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Project V6088 Final Report

A STUDY OF EVALUATION TECHNIQUES FOR LUNAR SCIENCE



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IIT RESEARCH INSTITUTE

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Project V6088 Final Report

A STUDY OF EVALUATION TECHNIQUES
FOR LUNAR SCIENCE

by

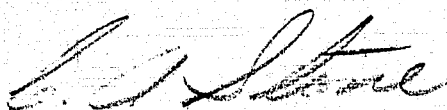
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for

Apollo Lunar Exploration Office
NASA Headquarters
Washington, D. C.

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October 1969

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SUMMARY

This report presents analysis which has been directed at the problem of quantifying the value of science in the lunar exploration program. NASA is on the verge of extensive lunar exploration following the successful Apollo program. However, the analysis which will lead to efficient and effective exploration plans, consistent with practical economic and technological constraints, is complex and difficult to define. It, however, was apparent that a major inadequacy of the existing methods was that it was not possible to make quantitative comparisons of the scientific value of proposed exploration programs.

The purpose of this preliminary study was to attempt to formulate a logic for quantifying the value of scientific measurements by using specific but representative examples. The two examples selected are (1) "Is there active volcanism on the moon?" and (2) "Does the moon exhibit natural seismicity?" The basic concept involved two major steps. The first is to express the scientific goals and objectives of exploration in terms of measurement requirements and the second is to determine the relative worth of possible experiments in fulfilling the measurement requirements.

Tables S1 and S2 summarize the key relationships for the two examples considered. It is readily apparent that the final value equations are somewhat cumbersome. They do demonstrate the feasibility of mathematically expressing scientific value but they are difficult to derive and do not easily demonstrate all the functional dependencies between parameters. Although such equations, if used with a computer, will allow trade-offs to be made between one technique and another and one instrument and another, the benefit would not seem to warrant

the effort. It is not recommended that this line of research be pursued until much more emphasis is placed on the quantitative evaluation of science by advanced mission planners.

Table S1

SUMMARY OF RELATIONSHIPS
FOR ACTIVE VOLCANISM*

Objective: Determine if there is active volcanism on the moon (value V_o).

Purpose: Relates to history of lunar interior and evolution of surface.

Measurement: Determine probability of a given measurement detecting an active volcano (value = V_{om}).

Value Parameter: Probability of detection of volcanoes.

Measurement Value Equation: $V_{meas} = 1 - (1 - V_{om})^n$

where

$$V_{om} = \frac{V_o \text{ fF } (t_e + \ln \theta_o / \theta_v) E_o}{\alpha e^{\gamma A/2}}$$

and

$$n = \frac{4\pi R_o^2 \sin i}{\beta a^2}$$

Instrument Equation:

$$\theta_v = \frac{hc}{k\lambda \ln \left(\frac{2hc^2 a^2 (\Delta\lambda) D^* \beta^2}{(S/N) \lambda^5} \right)}$$

*For symbols to equations see text.

Table S2

SUMMARY OF RELATIONSHIPS
FOR NATURAL SEISMIC ACTIVITY*

Objective: Determine if moon exhibits natural seismic activity (value = V_o).

Purpose: Comparisons with Earth and extension of seismological theory
Future engineering use.

Measurement: Define frequency distribution of moonquake using the meteoritic distribution

$$\log N = 5 - 0.8 m$$

as a datum (value = V_{om}).

Value Parameter: The confidence with which the lunar seismic distribution can be determined.

Measurement Value Equation: $V_{meas} = V_{om} \sqrt{T} (1 - 0.33m_{min} + 0.013m_{min}^2)^{-1/2} e^{-n/10}$

Instrument Equations:
$$n = \frac{3.8 \times 10^4}{(-\log A_{min} - 1.4)^{4.44}}$$

$$m_{min} = (\log A_{min} - \log A_o).$$

*For symbols to equations see text.

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Project V6088 Final Report

A STUDY OF EVALUATION TECHNIQUES FOR LUNAR SCIENCE

1. INTRODUCTION

The analysis and evaluation of the many options for lunar exploration have become increasingly important to NASA. Continued success in the Apollo program has brought the United States to the verge of extensive lunar exploration and has established the capability to conduct a wide range of investigations which can provide answers to many questions concerning the origin and evolution of the moon. However, the analysis leading to the selection of an exploration plan or plans which provide for maximum scientific return (consistent with the economic and technological constraints) represents a complex and difficult planning program. NASA has conducted a number of studies on ways to derive and compare the capability of spacecraft and launch vehicle combinations (MIMOSA 1967, Campbell et al. 1964, Prospectus 1965) which have resulted in operational methods of analysis. These methods are able to define in some detail the spacecraft and launch vehicle requirements needed to achieve a prescribed set of experiments conducted according to a specified exploration plan. They use as an input, somewhat arbitrary groupings of experiments designed to meet most or all of the conceived exploration objectives.

In discussions with NASA in the fall of 1966 it became apparent that the major inadequacy of the existing methods of evaluation was that it was not possible to make quantitative comparisons of the scientific value to be achieved by proposed

exploration programs. The evaluations that were possible required "total exploration" and assumed that each experiment fully satisfied the related objectives. Little or no consideration was given to the relative worth of individual measurements or to the influence of discovery and understanding upon subsequent experiments. If one is to be able to infer the true influence of program approaches and decisions, the evaluation of the science to be achieved is as essential as the evaluation of spacecraft-launch vehicle capabilities.

The problem of quantitatively measuring science is a difficult one at best and in many cases the relationship of measurements to questions is truly non-deterministic. The scientific community has been critical of attempts to numerically evaluate science and justifiably so because past approaches have been based on seemingly arbitrary opinions and as such have contained unidentifiable assumptions and approximations. The fact remains that a suitable approach to the quantitative expression of scientific value must be found if plans with decision options are to be evaluated. To be of practical value the methodology must clearly identify all assumptions and achieve conceptual and mathematical simplicity.

Prior attempts by the Astro Sciences Center of IIT Research Institute to characterize and rate science objectives and experiments had led to the belief that the logic by which subjective opinions are formed could be analytically expressed. The essential ingredients of the logic were felt to be a clear statement of the required measurable parameter(s) and of the influence of the quantity and quality of observations of these parameters on the attainment of the objectives. Thus the purpose of this preliminary study was to attempt to formulate this logic, for representative examples, by specifying the measurable parameters and by identifying their functional relationship to related instrument specifications.

The examples presented in this report demonstrate the validity of the concept. However, the study has shown that the effort required to analytically express the value of an experiment is relatively large. Section 2 discusses the methodology in more detail and Sections 3 and 4 present the results obtained for volcanism and for natural seismicity, respectively. Section 5 summarizes the conclusions which have been developed during this study.

2. STUDY CONCEPT

Although space missions will necessarily contribute to the national and international prestige of the United States, and ultimately have considerable impact upon the economy of the country, the scientific data which are obtained from measurements represent the most immediate return. However, not all experiments are equally valuable, nor do they contribute equally to the general body of science. The purpose of this study was to provide a scheme whereby the relative value of scientific experiments can be given, based upon the logic of experiment design.

The basic concept involves two major steps:

1. To express the scientific goals of exploration in terms of measurement requirements.
2. To determine the relative worth of possible experiments in fulfilling the measurement requirements.

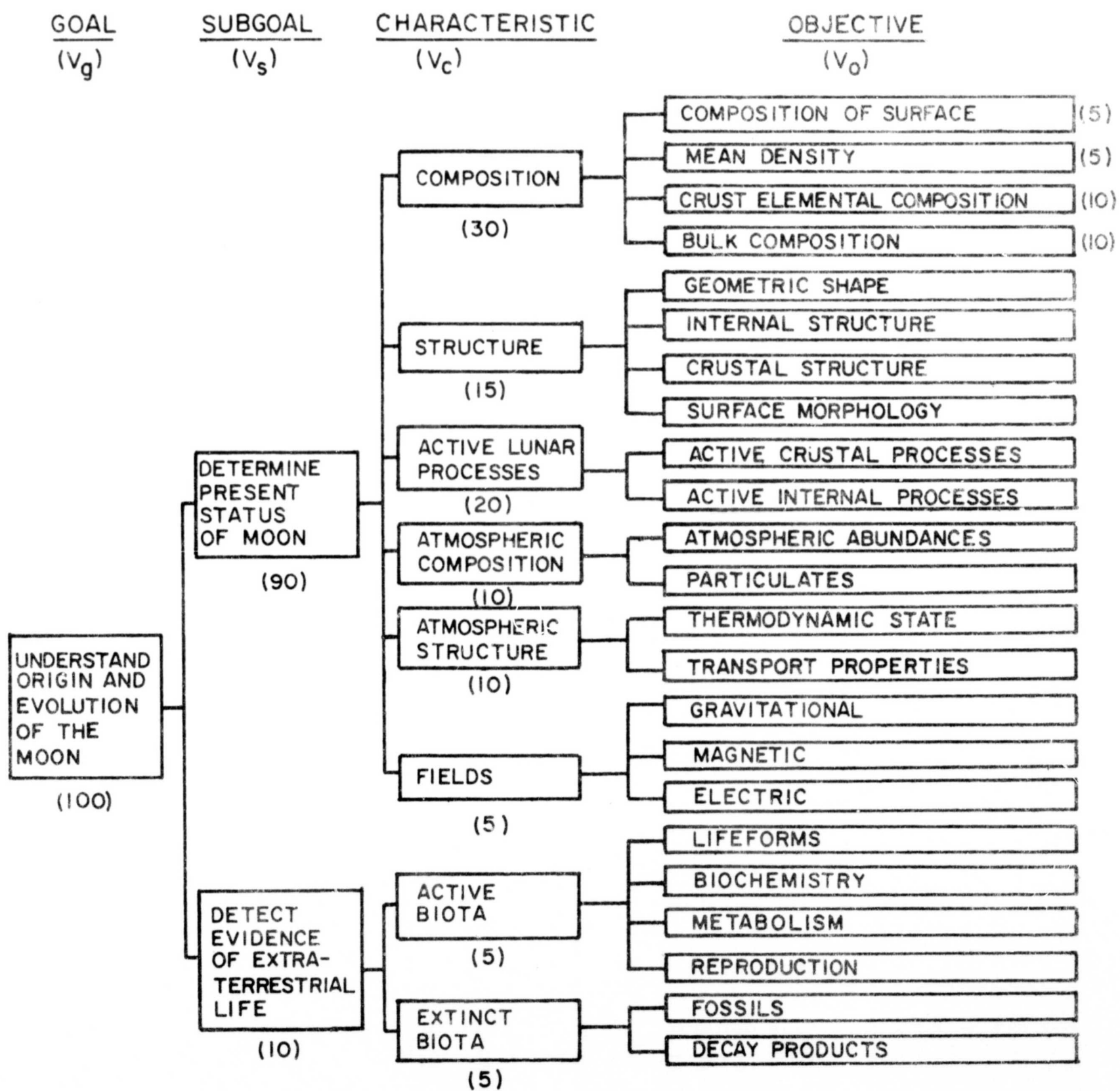
The scientific goals are the broad, long term and relatively unchanging ends to be sought in the exploration of, in this case, the moon. As such they are the starting point for the evaluation of experiments. A goal may give rise to a number of objectives, each of which may be fulfilled by a number of measurement requirements which will determine the

contribution to the goal.

By way of example, the goal of lunar exploration is "To understand the origin and evolution of the moon." This is indeed a valid goal but it does little by itself to define the measurements which should be made in order to fulfill it. The method used in the study to identify such measurements has been to further define the goal in successive stages of increasing detail, ultimately reaching a level at which individual measurements can be specified. Figure 1 shows the first four steps in such a breakdown. It can be seen that the goal can only be fulfilled if the subgoals are fully understood. However, each subgoal can by itself only be specified if each of the "Characteristics" are fully defined and so on.

Conceptually it should be possible to evaluate the relative worth of each of the measurements by utilizing the logic shown in Figure 1. It is possible, albeit subjectively, to define the relative contributions that each of the subgoals makes to the fulfillment of the goal. Similarly the relative contribution of the characteristics to each of the subgoals can be evaluated. Ultimately the relative contribution of each of the Objectives (V_o) to the goal can be defined. The results of this relative evaluation are indicated in Figure 1 by the numbers in parenthesis. This is the first major step in the evaluation of the worth measurements. The second step is to evaluate the relative worth of actual experiments in meeting the objectives. The first step requires an assumption that the subjective estimates at each level can be quantified satisfactorily through one of the many opinion survey methods available.

Regarding the second step the quality of a measurement can be defined in terms of the individual parameters of the measurement. Thus the "perfect" measurement can be defined by a set of measurement specifications and the quality of an



V_g = RELATIVE VALUE OF GOAL.

V_s = RELATIVE VALUE OF SUBGOAL

V_c = RELATIVE VALUE OF CHARACTERISTICS

V_o = RELATIVE VALUE OF OBJECTIVES

FIGURE 1. EXPANSION OF AN EXPLORATION GOAL

actual measurement can be determined by how nearly it meets the specifications. The value of a measurement can be expressed as

$$V_{\text{meas}} = V_{\text{om}} V_i (P_1, P_2 \dots P_n) \quad (1)$$

where V_{om} is the value of the perfect measurement to the objective and V_i is a function of measurement parameters (P_i) which defines the fractional value of the "actual" measurements.

For any given objective and its related "perfect" measurement specification, the relative value of an actual measurement will depend on the specification of the instrument(s) being used and on the parameters of the particular missions from which the measurements are made. Thus

$$V_i (P_1, P_2 \dots P_n) \equiv V_i (I_1, I_2 \dots I_n, M_1, M_2 \dots M_n) \quad (2)$$

where I_i are instrument parameters and M_i are mission parameters.

By reducing the variables to instrument specifications and mission parameters the problem of evaluation can be treated mathematically using the same basic terms and logic as are used in the normal design phase of scientific experiments.

In order that this concept be fully understood it is worthwhile discussing an example qualitatively. Figure 1 shows, as one of the objectives, "determining the composition of the lunar surface." If it is presumed that the application of the logic up to that point yields a value of V_o (surface composition) then it remains to evaluate a given experiment designed to meet the objective. In order to proceed, "composition" must be further defined as being either elemental, chemical, mineralogic, petrologic, lithologic or some combination thereof. For purposes of example let us select elemental composition. A reasonable statement of the measurement specifications might be, "Determine the Elemental Composition of the Major Elements in

Lunar Surface Material to an Accuracy Consistent with our Present Knowledge of the Earth's Crust." However, a number of even more precise measurement specifications are necessary to determine the "perfect" measurement. The selection of appropriate major elements, the number, as well as type and location of sites for measurement, and the desired accuracy of the measurement for each element all go to make up the desired experiment.

Table 1 summarizes the state of knowledge concerning the composition of the Earth's crust. Figure 2 illustrates the many possible options which would have to be investigated and analytically expressed to satisfy this single objective. Once the basic measurements have been identified, their value to the objectives (V_{om}) can be defined, assuming perfect measurements. This example requires 23 individual values of V_{om} and the same number of equations which will have limited commonality with each other. Clearly any technique suitable for practical application to the many objectives and the related instruments must be capable of rapid and simple exposition. The subtleties of definition and assumption associated with the objectives were clearly the factors which would determine the effort involved in the proposed method. Thus the study evolved into an examination of the expertise and time required to obtain value equations for selected questions. The following sections describe the results from the analysis of two objectives, active volcanism and natural seismicity.

3. ACTIVE VOLCANISM

The processes which have formed and modified the lunar topography have long been the subject for scientific discussion. The relative roles of tidal forces, meteoritic impact, solar wind and volcanism are important to understanding the varied surface features. Relatively recent observations of anomalous radiation from the moon have been ascribed to clouds of gas issuing from craters and have given further stimulus to volcanic

Table 1

AVERAGE COMPOSITION OF THE CONTINENTAL CRUST
WITH UNCERTAINTY*

Element	Weight Percent	Measurement Accuracy
Oxygen	48.2	4%
Silicon	28.7	4
Aluminum	8.2	1
Iron	4.4	11
Calcium	2.9	23
Potassium	2.8	7
Sodium	2.3	22
Magnesium	1.9	11

*Derived from Jacobs, Russell and Wilson (1959).

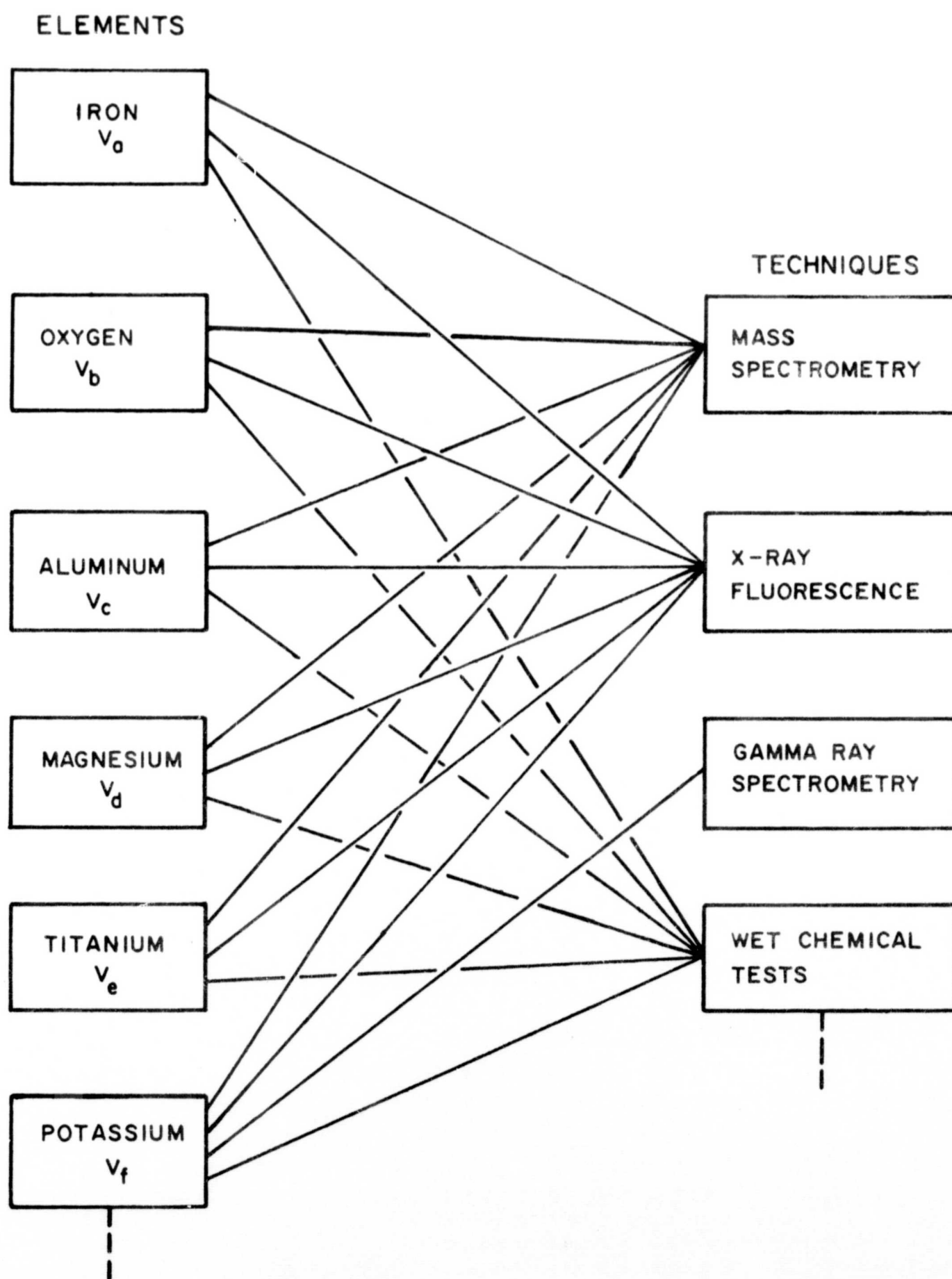


FIGURE 2. MEASUREMENTS RELATED TO ELEMENTAL ABUNDANCES.

theories. The Space Science Board specifically asked "Is there Active Volcanism on the Moon?" -- as a part of one of the questions resulting from the 1965 Summer Conference on Lunar Exploration and Science. Remote detection of active volcanism was selected as one of the objectives to be analyzed as part of this study.

3.1 Question Definition and Experiment Choice

Further definition of the question is required to proceed. We have chosen to interpret active as meaning occurring during the period of measurement. One could have defined active as including the past few months or years but this would enlarge the number of possible measurement types. Even the narrow definition employed -- occurring during the period of measurement -- admits of four different experiments. Terrestrial volcanism is characterized by seismic activity, emission of gas (predominantly water vapor), ejection of particulates and lava flow. Thus in principle, experiments to measure any one of these observables could be considered although one knows intuitively that they are not of equal value. Since molten rock (lava) is the most unique feature of volcanic activity and is the mechanism which usually provides the major topographic change, we have further selected the detection of molten rock as the experiment to be evaluated.

The distinguishing property of molten rock which is most easily observed is, of course, its temperature which is much higher than any environmental temperatures expected to be encountered. Three spectral regions can be considered for observation of thermal emissions: visible, infrared (IR), and mm wavelengths. Again a selection, detection of IR, has been made for purposes of this study.

The selections and alternatives are summarized in Figure 3. This example serves to further emphasize the potentially large number of options which might be pursued

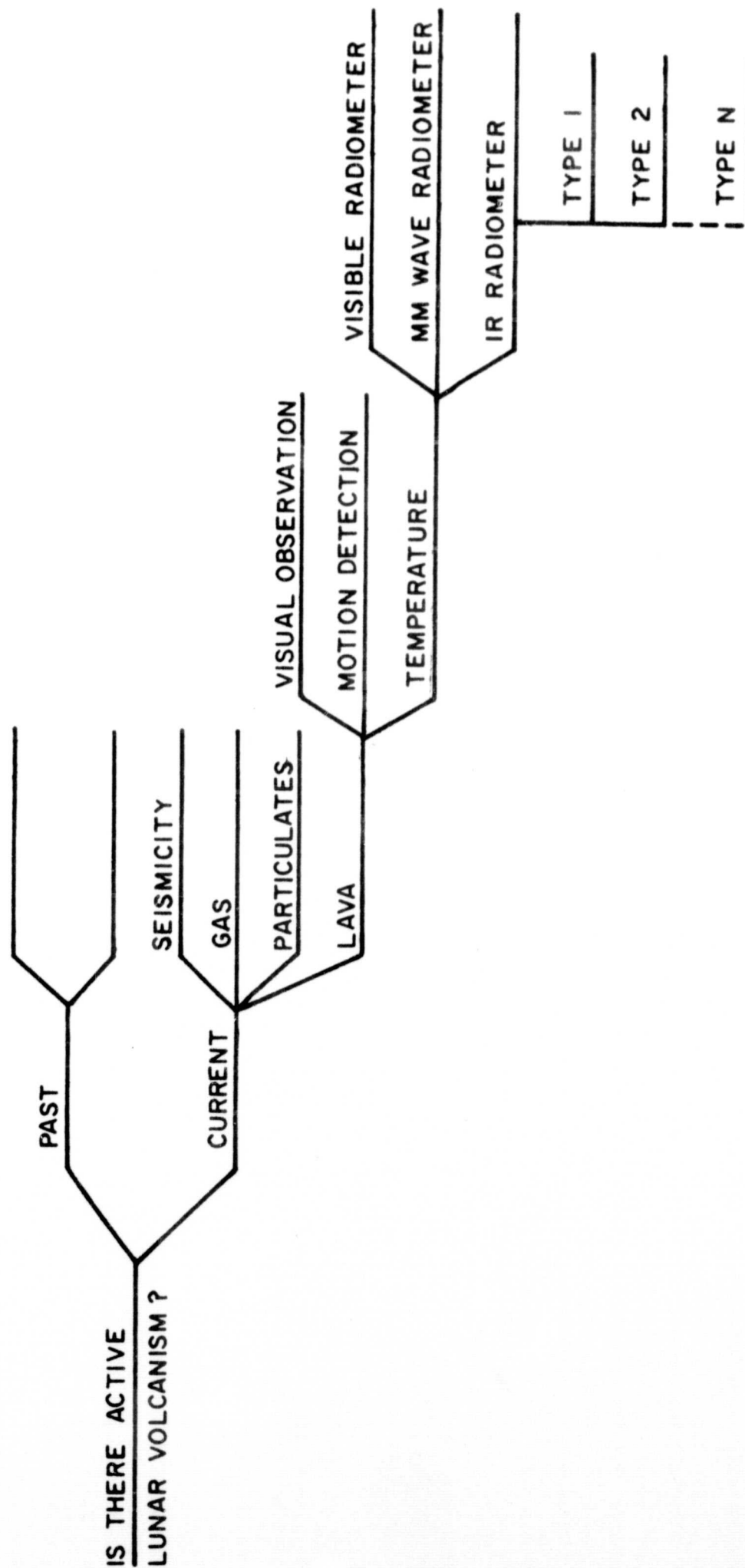


FIGURE 3. EXPERIMENT OPTIONS FOR ACTIVE VOLCANISM.

although it can be argued that some of the choices are easily made on the basis of first principles.

3.2 Development of the Value Equation

The question as stated -- "Is there Active Volcanism on the Moon?" -- requires a yes or no answer and the value of obtaining an answer (V_o) can be assessed using the logic discussed in Section 2. However, observational evidence to date, both earth and lunar, suggests that if volcanoes exist on the moon their number is small and their activity is low. Further, this example which is based on detection of very hot or molten rock requires the activity to be nearly concurrent with the measurement. The detection of such semi-random and infrequent phenomena is best treated on a probability basis and if the probability for detection is high, the absence of the observable can be taken as a satisfactory no answer (obviously detection provides a yes). Thus the probability of detection is a measure of value of the experiment (V_{om}) with a scale ranging from zero to one.

The first step is to express the probability that a single field of observation contains a volcano emitting (or, as is shown later, having recently emitted) lava. For purposes of example we have assumed a model of lunar volcanism which is similar to earth experience. Other models could be treated in an analogous manner. Terrestrial volcanoes are usually associated with conical peaks and, unless they are just forming, possess a central crater. The probability that a randomly positioned observation* includes such a feature is the fraction, f , of the lunar surface covered by potentially volcanic areas. This fraction is the estimated number of volcanic features multiplied by their average area of the active feature divided by the surface area of the moon. We know from earth experience

*A more sophisticated approach could be used invoking the latitude and longitude of regions containing features which were judged potentially volcanic.

that approximately 500 volcanoes have been active in historic time (Jacobs et al. 1959, Howell 1959). This represents a fraction F (one percent or less) of all volcano-like peaks. Therefore this model gives a probability, fF , that a spatially random observation will include a feature which is active in historical time.

Terrestrial volcanoes exhibit a changing rate of activity (eruption with lava flow) which decreases as the age of the volcano increases. Eruptions may occur a few hours or days apart as the volcano is born, subside for several years and finally occur tens to hundreds of years apart. While the data on terrestrial volcanic activity versus age are too sparse to permit accurate representation, a crude approximation is that of exponential decay wherein the number of eruptions per unit time E is given by

$$E = E_0 e^{-\gamma t} \quad (3)$$

where t is time measured from the birth of the volcano, E_0 is the average rate of eruption at birth and γ is a decay constant. E_0 and γ can be estimated from earth experience. The time T between eruptions for any given volcano can be approximated by

$$T = \frac{1}{E} = \frac{e^{\gamma \tau}}{E_0} \quad (4)$$

where τ is the age of the volcano.

Evaluation of the period of eruption is seen to require knowledge of the volcano age τ . Again we know from earth experience that the number of active volcanoes has not changed substantially in the past 10 centuries or so. This implies that the present birth and extinction rates for volcanoes are roughly equal and further, that the distribution of ages of active volcanoes is approximately uniform between the limits

of 0 and say 500 years. Thus all ages of volcanoes in that range are equally probable. The probable value of the age τ can be shown to be

$$\tau_p = \frac{1}{N} \sum_{i=0}^{A/N} \frac{A}{N} i = \frac{A}{2} \quad (5)$$

where N is the number of active volcanoes and A is the maximum age of interest. Thus the probable time between eruptions becomes

$$T = \frac{e^{\gamma A/2}}{E_0} \quad (6)$$

If the time period over which the volcano remains active is t_e then the probability that the observation will be made at the time of an eruption is just t_e/T .

The instrument postulated for this example is an IR radiometer system operating as a threshold temperature detector. For temperatures greater than some value θ_v , the instrument will record a yes answer and, for lower temperatures, a no answer. If θ_v is chosen sufficiently high in comparison to the maximum surface temperature expected from all other effects then a temperature of θ_v or greater can be said to be 100 per cent probability that a volcano has been observed. However, we have seen that the probability of observing a volcano at the time it is active is proportional to the time over which the volcano is active. In terms of a threshold detector, this is the time that the temperature is greater than θ_v . If we choose θ_v as low as possible consistent with the constraint that it be high compared to other possible surface temperatures then the period of "activity" is extended.

Typically a volcano having lava flow will exude molten rock for a period of hours or longer after which the rock will cool by radiation and conduction. The cooling rate is expected

to be approximately exponential

$$\theta = \theta_o e^{-\alpha t} \quad (7)$$

where α is a coefficient of cooling and θ_o is the melting temperature. Therefore the time required to cool to a temperature θ_v is

$$t_c = \frac{\ln \theta_o / \theta_v}{\alpha} . \quad (8)$$

Thus the effective time of the eruption is the sum of the eruption time and cooling time

$$t'_e = t_e + \frac{\ln \theta_o / \theta_v}{\alpha} . \quad (9)$$

If we combine the probability of seeing a volcano-like feature, the probability that it is historically active, the probability that it is erupting at the time of observation and the probability that we will see the eruption, the value of any given measurement is given by

$$V_{om} = \frac{V_o f F (t_e + \ln \theta_o / \theta_v) E_o}{\alpha e^{\gamma A/2}} \quad (10)$$

where

V_o = intrinsic value of the objective

f = fraction of lunar surface covered by potentially volcanic features,

F = fraction of potentially volcanic features thought to be historically active,

t_e = average period of lava flow,

θ_o = temperature at which lunar material is expected to melt,

θ_v = threshold temperature of detection,
 E_o = average rate of eruption for a new volcano,
 α = cooling coefficient for lunar lava,
 γ = decay constant for volcanic activity, and
 A = age of the oldest volcanoes of interest.

If the probability that a single measurement detects an erupting volcano is given by V_{om} , then the value of multiple measurements in the detection of volcanoes is given by

$$V_{meas} = 1 - (1 - V_{om})^n \quad (11)$$

where n = the number of different measurements made. The number of different measurements, ignoring overlap, can be defined in terms of orbital and instrumental parameters as

$$n = \frac{4\pi R_o^2 \sin i}{\beta a^2} \quad (12)$$

where

R_o = lunar radius,
 i = orbital inclination,
 β = instrumental field of view in steradians, and
 a = orbital altitude.

We can now express the value of a threshold detector orbiting the moon to detect active volcanoes as

$$V_{meas} = 1 - \left[1 - \frac{V_o}{\alpha} fF e^{-\gamma A/2} (t_e + \ln \theta_o/\theta_v) E_o \right]^{4\pi R_o^2 \sin i / \beta a^2} ; \quad (13)$$

this is assuming that volcanic-like features can be preselected and that some variation of earth experience concerning volcanic activity and age is applicable.

The connection of the value equation to detailed instrumental parameters is through θ_v and β . We have assumed that the instrument will measure temperatures equal to or greater than θ_v with 100 percent reliability. For some signal to noise ratio S/N this will be true. The signal to noise ratio for an IR detector is given by $D^* \times$ power received. The power received can be obtained from Planck's law and the instrument geometry as

$$\frac{2hc^2}{\lambda^5} e^{-hc/k\lambda} \theta_v (\Delta\lambda) \beta^2 a^2 \quad (14)$$

where

h = Planck's constant

c = velocity of light

λ = wavelength detected

k = Boltzmann's constant

θ_v = threshold temperature

$\Delta\lambda$ = instrument bandwidth

β = instrument field of view, and

a = orbital altitude.

Solving in terms of D^* and β we obtain

$$D^* \beta^2 = \frac{(S/N) \lambda^5 e^{hc/k\lambda} \theta_v}{2hc^2 (\Delta\lambda) a^2} \quad (15)$$

Or, in terms of instrument parameters, the threshold temperature is given by

$$\theta_v = \frac{hc}{k\lambda \ln \left(\frac{2hc^2 a^2 (\Delta\lambda) D^* \beta^2}{(S/N) \lambda^5} \right)} \quad (16)$$

The overall value equation is then given by substituting Eq. (16) in Eq. (13).

$$V_{\text{meas}} = 1 - \left[1 - \frac{V_o}{\alpha} fF e^{-\gamma A/2} \left(t_e + \frac{n\theta_o k\lambda}{hc} \ln \left(\frac{2hc^2 a^2 (\Delta\lambda) D^* \beta^2}{(S/N) \lambda^5} \right) \right) \right]^{4\pi R_o^2 \sin i / \beta a^2} \quad (17)$$

Equation 17 is clearly a complex and somewhat cumbersome expression. However all the terms, taken individually can be quantified and the answer calculated. Its major drawback is that it represents only one method of measurement of one of the aspects of volcanoes. To permit a proper evaluation of volcano detection experiments would require many such expressions (ref. Fig. 3, pg 11).

Table 2 provides a summary of the basic relationships for active volcanism.

4. NATURAL SEISMIC ACTIVITY

The scientific objective is "to determine if the moon exhibits natural seismic activity." Furthermore, natural seismicity will be assumed if seismic events are detected which are not due to meteoroid impact, that is, if they have nonsurface epicenters. As more knowledge of the internal structure of the moon is attained, it may be possible to use the ratio of P and S waves to differentiate between natural seismicity and meteoroid impacts. This is not feasible for the initial measurements because there is no present knowledge of the transmission characteristics of lunar material.

There are three major sources of seismic activity on the moon:

1. Natural activity
2. Meteoritic impacts
3. Tidal effects.

The latter two may be considered as a noise background to the desired measurement of (1).

Table 2

SUMMARY OF RELATIONSHIPS
FOR ACTIVE VOLCANISM

Objective: Determine if there is active volcanism on the moon (value V_o).

Purpose: Relates to history of lunar interior and evolution of surface.

Measurement: Determine probability of a given measurement detecting an active volcano (value = V_{om}).

Value Parameter: Probability of detection of volcanoes.

Measurement Value Equation:

$$V_{meas} = 1 - (1 - V_{om})^n$$

where

$$V_{om} = \frac{V_o fF (t_e + \ln \theta_o / \theta_v) E_o}{\alpha e^{\gamma A/2}}$$

and

$$n = \frac{4\pi R_o^2 \sin i}{\beta a^2}$$

Instrument Equation:

$$\theta_v = \frac{hc}{k\lambda \ln \left(\frac{2hc^2 a^2 (\Delta\lambda) D^* \beta^2}{(S/N) \lambda^5} \right)}$$

4.1 Internal Activity

The simplest model for the moon is that it is an undifferentiated, gravitationally compressed, sphere of ultrabasic rock such as dunite or eclogite. Using this assumption, mathematical models have been derived by assuming that the moon is made of material both chemically and physically very similar to the upper mantle of the Earth and a good fit of mass and density has been obtained.

Empirical data for the Earth defines the frequency distribution of natural earthquakes as

$$\text{Log } N = a - bm$$

where N = the number of earthquakes per unit time, m = magnitude, and a and b are empirical constants. An additional empirical relationship for the Earth is

$$\text{Log } E = c + dm$$

where E is the energy of the earthquake, m is its magnitude, and c and d are empirical constants.

If it is assumed that the moon and the Earth are similar, and if E is known for the moon, then the magnitude frequency distribution can be calculated. MacDonald (1960) using thermal histories has shown that one can expect about 10 to 100 detectable moonquakes each month on the moon. However, recent results from the lunar anchored IMP (Ness 1969) lead to the conclusion that the internal temperature of the moon may be as low as 1000°K , i.e., much below MacDonald's estimate. Thus the above 10-100 detectable moonquakes per month may be many orders too high.

The required experiment is to measure the magnitude of all natural seismicity on the moon as a function of time, in

anticipation of a relationship of the form

$$\text{Log } N = a - bm.$$

4.2 Meteoritic Impacts

The surface of the moon contains a record of meteoritic impacts over its lifetime. By observing the distribution of craters as a function of their size, and by relating their size to the energy of impact, Hartmann (1966) has developed an equivalent impact distribution:

$$\text{Log } N_m = 5 - 0.8 m$$

where N_m is the number of meteoric impacts per year and m is the magnitude of the resultant moonquake. Shoemaker has provided evidence that the present meteoritic flux is at present ten times that derived from the Hartmann data. In either event, the meteoritic influx can be used as a lower limit to the detection of natural seismic measurements since it represents a background noise level.

The seismic experiment will therefore be set up such that the equation $\log N = 5 - 0.8 m$ can be measured with a high degree of confidence. In fact, the level of confidence that all seismic events above the noise level can be detected is a measure of the quality of the seismic measurements.

4.3 Tidal Effects

Tidal forces on the moon have not apparently induced enough stress to cause rending of the moon, but they may well trigger natural seismic events at an unnatural rate. The eccentricity of the lunar orbit will induce monthly tidal fluctuations on the order of 2.5 meters (as shown in Figure 4) (MacDonald 1960). If the moon is perfectly elastic the amount of energy involved corresponds to approximately 1.9×10^{25} ergs.

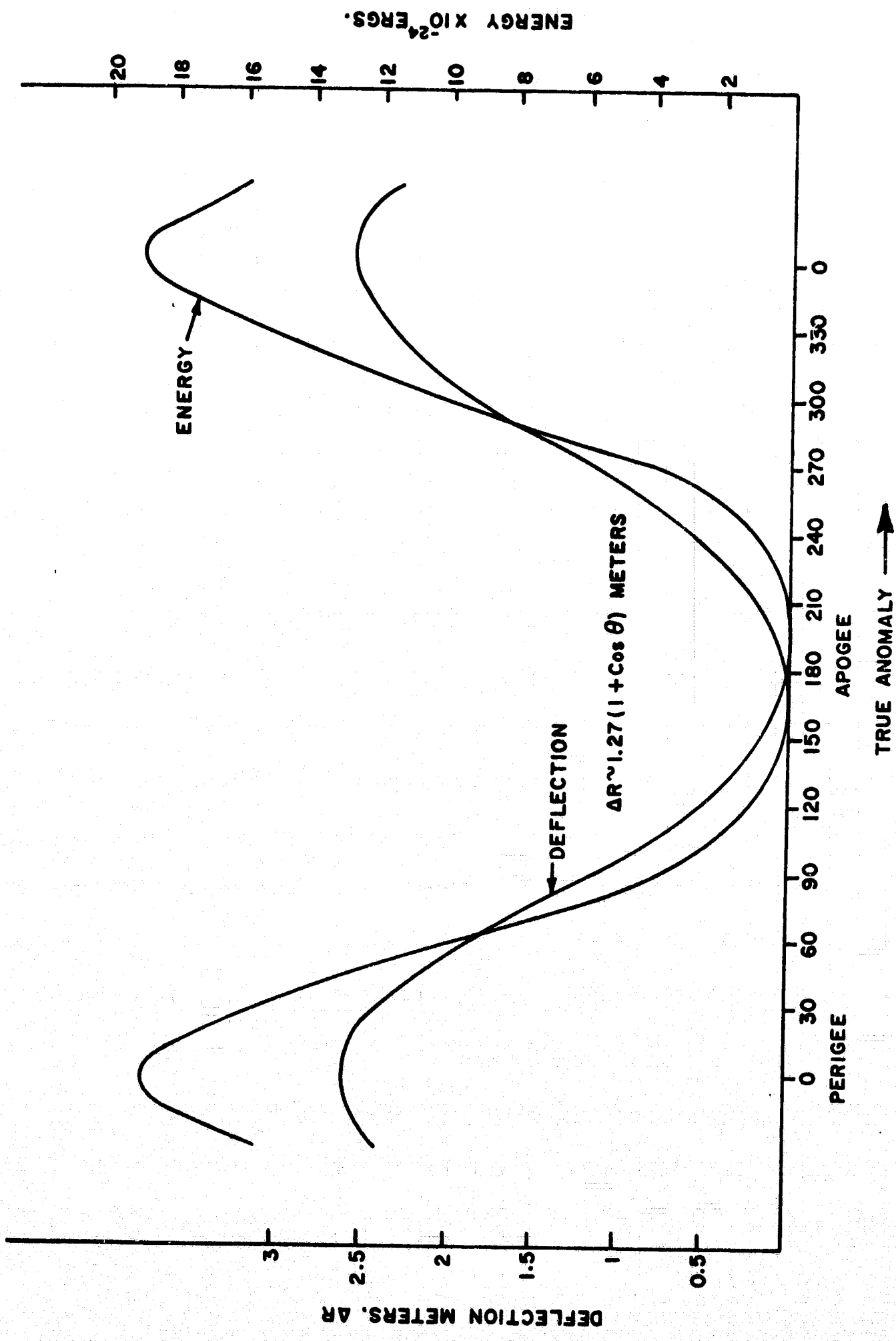


FIGURE 4. DEFORMATION OF THE MOON DUE TO TIDAL FORCES

Thus, each month the moon changes its stored internal energy by about as much as the Earth releases in earthquakes in a year. There is no way to estimate how much of this potential energy is released as tidal moonquakes. The most probable effect is simply a release mechanism for internally induced stresses. It acts therefore to limit the level of natural stress buildup, but it probably does not affect the magnitude of the stress release when it is triggered.

Thus no account has been made for tidal effects in specifying the required measurements.

4.4 Value Equations

The value of the seismic measurements can be assigned as the confidence with which the relationship

$$\text{Log } N = 5 - 0.8 m$$

can be determined from measurements of moonquakes of magnitude $m_{\min}$ or greater over a given time T .

Figure 5 shows the results of an error analysis on the given relationship. The size of the 3σ errors is a function of the limiting magnitude which can be detected ($m_{\min}$) and of the time (T) over which measurements are made.

Therefore the measurement value equation has the form

$$V_{\text{meas}} = V_{\text{om}} V_i (m_{\min}, T).$$

The error analysis from which Figure 3 has been drawn is included in Appendix 1. The error is proportional to the square root of time and is related to the minimum detected magnitude by the relation:

$$\sigma_N^2 = 0.004 m_{\min}^2 - 0.009 m_{\min} + 0.03.$$

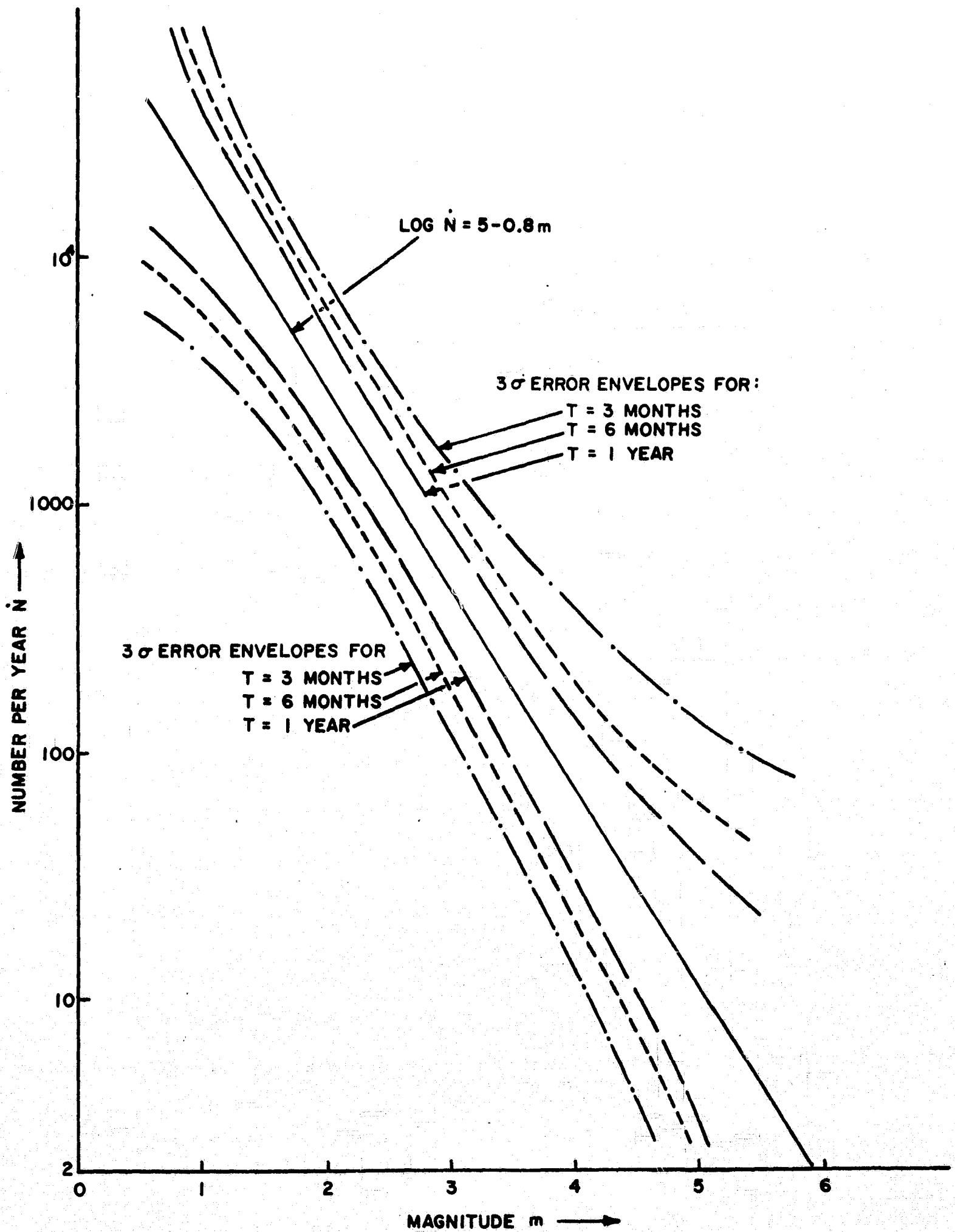


FIGURE 5. ERROR ENVELOPES FOR RELATION $\text{LOG } \dot{N} = 5 - 0.8m$

Thus the measurement value equation is

$$V_{\text{meas}} = \alpha V_{\text{om}} \sqrt{T} (1 - 0.03 m_{\text{min}} + 0.013 m_{\text{min}}^2)^{-1/2}$$

where α is a constant of proportionality. The error is minimum when $m_{\text{min}} = 2$ and if it is assumed that $V_{\text{meas}} = V_{\text{om}}$ when $T = 1$ year then

$$\alpha = 0.996 \approx 1.$$

The final step in the measurement logic is to express the measurement parameters in terms of instrument parameters and mission parameters. Time is in fact both a measurement and a mission parameter. The limiting magnitude m_{min} can be expressed in terms of the seismometer sensitivity A_{min} , in millimeters deflection, and the number of instruments with that sensitivity needed to detect all moonquakes of magnitude m_{min} or larger.

The relationship for the detection of quakes on the Earth is

$$m = \log A - \log A_0$$

where A is the detected deflection in mm at the detector from a magnitude m quake, and A_0 is the deflection from a magnitude 0 quake at a distance Δ . A_0 is given by an empirical relation

$$-\log A_0 = 0.2 (\Delta)^{0.45} + 1.4$$

where Δ is the distance of a quake from the detector in km. In order to detect a magnitude 0 quake across the moon, $A_0 \approx 10^{-8}$ mm. The distance Δ_0 at which a magnitude 0 quake can be detected with a sensitivity A_{min} can also be determined directly from the above equation. If a detector network is

set up so that detectors are no more than Δ_0 km apart, the number of seismometers required in the net is given by

$$n \approx \frac{\text{area of moon}}{\text{area of one loop in net}}$$

$$= \frac{3.8 \times 10^4}{(-\log A_{\min} - 1.4)^{4.44}}$$

Figure 6 shows the relationship between $A_{\min}$ and n and demonstrates the need for high sensitivity if the number of stations is to be kept low. The value of maintaining n small, from a mission standpoint, can be estimated to have the relation $e^{-n/10}$ where 10 stations is considered an acceptable number of seismometers which can be distributed over the lunar surface.

Thus the overall value equation is

$$V_{\text{meas}} = V_{\text{om}} \sqrt{T} \left[1 - 0.03(\log A_{\min} - \log A_0) + 0.013(\log A_{\min} - \log A_0)^2 \right]^{-1/2} e^{-n/10}$$

where

$$n = \frac{3.8 \times 10^4}{(-\log A_{\min} - 1.4)^{4.44}}$$

and

$$A_0 \approx 10^{-8} \text{ mm.}$$

Table 3 summarizes the relationships for natural seismic activity. Once again a relatively complex final equation results and it is difficult computationally because of the 4.44^{th} power. Nevertheless the major functional relationships are relatively clear.

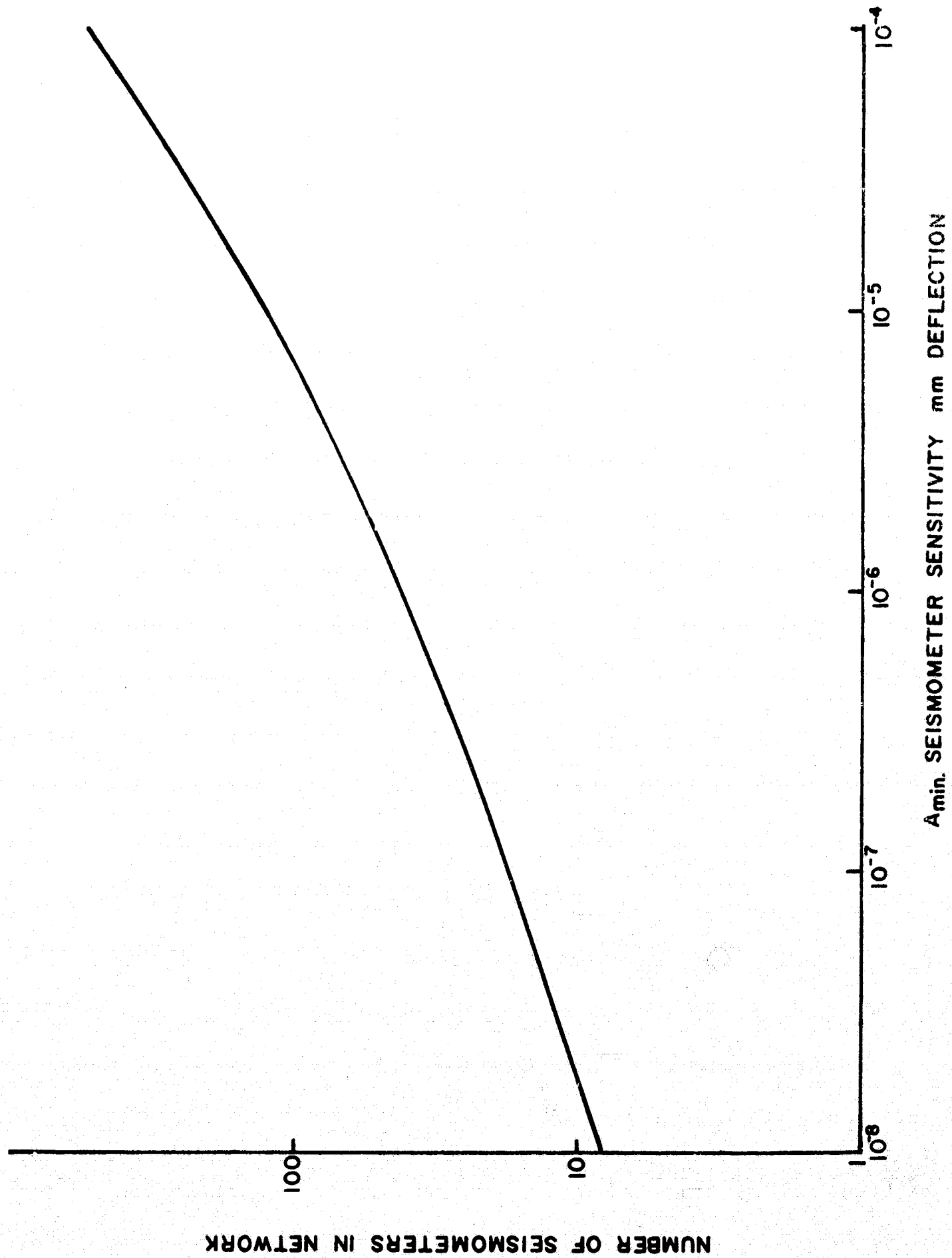


FIGURE 6: SEISMOMETERS REQUIRED IN LUNAR NETWORK VS. SENSITIVITY.

Table 3

SUMMARY OF RELATIONSHIPS
FOR NATURAL SEISMIC ACTIVITY

Objective: Determine if moon exhibits natural seismic activity (value = V_o).

Purpose: Comparisons with Earth and extension of seismological theory
Future engineering use.

Measurement: Define frequency distribution of moonquake using the meteoritic distribution

$$\log N = 5 - 0.8 m$$

as a datum (value = V_{om}).

Value Parameter: The confidence with which the lunar seismic distribution can be determined.

Measurement Value Equation: $V_{meas} = V_{om} \sqrt{T} (1 - 0.33m_{min} + 0.013m_{min}^2)^{-1/2} e^{-n/10}$

Instrument Equations:
$$n = \frac{3.8 \times 10^4}{(-\log A_{min} - 1.4)^{4.44}}$$

$$m_{min} = (\log A_{min} - \log A_o)$$

5. CONCLUSIONS

The two examples developed above adequately demonstrate the feasibility of the basic premise of the study. Mathematical equations have been derived which express the value of a measurement in terms of the parameters of the instrument, the operational conditions, and the intrinsic value of the objectives to which it relates.

The equations developed here are useful in that they express, on sight in some cases, the functional relationships which are important to the usefulness of a measurement. For example, in the case of seismic activity, the value is, among other things, proportional to the square root of the time for which the instrument is operational. However, for other parameters, such as $A_{\min}$ the detector sensitivity, the relationship is not obvious. This is even more true for the active volcanism equation since it is difficult to comprehend, on sight, any of the relationships in the equation.

The two examples used here are relatively straightforward in terms of the measurement techniques. Nevertheless lengthy equations had to be derived and only then after considerable effort. Furthermore the solutions presented are not unique. For any given information requirement there are usually three or more measurement techniques. The examples in Figure 2 and Figure 3 show upwards of twenty possible measurements all in support of a single objective. A similar situation prevails with active seismicity since devices other than seismometers may be used (i.e., geophones) and a number of lunar models should be considered in the derivation of the equations. Thus to develop the relevant equations completely for just two objectives can require a very significant effort from which a plethora of equations will result. Such equations, if developed, would indeed allow trade-offs to be made between one technique and another and between one instrument and another. However the benefit would not seem to warrant the effort.

The development of these equations could be eased if the scientists currently working in the respective areas were to develop them since they are already dealing with the same basic relationships. There would then be problems of uniformity between one set of values and another which would be difficult to overcome. On the other hand it is undeniable that if such equations could be developed by the experts they could be the basis of an excellent mission planning tool. To make use of the tool would require a computer but this is only a problem insofar as the mission analyst may lose sight of the complex array of relationships.

In summary it would appear that although value equations can be derived effectively, and do yield useful information, the effort involved in developing them comprehensively enough is excessive. Therefore it is not recommended that this line of research be pursued until such a time as more emphasis is laid on to quantitative evaluation of science experiments.

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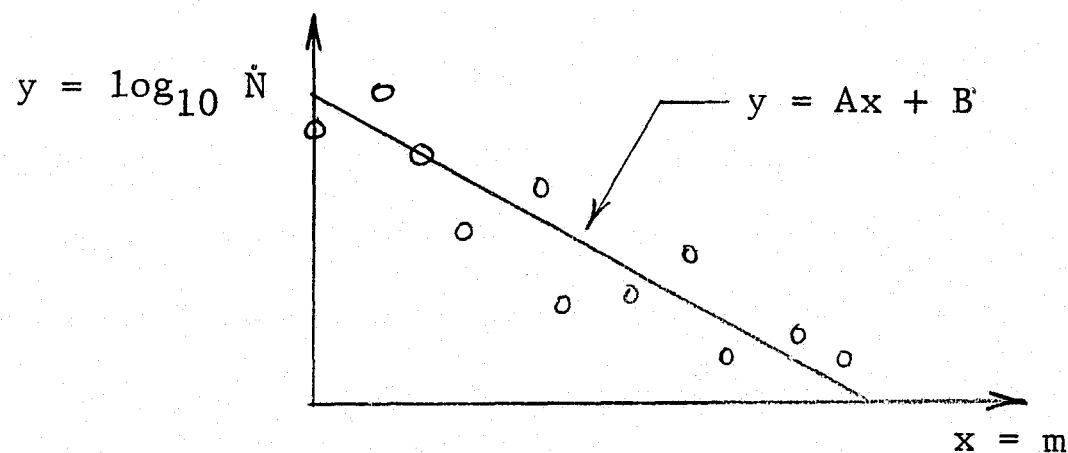
Appendix 1

LUNAR SCIENCE - SEISMIC ACTIVITY
CURVE FITTING ERROR ANALYSIS

Appendix 1

LUNAR SCIENCE - SEISMIC ACTIVITY CURVE FITTING ERROR ANALYSIS

The method of weighted least-squares is applied to the problem of determining the best linear graph to the set of data points (x_i, y_i) where each of the coordinates are subject to statistical error. This is illustrated by the sketch below.



The coordinate $x = m$ represents the magnitude of a seismic disturbance and $\dot{N} = 10^y$ is the frequency of occurrence (number of events per year). If $\sigma_{x_i}^2$ and $\sigma_{y_i}^2$ denote the coordinate variances of independent error sources, then each data point is properly weighted by the inverse of

$$\sigma_i^2 = A^2 \sigma_{x_i}^2 + \sigma_{y_i}^2 \quad (A1)$$

The least-squares determination of the linear slope A and the intercept B is given by

$$A = \frac{\left(\sum \frac{1}{\sigma_i^2}\right) \left(\sum \frac{x_i y_i}{\sigma_i^2}\right) - \left(\sum \frac{x_i}{\sigma_i^2}\right) \left(\sum \frac{y_i}{\sigma_i^2}\right)}{D} \quad (A2)$$

$$B = \frac{\left(\sum \frac{x_i^2}{\sigma_i^2}\right) \left(\sum \frac{y_i}{\sigma_i^2}\right) - \left(\sum \frac{x_i}{\sigma_i^2}\right) \left(\sum \frac{x_i y_i}{\sigma_i^2}\right)}{D} \quad (A3)$$

where

$$D = \left(\sum \frac{x_i^2}{\sigma_i^2}\right) \left(\sum \frac{1}{\sigma_i^2}\right) - \left(\sum \frac{x_i}{\sigma_i^2}\right)^2 \quad (A4)$$

$$i = 1, 2, \dots, n$$

Note that σ_i^2 is a function of A so that Eqs. (A1) to (A4) must be solved iteratively. Any reasonable initial guess of the slope obtained from the data would suffice.

The error variances in the estimates of A and B, and the statistical correlation ρ_{AB} between these variates are given by

$$\sigma_A^2 = \frac{\left(\sum \frac{1}{\sigma_i^2}\right)}{D} \quad (A5)$$

$$\sigma_B^2 = \frac{\left(\sum \frac{x_i^2}{\sigma_i^2}\right)}{D} \quad (A6)$$

$$\rho_{AB} \sigma_A \sigma_B = \frac{-\left(\sum \frac{x_i}{\sigma_i^2}\right)}{D} \quad (A7)$$

The following subsections examine the nature of the separate error sources.

Error in Seismic Rate of Occurrence

Let $\dot{N}_i$ be the number of events per year of magnitude m_i . The error in this quantity is due to the finite listening or observation time T . We assume that $\Delta\dot{N}_i$ has a Poisson distribution; i.e., if the data $\dot{N}_i$ is obtained over the interval of T years, where $\dot{N}_i = N_i/T$, then the variance of $\Delta\dot{N}_i$ is equal to the mean rate of occurrence divided by T .

$$\sigma_{\Delta\dot{N}_i}^2 = \frac{\dot{N}_i^*}{T} \quad (A8)$$

Since $\Delta y_i = \Delta\dot{N}_i/\dot{N}_i$, the variance in y_i is

$$\sigma_{y_i}^2 = \frac{1}{\dot{N}_i^* T} \quad (A9)$$

For example, if $\dot{N}_i^* = 10/\text{year}$ and $T = 2$ years, then

$$\sigma_{\Delta\dot{N}_i} = \sqrt{10/2} = \sqrt{5} = 2.24 \text{ per year, and } \sigma_{y_i} = 1/\sqrt{20} = 0.224.$$

Error in Seismic Magnitude

The magnitude is obtained from the equation

$$m = \log_{10} a + 3 \log_{10} d - 3 \quad (A10)$$

where a is the amplitude in millimeters and d is the distance in kilometers. Distance is given by wavefront differential time-history relationship

$$d = \frac{T_S - T_P}{\frac{1}{V_S} - \frac{1}{V_P}} \quad (A11)$$

Taking differentials of the above expressions yields

$$\Delta m = \frac{\Delta a}{a} + \frac{3\Delta d}{d} \quad (A12)$$

$$\frac{\Delta d}{d} = \frac{\Delta(T_S - T_P)}{T_S - T_P} - \frac{1}{V_P - V_S} \left(\frac{V_S}{V_P} \Delta V_P - \frac{V_P}{V_S} \Delta V_S \right) \quad (A13)$$

For $V_P = 7.9$ km/sec, $V_S = 4.4$ km/sec and $d \leq 3500$ km, it is found that $(T_S - T_P) \leq 350$ sec. In evaluating the above errors the following (1σ) magnitudes of the independent error are assumed:

$$\frac{\Delta a}{a} = 0.01$$

$$\Delta(T_S - T_P) = 0.1 \text{ sec}$$

$$\Delta V_P = 0.2 \text{ km/sec}$$

$$\Delta V_S = 0.1 \text{ km/sec}$$

The separate effects of the time and velocity term errors are given by the table below,

d	$T_S - T_P$	$\left(\frac{\Delta d}{d}\right)_{\text{time}}$	$\left(\frac{\Delta d}{d}\right)_{\text{velocity}}$
3500 km	350 sec	0.00029	0.06
1000	100	0.001	0.06
500	50	0.002	0.06
100	10	0.01	0.06
10	1	0.10	0.06

from which it is noted that the velocity error predominates through most of the distance region of interest. Hence, it will be assumed that the error in determining the seismic

magnitude m is a constant for all m , i.e.,

$$\sigma_{x_i} = \sigma_m = 1/6 . \quad (A14)$$

Combined Errors - An Example

For the sake of example assume that the actual linear relationship is

$$\begin{aligned} \log_{10} \dot{N} &= -0.8m + 5 \\ \text{or } y &= -0.8x + 5 \end{aligned} \quad \left. \begin{array}{l} A = -0.8 \\ B = 5 \end{array} \right\}$$

Suppose magnitude data is available at the discrete points $\{x_i\} = \{1, 2, 3, 4, 5\}$, and that $T = 1$ year. The problem variables are listed in the following table:

$\underline{x_i}$	$\underline{y_i}$	$\underline{\dot{N}_i}$	$\underline{\sigma_{y_i}}$	$\underline{A\sigma_{x_i}}$
1	4.2	15,800	0.00795	0.134
2	3.4	2,500	0.02	0.134
3	2.6	400	0.05	0.134
4	1.8	63	0.126	0.134
5	1.0	10	0.316	0.134

Evaluating Eqs. (A4) to (A7) gives

$$\begin{aligned} D &= 5.4 \times 10^4 \\ \left. \begin{array}{l} \sigma_A = 0.0605 \\ \sigma_B = 0.161 \end{array} \right\} & \quad \left. \begin{array}{l} |\sigma_A/A| = 7.6\% \\ |\sigma_B/B| = 3.2\% \end{array} \right\} \\ \rho_{AB} &= -0.90 \end{aligned}$$

The error bounds of the curve fit may be obtained from the expression

$$\sigma_y^2 = \sigma_A^2 x^2 + \sigma_B^2 + 2x \rho_{AB} \sigma_A \sigma_B$$

where it would be useful to plot the bounds $y \pm 3\sigma_y$ as a function of x as is done in Figure A1.

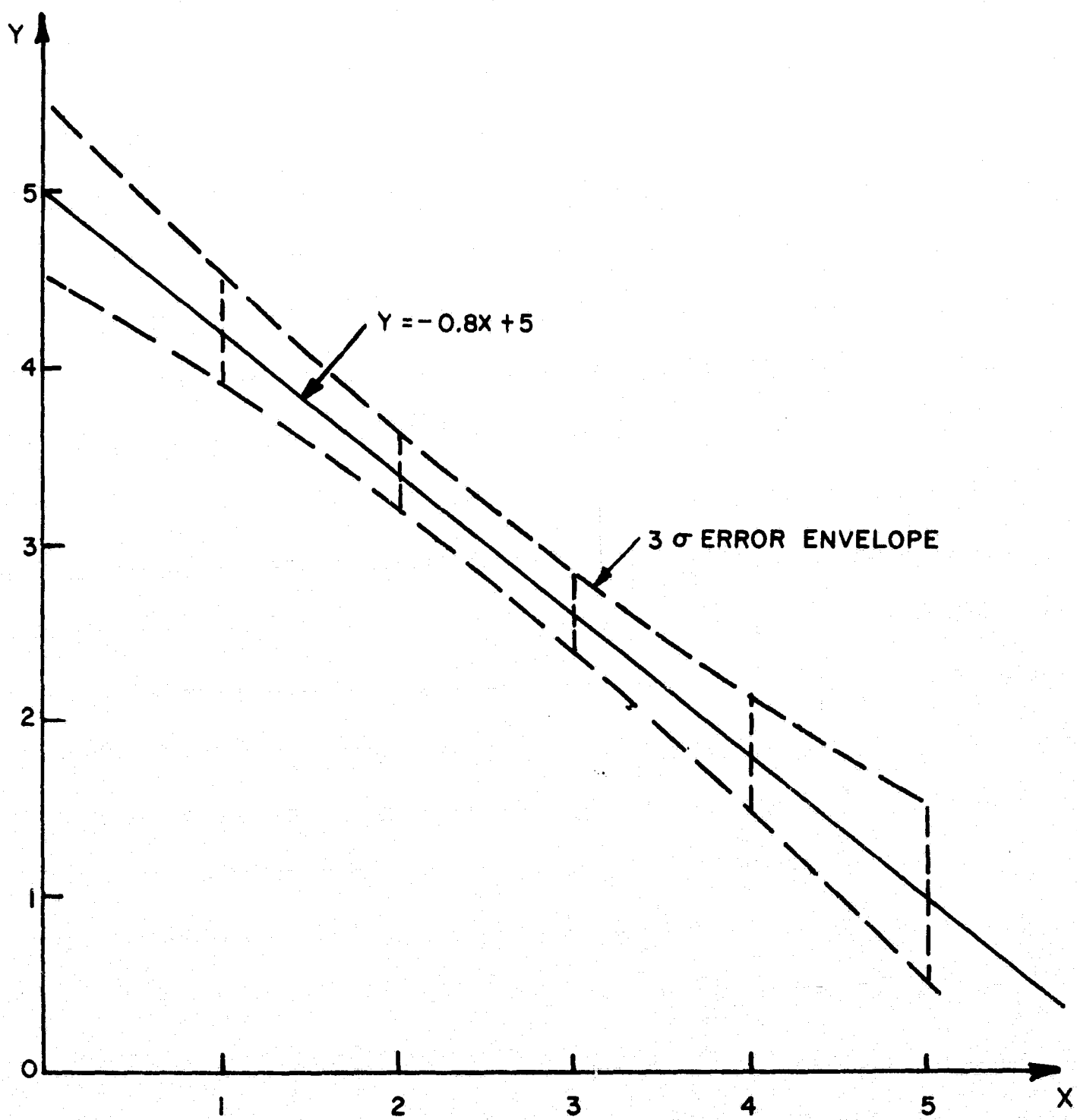


FIGURE A1. RESULTS OF ERROR ANALYSIS ON EQUATION $Y = 5 - 0.8m$.