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ON OPTIMAL SCHEDULING AND HOLDING STRATEGIES FOR THE AIR TRAFFIC CONTROL PROBLEM

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This report consists of the unaltered thesis of Lynn Willard Porter, submitted in partial fulfillment of the requirements for the degree of Master of Science at the Massachusetts Institute of Technology in September, 1969. This research was carried out at the M.I.T. Electronic Systems Laboratory with support extended by the National Aeronautic and Space Administration under Research Grant No. NGL-22-009-(124), M.I.T. DSR Project No. 76265.

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B.S., State University of New York at Buffalo
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September, 1969

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Chairman, Departmental Committee on Graduate Students

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ABSTRACT

The major problem of air traffic control is the guidance and scheduling of many aircraft converging to a terminal for landing. Each aircraft must maintain adequate separation from all other aircraft so as to minimize collision risk. Also, an acceptable airspeed must be maintained in order to avoid unintentional stalls and also to prevent one aircraft from overtaking another. A solution is effected by a linear, optimal, feedback controller that merges aircraft in several feeder guideways into a single string on final approach. Any deviation from the desired spacing or airspeed is quickly corrected by the linear controller. In the event such corrections would require unacceptable airspeeds, the proposed control algorithm directs the aircraft to perform a constant airspeed, time-optimal maneuver outside the guideway. Simulation of the findings were performed on an analog computer.

Thesis Supervisor: Michael Athans

Title: Associate Professor of Electrical Engineering

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CHAPTER I

ON OPTIMAL SCHEDULING AND HOLDING STRATEGIES FOR THE AIR TRAFFIC CONTROL PROBLEM

Growth has been an impressive characteristic of the national aviation industry. Equally impressive statistics tell us that airline traffic increased twenty percent last year. The civil air fleet adds thirteen new jets every ten days and the registry of private aircraft increases by seventeen every day.

Unfortunately, growing pains accompany this burgeoning population. As many airline passengers can attest, this has been most noticeable in the air traffic control (ATC) system. The basic ATC system was developed in the era of the DC-3, a piston-powered airliner with a top speed in the 200 mph range, but today's jet transports can approach the terminal at that speed. To accommodate such a speed differential and a much larger traffic volume, patchwork improvisations were deemed sufficient. But on 26 July 1968, a day now known as "Black Friday," airlines in the Golden Triangle of New York, Chicago and Washington experienced 2,029 delays costing one million dollars. The immediate cause was a rigid enforcement of previously ignored traffic control regulations by Federal Aviation Administration (FAA) air traffic controllers. Their purpose: to demonstrate serious manpower shortages and equipment deficiencies. Indeed, it is not uncommon for a controller to employ outmoded radar and handle twenty aircraft per minute.

Since Black Friday, numerous solutions to this critical situation have been voiced. Some are modifications of the existing system, whereas other schemes replace it entirely with more comprehensive concepts such as the highway in the sky or the conveyor belt proposal. In fact, one popular journal commented, "...some day, perhaps, the air will be fully automated, a three dimensional slot car track with computer-controlled aircraft shuffling around the sky without crowding or possible human error."

It is the objective, then, of this thesis to present an alternative technique for sequencing and control of air traffic in the near-terminal area (NTA). The NTA (an area approximately 40 miles in radius from the terminal) is chosen since the dangerous conditions and inefficiencies occur in the vicinity of the terminal. This approach also obviates the need for constant(and costly) monitoring and control during the take-off and en-route phases of flight. One other possible advantage is that the proposed control technique could be employed without wholesale revision of the present ATC system.

The control algorithm that is presented in this thesis is based upon the results of optimal control theory. One significant area is the contribution of Project Transport for the optimal regulation of the position and spacing of high speed ground vehicles. With these results and the results obtained from the application of the Minimum Principle to the minimum time problem, a system is developed that will control the position, velocity and spacing of aircraft entering the NTA in an optimal fashion. The result of the control application is a string of aircraft properly headed and spaced on final approach.

This novel ATC system is developed in succeeding chapters as follows:

Chapter II presents background information on the present ATC system and discusses the pertinent features that must be incorporated in the system model.

Chapter III details the system model and outlines its various features and benefits.

Chapter IV discusses the mathematical formulation of the system requirements, basically a quadratic cost optimization and a minimum time requirement.

Chapter V reveals the solution to the two problems proposed in Chapter IV.

Chapter VI presents the complete control algorithm as well as the results of simulation and their incorporation into the system model.

Chapter VII offers the conclusions of the thesis.

Chapter IIX presents suggestions for future work in this critical yet exciting area.

CHAPTER II

BACKGROUND ON AIR TRAFFIC CONTROL

As indicated in the previous section, the present ATC system is in an imperfect and unhappy state. FAA controllers have remarked to this author, ... "whatever you come up with, we'll take it ..., ... well, it would only be better than what we have now ... ." It is the purpose of this section, then, to present a general view of the ATC system, briefly indicate why the system provokes such adverse criticism and finally outline the essential features of the physical situation that must be incorporated in the system model.

The following discussion of the present ATC system is centered upon a single category of aircraft : commercial air carriers. This distinction is made to exclude a large group of airspace users, namely the many pilots and aircraft owners who rarely (or never) utilize the large metropolitan airports. Each air carrier initiates his participation in the ATC system by filing a flight plan: a document specifying destination, arrival time, altitude, airspeed, radio call sign and other pertinent data. This flight plan is filed with the local FAA Flight Service Station, a facility usually located at the terminal. By departure time, two groups at the terminal have received essential information from the filed flight plan. One is the control tower personnel who radio instructions for taxiing , runway selection and give final clearance for takeoff. After takeoff, the second group assumes control of the aircraft in the NTA. This group is comprised of departure controllers who observe radar displays while guiding the aircraft out of the NTA via voice commands. Once the aircraft reaches the boundary of the terminal's positive control area, the aircraft, i.e. its flight plan information and responsibility for control, is "handed-off" to the regional control center. This regional control center monitors traffic via radar and radio in its specified area and hands-off aircraft leaving the area to adjacent regions. Thus, a cross-country flight from Los Angeles to Boston would be controlled in sequence by Los Angeles, Albuquerque, Ft. Worth, Memphis, Indianapolis, Washington, New York and Boston regional control centers. The final hand-off performed by the regional control center is made to the approach controller at the terminal or destination. By radar and voice contact, the approach controller guides the incoming aircraft into sectors or "tubs" in the NTA. These "tubs" are regions of protected airspace located around the terminal.

Here, aircraft are vertically segregated in "stacks" and perform holding maneuvers until the approach controller permits the aircraft to proceed to the terminal for landing. At this point, tower controllers assign a runway, monitor the approach to the runway and give taxiing information after touchdown. The pilot's closing or cancellation of the flight plan removes the aircraft from any further consideration by the ATC system and indicates a safe arrival to the FAA Flight Service.

With the above overview in mind, it is proposed to point out why the ATC system has come under fire in recent days.

A first consideration that is partically divorced from the ATC system is the terminal facilities. The airport can accept a certain maximum number of aircraft every hour. This is called the acceptance rate and is dependent upon the number of runways and taxiways in operation, wind direction, visibility, navigational equipment available, runway surface condition, etc. For example, Logan International Airport in Boston, Massachusetts can nominally accept sixty aircraft per hour when using parallel runways, one runway for landings, the other for take-offs. This is halved when the wind direction shifts, and only a single runway may be used. This decreases further to twenty aircraft per hour in poor flying conditions. However, these figures are influenced by the ATC system in operation at Logan. One approach controller remarked that it would be possible to land an aircraft every thirty seconds or 120 per hour using parallel runways. He expressed the idea that this was an upper limit since the preceding aircraft would just be turning off the runway onto the taxiway as the next airliner was passing over the end of the runway for landing. Thus, advancements in ATC should raise the acceptance level but once determined becomes an inviolable constraint on the ATC system. A simple relation points this out:

$$D = \frac{V}{r} \qquad 2.1$$

where D = separation distance on final approach

V = approach airspeed

r = acceptance rate

With an approach speed of 160 knots and an acceptance rate of 120 aircraft per hour, $D = 4/3$ nautical miles. Any separation less than this would result in dangerously crowded conditions; exceeding this distance would represent inefficiency and unnecessary delay. In contrast to this figure, the FAA requires three mile planar separation of aircraft under instrument Flight Rules (IFR). The approach of two aircraft closer than a mile and a half is termed a "near-miss." Controllers as a matter of practice violate this three mile minimum in order to increase traffice flow.

In addition to FAA ATC regulations, other factors prevent the attainment of a maximum acceptance rate and accumulate to produce the delays now commonly experienced in air transportation. One of these is the nearly total dependence on voice communication. Clearly, during the (sometimes lengthy) period required to transmit, receive, understand, acknowledge and act upon a message, the pilot may not be flying in an optimal manner. In addition, voice channels are often crowded, and thus prone to distortion, sometimes requiring message repetition. Another cause of delay is the visual monitoring of position on radar. Any distance measurement is an approximation at best. With this inaccuracy in mind, it is not surprising to find controllers being conservative in their maneuvering and spacing of aircraft. One other characteristic of the ATC system is that human operators are in series with the flow of information. Aircraft identification and other data are passed from controller to controller. As noted above even the pilot is in the control loop. This, of course, tends to slow system operation with attendant possibility of human error. A more ideal situation would be to have human operators in parallel with the system i.e. employ an automatic information processing system with controllers standing by to make necessary decisions and correct emergency conditions.

Leaving the human considerations, the physical situation is now reviewed to justify the work that follows and give meaning to the results, the model to be proposed in the next chapter must adequately embody the characteristics of the real world.

Consider, therefore, the following points:

1. Aircraft vary widely in instrumentation and hence differ greatly in their ability to utilize the ATC system,

2. weather is a considerable influence on airport operations,

3. many different categories of aircraft may request runway use simultaneously,

4. larger airports possess a complex of runways, single or parallel runways for each major wind direction,

5. traffic density varies markedly during the day,

6. aircraft approaching the airport may have different priorities to land,

7. aircraft enter the system with a variety of headings, altitudes and airspeeds, and

8. jet transports operate in the lower end of their speed regime in the NTA, hence their acceleration capabilities are limited.

With these essential features in mind, a set of assumptions upon which the system model is based will be proposed in the next chapter.

In general, utilization of the proposed control system is expected to increase the acceptance rate, but specifically, the following benefits are to be realized:

1. Significant decrease in the time spent in the near terminal area,

2. substantial reduction in the maneuvers required to achieve the desired sequencing and spacing,

3. elimination of the need for stacking,

4. assurance that each aircraft is separated from all other aircraft by a minimum distance, and

5. airspeed in the NTA will vary only slightly about a specified velocity, V, thus preventing the possibility of inadvertent stalls or one aircraft overtaking another.

Other benefits that could be derived from an extended system might include:

1. execution of landing priorities, especially for emergency situations,

2. adaptation to a parallel runway configuration,

3. consideration of weather factors on the landing rate, especially wind direction changes,

4. control techniques for overlapping airspace of adjacent airports such as in the New York City area, and

5. scheduling a mix of several categories of aircraft.

CHAPTER III

A SYSTEM MODEL

In this chapter, basic assumptions are listed, and a system model is described. Due to the limitations inherent in the basic system model, corrective measures are also proposed.

The set of assumptions upon which the system model is constructed is listed below:

1. Aircraft have sufficient instrumentation to receive control information and to precisely and accurately execute maneuvers dictated by the ATC system,

2. as has been said concerning weather ... you can't do anything about it ... so, assume zero wind velocity and otherwise flyable weather conditions,

3. each aircraft in the system will be able to maintain an approach airspeed, V, and have a turning radius no smaller than R at that speed,

4. for this thesis, the sequencing and approach of aircraft will be for a single runway. Extension to multiple runways should not be difficult,

5. high density traffic is one of the major causes of the present situation; assume that the airspace is densely packed in the NTA,

6. all aircraft have equal priorities to land although this requirement may be easily relaxed, and

7. since height information is of questionable reliability, consider all the incoming aircraft to be flying on the same plane.

Assumptions (1) and (3) may be justified on the grounds that high accuracy inertial reference platforms are being installed in commercial aircraft. Also, private aircraft or those aircraft incapable of maintaining the desired approach airspeed V, say 160 knots (1 knot = 1 nautical mile per hour = 1.151 statute miles per hour), may be relegated to secondary airfields.

At this point, the problem requires the guidance of an arbitrary number of aircraft in a plane where each aircraft has an initial airspeed in the neighborhood of V, a minimum turning radius R at that speed and a non-zero distance separation from all other aircraft. The end result of the control process is each aircraft crossing the outer marker (5-7 miles from the terminal) with a specified heading, airspeed V and separated by distance D from leading and trailing aircraft. See Fig. 1 .

It is now proposed to assign each aircraft entering the NTA to one of several feeder guideways. Each guideway is a fixed, straight-line path from the boundary of the NTA to the outer marker. When approaching the NTA, each aircraft descends to a common altitude and enters its disignated guideway. While in the guideway, the aircraft experiences controls that properly adjust its spacing and velocity for

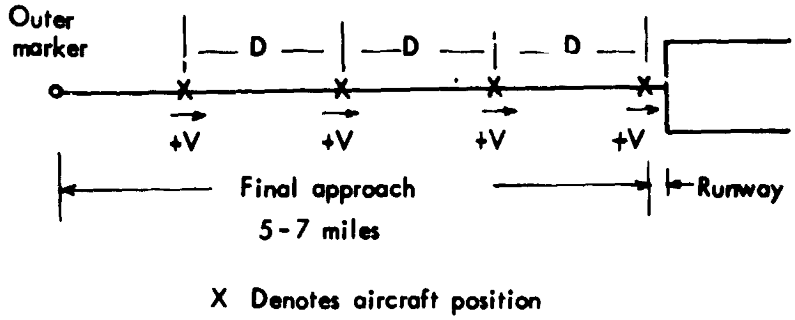


Fig. 1 Desired final state of ATC system in NTA

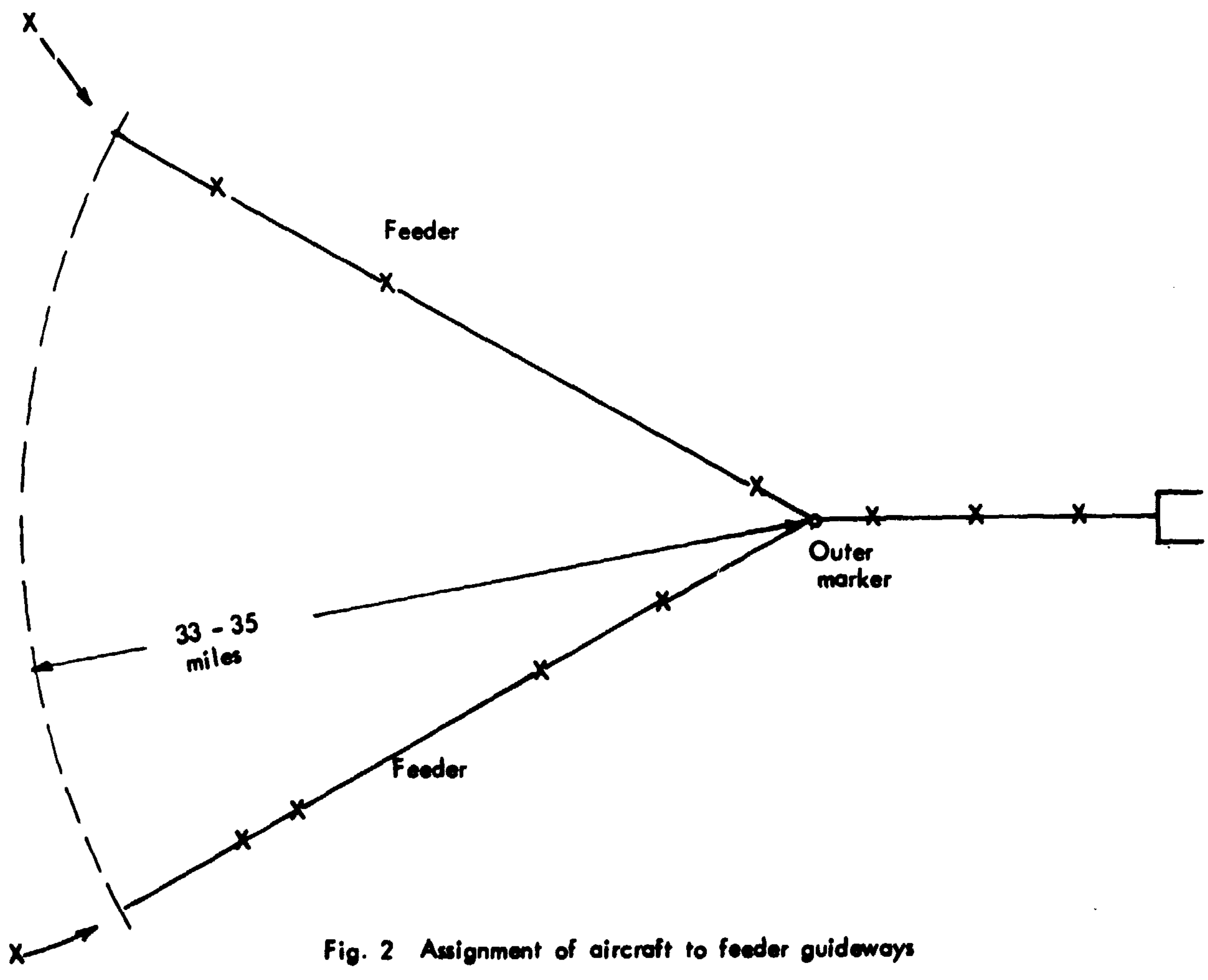


Fig. 2 Assignment of aircraft to feeder guideways

the upcoming merging process. At the outer marker, all aircraft are merged into the main guideway (final approach) with the desired inter-vehicle spacing, D, and approach airspeed, V. In Figure 2, the aircraft nearest the outer marker typify the spacing required for a safe and successful merging. Only two guideways are shown, but several more may be easily included. This extension to multiple guideways is simple due to the following observation of Levine and Athans[1]: the vehicle distributions required for merging are identical to those encountered in the superposition of each feeder onto a single guideway. To accomplish this, the feeders are "swung" together until they are collinear with the main guideway. This process is shown in Figure 3. The result is a single string of aircraft, and the problem of multiple guideway control is reduced to that of controlling spacing and velocity on a single guideway. The single string control problem is treated intensively in the references cited and thus, a complete transfer of theory and conclusions can be made from the single string case to the multiple-guideway situation. With this theory and its results, a control system is available that:

1. Performs a safe merging of the feeder guideways,

2. separates the aircraft by desired separation distance, D,

3. directs each aircraft to maintain an airspeed close to V, and

4. minimizes needless accelerations.

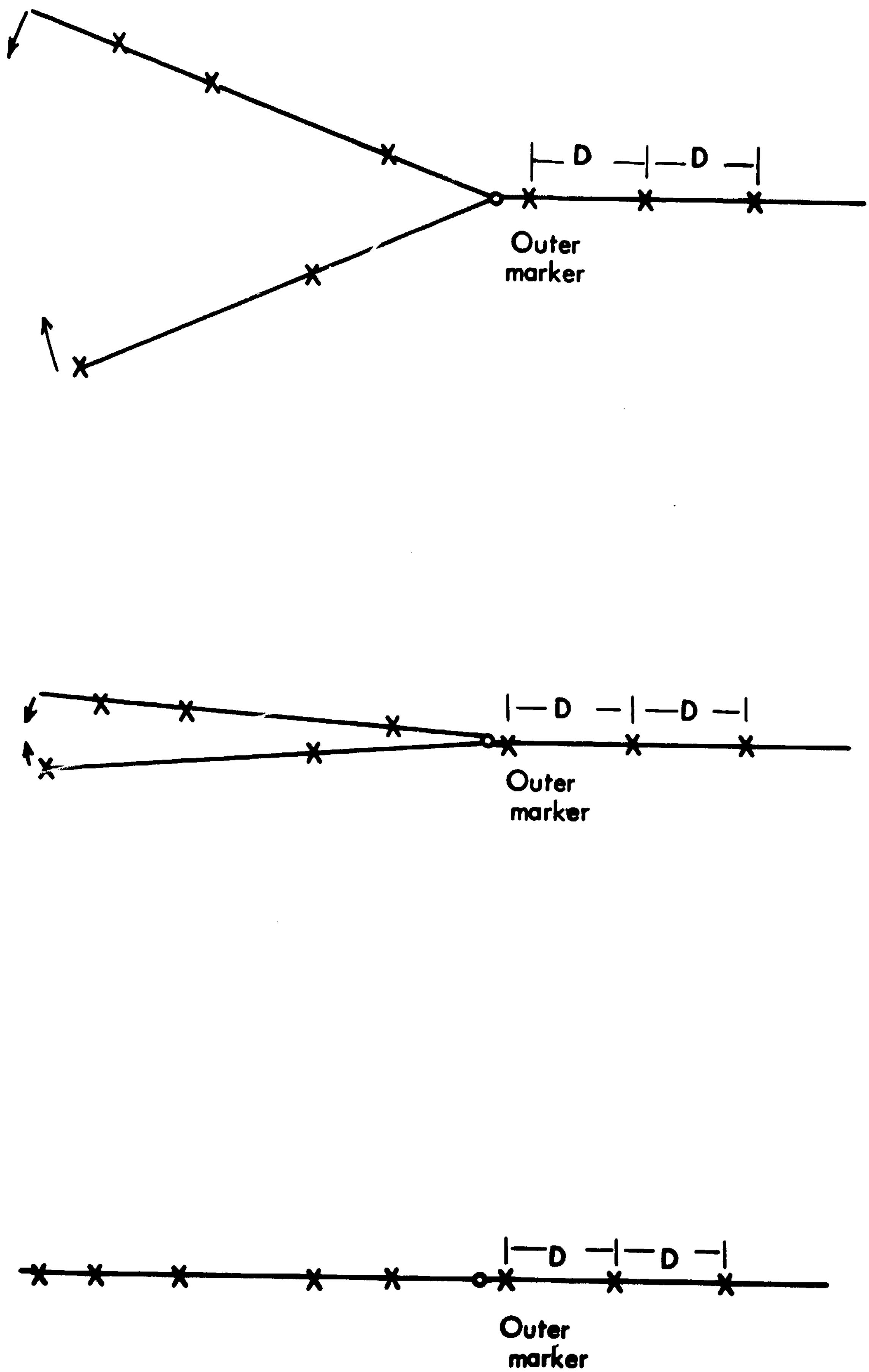


Fig. 3 Equivalent distributions of aircraft for merging

All these characteristics are desirable for both safety of aircraft operation and passenger comfort. These features arise from the development of a control system that seeks to minimize a quadratic cost functional. Through such a minimization, we obtain an optimal feedback control system and realize the features of the control system listed above.

The theory referred to above was developed for high-speed trains. Unfortunately, an airplane is not a train or any other ground vehicle. This obvious statement reveals the gap between the theory available and the problem under investigation. The remainder of this chapter is devoted, then, to overcoming this limitation.

This limitation is velocity control. It is very easy to attain a wide range of safe operating speeds with a ground vehicle, whereas an aircraft has a very limited airspeed range in the NTA. Airspeed below a certain point will result in a stalled condition i.e. , loss of wing lift followed by a plunge toward the earth. On the other hand, high airspeeds increase greatly the collision risk.

For the concession made to safety, we now have limited velocity control. To circumvent this restriction, we shall rely on maneuvers outside the guideway to correct gross deviations from the desired spacing. This, then, is the significant problem area and involves the following points:

1. A criterion must be found that determines those aircraft that require maneuvering,

2. a set of precautions must be devised so as to minimize collision risk between maneuvering aircraft,

3. what maneuvers can be performed subject to the constraints of a minimum turning radius and limited velocity variation,

4. how are control laws for these maneuvers formulated, and

5. how are aircraft controlled that reside in the guideway while others are maneuvering.

For maneuvers outside the guideway, the aircraft shall be restricted to a constant airspeed. This is done for the sake of simplicity in problem formulation and solution. Also, most normal maneuvers performed by aircraft are constant airspeed in nature. With this restriction and realizing that the aircraft must exit and re-enter the guideway tangentially, several classes of maneuvers may be immediately recognized. One type would be a "fly-around" maneuver in which the aircraft flies in a direction opposite to its initial heading as in Figure 3 a. The limiting case for this maneuver would be a circle. "Oscillation" of the aircraft about the guideway composes the other class of maneuvers. In this category, the aircraft would maintain its forward direction of flight. Some examples are shown in Figure 8 b.

There are many control programs that merge the aircraft trajectory with that of the target. The "best" trajectory shall be the one that accomplishes the rendezvous in minimum time. Minimum time is stipulated for several reasons:

1. elimination of needless maneuvering

2. to minimize collision risk by minimizing time spent outside the protected airspace of the guideway, and

3. to maximize landing rate by enabling the aircraft to adhere to the schedule imposed by the target string through an efficient set of "waste-time-and-let-target-catch-up" maneuvers.

To achieve a better understanding of the system operation and facilitate the solution of the problems listed above, the following addition is made to system model. A string of imaginary "target" aircraft is superimposed over the string of real aircraft with the positions of the first aircraft in each string coincident. These imaginary targets travel along the guideway with desired airspeed, V, and inter-vehicle spacing, D. Thus, each aircraft that is violating the separation constraint will appear to be ahead of its corresponding target. See Fig. 4. Also, the target string displays the desired final state of the merging control and provides a target for rendezvous for each aircraft maneuvering outside the guideway.

The mathematical formulation of these problems is presented in next chapter.

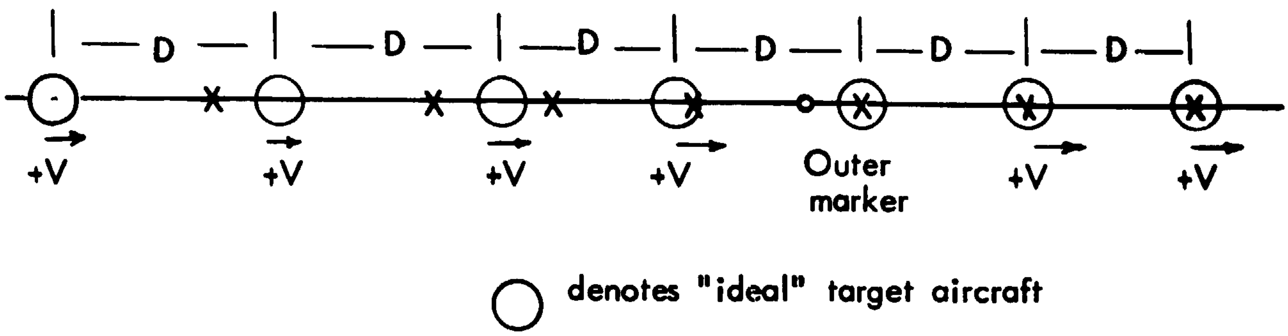


Fig. 4 Target string placed upon guideway

CHAPTER IV

MATHEMATICAL FORMULATION

Before proceeding with the primary intent of this chapter, one critical issue deserves attention: the initial placement of target aircraft. Not only does the target string exhibit the desired position, velocity and separation of the real aircraft for all time, but its initial placement is of crucial importance in selecting those aircraft which are to maneuver outside the guideway. The procedure is as follows: for one aircraft in the guideway, the target would be placed on the aircraft's initial position as shown in Figure 5 a. Adding another aircraft in the guideway, the target would be placed to the left of the first at the desired separation distance, D. See Figure 5 b. This process would then be repeated for every additional aircraft on the guideway. After a period of continuous operation, the guideway takes on the appearance of Figure 4. Also, since an aircraft achieves coincidence with its target by the time it reaches the outer marker and remains coincident on final approach, it can be seen that any aircraft on final approach uniquely determines and fixes the placement of the target string.

Using the coordinate and vehicle numbering systems as shown in Figure 6, the equation of motion for i^{th} target aircraft is:

$$z_i(t) = Vt + z_{io} \tag{4.1}$$

where

V is desired string velocity

z_{io} is position of i^{th} target at $t = 0$.

The result of differentiating 4.1 with respect to time is

$$\dot{z}_i(t) = V = \text{constant} \tag{4.2}$$

The observation is also made that

$$z_i - z_{i+1} = z_{io} - z_{i+1,o} = D \tag{4.3}$$

where D is the desired separation distance, a constant.

Thus, the target string moves along the guideway with the desired, constant approach airspeed, V, and each target is separated from leading and trailing targets by the desired, constant, separation distance, D.

The equations of motion for the i^{th} aircraft are

$$x_i(t) = v_i(t) \tag{4.4}$$

$$m_i v_i(t) = - g_i(v_i(t)) + f_i(t) \tag{4.5}$$

where $x_i(t)$ is the position of i^{th} aircraft

$v_i(t)$ is the velocity of i^{th} aircraft

m_i is the mass of i^{th} aircraft

$g_i(\cdot)$ is the drag function of i^{th} aircraft

$f_i(t)$ is the total thrust applied to the i^{th} aircraft

For the differential equations above and subsequent derivations, the following assumptions are made:

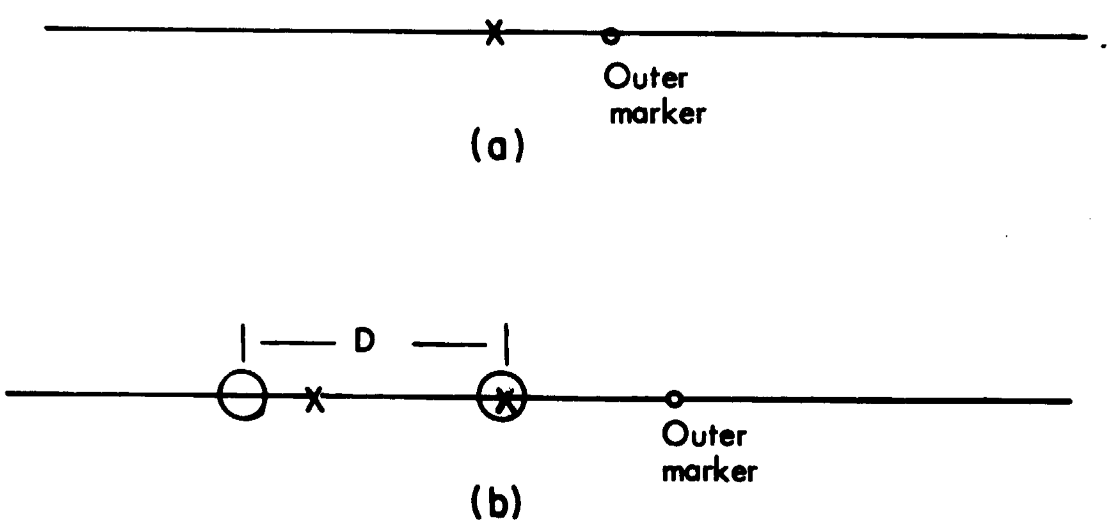


Fig. 5 Initial placement of targets

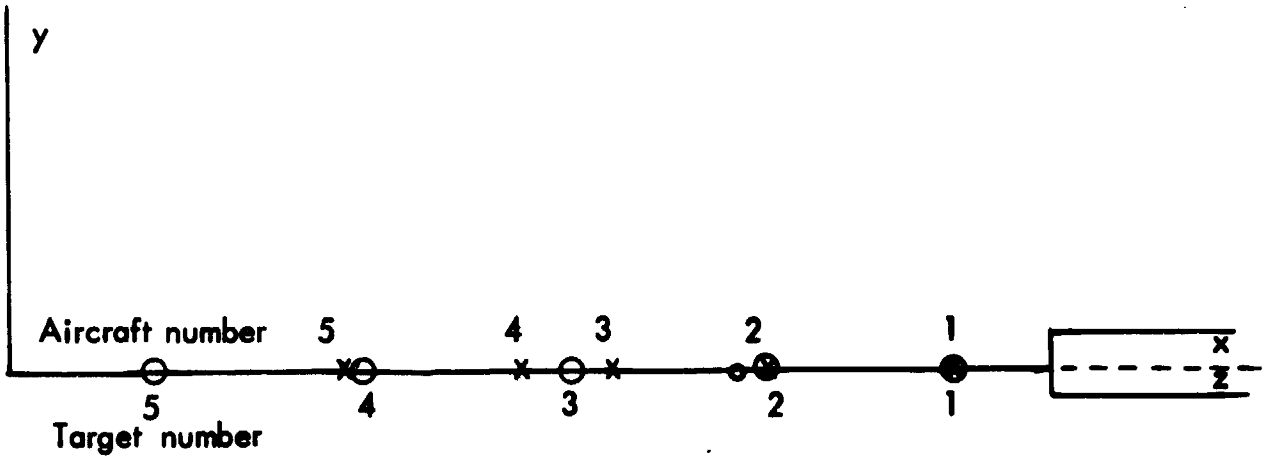


Fig. 6 Coordinate and numbering systems

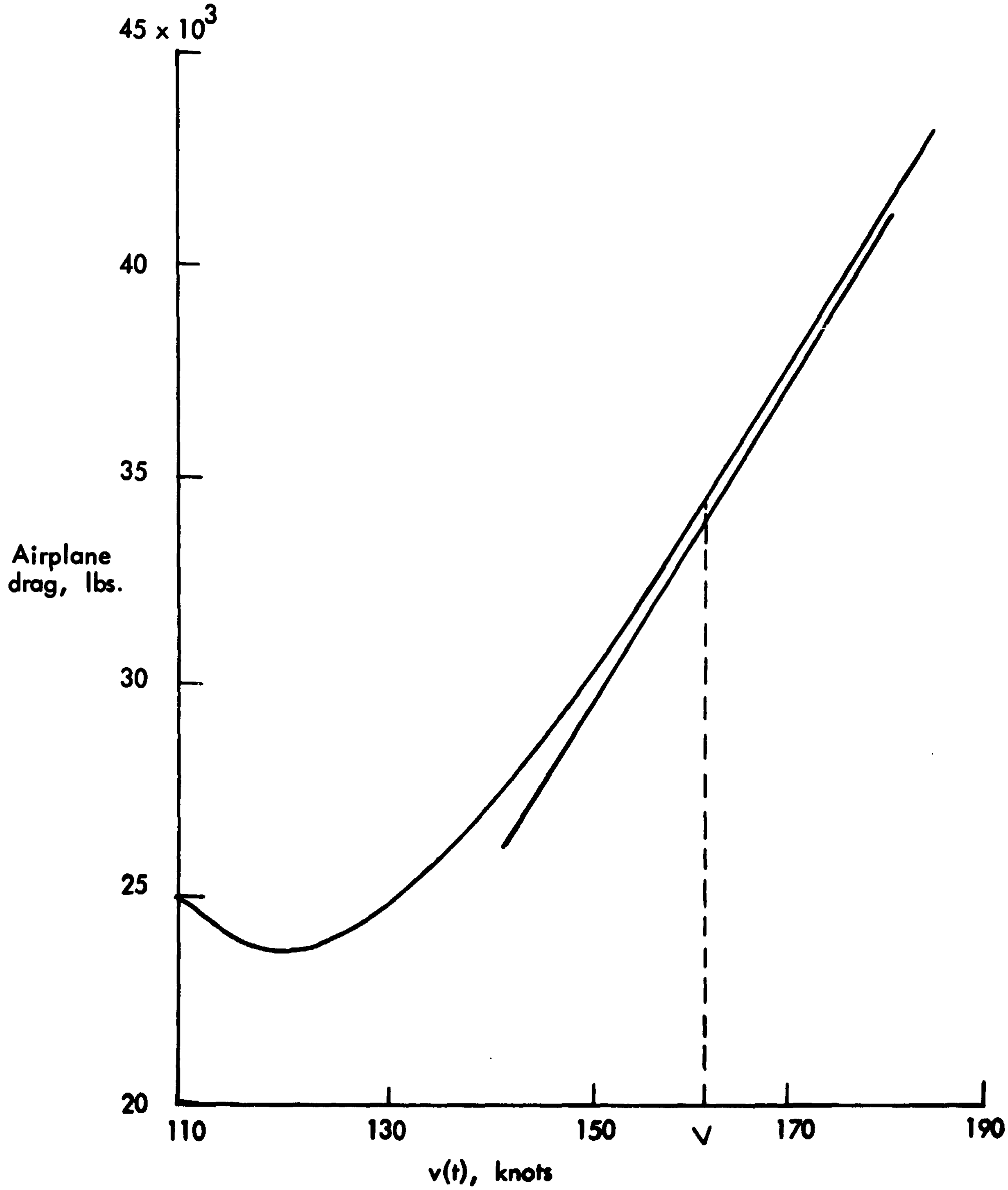


Fig. 7 Drag vs. Velocity for KC-135

1. The mass is assumed to be constant. For the relatively short time of operation in the NTA, it appears reasonable to neglect the decrease in mass due to fuel consumption.

2. Velocity deviations are assumed to be small thus allowing a truncated Taylor series expansion about V to be used in place of $g_i(\cdot)$. The approximate linearity of the drag function about V is evident from Figure 7. Ignoring higher order derivatives, and setting

$$\left.\frac{\partial g_i(\cdot)}{\partial v_i(t)}\right|_{v_i(t) = V} = a_{io} \tag{4.6}$$

the approximate expression for $g_i(\cdot)$ results

$$g_i(\cdot) \approx g_i(V) + a_{io}(v_i(t) - V) \tag{4.7}$$

Since the objective of the control system is to bring the real aircraft to their targets and maintain that coincidence, the target string may be thought of as the system's operating point. With this idea in mind, the following "error variables" are introduced

<u>position error</u> $$e_{ix}(t) = x_i(t) - z_i(t) \tag{4.8}$$

<u>velocity error</u> $$e_{iv}(t) = v_i(t) - V \tag{4.9}$$

<u>acceleration error</u> $$B_i u_i(t) = \frac{f_i(t) - g_i(V)}{m_i} \tag{4.10}$$

Thus, the following significance can be attached to these new quantities:

1. if $e_{ix}(t) > 0$, the aircraft is ahead of its target. Since densely packed airspace is assumed, the initial value of $e_{ix}(t)$ shall always be $e_{ix}(0) \geq 0$.

2. if $e_{ix}(t) < 0$, the aircraft is behind its target;

3. if $e_{iv}(t) > (<) \; 0$, the aircraft has an airspeed greater (less) than the desired string velocity, V;

4. if $B_i u_i(t) > (<) \; 0$, the total thrust is greater (less) than that required for straight and level flight i.e. the aircraft is accelerating (decelerating).

Substituting these "error variables" into Eqs. 4.4, 5, the state equations for i^{th} aircraft result

$$\dot{e}_{ix}(t) = e_{iv}(t) \tag{4.11}$$

$$\dot{e}_{iv}(t) = \frac{a_{io}}{m_i} \; e_{iv}(t) + B_i u_i(t) \tag{4.12}$$

Setting $a_{io}/m_i = a_i$, the system equations may be written for any number of aircraft, say n.

$$\begin{vmatrix} e_{1x} \\ e_{1v} \\ e_{2x} \\ e_{2v} \\ e_{3x} \\ e_{3v} \\ \vdots \\ e_{nx} \\ e_{nv} \end{vmatrix} = \left|\begin{array}{cc:cc:cc:c:cc} 0 & 1 & & & & & & & \\ 0 & -a_1 & \multicolumn{2}{c:}{\underline{0}} & \multicolumn{2}{c:}{\underline{0}} & \cdots & \multicolumn{2}{c}{\underline{0}} \\ \hdashline \multicolumn{2}{c:}{\underline{0}} & 0 & 1 & \multicolumn{2}{c:}{\underline{0}} & \cdots & \multicolumn{2}{c}{\underline{0}} \\ & & 0 & -a_2 & & & & & \\ \hdashline \multicolumn{2}{c:}{\underline{0}} & \multicolumn{2}{c:}{\underline{0}} & 0 & 1 & \cdots & \multicolumn{2}{c}{\underline{0}} \\ & & & & 0 & -a_3 & & & \\ \hdashline \vdots & & \vdots & & \vdots & & \cdots & \vdots & \\ \hdashline \multicolumn{2}{c:}{\underline{0}} & \multicolumn{2}{c:}{\underline{0}} & \multicolumn{2}{c:}{\underline{0}} & \cdots & 0 & 1 \\ & & & & & & & 0 & -a_n \end{array}\right| \begin{vmatrix} e_{1x} \\ e_{1v} \\ e_{2x} \\ e_{2v} \\ e_{3x} \\ e_{3v} \\ \vdots \\ e_{nx} \\ e_{nv} \end{vmatrix} + \begin{vmatrix} 000\ldots0 \\ \beta_1 00\ldots0 \\ 000\ldots0 \\ 0\beta_2 0\ldots0 \\ 000\ldots0 \\ 00\beta_3\ldots0 \\ \vdots \\ 000\ldots0 \\ 000\ldots\beta_n \end{vmatrix} \begin{vmatrix} u_1 \\ u_2 \\ u_3 \\ \vdots \\ u_n \end{vmatrix} \qquad (4.13)$$

The performance objectives outlined in chapter III are embodied in the minimization of a cost functional of the form

$$J = \frac{1}{2} \int_0^{\infty} \Big[\sum_{i=1}^{n} r_i\, u_i^2(t) + \sum_{i=2}^{n} q_i \left(e_{ix}(t) - e_{i-1,x}\right)^2 \qquad (4.14)$$

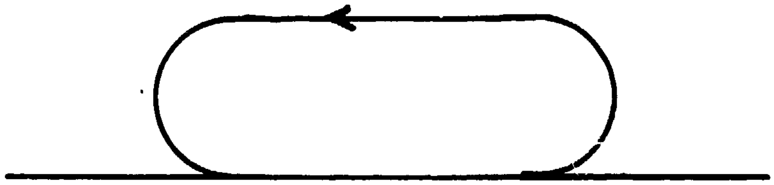
$$+ \sum_{i=1}^{n} p_i\, e_{iv}^2(t) \Big] dt$$

where $r_i > 0$

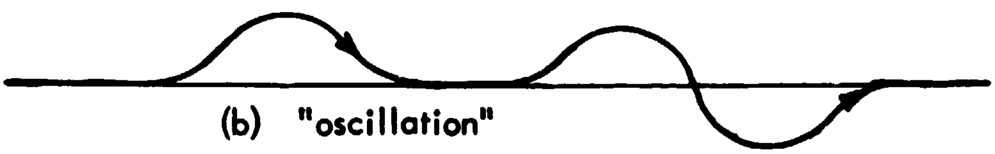
$p_i > 0$

$q_i > 0$

n = number of aircraft in system



(a) fly-around



(b) "oscillation"

Fig. 8 Typical candidates for extra-guideway maneuvers

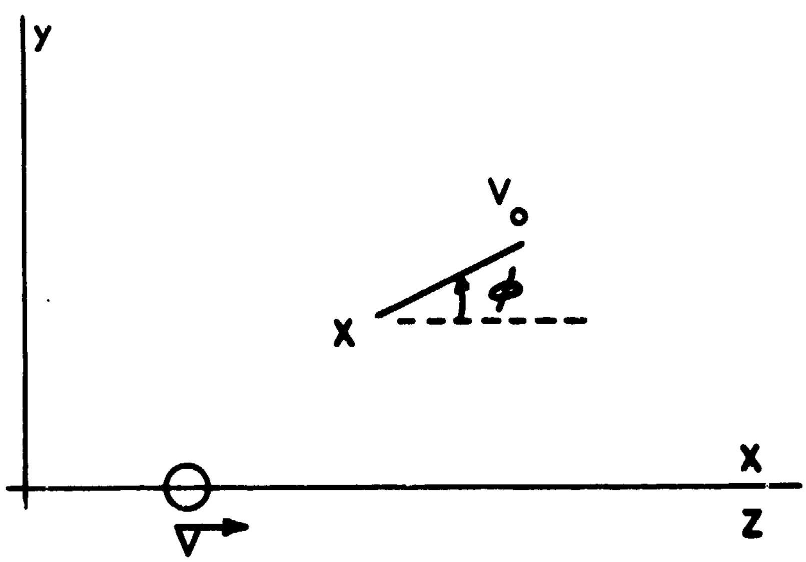


Fig. 9 Coordinate system for minimum time rendezvous

Examination of the cost functional reveals that this is so:

1. The quadratic form penalizes large controls and errors more strongly than small ones. This is important for minimizing needless accelerations and velocity deviations.

2. The integrands are zero when the desired spacing and velocity are achieved. Thus, the cost is zero for aircraft spaced at D intervals and maintaining an airspeed, V.

The control that drives the system to the desired state, $\underline{e}(t) = \underline{0}$, is presented in the next chapter.

For the minimum-time rendezvous problem, the equations of motion for the aircraft and target are based upon the coordinates shown in Figure 9 and are listed below:

<u>aircraft</u>

$$\dot{x}(t) = V_o \cos\phi(t) \tag{4.15}$$

$$\dot{y}(t) = V_o \sin\phi(t) \tag{4.16}$$

$$\dot{\phi}(t) = u\ , \quad |u| \leq A \tag{4.17}$$

where A is the maximum turn rate

<u>target</u>

$$z(t) = Vt + z_o \tag{4.18}$$

$$\dot{z}(t) = V \tag{4.19}$$

Boundary conditions are :

$$x(0) = x_o \qquad x(T) = z(T)$$
$$y(0) = 0 \qquad y(T) = 0$$
$$\phi(0) = z_o,\ x_o \neq z_o \qquad \phi(T) = 0$$

Where T, rendezvous time, is determined from the minimum value of the cost functional for a given set of boundary conditions

$$J = \int_o^T dt \tag{4.20}$$

Similar to the above approach to obtain error variable notation, the rendezvous equations will likewise be transformed. Also, the maximum turn rate, A, is normalized to unity as well as the airspeeds of the real aircraft and its target. The reasons for the normalization and identity between the two airspeeds are given in later chapters.

$$\dot{x}(t) = \cos\phi(t) \tag{4.21}$$
$$\dot{y}(t) = \sin\phi(t) \tag{4.22}$$
$$\dot{\phi}(t) = u\ ,\ |u| \leq 1 \tag{4.23}$$

Similarly, the target's coordinates are denoted by primes:

$$\dot{x}'(t) = z(t) = 1 \tag{4.24}$$
$$\dot{y}'(t) = 0 \tag{4.25}$$
$$\dot{\phi}'(t) = 0 \tag{4.26}$$

And defining the following error variables,

$$\Delta x(t) = x(t) - x'(t) \tag{4.27}$$

$$\Delta y(t) = y(t) - y'(t) = y(t) \tag{4.28}$$

$$\Delta \phi(t) = \phi(t) - \phi'(t) = \phi(t) \tag{4.29}$$

the following equations are obtained:

$$\Delta \dot{x}(t) = \cos\phi(t) - 1 \tag{4.30}$$

$$\Delta \dot{y}(t) = \sin\phi(t) \tag{4.31}$$

$$\Delta \dot{\phi}(t) = u \tag{4.32}$$

The transformed boundary conditions are:

$$\Delta x(0) > 0 \qquad \Delta x(T) = 0$$

$$\Delta y(0) = 0 \qquad \Delta y(T) = 0$$

$$\Delta \phi(0) = 0 \qquad \Delta \phi(T) = 0$$

The solution i.e. the minimum time trajectories is presented in the next chapter.

CHAPTER V

SOLUTION

At this point, the solutions to the problems posed in the previous chapter are presented.

The solution to the optimal merging problem is a straight-forward and standard result of optimal control theory. Given a linear, time-invariant, controllable system of the form:

$$\dot{\underline{x}}(t) = \underline{A}\,\underline{x}(t) + \underline{B}\,\underline{u}(t) \tag{5.1}$$

where:

$\underline{x}(t)$ is the n-valued state vector

$\underline{u}(t)$ is the m-valued control vector

$\underline{A}$ is the nxn system matrix

$\underline{B}$ is an nxm matrix

with the desired end-state

$$\underline{x}(t) = 0 \tag{5.2}$$

and a cost funtional of the form:

$$J(\underline{u}) = \frac{1}{2} \int_0^{\infty} \left[\underline{x}'(t)\,\underline{Q}\,\underline{x}(t) + \underline{u}'(t)\,\underline{R}\,u(t) \right] dt \tag{5.3}$$

where:

$\underline{Q}$ is an nxn, non-negative definite symmetrical matrix

$\underline{R}$ is an mxm, positive definite, symmetrical matrix, then there exists a solution of the optimal control problem of the form:

$$\underline{u}^*(t) = - \underline{R}^{-1} \underline{B}' \underline{K} x^*(t) \tag{5.4}$$

where:

$u^*(t)$ is the optimal control

$x^*(t)$ is the optimal trajectory

$\underline{K}$ is the solution to the following non-linear matrix equation

$$\underline{K}\,\underline{A} + \underline{A}'\underline{K} - \underline{K}\,\underline{B}\,\underline{R}^{-1}\underline{B}'\underline{K} + Q = 0 \tag{5.5}$$

where $\underline{K}$ is an nxn, positive definite, symmetrical matrix.

The cost functional proposed in chapter III is equivalent if the $\underline{Q}$ and $\underline{R}$ matrices are written as follws for say, n=4:

$$\underline{R} = \begin{vmatrix} r_1 & 0 & 0 & 0 \\ 0 & r_2 & 0 & 0 \\ 0 & 0 & r_3 & 0 \\ 0 & 0 & 0 & r_4 \end{vmatrix} \tag{5.6}$$

$$Q = \begin{vmatrix} q_1 & 0 & -q_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & p_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -q_1 & 0 & q_1+q_2 & 0 & 0 & -q_2 & 0 & 0 \\ 0 & 0 & 0 & p_2 & p_2 & 0 & 0 & 0 \\ 0 & 0 & -q_2 & 0 & q_2+q_3 & 0 & -q_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & p_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & -q_3 & 0 & q_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_4 \end{vmatrix} \tag{5.7}$$

Since $r_i > 0$, $\underline{R}$ is positive definite by inspection. $\underline{Q}$ is the summation of the terms shown in Eqn. 4.14:

$$p_i e_{iv}^{\,2}(t) \text{ and } q_i \left(e_{ix}(t) - e(t) \right)^2 .$$

Since p_i, $q_i > 0$ and each quadratic is greater than or equal to zero, $\underline{Q}$ is non-negative definite. Also, the expansion of $\underline{Q}$ and $\underline{R}$ to larger n is self-evident.

Since the above is a generalization of the merging control problem posed in chapter III, the solution to that problem is given by Eqn. 5.4, or in problem notation:

$$u^*(t) = - \underline{R}^{-1} \underline{B}' \underline{K} e^*(t) \tag{5.8}$$

The nature and implementation of this solution are discussed in the following chapter.

The remainder of this chapter is devoted to the soloution of the minimum-time rendezvous problem.

First, the Hamiltonian shall be analyzed to determine what conditions must be satisfied by the time-optimal aircraft trajectories. For this problem, the Hamiltonian is given to be:

$$H = 1 + (\cos\phi - 1)\, p_x + \sin\phi\, p_y + u p_\phi \tag{5.9}$$

where:

p_x, p_y, p_ϕ are the components of the co-state vector, and the following changes are made for notational convenience:

$$\Delta x = x$$

$$\Delta y = y$$

$$\Delta \phi = \phi$$

Since u is the only control available, the minimum value of H occurs in the following situations:

$$p_\phi < 0 \text{ and } u = +1$$

$$p_\phi > 0 \text{ and } u = -1$$

$$p_\phi = 0 \text{ and } u = 0$$

The last assertion shall be proven below; the other are obvious. An immediate consequence of the above is that the control is "bang-bang" in nature i.e. the aircraft is turning (to the left or right) at the maximum turn rate or flying in a straight line.

The time derivatives of co-state vector components are found from the relations:

$$\dot{p}_x = -\frac{\partial H}{\partial x} = 0 \qquad (5.10)$$

$$\dot{p}_y = -\frac{\partial H}{\partial y} = 0 \qquad (5.11)$$

$$\dot{p}\phi = -\frac{\partial H}{\partial \phi} = p_x \sin\phi - p_y \cos\phi \qquad (5.12)$$

Eqns. 5.10 and 5.11 relate that p_x and p_y are constant, this has the following consequence given that p_ϕ is zero over an interval: Thus,

$$\dot{p}_\phi = 0 \qquad (5.13)$$

Equation 5.12 now has the form:

$$p_x \sin\phi = p_y \cos\phi \qquad (5.14)$$

or,

$$\tan\phi = \frac{p_y}{p_x} \tag{5.15}$$

$$= \text{constant}$$

This results in the following chain of implications:

$$\phi = \text{constant} \tag{5.16}$$

$$\dot{\phi} = 0 \tag{5.17}$$

$$u(t) = 0 \tag{5.18}$$

Therefore, a staight line path is optimal, and u(t) is zero when p_ϕ is zero, thus proving the assertion above.

From the boundary conditions given in chapter IV, boundary conditions are determined for the co-state. Since H is zero along an optimal trajectory for all time:

$$H(0) = 0 = 1 + u(0) \tag{5.19}$$

Therefore, neither u nor p_ϕ are zero at the initial time.

$$p_\phi(0) = \frac{-1}{u(0)} \tag{5.20}$$

Thus, $p_\phi(0)$ has the possible values of 1 and -1.

Similarly,

$$p_\phi(T) = \frac{-1}{u(T)} \tag{5.21}$$

From Eqn 5.12, the boundary conditions for $\dot{p}_\phi$ can be obtainted.

$$\dot{p}_\phi(0) = p_x \cdot 0 - p_y \cdot 1 \tag{5.22}$$

$$\dot{p}_\phi(0) = -p_y \tag{5.23}$$

Likewise,

$$\dot{p}_\phi(T) = p_x \cdot 0 - p_y \cdot 1 \tag{5.24}$$

$$\dot{p}_\phi(T) = -p_y \tag{5.25}$$

The control law for the minimum time rendezvous may now be stated:

$$u(t) = -1 \cdot \operatorname{sgn}(p_\phi(t)) \tag{5.26}$$

where:

$$\operatorname{sgn}(x) = \begin{cases} +1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$$

And, where $p_\phi(t)$ satisfies the following conditions:

$$\dot{p}_\phi = p_x \sin_\phi - p_y \cos_\phi \tag{5.29}$$

$$p_\phi(0) = \frac{-1}{u(0)} \tag{5.30}$$

$$p_\phi(T) = \frac{-1}{u(T)} \tag{5.31}$$

$$\dot{p}_\phi(0) = p_\phi(T) = -p_y \tag{5.32}$$

where p_x and p_y are constants.

At this point, the method of attack will not be to attempt to determine a general solution for p_ϕ directly from the above equations. Rather, various trajectories shall be proposed as time-optimal candidates; their control histories constructed and tested by Eqns. 58-62. If all conditions are satisfied, the proposed trajectory shall then be considered as a time-optimal path.

The first trajectory to be considered is shown in Figure 10, a complete circle of minimum radius. For this and all other trajectories, ϕ shall be assumed to be increasing in the counter-clockwise (clockwise) direction for $u(t) > 0$ (< 0). Another observation applicable to all trajectories is that p_ϕ (Eqn. 5.29) may be easily rewritten as

$$\dot{p}_\phi = -A \sin(\phi - \phi_o) \tag{5.33}$$

where

$$A = (p_x^2 + p_y^2)^{1/2}$$

$$\phi_o = \tan^{-1} \frac{p_y}{p_x}$$

Thus, the form of p_ϕ will be that of a sinusoid. For this circling maneuver:

$$u(t) = +1 \qquad 0 \leq t \leq 2\pi \tag{5.34}$$

Which implies by Eqn. 4.23

$$\phi(t) = t \qquad 0 \leq t \leq 2\pi \tag{5.35}$$

These relations are shown in Figure 10. Since by Eqn. 5.32

$$p_\phi(0) = p_\phi(T) = -1 \tag{5.36}$$

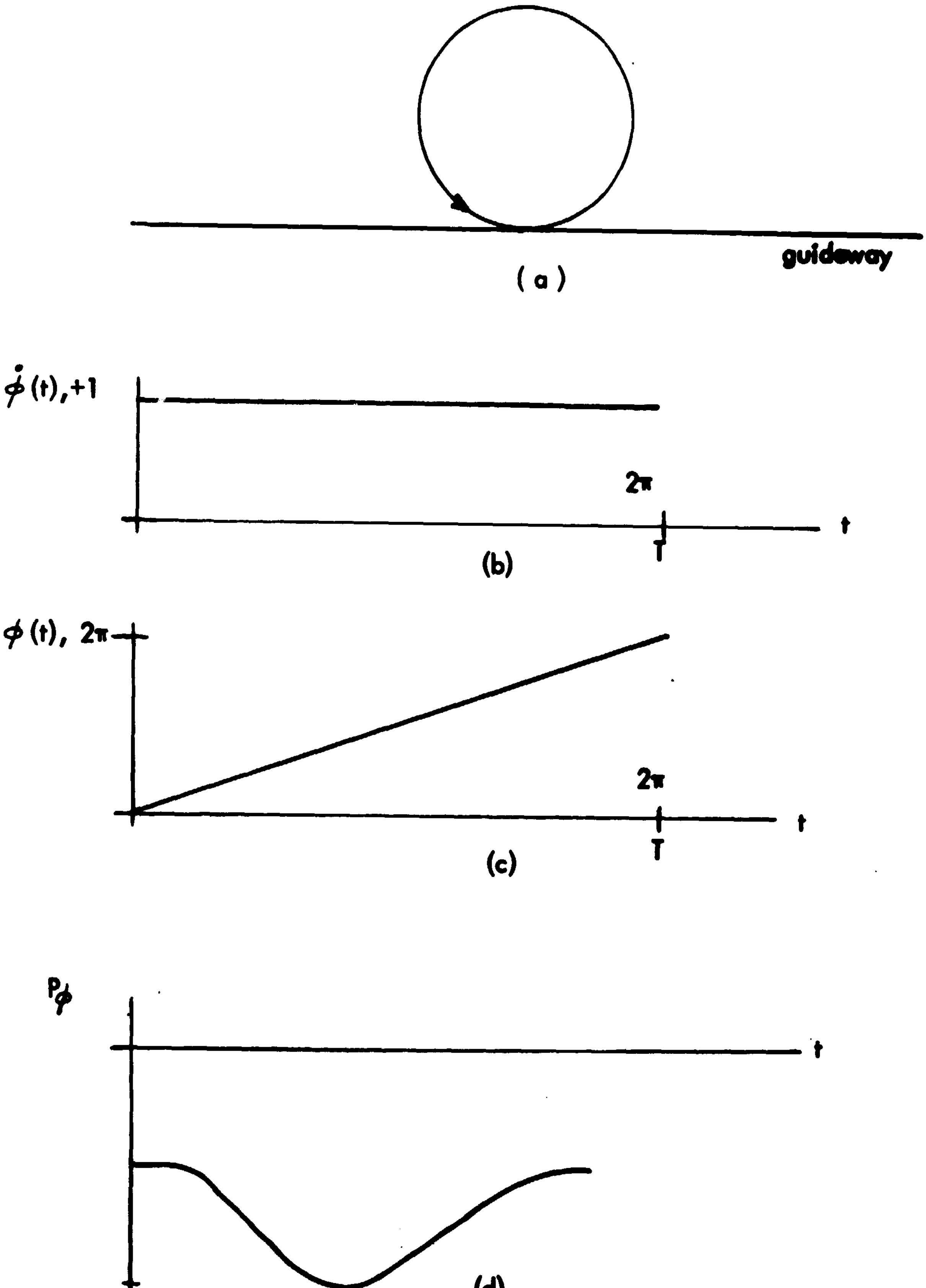


Fig. 10 Fly-around maneuver umax = 1 radian/minute

let

$$p_\phi = \cos\phi - 2 \tag{5.37}$$

$$= \cos t - 2 \tag{5.38}$$

Taking the time derivative,

$$\dot{p}_\phi = - \dot{\phi} \sin \phi \tag{5.39}$$

$$= -(+1) \sin\phi \tag{5.40}$$

$$= - \sin\phi \tag{5.41}$$

Comparing Eqn. 5.41 with Eqn. 5.29, the following equalities may be stated:

$$p_y = 0 \tag{5.42}$$

$$p_x = -1 \tag{5.43}$$

But, Eqn. 5.42 implies

$$\dot{p}_\phi(0) = \dot{p}_\phi(T) = 0 \tag{5.44}$$

This is verified by the choice that was made for p_ϕ. (See Figure 10d) Thus, the conclusion is made that Figure 10a represents a time-optimal rendezvous trajectory.

Figure 11a is the general case for "fly-around" maneuvers. For a period of time, $T_{st} \geq 0$, the aircraft flies anti-parallel to the guideway. Thus, the control switches as shown in Figure 11b i.e.

$$u(t) = \begin{cases} +1 & 0 \leq t \leq \pi \\ 0 & \pi < t \leq T_{st} + \pi \\ +1 & T_{st} + \pi < t \leq 2\pi + T_{st} \end{cases} \tag{5.45}$$

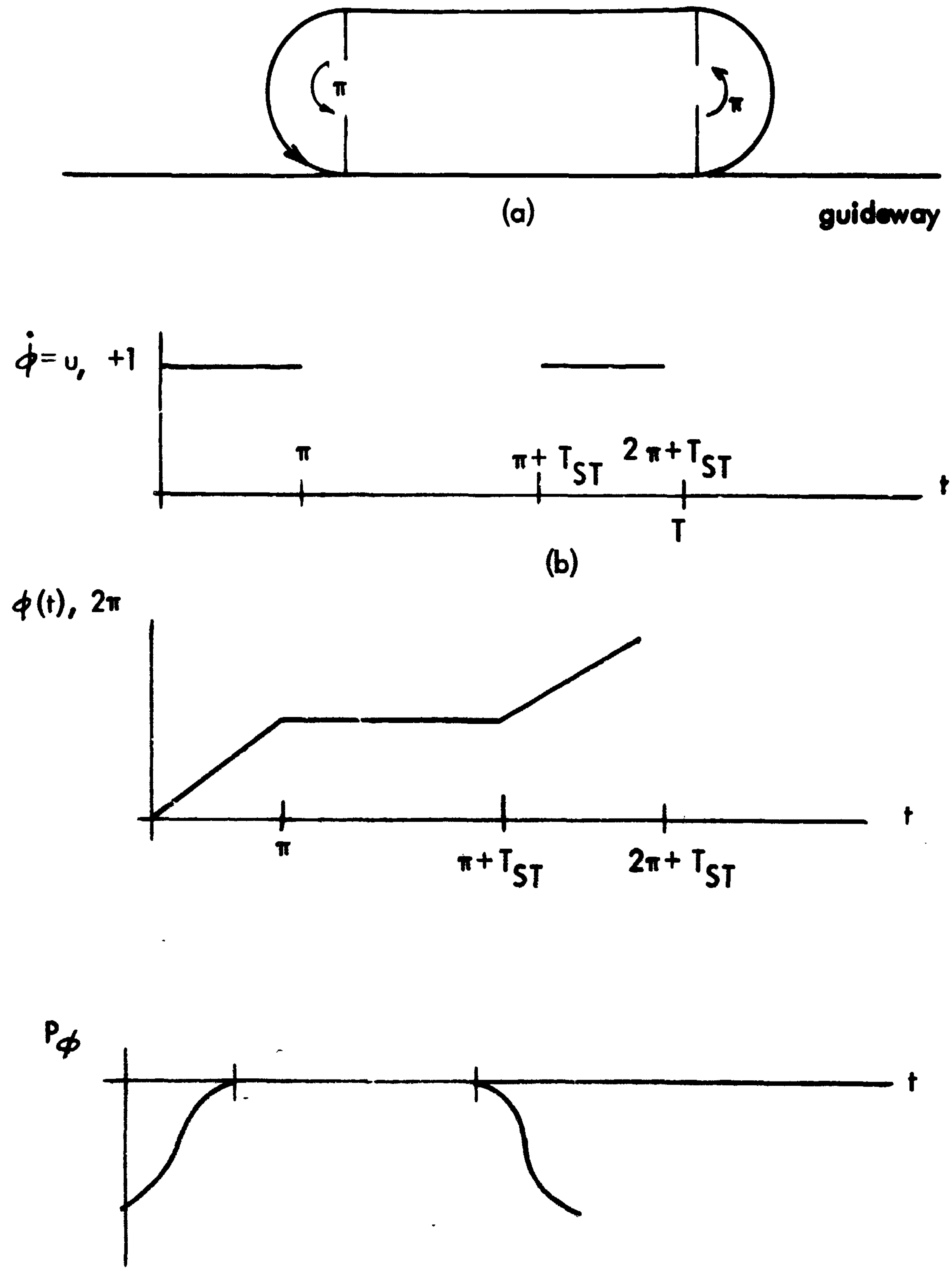


Fig. 11 Fly-around maneuver
u max = 1 radian/minute

which implies

$$\phi(t) = \begin{cases} t & 0 \le t \le \pi \\ \pi & \pi < t \le T_{st} + \pi \\ t - T_{st} & T_{st} + \pi < t \le T_{st} + 2\pi \end{cases} \qquad (5.46)$$

Since $\quad p_\phi(0) = -1 = p_\phi(T)$, let $\qquad (5.47)$

$$p_\phi = \frac{-1}{2} \cos\phi - \frac{1}{2}$$

Differentiating,

$$\dot{p}_\phi = \frac{1}{2} \cdot \dot{\phi} \sin\phi \qquad (5.48)$$

Noting the values taken on by $\phi\,(= u)$, the result is:

$$\dot{p}_\phi = \begin{cases} \frac{1}{2} \sin\phi & 0 \le t \le \pi \\ & \pi + T_{st} < t \le 2\pi + T_{st} \\ 0 & \pi < t \le \pi + T_{st} \end{cases}$$

Again, comparing Eqn. 5.49 with Eqn. 5.29, the value for p_y is zero. This confirms the choice for p_ϕ since

$$\dot{p}_\phi(0) = \dot{p}_\phi(T) = 0 \qquad (5.50)$$

$$p_\phi = 0\,, \quad \dot{p}_\phi = 0 \quad \Rightarrow \quad u = 0 \qquad (5.51)$$

Thus, Figure 11 a is likewise concluded to be a time-optimal rendezvous trajectory.

The final trajectory to be analyzed is the general form of the "oscillation" maneuver as shown in Figure 12 a. Here, the switches in the control appear as follows:

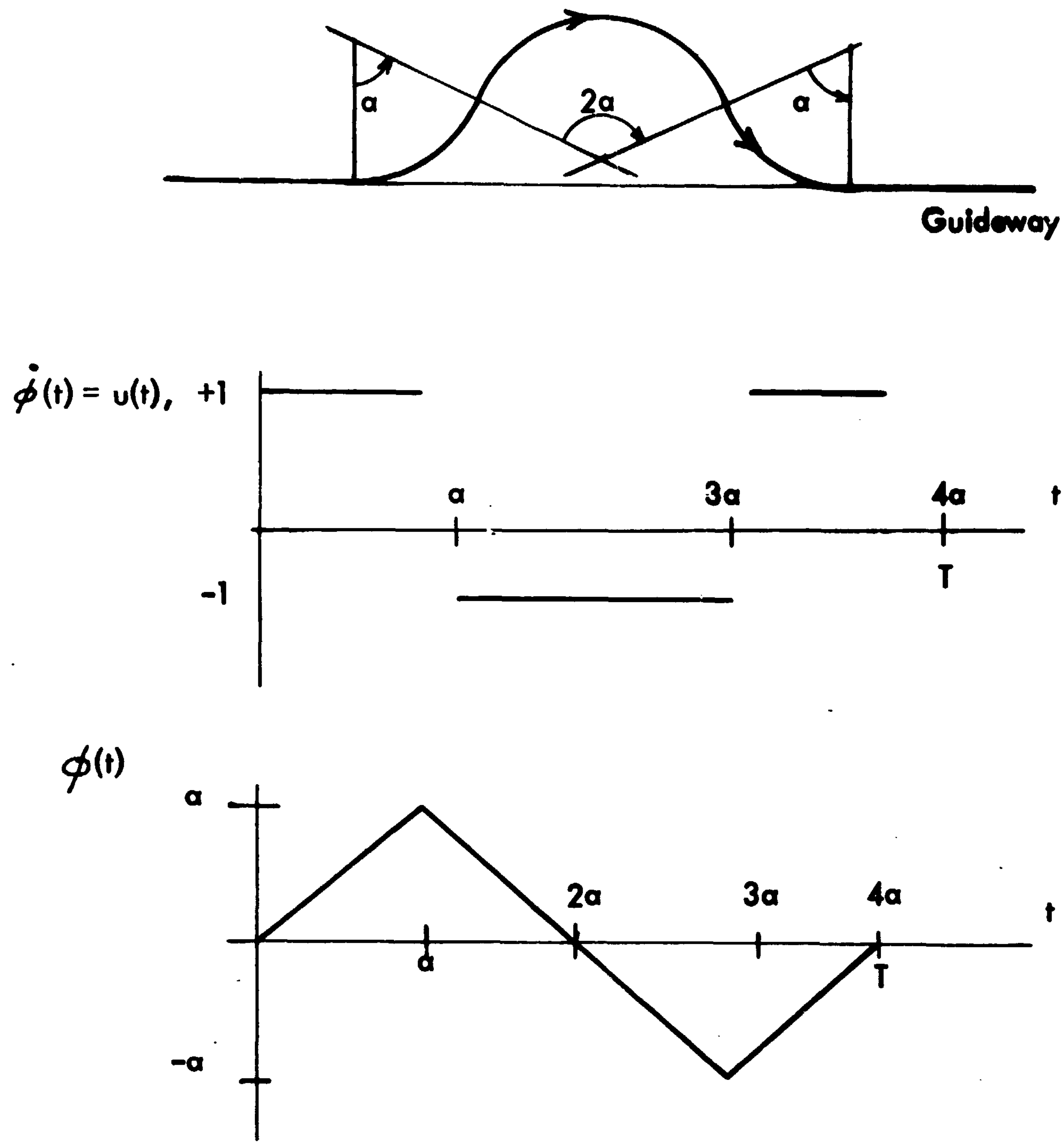


Fig. 12 Oscillation maneuver
u_{max} = 1 radian/minute

$$u(t) = \begin{cases} +1 & 0 \leq t \leq a \\ -1 & a \leq t \leq 3a \\ +1 & 3a < t \leq 4a \end{cases} \tag{5.52}$$

which results in the following relations for $\phi(t)$:

$$\phi(t) = \begin{cases} t & 0 \leq t \leq a \\ 2a - t & a < t \leq 3a \\ t - 4a & 3a < t \leq 4a \end{cases} \tag{5.53}$$

Since

$$p_\phi(0) = p_\phi(T) = -1 \tag{5.54}$$

it is obvious that some adjustments are required to allow $\cos\phi$ to take on these values. (See Figure 13 a). $\cos a$ is substracted in Figure 13 b to bring the graph down to the time axis. Multiplication by $-\dot{\phi}$ results in proper signs for switching, and division by $(1 - \cos a)$ insures that the corrected magnitude is obtained at the endpoints. (Figures 13 (c), (d) respectively). The claim is now made that the following is a valid function for p_ϕ:

$$p_\phi = \frac{-\dot{\phi}}{1 - \cos a} (\cos\phi - \cos a) \tag{5.55}$$

Taking the derivative,

$$\dot{p}_\phi = \frac{-1}{1 - \cos a} [-\dot{\phi}\,\dot{\phi} \sin\phi + \ddot{\phi} \cos\phi] \tag{5.56}$$

and noting that

$$\dot{\phi}^2(t) = 1 \text{ everywhere} \tag{5.57}$$

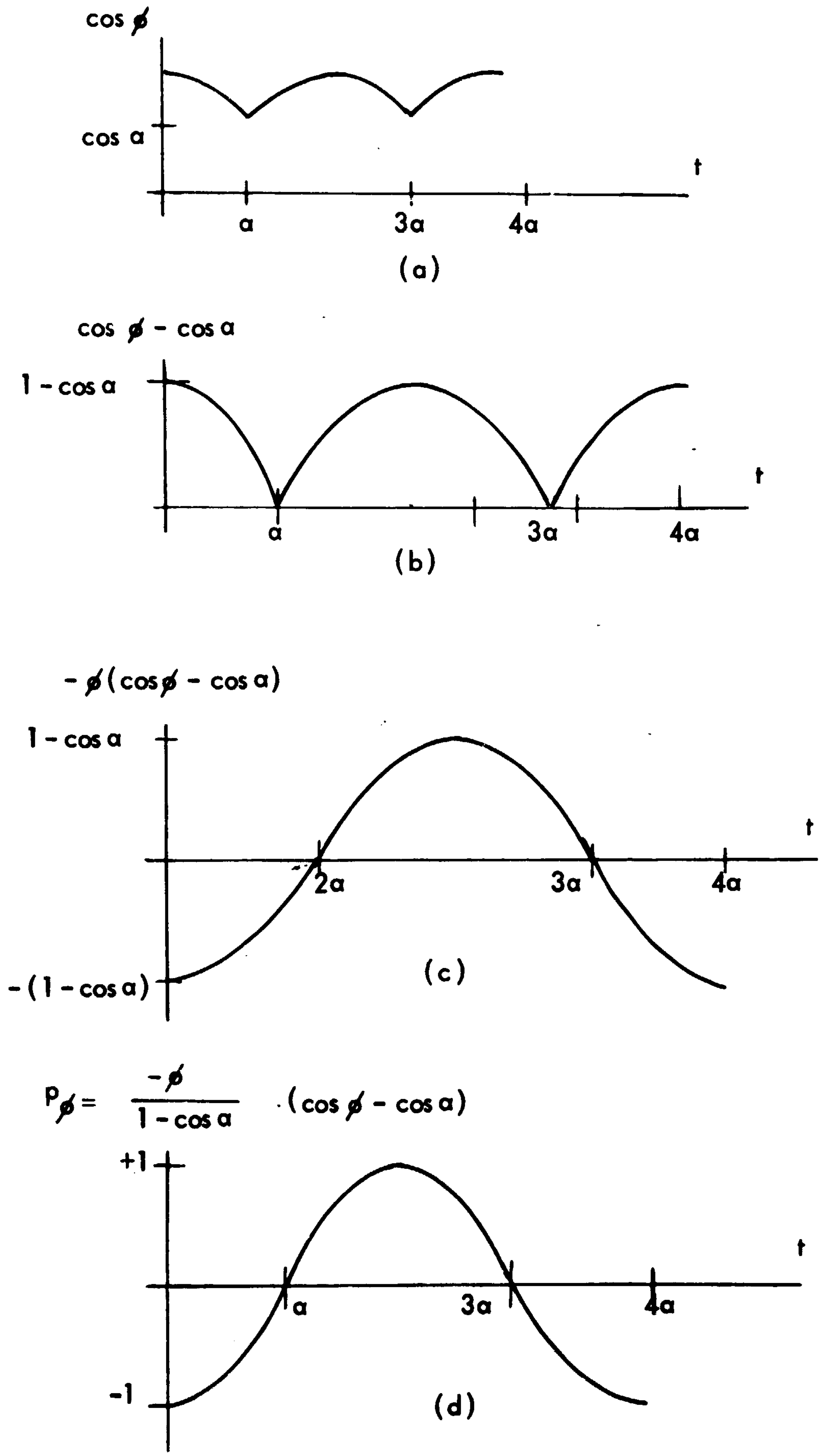


Fig. 13 Construction of $p_ø$ for oscillation maneuvers

and neglecting the impulses generated by $\ddot{\phi}$ (t) ($\ddot{\phi}$(t) is otherwise zero), then

$$\dot{p}_\phi = \frac{\sin\phi}{1 - \cos\phi} \tag{5.58}$$

comparing with Eqn. 5.29,

$$p_y = 0 \tag{5.59}$$

which implies

$$\dot{p}_\phi(0) = \dot{p}_\phi(T) = 0 \tag{5.60}$$

which again is shown by the function chosen for p_ϕ. Thus, the maneuver shown in Figure 12 a may be considered to be time optimal if the derivative of p_ϕ is valid. In its determination, the term $\ddot{\phi}$ (t) was neglected. This reservation may be cleared up in the following way. Form the derivative of $\dot{p}_\phi$ from Eqn. 5.29;

$$\begin{aligned} \dot{p}_\phi &= \dot{\phi}\, p_x \cos\phi + \dot{\phi} p_y \sin\phi \\ &= \dot{\phi}\,(p_x \cos\phi + p_y \sin\phi) \\ &= u(p_x \cos\phi + p_y \sin\phi) \end{aligned}$$

multiply by u(t), and since u(t) is never zero for this category of maneuver, $u^2 = 1$, the following is obtained

$$u\ddot{p}_\phi = p_x \cos_\phi + p_y \sin_\phi \tag{5.62}$$

Rewriting the Hamiltonian and grouping terms,

$$p_x \cos_\phi + p_y \sin_\phi + 1 - p_x + u\, p_\phi = 0 \tag{5.63}$$

The first two terms are given by Eqn. 5.62. Thus,

$$u\ddot{p}_\phi + up_\phi + 1 - p_x = 0 \tag{5.64}$$

Rearranging,

$$\begin{aligned}\ddot{p}_\phi + p_\phi &= u(p_x - 1) \\ &= -\operatorname{sgn}(p_\phi)[p_x - 1]\end{aligned}$$

The homogeneous portion of Eqn. 5.63 is simply the undamped harmonic oscillator. The driver, $-\operatorname{sgn}(p_\phi)[p_x - 1]$, is a constant term that changes sign as u. This serves to raise or lower the sinusoid by a constant amount. Forming the derivative of the derivative in question Eqn. 5.58:

$$\ddot{p}_\phi = \frac{\dot{\phi}\cos\phi}{1-\cos\alpha} \tag{5.66}$$

Substituting this and the accepted general function for p_ϕ into Eqn. 5.65:

$$\frac{-\dot{\phi}\,(\cos\phi - \cos\alpha)}{1-\cos\alpha} + \frac{\dot{\phi}\cos\phi}{1-\cos\alpha} = u\,(p_x - 1) \tag{5.67}$$

Writing u as $\dot{\phi}$, p_x as $1/(1-\cos\alpha)$ and unity as $(1-\cos\alpha)/(1-\cos\alpha)$

$$\frac{-\dot{\phi}\cos\phi + \dot{\phi}\cos\phi + \dot{\phi}\cos\alpha}{1-\cos\alpha} = \dot{\phi}\,\frac{(1+\cos\alpha - 1)}{\cos\alpha - 1} \tag{5.68}$$

the result is an identity,

$$\frac{\cos\alpha}{1-\cos\alpha} = \frac{\cos\alpha}{1-\cos\alpha} \tag{5.69}$$

thus testifying to the validity of neglecting $\dot{\phi}$ in evaluating $\dot{p}_{\phi}$. Therefore, the "oscillation" maneuver is a minimumtime rendezvous trajectory.

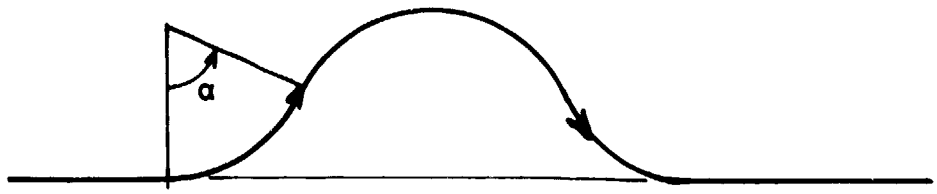
From Eqn. 5.65, the observation is made that p_x effectively determines the trajectory. This is shown in Figure 14. It is also noted that for all trajectories but the complete circle, p_x is determined by

$$p_x = \frac{1}{1 - \cos a}, \qquad 0 < a \leq \pi \tag{5.70}$$

and hence p_{ϕ} for all trajectories but the complete circle is given by

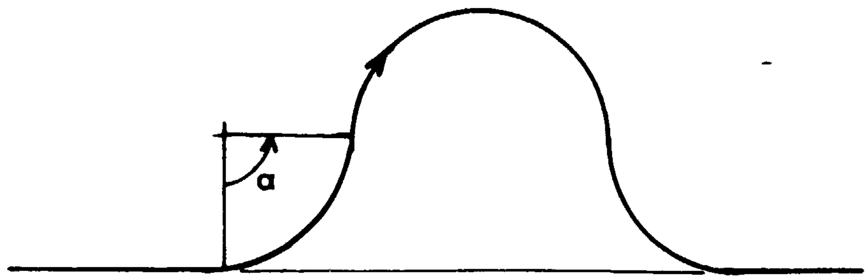
$$p_{\phi} = \frac{-\dot{\phi}}{1 - \cos a} \quad (\cos \phi - \cos a) \tag{5.71}$$

In the next chapter, these results will be combined into a complete algorithm for the sequencing and control of aircraft in the NTA.



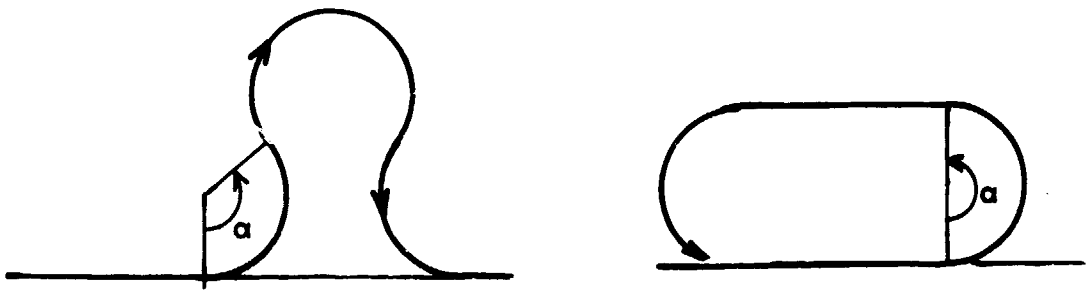
$p_x > 1, \quad 0 < \alpha < \pi/2$

(a)



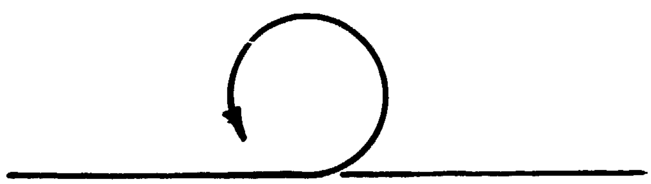
$p_x = 1, \quad \alpha = \pi/2$

(b)



$p_x < 1, \quad \pi/2 < \alpha \leq \pi$

(c)



$p_x = -1$

Fig. 14 Dependence of trajectories upon p_x

CHAPTER VI

RESULTS

At this point, the findings of the previous chapters are consolidated into an algorithm suitable for the control of flight paths in the NTA. Also included are the results from simulation and computation.

The following sequence of steps are now proposed as algorithm satisfying the requirements of the ATC problem:

1. Given an initial distribution of aircraft in the guideways, a numbering scheme tags each aircraft with a unique identifier. This identifier specifies the guideway in which the aircraft resides and its relative placement within the guideway. See Figure 15a for an example.

2. The feeder guideways are conceptually "swung" together to obtain an equivalent vehicle distribution on a single guideway.

3. Next, the order in which the aircraft will land is determined. An airborne emergency or any other set of priority requirements will determine this. If the aircraft all have equal priorities, their "natural" ordering is used, i.e., the ordering displayed by the equivalent vehicle distribution. Since initial velocity deviations are most likely to be small, an attempt to merge an "un-natural" sequence would require unacceptable velocity deviations or holding maneuvers. For this reason, the "natural" ordering is utilized unless priority requirements dictate otherwise. For example, in Figure 15b, the natural ordering of aircraft is explicitly shown.

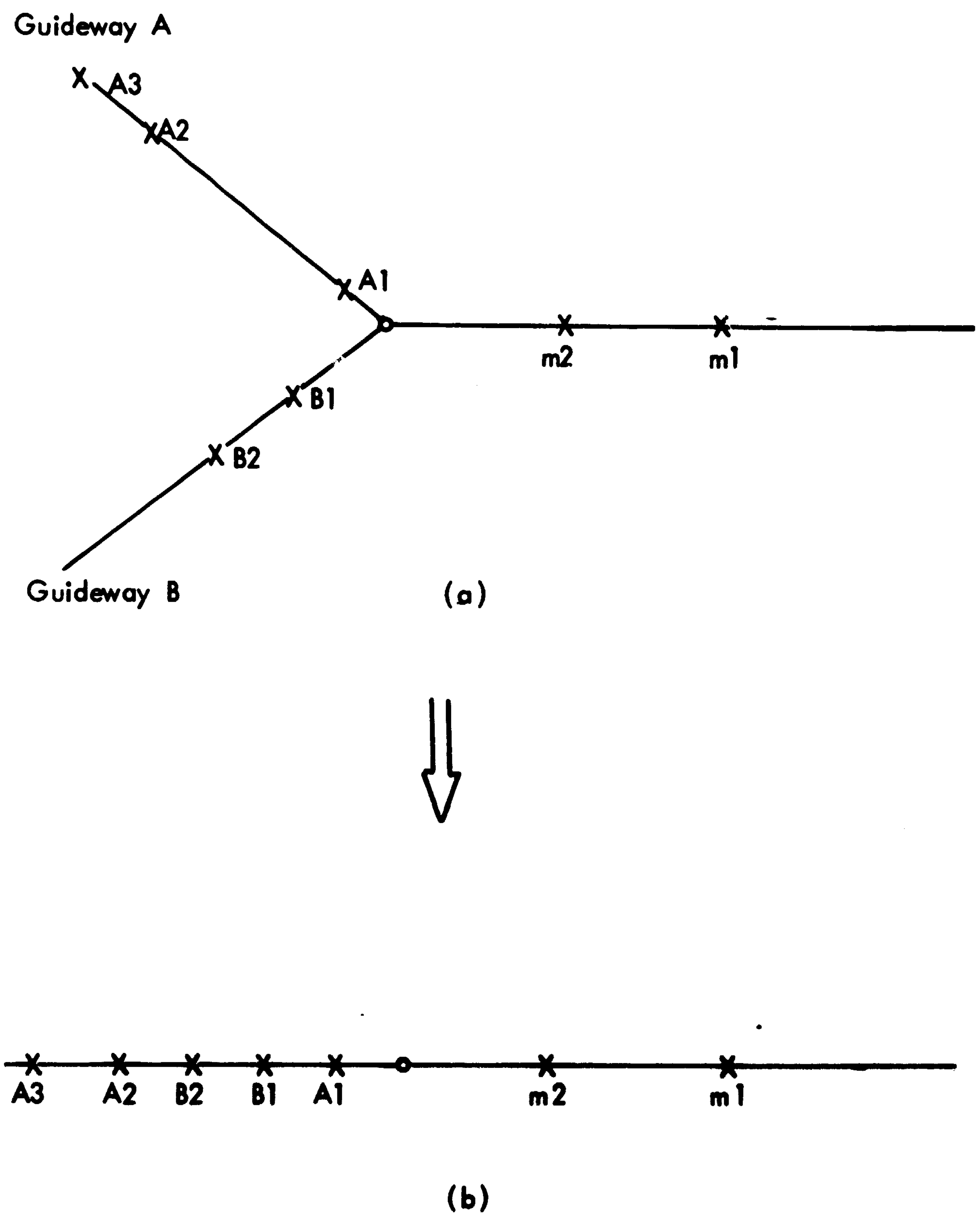
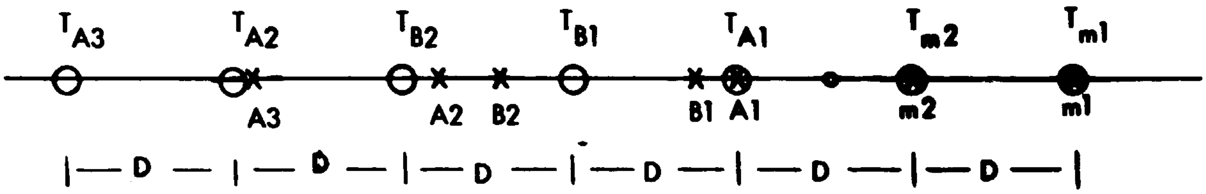


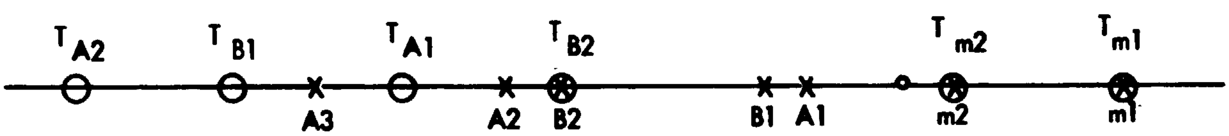
Fig. 15 Equivalent guideway distributions

4. The target string is now superimposed on the guideway. Here, the relative placement of targets within the string reflects the ordering determined in 2. If the "natural" ordering is utilized, the target string is placed as directed in Chapter III. An ordering directed by priorities is accomplished as shown in Figure 16b. Moving to the left from the outer marker or merge point, the next aircraft in the landing sequence has its target placed directly on it. The next aircraft in the ordering has its target placed at the desired separation distance, D, to the left of the first target. This process is repeated as shown in Figure 16b until all aircraft have been assigned targets. In this example, aircraft B2 is assigned to land before B1 and A1. Since no priority is given for B1 and A1, their targets are placed behind B2's target in their "natural" order.

5. The normal mode of operation of the system is to allow the optimal feedback controller described in Chapter V to bring the aircraft to their respective targets. However, the controller may require an aircraft to attain an unacceptable airspeed in doing so. Thus, this step in the control procedure requires the system to determine which aircraft will exceed a pre-determined bound on velocity. These aircraft will then be constrained to perform constant-airspeed maneuvers in order to rendezvous with their targets. This maneuver criterion is an exceedingly important portion of the ATC algorithm; several criteria are explained at length later in this section.



(a) " natural" assignment



(b) " priority" assignment

Fig. 16 Target assignment

6. Constant airspeed, time-optimal trajectories are determined for those aircraft specified in 5. Chapter V disclosed what classes of trajectories are available. Several methods of computing the required controls are given later in this section.

7. For every aircraft maneuvering outside the guideway the following changes are introduced to the state vector:

$$\begin{aligned} e_{ix}(0) &= 0 = e'_{ix}(0) \\ e_{iy}(0) &= e_{iy}(0) = e'_{iy}(0) \end{aligned} \tag{6.1}$$

i.e., the real aircraft is attempting to rendezous with its target at its initial airspeed. For all practical purposes, the state vector components listed above adequately represent the real aircraft during the maneuver and <u>are</u> the actual values of these variables at the time the maneuver is completed. For this reason, these componsents are held constant or "bound" until the maneuver is completed.

8. At this point, the optimal feedback controller is started after selection of the properly dimensioned K matrix, and extra-guideway maneuvers are initiated.

9. When a rendezvous is accomplished, the corresponding "bound" state variables are released to the corrective action of the optimal controller.

10. The above process is repeated for each aircraft entering the NTA.

Several maneuver criteria as described in 5 are discussed below:

1. Several attempts were made to derive an analytical expression for the maneuver criterion from the optimal controller's cost functional. A general form of the quadratic cost critetion appears promising. Using the familiar form:

$$J_m(n) = \underline{e}_n'(0)\ \underline{M}\ \underline{e}_n(0) \qquad (6.2)$$

Where $\underline{e}_n(0)$ is the initial state vector for the first n aircraft in the string, and $\underline{M}$ is a yet-to-be determined 2n x 2n matrix.

The maneuver criterion is employed as follows:

a. The first two aircraft in the string are "cost-tested" by Equation 6.2 (n=2).

b. If a certain threshold value of $J_m(n)$ is exceeded, then the trailing aircraft is placed on a holding maneuver.

c. The initial state vector components of the maneuvering aircraft are replaced by those in Equation 6.1. Otherwise, the state vector remains unchanged.

d. Steps 1-3 are repeated by including the next aircraft in the string one at a time until all aircraft have been so processed.

Thus, the desired end result of determining which aircraft must perform holding maneuvers is achieved, and the criterion also explicitly formulates the initial state vector of the linear, optimal controller.

2. An extensive simulation could provide the system with a locus of undesirable initial conditions. An empirical formulation of these results would allow a simple comparison of the given aircraft's initial conditions to suffice for Step 5.

3. A third alternative would be an on-line numerical simulation of the optimal controller. The state equations, Eqn 4.13, could be time-scaled and then transformed to discrete state equations

$$e[(k+1)T] = e^{\underline{A}T}\underline{e}(kT) + \left[\int_0^T e^{\underline{A}\tau} d\tau\right] \underline{B}\,\underline{u}(kT) \tag{6.3}$$

Making the following changes in notation

$$\underline{\Psi} = e^{\underline{A}T}, \quad \underline{\Delta} = \int_0^T e^{\underline{A}\tau}\, d\tau\, \underline{B} \tag{6.4}$$

The result is:

$$\underline{e}(k+1) = \underline{\Psi}(T)\underline{e}(k) + \underline{\Delta}(T)\,\underline{u}(k) \tag{6.5}$$

For a given choice of T and number of aircraft in the system, i.e., system dimension, the matrices $\underline{\Psi}$ and $\underline{\Delta}$ need only be evaluated once. Also, this simulation is almost completely exact and is faster than a comparable Runge-Kutta scheme. Since the system is time-scaled, only a brief run of several time constants duration is required to detect which aircraft exceeds the limits on velocity by the greatest amount. This aircraft is awarded the initial conditions in Eqn. 6.1, and the process is repeated until a run occurs in which all velocity deviations are acceptable. This one-at-a-time replacement of unacceptable initial conditions is necessary since the i^{th} aircraft's control is dependent upon the states of all the aircraft in the string. Thus, an aircraft with extremely undesirable initial conditions may cause its neighbors to exceed their velocity limitations even though their initial states may be perfectly acceptable. Thus, this criterion may disqualify itself on the amount of time required to process large strings. However, such a simulation technique may be used to generate the locus of acceptable initial conditions proposed above.

The next consideration is the maneuvers performed by the aircraft designated by the maneuver criterion. From Figure 17, an aircraft is performing a fly-around maneuver with (initial) constant airspeed Vo and minimum radius Ro. The rendezous point may be expressed as:

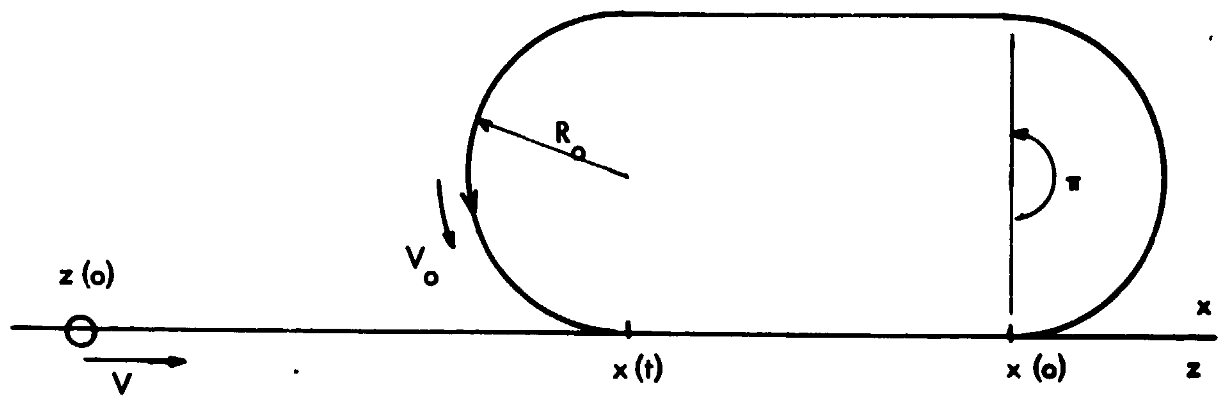
$$x(T) = z_o + VT \tag{6.6}$$

$$x(T) = x_o - V_o T_{st} \tag{6.7}$$

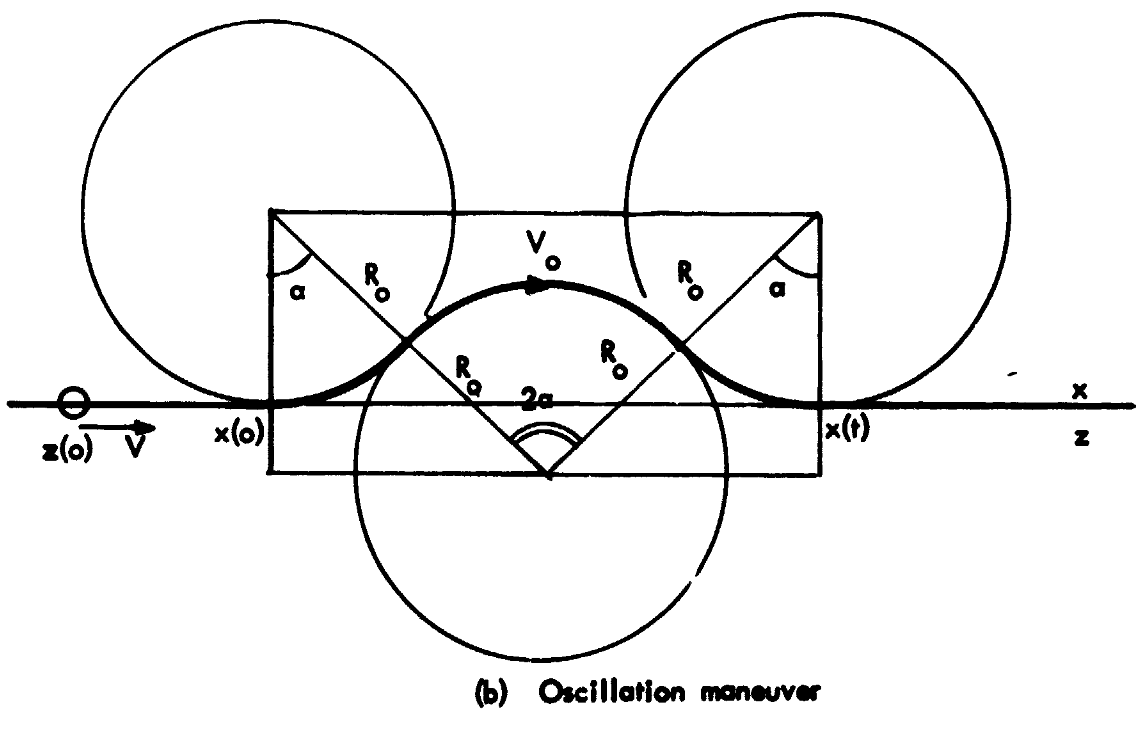
The expression for T_{st} is found as follows:

$$\begin{aligned} T &= T_{st} + T_{turn\ 1} + T_{turn\ 2} \\ &= T_{st} + \frac{\pi R_o}{V_o} + \frac{\pi R_o}{V_o} \\ &= T_{st} + \frac{2\pi R_o}{V_o} \end{aligned}$$

$$\therefore \quad T_{st} = T - \frac{2\pi R_o}{V_o} \tag{6.8}$$



(a) Fly-around maneuver



(b) Oscillation maneuver

Fig. 17 Trajectory geometry

Equating Equ. 6.6 and 6.7 and substituting T_{st}:

$$z_o + VT = x_o - V_oT + 2\pi R_o \qquad (6.9)$$

Solving for T,

$$T = \frac{(x_o - z_o) + 2\pi R_o}{(V + V_o)} \qquad (6.10)$$

This expression is valid for $T \geq T_{min}$ where T_{min} is time required to complete a full circle, i.e.,

$$T_{min} = \frac{2\pi R_o}{V_o} \qquad (6.11)$$

This implies a minimum separation for fly-around maneuvers:

$$T_{min} = \frac{2\pi R_o}{V_o} = \frac{(x_o - z_o)_{min} + 2\pi R_o}{(V + V_o)} \qquad (6.12)$$

or,

$$(x_o - z_o)_{min} = e_{ix}(0)_{min} = (V + V_o)\frac{2\pi R_o}{V_o} - 2\pi R_o \qquad (6.13)$$

Thus, a flyaround maneuver is required for rendezvous if the following condition is satisfied:

$$e_{ix}(0) \geq (V + V_o)\frac{2\pi R_o}{V_o} - 2\pi R_o \qquad (6.14)$$

or simply,

$$e_{ix}(0) \geq \frac{2\pi V}{A} \qquad (6.15)$$

where A is the maximum turn rate $(= \frac{V_o}{R_o})$. The control for fly-around maneuvers is simply stated:

1. if Equ. 6.15 is satisfied, then turn at maximum rate for $\frac{\pi}{A}$ minutes.

2. fly anti-parallel to the guideway for $\dfrac{e_{ix}(0) + 2\pi R_o}{V + V_o} - \dfrac{2\pi}{A}$ minutes, and

3. turn at maximum rate for $\frac{\pi}{A}$ minutes where all turns are made in the same direction.

If Equ. 6.15 is not satisfied, an oscillation maneuver is required. Referring to Figure 17b and expressing the location of the rendezvous point as:

$$x(T) = z_o + VT \tag{6.16}$$

$$x(T) = x_o + f(V_o, A, T) \tag{6.17}$$

The rendezvous time, T, may be described as the time required to tranverse an arc of $4\alpha R_o$ at a speed of V_o, or

$$T = \frac{4\alpha R_o}{V_o}$$

$$T = \frac{4\alpha}{A} \tag{6.18}$$

From Figure 17b, $f(V_o, A, T)$ is simply

$$f(V_o, A, T) = 2.2R_o \sin\alpha$$

$$f(V_o, A, T) = 4R_o \sin\alpha \tag{6.19}$$

Manipulation of the above equations results in an expression for the rendezvous time, T, in terms of the initial separation and velocity.

$$x_o - z_o = VT - 4R_o \sin\alpha \tag{6.20}$$

$$x_o - z_o = VT - \frac{4V_o}{A} \sin\frac{AT}{4} \tag{6.21}$$

This implicit relation can be solved for T and then by Equ. 6.18 for the control angle, α.

The control for the oscillation maneuver is now:

1. turn to left (right) at maximum rate for α/A minutes,
2. Turn in the opposite direction at maximum rate for $2\alpha/A$ minutes, and
3. turn to the left (right) at maximum rate for α/A minutes.

Computationally, both of these controls are easy to implement. Some results of a simple iterative routine are shown in Figure 18. One interesting result is the time discontinuity that occurs at the minimum separation distance for fly-around maneuvers. It is felt that this occurs since the target is too close to fly "backwards" to it. Flying "backwards" or anti-parallel to the guideway is, of course, the fastest way to close the separation between the target and aircraft. This freedom is lost when an "oscillation" maneuver is performed.

The first step in simulation of the system was the transformation of the system variables into a set of dimension-less quantities suitable for implementation on an analog computer. For Eqns. 4.11-12, the following transformations were used:

dimensionless distance error:

$$\chi_i = e_{ix} \Big/ \frac{m_i V}{a_{i_o}} \tag{6.22}$$

dimensionless velocity error:

$$\psi_i = e_{iv} / V \tag{6.23}$$

dimensionless thrust error:

$$\phi_i = (f_i - g_i(V)) / a_{io} \tag{6.24}$$

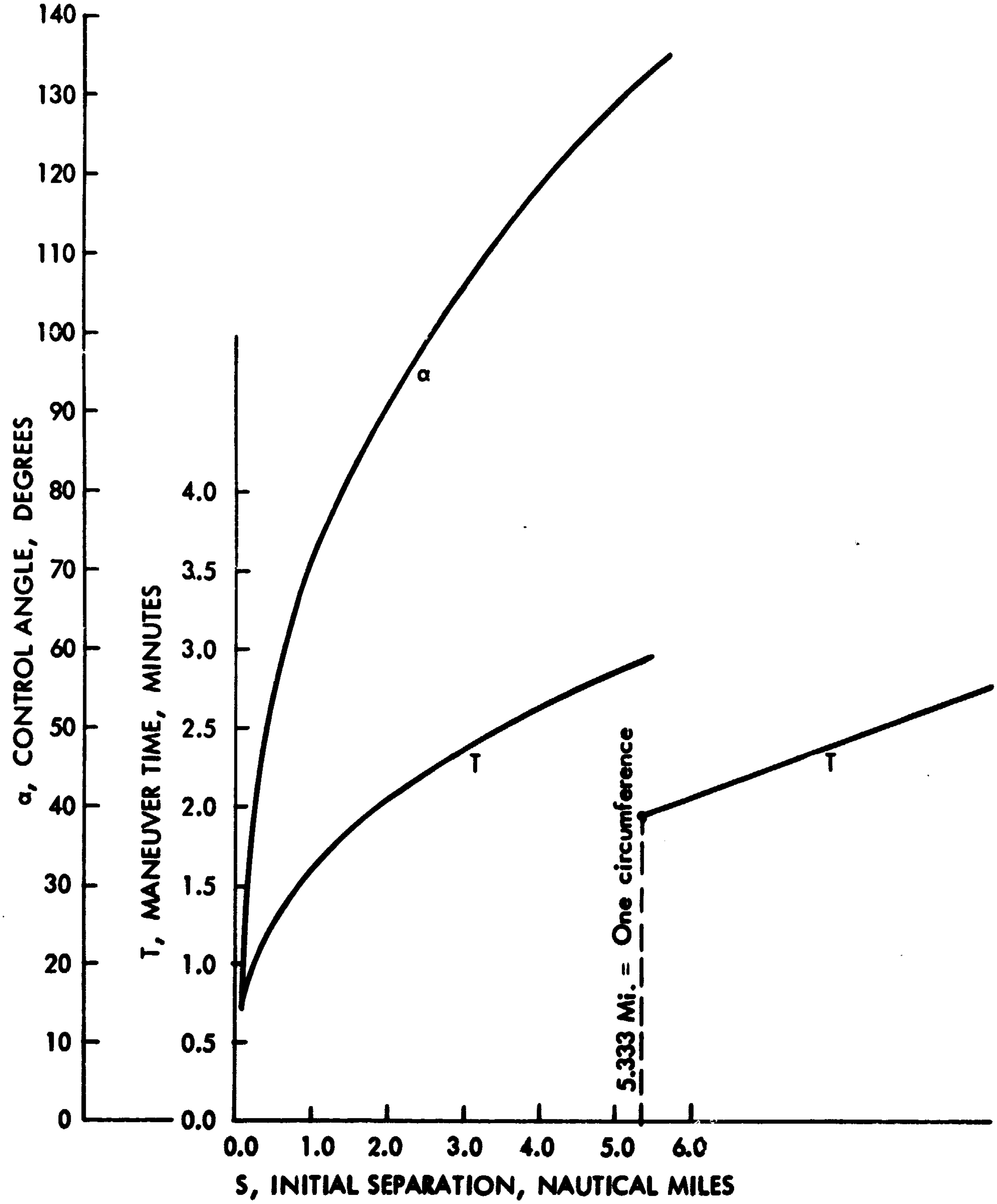


Fig. 18 Maneuver time and control angle vs initial separation at two minute turn rate and 160 knots for both target and aircraft

dimensionless time:

$$\tau = t / \frac{m_i}{a_{i_o}} \tag{6.25}$$

At this point, all aircraft were assumed to have identical mass and linearized drag functions. In particular, data was obtained for the KC-135 (Boeing 707):[3,4]

desired approach airspeed: $V = 160$ knots

gross weight: $m_i g = 297,000$ lbs.

first order drag coefficient: $a_{io} = 400$ lbs/knot

steady state drag: $g_i(V) = 36.10^3$ lbs.

maximum engine thrust: $f_{imax} = 13,750\ lb_f$

Substitution rendered the following equivalences:

$$1 \text{ unit } \chi_i = 1.5 \text{ nautical miles} \tag{6.26}$$

$$1 \text{ unit } \psi_i = 160 \text{ knots} \tag{6.27}$$

$$1 \text{ unit } \phi_i = 64.10^3 \text{ lbs} \tag{6.28}$$

$$1 \text{ unit } \tau = 22.2 \text{ seconds} \tag{6.29}$$

The aircraft equations now appear as:

$$\dot{\chi}_i(\tau) = \psi_i(\tau) \tag{6.30}$$

$$\dot{\psi}_i(\tau) = \psi_i(\tau) + \phi_i(\tau) \tag{6.31}$$

A string of four aircraft was simulated, and the following weights were assigned as penalties to the variations in airspeed, position and acceleration:

$r_i = 1$ for all i

$p_i = 100$ for all i

$q_i = 10$ for all i

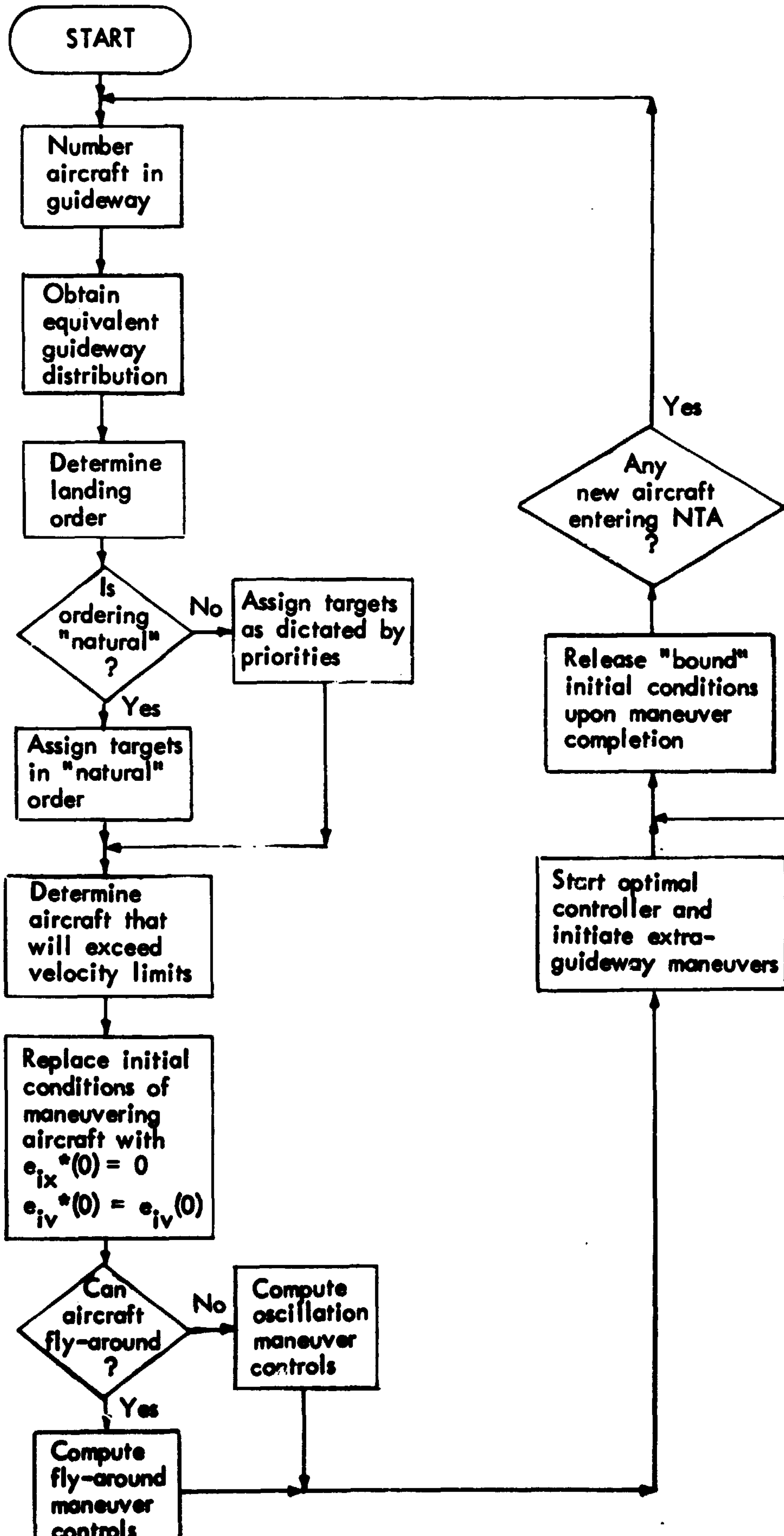


Fig. 19 Proposed ATC Algorithm

In matrix notation, this is expressed as:

$$\underline{A} = \left|\begin{array}{cc:cc:cc:cc} 0 & 1 & & & & & & \\ 0 & -1 & \multicolumn{2}{c:}{\underline{0}} & \multicolumn{2}{c:}{\underline{0}} & \multicolumn{2}{c}{\underline{0}} \\ \hdashline \multicolumn{2}{c:}{\underline{0}} & 0 & 1 & \multicolumn{2}{c:}{\underline{0}} & \multicolumn{2}{c}{\underline{0}} \\ & & 0 & -1 & & & & \\ \hdashline \multicolumn{2}{c:}{\underline{0}} & \multicolumn{2}{c:}{\underline{0}} & 0 & 1 & \multicolumn{2}{c}{\underline{0}} \\ & & & & 0 & -1 & & \\ \hdashline \multicolumn{2}{c:}{\underline{0}} & \multicolumn{2}{c:}{\underline{0}} & \multicolumn{2}{c:}{\underline{0}} & 0 & 1 \\ & & & & & & 0 & -1 \end{array}\right| \qquad \underline{B} = \begin{vmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \qquad (6.32)$$

$$\underline{R} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \qquad (6.33)$$

$$\underline{Q} = \begin{vmatrix} 10 & 0 & -10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 100 & 0 & 0 & 0 & 0 & 0 & 0 \\ -10 & 0 & 20 & 0 & -10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 100 & 0 & 0 & 0 & 0 \\ 0 & 0 & -10 & 0 & 20 & 0 & -10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 & 0 & 0 \\ 0 & 0 & 0 & 0 & -10 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 100 \end{vmatrix} \qquad (6.34)$$

With these values available, the steady state Riccati equation Eqn 5.5 was solved by a digital computer subroutine for $\underline{K}$. This non-iterative method was first proposed by J. E. Potter and is characterized by its speed and accuracy. This program is available upon request. $\underline{K}$ was found to be:

$$
\underline{K} = \begin{vmatrix}
26.88 & 2.58 & -18.30 & -1.72 & -5.18 & -0.51 & -3.46 & -0.34 \\
2.58 & 9.30 & -1.72 & -0.17 & -0.51 & -0.05 & -0.34 & -0.03 \\
-18.30 & -1.72 & 40.00 & 3.79 & -16.59 & -1.55 & -5.18 & -0.51 \\
-1.72 & -0.17 & 3.79 & 9.42 & -1.55 & -0.15 & -0.51 & -0.05 \\
-5.18 & -0.51 & -16.59 & -1.55 & 40.00 & 3.79 & -18.30 & -1.72 \\
-0.51 & -0.05 & -1.55 & -0.15 & 3.79 & 9.52 & -1.72 & -0.17 \\
-3.46 & -0.34 & -5.18 & -0.51 & -18.30 & -1.72 & 26.88 & 2.58 \\
-0.34 & -0.03 & -0.51 & -0.05 & -1.72 & -0.17 & 2.58 & 9.30
\end{vmatrix} \tag{6.35}
$$

The optimal control is given by:

$$\underline{\phi}(\tau) = -\underline{G}\ \underline{v}(\tau) = -\underline{R}^{-1}\ \underline{B}^{1}\ \underline{K}\ \underline{x}(\tau) \tag{6.36}$$

Thus, the gain matrix is constant and is independent of the merging order. Therefore, the gains in the feedback system need only be calculated once for a given number of aircraft. Even though the ordering determined in Step 3 may change, the structure and gains of the feedback system are invariant.

In Appendix A, the behavior of the simulated system is shown. As noted in Reference 5, the optimal feedback controller does indeed reduce airspeed deviations to zero, separates the aircraft by distance D, and each aircraft comes to rest slightly in front of its target. This steady-state error is undesirable and should be corrected. One possible corrective measure is a heavier penalty on positional errors. In fact, the weights selected in this simulation were chosen simply on the basis of reasonably reflecting the system's requirements of a hard constraint on airspeed deviation and the assurance of adequate separation between aircraft. If the system's performance is unsatisfactory in any respect, weights may be freely adjusted until acceptable behavior is obtained.

Completion of rendezvous maneuvers is shown in Figures 20-26. At the instant of rendezvous, the initial conditions of maneuvering aircraft are released to the action of the optimal controller. Again, performance of the system is satisfactory in that no undesirable transients are experienced by the system.

CHAPTER VII

CONCLUSIONS

A most welcome conclusion would be that the algorithm proposed in this thesis is an adequate solution for the ATC problem. Considering the amount of research and simulation performed, such a conclusion is, of course, not justified. However, several of the model's characteristics are those essential to an adequate ATC system. With this in mind, it is suggested that the proposed algorithm be the subject of additional research.

CHAPTER IIX

SUGGESTIONS FOR FUTURE RESEARCH

The topics listed below are several inter-related areas in which additional research may be very fruitful:

1. Is the proposed algorithm feasible? A full-scale simulation would determine if the system can adequately and safely process the traffic situation characteristic of the large metropolitan airports.

2. The proposed system can be extended in several respects:

   a. adaptation to handle both take-offs and landings for single and parallel runways,

   b. consideration of weather factors on the acceptance rate, especially wind direction changes,

   c. control techniques for overlapping airspace of adjacent airports such as in the New York City area, and

   d. scheduling a mix of heavy, medium and light aircraft.

3. What are requirements for actual implementation of the system? What airborne and ground-based sub-systems are required? To what extent may the present ATC system be modified to accept the porposed algorithm?

APPENDIX A

CAPTIONS

Figure 20

(a) Error in velocity and position of aircraft three vs time

(b) Error in velocity and position of aircraft two vs time

Initial conditions:

$e_{2x}(0) = 0.57$ nautical miles

all other state variables $= 0$

Minimum allowable velocity is achieved by aircraft two (140 knots).

Figure 21

(a) Error in velocity and position of aircraft three vs time

(b) Error in velocity and position of aircraft two vs time

Initial conditions:

$e_{2x}(0) = 1.50$ nautical miles

all other state variables $= 0$

Unacceptable initial conditions of aircraft two cause unacceptable velocity deviation of aircraft three.

Figure 22

(a) Error in velocity and position of aircraft three vs time

(b) Error in velocity and position aircraft two vs time

Initial conditions:

$e_{2x}(0) = 1.20$ nautical miles

$e_{2v}(0) = 12$ knots

all other state variables $= 0$

Forward velocity error aids in maintaining acceptable velocity deviation.

Figure 23

Error in position of all aircraft vs time

Initial conditions:

$e_{1x}(0) = 0.$ $\qquad$ $e_{3x}(0) = 0.45$ nautical miles

$e_{1v}(0) = 0.$ $\qquad$ $e_{3v}(0) = -9.60$ knots

$e_{2x}(0) = 3.00$ nautical miles $\quad e_{4x}(0) = 3.75$ nautical miles

$e_{2v}(0) = 16$ knots $\quad e_{4v}(0) = 0.$

Aircraft one and three reside in guideway A; two and four reside in guideway B.

Figure 24

Error in velocity of all aircraft vs time

Initial conditions: see initial conditions above

Figures 23-24 show unacceptable velocity deviations for given initial conditions, thus indicating need for holding maneuvers.

Figure 25

Error in position of all aircraft vs time

Initial conditions:

$e_{1x}(0) = 0.$ $\quad e_{3x}(0) = 1.00$ nautical mile

$e_{1v}(0) = -16$ knots $\quad e_{3v}(0) = 0.$

$e_{3x}(0) = 0.37$ nautical mile $\quad e_{4x}(0) = 4.00$ nautical miles

$e_{3v}(0) = 11$ knots $\quad e_{4v}(0) = 16$ knots

Holding maneuvers are required for aircraft three and four. Their "new" initial conditions are:

$e'_{3x}(0) = 0.$ $\quad e'_{4x}(0) = 0.$

$e'_{3v}(0) = 0.$ $\quad e'_{4v}(0) = 16$ knots

$T_3 = 1.60$ minutes $\quad T_4 = 2.85$ minutes

Aircraft one and three reside in guideway A, and aircraft two and four are relegated to guideway B.

Figure 26

Error in velocity of all aircraft vs time

Initial conditions: see initial conditions above

NOTE: the acceptable range of velocities for aircraft in the NTA is arbitrarily specified to be: 140 - 180 knots.

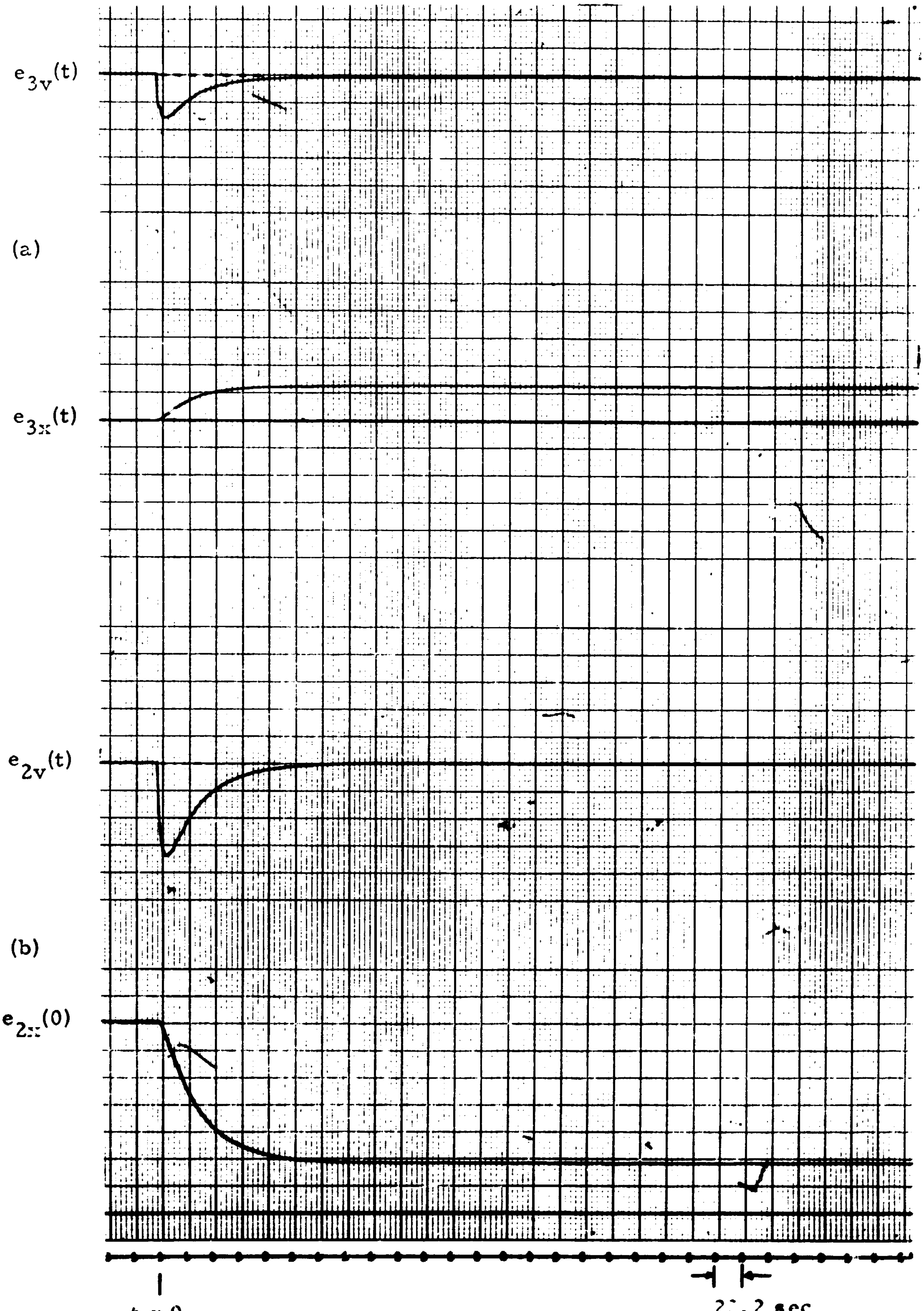


Figure 20

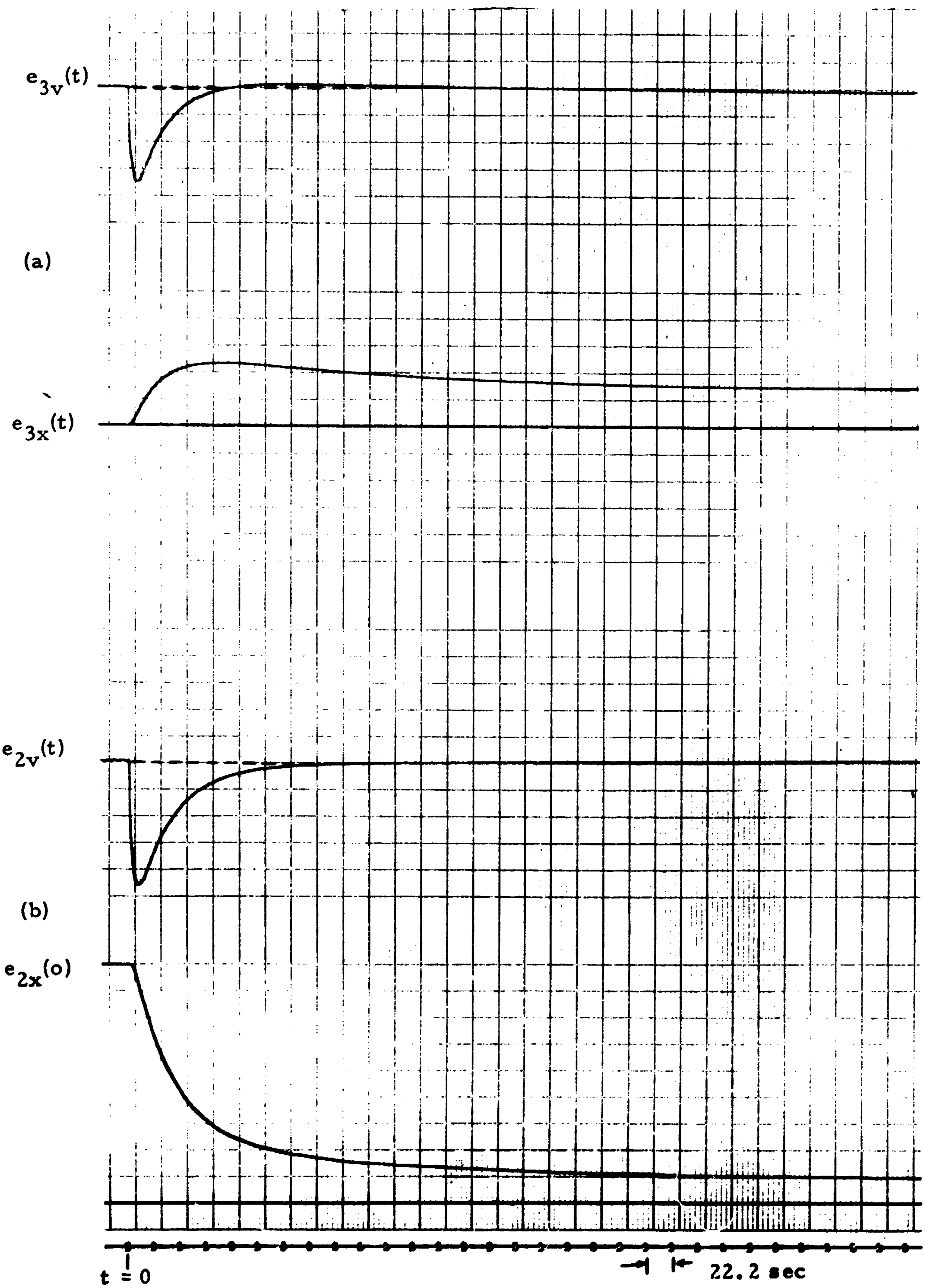


Figure 21

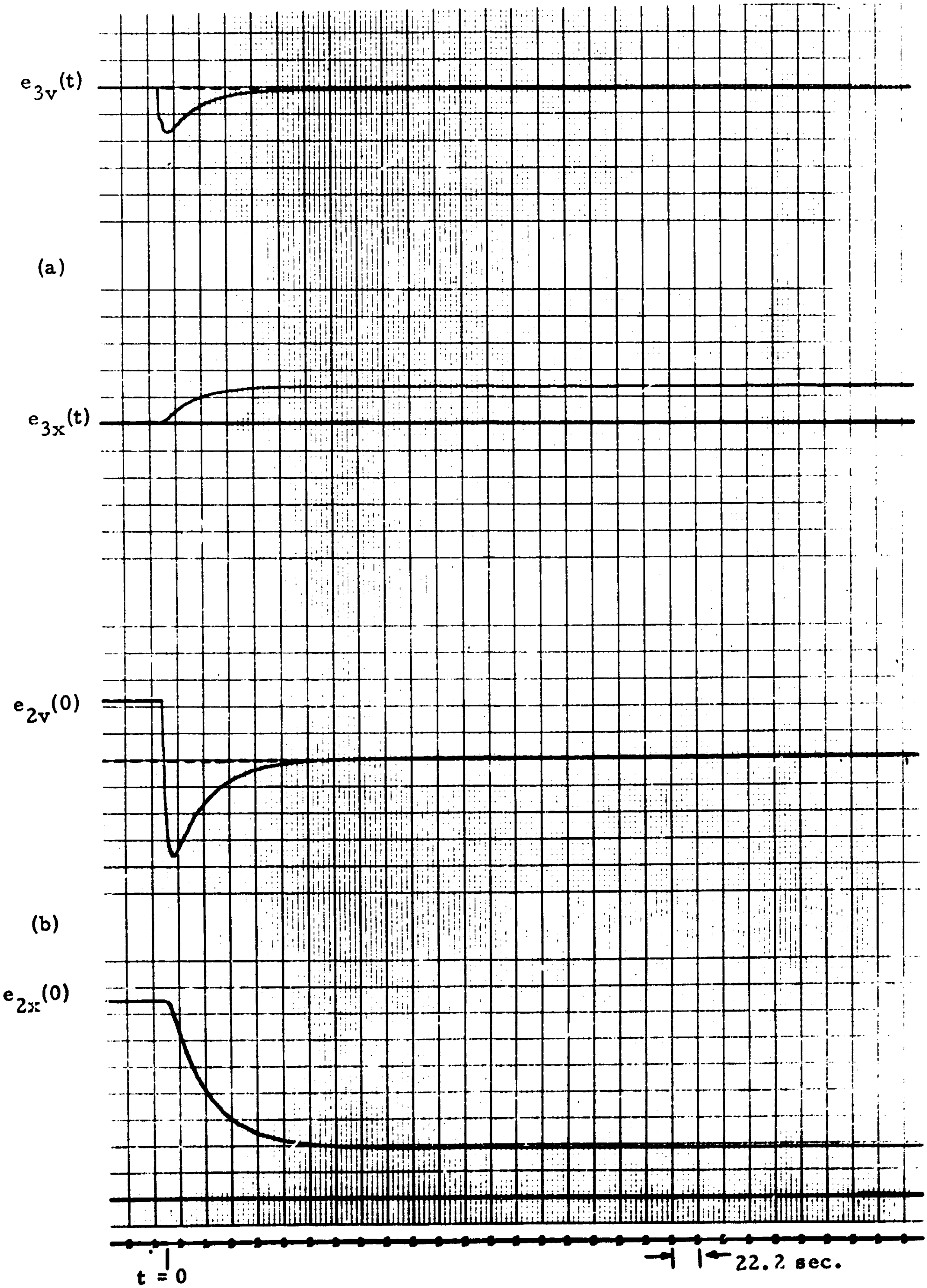


Figure 22

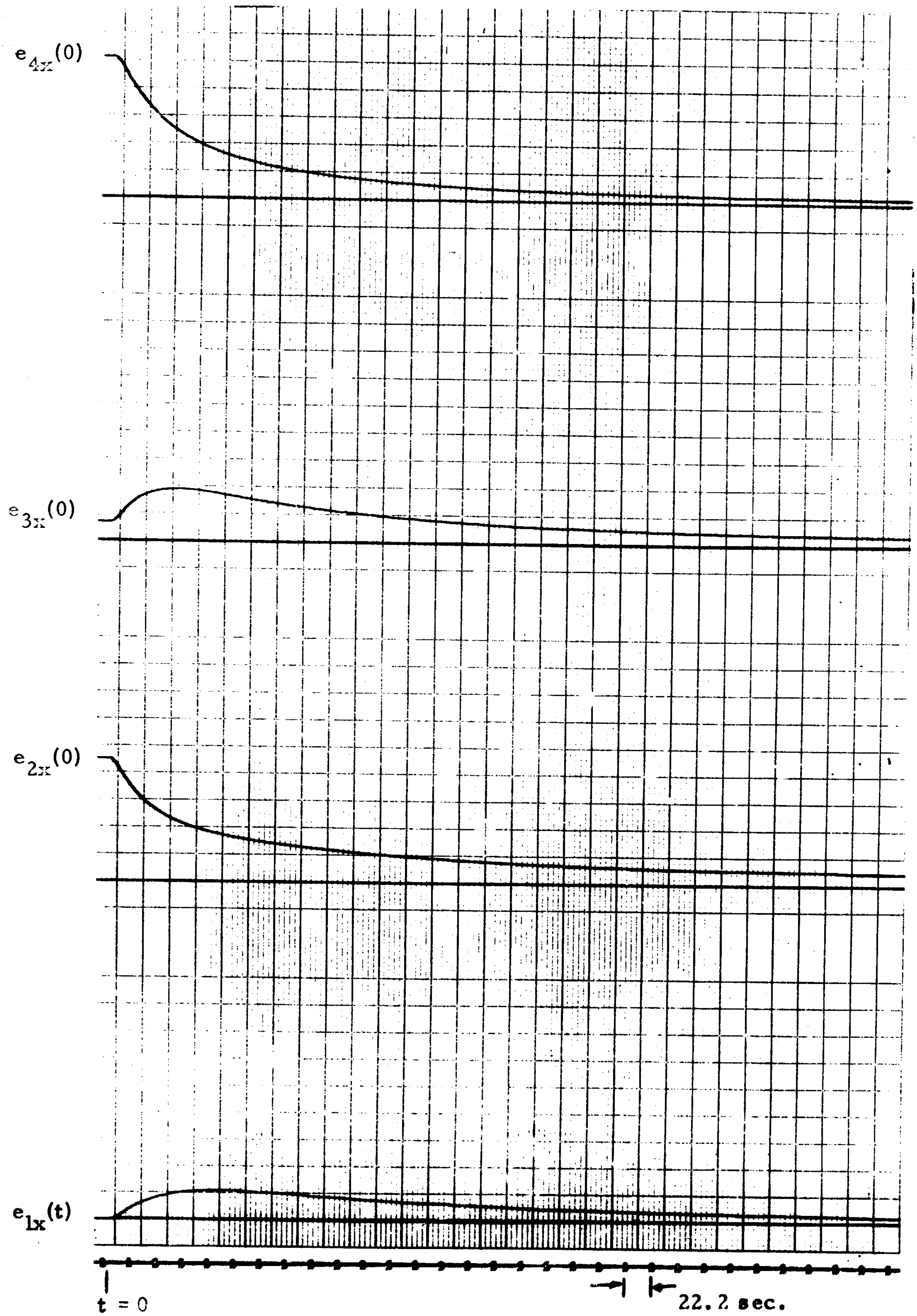


Figure 23

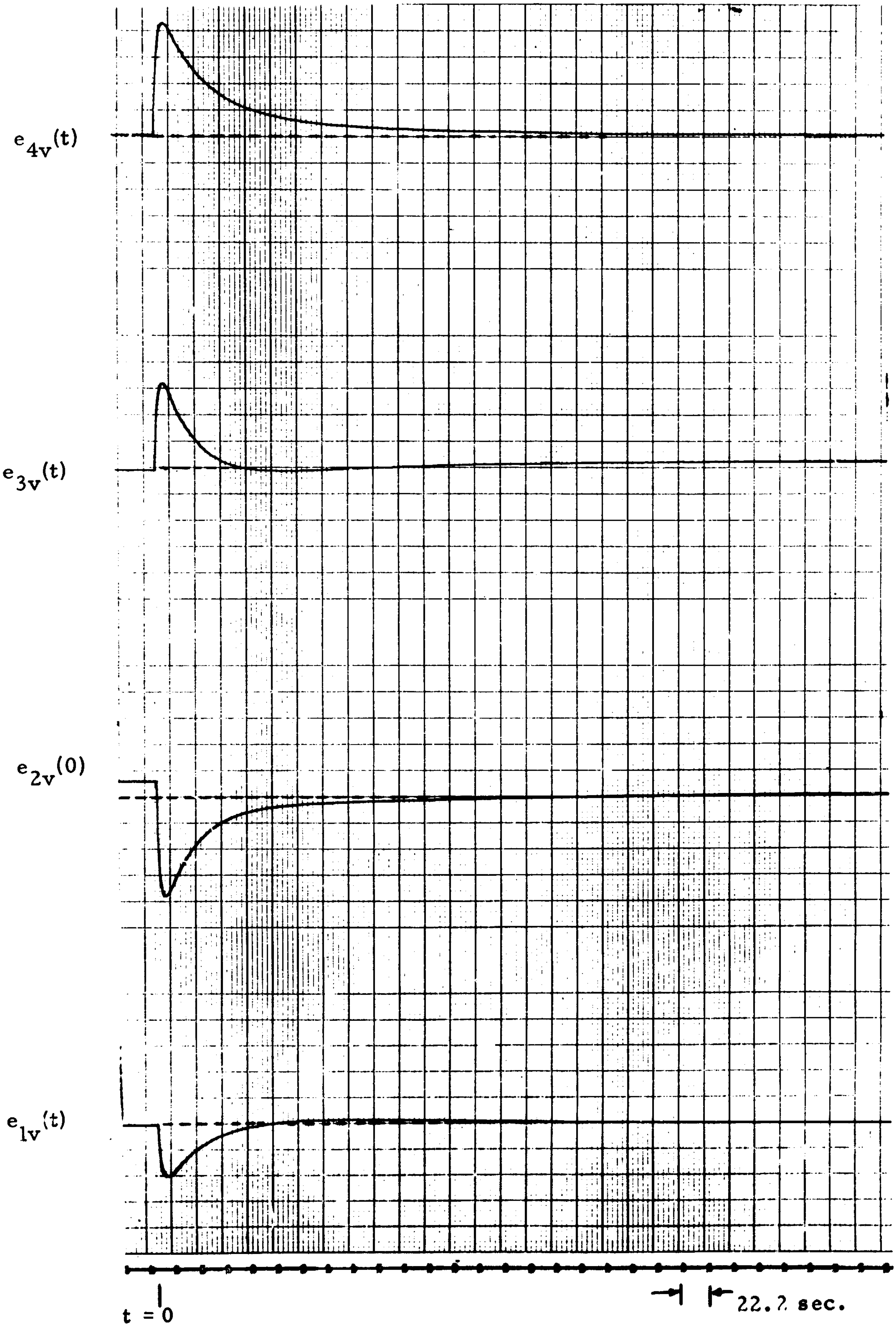


Figure 24

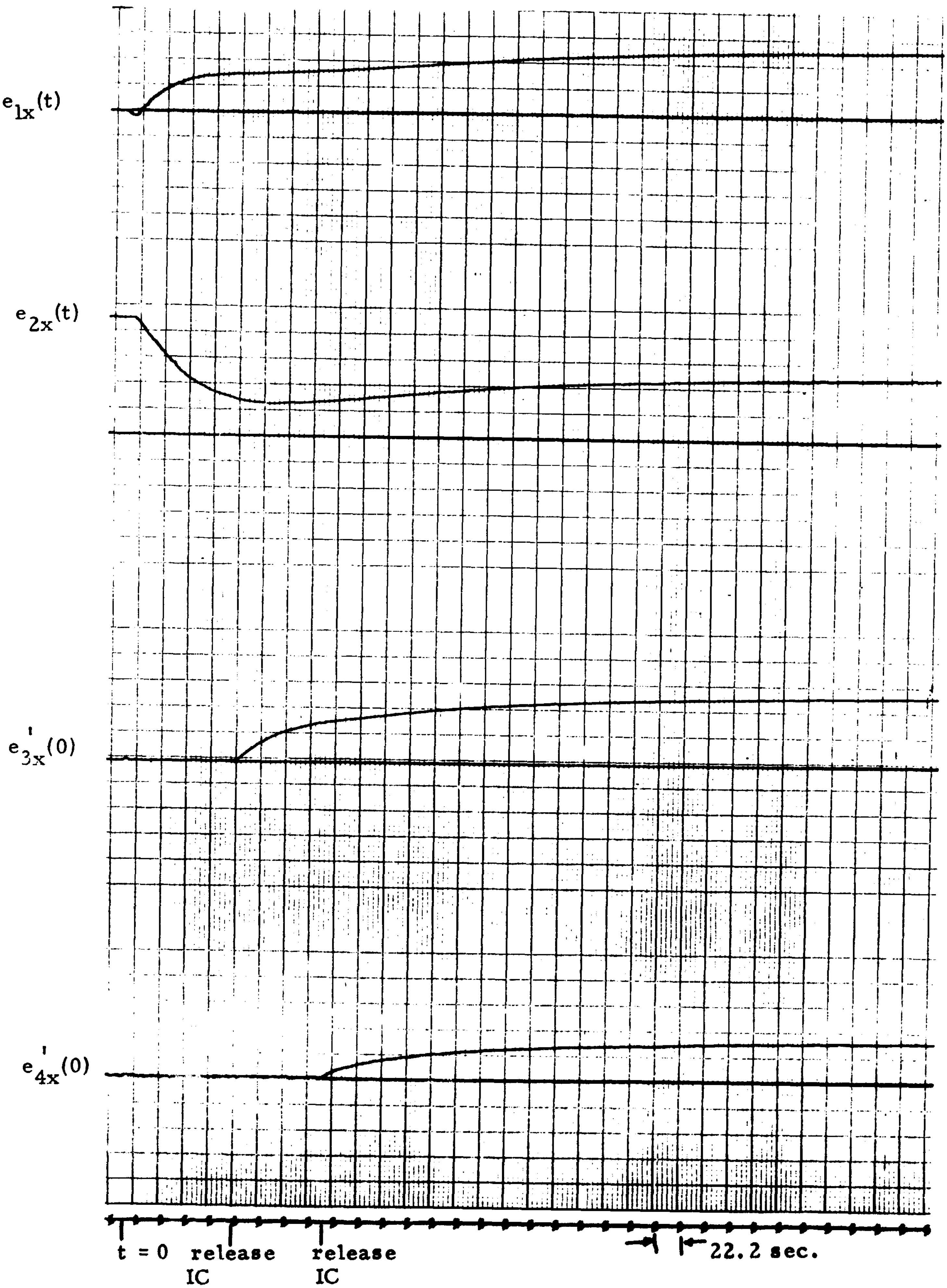


Figure 25

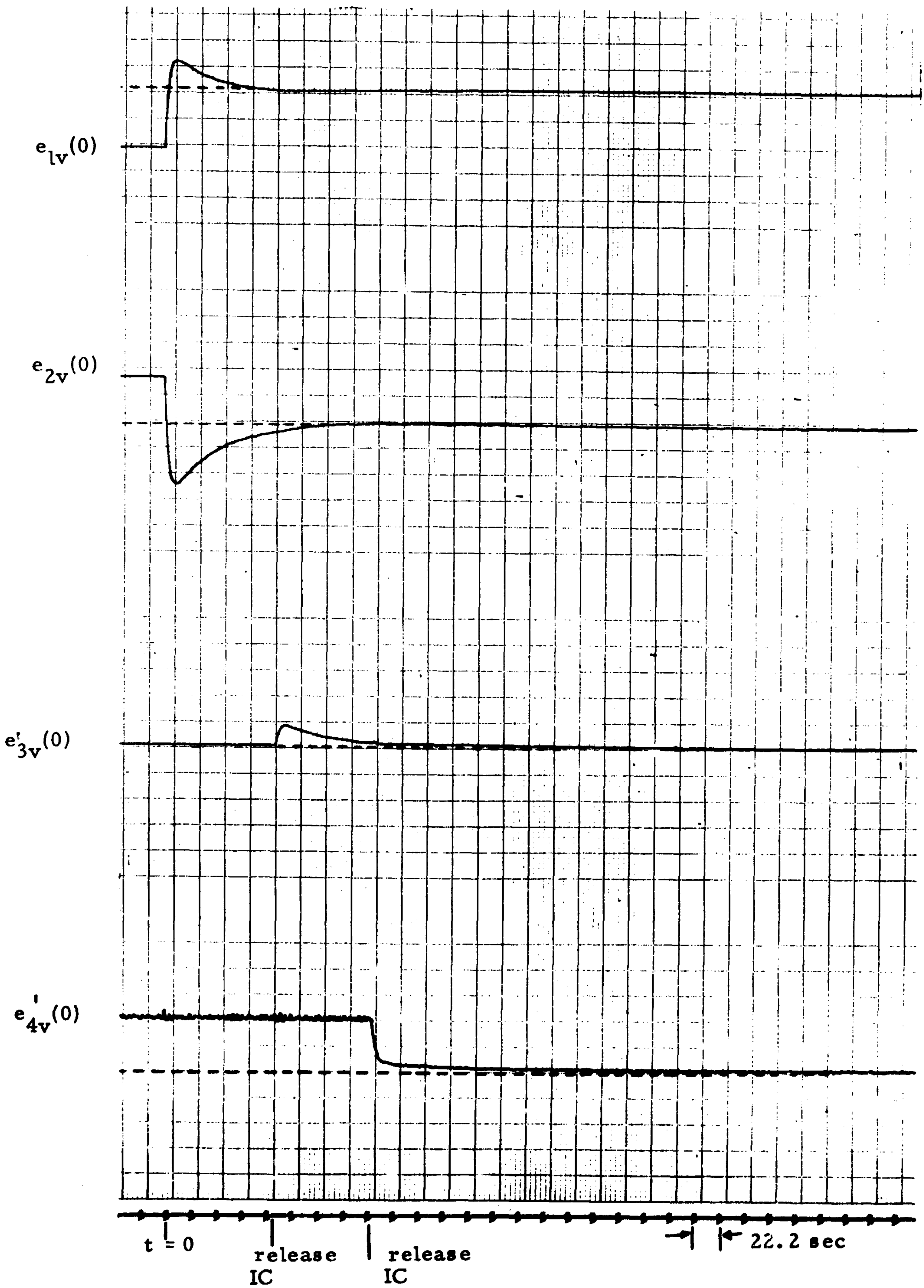


Figure 26

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