NASA TECH BRIEF

Derivation of a General Perturbation Solution: Its Application to Determination of Orbit

An analytical solution to three-body problems has been applied to the problem of predicting the orbit of a lunar satellite and determining the orbit of a near-Earth satellite. Using this solution, the state vector may be generated at any desired time without intermediate numerical extrapolation.

General perturbation theory is concerned with the development of analytical solutions to the equations of motion of a satellite moving under the influence of an arbitrary gravitational force. A first-order general perturbation solution has been developed for the motion of a satellite under the influence of an arbitrarily shaped primary body and a point-mass third body. A set of nonsingular orbit elements is used to describe the satellite's motion; thus the results are valid for all circular and elliptic motion. The solutions utilize the development of the disturbing function in terms of the Keplerian elements as given by Kaula. They are obtained by an extension of Ingram's work to include the third-body effects.

The general perturbation solutions are then incorporated into an orbit-determination scheme. The result is a rapid, accurate program that is valuable for preliminary design work. Its value as a research tool is demonstrated by its use in studying several problems associated with orbit determination.

A modified first-order general perturbation solution is developed for the motion of a point-mass satellite under the influence of an arbitrarily shaped primary body and a point-mass third body. Results yielded by these solutions are compared with numerically integrated trajectories for various orbital configurations. It is shown that the analytical solutions are accurate and are amenable to computer evaluation.

Linear estimation theory is reviewed, and a technique for recursive estimation of the observation-error covariance matrix is developed. The analytical solutions are used as the basis for an orbit-determination scheme that compares favorably with an orbit-determination program using numerical integration. The accuracy of transition matrices, formed by numeric partial differentiation, is studied briefly for justification of their use in the analytical program. The effects of the coordinates used in the estimation process are compared numerically, using both Keplerian elements and Cartesian coordinates to determine the orbit of a near-Earth satellite. In addition, the recursive scheme for estimating the radar covariance matrix is tested using the same sample problem.

For the numerical studies, the problem of estimating the orbit of a near-Earth satellite under the influence of the gravitational harmonics J_{20}, J_{30}, and J_{40} was considered. Data on range, azimuth, and elevation, as observed from four tracking stations, were simulated and corrupted with normally distributed random noise.

Note:

Requests for further information may be directed to:

Technology Utilization Officer
Manned Spacecraft Center, Code BM7
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(continued overleaf)

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No patent action is contemplated by NASA.

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