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AN APPROXIMATE SOLUTION OF ADDITIVE-DRAG COEFFICIENT AND MASS-FLOW RATIO FOR INLETS UTILIZING RIGHT CIRCULAR CONES AT ZERO ANGLE OF ATTACK

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16. Abstract

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SUMMARY

This paper presents an approximate solution of additive-drag coefficient and mass-flow ratio for inlets utilizing right circular cones at zero angle of attack. The results are in good agreement with the exact solution, especially at high Mach numbers and large cone half-angles. The present method uses the incompressible approximation of the conical flow field. This conical-flow-field approximation facilitates analytic integration of the pressures acting on the entering streamline. In addition, the flow-field properties obtained by the approximation are in good agreement with the results of the exact solution and other approximate solutions. The largest deviation between the exact solution and the present solution of additive-drag coefficient was $\Delta C_{D,a} = 0.039$.

INTRODUCTION

The calculation of additive-drag coefficient and mass-flow ratio for axisymmetric inlets is complex and time-consuming. The exact solution may be obtained by numerical integration of the pressures acting along the entering streamline. This procedure was used to obtain the results presented in reference 1. Using graphical or tabulated data for automated computation of installed performance of air-breathing engines requires an elaborate system of fitted curves over a wide range of conditions. This approach may limit computer storage, generality, and, most important, accuracy.

This paper presents an approximate method by which additive-drag coefficient and mass-flow ratio can be calculated directly. Direct calculation has been accomplished by using an approximate solution to represent the conical flow field. Use of this approximation permits the force on the curved streamline to be determined by analytic integration. The approximate expressions for additive-drag coefficient and mass-flow ratio for inlets utilizing right circular cones at zero angle of attack have been compared with the exact solution of reference 1.

SYMBOLS

a	nondimensional speed of sound, $\bar{a}/\bar{V}_l$
$\bar{a}$	speed of sound
A	cross-sectional area
$C_{D,a}$	additive-drag coefficient, $\frac{D_a}{qA_L}$
C_p	pressure coefficient
D_a	additive drag
M	Mach number, $\bar{V}/\bar{a}$
p	pressure
q	dynamic pressure, $\frac{\gamma}{2} p_\infty {M_\infty}^2$
u	nondimensional velocity along conical ray line in spherical coordinate system, $\bar{u}/\bar{V}_l$
v	nondimensional velocity normal to conical ray line in spherical coordinate system, $\bar{v}/\bar{V}_l$
$\bar{u},\bar{v}$	velocity components, dimensional
V	nondimensional resultant velocity at any ray line, $\bar{V}/\bar{V}_l$
$\bar{V}$	resultant velocity, dimensional
$\bar{V}_l$	limiting velocity due to adiabatic expansion into a vacuum
w	mass flow
w/w_∞	mass-flow ratio
γ	ratio of specific heat at constant pressure to specific heat at constant volume

θ conical ray angle measured from cone axis

Subscripts:

s cone surface

L location of inlet lip

t stagnation conditions

w conditions back of shock wave

∞ free-stream condition

METHOD OF SOLUTION

Conical-Flow Approximation

The purpose of this section is to derive a conical-flow-field approximation which permits the analytic integration of the pressures acting on the entering streamline. Also, the conical-flow-field approximation should provide good agreement with the exact solution over a wide range of conditions.

Two conical-flow approximations which satisfy the latter condition are presented in references 2 and 3. The method of reference 2 offers little advantage over the exact solution because it requires an iterative or semigraphical solution. In contrast, the method of reference 3 is a closed-form solution. Unfortunately, attempts to integrate the pressures acting on the entering streamline failed.

A conical-flow approximation has been found which satisfies the two requirements stated above. The differential equation which corresponds to this solution was first recognized in reference 4 as the incompressible solution to the supersonic conical flow field. This solution is more appropriately referred to as a constant-density solution as pointed out in reference 5. In reference 6, the solution to this equation has been applied to the hypersonic speed regime where the constant-density approximation is valid. However, it is not widely recognized that this solution gives good results even at the lower Mach numbers. The "Results and Discussion" section indicates that the results of the constant-density solution are comparable to those obtained by the approximation of reference 3.

The derivations of the basic equations for the conical-flow problem are presented in reference 4 and, therefore, will not be repeated herein. The second-order, nonlinear differential equation representing the conical flow field in the spherical coordinate system (fig. 1) is

$$\frac{d^2u}{d\theta^2} + u = \frac{a^2(u + v \cot \theta)}{v^2 - a^2} \tag{1}$$

where

$$v = \frac{du}{d\theta}$$

and

$$a^2 = \frac{\gamma - 1}{2}\left(1 - u^2 - v^2\right)$$

In these equations, the velocities had been nondimensionalized by dividing them by the limiting velocity attainable by adiabatic expansion into a vacuum. Rearranging equation (1) gives

$$\frac{d^2u}{d\theta^2} + u = \frac{u + v \cot \theta}{\frac{v^2}{a^2} - 1}$$

The equation is nonlinear because of the quantity v^2/a^2. The assumption of this paper is that $\frac{v^2}{a^2} << 1$ (constant-density solution).

The differential equation reduces to

$$\frac{d^2u}{d\theta^2} + \cot \theta \frac{du}{d\theta} + 2u = 0 \tag{2}$$

Equation (2) is linear and has as one solution $u = \cos \theta$. With this solution, the second solution can be found by quadratures, as follows:

$$u = \cos \theta\left\{C_1\left[\sec \theta + \log_e\left(\tan \frac{\theta}{2}\right)\right] + C_2\right\} \tag{3}$$

where C_1 and C_2 are constants of integration to be evaluated by the boundary conditions.

One boundary condition is at the conical surface when $\theta = \theta_s$ and is as follows:

$$v = \frac{du}{d\theta} = 0$$

This condition results in the relation

$$C_2 = C_1\left[\frac{\cos \theta_s}{\sin^2 \theta_s} - \log_e\left(\tan \frac{\theta_s}{2}\right)\right] \tag{4}$$

Substituting equation (4) into equation (3) gives

$$u = C_1 \cos\theta\left[\sec\theta + \log_e\left(\tan\frac{\theta}{2}\right) + \frac{\cos\theta_s}{\sin^2\theta_s} - \log_e\left(\tan\frac{\theta_s}{2}\right)\right] \tag{5}$$

The second boundary condition is behind the shock wave when $\theta = \theta_w$ and is as follows:

$$u_w = V_\infty \cos\theta_w$$

From reference 7,

$$V_\infty{}^2 = \frac{1}{1 + \frac{2}{\gamma - 1}\frac{1}{M_\infty{}^2}}$$

Therefore,

$$u_w = \sqrt{\frac{1}{1 + \frac{2}{\gamma - 1}\frac{1}{M_\infty{}^2}}}\cos\theta_w \tag{6}$$

Substituting equation (6) into equation (5) gives

$$C_1 = \frac{\sqrt{\frac{1}{1 + \frac{2}{\gamma - 1}\frac{1}{M_\infty{}^2}}}}{\sec\theta_w + \log_e\left(\tan\frac{\theta_w}{2}\right) + \frac{\cos\theta_s}{\sin^2\theta_s} - \log_e\left(\tan\frac{\theta_s}{2}\right)} \tag{7}$$

Substituting equation (7) into equation (5) gives

$$u = K\left\{1 + \cos\theta\left[\frac{\cos\theta_s}{\sin^2\theta_s} + \log_e\left(\frac{\tan\frac{\theta}{2}}{\tan\frac{\theta_s}{2}}\right)\right]\right\} \tag{8}$$

where

$$K = \frac{\sqrt{\frac{1}{1 + \frac{2}{\gamma - 1}\frac{1}{M_\infty{}^2}}}}{\sec\theta_w + \frac{\cos\theta_s}{\sin^2\theta_s} + \log_e\left(\frac{\tan\frac{\theta_w}{2}}{\tan\frac{\theta_s}{2}}\right)}$$

In addition,

$$v = \frac{du}{d\theta} = K\left\{\cot\theta - \sin\theta\left[\frac{\cos\theta_s}{\sin^2\theta_s} + \log_e\left(\frac{\tan\frac{\theta}{2}}{\tan\frac{\theta_s}{2}}\right)\right]\right\} \tag{9}$$

and

$$V = \sqrt{u^2 + v^2} = K\sqrt{\csc^2\theta + \left[\frac{\cos\theta_s}{\sin^2\theta_s} + \log_e\left(\frac{\tan\frac{\theta}{2}}{\tan\frac{\theta_s}{2}}\right)\right]^2} \tag{10}$$

Equations (8), (9), and (10) are the expressions for the flow velocities in terms of the cone half-angle, shock wave angle, free-stream Mach number, and ray angle.

It should be pointed out that the Rankine-Hugoniot conditions at the shock have not been satisfied. The second boundary condition merely specifies that the tangential velocity component is constant across the shock. The reason for this approach is to provide two methods for determining the shock wave angle:

(1) To impose the Rankine-Hugoniot condition and develop an expression for shock wave angle as a function of free-stream Mach number and cone half-angle

(2) To use the exact shock wave angle in tabular form to provide more precise results

With equations (8), (9), and (10) in their present form, either method may be used.

The Rankine-Hugoniot equations can be expressed as:

$$\tan\theta_w = \frac{\gamma - 1}{\gamma + 1}\,\frac{u_w^2 - 1}{u_w v_w} \tag{11}$$

Equations (8) and (9) evaluated at the shock wave become

$$u_w = \sqrt{\frac{1}{1 + \frac{2}{\gamma - 1}\frac{1}{M_\infty^2}}}\cos\theta_w \tag{12}$$

$$v_w = \sqrt{\frac{1}{1 + \frac{2}{\gamma - 1}\frac{1}{M_\infty^2}}}\left\{\frac{\cot\theta_w - \sin\theta_w\left[\frac{\cos\theta_s}{\sin^2\theta_s} + \log_e\left(\frac{\tan\frac{\theta_w}{2}}{\tan\frac{\theta_s}{2}}\right)\right]}{\sec\theta_w + \frac{\cos\theta_s}{\sin^2\theta_s} + \log_e\left(\frac{\tan\frac{\theta_w}{2}}{\tan\frac{\theta_s}{2}}\right)}\right\} \tag{13}$$

Substituting equations (12) and (13) into equation (11) and reducing gives

$$\frac{1}{M_\infty^2} = \sin^2\theta_w - \frac{\gamma + 1}{2}\left\{\frac{1}{1 + \cos\theta_w\left[\frac{\cos\theta_s}{\sin^2\theta_s} + \log_e\left(\frac{\tan\frac{\theta_w}{2}}{\tan\frac{\theta_s}{2}}\right)\right]}\right\} \tag{14}$$

Unfortunately, the shock angle cannot be calculated directly when cone half-angle and free-stream Mach number are specified. Since an iteration scheme is required to solve equation (12) when θ_s and M_∞ are specified, using the solution of the approximate shock angle offers little advantage over using a tabulation of the exact shock angle.

Additive-Drag Coefficient and Mass-Flow Ratio

By using the expressions for the velocity components in the conical flow field, the equations for additive-drag coefficient and mass-flow ratio can be derived. The detailed steps of the derivation are presented in the appendix. The resulting equation for the additive-drag coefficient is

$$C_{D,a} = C_p - \frac{w}{w_\infty} C_{p,w} - \frac{4K}{(\gamma - 1)M_\infty^2}\frac{p_{t,w}}{p_\infty}\frac{v\left(1 - V^2\right)^{\frac{1}{\gamma-1}}}{\sin\theta}\log_e\left(\frac{\tan\frac{\theta}{2}}{\tan\frac{\theta_w}{2}}\right) \tag{15}$$

where

$$C_p = \frac{2}{\gamma M_\infty^2}\left[\frac{p_{t,w}}{p_\infty}\left(1 - V^2\right)^{\frac{\gamma}{\gamma-1}} - 1\right]$$

and, from reference 7,

$$\frac{p_{t,w}}{p_\infty} = \left[\frac{\gamma + 1}{2\gamma M_\infty{}^2\sin^2\theta_w - (\gamma - 1)}\right]^{\frac{1}{\gamma-1}} \left\{\frac{(\gamma + 1)M_\infty{}^2\sin^2\theta_w\left[(\gamma - 1)M_\infty{}^2 + 2\right]}{2\left[(\gamma - 1)M_\infty{}^2\sin^2\theta_w + 2\right]}\right\}^{\frac{\gamma}{\gamma-1}}$$

The resulting equation for mass-flow ratio is

$$\frac{w}{w_\infty} = \frac{v}{v_w}\,\frac{\sin\theta_w}{\sin\theta}\left(\frac{1 - V^2}{1 - V_w{}^2}\right)^{\frac{1}{\gamma-1}} \tag{16}$$

From equations (15) and (16), the two obvious end results are satisfied:

When $\theta = \theta_w$

$$\frac{w}{w_\infty} = 1 \qquad C_{D,a} = 0$$

When $\theta = \theta_s$

$$v = 0 \qquad \frac{w}{w_\infty} = 0 \qquad C_{D,a} = C_{p,s}$$

RESULTS AND DISCUSSION

Conical-Flow Approximation

The purpose of this section is to provide some confidence in the use of the constant-density conical-flow solution over the Mach number range. Evidenced by reference 6, the constant-density solution has been successfully applied at high Mach numbers. However, no comparison of the results of the approximate methods with those of the exact solution was made at lower Mach numbers. Figure 2 presents the variation of the pressure coefficient with conical ray angle for four cones at zero angle of attack and at a free-stream Mach number of 2. The solid line corresponds to the exact solution of reference 8. The short dashed line corresponds to the approximate solution using equation (10) and the exact value of shock angle. The long dashed line corresponds to the approximate solution of reference 3. The results of the present solution agree reasonably well with the results of the exact solution and those obtained by the approximation of reference 3.

Additive-Drag Coefficient and Mass-Flow Ratio

Additive-drag coefficient and mass-flow ratio, equations (15) and (16), have been computed for an inlet cowl placed in the conical flow field at zero angle of attack. The results are presented for cone half-angles of 5°, 10°, 20°, and 30° in figures 3, 4, 5, and 6, respectively. Additive-drag coefficient and mass-flow ratio are plotted against ray angle for three Mach numbers: sonic flow on the cone surface, $M_{\infty} = 2$, and $M_{\infty} = 6$. The solid lines correspond to the exact solution obtained by numerical integration in reference 1. The dashed lines were computed from the approximate expressions (15) and (16). Additive-drag coefficient is in good agreement with the exact solution, especially at the higher Mach numbers and cone half-angles. The fact that the agreement in mass-flow ratio is very good over the full range of conditions indicates that the conical-flow approximation provides a good approximation of the entering streamline.

Figures 3 to 6 show that the maximum deviation in additive-drag coefficient occurs when the cowl lip is placed on the cone surface. Figure 7 shows the maximum deviation in additive-drag coefficient between the exact and approximate solutions. The difference between the exact and approximate additive-drag coefficient is plotted against Mach number for four cone half-angles. The largest deviation shown is $\Delta C_{D,a} = 0.039$ at a Mach number of 1.12 for $\theta_s = 10^{\circ}$. The importance of this error in terms of inlet and aircraft performance will vary with aircraft configuration and is beyond the scope of this paper.

CONCLUSION

An approximate method for calculating additive-drag coefficient and mass-flow ratio of inlets utilizing right circular cones at zero angle of attack has been presented. The solution uses the constant-density conical-flow-field equations. These equations are shown to provide good agreement with the exact cone solution over a wide range of variables. Results of the approximate expressions for additive-drag coefficient and mass-flow ratio are in good agreement with the results of the exact solution and other approximate solutions. The largest deviation between the exact solution and the present solution of additive-drag coefficient was $\Delta C_{D,a} = 0.039$.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., September 2, 1969.

APPENDIX

DERIVATION OF EQUATIONS FOR ADDITIVE-DRAG COEFFICIENT AND MASS-FLOW RATIO

Additive drag is defined as

$$D_a = \int_{\theta_w}^{\theta} (p - p_\infty) dA \tag{A1}$$

The additive-drag coefficient is given by

$$C_{D,a} = \frac{D_a}{qA_L} \tag{A2}$$

Substituting equivalent expressions for the parameters into equation (A2) and rearranging yields

$$C_{D,a} = \frac{2}{\gamma M_\infty^2 A_L} \int_{\theta_w}^{\theta} \left(\frac{p}{p_\infty} - 1\right) dA \tag{A3}$$

Because

$$\frac{p}{p_\infty} = \frac{p}{p_{t,w}} \frac{p_{t,w}}{p_\infty}$$

and

$$\frac{p}{p_{t,w}} = \left(1 - V^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$C_{D,a} = \frac{2}{\gamma M_\infty^2 A_L} \left[\frac{p_{t,w}}{p_\infty} \int_{\theta_w}^{\theta} \left(1 - V^2\right)^{\frac{\gamma}{\gamma-1}} dA - \left(A_L - A_w\right)\right] \tag{A4}$$

The mass-flow ratio captured by a cowl lip at position L is defined as follows:

$$\frac{w}{w_\infty} = \frac{\left(1 - V^2\right)^{\frac{1}{\gamma-1}} V}{C \sin \theta}$$

where

$$C = \frac{\left(1 - V_w^2\right)^{\frac{1}{\gamma-1}} v_w}{\sin \theta_w}$$

From reference 1,

$$A = \frac{\pi}{w/w_\infty} \tag{A5}$$

Substituting the equivalent expression for w/w_∞ into equation (A5) yields

$$A = \frac{\pi C \sin \theta}{\left(1 - V^2\right)^{\frac{1}{\gamma-1}} v}$$

Then

$$dA = \pi C \left[\frac{v \cos \theta \, d\theta - \sin \theta \, dv}{\left(1 - V^2\right)^{\frac{1}{\gamma-1}} v^2} + \frac{2 \sin \theta V \, dV}{(\gamma - 1)\left(1 - V^2\right)^{\frac{\gamma}{\gamma-1}} v}\right] \tag{A6}$$

Applying the expression for v from equation (9) gives

$$v \cos \theta \, d\theta - \sin \theta \, dv = \frac{2K}{\sin \theta} d\theta \tag{A7}$$

Applying the expressions for v and V from equations (9) and (10) gives

$$\frac{2 \sin \theta V \, dV}{v} = -\frac{2K}{\sin \theta} d\theta \tag{A8}$$

Substituting the equivalent expressions (A6), (A7), and (A8) into equation (A4) gives

$$C_{D,a} = \frac{2}{\gamma M_\infty{}^2 A_L} \left\{\frac{p_{t,w}}{p_\infty} \pi C \int_{\theta_w}^{\theta} \left[\frac{2K\left(1 - V^2\right)}{v^2 \sin \theta} - \frac{2K}{(\gamma - 1)\sin \theta}\right] d\theta - \left(A_L - A_w\right)\right\} \tag{A9}$$

The procedure for solving the first integral is as follows:

$$\int_{\theta_w}^{\theta} \frac{2K\left(1 - V^2\right)}{v^2 \sin \theta} d\theta = \int_{\theta_w}^{\theta} \frac{1 - K^2 \csc^2 \theta - K^2 \left[\frac{\cos \theta_s}{\sin^2 \theta_s} + \log_e\left(\frac{\tan \frac{\theta}{2}}{\tan \frac{\theta_s}{2}}\right)\right]^2}{K^2 \left\{\frac{\cos \theta}{\sin^2 \theta} - \left[\frac{\cos \theta_s}{\sin^2 \theta_s} + \log_e\left(\frac{\tan \frac{\theta}{2}}{\tan \frac{\theta_s}{2}}\right)\right]\right\}^2} 2K \csc^3 \theta \, d\theta \tag{A10}$$

If

$$R = K\left\{\frac{\cos\theta}{\sin^2\theta} - \left[\frac{\cos\theta_s}{\sin^2\theta_s} + \log_e\left(\frac{\tan\frac{\theta}{2}}{\tan\frac{\theta_s}{2}}\right)\right]\right\}$$

then

$$dR = -2K\csc^3\theta\, d\theta$$

Therefore,

$$\int_{\theta_w}^{\theta} \frac{2K\csc^3\theta\, d\theta}{K^2\left\{\frac{\cos\theta}{\sin^2\theta} - \left[\frac{\cos\theta_s}{\sin^2\theta_s} + \log_e\left(\frac{\tan\frac{\theta}{2}}{\tan\frac{\theta_s}{2}}\right)\right]\right\}^2} = -\int_{\theta_w}^{\theta}\frac{dR}{R^2} = \frac{1}{R}\bigg|_{\theta_w}^{\theta}$$

$$= \frac{1}{K\left\{\frac{\cos\theta}{\sin^2\theta} - \left[\frac{\cos\theta_s}{\sin^2\theta_s} + \log_e\left(\frac{\tan\frac{\theta}{2}}{\tan\frac{\theta_s}{2}}\right)\right]\right\}}\Bigg|_{\theta_w}^{\theta} \tag{A11}$$

If

$$S = -K^2\csc^2\theta - K^2\left[\frac{\cos\theta_s}{\sin^2\theta_s} + \log_e\left(\frac{\tan\frac{\theta}{2}}{\tan\frac{\theta_s}{2}}\right)\right]^2$$

then

$$dS = 2K^2\left\{\frac{\cos\theta}{\sin^2\theta} - \left[\frac{\cos\theta_s}{\sin^2\theta_s} + \log_e\left(\frac{\tan\frac{\theta}{2}}{\tan\frac{\theta_s}{2}}\right)\right]\right\}\csc\theta\, d\theta$$

Therefore,

$$\int S\, dR = SR - \int R\, dS$$

$$= -K\,\frac{\csc^2\theta + \left[\frac{\cos\theta_s}{\sin^2\theta_s} + \log_e\left(\frac{\tan\frac{\theta}{2}}{\tan\frac{\theta_s}{2}}\right)\right]^2}{\frac{\cos\theta}{\sin^2\theta} - \left[\frac{\cos\theta_s}{\sin^2\theta_s} + \log_e\left(\frac{\tan\frac{\theta}{2}}{\tan\frac{\theta_s}{2}}\right)\right]}\Bigg|_{\theta_w}^{\theta} - \int_{\theta_w}^{\theta} 2K\csc\theta\, d\theta \tag{A12}$$

Substituting equations (A11) and (A12) into equation (A10) gives the following equation for the first integral:

$$\int_{\theta_W}^{\theta} \frac{2K\left(1 - V^2\right)}{V^2 \sin \theta}\, d\theta = \left[\frac{1}{K\left\{\dfrac{\cos \theta}{\sin^2 \theta} - \left[\dfrac{\cos \theta_s}{\sin^2 \theta_s} + \log_e\left(\dfrac{\tan \frac{\theta}{2}}{\tan \frac{\theta_s}{2}}\right)\right]\right\}} \right.$$

$$\left. - K \frac{\csc^2 \theta + \left[\dfrac{\cos \theta_s}{\sin^2 \theta_s} + \log_e\left(\dfrac{\tan \frac{\theta}{2}}{\tan \frac{\theta_s}{2}}\right)\right]}{\dfrac{\cos^2 \theta}{\sin^2 \theta} - \left[\dfrac{\cos \theta_s}{\sin^2 \theta_s} + \log_e\left(\dfrac{\tan \frac{\theta}{2}}{\tan \frac{\theta_s}{2}}\right)\right]^2} \right]_{\theta_W}^{\theta}$$

$$- \int_{\theta_W}^{\theta} 2K \csc \theta \, d\theta$$

$$= \frac{\left(1 - V^2\right)\sin \theta}{V}\Bigg|_{\theta_W}^{\theta} - \int_{\theta_W}^{\theta} 2K \csc \theta \, d\theta \qquad \text{(A13)}$$

Substituting equation (A13) into equation (A9) gives

$$C_{D,a} = \frac{2}{\gamma M_\infty{}^2 A_L}\left\{\frac{p_{t,w}}{p_\infty}\pi C\left[\frac{\left(1 - V^2\right)\sin \theta}{V} - \frac{\left(1 - V_W{}^2\right)\sin \theta_W}{V_W}\right.\right.$$

$$\left.\left. - \frac{2K\gamma}{\gamma - 1}\int_{\theta_W}^{\theta} \csc \theta \, d\theta\right] - \left(A_L - A_W\right)\right\}$$

Solving the second integral and rearranging gives

$$C_{D,a} = \frac{2}{\gamma M_\infty^2}\left\{\frac{p_{t,w}}{p_\infty}\frac{\pi C}{A_L}\left[\frac{\left(1 - V^2\right)\sin\theta}{v} - \frac{\left(1 - V_w^2\right)\sin\theta_w}{v_w} - \frac{2K\gamma}{\gamma - 1}\log_e\left(\frac{\tan\frac{\theta}{2}}{\tan\frac{\theta_w}{2}}\right)\right] - 1 + \frac{A_w}{A_L}\right\}$$

Recognizing that

$$\frac{A_w}{A_L} = \frac{w}{w_\infty}$$

and

$$\frac{\pi C}{A_L} = \frac{v\left(1 - V^2\right)^{\frac{1}{\gamma-1}}}{\sin\theta}$$

$$C_{D,a} = \frac{2}{\gamma M_\infty^2}\left\{\frac{p_{t,w}}{p_\infty}\left[\left(1 - V^2\right)^{\frac{\gamma}{\gamma-1}} - \frac{w}{w_\infty}\left(1 - V_w^2\right)^{\frac{\gamma}{\gamma-1}}\right.\right.$$

$$\left.\left. - \frac{2K\gamma}{\gamma - 1}\frac{v\left(1 - V^2\right)^{\frac{1}{\gamma-1}}}{\sin\theta}\log_e\left(\frac{\tan\frac{\theta}{2}}{\tan\frac{\theta_w}{2}}\right)\right] - 1 + \frac{w}{w_\infty}\right\}$$

Since

$$C_p = \frac{2}{\gamma M_\infty^2}\left[\frac{p_{t,w}}{p_\infty}\left(1 - V^2\right)^{\frac{\gamma}{\gamma-1}} - 1\right]$$

then

$$C_{D,a} = C_p - \frac{w}{w_\infty}C_{p,w} - \frac{4K}{(\gamma - 1)M_\infty^2}\frac{p_{t,w}}{p_\infty}\frac{v\left(1 - V^2\right)^{\frac{1}{\gamma-1}}}{\sin\theta}\log_e\left(\frac{\tan\frac{\theta}{2}}{\tan\frac{\theta_w}{2}}\right) \tag{A14}$$

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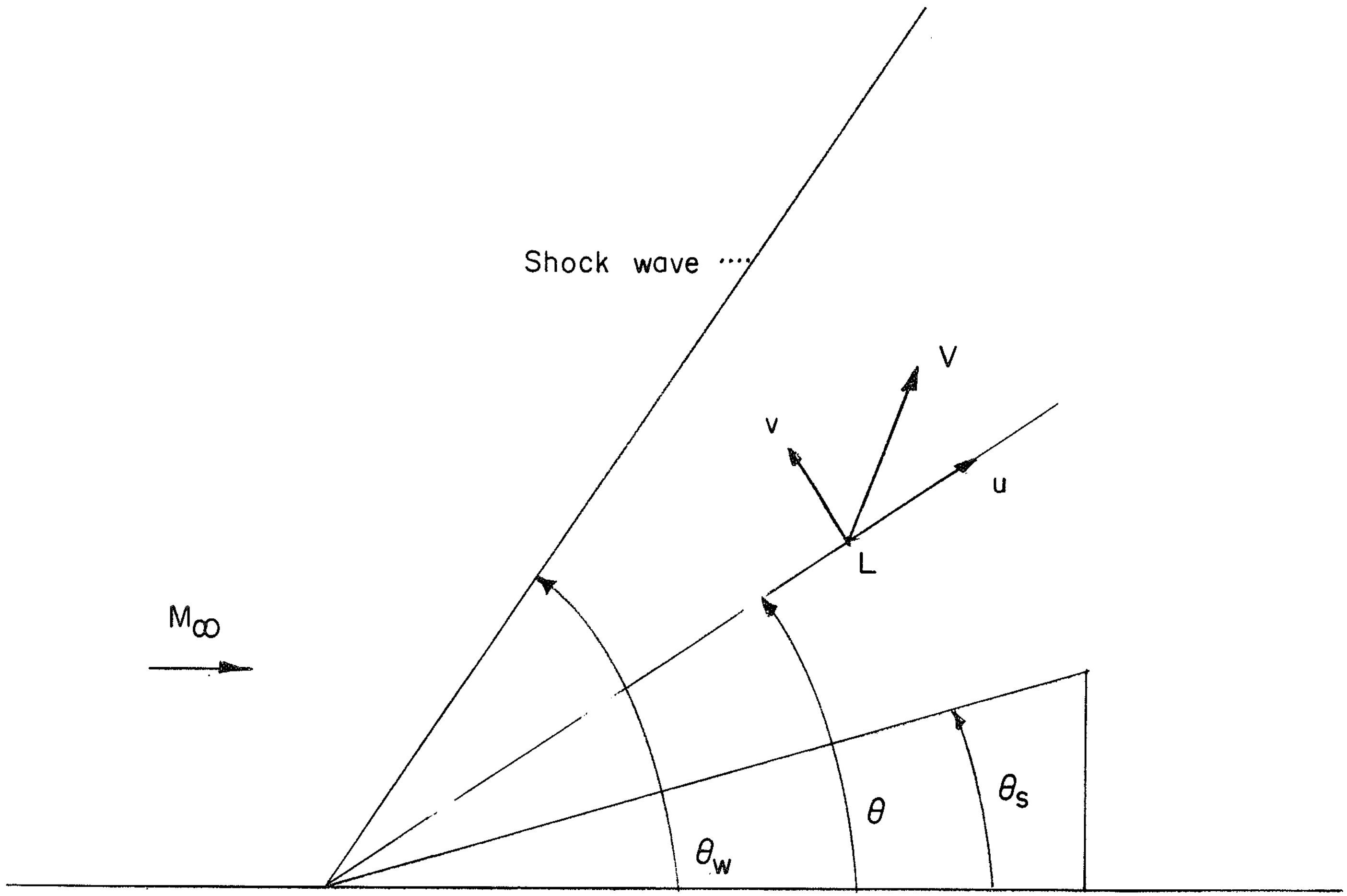

Figure 1.- Coordinate system.

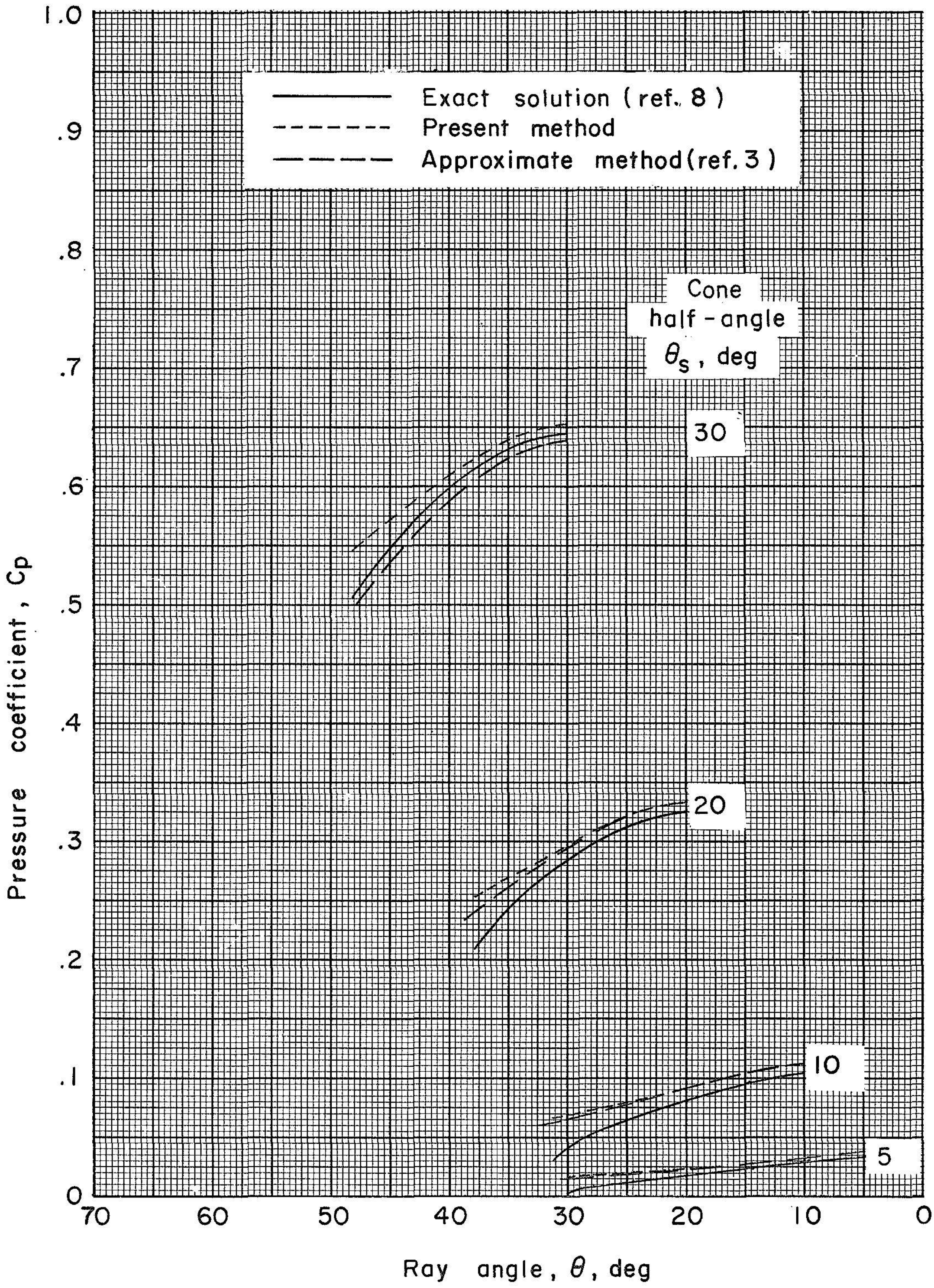

Figure 2.- Pressure coefficient in conical flow field. $M_\infty = 2$; $\gamma = 1.4$.

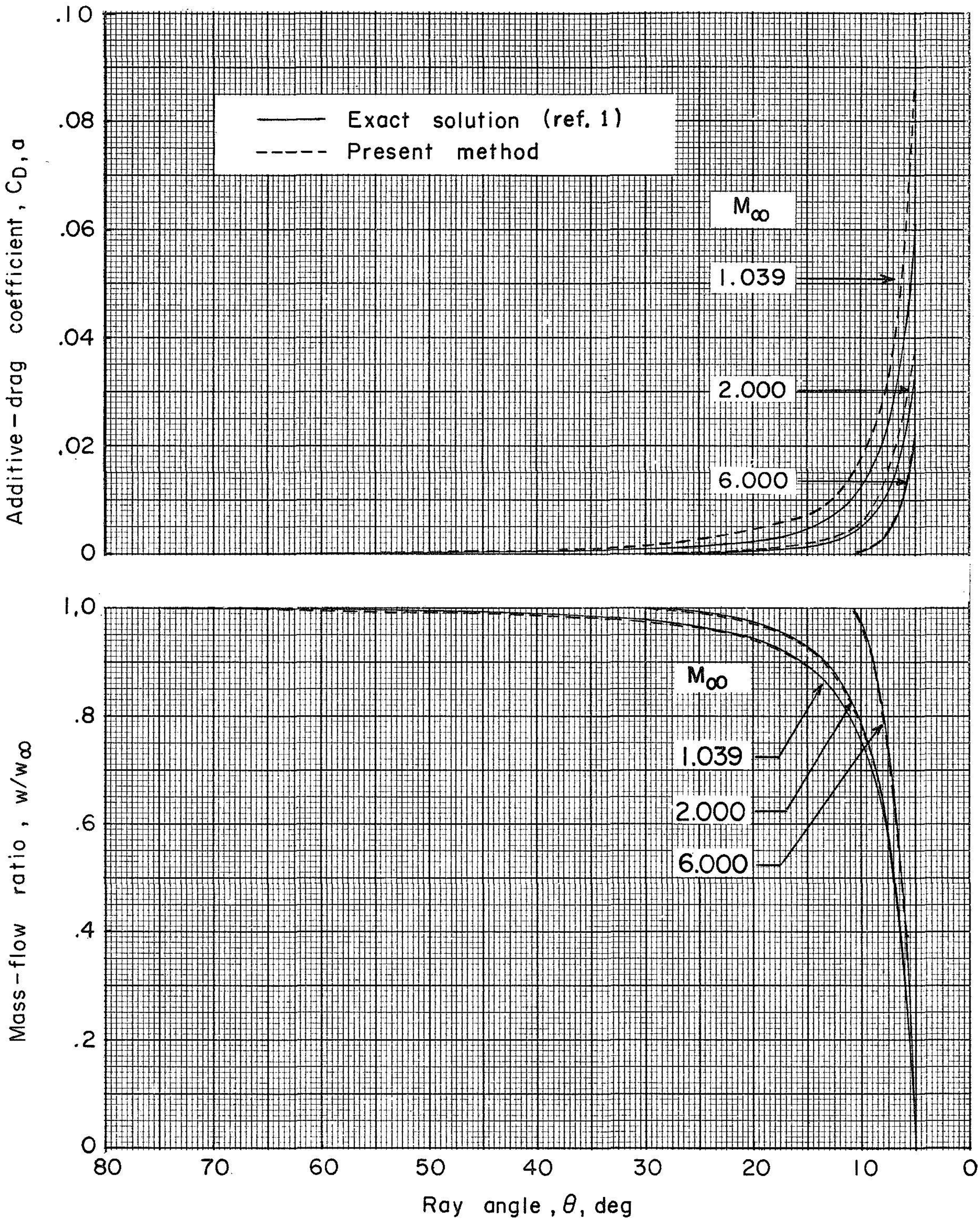

Figure 3.- Variation of additive-drag coefficient and mass-flow ratio with ray angle. $\theta_s = 5^0$; $\gamma = 1.4$.

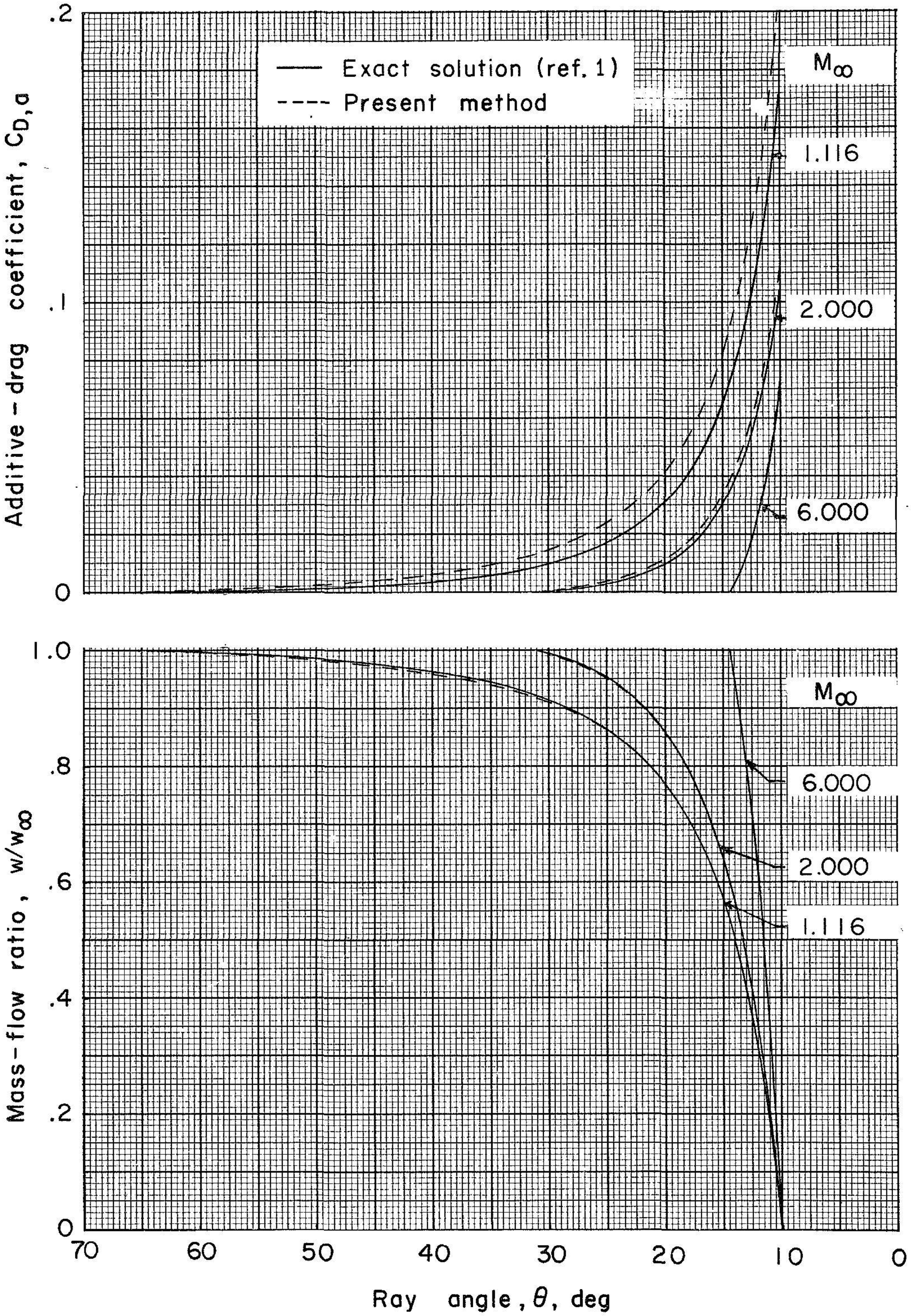

Figure 4.- Variation of additive-drag coefficient and mass-flow ratio with ray angle. $\theta_s = 10^0$; $\gamma = 1.4$.

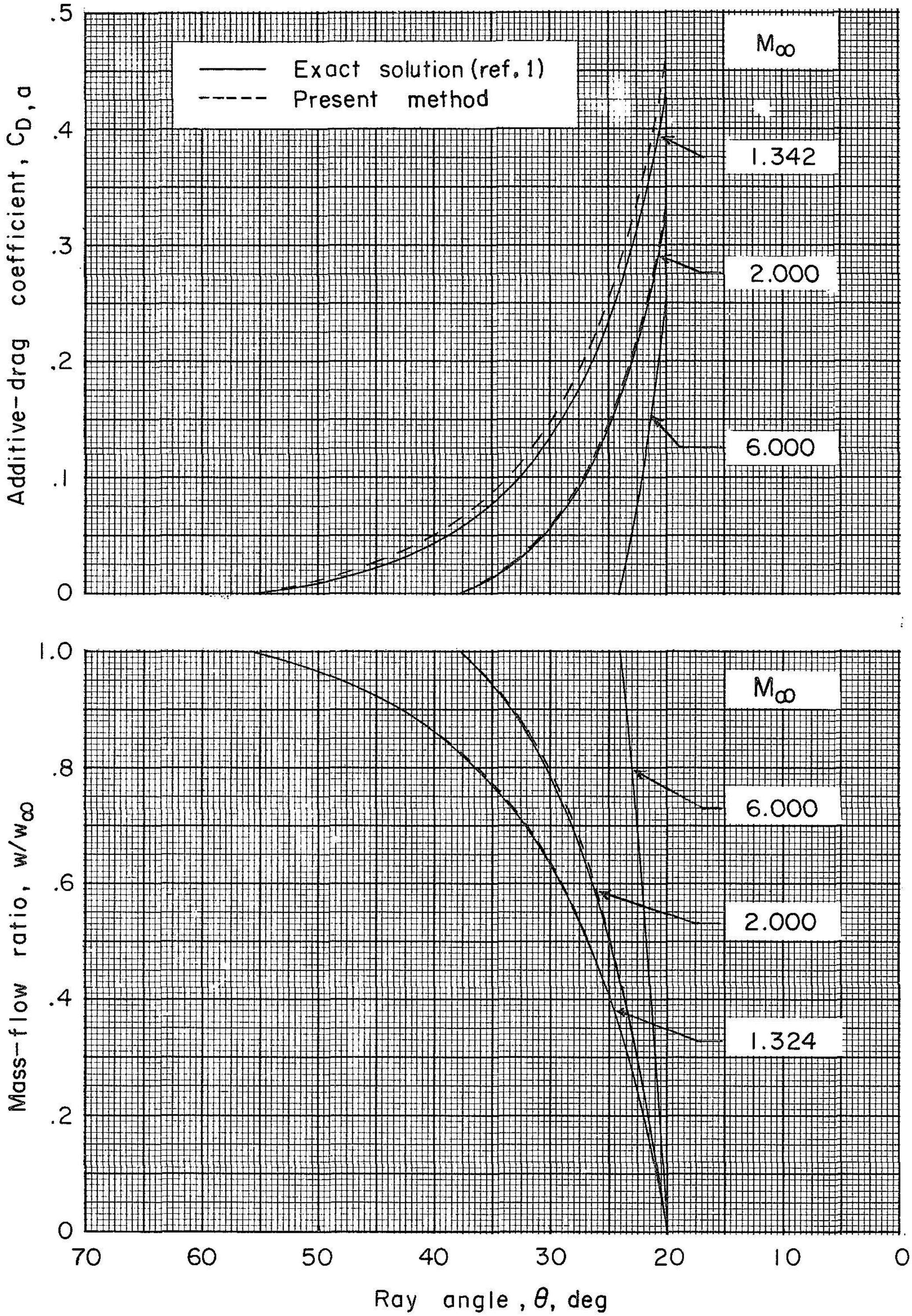

Figure 5.- Variation of additive-drag coefficient and mass-flow ratio with ray angle. $\theta_s = 20^0$; $\gamma = 1.4$.

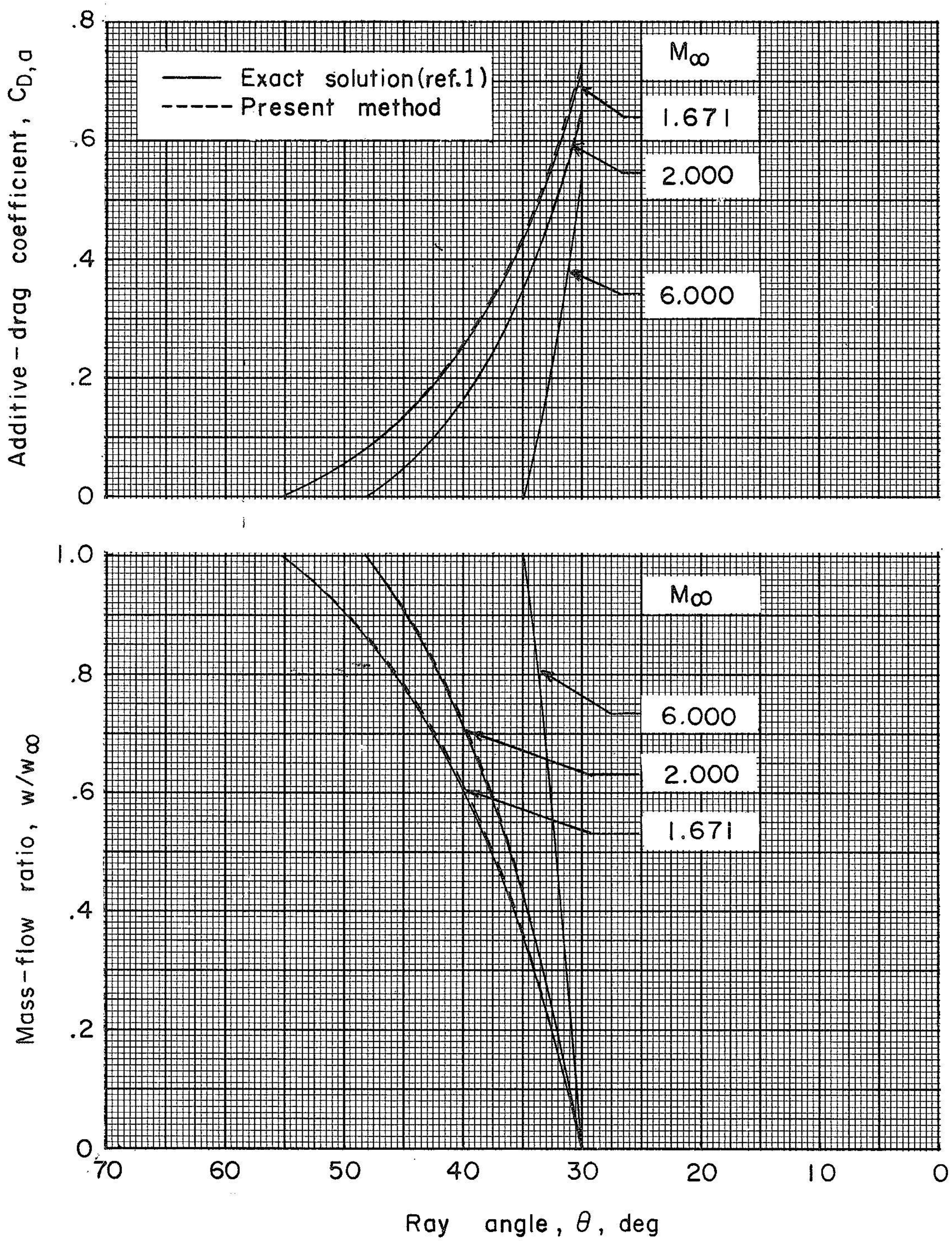

Figure 6.- Variation of additive-drag coefficient and mass-flow ratio with ray angle. $\theta_s = 30^0$; $\gamma = 1.4$.

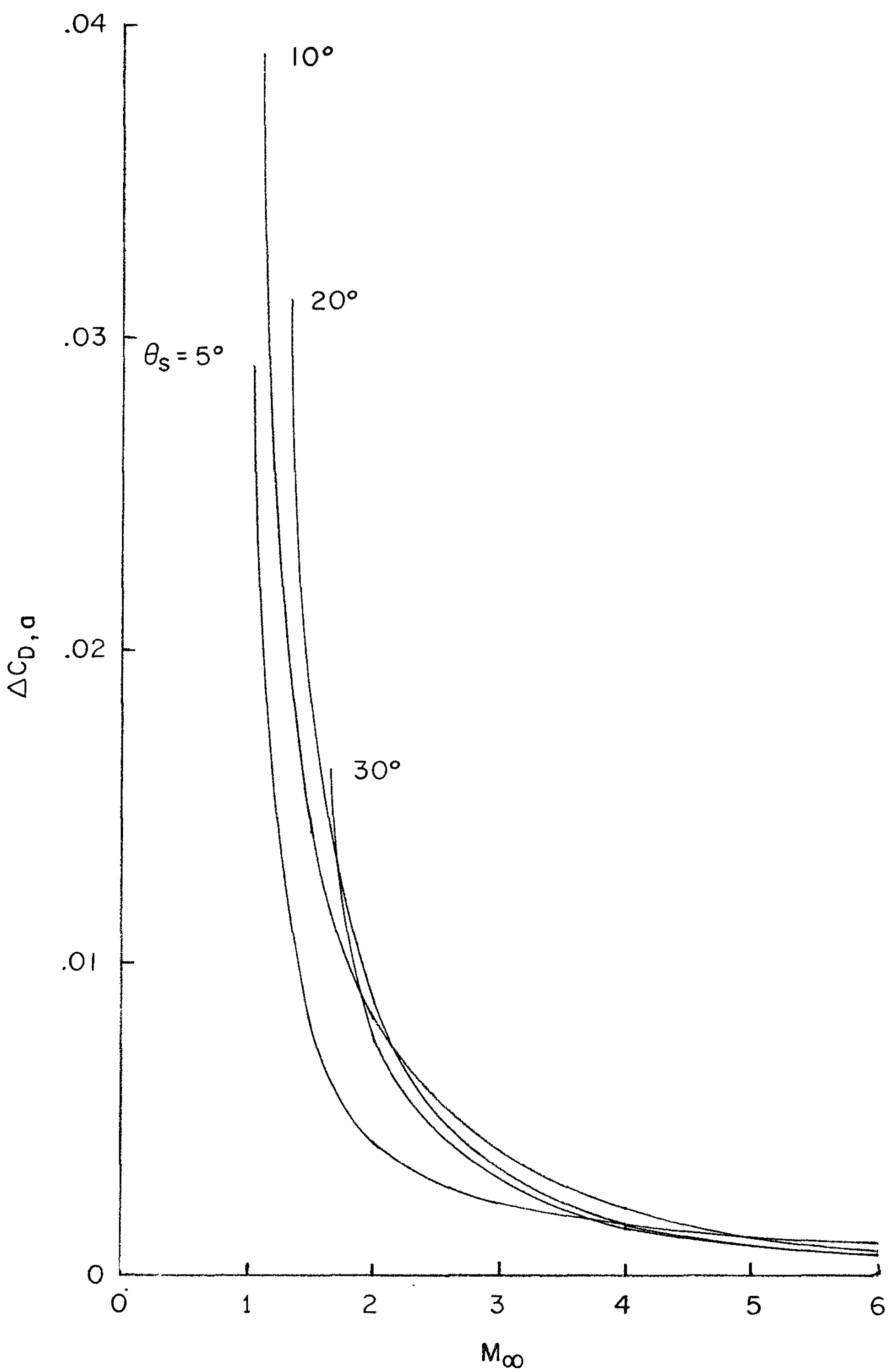

Figure 7.- Variation of maximum deviation in additive-drag coefficient with Mach number and cone half-angle. $\gamma = 1.4$.

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