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PASSIVE AND ACTIVE SHOCK ISOLATION

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SUMMARY

This paper discusses the state of the art of isolation from mechanical shock, particularly with regard to providing protection from aircraft and aerospace shock environments. Idealized forms of shock excitation are employed, and the performance capabilities of shock isolation systems are emphasized rather than mathematical analysis techniques. Both passive and active single-degree-of-freedom shock isolation systems are discussed, including the effects of isolator damping and nonlinear stiffness characteristics. The ranges of practical application of passive and active isolation systems are indicated, and the concept of optimum shock isolation analysis and synthesis is discussed. Problem areas requiring additional research are identified

INTRODUCTION

Shock excitation of a mechanical system causes the response of the system to change radically in a short period of time. It may be defined in terms of the sudden variation of force applied to the system, or by displacement, velocity, or acceleration shock pulses imposed upon a particular point in the system. Mitigation of the effects of shock excitation may be achieved by inserting an isolator having appropriate characteristics between the system being protected and the source of shock excitation.

Shock and vibration isolation systems are categorized as linear or nonlinear depending on whether or not their dynamic response is described by a set of linear time-invariant differential equations. They are further categorized as active or as passive, depending on whether or not power is required for the isolator to perform its function.

The essential features of a passive isolator are a resilient load-supporting means (stiffness) and an energy dissipating means (damping); typical passive isolators employ metallic springs, elastomers, wire mesh, pneumatic springs, elastomeric foams, and combinations of these or other cushioning devices. Typical active isolator mechanisms include servo motor actuated mechanical linkages, variable resilience devices containing conductive or magnetic fluids, and pneumatic or hydraulic valve-operated actuators. The active isolator mechanisms are power operated in accordance with command signals derived from feedback control signals.

For purposes of discussing the state of the art of shock isolation, it is convenient to make certain simplifying assumptions. The isolation system is considered to be a single-degree-of-freedom system comprised of a rigid mass supported on a rigid foundation by a single isolator undergoing unidirectional dynamic response. Schematic diagrams of idealized shock isolation systems are illustrated in Figure 1 where: (a) the shock excitation is represented by an acceleration $\ddot{a}(t)$ applied to the rigid foundation, and (b) the shock excitation is represented by a force $P(t)$ applied to the rigid mass. For shock excitation of the support structure, the purpose of the isolator is to reduce the magnitude of acceleration transmitted to the isolated mass whereas, for shock excitation of the isolated mass, the purpose of the isolator is to reduce the magnitude of force transmitted to the foundation.

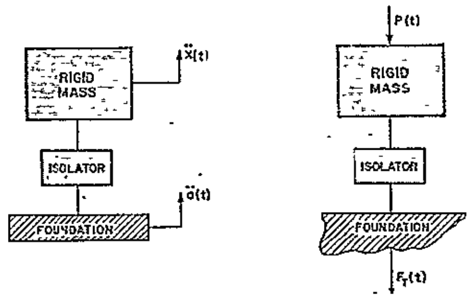

Figure 1.- Schematic diagrams of idealized isolation systems.

NATURE OF SHOCK ENVIRONMENTS

Shock excitation is generally described by a shock pulse, which specifies the time-history of the acceleration, velocity, displacement or force excitation for the time interval of the applied shock. Service shock conditions may be represented by shock pulses having complex time-histories which are analyzed by numerical analysis techniques or by mathematically modeling the shock environment using relatively simple analytical functions (Ref. 1). A variety of analytical techniques exist for determining the response of isolation systems subjected to complex shock excitation. This paper will deal only with idealized forms of shock excitation so that attention is focused on the performance of isolation systems rather than the mathematical techniques employed in their analysis.

Examples of typical idealized shock excitation time-histories are illustrated in Figure 2. In addition to the shape of the shock pulse, the other distinguishing characteristics are the peak magnitude of excitation velocity V_o or acceleration $A_o = \ddot{a}_o/g$*, and the characteristic time or duration of the shock pulse t_o. A discussion of typical mathematical models of shock excitation follows, with the remainder of the paper concerned solely with shock excitation specified in the form of acceleration shock pulses.

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* Acceleration in units of $[L/T^2]$ is indicated by the second-derivative double-dot notation, whereas A represents acceleration measured as a multiple of gravitational acceleration g.

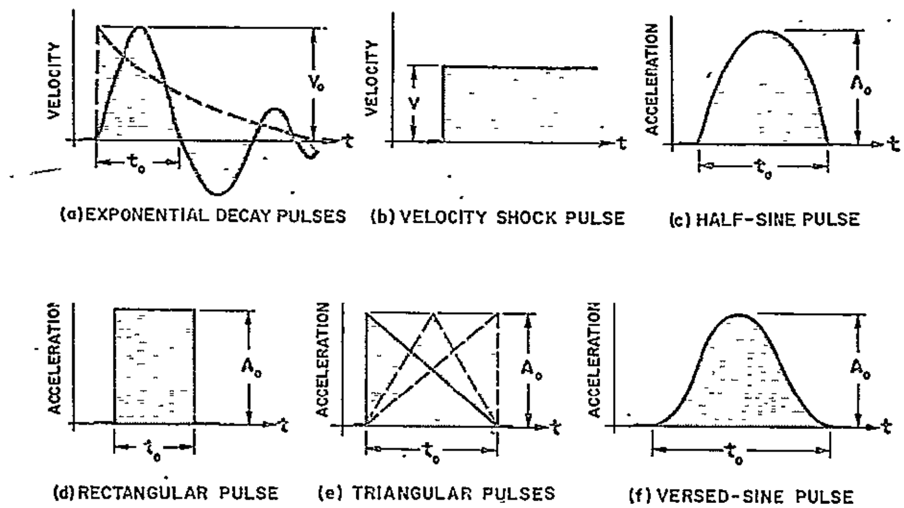

Figure 2.- Typical idealized shock excitation time-histories.

IMPULSIVE SHOCK

Shock excitation in the form of an applied shock force, which has a high peak magnitude and a time duration that is short relative to the natural period of vibration of the system, is usually defined as <u>impulsive shock</u>, where impulse I is given by

$$I = \int_0^{t_0} P(t)\,dt \tag{1}$$

and the force $P(t)$ is equal to zero before time $t = 0$ and after time $t = t_0$. For an infinitesimally short time duration t_0, the impulsive shock represents an impact condition wherein momentum transfer occurs instantaneously, during which the mass experiences an instantaneous velocity change V, as follows

$$V = I/m \tag{2}$$

where m is the mass of the body on which the impulsive force $P(t)$ acts.

EXPONENTIAL DECAY SHOCK

The sudden release of a large amount of energy, as in an explosion, or the effect of a shock being transmitted through an intermediate mechanical system, may result in exponential decay shock excitation time-histories of the type illustrated in Figure 2(a). The excitation velocity may be a decaying oscillation or a nonoscillatory decay of peak velocity to zero velocity, as described mathematically by various combinations of exponential and harmonic time functions.

VELOCITY SHOCK

If the excitation velocity undergoes an instantaneous change, the system is subjected to a velocity shock. The system could be initially at rest, as illustrated in Figure 2(b), be experiencing a velocity prior to being brought to rest, or experience an instantaneous change from one velocity to another. The velocity change V is the algebraic difference between the excitation velocity that exists after and before the shock excitation has taken place.

For an impulsive shock force, the velocity change of the isolated mass is given by Equation (1) whereas, for an acceleration impulse, the velocity change of the foundation is given by the area under the acceleration shock pulse time-history, as follows

$$V = \int_{0}^{t_0} \ddot{a}(t)\,dt \tag{3}$$

where the excitation acceleration $\ddot{a}(t)$ is zero before time $t = 0$ and after time $t = t_0$.

FREE FALL IMPACT

Impact resulting from free fall in the gravitational field imposes a velocity shock on the falling object if the impact time duration is short relative to the natural periods of vibration of the elements comprising the object. For impact with no rebound, the shock condition is defined as inelastic impact, and the velocity change V_i is given by

$$V_i = \sqrt{2gh} \tag{4}$$

whereas, for impact with full rebound, the shock condition is defined as <u>elastic impact</u>, and the velocity change V_e is given by

$$V_e = 2\sqrt{2gh} \tag{5}$$

where g is the acceleration of gravity and h is the drop height (Ref. 2).

ELASTIC AND INELASTIC IMPACT

Shock excitation for an elastic impact may be in the form of an acceleration half-sine shock pulse illustrated in Figure 2(c). For inelastic impacts, the shock excitation may be represented by acceleration shock pulses having half-sine, rectangular, triangular or versed-sine shapes, as illustrated in Figure 2(c) through (f), respectively. The velocity changes for these shock pulses are as follows:

$$\text{half-sine:} \quad V = (2g/\pi)\, A_o t_o \tag{6}$$

$$\text{rectangular:} \quad V = g\, A_o t_o \tag{7}$$

$$\text{triangular:} \quad V = g\, A_o t_o/2 \tag{8}$$

$$\text{versed-sine:} \quad V = g\, A_o t_o/2 \tag{9}$$

SUSTAINED ACCELERATION

Sustained acceleration exists in a dynamic environment when a constant level of acceleration, usually measured as a multiple of gravitational acceleration, is maintained for an extended length of time. The level of sustained acceleration may be reached instantaneously, over an infinite time duration, or over a finite time duration, where t_r represents the rise time of the leading edge of the sustained acceleration time-history.

Conditions of sustained acceleration are normally categorized as shock because of the sudden manner in which the acceleration changes from a reference magnitude to its maximum sustained magnitude $A_s = \ddot{a}_s/g$. Examples of typical idealized sustained acceleration time-histories are illustrated in Figure 3. The onset of a sustained acceleration condition may be considered to take place in an instantaneous step, as shown in Figure 3(a), or over an infinite time duration as shown in Figure 3(b). Alternately, the level of sustained acceleration may be reached over a finite time duration, as shown for ramp, exponential, versed-sine, and cycloidal leading edges in Figures 3(c) through 3(f), respectively. Since the instantaneous step represents the most abrupt shock condition, it is the type most frequently employed in the determination of shock and vibration isolation system performance characteristics:

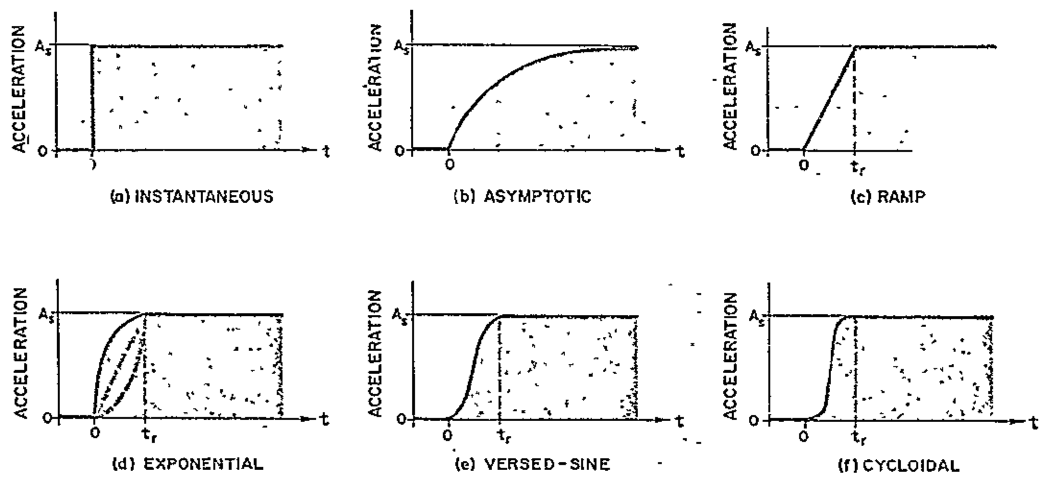

Figure 3.- Typical idealized sustained acceleration time-histories.

PHILOSOPHY OF SHOCK ISOLATION

Shock isolators are inserted between the source of shock excitation and the system requiring protection. A reduction in the magnitude of system response is provided because of the ability of the isolator to store energy at the relatively high rate associated with the shock excitation, and subsequently release it at a relatively low rate. Release of the strain energy stored in the isolator causes the isolated body to vibrate at the natural frequency of the isolation system, until the energy is dissipated by the isolator damping mechanism (Ref. 3).

The dynamic performance of a shock isolation system is illustrated qualitatively in Figure 4. The shock excitation is in the form of an acceleration time-history $\ddot{a}(t)/g$ represented by a pulse of a given shape having a peak magnitude of acceleration A_0 and a time duration t_0. The natural period τ_0 of the isolation system is selected to be a relatively high value compared to the shock pulse time duration t_0, so that the acceleration response $\ddot{x}(t)/g$ is represented by a low-frequency transient vibration having a maximum magnitude of acceleration A_{max} that is less than the peak magnitude of shock excitation A_0.

Shock transmissibility T_s, which is defined as

$$T_s = A_{max}/A_o \tag{10}$$

is a common measure of the shock transmission characteristics of an isolation system when the shock pulse is of a well-defined shape such that the peak magnitude of shock excitation A_o is accurately determined. For impulsive shock excitation, which is defined completely by the velocity change imposed upon the isolation system, the maximum acceleration response is the parameter of major interest.

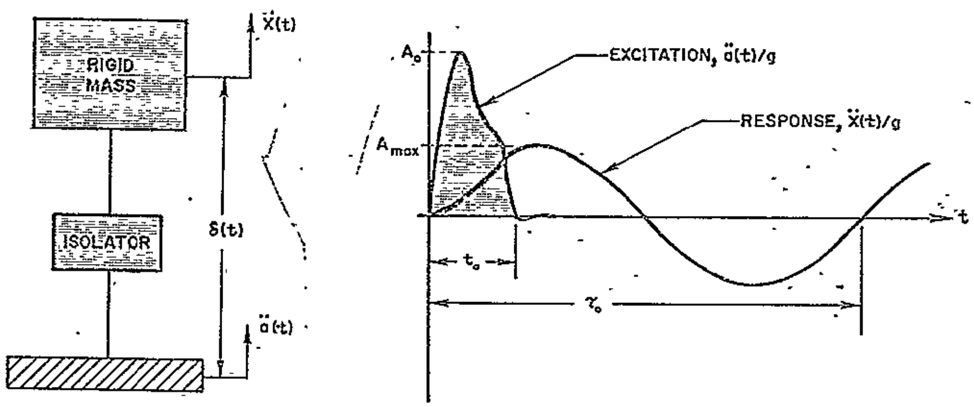

Figure 4.- Qualitative description of shock isolation system performance.

In addition to the maximum response acceleration, the maximum relative displacement $\delta = (x-a)$ is an important response parameter. The maximum stress created in the isolator and the clearance requirements for the isolation system are both a function of the maximum relative displacement δ_{max}. Relatively large static and dynamic displacements are normally associated with low-frequency systems experiencing vibration. Since the clearance available for response displacements is frequently limited, selection of an isolator design generally involves achieving an acceptable compromise between the maximum acceleration and relative displacement response magnitudes.

When evaluating shock isolation performance, consideration must be given to requirements for vibration isolation and the effect of sustained acceleration conditions. An isolator that exhibits good shock isolation does not necessarily provide acceptable vibration isolation. Furthermore, sustained acceleration may cause isolator snubbing or operation in its nonlinear (high-stiffness) region. Having established (1) the allowable magnitudes of response acceleration and relative displacement, (2) the requirements for shock and vibration isolation, and (3) the level of sustained acceleration anticipated, consideration can be given to a wide variety of linear and nonlinear isolators that may be either passive or active, depending upon the nature and degree of protection required.

RELATION BETWEEN SHOCK AND VIBRATION ISOLATION

The response of linear isolation systems for impulsive shock and harmonic vibration excitation are related mathematically since they are Fourier transform pairs (Ref. 4). The complex form of the vibration transmissibility T_v, which is defined as

$$T_v = \ddot{x}_o / \ddot{a}_o \tag{11}$$

where $\ddot{x}_o$ and $\ddot{a}_o$ represent the harmonic amplitudes of response and excitation acceleration, respectively, is equal to the Fourier transform of the impulse response of the isolation system (Ref. 5). Conversely, the impulse response of the isolation system is the inverse Fourier transform of the complex vibration transmissibility. Consequently, exact relationships exist between the response of a linear isolation system subjected to impulsive shock and harmonic vibration excitation.

Perhaps equally important in relating the shock response to the vibration transmissibility of linear isolation systems is recognition of the fact that attenuation of acceleration response is accomplished in both cases by employing low-frequency systems. Since the shock response is strongly dependent upon the natural frequency of the isolation system, the resonance characteristics exhibited by the vibration transmissibility curve can be employed as an indicator of the shock isolation performance. Specifically, the resonant frequency f_r is approximately equal to the natural frequency f_o for light damping, which provides a measure of the natural period $\tau_o = 1/f_o$, and the resonant transmissibility T_r, being an inverse function of damping, provides an indication of the degree of isolation system damping. Crede's text (Ref. 2) on vibration and shock isolation and the more recent documentation by Crede and the present author (Ref. 6) provide relevant information on vibration isolation characteristics.

PASSIVE ISOLATION SYSTEMS

To a great extent, passive shock isolation techniques have developed from Mindlin's basic monograph on package cushioning (Ref. 7). This work was later extended into useful design data by Crede (Ref. 2), with extensive work on the effect of shock pulse shape performed by Jacobson and Ayre (Ref. 8). Collections of data on the shock response of linear single-degree-of-freedom systems for a wide range of pulse shapes are available in the technical literature, including the effects of viscous damping (Ref. 7-16).

The differential equation of motion of a linear single-degree-of-freedom isolation system with viscous damping is given by

$$\ddot{x} + 2\zeta \omega_0 \dot{\delta} + \omega_0^2 \delta = 0 \tag{12}$$

where x is the absolute displacement of the isolated mass, $\delta = (x-a)$ is the relative displacement, and the shock excitation is assumed to be specified by the foundation motion a(t) or its derivatives. The undamped natural frequency f_0 (measured in Hz or cps) is given by

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{13}$$

where m is the mass of the isolated body and k is the linear stiffness of the isolator. The viscous damping ratio $\zeta = C/C_c$ is the ratio of the viscous damping coefficient C to the critical value of viscous damping $C_c = 2\sqrt{km}$. Consideration will first be given to linear undamped isolation systems, followed by discussions of the effects of isolation system damping and nonlinear isolator stiffness characteristics.

VELOCITY SHOCK OF LINEAR UNDAMPED ISOLATION SYSTEM

Under impulsive shock excitation, the instantaneous velocity change V defined by Equation (2) or (3) gives rise to sinusoidal acceleration and relative displacement responses having maximum magnitudes given by

$$A_{max} = \frac{\ddot{x}_{max}}{g} = \frac{f_0 V}{61.4} \tag{14}$$

$$\delta_{max} = \frac{V}{6.3 f_0} \tag{15}$$

These equations indicate that the transmitted acceleration is reduced by employing low values of the natural frequency f_0; however, use of a low natural frequency results in a large relative deflection for a given velocity change. Consequently, a compromise between the requirements of maximum allowable acceleration and relative displacement response is required in the selection of the isolation system natural frequency.

EFFECT OF SHOCK PULSE SHAPE

In evaluating the effect of shock pulse shape on the dynamic response of linear isolation systems, the concept of shock spectra is usually employed. Vigness recently presented a discussion of the history of the development of shock spectra and their use in the study of shock response problems (Ref. 16). Basically, a shock spectrum is a description of the manner in which the response maxima of single-degree-of-freedom systems vary with natural frequency and damping for a given shock excitation. The most common means of graphically presenting shock spectra is the four-coordinate logarithmic graph paper illustrated in Figure 5.

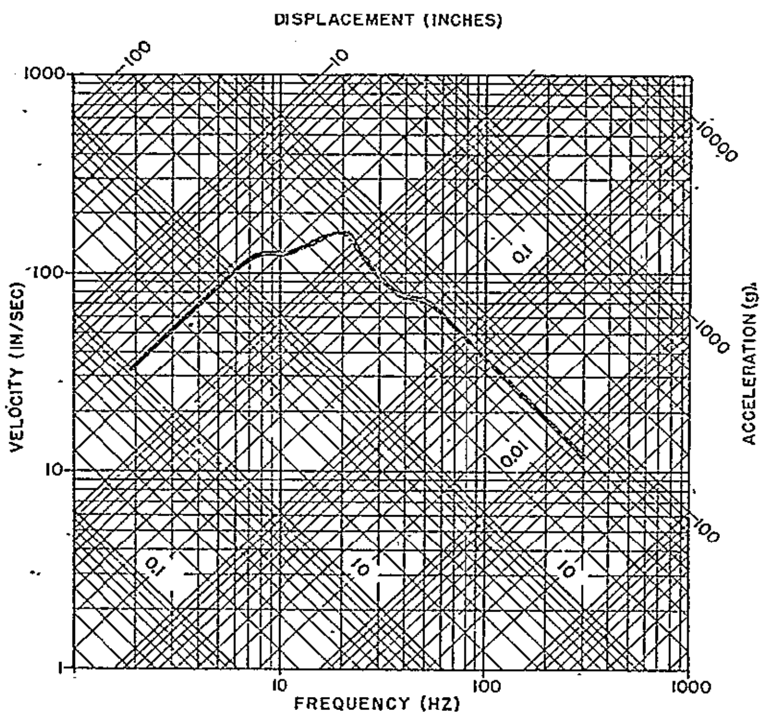

Figure 5.- Shock spectrum of a particular complex shock excitation.

Complex shock excitations are analyzed by analog or digital computation means to obtain a plot of the shock spectrum, an example of which is included in Figure 5 for a certain value of damping. For a given allowable maximum acceleration, the required isolation system natural frequency and the resulting maximum relative displacement is obtained directly from the shock spectrum curve. For example, if the maximum allowable acceleration response is 3 g's, the shock spectrum shown in Figure 5 indicates that a 3.2 Hz isolation system is required, and a maximum relative displacement of 2.8 inches will result. An optimum design can be attained by selecting various values of natural frequency until the best compromise between response acceleration and relative displacement is obtained.

With the data available on the shock spectra for analytically defined pulse shapes (such as rectangular, half-sine, triangular and versed-sine), a relatively simple design criteria for shock isolation can be established. For positive value shock pulses ($\ddot{a} > 0$) having a single peak of magnitude A_0, it is convenient to define the <u>shock pulse effective time duration</u> t_e, as follows (Ref. 13)

$$t_e = V/A_0 g \tag{16}$$

where V is the velocity change (area under the excitation acceleration time-history) and A_0 is the peak magnitude of excitation acceleration. For a rectangular pulse, the effective time duration t_e equals the actual pulse duration t_0; however, for all other pulse shapes, the effective time duration is less than the actual pulse time duration. For example, $t_e = (2/\pi)t_0$ for the half-sine pulse, and $t_e = t_0/2$ for the triangular and versed-sine pulses.

For purposes of comparison, the vibration and shock transmissibility for a linear isolation system are presented in Figure 6. For the case of harmonic

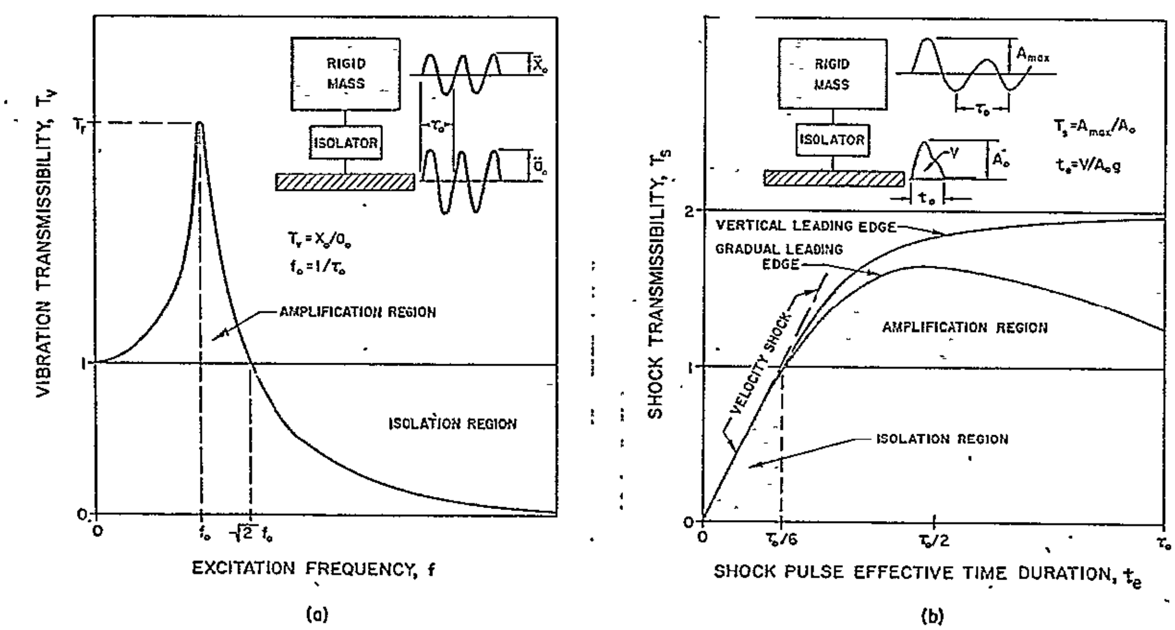

Figure 6.- Performance characteristics of linear isolation systems for (a) harmonic vibration excitation and (b) shock excitation.

vibration excitation depicted in Figure 6(a), the vibration transmissibility curve indicates that amplification of vibration occurs for low values of excitation frequency, and isolation of vibration occurs for excitation frequencies greater than approximately 1.4 times the natural frequency of the isolation system. Therefore, in order to provide isolation of vibration, the natural frequency of the isolation system f_0 is made less than approximately 0.7 times the vibration excitation frequency.

For the case of shock pulse excitation depicted in Figure 6(b), the shock transmissibility curves indicate that amplification of shock occurs for high values of the shock pulse effective time duration, and isolation of shock occurs for effective time durations t_e less than approximately one-sixth the natural period τ_0. The shape of the shock pulse is unimportant in the shock isolation region and, for all practical purposes, velocity shock conditions apply for the entire isolation region. Therefore, in order to provide isolation of shock, the natural frequency of the isolation system f_0 is made less than one-sixth the reciprocal of the shock pulse effective time duration. The maximum magnitude of acceleration response is slightly overestimated when determined on the basis of velocity shock excitation in the isolation region; however, calculation of isolation system performance on this basis is considered appropriate in that the small error involved is conservative in nature.*

Amplification of shock occurs for $t_e > \tau_0/6$, and the maximum shock transmissibility generally has a value between 1.5 and 2, depending on the shape of the shock pulse. For shock pulses having a leading edge with a gradual rise, the maximum shock transmissibility occurs in the region $\tau_0/3 < t_e < 2\,\tau_0/3$ and, for $t_e > 5\,\tau_0/4$, the shock transmissibility ranges between 1.0 and 1.2, regardless of the shape of the excitation shock pulse. For shock pulses having a vertical leading edge, the maximum shock transmissibility generally occurs when $t_0 \geq \tau_0/2$, and the shock transmissibility approaches a factor of 2.

SELECTION OF NATURAL FREQUENCY

A low natural frequency is required to provide isolation of vibration and shock. For vibration excitation $f_0 < 0.7f$ whereas, for shock excitation, $f_0 < 1/6\,t_e$, where f and t_e are the frequency and effective time duration characteristics of the dynamic excitation. Selection of natural frequency on this basis will ensure isolation of the dynamic excitation; however, large static and dynamic deflections may result.

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*In terms of the relative values of the actual shock pulse time duration t_0 and the isolation system natural period τ_0, velocity shock conditions prevail, with less than approximately ten percent error in response prediction, when $t_0 < \tau_0/4$ for a rectangular acceleration pulse and $t_0 < \tau_0/3$ for half-sine, triangular, and versed-sine acceleration pulses.

The static deflection δ_{st} of a linear isolation system is given by

$$\delta_{st} = 9.8/f_o^2 \tag{17}$$

and the maximum relative displacement during response to velocity shock excitation is determined from Equations (14), (15) and (17), as follows:

$$\delta_{max} = A_{max}\delta_{st} = 9.8\ A_{max}/f_o^2 \tag{18}$$

Selection of the isolation system natural frequency is made by determining the best compromise between the degree of isolation and the allowable static and dynamic displacements (as dictated primarily by clearance availability and isolator stress). Space limitations and undesirable properties (such as lateral instability and drift) of certain types of passive isolators having low stiffness, also influence the selection of natural frequency. Space for isolators and clearance for isolation system dynamic displacements are generally quite limited for most aircraft and aerospace vibration and shock problems. For these cases, the lowest practical natural frequency using passive isolation techniques ranges between 3-5 Hz.

EFFECT OF ISOLATION SYSTEM DAMPING

The technical literature contains erroneous information on the effect of directly coupled viscous damping on the shock response of a single-degree-of-freedom system. This situation has resulted from misinterpretation of data presented for the half-sine acceleration shock pulse by Mindlin and his co-workers (Ref. 7, 9). Data was originally presented in terms of the amplification factor defined as δ_{max}/Δ_{st} where δ_{max} is the maximum relative displacement that results from the half-sine acceleration shock excitation and Δ_{st} is the equivalent static deflection that would result if the peak excitation acceleration A_o were applied statically to the rigid mass m, as follows

$$\Delta_{st} = \frac{m A_o g}{k} = \frac{A_o g}{\omega_o^2} \tag{19}$$

For zero damping, Equation (12) indicates that $\omega_o^2 = \ddot{x}_{max}/\delta_{max} = A_{max}\, g/\delta_{max}$ and, consequently, the amplification factor δ_{max}/Δ_{st} is identical to the shock transmissibility $T_s = A_{max}/A_o$. However, when significant damping is present ($\zeta > 0.2$), the maximum acceleration response $\ddot{x}_{max}$ is not directly proportional to the maximum relative displacement response δ_{max} and, therefore, the shock transmissibility in general is not equal to the amplification factor (Ref. 17). Consequently, the curves of amplification factor for viscous damping that have been reproduced in the technical literature and relabled as shock transmissibility (Ref. 13) or dynamic load factor (Ref. 18) are incorrect. This conclusion can also be reached by considering the fact that, for very high values of viscous damping, the isolator would act as a rigid connection and cause shock transmissibility to be unity for all values of the pulse time duration t_o and the isolation system natural period τ_o, rather than providing increasingly greater degrees of isolation as indicated by the amplification factor curve. Similar erroneous results are obtained when using the generalized excitation and response notation employed by Ayre (Ref. 12); proper distinction between the generalized response parameters for undamped and damped systems has been made by Marous and Schell (Ref. 15, 19).

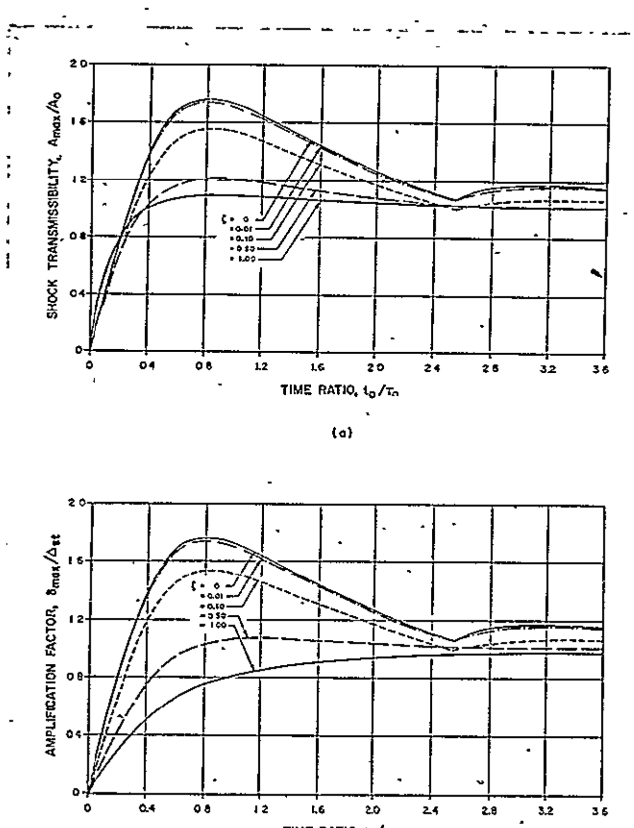

Figure 7.- Effect of isolation system damping for acceleration half-sine shock pulse excitation (after Luke. Ref. 11).

Damped shock spectra for both the shock transmissibility A_{max}/A_o and the amplification factor δ_{max}/Δ_{st} have been presented by Luke (Ref. 11), which correctly indicate the effect of viscous damping on the response for a wide range of shock pulse shapes. The applicable curves for acceleration half-sine shock pulse excitation are presented in Figure 7, where the viscous damping ratio ζ represents the fraction of critical damping.

For velocity shock conditions, the effect of viscous damping is to decrease the maximum acceleration response below that which exists for zero damping as specified by Equation (14), when the value of the viscous damping ratio ζ lies in the range $0 < \zeta < 0.5$ (Ref. 7). The optimum value of the viscous damping ratio for velocity shock is $\zeta_{op} = 0.25$, which provides a maximum acceleration magnitude that is 20 percent less than the corresponding value for zero damping. For values of $\zeta > 0.5$, the effect of damping is to increase the maximum acceleration response beyond that which would exist for zero damping. However, for all values of the viscous damping ratio, the maximum magnitude of relative displacement is decreased below that which exists for zero damping, as specified by Equation (15).

The response of linear isolation systems with directly coupled and elastically coupled viscous damping has been studied for displacement shock pulse excitation (Ref. 20). The effect of nonlinear damping on the shock response of isolation systems is a considerably more complex problem, for which general analytical methods (Ref. 21), graphical techniques (Ref. 10, 12, 22), and results presented in the form of shock spectra (Ref. 23) are available.

RESPONSE TO SUSTAINED ACCELERATION

For conditions of sustained acceleration, the maximum magnitudes of acceleration and relative displacement range from one to two times the values that would result if the level of sustained acceleration were applied statically to the mass. The values of the response maxima are determined solely by the ratio of leading-edge rise time t_r to the isolation system natural period τ_0 for the case of zero damping (Ref. 8, 10, 12). Both the shock transmissibility $T_s = A_{max}/A_s$ and the amplification factor δ_{max}/Δ_{st}, where Δ_{st} is given by Equation (19) with A_0 replaced by A_s, have a value of 2.0 for an instantaneous step. However, for finite leading-edge time durations, these dimensionless parameters always have values less than 2.0, regardless of the shape of the leading-edge characteristic.

The effect of isolation system damping is to decrease the response maxima of the isolation system below the values which exist for zero damping, and provide a faster decay of the transient vibration response. Since the maximum undamped dynamic response to sustained acceleration is only a factor of two times the values applicable for static application of the sustained acceleration, isolation system damping is seen to have appreciably less effect than for the case of steady-state vibration.

EFFECT OF NONLINEAR STIFFNESS

Isolators frequently exhibit nonlinear stiffness characteristics, the nature of which being dependent upon the type of stiffness mechanism employed. Advantage may be taken of isolator nonlinear stiffness characteristics to achieve a more acceptable compromise between the acceleration and relative displacement response maxima than that provided by a linear isolator.

Typical stiffness characteristics of passive isolators are illustrated in Figure 8, in the form of static force-deflection curves. The stiffness coefficient k for a linear isolator is $k = P/\delta$ whereas, for nonlinear isolators, the stiffness is given by $k = dP/d\delta$, which corresponds to the slope of the force-deflection curve at a specified equilibrium position.

Neglecting the effect of damping, the area under the isolator force-deflection curve represents the maximum potential energy stored as a result of an impulsive shock excitation which, in accordance with the conservation of energy, equals the maximum kinetic energy $mV^2/2$ associated with the velocity shock condition.

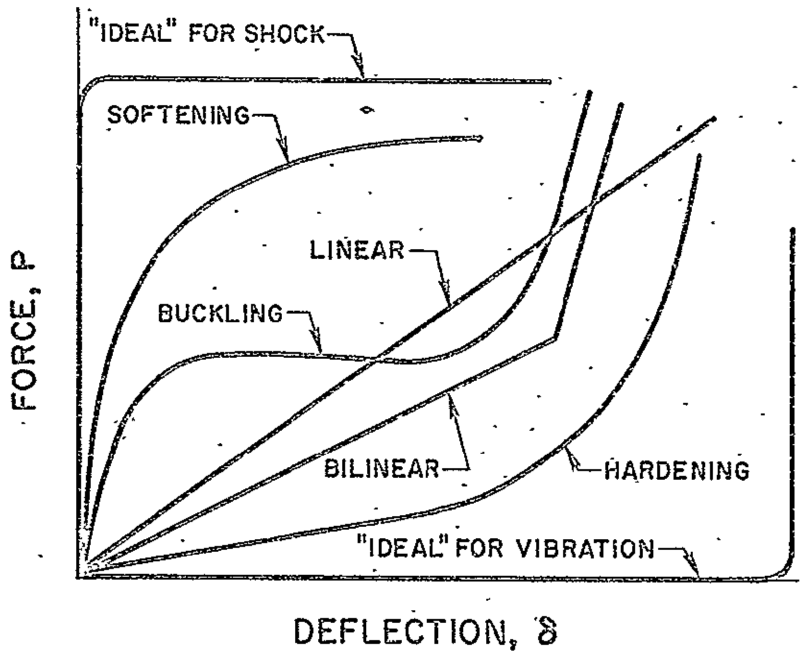

Figure 8.- Force-deflection characteristics of isolators.

The "ideal" stiffness characteristic for shock is one having a constant force for all displacements greater than zero, the magnitude of which is equal to the allowable value of $m\ddot{x}_{max}$. This isolator stiffness characteristic provides maximum storage of energy for a given deflection and, therefore, provides shock isolation with minimum isolator displacement. The ideal shock isolator, however, provides no vibration isolation and may require an external restoring force to return the system to the static equilibrium position.

The "ideal" stiffness characteristics for vibration is one having a constant force of zero magnitude for all allowable displacements. This is achieved physically by having no force transmitting element between the mass and the support foundation of the isolation system. The natural frequency is zero, thereby providing infinite isolation of vibration as long as the displacement is less than the maximum allowable value, at which the stiffness becomes infinite. The ideal vibration isolator, however, provides no shock isolation since it is incapable of storing potential energy; infinite shock amplification will result since the isolator exhibits infinite stiffness when the maximum allowable deflection is reached.

Isolator stiffness characteristics ranging between the "ideal" characteristics for shock and vibration include those identified as hardening, softening, buckling, linear and bilinear. The performance of linear isolation systems has been previously discussed. With regard to achieving optimum performance for a specified velocity change, various types of isolator nonlinear stiffness characteristics may be employed. If relative displacement is the most important response parameter, isolators with hardening or bilinear stiffness characteristics may be employed whereas, if acceleration is the most important response parameter, isolators with a softening stiffness characteristic may be selected. Alternately, a combination of hardening and softening stiffness characteristics (buckling) sometimes may be employed to provide improved isolation performance. The bilinear stiffness characteristic may be used to determine the effects of isolator snubbing or abrupt bottoming (Ref. 7).

The effect of hardening and softening stiffness characteristics on the performance of shock isolation system is demonstrated by the dynamic response time-histories illustrated in Figure 9, which presents the acceleration $\ddot{x}(t)$, velocity $\dot{x}(t)$ and relative displacement $\delta(t)$ for velocity shock excitation of the foundation with the system initially at rest. It is assumed that the isolators are undamped and that polar symmetry exists for the isolator stiffnesses; that is, $P(\delta) = -P(-\delta)$.

The acceleration and relative displacement time-histories for a linear isolator are sinusoidal, with maximum magnitudes given by Equations (14) and (15), respectively. The isolated mass acquires a velocity equal in value to the velocity change after a period of time equal to π/ω_0, when the magnitudes of acceleration and relative displacement response are zero.

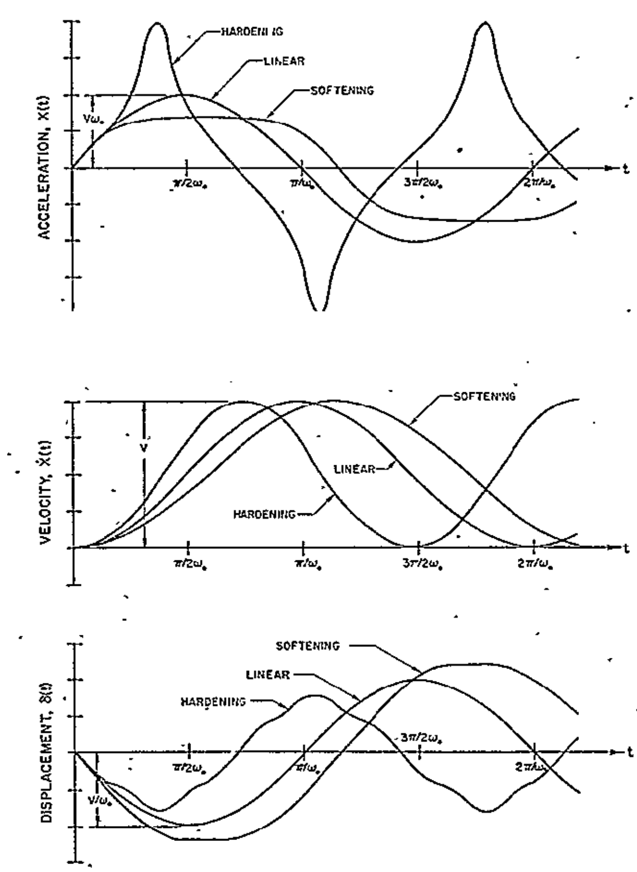

Figure 9.- Isolation system response for velocity shock excitation.

A hardening stiffness characteristic allows a greater magnitude of acceleration to be transmitted with less relative displacement. The isolated mass acquires a velocity equal in value to the velocity change in a shorter period of time. The response curves exhibit a "spike" and indicate a shorter natural period of vibration.

A softening stiffness characteristic allows a lower magnitude of acceleration to be transmitted with more relative displacement. The isolated mass is slower to acquire a velocity equal in value to the velocity change, and the "flattened" response curves indicate an increased natural period of vibration.

The use of nonlinear stiffness characteristics provides considerable flexibility in tailoring isolator designs for specific engineering applications. Basic techniques for analyzing and designing nonlinear shock isolators are available in the technical literature (Ref. 2, 7, 13, 24-27), including the effects of isolator damping (Ref. 23, 28, 29). More advanced analysis and automated design procedures have been recently developed with regard to the problem of mitigating the effects of air, ground and underwater blast shock (Ref. 30-32).

ACTIVE ISOLATION SYSTEMS

Active isolation of shock and vibration involves the application of automatic feedback control techniques to achieve the major performance objectives of a very stiff system for constant applied loads (static force or mass loading and sustained acceleration) and a very soft system for oscillatory dynamic excitations. Active isolation systems are servo-mechanisms comprised of excitation and/or response sensors, sensor signal processors, and actuators. The sensors provide signals proportional to dynamic excitation or response quantities. The signal processors modify and combine sensor signals to create a command signal. And, the actuators apply forces or induce motion in accordance with the command signal.

A wide variety of excitation and response sensors can be employed to provide feedback signals to form a closed loop control system. For example, feedback signals can be developed which are a function of jerk, acceleration, velocity, displacement, integral displacement, differential pressure or force. The signal processor may consist of an active electronic network which performs amplification, attenuation, differentiation, integration, addition, and compensation functions. Alternately, the signal processor can be in the form of a simple lever mechanism.

Power is required to operate the active isolator mechanisms and, in some cases, power is also required for the signal processor. The requirement of externally supplied power is the primary distinguishing characteristic between active and passive isolation systems. A major consideration in the design of active isolation systems is the achievement of an adequate margin of stability while providing a high speed of response.

Analytical studies and engineering applications of active isolation systems that are reported in the technical literature include those that employ electrically or mechanically activated pneumatic actuators (Ref. 33-43), force servos in conjunction with conventional passive isolators (Ref. 44-46), position servos in conjunction with levered passive isolators (Ref. 47), electrically activated fluid dampers in conjunction with conventional passive isolators (Ref. 48), and electrically or mechanically activated hydraulic actuators (Ref. 43, 49-54). To the author's knowledge, of all the active isolator mechanisms studied over the past decade, only the mechano-pneumatic isolator (Ref. 33, 34, 37-42), and the electro-hydraulic isolator (Ref. 52) have been reduced to a practical reality in terms of achievement of stable operational systems. Consequently, the remainder of the discussion of active isolation will center on these two active isolator mechanisms.

MECHANO-PNEUMATIC ISOLATION SYSTEMS

A schematic diagram of a mechano-pneumatic isolation system is shown in Figure 10, where mechanical displacement feedback controls the flow of a compressible gas to and from a double-acting actuator through a servovalve. The effect of displacement feedback actuation of the valve spool is to make the actuator output force a function of the time-integral of relative displacement (Ref. 35). The integral displacement control operates in a fashion such that no static deflection results from mass loading, and deflection during sustained acceleration conditions are initially reduced and eventually eliminated.

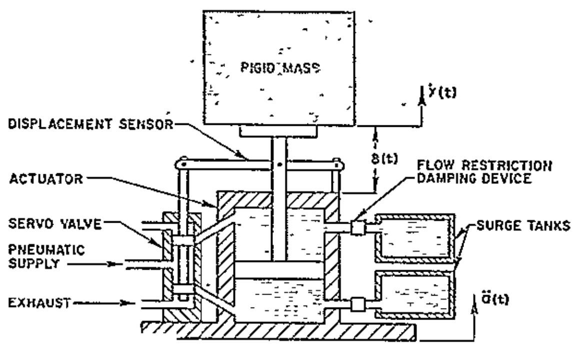

Figure 10.- Schematic diagram of a mechano-pneumatic isolation system.

Integral displacement control can be designed to be effective only at extremely low frequencies so as not to materially affect the isolation system natural frequency. Isolation is then provided essentially in accordance with the low stiffness of the passive pneumatic actuator, with damping provided by use of an external damping mechanism or by use of a flow-restriction damping device inserted between the pneumatic actuator and surge tank, as illustrated in Figure 10. The use of surge tanks is advantageous inasmuch as they allow lower natural frequencies to be achieved, provide excellent resonant vibration control, and offer a 12 db/octave high-frequency attenuation rate regardless of the degree of isolator damping employed (Ref. 35, 55).

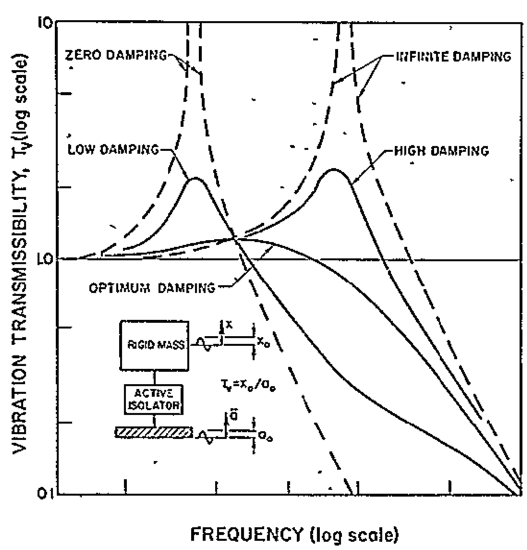

Figure 11.- Harmonic vibration response of mechano-pneumatic isolation system.

The vibration transmissibility of the mechano-pneumatic isolation system with surge tanks is shown in Figure 11. Zero and infinite damping are achieved by providing no restriction and infinite restriction, respectively, of the cyclic flow of gas between the actuator and the surge tanks. An optimum degree of damping is required to minimize the resonant transmissibility of the isolation system. However, optimum damping is not of a critical nature since a fairly large deviation from optimum damping results in a relatively small increase in resonant transmissibility. The vibration isolation characteristics of the mechano-pneumatic isolation system shown in Figure 10, when a capillary flow-restriction device is employed, are similar to those exhibited by an isolation system employing an elastically coupled viscous damper (Ref. 56).

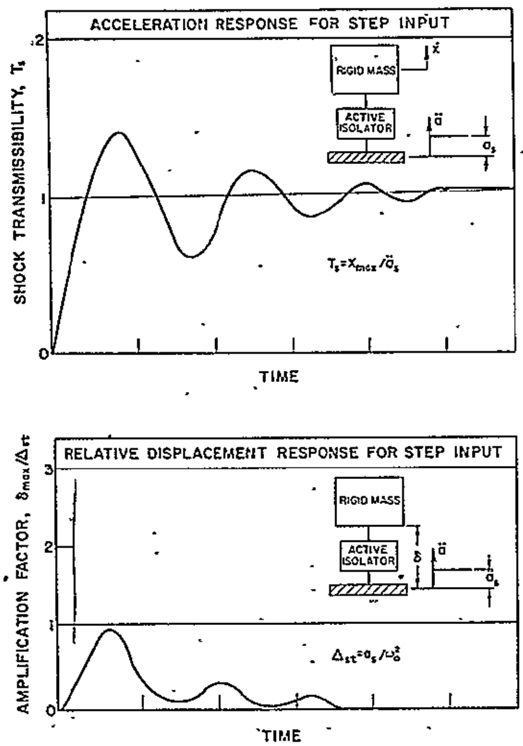

Figure 12.- Transient response of mechano-pneumatic isolation system for acceleration step input.

Typical transient response characteristics of the mechano-pneumatic isolation system subjected to acceleration step excitation are shown in Figure 12. The response is essentially that of a conventional passive system with an equal natural frequency, except that the magnitude of relative displacement is substantially reduced and eventually becomes zero (Ref. 35). Consequently, because of the integral displacement control, the isolated body ultimately is returned to its neutral position, and isolation of vibration can be provided during the sustained acceleration excitation condition.

Having eliminated the static deflection and reduced the dynamic displacements under shock by use of integral displacement control, it becomes possible to construct mechano-pneumatic isolation systems having natural frequencies less than conventional passive systems. For example, natural frequencies somewhat less than 1 Hz can be provided in practical installations. However, because of the compressibility of the gas and the necessity of having relatively low-gain in the feedback loop to ensure system stability, the speed of response of the mechano-pneumatic isolation system is relatively slow.

ELECTRO-HYDRAULIC ISOLATION SYSTEMS

A schematic diagram of an electro-hydraulic isolation system is shown in Figure 13, where multiple electronic feedback signals are processed through a servoamplifier to create a command signal that controls the flow of a relatively incompressible fluid to and from a cylinder through a servovalve. Sensors are employed to provide acceleration and relative displacement feedback signals, which are modified in the servoamplifier such that the flow through the valve is made a function of acceleration, relative velocity, relative

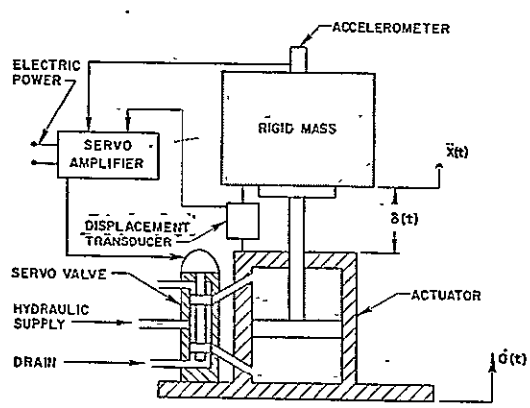

Figure 13.- Schematic diagram of an electro-hydraulic isolation system.

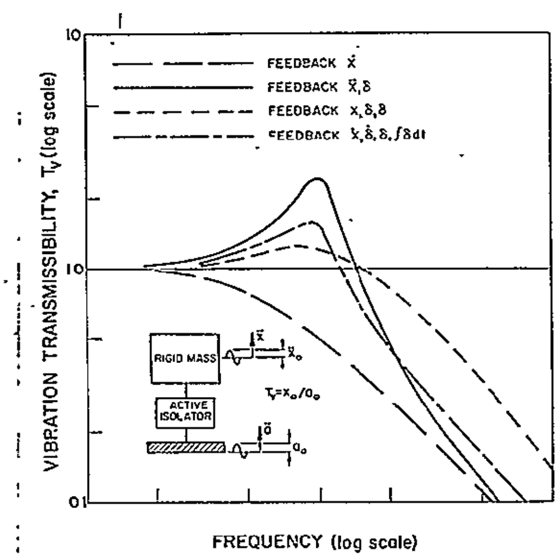

Figure 14.- Low-frequency harmonic vibration response of electro-hydraulic isolation system.

displacement, and the time-integral of relative displacement, where the effect of each feedback parameter is independently controlled by adjustable loop gains.

The stiffness of the passive hydraulic actuator is high, which provides a high natural frequency system in the open loop. Upon closing the loop, extremely low natural frequencies can be provided, at values substantially lower than 1 Hz, since the isolation system resonance is created electronically. The typical vibration transmissibility characteristics of the electro-hydraulic isolation system with various combinations of feedback parameters shown in Figure 14 are applicable in the low-frequency region where the effect of hydraulic component dynamics is negligible. Pure acceleration ($\ddot{x}$) feedback provides a resonance-free transmissibility characteristic that is a unit lag function $1/(\tau_a^2 \omega^2 + 1)$ where the time constant $\tau_a = C_a/A_p$, C_a is the acceleration feedback gain, and A_p is the effective actuator piston area. The combination of acceleration ($\ddot{x}$) and relative displacement (δ) feedback provides a response identical to a conventional viscous damped isolation system (Ref. 53), in accordance with solutions to the governing equation of motion given by Equation (12). In this case, the natural frequency is given by

$$f_o = \frac{\omega_o}{2\pi} = \frac{1}{2\pi} \sqrt{C_d/C_a} \tag{20}$$

and the effective viscous damping ratio is

$$\zeta = \frac{A_p}{2\sqrt{C_a C_d}} \tag{21}$$

where C_d is the relative displacement gain. Relative velocity ($\dot{\delta}$) and integral displacement ($\int \delta dt$) feedback can be introduced to provide additional effective

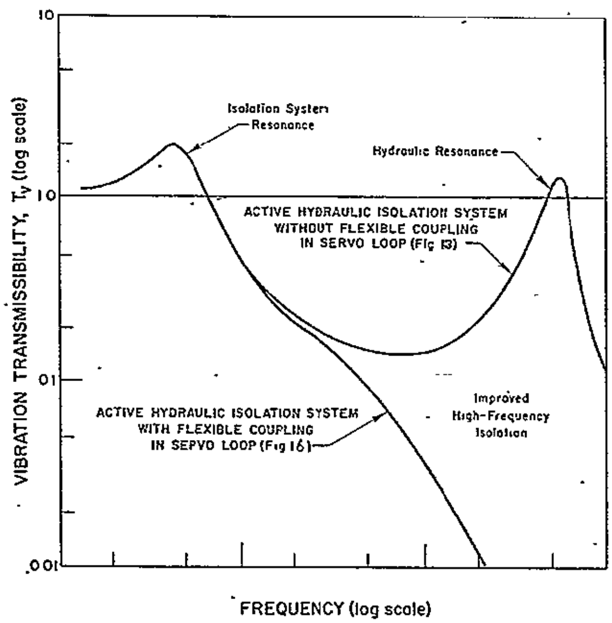

Figure 15.- Broad-band harmonic vibration response of electro-hydraulic isolation system.

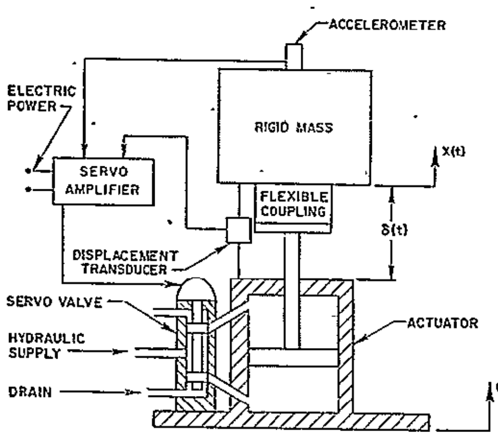

Figure 16.- Schematic diagram of an electro-hydraulic isolation system with passive isolator (flexible coupling) in the servo loop.

damping at resonance and displacement control, respectively, without materially affecting the high-frequency isolation characteristics.

The electro-hydraulic isolation system shown in Figure 13 is effective over a limited frequency bandwidth. Degradation of high-frequency vibration isolation occurs because of the effects of hydraulic resonances, which are determined by the dynamic characteristics of the hydraulic components and the electronic networks. This is illustrated in Figure 15, which indicates a low isolation system natural frequency and a hydraulic resonance in the high-frequency region. The transmissibility increases to a value of unity and greater, and exhibits isolation characteristics associated with a passive isolation system having a natural frequency associated with the hydraulic resonance condition.

Improved high-frequency isolation can be provided, however, by introducing a suitable passive isolator in the form of a flexible coupling in the servo loop, as illustrated in Figure 16. The effect of the flexible coupling is to provide mechanical compensation which results in reshaping the high-frequency response characteristics. As indicated by the transmissibility curves shown in Figure 15, broad-band isolation is provided with the combined active/passive isolation system. Isolation of vibration is provided by active means in the low-frequency region where the passive isolator has a unity transfer function, while the passive isolator provides isolation of high-frequency vibration, where the isolated body is effectively decoupled from the hydraulic actuator and the excitation frequency is beyond the frequency band over which the active control system is operative.

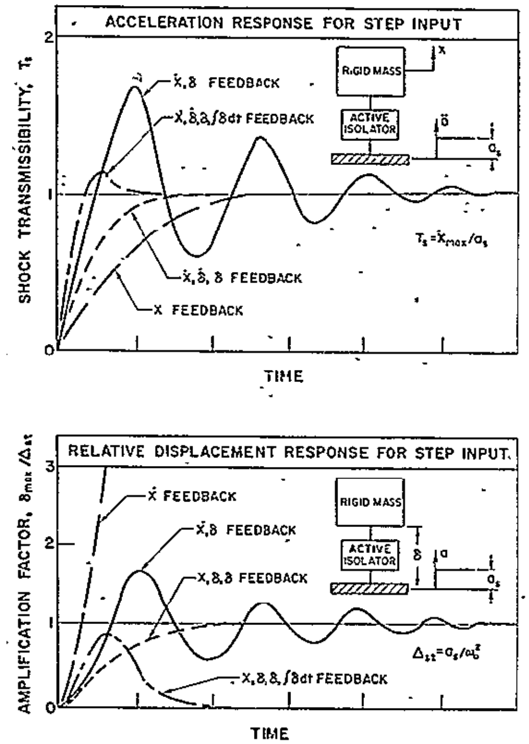

Figure 17.- Transient response of electro-hydraulic isolation system for acceleration step input.

Typical transient response characteristics of the electro-hydraulic isolation system subjected to acceleration step excitation are shown in Figure 17. Pure acceleration feedback ($\ddot{x}$) provides excellent control of acceleration, but requires an infinite relative displacement. As discussed previously, the combination of acceleration ($\ddot{x}$) and relative displacement feedback (δ) provides performance comparable to a conventional passive isolation system, with the capability of electronically creating an extremely low natural frequency. By adding velocity and integral displacement feedback, excellent control of both acceleration and relative displacement response is achieved. There is little overshoot in acceleration, and the system exhibits high speed response in eliminating the relative displacement. Consequently, because of the wide selection of feedback signals and loop gains available and the relative incompressibility of the hydraulic fluid, ultra low-frequency isolation can be provided by the electro-hydraulic isolation system even during conditions of sustained acceleration, with zero static, high speed of response and extreme flexibility in shaping the overall frequency response characteristics.

NONLINEAR STIFFNESS AND FEEDBACK EFFECTS

In response to shock excitation, large dynamic displacements of linear active isolation systems may occur because of the extremely low natural frequencies that can be achieved. Consequently, hardening effective stiffness characteristics are generally desirable, as concluded from the previous discussion of nonlinear passive isolation systems.

This is accomplished automatically in the mechano-pneumatic isolation system, since the isolator stiffness has a hardening characteristic that is described approximately by a quadratic force-deflection characteristic. For the electro-hydraulic isolation system, nonlinear electronic circuits can be introduced into the displacement feedback loop to substantially increase the loop gain for relative displacements exceeding an established linear range of operation (Ref. 52). This has the effect of providing a hardening stiffness characteristic with greatly increased feedback control operating to rapidly reposition the isolator in its region of linear operation.

The hardening stiffness and feedback effects of the active isolation systems result in a higher acceleration being transmitted to the isolated body during severe shock excitation. In the case of the electro-hydraulic isolation system, however, the variety of feedback parameters provides considerably more flexibility than is available with passive isolation systems in achieving an acceptable compromise between the acceleration and relative displacement response maxima.

RANGES OF PRACTICAL APPLICATION

A summary of the ranges of practical application of passive and active isolation systems, in terms of the natural frequency that can be achieved in practical installations, is presented in Figure 18 based on experience with actual hardware systems. The range of equivalent passive static deflection associated with the natural frequencies included in the comparison are also presented.

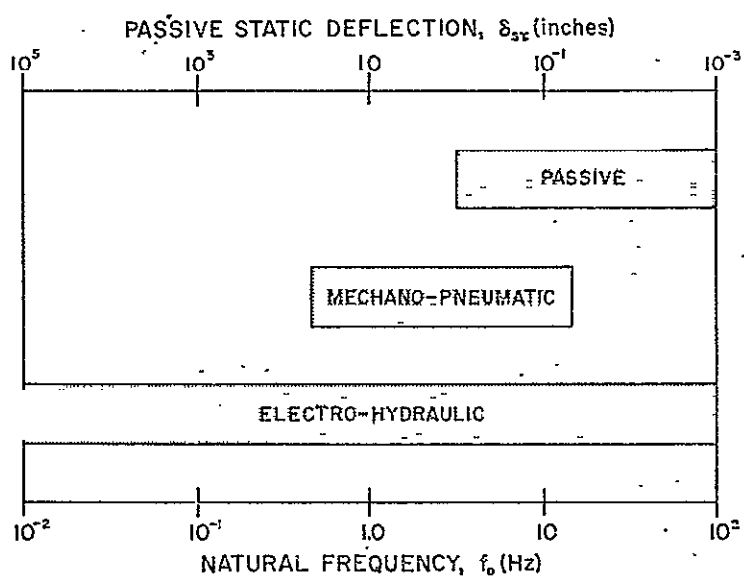

Figure 18.- Range of practical natural frequencies achievable with passive and active isolation systems

Passive isolators provide natural frequencies nominally in the range of 3 Hz and above. Mechano-pneumatic isolators have provided natural frequencies ranging from 0.7 Hz to 20 Hz in practical installations. Electro-hydraulic isolators have been constructed which exhibit natural frequencies of 0.01 Hz, where comparable passive isolators would have static deflections in the order of 8,000 feet.* The mechano-pneumatic isolation systems tend to become large as the natural frequency approaches the lower limit, whereas the size of the electro-hydraulic isolation system with nonlinear displacement feedback is practically unvariant with natural frequency, but would generally represent the more complex and higher cost active isolator mechanism.

OPTIMUM ISOLATION ANALYSIS AND SYNTHESIS

While the concept of optimal design has been employed successfully with regard to the optimization of parameters such as weight, stiffness and energy storage in mechanical systems and electronic networks in modern control systems, it is only recently that techniques have been developed and applied to the optimization of the time-dependent response of shock and vibration isolation systems (Ref. 45, 57-63). A typical problem in optimizing the design of a shock isolation system is that of minimizing the maximum acceleration response for a specified allowable relative displacement. Alternately, the problem can be stated as minimizing the maximum relative displacement for a specified allowable acceleration response. Actually, these two formulations of the optimal design problem for shock isolation systems are reciprocal in that they both lead to the same minimax solutions (Ref. 60, 62).

In addition to the constraints placed on maximum acceleration or relative displacement response, the performance criteria may include considerations such as weight, strength, stiffness, size, cost, etc. The performance criteria is frequently expressed in terms of a penalty function (Ref. 45), merit function (Ref. 57), index of shock severity (Ref. 59), performance index (Ref. 63), or objective function (Ref. 64), which is to be minimized.

The following discussion is intended to distinguish between optimum isolation analysis and synthesis, as represented in the functional block diagram presented in Figure 19.

OPTIMUM SHOCK ISOLATION ANALYSIS

The process of optimum shock isolation analysis, as represented in Figure 19(a), involves imposing a specified shock excitation and a performance

* Reference 52 provides experimental data on a broad-band electro-hydraulic isolation system having a nominal natural frequency of 0.45 Hz, with an effective passive static deflection of approximately four feet. Data for the ultra-low natural frequencies cited have been collected in corporate research programs under the direction of the author, and will be published in the near future.

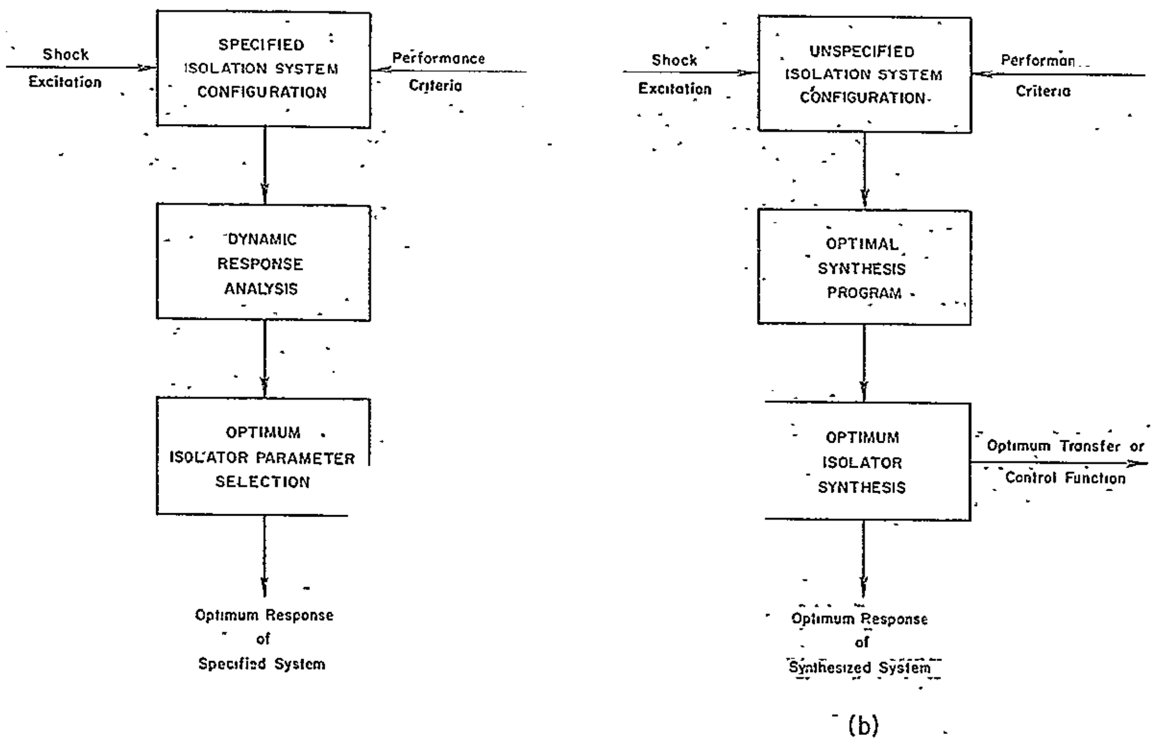

Figure 19.- Functional block diagram of optimum shock isolation (a) analysis and (b) synthesis.

criteria on an isolation system of a specified configuration having a known transfer response function. A standard dynamic response analysis is performed by writing the equations of motion and other applicable system performance equations. A parametric variation analysis is then conducted, using variational calculus, analog computer or digital computer search techniques, to arrive at the selection of optimum isolator parameter values that provide the optimum response of the specified isolation system configuration. Shock spectra for wide variations in isolation system parameters provides a relatively simple graphical means of achieving an optimum design with regard to establishing the best compromise between the acceleration and relative displacement response maxima. Such analyses provide an optimum design within the capabilities of the specified system; however, they do not indicate whether some other system could have provided even better performance, or the magnitude of the potential margin of improvement.

OPTIMUM SHOCK ISOLATION SYNTHESIS

The process of optimum shock isolation synthesis, as represented in Figure 19(b), involves imposing a specified shock excitation and a performance criteria on an isolation system of an unspecified configuration. The specified shock excitation and performance criteria are then processed in an optimal synthesis digital program, which typically involves variational calculus, gradient analyses, and linear, nonlinear, or dynamic programming computational methods (Ref. 45, 57-65). The computation results in the optimum isolator synthesis, as expressed by the optimum response of the synthesized system, and either the optimum transfer function or the optimum control function, depending upon the performance criteria specified and the computational method employed.

The form of the optimum transfer or control functions provides an indication of the active isolator mechanism (type of feedback, compensation, component dynamics, etc.) that is required to provide the optimum response. Mechanization of the synthesized system may be impossible, but determination of the optimum response of the synthesized system provides a statement of the absolute optimum performance, against which the performance of an imperfectly mechanized (but practically realizable) isolator or that of any other specified isolator configuration can be compared.

CONCLUSIONS

This review of the state of the art of shock isolation indicates that passive shock isolation techniques are developed to a relatively high level of refinement. The active pneumatic and hydraulic shock isolation techniques that have been reduced to a practical reality, and the optimum shock isolation synthesis techniques currently being studied, represent promising solutions to the more difficult problems of shock mitigation. Problem areas in which additional research should be conducted include:

(1) the effect of damping on the shock response of passive isolation systems, particularly with regard to acceleration shock pulse excitation and elastically coupled linear and nonlinear damping mechanisms;

(2) the performance of practical active shock isolation systems, such as those employing mechano-pneumatic and electro-hydraulic isolator mechanisms, for impulse and acceleration shock pulse excitation and various combinations of feedback parameters;

(3) the absolute optimum response and associated transfer or control functions of optimum synthesized shock isolation systems for classes of shock excitation of interest, including the sensitivity of the optimum solution to changes in the character of the shock excitation;

(4) the relative advantage of practical passive and active shock isolation systems tailored to provide performance approaching that of the absolute optimum synthesized response.

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