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High-Frequency Beam-Plasma Interaction at Finite Temperatures

by
M. Seidl

June 1969

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Abstract

The dispersion equation corresponding to the interaction of an electron beam with a Maxwellian plasma has been solved in quasi-static and infinite geometry approximations.

Two representative beams are considered: a parallel beam with isotropic velocity spread and a helical beam with different longitudinal and transverse temperatures. The aim of the work is to present an "Experimenter's Guide" to predict the frequency, growth rate, and wave numbers of the most unstable waves as functions of the basic beam and plasma parameters. The results of the calculations are presented in graphs suitable for comparison with the experiments. The effect of plasma and beam temperatures on the spectrum of the instabilities is discussed in detail. It is shown that the plasma temperature reduces the growth rate of the instabilities at cyclotron harmonic frequencies while a velocity spread in the helical beam suppresses the instabilities everywhere, except in the vicinity of the cyclotron harmonics.

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1. Introduction

The interaction of an electron beam with a cold or moderately warm plasma has been investigated with growing interest since more attention has been directed towards basic understanding of collective processes in plasmas. This is due to the fact that beam-plasma interaction represents the simplest system in which velocity-space instabilities occur in pure form.¹⁻³ Beam-plasma interaction is useful for studying linear and non-linear properties of these instabilities as well as for simulating the closely related instabilities of a hot plasma. Further interest in this field has been stimulated by the use of beam-plasma interaction for plasma production and heating⁴ and by observations of rf emission from laboratory plasmas⁵ as well as from outer space.

This paper is concerned with high frequency instabilities ($\omega > \sqrt{\omega_{ci}\omega_{ce}}$) of beam-plasma systems in a uniform magnetic field. Such instabilities have been investigated in numerous experiments. In typical beam-plasma experiments, the electrons are injected into the plasma from a gun with velocities directed either parallel⁶⁻¹⁴ or at an angle¹¹⁻¹⁷ to the magnetic field. The plasma is usually produced by the beam, although cleaner measurements are possible when the plasma is produced independently of the beam.¹³ In another group of experiments, started by Landauer¹⁸ and followed by many others,⁵ noise emission at electron cyclotron harmonics has been observed from discharge plasmas in a magnetic field. There is strong evidence⁵ that this emission is due to instabilities excited by groups of superthermal electrons produced in the discharge.

The linear theory of high-frequency interaction of unbounded, weak electron beams with plasma has also been extensively treated, assuming

either parallel^{1,2,19} or helical²⁰⁻²³ beams. The effect of finite transverse dimensions of cold²⁴⁻²⁶ or warm²⁷⁻²⁹ beams has also been investigated.

In spite of all this work, it is often very tedious to make even rough comparisons of the frequency observed in the experiments, with theoretical predictions. This is not due to an inadequacy of linear theory, but on the contrary, many authors came to the conclusion that it is the wave with the largest linear growth rate which is experimentally observed.^{8,10,12,25} The difficulty is that many instability modes are possible in a beam-plasma system, and their frequency, wavelength, and growth rate depend on a large number of parameters. Most of the general theories are not explicit enough to enable selection of the mode with the largest growth rate for given conditions.

This paper is an attempt to present an "Experimenter's Guide" to predict the frequency, wavelength, and growth rate of the most unstable mode for a wide range of plasma and beam parameters. In order to keep the problem tractable, several drastic approximations have been made at the start of the analysis. Quasi-static theory of an infinite beam-plasma system has been used assuming the beam density much smaller than the plasma density. Transverse boundary conditions have been taken into account only by assuming some perpendicular wave number $k_{\perp}$. Longitudinal boundary conditions have been neglected altogether. The same approach has been shown to be useful previously¹² when evaluating experimental results, although the theory¹² has considered only cold beam and cold plasma interaction. In this work, non-zero temperatures of both beam and plasma are taken into account. The close connection between pure beam-plasma experiments and noise emission experiments will emerge from this analysis.

2. Dispersion Equation

The model used for analysis consists of an infinite homogeneous plasma and electron beam in a uniform magnetic field. Ion motion is neglected so that only high-frequency instabilities are considered. The perturbations are assumed quasi-static and in the form of plane waves

$$\exp[i(k_z z + k_\perp y - \omega t)]$$

with real wave numbers $k_z, k_\perp$. The static magnetic field is in the z direction. Finite transverse dimensions of a practical beam-plasma system are taken into account by choosing for the transverse wave number, $k_\perp$, any value satisfying the transverse boundary conditions. The smallest value of $k_\perp$ (approximately equal to $2.4/R$) will simply be considered as a given parameter characterizing the system radius, R . Assuming that the plasma is much longer than the axial wavelength, the longitudinal wave number, k_z , will be allowed to take on a continuous spectrum of real values. In general, the boundary conditions at the ends of the plasma column can be satisfied only by a set of waves with complex k_z . However, if the rather singular case of total wave absorption on the ends of the plasma column is excluded, the imaginary part of k_z is small because the reflected waves are only slightly damped. If the beam density sufficiently exceeds the threshold value for the onset of the instability, the dispersion equation can be satisfied only with complex frequency, ω , and the influence of the small imaginary part of k_z is unimportant.

With these assumptions, the starting point of the analysis is the quasi-static dispersion equation which can be written in the form

$$1 + \epsilon_p + \epsilon_b = 0, \quad (1)$$

where ϵ_p , ϵ_b are the equivalent susceptibilities of the plasma and beam, respectively, defined by³⁰

$$\epsilon_{p,b} = \frac{\omega_{p,b}^2}{k^2} \sum_{n=-\infty}^{+\infty} \int_L \frac{2\pi v_{\perp} dv_{\perp} dv_z}{\omega - n\omega_c - k_z v_z} \left[J_n^2 \left(\frac{k_{\perp} v_{\perp}}{\omega_c} \right) \left(n\omega_c \frac{\partial f_{p,b}}{\partial (v_{\perp}^2/2)} + k_z \frac{\partial f_{p,b}}{\partial v_z} \right) \right], \quad (2)$$

Here $f_{p,b}(v_z, v_{\perp})$ are the velocity distribution functions; $\omega_{p,b}$ are the electron plasma frequencies, ω_c is the electron cyclotron frequency, and $k^2 = k_z^2 + k_{\perp}^2$. The integration with respect to v_z is to be carried out along the Landau contour.

Assuming a Maxwellian velocity distribution for the plasma electrons,

$$f_p = n_p \left(\frac{1}{\pi v_t^2} \right)^{3/2} \exp \left[- \frac{v_z^2 + v_{\perp}^2}{v_t^2} \right], \quad (3)$$

the plasma susceptibility becomes

$$\epsilon_p = \frac{2\omega_p^2}{k^2 v_t^2} \left[1 + \frac{\omega}{|k_z| v_t} \sum_{n=-\infty}^{+\infty} Z \left(\frac{\omega - n\omega_c}{|k_z| v_t} \right) \exp[\lambda] I_n(\lambda) \right], \quad (4)$$

where λ has been written for $(k_{\perp}^2 v_t^2 / 2\omega_c^2)$; $I_n(\lambda)$ are modified Bessel functions of the first kind, and

$$Z(x) = 2i \exp(-x^2) \int_{-\infty}^{ix} \exp(-t^2) dt \quad (5)$$

is the plasma dispersion function.³¹

The distribution of the electrons in the beam is assumed to have the form

$$f_b = n_b f_{\perp}(v_{\perp}) \times \left(\frac{1}{\pi v_b^2} \right)^{1/2} \exp \left[- \frac{(v_z - v)^2}{v_b^2} \right], \quad (6)$$

where the distribution in transverse velocities, $f_{\perp}(v_{\perp})$, is arbitrary except for the normalization $2\pi \int v_{\perp} f(v_{\perp}) dv_{\perp} = 1$. The susceptibility of the beam becomes

$$\epsilon_b = \frac{2\omega_b^2}{k_z^2 v_b^2} \sum_{m=-\infty}^{+\infty} \left[S_m + \left(S_m \frac{\omega - m\omega_c - k_z V}{|k_z| v_b} + T_m \frac{m\omega_c}{|k_z| v_b} \frac{1}{2} \frac{k_{\perp}^2 v_b^2}{\omega_c^2} \right) Z \left(\frac{\omega - m\omega_c - k_z V}{|k_z| v_b} \right) \right], \quad (7)$$

where ω_b is the plasma frequency of the beam, and the parameters

$$S_m = 2\pi \int_0^{\infty} v_{\perp} f_{\perp} J_m^2 \left(\frac{k_{\perp} v_{\perp}}{\omega_c} \right) dv_{\perp}, \quad T_m = -2\pi \frac{\omega_c^2}{k_{\perp}^2} \int_0^{\infty} \frac{\partial f_{\perp}}{\partial v_{\perp}} J_m^2 \left(\frac{k_{\perp} v_{\perp}}{\omega_c} \right) dv_{\perp} \quad (8)$$

are real numbers determined by the transverse distribution, $f_{\perp}(v_{\perp})$.

Table 1 gives the values of S_m and T_m for three simple transverse distributions. If the beam has a small spread in longitudinal velocities,

$|k_z v_b| \ll |\omega - m\omega_c - k_z V|$, Eq. (7) simplifies to

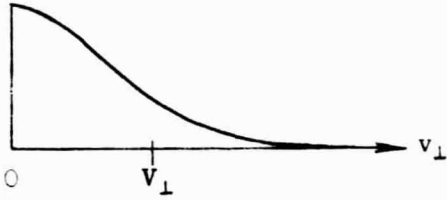
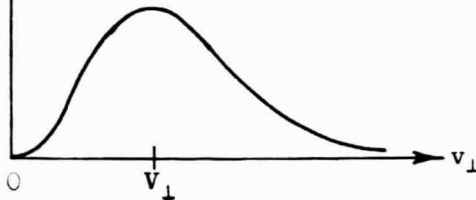
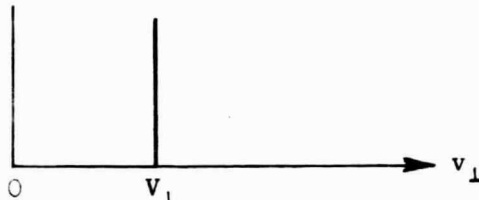
$$\epsilon_b = -\frac{\omega_b^2}{\omega_c^2} \sum_{m=-\infty}^{+\infty} \left[\frac{S_m \omega_c^2}{(\omega - m\omega_c - k_z V)^2} \frac{k_z^2}{k^2} + \frac{m T_m \omega_c}{\omega - m\omega_c - k_z V} \frac{k_{\perp}^2}{k^2} \right]. \quad (9)$$

This result may be obtained either from Eq. (7) using the asymptotic expansion³¹ $Z(x) = -x^{-1}(1 + 1/2x^2 + \dots)$, or from Eq. (2) assuming a delta function distribution in longitudinal velocities, $f_{\parallel}(v_z) = \delta(v_z - V)$.

3. Solution of the Dispersion Equation

The solution to Eq. (1) is simplified considerably if the beam density is appreciably smaller than the plasma density, $\omega_b^2 \ll \omega_r^2$, and if the beam velocity spread in the longitudinal direction is smaller than the directed velocity, $|v_b/V| < 1$. In this case, the main part of the dispersion equation is the dispersion equation of the plasma alone,

Table 1. Examples of Transverse Distributions.

Case	$f_{\perp}(v_{\perp})$	S_m, T_m
1	$f_{\perp} = \frac{1}{\pi V_{\perp}^2} \exp\left[-v_{\perp}^2/V_{\perp}^2\right]$ 	$S_m = e^{-\nu} I_m(\nu)$ $T_m = \frac{1}{\nu} S_m$ $\nu = \frac{1}{2} \left(\frac{k_{\perp} V_{\perp}}{\omega_c} \right)^2$
2	$f_{\perp} = \frac{1}{\pi V_{\perp}^2} \left(\frac{v_{\perp}}{V_{\perp}} \right)^2 \exp\left(-v_{\perp}^2/V_{\perp}^2\right)$ 	$S_m = \frac{d}{d\nu} \left(\nu e^{-\nu} I_m(\nu) \right)$ $T_m = \frac{d}{d\nu} \left(e^{-\nu} I_m(\nu) \right)$ $\nu = \frac{1}{2} \left(\frac{k_{\perp} V_{\perp}}{\omega_c} \right)^2$
3	$f_{\perp} = \frac{1}{2\pi V_{\perp}} \delta(v_{\perp} - V_{\perp})$ 	$S_m = J_m^2(\mu)$ $T_m = \frac{2}{\mu} J_m(\mu) J_m'(\mu)$ $\mu = \frac{k_{\perp} V_{\perp}}{\omega_c}$

$$1 + \epsilon_p = 0 , \quad (10)$$

which is slightly perturbed by the presence of the beam. Because of the second assumption, the individual terms of the sum in Eq. (7) are decoupled so that, for any integer m , only the term with

$$\omega - m\omega_c - k_z V \approx 0 , \quad (11)$$

may have a significant influence. The instabilities will have the highest growth rate in the vicinity of $\omega = \omega_m$ and $k_z = k_{zm}$ satisfying Eqs. (10) and (11). They correspond to the intersections of the plasma dispersion curves with the beam resonance lines, as shown in Fig. 1. Eliminating k_z from Eqs. (10) and (11), one obtains the frequency ω_m as a function of only the plasma parameters, $k_\perp$, and the mode number m . Figure 4 shows that the growth rate of any instability mode is sharply peaked at values of k_z approximately satisfying Eqs. (10) and (11). In this paper, only the highest growth rates of any instability mode (defined by the integer m) will be calculated.

If the plasma is cold, ω_m is real, and the dispersion equation described by Eq. (10) has two real branches: the L wave ($\omega < \omega_c$), and the H wave ($\omega > \omega_c$). For a warm plasma, the H wave is decomposed into an infinite number of branches which we denote by $H_1, H_2, H_3, \dots$ understanding by H_n the branch corresponding to the passband $n\omega_c < \omega < (n+1)\omega_c$. In addition, the frequency ω_m has a negative imaginary part.

In general, any of the L, H_n waves can be excited by the resonances expressed by Eq. (11). Accordingly, there is a doubly infinite set of possible instability modes which we denote by L^m, H_n^m (see Fig. 1).

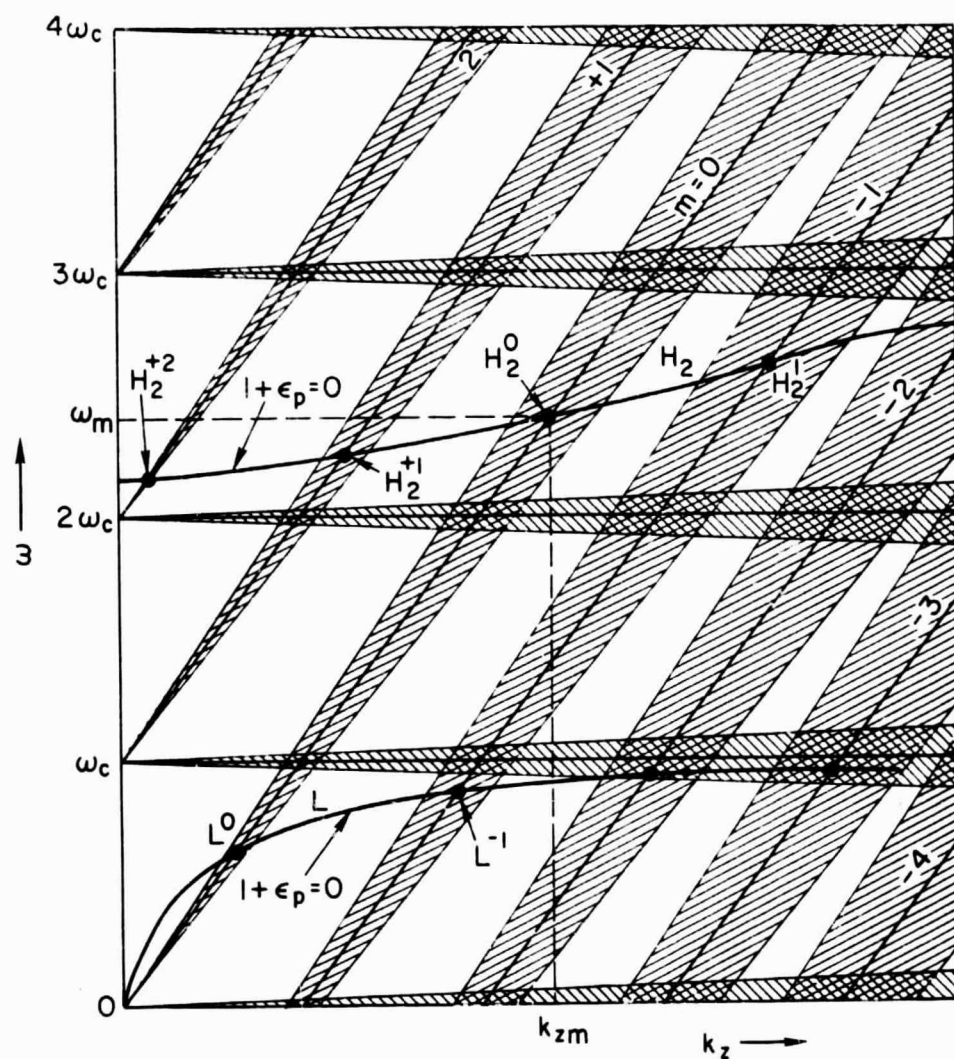


Fig. 1. Intersections of the plasma dispersion curves with the beam resonance lines. Classification of the instability modes is indicated.

Here m is a positive or negative integer including zero, and the integer $n \geq 1$. Essentially the same notation has been used previously.¹²

The frequency and highest growth rate of these instability modes have been found in the following way. First, solutions to the dispersion equation (1) have been calculated for the case of a cold plasma ($v_t = 0$) and a beam with no longitudinal velocity spread ($v_o = 0$). In this case, the plasma susceptibility is

$$\epsilon_p = -\frac{\omega_p^2}{\omega^2} \frac{k_z^2}{k^2} - \frac{\omega_p^2}{\omega^2 - \omega_c^2} \frac{k_\perp^2}{k^2} \quad (12)$$

while the beam susceptibility is given by Eq. (9). Approximate roots of Eq. (1) can be found analytically. Eliminating k_z from Eq. (10) and (11) gives

$$\frac{\omega_p^2}{\omega_c^2} = \frac{1 + \frac{g^2}{(x-m)^2}}{\frac{1}{x^2} + \frac{g^2}{(x^2-1)(x-m)^2}}, \quad (13)$$

$$g = \frac{k_\perp V}{\omega_c}, \quad x = \frac{\omega_m}{\omega_c}.$$

This equation relates, for a given parameter g and integer m , the plasma parameter (ω_p/ω_c) to the frequency ω_m corresponding to the highest growth rate of the m^{th} instability mode.

The growth rate $\text{Imag}(\omega)$ may be approximately obtained by writing the solution of Eq. (1) in the form

$$\omega = \omega_m + \omega_c \delta \quad (\delta \ll \omega_m/\omega_c) \quad (14)$$

and approximating $(1 + \epsilon_p)$ by the first two terms of the Taylor expansion at $\omega = \omega_m$:

$$1 + \epsilon_p = \omega_c \delta \cdot \left. \frac{\partial \epsilon_p}{\partial \omega} \right|_{\omega=\omega_m} = \frac{1}{G k^2 v^2 / \omega_c^2} \delta \quad (15)$$

with

$$G = \frac{x}{2} \frac{x^2 - 1}{(x^2 - 1)(x - m)^2 + g^2 x^4} \frac{(x - m)^2 (x^2 - 1) + g^2 x^2}{(x - m)^2 + g^2} \quad (16)$$

The RHS of Eq. (15) has been obtained by expressing $(\omega_p / \omega_c)^2$ and k_z in terms of $x = \omega_m / \omega_c$ using Eq. (13) and (11). Retaining in the summation (9) only the two terms whose denominator approximately satisfies the resonance condition (11), the dispersion equation (1) reduces to a cubic equation in δ

$$\delta^3 - \left(\frac{\omega_b^2}{\omega_c^2} g^2 G m T_m \right) \delta - \left(\frac{\omega_b^2}{\omega_c^2} (x - m)^2 G S_m \right) = 0 \quad (17)$$

which will yield two complex roots, one corresponding to instability ($\text{Imag } \delta > 0$), if

$$\left[(x - m)^2 S_m \right]^2 > \frac{4}{27} \left(\frac{\omega_b}{\omega_c} \right)^2 G (g^2 m T_m)^3 \quad (18)$$

Since $G > 0$ for all values of x consistent with Eq. (13) (with $(\omega_p / \omega_c)^2 > 0$), the sufficient, but not necessary, condition for satisfying inequality (18) is

$$m T_m \leq 0. \quad (19)$$

Thus, the $m = 0$ mode (Čerenkov interaction, $\omega / k_z \approx V$) will always be unstable regardless of the shape of the transverse velocity distribution $f_\perp(v_\perp)$ in the beam. On the other hand, the stability of the $m \neq 0$ modes is determined by the shape of $f_\perp(v_\perp)$. In particular, if

$\partial f_{\perp} / \partial v_{\perp} < 0$ for all $v_{\perp}$, then, according to Eq. (8), all T_m 's are positive, and condition (19) will also be satisfied for $m < 0$ (anomalous Doppler effect, $\omega/k_z < V$). In addition, some of the fast modes with $m > 0$ (normal Doppler effect, $\omega/k_z > V$) may also be unstable for a limited range of parameters. However, the fastest wave, having $k_z \approx 0$, remains stable as long as $T_m > 0$. According to the synchronism condition (11), $k_z \approx 0$ is equivalent to $(x-m) \approx 0$ and the inequality (18) cannot be satisfied when the RHS is positive. Such a wave propagating nearly perpendicularly to the magnetic field will be unstable if the corresponding T_m is negative. This happens when $\partial f_{\perp} / \partial v_{\perp}$ is positive in regions where $J_m^2(k_{\perp} v_{\perp} / \omega_c)$ is large such that $f_{\perp}(v_{\perp})$ is peaked at some sufficiently large velocity. Examples of such distribution are cases 2 and 3 in Table 1.

The complex root of Eq. (17) having $\text{Imag } \delta > 0$ determines the maximum growth rate of the m^{th} instability mode for a cold plasma ($v_t = 0$) and a beam with no longitudinal velocity spread ($v_b = 0$). In the general case, $v_t > 0$, $v_b > 0$, the plasma and beam permittivities are given by the complicated expressions (4) and (7), respectively. In this case, the dispersion equation (1) has been solved numerically using the following iterative procedure. Starting from $v_t = 0$, $v_b = 0$, the thermal spreads have been varied in small steps Δv_t , Δv_b , so that in the n^{th} step $(v_t)_n = n\Delta v_t$, $(v_b)_n = n\Delta v_b$. For each step, the complex root ω with a positive imaginary part has been calculated by Newton's interpolation formula, using for the starting value the root obtained in the previous step. The starting value for the first step has been the solution of Eq. (17). In the course of the computation, k_z also has been varied until the maximum growth rate $\text{Imag } \omega$ has been found. It turned out

that the corrected k_z deviated from the resonant value (11) by less than 10 percent for $\omega_b/\omega_c = 0.1$ and $v_b/V < 0.3$.

In the following, the results of these computations will be presented for two representative beams with different transverse velocity distributions. The first beam has a distribution with negative slope $\partial f_{\perp}/\partial v_{\perp}$ so that all coefficients T_m are positive. The distribution of the second beam is peaked at some large $v_{\perp}$, making some of the coefficients T_m negative. It will be seen that longitudinal velocity spread in the beams affects the instabilities differently in these two cases.

4. Parallel Beam with Isotropic Velocity Spread

In most experiments, electrons are injected into the plasma with some directed velocity parallel to the magnetic field lines. Imperfections of the injector as well as electric fields due to the instabilities will partially randomize the beam. Such a beam may be represented by the distribution

$$f_b = n_b \left(\frac{1}{\pi v_b^2} \right)^{3/2} \exp \left[- \frac{(v_z - v)^2 + v_{\perp}^2}{v_b^2} \right] \quad (20)$$

which corresponds to case 1 in Table 1 with $V_{\perp} = v_b$. In the following, the interaction of this beam with a Maxwellian plasma (3) will be studied, starting from the limiting case, $v_t = 0$, $v_b = 0$, and gradually increasing the thermal velocities, v_t , v_b , of the plasma and beam, respectively.

4.1. Cold Plasma - Cold Beam Interaction

Solutions corresponding to $v_t = 0$, $v_b = 0$ are contained in the more general Eqs. (13) and (17). Since the beam is cold, $v = 0.5 (k_{\perp} v_b / \omega_c)^2 \approx 0$,

all coefficients S_m, T_m in Eq. (17) are equal to zero except those corresponding to $m = 0, \pm 1$. Since $T_{+1} > 0$, the only unstable modes are $m = 0, -1$, a well-known result.^{1,2} The frequency, ω_m , giving the highest growth rate (i.e. the frequency which should be observed in an experiment) depends on two parameters, (ω_p/ω_c) and $g = k_\perp V/\omega_c$, according to Eq. (13) with $m = 0$ and $m = -1$.

For the $m = 0$ interaction, Eq. (17) gives the growth rate

$$\frac{\omega_i}{\omega_o} = \frac{\sqrt{3}}{2} \left(\frac{\omega_b^2}{2\omega_c^2} \right)^{1/3} \left[\frac{x^2 - 1}{(x^2 - 1)^2 + g^2 x^2} \cdot \frac{x^2 - 1 + g^2}{x^2 + g^2} \right]^{1/3} \quad (21)$$

where $x = \omega_o/\omega_c$. For any real frequency, ω_o , Eq. (13) (with $m = 0$) gives the plasma density, ω_p/ω_c , needed for excitation of the frequency ω_o , while Eq. (21) gives the corresponding growth rate. These equations are plotted in Fig. 2. The right-hand side shows the dependence of the excited frequency, $\omega_r \approx \omega_o$, on plasma density. The left-hand side shows the largest growth rate, ω_i , as a function of the excited frequency. The two branches L^0 and H^0 are indicated. It may be noted that the L^0 mode can be excited only if $g < 1$, and it exists at densities satisfying the relation $(\omega_p/\omega_c)^2 > g^2/(1-g^2)$. The growth rate increases with decreasing g , reaching the largest value as $g \rightarrow 0$.

For the $m = -1$ interaction, the growth rate is

$$\frac{\omega_i}{\omega_{-1}} = \frac{1}{2} \frac{\omega_b}{\omega_c} g \left[\frac{1}{x} \frac{x^2 - 1}{(x^2 - 1)^2 (x+1)^2 + g^2 x^4} \cdot \frac{(x+1)^2 (x^2 - 1) + g^2 x^2}{(x+1)^2 + g^2} \right]^{1/2} \quad (22)$$

where $x = \omega_{-1}/\omega_c$. Again, the real frequency ω_{-1} can be freely chosen and the corresponding plasma density and growth rate may be calculated from Eq. (13) (with $m = -1$) and Eq. (22), respectively.

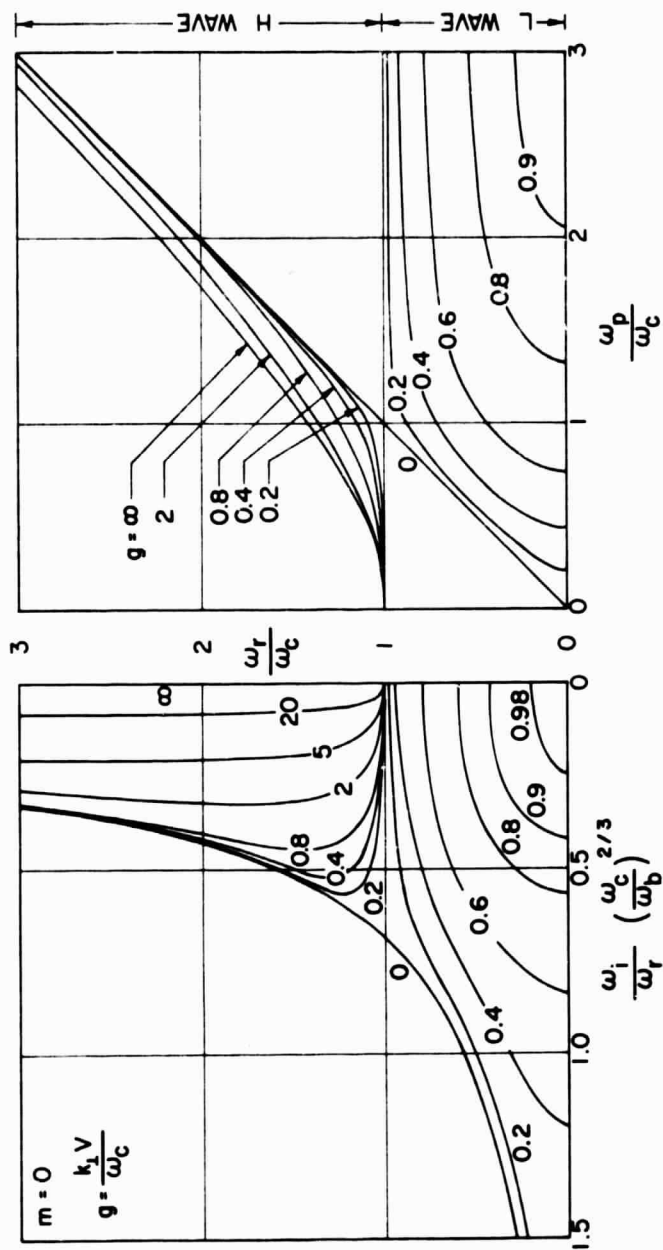


Fig. 2. Dependence of the excited frequency ω_r on the plasma density ω_p and of growth rate ω_i on ω_r for the cold plasma - cold parallel beam interaction at the $m = 0$ mode.

Figure 3 shows a plot of the excited frequency and growth rate as a function of the parameters (ω_p/ω_c) and g . In contrast to the $m = 0$ interaction, the growth rate increases with g , reaching the largest value as $g \rightarrow \infty$.

Equations (21) and (22) do not contain k_z since the latter has been selected, according to the resonance condition (11), to maximize the growth rate. Figure 4 shows how quickly the growth rate decreases with deviation of k_z from the resonance value (11).

To summarize, a cold electron beam can excite only four instability modes L^0 , H^0 , L^{-1} , H^{-1} in a cold plasma. Their frequencies and highest growth rates are determined by the three parameters (ω_p/ω_c) , (ω_b/ω_c) , g . As long as the plasma radius and density are sufficiently large to allow $g < 1$ and $(\omega_p/\omega_c)^2 > g^2/(1-g^2)$, the $m = 0$ interaction will be dominant because $\omega_i \sim \omega_b^{2/3}$, in contrast to $\omega_i \sim \omega_b$ for $m = -1$ interaction. On the other hand, the $m = -1$ interaction will dominate if the plasma is thin, $g > 1$. The L^0 mode cannot be excited at all and the H^{-1} mode will have a higher growth rate than the H^0 mode if $g^2 > (\omega_c/\omega_b)$.

4.2. Effect of Plasma Temperature

When the plasma has a non-zero temperature, $v_t > 0$, but the beam can still be considered as cold, Eq. (1) has to be solved with ϵ_p , ϵ_b given by Eqs. (4) and (9), respectively. Inserting into Eq. (9) the values of $S_m(\nu)$, $T_m(\nu)$ corresponding to $\nu \rightarrow 0$, ϵ_b becomes

$$\epsilon_b = -\frac{\omega_b^2}{\omega_c} \left[\frac{\omega_c^2}{(\omega - k_z v)^2} \frac{k_z^2}{k^2} + \frac{1}{2} \left(\frac{\omega_c}{\omega - k_z v - \omega_c} - \frac{\omega_c}{\omega - k_z v + \omega_c} \right) \frac{k_\perp^2}{k^2} \right] \quad (23)$$

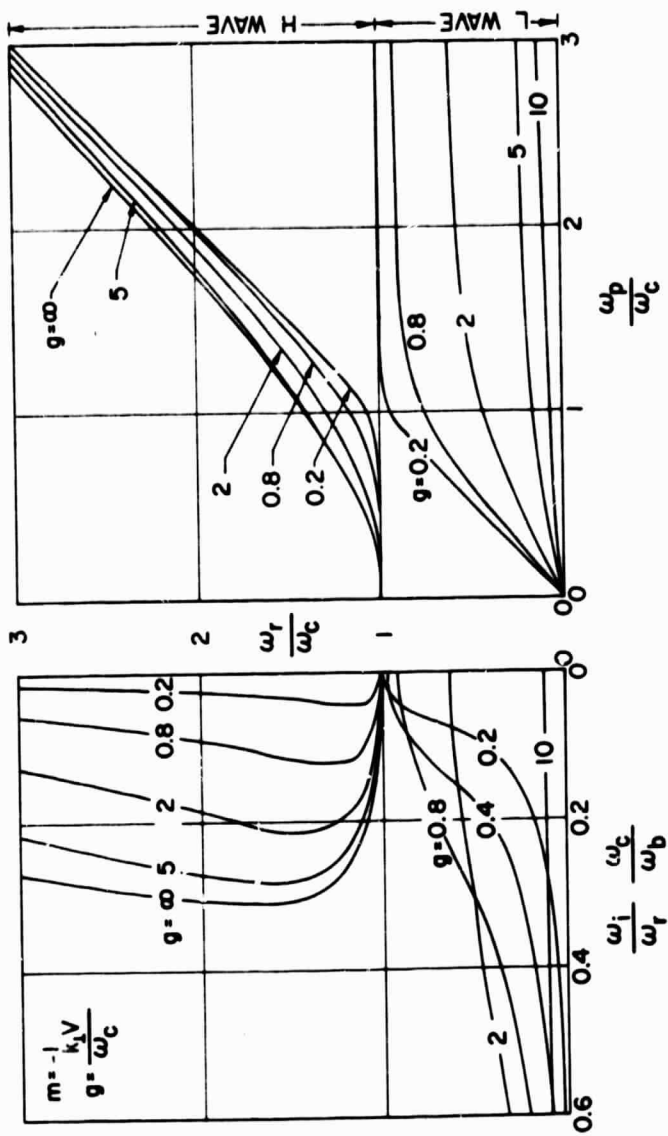


Fig. 3. Cold plasma - cold parallel beam interaction at the $m = -1$ mode.

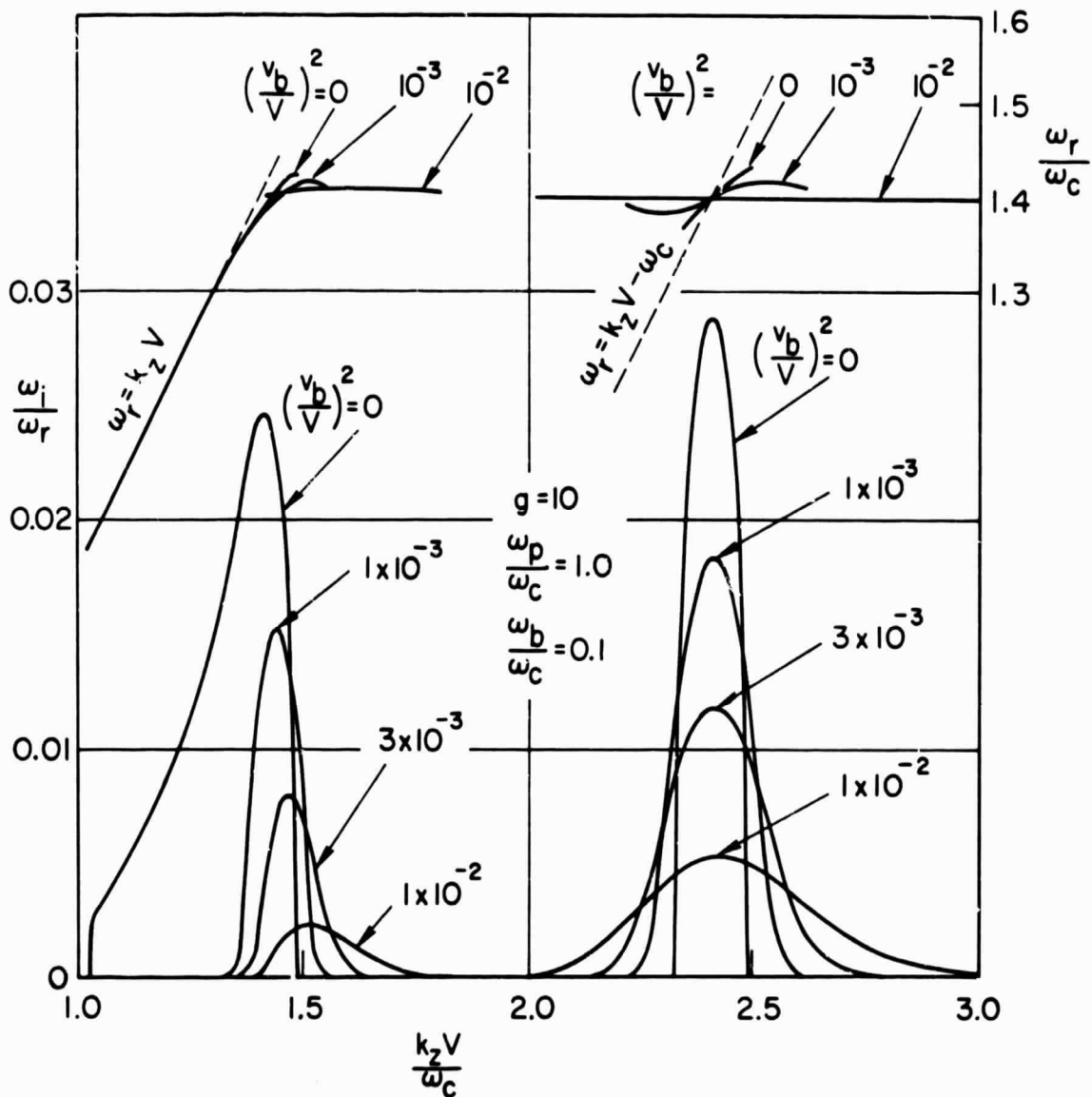


Fig. 4. Dependence of the real (upper curves) and the imaginary (lower curves) part of the frequency on $k_z V / \omega_c$ for the cold plasma - warm parallel beam interaction.

Again, only the interaction modes $m = 0, -1$ can lead to growing waves.

The warm plasma dispersion equation (10) with no beam has been discussed often in various approximations;³²⁻³⁴ a detailed numerical analysis has been given by Tataronis.³⁵ The main effect of plasma temperature is the decomposition of the H wave into an infinite set of branches H_n (cyclotron harmonic waves). It would appear that any of these waves could be excited by the beam. This is not so because most of these waves are strongly damped at phase velocities satisfying the resonance condition (11).

Numerical solutions to the dispersion equation are summarized in Figs. 5 and 6. Figure 5 corresponds to the case $g < 1$ when the $m = 0$ interaction is dominant and Fig. 6 gives a representative example of the case $g > 1$ when the $m = -1$ interaction is dominant. The complex solution $(\omega_r + i\omega_i)$ of Eq. (1), corresponding to growing waves, has been plotted as a function of the plasma parameter (ω_p/ω_c) for fixed g , ω_b/ω_c , and for several plasma temperatures expressed by the parameter $(v_t/V)^2$. For each solution, k_z has been varied in the vicinity of the resonance (11), corresponding to the dominant interaction, until the largest growth rate has been obtained. Hence, the frequencies and growth rates shown in the figures correspond to the strongest instability. The deviation of optimum k_z from the resonant value given by Eq. (11) is less than 5 percent for Fig. 5 and less than 0.5 percent for Fig. 6.

The nature of these solutions can be understood from the form of Eq. (4). When $g < 1$ and $(v_t/V) < 1$, the argument, $\lambda = k_1^2 v_t^2 / 2\omega_c^2$, of the Bessel functions is small and all higher order terms, $|n| > 1$, can be neglected in the summation (4). The solution will differ markedly from the cold plasma approximation only in regions $|\omega/k_z v_t| < 2$ and

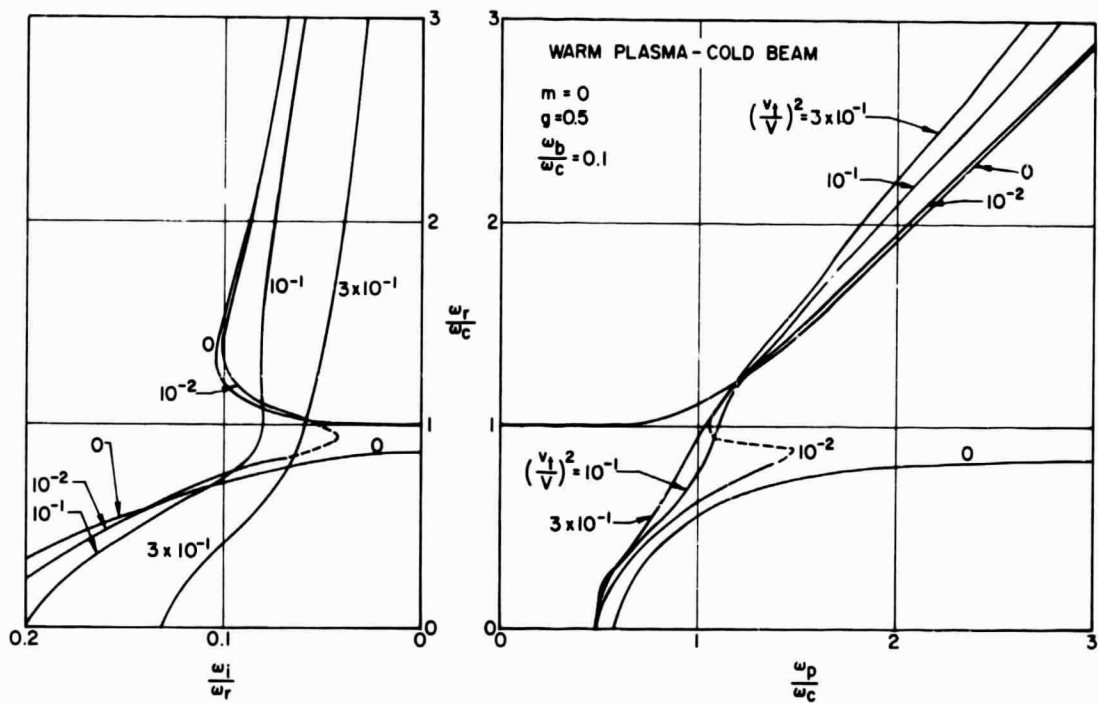


Fig. 5. Warm plasma - cold parallel beam interaction at the $m = 0$ mode.

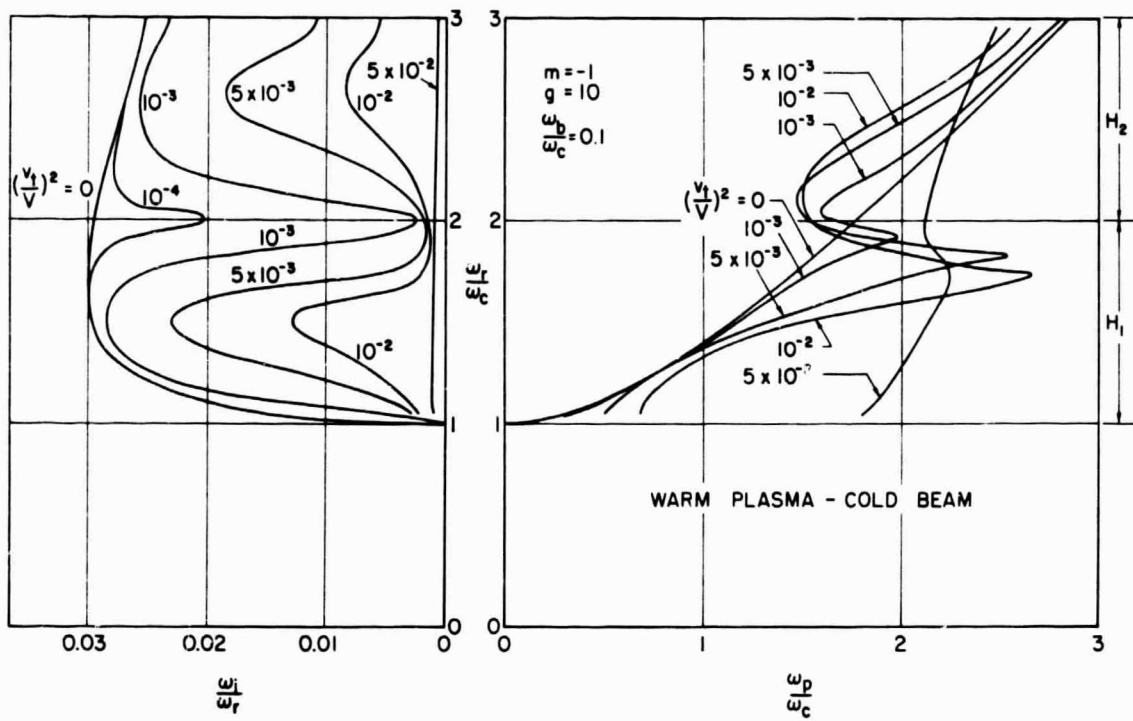


Fig. 6. Warm plasma - cold parallel beam interaction at the $m = -1$ mode.

$|(\omega - \omega_c)/k_z v_t| < 2$ where the functions $Z(x)$ cannot be replaced by the asymptotic form $Z(x) \sim x^{-1}(1 + 1/2 x^2 + \dots)$ used in the cold plasma theory. As a result, the main effect of finite plasma temperature, as long as $v_t/V < 1/2$, is the disappearance of the stopband just below the cyclotron frequency which separated the L wave from the H wave in the cold plasma. At higher plasma temperature, $v_t/V > 1/2$, Landau damping reduces the growth rate equally in the whole frequency band.

When $g \gg 1$, the argument, λ , of the Bessel functions is no longer small even in a moderately warm plasma. Higher order terms in the summation (4) may appreciably affect the dispersion equation in regions $|\omega - n\omega_c| \leq |k_z v_t|$ near the cyclotron harmonics. The main effect, as shown in Fig. 6, is a reduction of the growth rate at cyclotron harmonics due to cyclotron damping, and a multivalued dependence of the excited frequency on the plasma parameter (ω_p/ω_c) . This distortion, which may be viewed as a consequence of cyclotron harmonic waves, disappears at higher frequencies, e.g. for $\omega > 4\omega_c$ if $\lambda \sim 1$. The reason for this is simply that the distortion near the n^{th} harmonic is caused by the n^{th} term of the sum (4) which becomes small in comparison with the lower order terms. This is because $I_n(\lambda)$ is small and the function $Z(x)$ is bounded in the $\text{Imag}(x) > 0$ half plane, $|Z(x)| < \sqrt{\pi}$.

To summarize, the main effect of plasma temperature is a reduction of the growth rate of the instabilities. Since the growth rate decreases with increasing $k_\perp$, waves with the lowest possible transverse wave number are most likely to be excited. If the plasma radius is so large that $g = k_\perp V/\omega_c < 1$, the growth rate is not very sensitive to plasma temperature and to plasma density. On the other hand, when the plasma radius is so small that $g \gg 1$ the growth rate is strongly affected by

plasma temperature, especially near the low order cyclotron harmonics, reaching maximum values at frequencies $\omega \approx (n + 1/2)\omega_c$, $n = 1, 2, 3$. As a result, frequencies between cyclotron harmonics are most likely to be excited.

4.3. Effect of Beam Temperature

The susceptibility of the warm beam is given by Eq. (7). Inserting for $S_m(\nu)$, $T_m(\nu)$ the expressions given in Table 1 for the distribution (20), the beam susceptibility becomes

$$\epsilon_b = \frac{2 \omega_b^2}{k_z^2 v_b^2} \left[1 + \frac{\omega - k_z V}{|k_z| v_b} \sum_{m=-\infty}^{+\infty} Z \left(\frac{\omega - k_z V - m \omega_c}{|k_z| v_b} \right) e^{-\nu} I_m(\nu) \right]. \quad (24)$$

As long as $\nu = (1/2)(k_\perp v_b / \omega_c)^2 < 0.1$ the only significant terms in the summation correspond to $m = 0, \pm 1$, and the instability is again possible only for the $m = 0$ and $m = -1$ interactions. For larger ν , higher order interactions also become possible in a limited range of parameters satisfying inequality (18). The calculation has been limited to the vicinity of the two lowest modes.

Figures 7 and 8 represent solutions of the dispersion equation (1) for the case of a cold plasma, having a susceptibility given by Eq. (12), and a warm beam with ϵ_b given by Eq. (24). The results are shown in the same way as in Figs. 5 and 6 except that the variable parameter is the ratio $(v_b/V)^2$ instead of $(v_t/V)^2$. Since the beam temperature has a very small effect on the real part of the frequency for the $m = -1$ interaction, Fig. 8 shows only the dependence of ω_i on ω_r .

The beam velocity spread reduces the growth rate in comparison with the cold beam when

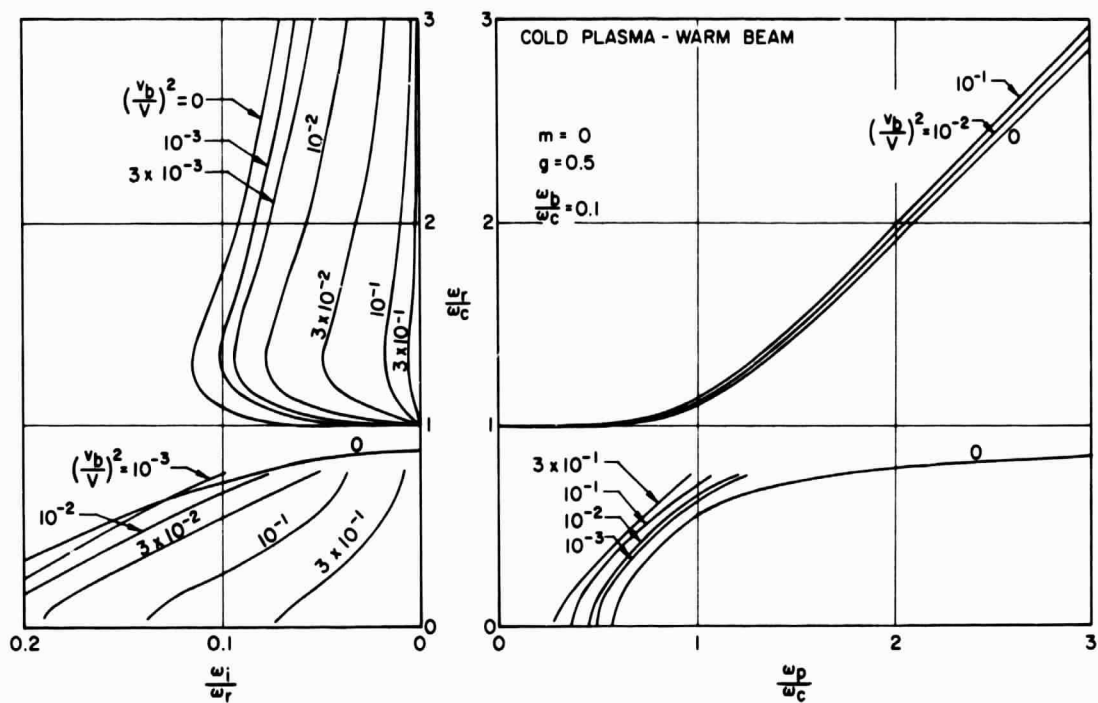


Fig. 7. Cold plasma - warm parallel beam interaction at the $m = 0$ mode.

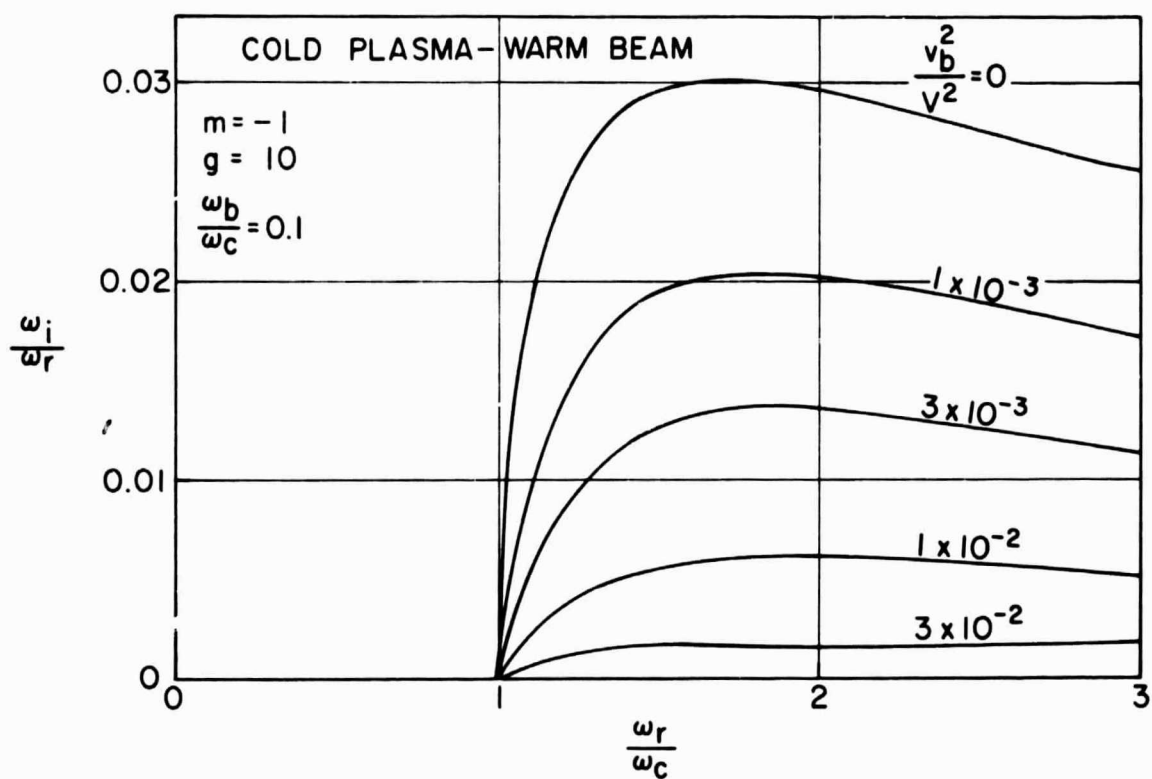


Fig. 8. Cold plasma - warm parallel beam interaction at the $m = -1$ mode.

$$\left| \frac{\omega - k_z V - m\omega_c}{k_z v_b} \right| \approx \left| \frac{\omega_i}{k_z v_b} \right| \approx \left| \frac{\omega_i}{\omega - m\omega_c} \frac{V}{v_b} \right| < 1 \quad (25)$$

where ω_i is the growth rate for cold beam-cold plasma interaction given by Eqs. (21) and (22). Since under usual experimental conditions $(\omega_i/\omega) \approx 0.01$ to 0.1 , the beam velocity spread becomes important for the $m = 0, -1$ interactions when $(v_b/V) > 0.01$ to 0.1 . It may be pointed out that beam temperature does not affect the overall dependence of the growth rate on frequency for these slow modes.

In Figs. 9 and 10 the plasma temperature has also been assumed to be finite. Since the real part of the frequency changes with (ω_p/ω_c) identically as in Figs. 5 and 6, only the dependence of ω_i on ω_r has been shown. In contrast to the case of a cold beam, in which the growth rate goes asymptotically to zero as the plasma temperature increases, the growth rate can become negative near the cyclotron harmonics when both the plasma and the beam have non-zero temperatures.

When the temperatures are so high that the dispersion equation can be solved in the kinetic approximation, the resultant growth rate is simply the growth rate due to the beam minus the damping rate due to the plasma.³⁶

5. Helical Beam

The second prototype beam to be analyzed is a helical beam which has a distribution in transverse velocities peaked at some velocity $V_\perp$, comparable to the directed longitudinal velocity V . Such beams have usually a large spread in longitudinal velocities, $v_b/V \sim 0.1$, even when produced by specially designed guns.¹²

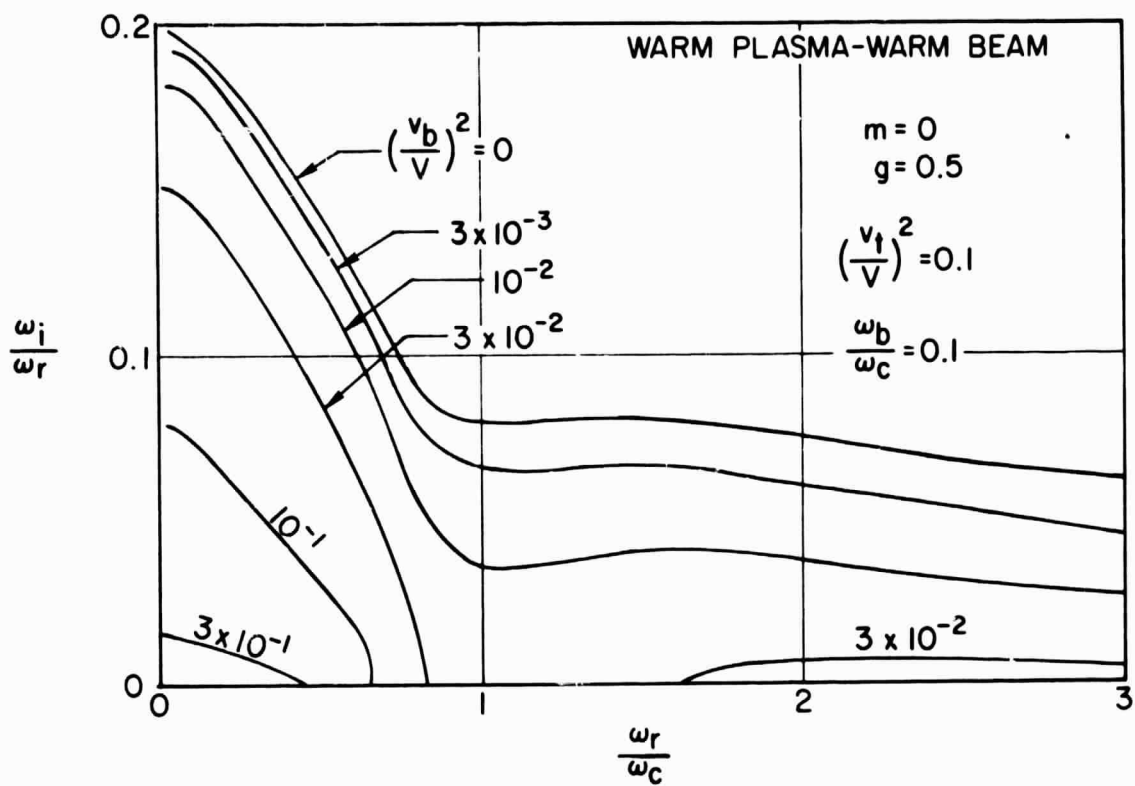


Fig. 9. Warm plasma - warm parallel beam interaction for the $m = 0$ mode and $g = 0.5$.

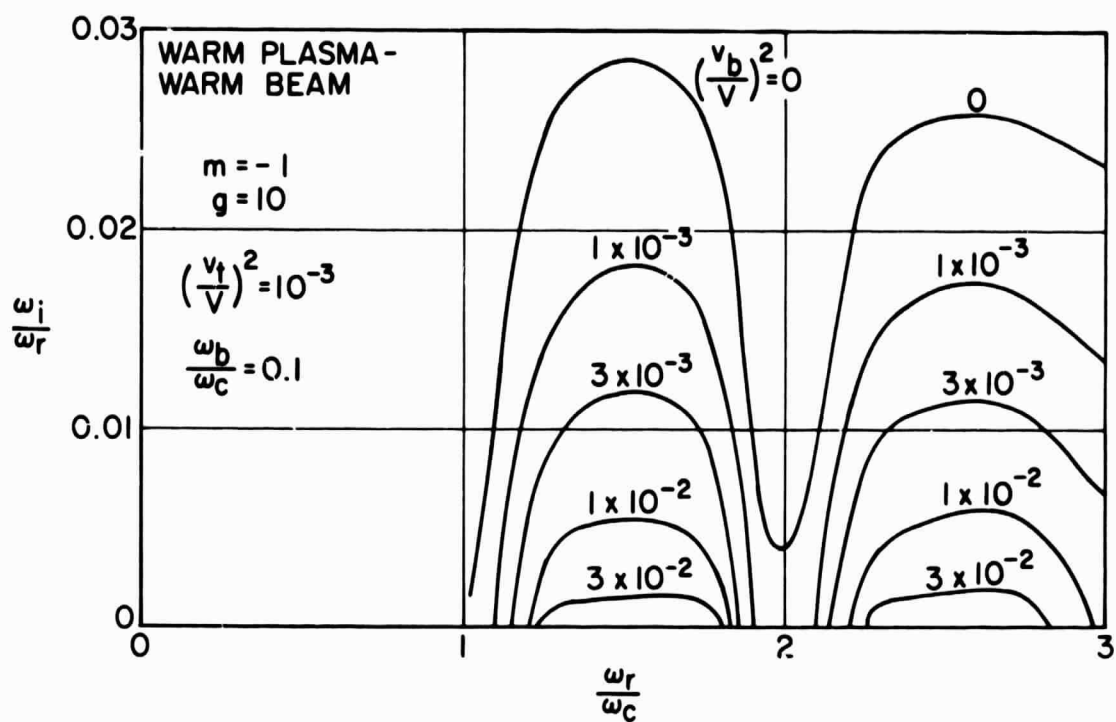


Fig. 10. Warm plasma - warm parallel beam interaction for the $m = -1$ mode and $g = 10$.

Considerable work has been devoted to stability analysis of helical beams with delta function velocity distribution either in the absence³⁷⁻³⁹ or presence²¹⁻²³ of a background plasma. A simpler and more realistic model of a beam with large transverse energy is the distribution given by Eq. (6) with $f_{\perp}(v_{\perp})$ corresponding to case 2 of Table 1. This is a special case of a class of distributions

$$f_{\perp}(v_{\perp}) = \frac{1}{\pi v_{\perp}^2 s!} \left(\frac{v_{\perp}}{V_{\perp}} \right)^{2s} \exp(-v_{\perp}^2/V_{\perp}^2) \quad (26)$$

analyzed by Dory et al⁴⁰ for stability with respect to waves propagating transverse to the magnetic field. Since no background plasma was assumed, the system was found to be stable for $s < 3$. For the present analysis, the distribution corresponding to $s = 1$ has been chosen in order to demonstrate that the presence of background plasma makes the beam more unstable. Thus, the beam distribution is assumed in the form

$$f_b = n_b \left(\frac{1}{\pi v_b^2} \right)^{\frac{1}{2}} \frac{1}{\pi v_{\perp}^2} \left(\frac{v_{\perp}}{V_{\perp}} \right)^2 \exp \left[-v_{\perp}^2/V_{\perp}^2 - (v_z - v)^2/v_b^2 \right] \quad (27)$$

The beam susceptibility is given by Eq. (7) in which the parameters S_m, T_m have the values given in Table 1, case 2. The interaction of this beam with a cold plasma will be investigated starting from the limit $v_b = 0$ and gradually increasing the longitudinal velocity spread, v_b , of the beam. Finally the effect of finite plasma temperature will be briefly discussed.

5.1. Cold Helical Beam-Cold Plasma Interaction

If the beam has no spread in longitudinal velocities, $v_b = 0$, the susceptibility ϵ_b is given by Eq. (9) and the dispersion equation (1) can be solved analytically. The approximate solution for the m^{th} instability mode is

$$\omega = \omega_m + \omega_c \delta \quad (28)$$

The main contribution to the real part of the frequency, ω_m , is determined by Eq. (13) as a function of the two parameters ω_p/ω_c and g . If $g^2 \gg (x-m)^2$ and $\omega_m > \omega_c$, Eq. (13) reduces to $\omega_m \approx (\omega_p^2 + \omega_c^2)^{1/2}$, the upper hybrid frequency of the plasma.

The imaginary part of the frequency is determined by the roots, δ of the cubic equation (17) in which

$$S_m(\nu) = \frac{d}{d\nu} \left(\nu e^{-\nu} I_m(\nu) \right) \quad (29)$$

$$T_m(\nu) = \frac{d}{d\nu} \left(e^{-\nu} I_m(\nu) \right) \quad (30)$$

with $\nu = k_{\perp}^2 V_1^2 / 2\omega_c^2$. It may be noted that $T_m(\nu)$ becomes negative if

$$\nu > m^2 + \frac{1}{2} + \frac{1}{4m^2} + \dots \quad (31)$$

Figure 11 shows plots of $T_m(\nu)$ for ν satisfying this inequality.

When $T_m < 0$ for some chosen mode number $m > 0$, Eq. (17) has complex solutions even at $\omega - m\omega_c \approx 0$ where $k_z \approx 0$. This means that a helical beam can excite waves propagating nearly transverse to the magnetic field. For any mode number $m > 0$, the growth rate is the smallest when $r = m\omega_c$, as shown in Fig. 12. Nevertheless, waves with $k_z \approx 0$ are of considerable interest since their growth rate is little affected by a longitudinal velocity spread in the beam.

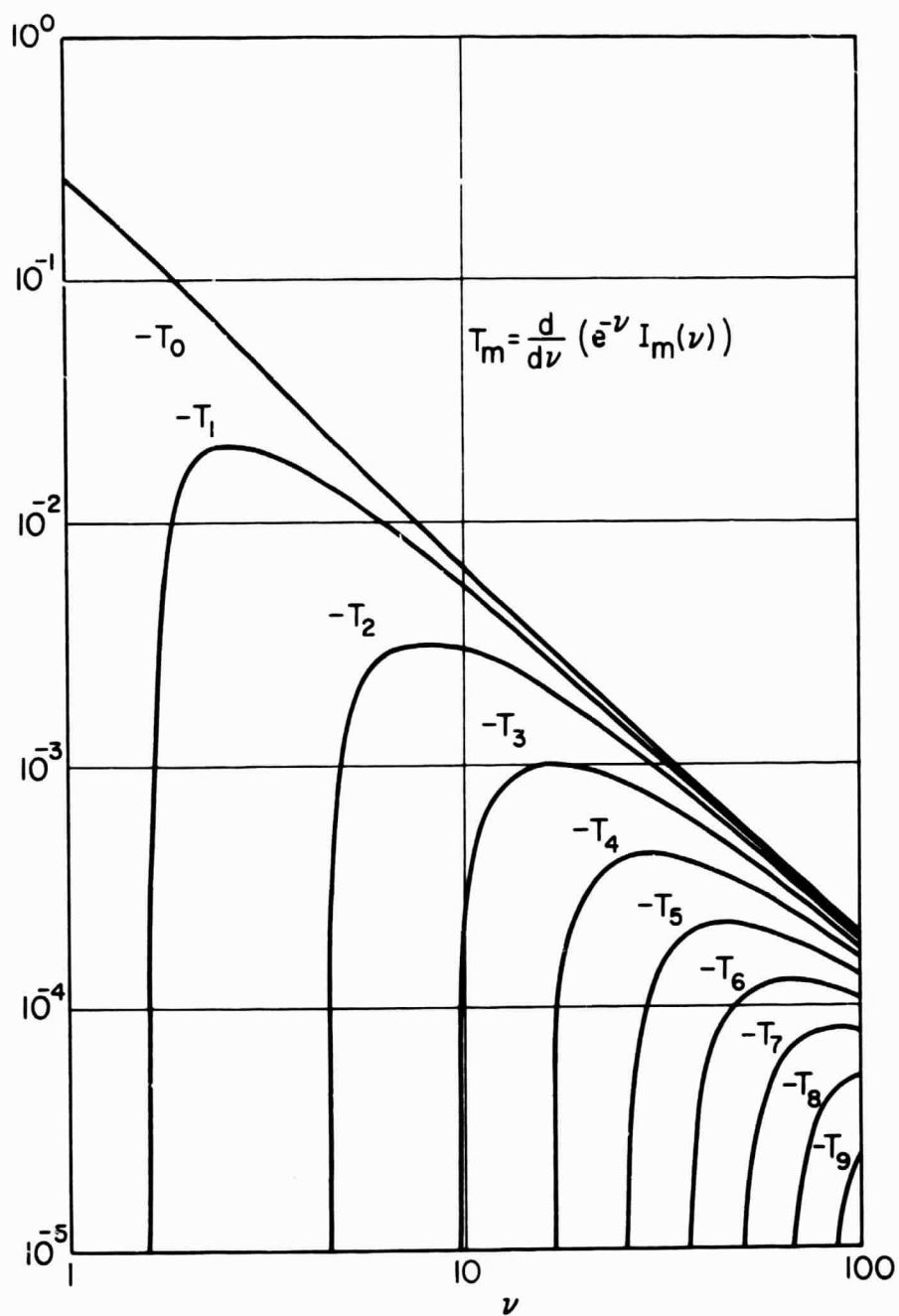


Fig. 11. Function $T_m(\nu) = \frac{d}{d\nu} (e^{-\nu} I_m(\nu))$.

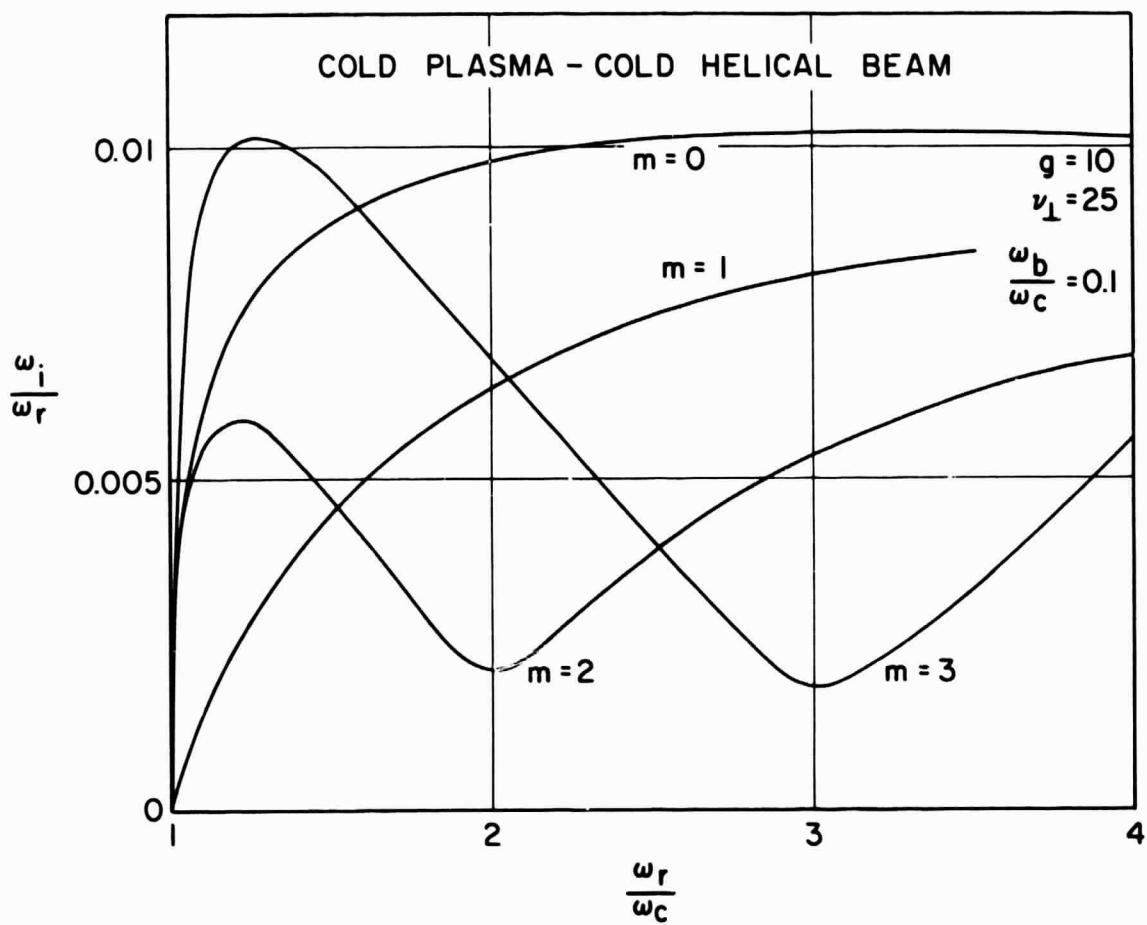


Fig. 12. Cold plasma - cold helical beam interaction.

5.2. Effect of Beam Temperature

Assuming again a cold plasma but a non-zero longitudinal velocity spread, v_b , in the beam distribution (27), the dispersion equation (1) has been solved numerically with ϵ_p , ϵ_b given by Eqs. (12) and (7), respectively. Figures 13 to 16 show the dependence of the imaginary part, ω_i , of the unstable root on the real part, ω_r , of the root for four modes, $m = 0, 1, 2, 3$ and for several beam velocity spreads, expressed by the parameter $(v_b/V)^2$. The real part of the frequency, ω_r , is very nearly equal to the upper hybrid frequency since $g \gg 1$. In these calculations, k_z has been replaced by the value determined by the synchronism condition (11) with $\omega = \omega_r$.

The important feature of these figures is that the growth rate, ω_i , of the $m = 2$ and $m = 3$ instability modes decreases with increasing v_b at all frequencies except for those near $\omega_r = m\omega_c$ where it is little affected by the beam spread. No such resonance exists for the $m = 0$ mode and only a weak one for the $m = 1$ mode (because $G \approx 0$ as $\omega_r \approx \omega_c$).

These results may be easily understood by remembering that the cold beam approximation breaks down when the argument $(\omega - m\omega_c - k_z V)/|k_z|v_b$ of the plasma dispersion function Z in Eq. (7) becomes small enough such that the inequality (25) is satisfied. This will happen, for moderately large v_b , at all frequencies except at $\omega - m\omega_c - k_z V \approx 0$ where the argument remains large, $|\omega_i/k_z v_b| > 1$, so that the beam velocity spread has no effect.

As a result, a helical beam with sufficiently large velocity spread will excite only cyclotron harmonic waves propagating almost perpendicularly

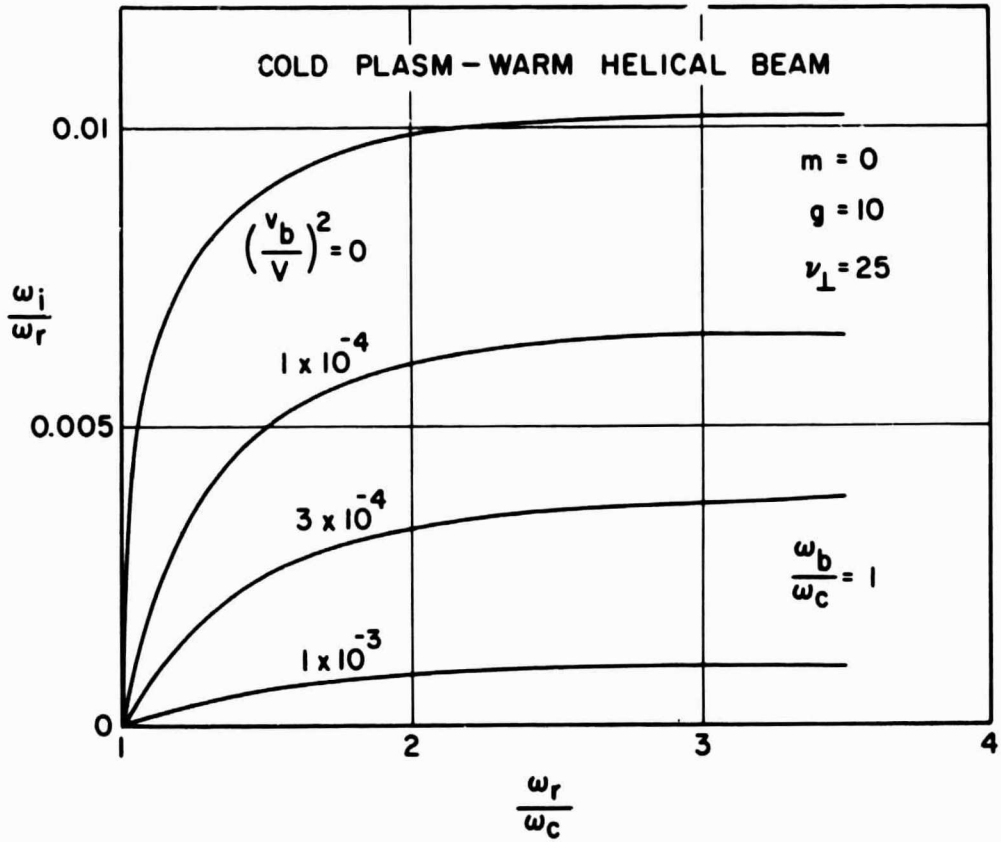


Fig. 13. Effect of the beam temperature on the growth rate of the $m = 0$ mode.

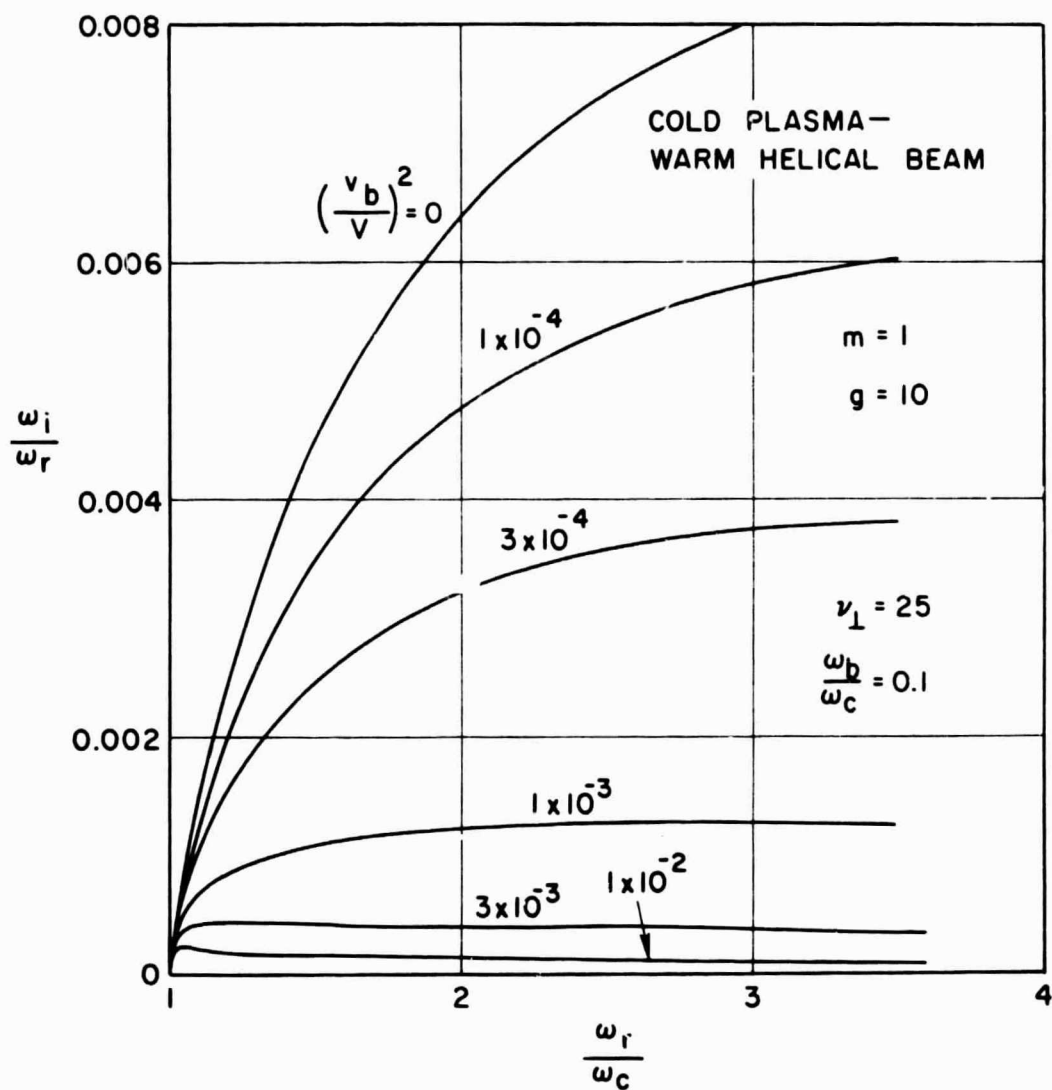


Fig. 14. Effect of the beam temperature on the growth rate of the $m = 1$ mode.

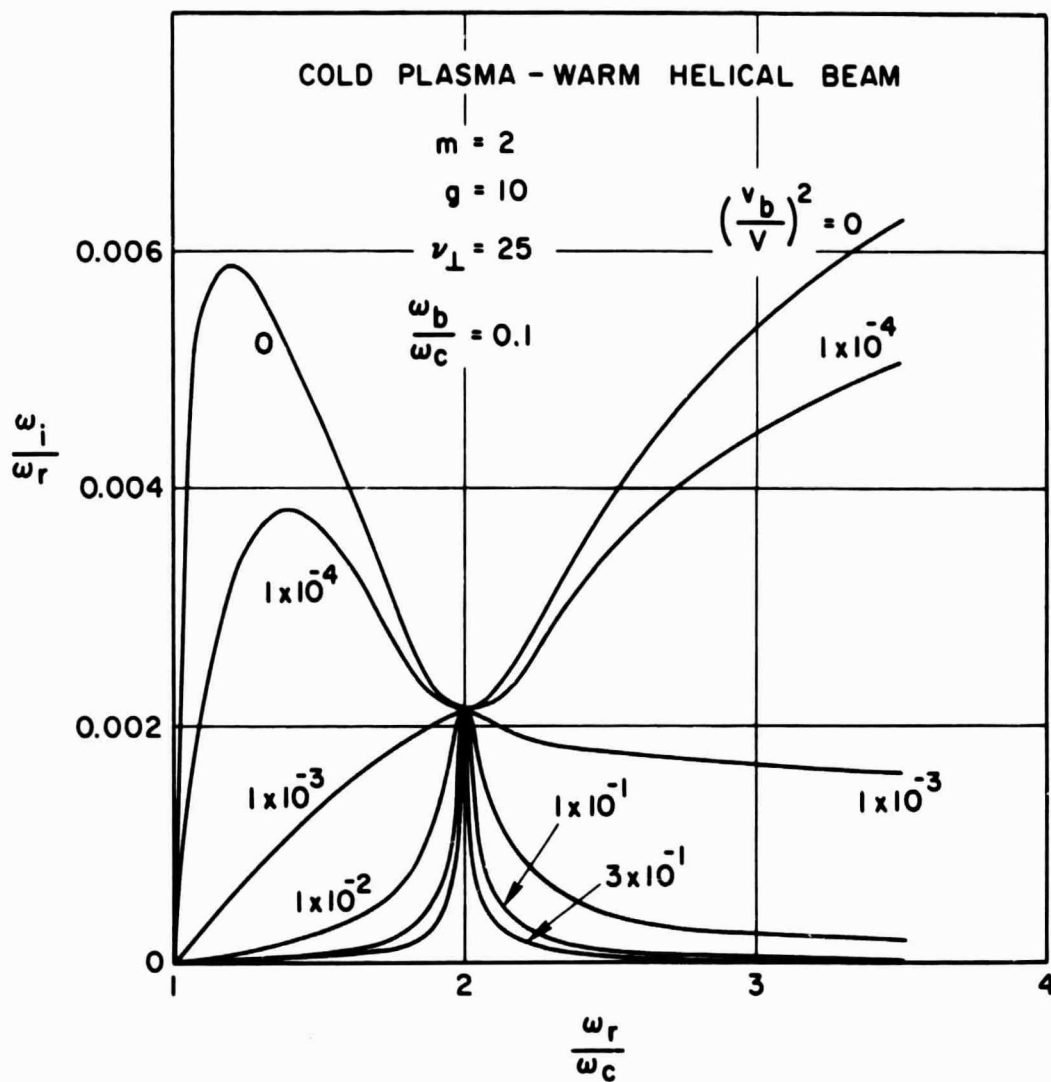


Fig. 15. Effect of the beam temperature on the growth rate of the $m = 2$ mode.

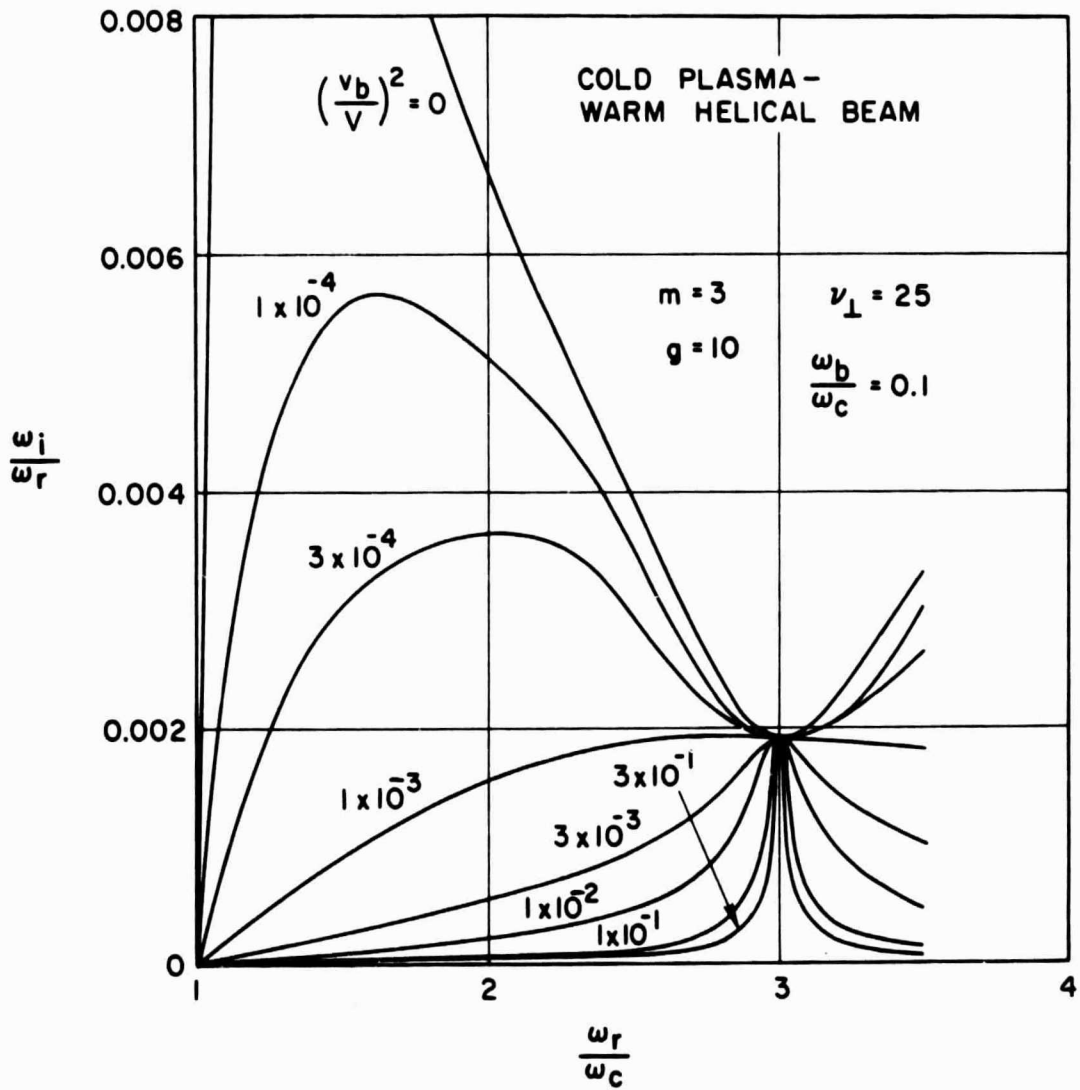


Fig. 16. Effect of the beam temperature on the growth rate of the $m = 3$ mode.

to the magnetic field. Their growth rate may be calculated from Eq. (17) corresponding to $v_b = 0$. Since $x-m \approx 0$, Eq. (17) gives

$$\delta = i \frac{\omega_b}{\omega_c} (-g^2 G m T_m)^{\frac{1}{2}} \quad (32)$$

Assuming $g^2 \gg (x-m)^2$, $\omega > \omega_c$, the expression (16) for $G(x)$ can be simplified to

$$g^2 G \approx \frac{1}{2} \frac{x^2-1}{x} \approx \frac{1}{2} \frac{m^2-1}{m} \quad (33)$$

where the mode number m also defines the real part of the excited frequency, $\omega \approx m\omega_c$. The growth rate becomes

$$\frac{\omega_i}{\omega_r} = \frac{\omega_b}{\omega_c} \sqrt{-\frac{1}{2} \frac{m^2-1}{m^2} T_m(\nu)} \quad (34)$$

This is a function of $\nu = k_1^2 v_1^2 / 2\omega_c^2$.

Since the wave with the highest growth-rate is most likely to be excited, it is of interest to find the value of ν which maximizes ω_i . Although Fig. 11 gives this information directly, simple formulas may be obtained using the first two terms of the asymptotic expansion of $T_m(\nu)$. The growth rate near the m^{th} cyclotron harmonic frequency reaches the largest value

$$\frac{\omega_i}{\omega_r} \approx \frac{1}{10} \frac{\omega_b}{\omega_c} \sqrt{\frac{m^2-1}{m^5}} \quad (35)$$

when

$$\nu \approx \frac{5}{8} (4m^2-1) \quad (36)$$

Due to finite length of the plasma column, waves with $k_z = 0$ are not possible. However, longitudinal velocity spread in the beam can be neglected, as long as $|\omega_i / k_z v_b| \geq 1$, a condition which allows a finite k_z .

This condition also determines the bandwidth

$$\left| \frac{\omega_r - m\omega_c}{\omega_r} \right| \leq \frac{\omega_i}{\omega_r} \frac{V}{v_b} \quad (37)$$

in which the growth rate is approximately given by Eq. (34). Outside this frequency band the growth rate is very small, as is illustrated in Figs. 15 and 16.

5.3. Effect of Plasma Temperature

Although no numerical calculations have been made for the case when the plasma is warm, it is of considerable interest to estimate the effect of plasma temperature on the instability occurring when the beam has a large spread in longitudinal velocities.

According to the previous section, the largest growth rate corresponds to a wave which has a frequency in the frequency band given by Eq. (37) a large transverse wave number, given by Eq. (36), and a small longitudinal wave number $k_z \approx (\omega_r - m\omega_c)/V$. For such a wave, the plasma susceptibility (4) can be approximately written as

$$\epsilon_p = - \frac{\omega_p^2}{\omega_c^2} \sum_{n=-\infty}^{+\infty} \frac{n e^{-\lambda} I_n(\lambda)}{\lambda(\omega - n\omega_c)} \quad (38)$$

An appropriate solution of the dispersion equation (1) discloses two effects of non-zero plasma temperature. A minor effect is a modification of Eq. (13) determining the relationship between the excited frequency and the plasma density. Although this relation has not been calculated explicitly, its general feature is similar to that shown on the right hand side of Fig. 6. A more important effect is the reduction of the

growth rate by plasma temperature. However, the cold plasma formula (34) for the growth rate remains approximately valid as long as the plasma temperature is sufficiently small, such that

$$\frac{1}{\lambda} e^{-\lambda} I_m(\lambda) << \left(\frac{\omega_b^2}{\omega_p} \right) (-T_m(\nu)) \frac{\sqrt{(\omega_r - m\omega_c)^2 + \omega_i^2}}{\omega_i} \quad (39)$$

where $\lambda = k_{\perp}^2 v_t^2 / 2\omega_c^2 = \nu (v_t / v_{\perp})^2$ and ω_i is given by Eq. (34).

It may be noted that the upper bound on plasma temperature, fixed by this inequality, depends on the difference $(\omega_r - m\omega_c)$ between the excited frequency and the cyclotron harmonic frequency. The permitted plasma temperature has the lowest value when $\omega_r - m\omega_c = 0$ and it increases with increasing $(\omega_r - m\omega_c)$ until the latter reaches the limit given by Eq. (37). Replacing the LHS of the inequality (39) by the first term of the power series in λ and using Eq. (37), the inequality (39) becomes

$$\left(\frac{v_t}{v_{\perp}} \right)^{2(m-1)} << \left(\frac{\omega_p}{\omega_b} \right)^2 \frac{2^m m!}{\nu^{m-1}} (-T_m(\nu)) \frac{\nu}{v_b} \quad (40)$$

This gives the largest value of the permitted plasma temperature. For the parameters corresponding to Fig. 16 and assuming $(v_b / V) = 0.1$ one obtains $(v_t / v_{\perp})^2 << 10^{-3}$ at the boundary $(\omega_r - m\omega_c) / \omega_r \approx 0.02$ and $(v_t / v_{\perp})^2 << 3 \times 10^{-4}$ in the center of the allowed frequency band (37).

Summarizing, the main effect of moderate plasma temperature is a splitting of the instability peak into a doublet which has a small growth rate at the cyclotron harmonic frequency and the largest growth rate at the boundaries of the frequency band (37).

6. Discussion

The two beams analyzed in the previous sections are relevant for two distinct types of experiments. The theory is aimed at predicting the rf spectrum as a function of experimental parameters. Comparison of the results of linear theory with experiments operating predominantly in the nonlinear regime is only possible with the additional assumption that it is the mode with the highest linear growth rate which reaches the nonlinear limit.

The beam considered in Section 4 is a model for an essentially parallel beam used in beam-plasma experiments in which the electron gun is immersed in the uniform magnetic field.⁶⁻¹⁴ According to the theory, the wave with the lowest transverse wave number, compatible with the transverse dimensions of the system, is most likely to be excited.

If this transverse wave number is so small that the parameter $g = k_{\perp} V / \omega_c \ll 1$, the $m = 0$ interaction is dominant. The excited frequency is close to the plasma frequency $\omega \approx \omega_p$. When the plasma is cold, there is a stopband just below the cyclotron frequency as shown in Fig. 2. When the plasma is warm, this stop band disappears and the excited frequency can be continuously tuned by varying the plasma density as illustrated in Fig. 5. Finite temperature of the plasma and of the beam make the growth rate smaller, but do not change the excited frequency much. On the other hand, the $m = -1$ interaction is dominant if $g \gg 1$. In this case, only the H wave can be excited and the frequency of the instability is near to the upper hybrid frequency when the plasma is cold. When the plasma is warm, the excited frequency becomes a multivalued function of the plasma density and the growth rate is substantially reduced at the lower

cyclotron harmonics, as illustrated in Fig. 6. As a result, frequencies in between cyclotron harmonics, $\omega \approx (m + 1/2)\omega_c$ are most likely to be excited.

These conclusions are in agreement with experimental data in the case $g < 1$. Somewhat controversial is the case $g \gg 1$. Idehara,¹³ Landauer and Müller,¹⁴ Lustig et al.,¹⁵ Rognlien and Seidl⁴¹ indeed observed instabilities at $\omega \approx (m + 1/2)\omega_c$ when using a parallel beam. On the other hand, Seidl and Šunka¹² consistently saw the strongest instabilities just below the cyclotron harmonic frequencies even when the beam had no transverse energy. However, the experiments^{12, 14, 15, 41} must be interpreted cautiously because the plasma density was caused by the unstable waves. It may be argued that the energy available in the beam of the experiment¹² was so large that the waves with the largest amplitude quickly changed the plasma density to a value where the growth rate versus the density curve had a negative slope. The dependence of the excited frequency on the plasma density as measured by Seidl and Šunka¹² resembles the theoretical curves shown in Fig. 6.

The beam treated in the Section 5 models helical beams which have been used either deliberately^{12, 14-17} or incidentally in PIG discharges.^{5, 18} The spread in the longitudinal velocities of these beams is large, $v_b/V \sim 0.1$, even when they are produced by specially designed guns.¹² An even larger spread may be expected for the beams occurring in the PIG discharges.

Due to this spread, the largest growth rate of the instabilities occurs on the narrow frequency bands (37) centered at cyclotron harmonics. The harmonic number m is determined by the plasma density, approximately

by $m = \omega_p / \omega_c$. The corresponding wave has a large transverse wave number, given by Eq. (36), and a small longitudinal wave number. A moderate plasma temperature satisfying the inequality (40) shifts the peak of the instability from the center to the boundaries of the frequency band (37).

These conclusions, as far as the spectrum and longitudinal wave number are concerned, are in good agreement with both the beam-plasma and the discharge experiments.^{5,12,14-18}

It may be concluded that while finite plasma temperature causes maximum growth rates in the center of the passbands between cyclotron harmonics, the finite beam temperature sharply peaks the growth rates in the vicinity of the cyclotron harmonics. In other words, the cyclotron harmonic excitation is primarily a hot beam effect rather than a hot plasma effect.

Acknowledgment

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