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SIMPLIFIED BALLISTIC-LIMIT EXPRESSIONS FOR THIN SHEETS

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SUMMARY

Simple equations have been derived to predict the ballistic-limit thickness of thin sheets. Perforation of the plates was assumed to be caused by a shearing failure of the plate material due to impact by a rigid circular cylindrical projectile. Simplified equations for predicting the ballistic-limit velocity of the projectiles have also been presented. The plate was assumed to behave as a visco-plastic solid. The equations were based on a simplification of a more accurate series solution dealing with perforation of a visco-plastic solid. Numerical results were compared with both experimental results and calculations based on the more accurate analysis. These comparisons indicate that the equations are valid for short cylindrical projectiles.

INTRODUCTION

Knowledge of the resistance to perforation or ballistic limit of thin sheets is needed for design of space structures which are subjected to the meteoroid environment. The ballistic-limit thickness is defined as the minimum thickness of a given material that will completely stop perforation of the sheet by a given meteoroid.

Present experimental techniques are not capable of accelerating projectiles of substantial mass to speeds much beyond the lower end of the meteoroid velocity range. For structural design purposes, a method is needed for extrapolating the experimental data over the entire meteoroid velocity range. One such method, developed in reference 1, has been compared with experimental data for disk-shaped projectiles (ref. 2) with satisfactory results. The method of solution in reference 1 uses a series solution, Laplace transforms, and asymptotic expansions and involves extensive numerical calculations for specific values of the projectile parameters. This approach does not result in an explicit relationship between ballistic-limit thickness and projectile properties. The large amount of computational detail makes prediction of trends difficult and limits the usefulness of the method as a design tool.

The purpose of the present paper is to develop an explicit relationship for the ballistic-limit thickness as a function of the projectile parameters. This relationship is based on a simplification of the analysis of reference 1 and allows general parametric variations to be examined easily. The accuracy and range of applicability of the equations derived herein are ascertained by comparisons with the more accurate results from reference 1 and the experimental results of reference 2.

SYMBOLS

The units used for physical quantities in this paper are given in the International System of Units (SI). Factors relating this system to U.S. Customary Units are presented in reference 3.

a	projectile radius
C_1,C_2	dynamic critical strain-rate and strain factors, respectively
$\bar{C}_1,\bar{C}_2$	nondimensional factors defined by equation (9)
$f(\alpha)$	function of α defined by equation (19)
g_o	initial projectile velocity
h	target plate thickness
H	mass-ratio factor, $2m_{pl}/\left(m_p + m_{pl}\right)$
k	target yield stress in shear
K	nondimensional parameter, $\dfrac{\bar{C}_2}{\bar{C}_1} = \dfrac{C_2\mu^2}{C_1 k\rho a^2}$
l	projectile length
m_p	mass of projectile
m_{pl}	mass of plug of target plate, $\pi a^2\rho h$
r	radial coordinate
t	time

$\bar{t}$	nondimensional time, $\frac{\mu t}{\rho a^2}$
V	velocity of target plate
V_o	initial velocity of target plate
w	deflection of target plate
z	axial coordinate
α	parameter, $(\pi K)^{-1}\left(H - \frac{1}{2}\right)^{-2}$
γ_i	coefficient of proportionality (i = 1,2)
μ	dynamic viscosity coefficient of target material
ρ	mass density of target plate

ANALYTICAL APPROACH

Governing Equations

The mathematical model of the impact and target plate response used in the present investigation is the same as that of reference 1. A rigid circular cylindrical projectile of radius a, length l, and mass m_p is considered to impact upon a thin infinite plate of thickness h as shown in figure 1. The plate is considered to be thin; that is, the plate thickness is small in comparison with the projectile diameter. The resulting perforation is assumed to be a shear-plugging perforation in which only the transverse shear stresses act to resist the inertia of the impacting projectile. The perforation is considered to be axially symmetrical and the shear stress is taken to be constant through the plate thickness.

The plate material is assumed to be a visco-plastic Bingham solid. Such a material is considered to remain rigid until the shear stress reaches a critical value k. After this stress value is reached, the material flows as a viscous fluid. A more complete discussion of this type of material is presented in reference 4.

Upon initial impact of the projectile on the target plate, an inelastic momentum exchange occurs between the projectile and a plug of plate material. Through conservation of momentum, the initial velocity of the target plate V_o can be related to the velocity of the projectile g_o by

$$
\left.
\begin{aligned}
V_o &= g_o \frac{2 - H}{2} && (\text{for} \quad r \leqq a) \\
V_o &= 0 && (\text{for} \quad r > a)
\end{aligned}
\right\} \tag{1}
$$

where H is a mass-ratio factor defined by

$$
H = \frac{2}{1 + \dfrac{m_p}{\pi a^2 \rho h}} \tag{2}
$$

If the mass of the projectile is small in comparison with the mass of the plug of target plate material, then

$$
H \approx 2\left(1 - \frac{m_p}{\pi a^2 \rho h}\right) \tag{3}
$$

The initially discontinuous velocity distribution propagates through the Bingham plate and causes stresses, strains, and displacements. The resulting strain rate $\left|\frac{\partial V}{\partial r}\right|$ and radial strain $\left|\frac{\partial w}{\partial r}\right|$ in the plate as a function of time t were derived in series form in reference 1. In order to determine the size of the hole created in the thin sheet, the separation criterion developed in reference 1 was used. This criterion states that a hole will occur if the strain rate and radial strain simultaneously satisfy the following failure conditions:

$$
\left.
\begin{aligned}
\left|\frac{\partial V}{\partial r}\right| &\leqq C_1 \frac{k}{\mu} \\
\left|\frac{\partial w}{\partial r}\right| &\geqq C_2
\end{aligned}
\right\} \tag{4}
$$

In these expressions, $C_1 \frac{k}{\mu}$ represents the dynamic yield strain-rate factor under the conditions of impact and C_2 is the dynamic yield strain factor.

The minimum projectile velocity at which a hole would be formed (i.e., the ballistic-limit velocity) was found to occur when the radius of the hole was equal to the radius of the projectile. A hole of larger radius was formed for higher velocities, but no hole smaller than the projectile was formed. Therefore, at the ballistic limit, the strain rate and the radial strain are equal to their critical values, and the radius of the hole is equal to the radius of the projectile. Thus, from reference 5, with $r = a$, the following two series are obtained for the strain rate and the plate radial strain:

$$\left(-\frac{\partial V}{\partial r}\right)_{r=a} = \frac{(2 - H)g_o}{2a}\left\{\frac{1}{\sqrt{\pi}}\bar{t}^{-1/2} - \left(H - \frac{1}{2}\right) - \frac{2}{\sqrt{\pi}}\left[\frac{1}{8} + (H - K)(1 - H)\right]\bar{t}^{1/2}\right.$$

$$\left. + \left[\frac{1}{2}\left(3H^2 - 2H^3 + \frac{1}{4}\right) + K(1 - H)\left(\frac{1}{2} - H\right)\right]\bar{t} + ... - K\bar{t}\right\} \tag{5}$$

$$\left(-\frac{\partial w}{\partial r}\right)_{r=a} = \frac{(2 - H)g_o\rho a}{2\mu}\left\{\frac{2}{\sqrt{\pi}}\bar{t}^{1/2} - \left(H - \frac{1}{2}\right)\bar{t} - \frac{4}{3\sqrt{\pi}}\left[\frac{1}{8} + (H - K)(1 - H)\right]\bar{t}^{3/2}\right.$$

$$\left. + \frac{1}{2}\left[\frac{1}{2}\left(3H^2 - 2H^3 + \frac{1}{4}\right) + K(1 - H)\left(\frac{1}{2} - H\right)\right]\bar{t}^2 + ... - K\frac{\bar{t}^2}{2}\right\} \tag{6}$$

These expressions differ slightly from those in reference 5 as a consequence of including all terms consistent with four-term series expansion. These differences occur in the fourth-order term and are considered to be small. The failure criterion (eqs. (4)), with the equality sign, may be substituted into equations (5) and (6) to obtain

$$\overline{C}_1 = \frac{1}{\sqrt{\pi}}\bar{t}^{-1/2} - \left(H - \frac{1}{2}\right) - \frac{2}{\sqrt{\pi}}\left[\frac{1}{8} + (H - K)(1 - H)\right]\bar{t}^{1/2}$$

$$+ \left[\frac{1}{2}\left(3H^2 - 2H^3 + \frac{1}{4}\right) + K(1 - H)\left(\frac{1}{2} - H\right)\right]\bar{t} + ... - K\bar{t} \tag{7}$$

$$\overline{C}_2 = \frac{2}{\sqrt{\pi}}\bar{t}^{1/2} - \left(H - \frac{1}{2}\right)\bar{t} - \frac{4}{3\sqrt{\pi}}\left[\frac{1}{8} + (H - K)(1 - H)\right]\bar{t}^{3/2}$$

$$+ \frac{1}{2}\left[\frac{1}{2}\left(3H^2 - 2H^3 + \frac{1}{4}\right) + K(1 - H)\left(\frac{1}{2} - H\right)\right]\bar{t}^2 + ... - \frac{K\bar{t}^2}{2} \tag{8}$$

where

$$\overline{C}_1 = \frac{2C_1 ka}{\mu g_o(2 - H)} \tag{9a}$$

$$\overline{C}_2 = \frac{2C_2\mu}{\rho g_o a(2 - H)} \tag{9b}$$

and

$$\bar{t} = \frac{\mu t}{\rho a^2} \tag{10}$$

The ballistic-limit velocity g_o is determined from equations (7) and (8) by eliminating $\bar{t}$ between them.

Closed-Form Expressions for Ballistic-Limit Thickness and Velocity

If all the terms are retained in equations (7) and (8), an explicit relationship for ballistic-limit thickness is impossible. An explicit expression, however, can be derived if the series expressions in equations (7) and (8) are truncated to either one or two terms in each series.

If only one term in each of the equations is retained and if the time $\bar{t}$ is eliminated between the equations, then

$$\bar{C}_1 = \frac{2}{\pi \bar{C}_2} \tag{11}$$

By using the definitions of $\bar{C}_1$ and $\bar{C}_2$ from equations (9a) and (9b), an explicit relationship for the ballistic-limit velocity g_o can be derived:

$$g_o = \frac{\sqrt{2\pi C_1 C_2}}{2 - H} \sqrt{\frac{k}{\rho}} \tag{12}$$

Thus, when the target thickness and material properties and the mass and diameter of the projectile are known, the ballistic-limit velocity is directly calculable from equation (12). Often, the velocity of the projectile is known, and the thickness is desired that will just defeat the given projectile (ballistic-limit thickness). With the use of equation (2), an expression for the ballistic-limit thickness h can be derived from equation (12) as

$$h = \left(\frac{2 g_o \sqrt{\rho}}{\sqrt{2\pi C_1 C_2 k}} - 1 \right) \frac{m_p}{\pi \rho a^2} \tag{13}$$

If the mass of the projectile is small in comparison with the mass of the plug of target plate material (eq. (3)), then the second term in parentheses in equation (13) is negligible, and the ballistic-limit thickness is simply

$$h = \gamma_1 \frac{g_o m_p}{a^2} \tag{14}$$

where

$$\gamma_1 = \frac{2}{\pi\sqrt{2\pi C_1 C_2 \rho k}} \tag{15}$$

If two terms are retained in each of equations (7) and (8), the equations can be solved by eliminating the time parameter to obtain

$$\overline{C}_1^{\,3} + (2H - 1)\overline{C}_1^{\,2} + \left[\left(H - \frac{1}{2}\right)^2 - \frac{2}{\pi K}\right]\overline{C}_1 - \frac{H - \frac{1}{2}}{\pi K} = 0 \tag{16}$$

where

$$K = \frac{C_2\mu^2}{C_1 k\rho a^2}$$

This cubic equation can be solved for $\overline{C}_1$. Two of the three roots yield physically meaningless negative values of $\overline{C}_1$. The remaining root is

$$\overline{C}_1 = \frac{2}{3}\left(H - \frac{1}{2}\right)\left[-1 + f(\alpha)\sqrt{1 + 6\alpha}\right] \tag{17}$$

where

$$\alpha = \frac{1}{\pi K\left(H - \frac{1}{2}\right)^2} \tag{18}$$

and

$$f(\alpha) = \cos\left[\frac{1}{3}\cos^{-1}\frac{1 - \frac{9}{2}\alpha}{(1 + 6\alpha)^{3/2}}\right] \tag{19}$$

From the definition of $\overline{C}_1$ (eq. (9a)), the ballistic-limit velocity becomes

$$g_o = \frac{3kC_1 a}{\mu(2 - H)}\left(\frac{1}{H - \frac{1}{2}}\right)\left(\frac{1}{-1 + f(\alpha)\sqrt{1 + 6\alpha}}\right) \tag{20}$$

Equation (20) represents an approximate expression by which the ballistic-limit velocity can be calculated for a given projectile and plate material. A direct solution of this equation for the ballistic-limit thickness is impossible. For projectiles of small mass for which $\frac{m_p}{\pi a^2 \rho h} < 1$, H is close to 2, and equation (3) can be used as in the one-term expression. This simplified form of equation (20) can then be solved. The resulting expression for the ballistic-limit thickness is

$$h = \gamma_2 \frac{g_o m_p}{a^2} \tag{21}$$

where

$$\gamma_2 = \frac{1}{\pi}\sqrt{\frac{K}{C_1 C_2 \rho k}}\left\{-1 + \sqrt{1 + \frac{8}{3\pi K}} \cos\left[\frac{1}{3} \cos^{-1} \frac{1 - \frac{2}{\pi K}}{\left(1 + \frac{8}{3\pi K}\right)^{3/2}}\right]\right\} \tag{22}$$

ACCURACY OF APPROXIMATE EXPRESSIONS

In order to evaluate the accuracy of the approximate expressions, comparisons were made between the ballistic-limit thicknesses and velocities calculated from equations (13), (14), (20), and (21) and those determined in reference 2. The materials used in reference 2 were disk projectiles of thin plastic and targets of aluminum. The values of the physical parameters used were as follows:

$$k = 0.689 \text{ GN/m}^2$$

$$\mu = 15 \text{ kN-s/m}^2$$

$$\rho = 2.77 \text{ Mg/m}^3$$

$$C_1 = 1$$

$$C_2 = 0.02$$

These values of C_1 and C_2 have been used previously with some success for aluminum and steel. (For examples, see references 1, 2, and 6.) The specific values of k and μ were evaluated in reference 2 from experimental results.

In figure 2, the ballistic-limit thickness calculated by the method of solution of reference 1 (and presented in ref. 6) is compared with the thickness calculated by the one- and two-term simplified equations for a range of ballistic-limit velocities and for two projectile radii ($a = 3.2$ mm and $a = 4.75$ mm). For the sake of completeness, the experimental data obtained in reference 2 are also shown. The solid lines represent the results that were obtained from equations applicable to all projectile masses (eqs. (13) and (20)). The dashed lines represent the results obtained from the equations applicable to small projectile masses (eqs. (14) and (21)).

Comparison of the solid and dashed curves shows that for both radii the small-mass approximation has little effect on the ballistic-limit results for disk projectiles. The one-term solution overestimates the required ballistic-limit thickness and, therefore, should not be used. The two-term approximation, however, agrees quite well with the more accurate series solution obtained on an automatic digital computer (ref. 6). The magnitude of the error increases with velocity; the percent of error, however, is about 5 percent over the velocity range shown.

Examination of equation (21) shows that the ballistic-limit thickness is a function of the momentum per unit area of the impacting projectile and the parameter γ_i. For the one-term approximation (eq. (15)), γ_1 is a function of target material only. This result is the same as that obtained in reference 7 for an elastic failure of a plate subjected to an impact by a projectile. The two-term approximation, however, indicates that γ_2 is a function of the projectile radius. The dependence on the radius is shown in figure 3 where γ_i is plotted for a range of radii. Also shown in figure 3 is a plot of the value of γ_i calculated from the solution of reference 6. The results of the two-term approximations and the solution of reference 6 show good agreement for a range of projectile radii above a certain small value that varies with projectile parameters. Below this value, the two-term solution is not adequate.

CONCLUDING REMARKS

Simple equations have been derived to predict the ballistic-limit thickness of thin-plate targets. The analysis is based on a visco-plastic target material. The target is assumed to fail by a shear-plugging perforation. The simplified equations agree reasonably well with results from a more complete solution obtained on an automatic digital computer and with experimental data obtained for disk projectiles only. These comparisons indicate that the equations are valid for short cylindrical projectiles. The equations show that, for a given target material, the ballistic-limit thickness is a function of the projectile momentum and the projectile radius.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., September 24, 1969.

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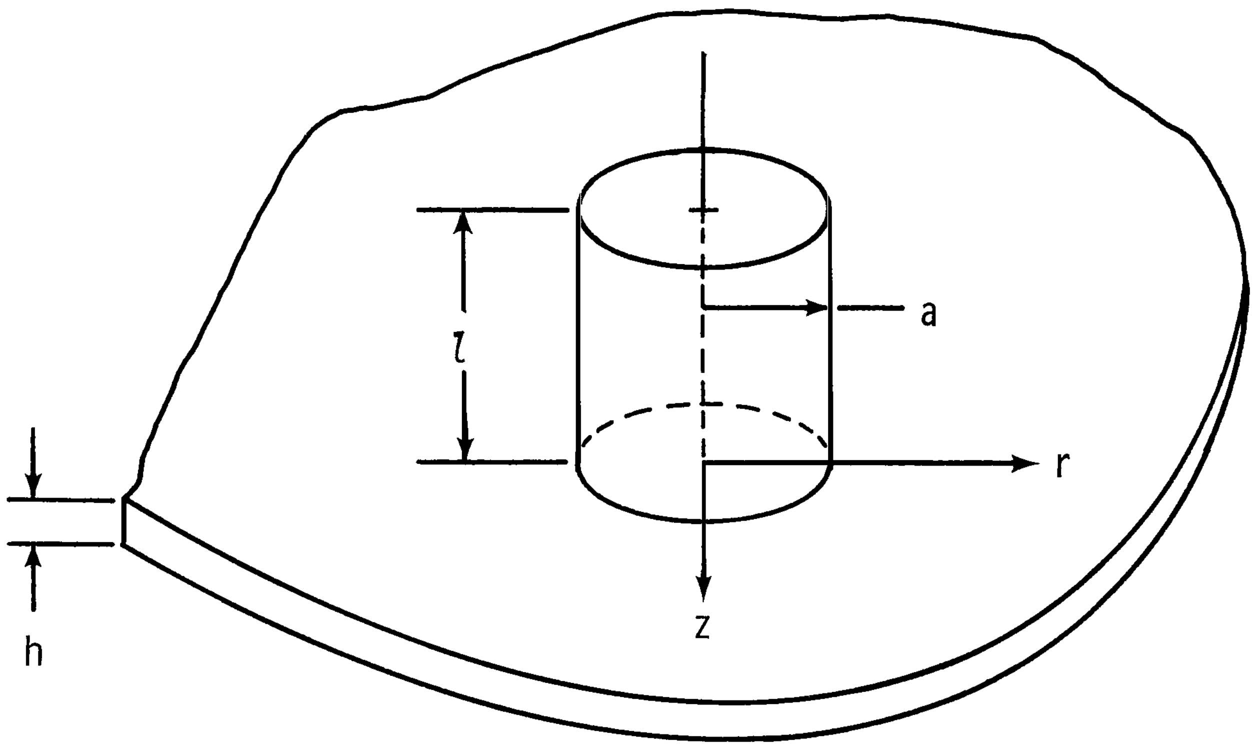
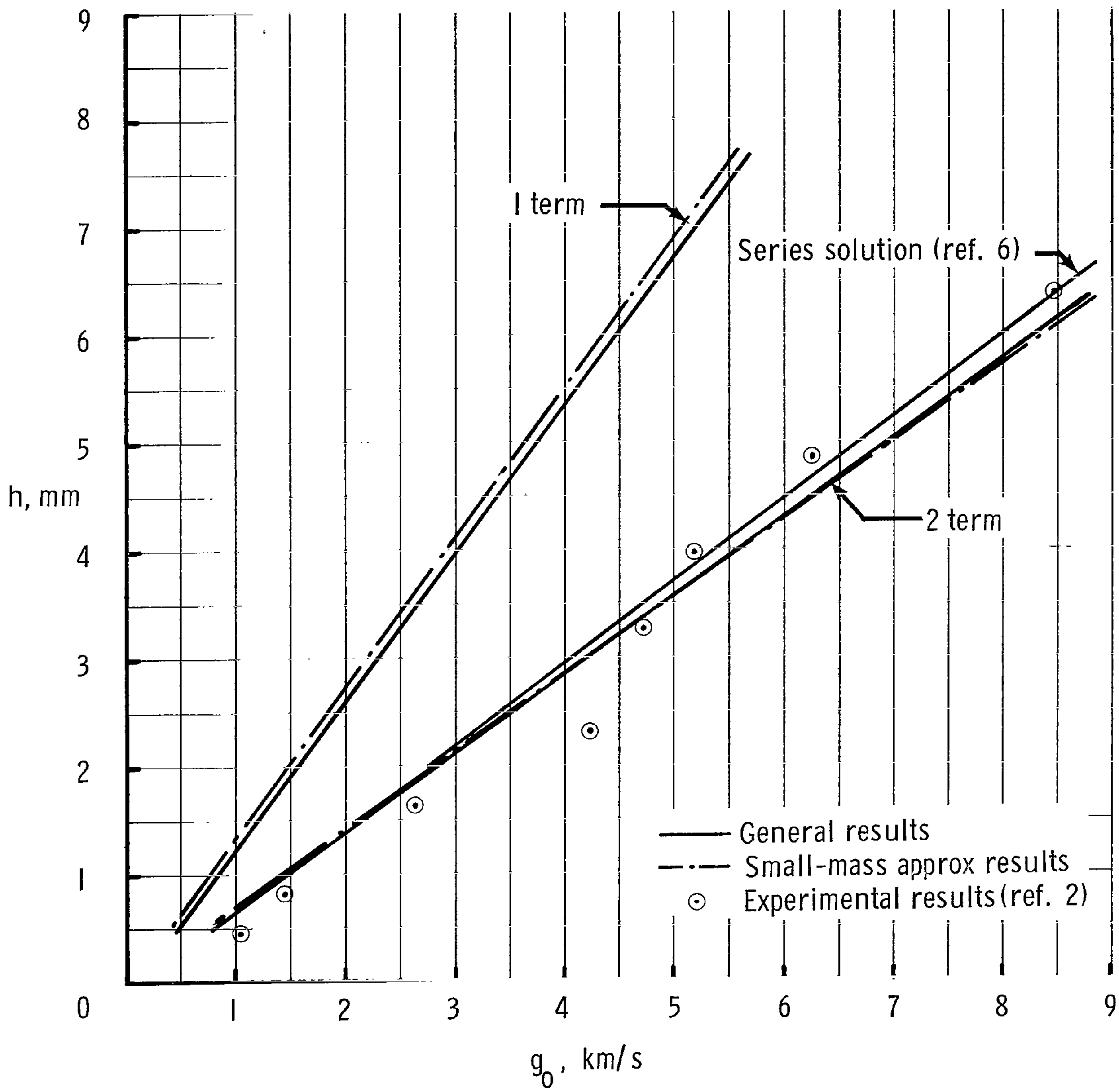
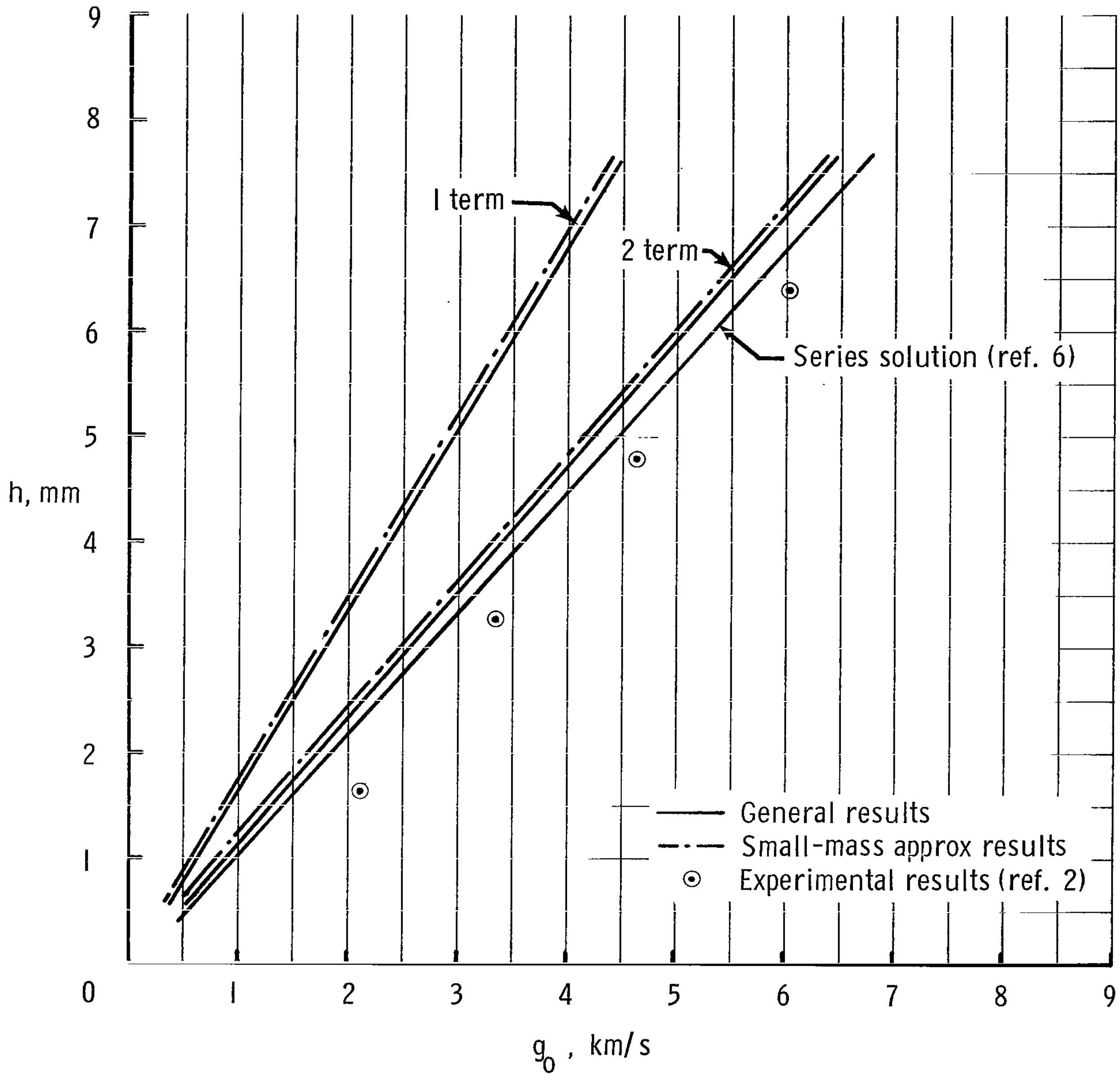


Figure 1.- Coordinate system and dimensions.



(a) 3.2-mm radius; 10-mg plastic projectile.

Figure 2.- A comparison of computed ballistic-limit results with experimental results for disk projectiles. Targets were aluminum.



(b) 4.75-mm radius; 30-mg plastic projectile.

Figure 2.- Concluded.

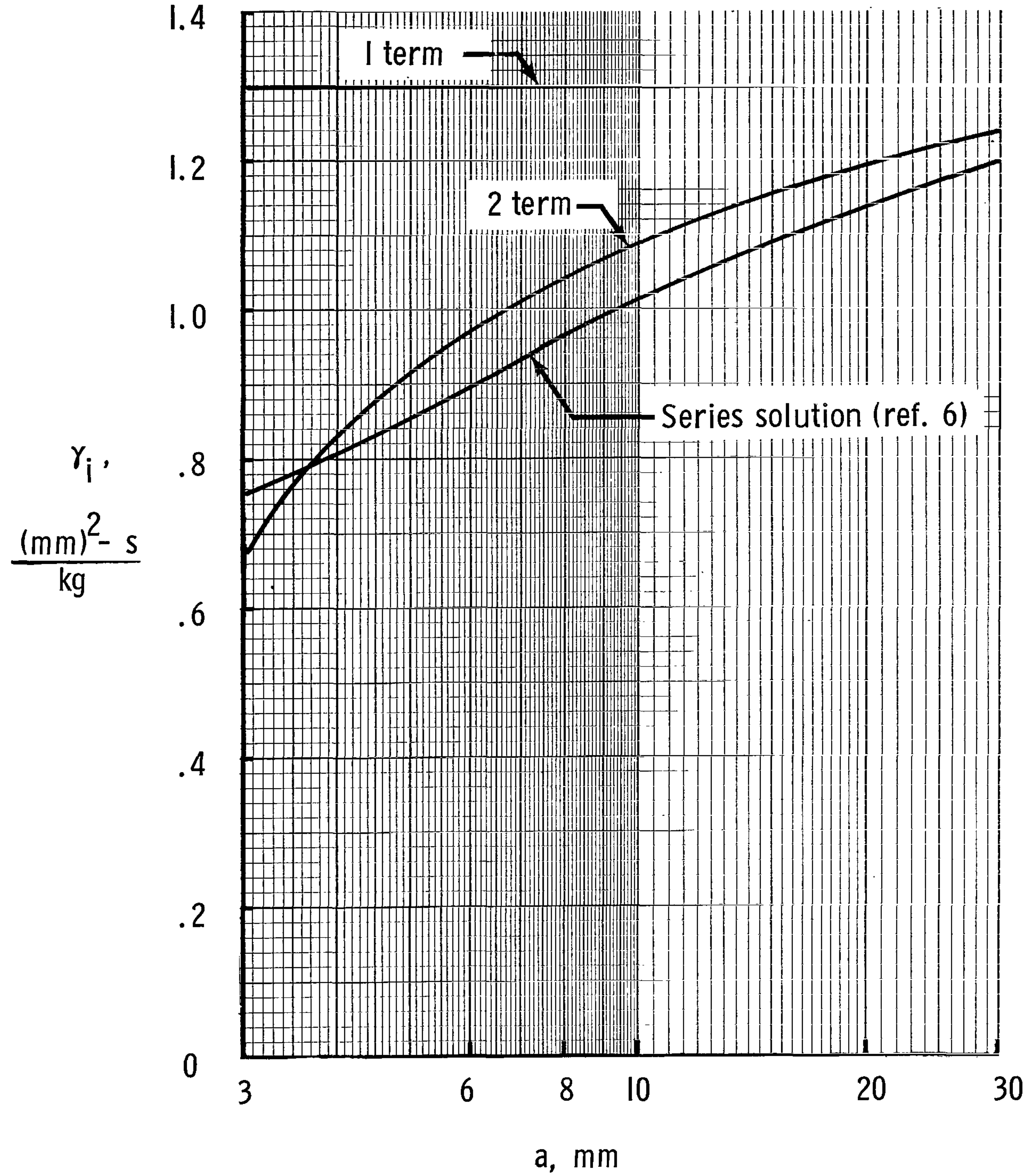


Figure 3.- The variation of the proportionality factor γ_i with projectile radius. Targets were aluminum.

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