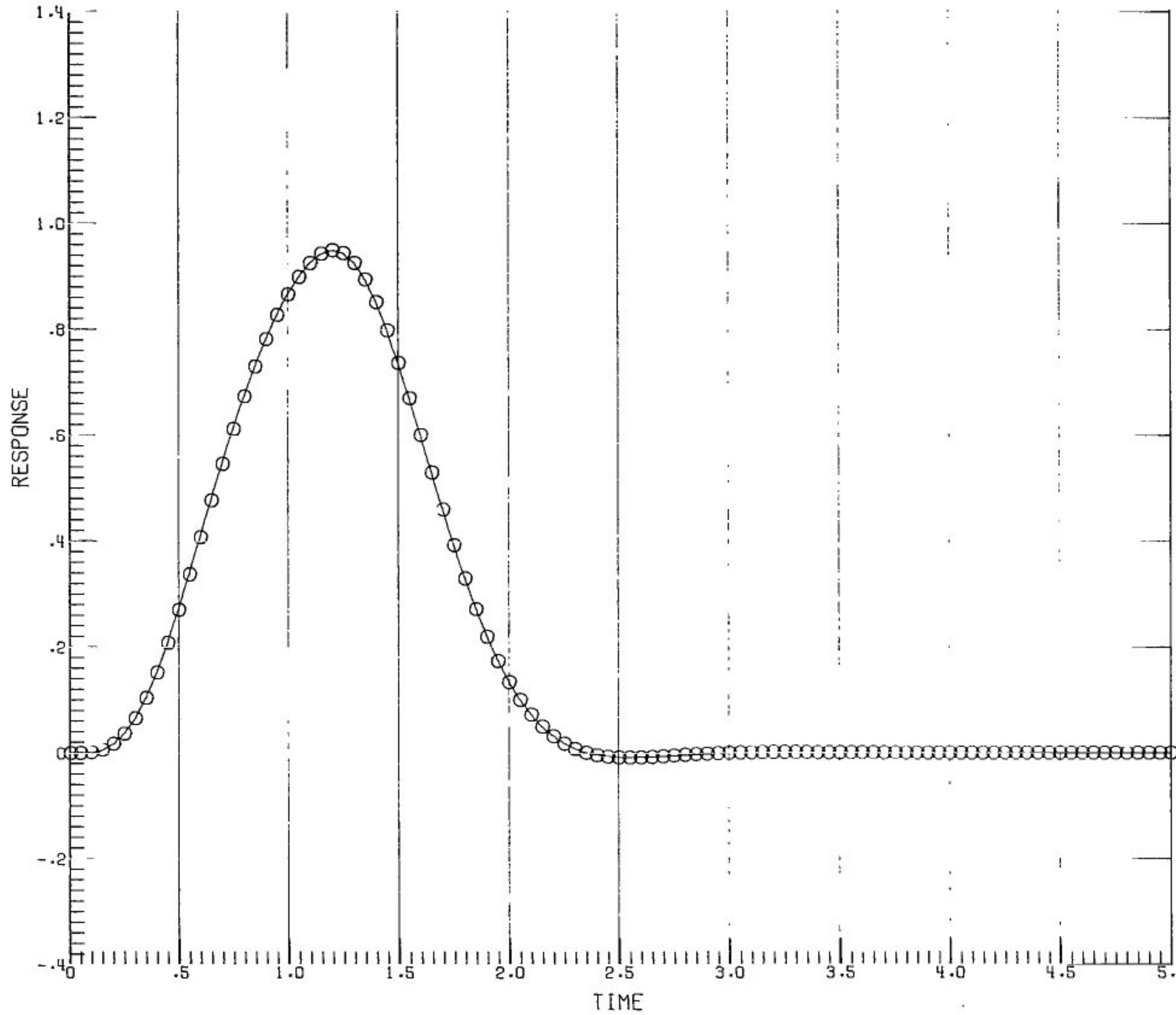

Figure 73.- Response of fourth-order Bessel filter with BT = 0.5.

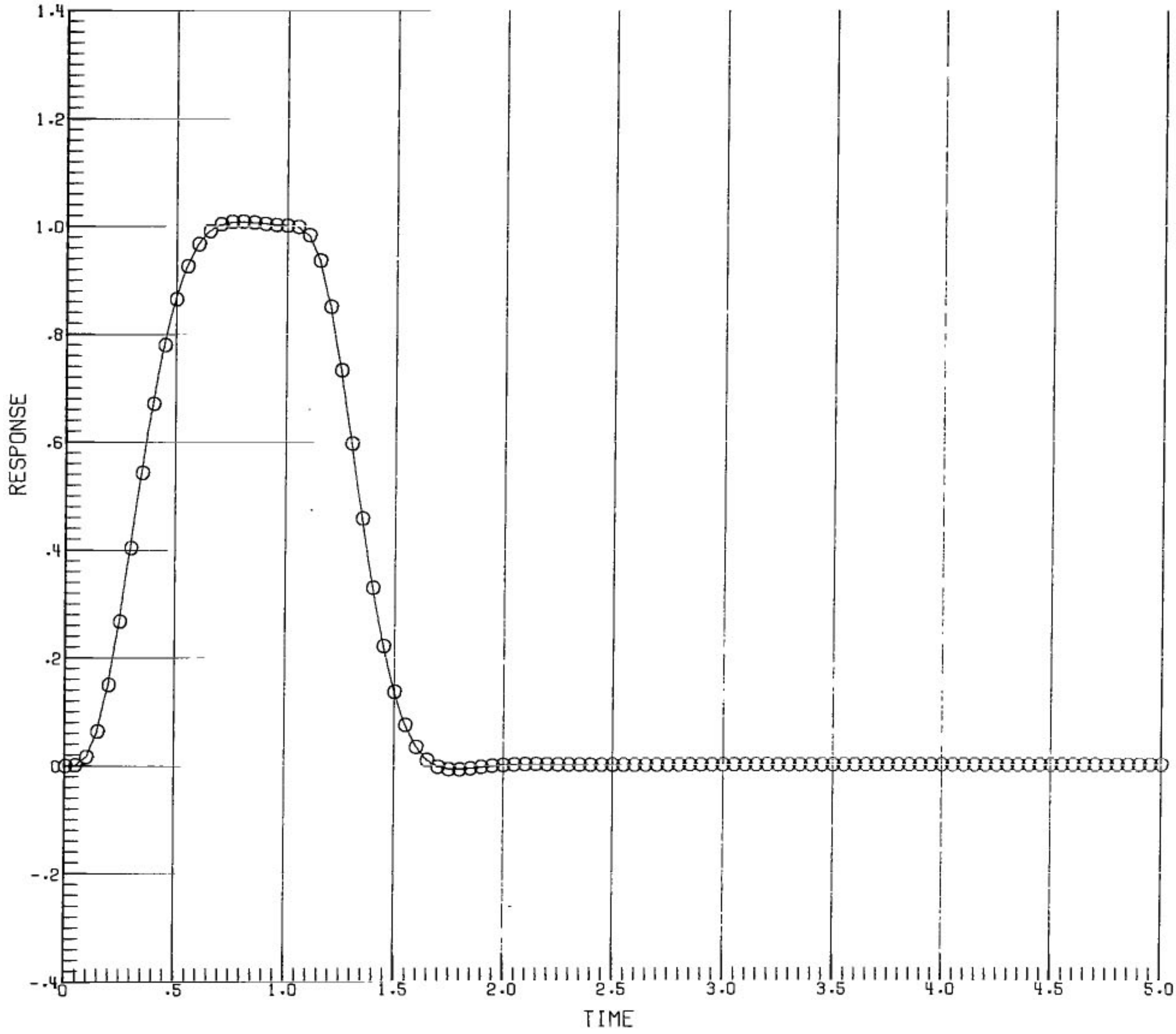

Figure 74.- Response of fourth-order Bessel filter with BT = 1.

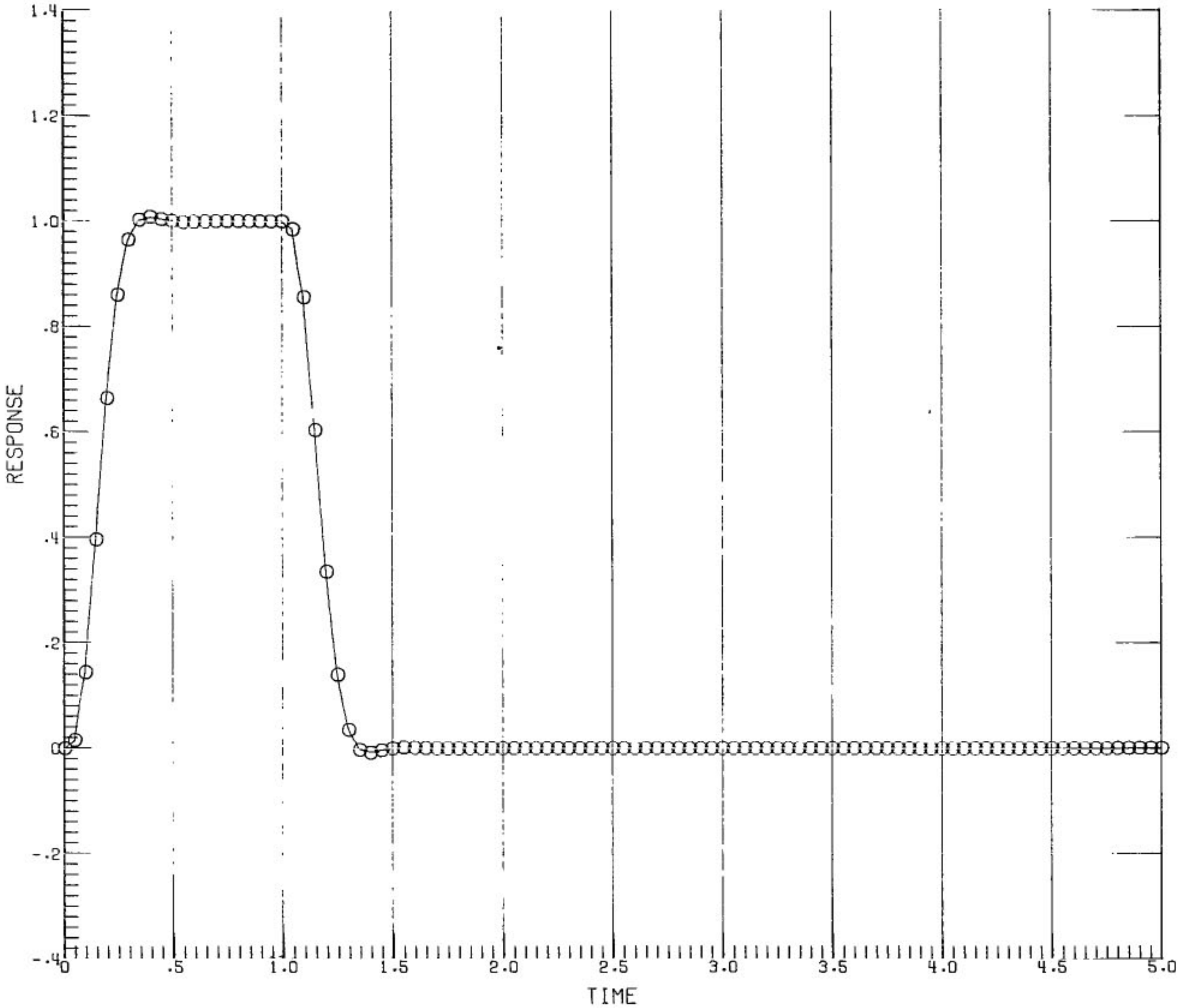

Figure 75.- Response of fourth-order Bessel filter with BT = 2.

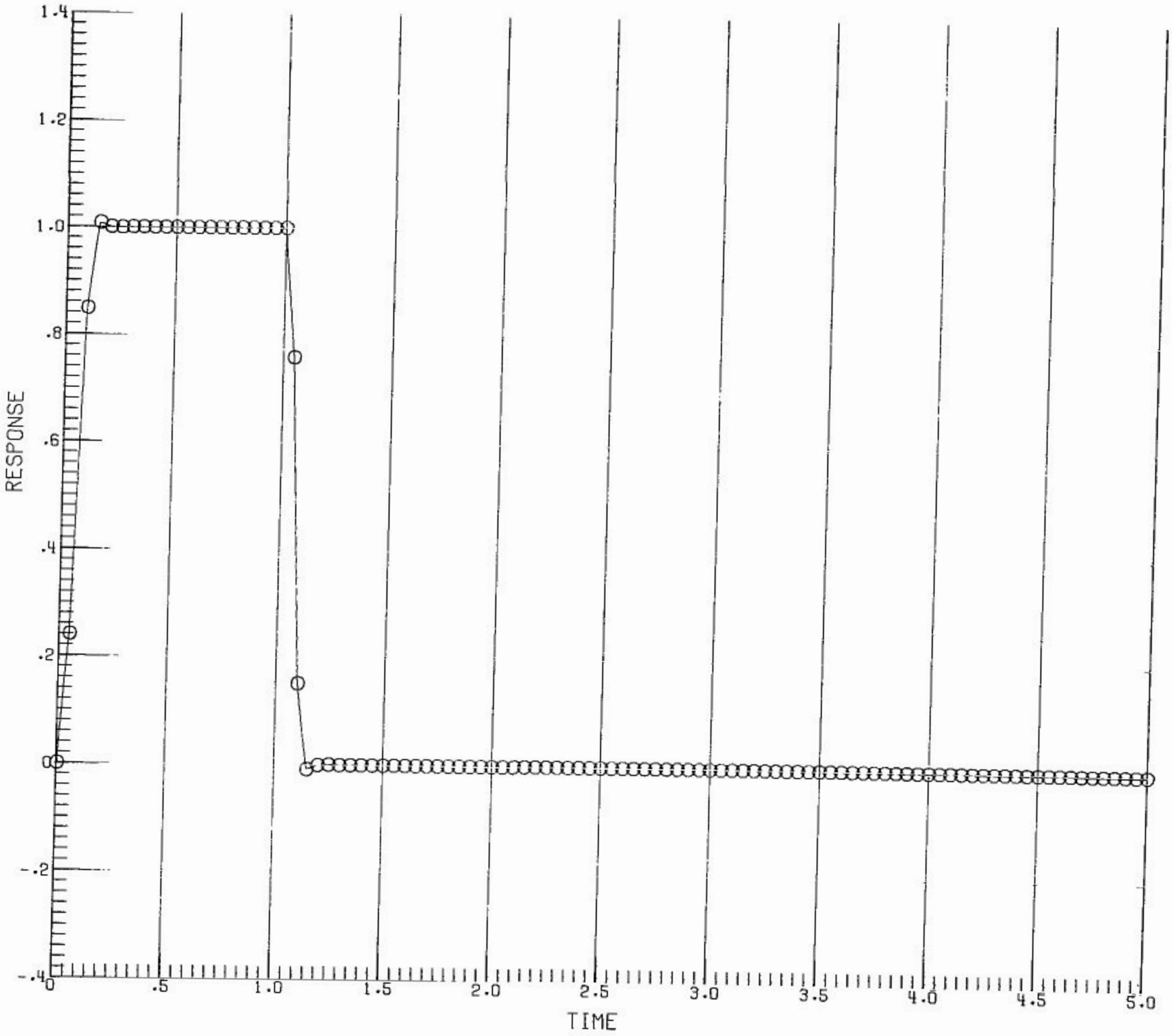

Figure 76.- Response of fourth-order Bessel filter with BT = 5.

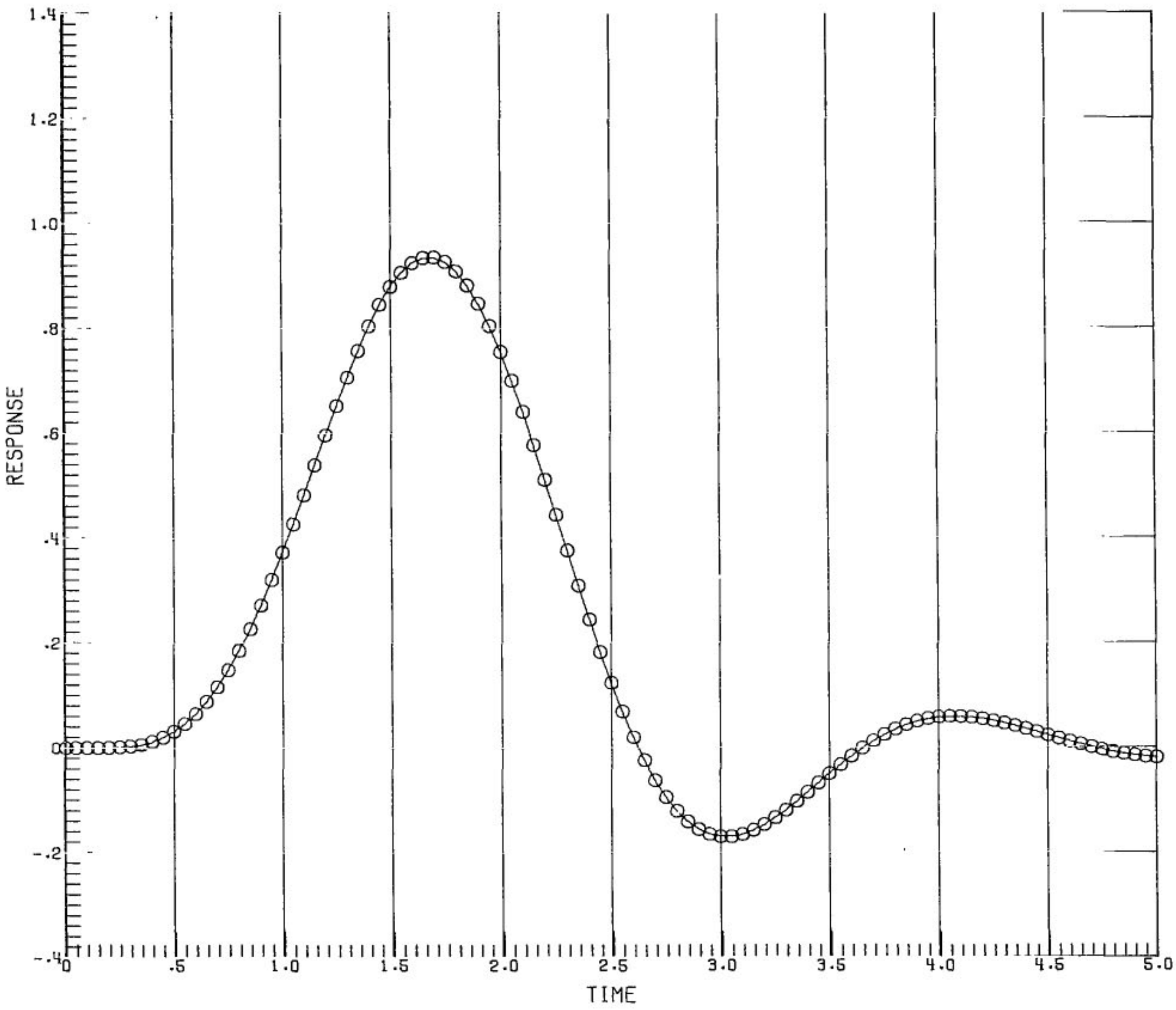

Figure 77.- Response of fifth-order Butterworth filter with BT = 0.5.

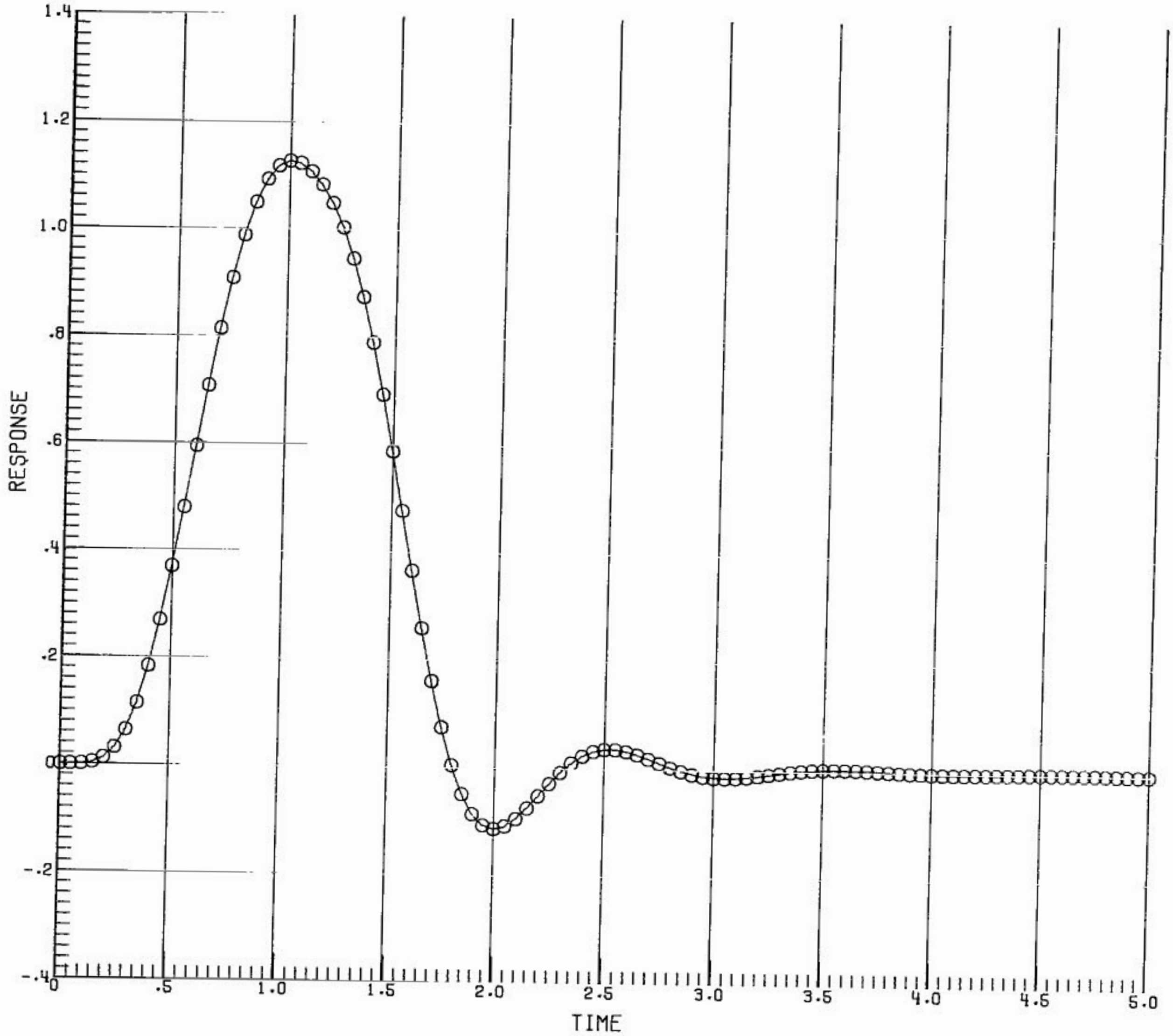

Figure 78.- Response of fifth-order Butterworth filter with BT = 1.

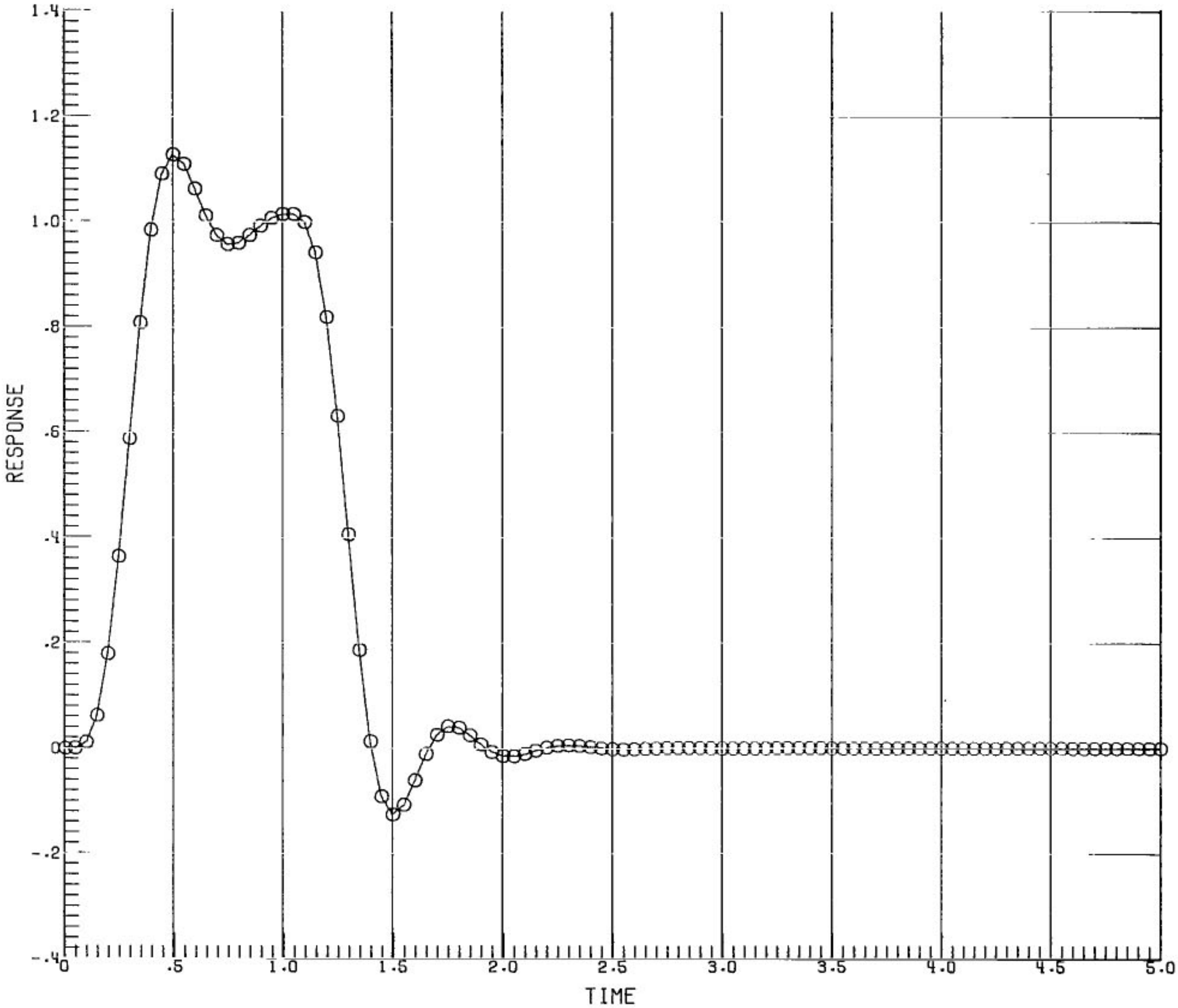

Figure 79.- Response of fifth-order Butterworth filter with BT = 2.

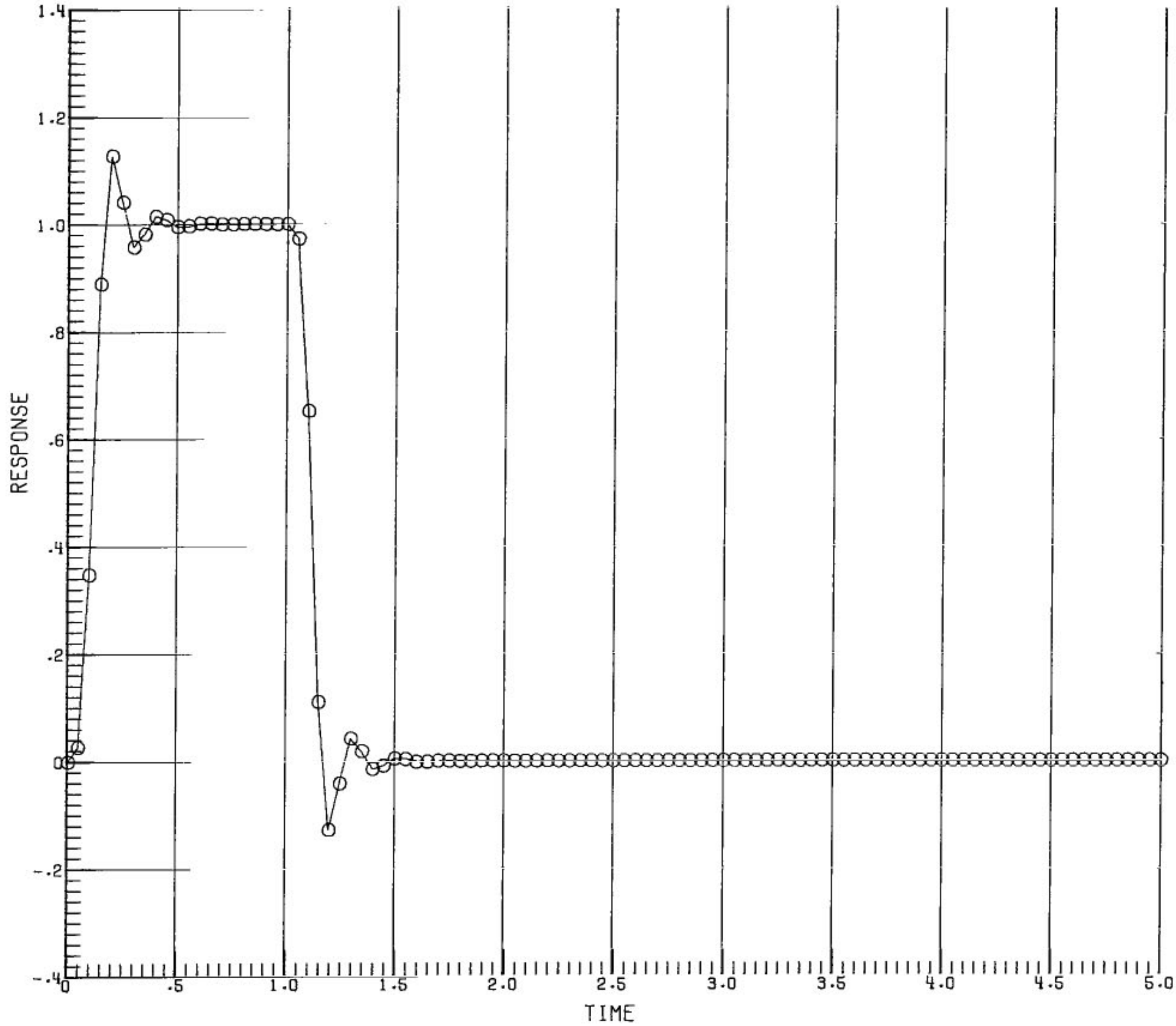

Figure 80.- Response of fifth-order Butterworth filter with BT = 5.

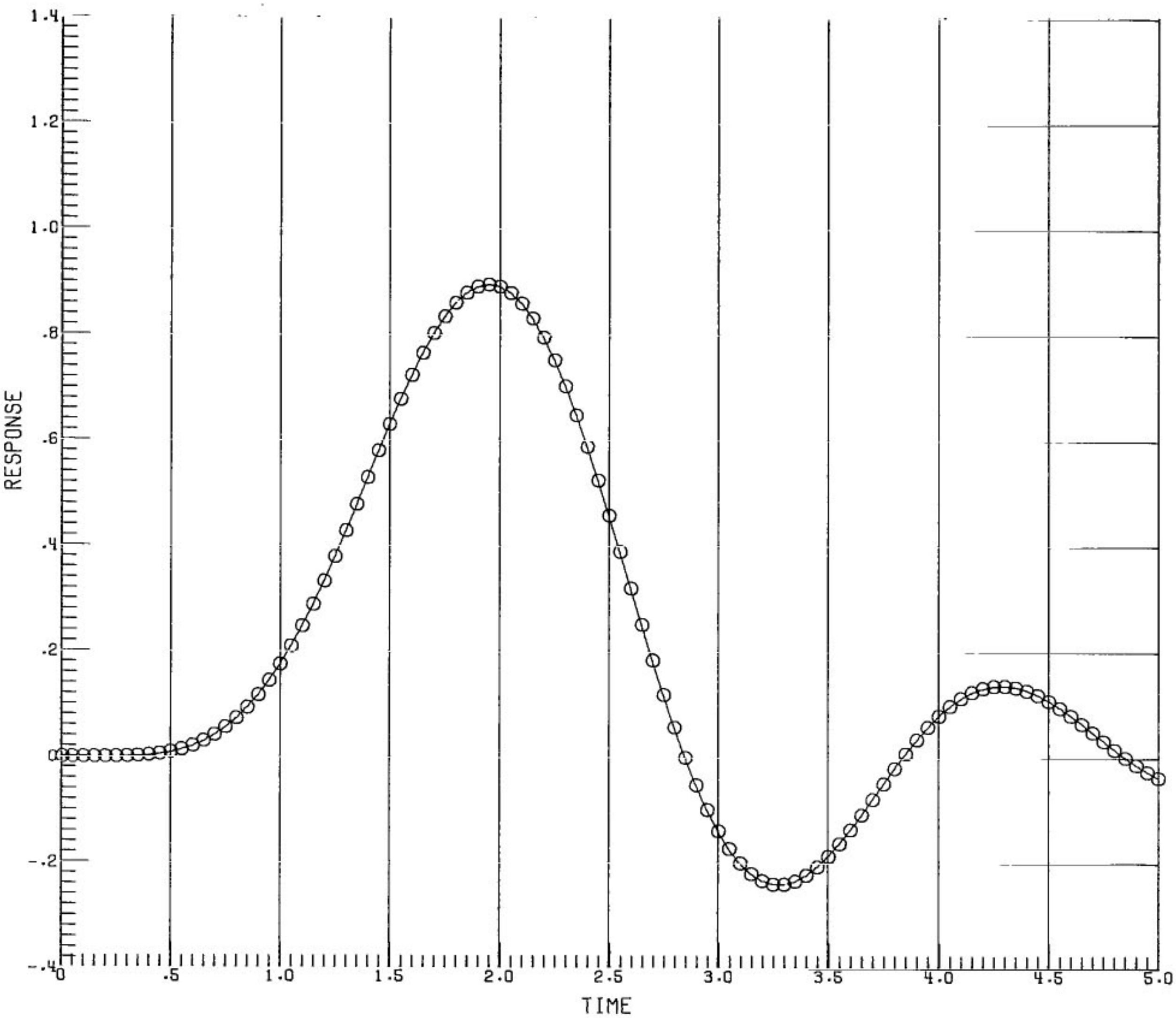

Figure 81.- Response of fifth-order Chebyshev (0.5-dB ripple) filter with BT = 0.5.

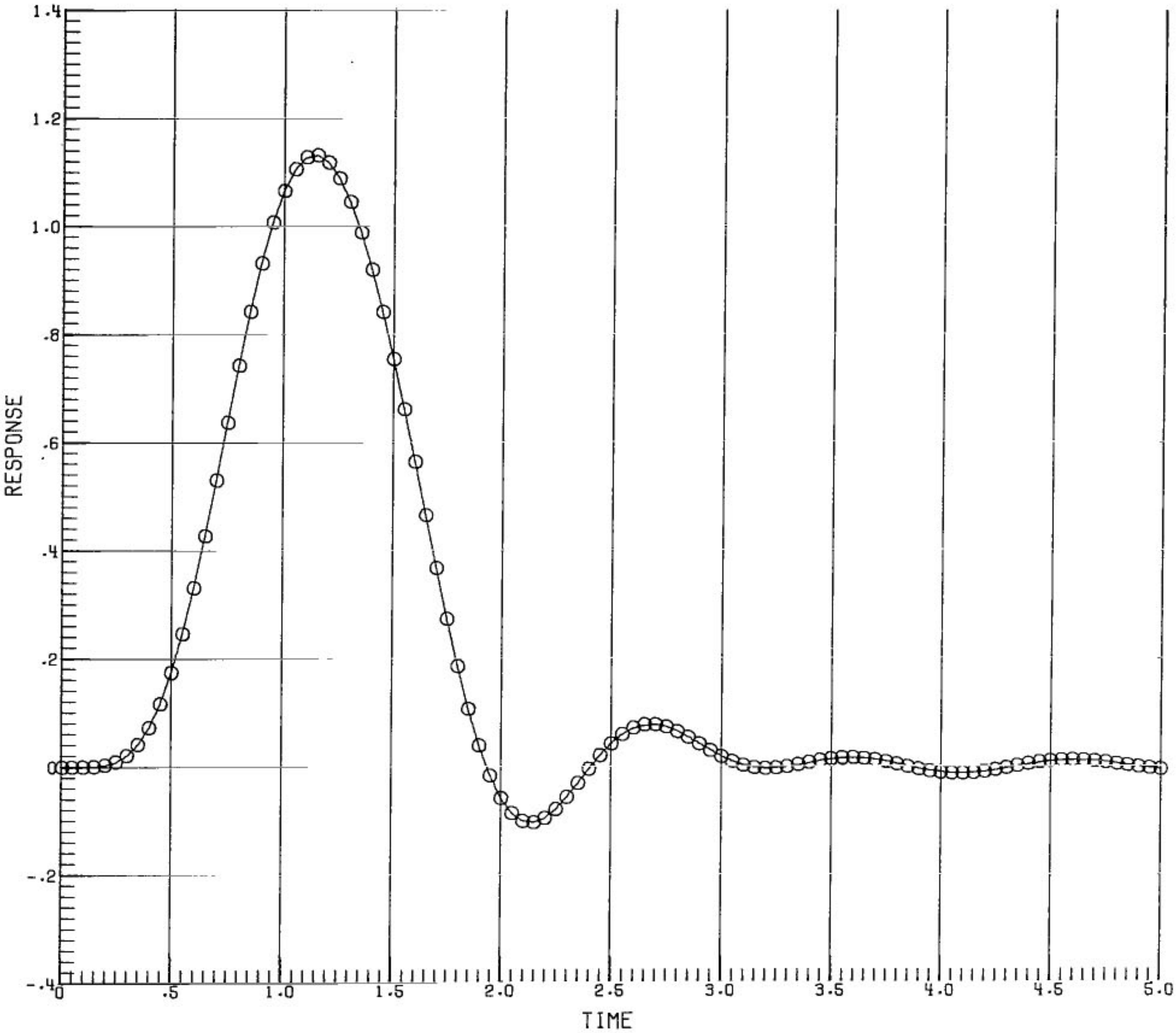

Figure 82.- Response of fifth-order Chebyshev (0.5-dB ripple) filter with BT = 1.

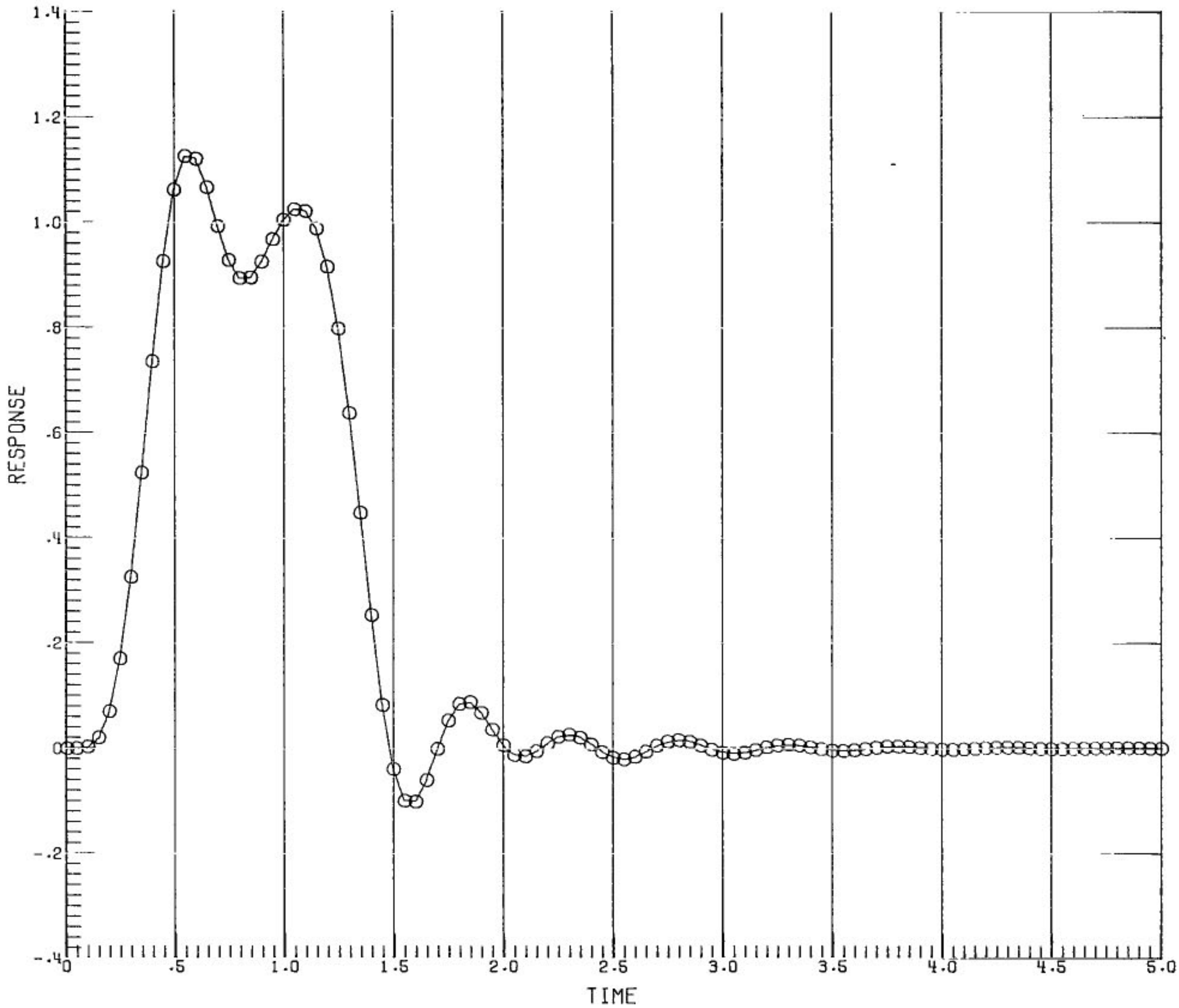

Figure 83.- Response of fifth-order Chebyshev (0.5-dB ripple) filter with BT = 2.

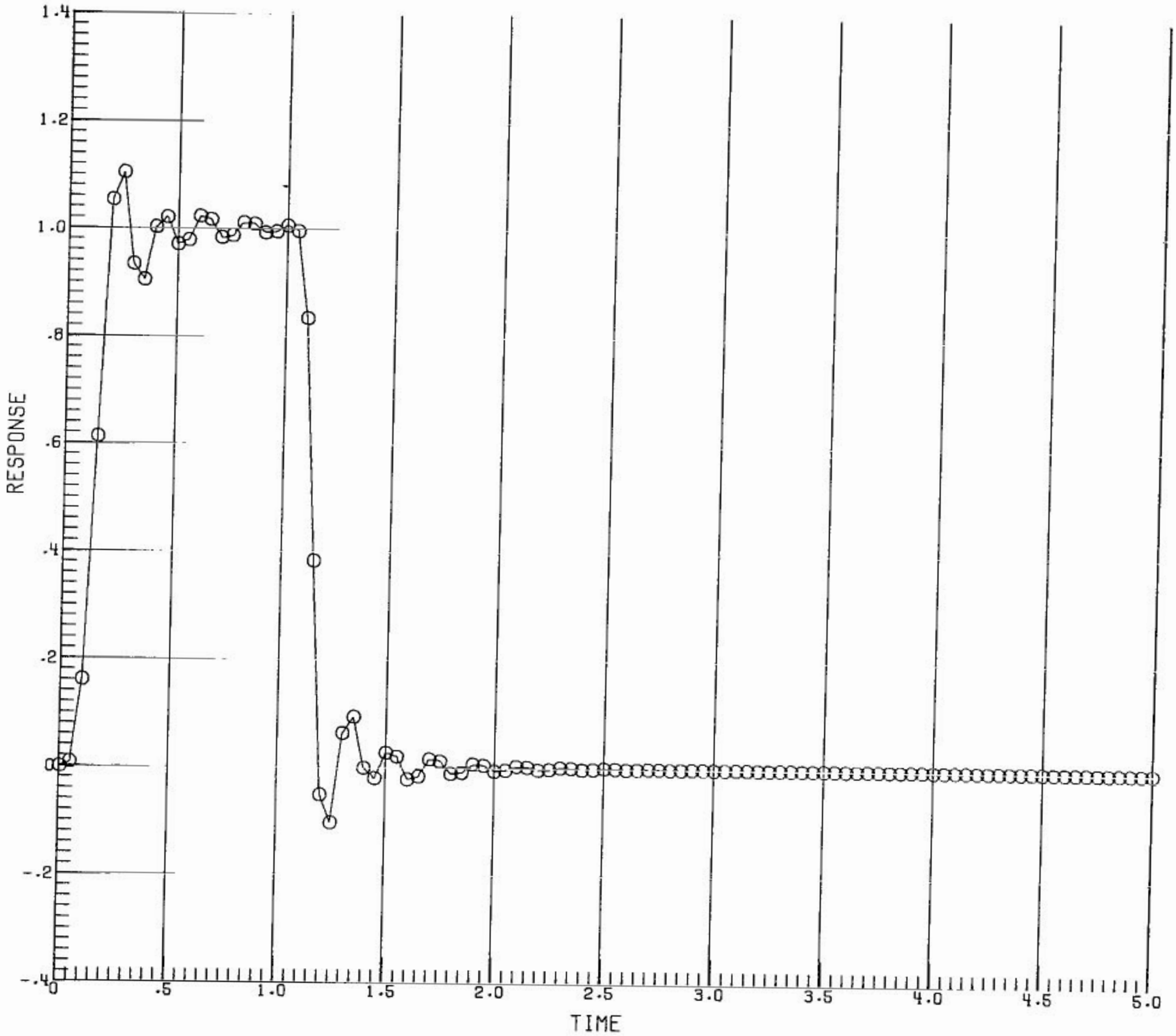

Figure 84.- Response of fifth-order Chebyshev (0.5-dB ripple) filter with $BT = 5$.

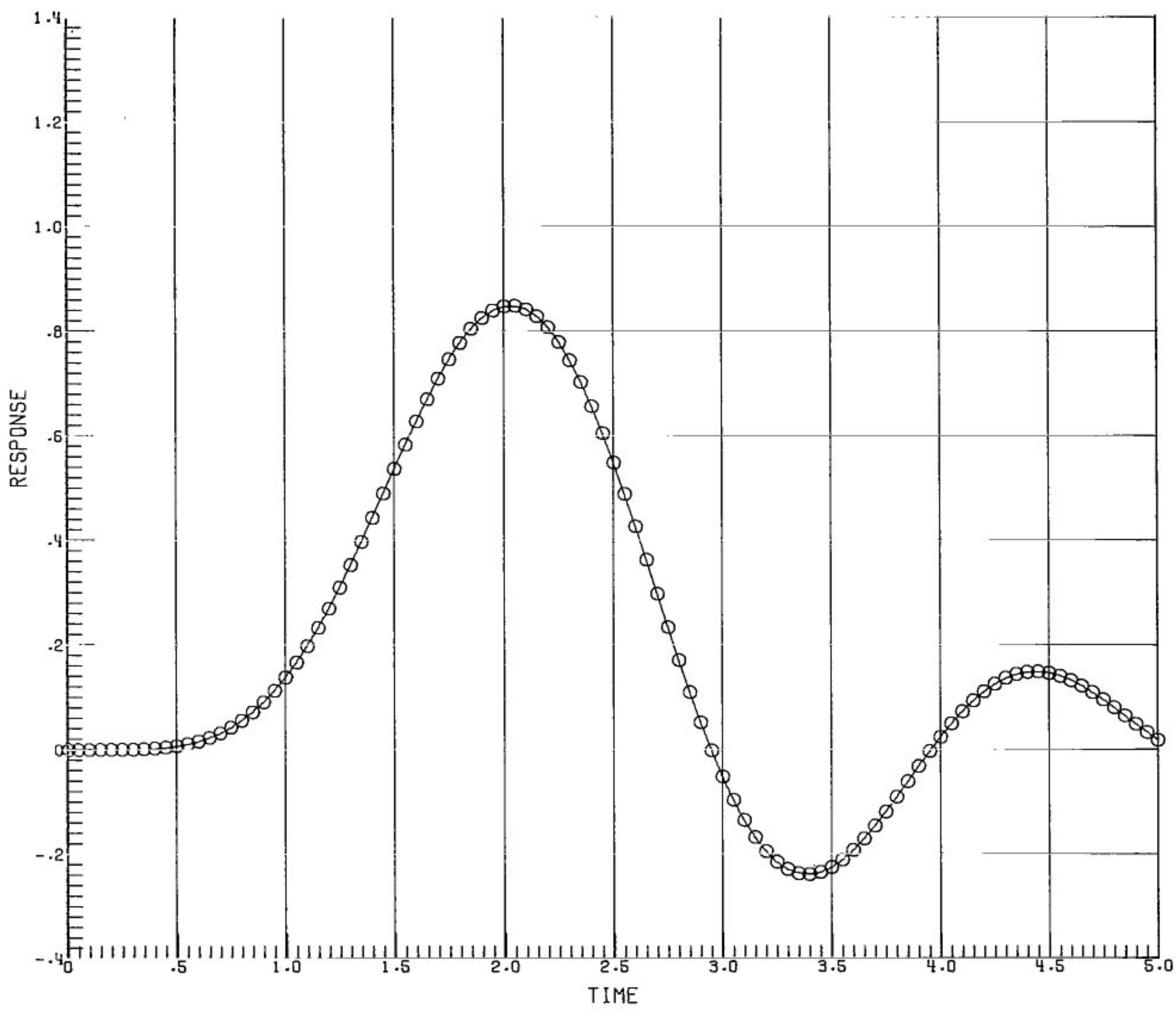

Figure 85.- Response of fifth-order Chebyshev (1-dB ripple) filter with BT = 0.5.

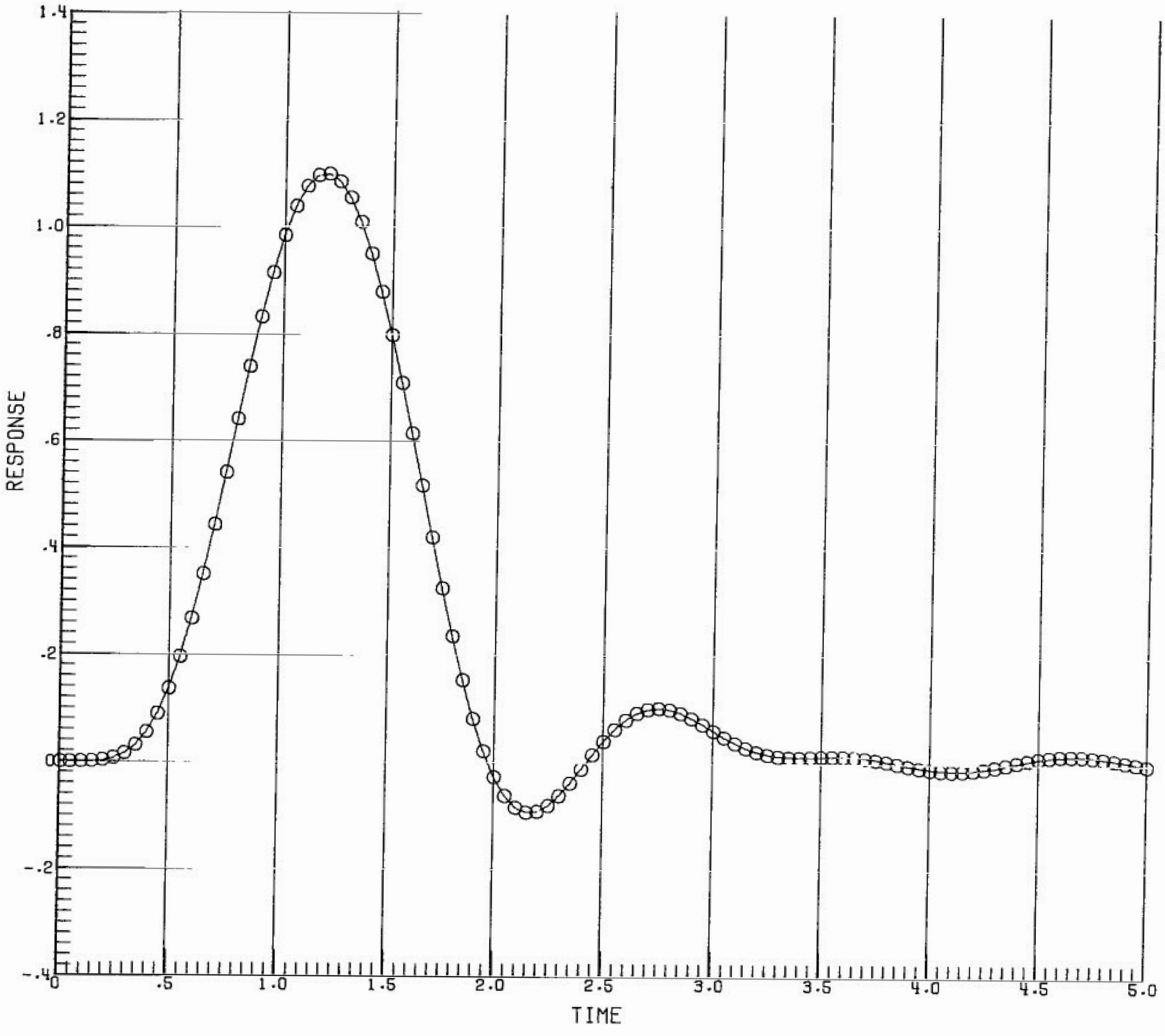

Figure 86.- Response of fifth-order Chebyshev (1-dB ripple) filter with BT = 1.

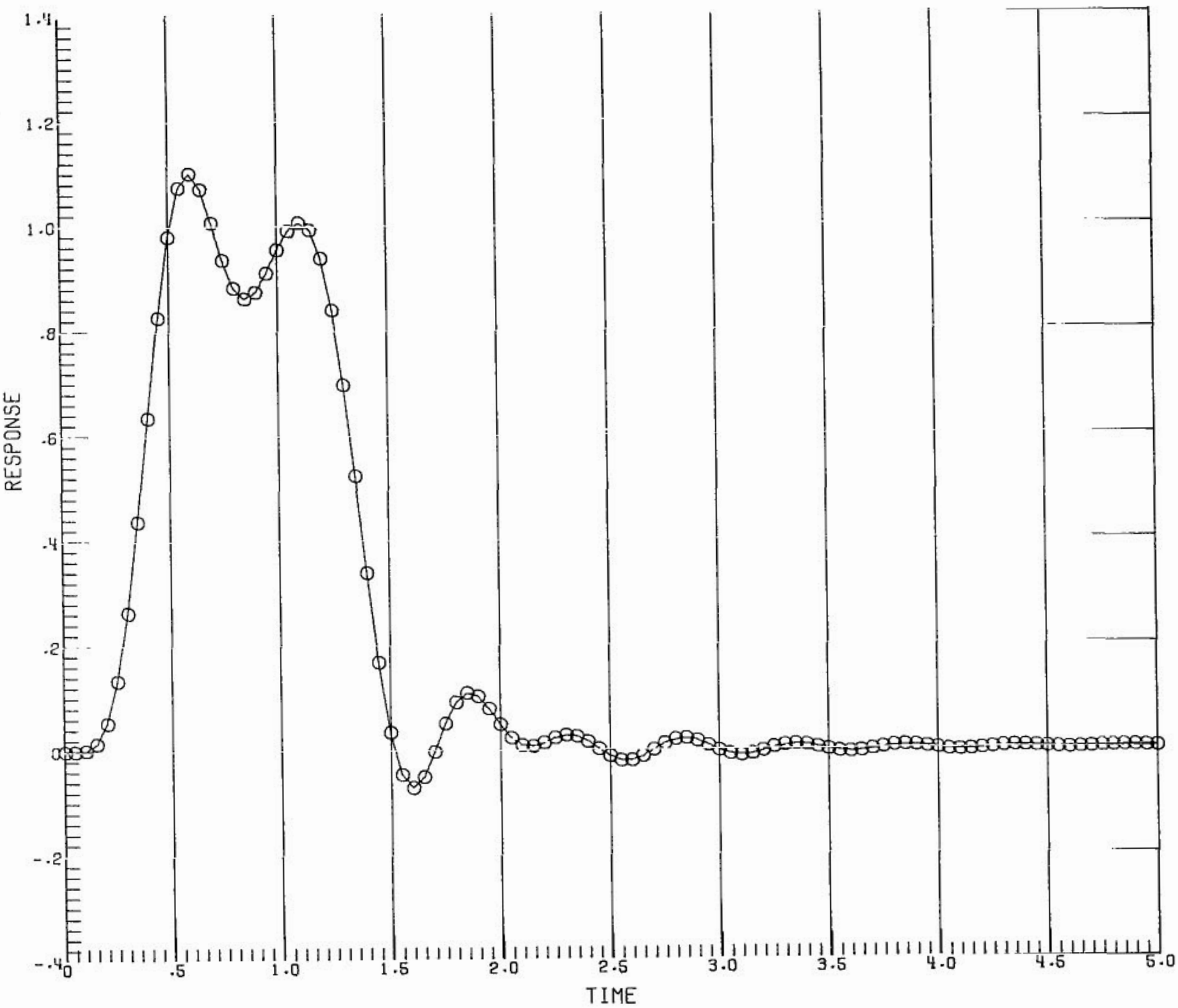

Figure 87.- Response of fifth-order Chebyshev (1-dB ripple) filter with BT = 2.

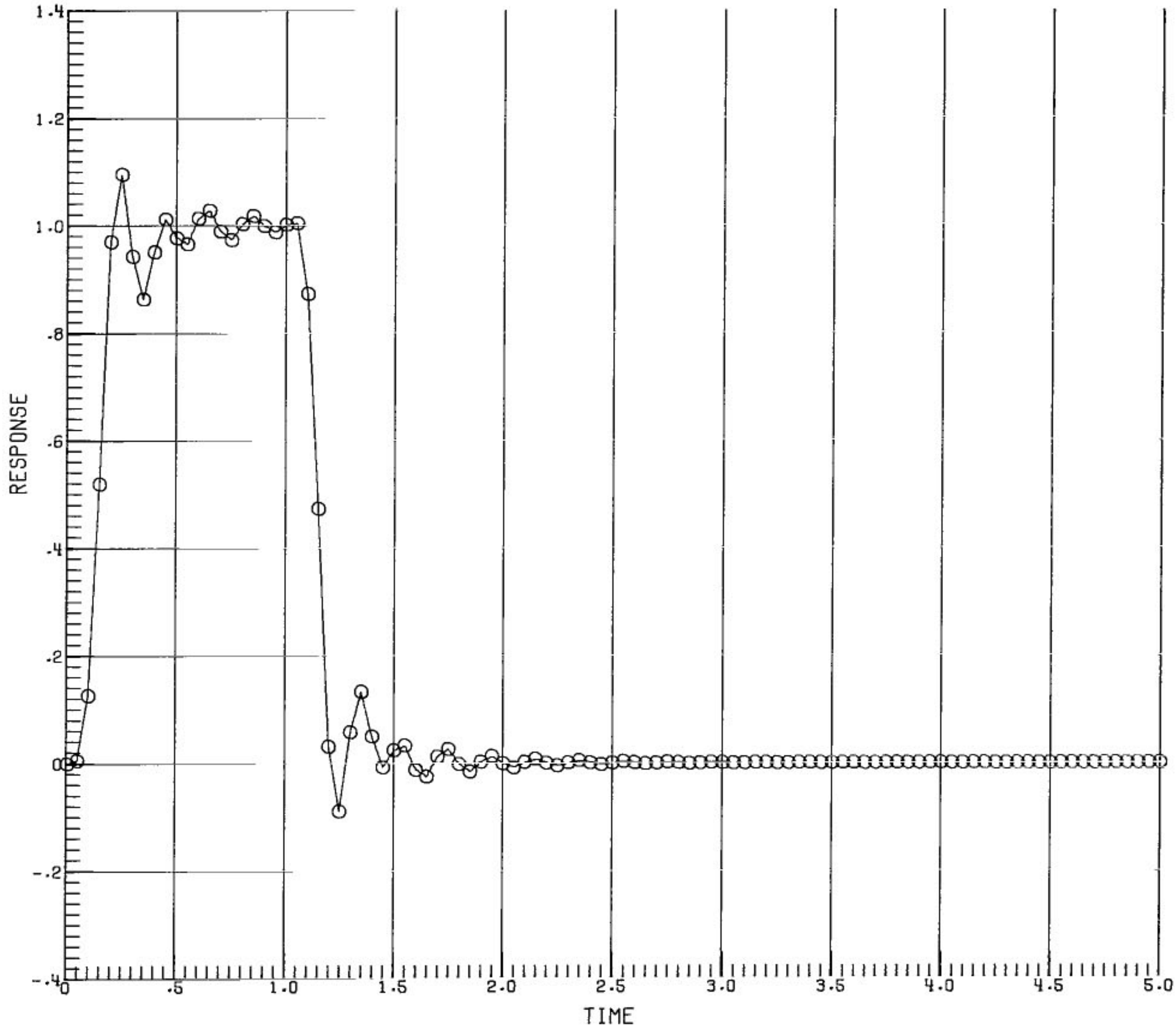

Figure 88.- Response of fifth-order Chebyshev (1-dB ripple) filter with BT = 5.

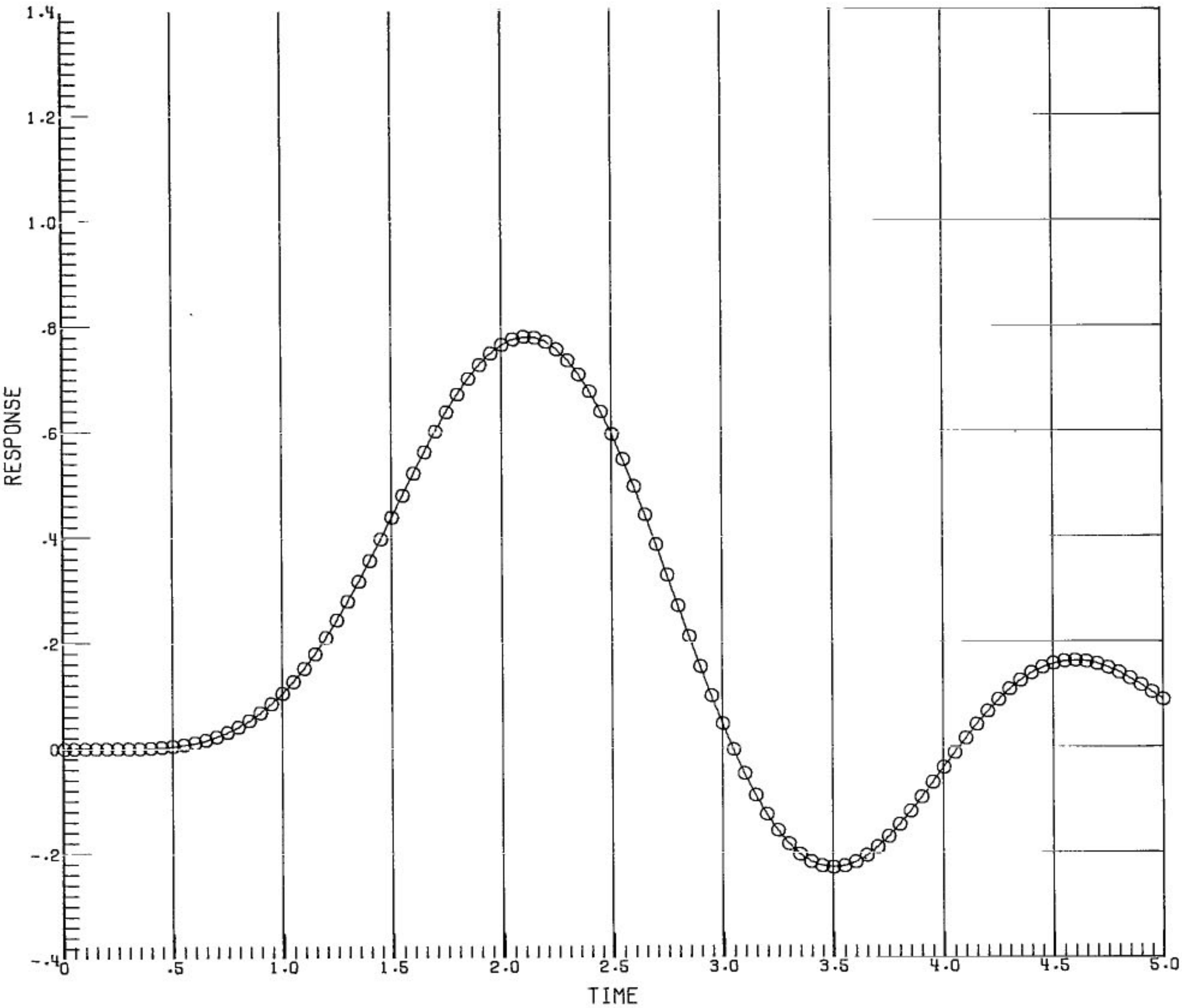

Figure 89.- Response of fifth-order Chebyshev (2-dB ripple) filter with BT = 0.5.

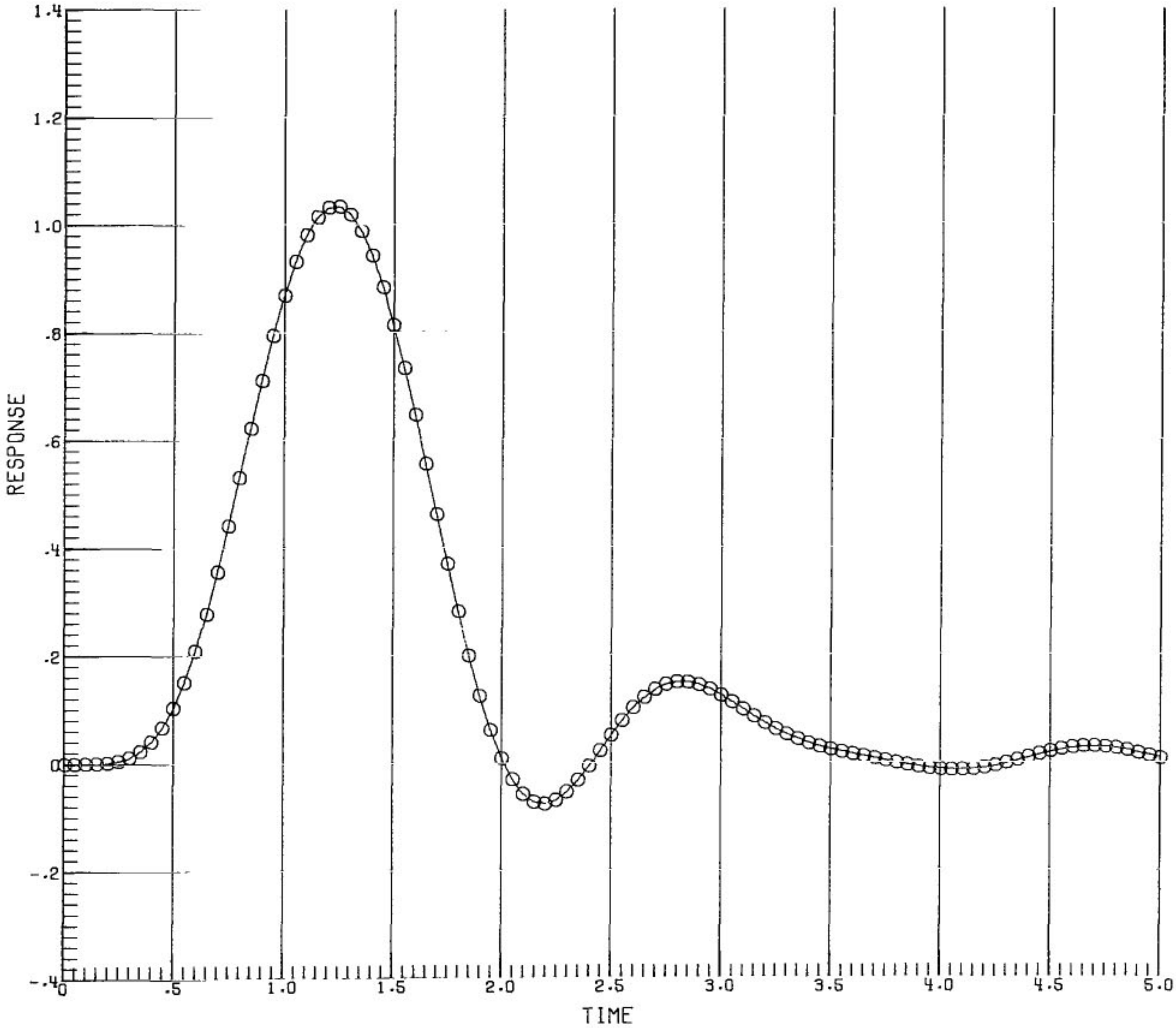

Figure 90.- Response of fifth-order Chebyshev (2-dB ripple) filter with BT = 1.

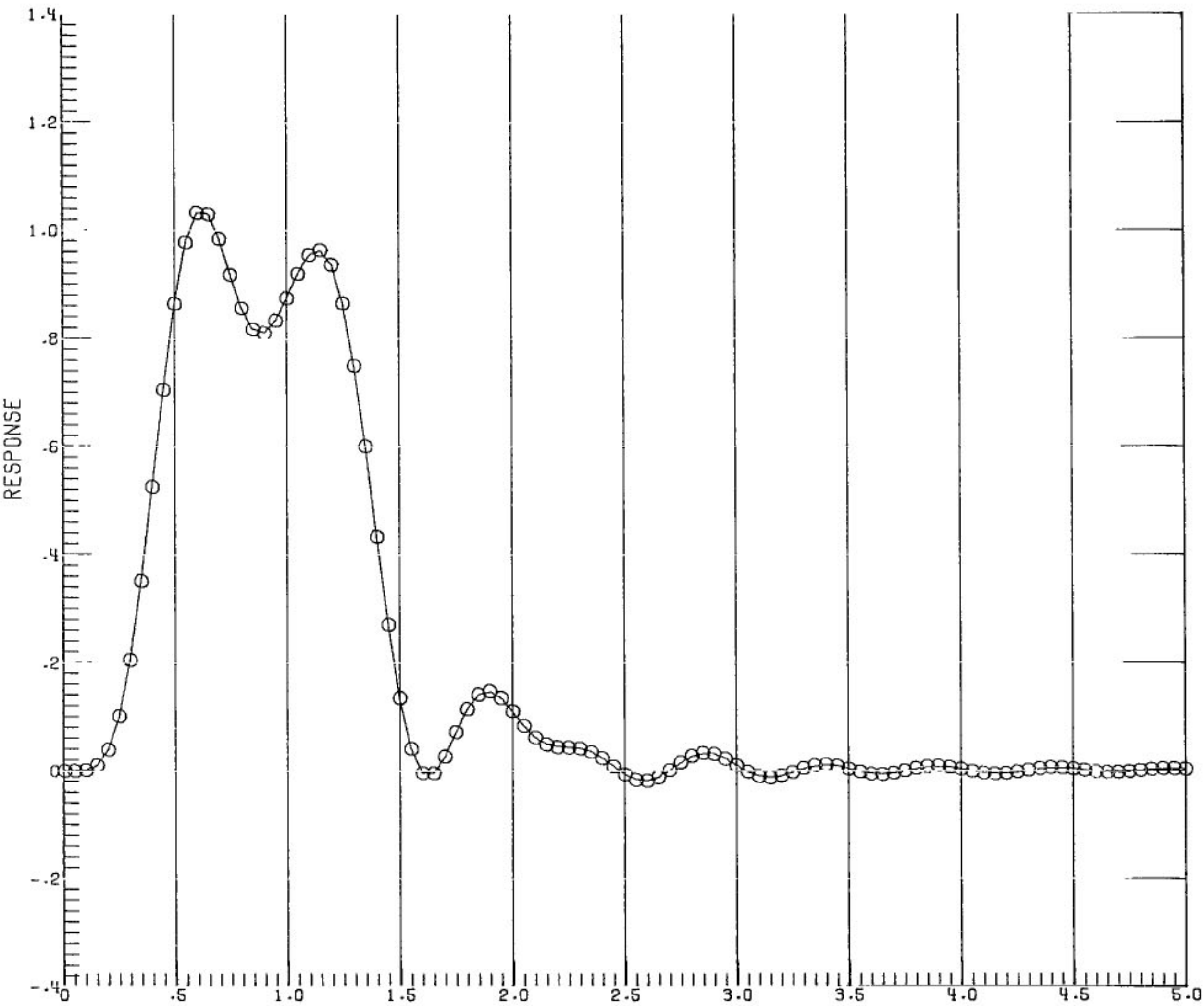

Figure 91.- Response of fifth-order Chebyshev (2-dB ripple) filter with BT = 2.

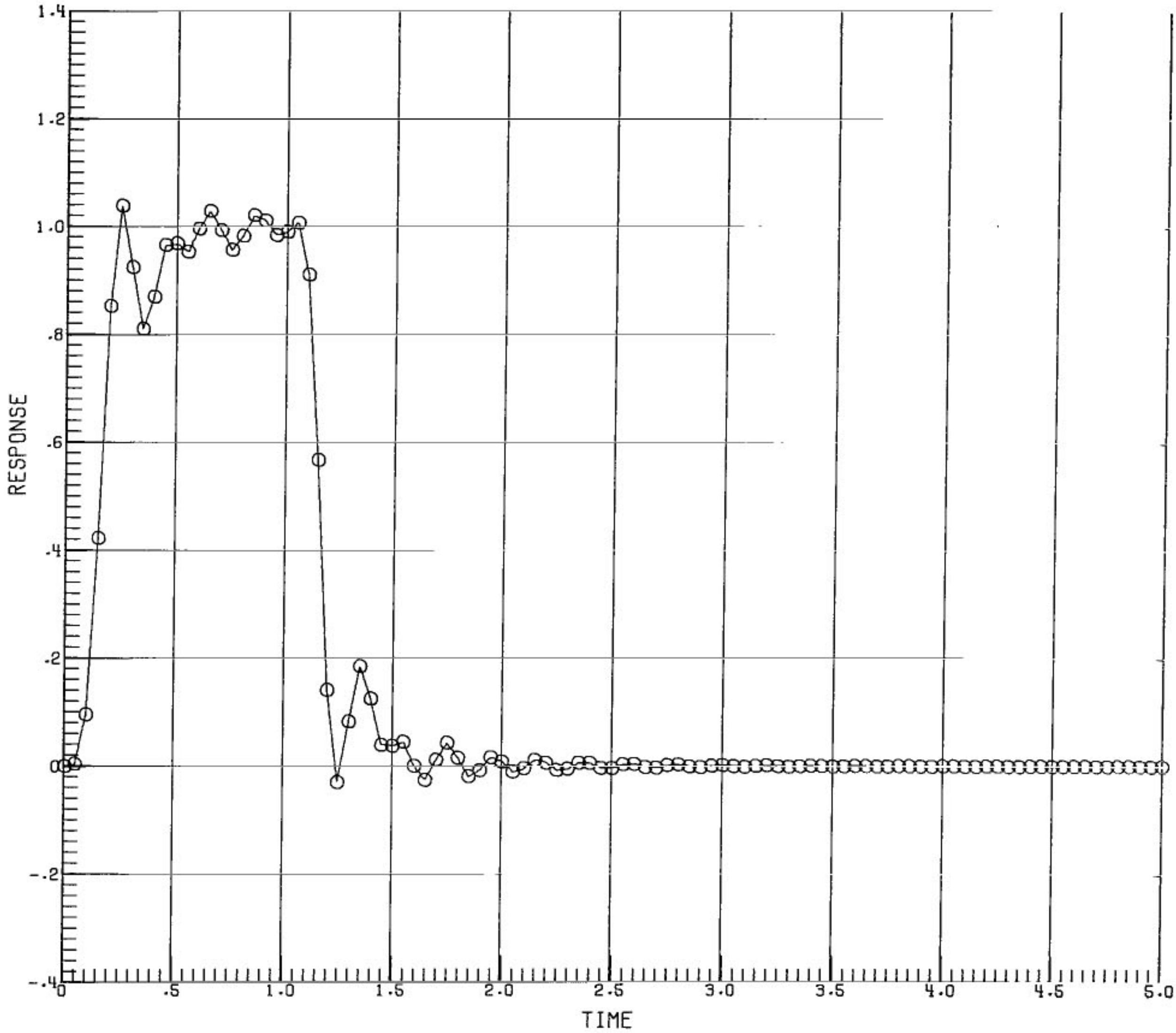

Figure 92.- Response of fifth-order Chebyshev (2-dB ripple) filter with $BT = 5$.

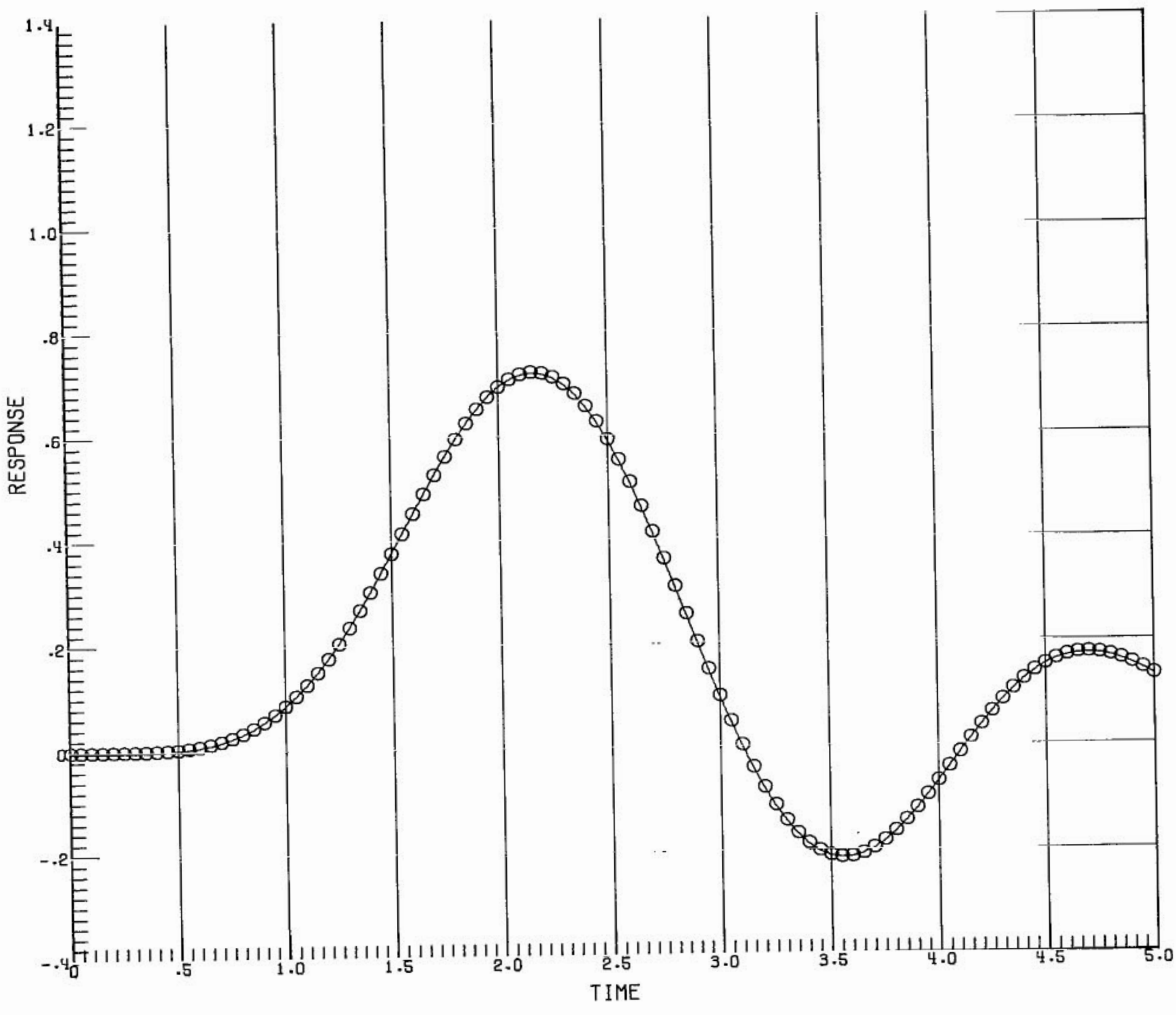

Figure 93.- Response of fifth-order Chebyshev (3-dB ripple) filter with BT = 0.5.

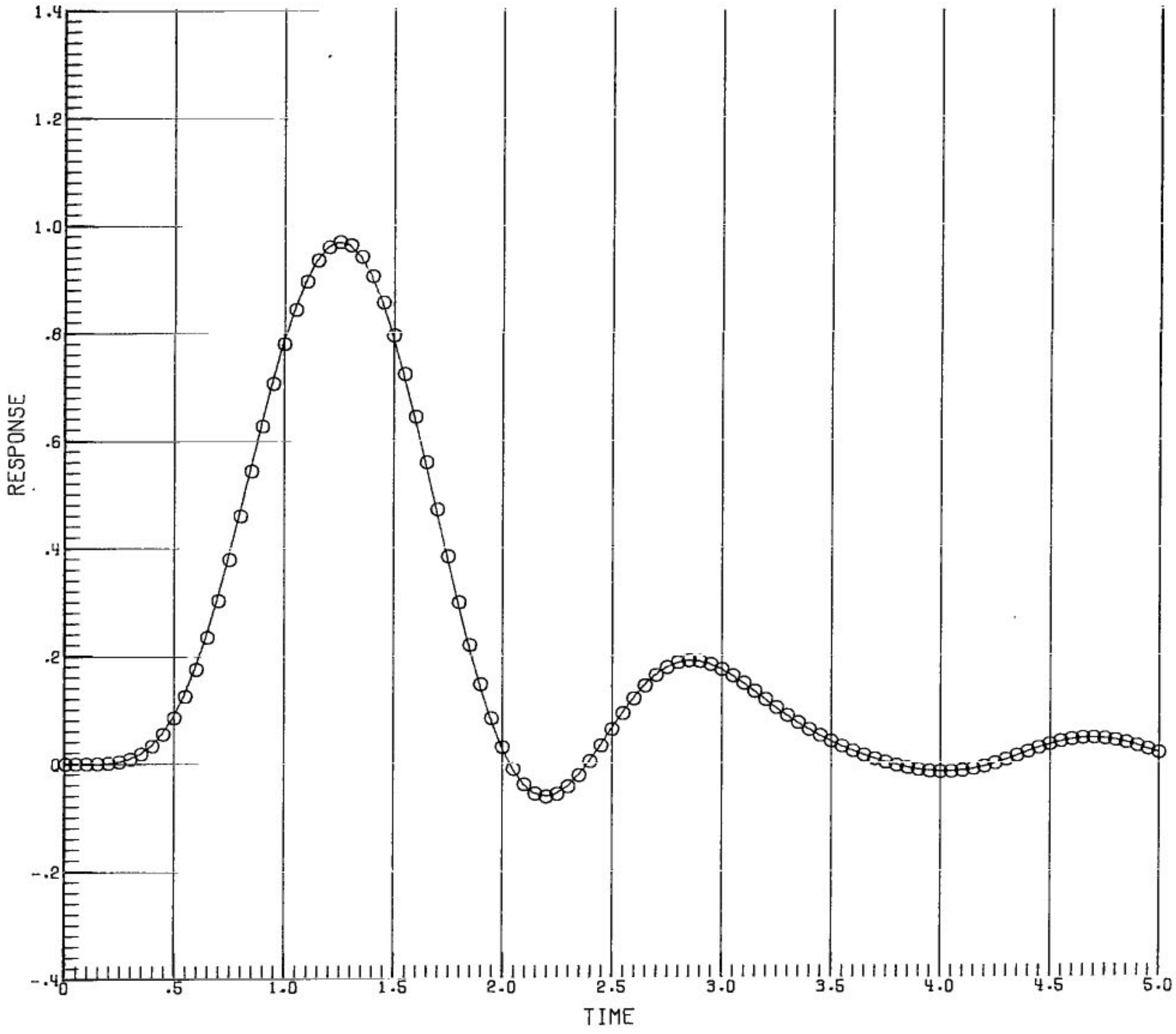

Figure 94.- Response of fifth-order Chebyshev (3-dB ripple) filter with BT = 1.

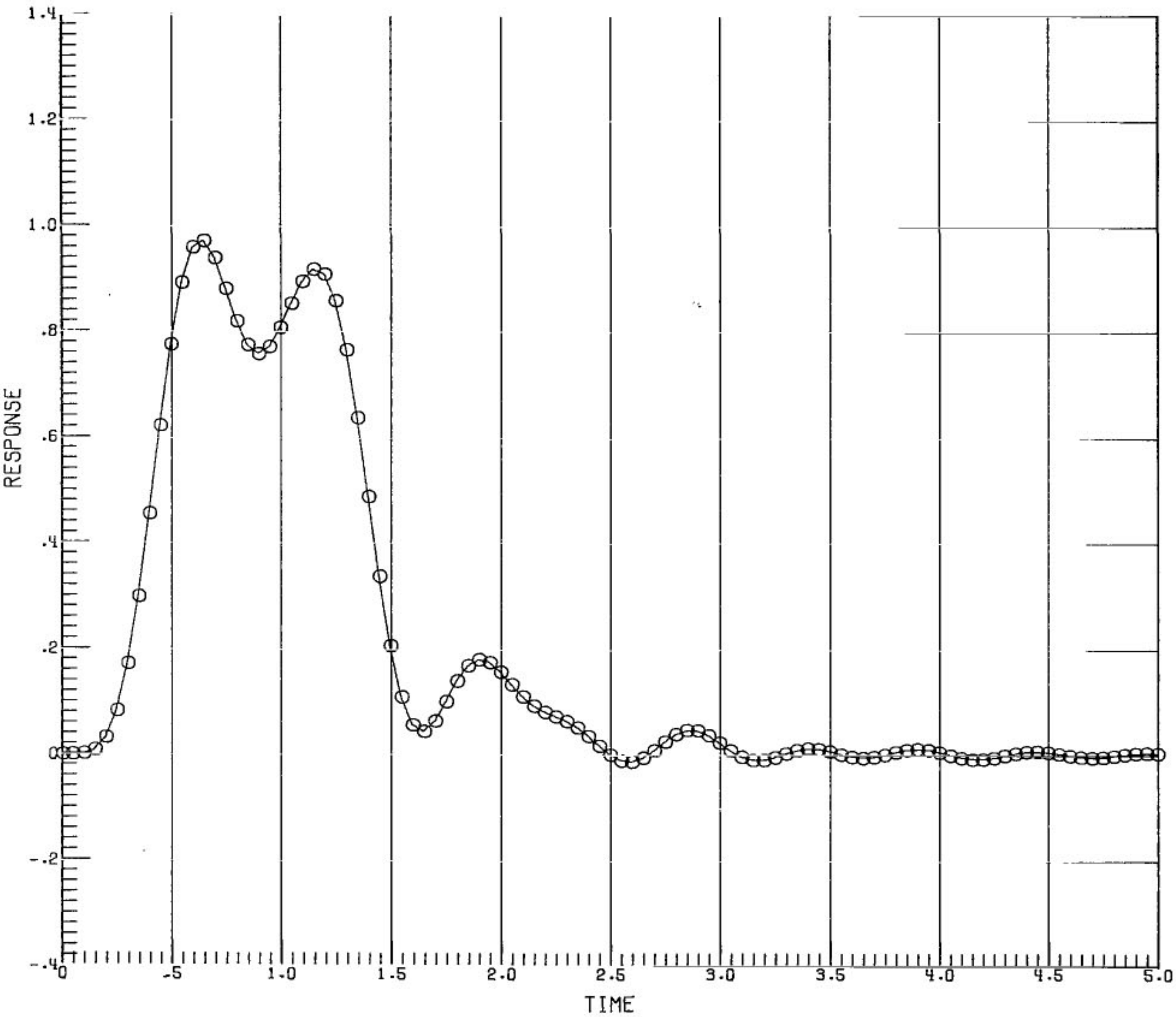

Figure 95.- Response of fifth-order Chebyshev (3-dB ripple) filter with BT = 2.

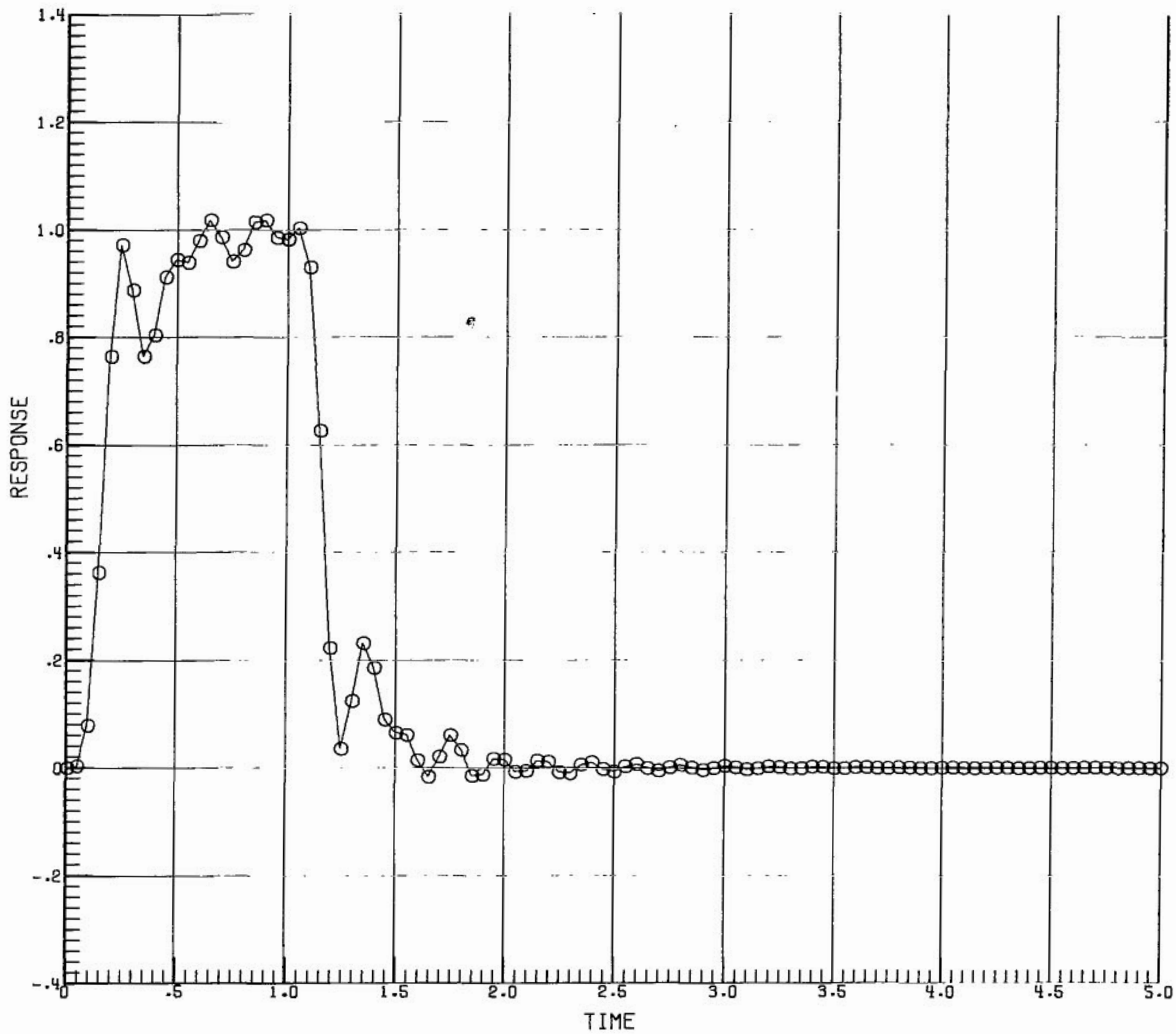

Figure 96.- Response of fifth-order Chebyshev (3-dB ripple) filter with BT = 5.

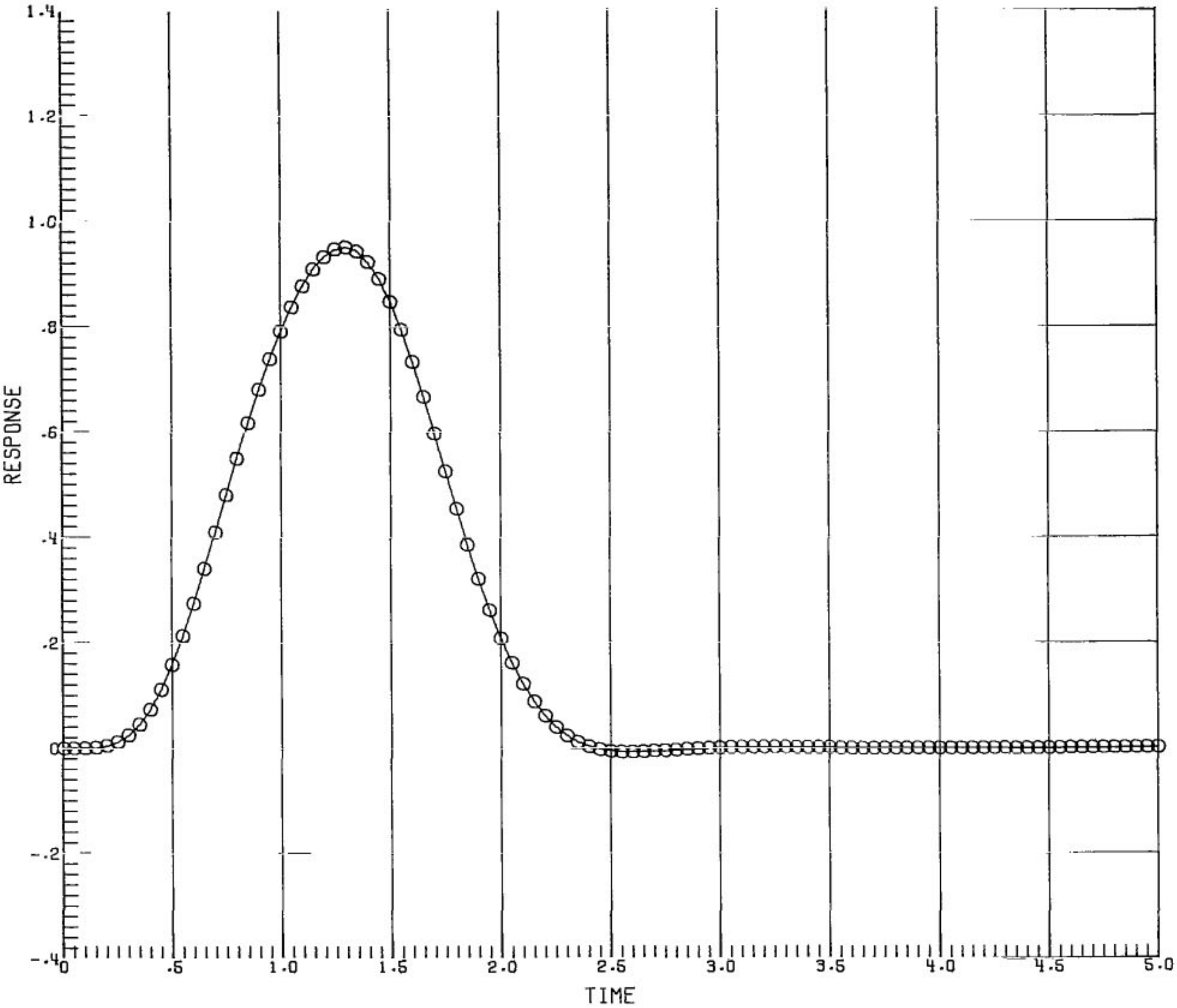

Figure 97.- Response of fifth-order Bessel filter with BT = 0.5.

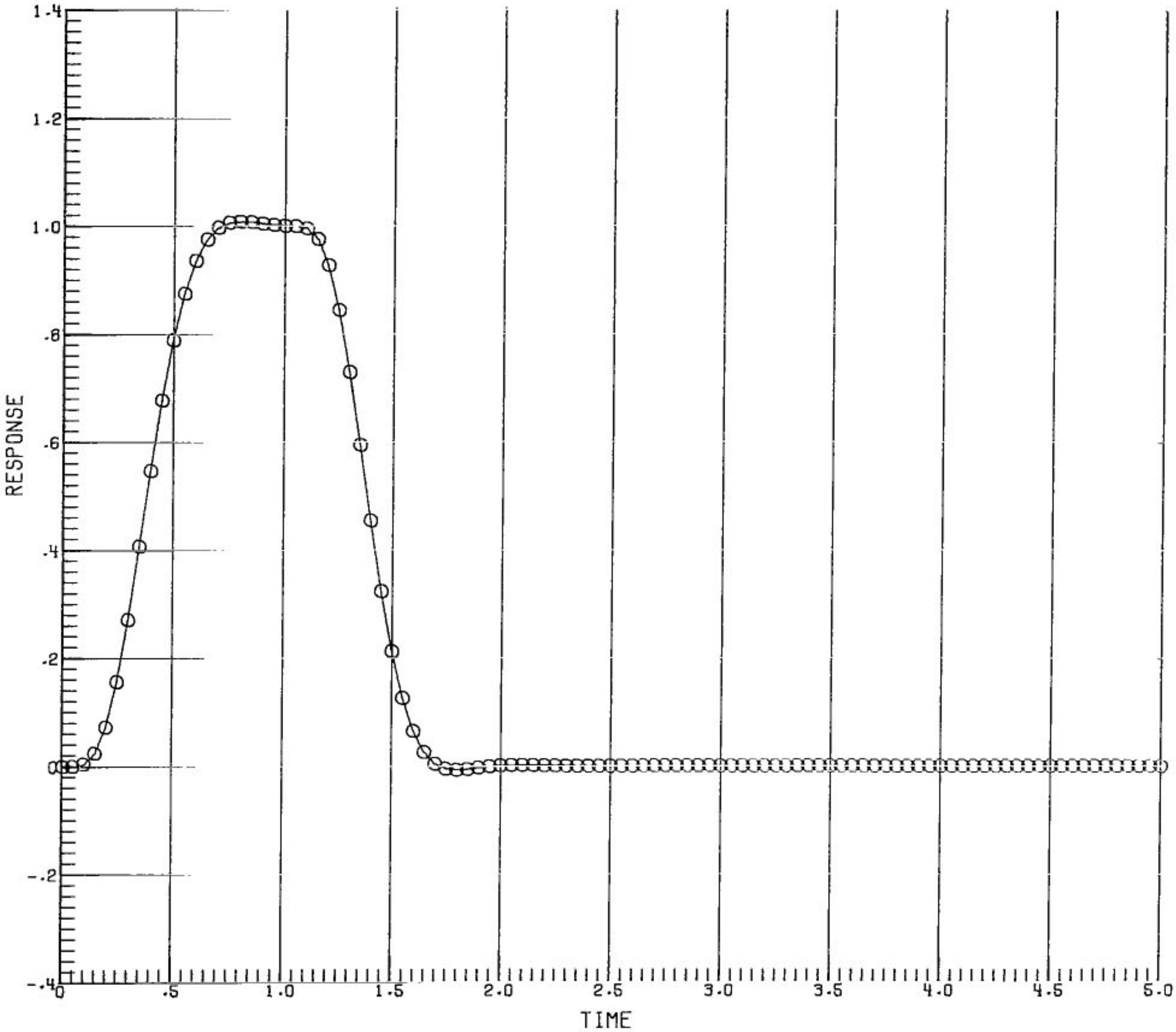

Figure 98.- Response of fifth-order Bessel filter with BT = 1.

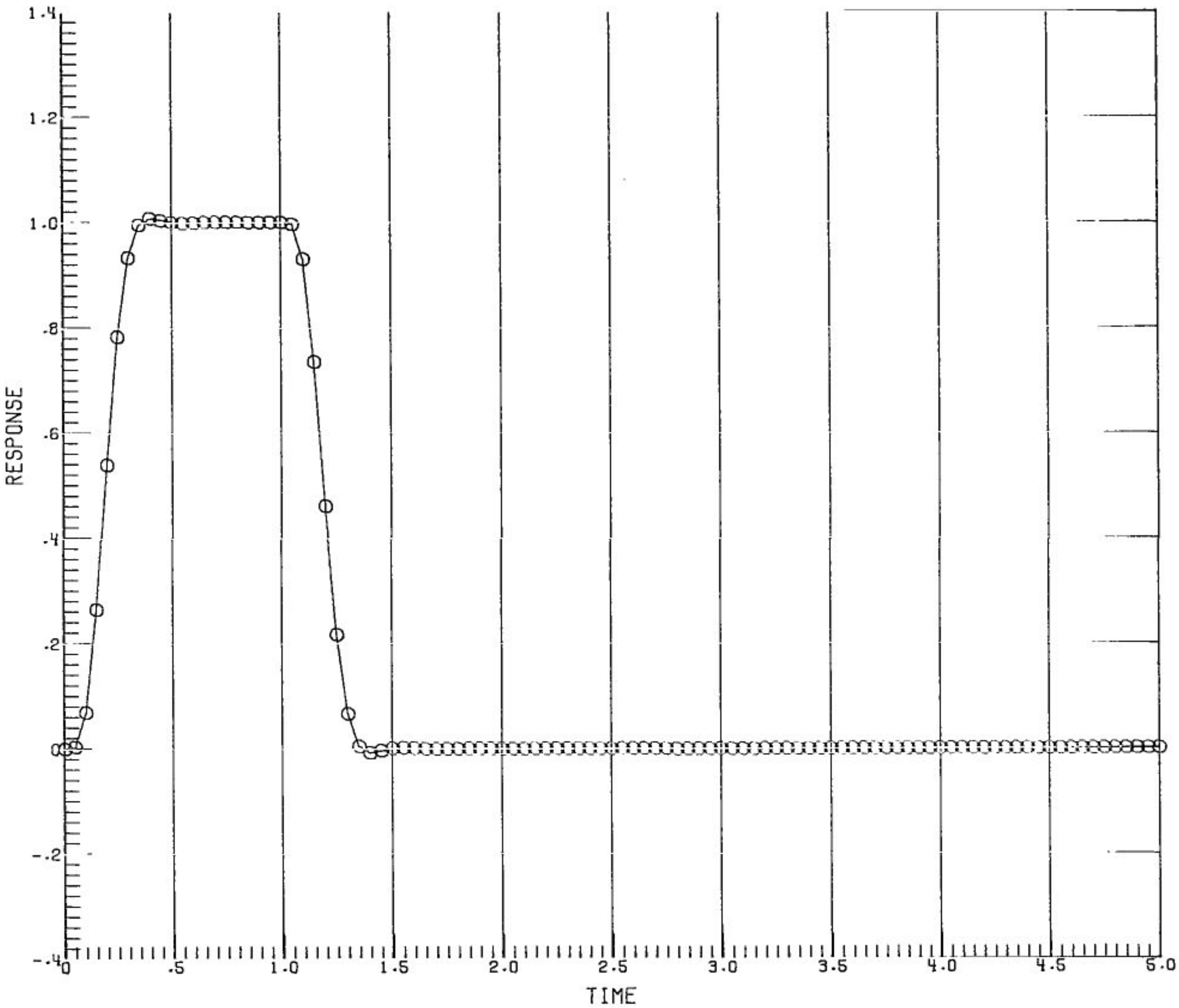

Figure 99.- Response of fifth-order Bessel filter with BT = 2.

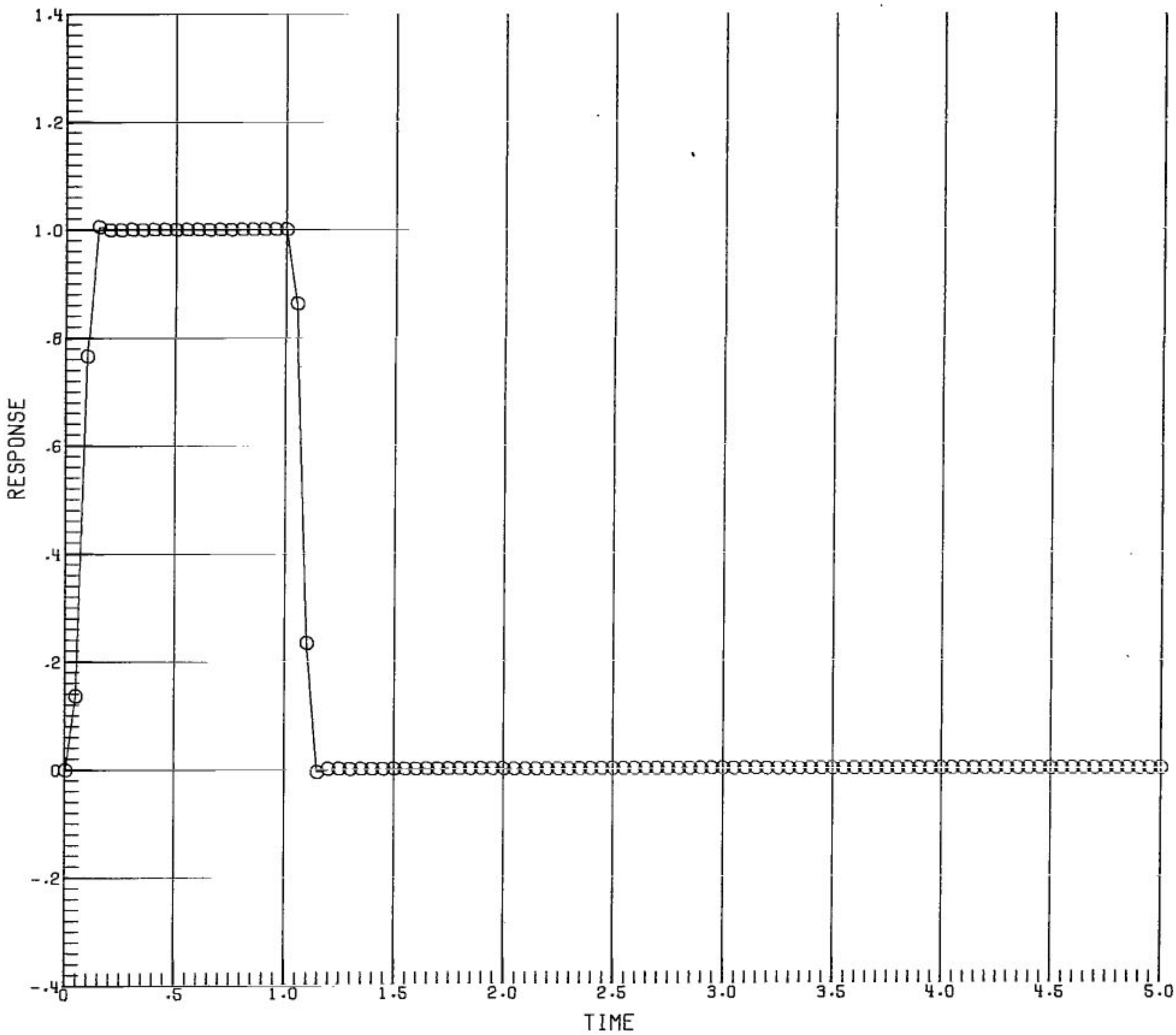

Figure 100.- Response of fifth-order Bessel filter with BT = 5.

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