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DIRECTIVE GAIN OF CIRCULAR TAYLOR PATTERNS

D.C.F. Wu and R.C. Rudduck

The Ohio State University

ElectroScience Laboratory

(formerly Antenna Laboratory)

Department of Electrical Engineering

Columbus, Ohio 43212

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NOTICES

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ABSTRACT

The directive gain of circular Taylor patterns is determined. It is shown that so-called optimum patterns, i.e., uniform sidelobes in all planes, are severely limited for planar apertures because of excessive sidelobe power. It is shown that optimum directive gain of the circular Taylor pattern with a given sidelobe level can be obtained by appropriate design. Two parameters, A, a quantity uniquely related to the design sidelobe level and $\bar{n}$ a number controlling the degree of uniformity of the sidelobes have been determined to achieve the optimum directive gain.

CONTENTS

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I. INTRODUCTION

The directive gain of the circular Taylor pattern is examined in this publication. The radiation pattern of the Taylor distribution has a specified number of equal level sidelobes followed by tapered lobes. This research was motivated by consideration of the design of large ground based antennas for space communications.

Over the past two decades of antenna research, considerable attention has been devoted to the design of high gain, narrow beam antennas. It is well known that the Chebyshev pattern gives the optimum beamwidth to sidelobe level relationship. The true Chebyshev patterns are applicable only to arrays, not to continuous aperture distributions. Furthermore, the continuous aperture equivalent of the Chebyshev or the ideal Taylor pattern is not physically realizable. However, Taylor[1] introduced a physically realizable pattern which is obtained by approximating the Chebyshev or ideal Taylor pattern by nearly equal sidelobe level out to a transition point $\bar{n}$, beyond which the sidelobe level is tapered as sin (x)/x. As Taylor pointed out, the ideal space factor can be practically approximated by choosing the transition point $\bar{n}$ within the visible region, thus avoiding supergain. However, patterns with nearly uniform sidelobes throughout the visible region are not optimum with respect to directive gain.

Hansen[2] has determined the directive gain for Taylor line source patterns, and has shown the limitations of long line sources. His results are also applicable to rectangular planar apertures for which the pattern is the product of two perpendicular Taylor line sources. However, the latter do not have the optimum beamwidth to sidelobe level relationship, because even the near sidelobes are depressed outside the principal planes. The circular Taylor pattern by its circular symmetry approximates the optimum beamwidth to sidelobe level relationship.[3,4] Tseng and Cheng[5] have also derived the uniform sidelobe pattern (optimum Chebyshev pattern) for a rectangular planar aperture which is optimum with respect to the beamwidth to sidelobe level relationship.

In this paper directive gain is determined for the circular Taylor patterns, including both approximate and ideal Taylor space factors, i.e., only the omnidirectional element pattern is considered. It is shown that the optimum Chebyshev (uniform sidelobes in all planes) or the approximation to the ideal Taylor space factor (nearly uniform visible sidelobes) is more severely limited for planar apertures, either rectangular or circular, than for line source apertures. For example, a 20 dB sidelobe level design with transition point near the edge of the visible region gives poor directive gain for circular apertures which exceed 10λ in diameter whereas a line source may be useable to 100λ. Further, it is seen that maximum directive gain of the circular Taylor pattern is achieved for a given sidelobe level by appropriate choice of transition point $\bar{n}$.

II. DIRECTIVE GAIN ANALYSIS FOR CIRCULAR APERTURES

A. Radiation Pattern Functions

The normalized radiation pattern of the ideal Taylor distribution[3,4] (all sidelobes uniform) for circular apertures is given by

$$(1) \qquad F(u,A) = \begin{cases} \dfrac{\cosh \pi\sqrt{A^2 - u^2}}{\eta} & \text{main beam} \\[2ex] \dfrac{\cos \pi\sqrt{u^2 - A^2}}{\eta} & \text{sidelobes} \end{cases}$$

where $u = \frac{2a}{\lambda} \sin \theta$ and a is the radius of the aperture. Parameter A is related with the design sidelobe ratio η by

$$(2) \qquad \eta = \cosh \pi A$$

Taylor[3] introduced the following pattern function to approximate the ideal pattern:

$$(3) \qquad F(u,A,\bar{n}) = \begin{cases} \dfrac{\cosh \pi\sqrt{A^2 - \left(\frac{u}{\sigma}\right)^2}}{\eta} & \text{main beam} \\[2ex] \dfrac{\cos \pi\sqrt{\left(\frac{u}{\sigma}\right)^2 - A^2}}{\eta} & \text{uniform sidelobes} \\[2ex] \left(\dfrac{2}{\pi}\right)^{\frac{3}{2}} \dfrac{\sin \pi\left(u-\frac{1}{4}\right)}{\eta \; u^{\frac{3}{2}}} \displaystyle\prod_{n=1}^{\bar{n}-1} \dfrac{u_n^2}{\sigma^2\left[A^2 + \left(n-\frac{1}{2}\right)^2\right]} & \text{tapered sidelobes} \end{cases}$$

where $\bar{n}$ is a design parameter with $\bar{n}-1$ uniform sidelobes, u_n is the n^{th} zero of the first order Bessel function and σ, the beam broadening factor is given by

(4) $$\sigma = \frac{u_{\overline{n}}}{\sqrt{A^2 + \left(\overline{n} - \frac{1}{2}\right)^2}}$$

B. Directive Gain Calculation

The directive gain of a uniformly illuminated circular aperture distribution is given by

(5) $$G_o = \frac{4\pi^2 a^2}{\lambda^2}$$

The directive gain of a circularly symmetric aperture is given by

(6) $$G = \frac{\lambda^2}{\pi a^2 \int_0^{2\pi} \int_0^{\pi/2} |F(u)|^2 \sin\theta \, d\theta \, d\phi}$$

$$= \frac{\lambda}{\pi^2 a \int_0^{2a/\lambda} |F(u)|^2 \frac{u}{\sqrt{\left(\frac{2a}{\lambda}\right)^2 - u^2}} \, du}$$

1. Ideal Taylor Distribution (Dolph-Chebyshev)

The integral of Eq. (6) is separated into two parts as given in Eqs. (7) and (8). The main beam contribution can be evaluated in closed form, giving

(7) $$I_1 = \int_0^A \frac{\cosh^2 \pi\sqrt{A^2 - u^2}}{\eta^2} \frac{u}{\sqrt{\left(\frac{2a}{\lambda}\right)^2 - u^2}} \, du$$

$$\approx \frac{\lambda}{2a\pi^2\eta^2} \int_0^{\pi A} x \cosh^2 x \, dx$$

$$= \frac{\lambda}{2a\pi^2\eta^2} \left\{ \frac{\pi A}{4} \sinh 2\pi A - \frac{1}{8} \cosh 2\pi A + \frac{\pi^2 A^2}{4} + \frac{1}{8} \right\}$$

The sidelobe contribution is given by

$$(8) \qquad I_2 = \int_A^{2a/\lambda} \frac{\cos^2 \pi\sqrt{u^2 - A^2}}{\eta^2} \frac{u}{\sqrt{\left(\frac{2a}{\lambda}\right)^2 - u^2}} du$$

$$= \int_0^{\sqrt{\left(\frac{2a}{\lambda}\right)^2 - A^2}} \frac{x \cos^2 \pi x}{\eta^2 \sqrt{\left(\frac{2a}{\lambda}\right)^2 - A^2 - x^2}} dx$$

$$= \frac{1}{\eta^2} \left\{ \sqrt{\left(\frac{2a}{\lambda}\right)^2 - A^2} - \int_0^1 \pi \left[\left(\frac{2a}{\lambda}\right)^2 - A^2\right] \sqrt{1 - y^2} \sin 2\pi \sqrt{\left(\frac{2a}{\lambda}\right)^2 - A^2}\, y \, dy \right\}$$

The last integral can be readily evaluated since $\sqrt{1-y^2}$ is a well defined, slowly varying function. The numerical technique used consists basically of summing maximum values of the slowly varying modulus of the rapidly oscillating sinusoidal function times the area under one lobe of the sine function. Hence the directive gain of the circular Taylor distribution is given by

$$(9) \qquad G = \frac{\lambda}{\pi^2 a \, (I_1 + I_2)}$$

2. Taylor distribution

The gain of the circular Taylor distribution can be calculated in a similar manner. The main beam contribution can be accurately approximated by using the beam broadening factor σ, giving

$$(10) \qquad T_1 = \int_0^{\sigma A} \frac{u \cosh^2 \pi\sqrt{A^2 - \left(\frac{u}{\sigma}\right)^2}}{\eta^2 \sqrt{\left(\frac{2a}{\lambda}\right)^2 - u^2}} du$$

$$\approx \sigma^2 I_1 .$$

The contribution of the uniform sidelobe region can be approximated in closed form if less than 1/2 of the visible sidelobes are uniform; i.e., $\bar{n} < a/\lambda$. This gives

$$(11) \qquad T_2 = \int_{\sigma A}^{u_{\bar{n}}} \frac{\cos^2 \pi \sqrt{\left(\frac{u}{\sigma}\right)^2 - A^2} \; u \, du}{\eta^2 \sqrt{\left(\frac{2a}{\lambda}\right)^2 - u^2}}$$

$$\approx \frac{\lambda \sigma^2}{2a\pi^2 \eta^2} \int_0^{\pi(\bar{n}-0.5)} x \cos^2 x \, dx$$

$$= \frac{\lambda \sigma^2}{2a\pi^2 \eta^2} \left\{ \frac{[(\bar{n}-0.5)\pi]^2}{4} - \frac{1}{4} \right\}$$

The contribution of the tapered sidelobe region is given by

$$(12) \qquad T_3 = B(\bar{n},A) \int_{\bar{n}+\frac{1}{4}}^{2a/\lambda} \frac{\sin^2 \pi \left(u - \frac{1}{4}\right) \; du}{\eta^2 u^2 \sqrt{\left(\frac{2a}{\lambda}\right)^2 - u^2}}$$

$$= -\left(\frac{\lambda}{2a}\right)^2 B(\bar{n},A) \int_{\bar{n}+\frac{1}{4}}^{2a/\lambda} \frac{\sqrt{\left(\frac{2a}{\lambda}\right)^2 - u^2}}{\eta^2 u} \; \pi \cos 2\pi u \, du$$

where

$$(13) \qquad B(\bar{n},A) = \left(\frac{2}{\pi}\right)^3 \left(\prod_{n=1}^{\bar{n}-1} \frac{u_n^2}{\sigma^2 \left[A^2 + \left(n - \frac{1}{2}\right)^2 \right]} \right)^2$$

The integrals of Eq. (12) can also be accurately evaluated by numerical integration. Hence the gain of the Taylor distribution is given by

$$(14) \qquad G = \frac{\lambda}{\pi^2 a (T_1 + T_2 + T_3)} ,$$

where T_1, T_2 and T_3 are given by Eqs. (10) to (12) respectively.

C. Results

Values of normalized directive gain for circular Taylor patterns are given in Table I for various sidelobe ratios and for various transition values $\bar{n}$ ranging from 3 to 10. It is to be noted that the normalized gain does not vary for large circular apertures, say $a \geq 10\lambda$. The transition point $\bar{n}$ is chosen to be less than a/λ (less than half of the sidelobes are uniform) due to the approximation in Eq. (11). However, for $\bar{n} > a/\lambda$ (more than half of the sidelobes are uniform), the normalized directive gain approaches that of the Taylor Ideal circular patterns.

The normalized directive gain of various sidelobe ratios of the ideal circular Taylor patterns is plotted in Fig. 1 as a function of aperture radius a. It is noted that the directive gain of the ideal pattern (or practical distributions with more than 1/2 of the visible sidelobes uniform) is severely limited for large planar apertures. For example, the directive gain is less than 30% of that available from a uniform aperture distribution for aperture diameters $2a > 10\lambda$ (20 dB sidelobes or higher), or $2a > 30\lambda$ (30 dB sidelobes or higher). As evident from the figure the normalized directive gain for a fixed sidelobe level decreases rapidly with increasing aperture size. The useable range is considerably greater for the line source case, e.g., the normalized directive gain of an ideal Taylor line source with 20 dB sidelobes is greater than 50% for lengths up to 100λ.[2]

As seen from Table I, appropriate choice of the transition point $\bar{n}$ yields a maximum value for the directive gain of the circular Taylor pattern with a given sidelobe design. These optimum directive gain values and corresponding $\bar{n}$ values are given in Table II. Although the directive gain is not particularly sensitive to choice of $\bar{n}$, substantial departure from optimum $\bar{n}$ will result in decreased gain. Optimum directive gain represents a tradeoff between beam width and sidelobe power. As $\bar{n}$ is increased the beamwidth decreases, tending to increase gain; however, sidelobe power also increases, thus limiting the maximum value of directive gain.

For comparison the optimum directive gain for the rectangular planar aperture (two perpendicular Taylor line sources) as determined by Hansen[2] is also shown in Table II. It is noted that the optimum directive gain for the circular aperture is higher than that of the rectangular aperture, especially for larger sidelobe ratios.

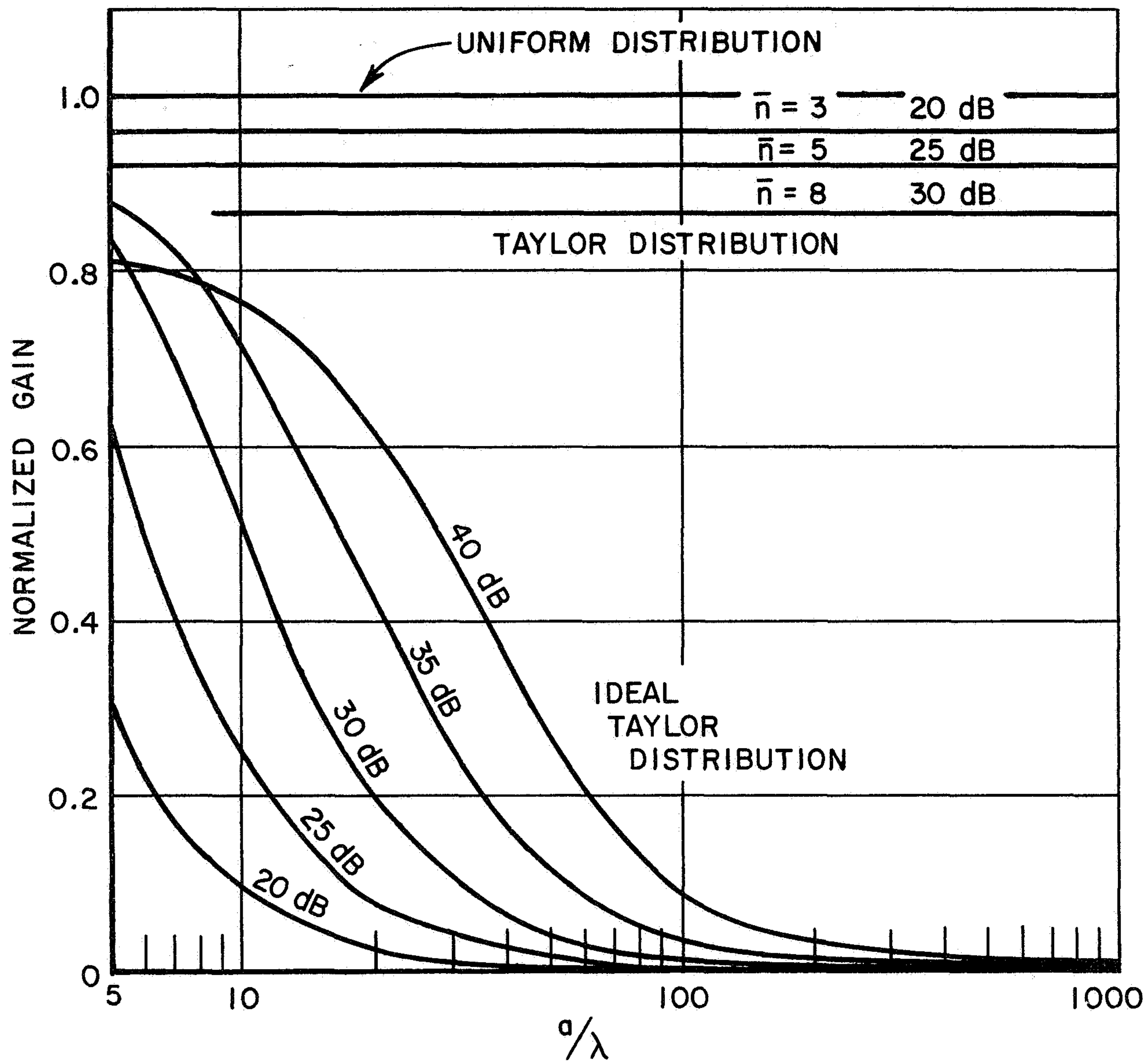

Fig. 1. Normalized gain vs radius of circular aperture.

TABLE I

Sidelobe level in dB	$\bar{n}$ = 3	4	5	6	8	10
15	0.8752	0.7266	0.5838	0.4671	0.3074	0.2130
18	0.9552	0.8839	0.7832	0.6782	0.4978	0.3683
20	0.9614	0.9327	0.8713	0.7957	0.6325	0.4960
22	0.9455	0.9441	0.9163	0.8691	0.7481	0.6247
25	0.9032	0.9185	0.9210	0.9090	0.8536	0.7757
30	0.8273	0.8378	0.8525	0.8631	0.8679	0.8550
35		0.7602	0.7695	0.7806	0.7982	0.8072
40		0.6985	0.6987	0.7052	0.7203	0.7322

TABLE II

Sidelobe level in dB	Circular Optimum Gain	Circular $\bar{n}$	Rectangular Optimum Gain	Rectangular $\bar{n}$
20	0.9614	3	0.941	9
25	0.9210	5	0.81	18
30	0.8679	8	0.5625	36

III. CONCLUSIONS

The directive gain is evaluated for circular Taylor distributions, both ideal and tapered space factors. The circular Taylor patterns offer a practical approximation to the optimum beamwidth-sidelobe relationship for planar apertures.

It is shown that the so-called optimum patterns, i.e., with all visible sidelobes uniform, are severely limited for planar apertures because of excessive sidelobe power. It is also shown that maximum directive gain can be achieved for the circular Taylor pattern by appropriate choice of the number of uniform sidelobes. The optimum directive gain is found to be higher for the circular Taylor than for the square Taylor pattern since the off-principle plane sidelobes are lower for the square Taylor.

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