

General Disclaimer

One or more of the Following Statements may affect this Document

- This document has been reproduced from the best copy furnished by the organizational source. It is being released in the interest of making available as much information as possible.
- This document may contain data, which exceeds the sheet parameters. It was furnished in this condition by the organizational source and is the best copy available.
- This document may contain tone-on-tone or color graphs, charts and/or pictures, which have been reproduced in black and white.
- This document is paginated as submitted by the original source.
- Portions of this document are not fully legible due to the historical nature of some of the material. However, it is the best reproduction available from the original submission.

Blanked

QSL-9229-3
17 October 1969

ADVANCED FEED SYSTEM
COMPONENT INTERACTION ANALYSIS
NAS7-749

QUARTERLY STATUS LETTER
PERIOD ENDING 30 SEPTEMBER 1969

Prepared By

LESack

L. E. Sack
Principal Engineer
System Dynamics

N69-40946

FACILITY FORM 802

(ACCESSION NUMBER)	(THRU)
<i>61</i>	<i>1</i>
(PAGES)	(CODE)
<i>CR-106450</i>	<i>12</i>
(NASA CR OR TMX OR AD NUMBER)	(CATEGORY)

Approved By

J. Friedman

J. Friedman
Program Manager
Small Engine Division

TABLE OF CONTENTS

FOREWORD		1
SUMMARY		1
INTRODUCTION		2
DISCUSSION		5
DEVELOPMENT OF REGULATOR EQUATIONS		5
DEVELOPMENT OF FLOW BENCH EQUATIONS		17
COMPUTER MECHANIZATION		19
SIMPLIFIED WATER EXPULSION MODEL		25
RESULTS		36
ANALOG MODEL RESULTS		36
SIMPLIFIED WATER EXPULSION MODEL		41
CONCLUSIONS		43
EXPENDITURES		43
PLANNED EFFORT		44
REFERENCES		45
APPENDIX I -		
REGULATOR EQUATIONS		AI-1
APPENDIX II -		
ANALOG DIAGRAMS FOR MM'69 REGULATOR		AII-1
APPENDIX III -		
SIMPLIFIED WATER EXPULSION MODEL		AIII-1

FOREWORD

This is the first of two quarterly status letters in compliance with Article II, Paragraph C, of Contract NAS7-749. This letter covers the period from 30 June 1969 to 30 September 1969.

SUMMARY

A dynamic model of the MM'69 pressure regulator has been developed. The development of the equations and their mechanization on the analog computer are presented herein. This model has been used to generate transient performance data with high, constant, upstream pressure, and a simplified downstream flow system and accumulator. This data indicates that poppet oscillations are present whenever the spring is operating in the region of large negative slope.

A simplified model of the regulator and water expulsion system has also been developed. This model and preliminary results obtained with it are presented herein.

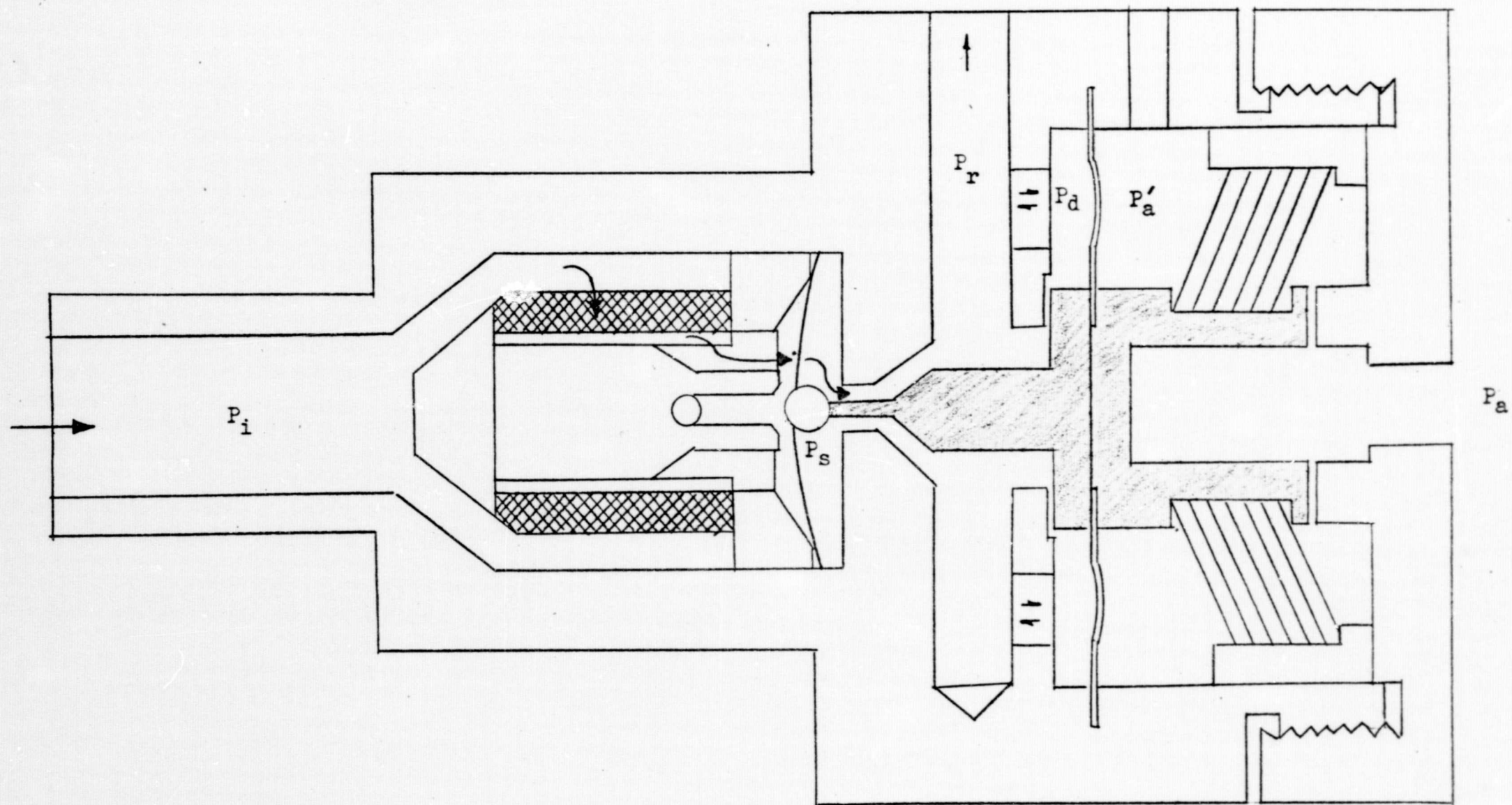
Dynamic equations of the flow bench system have also been developed. The basic equations are presented.

INTRODUCTION

The Mars Mariner '69 pneumatic pressure regulator, shown in Fig. 1, is the first example case considered in the Advanced Feed Systems Component Interaction Study. The first objective of the study is to model this regulator, together with two of its test systems, and to demonstrate good correlation with available test data. The modeling of this regulator, and to a lesser extent its test systems, is covered in this report, and represents the technical effort expended during the first quarter of the study.

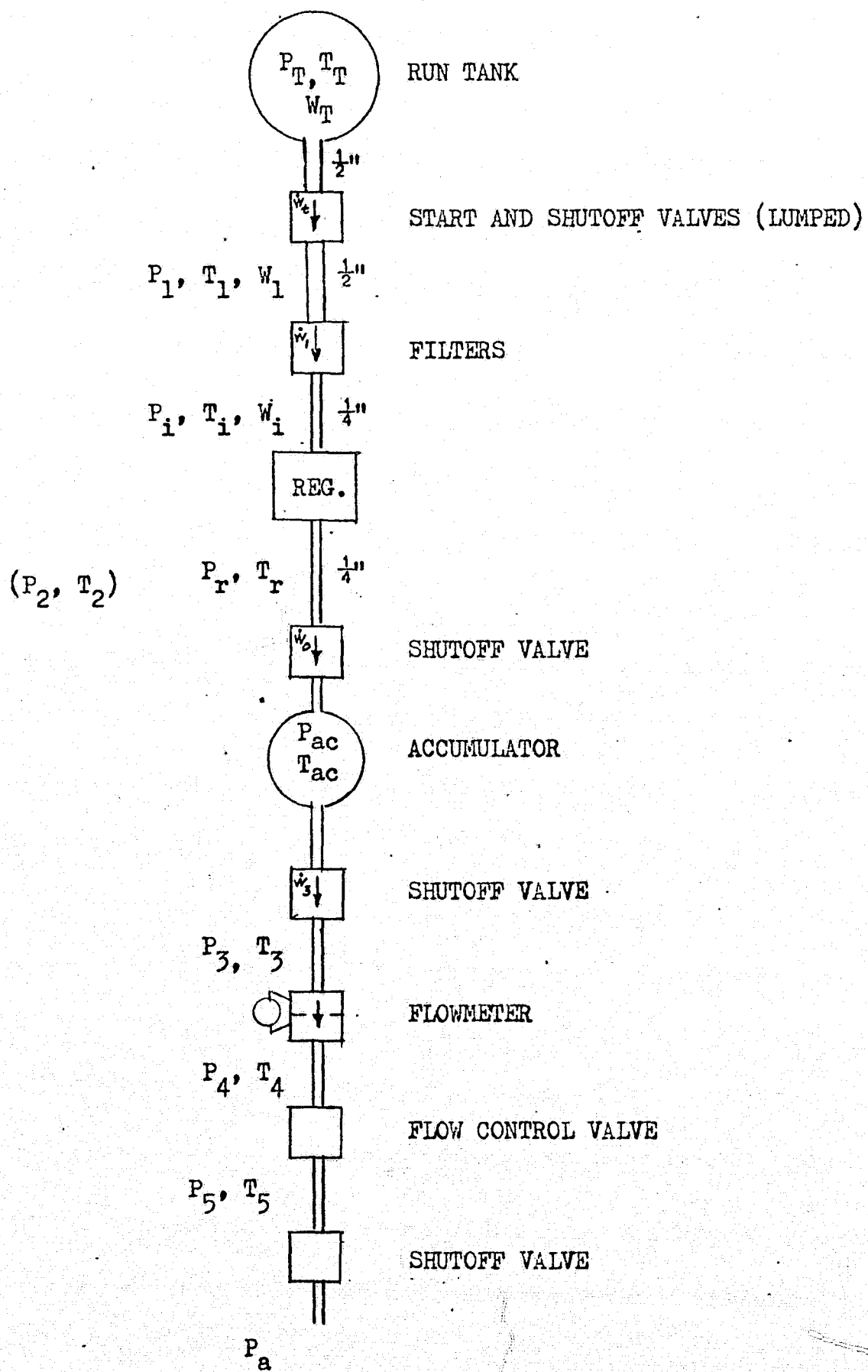
The regulator in Fig. 1 is a single stage design. The reference pressure, P_r , is maintained by flow modulation at the seat in response to position of the poppet assembly. Pressure force on the diaphragm is balanced by the compression of a set of Belleville springs. It is notable that since the regulator does not employ a separate sense line for reference pressure, the regulated quantity is the pressure in the flow system near the regulator exit, and not tank pressure.

The flow bench test system for the MM'69 regulator is shown schematically in Fig. 2. It is intended to develop a dynamic model of this system to be integrated with the regulator model. The dynamic equations for this system are presented herein.



SIMPLIFIED CROSS-SECTION OF MM '69 PRESSURE REGULATOR

FIGURE 1



FLOW BENCH TEST SYSTEM FOR MM '69 REGULATOR

FIGURE 2

DISCUSSION

DEVELOPMENT OF REGULATOR EQUATIONS

The MM'69 regulator shown in Fig. 1 consists of a gas flow system and an interacting mechanical system, i.e., the poppet. The dynamic model of the regulator can be obtained by developing a model of each of these systems separately, and joining them by coupling variables.

Gas System Equations

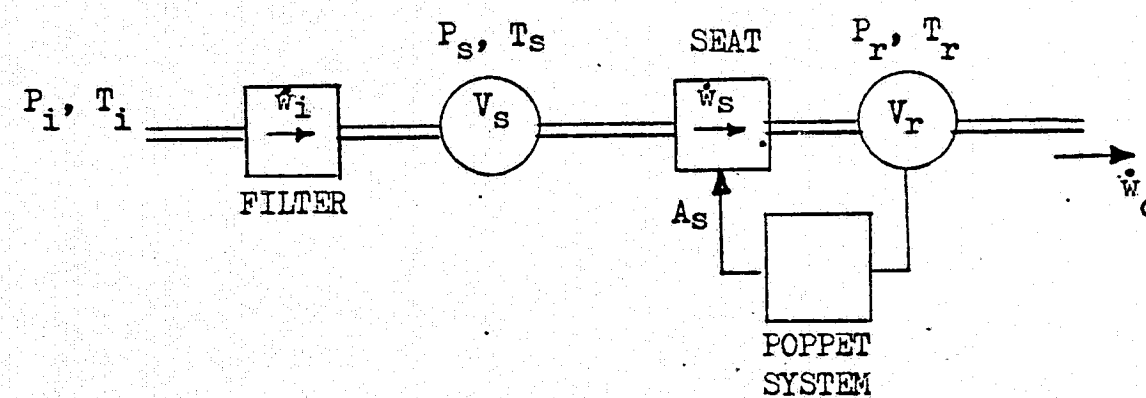
A gas flow system can be represented by a series of compressible volumes separated by localized restrictions. In the MM'69 regulator, the only significant points of restriction are the filter and the seat, and therefore the model shown in Fig. 3 is representative.* Here, the volume V_s represents the volume between the filter and seat. The volume V_r represents the volume within the regulator downstream of the seat, plus that of the outlet tube to the next point of restriction.

The pressures and temperatures at the lumped volume or nodal points can be determined from the equations of conservation of mass and energy. The flowrate between nodal points can be determined by a flow equation as indicated by experimental data. Data for the filter, for example, indicate that laminar flow exists and the flowrate is related to pressure by

$$\dot{w}_i = K P_i (P_i - P_s) / T_i \quad (1)$$

This equation assumes perfect gas behavior.

* At points other than the seat, gas velocities in the regulator are low. The annular region downstream of the seat represents the next most important point of restriction, and amounts to about 1.3 psi at nominal flow conditions.



LUMPED PARAMETER REPRESENTATION OF MM '69 REGULATOR

FIGURE 3

Flowrate through the seat follows the compressible flow orifice equation.

$$\dot{w}_s = C_s A_s P_s \phi_s / \sqrt{T_s} \quad (2)$$

In this equation C_s is the discharge coefficient, which can be assumed to be constant due to the small changes in ball position. The factor ϕ_s is a function of upstream and downstream pressure given approximately by

$$\begin{aligned} \phi &= 1.06 \left[\frac{P_r}{P_s} \left(1 - \frac{P_r}{P_s} \right) \right] ; \frac{P_r}{P_s} \geq .5 \\ &= 0.53 ; \frac{P_r}{P_s} < .5 \end{aligned} \quad (3)$$

Equation (3) introduces less than 2 percent error over the full range of pressure ratio. The actual function and points calculated from Eq. (3) are shown in Fig. 4.

The stored weight of gas in the lumped volumes is found from the conservation of mass equation,

$$w_s = \int (\dot{w}_i - \dot{w}_s) dt \quad (4)$$

The same equation is applicable for the stored weight downstream of the seat, w_r .

The temperature within each lumped volume can be calculated from the conservation of energy. Assuming perfect gas behavior and an adiabatic process,

$$T_s = \int \left[\dot{w}_i (\gamma T_i - T_s) - \dot{w}_s T_s (\gamma - 1) \right] / \dot{w}_s dt \quad (5)$$

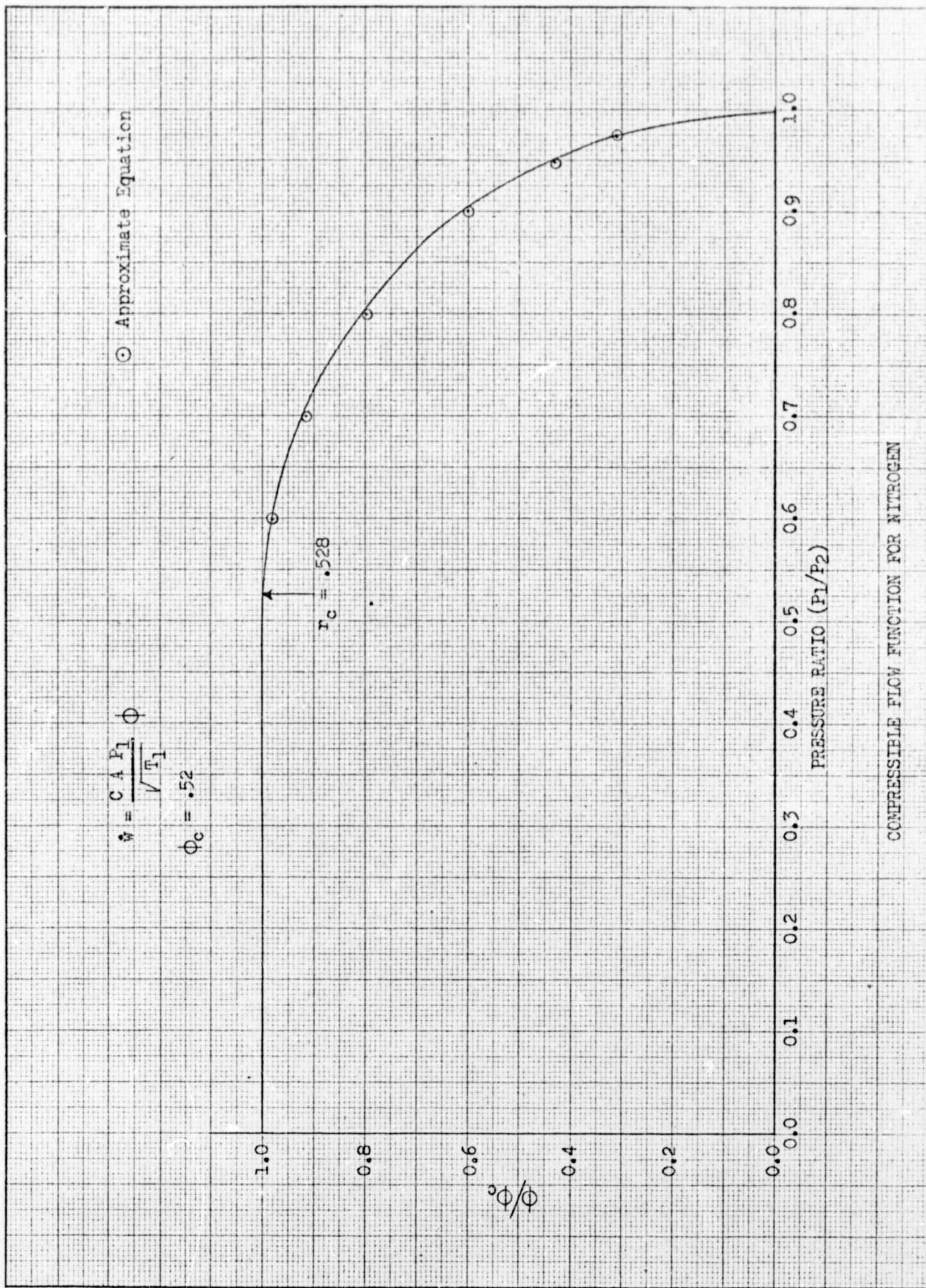


FIGURE 4

Once the stored weight and temperature within each volume have been determined, the local pressure can be calculated from the perfect gas law. That is

$$P_s = w_s R T_s / V_s \quad (6)$$

Equations (1) through (6) represent all forms necessary to construct a complete model of the regulator gas flow system. Equations (4), (5), and (6) can be applied at the volume downstream of the seat. The outflow, $\dot{w}_o$, must be determined from the downstream flow system, i.e., the flow bench, water expulsion, or propulsion system. The inlet pressure and temperature P_i and T_i are determined from the supply system, and are therefore inputs to the regulator model. The complete set of gas flow equations for the regulator are presented in Appendix I, along with the definitions of symbols.

The lumped parameter model of the gas flow equations shown in Fig. 3 does not have a separate volume representing the diaphragm cavity. An earlier model included the diaphragm cavity and accounted for flow in and out through the provided passages. However, due to the large flow areas of the passages, the pressure difference between the cavity and the reference volume was found to be negligible (less than 1 psi) even during high-frequency oscillation of the diaphragm. The model was therefore simplified to that shown in Fig. 3 by elimination of the diaphragm cavity. In this model the diaphragm pressure is assumed to be the reference pressure. It is notable that if the passages to the diaphragm cavity were smaller, this simplification would not be possible.

Mechanical System

The mechanical system consists of the poppet diaphragm, spring assembly, and ball. The forces acting on the poppet are the diaphragm pressure force, the balancing spring force, and the force transmitted through the ball. The sum of these forces determine the instantaneous acceleration, integration of which yields the velocity and position. Position

of the poppet is limited by mechanical stops, and that of the ball by the seat.

If it is assumed that the ball follows the poppet perfectly, there is only one equation of motion. Assuming a position variable x_p which is positive as the ball is lifted from its seat, the equation of motion is,

$$m_p \ddot{x}_p = F_{ps}(\xi, \dot{x}_p) - A_d(P_r - P_a) - B_p \dot{x}_p - (F_{bs} + F_{pr})f_1(x_p) \quad (7)$$

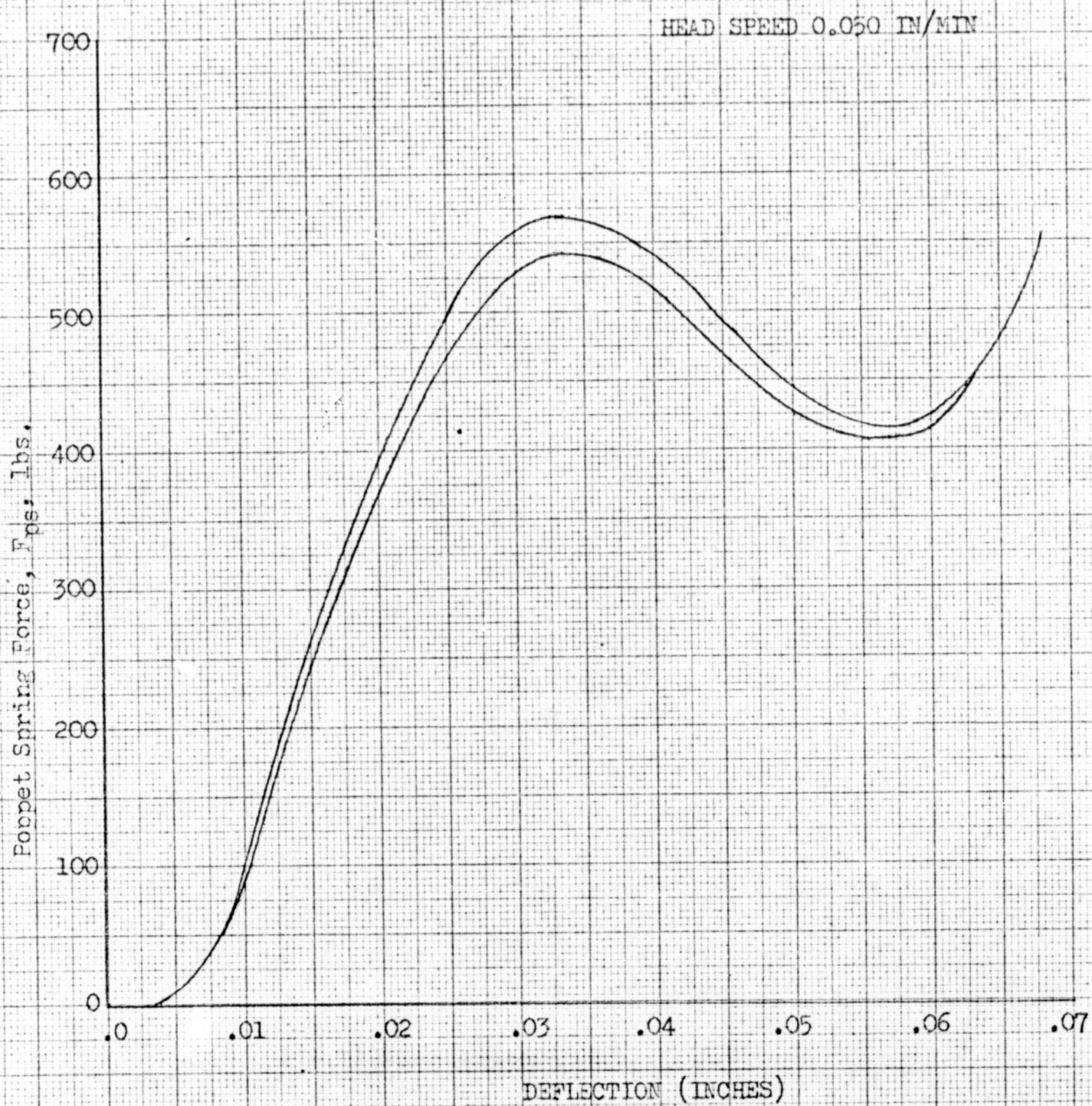
$$x_{p_{\min}} \leq x_p \leq x_{p_{\max}}$$

$$\dot{x}_p = 0 \quad \text{for } x_p = x_{p_{\max}}, x_{p_{\min}}$$

In this equation F_{ps} is the poppet spring force, which is a function of spring deflection and rate of change of deflection. The dependence on rate of change is indicated by the hysteresis of the experimental force-deflection curve. The diaphragm pressure force is the effective area times the pressure difference across the diaphragm. A damping term, $B_p \dot{x}_p$, is included, although the coefficient is not easily determined and will most likely be small. The force transmitted through the ball is the sum of the ball retaining spring force and the pressure forces which act on the ball, F_{bs} and F_{pr} respectively. The function $U(x_p)$ is a unit step function which removes the ball forces from the poppet when the ball is resting on the seat.

A typical poppet spring force characteristic is shown in Fig. 5. The non-linear characteristic derives from the fact that the assembly contains a number of Belleville springs. The hysteresis exhibited in the characteristic represents an irreversible conversion of mechanical energy to internal energy in the spring material. The characteristic can be approximated, including the hysteresis, by considering the force to be the

KEUFFEL & ESSER CO.
10 X 10 TO THE CENTIMETER
MADE IN U.S.A.
40 1211



TYPICAL POPPET SPRING CHARACTERISTICS
S/N 001 SPRING ASSEMBLY WITH S/N 6 SPRING REMOVED

FIGURE 5

sum of two terms, one dependent only on deflection and the other dependent only on velocity. That is,

$$F_{ps}(\delta, \dot{x}_p) = F_1(\delta) + F_2(\dot{x}) \quad (8)$$

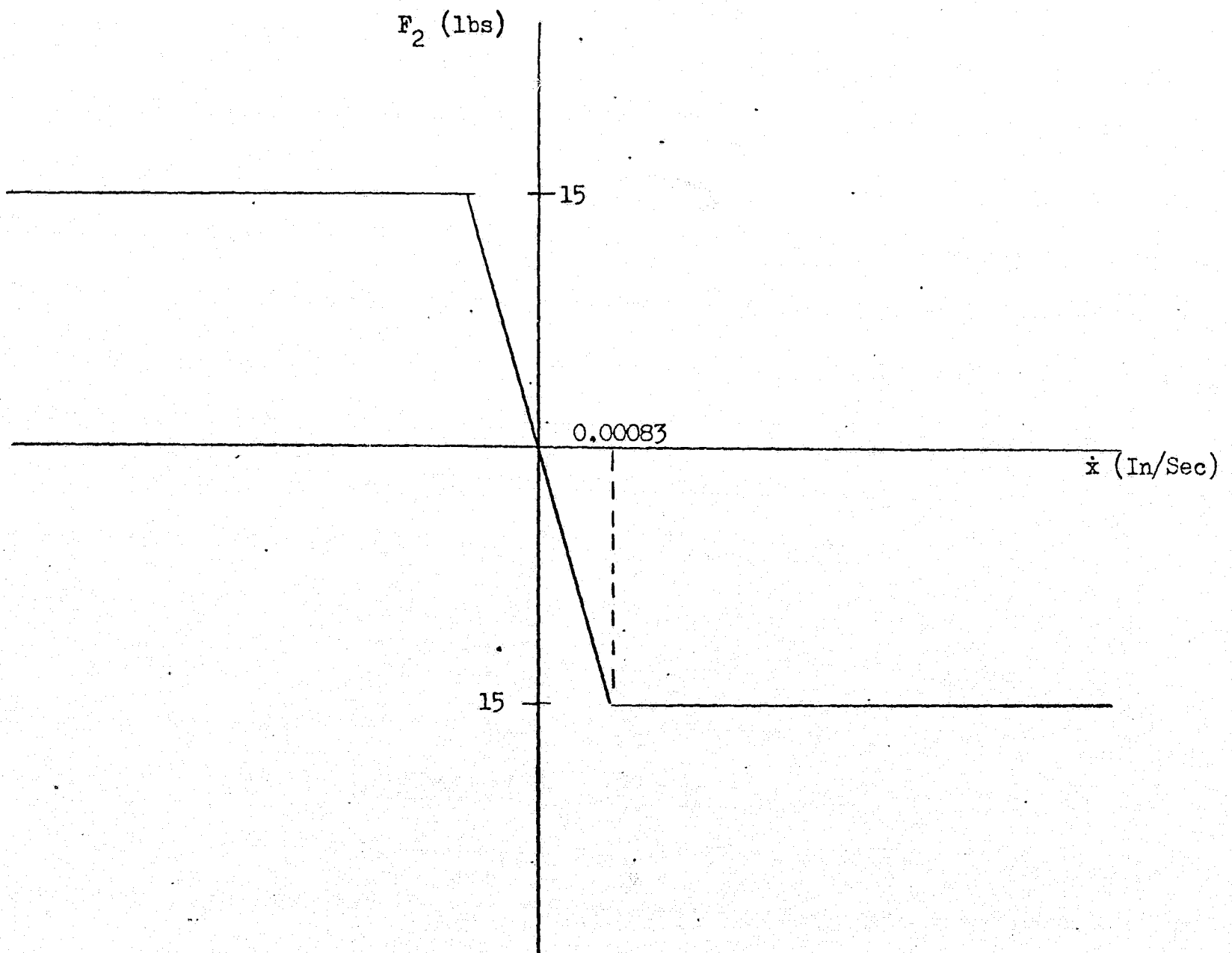
The deflection function must be assumed to be the mean curve in the absence of contrary evidence. Ideally, one could generate $F_1(\delta)$ by measuring steady-state forces, as opposed to the curves in Fig. 5 which were generated at a finite velocity. Using the mean curve for $F_1(\delta)$, the velocity-dependent term $F_2(\dot{x})$ is then an odd function. A function which fits the available data is shown in Fig. 6. This function is based upon the observed height of the hysteresis of ± 15 pounds and the given head speed of the tester of 0.05 in/min. It assumes that the height of the hysteresis loop has reached its maximum at this velocity. It is notable that for small oscillations of the poppet velocity, this representation of hysteresis amounts to extremely high damping. Also, if poppet motion is reduced to zero gradually, as in the case of a spring tester, there is a smooth convergence to the steady-state curve, as is suggested in Fig. 5.

The spring deflection, δ , is the sum of the deflection of each end. Noting that the poppet position coordinate, x_p , is positive in the opening direction, the total spring deflection is then

$$\delta = \delta_o - x_p \quad (9)$$

where δ_o is the deflection effected by rotation of the adjustment cap. This equation permits the set point to be variable parameter in the model.

The effective diaphragm area is less than the total area since the outer edge is constrained. The effective area of the diaphragm to be used in Eq. (7) can be determined by equating the work done by the gas moving into the elemental volume caused by an elemental change in x_p to the work done moving the resisting force through the elemental change in x_p .



PORTION OF POPPET SPRING FORCE DUE TO VELOCITY

FIGURE 6

That is,

$$(P_r - P_a) dv = A_d(P_r - P_a) dx \quad (10)$$

It is seen then that

$$A_d = \frac{dv}{dx} \quad (11)$$

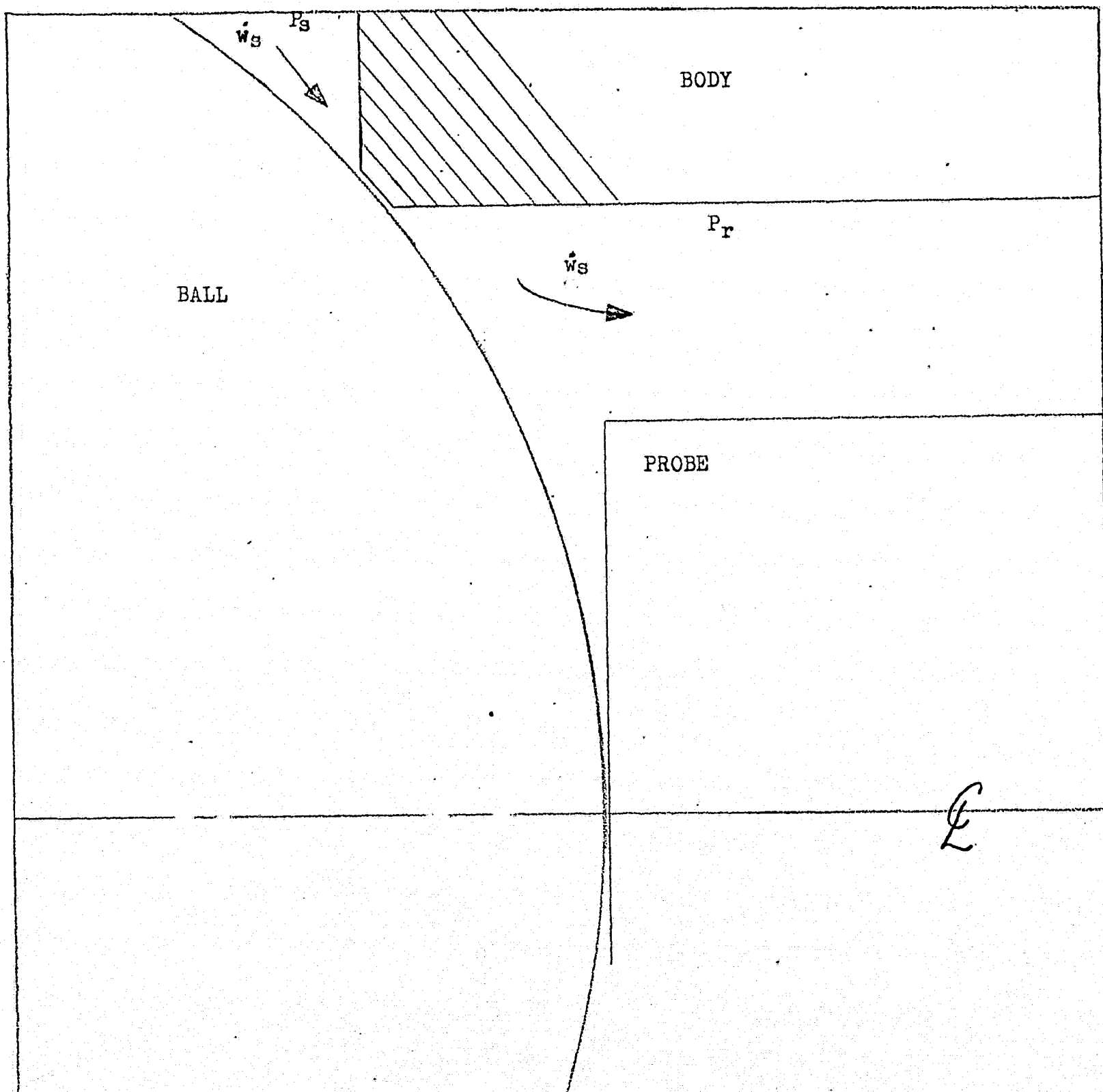
By careful calculations using the regulator dimensions and assuming the diaphragm to deflect in the shape of a cone, it can be shown that $A_d = 1.665 \text{ in}^2$.

The ball retaining spring is Belleville-type spring with deep cut-outs around the outer circumference, making it star-shaped. In its nominal position it is compressed beyond its flat position. In the absence of actual force-deflection data for this spring, and in view of the small range in ball movement, the ball spring force is approximated by

$$F_{bs} = F_{bso} + K_5 x_p \quad (12)$$

In this equation F_{bso} is the ball spring force when the ball is resting on the seat and K_5 is the local slope of its characteristic curve. It is notable that K_5 is most likely small since the spring is of the Belleville type and is deflected beyond its flat position.

Pressure forces acting on the ball are also transmitted to the poppet when the ball is not on the seat. The pressure force on the ball can be estimated by considering the geometry of the constriction of flow between the ball and the seat. This is shown at 100 times actual size in Fig. 7. Due to the small size of flow passage relative to the ball and bore diameters, it is apparent that downstream static pressure, P_r , acts on the ball over the entire bore diameter. The corresponding area on the upstream side of the ball is acted upon by upstream static pressure.



GEOMETRY OF BALL - SEAT FLOW CONSTRICTION
(100 TIMES ACTUAL SIZE)

FIGURE 7

Therefore the ball pressure force is

$$F_{pr} = A_b (P_s - P_r) \quad (13)$$

At nominal conditions, i.e., with $P_s = 3570$ and $P_r = 308$ psia, $F_{pr} = 15.65$ pounds. This is approximately 3 percent of the nominal force applied to the poppet by the poppet spring.

Equations (7) through (13) above are sufficient to compute ball and poppet position as a function of time, provided that there is always contact pressure between the poppet and ball. Due to the high pressure force on the ball, and the low mass of the ball, this will most likely be the case when upstream pressure is high. When upstream pressure drops, the pressure force falls to nearly zero, leaving the ball with only its retaining spring to hold it firmly against the poppet. Under these conditions the ball might "float" at high frequencies of the poppet. The model as presented herein will not reflect this phenomenon, so further investigation may be required if indicated by the test data.

Coupling and Interface Equations

The flow system equations and the mechanical system equations are coupled by having certain common variables and by a coupling equation. For example, Eq. (7) contains pressure P_r , which is determined in the flow system equations. The coupling equation is

$$A_s = K_2 x_p ; A_s \geq 0 \quad (14)$$

which relates poppet position to seat flow area. Limiting the seat area positive represents the ball coming to rest on the seat while the poppet retracts to a negative position.

The total set of regulator system equations are summarized in Appendix I. There are 16 independent equations and 19 system variables in this set. Three of these system variables, P_i , T_i , and $\dot{w}_o$, are input quantities

which must be supplied by a separate flow system model. All variables can then be determined uniquely as functions of time by simultaneous solution of these equations.

DEVELOPMENT OF FLOW BENCH EQUATIONS

The flow bench test system for the MM'69 regulator is shown in schematic form in Fig. 2. Aside from the regulator, it can be represented as a series of storage volumes separated by flow restrictions. Due to the relatively low gas velocities, distributed line losses are small and can be lumped with the valves. Volumes between the valves, determined from line diameters and lengths, are represented as storage volumes in the system.

The entire flow bench system can be modeled by repeated application of equations similar to Eq. (1) through (6). For example, Fig. 8 represents a general restriction and downstream volume in the flow bench system. The flowrate through the i th restriction is

$$\dot{w}_i = A_i P_{i-1} \phi_i / \sqrt{T_{i-1}} \quad (15)$$

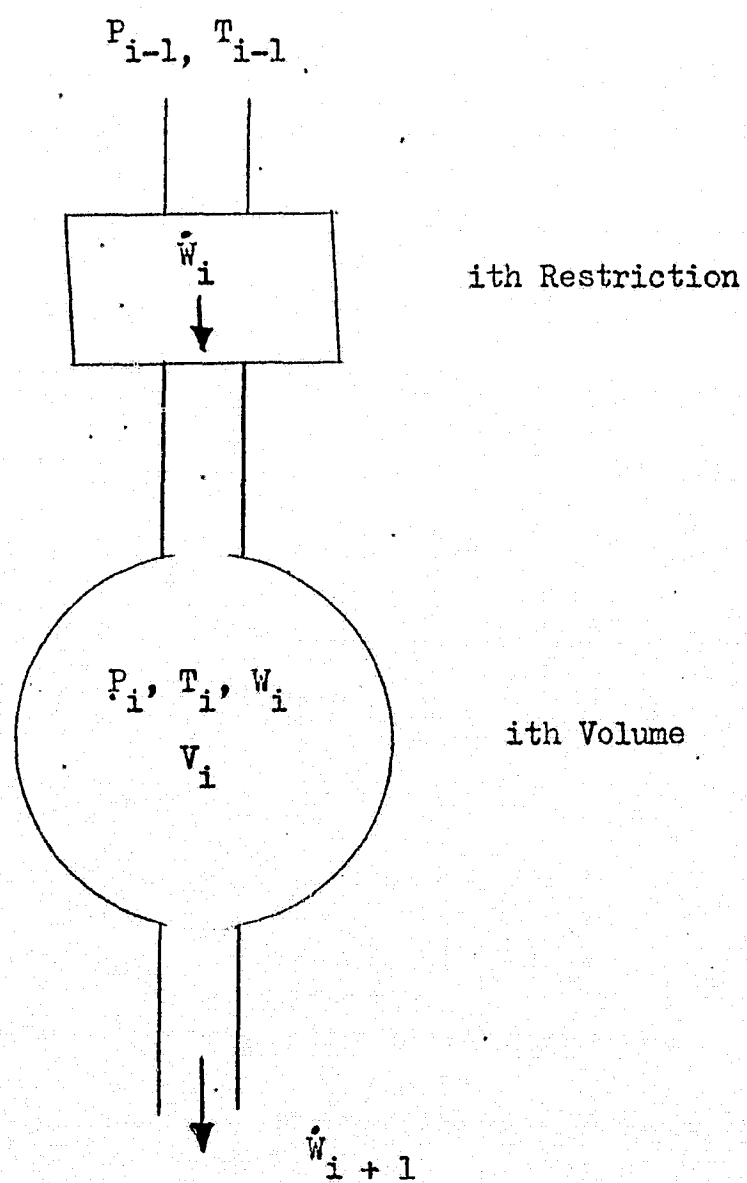
where

$$\begin{aligned} \phi_i &= 1.06 \left[P_i/P_{i-1} (1 - P_i/P_{i-1}) \right]^{\frac{1}{2}} ; P_i/P_{i-1} \geq .5 \\ &= 0.53 ; P_i/P_{i-1} < .5 \end{aligned} \quad (16)$$

The effective area, A_i , is the valve area times the discharge coefficient. For a valve, this area is an arbitrary function of time.

The stored weight in the i th volume is

$$w_i = \int (\dot{w}_i - \dot{w}_{i+1}) dt \quad (17)$$



TYPICAL RESTRICTION AND DOWNSTREAM
VOLUME OF FLOW BENCH

and the temperature is

$$T_i = \int \left[\dot{w}_i (T_{i-1} - T_i) - \dot{w}_{i+1} T_i (\gamma - 1) \right] / w_i dt \quad (18)$$

Then from the perfect gas law, the pressure is

$$P_i = w_i R T_i / V_i \quad (19)$$

Equations (15) through (19), together with the regulator model and valve schedules, permit calculation of temperatures, pressures, and flow-rates throughout the flow bench system.

Computer Mechanization

Simultaneous solution of the dynamic regulator and flow bench equations is easily accomplished on an analog computer. In order to do so, it is necessary to determine numerical values for the equation constant, and to scale the system variables to the voltage range of the computer to be employed.

The regulator equations, Appendix I, have been scaled and mechanized on one of the Applied Dynamics AD-256 analog computers at the Rocketdyne Analog Simulation Facility. The system constants and nominal values of system variables employed in this mechanization are presented in Appendix I. These values were determined from the regulator drawings. The operating point selected as nominal corresponds to a pressure of 3600 psia at the regulator inlet and a flowrate of 0.006 lbs/sec of nitrogen.

The AD-256 computer has a voltage range of ± 100 volts. The over-voltage tolerance is very small due to the type of non-linear components employed. For this reason, the regulator equations were scaled such that analog voltages of the system variables never approach 100 volts. This was done by selecting scale factors (s.f.) of convenient fractions of the nominal values of the actual variables. For example, the system

flowrate variables, $\dot{w}$, are represented on the computer by $10\bar{w}$, where

$$\bar{w} = \dot{w}/\dot{w}_n \quad (20)$$

The subscript n refers to the nominal point, arbitrarily selected as the steady state operating point with an upstream pressure of 3600 psia. Then by definition over-barred quantities (normalized variables) are all numerically equal to 1.0 at the nominal conditions. This results in machine variables having nominal values of 10 volts or 50 volts, depending upon the desired scaling. For example, flowrates can go up to 7 - 9 times their nominal values, so by letting the machine variable be $10\bar{w}$ the voltage goes from 10 to 70 or 90 volts. On the other hand, pressures tend to go down from the nominal value, so a voltage of 50 $\bar{p}$ is employed to represent pressures.

Using the above scaling method, the general analog variable, y , is related to the general system variable by the formula

$$Y = a y/y_n = a \bar{y} \quad (21)$$

where y_n is the nominal value of y , and "a" is a voltage between 10 and 100 volts (usually 10 or 50). The conventional scale factor is then

$$s.f = y_n/a \quad (22)$$

The scale factors for all regulator variables are shown in the table of symbols in Appendix I.

This method of scaling or normalizing the system equations has the additional advantage of generalizing the problem. After normalizing all system variables, the system equations are dimensionless, and results obtained are applicable to other similar systems regardless of the absolute values of the variables. Similarity is automatically revealed by the values of the dimensionless coefficients in the normalized equations.

To see this, consider the equation for stored weight, Eq. (17). In normalized form it reduces to

$$\bar{w}_i = \frac{\dot{w}_n}{w_n} a_t \int (\bar{w}_i - \bar{w}_{i+1}) d\gamma \quad (23)$$

The coefficient

$$\frac{\dot{w}_n a_t}{w_n}$$

is the important parameter, and any system with the same value of this parameter will have the same dynamic characteristics. That is, a system with twice the flowrate will have the same characteristics as the one under consideration if the nominal stored weight (cavity size) is twice as large. Conversely, this coefficient is a meaningful quantity to be systematically varied in a parametric analysis. All other system equations normalize to a similarly convenient form, and reveal other important dimensionless groups.

Most of the system equations are scaled in the above manner, and mechanize on the analog computer in a straightforward manner. However, the poppet equation of motion and the poppet spring mechanization warrant specific discussion. First, it is found convenient to define the poppet position variable as

$$50 \bar{x}_p = 50 \left[\frac{x_p - x_{pn}}{x_{p_{max}} - x_{pn}} \right] \quad (24)$$

This machine variable then has a nominal value of zero, and a maximum value of 50 volts, thus taking maximum advantage of the available voltage range. This differs from other machine variables in that it is nominally zero.

The poppet position limits are easily mechanized on the AD-256 computer

by use of the logical elements, and the ability to control integrator modes individually. By mechanizing the logical equations below, the velocity integrator can be reset to the initial condition mode (zero) upon encounter of the stops. Upon reversal of the poppet forces, the velocity integrator is restored to the operate mode, allowing the poppet to come off of the stop. The logical variables A, B, C, D, and E are by definition either 0 or 1. The output signal, E, controls the mode of the velocity integrator.

$$A = 1 \quad \text{if } x_p \geq x_{p_{\max}} \quad (25)$$

$$B = 1 \quad \text{if } -x_p \geq -x_{p_{\min}} \quad (26)$$

$$C = 1 \quad \text{if } x_p \geq 0 \quad (27)$$

$$D = 1 \quad \text{if computer is in "operate"} \quad (28)$$

The logical equation for E is then

$$\begin{aligned} E &= (\bar{A} + \bar{B} + D) \cdot (A + \bar{B} + \bar{C} + D) \cdot (\bar{A} + B + C + D) \quad (29) \\ &= (\bar{A} + \bar{B}) \cdot (A + \bar{B} + \bar{C}) \cdot (\bar{A} + B + C) + D \end{aligned}$$

The AD-256 computer has comparators which produce logical variables such as A - C, and logical "or/nor" gates for implementing the logical equations. By use of DeMorgan's Law, E can be generated entirely with these gates.

Mechanization of the poppet spring characteristic is best achieved by means of a curve-fit of the experimental curve. For example, the characteristic shown on the spring assembly drawing, JPL 10000976, is approximated by

$$\bar{\phi}_s = 0.5418 + 2.966 \Delta - 13.81 \Delta^2 + 12.97 \Delta^3 \quad (30)$$

where

$$\bar{\phi}_s = (F_{ps} - 480)/120 \quad (31)$$

$$\Delta = (\xi - .02)/.04 \quad (32)$$

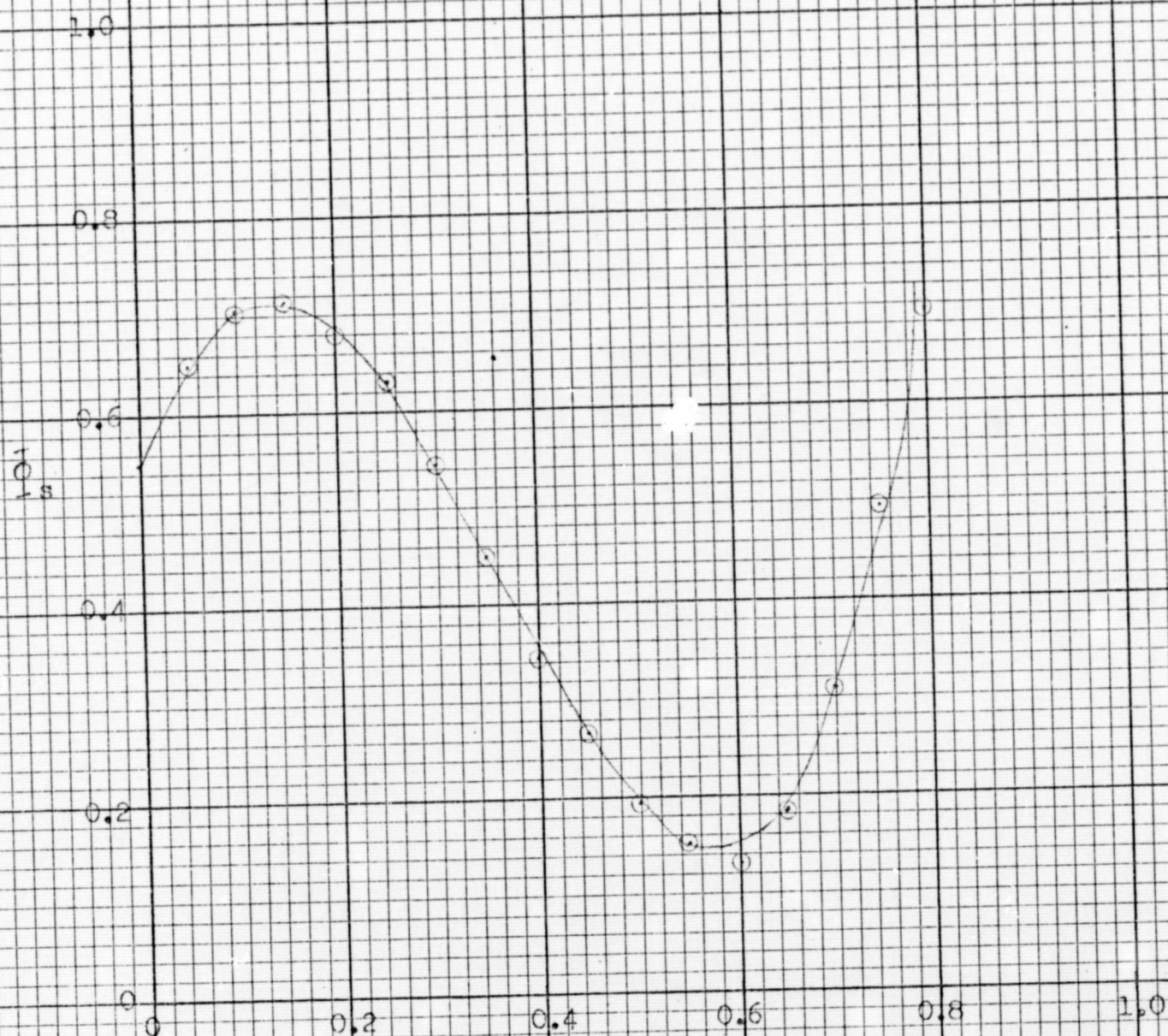
This fit was determined by a digital computer program using a least-squares method. The input data were $\bar{\phi}_s$ values computed from Eq. (31) and the average of the upper and lower force values from the spring assembly drawing mentioned above. The accuracy of the fit can be seen in Fig. 9. The largest error is 11 percent in $\bar{\phi}_s$, which amounts to 0.4 percent error in the spring force. This equation is easily mechanized on the AD-256 computer by the use of two multipliers and several summers.

It should be noted that while Eq. (30) was employed in the generation of all results during the first quarter, it does not well represent the actual spring assembly characteristics. The characteristics received late in the quarter, Fig. 5, has a greater negative slope and this region extends to greater deflections than that given as "typical" on JPL drawing 10000976. A new curve fit will have to be generated at a later date.

The hysteresis effect was mechanized by adding a force increment to the force calculated from Eqs. (30) through (32). The added force is a function of poppet velocity, as explained in the development of the poppet equations of motion. The function shown in Fig. 6 is easily generated on the computer.

In order to check the performance of the regulator analog model, it was necessary to mechanize a simplified flow system. This flow system consisted of a downstream accumulator discharging to the atmosphere. This permitted generation of an outlet flow which is required for solution of the regulator equations. Upstream pressure and temperature were represented by constant voltages.

$$\begin{aligned}
 \delta &= .5418 + 2.966\Delta - 13.81\Delta^2 + 12.97\Delta^3 \\
 \odot &\text{ Actual Avg. Force Data} \\
 \odot &= (Fps - 480) / 120 \\
 \Delta &= (\delta - 0.020) / 0.04
 \end{aligned}$$



CURVE FIT OF SPRING CHARACTERISTIC
 (JPL DRWG. 10000916)

FIGURE 9.

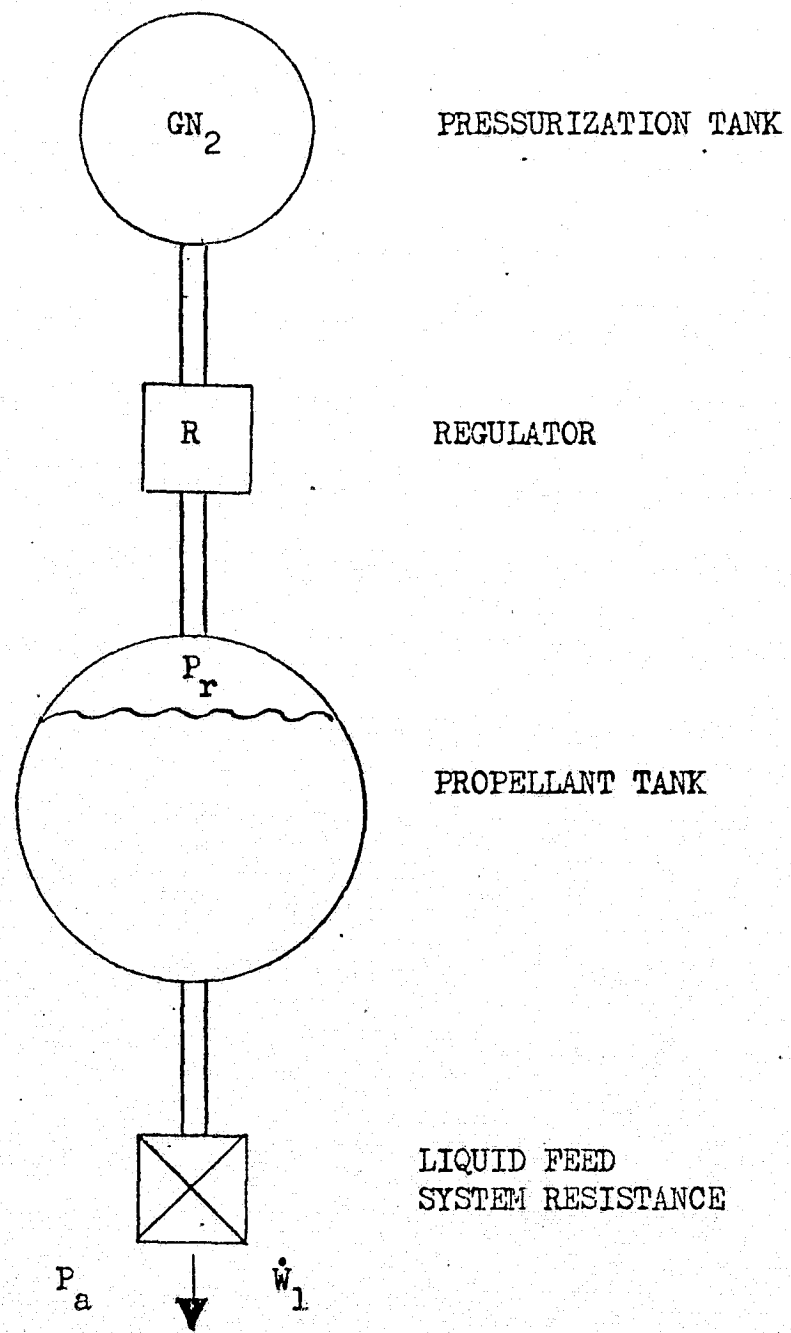
The complete analog computer diagrams are shown in Appendix II. The symbols employed are identified on the diagrams. It should be noted that the amplifiers are bi-polar on the AD-256 computer, i.e., both + and - voltages are available at the output of every amplifier. Also, the multipliers require both polarities of both inputs. The multipliers themselves have current outputs and require an output amplifier to generate a voltage. Division is accomplished by multiplication in the feedback path of a high-gain amplifier.

SIMPLIFIED WATER EXPULSION MODEL

The MM'69 regulator is tested as part of the propulsion system in the Water Expulsion Tests. In this test, the propellant tank is filled with water, and an engine burn is simulated by expelling the water under the controlled pressure provided through the calibrated regulator. Although the modeling of this test system is scheduled later in the Advanced Feed System Study, it was decided to perform a preliminary analysis during the first quarter. This was due to certain of these tests, on particular regulators, exhibiting wide pressure excursions. It was felt that the cause of these fluctuation might be revealed by a simple model of this system and regulator. If so, the results would indicate where emphasis might best be placed in the development of the complete, detailed regulator model. Further, comparison of results of the simplified and detailed models might indicate the degree of sophistication required in order to accurately predict the performance of future regulators.

A simplified water expulsion system is shown schematically in Fig. 10. Only the most significant elements are included. Further simplifications are afforded by the following assumptions:

1. The pressurization tank is sufficiently large that its pressure is constant over the time period considered.
2. Gas flow is isothermal.
3. There is no pressure drop between the regulator and the propellant tank.



SIMPLIFIED SCHEMATIC OF WATER EXPULSION SYSTEM

4. Perfect gas behavior.
5. Poppet forces consist of diaphragm pressure and spring, i.e., ball forces are negligible.
6. Changes in downstream (regulated) pressure are small, so that linearization is permissible.

Under assumptions 1 through 5, the system equations presented in Appendix III can be developed. In these equations, the poppet and ball positions, x_p and x_b , are measured from the seated position, with the opening direction being positive. The spring force, F_{ps} , is shown only as a function of poppet position, which implies that the free end is fixed. Changes in the set-point must be accomplished by using different functions.

These equations can be normalized and linearized to permit easier analysis. After considerable simplification, there results

$$\ddot{\bar{x}}_p + \gamma \dot{\bar{x}}_p - \beta K_2 \bar{x}_p = -K_2 \bar{P}; \quad a \leq \bar{x}_p \leq 1 \quad (33)$$

$$\dot{\bar{x}}_p = 0 \text{ for } \bar{x}_p = a, \bar{x}_p = 1$$

$$\dot{\bar{P}} + (1/T_1) \bar{P} = K_1 \bar{x}_b \quad (34)$$

$$\bar{x}_b = \bar{x}_p; \quad \bar{x}_b \geq b \quad (35)$$

Equation (33) is the linear second order equation of poppet/ball motion, with regulated pressure as a driving function. Equation (34) is the linear first order equation of regulated pressure with poppet position as a driving function. They are obviously coupled by $\bar{P}$ and Eq. (35). Due to the linearization about the nominal point, the equilibrium point is $\bar{x}_p = \bar{x}_b = \bar{P} = 0$. The constants are

	<u>Approx. Mag.</u>
$\beta = \frac{\partial F_{ps}}{\partial x_p}$ (note: $\beta = -\frac{\partial F_{ps}}{\partial \delta}$)	-0.2, +0.1
$\gamma = B/m$	(?) sec^{-1}
$K_1 = (x_{p_{\max}}/x_{pn} - 1) \dot{v}_{ln}/(1 - P_a/P_{rn}) v_{an}$	.8 sec^{-1}
$K_2 = (P_{rn} - P_a) A_d/m(x_{p_{\max}} - x_{pn})$	$0.5 \cdot 10^9 \text{ sec}^{-2}$
$1/T_1 = \left[1 + \frac{1}{2} \left(\frac{1}{1 - P_a/P_{rn}} \right) \right] \dot{v}_{ln}/v_{gn}$	0.08 sec^{-1}
$a = (x_{p_{\min}} - x_{pn})/(x_{p_{\max}} - x_{pn})$	-1.7
$b = -x_{pn}/(x_{p_{\max}} - x_{pn})$	-0.14

As a result of normalization, the variables are dimensionless and the time derivatives have units of reciprocal seconds raised to the appropriate power.

Stability of these equations in the presence of small disturbances can be determined by examination of the characteristic equation. In order to determine the characteristic equation, define the state variables

$$\begin{aligned} x_1 &= \bar{x}_p \\ x_2 &= \dot{\bar{x}}_p \\ x_3 &= \bar{p} \end{aligned}$$

Then Eq. (33) through (35) can be written

$$\dot{\underline{x}} = A \underline{x} \quad (36)$$

where $\underline{x}$ is the vector of state variables and

$$A = \begin{bmatrix} 0 & 1 & 0 \\ \beta K_2 & -\gamma & -K_2 \\ K_1 & 0 & -1/T_1 \end{bmatrix} \quad (37)$$

The characteristic equation is then

$$|A - \lambda I| = 0 \quad (38)$$

Upon expansion this results in

$$\lambda^3 + (\gamma + 1/T_1)\lambda^2 + (\gamma/T_1 - \beta K_2)\lambda + (K_1 K_2 - \beta K_2/T_1) = 0 \quad (39)$$

By application of Routh's stability criterion (Ref. 1), the following necessary and sufficient conditions for stability can be determined:

$$\gamma > \beta K_2 T_1 \quad (40)$$

$$K_1 > \beta/T_1 \quad (41)$$

$$(\gamma/T_1 + \beta K_2)(\gamma + 1/T_1) > (K_1 - \beta/T_1) K_2 \quad (42)$$

A slight rearrangement of Eq. (42) gives

$$K_2 < \frac{\gamma}{T_1} \frac{(\gamma + 1/T_1)}{(K_1 + \beta\gamma)} \quad (43)$$

From these inequalities it is apparent that stability exists for at least some values of K_2 when the spring slope, β , is negative, since this would satisfy Eq. (40) and (41) automatically. However, when β is

positive, as it is when the reference setting is in the negative-slope region of the spring characteristic, stability depends upon damping and downstream volume. Equation (4) shows the required damping. The required downstream volume is indicated by Eq. (41) since K_1 varies inversely with volume.

By considering the approximate values of the constants, given above, further insight is possible. Assuming that $\beta = +0.1$

$$\begin{aligned}\gamma &> 0.63 \cdot 10^9 \text{ sec}^{-1} \\ K_1 &> 0.008 \text{ sec}^{-1} \\ K_2 &< \frac{\gamma}{\beta T_1} \text{ sec}^{-2}\end{aligned}$$

It is noted that for large values of damping, γ , the inequality Eq. (42) reduces to Eq. (40). Also, Eq. (41) is satisfied even for the large volume of the propellant tank ullage, 100 in^3 . It, therefore, appears that large damping is the only requirement for stability in the $+\beta$ region.

The only indication of the actual damping present in the mechanical system is the hysteresis in the spring characteristic, Fig. 5. If it is assumed that the hysteresis loop is a result of a force proportional to velocity, then the damping coefficient can be calculated from Fig. 5. This method gives

$$\gamma \approx 0.715 \cdot 10^8 \text{ sec}^{-1}$$

which is approximately one order of magnitude less than that required for stability by Eq. (40)

The above stability analysis assumed small changes in the state variables from their nominal value, zero. An indication of instability automatically ensures that any small disturbance will immediately begin to grow, and in time will result in violation of the small amplitude assumption. Large

amplitude oscillations drive the system into non-linear regions, and there may be convergence to a stable limit cycle, of either large or small amplitude. This is best studied with the detailed model.

It must also be noted that some degree of positive β is permitted by the above development. For example, Eq. (40) can be re-arranged to give

$$\beta < \frac{\gamma}{K_2 T_1}$$

Thus for

$$\gamma = 0.7 \cdot 10^8 \text{ sec}^{-1}$$

$$K_2 = 0.5 \cdot 10^9 \text{ sec}^{-2}$$

$$1/T_1 = 0.08 \text{ sec}^{-1}$$

the maximum tolerable slope is given by

$$\beta < 1.12 \cdot 10^{-2}$$

Further, if the downstream volume is smaller, even a larger slope could be tolerated.

In order to show the effect of the non-linear spring characteristic, the equations in state-variable form, Eq. (36), can be examined graphically. The case most easily examined is when the mechanical system is lightly damped and the downstream volume is large. In this case the poppet moves extremely fast relative to regulated pressure, and the system can be represented in two-dimensional state space, i.e., the phase plane.

Figure 11 shows the phase plane of system downstream pressure versus poppet position. In this plane, the spring characteristic appears as a switching line. That is, above this line the poppet moves to the left,

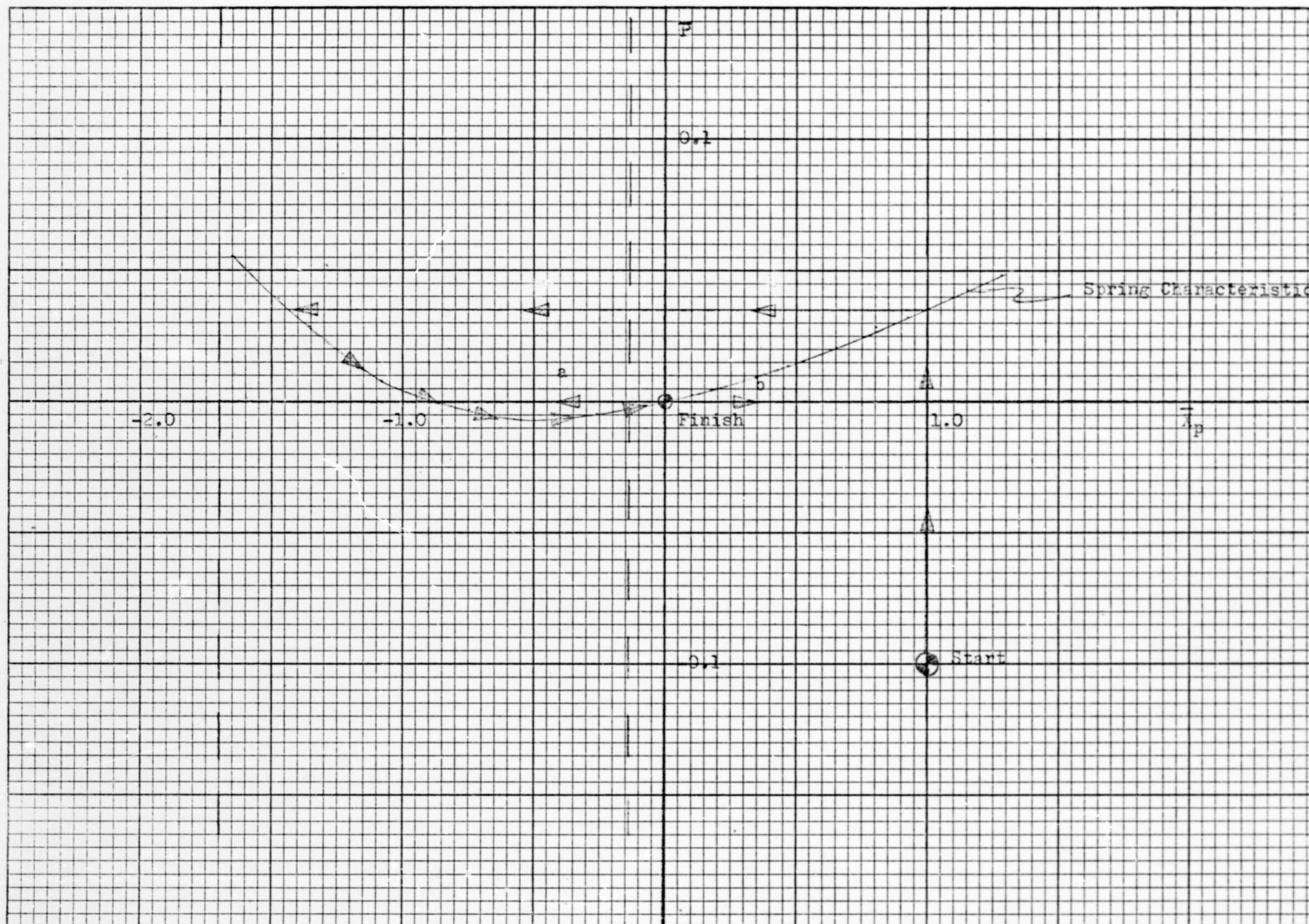
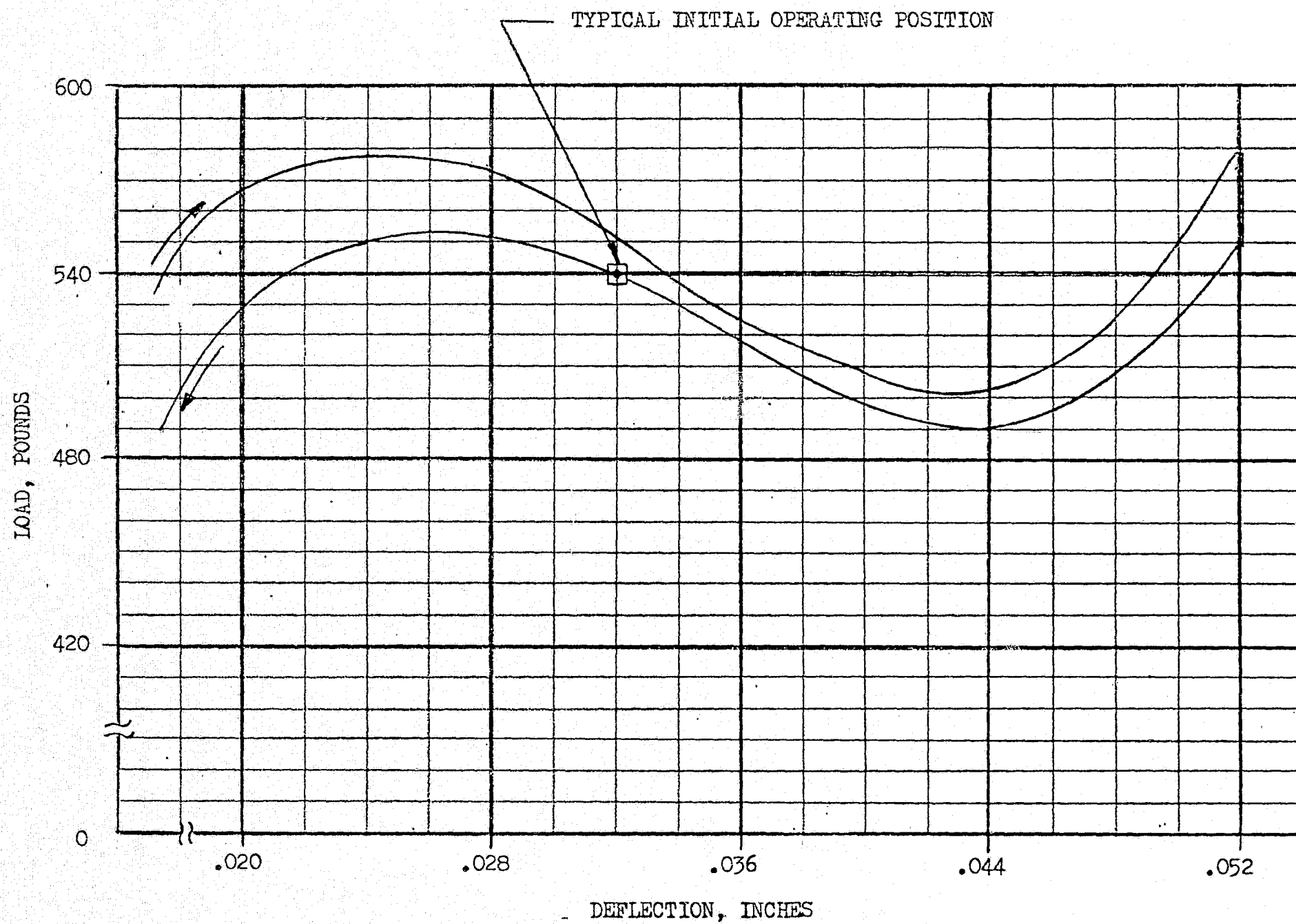


Figure 11. PHASE PLANE OF PRESSURIZATION OF SIMPLIFIED WATER EXPULSION SYSTEM.

and below it the poppet moves to the right. The forward poppet stop, full open, appears as boundary at $\bar{x}_p = 1.0$, and the retraction stop appears as a boundary at $\bar{x}_p = -1.7$. When $\bar{x}_p = -0.14$, the ball is on the seat. The spring characteristic is shown for a nominal setting with a small positive ℓ , i.e., just to the left of the minimum force point on the deflection curve, Fig. 12.

The path marked with arrows shows a typical pressurization transient. The starting point shown corresponds to an initial reference pressure of about 277 psia, which is 10 percent low. Since the pressure is low, the poppet is in the fully opened position. As the upstream valve is opened, flow begins through the seat and downstream pressure begins to rise. The poppet remains on the forward stop until the pressure force on the diaphragm equals the spring force, i.e., until the spring curve is reached. At this point the rate of change of pressure, $\bar{P}$, is still large, as can be seen from Eq. (34), which results in the pressure rising above the spring curve. Once the trajectory enters the region above the spring curve, the poppet is driven in the negative direction. In the absence of significant damping, the trajectory will be almost parallel to the $\bar{x}_p$ axis due to the extremely fast motion of the poppet relative to the pressure change. This trajectory intersects the spring curve in the second quadrant, where the pressure derivative is negative, resulting in a tendency to cross the spring curve. Crossing the spring curve introduces positive poppet velocity, which drives the trajectory back to the curve. It will be observed that ℓ is negative in this region, making the poppet equation of motion stable, so that the trajectory follows the spring curve. The system comes to rest at the origin since all time derivatives are zero at this point. If the ℓ at this point is sufficiently small, i.e., if Eq. (40) is satisfied locally, this is a stable equilibrium point and satisfactory operation is achieved.

In the event that the set-point on the spring characteristic curve is such that Eq. (40) is not met at the origin, the origin is an unstable equilibrium point. In this event, a small disturbance in either pressure or position which causes the operating point to shift up or to the left



TYPICAL LOAD vs DEFLECTION CURVE OF BELLEVILLE SPRING PACKAGE FOR
308 PSIG REGULATION (JPL DRAWING NO. 10000976)

FIGURE 12

will result in trajectory "a". Similarly, a shift down or to the right will result in trajectory "b". The excursion along "a" would result in an extremely small pressure excursion, less than 1 percent. The excursion along "b" would cause a complete re-cycle of the start-up transient. It is conceivable that, with a certain set point, either of these excursions could repeat indefinitely.

The above arguments could be extended to include the effects of damping and hysteresis. Large damping would make the upper portion of the trajectory curve upward. Hysteresis represented as discussed previously would have similar effects, since it is a manifestation of internal damping.

A smaller downstream volume would have the effect of increasing the rate of change of pressure. This would also cause the upper trajectory to curve upward.

The findings above, i.e., the stability criteria and the phase trajectories, indicate the importance of certain parameters on system performance. First, required damping is a function of the magnitude and sign of the spring characteristic slope, and of the size of the downstream volume. This suggests that the detailed model must reflect the actual regulator precisely in these areas. The importance of β , and the shape of the spring curve in the region of the set point indicate that the set point must be a variable parameter in the detailed model.

RESULTS

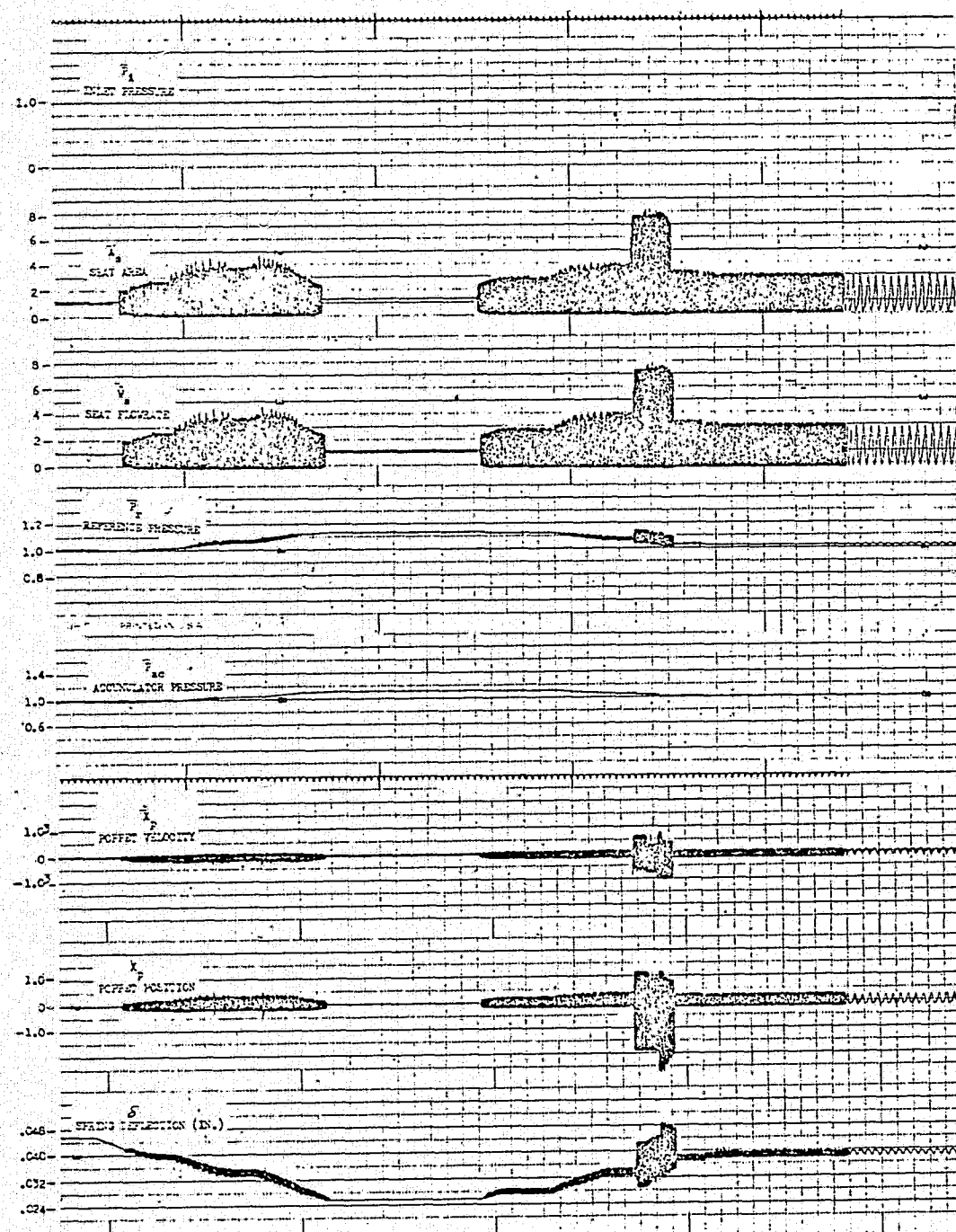
Results obtained during the first quarter include some transient performance data generated by the detailed analog model, and certain predictions of the simplified water expulsion system model. These results must be interpreted as representative rather than accurate, since certain portions of the input data were preliminary. In particular, the spring characteristic curve used in all work of the first quarter was obtained from JPL drawing 1000976. Actual spring data, such as Fig. 5, was obtained late in the quarter and found to differ considerably from the curve employed. Also, the actual mass of the moving assembly was found to be 40 percent higher than early estimates which were used in obtaining the results below.

ANALOG MODEL RESULTS

Typical analog model results are shown in Fig. 13 and 14. These results show the effect of moving the reference set-point through the region of negative slope of the spring characteristic. Also, the downstream geometry is varied. Figure 13 shows two computer runs, and Fig. 14 shows one.

The system variable recorded on each strip is indicated at the left. For example, regulator inlet pressure, $\bar{P}_i$, is recorded on the top strip, followed by seat area, $\bar{A}_s$, seat flowrate $\bar{W}_s$, and so on. All variables except spring deflection are recorded in normalized form, with the scale as indicated at the left. For example, inlet pressure is held constant at its nominal value so that $\bar{P}_i = 1.0$. The seat area scale allows this variable to increase up to 8 times its nominal value. The downstream pressures $\bar{P}_r$ and $\bar{P}_{ac}$ are nominally 1.0, and their scale allows for variations of ± 20 percent and ± 40 percent, respectively. Poppet velocity $\dot{\bar{x}}_p$, nominally zero and is scaled to vary between $\pm 10^3$. Poppet position, $\bar{x}_p$, is also nominally zero and is scaled between ± 1.0 .

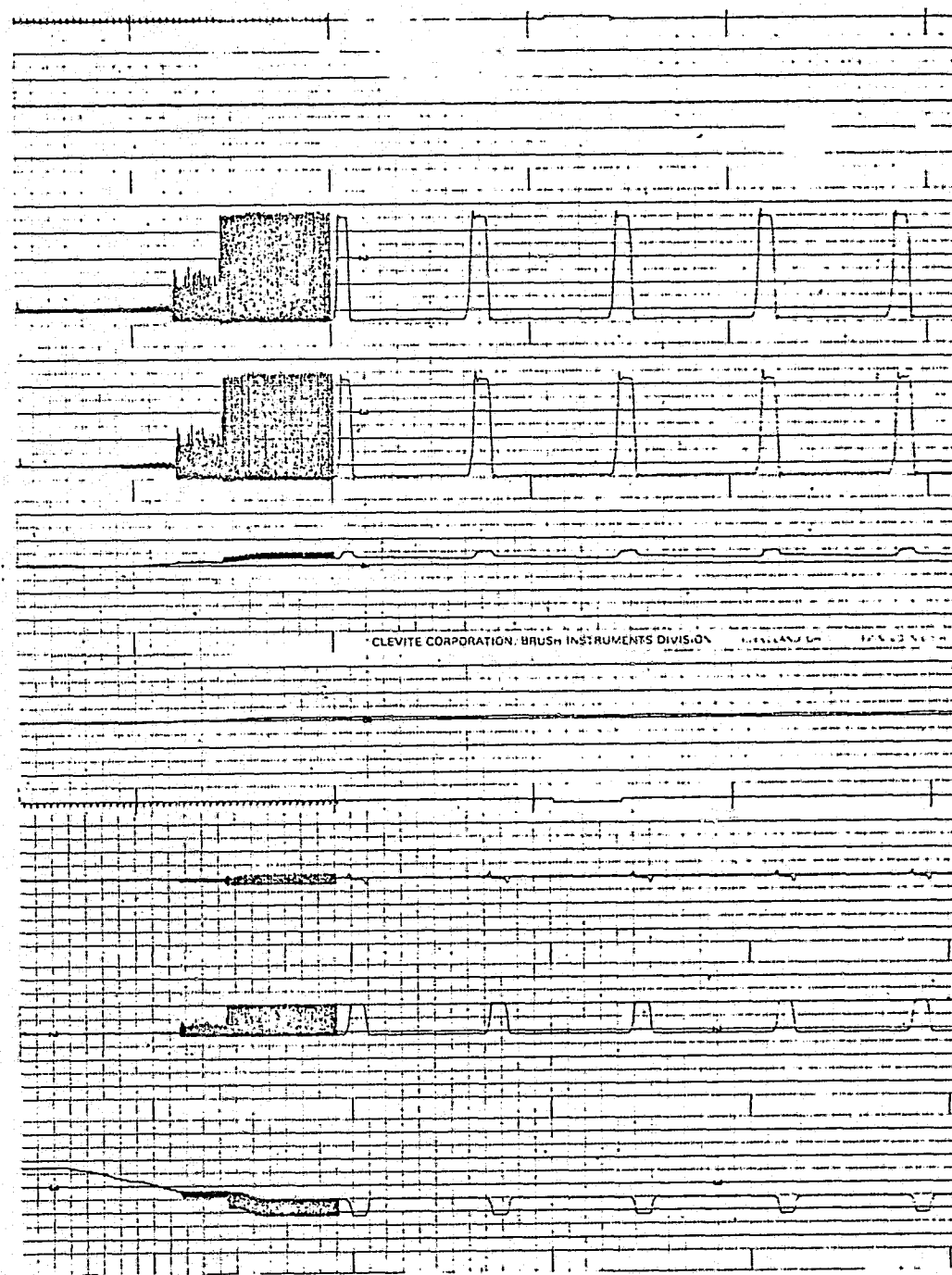
All of the normalized variables recorded in Fig. 13 and 14 can be con-



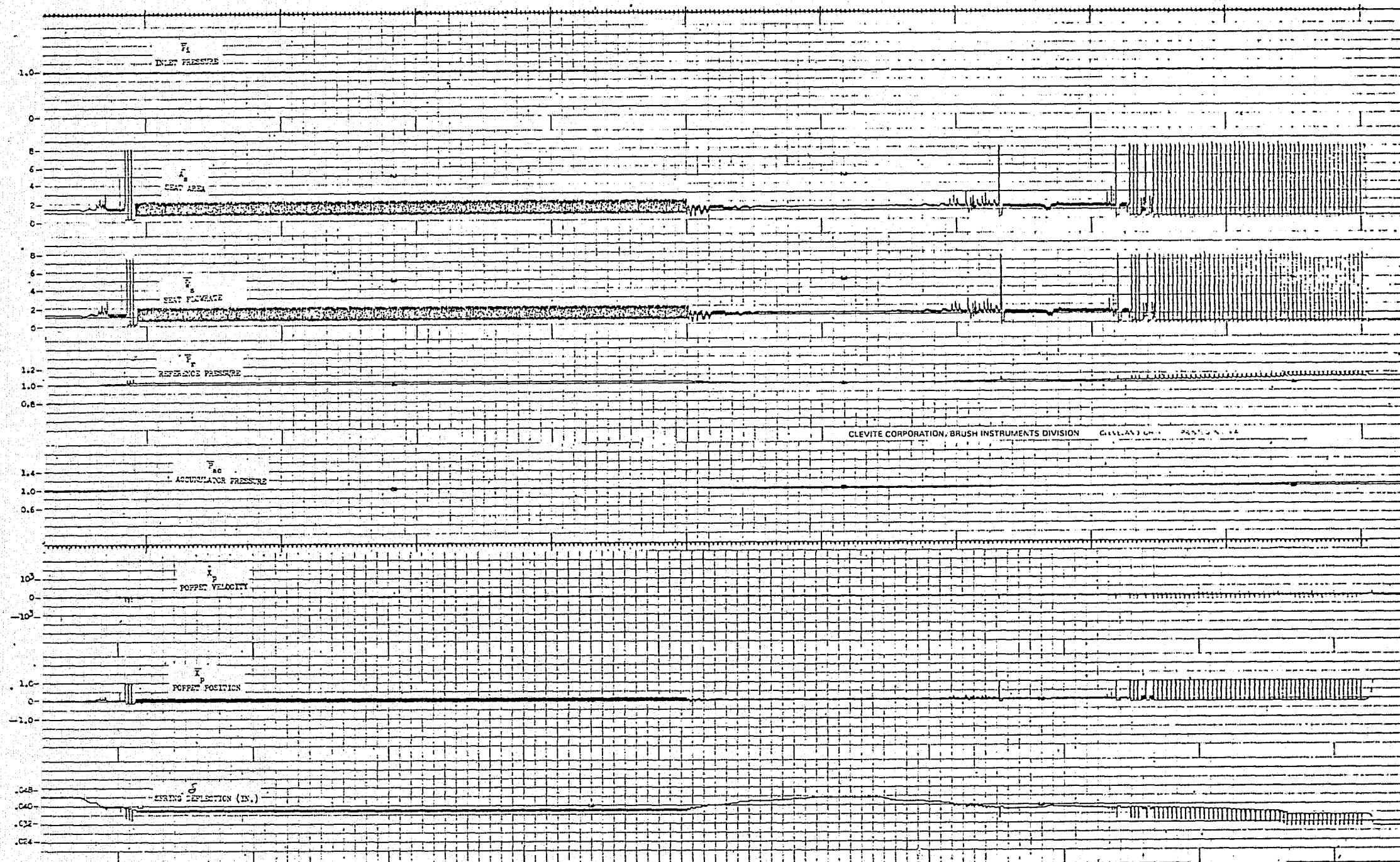
Damping Coefficient (Scaled Equations) = .06
Hysteresis = 1.5 v
Downstream ΔP = 8 psi
Downstream Volume = 68 in³

OPERATION OF M-69 PRESSURE REGULATOR AT CONSTANT INLET PRESSURE
WHILE REFERENCE SET POINT IS MOVED THROUGH ITS RANGE

FIGURE 13



Damping Coefficient (Scaled Equations) = .06
Hysteresis = 1.5 v
Downstream ΔP = 2 psi
Downstream Volume = 68 in³



Damping Coefficient (Scaled Equations) = .06
 Hysteresis: ± 1.5 V
 Downstream $\Delta P = 2$ psi
 Downstream Volume 650 in³

OPERATION OF PM '69 PRESSURE REGULATOR AT CONSTANT INLET PRESSURE, WITH LARGE DOWNSTREAM VOLUME,
 WHILE REFERENCE SET POINT IS MOVED THROUGH ITS RANGE

FIGURE 14

verted to actual values by multiplying by the nominal values. The spring deflection, δ , is recorded as its actual value in the last strip. This permits easy comparison with the spring deflection curve.

The results shown in Fig. 13 and 14 were generated in real time, so that the time scale is indicated by the one-second marker at the top of the charts. Thus, each small division of the chart represents 0.5 second and each major division represents 2.5 seconds. In some places, such as the right-hand side of Fig. 13, the paper speed was increased by 100 times in order to expand the time scale. In these areas each small division represents 0.005 seconds.

The results shown in the left hand portion of Fig. 13 show the effect of moving the set point through its range. The downstream volume for this run was 68 in^3 , which corresponds to the nominal flow bench test system (Ref. 3). The nominal pressure drop between the regulator and accumulator was assumed to be 8 psi. The initial set-point corresponded to a nominal spring deflection of 0.0456 inch. As can be seen from the bottom strip, the deflection was changed to 0.0256 inch over a period of about 35 seconds. When the deflection reached about 0.042, the poppet began to oscillate. This deflection puts the operating point slightly into the negative slope region, as can be seen in Fig. 12. The oscillations continued until the deflection was reduced to 0.0256 inch, which corresponds to the peak of the deflection curve. It is seen that during these oscillations the seat area varies from almost zero to over 3 times its nominal value. The reference pressure oscillations are about 3 percent peak-to-peak. The mean value of reference pressure increases due to the higher force of the spring at the lower deflections.

Upon increasing the deflection, the oscillations resumed. At a deflection of about 0.034 inch, the amplitude of the oscillations increased to the point where the poppet was cycling from one stop to the other. This persisted until a deflection of 0.038 inch. During these

high-amplitude oscillations, reference pressure varied with an amplitude of about 10 percent peak-to-peak.

It is notable that the oscillations cannot be seen in the accumulator pressure. This is obviously due to the large volume. In an actual test, it is entirely possible that oscillations of the poppet would go unnoticed if only the accumulator pressure was monitored.

In order to see the influence of pressure drop between the regulator and accumulator, this parameter was varied from 8 psi to 2 psi. The results of this run are shown in the right-hand portion of Fig. 13. Again, the spring deflection was reduced into the negative slope region. Steady operation existed until deflection reached 0.0384 inch, whereupon poppet oscillations began. Stop-to-stop oscillations began at about 0.027 inch. At this point the time scale was expanded by increasing the chart speed by 100 times. In the expanded scale, it can be seen that the poppet goes to its full-open position about 5-1/2 times per second. This cycling is somewhat slower than in the previous run. Apparently, the lower pressure drop results in interaction with the downstream volume. Also, it has increased the systems tolerance to negative spring slope, evidenced by the lower deflection before the onset of oscillations.

In order to prove that the cycle frequency is indeed influenced by downstream volume, the accumulator volume was increased by a factor of 10. This makes it correspond approximately to that of the ullage of the propellant tank. The results of this run are shown in Fig. 14.

The first portion of Fig. 14 shows that oscillations begin at a deflection of about 0.0384 inch. Returning to the nominal deflection restores steady operation. When the deflection is reduced to slightly below 0.0384 inch, the stop-to-stop cycling begins. Now, however, the frequency is about 1 cycle per second. This suggests that the cycle frequency is a result of interaction with the accumulator.

The above results are the most meaningful of those obtained with the detailed analog model during the first quarter. They are not conclusive, but do suggest the importance of spring slope at the operating point and the downstream geometry. It appears that the observed cycling in the water expulsion tests could be duplicated by the combination of large downstream volume separated from the regulator by a low resistance.

SIMPLIFIED WATER EXPULSION MODEL

The results obtained from the simplified water expulsion system model consist of the stability inequalities, Eq. (40), (41), and (43), and the phase plot, Fig. 11.

The stability inequalities are

$$\gamma > \beta K_2 T_1 \quad (40)$$

$$K_1 > \beta / T_1 \quad (41)$$

$$K_2 < \frac{\gamma}{T_1} \frac{(\gamma + 1/T_1)}{(K_1 + \beta \gamma)} \quad (43)$$

These relationships indicate that a negative spring force-deflection slope (positive $\mathcal{Q}$) requires a large amount of damping for stable operation. From the spring hysteresis data, it was shown that high damping does exist, at least for small velocities. Approximate values for the other system constants indicate that the maximum allowable positive $\mathcal{Q}$ is about 0.08. This suggests that instability should occur at points slightly inside the negative slope region of the deflection curve, which has been borne out by the analog results.

The inequalities also show the influence of downstream volume. From

Eq. (40) it is apparent that larger downstream volumes decrease the tolerable positive β , since T_1 goes up with this volume.

The phase plane plot, Fig. 11, shows a reasonable explanation for the limit cycle observed in some water expulsion tests. The linearized stability model only indicates the conditions for unstable oscillation. The phase plane plot shows how the non-linear spring curve and poppet stops can limit the amplitude of this oscillation.

CONCLUSIONS

Due to the preliminary input data employed in the analyses during the first quarter, no firm conclusions are possible at this time. There are strong indications, however, that operation in the negative slope region of the spring force-deflection curve can lead to oscillations in poppet position, and possibly downstream pressure. The nature of these oscillations depend upon the spring characteristic curve, the set point, and the downstream flow system. The detailed model and the simplified model both indicate that a set point at which the spring rate is excessively negative can cause unsatisfactory regulation.

EXPENDITURES

During the month from 1 September 1969 to 30 September 1969 expenditures totaled \$1914. This includes 110 man-hours of labor.

Total expenditures for the quarter amount to \$8012, which includes 461 man-hours of labor. This represents approximately 75 percent of the planned expenditures and 80 percent of the planned man-hours for the first quarter (Ref. 2).

PLANNED EFFORT

There have been no developments during the first quarter which require changes in the Work Plan as presented in Ref. 2. There has been a slight slippage in schedule, due primarily to initial delays in obtaining regulator description and flow bench data. This is also reflected in the man-hours and expenditures above, which are less than planned.

The next step in the analysis will be to wire the analog model of the flow bench system. This will permit correlation of the flow bench test data which has been transmitted to Rocketdyne. The regulator model constants will first be updated to reflect recently obtained spring and mass data.

REFERENCES

1. Ogata, K., State Space Analysis of Control Systems, Prentice-Hall Inc., Englewood Cliffs, New Jersey, 1967 pg. 442
2. Sack, L. E., Advanced Feed Systems Component Interaction Analysis NAS7-749, Program Work Plan and Schedule, PWP-9229, 28 July 1969, Rocketdyne, a Division of North American Rockwell Corporation, Canoga Park, California
3. Type Approval and Flight Acceptance Test, MM'69 Propulsion Subsystem Pneumatic Regulator, JPL Specification No. TS503503A, Revised 15 September 1969.

APPENDIX I

Regulator Equations

Symbols

APPENDIX I
REGULATOR EQUATIONS
SYMBOLS

Symbol	Definition	Units	Nom. Value	S. F.
A_d	Effective diaphragm area	in. ²	1.665	--
A_s	Seat flow area	in. ²	1.211×10^{-4}	$A_{sn}/10$
A_b	Area of ball seat bore	in. ²	4.78×10^{-3}	--
B_p	Damping Coefficient for poppet	lb-sec/in.		--
C_s C_s	Seat flow coefficient		0.6	--
F_{bs}	Ball retaining spring force	lbs	2	$F_{bsn}/50$
F_{pr}	Pressure force on ball	lbs	15.65	$F_{prn}/50$
F_{ps}	Poppet spring force	lbs	507	$F_{ps}/50$
K_1	Empirical constant in filter flowrate equation	$\frac{\text{in.}^4 - R}{\text{lb-sec}}$	2.945×10^{-5}	--
K_2	Constant ratio of seat flow area to ball position	in.	0.1916	
K_5	Ball spring rate	lbs/in.	4	--
m_p	Mass of poppet moving assembly, including 1/2 of springs	$\frac{\text{lbs-sec.}^2}{\text{in.}}$	2.50×10^{-4}	--
P_a	Ambient pressure	lbs/in. ²	14.7	--
P_i	Inlet pressure, up-stream of filter	lbs/in. ²	3600	$P_{in}/50$

Symbol	Definition	Units	Nom. Value	S. F.
P_s	Pressure upstream of seat	lbs/in. ²	3570	$P_{sn}/50$
P_r	Regulated pressure, downstream of seat	lbs/in. ²	308	$P_{rn}/50$
T_i	Inlet temperature, upstream of filter	°R	530	$T_n/50$
T_s	Temperature upstream of seat	°R	530	$T_n/50$
T_r	Temperature downstream of seat	°R	530	$T_n/50$
U	Unit step function	--	--	--
V_s	Volume from filter to seat	in. ³	0.0404	--
V_r	Volume between the seat and the downstream restriction	in. ³	1.30	
W_s	Stored weight of gas between filter and seat	lbs	4.12×10^{-4}	$W_{sn}/50$
W_r	Stored weight of gas between seat and downstream restriction	lbs	6.70×10^{-5}	$W_{rn}/50$
$\dot{W}_i$	Inlet flowrate	lbs/sec	0.006	$\dot{W}_n/10$
$\dot{W}_s$	Seat flowrate	lbs/sec	0.006	$\dot{W}_n/10$
$\dot{W}_o$	Outlet flowrate	lbs/sec	0.006	$\dot{W}_n/10$
X_p	Poppet position, measured from the point at which the plunger contacts the seated ball. The opening direction is positive	in.	6.33×10^{-4}	$\frac{X_{pmax} - X_{pn}}{50}$

Symbol	Definition	Units	Nom. Value	S. F.
x_{pmax}	Full open position of poppet	in.	0.005	--
x_{pmin}	Fully retracted position of poppet	in.	-0.007	--
δ	Poppet spring deflection measured from free position	in.	variable	
δ_0	Poppet spring deflection when poppet is contacting seated ball	in.	Variable input	--
ϕ	Compressible flow function	--	0.53	--

REGULATOR EQUATIONS

$$(1) \quad \dot{W}_i = K_1 P_i (P_i - P_s)/T_i \quad (P_i, T_i = \text{inputs}, K_1 = \text{const.})$$

$$(2) \quad W_s = \int (\dot{W}_i - \dot{W}_s) dt$$

$$(3) \quad T_s = \int \left[\dot{W}_i (\gamma T_i - T_s) - \dot{W}_s T_s (\gamma - 1) \right] / W_s dt \quad (\gamma = \text{const.})$$

$$(4) \quad P_s = W_s R T_s / N_s \quad (R, V_s = \text{const.})$$

$$(5) \quad W_s = C_s A_s P_s \phi_s / \sqrt{T_s} \quad (C_s = \text{const.})$$

$$(6) \quad \phi_s = 1.06 \left[P_r / P_s (1 - P_r / P_s) \right]^{1/2} \quad P_r / P_s \geq 0.5 \quad (\text{Fig. 4})$$

$$= 0.53 \quad P_r / P_s < 0.5$$

$$(7) \quad A_s = K_2 X_p \quad A_s \geq 0 \quad (K_2 = \text{const.})$$

$$(8) \quad T_r = \int \left[\dot{W}_s (\gamma T_s - T_r) - \dot{W}_o T_r (\gamma - 1) \right] / W_r dt \quad (\dot{W}_o = \text{input})$$

$$(9) \quad W_r = \int (\dot{W}_s - \dot{W}_o) dt$$

$$(10) \quad P_r = W_r R T_r / V_r \quad (V_r = \text{const.})$$

$$(11) \quad m_p \ddot{X}_p = F_{ps} (\delta, \dot{X}_p) - A_d (P_r - P_a) - B_p \dot{X}_p - (F_{bs} + F_{pr}) U(X_p)$$

$$X_{pmin} \leq X_p \leq X_{pmax} \quad \dot{X}_p = 0 \text{ for } X_p = X_{pmax}, X_{pmin}$$

$$m_p, A_d, P_a, B_p = \text{const.}$$

$$(12) \quad U(X_p) = \begin{cases} 0 & X_p \leq 0 \\ 1 & X_p > 0 \end{cases}$$

$$(13) \quad F_{ps} (\delta, \dot{X}_p) = \text{curve fit equation of spring characteristics (print No. 10000976)}$$

$$(14) \quad \delta = \delta_o - x_p$$

($\delta_o = \text{input}$)

$$(15) \quad F_{bs} = F_{bso} + K_5 x_p$$

($F_{bso}, K_5 = \text{const.}$)

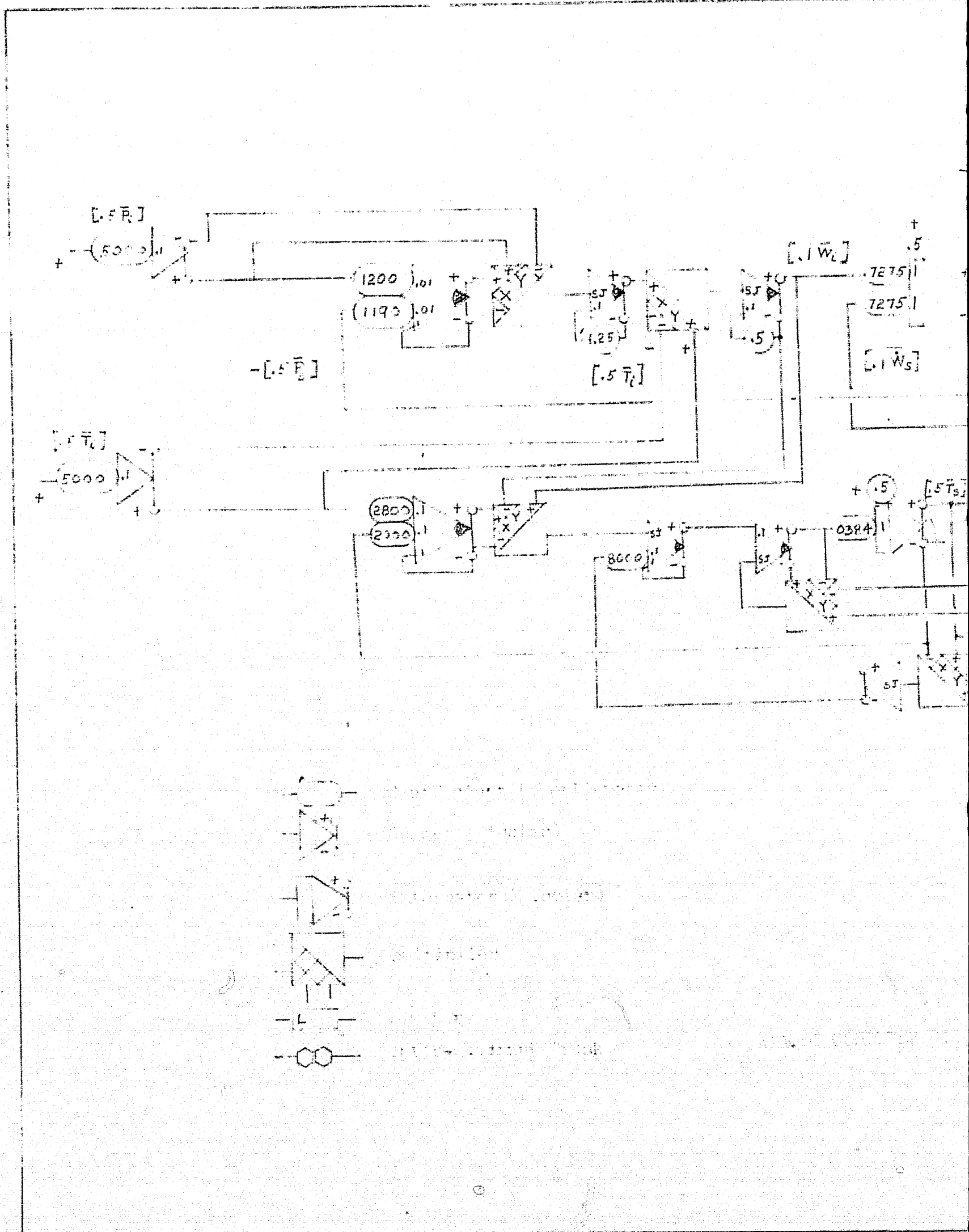
$$(16) \quad F_{pr} = A_b (P_s - P_r)$$

($A_b = \text{const.}$)

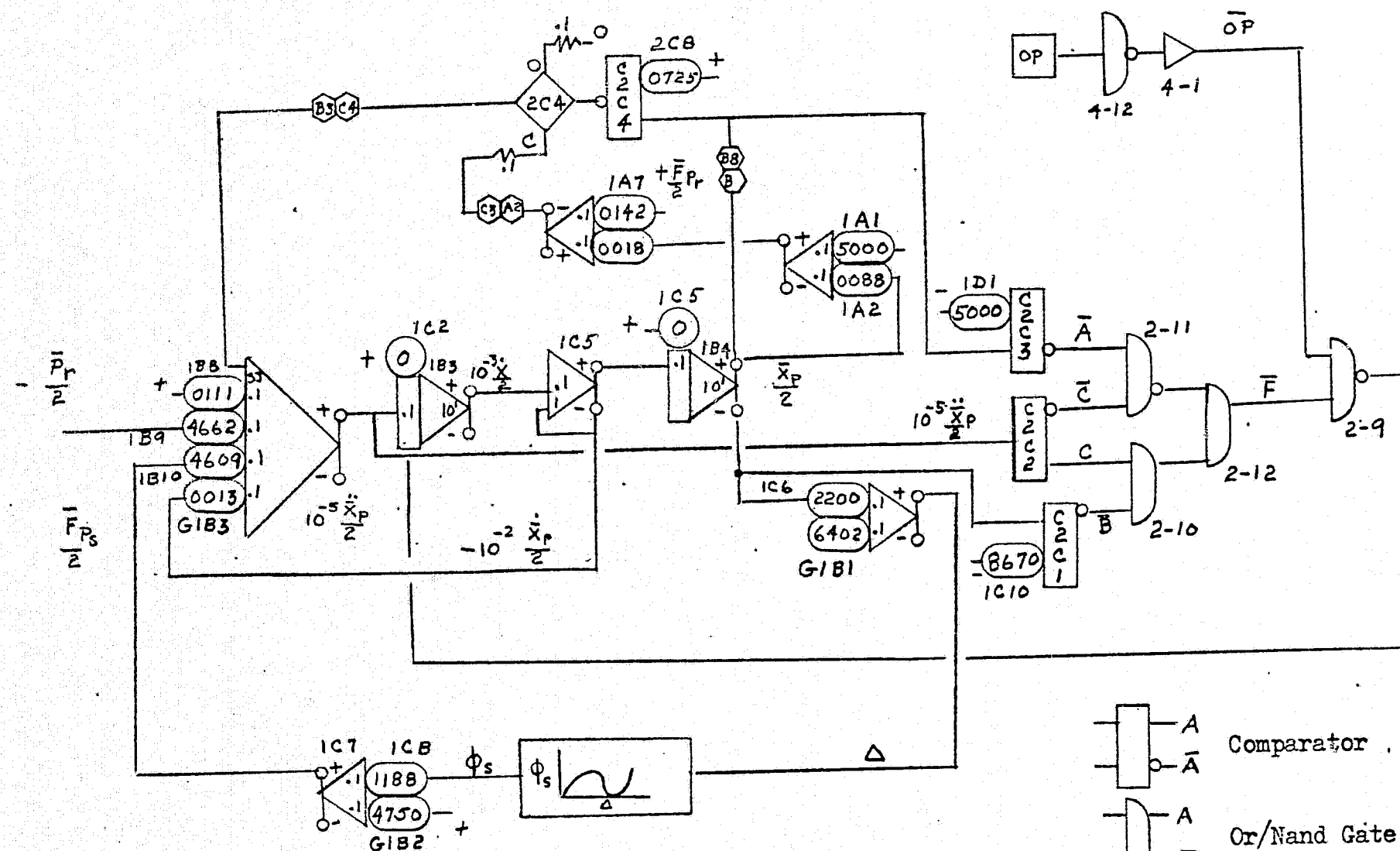
APPENDIX II

Analog Diagrams
for MM'69 Regulator

REPRODUCIBILITY OF THE ORIGINAL PAGE IS POOR.



Foldover FRAME 1



ANALOG COMPUTER DIAGRAM OF MM '69 MECHANICAL SYSTEM EQUATIONS

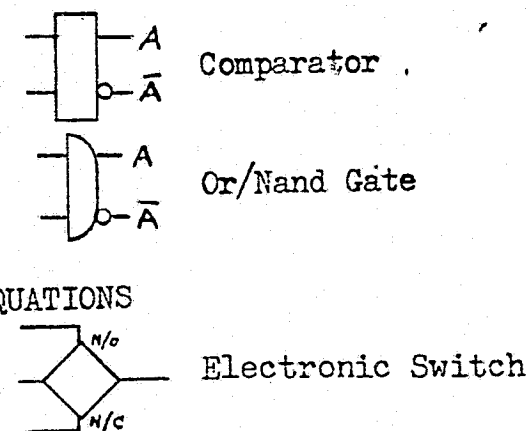
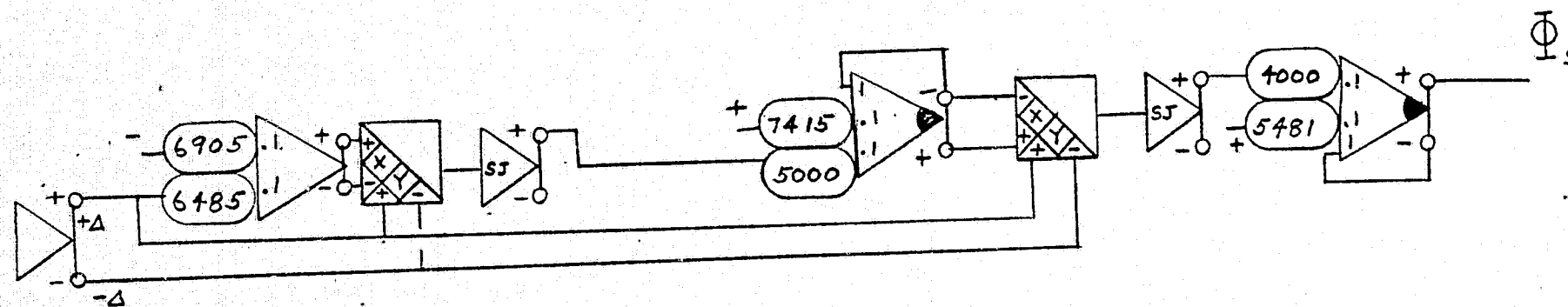


FIGURE II-2



SPRING CHARACTERISTIC GENERATION

FIGURE II-3

APPENDIX III

Simplified Water Expulsion Model

Symbols

Equations

APPENDIX III
Simplified Water Expulsion Model
Symbols

A_d	-	diaphragm area
A_s	-	seat area
B	-	damping coefficient
C_s	-	seat discharge coefficient
F_{ps}	-	spring force, a function of spring deflection
K	-	ratio of seat flow area to ball position
m	-	moving mass of poppet
P_a	-	ambient pressure
P_r	-	liquid tank pressure
P_t	-	pressurization tank pressure
R_l	-	liquid system resistance
T	-	uniform gas temperature throughout
V_g	-	gas volume in liquid tank
$\dot{w}_g$	-	gas weight flowrate
$\dot{w}_e$	-	liquid outflow
X_b	-	ball position
X_p	-	poppet position (positive in opening direction)
$X_{p_{max}}$	-	poppet position
$X_{p_{min}}$	-	poppet position
β	-	slope of normalized spring force-deflection curve
ρ_g	-	gas density in liquid tank
ρ_l	-	liquid density
ϕ	-	compressible flow function

APPENDIX III
System Equations

- (1) $\dot{w}_1 = \left[(P_r - P_a) / R_1 \right]^{1/2}$
- (2) $\dot{P}_r = RT (\dot{w}_g / V_g - \rho_g \dot{V}_g / V_g)$
- (3) $\dot{V}_g = \dot{w}_1 / \rho_1$
- (4) $m \ddot{X}_p = F_{ps}(X_p) - B \dot{X}_p - (P_r - P_a) A_d ; X_{p_{\max}} \leq X_p \leq X_{p_{\min}}$
- (5) $X_b = X_p ; X_p \geq 0$
 $= 0 ; X_p < 0$
- (6) $\dot{w}_g = C_s P_t A_s \phi / \sqrt{T_1}$
- (7) $C_s \phi / \sqrt{T_1} = \text{CONST}$
- (8) $A_s = K X_b$