
APOLLO COMMUNICATIONS SYSTEMS
TASK E-59C
TECHNICAL REPORT

LUNAR MULTIPATH
SIGNAL CHARACTERISTICS

NAS 9-8166

15 October 1969

Prepared for
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
MANNED SPACECRAFT CENTER
HOUSTON, TEXAS

Prepared by
Communications and Sensor Systems Department
Electronic Systems Laboratory

FACILITY FORM 602	N69-40569	
	(ACCESSION NUMBER)	(THRU)
	29	1
	(PAGES)	(CODE)
	CR-101974	30
	(NASA CR OR TMX OR AD NUMBER)	(CATEGORY)

TRW
SYSTEMS GROUP

11176-H368-R0-00

APOLLO COMMUNICATIONS SYSTEMS
TASK E-59C
TECHNICAL REPORT

LUNAR MULTIPATH
SIGNAL CHARACTERISTICS

NAS 9-8166

15 October 1969

Prepared for
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
MANNED SPACECRAFT CENTER
HOUSTON, TEXAS

Prepared by
H. W. Belles

Approved by H. D. Freeman
H. D. Freeman, Manager
Task E-59C

Approved by John DeVillier
John DeVillier, Manager
Communication and Sensor
Systems Department

TRW SYSTEMS GROUP

ACKNOWLEDGEMENTS

The data used in this report were obtained from J. D. Osborn and Michael D. Zuteck who are working on antenna problems caused by multipath under TRW Task E53. Credit should also be given to Richard Chan for checking the derivations.

CONTENTS

	Page
1.0 INTRODUCTION	1
2.0 LUNAR MULTIPATH FORMULAS	2
3.0 AMPLITUDE, PHASE, AND FREQUENCY MODULATION DUE TO MULTIPATHS.	8
4.0 SIMULATION AND MEASUREMENTS OF LUNAR MULTIPATH SIGNALS	21
REFERENCES	23
APPENDIX.	A-1

ILLUSTRATIONS

Figure Number		Page
1	Lunar Multipath Geometry for Specular Reflections	5
2	Multipath Beat Frequency Versus Spacecraft Position for 60 NM Circular Orbit	6
3	Reflected Power Versus Spacecraft Position for 60 NM Circular Orbit	7
4	Amplitude Modulation Versus Time from AOS for 60 NM Circular Orbit	13
5	Phase Modulation Versus Time from AOS for 60 NM Circular Orbit	14
6	Frequency Modulation Versus Time from AOS for 60 NM Circular Orbit	15
7	Multipath Beat Frequency Versus Time from AOS for Apollo 12 Descent Trajectory	16
8	Reflected Power Versus Time from AOS for Apollo 12 Descent Trajectory	17
9	Amplitude Modulation Versus Time from AOS for Apollo 12 Descent Trajectory	18
10	Phase Modulation Versus Time from AOS for Apollo 12 Descent Trajectory	19
11	Frequency Modulation Versus Time from AOS for Apollo 12 Descent Trajectory	20
12	Simulation and Measurement of Lunar Multipath Signals	22

1.0 INTRODUCTION

The purpose of this report is to determine the characteristics of signals which may occur due to lunar multipaths when the LM is in lunar orbit, and to devise a method of detecting and measuring lunar multipath signals in real-time.

The procedure is to determine the specular signal reflections which could be obtained from the lunar surface for the LM in a 60 NM circular orbit using the steerable antenna for communications. The characteristics of the signals which would be received at a ground station are then determined. Of particular interest are the phase or frequency modulations which could be detected in real-time.

Predictions of the amount and characteristics of multipath signals for the Apollo 12 spaceflight are also included in this report.

It is concluded as a result of the analysis in this report that lunar multipath signals may readily be detected at a ground station with a suitable FM discriminator.

2.0 LUNAR MULTIPATH FORMULAS

The lunar multipath formulas for a spacecraft orbiting the moon have been previously determined (References 1, 2). The derived equations are repeated below for the purpose of completeness. These formulas assume specular reflection.

The phase relationships of the direct and reflected waves will be determined by the multipath geometry as shown in Figure 1. The differences in the path lengths, p_1 and p_2 , for the reflected and direct waves as a function of the spacecraft angular position, ϕ_c , are determined from the following equations:

$$s = r_c \cos \phi_c - r_m \cos \phi_r$$

$$p_1 = s / \sin(2\phi_r)$$

$$p_2 = s / \tan(2\phi_r)$$

$$\Delta D = p_1 - p_2 = s \left(\frac{1}{\sin(2\phi_r)} - \frac{1}{\tan(2\phi_r)} \right)$$

$$2\phi_r = \phi_c + \cos^{-1} \left(\frac{r_m}{r_c} \cos \phi_r \right)$$

where

ϕ_c = angular position of spacecraft

s = distance between direct signal and reflected signal paths

ϕ_r = angular position of reflection point

r_m = radius of moon (938.5 nautical miles)

r_c = radius of spacecraft orbit

ΔD = difference in path lengths

The multipath "fade" or "beat" frequencies due to movement of the spacecraft will be

$$ff = \frac{1}{\lambda} \left| \frac{d(\Delta D)}{dt} \right| \quad \lambda = 5.6 \text{ inches}$$

The above equations are readily adaptable to computer solution with ff being determined as a function of time.

The relative amplitude of the reflected wave with respect to the direct wave for specular reflection is dependent upon the dielectric constant, ϵ_r , of the moon and the spherical divergence factor, D . In addition, the polarization of the incident wave will be affected, i.e., the right circular polarization of the incident wave may be changed to left circular. The equations for determining relative reflected amplitudes are as follows:

$$\frac{A'}{A} = - \frac{\tan(\phi - \phi')}{\tan(\phi + \phi')} \quad \vec{E} \text{ in incidence plane}$$

$$\frac{A'}{A} = - \frac{\sin(\phi - \phi')}{\sin(\phi + \phi')} \quad \vec{E} \text{ perpendicular to incidence plane}$$

$$\phi' = \sin^{-1} \frac{\sin \phi}{\epsilon_r}$$

where

A = amplitude of incident wave

A' = amplitude of reflected E vector component

ϕ = angle of incidence

ϕ' = angle of refraction (into medium)

ϵ_r = dielectric constant (3.0 for moon)

The spherical divergence factor, D , may be determined from the following:

$$D = \frac{r_m \sqrt{\cos(\phi_r) \sin(\phi_r)}}{\sqrt{[r_c \sin(2\phi_r - \phi_c) + p_1] r_c \cos(\phi_c)}}$$

Plots of fade frequency and reflected signal levels as a function of spacecraft angular position, ϕ_c , are shown in Figures 2 and 3 for a 60 NM circular orbit.

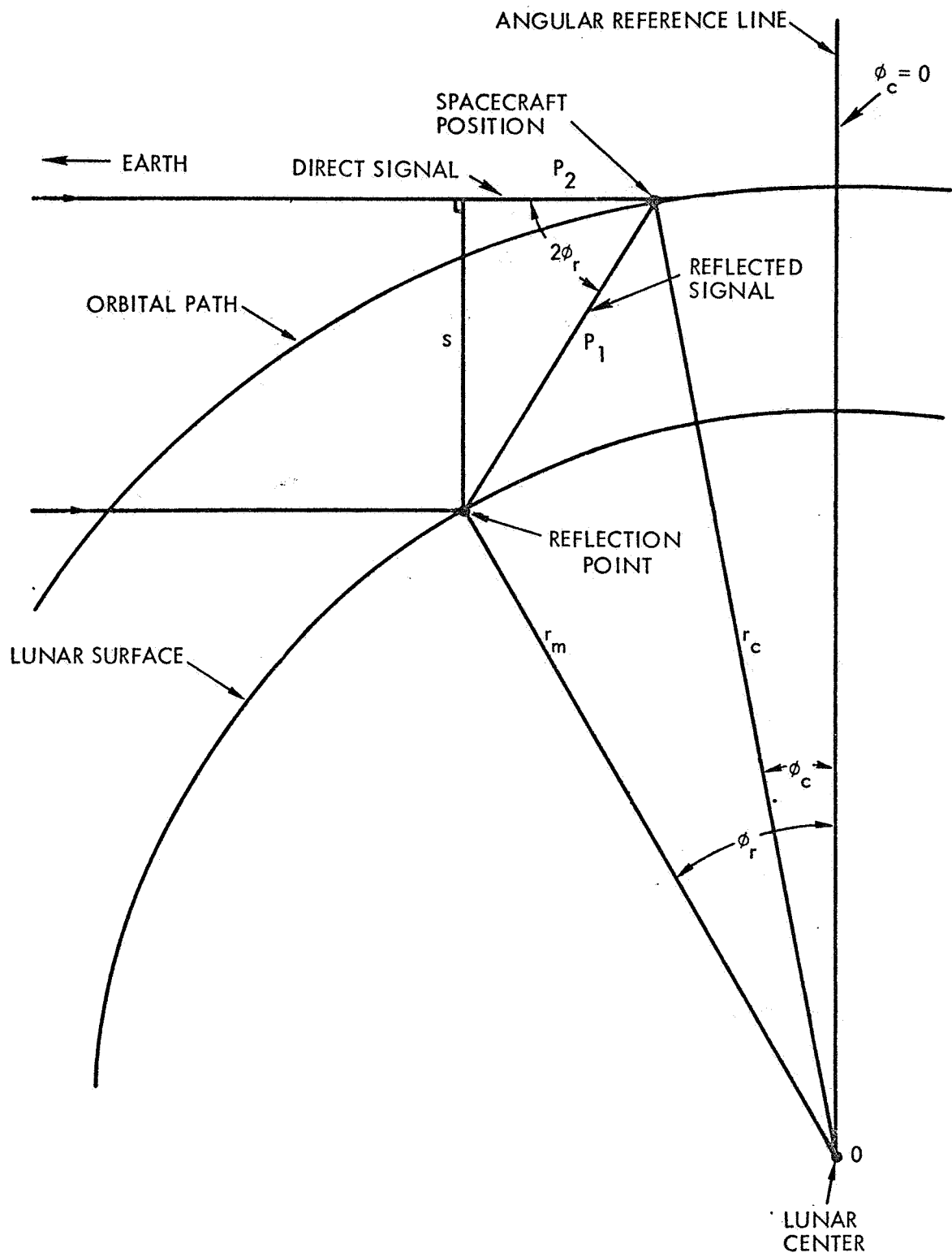


Figure 1. Lunar Multipath Geometry for Specular Reflections

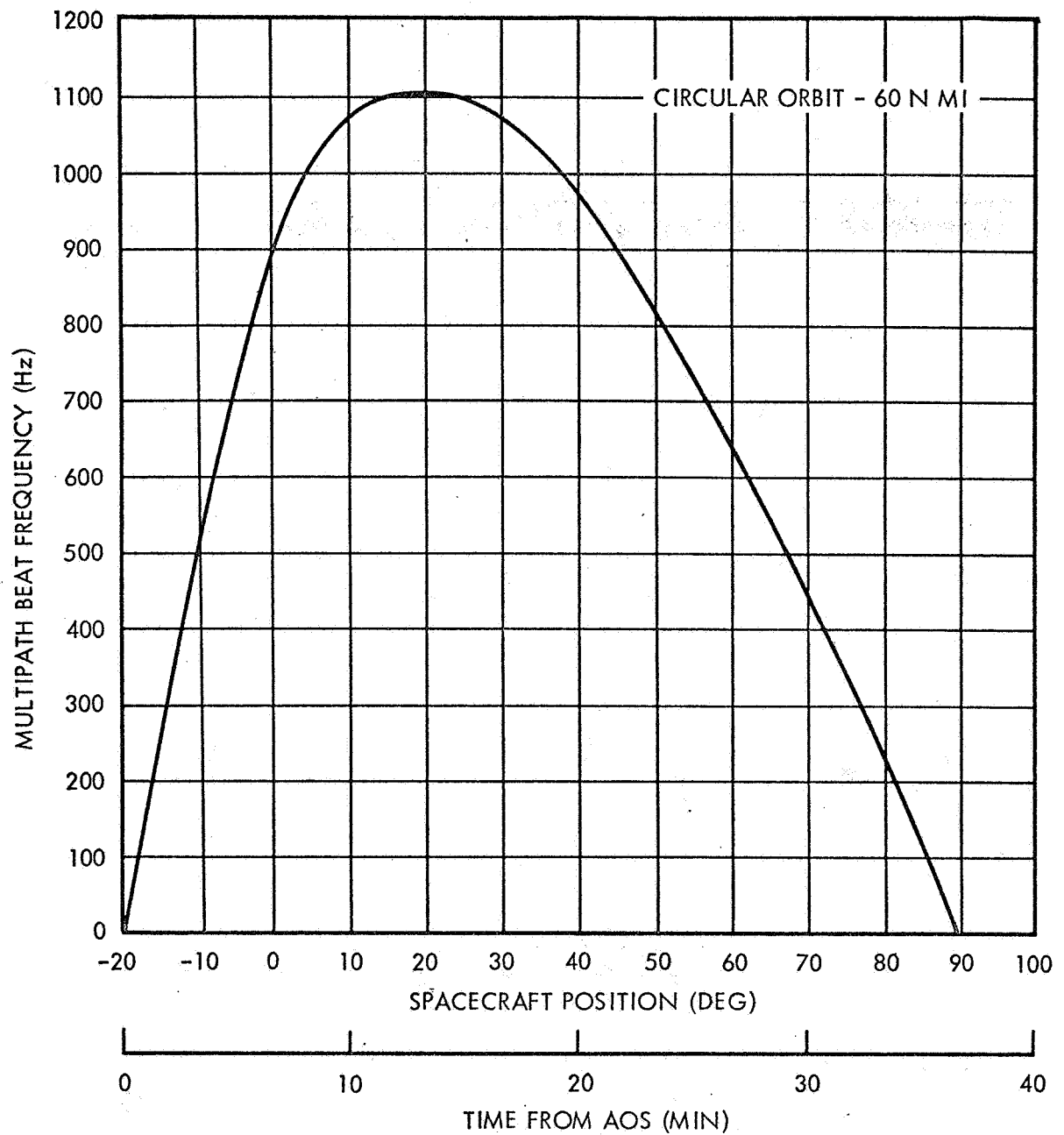


Figure 2. Multipath Beat Frequency Versus Spacecraft Position for 60 NM Circular Orbit

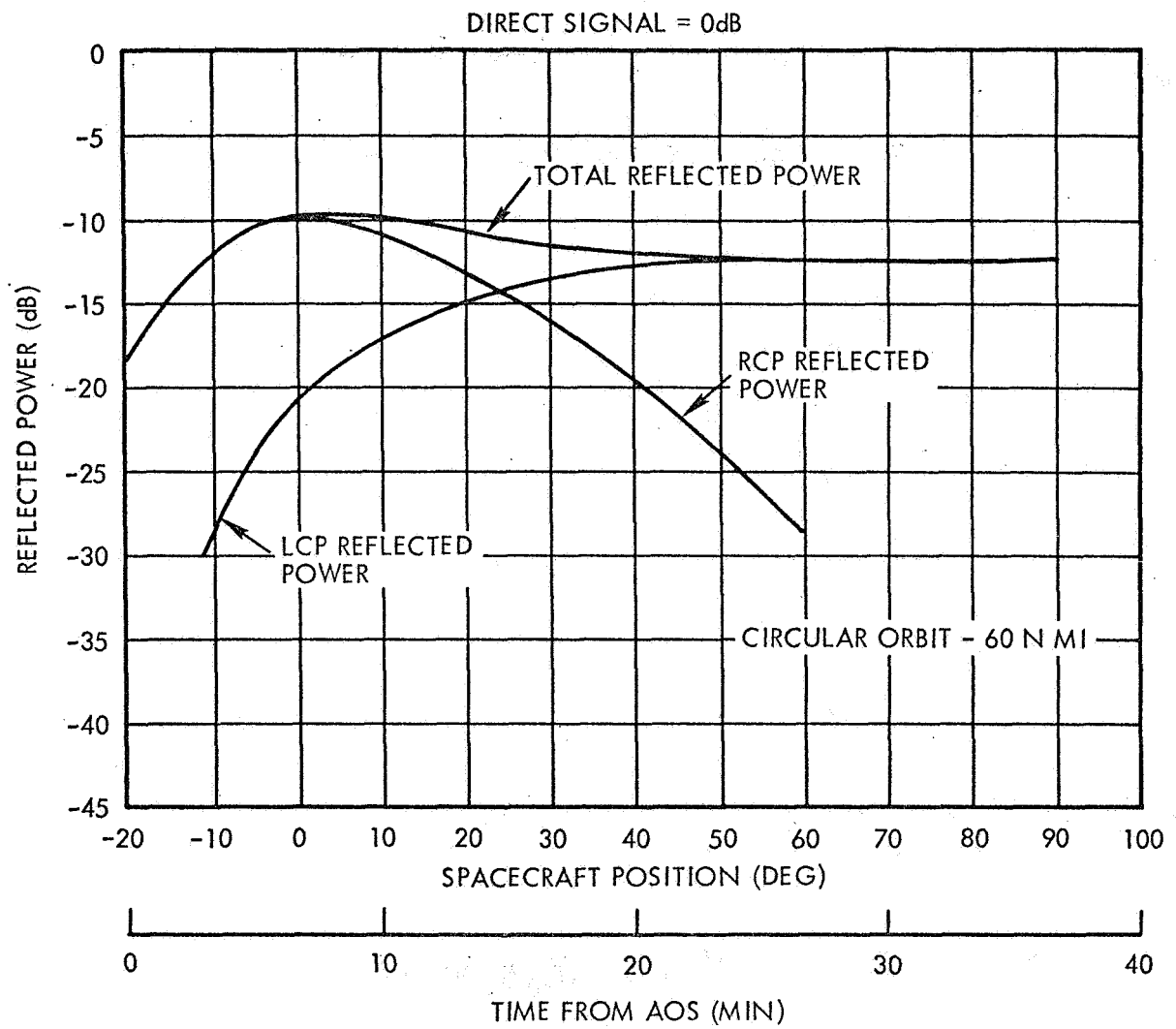


Figure 3. Reflected Power Versus Spacecraft Position for 60 NM Circular Orbit

3.0 AMPLITUDE, PHASE, AND FREQUENCY MODULATION DUE TO MULTIPATHS

The combined signal at the input to the spacecraft receiver will consist of the direct and reflected waves superimposed. The difference in path lengths, and thus the phase of the reflected signal relative to the direct signal, will increase from AOS until the spacecraft reaches a position directly between the earth and moon. The equation for the signal at the input to the spacecraft receiver may be expressed as follows:

$$e(t) = V_D \sin \omega t + V_R \sin (\omega t + m\phi t)$$

where

V_D = direct signal amplitude

V_R = reflected signal amplitude

m = phase modulation coefficient

ϕ = difference in path lengths in degrees

The amplitude of the reflected wave, V_R , will depend primarily on the dielectric constant of the moon, ϵ_R , the spherical divergence, D , the position of the spacecraft, and the antenna pattern of the spacecraft.

The phase modulation coefficient, m , is a function of the rate of change of the difference between the path lengths; i.e. $\frac{d(\Delta D)}{dt}$.

The above equation may also be written as

$$e(t) = V_D \sin \omega t + V_R \sin \left[2\pi \left(f + \frac{m\phi}{2\pi} \right) t \right]$$

Let

$$f_R = f + \frac{m\phi}{2\pi} \text{ and } \omega_R = 2\pi(f + \frac{m\phi}{2\pi})$$

Thus f_R will cause a heterodyne "fade" or "beat" frequency effect when combined with the carrier frequency.

Continuing with the trigonometric manipulations

$$\begin{aligned} e(t) &= V_D \sin \omega t + V_R \sin \omega_R t \\ &= V_D \sin \omega t + V_R \sin [\omega - (\omega - \omega_R)]t \\ &= V_D \sin \omega t + V_R \cos (\omega - \omega_R)t \sin \omega t - V_R \sin (\omega - \omega_R)t \cos \omega t \\ e(t) &= [V_D + V_R \cos (\omega - \omega_R)t] \sin \omega t - V_R \sin (\omega - \omega_R)t \cos \omega t \end{aligned}$$

The first term of the above equation is the "in phase" modulation component and the second term is the "quadrature" modulation component.

This signal may also be expressed as

$$e(t) = A(t) \sin [\omega t + \theta(t)]$$

where

$$\begin{aligned} A(t) &= \sqrt{[V_D + V_R \cos (\omega - \omega_R)t]^2 + V_R^2 \sin^2 (\omega - \omega_R)t} \\ &= \sqrt{V_D^2 + 2V_R V_D \cos (\omega - \omega_R)t + V_R^2} \\ &= V_D \sqrt{1 + 2V_R/V_D \cos (\omega - \omega_R)t + (V_R/V_D)^2} \\ \theta(t) &= \tan^{-1} \frac{-V_R \sin (\omega - \omega_R)t}{V_D + V_R \cos (\omega - \omega_R)t} \end{aligned}$$

The percentage amplitude modulation may be obtained by noting that

$$A_{\max} = V_D + V_R$$

$$A_{\min} = V_D - V_R$$

$$\begin{aligned} \% \text{ AM} &= \frac{\text{Max} - \text{Min}}{\text{Max} + \text{Min}} \times 100 = \frac{(V_D + V_R) - (V_D - V_R)}{(V_D + V_R) + (V_D - V_R)} \times 100 \\ &= \frac{V_R}{V_D} \times 100 \end{aligned}$$

The peak phase modulation can be shown to be

$$\Delta\theta = \theta(t)_{\max} = \sin^{-1}(V_R/V_D) \quad (\text{See Appendix})$$

Since the reflected signal amplitude is more than 13 dB less than the direct signal amplitude the amount of frequency modulation may be obtained from the equation*

$$\Delta f \approx f_m \Delta\theta$$

where

Δf = maximum frequency deviation

$f_m = f - f_R$ = modulating frequency

= "fade" or "beat" frequency

It is of interest to determine the amount of amplitude and phase modulation that would exist at the input to the spacecraft receiver in a typical LM lunar orbit, assuming that the steerable antenna were being used under ideal operating conditions. Curves showing these as a function

*As the amplitude of an interfering signal approaches the amplitude of a carrier signal the resultant frequency deviation becomes unsymmetrical and distorted. (Reference 3, page 359).

of time are shown in Figures 4 and 5 for a 60 nautical mile circular orbit assuming that the steerable antenna is maintained on track towards an earth station. These data are shown for 7 minutes after AOS. After this time the reflected signal is over 20° off the main lobe of the antenna and left circular polarization effects begin to occur. Since the side lobe sensitivity of the steerable antenna is not known for LCP only a gross estimate can be made that the reflected signal will be approximately 33 dB less than the direct signal. If so, this would cause 2 to 3% AM and 0.02-0.03 radians peak phase deviation. The curves in Figures 4 and 5 were obtained by a computer computation of the equations given in Sections 2 and 3. The peak phase deviation of 0.18 radians at 250 Hz due to multipath is approximately 10 to 15 dB below the peak phase deviation of the uplink signal and thus could cause significant interference. One effect of the multipath signal is that it phase modulates the VCO in the spacecraft receiver phase lock loop.

The VCO in the phase lock loop operates at a frequency of 19.02 MHz. Due to the PLL action the VCO will follow the frequency of the incoming signal up to a rate of 35 KHz per second. After converting the phase modulation due to multipath to frequency modulation it is seen that the VCO will be frequency modulated by the multipath beat frequency over virtually the entire range of multipath beat frequencies. (The specified noise bandwidth of the PLL is 1100 Hz). Since the VCO frequency is multiplied by 120 and thence transmitted to the earth station, the frequency and phase modulation of the downlink carrier are also multiplied by 120. Curves showing frequency modulation due to multipaths for a 60 nautical mile circular orbit are shown in Figure 6.

The downlink signal, in addition to having magnified frequency modulation due to multipath effects from the turned around uplink, will also have amplitude and phase modulations due to downlink multipath reflections. These will be superimposed on the spacecraft transmitted signals at the ground receiver. The downlink reflections will have the same amplitude as the uplink reflections. However, the fade frequencies will be slightly different from the uplink fade frequencies since the uplink and downlink carrier frequencies are 2101 MHz and 2282.5 MHz, respectively.

Predicted data for the Apollo 12 spaceflight are shown in Figures 7 through 11.

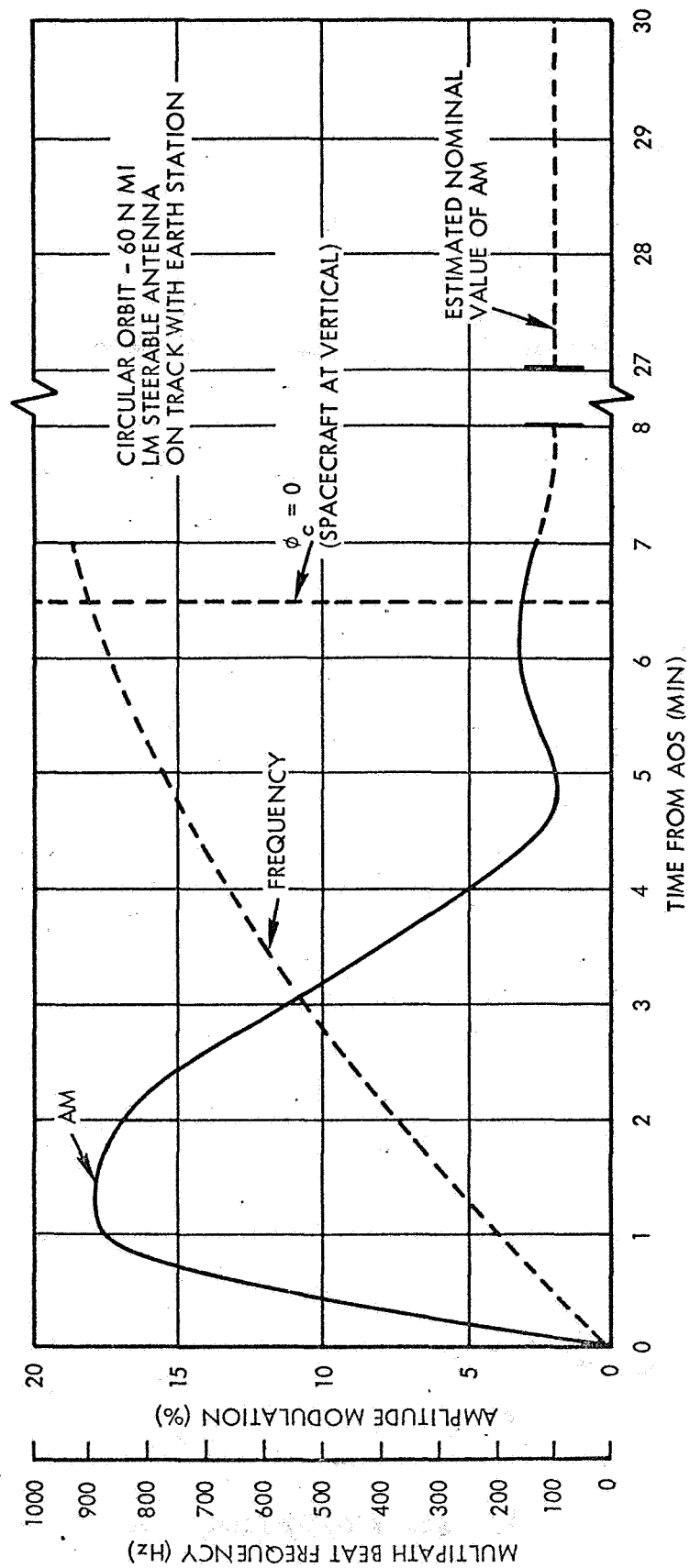


Figure 4. Amplitude Modulation Versus Time from AOS for 60 NM Circular Orbit

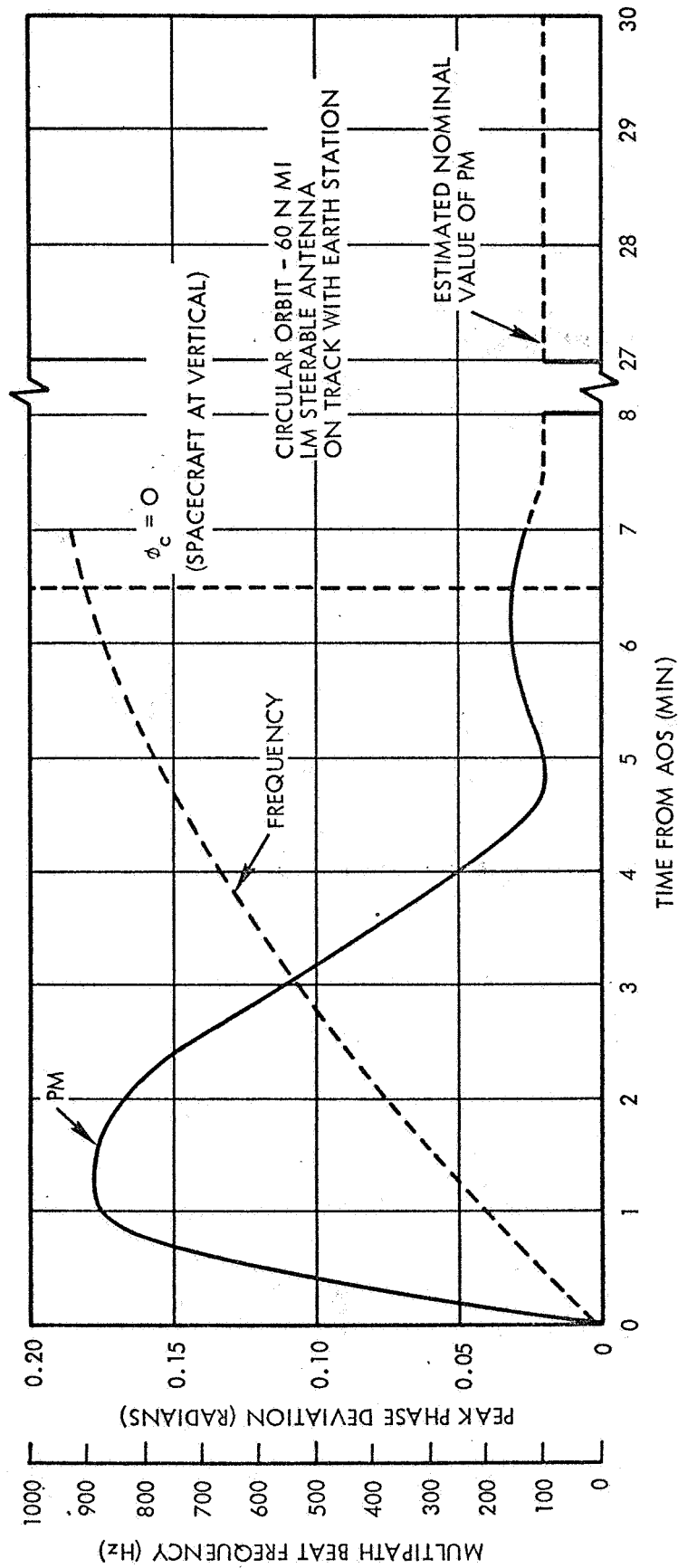


Figure 5. Phase Modulation Versus Time from AOS for 60 NM Circular Orbit

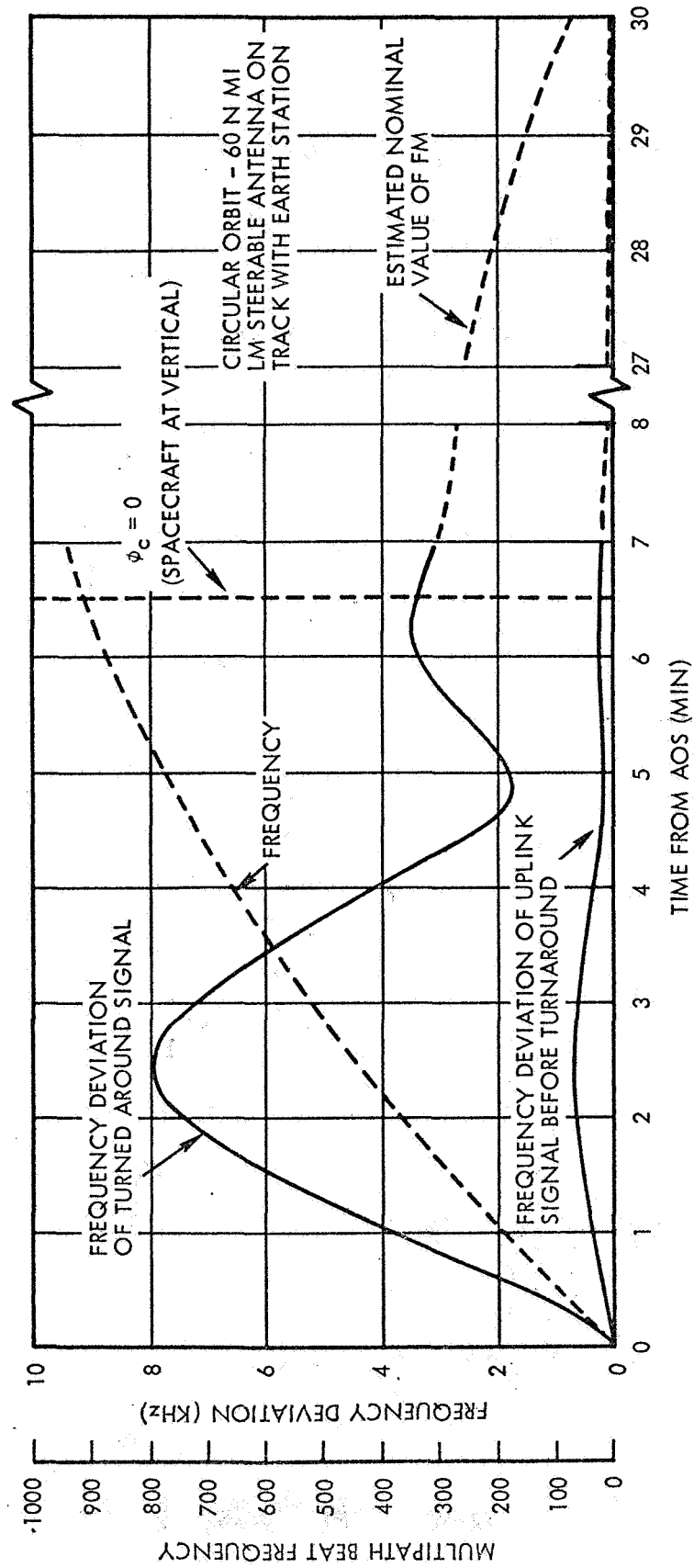


Figure 6. Frequency Modulation Versus Time from AOS for 60 NM Circular Orbit

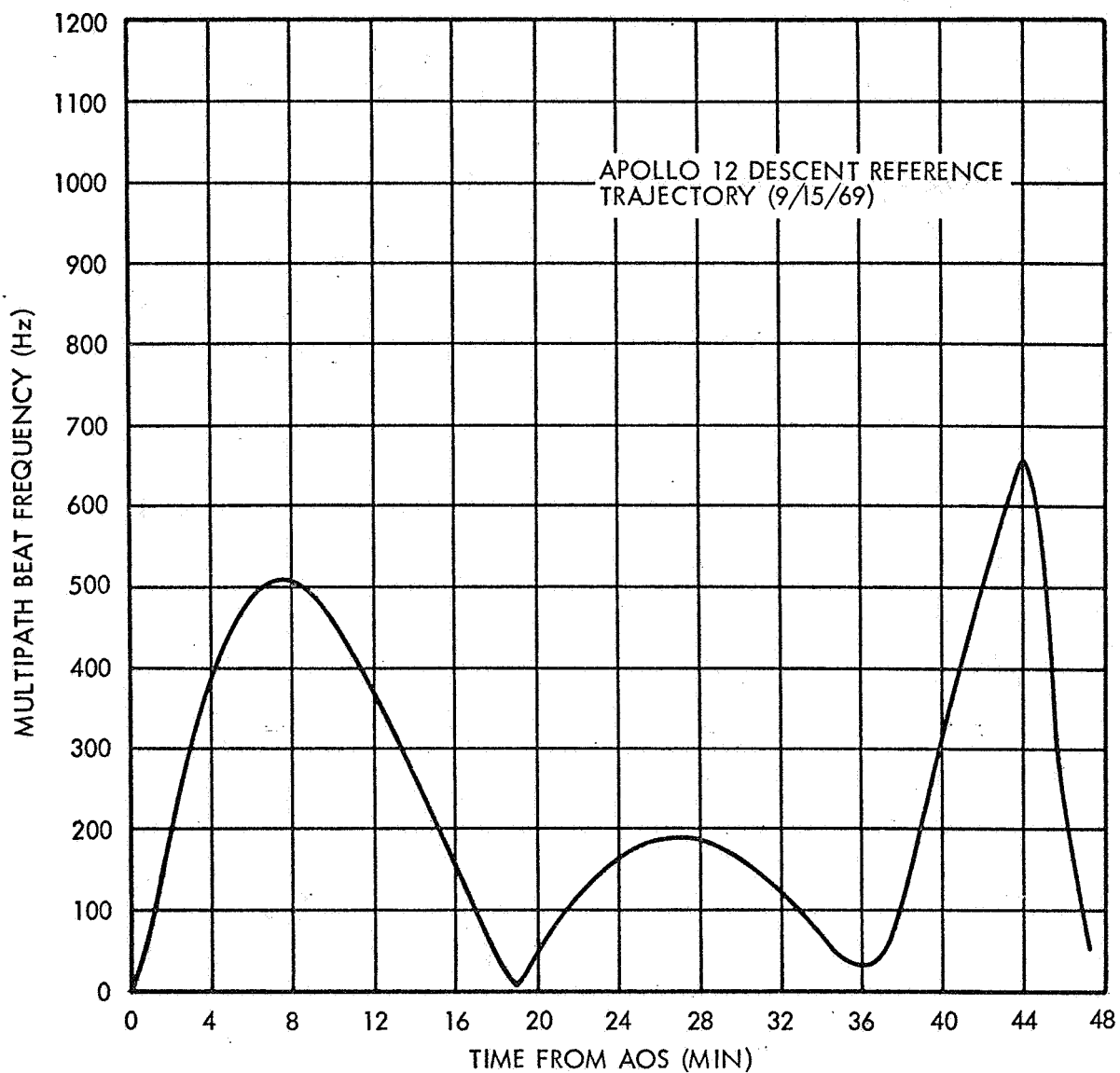


Figure 7. Multipath Beat Frequency Versus Time from AOS for Apollo 12 Descent Trajectory

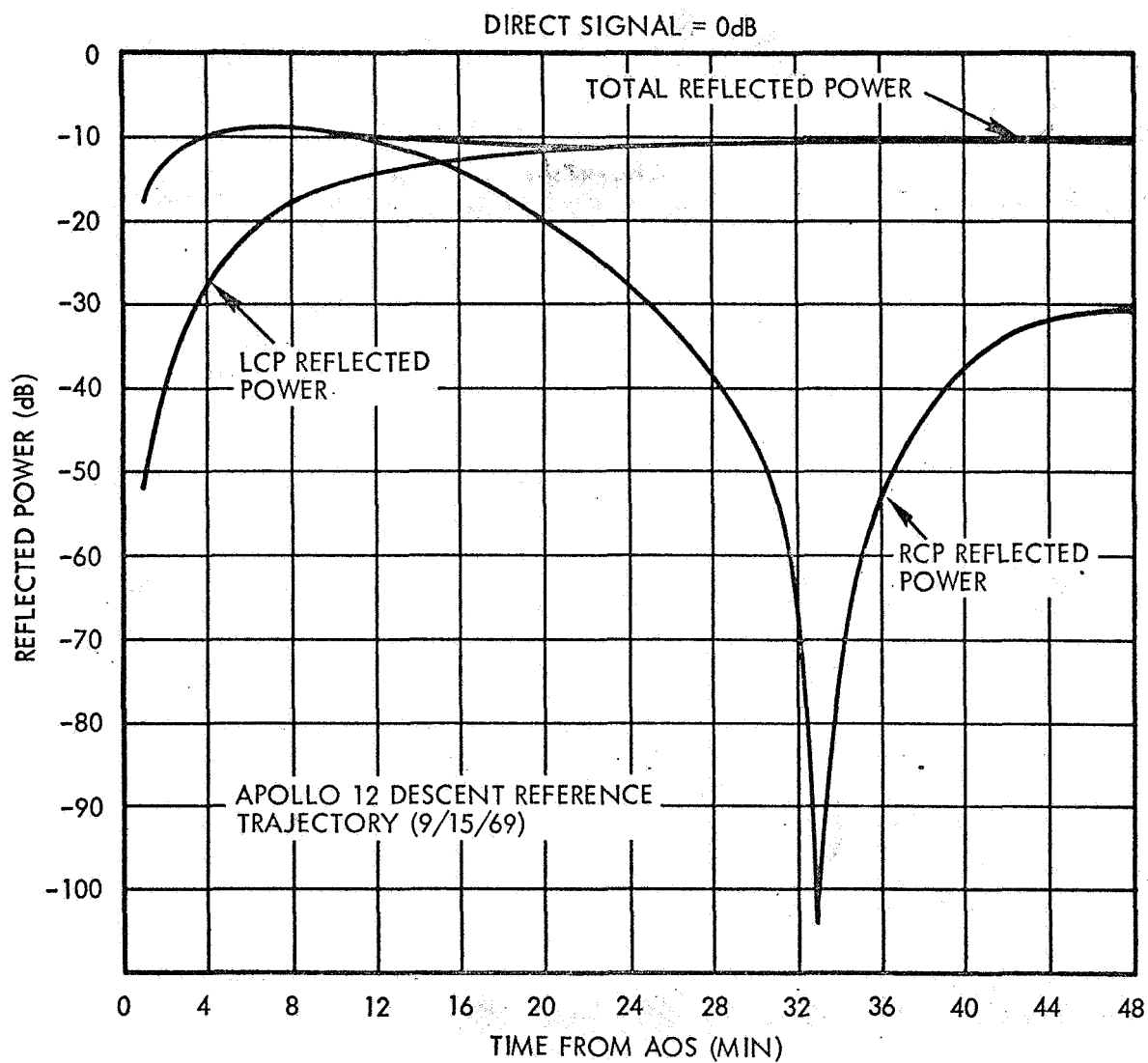


Figure 8. Reflected Power Versus Time from AOS for Apollo 12 Descent Trajectory

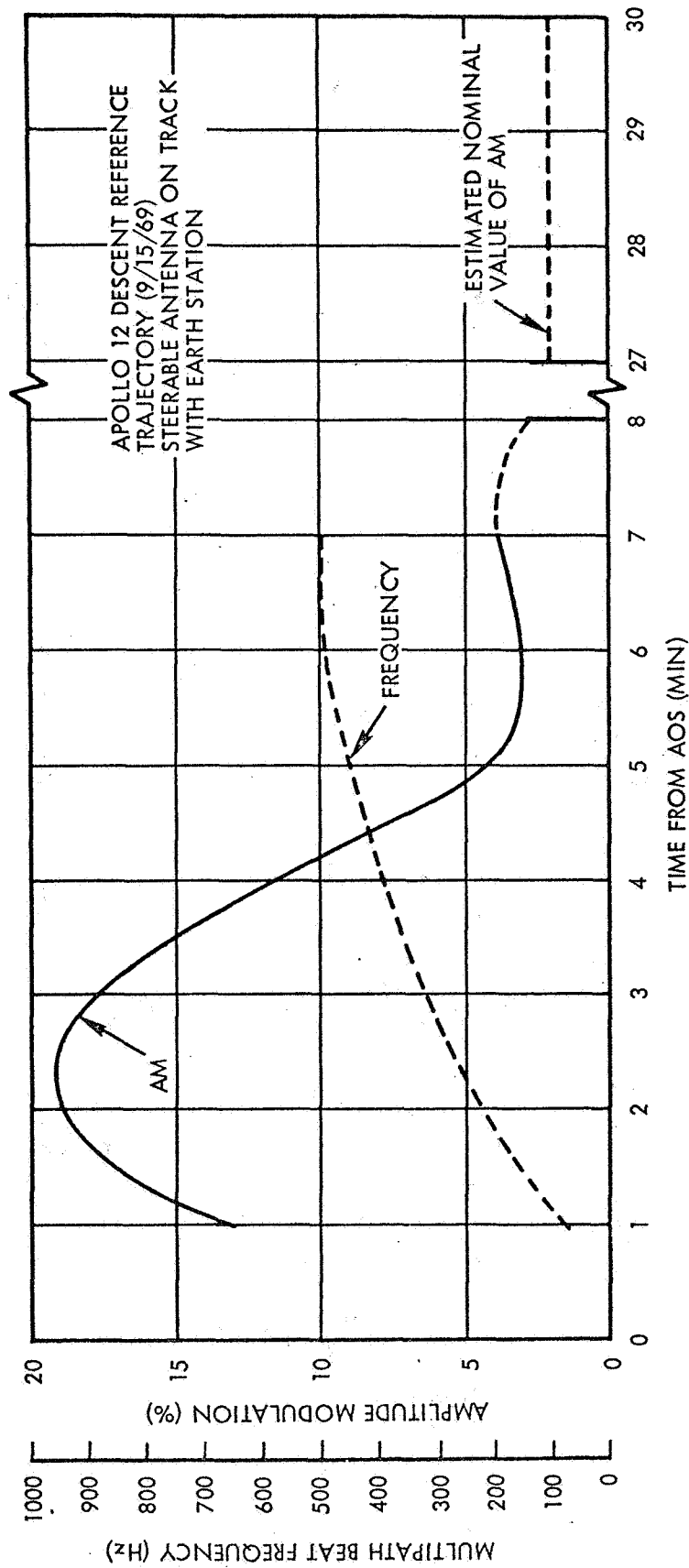


Figure 9. Amplitude Modulation Versus Time from AOS for Apollo 12 Descent Trajectory

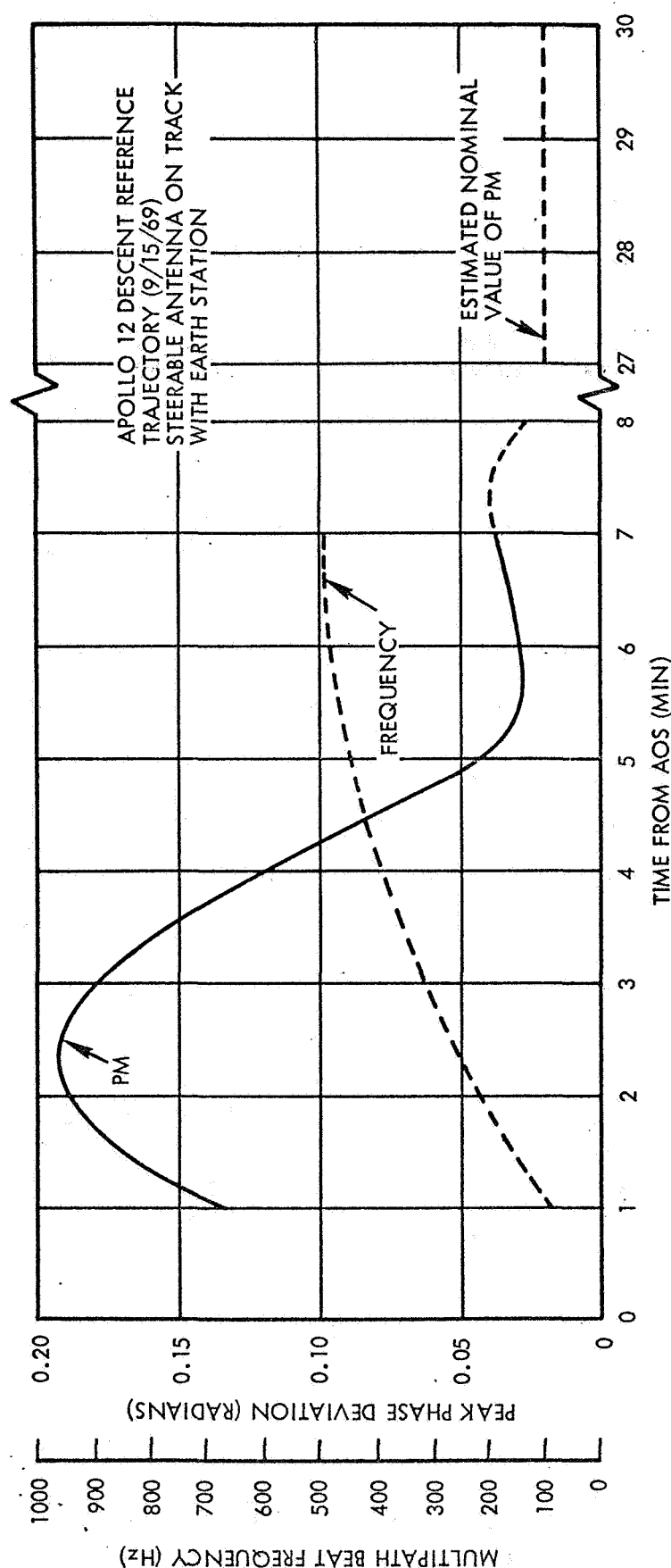


Figure 10. Phase Modulation Versus Time from AOS for Apollo 12 Descent Trajectory

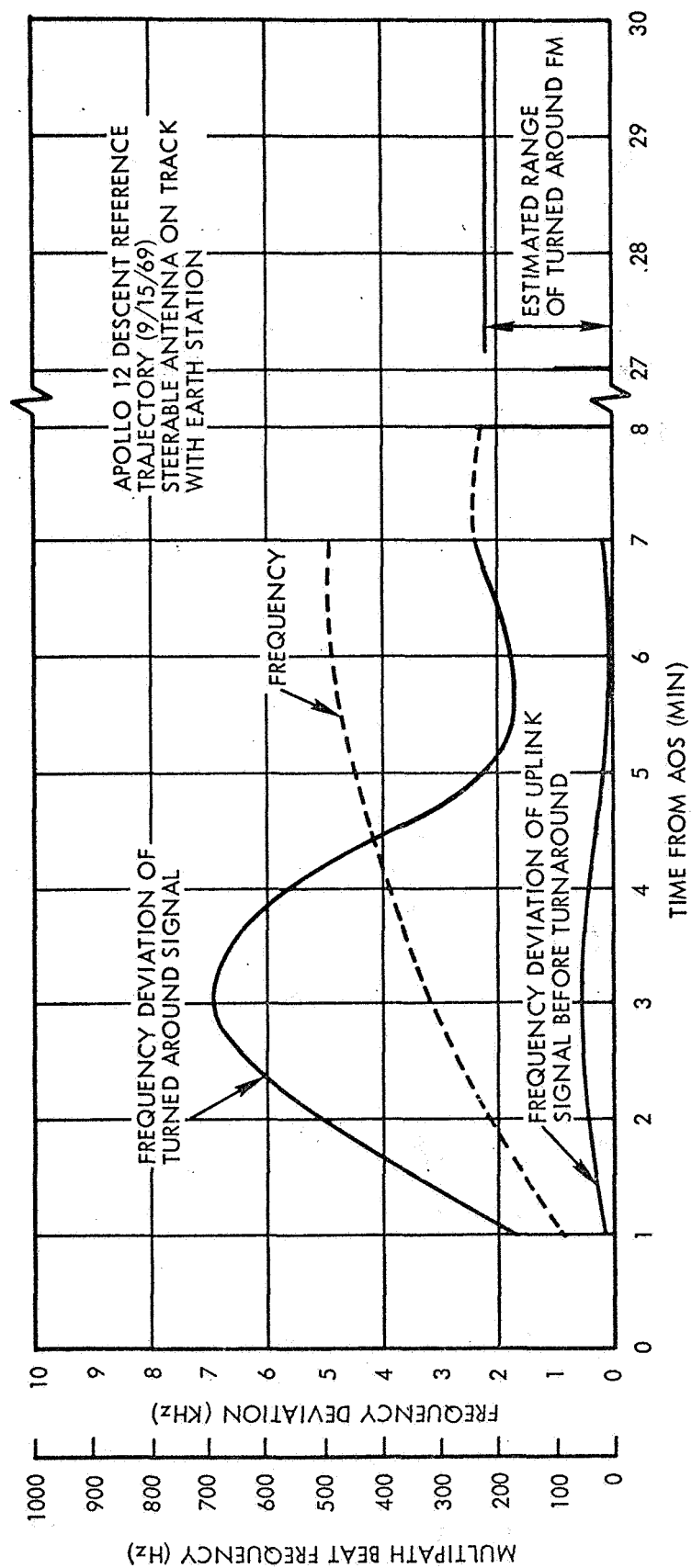


Figure 11. Frequency Modulation Versus Time from AOS for Apollo 12 Descent Trajectory

4.0 SIMULATION AND MEASUREMENTS OF LUNAR MULTIPATH SIGNALS

As shown in the sections above, a lunar multipath signal will have characteristics identical to those of a heterodyne signal and will cause frequency modulation with deviations up to 8 KHz on the transponder turned around signal.

Multipaths may be simulated by a beat frequency signal generator and may be detected by a frequency discriminator as shown in the diagram in Figure 12.

It is recommended that the configuration shown in Figure 12 be set up and measurements of frequency response, frequency deviation, and other significant characteristics be made.

It should be noted that the ground receiver phase lock loop will track out a portion of the received frequency modulation. How much will depend on the phase lock loop bandwidth setting and signal strength. The extent of ground receiver phase lock loop tracking under various conditions should also be ascertained.

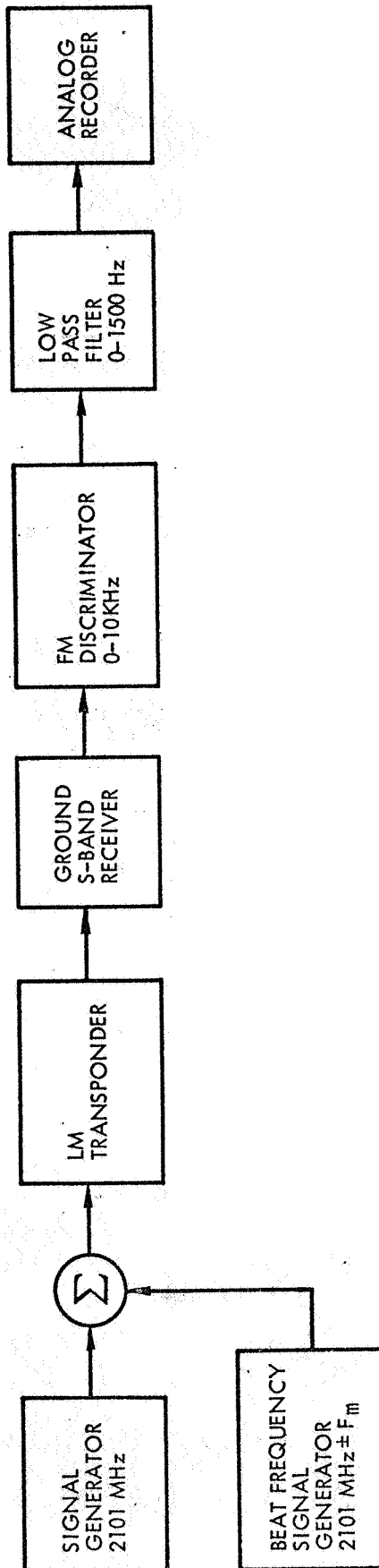


Figure 12. Simulation and Measurement of Lunar Multipath Signals

REFERENCES

1. Zuteck, Michael F., "An Investigation of the Lunar Multipath Problems Associated with the Apollo Spacecraft S-Band Tracking Antennas," TRW Systems Technical Report No. 11176-H238-R0-00, 29 May 1969 (Addendum 11 July 1969).
2. Osborn, J. D., Lee, P. H., "Analysis of the LM Steerable Antenna Tracking Anomaly on the Apollo 11 Mission," TRW Systems Report No. 11176-H309-R0-00, 8 August 1969.
3. Panter, Philip F., "Modulation, Noise, and Spectral Analysis", McGraw-Hill, 1965.

APPENDIX

DETERMINATION OF THE MAXIMUM PHASE DEVIATION FOR AN INTERFERING SIGNAL

Given the phase deviation

$$\theta(t) = \tan^{-1} \frac{V_R \sin(\omega - \omega_R)t}{V_D + V_R \cos(\omega - \omega_R)t}$$

the maximum phase deviation may be found by using the formula

$$\frac{d(\tan^{-1}x)}{dt} = \frac{1}{1+x^2} \frac{dx}{dt} = 0$$

Since $x^2 < \infty$

$$\frac{dx}{dt} = 0$$

and

$$\frac{d}{dt} \left[\frac{V_R \sin(\omega - \omega_R)t}{V_D + V_R \cos(\omega - \omega_R)t} \right] = 0$$

which gives

$$\cos(\omega - \omega_R)t = -\frac{V_R}{V_D}$$

from which

$$\sin(\omega - \omega_R)t = \frac{\pm \sqrt{V_D^2 - V_R^2}}{V_D}$$

Substituting in the original equation

$$\theta(t)_{\max} = \tan^{-1} \frac{V_R}{\sqrt{V_D^2 - V_R^2}} = \sin^{-1} \frac{V_R}{V_D}$$