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RADIAL GRADIENTS AND ANISOTROPIES OF COSMIC RAYS IN THE INTERPLANETARY MEDIUM

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Cosmic Rays in the Interplanetary Medium

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ABSTRACT

Approximate equations which describe the behavior of cosmic rays in the interplanetary medium under suitable conditions are used to make comparisons between observations and theoretical predictions of radial gradients and radial anisotropies. In the high energy region there appear to be no inconsistencies between theory and observations. In the low energy region it is shown that theoretical predictions of the radial anisotropy expected from large radial gradients of the intensity are not inconsistent with observed radial anisotropies. However, in the latter case there are other inconsistencies, which suggest that some aspects of the observations or of the theory (or both) are unsatisfactory.

1. Introduction

The modulation of cosmic ray particles in the interplanetary medium can be discussed in terms of the quasi-steady, spherically-symmetric model developed by Parker (1965, 1966), Gleeson and Axford (1967, 1968a), and Jokipii and Parker (1967). In this paper observations of radial gradients and radial anisotropies of both high energy and low energy cosmic rays are compared with the predictions of the approximate equations derived for this model by Gleeson and Axford (1968b,c) and by Fisk and Axford (1969).

It has been shown by Gleeson and Axford (1967) that the cosmic ray number density $U(r,T)$ and streaming (or radial current density) $S(r,T)$, per unit interval of kinetic energy T, satisfy the equations:

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 S) = \frac{\partial S}{\partial r} + \frac{2S}{r} = -\frac{V}{3}\frac{\partial^2}{\partial r \partial T}(\alpha TU) \qquad (1)$$

and

$$S = VU - \frac{V}{3}\frac{\partial}{\partial T}(\alpha TU) - \kappa\frac{\partial U}{\partial r} = CVU - \kappa\frac{\partial U}{\partial r} \qquad (2)$$

where

$$C = [1 - \frac{1}{3U}\frac{\partial}{\partial T}(\alpha TU)] \qquad (3)$$

is the Compton-Getting factor (Gleexon and Axford, 1968a). Here r is the heliocentric distance, κ (r,T) is the diffusion coefficient, V(r) is the solar wind speed, and $\alpha(T) = (T + 2T_o)/(T + T_o)$ where T_o is the rest energy of a particle. When V is a constant, S can also be expressed as:

$$S = -\frac{r}{2}\frac{\partial}{\partial r}(VU - \kappa\frac{\partial U}{\partial r}) \qquad (4)$$

Analytic solutions of these equations have been found for certain simple forms of κ (r,T), with α assumed constant, and (usually) with the unmodulated cosmic ray spectrum a power law in kinetic energy (see Fisk and Axford, 1969). Numerical solutions are available which do not suffer from these restrictions, although they must inevitably involve arbitrary assumptions about the variation of κ with r and the form of the unmodulated spectrum (Fisk, 1969). Unfortunately, while these solutions are valuable in a qualitative sense, they are not directly useful in interpreting observations of cosmic rays in the vicinity of the earth. It is more convenient for this purpose to use simpler, more manageable approximate equations which are valid in some limited energy range and over some part, if not all, of the modulating region. Useful approximations are possible in cases where the dimensionaless number $\mathcal{R} \equiv \tilde{V}\,\tilde{r}/\tilde{\kappa}$ (with the tilde denoting 'characteristic value') is either sufficiently small or sufficiently large (Gleeson and Axford, 1968b,c; Fisk and Axford, 1969). The analytic and numerical solutions described above are useful in that they permit the range of validity of these approximate equations to be estimated.

In this paper we discuss these approximate equations further, emphasizing in particular their usefulness for determining the behavior of the radial anisotropy. In Section 2 we discuss the approximate equation for the number density which is valid when $\mathcal{R} \lesssim 1$, a condition that should be satisfied in the vicinity of the earth at energies above a few hundred MeV/nucleon. We find that solutions to this equation can be used to determine corresponding approximations to the streaming and hence the radial anisotropy only to limited accuracy. In Section 3 we discuss the approximate equations relating the anisotropy to the spectrum when $\mathcal{R}$ is large, which is likely to be the case at energies below 50 - 75 MeV/nucleon. Assuming several possible forms for the low energy proton spectrum, we use these equations to determine the corresponding anisotropies, which are then compared with the anisotropies observed at low energies by Rao, et al. (1967). We find that there is an acceptable spectrum for which the observed and predicted anisotropies agree.

Finally, in Section 4 we review earlier studies of the behavior of the anisotropy.

2. Approximations for Intermediate and High Energies

It has been argued by Gleeson and Axford (1967, 1968c) and by Fisk and Axford (1969) that if only galactic cosmic rays are considered, the streaming S can be neglected on the left side of Equation (2) when $|(1/2)\ C\ (C - 1)\mathcal{R}| \ll 1$. In this case the number density satisfies the 'force-field' equation:

$$CVU \simeq \kappa \frac{\partial U}{\partial r} \qquad (5)$$

It is believed that this approximation is valid locally (i.e. in the vicinity of the orbit of the earth) for particles with energies exceeding a few hundred MeV/nucleon, and beyond the orbit of the earth for particles with even lower energies. In the energy range for which this approximation is valid, Equation (5) relates the radial gradient of the number density directly to the spectrum of the cosmic ray particles and to the diffusion coefficient observed locally. It should be noted that the factor C in the expression for the gradient has an important effect at energies below about 1 GeV/nucleon, and makes the gradient in this energy range much smaller than would be expected on the basis of the simple convection-diffusion model (which corresponds formally to C = 1).

An approximate expression for the radial anisotropy, $\xi = 3S/vU$ (where v is particle speed), can be obtained by substituting Equation (5) into

Equation (4); thus

$$S \simeq -\frac{Vr}{6}\frac{\partial^2}{\partial r \partial T}(\alpha TU), \tag{6}$$

and hence

$$\xi \simeq -\frac{Vr}{2vU}\frac{\partial^2}{\partial r \partial T}(\alpha TU) \simeq -\frac{V}{2v}\frac{1}{U}\frac{\partial}{\partial T}\left[\alpha TCU\left(\frac{Vr}{\kappa}\right)\right] \tag{7}$$

In fact, this is only an order-of-magnitude result and should generally be expected to hold to within a factor ~ 2. The reason for this numerical uncertainty can be seen by comparing Equation (6) with Equation (1). It is evident that Equations (6) and (7) can be expected to be good approximations only if $\partial S/\partial r << S/r$; however, it can be shown from analytic solutions that $\partial S/\partial r$ is in general of the same order as S/r when $\mathcal{R} \lesssim 1$, and hence an error of a factor ~ 2 can be expected. For the very simple solution in which $\kappa = \kappa_o r$, Equation(6) yields the correct asymptotic result in the limit $\mathcal{R} << 1$, but when $\kappa \propto r^b$, the error appears to be a factor $2/(3 - b)$.

To determine ξ to lowest order in $\mathcal{R}$ it is clear from Equation (7) that it is sufficient to use the unmodulated spectrum for C and U. Thus if we take $U \propto T^{-\mu}$ at high energies, then $C = [1 + \alpha(\mu - 1)/3]$. Furthermore, there is some observational evidence which suggests that for high energies the diffusion coefficient has the form $\kappa = \beta P \kappa_1(r)$, where $\beta = v/c$, c is the speed of light, and P is the particle rigidity (Gloeckler and Jokipii, 1966). Noting that $\beta P \propto \alpha T$ we find that with these forms for U, C, and κ,

$$\xi \simeq \mu\alpha \left(\frac{V}{2v}\right) [1 + \alpha(\mu - 1)/3] \left(\frac{Vr}{\kappa}\right) \simeq \mu\alpha\left(\frac{V}{2v}\right) \left(\frac{r}{U}\frac{\partial U}{\partial r}\right) \qquad (8)$$

According to O'Gallagher (1967) the radial gradient of protons with $T \sim 5$ GeV in the heliocentric distance range 1 - 1.57 A. U. was $(r/U)(\partial U/\partial r) \simeq +9\%$ during 1965. On taking $V = 400$ km. sec.$^{-1}$ $= 0.0014v$, $\alpha = 1.17$, and $\mu = 2.65$, we find from Equation (8) that $\xi \simeq +0.02\%$. This small positive anisotropy at high energies is in agreement with earlier predictions by Gleeson and Axford (1968b,c). It is sufficient to shift the direction of maximum intensity associated with the normal diurnal variation from 90° east to about 88° east of the sun, in accordance with observations described by McCracken and Rao (1966).

In Figure 1 we have plotted the radial gradient of the intensity determined by Equation (5), and the radial anistropy determined by Equation (7), using the spectrum for protons given Gloeckler and Jokipii (1967) and assuming that $\kappa \propto \beta P$.

3. Approximations for Low Energies

It seems well-established that in the vicinity of the earth the diffusion coefficient decreases with decreasing energy at least down to energies of the order of a few hundred MeV/nucleon (e.g. Ormes and Webber, 1968; Sari and Ness, 1969). If this behavior continues we anticipate that locally $\mathcal{R}$ attains quite large values (i.e. $\mathcal{R} \gtrsim 3$) at energies below about 50 - 75 MeV/nucleon.

Fisk and Axford (1969) have shown that two distinct sets of approximate equations are possible in the limit of large $\mathcal{R}$: one is applicable if the magnitudes of the radial gradients of the number density and streaming are of order $\mathcal{R}$, and the other, for substantially smaller gradients. Unfortunately, it is not possible to decide at present which of the two sets should be used since radial gradients of the intensity with both large and small magnitudes have been observed at low energies (e.g. O'Gallagher, 1967; Anderson, 1968; Krimigis and Venkatesan, 1968). We will discuss the local particle behavior in terms of both sets of possible approximate equations valid for large $\mathcal{R}$.

Fisk and Axford (1969) showed that if the gradients at low energies are as large as observations by O'Gallagher (1967) indicate, then the local behavior of galactic cosmic rays at energies below 50 - 75 MeV/nucleon can be described by an approximate equation which relates the streaming directly to the local spectrum:

$$S \simeq - \frac{V}{3} \frac{\partial}{\partial T} (\alpha T U) \qquad (9)$$

This equation can be obtained directly from Equation (1) by dropping the term 2S/r and integrating, assuming V to be constant. The integration produces an additional function of T, which we interpret as representing a source term (i.e. solar particles), and accordingly neglect. (The term is indeed absent in our analytic solutions (Fisk and Axford, 1969).) In terms of the radial anisotropy ξ and the mean differential intensity $j_o = vU/4\pi$, Equation (9) can be written

$$\xi \simeq - \frac{V}{v} (\alpha \frac{\partial \ln j_o}{\partial \ln T} + 1) \qquad (10)$$

In this case the number density of galactic cosmic rays satisfies the simple convection-diffusion equation:

$$VU \simeq \kappa \frac{\partial U}{\partial r} \tag{11}$$

provided the spectrum does not have an extremely large slope, that is, provided

$$\eta \equiv \left| \frac{1}{3U} \frac{\partial}{\partial T} (\alpha TU) \right| = \frac{1}{3} \left| \alpha \frac{\partial \ln j_o}{\partial \ln T} + 1 \right| \lesssim 1 \tag{12}$$

If the radial gradients at low energies are small (as suggested by Anderson (1968) and by Krimigis and Venkatesan (1968), for example), then if R is large, it is permissible to neglect the term $\kappa\, \partial U/\partial r$ on the right side of Equations (2) and (4), thus yielding:

$$S \simeq CVU \tag{13}$$

and

$$S \simeq -\frac{r}{2} \frac{\partial}{\partial r} (VU) \tag{14}$$

In terms of the radial anisotropy and the mean differential intensity, Equation (13) can be written:

$$\xi \simeq -\frac{V}{v} \left(\alpha \frac{\partial \ln j_o}{\partial \ln T} - 2\right) \tag{15}$$

On eliminating S between Equations (13) and (14) we obtain a simple expression for the radial gradient:

$$\frac{r}{j_o} \frac{\partial j_o}{\partial r} \simeq \frac{2}{3} \left(\alpha \frac{\partial \ln j_o}{\partial \ln T} - 2\right) \tag{16}$$

This approximation corresponds to the case in which the scattering is so effective that it keeps the cosmic ray particles "frozen" in the expanding solar wind, and hence they undergo severe deceleration. Equation (15) accordingly corresponds to the anisotropy associated with a bulk speed V, allowing for the Compton-Getting effect (Gleeson and Axford, 1968a, Forman, 1969), while Equation (16) corresponds to pure adiabatic expansion. These approximate equations can be used to treat galactic cosmic rays (Fisk and Axford, 1969), and also presumably solar cosmic rays (e.g. Forman, 1968a) provided $|(r/j_o)\ \partial j_o/\partial r| \ll \mathcal{R}$.

Having derived these two sets of approximate equations for the case of large $\mathcal{R}$, we are in a position to discuss the implications of observations of radial gradients and anisotropies at low energies. In Figure 2 we have plotted a spectrum which provides a reasonable fit to the available observations of low energy proton intensities in 1966. This spectrum could presumably result from (a) galactic protons for which the gradients are large, (b) a combination of galactic protons, for which the gradients are small, together with solar protons, or (c) a combination of galactic protons, for which the gradients are large, and solar protons. To determine which, if any, of these three possible cases is acceptable (assuming that locally $\mathcal{R}$ is large at low energies), we can compare the anisotropy determined for each of the cases by Equations (10) and (15) with the low energy anisotropy observed by Rao, et al. (1967). The particles observed by Rao, et al. in the vicinity of the earth ($r = 0.8 - 1.1$ A. U.) in 1966 are presumably mainly protons, and have essentially radial mean anisotropies of $\bar{\xi} = 0.19 \pm 0.06\%$ in the energy range (7.5 - 45 MeV), and of $\bar{\xi} = 0.2 \pm 0.2\%$ in the range (45 - 90 MeV). The approximate equations are unlikely to be valid at energies above 50 - 75 MeV, and hence a comparison between predicted and observed anisotropies in the (45 - 90 MeV) energy range is probably not meaningful. We shall, however, give the predicted anisotropies in this upper energy range for completeness.

Case (a): <u>Galactic protons for which the gradients are large</u>.

The radial anisotropy determined by Equation (10) for the form of the spectrum shown in Figure 2 is plotted in Figure 3. To determine the predicted mean anisotropies, we average the anisotropy shown in Figure 3 over each of the two observed energy ranges using the intensity as a weighting function:

$$\bar{\xi} = \int_{T}^{T_2} \xi \, j_o \, dT / \int_{T_1}^{T_2} j_o \, dT \qquad (17)$$

where T_2 and T_1 are the upper and lower limits, respectively, of a given energy range. For this case the predicted and observed anisotropies agree in the (7.5 - 45 MeV) energy range where the predicted mean anisotropy is found to be $\bar{\xi} = 0.25\%$. In the (45 - 90 MeV) energy range the predicted mean anisotropy of $\bar{\xi} = -1.14\%$ is much larger in magnitude than the observed anisotropy, but, as we indicated above, close agreement should not be expected in this energy range. In Figure 4 we have plotted the quantity η (Equation (12)) for this case. As is evident in this figure, η is generally of the order unity or less; hence the number density corresponding to this form of the spectrum should satisfy a simple convection-diffusion equation at low energies, and V/K can be estimated from the radial gradient.

Case (b): <u>Galactic protons for which the gradients are small, together with solar protons.</u>

The radial anisotropy determined by Equation (15) for this case is also plotted in Figure 3. Here the predicted mean anisotropies are found to be $\bar{\xi}$ = 2.30% in the (7.5 - 45 MeV) energy range, and $\bar{\xi}$ = 0.0001% in the (45 - 90 MeV) range. The predicted and observed anisotropies do not agree in the lower energy range, and such agreement as exists in the upper energy range is probably fortuitous. In Figure 5 we have plotted the radial gradient of the intensity determined by Equation (16) for this form of the spectrum. The pronounced negative gradient at energies below 65 MeV shown in Figure 5, and the large positive anisotropy shown in Figure 3 at these energies presumably indicate that this form of the proton spectrum could only be realistic if the protons at low energies were predominantly of solar origin. Since the observed and predicted anisotropies do not agree in the lower energy range, this is apparently not the case. It should be noted that at very low energies (< 12 MeV) the magnitude of the radial gradient shown in Figure 5 is large, and hence it is not certain whether the condition required for Equations (15) and (16) to be valid, viz. $(1/\Re)|(r/j_0)(\partial j_0/\partial r)| \ll 1$, is satisfied. We assume, however, that these equations indicate at least the general features of the particle behavior predicted by this form of the spectrum.

Case (c): Galactic protons for which the gradients are large, together with solar protons.

For this case we assume that the spectrum shown in Figure 2 can be decomposed into solar and galactic proton spectra, each of which is also shown in Figure 2. The solar proton anisotropy is determined by Equation (15), and the galactic proton anisotropy by Equation (10). The predicted anisotropy shown in Figure 3 is then obtained by adding each of the anisotropies, weighting them according to the relative number of particles of each species present at a given energy. The predicted mean anisotropies of $\bar{\xi} = 1.27\%$ in the (7.5 - 45 MeV) energy range, and of $\bar{\xi} = -1.12\%$ in the (45 - 90 MeV) range are much larger in magnitude than the corresponding observed anisotropies.

According to the above discussion, of the three cases considered, only the anisotropy corresponding to the form of the low energy proton spectrum given in case (a) (galactic protons for which the gradients are large) fits the observed anisotropy. Of course, the predicted anisotropies are sensitive to the shape of the curve used for the spectrum, which, since there are only a few data points available in the energy range considered (7.5 - 90 MeV), is not well established. However, with reasonable variations of the spectrum it is unlikely that the anisotropies in cases (b) and (c) would agree with the observed anisotropy. The mean anisotropy in case (a), even with some variation of the spectrum, should be small and positive in the (7.5 - 45 MeV) energy range, in agreement with the observations.

We conclude, therefore, that (i) a low energy proton spectrum which is the result primarily of galactic protons and which resembles the spectrum shown in Figure 2, (ii) a radial gradient of the intensity of galactic protons which locally has a large magnitude at low energies (i.e. a gradient as large as that observed by O'Gallagher (1967)), and (iii) the small low energy anisotropy observed by Rao, et al. (1967) are all mutually consistent with one another. It is important to note that we do not conclude that gradients in the vicinity of the earth are in fact large at low energies, but only that on the basis of the model used large gradients and small anisotropies are not inconsistent with each other.

Since there is some evidence that low energy protons ($T \lesssim 20$ MeV) might be of solar origin (e.g. Kinsey, 1967), which is not consistent with the above result, we point out that there are other possibilities. For example, (i) the model we have used might be inadequate, (ii) $\mathcal{R}$ might not be large in the energy range in question, and (iii) the observations we have used might, for some reason, be incorrect. It is evident that to establish that the low energy particles are indeed of solar origin is not an easy matter, and if we are to use arguments similar to those developed in this paper, observations of the local spectrum, anisotropies, and radial gradients should be performed simultaneously and with considerable care.

4. Discussion

Previous studies of the relationship between the radial gradient and radial anisotropy of cosmic rays have been carried out by Gleeson and Axford (1968b), Jokipii and Parker (1968), and Forman (1968b). In each case an attempt was made to deal directly with Equations (1) and (2), or with equivalent forms, rather than with asymptotically valid approximations as we have done in the present paper.

The first such study to be carried out was that of Gleeson and Axford (1968b), who used Equation (2) together with observations of the local spectrum and radial gradient of cosmic rays, and also with an estimate for κ based on the spectrum of interplanetary magnetic field fluctuations (e.g. Jokipii, 1966), to predict the behavior of the radial anisotropy as a function of particle kinetic energy. They found that the radial anisotropy should be small and positive for kinetic energies greater than about 600 MeV/nucleon, negative in the range 40 - 600 MeV/nucleon, and positive for low energies. In fact this prediction is qualitatively in agreement with observations (McCracken and Rao, 1966; Rao, et al., 1967). Since the argument is based on an exact, not an approximate, equation involving quantities which can be determined locally, it should be possible in principle to make correct predictions provided the equation itself is a valid representation of the behavior of the cosmic rays.

For high energies Gleeson and Axford (1968b) used essentially the same arguments as we have used in Section 2. For intermediate and low energies they had to estimate the term $\kappa\, \partial U/\partial r$ using the gradients determined by O'Gallagher (1967), assuming $\kappa \propto \beta P$ throughout the entire energy range and matching κ to the high energy value obtained from Equation (5). The resulting value of $\kappa\, \partial U/\partial r$ is small compared with CVU (c.f. case (b) of Section 3) and hence the general nature of the predicted anisotropy is rather insensitive to errors in κ and $\partial U/\partial r$ unless κ is actually very much larger than estimated. The predicted magnitude of the anisotropy does not however agree with that found by Rao, et al. (1967) for the energy range (7.5 - 45 MeV).

Jokipii and Parker (1968) used Equation (1) in an integral form in their discussion of this problem. If Equation (1) is multiplied by r^2 and integrated with respect to r successively by parts in the range (0, r), one obtains:

$$\xi = \frac{2V}{3U} \frac{\partial}{2T} \left(\alpha T \sum_{i=1}^{\infty} \frac{(-r)^i}{(i+2)!} \frac{\partial^i U}{\partial r^i} \right) \tag{18}$$

on putting $S = 3vU\xi$, and using the condition $r^2 S \rightarrow 0$ as $r \rightarrow 0$ which corresponds to having no sources or sinks at the sun (i.e. no solar cosmic rays). Jokipii and Parker make the assumption that the gradient in the vicinity of the earth can be extrapolated all the way back to the sun, so that the higher derivatives vanish and the anisotropy is given by:

$$\xi = - \frac{Vr}{3vU} \frac{\partial}{\partial T} \left(\alpha T \frac{\partial U}{\partial r} \right) \tag{19}$$

It is clear, however, that such an assumption cannot be valid if the gradient in the vicinity of $r = 1$ A.U. exceeds 100% A.U. as reported by O'Gallagher (1967). It is obvious from simple order of magnitude arguments and from consideration of analytic solutions (Fisk and Axford, 1969) that in situations where the radial gradient is not small, the terms in the series on the right side of Equation (18) becoming increasingly large in magnitude as i increases. Accordingly, one cannot expect Equation (19) to correctly predict the radial anisotropy using the large radial gradients reported by O'Gallagher (1967), and one cannot assert on the basis of such a prediction that there is any inconsistency between these radial gradients and the radial anisotropies observed by Rao, et al. (1967).

At high energies, where the radial gradients can be expected to be small, Equation (19) is a useful result and indeed is the same as Equation (7) to within a small numerical factor. It is evident from consideration of analytic solutions (Fisk and Axford, 1969) that this numerical uncertainty arises because the terms $r^i \partial^i U/\partial r^i$ are in general of comparable magnitude when $\Re \lesssim 1$ and hence one depends on the coefficients $(-1)^i/(i+2)!$ for convergence of the series in Equation (18). In some simple cases the series can be summed (Fisk and Axford, unpublished), and in particular if $\kappa = \kappa_0 r$, Equations (7) and (18) (but not (19)) yield the same result for $\Re \ll 1$.

Forman (1968b) has approached the problem from a somewhat different point of view, and points out that if the radial gradient of the intensity is large at low energies as suggested by the observations of O'Gallagher (1967), then the radial gradient of the anisotropy, $(r/\xi)(\partial\xi/\partial r)$, must also be large at these energies. This result can be deduced easily from the approximate equations derived in Section 3. Thus, unless there was a chance cancellation, one might have expected a noticeable variation in ξ even within the small variation of heliocentric distance (0.8 - 1.1 A.U.) involved in the observations of Rao, et al. (1967). On this basis the predictions of the model and the observations of the anisotropy and of the radial gradient are mutually inconsistent. There is no contradiction with the results given in Section 3 of this paper, however, since the comparisons involve different quantities. In effect, Forman's argument confirms our use of Equation (9); our neglect of $2S/r$ compared with $\partial S/\partial r$ is essentially equivalent to assuming that $\partial\xi/\partial r \gg \xi/r$.

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FIGURE CAPTIONS

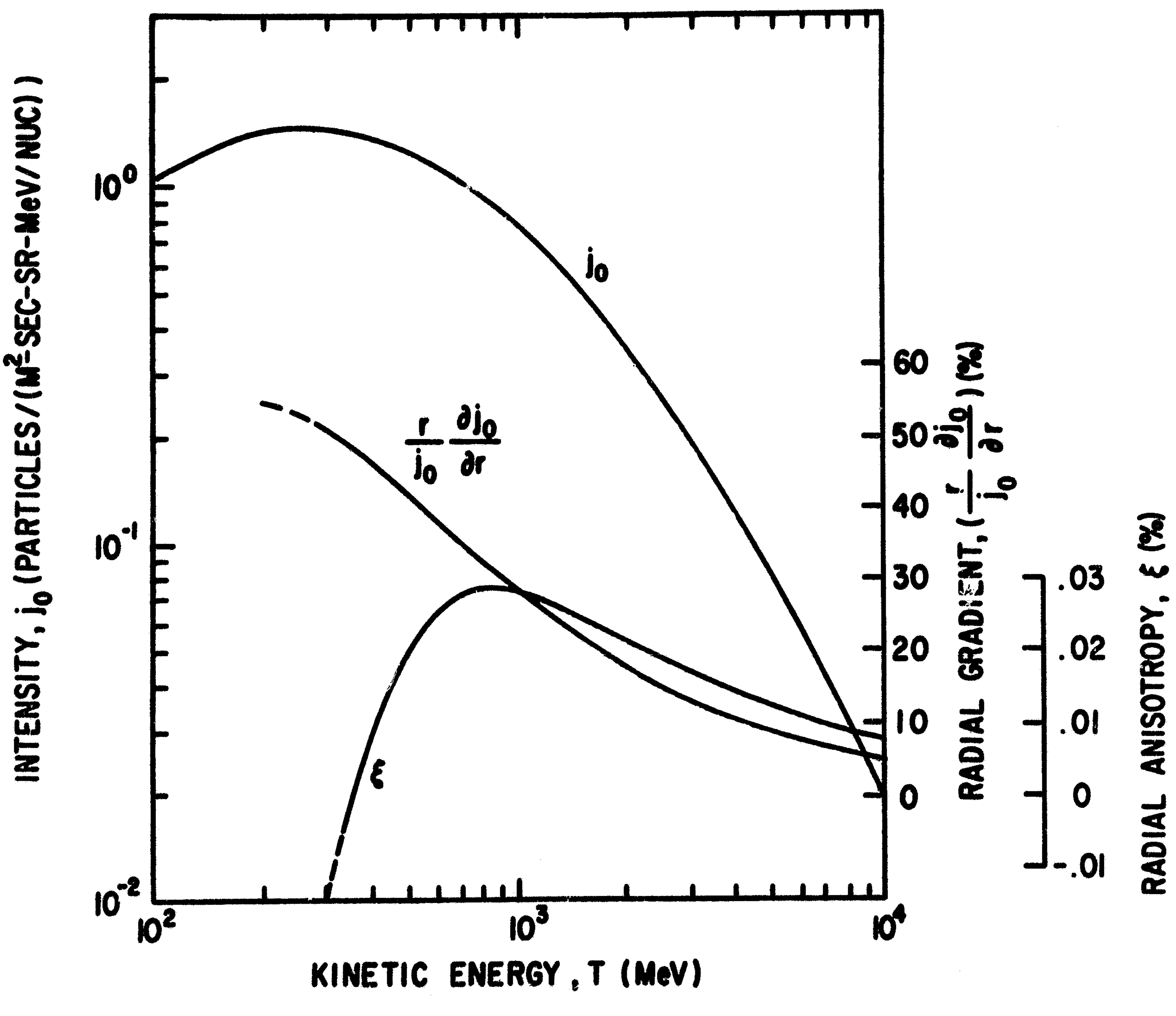
Fig. 1 A plot of the radial gradient of the intensity determined by Equation (5) and the radial anisotropy determined by Equation (7), using the spectrum for protons given by Gloeckler and Jokipii (1967). κ is taken to be proportional to βP and is chosen to have a value $\kappa = 9.3 \times 10^{21}$ $cm^2\ sec^{-1}$ for protons with $T \sim 5$ GeV in order that the predicted gradient at this energy is ~ 9%, in agreement with the observations of O'Gallagher (1967). V is taken to be 400 km sec^{-1}. The Compton-Getting factor C corresponding to this spectrum is shown by Gleeson and Axford (1968b).

Fig. 2 A spectrum which provides a reasonable fit to the available observations of low energy proton intensities in 1966. The symbol Θ is used to represent the observations of Fan, et al. (1968), and the symbol Δ, the observations of Badhwar, et al. (1968). In case (c) discussed in the text, the spectrum is assumed to result primarily from solar protons at energies below 10 MeV, and primarily from galactic protons at energies above 60 MeV; the two dashed curves shown in this figure represent solar and galactic proton spectra, into which the total spectrum can be decomposed in the (10 - 60 MeV) energy range.

Fig. 3 A plot of the radial anisotropy determined by Equation (10) or (13) for each of the three cases discussed in the text, vs T.

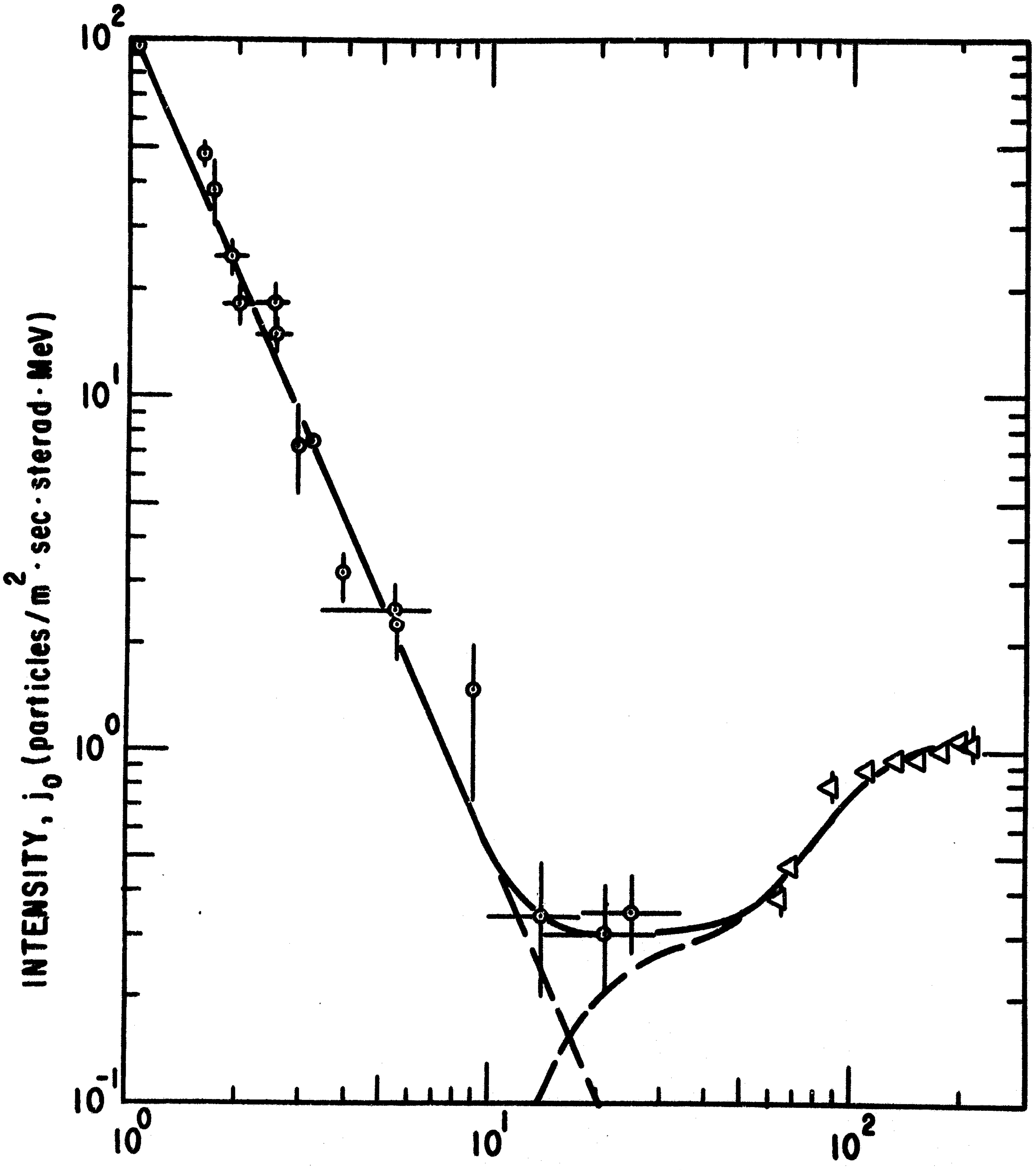
Fig. 4 A plot of the quantity η determined by Equation (12) for case (a) discussed in the text, vs T.

Fig. 5 A plot of the radial gradient of the intensity determined by Equation (16) for case (b) discussed in the text, vs T.



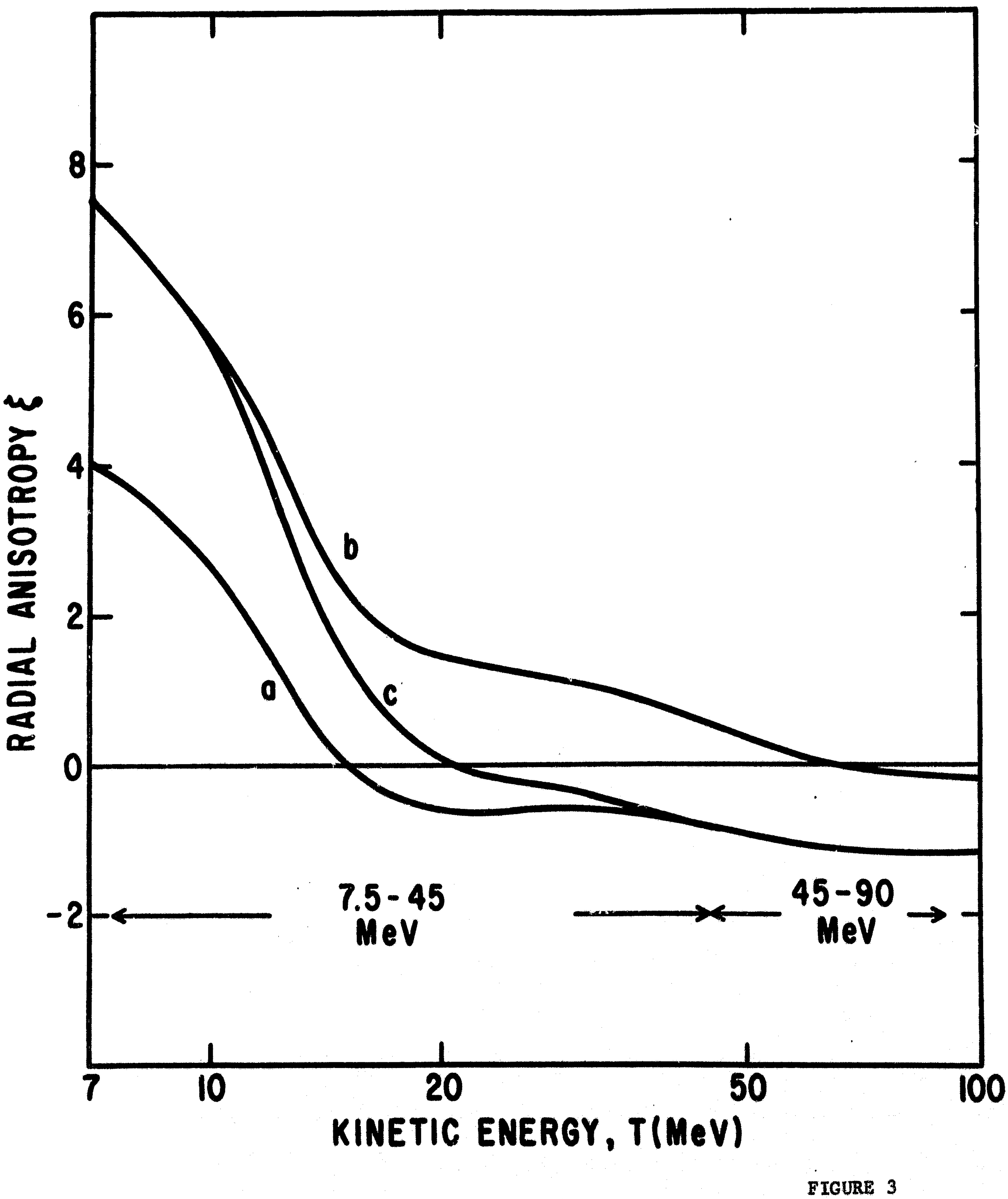
INTENSITY, j_0 (PARTICLES/(M^2-SEC-SR-MeV/NUC))
10^0
10^{-1}
10^{-2}
j_0
$\frac{r}{j_0}\frac{\partial j_0}{\partial r}$
ε
60
50
40
30
20
10
0
RADIAL GRADIENT, $(\frac{r}{j_0}\frac{\partial j_0}{\partial r})$ (%)
.03
.02
.01
0
-.01
RADIAL ANISOTROPY, ε (%)
10^2
10^3
10^4
KINETIC ENERGY, T (MeV)

FIGURE 1



10^2
10^1
10^0
10^{-1}
INTENSITY, j_0 (particles/m^2 · sec · sterad · MeV)
10^0
10^1
10^2
KINETIC ENERGY, T (MeV)

FIGURE 2



RADIAL ANISOTROPY ξ
8
6
4
2
0
-2
b
a
c
7.5 - 45
MeV
45 - 90
MeV
7
10
20
50
100
KINETIC ENERGY, T(MeV)

FIGURE 3

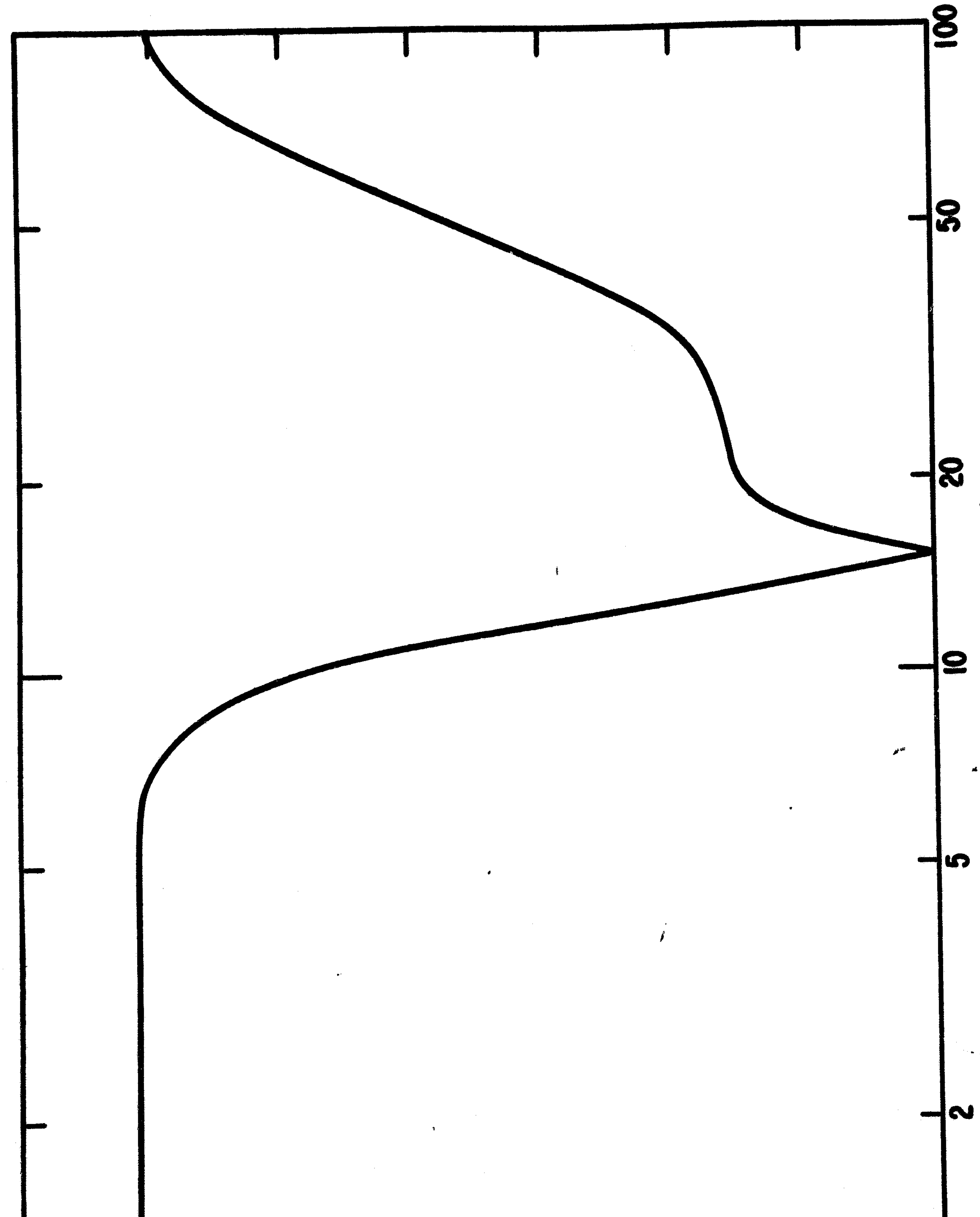


FIGURE 4

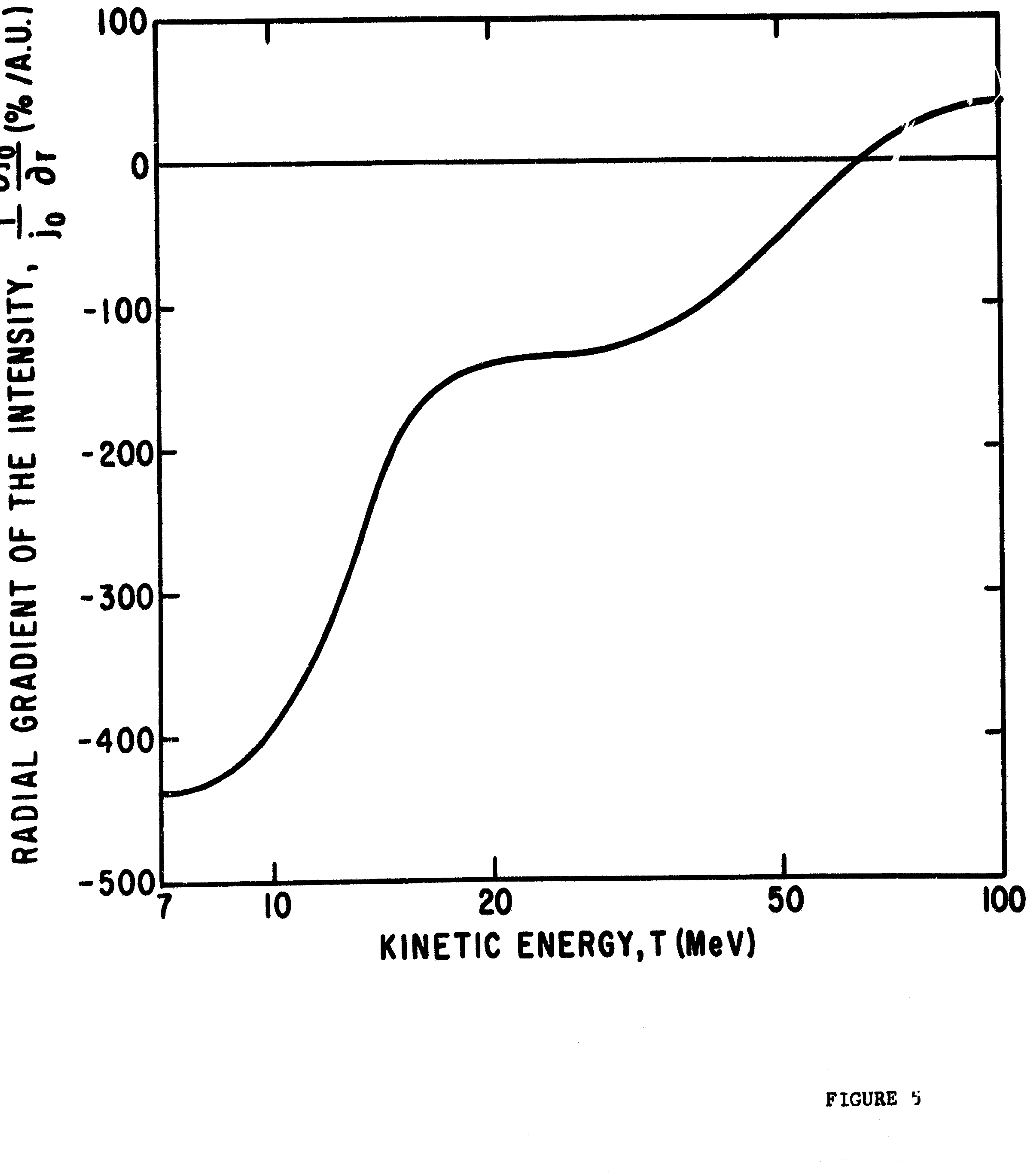

RADIAL GRADIENT OF THE INTENSITY, $\frac{1}{j_0}\frac{\partial j_0}{\partial r}$ (% /A.U.)
100
0
-100
-200
-300
-400
-500
7
10
20
50
100
KINETIC ENERGY, T (MeV)

FIGURE 5

ABSTRACT

Approximate equations which describe the behavior of cosmic rays in the interplanetary medium under suitable conditions are used to make comparisons between observations and theoretical predictions of radial gradients and radial anisotropies. In the high energy region there appear to be no inconsistencies between theory and observations. In the low energy region it is shown that theoretical predictions of the radial anisotropy expected from large radial gradients of the intensity are not inconsistent with observed radial anisotropies. However, in the latter case there are other inconsistencies, which suggest that some aspects of the observations or of the theory (or both) are unsatisfactory.

1. Introduction

The modulation of cosmic ray particles in the interplanetary medium can be discussed in terms of the quasi-steady, spherically-symmetric model developed by Parker (1965, 1966), Gleeson and Axford (1967, 1968a), and Jokipii and Parker (1967). In this paper observations of radial gradients and radial anisotropies of both high energy and low energy cosmic rays are compared with the predictions of the approximate equations derived for this model by Gleeson and Axford (1968b,c) and by Fisk and Axford (1969).

It has been shown by Gleeson and Axford (1967) that the cosmic ray number density $U(r,T)$ and streaming (or radial current density) $S(r,T)$, per unit interval of kinetic energy T, satisfy the equations: