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SHOCK-TUBE FLOW ANALYSIS WITH A DIMENSIONLESS VELOCITY NUMBER

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16. Abstract

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VELOCITY NUMBER

By N. Muniswamy Reddy*

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SUMMARY

The classical shock-tube equation, for a constant-area shock tube, is rederived in terms of a dimensionless velocity. The equation reduces to a universal form so that a single graphical plot gives the solution of the shock-tube equation for all combinations of pressures and temperatures in the driver. Real-gas effects behind the shock wave are included in the solution, but the driver gas is assumed to be perfect with a constant ratio of specific heats. Specific solutions for perfect and real driven gases are discussed. All the thermodynamic quantities behind the unsteady rarefaction wave, ratios across the contact surface, wave diagram parameters and testing time in a shock tube are expressed in terms of the dimensionless velocity. Twin tailoring constants are obtained from the universal shock-tube equation, one giving the tailoring Mach number and the other giving the loading pressure ratio and speed-of-sound ratio (in the driver-driven tubes) required to achieve the tailoring Mach number. Conditions for matching the thermodynamic quantities across the contact surface are given, in terms of the dimensionless velocity, and their significance in shock-tube performance is discussed.

INTRODUCTION

The problem of predicting the strength of the shock wave produced by instantaneously opening the diaphragm that separates a high and low pressure region has been considered extensively in reference 1. By using the conditions of equal pressure and equal velocity across the contact surface (C in fig. 1), one can derive an equation for the pressure ratio (P_{21}) across the moving shock in terms of known quantities[1] P_{41}, a_{41}, γ_1,

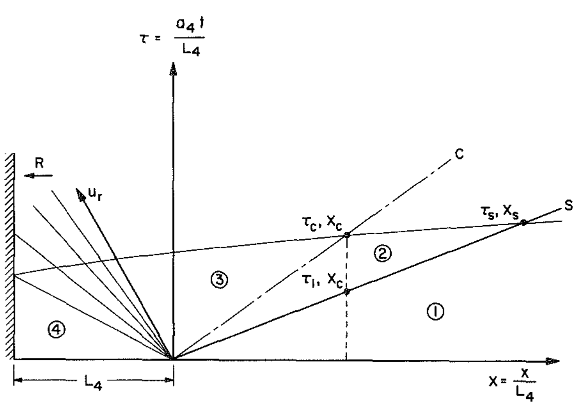

Figure 1.- Wave system in an ideal shock tube.

*This report was prepared while the author was a National Academy of Sciences' Resident Research Associate at the Ames Research Center. Dr. Reddy is now at Aerospace Research Laboratories, Ohio State University Research Foundation, Columbus, Ohio.

[1]The notation used here is the same as in reference 1; definitions of all parameters are given in the Notation in the appendix. Regions (1), (2), (3), and (4) are identified in figure 1.

and γ_4. For example, the equation derived in reference 1, for a perfect driver and driven gas combination, in a constant-area shock tube, is

$$(P_{14}P_{21})^{\beta_4} + \frac{\gamma_{41}\beta_4 a_{14}(P_{21} - 1)}{[\beta_1(\alpha_1 P_{21} + 1)]^{1/2}} - 1 = 0 \tag{1}$$

To evaluate P_{21} from equation (1) a graphical plot is necessary, since P_{21} is an implicit function of known quantities. Therefore, many graphical plots are given in reference 1 for various combinations of speed of sound and specific heat ratio. However, if a solution for a specific combination of a_{41}, P_{41}, and γ_{41} is desired, a new graphical plot must be generated. This problem becomes aggravated in combustion driven and electric arc heated shock tubes because different effective values of a_4, P_4, and γ_4 are obtained for each different run condition. Furthermore, consideration of real-gas effects (ref. 2) on the performance of the shock tube requires a separate graphical interpolation for each case. In addition, all other thermodynamic quantities in region (3) depend not only on the shock Mach number but also on a_4 and γ_4. Hence, a large series of curves for thermodynamic quantities in this region is needed (ref. 1).

Hall and Russo (refs. 3 and 4) recognized that equation (1) could be expressed in terms of two normalized parameters such that the functional relation between them is weakly dependent on just the quantity γ_4. Their work was limited to an ideal gas in both the driver and driven tubes. It is the purpose of this report to show that the shock-tube equation can be expressed in terms of a slightly different set of dimensionless numbers that permits one to include the effects of real driven gas, and moreover that permits all the flow quantities in different regions in the shock tube to be reduced to relatively simple functions of one dimensionless velocity number. In a sense, then, the work of Hall and Russo is generalized and extended here, although the present report pertains only to an area ratio of unity between driver and driven chambers, while theirs includes the effect of different area ratios in an ideal-gas analysis.

In the present analysis, we rederive the shock-tube equation in such a form that a single graphical plot gives the solution for all combinations of P_{41} and a_{41}. Furthermore, no assumption as to the thermodynamic state of the gas behind the shock wave (region (2)) is made in deriving this universal shock-tube equation, so that it also gives the solution for a real gas behind the shock wave. Specific ways of obtaining the shock Mach numbers for a perfect and real gas are discussed. In each case, the driver gas is treated as a perfect gas. However, an effective γ_4 can be chosen to account for some of the real-gas effects in the driver.

The following aspects of shock-tube flow are also discussed in terms of the new dimensionless velocity number:

(1) Functional dependence of all thermodynamic quantities in regions (2) and (3), and of the wave diagram parameters.

(2) Testing time.

(3) Tailored operation.[2] The universal shock-tube equation gives twin universal tailoring constants.

(4) Significance of matching the contact surface temperature.

THE SHOCK-TUBE EQUATION

In a shock tube, the unsteady rarefaction wave that travels into the driver converts the pressure and thermal energy into kinetic energy, and large particle velocities result behind the rarefaction wave (region (3)). The normal shock wave traveling into the driven gas also creates large particle velocities in region (2) and these must match the velocity at the contact surface with region (3). A general expression for this velocity, for perfect driver gas, is the well-known relation (ref. 1)

$$u_2 \equiv u_3 = \frac{2a_4}{\gamma_4 - 1}\left[1 - (P_{24})^{\beta_4}\right] \tag{2}$$

where the pressure, as well as the velocity, has been matched at the contact surface ($p_2 = p_3$). The maximum velocity (usually called the escape velocity) that can be achieved in region (3) is simply the limit of equation (2) when $P_{24} \to 0$

$$\hat{u}_3 = \frac{2a_4}{\gamma_4 - 1} \tag{3}$$

We shall define a dimensionless velocity

$$R_n = \frac{u_2}{\hat{u}_3} = \frac{u_3}{\hat{u}_3} \tag{4}$$

and in terms of this number, equation (2) becomes

$$P_{24}^{\beta_4} \equiv (P_{14}P_{21})^{\beta_4} = 1 - R_n \tag{5}$$

Now, according to conservation of mass and momentum across the shock wave, the pressure and velocity can be expressed in terms of the shock Mach number M_s and the density ratio $\epsilon = \rho_1/\rho_2$

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[2]Tailoring concept is defined elsewhere in this report.

$$P_2 = P_1[1 + \gamma_1(1 - \epsilon)M_s^2] \tag{6}$$

$$u_2 = (1 - \epsilon)U_s = (1 - \epsilon)a_1M_s \tag{7}$$

Equation (6) may be put in the form

$$P_{21} = \gamma_1 M_s^2 \left(1 - \epsilon + \frac{1}{\gamma_1 M_s^2}\right) \tag{8}$$

and from equations (7), (3), and (4)

$$M_s = \frac{2a_{41}R_n}{(1 - \epsilon)(\gamma_4 - 1)} \tag{9}$$

or

$$U_s/a_4 = 2R_n/[(1 - \epsilon)(\gamma_4 - 1)]$$

Then the shock-tube equation can be written

$$\left(\frac{2\gamma_1}{\gamma_4 - 1}\ \frac{a_{41}R_n}{\sqrt{\gamma_1 P_{41}}}\right)^{2\beta_4} \left[\frac{\left(1 - \epsilon + \dfrac{1}{\gamma_1 M_s^2}\right)^{\beta_4}}{(1 - \epsilon)^{2\beta_4}}\right] + (R_n - 1) = 0 \tag{10}$$

which when expanded in powers of quantities less than unity becomes

$$\left(\frac{2\gamma_1}{\gamma_4 - 1}\ \frac{a_{41}R_n}{\sqrt{\gamma_1 P_{41}}}\right)^{2\beta_4} \left(1 + \beta_4\epsilon + \frac{\beta_4}{\gamma_1 M_s^2} - 2\beta_4^2\epsilon^2 + \ldots\right) + (R_n - 1) = 0 \tag{11}$$

If quantities the order of $\beta_4\epsilon$ and $\beta_4/\gamma_1 M_s^2$ are neglected compared with unity, which is a good approximation for Mach numbers greater than about 4, the shock-tube equation becomes

$$S_n = \frac{(1 - R_n)^{1/2\beta_4}}{R_n} \tag{12}$$

where S_n is the dimensionless quantity $(\gamma_1/\gamma_4\beta_4)$ $(a_{41}/\sqrt{\gamma_1 P_{41}})$, which we shall call the shock-tube number. When $R_n \rightarrow 0$, $S_n \rightarrow \infty$, or, in other words, $a_{41} \rightarrow \infty$. When $R_n \rightarrow 1$, $S_n \rightarrow 0$, or $P_{41} \rightarrow \infty$. Thus, all possible shock Mach numbers are represented by R_n between 0 and 1. The approximation neglecting terms of order $\beta_4\epsilon$, that was used in deriving equation (12) becomes invalid at very low Mach numbers; but normally these are not the Mach numbers of interest. A plot of S_n vs. R_n is necessary to obtain R_n for any given conditions, but a single universal plot (fig. 2) is sufficient to solve the shock-tube equation for a given γ_4.

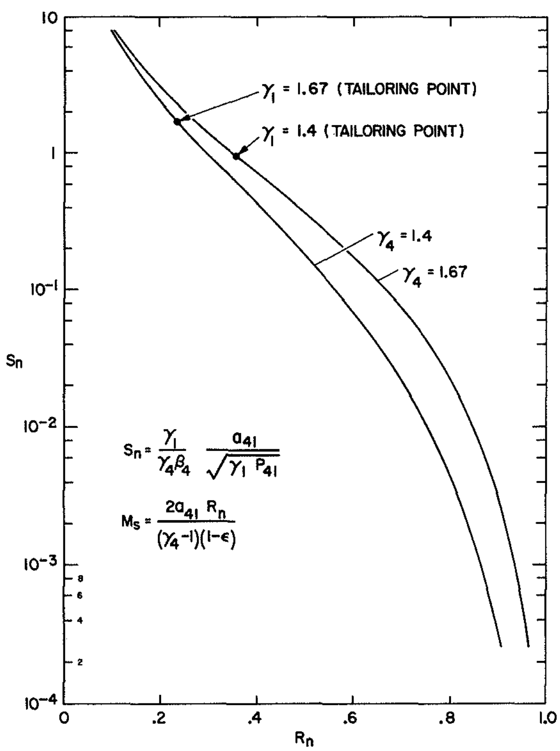

Figure 2.- Variation of S_n vs. R_n.

In the derivation of equation (12) no assumption has been made about the thermodynamic state of the driven gas behind the shock wave (region (2)), but the driver gas has been assumed to be perfect. Hence, the value of R_n obtained from equation (12) is unique in the sense that it is the same irrespective of the thermodynamic state of the gas behind the shock wave.

SOLUTIONS OF R_n

Limiting Cases

Perfect driven gas.- Invoking $a_2{}^2 = \gamma_1 P_2/\rho_2$ and using conservation of energy across the shock wave, one obtains

$$1 - \epsilon = \frac{2}{\gamma_1 + 1}\left(1 - \frac{1}{M_s{}^2}\right) \tag{13}$$

Then, the relation between R_n and M_s (eq. (9)) becomes

$$M_s\left(1 - \frac{1}{M_s{}^2}\right) = \delta a_{41} R_n \tag{14}$$

or

$$M_s \approx \delta a_{41} R_n \qquad \text{for} \qquad M_s{}^2 \gg 1$$

Hence, the problem of finding M_s or P_2 from the shock-tube equation is solved.

Driven gas with infinite heat capacity, $\epsilon \to 0(\gamma_2 \to 1)$.- In this limit

$$M_s = \frac{2a_{41}}{\gamma_4 - 1} \tag{15}$$

Real Driven Gas

The solution for M_s in the case of a real driven gas lies between the preceding two limits. The change in M_s between the two limits is

$$\Delta M_s = \frac{1}{\alpha_1}(M_s{}^*) \tag{16}$$

where $(M_s{}^*)$ corresponds to the perfect gas case. Thus, the change in M_s due to real-gas effects is typically the order of 15 percent or less. The relatively small size of the effect is due to the fact that the pressure and particle velocity behind a shock wave are not significantly affected by real-gas effects. The relatively strong effects are limited to temperature and density.

Exact solutions for M_s can be obtained by making use of existing charts of real-gas normal shock properties (such as given in ref. 5), along with the solution for R_n obtained from our universal curve (fig. 2). The value of R_n determines u_2 from equation (4) and the value of M_s can be determined from a plot of u_2 vs. M_s for a given P_1.

THE SHOCK-TUBE NUMBER S_n

The parameter S_n in equation (12) is a function of the two important parameters a_{41} and P_{41} in a shock tube. Note that the diaphragm pressure ratio P_{41} has far less effect on the shock-tube number than the speed-of-sound ratio a_{41}, since S_n varies as the square root of the pressure ratio but as the first power of the sound-speed ratio. This point has been discussed in reference 1, but it becomes immediately evident with the present universal form of the shock-tube equation (eq. (12)). It can be shown that S_n is a ratio of two limiting shock Mach numbers,

$$S_n = \frac{(M_s)_{P_{41}\to\infty}}{(M_s)_{a_{41}\to\infty}} \tag{17}$$

where

$$(M_s)_{P_{41}\to\infty} = \frac{2a_{41}}{(\gamma_4 - 1)(1 - \epsilon)} ; \qquad (M_s)_{a_{41}\to\infty} = \frac{\sqrt{\gamma_1 P_{41}}}{\gamma_1(1 - \epsilon)}$$

In addition, so long as the gases in both regions (1) and (4) obey the perfect gas law, it follows from the definition of S_n that

$$S_n = \frac{1}{\beta_4} \sqrt{\frac{\rho_{14}}{\gamma_4}} \tag{18}$$

Thus, the fundamental parameters in the analysis of shock-tube flow are the density ratio across the diaphragm ρ_{14} (eq. (18)) and the ratio of shock velocity to speed of sound in the driver gas U_s/a_4 (eq. (9)). These are the basic quantities represented by our dimensionless numbers S_n and R_n, and these basically determine the complete shock-tube performance.

It is interesting to compare these results with those of reference 3. The number S_n is essentially the same as Hall and Russo's Γ_{14}, the relation between them being

$$S_n = \frac{2}{\gamma_4 - 1} \sqrt{\frac{\gamma_4}{\Gamma_{14}}} \tag{18a}$$

In the ideal gas this is simply proportional to the square root of the initial density ratio across the diaphragm, as Hall and Russo have pointed out The dimensionless velocity R_n is somewhat different from Hall and Russo's parameter $M_s/\sqrt{P_{41}}$, however, even for an ideal gas:

$$R_n = \frac{\gamma_4 - 1}{2} (1 - \epsilon) \frac{M_s}{a_{41}} \xrightarrow[\text{gas}]{\text{ideal}} \frac{2}{\gamma_1 + 1} \sqrt{\frac{\gamma_1 \Gamma_{41}}{\gamma_4}} \left(\frac{M_s}{\sqrt{P_{41}}}\right) \tag{18b}$$

It is this difference that permits us to extend the analysis to nonideal driven gases and to derive simple expressions for all the shock-tube properties in terms of R_n. Thus, it appears that the parameter R_n is more useful in the shock-tube flow analysis than other dimensionless combinations.

ACCURACY OF SHOCK-TUBE EQUATION (EQ. (12))

In the derivation of equation (12), terms of the order $\beta_4 \epsilon$ and smaller were neglected. The error between the exact analysis (eq. (1)) for a perfect

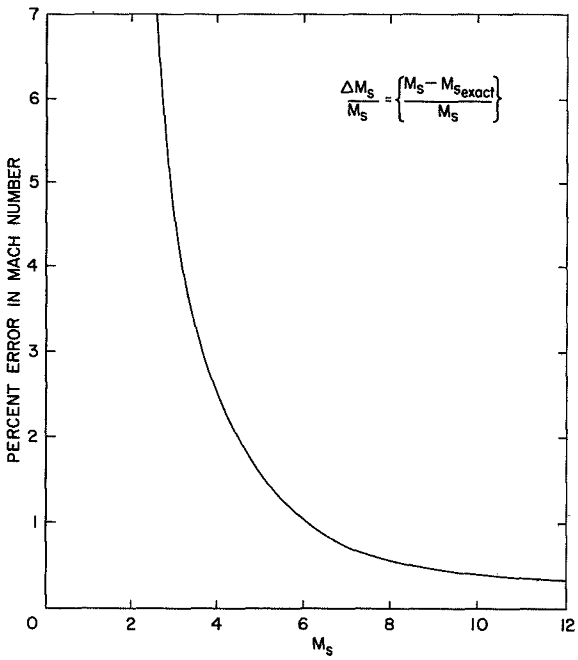

Figure 3.- Variation of percentage error with shock Mach number.

gas and the universal solution (eq. (12)) was computed at different shock Mach numbers and is shown in figure 3. The error is about 4.5 percent at $M_s = 3$ and reduces to 0.5 and 0.25 percent at $M_s = 8$ and 15, respectively, regardless of the value of β_4. The magnitude of error will be still less for a real gas because ϵ is smaller in this case than for the ideal gas.

SHOCK-TUBE FLOW QUANTITIES WITH R_n

Behind the Unsteady Rarefaction Wave

For a perfect driver gas (driven gas state arbitrary) the following thermodynamic quantities in region (3) can be expressed in terms of R_n.[3]

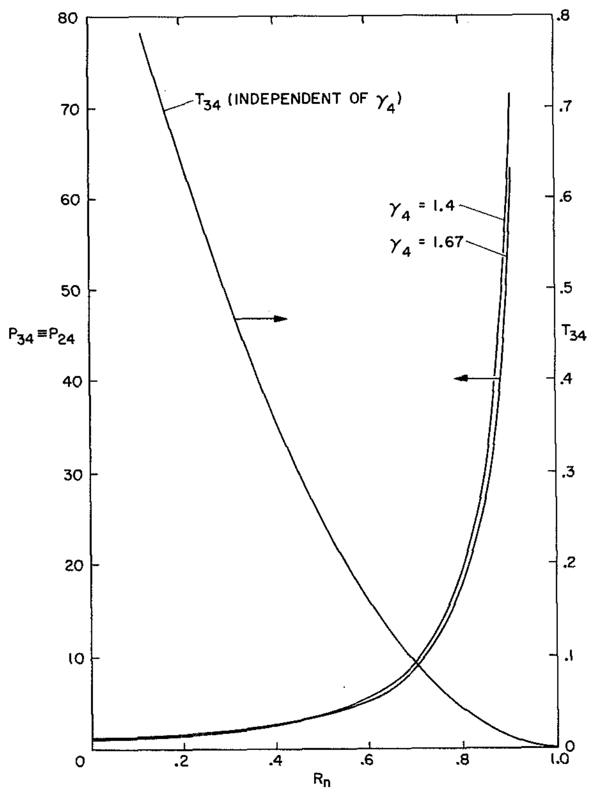

Figure 4.- Variation of P_{34} and T_{34} with R_n.

$$a_{34} = (1 - R_n) \tag{19}$$

$$T_{34} = (1 - R_n)^2 \tag{20}$$

$$\rho_{34} = (1 - R_n)^{\frac{1-2\beta_4}{\beta_4}} \tag{21}$$

$$P_{34} \equiv P_{24} = (1 - R_n)^{\frac{1}{\beta_4}} \tag{22}$$

T_{34} and P_{34} are plotted in figure 4. It can be seen that the effect of γ_4 is not very important as far as these quantities are concerned.

Across the Contact Surface

The ratios of thermodynamic quantities across the contact surface for

[3]The basic expressions for these quantities were taken from reference 1.

perfect driver and driven gases with $M_s^2 \gg 1$ are given below with only R_n as the variable:

$$E_{32} = \beta_4 \left(\frac{1 - R_n}{R_n}\right)^2 \tag{23}$$

$$T_{32} = \mu_{41}\left(\frac{1 - R_n}{R_n}\right)^2 \left(\frac{\beta_4 \alpha_1}{\delta}\right) \tag{24}$$

$$\rho_{32} = \left(\frac{R_n}{1 - R_n}\right)^2 \left(\frac{\delta}{\beta_4 \alpha_1}\right) \tag{25}$$

$$(\rho a)_{32} = \left(\frac{R_n}{1 - R_n}\right) \left(\frac{\delta \gamma_{41}}{\beta_4 \alpha_1}\right)^{1/2} \tag{26}$$

$$a_{32} = \left(\frac{1 - R_n}{R_n}\right) \left(\frac{\gamma_{41} \beta_4 \alpha_1}{\delta}\right)^{1/2} \tag{27}$$

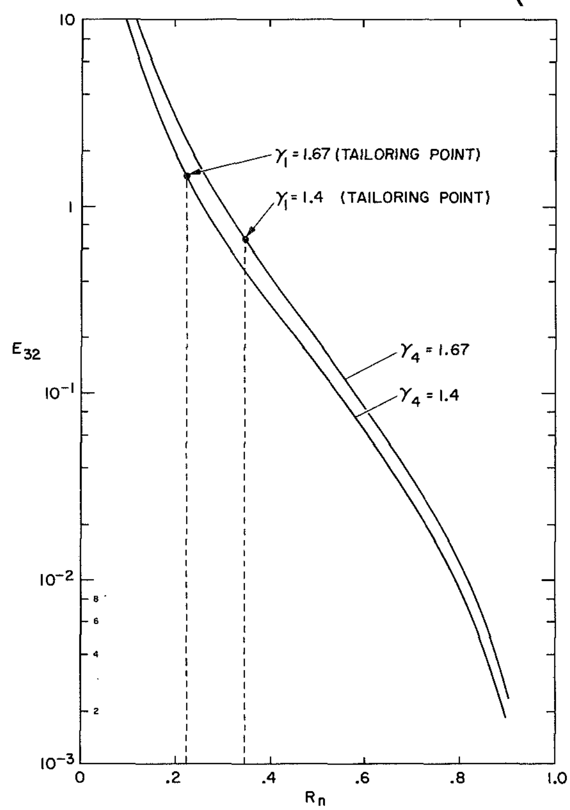

Figure 5.- Variation of E_{32} with R_n.

The internal energy ratio E_{32}, which is important in tailoring is plotted in figure 5. In this case, the value of γ_4 has a small but noticeable effect.

Shock-Tube Wave Diagram Parameters

Speed of the tail of rarefaction wave.- For a perfect driver gas and with arbitrary driven gas thermodynamics, the velocity of the tail of the rarefaction wave is given as

$$\frac{u_r}{a_4} = \alpha_4 R_n - 1 \tag{28}$$

If $R_n = 1/\alpha_4$ the velocity of the tail of the unsteady rarefaction wave becomes zero and hence the rarefaction wave remains inside the driver for all times. The corresponding shock-tube number is

$$\frac{2\gamma_1 a_{41}}{(\gamma_4 - 1)\sqrt{\gamma_1 P_{41}}} \equiv (S_n)_{u_r=0} = \frac{(\alpha_4 - 1)^{\frac{1}{2\beta_4}}}{\alpha_4^{\left(\frac{1-2\beta_4}{2\beta_4}\right)}} \tag{29}$$

Equation (29) gives the relation between a_4 and P_4 to keep the rarefaction wave always inside the driver.

τ_c, X_c, τ_s, and X_s.- Using the basic equations in reference 1, we express the quantities τ_c, X_c, τ_s, and X_s (which are defined in fig. 1) in terms of R_n:

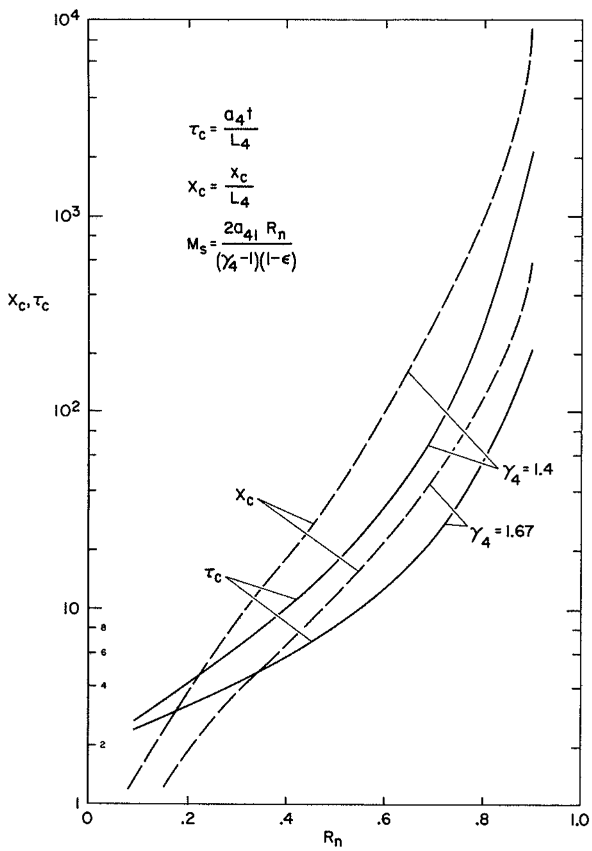

Figure 6.- Variation of τ_c and X_c with R_n.

Perfect driver gas assumption

$$\tau_c = 2(1 - R_n)^{-\alpha_4/2} \tag{30}$$

$$X_c = (\alpha_4 - 1)\tau_c R_n \tag{31}$$

Perfect driver and driven gases $(M_s^2 \gg 1)$

$$\tau_s = \frac{2}{1 - \beta_1^{1/2}} (1 - R_n)^{-\alpha_4/2} \tag{32}$$

$$X_s = \delta R_n \tau_s \tag{33}$$

These quantities are plotted in figures 6, 7, and 8, for various combinations of γ_4 and γ_1.

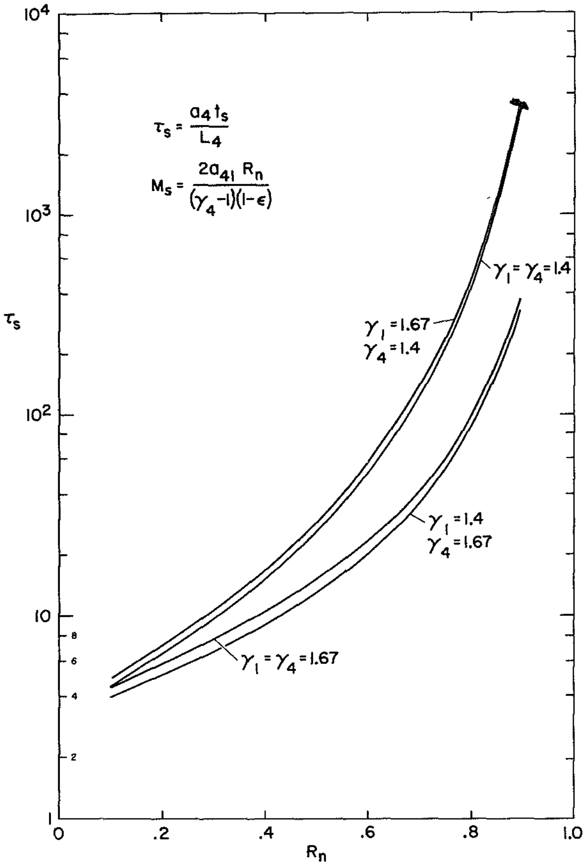

Figure 7.- Variation of τ_s with R_n.

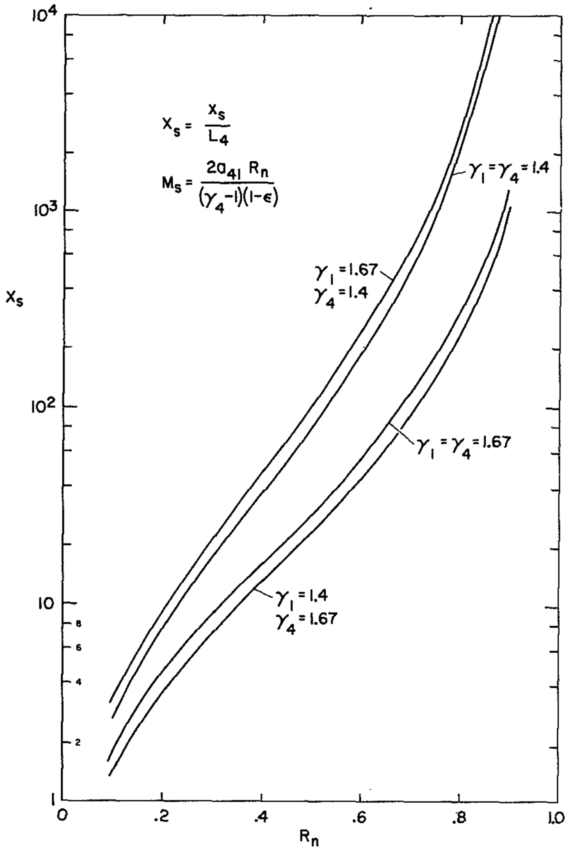

Figure 8.- Variation of X_s with R_n.

Testing Time in a Shock Tube

Testing time Δt in a shock tube is defined as the time elapsed between the arrival of the shock wave and contact surface at a given station along the driven tube. Using the relation between u_2 and U_s (eq. (7)), one can express Δt in terms of R_n as

$$\frac{a_4 \Delta t}{x_t} = \frac{\epsilon \alpha_1 \beta_1 \gamma_1}{\delta R_n} \tag{34}$$

where x_t is the testing station along the shock tube. Once R_n is obtained from the universal plot (fig. 2), Δt is known. It should be noted that Δt becomes very small for a real gas since $\epsilon \to 0$, especially at high shock Mach numbers. The location for maximum testing time from figure 1 should be equal to X_c. Then, using equations (9) and (31), we have

$$(\Delta\tau)_{max} = (\tau_c - \tau_1) = \epsilon\tau_c \tag{35}$$

where

$$(\Delta\tau)_{max} = \frac{\Delta t a_4}{L_4}$$

TAILORED OPERATION IN A SHOCK TUBE

If the reflected wave after the interaction of a shock wave with a contact surface is a Mach wave, then the contact surface is tailored (refs. 1 and 6). For tailored operation, a condition of the type given below can easily be derived for the perfect driver and driven gas case (refs. 1 and 6).

$$(E_{32})_T = \frac{\alpha_4 + P_{25}}{\alpha_1 + P_{25}} \tag{36}$$

and

$$P_{25} = \frac{2 + (\gamma_1 - 1)M_s^2}{(3\gamma_1 - 1)M_s^2 - 2(\gamma_1 - 1)} \tag{37}$$

for

$$M_s^2 \gg 1 \ , \quad (E_{32})_T \approx \alpha_{41}$$

According to equation (23) for E_{32}, the tailoring constant is given by

$$(R_n)_T = \frac{(\alpha_1\beta_4)^{1/2}}{\alpha_4^{1/2} + (\alpha_1\beta_4)^{1/2}} \tag{38}$$

In terms of M_s,

$$(M_s)_T = \delta a_{41}\left[\frac{(\alpha_1\beta_4)^{1/2}}{\alpha_4^{1/2} + (\alpha_1\beta_4)^{1/2}}\right] \tag{39}$$

In addition, the shock-tube constant for tailoring is (from eq. (12))

$$(S_n)_T \equiv \frac{2\gamma_1 a_{41}}{(\gamma_4 - 1)\sqrt{\gamma_1 P_{41}}} = \frac{(\alpha_{41}/\beta_4)^{\frac{1}{4\beta_4}}}{\left[1 + (\alpha_{41}/\beta_4)^{1/2}\right]^{\frac{1-2\beta_4}{2\beta_4}}} \tag{40}$$

The twin tailoring conditions (eqs. (38) and (40)) can be represented by a unique point on the universal plot of the shock-tube equation as shown in figure 2. The basic condition for tailoring is given by S_n. This gives an explicit relation between a_{41} and P_{41} for tailoring which has been made possible by the universal form of the shock-tube equation.

CONTACT SURFACE MATCHING

In the tailored operation the contact surface is not totally matched in the sense that neither internal energy nor any of the other thermodynamic quantities (except P and u) are equated across the contact surface.

(1) For a tailored operation ($E_{32} \approx \alpha_{41}$)

$$T_{32} = \mu_{41}\left(\frac{\gamma_1 + 1}{\gamma_4 + 1}\right) \tag{41}$$

If acoustic impedance (ρa), density, and temperature have to be matched across the contact surface, the following conditions in terms of temperature ratio must be satisfied.

(2) For acoustic impedance matching ($\rho a_{32} = 1$)

$$T_{32} = \gamma_{41}\mu_{41} \tag{42}$$

(3) For density matching ($\rho_{32} = 1$)

$$T_{32} = \mu_{41} \tag{43}$$

(4) For temperature matching ($T_{32} = 1$)

$$T_{32} = 1 \tag{44}$$

It is interesting that

$$(T_{32})_T \approx (T_{32})_{\rho a_{32}=1} \approx (T_{32})_{\rho_{32}=1}$$

Thus, in a tailored operation, density and acoustic impedance across the contact surface are nearly matched but not the temperature ratio.

For contact surface temperature matching ($T_3 = T_2$) the dimensionless number R_n should be, according to equation (24),

$$(R_n)_{TM} = \frac{(\alpha_1\beta_4)^{1/2}}{(\alpha_1\beta_4)^{1/2} + (\delta\mu_{14})^{1/2}} \tag{45}$$

The corresponding shock-tube number is

$$(S_n)_{TM} = \frac{(\mu_{14}\delta/\alpha_1\beta_4)^{\frac{1}{4\beta_4}}}{[1 + (\mu_{14}\delta/\alpha_1\beta_4)^{1/2}]^{\frac{1-2\beta_4}{2\beta_4}}} \tag{46}$$

It should be noted that R_n and S_n for tailoring (eqs. (38) and (40)) do not depend on the molecular weights, but R_n and S_n for contact surface temperature matching do.

For simultaneous tailoring and temperature matching

$$(R_n)_T = (R_n)_{TM}$$

hence,

$$\mu_{14} = \frac{\gamma_4 + 1}{\gamma_1 + 1} \equiv \frac{\alpha_4}{\delta} \tag{47}$$

Then $(S_n)_T$ becomes equal to $(S_n)_{TM}$.

Even for different γ combinations α_4/δ is not very different from unity; hence μ_{14} (molecular weight ratio) should be nearly equal to unity for simultaneous tailoring and temperature matching.

It is well recognized that in a shock-tube operation, contact surface mixing significantly reduces the testing time, particularly at high shock Mach numbers. This mixing is probably due to unequal temperature and density across the contact surface. Thus, it is worthwhile to match the thermodynamic quantities across the contact surface. This requires, according to the preceding discussion, that driver and driven gas molecular weights be nearly equal.

CONCLUDING REMARKS

The problem considered is that of predicting the strength of a shock wave produced when the diaphragm in a shock tube is opened. Without making any assumptions as to the thermodynamic state of gas behind the shock wave, but assuming that the driver gas behaves as perfect gas, we rederive the shock-tube equation in terms of two new dimensionless numbers. The equation reduces to a universal form so that a single graphical plot suffices for all combinations of initial conditions. The fundamental parameters that influence the performance of a shock tube are the density ratio across the diaphragm, the shock velocity, and the speed of sound in the driver gas. All the flow parameters in a shock tube can be expressed in terms of these dimensionless numbers for all possible shock Mach numbers. The present universal shock-tube equation provides twin universal tailoring constants and puts the tailoring operation of a shock tube in clear perspective. It is hoped that the present analysis will enhance the understanding of the role of many variables in a shock-tube flow.

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APPENDIX

NOTATION

a	speed of sound
E	internal energy
M_s	shock Mach number, $\frac{U_s}{a_1}$
P	pressure
R_n	dimensionless velocity (defined in eq. (12))
S_n	dimensionless shock-tube number (eq. (12))
U_s	shock velocity
u	particle velocity
u_r	speed of the tail of rarefaction wave
ρ	density
μ	molecular weight
Δt	testing time in shock tube
γ	ratio of specific heats
ϵ	$\frac{\rho_1}{\rho_2}$ (density ratio across the shock wave)
α	$\frac{\gamma + 1}{\gamma - 1}$
β	$\frac{\gamma - 1}{2\gamma}$
δ	$\frac{\gamma_1 + 1}{\gamma_4 - 1}$
ψ_{jk}	$\frac{\psi_j}{\psi_k}$ (unless otherwise defined)

Regions (1), (2), (3), and (4) are identified in figure 1.

Subscripts 1, 2, 3, and 4 refer to regions (1), (2), (3), and (4) as identified in figure 1.

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