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ABSTRACT

A very rapid variation of parameters method has been programmed to determine lifetimes of satellites in orbit about Venus. The perturbing function contains only the gravitational potential of the sun. The solar potential is expanded in a trigonometric series and then averaged over not only the period of the satellite but also over the period of Venus. The equation for variation of eccentricity uses only the singly averaged function so as to include medium period effects, however, for inclination and argument of pericenter the doubly averaged function is used.

It appears from the results of several sample cases that not only is this method considerably faster than a numerical integration solution of the three body problem (less than 15 seconds as opposed to one hour of machine time for an orbit study of 500 days) but is also highly accurate. Although this method contains only third body terms it should be very useful in first approximation studies.

CONTENTS

VARIATION OF PARAMETERS FOR LONG PERIODIC TERMS IN PREDICTING LIFETIMES OF VENUS ORBITERS

INTRODUCTION

One of the determining factors in considering a Venus orbiter is the length of time the satellite will remain above the dense atmosphere of the planet. The orbit must be chosen so as to allow a sufficient lifetime to conduct the required scientific experiments, but also to meet stringent quarantine conditions that have been imposed for Venus.

In the present study, the atmosphere and oblateness of Venus is not considered in the equations of motion. It is assumed that the gravitational attraction of the sun is the only disturbing force acting on the satellite and that if the closest approach of the orbit is below a certain limit, atmospheric drag will cause the satellite to impact within a short time period. The radius of closest approach (periapsis r_p) is defined as

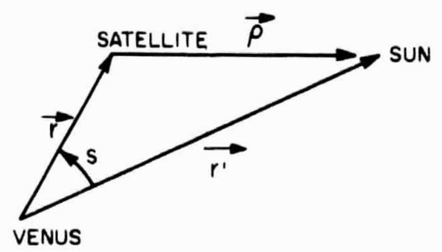
$$r_p = a(1 - e) \tag{1}$$

where a is the semi major axis and e is the eccentricity of the orbit. It will be shown later that under long period perturbations due to the gravitational presence of a third body, the initial value of the semi major axis remains constant. Therefore by choosing a limiting value for r_p, equation (1) may be solved for a so-called critical value, e_{cr}, of the eccentricity. A value of e larger than e_{cr} then will be an impact orbit.

The above outlined problem can of course be easily solved using a precision integration trajectory program to solve either Cowell's or Encke's equations of motion. However, past experience has shown that for small orbits, the computer time is prohibitive. This report investigates a variation of parameters solution that is intended to yield accurate but fast results.

The Solar Potential Function: $R_\odot$

With reference to Figure A the solar potential $R'_\odot$ may be defined as



$$R'_{\odot} = \mu_{\odot} \left[\frac{1}{\rho} - \frac{\vec{r}' \cdot \vec{r}}{r'^3} \right] \tag{2}$$

where

$$\vec{\rho} = \vec{r}' - \vec{r}$$

and

$$\rho^2 = (\vec{r}' - \vec{r})^2 = r'^2 + r^2 - 2 r r' \cos S$$

or

$$\rho^2 = r'^2 \left(1 + \frac{r^2}{r'^2} - \frac{2r}{r'} \cos S\right)$$

Then equation (2) becomes

$$R'_{\odot} = \frac{\mu_{\odot}}{r'} \left[\left(1 + \frac{r^2}{r'^2} - \frac{2r}{r'} \cos S\right)^{-1/2} - \frac{r \cos S}{r'} \right] \tag{3}$$

In this equation we may assume that $r/r' \ll 1$ and expand the square to include only terms of order 2. We then obtain

$$R'_{\odot} = \frac{\mu_{\odot}}{r'}\left[1 - \frac{1}{2}\left(\frac{r^2}{r'^2} - \frac{2r}{r'}\cos S\right) + \frac{3}{8}\left(\frac{4r^2}{r'^2}\cos^2 S\right) - \frac{r}{r'}\cos S\right]$$

$$= \frac{\mu_{\odot}}{r'}\left[1 + \frac{r^2}{r'^2}\left(\frac{3}{2}\cos^2 S - \frac{1}{2}\right)\right]$$

or

$$R'_{\odot} = \frac{\mu_{\odot}}{r'}\left\{1 + \frac{r^2}{r'^2}\left[\frac{3}{2}\left(\frac{\vec{r}\cdot\vec{r}'}{r r'}\right)^2 - \frac{1}{2}\right]\right\} \tag{4}$$

In the usual form of the equations of motion the gradient of the above equation would yield the disturbing function. However, the gradient is taken with respect to the state of the satellite and the first term of equation (4) contains only elements of the sun. Therefore we have

$$\vec{\nabla}\frac{\mu_{\odot}}{r'} \equiv 0$$

and we can redefine the solar potential as

$$\left.\begin{aligned} R_{\odot} &= \frac{\mu_{\odot} r^2}{2r'^3}\left[3\left(\frac{\vec{r}\cdot\vec{r}'}{r r'}\right)^2 - 1\right] \\ &\text{or} \\ R_{\odot} &= \frac{\mu_{\odot} r^2}{2r'^3}[3\cos^2 S - 1] \end{aligned}\right\} \tag{5}$$

Letting $\mu_{\odot} = n'^2 a'^3$ where n' is the mean motion of the sun and a'^3 is the semi-major axis we can write equation (5) as

$$R_{\odot} = \frac{a^2 n'^2}{2}\left(\frac{r}{a}\right)^2\left(\frac{a'}{r'}\right)^3[3\cos^2 S - 1]. \tag{6}$$

Now

$$\cos S = \frac{\vec{r} \cdot \vec{r}'}{r r'} = \vec{r}^{\circ} \cdot \vec{r}^{\circ\prime} \tag{7}$$

and

$$\vec{r}^{\circ} = \vec{P} \cos \theta + \vec{Q} \sin \theta \tag{8}$$

where θ is the true anomaly and

$$\vec{P} = \begin{bmatrix} \cos \Omega \cos \omega - \sin \Omega \sin \omega \cos i \\ \sin \Omega \cos \omega + \cos \Omega \sin \omega \cos i \\ \sin i \sin \omega \end{bmatrix}$$

$$\vec{Q} = \begin{bmatrix} -\cos \Omega \sin \omega - \sin \Omega \cos \omega \cos i \\ -\sin \Omega \sin \omega + \cos \Omega \cos \omega \cos i \\ \sin i \cos \omega \end{bmatrix} \tag{9}$$

$\vec{P}$ is a unit vector in the satellite's orbital plane from the center of Venus to pericenter, $\vec{Q}$ is a unit vector in the orbit plane perpendicular to $\vec{P}$ in the direction of satellite motion. The position of the sun in Venus' orbit plane may be written as

$$\vec{r}^{\circ\prime} = \begin{pmatrix} \cos \Omega_{\odot} \\ \sin \Omega_{\odot} \\ 0 \end{pmatrix} \tag{10}$$

Appendix A describes a venus centered coordinate system where the fundamental plane is Venus' orbital plane and also develops the transformation from the ecliptic plane to the new plane.

Equation (7) may now be written as

$$\cos S = (\vec{P} \cdot \vec{r}'^{\circ}) \cos \theta + (\vec{Q} \cdot \vec{r}'^{\circ}) \sin \theta$$

denoting

$$\left.\begin{aligned} \vec{P} \cdot \vec{r}'^{\circ} &= \alpha \\ \vec{Q} \cdot \vec{r}'^{\circ} &= \beta \end{aligned}\right\} \qquad (11)$$

then

$$\cos S = \alpha \cos \theta + \beta \sin \theta \qquad (12)$$

where it should be pointed out that α and β are independent of θ. Substituting equation (12) into equation (6)

$$R_{\odot} = \frac{a^2 n'^2}{2} \left(\frac{r}{a}\right)^2 \left(\frac{a'}{r'}\right)^3 [3 (\alpha^2 \cos^2 \theta + 2 \alpha \beta \sin \theta \cos \theta + \beta^2 \sin^2 \theta) - 1]$$

which after some manipulations may be written as

$$R_{\odot} = \frac{a^2 n'^2}{2} \left(\frac{r}{a}\right)^2 \left(\frac{a'}{r'}\right)^3 \left[\left(\frac{3}{2}\alpha^2 + \frac{3}{2}\beta^2 - 1\right) + \frac{3}{2}(\alpha^2 - \beta^2)\cos 2\theta + 3\alpha\beta \sin 2\theta\right] \qquad (13)$$

Since we are mainly interested in long period terms it would be advantageous to eliminate any dependence of the above disturbing function on the mean anomaly ℓ. This is done by means of the averaging process where the equation is integrated over the period of the satellite. The details of this process may be found in reference 1 and only the results are presented here. Equation (13) contains three terms which are dependent on the anomaly and the averages of these terms are:

$$\frac{1}{2\pi}\int_0^{2\pi}\left(\frac{r}{a}\right)^2 d\ell = 1 + \frac{3}{2}e^2 \tag{14}$$

$$\frac{1}{2\pi}\int_0^{2\pi}\left(\frac{r}{a}\right)^2 \cos 2\theta \, d\ell = \frac{5e^2}{2} \tag{15}$$

$$\frac{1}{2\pi}\int_0^{2\pi}\left(\frac{r}{a}\right)^2 \sin 2\theta \, d\ell = 0 \tag{16}$$

Substituting the above equations into equation (13) we obtain

$$R_\odot = \frac{a^2 n'^2}{2}\left(\frac{a'}{r'}\right)^3 \left[\left(\frac{3}{2}\alpha^2 + \frac{3}{2}\beta^2 - 1\right)\left(1 + \frac{3e^2}{2}\right) + (\alpha^2 - \beta^2)\frac{15\,e^2}{4}\right] \tag{17}$$

a result which is considerably easier to use than is equation (13).

Variation of Parameters

The development of equations for the method of variation of parameters may be found in any good textbook on celestial mechanics and therefore will only be listed here. For completeness all six equations will be listed although it will be shown that not all of them are needed.

$$\left.\begin{aligned}
\frac{da}{dt} &= \frac{2}{na}\frac{\partial R_\odot}{\partial \ell} \\
\frac{de}{dt} &= \frac{(1-e^2)}{ena^2}\frac{\partial R_\odot}{\partial \ell} - \frac{\sqrt{1-e^2}}{ena^2}\frac{\partial R_\odot}{\partial \omega} \\
\frac{di}{dt} &= -\frac{\csc i}{na^2\sqrt{1-e^2}}\left[\frac{\partial R_\odot}{\partial \Omega} - \cos i\,\frac{\partial R_\odot}{\partial \omega}\right] \\
\frac{d\omega}{dt} &= \frac{\sqrt{1-e^2}}{ena^2}\frac{\partial R_\odot}{\partial e} - \frac{\cos i}{na^2\sqrt{1-e^2}\sin i}\frac{\partial R_\odot}{\partial i} \\
\frac{d\Omega}{dt} &= \frac{1}{na^2\sqrt{1-e^2}\sin i}\frac{\partial R_\odot}{\partial i} \\
\frac{\partial \ell}{\partial t} &= n - \frac{2}{na}\frac{\partial R_\odot}{\partial a} - \frac{1-e^2}{nea^2}\frac{\partial R_\odot}{\partial e}
\end{aligned}\right\} \tag{18}$$

Variation in Eccentricity: Long and Medium Period Terms

The first of the above equations yields an extremely useful result. Since we have elliminated any dependence of $R_{\odot}$ upon ℓ we have

$$\frac{da}{dt} \equiv 0 \tag{19}$$

which means that there are no long term or secular variations in the semi major axis. This means we need only to compute the variations in eccentricity in order to obtain a time history of periapsis. We therefore need to look at the second of equations (26) and note that again

$$\frac{\partial R_{\odot}}{\partial \ell} \equiv 0$$

and therefore

$$\frac{de}{dt} = - \frac{\sqrt{1 - e^2}}{nea^2} \frac{\partial R_{\odot}}{\partial \omega} \tag{20}$$

With reference to equation (17) we may write

$$\frac{\partial R_{\odot}}{\partial \omega} = \frac{\partial R_{\odot}}{\partial \alpha} \frac{\partial \alpha}{\partial \omega} + \frac{\partial R_{\odot}}{\partial \beta} \frac{\partial \beta}{\partial \omega} \tag{21}$$

recalling the definitions of α, β, $\vec{P}$ and $\vec{Q}$ we have

$$\frac{\partial \alpha}{\partial \omega} = \frac{\partial \vec{P}}{\partial \omega} \cdot \vec{r}'^{\circ}$$

and from equation (9) we have

$$\frac{d\vec{P}}{\partial\omega} = \vec{Q}\,.$$

Therefore

$$\frac{\partial\alpha}{\partial\omega} = \vec{Q}\cdot\vec{r}'^{\circ} = \beta \tag{22}$$

Similarly we find

$$\frac{\partial\beta}{\partial\omega} = -\,\alpha \tag{23}$$

Then equation (21) becomes

$$\frac{\partial R_\odot}{\partial\omega} = \frac{\partial R_\odot}{\partial\alpha}\beta - \frac{\partial R_\odot}{\partial\beta}\,\alpha \tag{24}$$

The derivatives are easily found to be

$$\begin{aligned} \frac{\partial R_\odot}{\partial\alpha} &= \frac{a^2\,n'^2}{2}\left(\frac{a'}{r'}\right)^3 (3\alpha)\,(1+4e^2) \\ \frac{\partial R_\odot}{\partial\beta} &= \frac{a^2\,n'^2}{2}\left(\frac{a'}{r'}\right)^3 (3\beta)\,(1-e^2) \end{aligned} \tag{25}$$

Substituting (25) into (24) and then into (20) we obtain, after some algebraic manipulations:

$$\frac{de}{dt} = -\sqrt{1-e^2}\,\frac{15}{2}\,\gamma^2\,ne\,\alpha\beta\left(\frac{a'}{r'}\right)^3 \tag{26}$$

where

$$\gamma = \frac{n'}{n}$$

The above equation represents the variation in eccentricity due to the gravitational presence of the Sun. In the case of Venus, we have a near circular orbit for the sun and therefore we may simplify equation (26) further by setting

$$\frac{a'}{r'} \equiv 1 .$$

Equation (26) also indicates that in order to completely solve the equation we also need a time variation of Ω, ω and i which appear in α and β. Before developing these equations we will make a further simplification of $R_\odot$ which will reduce the system to one of only 3 coupled equations.

Medium Period Terms of $R_\odot$.

Lorell and Anderson (Reference 2) have made a further approximation by averaging over the period of the third body about the central body. In our case this would mean eliminating dependence upon $\Omega_\odot$ defined earlier as the longitude of the sun measured in Venus' orbital plane. From equation (17) and the definitions of α and β we need only consider the two terms

$$\frac{3}{2}\alpha^2 + \frac{3}{2}\beta^2 - 1 \tag{27}$$

and

$$\alpha^2 - \beta^2 \tag{28}$$

we consider first (27)

$$\begin{aligned}\alpha^2 + \beta^2 = {} & \cos^2 \Omega \cos^2 \Omega_\odot + \sin^2 \Omega \cos^2 i \cos^2 \Omega_\odot \\ & + \sin^2 \Omega \sin^2 \Omega_\odot + \cos^2 \Omega \cos^2 i \sin^2 \Omega_\odot \\ & + 2 \sin \Omega \cos \Omega \sin \Omega_\odot \cos \Omega_\odot \\ & - 2 \sin \Omega \cos \Omega \cos^2 i \sin \Omega_\odot \cos \Omega_\odot\end{aligned}$$

$$= 1 - \sin^2 \Omega_{\odot} \cos 2\Omega \sin^2 i + \sin^2 i \left[\frac{1}{2} \sin 2\Omega \sin 2\Omega_{\odot} - \sin^2 \Omega \right]$$

$$= 1 - \frac{\sin^2 i}{2} \left[1 - \cos 2\Omega \cos 2\Omega_{\odot} \right] + \frac{\sin^2 i}{2} \left[\sin 2\Omega \sin 2\Omega_{\odot} \right]$$

$$= 1 - \frac{\sin^2 i}{2} + \frac{\sin^2 i}{2} \cos 2 (\Omega_{\odot} - \Omega)$$

Therefore

$$\frac{3}{2}\alpha^2 + \frac{3}{2}\beta^2 - 1 = \frac{1}{2} - \frac{3}{4}\sin^2 i + \frac{3}{4}\sin^2 i \cos 2 (\Omega_{\odot} - \Omega) \tag{29}$$

In a similar manner we find that

$$\alpha^2 - \beta^2 = \frac{1}{2}\sin^2 i \cos 2\omega + \cos^4 \left(\frac{i}{2}\right) \cos 2 (\Omega_{\odot} - \omega - \Omega) + \sin^4 \left(\frac{i}{2}\right) \cos 2 (\Omega_{\odot} + \omega - \Omega) \tag{30}$$

when we carry out the averaging process we find that all terms containing $\Omega_{\odot}$ may be dropped. We may therefore redefine equation (17) as

$$R_{\odot} = a^2 n'^2 \left[\left(\frac{1}{4} - \frac{3}{8} \sin^2 i \right) \left(1 + \frac{3e^2}{2} \right) + \frac{15 e^2}{16} (\sin^2 i \cos 2\omega) \right] \tag{31}$$

where we have let $\frac{a'}{r'} = 1$

Variation in Eccentricity: Long Periodic Terms Only.

With the new definition of $R_\odot$ above we return now to the second of equations (18) and calculate again de/dt. Again we need only calculate $\partial R_\odot / \partial \omega$

$$\frac{\partial R_\odot}{\partial \omega} = -\frac{15}{8} a^2 n'^2 e^2 \sin^2 i \sin 2\omega \tag{32}$$

$$\frac{de}{dt} = -\frac{(1-e^2)^{1/2}}{ena^2}\left[-\frac{15}{8} a^2 n'^2 e^2 \sin^2 i \sin 2\omega\right]$$

or

$$\frac{de}{dt} = \frac{15}{8}\frac{n'^2}{n} e (1-e^2)^{1/2} \sin^2 i \sin 2\omega \tag{33}$$

This equation is considerably simpler than the previous equation (26) and it is noted that the dependence upon Ω has been eliminated. However, the dependence upon i and ω is still present and we must now develop the equations for these terms.

Variation in Inclination and Argument of Periapsis

Using the new definition of $R_\odot$ above we return now to the third of equations (18)

$$\frac{di}{dt} = -\frac{1}{na^2\sqrt{1-e^2}\sin i}\left[\frac{\partial R_\odot}{\partial \Omega} - \cos i \frac{\partial R_\odot}{\partial \omega}\right]$$

We observe immediately that

$$\frac{\partial R_\odot}{\partial \Omega} \equiv 0$$

a result that was not true before the elimination of medium period terms. Also from equation (32)

$$\frac{\partial R_\odot}{\partial \omega} = -\frac{15}{8} a^2 n'^2 e^2 \sin^2 i \sin 2\omega$$

substituting this we obtain

$$\frac{di}{dt} = -\frac{15}{8} \frac{n'^2}{n} \frac{e^2}{\sqrt{1 - e^2}} \sin i \cos i \sin 2\omega$$

or

$$\frac{di}{dt} = -\frac{15}{16} \frac{n'^2}{n} \frac{e^2}{\sqrt{1 - e^2}} \sin 2i \sin 2\omega \tag{34}$$

The equation for the variation in argument ofperiapsis requires $\partial R_\odot/\partial e$ and $\partial R_\odot/\partial i$.

$$\frac{\partial R_\odot}{\partial e} = a^2 n'^2 e \left[3 \left(\frac{1}{4} - \frac{3}{8} \sin^2 i\right) + \frac{15}{8} (\sin^2 i \cos 2\omega)\right]$$

$$\frac{\partial R_\odot}{\partial i} = a^2 n'^2 \left[-\frac{3}{4} \sin i \cos i \left(1 + \frac{3e^2}{2}\right) + \frac{15}{8} e^2 (\sin i \cos i \cos 2\omega)\right]$$

Substituting these into the fourth of equations (18)

$$\frac{d\omega}{dt} = \frac{n'^2}{n} (1 - e^2)^{1/2} \left[\frac{3}{4} - \frac{9}{8} \sin^2 i + \frac{15}{8} (\sin^2 i \cos 2\omega)\right]$$

$$- \frac{n'^2}{n} \frac{(1 - e^2)^{1/2}}{(1 - e^2)} \left[-\frac{3}{4} \cos^2 i \left(1 + \frac{3e^2}{2}\right) + \frac{15}{8} e^2 \cos^2 i \cos 2\omega\right]$$

$$= \frac{n'^2}{n} (1 - e^2)^{1/2} \left\{ \frac{3}{4} - \frac{3}{8} \sin^2 i \, (3 - 5 \cos 2\omega) \right.$$

$$\left. + \frac{3}{4} \frac{\cos^2 i}{(1 - e^2)} + \frac{9}{8} \frac{e^2 \cos^2 i}{(1 - e^2)} - \frac{15}{8} e^2 \frac{\cos^2 i}{(1 - e^2)} \cos 2\omega \right\}$$

$$= \frac{n'^2 (1 - e^2)^{1/2}}{n} \left\{ \frac{3}{4} - \frac{3}{4} \sin^2 i \, (5 \sin^2 \omega - 1) \right.$$

$$\left. + \frac{3}{4} \frac{\cos^2 i}{(1 - e^2)} + \frac{3}{4} \frac{e^2 \cos^2 i}{(1 - e^2)} (5 \sin^2 \omega - 1) \right\}$$

$$= \frac{3}{4} \frac{n'^2}{n} (1 - e^2)^{1/2} \left\{ 1 - 5 \sin^2 i \sin^2 \omega + \sin^2 i \right.$$

$$\left. + \frac{1 - 5 e^2 \sin^2 \omega - e^2}{(1 - e^2)} + \frac{e^2 \sin^2 i - \sin^2 i - 5 e^2 \sin^2 i \sin^2 \omega}{(1 - e^2)} \right\}$$

$$= \frac{3}{4} \frac{n'^2}{n} (1 - e^2)^{1/2} \left\{ 5 \sin^2 \omega \left[\frac{e^2 - e^2 \sin^2 i}{1 - e^2} - \sin^2 i \right] + 1 + \sin^2 i \right.$$

$$\left. + \frac{1 - e^2}{1 - e^2} + \frac{e^2 \sin^2 i - \sin^2 i}{1 - e^2} \right\}$$

and finally

$$\frac{d\omega}{dt} = \frac{3}{2} \frac{n'^2}{n} (1 - e^2)^{1/2} \left\{ 1 + \frac{5}{2} \sin^2 \omega \frac{(e^2 - \sin^2 i)}{(1 - e^2)} \right\} \qquad (35)$$

Equations (33), (34) and (35) represent a system of three equations in e , i and ω only and it is now possible to solve for a time history of the eccentricity without having to solve for ℓ or Ω. Actually one may use equation (26) instead of equation (33) by merely holding Ω constant at $\Omega = \Omega_0$. In the computer program, equation (26) was utilized instead of (33). The results will not differ significantly. It is to be noted that the above three equations are identical to those given by Lorrel and Anderson in reference 2.

Results

Figures 1 through 5 show comparisons between the above outlined method and a precision n body Encke integration program. As can be seen from these graphs there are no significant differences between the results obtained. However, it must be pointed out that the n body program took approximately one hour to integrate the orbit for 500 days and the variation of parameters method took less than 15 seconds.

Figures 1 through 3 are of similar size orbits with only the inclinations being changed. The apparent secular variation in eccentricity appears in all three graphs, however for an inclination of 5°, this variation is very small. Figures 4 and 5 show trajectories that impact the planet after 500 and 217 days respectively. Impact is defined as that point where the Venusian atmosphere will drastically alter the orbit such that virtual impact will shortly occur. For this study an altitude of 200 km was used for impact.

Figure 6 through 9 are for orbits with periapses of 6575 km and apoapses of 46025 km. Here the parameters that were varied are inclination and argument of pericenter. All four of these orbits impact in less than one terrestial year.

Figures 10 through 14 represent an orbit with periapsis radius of 7550 km and apoapsis radius of 46050 km. Again the parameters that were varied are inclination and argument of pericenter. The first three of these orbits have not yet impacted after 800 days but all indications are that they will. The last two will impact within a very short time after 800 days.

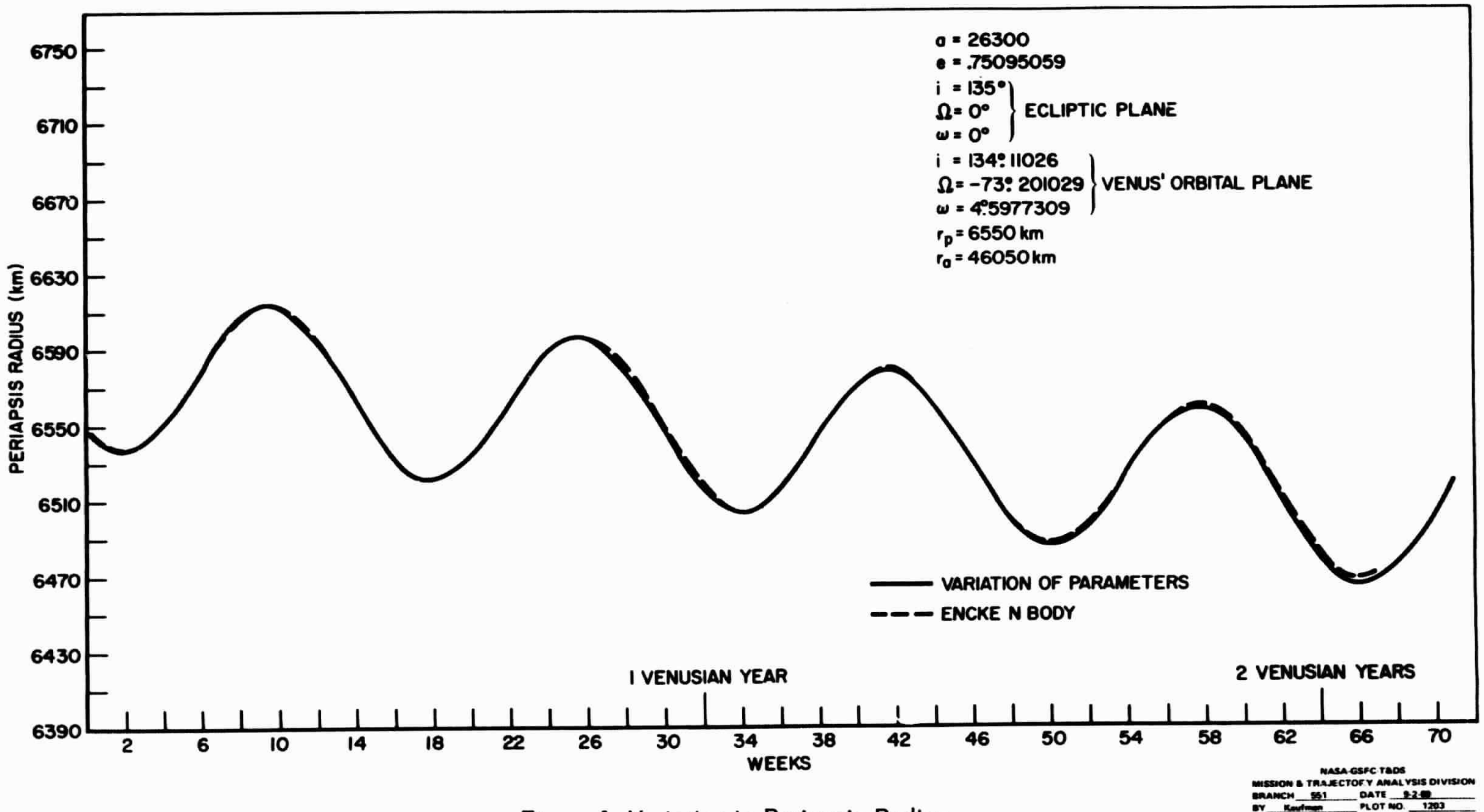


Figure 1—Variation in Periapsis Radius.

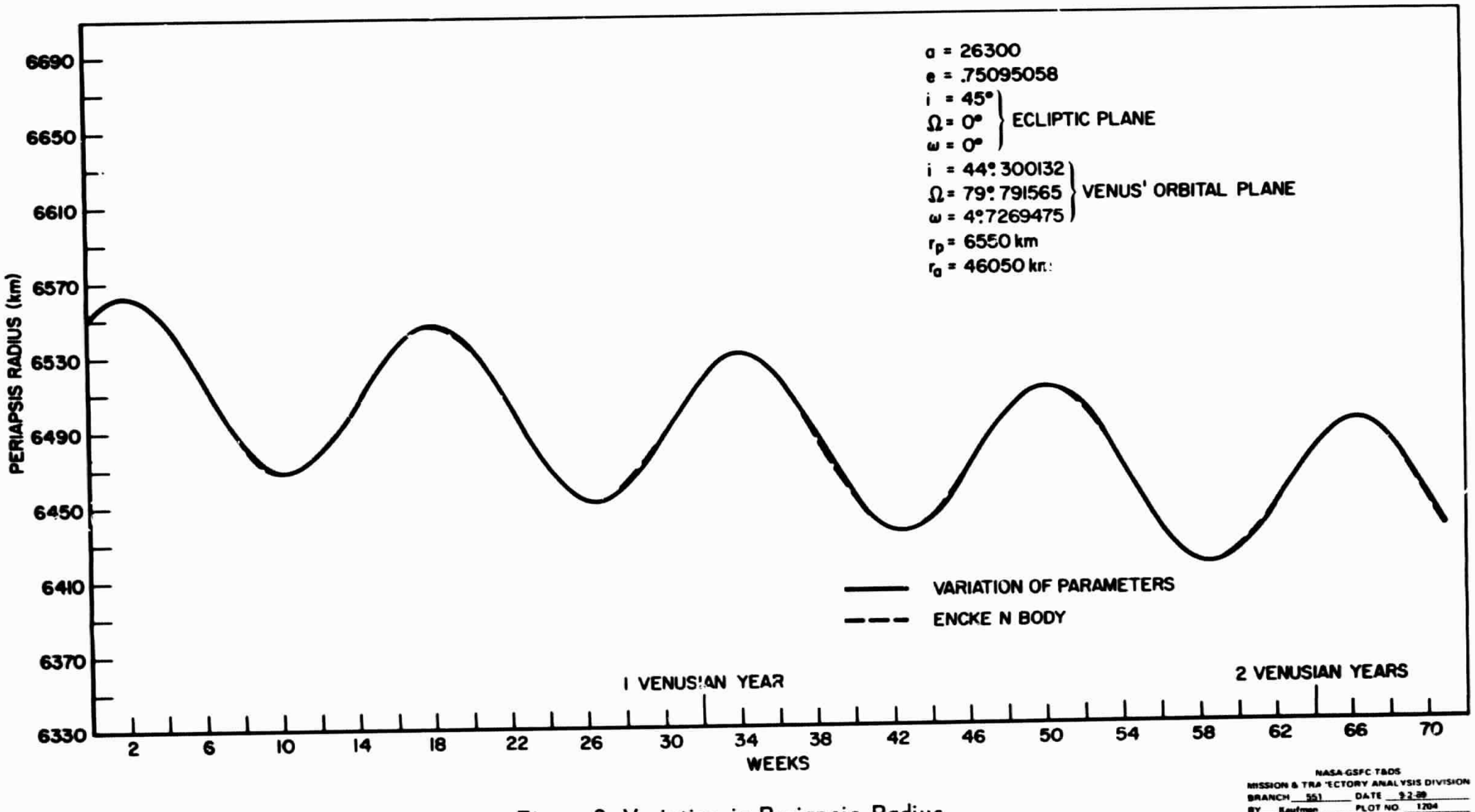


Figure 2—Variation in Periapsis Radius.

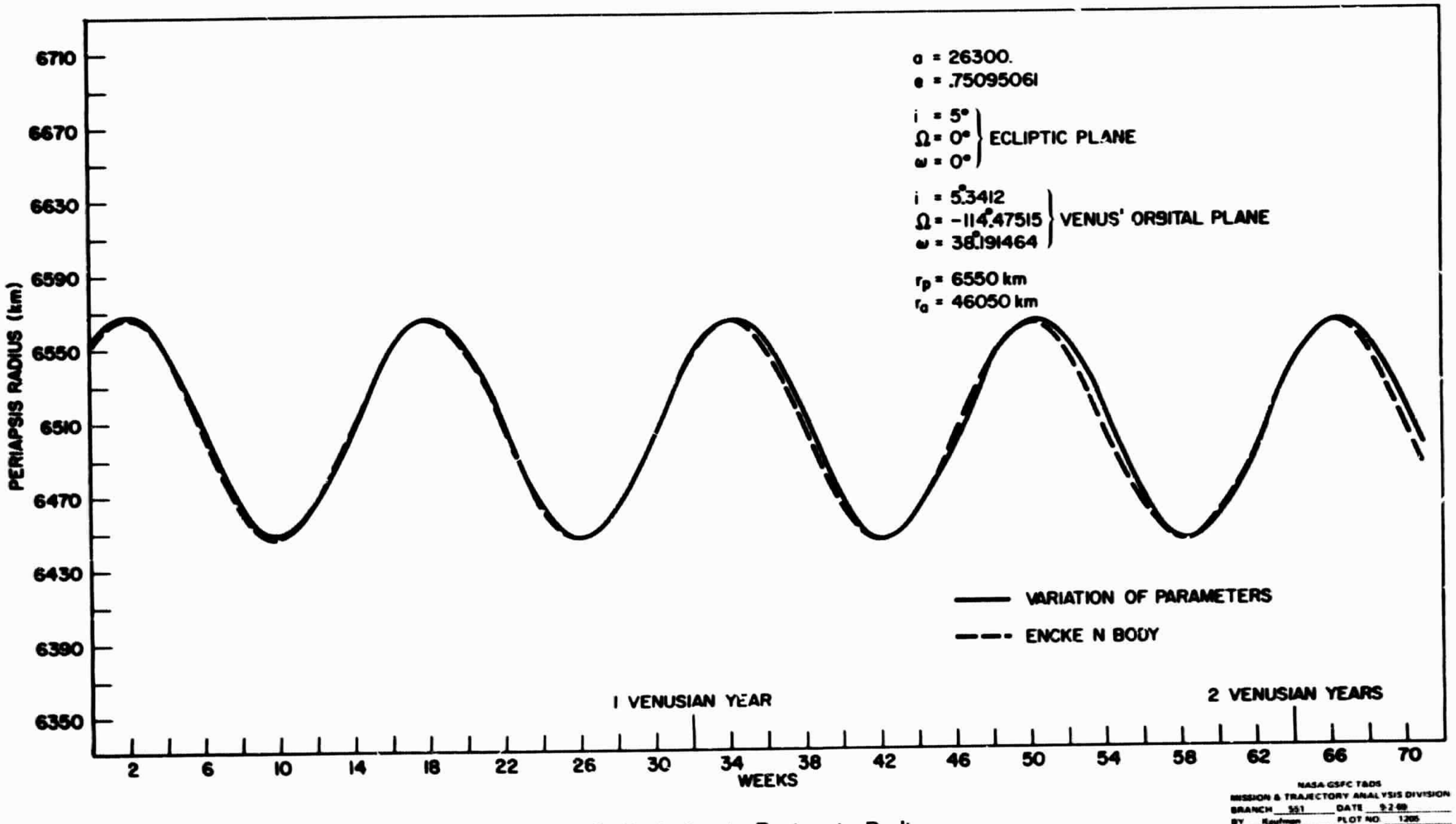


Figure 3—Variation in Periapsis Radius.

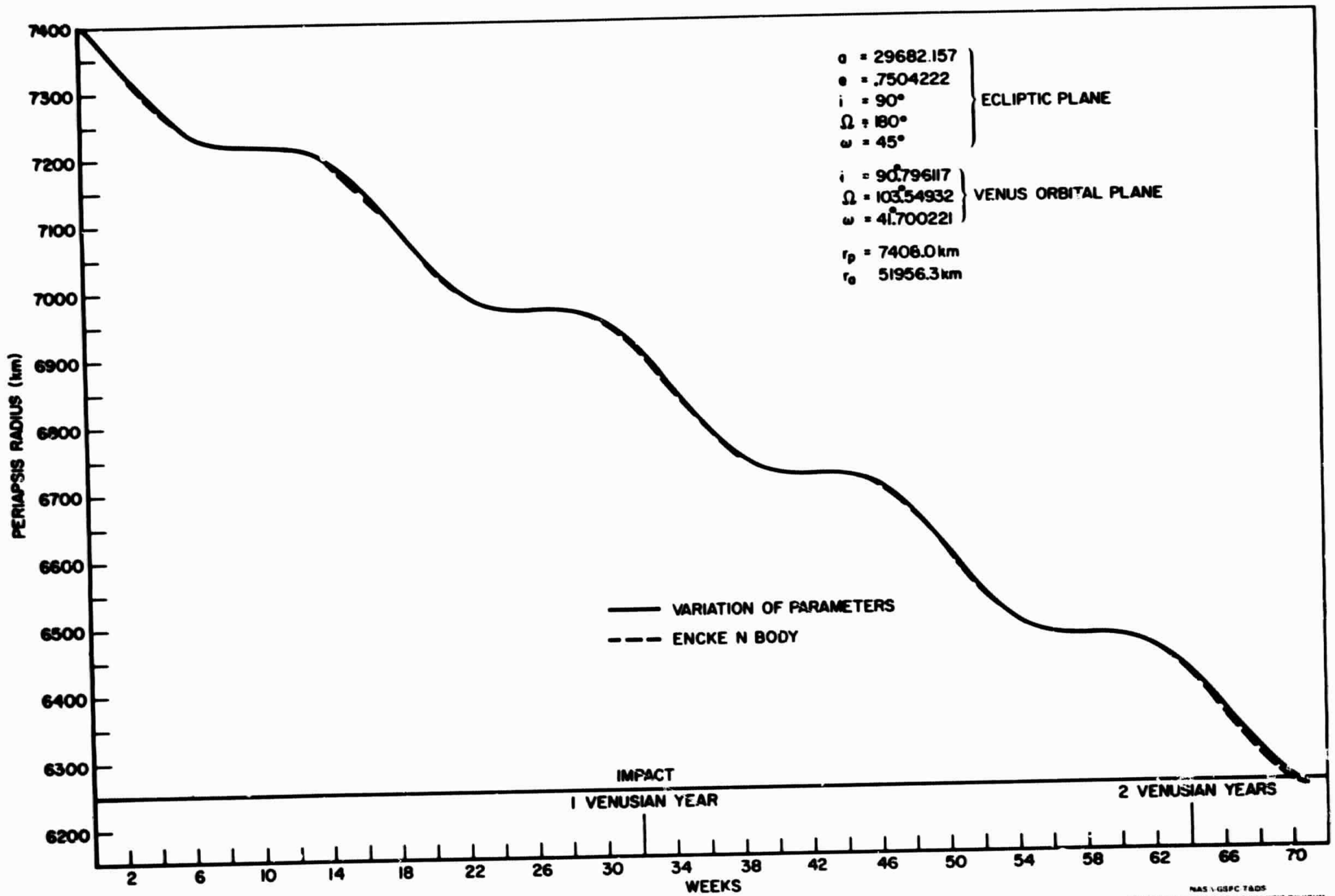


Figure 4—Variation in Periapsis Radius.

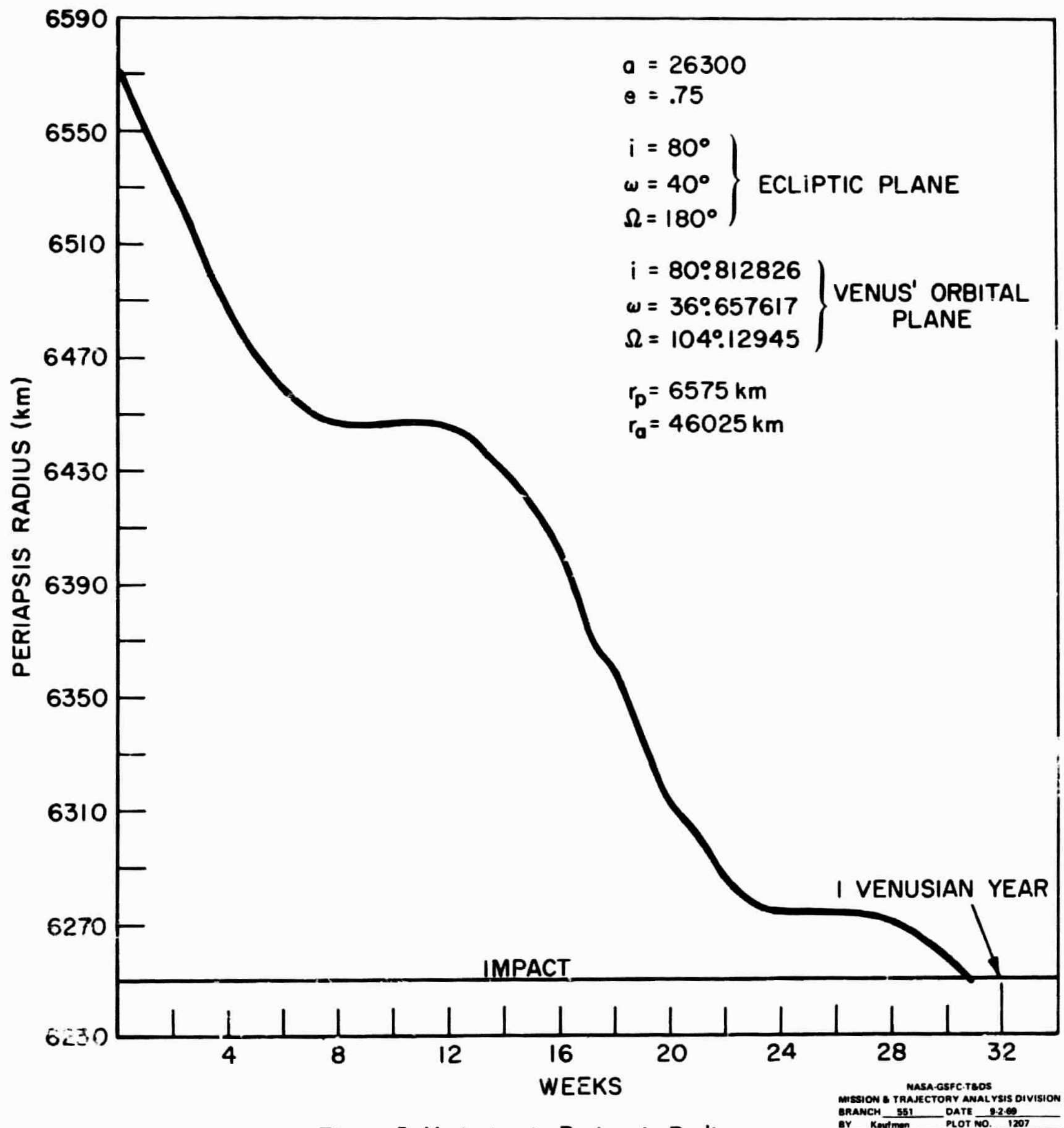


Figure 5—Variation in Periapsis Radius.

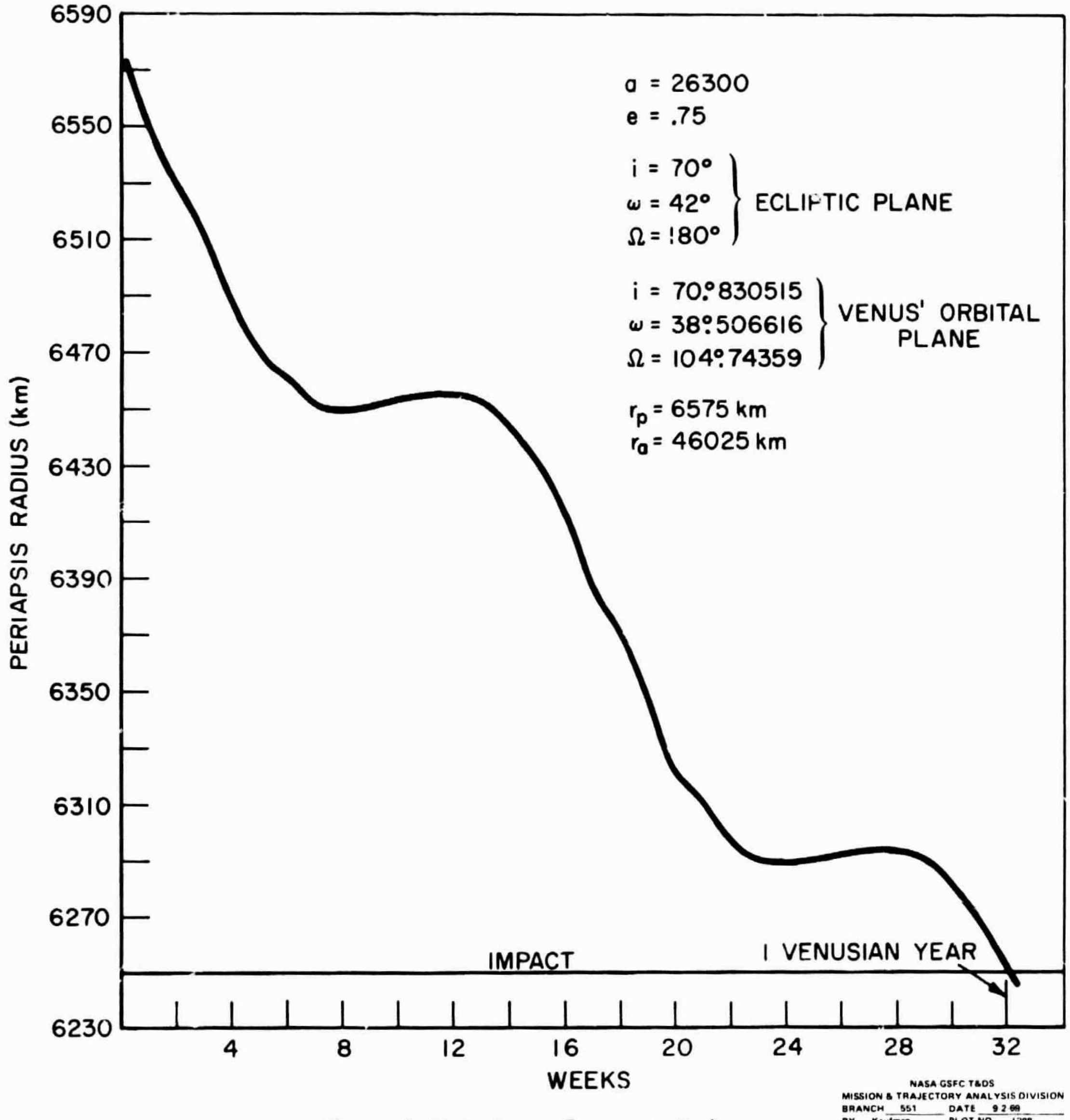


Figure 6—Variation in Periapsis Radius.

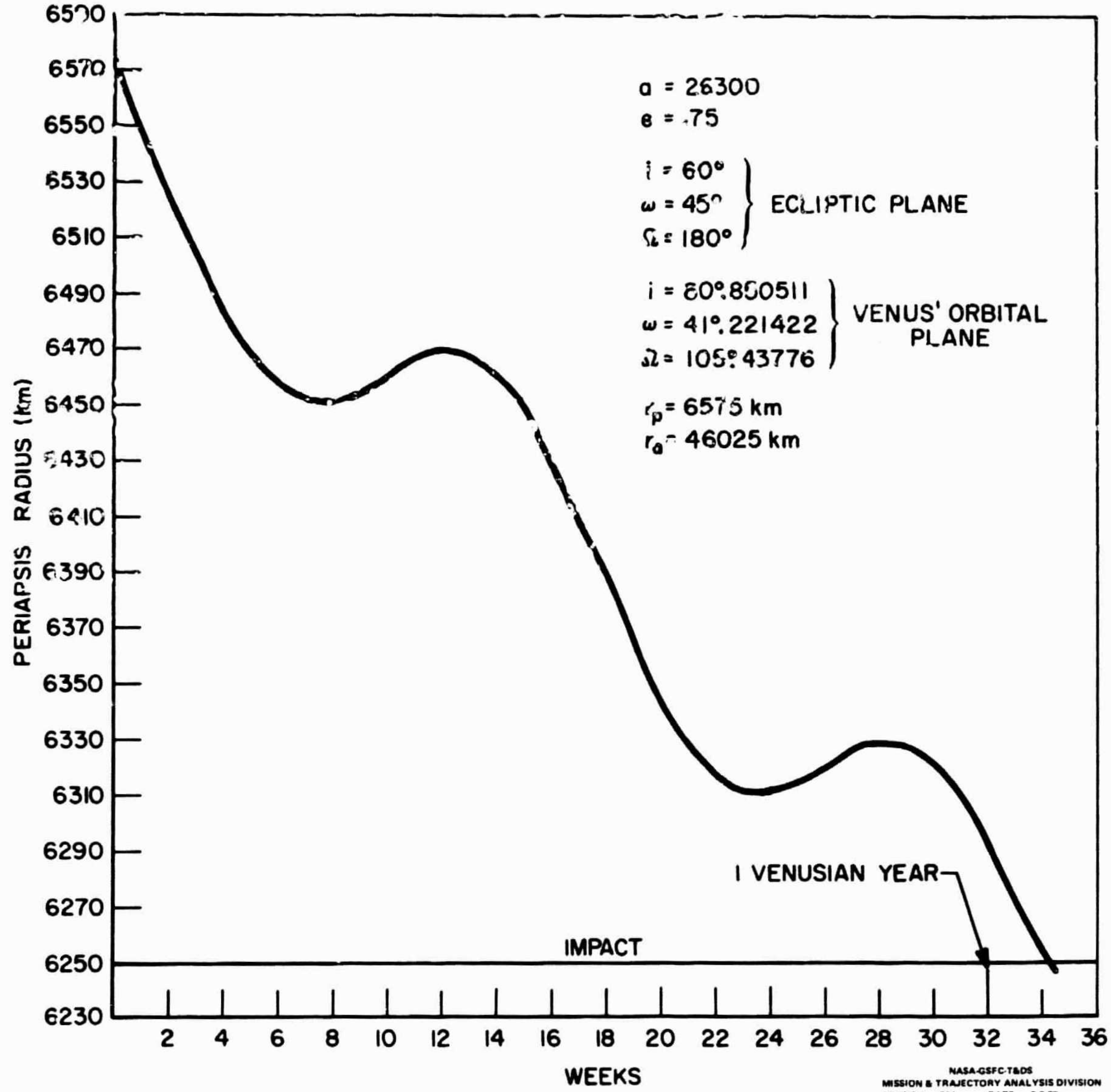


Figure 7–Variation in Periapsis Radius.

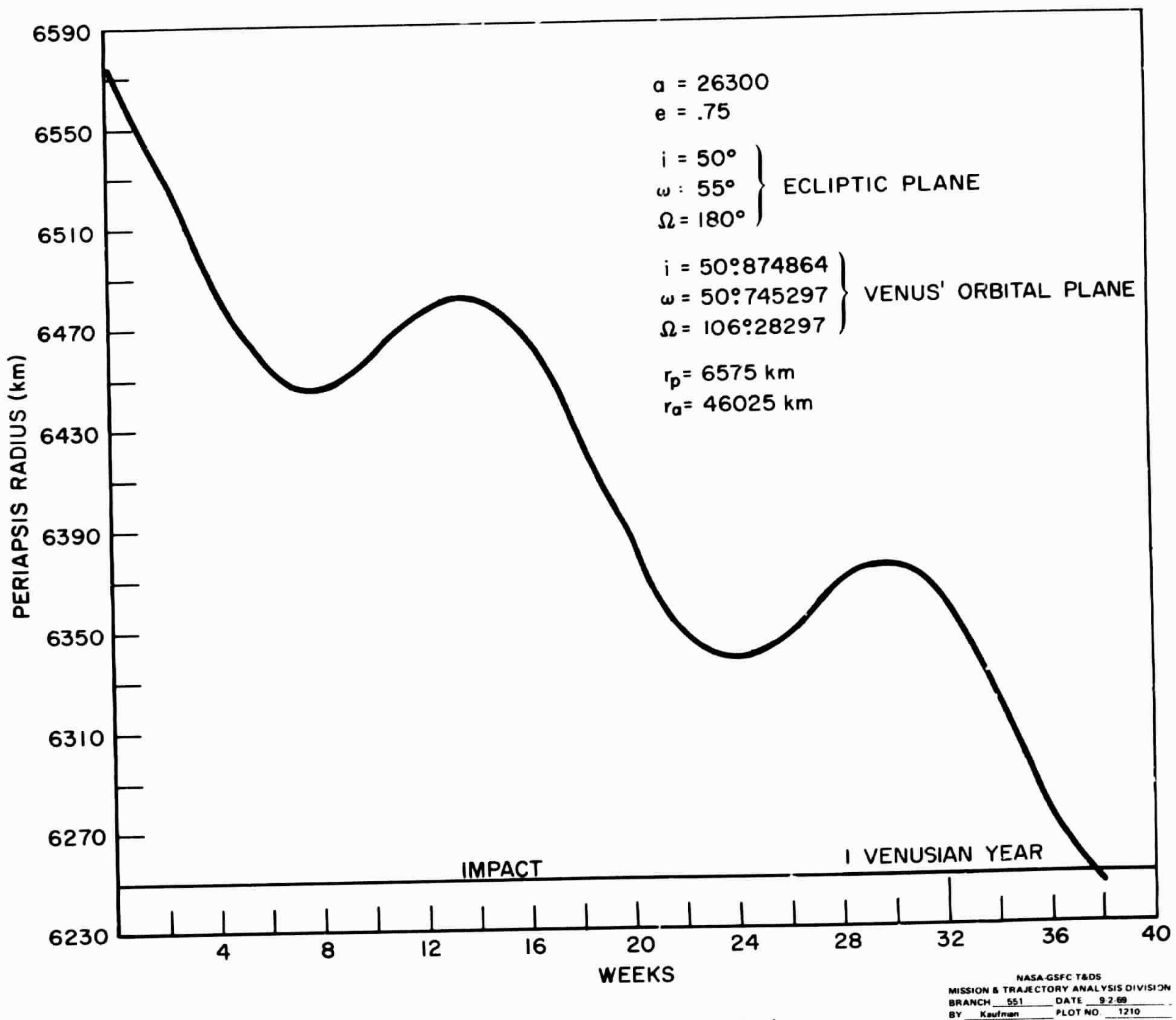


Figure 8—Variation in Periapsis Radius.

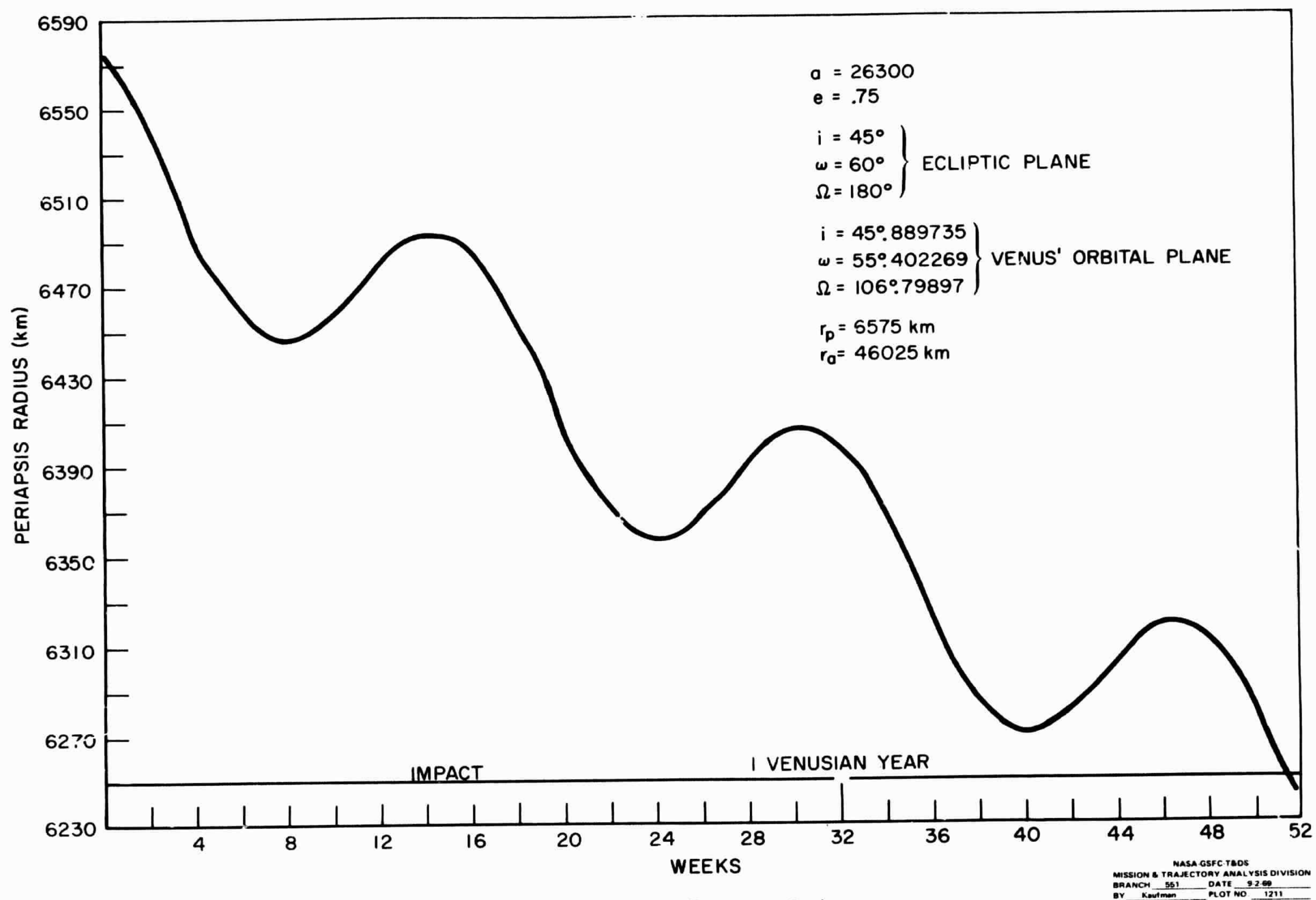


Figure 9—Variation in Periapsis Radius.

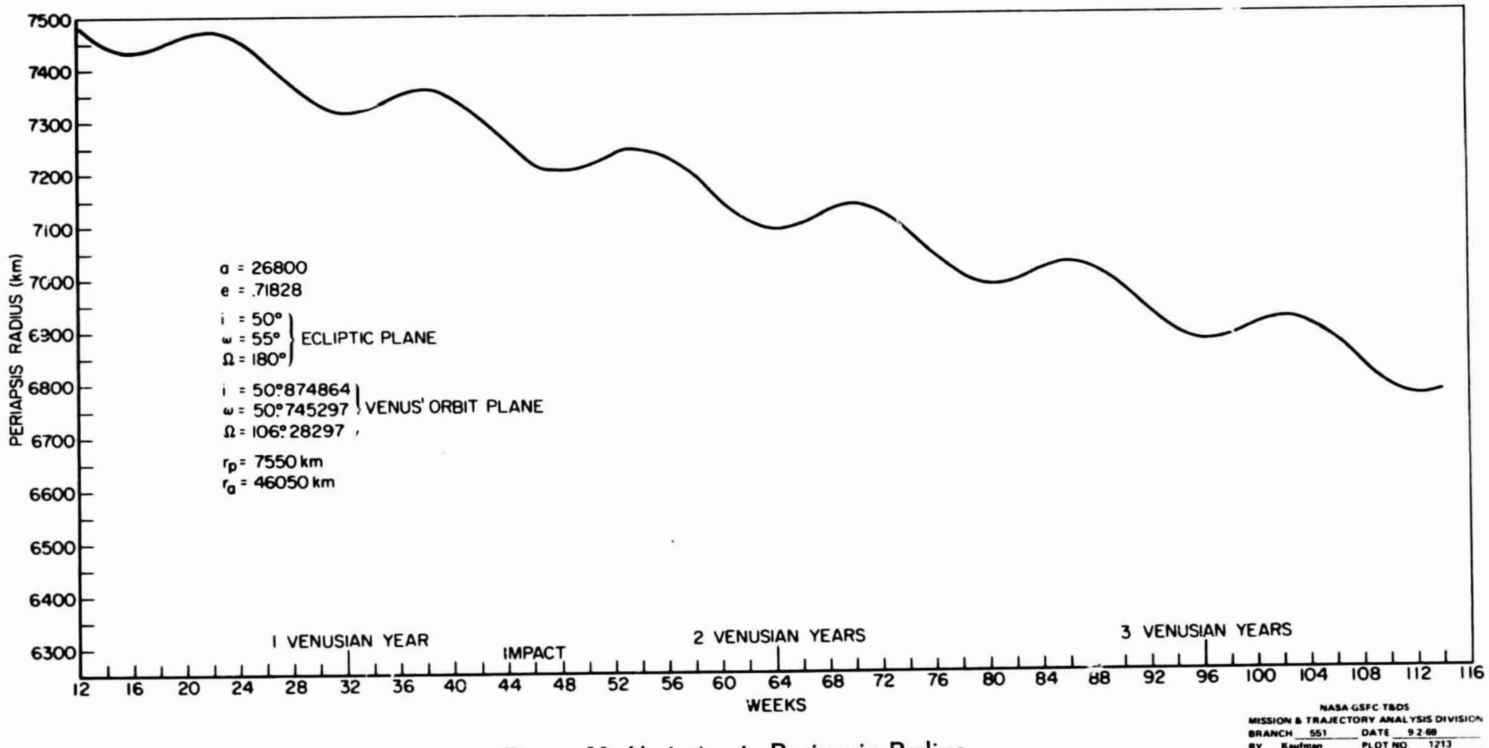


Figure 11—Variation in Periapsis Radius.

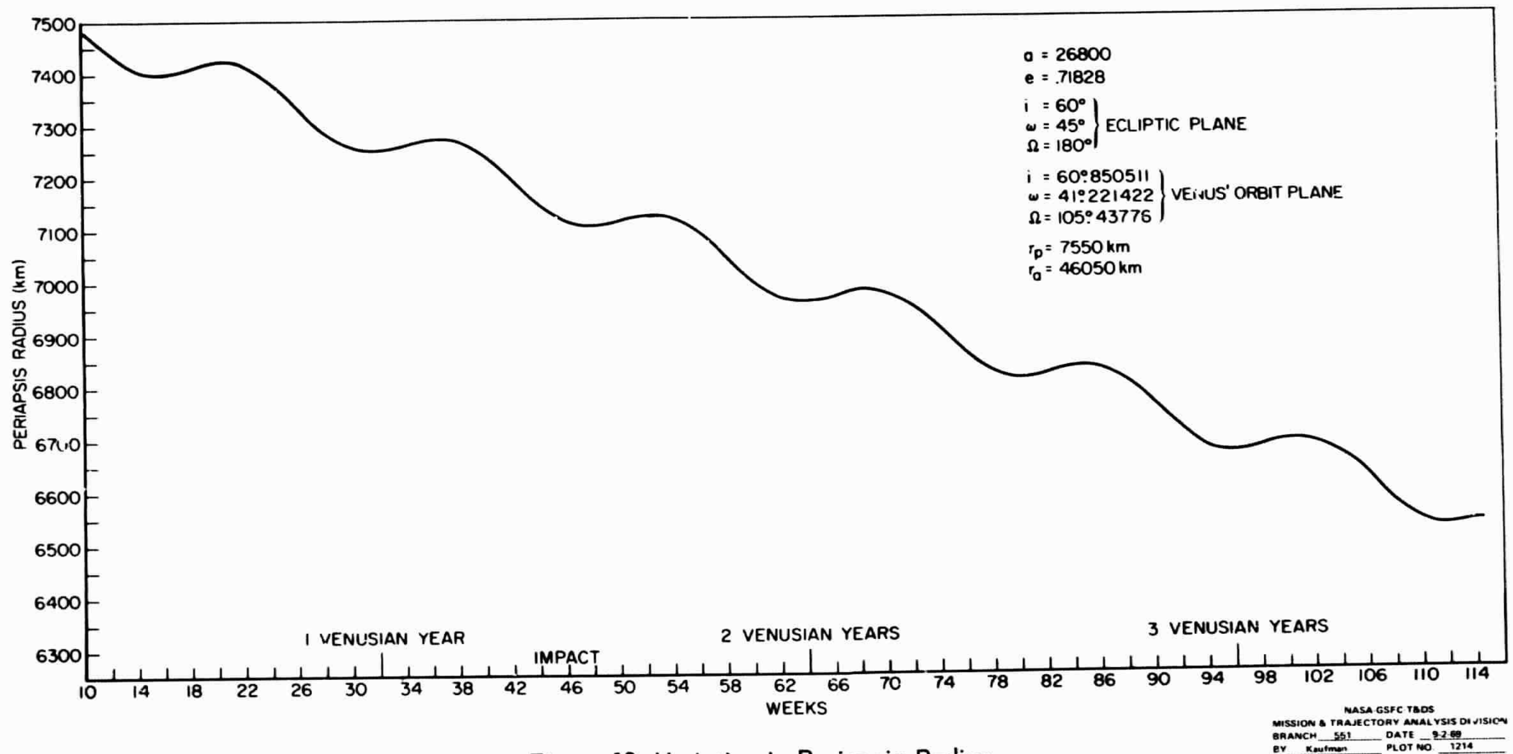


Figure 12—Variation in Periapsis Radius.

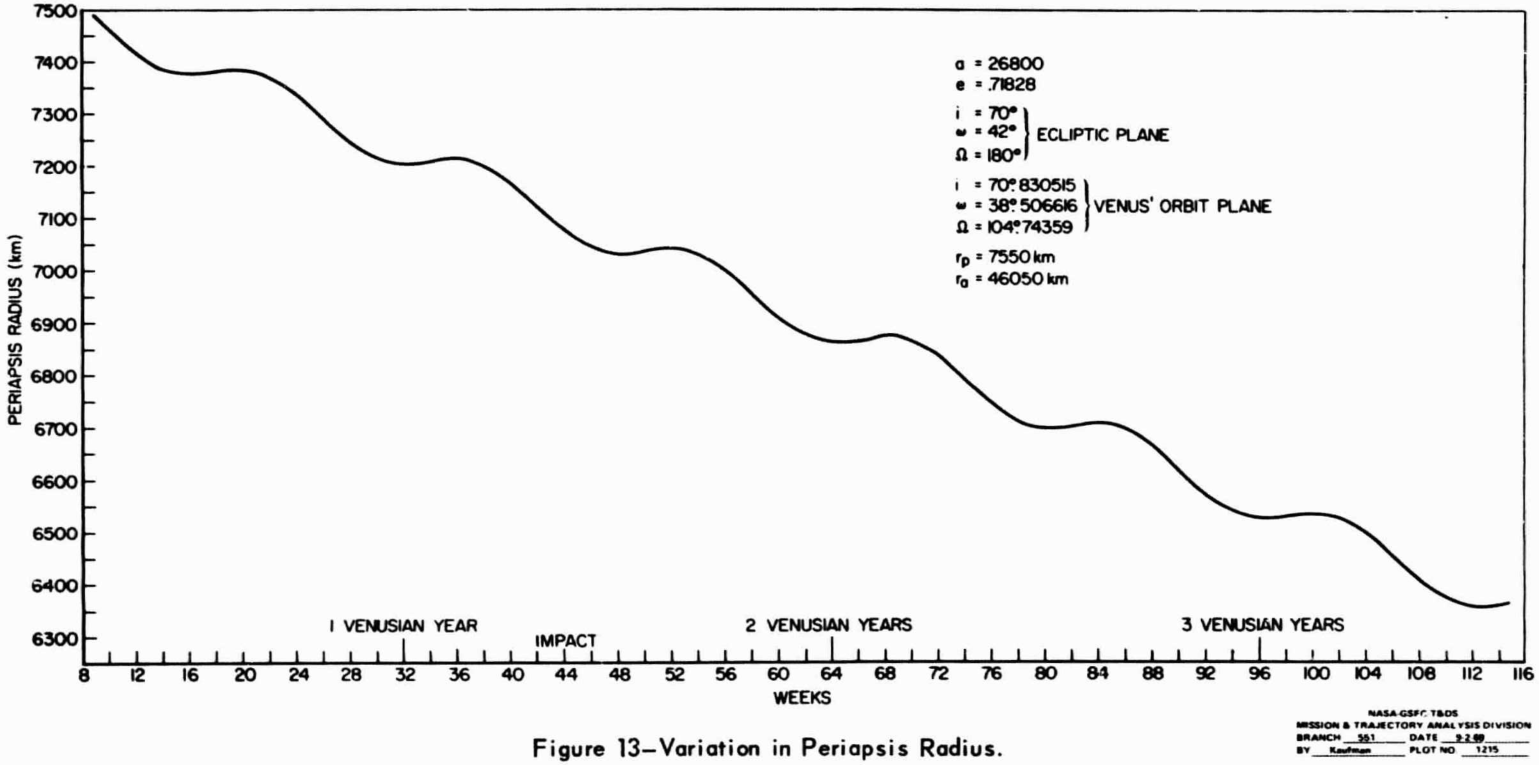


Figure 13–Variation in Periapsis Radius.

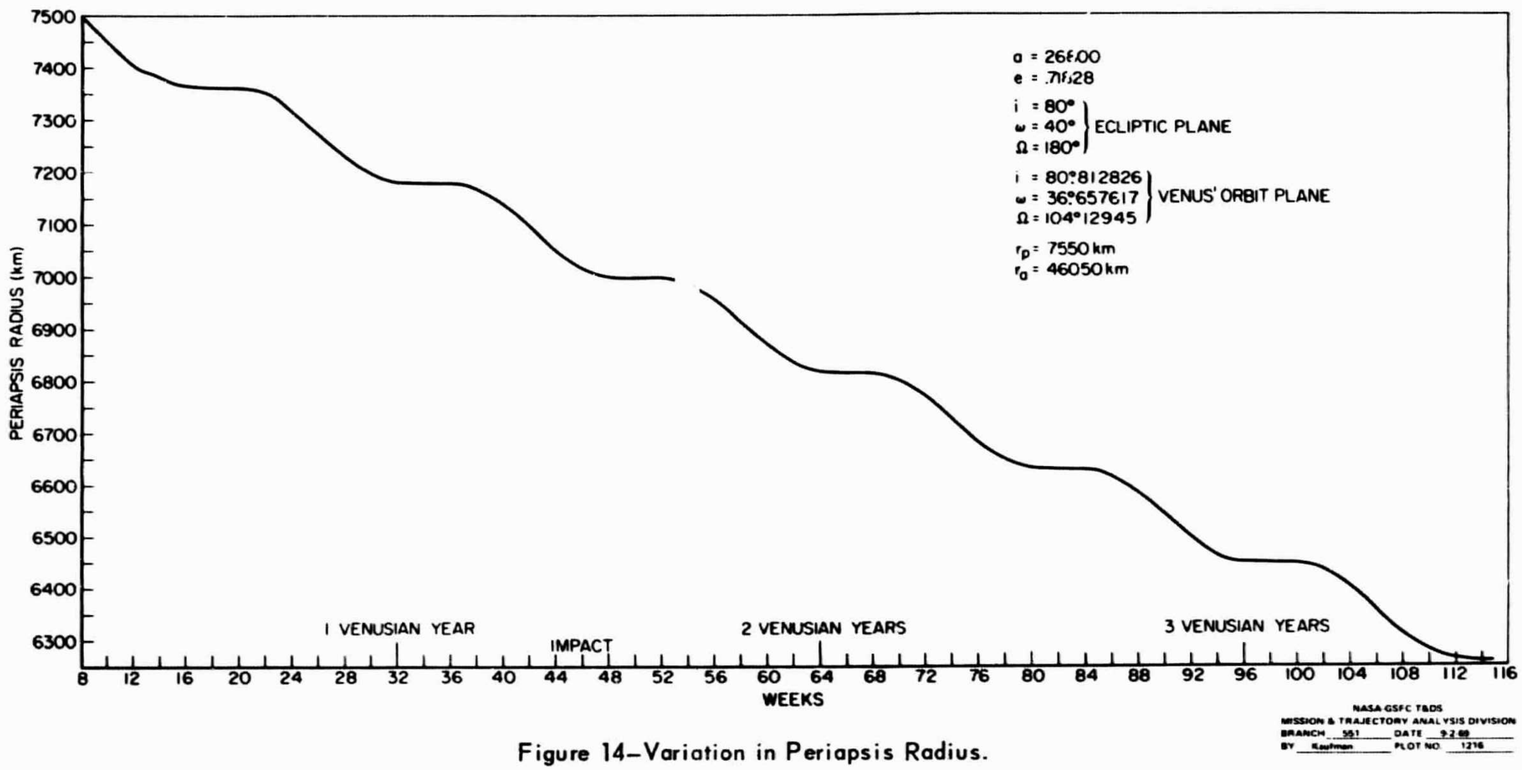


Figure 14—Variation in Periapsis Radius.

APPENDIX A

TRANSFORMATION FROM ECLIPTIC TO VENUS' ORBITAL PLANE

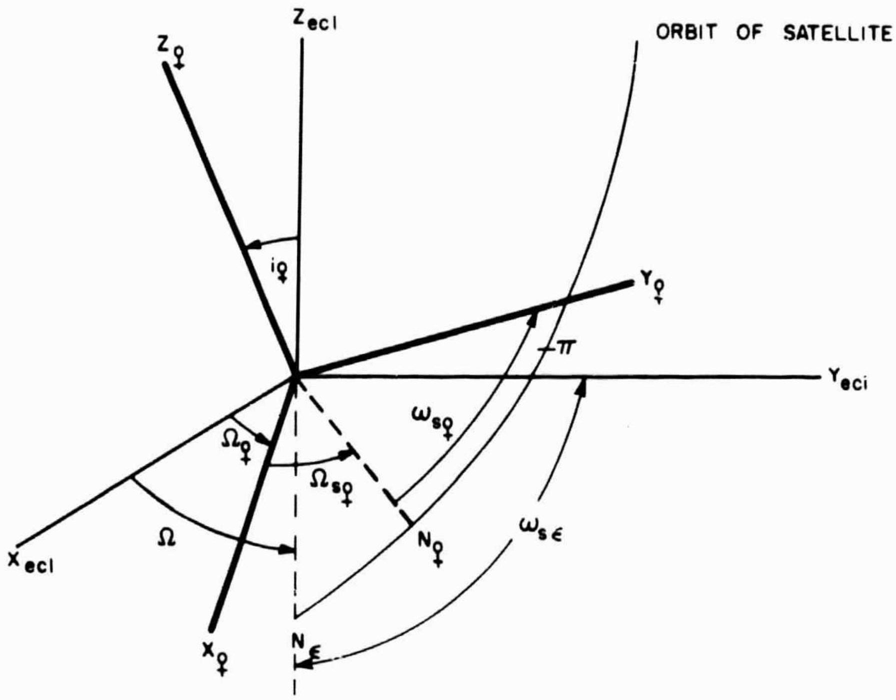


Figure A-1–Venus Coordinate System

Define:

$i_{♀}$ = inclination of Venus' orbital plane to ecliptic

$\Omega_{♀}$ = right ascension of ascending node of Venus' orbit upon the ecliptic

$\Omega_{s\epsilon}$ = right ascension of ascending node of satellite orbit upon ecliptic

$\omega_{s\epsilon}$ = argument of periapsis of satellite orbit referenced to ecliptic

N_{ϵ} = ascending node of satellite referenced to ecliptic

$\Omega_{s♀}$ = right ascension of ascending node of satellite orbit upon orbital plane of Venus

$\omega_{s♀}$ = argument of periapsis of satellite orbit referenced to orbital plane of Venus

$N_{♀}$ = ascending node of satellite referenced to orbital plane of Venus.

Let $\vec{r}^{\,o}_{\odot}$ be the Venus centered position of the Sun in mean ecliptic and equinox of Date Jan. 1, 1972 at 0^{hr}.

$$\vec{r}^{\,o}_{\odot} = \begin{pmatrix} \cos \Omega'_{\odot} \ \cos \phi'_{\odot} \\ \sin \Omega'_{\odot} \ \cos \phi'_{\odot} \\ \sin \phi'_{\odot} \end{pmatrix} \tag{A1}$$

where at the above date

$$\Omega'_{\odot} = 175\overset{\circ}{.}2756; \ \Omega_{♀} = 76\overset{\circ}{.}427751766$$

$$\phi'_{\odot} = 3\overset{\circ}{.}3542; \quad i_{♀} = 3\overset{\circ}{.}394354251$$

Let

$$T = x_{ecl} \, (i_{♀}) \, z_{ecl} \, (\Omega_{♀}) \tag{A2}$$

denote a rotation first about the z axis of the ecliptic through the angle $\Omega_{♀}$ followed by a rotation about the new x axis of the ecliptic through the angle $i_{♀}$. T is then the transformation matrix from the ecliptic plane to the orbital plane of Venus with the x axis in the direction of the ascending node of Venus' orbit upon the ecliptic and the Y axis 90° in the direction of planetary motion.

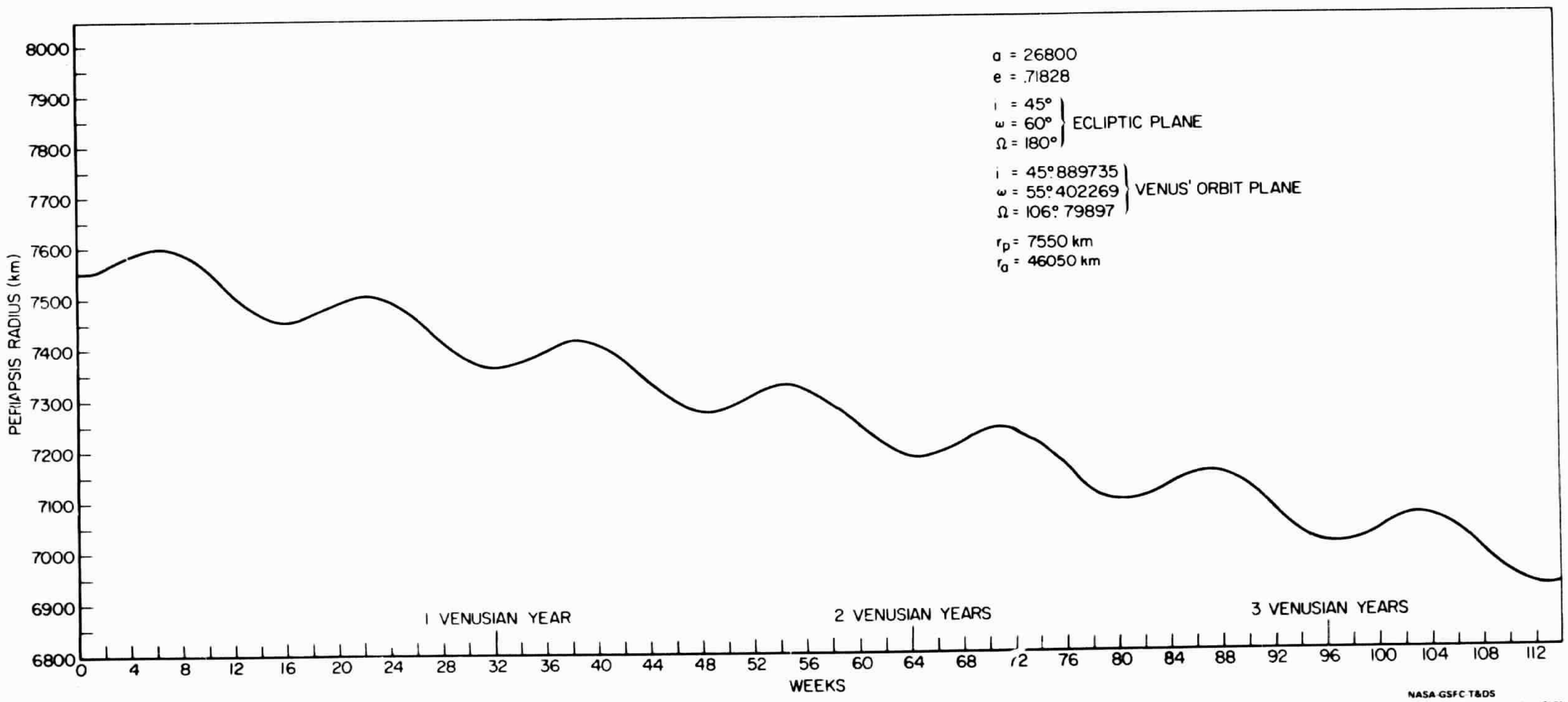


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Figure 10—Variation in Periapsis Radius.

Then $\vec{r}^{\,o}_{♀\odot}$, the position of the Sun in Venus' orbital plane at the above date, is

$$\vec{r}^{\,o}_{♀\odot} = T\,\vec{r}^{\,o}_{\odot} = \begin{pmatrix} \cos \Omega''_{\odot} \\ \sin \Omega''_{\odot} \\ 0 \end{pmatrix} \tag{A3}$$

also

$$\Omega''_{\odot}\,(1972) = \tan^{-1}\left(\frac{\sin \Omega''_{\odot}}{\cos \Omega''_{\odot}}\right)$$

To obtain $\Omega''_{\odot t}$ at any time t we must have then

$$\Omega''_{\odot t} = \Omega''_{\odot}\,(1972) + 1.6\,\Delta t \tag{A4}$$

where Δt is days from Jan. 1, 0^{hr} 1972. Equation (A4) is needed if the form of the disturbing function in equation (17) is used.

The orbital elements of the satellite referenced to Venus' orbital plane may now be found as follows:

The position $\vec{r}^{\,o}_{\epsilon}$ of the satellite referenced to the ecliptic is

$$\vec{r}^{\,o}_{\epsilon} = \vec{P}'\cos\theta + \vec{Q}'\sin\theta$$

where

$$\vec{P}' = \begin{bmatrix} \cos\Omega_{s\epsilon}\cos\omega_{s\epsilon} - \sin\Omega_{s\epsilon}\sin\omega_{s\epsilon}\cos i_{s\epsilon} \\ \sin\Omega_{s\epsilon}\cos\omega_{s\epsilon} + \cos\Omega_{s\epsilon}\sin\omega_{s\epsilon}\cos i_{s\epsilon} \\ \sin i_{s\epsilon}\sin\omega_{s\epsilon} \end{bmatrix}$$

$$\vec{Q}' = \begin{bmatrix} -\cos\Omega_{s\epsilon}\sin\omega_{s\epsilon} - \sin\Omega_{s\epsilon}\cos\omega_{s\epsilon}\cos i_{s\epsilon} \\ -\sin\Omega_{s\epsilon}\sin\omega_{s\epsilon} + \cos\Omega_{s\epsilon}\cos\omega_{s\epsilon}\cos i_{s\epsilon} \\ \sin i_{s\epsilon}\cos\omega_{s\epsilon} \end{bmatrix} \tag{A5}$$

and θ is the true anomaly of the satellite. Then $\vec{r}^{\,\circ}$, the position of the satellite referenced to Venus' orbital plane is

$$\vec{r}^{\,\circ} = T\,\vec{r}^{\,\circ}_{\epsilon} = (T\,\vec{P}')\cos\theta + (T\,\vec{Q}')\sin\theta$$

Letting $T\vec{P}' = \vec{P}$; $T\vec{Q}' = \vec{Q}$

$$\vec{r}^{\,\circ} = \vec{P}\cos\theta + \vec{Q}\sin\theta \tag{A6}$$

where $\vec{P}$ and $\vec{Q}$ are defined as above in (A5) with the appropriate change in notation to signify that the reference plane is now the orbital plane of Venus. Using the notation

$$\vec{P} = \begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix}$$

$$\vec{Q} = \begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \end{pmatrix}$$

and

$$\vec{P} \times \vec{Q} = \vec{R} = \begin{pmatrix} R_1 \\ R_2 \\ R_3 \end{pmatrix}$$

we have

$$\omega_{s♀} = \mathrm{Tan}^{-1}\left(\frac{P_3}{Q_3}\right) \tag{A7}$$

$$i_{s♀} = \cos^{-1}(P_1 Q_2 - P_2 Q_1) = \cos^{-1}(R_3) \tag{A8}$$

and

$$\Omega_{s♀} = \mathrm{Tan}^{-1}\left(\frac{P_2 Q_3 - P_3 Q_2}{P_1 Q_3 - P_3 Q_1}\right) = \mathrm{Tan}^{-1}\left(\frac{R_x}{-R_y}\right) \tag{A9}$$

Finally we write the full expansion for

$$T = \begin{bmatrix} \cos \Omega_♀ & \sin \Omega_♀ & 0 \\ -\sin \Omega_♀ \cos i_♀ & \cos \Omega_♀ \cos i_♀ & \sin i_♀ \\ \sin \Omega_♀ \sin i_♀ & -\cos \Omega_♀ \sin i_♀ & \cos i_♀ \end{bmatrix} \qquad \text{(A10)}$$

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1. Kaufman, B., "A Semi Analytic Method of Predicting the variation in Periapsis of a Mars Orbiter," GSFC Document X-551-69-41, February 1969.

2. Williams, R. R., Lorell, J., "The Theory of Long-Term Behavior of Artificial Satellite Orbits Due to Third-Body Perturbations," JPL Technical Report No. 32-916, February 15, 1966.

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