November 1970 Brief 70-10446

NASA TECH BRIEF

Error Compensation for Hybrid-Computer Solution of Linear Differential Equations

The problem:

Present methods for hybrid-computer solution of linear differential equations lack compensation for digital transport delay and digital-to-analog hold. Previous methods for compensating transport and hold errors extrapolated the variable involved using a multi-step scheme. The distance extrapolated is usually 1.5 time steps, based on a graphical argument that introduces additional extraneous terms in the hybrid solution, one per term in the extrapolation formula. Such extraneous terms may degrade both accuracy and stability in the hybrid solution.

Proposed solution:

A simple and straightforward Z-transform technique determines the best values for the compensation constants, both in multi-step and Taylor series projections. The technique also provides hybrid-calculation error compared to a continuous exact solution, plus system stability properties.

How it's done:

First, convert the differential equations to difference equations by solving them from one sampling instant to the next. The digital variables, sent from the digital to the analog, are held fixed between sampling periods. The difference equations relate the values at one sampling instant to those at the previous instant. Z-transform of these difference equations yield equations for the Z-transform of each dependent variable in terms of the digital variable. Finally, close the system by expressing the digital variable in terms of the dependent variable, using either a 3-term multi-step extrapolation of the digital variable, or Taylor series expansions of the dependent variables. In the latter case, values available determine expansion length. For example, in a second order equation for $x(t)$, expand x by 3 terms through the $\ddot{x}$ term. Expand $\dot{x}$ by two terms, also through $\ddot{x}$. Take the Z-transform of this relation for the digital variable.

Then solve the Z-transformed system for the dependent variables algebraically. Perform inversion by finding the poles of the Z-transforms as power series in the sampling time. This results in solutions in the form of the same type of power series, arising from the residues at these poles. The compensation constants, which are coefficients of either the multi-step extrapolation formula or the Taylor series, appear in the solution power series and are selected to have the desired effect on solution accuracy and stability.

Note:

Requests for further information may be directed to:

Technology Utilization Officer
Headquarters
National Aeronautics
and Space Administration
Washington, D.C. 20546
Reference: TSP70-10446

Patent status:

No patent action is contemplated by NASA.

Source: Nelson H. Kemp of
Wolf Research and Development Corp.
under contract to
Electronics Research Center
(ERC-10262)

Category 09