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NON-LINEAR LIMB-DARKENING FOR EARLY TYPE STARS

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by

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ABSTRACT

A set of coefficients has been obtained by fitting an empirical non-linear limb-darkening law of the form

$$\frac{I(\mu)}{I(1)} = 1 - A_\lambda(1-\mu) - B_\lambda\, \mu \log \mu$$

to values of $I(\mu)/I(1)$ computed from a grid of hydrogen line-blanketed model atmospheres. In addition, a set of coefficients for the conventional linear law has been calculated. The comparison with previous theoretical values for the linearized law shows good agreement with Grygar's (1965) results but systematically smaller values than those found by Hosokawa (1967).

I. INTRODUCTION

A number of detailed model atmospheres have been re-analyzed by Grygar (1965) to derive improved limb-darkening coefficients for early type stars. He noted that, although models from several different authors were used, good internal consistency existed among the separate determinations of the coefficients. A tabulation of mean coefficients is given for wavelengths ranging from λ1000Å to 9000Å. This tabulation is restricted to surface gravities with log (g) = 4.0 but additional coefficients from three of Strom's (1964) log(g) = 3.0 models are also depicted. More recently, Hosokawa (1967) has determined the darkening coefficients for a wider range of effective temperatures by analyzing the model atmospheres of Strom and Avrett, (1966) for early type stars and of Gingerich

(1966) for cooler stars. Again the determinations are only for log (g) = 4.0. This work differs from Grygar's in that the linearized darkening coefficient is defined by a flux equivalent relation rather than a least squares representation with unequal weights. We shall adopt Hosokawa's definition for u_1 and our results will be directly comparable.

It is well known that except for the case of the simple gray atmosphere solution, the direct respresentation of the emergent intensity distribution is markedly non-linear. To obtain observational confirmation of this relation by the study of eclipsing binaries is difficult. Nevertheless, Grygar (1963) found that the photometric solution for the eclipsing system AR Aurigae was improved, as indicated by a reduction in the residuals, when a non-linear darkening law was applied. This is not to say that solutions for all types of eclipsing binaries can be improved by adopting a non-linear law. Most likely those systems which exhibit photometric (and spectrographic) complications, large outside eclipse variations, or are comprised of tidally or rotationally distorted components will not respond to this elaboration. Conversely, it cannot be expected that these systems will provide observational verification of the computed coefficients.

In this present work a set of coefficients appropriate to the linear law of darkening (Hosokawa, 1967) and a set for an empirical non-linear law are calculated for early type stars. The latter set of coefficients is suitable for application in direct computer analysis of binary systems. Finally, the newly computed coefficients are compared both with previous theoretical values and with observed values.

II. MODEL CALCULATIONS

A grid of flux corrected radiative and hydrostatic equilibrium model atmospheres with hydrogen line-blanketing has been used to compute $I_\lambda(\mu)$ at wavelengths of astrophysical interest. The models cover the range 10000K $\leq T_{eff} \leq$ 40000K;

$2.5 \leq \log g \leq 4.5$ and have a composition (by mass) of $X = 2/3$, $Y = 1/3$. Flux is constant to within a few tenths of a percent. The opacity sources included are the bound-free and the free-free transitions of hydrogen, helium and their ions both positive and negative (Mihalas, 1965; Vardya, 1963; Geltman, 1962; Fischel, 1963 and McDowell, et al., 1966), electron scattering, and the bound-bound transitions of the Lyman and Balmer series of hydrogen (Griem, 1960 and Underhill, 1962). These models are currently being prepared for publication. No metal line blanketing has been included in these calculations hence, the results in the far UV must be considered of lower weight than those in the near UV and visible.

The emergent intensity as a function of μ (Chandrasekhar, 1950) is defined as

$$I_\lambda(\mu, \tau = 0) = \int_0^\infty S_\lambda(t_\lambda) e^{-t_\lambda/\mu} \frac{dt_\lambda}{\mu} \qquad (1)$$

where μ is the direction cosine of the line of sight to the surface normal and $S_\lambda(t_\lambda)$ is the monochromatic source function including the scattering term (Kourganoff, 1963). A ten point Gauss Laguerre quadrature formula was used to evaluate equation 1 in which $x = t_\lambda/\mu$ was taken as the independent variable. In order to verify the accuracy of the calculations, values of μ were chosen to permit a Gaussian integration of the moment equation

$$F_\lambda(\tau_\lambda = 0) = 2 \int_0^1 I_\lambda(\mu) \, \mu \, d\mu \qquad (2)$$

which could then be compared with the net emergent flux, F_λ (Kourganoff, 1963) computed directly from

$$F_\lambda(\tau_\lambda = 0) = 2 \int_0^\infty S_\lambda(t_\lambda) E_2(t_\lambda) dt_\lambda \qquad (3)$$

In all cases the difference between the emergent flux as computed by equations 2 and 3 is less than 1/2%.

A linearized representation of the emergent intensity as a function of μ is commonly assumed. For this present work, Hosokawa's (1967) relation defining the linear coefficient u_1 is adopted. The coefficient is found from the equivalence relation

$$\int_0^1 \frac{I(\mu)}{I(1)} \mu d\mu = \int_0^1 (1 - u_1 + u_1 \mu) \mu d\mu \qquad (4)$$

which after integration yields

$$u_1 = 3-6 \int_0^1 \frac{I(\mu)}{I(1)} \mu d\mu . \qquad (5)$$

In addition to the linearized form, the following empirical equations which express the intrinsic non-linearity of the emergent intensity with μ were assumed. The first can be recognized as that due to van't Veer (1960).

$$\text{I:} \quad I_\lambda(\mu)/I_\lambda(1) = 1 - A_\lambda(1-\mu) - B_\lambda(1-\mu)^3$$

$$\text{II:} \quad I_\lambda(\mu)/I_\lambda(1) = 1 - A_\lambda(1-\mu) - B_\lambda \mu \log_{10} \mu$$

The coefficients, A_λ and B_λ, were found by the method of least squares. Probable errors ranged from 0.001 to 0.01 with consistently smaller errors obtaining with the second empirical law. Tables 1 to 5 list the values of A_λ, B_λ pertaining to the second law and the linear coefficients u_1 for each wavelength and each model. Separate evaluations were made at each surface gravity. Effective temperature varies across the table and wavelength varies down the table.

III. DISCUSSION

It can be seen from the tabulated results that the limb darkening coefficients decrease smoothly with increasing surface gravity for all wavelengths.
The magnitude of the effect is rather small. On the other hand, the u_1 values vary markedly with temperature and wavelength in a manner similar to that found by previous investigators. It should be noted that the values of u_1 at λ = 3362Å deviate from an apparent smooth wavelength relation. At this particular wavelength, the wings of the Balmer hydrogen lines alter the continuum opacity and produce the anamolous result. At the present time, very few high quality limb

darkening determinations for early type stars are available (e.g., refer to Wood (1963)). The general tendency that the observed limb darkening coefficients are larger than the computed is strengthened by these present results.

Although the coefficients are not tabulated here, several models were solved to obtain linearized limb darkening coefficients in the manner described by Grygar. Excellent agreement between the two sets of coefficients was found indicating an equivalence for the respective model atmospheres used in the determination. However, a comparative study with Hosokawa's tabulation of mean limb darkening coefficients, where identical definitions for the linearization law were assumed, shows a systematic difference in the coefficients nearly independent of wavelength. The sense of the difference is that the present coefficients are of the order of .04 units smaller than Hosokawa's. This difference is probably not significant for the inverse observational problem.

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TABLE 1

LINEAR AND NON-LINEAR COEFFICIENTS OF LIMB DARKENING

LOG (G) = 4.0

T_{EFF} (°K)	10000			12000			14000			16000			18000		
λ (Å)	A_λ	B_λ	V_1	A_λ	B_λ	V_1	A_λ	B_λ	V_1	A_λ	B_λ	V_1	A_λ	B_λ	V_1
2000	0.516	-0.327	1.012	0.504	0.177	0.853	0.821	0.388	0.769	0.851	0.513	0.703	0.831	0.630	0.681
2500	0.611	0.219	0.752	0.782	0.413	0.666	0.757	0.524	0.609	0.727	0.581	0.563	0.712	0.652	0.520
3000	0.687	0.447	0.564	0.656	0.515	0.513	0.635	0.556	0.481	0.608	0.573	0.450	0.603	0.622	0.430
3647	0.549	0.540	0.358	0.522	0.537	0.373	0.507	0.540	0.356	0.450	0.552	0.338	0.452	0.555	0.338
3662	0.652	0.732	0.447	0.622	0.783	0.403	0.553	0.780	0.375	0.565	0.759	0.353	0.554	0.780	0.336
4400	0.689	0.639	0.452	0.627	0.796	0.405	0.590	0.778	0.373	0.558	0.749	0.349	0.541	0.761	0.329
5500	0.577	0.707	0.378	0.522	0.665	0.331	0.491	0.663	0.306	0.462	0.625	0.288	0.451	0.634	0.275
7000	0.452	0.574	0.253	0.412	0.556	0.258	0.388	0.534	0.240	0.366	0.511	0.225	0.361	0.518	0.218
8200	0.382	0.507	0.242	0.350	0.485	0.216	0.331	0.468	0.202	0.313	0.449	0.189	0.307	0.429	0.188
8400	0.444	0.619	0.271	0.359	0.579	0.236	0.375	0.549	0.223	0.352	0.515	0.210	0.346	0.517	0.203
8800	0.425	0.590	0.260	0.382	0.557	0.228	0.359	0.527	0.214	0.338	0.496	0.201	0.332	0.457	0.194
12500	0.304	0.440	0.183	0.278	0.415	0.163	0.262	0.397	0.153	0.246	0.381	0.142	0.238	0.349	0.141

T_{EFF} (°K)	20000			25000			30000			35000			40000		
λ (Å)	A_λ	B_λ	V_1	A_λ	B_λ	V_1	A_λ	B_λ	V_1	A_λ	B_λ	V_1	A_λ	B_λ	V_1
2000	0.802	0.653	0.616	0.755	0.738	0.545	0.665	0.741	0.476	0.584	0.792	0.361	0.502	0.718	0.302
2500	0.686	0.671	0.457	0.657	0.758	0.445	0.556	0.702	0.399	0.517	0.705	0.320	0.450	0.666	0.260
3000	0.581	0.625	0.407	0.572	0.710	0.375	0.517	0.650	0.336	0.453	0.638	0.273	0.388	0.627	0.216
3647	0.487	0.581	0.326	0.469	0.655	0.309	0.441	0.589	0.278	0.366	0.566	0.229	0.329	0.565	0.177
3662	0.534	0.757	0.323	0.515	0.784	0.297	0.479	0.715	0.281	0.430	0.659	0.237	0.376	0.670	0.192
4400	0.521	0.739	0.315	0.455	0.744	0.289	0.456	0.672	0.270	0.403	0.654	0.222	0.345	0.618	0.176
5500	0.433	0.617	0.262	0.422	0.637	0.247	0.385	0.585	0.224	0.338	0.554	0.165	0.283	0.517	0.142
7000	0.345	0.453	0.208	0.351	0.553	0.199	0.316	0.493	0.161	0.273	0.447	0.151	0.228	0.422	0.114
8200	0.298	0.426	0.181	0.313	0.456	0.177	0.260	0.440	0.159	0.239	0.389	0.132	0.156	0.363	0.098
8400	0.332	0.501	0.194	0.331	0.542	0.163	0.298	0.480	0.166	0.255	0.427	0.138	0.208	0.390	0.102
8800	0.322	0.454	0.166	0.320	0.526	0.176	0.287	0.463	0.160	0.244	0.408	0.132	0.158	0.373	0.098
12500	0.241	0.388	0.135	0.248	0.420	0.133	0.217	0.354	0.120	0.175	0.266	0.097	0.140	0.263	0.069

TABLE 2

LINEAR AND NON-LINEAR COEFFICIENTS OF LIMB DARKENING

LOG (G) = 4.0

T_{EFF} (°K)	10000			12000			14000			16000			18000		
λ(Å)	A_λ	B_λ	V_1	A_λ	B_λ	V_1	A_λ	B_λ	V_1	A_λ	B_λ	V_1	A_λ	B_λ	V_1
2000	0.515	-0.308	1.004	0.855	0.145	0.852	0.870	0.339	0.771	0.828	0.420	0.705	0.816	0.570	0.655
2500	0.807	0.194	0.754	0.770	0.366	0.666	0.746	0.478	0.611	0.697	0.486	0.559	0.704	0.604	0.531
3000	0.677	0.402	0.566	0.639	0.461	0.511	0.621	0.505	0.460	0.572	0.479	0.439	0.550	0.562	0.432
3647	0.530	0.460	0.401	0.459	0.475	0.367	0.489	0.493	0.352	0.451	0.475	0.321	0.472	0.475	0.340
3862	0.652	0.781	0.472	0.646	0.790	0.424	0.614	0.781	0.358	0.578	0.728	0.373	0.576	0.756	0.354
4400	0.654	0.846	0.455	0.631	0.785	0.411	0.555	0.765	0.380	0.554	0.705	0.356	0.550	0.760	0.337
5500	0.581	0.724	0.378	0.623	0.676	0.334	0.452	0.652	0.310	0.451	0.578	0.290	0.456	0.628	0.281
7000	0.456	0.582	0.254	0.409	0.538	0.260	0.387	0.520	0.242	0.350	0.461	0.222	0.360	0.497	0.222
8200	0.384	0.506	0.244	0.344	0.462	0.216	0.327	0.445	0.203	0.255	0.404	0.183	0.307	0.421	0.191
8400	0.450	0.643	0.271	0.403	0.582	0.242	0.379	0.550	0.227	0.344	0.483	0.211	0.353	0.519	0.209
8800	0.431	0.613	0.260	0.386	0.567	0.231	0.363	0.528	0.217	0.329	0.463	0.201	0.338	0.457	0.200
12500	0.309	0.447	0.165	0.275	0.401	0.164	0.260	0.389	0.153	0.232	0.350	0.136	0.240	0.347	0.144

T_{EFF} (°K)	20000			25000			30000			35000			40000		
λ(Å)	A_λ	B_λ	V_1	A_λ	B_λ	V_1	A_λ	B_λ	V_1	A_λ	B_λ	V_1	A_λ	B_λ	V_1
2000	0.769	0.581	0.622	0.736	0.660	0.545	0.669	0.656	0.470	0.579	0.770	0.363	0.542	0.711	0.343
2500	0.682	0.636	0.503	0.651	0.720	0.445	0.558	0.685	0.405	0.529	0.750	0.319	0.506	0.737	0.381
3000	0.578	0.593	0.411	0.572	0.721	0.372	0.527	0.664	0.342	0.473	0.724	0.272	0.452	0.722	0.283
3647	0.475	0.528	0.328	0.493	0.653	0.313	0.458	0.619	0.287	0.416	0.670	0.231	0.354	0.681	0.207
3862	0.557	0.769	0.342	0.536	0.772	0.321	0.512	0.752	0.303	0.470	0.758	0.250	0.448	0.774	0.235
4400	0.529	0.720	0.325	0.508	0.742	0.302	0.461	0.718	0.282	0.440	0.763	0.229	0.414	0.742	0.211
5500	0.444	0.630	0.265	0.435	0.678	0.245	0.413	0.642	0.225	0.377	0.673	0.193	0.350	0.665	0.189
7000	0.354	0.511	0.213	0.364	0.578	0.206	0.342	0.552	0.191	0.313	0.564	0.159	0.251	0.578	0.135
8200	0.307	0.444	0.164	0.326	0.540	0.178	0.303	0.458	0.167	0.275	0.458	0.139	0.253	0.516	0.114
8400	0.342	0.517	0.159	0.347	0.574	0.190	0.325	0.546	0.176	0.293	0.543	0.145	0.266	0.545	0.121
8800	0.329	0.457	0.151	0.336	0.560	0.183	0.313	0.528	0.169	0.282	0.521	0.140	0.257	0.526	0.115
12500	0.238	0.364	0.137	0.260	0.447	0.138	0.236	0.403	0.125	0.204	0.374	0.103	0.187	0.395	0.081

TABLE 2

LINEAR AND NON-LINEAR COEFFICIENTS OF LIMB DARKENING

LOG (G) = 3.5

T_{EFF} (°K)	10000			12000			14000			16000			18000		
λ(Å)	A_λ	B_λ	V_1	A_λ	B_λ	V_1	A_λ	B_λ	V_1	A_λ	B_λ	V_1	A_λ	B_λ	V_1
2000	0.511	-0.309	0.555	0.891	0.112	0.857	0.662	0.306	0.771	0.626	0.401	0.710	0.604	0.475	0.666
2500	0.802	0.167	0.756	0.774	0.347	0.676	0.749	0.456	0.619	0.717	0.503	0.573	0.659	0.544	0.543
3000	0.667	0.350	0.570	0.643	0.419	0.525	0.626	0.483	0.450	0.605	0.516	0.460	0.558	0.549	0.443
3647	0.513	0.398	0.401	0.454	0.410	0.375	0.489	0.449	0.363	0.474	0.481	0.348	0.479	0.452	0.351
3862	0.718	0.807	0.450	0.670	0.801	0.445	0.638	0.791	0.415	0.612	0.769	0.355	0.559	0.762	0.379
4400	0.696	0.831	0.462	0.642	0.766	0.421	0.607	0.770	0.390	0.579	0.744	0.369	0.565	0.753	0.354
5500	0.585	0.728	0.381	0.537	0.657	0.342	0.508	0.673	0.320	0.483	0.642	0.304	0.475	0.651	0.294
7000	0.461	0.588	0.257	0.424	0.557	0.265	0.402	0.543	0.251	0.382	0.521	0.237	0.382	0.539	0.232
8200	0.388	0.505	0.247	0.357	0.472	0.225	0.340	0.464	0.211	0.327	0.457	0.201	0.332	0.476	0.208
8400	0.458	0.663	0.274	0.421	0.621	0.245	0.359	0.589	0.236	0.360	0.559	0.225	0.373	0.555	0.219
8800	0.435	0.634	0.263	0.404	0.555	0.239	0.362	0.565	0.226	0.364	0.537	0.215	0.360	0.541	0.210
12500	0.315	0.457	0.185	0.288	0.421	0.172	0.273	0.407	0.161	0.260	0.395	0.151	0.259	0.392	0.151

T_{EFF} (°K)	20000			25000			30000			40000		
λ(Å)	A_λ	B_λ	V_1	A_λ	B_λ	V_1	A_λ	B_λ	V_1	A_λ	B_λ	V_1
2000	0.770	0.478	0.631	0.725	0.545	0.566	0.643	0.646	0.458	0.557	0.606	0.424
2500	0.678	0.582	0.513	0.661	0.683	0.465	0.599	0.686	0.404	0.581	0.675	0.390
3000	0.581	0.565	0.421	0.553	0.671	0.406	0.548	0.703	0.351	0.548	0.737	0.342
3647	0.478	0.510	0.336	0.520	0.651	0.335	0.491	0.686	0.300	0.506	0.773	0.292
3862	0.581	0.764	0.367	0.574	0.786	0.353	0.548	0.790	0.327	0.545	0.780	0.331
4400	0.545	0.728	0.341	0.542	0.773	0.327	0.520	0.782	0.302	0.524	0.804	0.300
5500	0.463	0.653	0.282	0.475	0.705	0.260	0.460	0.743	0.255	0.473	0.814	0.249
7000	0.372	0.538	0.223	0.401	0.640	0.225	0.393	0.667	0.210	0.420	0.790	0.204
8200	0.322	0.473	0.152	0.359	0.601	0.155	0.353	0.614	0.165	0.380	0.754	0.175
8400	0.368	0.574	0.210	0.369	0.647	0.211	0.378	0.668	0.196	0.356	0.776	0.184
8800	0.352	0.547	0.202	0.375	0.640	0.200	0.366	0.650	0.188	0.384	0.764	0.176
12500	0.254	0.401	0.144	0.290	0.503	0.152	0.275	0.506	0.141	0.301	0.652	0.125

TABLE 4

LINEAR AND NON-LINEAR COEFFICIENTS OF LIMB DARKENING

LOG (G) = 3.0

T_{EFF} (°K)	10000			12000			14000			16000			18000		
λ(Å)	A_λ	B_λ	V_1	A_λ	B_λ	V_1	A_λ	B_λ	V_1	A_λ	B_λ	V_1	A_λ	B_λ	V_1
2000	0.502	-0.018	0.592	0.274	0.059	0.654	0.839	0.226	0.771	0.803	0.287	0.716	0.773	0.343	0.671
2500	0.752	0.112	0.761	0.766	0.293	0.663	0.742	0.405	0.624	0.712	0.446	0.584	0.696	0.457	0.552
3000	0.655	0.255	0.571	0.639	0.391	0.525	0.627	0.455	0.458	0.606	0.473	0.472	0.604	0.525	0.455
3647	0.455	0.337	0.401	0.451	0.387	0.382	0.494	0.439	0.371	0.485	0.438	0.363	0.459	0.487	0.382
3662	0.727	0.782	0.505	0.675	0.759	0.463	0.649	0.744	0.438	0.627	0.726	0.421	0.814	0.732	0.407
4400	0.654	0.753	0.470	0.644	0.757	0.430	0.613	0.740	0.405	0.592	0.723	0.388	0.581	0.733	0.374
5500	0.590	0.720	0.387	0.547	0.689	0.354	0.523	0.682	0.332	0.506	0.665	0.320	0.501	0.683	0.310
7000	0.468	0.585	0.303	0.436	0.571	0.277	0.420	0.572	0.261	0.407	0.559	0.251	0.410	0.589	0.247
8200	0.393	0.503	0.253	0.367	0.490	0.231	0.357	0.456	0.220	0.348	0.484	0.214	0.356	0.518	0.213
8400	0.470	0.681	0.261	0.440	0.644	0.261	0.423	0.632	0.248	0.411	0.618	0.241	0.412	0.637	0.236
8800	0.452	0.654	0.270	0.422	0.620	0.250	0.406	0.610	0.238	0.355	0.596	0.230	0.397	0.617	0.226
12500	0.325	0.472	0.154	0.303	0.451	0.178	0.293	0.452	0.169	0.283	0.440	0.182	0.288	0.460	0.182

T_{EFF} (°K)	20000			25000			30000		
(Å)	A_λ	B_λ	V_1	A_λ	B_λ	V_1	A_λ	B_λ	V_1
2000	0.744	0.335	0.644	0.693	0.453	0.561	0.629	0.517	0.479
2500	0.676	0.502	0.532	0.665	0.630	0.486	0.606	0.596	0.436
3000	0.558	0.559	0.440	0.627	0.709	0.428	0.580	0.683	0.387
3647	0.506	0.548	0.354	0.573	0.716	0.372	0.548	0.745	0.339
3662	0.604	0.708	0.403	0.607	0.744	0.356	0.553	0.710	0.391
4400	0.571	0.714	0.370	0.585	0.773	0.368	0.575	0.758	0.361
5500	0.457	0.687	0.306	0.537	0.798	0.316	0.537	0.814	0.309
7000	0.414	0.621	0.243	0.478	0.759	0.266	0.486	0.814	0.261
8200	0.365	0.586	0.210	0.438	0.730	0.235	0.453	0.803	0.232
8400	0.413	0.658	0.232	0.467	0.781	0.251	0.475	0.828	0.246
8800	0.359	0.642	0.222	0.456	0.772	0.242	0.464	0.822	0.237
12500	0.255	0.582	0.158	0.365	0.627	0.192	0.379	0.736	0.176

TABLE 8

LINEAR AND NON-LINEAR COEFFICIENTS OF LIMB DARKENING

LOG (g) = 2.5

T_{EFF} (°K)	10000			12000			14000			16000			18000		
λ (Å)	A_λ	B_λ	V_1	A_λ	B_λ	V_1	A_λ	B_λ	V_1	A_λ	B_λ	V_1	A_λ	B_λ	V_1
2000	0.863	-0.351	0.981	0.845	-0.046	0.854	0.804	0.066	0.774	0.766	0.149	0.719	0.729	0.183	0.672
2500	0.780	0.063	0.763	0.756	0.222	0.691	0.730	0.308	0.639	0.709	0.382	0.598	0.682	0.399	0.566
3000	0.647	0.258	0.575	0.637	0.342	0.540	0.628	0.395	0.515	0.623	0.456	0.494	0.616	0.497	0.475
3647	0.488	0.320	0.398	0.492	0.359	0.392	0.459	0.356	0.388	0.514	0.448	0.369	0.533	0.532	0.385
3862	0.721	0.697	0.520	0.678	0.680	0.488	0.654	0.652	0.466	0.637	0.634	0.455	0.626	0.625	0.447
4400	0.687	0.726	0.430	0.646	0.691	0.449	0.623	0.681	0.429	0.609	0.674	0.418	0.600	0.670	0.410
5500	0.594	0.655	0.398	0.562	0.681	0.371	0.547	0.678	0.356	0.542	0.696	0.347	0.540	0.705	0.343
7000	0.478	0.594	0.313	0.458	0.599	0.291	0.450	0.609	0.281	0.455	0.644	0.276	0.462	0.675	0.276
8200	0.404	0.516	0.261	0.390	0.526	0.244	0.387	0.540	0.238	0.397	0.578	0.237	0.414	0.636	0.239
8400	0.489	0.701	0.293	0.470	0.686	0.279	0.462	0.684	0.272	0.464	0.707	0.269	0.470	0.726	0.268
8800	0.471	0.679	0.282	0.453	0.668	0.267	0.446	0.668	0.261	0.449	0.693	0.258	0.456	0.717	0.258
12500	0.343	0.507	[illegible]	0.331	0.568	0.151	0.328	0.519	0.165	0.334	0.551	0.163	0.349	0.603	0.185

T_{EFF} (°K)	20000		
(Å)	A_λ	B_λ	V_1
2000	0.702	0.182	0.647
2500	0.668	0.384	0.557
3000	0.618	0.507	0.475
3647	0.555	0.584	0.392
3862	0.623	0.587	0.455
4400	0.601	0.642	0.419
5500	0.550	0.702	0.354
7000	0.482	0.712	0.265
8200	0.441	0.698	0.249
8400	0.486	0.753	0.278
8800	0.474	0.745	0.267
12500	0.375	0.670	0.193