



THE PENNSYLVANIA  
STATE UNIVERSITY

# IONOSPHERIC RESEARCH

Scientific Report No. 342

## AMPLITUDE AND PHASE SCINTILLATION OF HIGH FREQUENCY SATELLITE SIGNALS DUE TO IONOSPHERIC INHOMOGENEITIES

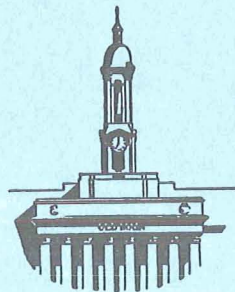
by

M. L. Rafael

November 1, 1969

**CASE FILE  
COPY**

IONOSPHERE RESEARCH LABORATORY



University Park, Pennsylvania NASA Grant NGR-39-009-002

Ionospheric Research

NASA Grant NGR-39-009-002

Scientific Report

on

"Amplitude and Phase Scintillation of High Frequency  
Satellite Signals due to Ionospheric Inhomogeneities"

by

M. L. Rafael

November 1, 1969

Scientific Report No. 342

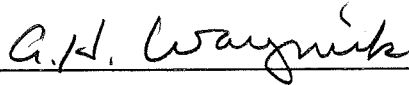
Ionosphere Research Laboratory

Submitted by:



William J. Ross, Professor of Electrical Engineering  
Project Supervisor

Approved by:



A. H. Waynick, Director  
Ionosphere Research Laboratory

The Pennsylvania State University

College of Engineering

Department of Electrical Engineering

## TABLE OF CONTENTS

|                                                                                                                |    |
|----------------------------------------------------------------------------------------------------------------|----|
| Abstract . . . . .                                                                                             | i  |
| I. INTRODUCTION . . . . .                                                                                      | 1  |
| 1.1 Historical Background . . . . .                                                                            | 1  |
| 1.2 General Statement of the Problem . . . . .                                                                 | 3  |
| 1.3 Specific Statement of the Problem . . . . .                                                                | 4  |
| II. COSINUSOIDAL SCREEN DIFFRACTION THEORY . . .                                                               | 6  |
| 2.1 Introduction . . . . .                                                                                     | 6  |
| 2.2 The One Dimensional Cosinusoidal Screen . . .                                                              | 8  |
| 2.2.1 The Form of the Solution . . . . .                                                                       | 8  |
| 2.2.2 Scintillation Indexes and Pattern<br>Scale Sizes . . . . .                                               | 15 |
| 2.2.3 The Limits of the Ray Optical Region .                                                                   | 22 |
| 2.2.4 The Special Case of Small $\Delta\phi$ . . . .                                                           | 23 |
| 2.3 The Effects of Additional Spatial Components . .                                                           | 25 |
| 2.4 The Effect of Oblique Incidence . . . . .                                                                  | 29 |
| 2.5 Point Source Effects . . . . .                                                                             | 30 |
| III. RANDOM SCREENS . . . . .                                                                                  | 34 |
| 3.1 Introduction . . . . .                                                                                     | 34 |
| 3.2 The Angular Spectrum of Complex Amplitude . .                                                              | 35 |
| 3.3 The Special Case of Small $\phi_0$ . . . . .                                                               | 38 |
| 3.4 The Case of Large $\phi_0$ . . . . .                                                                       | 46 |
| 3.4.1 The Amplitude Pattern . . . . .                                                                          | 46 |
| 3.4.2 The Phase Pattern . . . . .                                                                              | 50 |
| 3.5 The Relation Between a Thick Region of<br>Inhomogeneities and an Infinitely Thin<br>Phase Screen . . . . . | 51 |
| 3.6 The Limit of the Ray Optical Region . . . . .                                                              | 57 |

|     |                                                                                                           |     |
|-----|-----------------------------------------------------------------------------------------------------------|-----|
| IV. | THE MEASUREMENT OF IRREGULARITY PARAMETERS .                                                              | 62  |
| 4.1 | Introduction . . . . .                                                                                    | 62  |
| 4.2 | The Average Height of the Irregularity<br>Region as Deduced from Amplitude Measure-<br>ments . . . . .    | 62  |
| 4.3 | The Determination of Amplitude Pattern<br>Parameters by the Full Correlation Analysis<br>Method . . . . . | 66  |
| 4.4 | The Relations Between $V_c$ and the Thickness<br>of the Irregularity Region <sup>c</sup> . . . . .        | 71  |
| 4.5 | The Phase Pattern on the Ground . . . . .                                                                 | 73  |
| 4.6 | The One Dimensional Amplitude Pattern . . . . .                                                           | 74  |
| 4.7 | The One Dimensional Phase Pattern . . . . .                                                               | 78  |
| 4.8 | The Measurement of Phase Parameters by Means<br>of Differential Doppler Measurements . . . . .            | 82  |
| V.  | INSTRUMENTATION . . . . .                                                                                 | 85  |
| 5.1 | Introduction . . . . .                                                                                    | 85  |
| 5.2 | The Satellite . . . . .                                                                                   | 85  |
| 5.3 | The Spaced Station Locations . . . . .                                                                    | 86  |
| 5.4 | The Receiving Antennas . . . . .                                                                          | 86  |
| 5.5 | The Spaced Station Receiving Apparatus . . . . .                                                          | 89  |
| 5.6 | Spaced Station Recording Apparatus . . . . .                                                              | 92  |
| 5.7 | Space Station Playback Instrumentation . . . . .                                                          | 98  |
| 5.8 | Differential Doppler Instrumentation . . . . .                                                            | 100 |
| VI. | METHOD OF ANALYSIS . . . . .                                                                              | 118 |
| 6.1 | Introduction . . . . .                                                                                    | 118 |
| 6.2 | Spaced Station Correlation Analysis . . . . .                                                             | 118 |
| 6.3 | The Average Height of the Scintillating<br>Region and the Orientation of the Ground<br>Pattern . . . . .  | 121 |
| 6.4 | The Estimation of Power Spectral Densities . . . . .                                                      | 122 |

|       |                                                                                                                                |     |
|-------|--------------------------------------------------------------------------------------------------------------------------------|-----|
| VII.  | EXPERIMENTAL RESULTS . . . . .                                                                                                 | 125 |
| 7.1   | Introduction . . . . .                                                                                                         | 125 |
| 7.2   | The Height of the Scintillating Region<br>and Orientation of the Ground Pattern using<br>the Method of Similar Fades . . . . . | 125 |
| 7.3   | Correlation Analysis Results . . . . .                                                                                         | 136 |
| 7.3.1 | Correlogram Characteristics . . . . .                                                                                          | 136 |
| 7.3.2 | Ground Pattern Characteristics . . . . .                                                                                       | 137 |
| 7.3.3 | Ionospheric Parameters . . . . .                                                                                               | 146 |
| 7.4   | Differential Doppler Results . . . . .                                                                                         | 150 |
| VIII. | DISCUSSION AND CONCLUSIONS . . . . .                                                                                           | 160 |
| 8.1   | Introduction . . . . .                                                                                                         | 160 |
| 8.2   | Comparison with Other Work . . . . .                                                                                           | 160 |
| 8.2.1 | Amplitude Pattern Measurements . . . . .                                                                                       | 160 |
| 8.2.2 | Magnitude of Phase Deviations . . . . .                                                                                        | 162 |
| 8.2.3 | Spectrum of Irregularity Sizes . . . . .                                                                                       | 163 |
| 8.2.4 | Discussion . . . . .                                                                                                           | 165 |
| 8.3   | Sources of Error . . . . .                                                                                                     | 167 |
| 8.4   | Summary . . . . .                                                                                                              | 169 |
| 8.5   | Conclusions . . . . .                                                                                                          | 172 |
| 8.6   | Suggestions for Further Work . . . . .                                                                                         | 173 |
|       | BIBLIOGRAPHY . . . . .                                                                                                         | 175 |

## ABSTRACT

This investigation is concerned with the diffraction of high frequency radio waves by inhomogenities of electron density in the ionosphere with particular emphasis on the effects of the inhomogeneities on the phase of the received signals.

Instrumentation is described which measures the amplitude and phase variations of the received signals at three stations spaced approximately 2.5 km apart on the ground. Analytic techniques are discussed which can yield measures of the ground pattern velocity size, and orientation from these measurements.

To determine the relationship between the ground parameters and the form of the ionospheric disturbances causing them, the problem of the diffraction of a wave by a thin phase screen is investigated. Both a cosinusoidal phase variation, and a random variation of phase with position are considered. It is shown that close to the screen, the phase pattern will maintain the form of the variations in the screen more closely than will the amplitude pattern, especially when the phase variations introduced by the screen are large.

The results of the experiments show that the irregularities responsible for the scintillation of satellite signals observed during the hours of darkness are located at heights of 400 to 700 km above the Earth's surface. The height structure is very complex. The amplitude and phase patterns on the ground are both elongated in the direction which would be expected if the irregularities which are responsible are greatly elongated along the Earth's magnetic field. The irregularities have scale sizes in the direction transverse to the magnetic field of approximately 2 to 3 km as obtained from phase

measurements. The total rms phase shifts in the incident wave due to the irregularities are characteristically 1.5 to 3 radians. The transverse scale sizes as deduced from amplitude measurements are typically a factor of 4 to 7 times smaller than the sizes obtained from the phase measurements. This condition would be expected in view of the high phase deviations observed, and in view of the fact that the amplitude pattern is largely caused by smaller size components of the spectrum of inhomogeneities.

## I. INTRODUCTION

### 1.1 Historical Background

It was recognized at an early date that the ionosphere could not always be considered to be a medium whose properties changed only slowly with distance. Among the first to recognize this fact were Ratcliffe and Pawsey (1933), who showed that some of the irregular fluctuations which were commonly observed in the amplitude of signals reflected from the ionosphere could be attributed to scattering from inhomogeneities in ionospheric refractive index. They suggested that simultaneous reception of signals at receivers spaced some distance apart on the ground was a valuable technique in the study of these inhomogeneities. It was soon realized that measurements of inhomogeneity movements could provide a method of determining the characteristics of ionospheric winds, and Mitra (1949) established a simple mathematical technique for deducing wind velocities from measurements made at three noncolinear ground stations.

Little and Lovell (1950) discovered that fluctuations in the amplitude of radio signals received from the radio star Cygnus were due to ionospheric inhomogeneities. They postulated that the irregularities which were responsible for the fluctuations were situated in the F region of the ionosphere and had dimensions on the order of a kilometer. Hewish (1951) attempted to investigate

theoretically the amplitude and phase fluctuations which would be imposed on a plane wave after passing through a sinusoidal phase modulating screen. As a result of these studies and experimental measurements performed at Cambridge, England, he estimated (Hewish, 1952) that the pertinent irregularities were located at a height of 400 km above the Earth's surface and had a spatial period of about 5 km.

Briggs, Phillips, and Shinn (1950) developed a fairly general technique for analyzing diffraction patterns observed on the ground under the assumption that the ground pattern was isotropic. Phillips and Spencer (1955) generalized this technique to anisotropic patterns. Using the generalized technique, Phillips (1955) was able to show that the irregularities were elongated along the Earth's magnetic field with axial ratios of five to one or more. The mathematical techniques involved, and a summary of the results which had been obtained up to that time were reviewed in an article by Ratcliffe (1956).

With the launching of artificial Earth satellites in 1957, it became possible to study the irregularity characteristics in greater detail. Due to the high velocity of the satellite, any motions of the irregularities themselves could be neglected, and the irregularity distribution could be considered to be "frozen" during the period of observation. The predictable nature of the satellite orbit and the monochromatic nature of the satellite radio frequency sources enabled measurements to be performed to determine the irregularity heights and shapes to a greater degree

of accuracy than had previously been possible. Representative measurements by Kent (1959), DeBarber (1962), Jespersen and Kamas (1964), McClure and Swenson (1964), Liszka (1968) and others are discussed in detail in Chapter VIII of this report where they are compared to the results obtained in this study.

## 1.2 General Statement of the Problem

As early as 1952, Jones, Millman, and Nertney (1953) had concluded that the phase variations in the received signal might be more directly related to the properties of the diffracting medium than would the amplitude variations of the received signal, especially when the phase variations introduced by the inhomogeneities in the waves propagating through them were large. However, from an experimental point of view, the measurement of the amplitude of the received signal was far easier than the measurement of its phase, and so, with the exception of Hewish's early work, nearly all measurements of irregularity parameters performed to date with either satellites or radio stars have been restricted to measurements of the amplitude of the received signal.

The few measurements which did take into account the phase, as well as the amplitude, of the signals received on the ground appear to yield results which are somewhat contradictory. In 1957, Gruber (1961) obtained measurements on a phase switched interferometer which he claimed could be interpreted in terms of the amplitude and phase of the incident wave. He observed no

significant differences between the amplitude and phase results. DeBarber, Ross, and Chisholm (1963) reported that in 1961 few significant phase effects were observed between two stations 600 meters apart on the ground. Yet in 1965, Lansinger (1966) obtained significant phase variations with a 300 meter antenna spacing. He also observed that the phase fluctuations had more low frequency components than did the amplitude fluctuations observed simultaneously.

Although part of the variation in results is probably due to different forms of instrumentation, different geographical station locations, and different solar cycle conditions, it is still felt that the measurement of the phase scintillation of satellite signals passing through the ionosphere is of sufficient interest to warrant further investigation.

### 1.3 Specific Statement of the Problem

The purpose of this investigation was to determine the amplitude and phase characteristics at the ground of a satellite signal which had propagated through a layer of ionospheric inhomogeneities. From these characteristics, it was desired to determine as far as possible the parameters of the inhomogeneities themselves. It was of special interest to determine whether the amplitude and phase measurements gave results which were consistent with each other, and which were also consistent with theoretical descriptions of the diffraction effects involved.

To this end, instrumentation was developed to produce experimental data relating to the amplitude and phase of the satellite signals received at three stations spaced approximately 2.5 km apart on the ground. The analysis necessary to convert these data into measures of inhomogeneity parameters will be developed. The problem of the diffraction of an incident wave by a phase modulating screen and by a thick region of inhomogeneities will be considered theoretically, and the relationship of the amplitude fluctuations to the phase fluctuations of the scattered signal will be considered.

In the following chapters, the theoretical developments will be undertaken first, and the experimental considerations will follow them.

## II. COSINUSOIDAL SCREEN DIFFRACTION THEORY

### 2.1 Introduction

To establish a framework within which the experimental data can be interpreted, it is convenient to investigate the problem of a wave propagating through a medium with small variations of refractive index. Since the wave frequency of the satellite signal is much larger than the collision frequency of the medium, absorption can be ignored, and the major effect of the refractive index variations will be to modulate the phase of the incident wave. It is initially convenient to examine the problem of a wave diffracted by an infinitely thin phase modulating screen. The electric field of the incident wave immediately upon leaving the screen may be mathematically decomposed into an angular spectrum of plane waves propagating away from the screen in different directions. The field at any distance away from the screen may then be found by reconstituting the plane waves in this spectrum, taking into account the phase shifts incurred by the individual waves due to the distances they have traversed. Ratcliffe (1956) has reviewed this technique in some detail and has shown that such a technique is exactly equivalent to the solution of the diffraction problem by means of the scalar Fresnel-Kirchhoff formula.

In this chapter, the case of a one dimensional cosinusoidal screen will be treated in some detail. Although this model may

seem physically unrealistic, it nevertheless provides great insight into the diffraction mechanism. The case of normal plane wave illumination on a screen of arbitrary phase deviation will be considered, followed by the special case of small phase deviation. The effect of additional spatial components on the diffraction pattern will be discussed, and the effects of spherical wave illumination and oblique incidence will be treated. In chapter III, the case of random screens will be considered and the connection between a three dimensional diffracting medium and the thin screen will be established.

## 2.2 The One Dimensional Cosinusoidal Screen

### 2.2.1 The Form of the Solution

It will be shown in chapter VII that the irregularities pertinent to this study are highly elongated along the Earth's magnetic field. It will be shown in chapter III that such irregularities will produce equivalent irregularities in the thin screen which are also highly elongated. The diffraction theory developed in this chapter will show that the power scattered by an irregularity structure size subtends an angle inversely proportional to the size of the scatterer. It would then be expected that except along directions closely parallel to the direction of elongation, the angular extent of the scattered power would be primarily determined by the smaller dimension of the irregularity. Since

observation of the irregularity pattern is usually performed in directions other than along the direction of elongation, such a restriction is not serious, and a one dimensional representation of the irregularities will be adopted to simplify the theoretical discussion. Further justification for this representation will be given in chapter III.

It is, therefore, appropriate to consider a monochromatic plane wave normally incident on an infinitely thin phase modulating screen lying along the x axis of a two dimensional cartesian frame oriented such that the wave normal is in the positive z direction. Following Ratcliffe (1956), the form of the incident wave will be taken as

$$\underline{E}(x, z, t) = \underline{E}(x, z) \exp (j\omega t + j2\pi z/\lambda) \quad (2.1)$$

where a bar under a quantity denotes a complex quantity, and where  $\omega$  and  $\lambda$  are the angular frequency and wavelength of the incident radiation. The radiation leaving the screen may be decomposed into an angular spectrum of plane waves,  $\underline{P}_0(s)$ , with

$$\underline{P}_0(s) = \int \underline{E}(x_0) \exp \left[ \frac{2\pi j}{\lambda} x_0 \sin (s) \right] dx_0 \quad (2.2)$$

where  $s$  is the angle between the scattered wave normal and the z axis, and  $j$  is the square root of minus one. In this equation, and in all the integrals which follow in this work, the limits will be

taken as infinite unless otherwise stated and will not be written explicitly.

At a distance  $z$  from the screen, the phase of the scattered wave with respect to the incident component is

$$\frac{2\pi}{\lambda} [\cos (s) - 1]. \quad (2.3)$$

The angular spectrum at a distance  $z$  then becomes

$$\underline{P}(s) = \int \underline{E}(x_0) \exp \left\{ \frac{2\pi i}{\lambda} [x_0 \sin (s) + z [1 - \cos (s)]] \right\} dx_0. \quad (2.4)$$

For the small scattering angles pertinent to this study,  $\sin (s) \approx s$ ,  $1 - \cos (s) \approx s^2/2$ , and  $\underline{P}(s)$  can be written

$$\underline{P}(s) = \int \underline{E}(x_0) \exp \left\{ \frac{-2\pi i}{\lambda} [(sx_0 + s^2 z/2)] \right\} dx_0. \quad (2.5)$$

$\underline{E}(x, z)$  is given by the Fourier transform of  $\underline{P}(s)$  divided by  $\lambda$ .

$$\underline{E}(x, z) = \frac{1}{\lambda} \iint \underline{E}(x_0) \exp \left\{ \frac{-2\pi i}{\lambda} [(sx_0 - sx) + s^2 z/2] \right\} dx_0 ds \quad (2.6)$$

For a screen which modulates only the phase of the incident wave,

$$\underline{E}(x_0) = A \exp [j \phi (x_0)] \quad (2.7)$$

where A is the amplitude of the incident wave and  $\phi(x_0)$  is the phase modulating function of the screen. Taking A as unity for convenience, and taking  $\phi(x_0)$  as

$$\phi(x_0) = \Delta\phi \cos (2\pi x_0 / L_s), \quad (2.8)$$

$\underline{E}(x_0)$  may be expanded to yield

$$\underline{E}(x_0) = \sum_{n=-\infty}^{\infty} j^n J_n(\Delta\phi) \exp (2 j n x_0 / L_s) \quad (2.9)$$

where  $J_n(\Delta\phi)$  is a nth order Bessel function of the first kind with argument  $\Delta\phi$ .

Substituting  $\underline{E}(x)$  into equation 2.6 yields

$$\underline{E}(x, z) = \frac{1}{\lambda} \int \int \exp \frac{-2\pi j}{\lambda} [s(x_0 - x) + z s^2 / 2] \sum_{n=-\infty}^{\infty} j^n J_n(\Delta\phi) \exp (2\pi j x n / L_s) dx_0 ds \quad (2.10)$$

Interchanging the order of summation and integration, and integrating first on  $s$  and then on  $x$  yields after simplification

$$\underline{E}(x, z) = \sum_{n=-\infty}^{\infty} j^n J_n(\Delta\phi) \exp(2\pi j x n / L_s - j \pi z n^2 / L_s^2) \quad (2.11)$$

Expanding the exponential into real and imaginary parts, expanding the summation, using the identity  $J_{-n}(x) = (-1)^n J_n(x)$ , and resumming the result yields finally

$$\begin{aligned} \underline{E}(x, z) = & J(\Delta\phi) + 2 \sum_{n=1,3,5\dots}^{\infty} J_n(\Delta\phi) (-1)^{(n+3)/2} \cos(nB) \sin(n^2U) \\ & + 2 \sum_{n=2,4,6\dots}^{\infty} J_n(\Delta\phi) (-1)^{n/2} \cos(nB) \cos(n^2U) \\ & + 2j \left\{ \sum_{n=1,3,5\dots}^{\infty} J_n(\Delta\phi) (-1)^{(n+3)/2} \cos(nB) \cos(n^2U) \right. \\ & \left. + \sum_{n=2,4,6\dots}^{\infty} J_n(\Delta\phi) (-1)^{(n+2)/2} \cos(nB) \sin(n^2U) \right\} \end{aligned} \quad (2.12)$$

where  $U = \pi\lambda z/L_s^2$  and  $B = 2\pi x/L_s$ .

Denoting the four sums making up the real and imaginary parts of  $\underline{E}(x, z)$  as  $\text{Sum}_1$ ,  $\text{Sum}_2$ ,  $\text{Sum}_3$ , and  $\text{Sum}_4$ , respectively,  $\underline{E}(x, z)$  can be written in polar form as

$$\underline{E}(x, z) = A(x, z) \exp \{ j \theta(x, z) \} \quad (2.13)$$

where

$$A(x, z) = \{ [J_0(\Delta\phi) + \text{Sum}_1 + \text{Sum}_2]^2 + [\text{Sum}_3 + \text{Sum}_4]^2 \}^{1/2} \quad (2.14)$$

$$\theta(x, z) = \tan^{-1} \left\{ \frac{\text{Sum}_3 + \text{Sum}_4}{J_0(\Delta\phi) + \text{Sum}_1 + \text{Sum}_2} \right\} \quad (2.15)$$

The ambiguity of plus or minus  $n\pi$  radians, where  $n$  is an integer, which is implicit in the definition of the inverse tangent function must be removed by noting the actual phase variations at the screen and requiring that the phase for any value of  $x$  change continuously as the point of observation moves from the screen.

At the screen, the wave is purely phase modulated, and the amplitude does not vary with  $x$ . As  $z$  increases from zero, fluctuations begin to occur in the wave amplitude. Close to the screen, the  $\cos(n^2U)$  terms will remain near unity, and the amplitude pattern will build up primarily due to the  $\sin(n^2U)$  terms, which will be

dominated by the term whose Bessel function coefficient is of highest order while still having appreciable value. As  $\Delta\phi$  increases above unity, the Bessel functions  $J_n(\Delta\phi)$  will have appreciable value up to a value of  $n$  approximately of order  $\Delta\phi$ . This implies that the amplitude pattern should develop at closer distances to the screen as  $\Delta\phi$  increases. Furthermore, due to the large number of cross-products in the expression for the amplitude deviations, for all  $z$  larger than zero, the amplitude pattern will contain spatial components with shorter periods than that of the screen itself. Since the phase pattern is essentially determined by a ratio of terms, it can be expected that the phase pattern will change at a slower rate with respect to both  $x$  and  $U$  than the amplitude pattern, since contributions in the numerator will be partially offset by terms in the denominator.

To examine the nature of the phase and amplitude deviations as a function of  $U$ , numerical values were obtained for the amplitude and phase of the diffracted field for several values of  $\Delta\phi$  by means of a high speed digital computer. In this calculation, equations 2.14 and 2.15 were employed, and all Bessel coefficients up to 16th order were included. Some of the amplitude and phase patterns obtained for  $\Delta\phi = 3.5$  radians are shown in Figure 2.1. Other examples may be found in the results of Hewish (1951). These results show that close to the screen, the phase pattern more closely maintains the form of the screen variations, when  $\Delta\phi$  is large, than does the

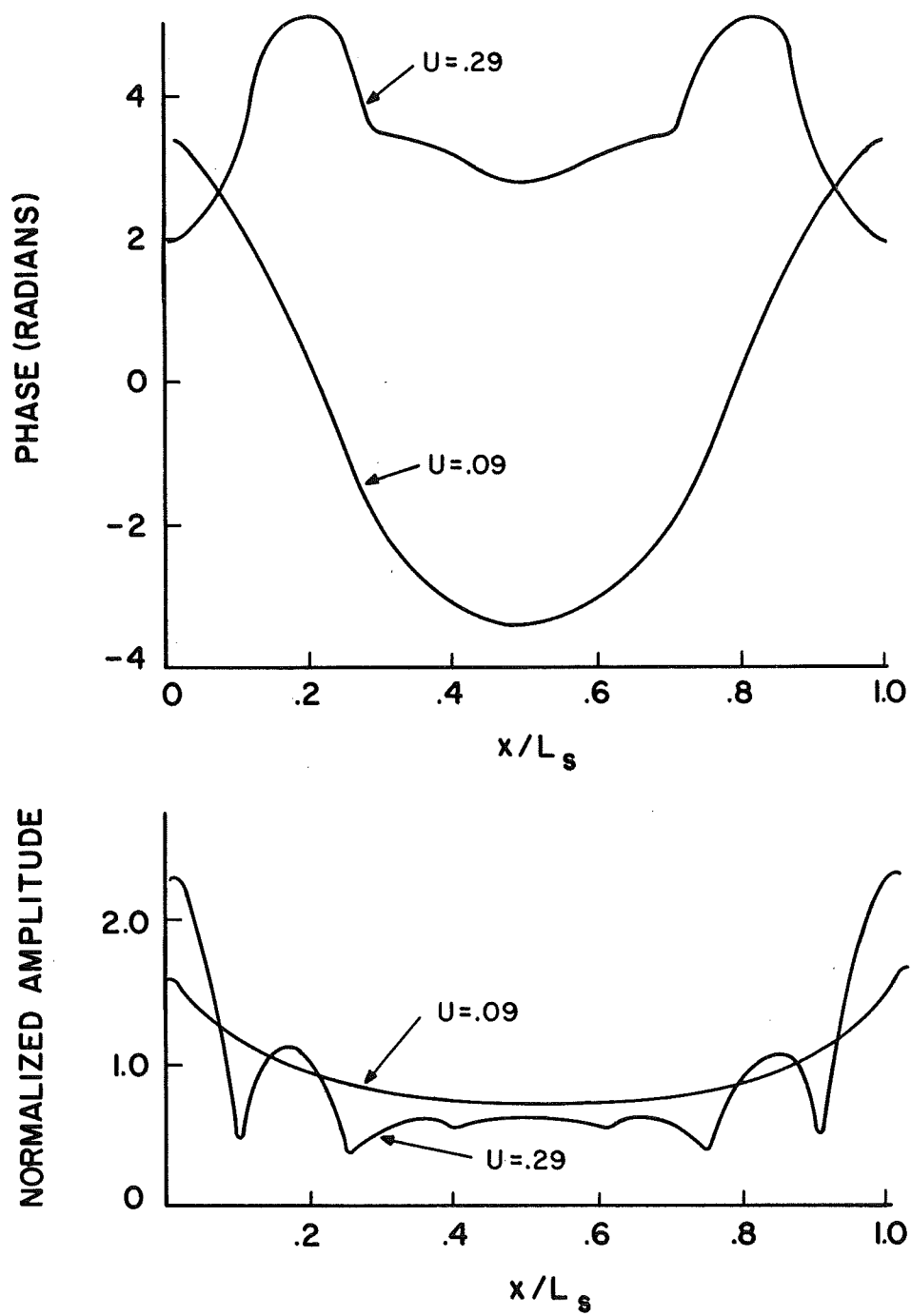


Figure 2.1. Phase and amplitude diffraction patterns for  $\Delta\phi=3.5$  radians.

amplitude pattern. Far from the screen, neither the phase nor the amplitude pattern are good representations of the phase variations.

### 2.2.2 Scintillation Indexes and Pattern Scale Sizes

To obtain quantitative measures of the diffraction effects it is convenient to define an amplitude scintillation index  $S_a(z)$ , and a phase scintillation index  $S_p(z)$  as:

$$S_a(z) = \{ [A(x, z) - \bar{A}(x, z)]^2 / \bar{A}(x, z)^2 \}^{1/2} \quad (2.16)$$

$$S_p(z) = [\theta(x, z) - \bar{\theta}(x, z)]^{1/2} \quad (2.17)$$

where a bar over a quantity denotes a spatial average over  $x$ .

Correlation functions of amplitude and phase variations may be defines as:

$$\rho_a(\Delta x, z) = \frac{[A(x, z) - \bar{A}(x, z)][A(x + \Delta x, z) - \bar{A}(x, z)]}{[A(x, z) - \bar{A}(x, z)]^2} \quad (2.18)$$

$$\rho_p(\Delta x, z) = \frac{[\theta(x, z) - \bar{\theta}(x, z)][\theta(x + \Delta x, z) - \bar{\theta}(x, z)]}{[\theta(x, z) - \bar{\theta}(x, z)]^2} \quad (2.19)$$

Since  $S_a(z)$ ,  $S_p(z)$ ,  $\rho_a(\Delta x, z)$  and  $\rho_p(\Delta x, z)$  will be evaluated at fixed values of  $z$  no confusion will result if the explicit  $z$  dependence is dropped from the notation, and so the scintillation indexes and correlations will be henceforth denoted simply as  $S_a$ ,  $S_p$ ,  $\rho_a(\Delta x)$ , and  $\rho_p(\Delta x)$ .

For a periodic screen,  $\rho_a(\Delta x)$  and  $\rho_p(\Delta x)$  will vary periodically as  $\Delta x$  is varied. However, for comparison with the results which will be obtained for random screens, it is convenient to fit in the least mean square sense the values of the actual correlations in the vicinity of small  $x$  with equivalent functions of the form

$$\rho(\Delta x) = \exp [-(\Delta x)^2 / 2L_c^2] . \quad (2.20)$$

The standard deviations of the functions fitted to the amplitude and phase correlograms will be denoted as  $L_{ca}$  and  $L_{cp}$  respectively and will be defined as the equivalent scale sizes of the amplitude and phase diffraction patterns. With these definitions, the scale size  $L_{co}$  of a cosinusoidal screen with spatial period  $L_s$  is given by:

$$L_{co} = L_s / 2\pi \quad (2.21)$$

In Figures 2.2 through 2.5 the quantities  $S_a$ ,  $S_p$ ,  $L_{ca}/L_{co}$  and  $L_{cp}/L_{co}$  are plotted versus  $U$  for several values of  $\Delta\phi$ . As

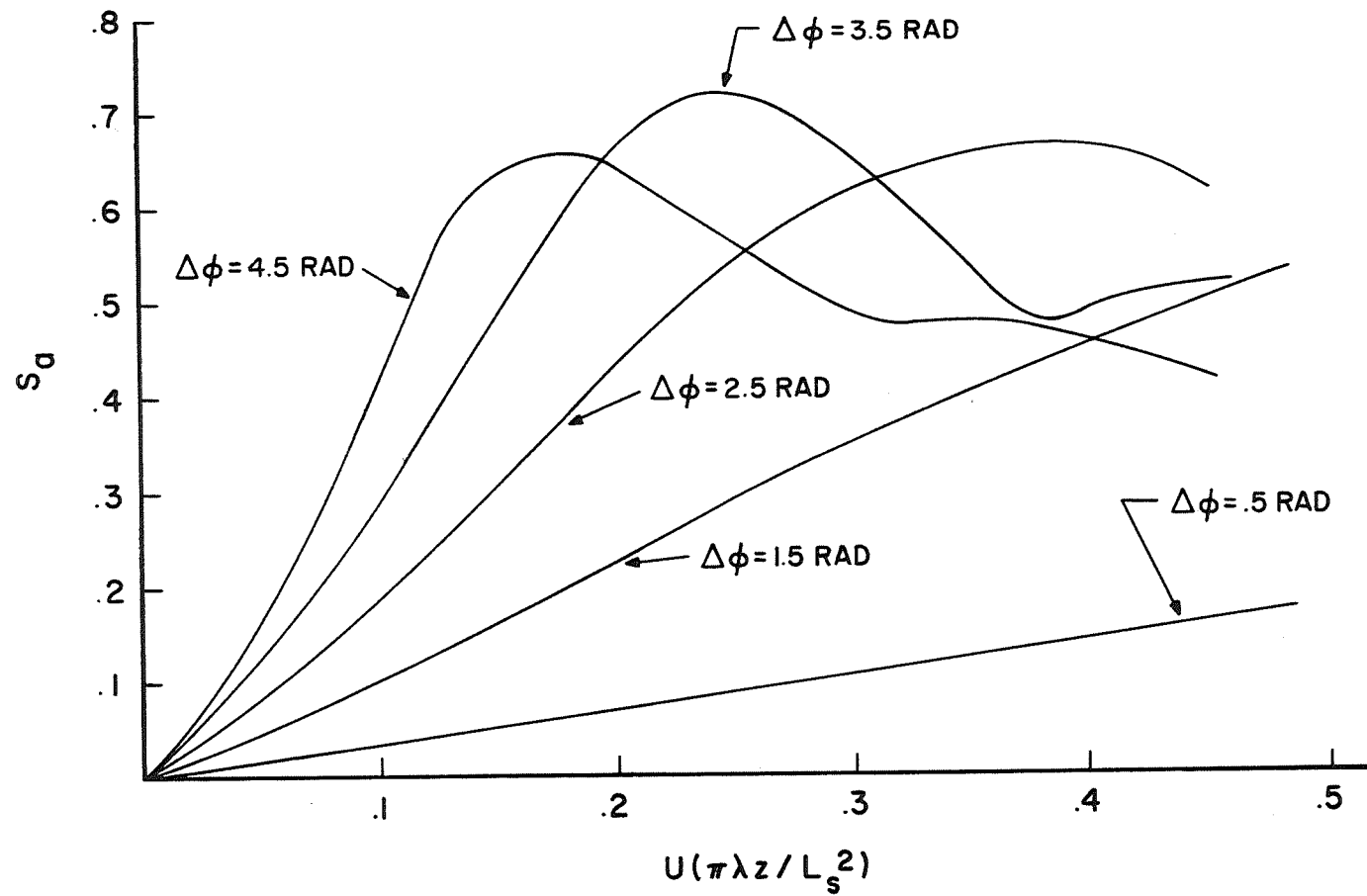


Figure 2.2.  $S_a$  versus  $U$  for several values of  $\Delta\phi$ .

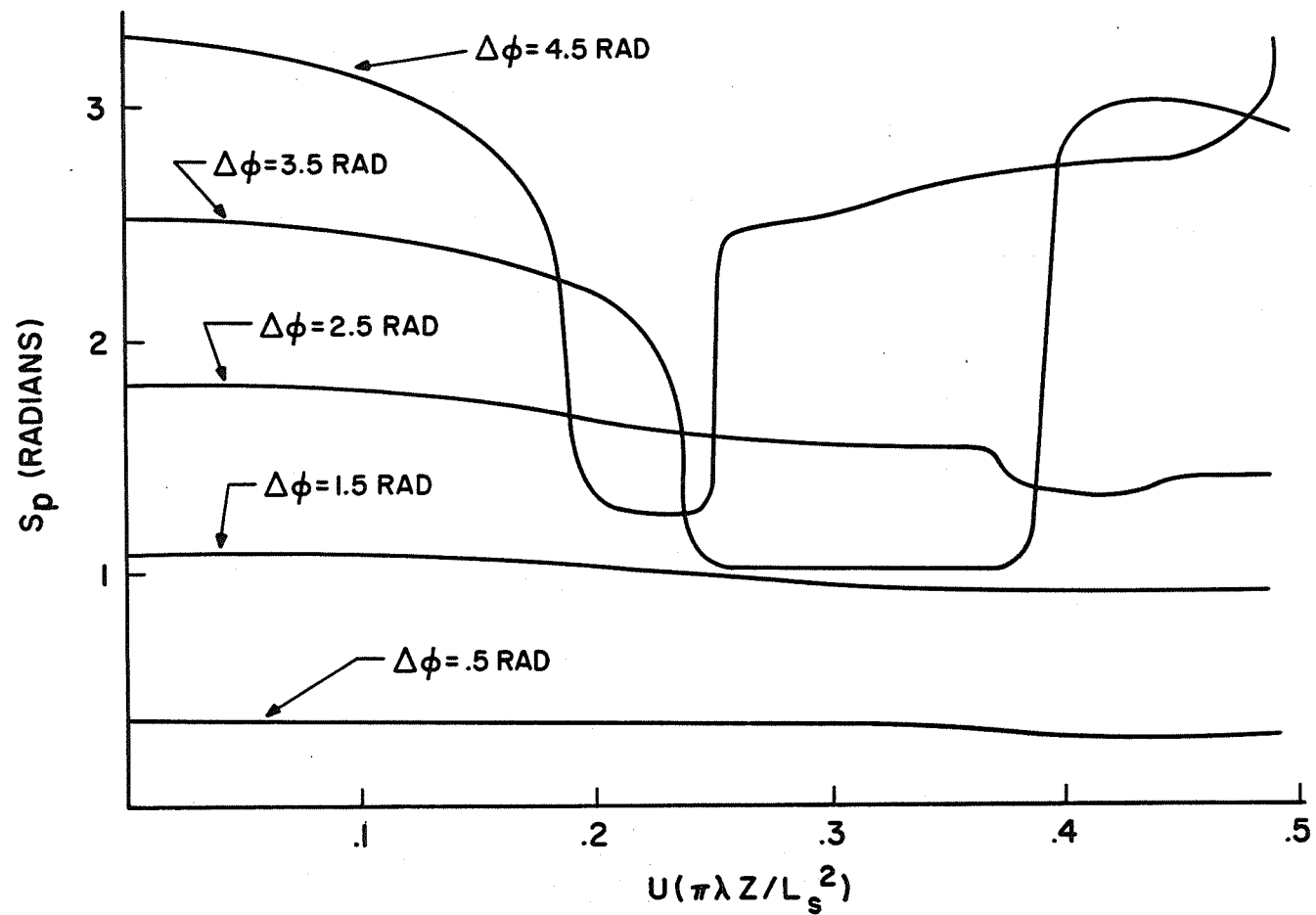


Figure 2.3.  $S_p$  versus  $U$  for several values of  $\Delta\phi$ .

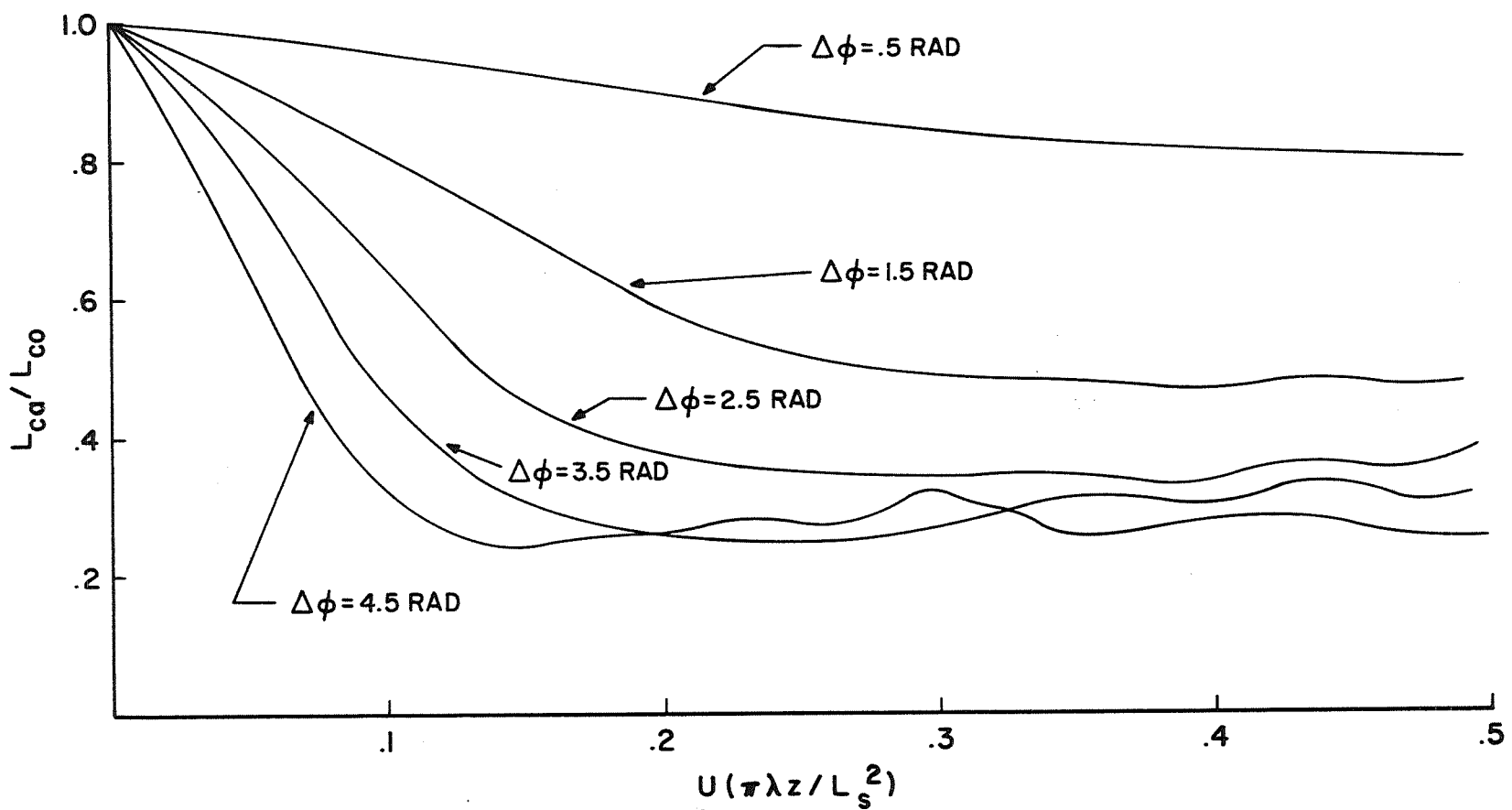


Figure 2.4. Relative scale size of amplitude pattern for several values of  $\Delta\phi$ .

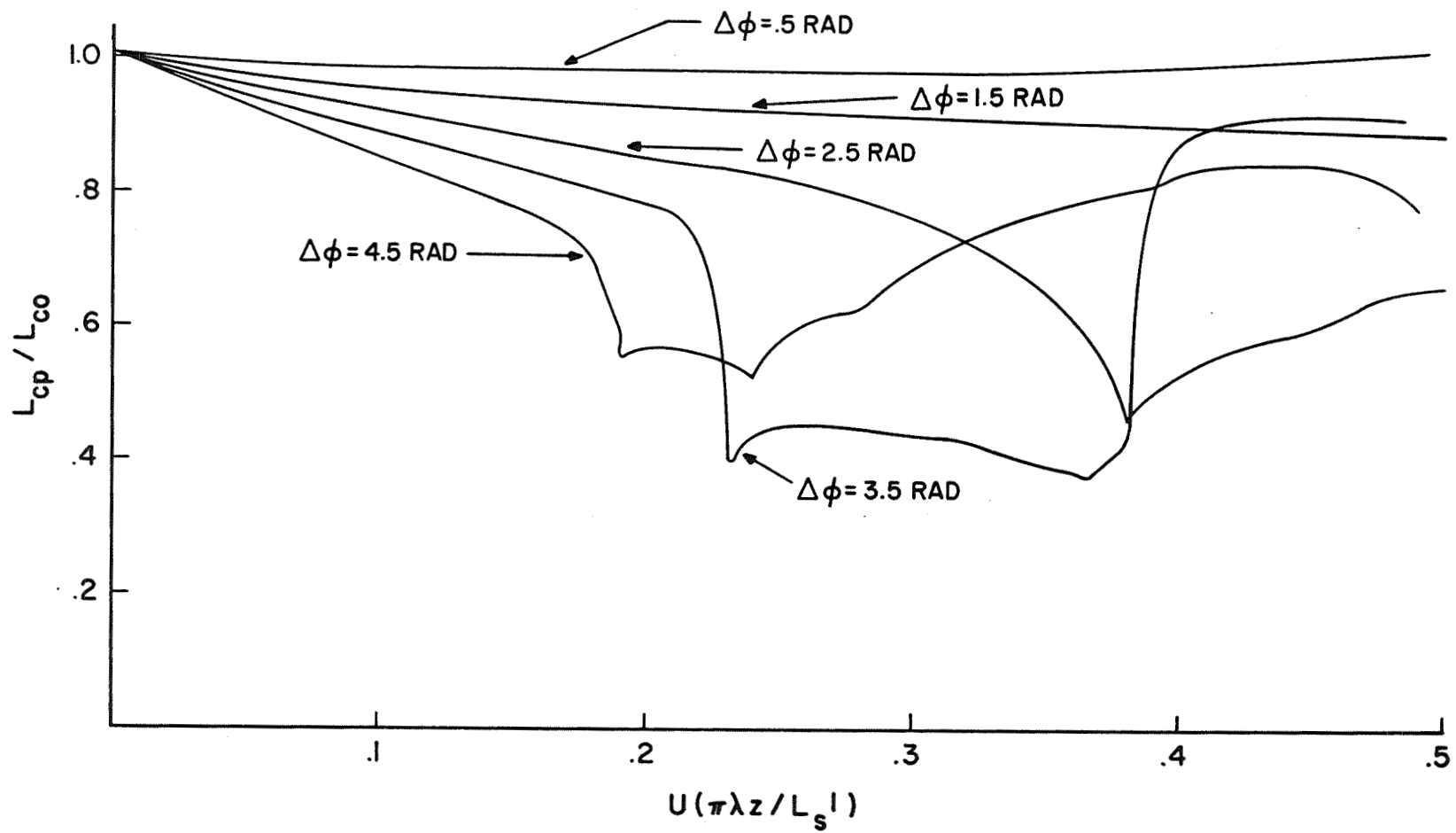


Figure 2.5. Relative scale size of phase pattern for several values of  $\Delta\phi$ .

expected,  $S_a$  rises to large values sooner as  $\Delta\phi$  is increases. The sudden changes in the behavior of the phase pattern for large values of  $U$  is a consequence of the fact that for  $\Delta\phi$  greater than unity, the higher order Bessel functions will have appreciable magnitude and result in destructive interference such that the real and imaginary parts of  $E(x, z)$  can change algebraic sign with only moderate values of  $U$ . The rate at which these terms vary will determine the form of the phase variations, and particularly when both terms are small, small changes in the value of one or the other can drastically affect the form of the phase pattern. Due to the squaring process involved in determining the amplitude, the amplitude is insensitive to the algebraic sign of the real and imaginary parts of  $E(x, z)$ . Consequently, values of  $U$  and  $x$  which simply create notches in the amplitude pattern can completely change the form of the phase pattern.

However, it is still seen that the phase scale sizes are always somewhat greater than those of amplitude, and that close to the screen, both  $L_{cp}$  and  $S_p$  stay reasonably close to their values at the screen even in the case of fairly large values of  $\Delta\phi$ . If diffractive effects are ignored, ray optics would predict that close to the screen, the phase perturbations in the emerging wavefront would be identical to  $\phi(x_0)$ . The results shown in Figures 2.2 through 2.5 indicate that although the ray optical approximation is fairly valid for describing phase effects close to the screen, it is nevertheless a poor approximation in the case of amplitude

when  $\Delta\phi$  is large. Physically, this is reasonable since the amplitude pattern is primarily a manifestation of the diffractive effects which are ignored in the ray optical approximation.

### 2.2.3 The Limits of the Ray Optical Region

In order for the perturbations in the phase of the incident wave to conform to the shape of  $\phi(x_0)$ , the sidewise deviation of the rays making up the wave must be small. If the expansion for  $\underline{E}(x_0)$  given in equation 2.9 is substituted into the expression for  $\underline{P}_0(s)$  given in equation 2.2, the order of summation and integration interchanged, and the integration performed, it can be shown that the resulting spectrum will consist of a series of delta functions which represent waves propagating at angles of plus or minus  $n\lambda/L_s$ . Denoting the sidewise deviation by  $x_s$ , it is seen that

$$x_s = n\lambda z/L_s. \quad (2.22)$$

Replacing  $n$  by  $\Delta\phi$  for large  $\Delta\phi$  and requiring that  $x_s$  be small compared to  $L_s$  for the ray optical approximation to be valid yields as requirements on  $z$

$$z \ll L_s^2/\lambda \quad \Delta\phi < 1 \quad (2.23)$$

$$z \ll L_s^2/\lambda\Delta\phi \quad \Delta\phi > 1 \quad (2.24)$$

It is seen that equation 2.23 is the usual condition for the validity of the ray optical approximation derived from single scatter considerations.

If an attempt is made to apply the results of an analysis based on a cosinusoidal screen to a situation in which the phase at the screen is a random variable of position, correspondence between the two cases can only be expected when the region of the screen contributing to the pattern is less than the size of features in the screen. For contributing regions greater than this the cosinusoidal analysis predicts an ordered relationship between the scatters which certainly will not exist in a random model, but for regions less than this the regions contributing to the pattern will be less than the size of a single irregularity and the difference between a random screen and a periodic screen disappears. It is seen that the region of the screen contributing to the pattern at a distance  $z$  from the screen is bounded by  $x_s$  and so when the ray optical approximation holds, the phase characteristics observed may be expected to be similar for either a random or periodic screen model.

#### 2.2.4 The Special Case of Small $\Delta\phi$

For small  $\Delta\phi$ , the only Bessel functions of appreciable magnitude in equation 2.12 are  $J_0(\Delta\phi)$  and  $J_1(\Delta\phi)$ . Expanding these in power series:

$$J_0(\Delta\phi) = 1 - (\Delta\phi)^2/2^2 + (\Delta\phi)^4/2^2 4^2 + + \quad (2.25)$$

$$J_1(\Delta\phi) = \Delta\phi/2 - (\Delta\phi)^3/2^4 = (\Delta\phi)^5/(3)(2)^7 + + . \quad (2.26)$$

Keeping only first order terms,  $\Delta(x, z)$  and  $\theta(x, z)$  become

$$A(x, z) = [1 + 2\Delta\phi \cos(2\pi x/L_s) \sin(\lambda\pi z/L_s^2)]^{1/2} \quad (2.27)$$

$$\theta(x, z) = \tan^{-1}\{\Delta\phi \cos(2\pi x/L_s) \cos(\pi\lambda z/L_s^2)\} . \quad (2.28)$$

Since  $\Delta\phi$  has been assumed small, the arc tangent may be replaced with its argument, and the square root may be replaced with its binomial expansion to first order in  $\Delta\phi$ . When the resulting values of  $A(x, z)$  and  $\theta(x, z)$  are substituted into equations 2.16 and 2.17, the resulting scintillation indexes are

$$S_a = .707 \Delta\phi \sin(\pi\lambda z/L_s^2) \quad (2.29)$$

$$S_p = .707 \Delta\phi \cos(\pi\lambda z/L_s^2). \quad (2.30)$$

Thus  $S_a$  and  $S_p$  are related to the real and imaginary parts of  $E(x, z)$  by

$$S_a = \{[\text{Re} [\underline{E}(x, z) - 1]]^2\}^{1/2} \quad (2.31)$$

$$S_p = \overline{\{[\text{Im } [\underline{E}(x, z)]]^2\}^{1/2}} . \quad (2.32)$$

This identification will be useful in the case of a random screen.

### 2.3 The Effect of Additional Spatial Components

If the phase modulating function  $\phi(x_0)$  is composed of many spatial components and the total phase deviation introduced by the screen is less than about one third radian, the response on the ground may be found by simply superimposing the responses of the components acting individually (Ratcliffe, 1956). Thus if  $\phi(x_0)$  is represented in the form of a Fourier Transform

$$\phi(x_0) = \int \phi(k_0) \cos [k_0 x_0 + \phi(k_0)] dk_0 , \quad (2.36)$$

with wavenumber  $k_0$  defined as  $2\pi$  divided by the irregularity spatial wavelength, equations 2.32 and 2.33 become

$$S_a^2 = .5 \int \phi(k_0) \sin^2 (\lambda k_0 z/4\pi) dk_0 \quad (2.37)$$

$$S_p^2 = .5 \int \phi(k_0) \cos^2 (\lambda k_0 z/4\pi) dk_0 . \quad (2.38)$$

Physically, it can be expected that  $\phi(k_0)$  will decrease as  $k_0$  becomes large. This circumstance together with the behavior of

the trigonometric terms in equations 2.37 and 2.38 will cause the phase response close to the screen to be weighted with a low pass filter like function, while the amplitude response will be analogous to that of a band pass filter. To illustrate this band pass action of the amplitude response, it may be assumed for example that  $\phi(k_0)$  is directly proportional to  $L_s$ . Making this substitution into equation 2.37 and differentiating inside the integral shows that the maximum contribution to the integrand comes from values of  $L_s$  close to  $(\pi\lambda z/1.17)^{1/2}$  kilometers. To further illustrate this effect, effect, the value of the integrand of equation 2.37 was calculated for several values of  $L_s$  for  $z$  equal to 250 km, and an incident wavelength of seven meters. The results are shown in Figure 2.6 where the scale sizes are expressed in terms of  $L_{c_0}$  rather than  $L_s$  by the use of equation 2.24. It is noted that the amplitude response is essentially determined by correlation scales of order one third kilometer, the same order of magnitude as is commonly observed in practice.

When the total phase deviation is larger than one third radian, superposition is no longer valid. As is known from the study of multitone frequency modulation (Crosby, 1938) the spectrum which results from wideband frequency modulation when more than one modulating frequency is present contains components not only at the modulating frequencies, but at frequencies corresponding to sums and differences of these frequencies and their harmonics. The net effect is to create more terms in the expressions for the amplitude

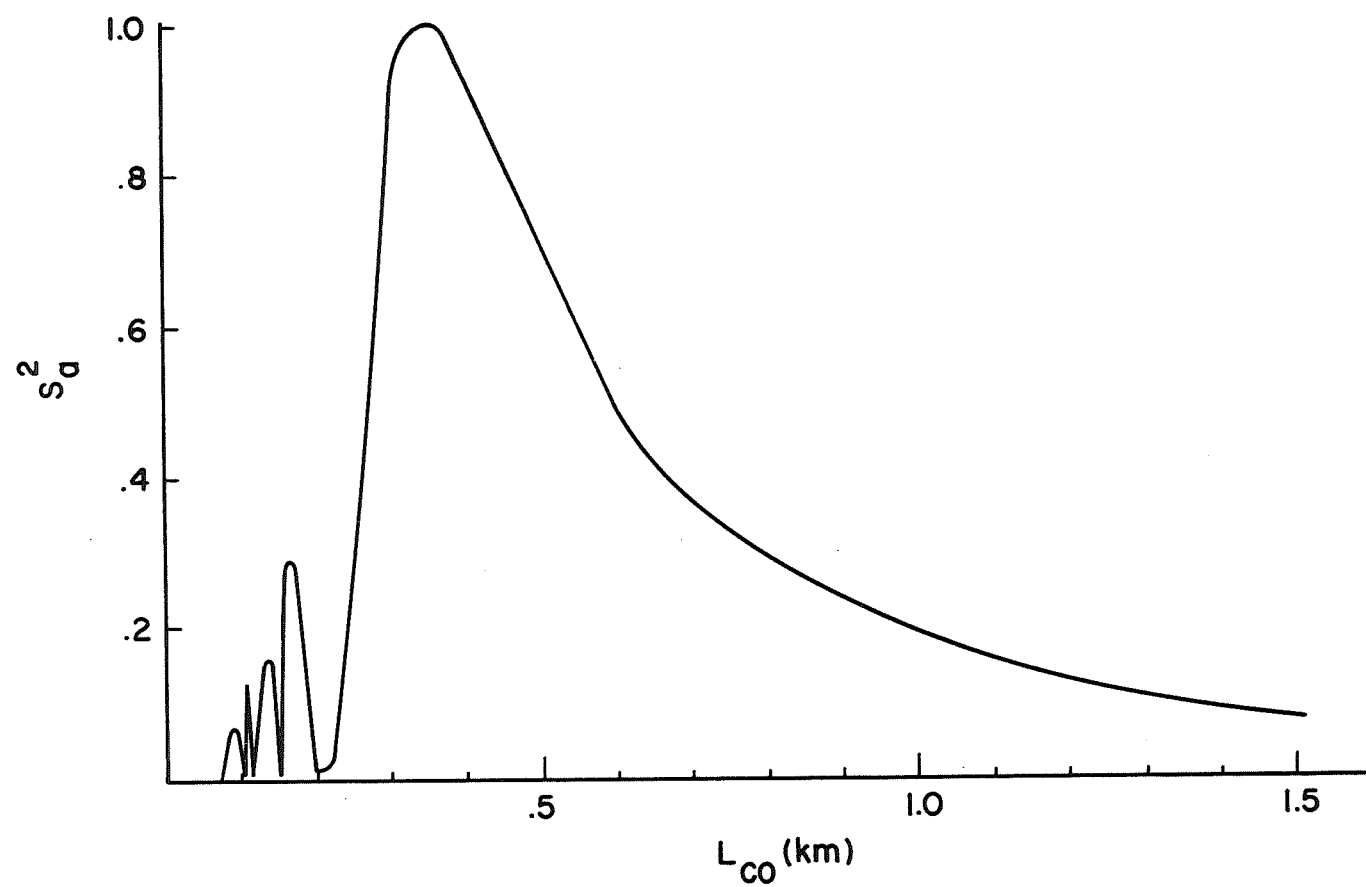


Figure 2.6. Amplitude pattern spatial filtering effect for small  $\phi_0$  with  $\lambda=7$  meters, and  $z=250$  km.

and phase patterns and thereby make further quantitative analysis difficult.

Physically, however, the lower wavenumbers, associated with the spatial components of highest phase deviation, will place the observer in the ray optical region, and since these terms contribute most of the phase shift, the form of the phase pattern will not change drastically at distances fairly close to the screen. The amplitude effects will predominately be caused by diffractive effects of high wavenumber, and therefore will be dominated by terms of high spatial frequency.

Within the ray optical region superposition of the phase effects is valid, and so the phase pattern would be expected to be a good representation of at least those components of low wavenumber. Since for large  $\Delta\phi$  the amplitude pattern is not similar to the pattern of phase in the screen, even for a single spatial component, the amplitude pattern cannot be expected to be a good representation of the pattern in the screen when more than one spatial component is present, and when the total screen phase deviation is larger than one radian.

It should be noted that Beckmann (1965) and Hodora (1968) have reached similar conclusions concerning the importance of the lower spatial frequencies in producing the phase pattern, and in the validity of applying ray optical techniques to these components in their studies of laser propagation through the turbulent atmosphere.

## 2.4 The Effect of Oblique Incidence

If the plane wave is incident on the screen with its wave-normal making an angle of  $\theta_z$  with the z axis, the incident phase is

$$\frac{2\pi}{\lambda} (x \sin \theta_z + z \cos \theta_z) \quad . \quad (2.36)$$

The angular spectrum at the screen is

$$\underline{P}_0(s) = \int e^{j\phi(x_0)} \exp \left\{ \frac{-2\pi j}{\lambda} [x_0 \sin(s) - x_0 \sin \theta_z] \right\} dx_0 \quad . \quad (2.37)$$

The scattered field relative to the incident field is then

$$\underline{E}(x, z) = \frac{1}{\lambda} \iint e^{j\phi(x_0)} \exp \left\{ \frac{-2\pi j}{\lambda} [x_0 [\sin(s) - \sin \theta_z] - z[1 - \sin^2(s)]^{1/2} - z \cos \theta_z] \right\} dx_0 d[\sin(s)] \quad . \quad (2.38)$$

Defining  $\sin(s) - \sin \theta_z$  as  $m_1$ , and  $[1 - \sin^2(s)]^{1/2} - \cos \theta_z$

as  $m_2$ ,  $m_2$  may be expressed in terms of  $m_1$  by the use of a binomial expansion. For small scattering angles and  $\theta_z$  not close to 90 degrees,  $m_2$  becomes to second order

$$m_2 = -m_1^2 \sec \theta_z/2 - m_1 \tan \theta_z \quad (2.39)$$

Since  $\theta_z$  is assumed fixed,  $d[\sin(s)] = dm_1$ , and  $\underline{E}(x, z)$  may be written as

$$\underline{E}(x, z) = \frac{1}{\lambda} \iint e^{j\phi(x_0)} \exp \left\{ \frac{-2\pi j}{\lambda} [m_1 [x_0 - (x-z \tan \theta_z)] - zm_1^2 \sec \theta_z/2] \right\} dx_0 dm_1 \quad (2.40)$$

Comparing equation 2.40 to equation 2.6 shows that the effect of oblique incidence is to translate the pattern in the x direction by  $z \tan \theta_z$ , and to increase the effective distance to the screen by a factor of  $\sec \theta_z$ . Similar results have been obtained by DeBarber and Ross (1966) for the case of small phase deviation. It is shown here that these results apply for the case of large phase deviation as well.

## 2.5 Point Source Effects

When the source of the incident radiation is at a finite distance from the screen the incident wave is not plane. The

resulting effects of the diffraction pattern are most simply derived at normal incidence. The phase of the incident wave is then

$$\frac{2\pi}{\lambda} (z_0^2 + x_0^2)^{1/2} \quad (2.41)$$

where  $z_0$  is the distance from the source to the screen. The phase of the diffracted component a distance  $z_1$  from the screen will be, for small scattering angles,

$$\frac{2\pi}{\lambda} \{ z_1(1 - s^2/2) + (z_0^2 + x_0^2)^{1/2} \} . \quad (2.42)$$

The relative phase of the scattered radiation with respect to the specular component will be

$$\frac{2\pi}{\lambda} \{ z_1(1 - s^2/2) + (z_0^2 + x_0^2)^{1/2} - [(z_1 + z_0)^2 + x^2]^{1/2} \} . \quad (2.43)$$

The scattered field relative to the incident field then is given by

$$\begin{aligned} \underline{E}(x, z) = \frac{1}{\lambda} \iiint \underline{E}(x_0) \exp \left\{ \frac{-2\pi j}{\lambda} [s(x_0 - x) - z_1(1 - s^2/2) \right. \\ \left. -(z_0^2 + x_0^2)^{1/2} + [(z_0 + z_1)^2 + \right. \\ \left. x^2]^{1/2} ] \right\} dx_0 ds . \end{aligned} \quad (2.44)$$

For  $z_0^2 \gg x_0^2$ , and  $(z + z_0)^2 \gg x$ , the quadratic terms can be expanded to second order in  $x$  and  $x_0$  yielding

$$\begin{aligned} \underline{E}(x, z) = \frac{1}{\lambda} \iiint \underline{E}(x_0) \exp \left\{ \frac{-2\pi j}{\lambda} \left[ \frac{x_0^2}{2z_0} - \frac{x^2}{2(z_0 + z_1)} \right. \right. \\ \left. \left. - zs^2/2 \right] \right\} dx_0 ds . \end{aligned} \quad (2.45)$$

To keep the development general,  $\underline{E}(x_0)$  may be formally represented by a Fourier integral as

$$\underline{E}(x_0) = \int C(k) \exp(-jk_0 x_0) dk \quad (2.46)$$

where  $k_0$  is the irregularity wavenumber. Substituting this value of  $\underline{E}(x_0)$  into equation 2.45, interchanging the order of integration, and integrating on  $s$  and then  $x_0$  yields

$$\underline{E}(x, z) = [z_0 / (z_0 + z_1)]^{1/2} \int C(k_0) \exp \left\{ \frac{j2\pi x z_0}{L_s(k_0)(z_0 + z_1)} - \frac{j\pi z_0 z_1}{L_s^2(k_0)(z_0 + z_1)} \right\} dk_0 \quad (2.47)$$

where  $k_0$  in the integrand has been replaced by its value  $2\pi/L_s(k_0)$  to demonstrate the effect of the finite value of  $z_0$  on the spatial wavelengths in the pattern. Defining  $z_0/(z_0 + z_1)$  as  $F$ , a comparison of the integrand of equation 2.49 for the case of  $z_0$  approaching infinity ( $F = 1$ ), and for the case of finite  $z_0$  shows that effects of a finite  $z_0$  is threefold. The magnitude of  $\underline{E}(x, z)$  is reduced by  $F^{1/2}$ . The scale sizes on the ground are increased by the factor  $1/F$ . The apparent distance of the screen from the ground is reduced from  $z_1$  to  $Fz_1$ . Similar results have been obtained by Ratcliffe (1956) by employing ray optical arguments, and by Bowhill (1961) for the case of a screen with small  $\Delta\phi$ . It is shown here that these results apply to a screen of arbitrary description even when diffractive effects are included.

### III. RANDOM SCREENS

#### 3.1 Introduction

Physically it seems more natural to assume that the irregularities are actually characterized by random or at best quasiperiodic functions of position. This complicates the problem, making it necessary to describe the fields in terms of statistical parameters. It is therefore necessary to assume some form for the probability distribution of phase and the autocorrelation of phase at the screen before the characteristics of the pattern at the ground can be obtained. As a convenient mathematical model, it will be assumed that both the probability distribution, and the phase correlation are adequately described by Gaussian functions.

It will be shown in section 3.5 that the phase variations in the equivalent thin screen may be regarded as the integrated effect of the passage of the incident wave through a thick layer of irregularities. If the individual irregularities can be assumed to be statistically independent and sufficiently numerous, then the Central Limit Theorem may be reasonably invoked to justify the Gaussian assumption for the phase distribution at the screen. Unfortunately, no such physical justification may be made for the form of the correlation function employed. However, the simplification of the mathematics which results from the Gaussian assumption is such that many results are obtainable which would be difficult

to obtain in any other way. To obtain a feeling for the physics involved, it will therefore be assumed that the actual correlation of phase at the screen can adequately be described by the Gaussian form. The probability distribution and the autocorrelation of screen phase will therefore be taken as

$$p(\phi) = \frac{\exp(-\phi^2/2\phi_0^2)}{(2\pi\phi_0^2)^{1/2}} \quad (3.1)$$

$$\rho_\phi(x_0) = \exp[-(\Delta x_0)^2/2L_{c_0}^2] \quad (3.2)$$

where  $\phi_0$  is the root mean square phase deviation and  $L_{c_0}$  is the scale size of the irregularities in the screen.

### 3.2 The Angular Spectrum of Complex Amplitude

The angular power spectrum of complex amplitude will be defined as

$$W_c(s) = \overline{\underline{P}(s) \underline{P}^*(s)} \quad (3.3)$$

where  $W_c(s)$  is the angular power spectrum,  $\underline{P}(s)$  is as defined in equation 2.4, and a star above a quantity denotes the complex conjugate of the quantity. The autocorrelation of complex amplitude,  $\rho_c(x)$ , may be defined as

$$\rho_c(\Delta x) = \frac{\overline{E(x, z) E^*(x + \Delta x, z)}}{\overline{E(x, z) E^*(x, z)}} \quad (3.4)$$

Assuming unity amplitude for the incident wave, and evaluating  $\rho_c(\Delta x)$  at the screen,

$$\rho_c(\Delta x) = e^{j\phi(x_0)} e^{-j\phi(x_0 + \Delta x_0)} \quad (3.5)$$

Since  $\phi(x_0)$  and  $\phi(x_0 + \Delta x_0)$  are random variables having zero mean values and are described by a joint Gaussian probability distribution (in view of equation 3.1), it is possible to show (Mercier, 1962)

$$\overline{e^{j\phi(x_0)} e^{-j\phi(x_0 + \Delta x_0)}} = \exp \{-0.5[\overline{\phi(x_0) - \phi(x_0 + \Delta x_0)}]^2\} \quad (3.6)$$

By definition

$$\rho_\phi(\Delta x_0) = \overline{\phi(x_0) \phi(x_0 + \Delta x_0)} / \phi_0^2 \quad (3.7)$$

For a statistically stationary screen

$$\overline{\phi(x_0) \phi(x_0)} = \overline{\phi(x_0 + x_0) \phi(x_0 + \Delta x_0)} = \phi_0^2 \quad (3.8)$$

The autocorrelation of complex amplitude at the screen is therefore given by

$$\rho_c(\Delta x_0) = \exp \{ -\phi_0^2 [1 - \rho_\phi(\Delta x_0)] \} \quad (3.9)$$

By virtue of the Wiener Kintchine theorem,  $W_c(s)$  at the screen is the Fourier Transform of  $\rho_c(\Delta x_0)$ . For plane wave illumination therefore,

$$W_c(s) = \int \exp(-2\pi j s \Delta x_0 / \lambda) \exp \{ -\phi_0^2 [1 - \rho_\phi(\Delta x_0)] \} d(\Delta x_0). \quad (3.10)$$

Expanding the second exponential term into a Taylor series, and replacing  $\rho_\phi(\Delta x_0)$  by its value from equation 3.2 yields

$$W_c(s) = e^{-\phi_0^2} [e^{-2\pi j s \Delta x_0 / \lambda} \sum_{n=0}^{\infty} \{\phi_0^{2n} \exp[-n(\Delta x_0)^2 / 2L_{co}^2]\} d(\Delta x_0). \quad (3.11)$$

Interchanging the order of summation and integration, and integrating, yields finally for the spectrum of complex amplitude:

$$W_c(s) = e^{-\phi_0^2} \{ \delta(s) + L_{c_0} (2\pi)^{1/2} \sum_{n=0}^{\infty} [(\phi_0^2)^n n^{-1/2} \exp(-2\pi^2 L_{c_0}^2 s^2 n^{-2})] \} \quad (3.12)$$

where  $\delta(s)$  is a Dirac delta function and represents the undiffracted or specular component.

Essentially similar results have been obtained by Fejer (1953), Bramley (1954), and Feinstein (1954). Fejer pointed out that by approximating the factorial term with Stirling's formula it can be shown that for large  $\phi_0$ , the major contribution to  $W_c(s)$  comes from terms up to  $n$  approximately equal to  $\phi_0^2$ , a situation analogous to that found for the cosinusoidal screen. It should also be noted that due to the complex conjugate involved in the definition of  $W_c(s)$  and  $\rho_c(\Delta x)$ , for plane wave illumination both  $W_c(s)$  and  $\rho_c(\Delta x)$  are invariant with distance and equal to their values at the screen.

### 3.3 The Special Case of Small $\phi_0$

If  $\phi_0$  is small, only the first term in the series expansion in equation 3.12 is important. If the specular component is ignored, and the spectrum representing the complex scattered power is denoted by  $W_{cs}(s)$ , then the normalized value of  $W_{cs}(s)$  is given by

$$W_{cs}(s) = \exp(-2\pi^2 L_c^2 s^2 / \lambda^2). \quad (3.13)$$

It should be noted that  $W_{cs}(s)$  has the same form as the power spectrum of phase at the screen obtained by taking the Fourier transform of  $\rho_\phi(x_0)$ .

The amplitude and phase of  $\underline{E}(x, z)$  rather than  $\underline{E}(x, z)$  itself are the quantities most easily measured at the ground. By analogy with equations 2.34 and 2.35, Bowhill (1961) has expressed the amplitude and phase of the scattered field in terms of the real and imaginary parts of  $\underline{E}(x, z)$ , and related these to  $W_{cs}(s)$ . In fact Bowhill solved the problem for a two dimensional screen. If the one dimensional approximation used in this research is valid, it would be expected that the two dimensional solutions rapidly approach the one dimensional solutions as the scale size in one direction was lengthened with respect to the scale size in the other direction. Such is found to be the case. The two dimensional correlations of amplitude and phase for example, show equivalent scale sizes along the smaller direction which are within 12 percent of the one dimensional values at all distances from the screen, if the two dimensional pattern elongation is only four to one. This point is discussed further by Briggs and Parkin (1963), who also concluded that for elongated two dimensional patterns, the one dimensional approximation is valid, except in the direction of elongation.

The relations derived by Bowhill in one dimensional form are

$$W_a(s) = W_{cs}(s) \sin^2(\pi s^2 z / \lambda) \quad (3.14)$$

$$W_p(s) = W_{cs}(s) \cos^2(\pi s^2 z / \lambda) \quad . \quad (3.15)$$

where  $W_a(s)$  and  $W_p(s)$  are the power spectra of amplitude and phase fluctuations respectively. As in the case of the cosinusoidal screen, the trigonometric terms act as spatial filters on the shape of the spectra, and close to the screen the phase spectrum  $W_p(s)$  is a better representation of  $W_{cs}(s)$ , and consequently of the spectrum of the screen phase, than is the amplitude spectrum  $W_a(s)$ . This is illustrated in Figure 3.1 in which the normalized values of  $W_{cs}(s)$ ,  $W_a(s)$ , and  $W_p(s)$  are plotted versus  $s$  for a  $\lambda$  of seven meters, a distance from the screen of 250 kilometers, and an  $L_{c0}$  of 1.5 kilometers.

The actual forms of the amplitude and phase correlations at the ground may be found by taking the inverse Fourier transforms of  $W_a(s)$  and  $W_p(s)$ . Define the following quantities:

$$a = \lambda z / \pi L_c^2 \quad (3.16)$$

$$\beta = 1 + a^2 \quad (3.17)$$

$$\gamma = 1/2 \tan^{-1}(a) \quad . \quad (3.18)$$

Then  $S_a$  and  $S_p$  are given by

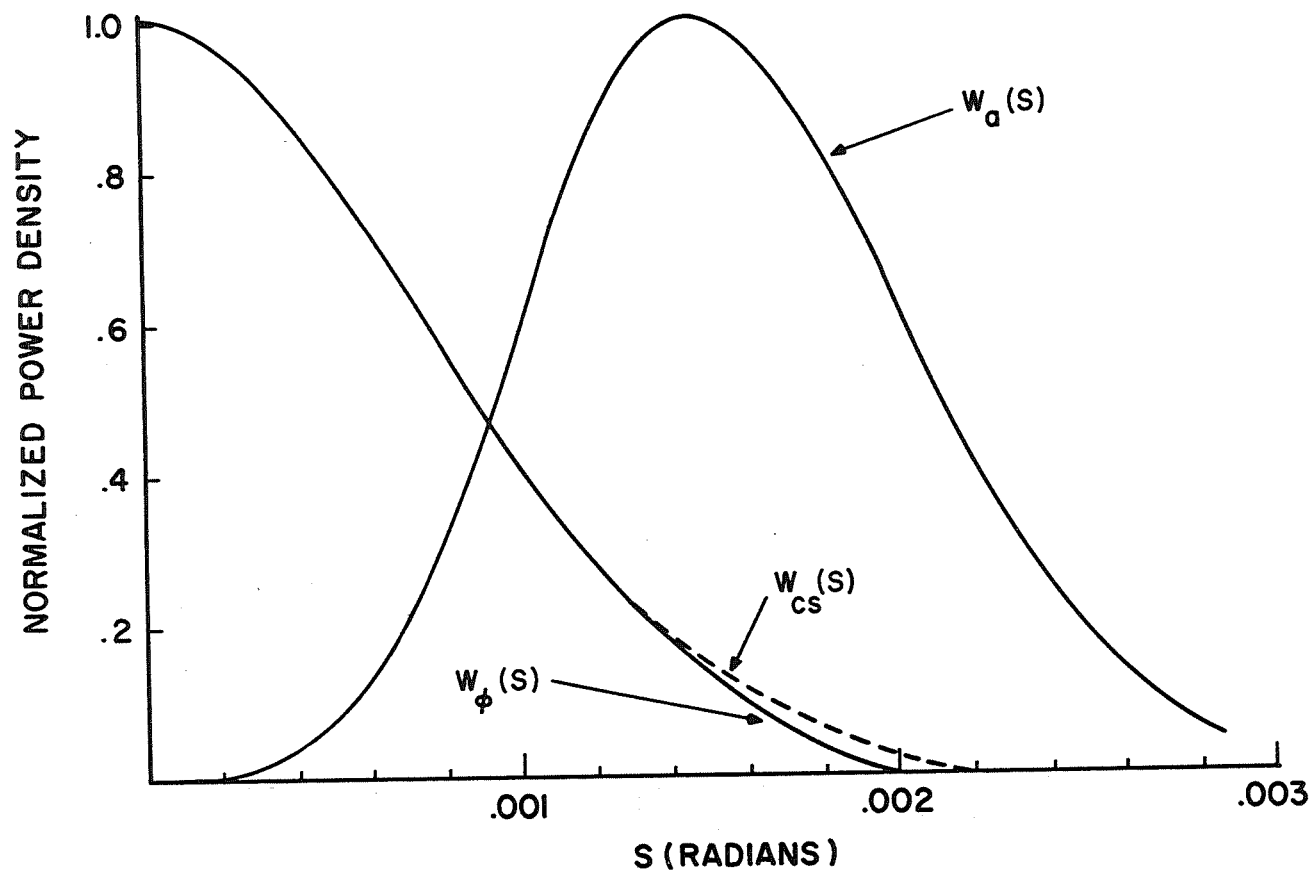


Figure 3.1. Amplitude and phase spectral shapes for small  $\phi_0$  with  $L_c = 1.5$  km,  $\lambda = 7$  meters, and  $z = 250$  km.

$$S_a = \frac{\phi_0^2}{2} (1 - \beta^{1/4} \cos \gamma)^{1/2} \quad (3.19)$$

$$S_p = [\phi_0^2 - S_a^2]^{1/2} . \quad (3.20)$$

Bowhill's (1961) results for  $\rho_a(\Delta x)$ , the correlation of amplitude, and  $\rho_p(\Delta x)$ , the correlation of phase, then become:

$$\begin{aligned} \rho_a(\Delta x) = \frac{\phi_0^2}{2S_a^2} \exp -[(\Delta x)^2/2L_{c_0}^2] - \beta^{-1/4} \exp [ \\ - (\Delta x)^2\beta/2L_{c_0}^2] \cos [a(\Delta x)^2\beta/2L_{c_0}^2 - \gamma] \end{aligned} \quad (3.21)$$

$$\begin{aligned} \rho_p(\Delta x) = \frac{\phi_0^2}{2S_p^2} \exp -[(\Delta x)^2/2L_{c_0}^2] + \beta^{-1/4} \exp - [ \\ (\Delta x)^2\beta/2L_{c_0}^2] \cos [a(\Delta x)^2\beta/2L_{c_0}^2 - \gamma] . \end{aligned} \quad (3.22)$$

Due to the filtering effect of the trigonometric terms on the spectra, it may be seen that  $\rho_a(\Delta x)$  and  $\rho_p(\Delta x)$  do not have the same form. For the sake of obtaining a measure of the relative sizes of both correlograms, Bowhill has suggested that the actual correlograms be fitted in the vicinity of their origins by Gaussian functions, whose standard deviations may be considered as being

equivalent scale sizes. In these terms, Bowhill has shown that the equivalent scale sizes of amplitude,  $L_{ca}$ , and of phase  $L_{cp}$  are related to  $L_{c_0}$  by

$$L_{ca} = L_{c_0} \left\{ \frac{1 + \beta^{-1/4} \cos \gamma}{1 + \beta^{-5/4} [\cos \gamma - a(\sin \gamma)]} \right\}^{1/2} \quad (3.23)$$

$$L_{cp} = L_{c_0} \left\{ \frac{1 - \beta^{-1/4} \cos \gamma}{1 + \beta^{-5/4} [\cos \gamma - a(\sin \gamma)]} \right\}^{1/2} \quad (3.24)$$

The ratios of  $L_{ca}$  and  $L_{cp}$  over  $L_{c_0}$  are plotted in Figure 3.2. As expected, for small or intermediate values of "a" the phase scale size is weighted somewhat toward larger spatial components, and the amplitude scale size is weighted very heavily toward smaller components of the spatial variation in the screen. It must be emphasized that the relations developed in this chapter hold only for screens which have truly Gaussian phase correlations for all values of  $\Delta x_0$ . Unless the actual form of the screen phase correlation is known a priori so that the relation between amplitude scale size and screen parameters can be computed, measurements on the amplitude pattern alone cannot be expected, in general, to yield valid information on the scale size of the screen variations. This implies that many of the measurements of irregularity parameters which have been limited to amplitude measurements, such as those discussed in Chapter VIII; may have limited value.

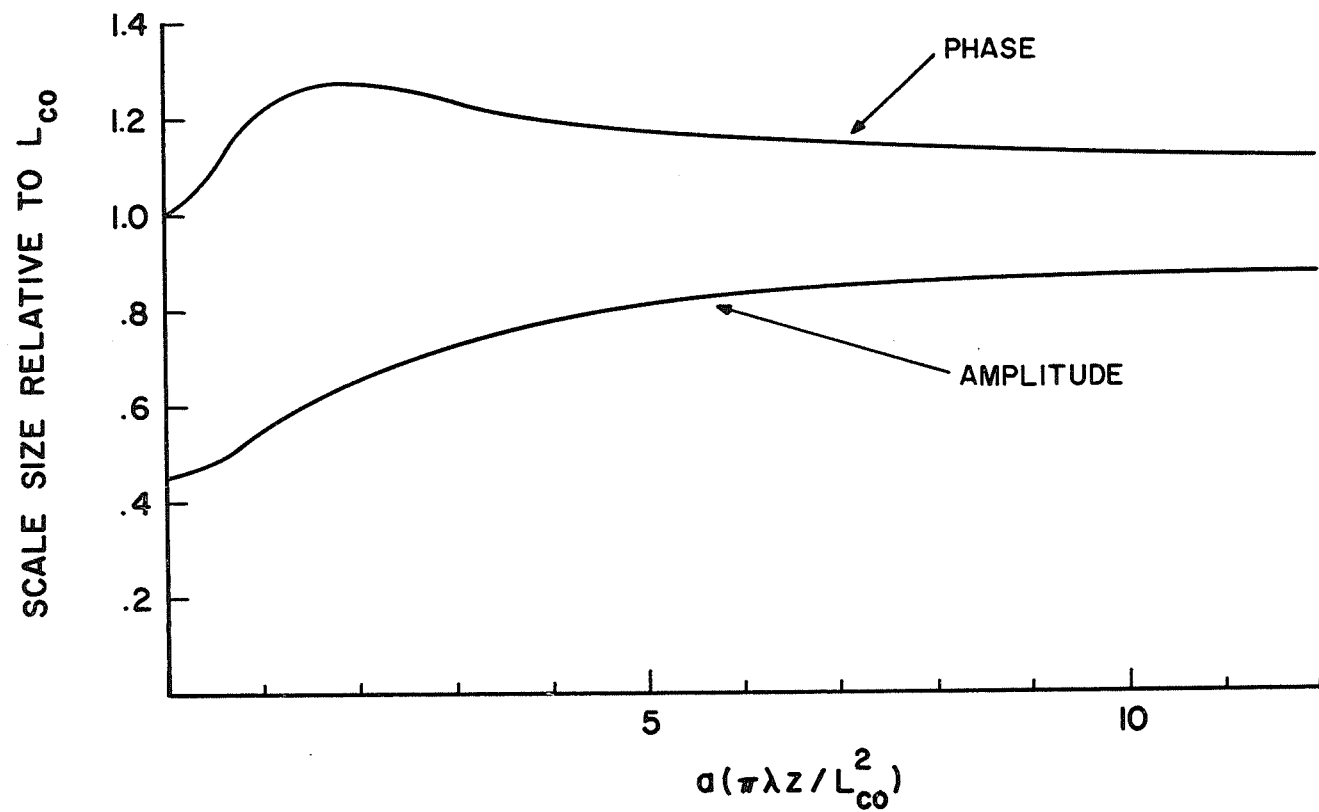


Figure 3.2. Amplitude and phase scale size reduction factor for small  $\phi_0$ .

It is interesting to expand equation 3.19 for small "a" and compare the value of  $S_a$  so obtained to that obtained for a cosinusoidal screen at small distances from the screen. For a random screen the result is

$$S_a^2 = \frac{3\phi_0^2}{8} (\lambda z / \pi L_{c_0}^2)^2 \quad (3.25)$$

Equation 2.32 yields for a cosinusoidal screen,

$$S_a^2 \approx \frac{\phi_0^2}{32} (\lambda z / \pi L_{c_0}^2)^2 \quad (3.26)$$

The difference between the factor of three eighths for a Gaussian screen as compared to one thirty-second for a cosinusoidal screen reflects the fact that a Gaussian screen has more high frequency components than a cosinusoidal screen of the same equivalent correlation scale size and so the pattern builds up faster.

The behavior of the Gaussian and cosinusoidal screens should also be examined at large distances from the screen. For a Gaussian screen with small  $\phi_0$ , both  $S_a^2$  and  $S_p^2$  approach a limiting value of  $\phi_0^2/2$  at large distances from the screen, while in the case of a cosinusoidal screen with small phase deviation,  $S_a^2$  and  $S_p^2$  continue to oscillate between 0 and  $\phi/2$  as  $z$  approaches infinity. This difference is due to the lack of coherence between scatters separated by large distances in a random screen, as compared to the regular behavior of widely separated scatters in a single component periodic screen.

### 3.4 The Case of Large $\phi_0$

#### 3.4.1 The Amplitude Pattern

Unfortunately for the case of large  $\phi_0$ , no simple identification can be made between the desired spectra of amplitude and phase, and the known spectrum  $W_c(s)$ . Since the screen phase deviation is random, the fields cannot be obtained explicitly as was the case for the deterministic cosinusoidal screen, and consequently only results in certain special cases are available for the general case of large  $\phi_0$ .

Mercier (1962) was able to obtain an expression for the correlation of square amplitude (intensity) of the diffracted field under the assumption of a Gaussian probability distribution of screen phase. He also assumed normal incidence, and a two dimensional correlation function for the screen phase deviations in which the contours of equal correlation were circles. Bramley (1967) extended this work to oblique incidence, and anisotropic correlation contours. Essentially the method employed consisted of expressing the electric field at the ground by the Kirchhoff integral, which may be shown to be identical to the integral which remains in the angular spectrum formulation after the integration on  $s$  only has been performed. The quantities  $\underline{E}(x, z)$ ,  $\underline{E}^*(x, z)$ ,  $\underline{E}(x + \Delta x, z)$ , and  $\underline{E}^*(x + \Delta x, z)$  may then be expressed as four integrals, and the correlation of square amplitude may be obtained by suitably

multiplying and averaging the product of the four integrals after suitable change of variable. In this formulation, the correlation of square amplitude is defined as

$$\rho_a^2(\Delta x) = \frac{\overline{\underline{E}(x, z) \underline{E}^*(x, z) \underline{E}(x + \Delta x, z) \underline{E}^*(x + \Delta x, z)}}{[\overline{\underline{E}(x, z) \underline{E}^*(x, z)}]^2} . \quad (3.27)$$

Unfortunately, the resulting integral obtained by Bramley can not be integrated analytically in general. If it is assumed that the correlation falls off with  $x_0$  with at least a Gaussian rate, and if the integral is evaluated only in the limit as  $z$  approaches infinite, the result becomes:

$$\rho_a^2(\Delta x, \infty) = \frac{\exp [2\phi_0^2 \rho_\phi(\Delta x, z = 0)] - 1}{\exp (2\phi_0^2) - 1} . \quad (3.28)$$

For large  $\phi_0$  and small  $\Delta x$ , the unity factors are much smaller than the exponentials and may be neglected. If equation 3.2 for  $\rho_\phi(\Delta x_0)$  is expanded into a Taylor series to second order in  $\Delta x_0$ , and the result substituted into equation 3.28, the result is

$$\rho_a^2(\Delta x, \infty) = \exp (-\Delta x \phi_0 / L_c)^2 \quad (3.29)$$

As  $z$  approaches infinity, the total field at a point in the ground is composed of a large number of components originating from points on the screen which are essentially uncorrelated. Under these conditions the spectrum is randomly phased in the sense of Ratcliffe (1956) and the situation is analogous to the problem in noise theory in which the amplitude of the sum of independent noise vectors is desired. Under these conditions it is expected that the probability distribution of the amplitude will be Rice Rayleigh in form, as has actually been established for the diffraction problem by Mercier (1962). The correlation of square amplitude,  $\rho_a^2(\Delta x, \infty)$  may then be shown to be approximately equal to the correlation of amplitude  $\rho_a(\Delta x, \infty)$  if the specular component of the diffracted field is small (Bramley, 1951). Since the specular component is given by the factor  $e^{-\phi_0^2} \delta(s)$  from equation 3.12, this condition is fulfilled for large  $\phi_0$ , and consequently, at least for small values of  $\Delta x$ ,  $\rho_a(\Delta x, \infty)$  is given by equation 3.29.

The correlation scale size for amplitude far from the screen is then seen to be approximately that of the phase at the screen divided by the factor  $2^{1/2}\phi_0$ . This relation, even though developed on the assumption of a random screen, yields the approximate order of magnitude of decrease of scale size with increasing mean square phase deviation as that displayed graphically in Figure 2.3 for a cosinusoidal screen. Furthermore, Figure 2.3 indicates that the amplitude scale size does not depend strongly on distance for moderate  $z$ , and this might be taken to imply that the reduction

factor developed on the random screen far field theory might be valid even at moderate distances from the screen. However, in an actual random screen, the filtering effect of the diffraction process on the scale sizes responsible for the development of the amplitude pattern must not be overlooked, and it is therefore likely that the actual reduction factor close to the screen is even greater than that obtained above.

Although Mercier (1962) and Bramley (1967) were not able to evaluate their solution analytically for small values of  $z$ , it is still possible that a numerical evaluation may be feasible. Mercier attempted a numerical evaluation with a Gaussian form of screen autocorrelation, but unfortunately the numerical methods employed broke down at a mean square phase deviation of only two radians. However, the results obtained showed no radical difference from the small phase shift formulation except that a small maximum in pattern intensity appeared at moderate values of distance from the screen, in the same way as found for a cosinusoidal screen. Mercier pointed out that this peaking effect was due to the fact that high phase deviations will produce a point of focus in the ray optical sense which is so close to the screen that the contributing area of the screen is not entirely incoherent and so constructive interference effects can occur.

As a check on the far field amplitude development, equation 3.28 may be examined for the case of small  $\phi_0$ . If  $\phi_0$  is small, a

Taylor series expansion of the exponentials in the numerator and denominator need only go to first order in  $\phi_0^2$  yielding

$$\rho_a^2(\Delta x, \infty) = \rho_\phi(\Delta x, z = 0) \quad (3.30)$$

For small  $\phi_0$ , Bramley (1950) shows that  $\rho_a^2(\Delta x, \infty)$  is again equal to  $\rho_a(\Delta x, \infty)$  so that in the far field  $\rho_a(\Delta x) = \rho_\phi(\Delta x, z = 0)$ , in accordance with equation 3.24.

### 3.4.2 The Phase Pattern

Unfortunately, results similar to those obtained for the amplitude pattern do not seem possible for the phase pattern when  $\phi_0$  is large. The definition of phase developed in Chapter II defines phase over the whole range of minus infinity to plus infinity, and not over the range minus  $\pi$  to plus  $\pi$  radians. As the point of observation moves away from the screen uncorrelated areas of the screen contribute to the pattern and in the far field the phase correlation scale becomes small. The probability distribution of the principal value of the phase becomes uniform over the interval minus  $\pi$  to plus  $\pi$ . However, the relation of the phase pattern and statistics when the phase is defined over an infinite interval to the pattern and statistics of the principal value of the phase is not clear when the amplitude of the pattern goes to zero, as occurs for values of  $x$  and  $z$  far from the screen.

If attention is given, however, to distances so close to the screen that ray optics holds, then the phase scintillation index and scale size will be reasonably close to the values at the screen itself, and the case of the phase pattern for large  $\phi_0$  can be treated similarly to the case of small  $\phi_0$ .

### 3.5 The Relation Between a Thick Region of Inhomogeneities and Infinitely Thin Phase Screen

Following the method of Bramley (1967) consider a plane wave incident on a slab of inhomogeneities in electron density bounded by the planes  $z = 0$ , and  $z = T$  defined by a cartesian coordinate system oriented with positive  $z$  facing down. Let the direction cosines of the incident wave normal be given by  $m_1, m_2, m_3$ . If the wave frequency is sufficiently high compared to the medium collision frequency, absorption in the medium can be ignored. If  $T$  is less than a critical distance which will be discussed later, ray optics can be assumed to hold, and the differential phase shifts along two parallel paths through the medium are given by

$$\phi(p_1) = k \int_0^{p_0} \mu(p) dp \quad (3.31)$$

$$\phi(p_2) = k \int_0^{p_0} \mu(p) dp \quad (3.32)$$

where  $p_1$  and  $p_2$  denote the two paths,  $p_0$  is the total path length,  $\mu(p_1)$  and  $\mu(p_2)$  are the perturbations in the refractive index along the two paths, and  $k$  is the propagation constant  $2\pi/\lambda$ .

The autocovariance of phase then becomes

$$\begin{aligned} \overline{\phi(p_1)\phi(p_2)} &= k^2 \int_0^{p_0} \mu(p_1) dp_1 \int_0^{p_0} \mu(p_2) dp_2 \\ &= k^2 \int_0^{p_0} \int_0^{p_0} \overline{\mu(p_1) \mu(p_2)} dp_1 dp_2 . \end{aligned} \quad (3.33)$$

The correlation of the refractive index variations may be defined as

$$\rho_{\mu}(p_1 - p_2) = \overline{\mu(p_1) \mu(p_2)} / \mu_0^2 \quad (3.34)$$

where  $\mu_0$  is the root mean square value of the refractive index variations. If a formal change of variable is performed such that

$$Q_1 = p_1 + p_2 \quad (3.35)$$

$$Q_2 = p_1 - p_2 \quad (3.36)$$

and the resulting Jacobian is evaluated, then equation 3.33 may be expressed in terms of  $Q_1$  and  $Q_2$ . Substituting the value for  $\rho_{\mu}(p_1 - p_2)$  from equation 3.34 into the resulting expression, and recognizing that  $\rho_{\mu}(p_1 - p_2)$  is an even function of  $Q_2$  yields

$$\rho_{\mu}(Q_2) = k^2 \int_{Q_2=0}^{p_0} \int_{Q_1=2Q_2}^{2p_0} \mu_0^2 \rho_{\mu}(Q_2) dQ_1 dQ_2 . \quad (3.37)$$

If the irregularity distribution with height is constant over a distance large in comparison to the correlation scale in the direction of propagation,  $\mu_0^2$  may be replaced with its average value and factored out of the integral. Since the remaining integrand does not depend on  $Q_1$ , and the  $Q_1$  integration may be carried out directly, yielding

$$\overline{\phi(p_1)\phi(p_2)} = 2k^2 \mu_0^2 \int_0^{p_0} \rho_\mu(Q_2)(p_0 - Q_2) dQ_2. \quad (3.38)$$

If the path length  $p_0$  is much greater than the correlation scale in the direction of propagation,  $\rho_\mu(Q_2)$  will approach zero small while  $Q_2$  is still small compared to  $p_0$ . Consequently, the  $(p_0 - Q_2)$  term may be replaced by  $p_0$  and factored out of the integral. For the same reason, the limits of integration may be replaced by 0 and  $\infty$ , and since the integral is even,

$$\overline{\phi(p_1)\phi(p_2)} = k^2 p_0 \mu_0^2 \int_{-\infty}^{\infty} \rho_\mu(Q_2) dQ_2. \quad (3.39)$$

$\overline{\phi(p_1)\phi(p_2)}$  will normally be evaluated in a plane of constant  $z$ , and therefore equation 3.39 states that autocovariance of phase at the base of the layer may be found simply by integrating the autocorrelation of electron density variations along straight lines (in the ray optical approximation) through the layer. Thus, the irregularities in the layer may be imagined to be optically projected onto a plane at the base of the layer.

The correlation of refractive index variations will be assumed to be an ellipsoid of revolution with the long axis along the Earth's magnetic field. Along any radius the correlation will be taken as Gaussian with scale size  $L_{c_0}$  in a direction transverse to the field, and  $L_{c_0}/h$  along the field line. For convenience the magnetic field lines will be taken as lying in the  $y, z$  plane with dip angle  $I$ .  $\rho_\mu$  will then have the form with respect to the  $x, y, z$  axis.

$$\begin{aligned} \rho_\mu(x, y, z) = \exp \{ -1/2 L_{c_0}^{-2} [x^2 + y^2 (\sin^2 I + h^2 \cos^2 I) \\ + z^2 (\cos I + h \sin I)^{-2} (1-h)^2 \sin^2 I \\ \cos I yz] \} \end{aligned} \quad (3.40)$$

Since  $p_1$  and  $p_2$  are parallel, if the distance between  $p_1$  and  $p_2$  in the plane  $x, y, T$  has components  $\Delta x, \Delta y$ , the coordinates of a point on  $p_2$  relative to those on  $p_1$  will have components  $m_1 Q_2 + \Delta x, m_2 Q_2 + \Delta y, m_3 Q_2$ . In terms of  $Q_2$ ,  $\rho_\mu(x, y, z)$  then becomes

$$\begin{aligned} \rho_\mu(Q_2) = \exp \{ -.5 L_{c_0}^{-2} [(m_1 Q_2 + \Delta x)^2 + (m_2 Q_2 + \Delta y)^2 F_1 \\ + (m_2 Q_2 + \Delta y)(m_3 Q_2 F_2) + m_3^2 Q_2^2 F_3] \} \end{aligned}$$

where

$$\begin{aligned}
 F_1 &= \sin^2 I + h^2 \cos^2 I \\
 F_2 &= -2 (1-h^2) \sin I \cos I \\
 F_3 &= \cos^2 I + h^2 \sin^2 I
 \end{aligned} \tag{3.41}$$

Expanding equation 3.41, substituting the result into equation 3.37 and integrating yields

$$\begin{aligned}
 \phi(p_1) \phi(p_2) &= \frac{4\pi^2 (2\pi)^1 {}^2 L_{c_0} p_0 \mu^2}{\lambda^2 {}_1 {}_2} \{ \exp -1/(2F_7 L_{c_0}^2) [ \\
 &F_4 (\Delta x)^2 + F_5 (\Delta y)^2 + F_6 (\Delta x)(\Delta y)] \}
 \end{aligned}$$

where

$$\begin{aligned}
 F_4 &= 1 - m_1^2/F_7 \\
 F_5 &= F_1^2 - (m_2^2 F_1^2 + m_2 m_3 F_1 F_2 + m_3^2 F_2^2/4)/F_7 \\
 F_6 &= (m_1 m_2 F_1 + .5 m_1 m_3 F_2)/F_7 \\
 F_7 &= m_1^2 + m_2^2 F_1^2 + m_3 F_3 + m_2 m_3 F_2
 \end{aligned} \tag{3.42}$$

The exponential factor is seen to be the correlation of phase in the base plane  $x, y, T$ , while the coefficient of the exponential gives the mean square phase deviation. These expressions do not agree with those in Bramley's work, which seem to be in error. If equation 3.42 is examined, it is seen that the correlation of phase in the base plane is Gaussian, and has contours of equal correlation which are elliptical. For angles of incidence not too close to the magnetic field direction, the ellipse will be elongated, with major axis in the approximate direction of the plane of incidence.

If the magnetic field is assumed to be completely vertical, the declination of the field will be zero, and the  $x, y, z$  frame may be rotated so that the  $y$  axis lies in the direction of incidence. Then " $m_1$ " is zero, and equation 3.42 reduces to

$$\frac{\phi(p_1)\phi(p_2)}{\lambda^2(\sin^2\theta_z + h^2\cos^2\theta_z)^{1/2}} = \frac{4\pi(2\pi)^{1/2}T\sec\theta_z\mu_0^2L_{c0}}{\lambda^2(\sin^2\theta_z + h^2\cos^2\theta_z)^{1/2}} \exp\{-.5[(\Delta x/L_{c0})^2 + (\Delta y h/L_{c0})^2(h^2 + \tan^2\theta_z)]\} \quad (3.43)$$

where  $\theta_z$  is the zenith angle of the incident ray. This expression is similar to one obtained by DeBarber (1962) for this special case.

It is therefore seen that if ray optics can be applied over the thickness of the inhomogeneity region, the thick layer can be replaced by a thin screen located at the plane  $x, y, T$ . The correlation of phase in this screen will be the geometrical

projection of the three dimensional correlation of refractive index variations, and will be elongated in the approximate direction of the plane of incidence if the original irregularities were elongated, and if the incident ray is not nearly parallel to the magnetic field direction. Furthermore, if the elongation is great,  $h$  will approach zero and the contours of constant correlation will quickly approach straight lines except along directions nearly parallel with the  $y$  axis, as can be seen from equation 3.43.

For example, for a zenith angle of 45 degrees or less, the error introduced by using the straight line approximation for the contours of constant phase is less than four percent for a four to one elongation along points lying within 45 degrees of the  $x$  axis, and the error increases to only 20 percent for points 70 degrees from the  $x$  axis. This is a further justification for using a one dimensional diffraction theory to investigate diffraction effects in directions transverse to that of pattern elongation.

### 3.6 The Limit of the Ray Optical Region

If the thickness of the inhomogeneity region in the direction of propagation is less than the distance defining the limit of validity the ray optical approximation, the thick region can be collapsed into an equivalent thin screen. If the distance from the top of the irregularity region to the ground is less than this critical distance, ray optics can be applied over the whole range from the top of the irregularity layer to the ground. To find this

critical distance in analogy with the method used in section 2.2.3, the scattering angle for a random screen is needed.

By employing ray optics, Beckmann (1965) has shown that the angular deflection of the incident ray has mean square value  $\theta_s^2$  where

$$\theta_s^2 = \frac{\lambda^2 \phi_0^2}{4\pi^2 L_c^2} \quad (3.44)$$

where  $L_c$  is the scale size in the direction transverse to that of propagation. Since the smallest scale size will yield the largest scattering angle and consequently the smallest limit on the distance over which the ray optical limit holds,  $L_{c_0}$  can be substituted for  $L_c$  without adversely effecting the estimation of the critical distance. The root mean square beam deflection must be less than  $L_{c_0}$  or for the incident wave with zenith angle  $\theta_z$

$$z < \cos \theta_z \frac{2\pi L_{c_0}^2}{\lambda \phi_0} \quad (3.45)$$

It can be seen that this limit is more conservative than the one obtained from that for a cosinusoidal screen given in equation 2.26 with  $L_s$  replaced by  $2\pi L_c$ . This is due to the fact that the ray displacement here has been required to be small compared to  $L_c$ , not  $L_s$ .

These calculations have assumed plane wave illumination. For the satellite at a finite distance above the irregularity layer,

section 2.5 shows that irregularities at different heights within the layer will project to the ground with different point source magnifications. If the irregularity height distribution is symmetrical about some average value, the larger projections of the irregularities high in the layer would be expected to be offset by the smaller projections of irregularities low in the layer. This suggests that an adequate description of the point source effects can be obtained by applying the point source correction calculated at the average height of the layer to the layer description obtained from the plane wave analysis.

Numerical calculations for a satellite at 1000 km, an average layer height of 500 km, and ionospheric scale sizes of .25 to 4 km have been performed to determine the error involved in this approximation. They have shown that for a uniform irregularity distribution with height, the difference between the actual phase autocorrelation on the ground, and the Gaussian one obtained by applying the point source magnification evaluated at the middle of the layer is less than three percent for all values of correlation greater than one third for layer thicknesses up to 300 km. Similar calculations on layers with Gaussian height distributions with standard deviations of up to 75 km yielded errors of less than seven percent over the same range of correlations. Since this approximate method preserves the ease of physical interpretation of the ray optical formulation,

since no adequate description of the actual irregularity height distribution is available, and since the errors should be small in any case, this approximate procedure will therefore be adopted.

The limit of the ray optical region then becomes

$$z \ll 2\pi \cos \theta_z L_{c_0}^2 (z_i + z_0) / (\lambda z_0 \phi_0) \quad (3.46)$$

where  $z_i$  and  $z_0$  are the distances from the ground to the average height of the irregularity layer, and from the average height of the irregularity layer to the satellite, respectively. Taking typical values of 45 degrees for  $\theta_z$ , 1.5 radians for  $\phi_0$ , 500 km for  $z_i$  and  $z_0$ , 2 km for  $L_{c_0}$ , and 7 meters for  $\lambda$ , yields a limit on  $z$  of approximately 3300 km. Since this distance is much greater than that from the top of the irregularity layer to the ground, it would seem that ray optics should hold, at least as far as a description of the phase pattern is concerned, for all but the smallest, most dense, irregularities.

Feinstein (1954) has shown that  $W_c(s)$  for a thick inhomogeneity region is given by the thin screen expression, equation 3.10, where  $\rho_\phi(\Delta x_0)$  is found by integrating the electron density autocorrelation along straight lines in the medium as given in the exponential term in equation 3.42. He has shown that this straight line integration is correct even if the pertinent irregularities are so small that ray optics cannot be applied throughout the whole layer. Since  $W_c(s)$  therefore has the same form for a thick region

as for a thin screen, the far field expressions for the amplitude pattern discussed in section 2.4.1 are also correct for the pattern caused by a thick inhomogeneity layer. Since a knowledge of  $W_c(s)$  still does not yield information on the manner or rate at which the amplitude pattern builds up within or close to the inhomogeneity layer, a complete solution of the three dimensional diffraction problem is necessary to adequately treat the near field amplitude pattern.

#### IV. THE MEASUREMENT OF IRREGULARITY PARAMETERS

##### 4.1 Introduction

As the satellite moves, the diffraction pattern on the ground will translate and rotate analogously to the manner in which a sundial gnomon's shadow moves due to the motion of the sun, an analogy which seems to have first appeared in the literature in the paper of McClure and Swenson (1964). Without a prohibitive number of ground stations, it is impossible to obtain the complete spatial characteristics of the pattern on the ground at any one instant of time. However, by measuring the time variations of the pattern at a limited number of closely spaced stations, the average size and height of the irregularities may still be obtained if the drifting pattern does not change too much as it drifts from one station to another. It is convenient to treat the amplitude and phase patterns separately.

##### 4.2 The Average Height of the Irregularity Region as Deduced From Amplitude Measurements

Over short time intervals the motion of the ground pattern will be essentially translational, and a measurement of this translational velocity can yield a measure of the average height of the irregularity region. As shown in Figure 4.1, a thin

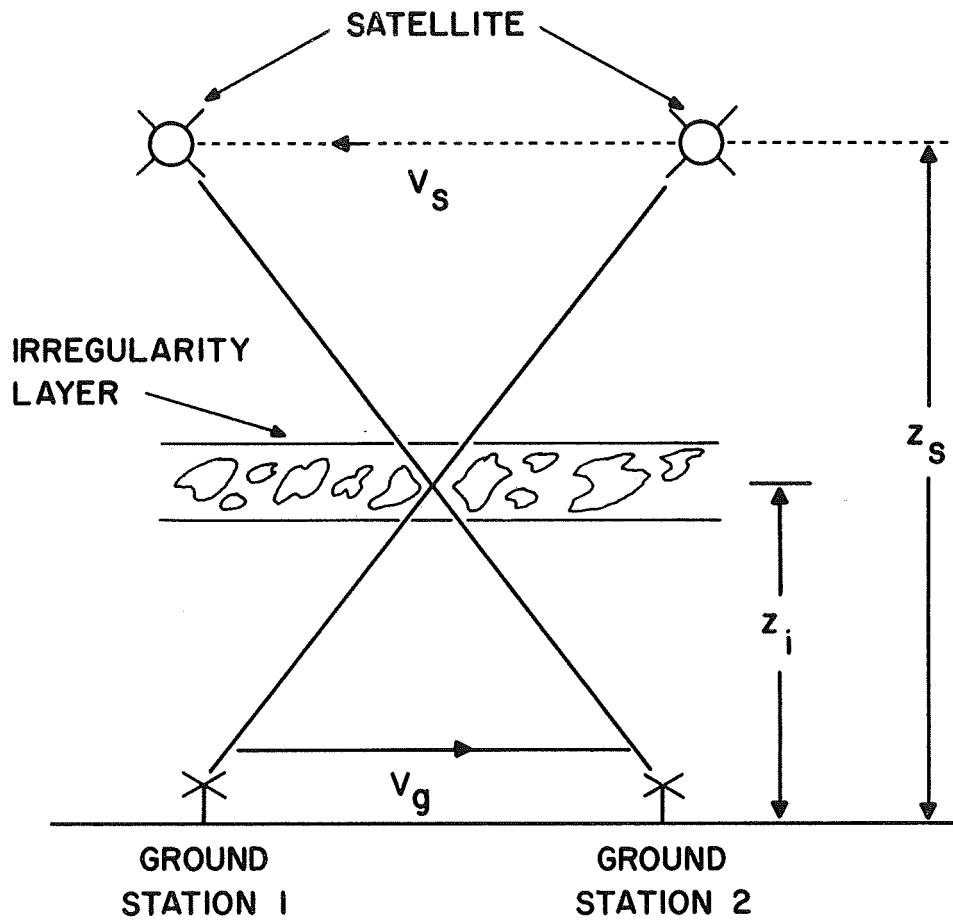


Figure 4.1. Geometry of moving pattern.

irregularity layer at height  $z_i$  produces a pattern on the ground which moves with velocity  $V_g$  such that

$$-V_g/z_i = V_s/(z_s - z_i) \quad (4.1)$$

where  $z_s$  is the satellite height and  $V_s$  is the satellite velocity.

If the satellite tangential velocity is not horizontal,  $\underline{V}_s$  must be replaced with

$$\underline{V}_s' = \underline{V}_t [\cos \gamma_s \pm \tan \gamma_s / \sin \gamma_s] \quad (4.2)$$

where  $V_t$  is the magnitude of the tangential satellite velocity and  $\gamma_s$  is the angle between the tangential velocity and the horizontal. For a satellite in a near polar orbit, the plus sign is employed for northbound passes when the satellite is south of the observing station, and vice versa. It is also necessary to replace  $z_s$  by  $z_s'$ , the average value of  $z_s$  during the measurement period. In these terms

$$z_i = -V_g z_s' / (V_s' - V_g) \quad (4.3)$$

The methods which will presently be described to obtain  $V_g$  essentially determine the speed of the pattern on the ground by measurement of the times of maximum crosscorrelation of the amplitude

patterns observed at receiving stations spaced a known distance apart on the ground. In the case of an irregularity layer of appreciable thickness, these methods will therefore yield ground velocities, and consequently heights, which correspond to the condition of maximum cross correlation between the integrated irregularity distributions throughout the irregularity layer.

Since the irregularity region contributing to the ground pattern is limited in transverse extent, maximum pattern cross-correlation will occur when the contributing regions share the greatest common volume, and consequently for irregularity height distributions symmetrical about some common height, the cross-correlation methods will yield this average height. For nonsymmetrical height distributions, however, the height of the layer found by crosscorrelation techniques will be skewed toward the region of higher irregularity density. Therefore, a change in height as indicated by equation 4.3 may simply be indicative of a change in the irregularity height distribution and cannot a priori be interpreted as a shift in the height of the whole layer.

The plane Earth heights obtained from equation 4.3 may be converted into heights above the Earth's surface where

$$h_i = (z_r + z_i) \cos \gamma_c - z_r \quad (4.4)$$

where  $h_i$  is the geocentric height,  $z_r$  the radius of the Earth

and  $\gamma_c$  is the angle at the Earth's center subtended by the satellite and the point of observation, and where it has been assumed that the Earth is spherical.

#### 4.3 The Determination of Amplitude Pattern Parameters by the Full Correlation Analysis Method

Briggs, Phillips, and Shinn (1950) were the first to develop a general method for treating moving amplitude diffraction patterns. They assumed that the ground amplitude pattern could be represented by a correlation function whose equicorrelation contours formed an ellipsoid of revolution in three dimensions  $x$ ,  $y$ , and  $t$ , where  $x$  and  $y$  represented the ground plane coordinates, and  $t$  represented time. They also assumed that the temporal variations had the same functional form as the spatial variations. This would be expected, for example, if the pattern motion consisted only of a steady drift. Briggs, Phillips, and Shinn also allowed for the presence of random motion in the pattern which they characterized by a velocity  $V_c$  defined as the scale size of the pattern on the ground divided by the lifetime of features of such size. For  $V_c$  non zero, the assumption that the temporal and spatial characteristics have the same form seems doubtful, but experiments by Brennan (1960) and Kelleher (1966) showed that this condition is approximately fulfilled for patterns in which  $V_c$  is less than drift velocity.

The Briggs, Phillips, and Shinn method is essentially graphical. Kent and Koester (1966) have developed a completely

analytical technique based on their method. They assumed that the equicorrelation contours form an ellipsoid of revolution as before, and further require that the correlation fall off at a Gaussian rate with distance in any direction. Again, the Gaussian assumption is not justified physically, but the experiments of Awe (1964b), and our own results show that this is a good approximation for values of correlation higher than 0.5. With this assumption, the amplitude correlation in terms of distance on the ground and time is given by

$$\rho_a(\Delta x, \Delta y, t) = \exp \{-1/2[A\Delta x^2 + 2H\Delta x\Delta y + B\Delta y^2 + 2F\Delta xt + 2G\Delta yt + Ct^2]\} \quad (4.5)$$

If three noncolinear stations are established on the ground, then three records of the variation of received amplitude with time may be obtained. From these records, it is possible to obtain three crosscorrelation functions and one average autocorrelation function. These four functions are sufficient to determine the coefficients of equation 4.5 and from these coefficients values for the pattern size, orientation, drift velocity, and random velocity may be obtained. To this end a value of correlation is picked which is slightly below the lowest peak value of the three crosscorrelation functions. The two times at which the cross-correlation between ground stations *i* and *j* reaches this value will

be denoted as  $t_{ij}$  and  $t_{ij}'$  respectively. The time that the average autocorrelation reaches this value will be denoted as  $t_0$ , and the distance between the  $i$ th and  $j$ th ground stations will be denoted as  $d_{ij}$ .

Define  $a_{ij}$ ,  $b_{ij}$ , and  $[k]$  as

$$a_{ij} = (1/d_{ij})(1 + t_{ij}t_{ij}'/t_0^2) \quad (4.6)$$

$$b_{ij} = -(t_{ij} + t_{ij}')/(2t_0^2d_{ij}) \quad (4.7)$$

$$[k] = \begin{bmatrix} \cos^2 \theta_{12} & \sin \theta_{12} & \cos \theta_{12} & \sin^2 \theta_{12} \\ \cos^2 \theta_{13} & \sin \theta_{13} & \cos \theta_{13} & \sin^2 \theta_{13} \\ \cos^2 \theta_{23} & \sin \theta_{23} & \cos \theta_{23} & \sin^2 \theta_{23} \end{bmatrix}$$

where  $\theta_{ij}$  is the angle between local east and the line joining the  $i$ th and  $j$ th stations. Kent and Koester show that the coefficients of equation 4.5 are then given by

$$\begin{bmatrix} A \\ 2H \\ B \end{bmatrix} = \begin{bmatrix} k \end{bmatrix}^{-1} \begin{bmatrix} a_{12} \\ a_{13} \\ a_{23} \end{bmatrix} \quad (4.9)$$

$$F = (h_{12} \sin \theta_{13} - h_{13} \sin \theta_{12}) / \sin (\theta_{13} - \theta_{12}) \quad (4.10)$$

$$G = (h_{12} \cos \theta_{13} - h_{13} \cos \theta_{12}) / \sin (\theta_{13} - \theta_{12}) \quad (4.11)$$

$$C = 1/t_0 \quad (4.12)$$

The axes of the ellipse on the ground are given by

$$Ax_1 = 2^{1/2} \{A + B + [(A-B)^2 + 4H^2]^{1/2}\}^{1/2} \quad (4.13)$$

$$Ax_2 = 2^{1/2} \{A + B - [(A-B)^2 + 4H^2]^{1/2}\}^{1/2} \quad (4.14)$$

The orientation of the major axis,  $\Omega_a$ , is given by

$$\Omega_a = 1/2 \tan^{-1} [2H/(A-B)] \quad (4.15)$$

The magnitude,  $V_g$ , and direction,  $\xi_g$ , of the drift velocity are given by

$$V_g = \{(BF-HG)^2 + (AG-HF)^2\}^{1/2} / (AB-H^2) \quad (4.16)$$

$$\xi_g = \tan^{-1} \{(AG-HF)/(BF-HG)\} \quad (4.17)$$

An approximate value for the component of drift velocity in the  $(90 - \Omega_a)$  direction is

$$V_{gm} = 1/2 [G/(B^2 + H^2)^{1/2} + F/(A^2 + H^2)]^{1/2} \quad (4.18)$$

The value of  $V_c$  along the  $\xi_g$  direction is given by

$$V_c = V_g \left\{ \frac{C(AB-H^2) - (AG^2-2FGH + BF^2)}{AG^2-2FGH+BF^2} \right\}^{1/2} \quad (4.19)$$

Kent and Koester also showed that the apparent value of the ratio  $V_c/V_g$  as measured in any direction other than that of pattern drift is larger than the actual value of  $V_c/V_g$  as measured in the  $\xi_g$  direction. In particular, if  $V_c/V_g$  is evaluated along a baseline direction,

$$V_c/V_g \text{ apparent} = t_{00}/t_{ij}'' \quad (4.20)$$

where  $t_{ij}''$  is the time lag necessary for maximum crosscorrelation between the  $i$ th and  $j$ th stations, and  $t_{00}$  is the time necessary for the average autocorrelation function to fall to the value of the crosscorrelation at its maximum.

#### 4.4 The Relation between $V_c$ and the Thickness of the Irregularity Region

If the region containing the irregularities has appreciable vertical extent, it can conceptually be subdivided into many thin regions. Each region will create a pattern which drifts across the ground at a different velocity in accordance with equation 4.1, and so the form of the ground pattern will gradually change in form. By assuming that the effects of the individual regions can be superimposed, and by assuming a vertical irregularity distribution of a Gaussian form, James (1962) has shown that

$$\Delta_t = \frac{V_c (z_s - z_i) z_i}{V_g z_i} \quad (4.21)$$

where  $2^{1/2} \Delta_t$  is the standard deviation of the layer height distribution. McClure and Swenson (1964) have also shown that a similar relation holds for a layer with a uniform height distribution of thickness  $2\pi^{1/2}$ .

Due to the assumption that superposition of the individual regions is valid, such an analysis cannot be used for the amplitude pattern when the mean square ionospheric phase deviation is large. However, as long as the ray optical approximation is valid, the James result is entirely applicable for arbitrary phase deviations to measurements of the phase pattern. Unfortunately, no measure of  $V_c$  is directly obtained from phase difference measurements. If

it is assumed that pattern lifetime is proportional to pattern scale size, then the value of  $V_c$  obtained from a phase analysis should agree with that obtained from the corresponding amplitude analysis if it is assumed that the same irregularities are responsible for both patterns.

Although this proportionality cannot be demonstrated rigorously, it is physically reasonable since any reduction in the apparent scale size of the diffraction pattern due to multiple scattering will produce a pattern which is very sensitive to satellite position. Similarly, any reduction in the pattern size due to the spatial filtering effect of the diffraction process will result in a pattern which is predominately caused by smaller irregularities and these irregularities will be part of the common scattering geometry over a smaller range of satellite positions than larger irregularities, and so the resulting pattern will have a correspondingly shorter lifetime. It should be noted here that Fooks (1965) has also found it necessary to postulate that for a statistically stationary pattern both the pattern size and lifetime would be altered by the same factor in the diffraction process in his discussion of the physical interpretation of  $V_c$ .

In this work, it will therefore be assumed that equation 4.21 holds at least approximately for large as well as small values of  $\phi_0$ . If this is not true, the values of layer thickness so derived will still be of value for comparison to values similarly obtained by others without consideration of the value of phase deviation actually present.

#### 4.5 The Phase Pattern on the Ground

Since the exact phase of the signal transmitted from the satellite is unknown, it is difficult to obtain direct measurements of the phase pattern on the ground. However, it is possible to measure differences of phase. The covariance of the phase difference between a station located at  $x, y$ , and a station at coordinates  $x + d_x, y + d_y$  is

$$\overline{\phi_d(x, y) \phi_d(x + \Delta x, y + \Delta y)} = \overline{[\phi(x + d_x, y + d_y) - \phi(x, y)] [\phi(x + d_x + \Delta x, y + d_y + \Delta y) - \phi(x + \Delta x, y + \Delta y)]}.$$

(4.22)

Expanding the product, and replacing the covariances of phase with the product of mean square phase deviation times correlation yields for the mean square value of phase difference  $\phi_d^2$ , and for the value of phase difference correlation  $\rho_{\phi d}(\Delta x, \Delta y)$

$$\phi_d^2 = 2\phi_0^2 [1 - \rho_{\phi}(d_x, d_y)] \quad (4.23)$$

$$\rho_{\phi d} = [2\rho_{\phi}(\Delta x, \Delta y) - \rho_{\phi}(\Delta x + d_x, \Delta y + d_y)$$

$$- \rho_{\phi}(\Delta x - d_x, \Delta y - d_y)] \phi_0^2 / \phi_d^2. \quad (4.24)$$

The relation between these quantities and those actually observed for a moving pattern is complicated by the two dimensional nature of the expressions. Since measurements show that the ground pattern is essentially one dimensional, more explicit expressions for the case of a moving pattern will be deferred to the section on the one dimensional phase pattern characteristics.

#### 4.6 The One Dimensional Amplitude Pattern

If the actual pattern elongation is great, the calculations may be simplified by considering a one dimensional model. If the station spacing is close enough so that the pattern does not change appreciably in moving from one station to another, then random changes can be ignored and the motion can be considered to be purely translational. The patterns at the two stations are then similar and the times of maximum pattern crosscorrelation can be determined by eye. It can be shown that the locus of such points of maximum crosscorrelation form a straight line known as the line of maximums. This line has been discussed more fully by Sales (1956), and is determined by the points of tangency between projections of the station positions in the direction of pattern drift, and the family of ellipses of constant correlation as shown in Figure 4.2. For highly elliptical contours, the line of maximums will lie very close to the major axis of the pattern unless the direction of drift velocity is nearly parallel to the major axis.

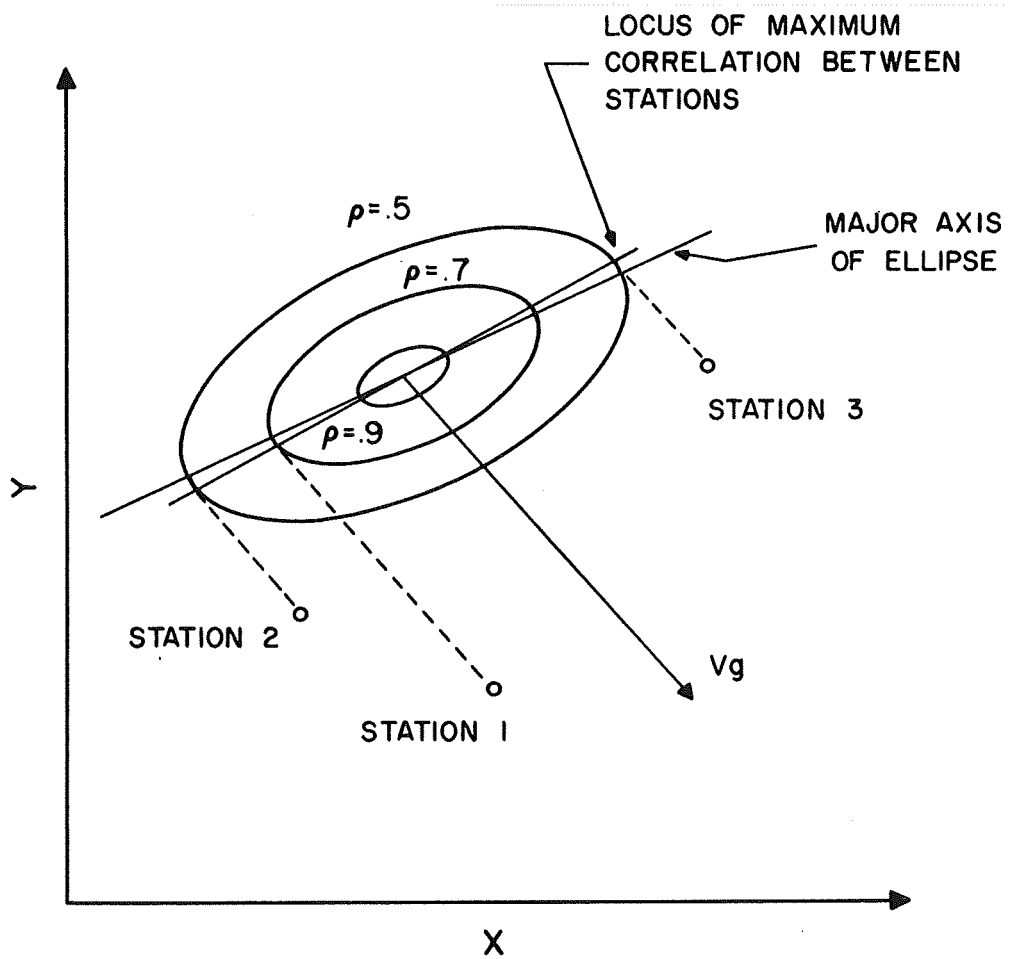


Figure 4.2. Geometry of line maximums.

In general, three stations are necessary to determine the magnitude of the pattern drift velocity and the orientation of the line of maximums. The method employed is essentially the so called method of similar fades of Mitra (1949), as developed by DeBarber (1962).

Define the following quantities with respect to an x, y frame tangent to the Earth's surface at receiving location one:

$$\underline{d}_{ij} = d_{ijx} \underline{x} + d_{ijy} \underline{y} \quad (4.25)$$

$$\underline{V}_g = V_{gx} \underline{x} + V_{gy} \underline{y} \quad (4.26)$$

$$\underline{V}_s = V_{sx} \underline{x} + V_{sy} \underline{y} \quad (4.27)$$

$$\underline{S} = S_x \underline{x} + S_y \underline{y} \quad (4.28)$$

where  $\underline{x}$  and  $\underline{y}$  are unit vectors and  $\underline{d}_{ij}$ ,  $\underline{V}_g$ ,  $\underline{V}_s$ , and  $\underline{S}$  are vectors representing the distance on the ground between stations i and j, the pattern ground velocity, the satellite horizontal velocity, and the line of maximums respectively. Define the time of maximum crosscorrelation in amplitude between the patterns at station i and j as  $t_{ij}$  as in equation 4.20. Define the orientations of  $\underline{S}$  and  $\underline{V}_g$  with respect to the x axis as  $\Omega_a$  and  $\xi_g$ . Then

$$\Omega_a = \arctan (S_y/S_x) \quad (4.29)$$

$$\xi_g = \arctan (v_{gy}/v_{gx}) = \arctan (v_{sy}/v_{sx}) \quad (4.30)$$

DeBarber then shows that

$$t_{ij}'' = [d_{ijy} - S_x/S_y d_{ijx}] / [v_g (\cos \xi_g - S_y/S_x \sin \xi_g)] \quad (4.31)$$

If the times of maximum crosscorrelation are determined between stations one and two and one and three, the resulting two equations can be solved for  $v_g$  and  $\Omega_a$  in terms of  $\xi_g$  which is known from satellite orbital data yielding

$$\Omega_a = \arctan \{ (d_{13x} t_{12}'' - d_{12x} t_{13}'') / (d_{13y} t_{12}'' - d_{12y} t_{13}'') \} \quad (4.32)$$

$$v_g = (d_{12y} - d_{12x} \cotan \Omega_a) / (\cos \xi_g - \cotan \Omega_a \sin \xi_g) \quad (4.33)$$

Equations 4.3 and 4.4 can then be used to find the irregularity height, since the satellite height and speed are known from orbital data.

If the autocorrelation of amplitude at a single receiving station is computed versus time, the equivalent autocorrelation versus distance may be obtained by multiplying the time scale by  $V_g$ . Denoting the correlation scale of this autocorrelgram as  $L_{ca}$ , simple geometry shows that the corresponding scale size,  $L_{cag}$ , of the minor axis of the two dimensional ellipse on the ground is

$$L_{cag} = [L_{ca} / \epsilon] [\epsilon^2 \sin^2(\xi_g - \Omega_a) + \cos^2(\xi_g - \Omega_a)]^{1/2} \quad (4.34)$$

where  $\xi$  is its ellipticity. As  $\xi$  approaches infinity, the approximate one dimensional relation becomes

$$L_{cag} = L_{ca} \left| \sin(\xi_g - \Omega_a) \right| \quad (4.35)$$

#### 4.7 The One Dimensional Phase Pattern

In the case of a one dimensional phase pattern, it is convenient to orient the y axis along the 1-2 station baseline. Then for a Gaussian form of phase autocorrelation, the phase difference

mean square value and autocorrelation become, from equations 4.23 and 4.24

$$\phi_d^2 = 2\phi_0^2 (1 - e^{-d_{12}^2/2L_{cp12}^2}) \quad (4.35)$$

$$\rho_{\phi d}(\Delta y) = \exp[-(\Delta y)^2/2L_{cp12}^2] \{1 - [\exp(-d_{12}^2/2L_{cp12}^2)] \cosh(yd_{12}/L_{cp12}^2)\} \quad (4.37)$$

where  $L_{cp12}$  is the scale size of  $\rho_{\phi}(y)$  as seen on the ground. If the random motions in the pattern can be ignored,  $\rho_{\phi d}(\Delta x)$  can be obtained from measurements of the phase difference between the two stations taken as a function of time by multiplying the time scale by the pattern velocity in the baseline direction which was deduced from the amplitude measurements. This implies that the irregularities causing both the amplitude and phase fluctuations are at approximately the same height.

Once  $\rho_{\phi d}(\Delta y)$  has been determined, numerical techniques can be employed to invert equation 4.37 and determine that value of  $L_{cp12}$  which best fits the observed data. The actual scale size of the phase pattern in the direction perpendicular to that of elongation,  $L_{cpg}$ , is given by an expression analogous to equation 4.36:

$$L_{cpg} = L_{cp12} |\cos \Omega_p| \quad (4.38)$$

where  $\Omega_p$  is the angle between the x axis and the direction of elongation of the phase pattern.

An examination of equation 4.22 will show that if  $L_{p12}$  is large compared to  $d_{12}$ , the magnitude of  $\phi_d$  is very small. Thus, a phase difference measuring system acts as a spatial filter in that it cannot detect very large irregularities. For a minimum system resolution of 3 degrees, r.m.s., the maximum scale size which would be observed on a 2.5 kilometer baseline would be, for a Gaussian autocorrelation, 25 kilometers if the associated value of  $\phi_0$  was one radian. Thus, this limitation does not seem too severe with a sufficiently long baseline, and good system resolution.

An examination of equation 4.37 shows that although  $\rho_{\phi_d}(\Delta y)$  does not decrease to zero as fast as  $\rho_{\phi}(\Delta y)$  for small  $\Delta y$   $\rho_{\phi}(\Delta y)$  may be expanded to second order in  $\Delta y d_{12}/L_{cp12}^2$  and  $d_{12}/L_{cp12}^2$  to yield

$$\rho_{\phi_d}(\Delta y) = \rho_{\phi}(\Delta y) [1 - (\Delta y)^2/L_{cp12}^2]. \quad (4.39)$$

Thus, for small time lags, the autocorrelation of phase difference falls off faster than the autocorrelation of phase itself. Therefore, the lack of high frequency components in the phase difference patterns implies that  $L_{cp12}$  is very large.

The value of  $\Omega_p$  may be determined if measurements of  $\phi_d$  are performed using a third station. If the phase statistics are stationary, equation 4.23 may be applied to each station yielding

$$\phi_0^2 = \phi_{12}^2 \{2[1 - \exp(-d_{12}^2/2L_{cp12}^2)]\} = \phi_{13}^2 / \{2[1 - \exp(-d_{13}^2/2L_{cp13}^2)]\} \quad (4.40)$$

where  $\phi_{12}$  and  $\phi_{13}$  are the values of  $\phi_d$  obtained along the 1-2 and 1-3 baselines. In the experiment actually conducted in this research, the 1-2 and 1-3 baselines are perpendicular to within a few degrees, so that for a one dimensional pattern

$$\tan \Omega_p \simeq L_{cp12}/L_{cp13} \quad (4.41)$$

Eliminating  $L_{cp13}$  between equations 4.40 and 4.41 and solving for  $\Omega_p$  yields

$$\Omega_p = \arctan \left\{ - \left[ \log_e \left( 1 - \frac{\phi_{13}^2}{\phi_{12}^2} \left( 1 - \exp(-d_{12}^2/2L_{cp12}^2) \right) \right) \right] \frac{2L_{cp12}^2}{d_{13}^2} \right\}^{1/2} . \quad (4.42)$$

Similarity between this value of  $\Omega_p$ , and the value of  $\Omega_a$  obtained from the amplitude analysis not only supports the view that the amplitude and phase patterns are similarly caused by field aligned irregularities, but also provides a check on the stationarity of the phase statistics and on the general self consistency of the whole analysis.

#### 4.8 The Measurement of Phase Parameters by Means of Differential Doppler Measurements

If two harmonically related frequencies are transmitted by the satellite, the dispersive nature of the ionospheric refractive index will cause each frequency to traverse a path of slightly different electrical wavelength so that a measurement of the difference in phase between the two signals received on the ground can yield information on the properties of the ionospheric medium traversed by the signals. In the ray optical sense, the difference in path length expressed in radians between a signal transmitted at frequency  $f_1$  and that of frequency  $f_2$  is given by

$$\Delta P = \frac{2\pi}{\lambda_1} \int_0^{L_1} \mu(f_1) dp_1 - \frac{2\pi}{\lambda_2} \int_0^{L_2} \mu(f_2) dp_2 \quad (4.43)$$

where  $L_1$  and  $L_2$  are the end points of paths  $P_1$  and  $P_2$ , and where  $\mu(f_1)$  and  $\mu(f_2)$  are the refractive indexes at frequencies  $f_1$  and

$f_2$ . If  $f_2$  is one half  $f_1$ , and the frequency of  $f_2$  is electronically doubled upon reception, then  $\lambda_2$  in equation 4.33 may be replaced with  $\lambda_1$ . The high frequency approximation for the refractive index will be deemed sufficiently accurate for the case of interest or

$$\mu(f) \approx 1 - f_n^2/2f^2 \approx 1 - 40.25 N/2f^2 \quad (4.44)$$

where  $f_n$  is the medium plasma frequency,  $f$  the r.f. frequency expressed in Hz, and  $N$  the medium electron density in electrons per cubic meter.

If both paths  $p_1$  and  $p_2$  are taken as coincident straight lines through the medium,  $\Delta P$  becomes

$$\Delta P = \frac{2}{\lambda_1} \int_0^{L_1} 3 \frac{40.25}{f_1^2} N(p_1) dp_1 \quad (4.45)$$

The phase shift incurred by the signal at  $f_1$  due to the ionosphere is

$$\phi = \frac{2\pi}{\lambda_1} \int_0^L \frac{40.25}{f_1^2} N(p_1) dp_1 \quad (4.46)$$

If  $f_1$  is 40 MHz, it is seen that phase shifts as observed on the spaced receiver apparatus will have one third the magnitude of the differential phase shifts observed between the 20 MHz and 40 MHz

signals. Measurements of the autocorrelation function of  $P$  on the ground may then be divided by a factor of three and compared to the result of the spaced receiver studies.

In practice, however,  $N(p_1)$  is likely to consist of an ambient contribution which varies slowly with distance, as well as contributions with relatively small spatial periods corresponding to the irregularities whose properties are desired. In the spaced receiver method, the finite baseline, and the elaborate discriminator circuitry used, suppresses the effects of structures with large spatial scales. In the differential Doppler case, this suppression will have to be done numerically, and for a disturbed ionosphere, the separation of the effects of tilts and other large scale disturbances from the effects of smaller field aligned structures may be difficult. For this reason, the results of the Doppler analysis would not be expected to agree too well with the results of the spaced station study, and should be regarded simply as a check on the proper order of magnitude of the spaced receiver results.

## V. INSTRUMENTATION

### 5.1 Introduction

In order to obtain data which can be analyzed to yield the irregularity parameters, a satellite, and the proper recording and playback equipment are necessary. Because the primary interest in this study was directed toward the spaced receiver results, the pertinent circuitry will be discussed in some detail. The circuitry necessary for the Doppler studies will be reviewed separately.

### 5.2 The Satellite

The experiment employed signals from the S-66 Beacon Satellite launched by the National Aeronautics and Space Administration October 10, 1964. Typical orbital parameters are given below:

|                                                  |               |
|--------------------------------------------------|---------------|
| inclination of orbital plane to equatorial plane | 79.69°        |
| apogee                                           | 1079 km       |
| perigee                                          | 878 km        |
| orbital period                                   | 104.7 minutes |

The satellite radiated c.w. signals on frequencies of 20.005, 40.01, and 41.01025 megahertz with a nominal radiated power of 250 milliwatts at each frequency. All signals were derived from a common ultrastable crystal oscillator. The satellite orientation was magnetically stabilized so that the transmitting antenna orientation was essentially constant. The 40 MHz signal was employed

in the spaced receiver study, and in addition the 20 MHz signal was used for the differential Doppler measurement.

### 5.3 The Spaced Station Locations

Three receiving locations were established as shown in Figure 5.1. The locations were determined primarily by the availability of power and phone lines, and the desire to establish a baseline roughly parallel to the satellite track on the ground. Also previous measurements by DeBarber (1962) had shown that the scale size of the amplitude pattern on the ground was on the order of a kilometer in the direction perpendicular to that of pattern elongation, and so a spacing of 2.5 kilometers seemed reasonable.

### 5.4 The Receiving Antennas

Horizontal turnstiles to receive the 40 MHz signals were employed at each site. The antennas were constructed of 300 ohm ribbon supported by nylon ropes to a wooden framework. The height of the antennas was chosen to be about one third wavelength above the ground to provide a broad vertical antenna pattern. As shown in Figure 5.2, small capacitors were inserted into the ends of the antenna as suggested in an early edition of the A.R.R.L. Antenna Book (1949) to provide an improved match to the transmission line and to compensate for the different electrical lengths of the 300 ohm ribbon in the radiating and transmission line modes existing in a folded dipole.

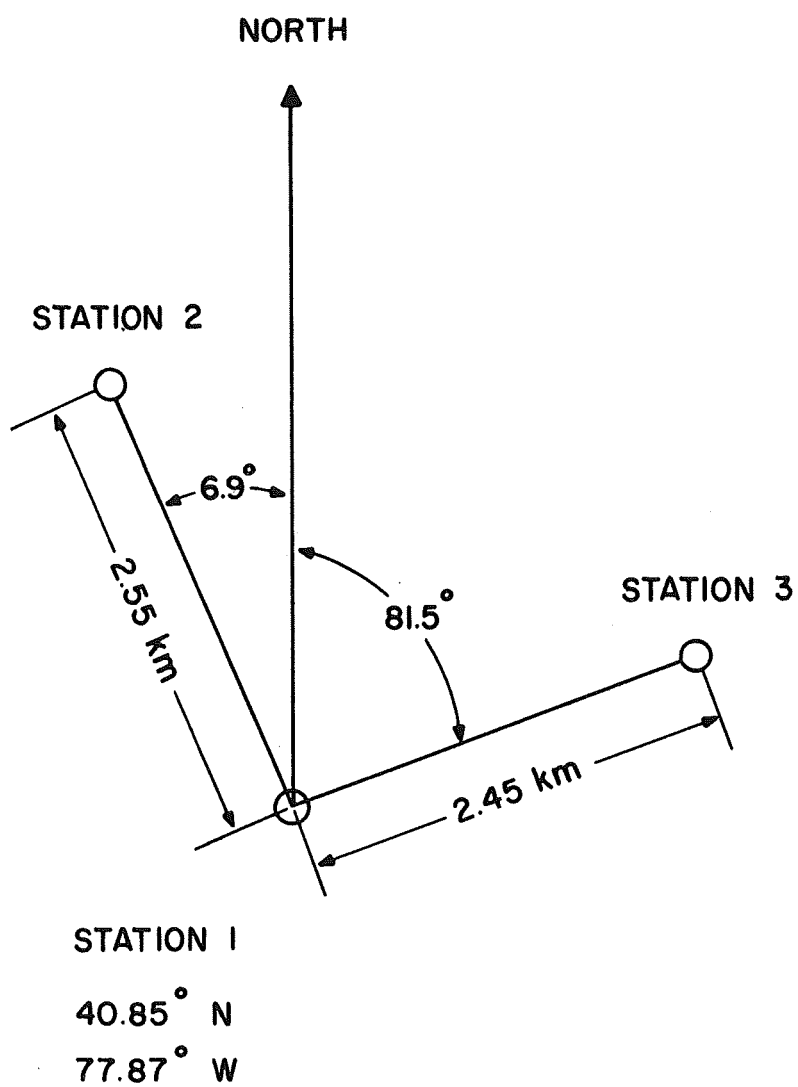


Figure 5.1. Receiving station geometry.

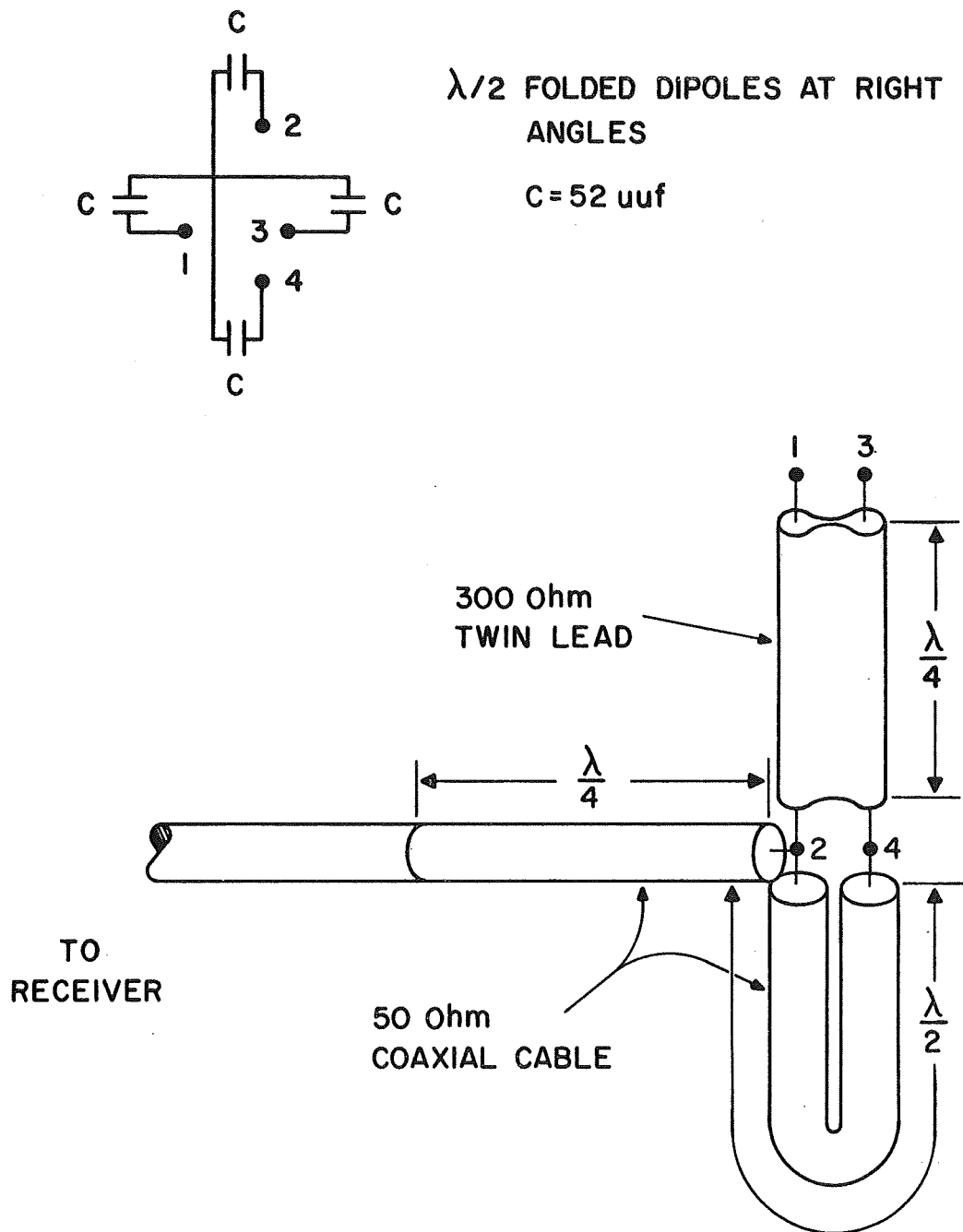


Figure 5.2. Receiving antenna phasing.

It was found necessary to carefully ground the coaxial shield of the feed line by running a heavy copper braid down from the center of the antenna to a ground stake driven at least three feet into the ground. Failure to do so resulted in a deterioration of the circular polarization of the antennas. The antenna at station number one was fed from 75 ohm coaxial cable through a quarter wave matching transformer of 52 ohm cable as shown in Figure 5.2. At stations two and three, the feed cable consisted of 52 ohm coax so that a slight mismatch existed. No appreciable difference in results due to the two methods of feed was observed in the study.

#### 5.5 The Spaced Station Receiving Apparatus

The receiver at each station consisted of a Tapetone converter model TC-40 in conjunction with a Specific Products WWV receiver model SR-7. The converter converted the 40 MHz satellite signal to a frequency of approximately 14 MHz. The WWV receiver was retuned so as to receive this frequency. The receiver was also modified in that the AGC loop had been replaced with a source of fixed bias, and the audio output transformer had been replaced with a parallel tuned circuit resonating at 500 Hz with a bandwidth of approximately 100 Hz. The resulting system had an r.f. sensitivity of better than one microvolt, and a predetection bandwidth of approximately 2.5 kHz which was sufficient to accomodate the free space Doppler shift on the received signal due to the motion of the satellite.

The audio output of the receiver was fed into a frequency discriminator which produced an output proportional to the difference between the input frequency and 500 Hz. This voltage was filtered and fed into a commercial voltage controlled oscillator manufactured by the Hill Manufacturing Company whose nominal center frequency was 1.000260 MHz. The output of the oscillator was multiplied in frequency 40 times and injected into the antenna terminal of the converter. The resulting heterodyne in the receiver second detector between the injected VCO signal and the satellite signal produced the receiver audio output. The resulting system formed an automatic frequency control which held the receiver audio output close to 500 Hz. The time constant of the filter preceeding the VCO was approximately 100 seconds, and so the frequency variations of the received signal due to Doppler shift caused by the high satellite velocity were removed, and the faster frequency deviations due to the inhomogeneities were not effected. The amplitude of the injected VCO signal was high enough to ensure linear mixing so that the amplitude as well as the phase of the signal received from the satellite were preserved. A block diagram of the receiving system is shown in Figure 5.3. Since the essentials of the receiver and VCO circuitry have appeared in DeBarber's report (1962), they will not be repeated here.

A signal derived from the frequency multiplier following the VCO was radiated with sufficient intensity to heterodyne with the satellite signals received at the other two sites and provide

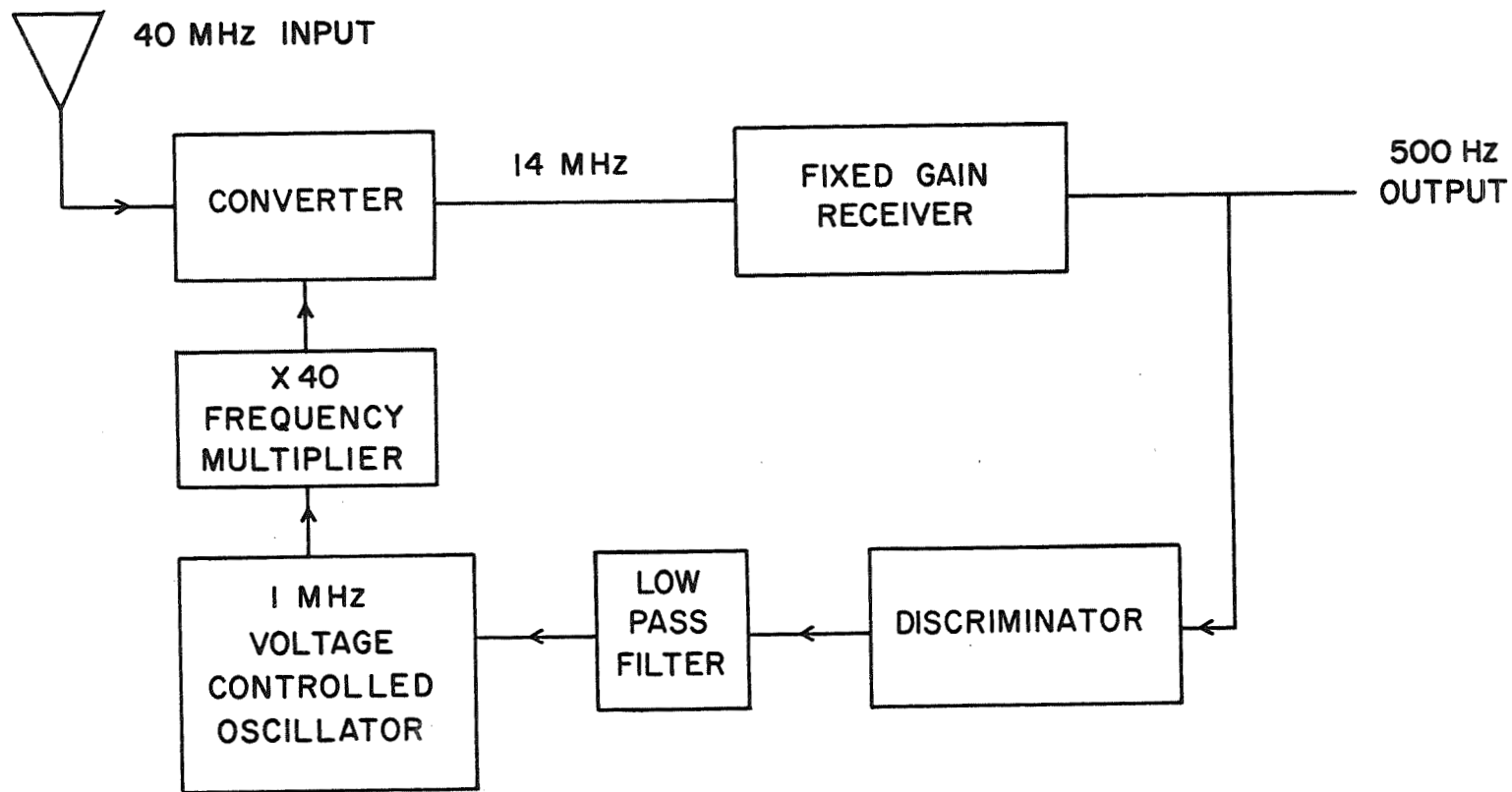


Figure 5.3. Receiving system block diagram.

linear detection. Three audio tones were thereby obtained whose amplitudes were proportional to the received satellite signal amplitudes, and whose phases were equal to the differences in phase between the satellite signals and the reference signal from the VCO. Since these tones had frequencies near 500 Hz they were sent over ordinary leased telephone lines to a central location for further processing and recording.

#### 5.6 Spaced Station Recording Apparatus

On arrival at the main processing point, the signals were further filtered through 500 Hz filters with 100 Hz bandwidths to remove any noise generated in the telephone lines. They were then processed to obtain amplitude and phase information, and the resulting signals were then recorded on a conventional seven track magnetic tape recorder, the Ampex model FR 1100. To minimize the effects of signal dropout due to minute imperfections in the magnetic tape surface, some form of frequency modulated recording was desirable. To minimize phase errors due to sidewise tape slippage, it was desirable to record the two signals being phase compared on the same tape track.

To obtain the amplitude information, the incoming signals were detected in conventional diode detectors. The resulting slowly varying outputs were fed into a commercial F.M. recording adapter, the A.R. Vetter Company Model 3. This unit, a block diagram of which is shown in Figure 5.4, consists of three voltage

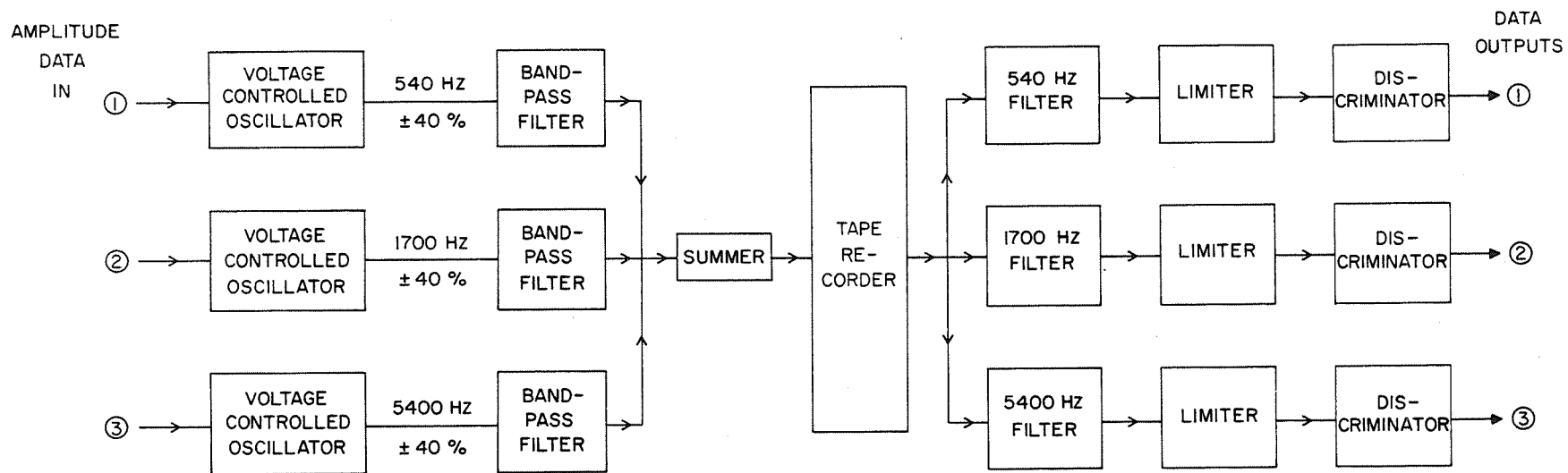


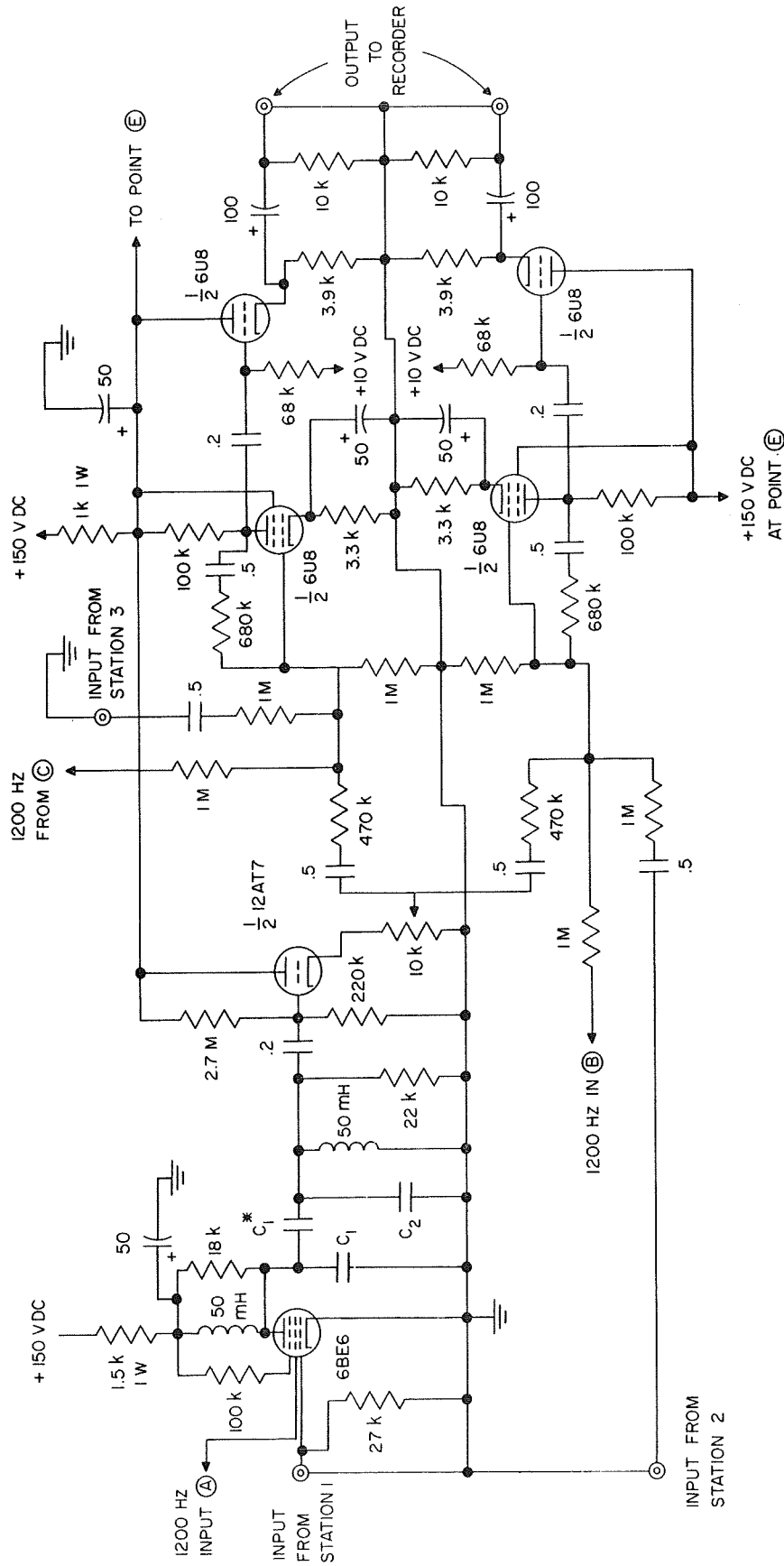
Figure 5.4. Model 3 F.M. recording adapter.

controlled oscillators with different center frequencies and the necessary filtering and summing electronics to frequency modulate three slowly varying signals within an 8000 Hz bandwidth and thereby permit their recording on a single tape track.

To obtain the phase information, the three input signals were limited and filtered. The resulting signal from station one was heterodyned against a 1200 Hz reference signal derived from the 60 Hz power line. The resulting 1700 Hz signal was then added to the 1200 Hz reference and to the 500 Hz signal from station two and recorded on a single tape track. The same process was performed with stations one and three, and the results recorded on still a third track. The circuitry involved is shown in Figures 5.5 and 5.6. In these schematic diagrams, and those that follow, all capacitances are in microfarads, all resistances are in ohms, and all inductances are in henries unless otherwise stated.

Time signals derived from WWV were then recorded on still another tape track. The remaining tracks were used for recording signals derived from the 20, 40, and 41 MHz transmissions which were used in the regular program of Doppler shift and Faraday rotation measurements performed by The Pennsylvania State University Ionosphere Research Laboratory. Since all signals were at relatively low audio frequencies, a recording speed of 3.75 inches per second was sufficient. A block diagram of the spaced receiver experiment recording instrumentation is shown in Figure 5.7.





$C_1, C_2$  ADJUSTED FOR RESONANCE  
 AT 1700 HZ  
 $C_1^*$  CHOSEN FOR CRITICAL COUPLING  
 AT RESONANCE

Figure 5.6. Mixer-summer.

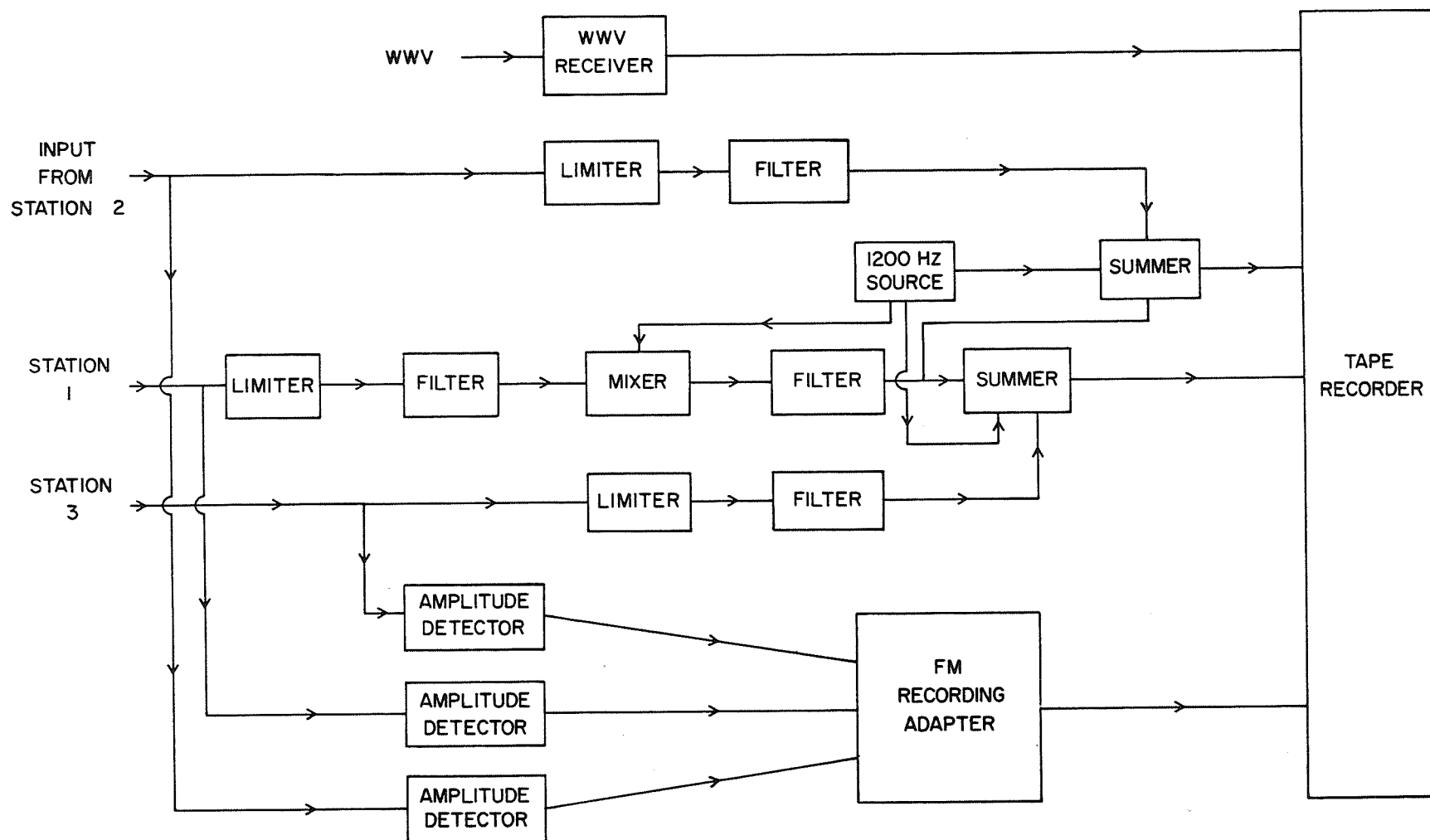


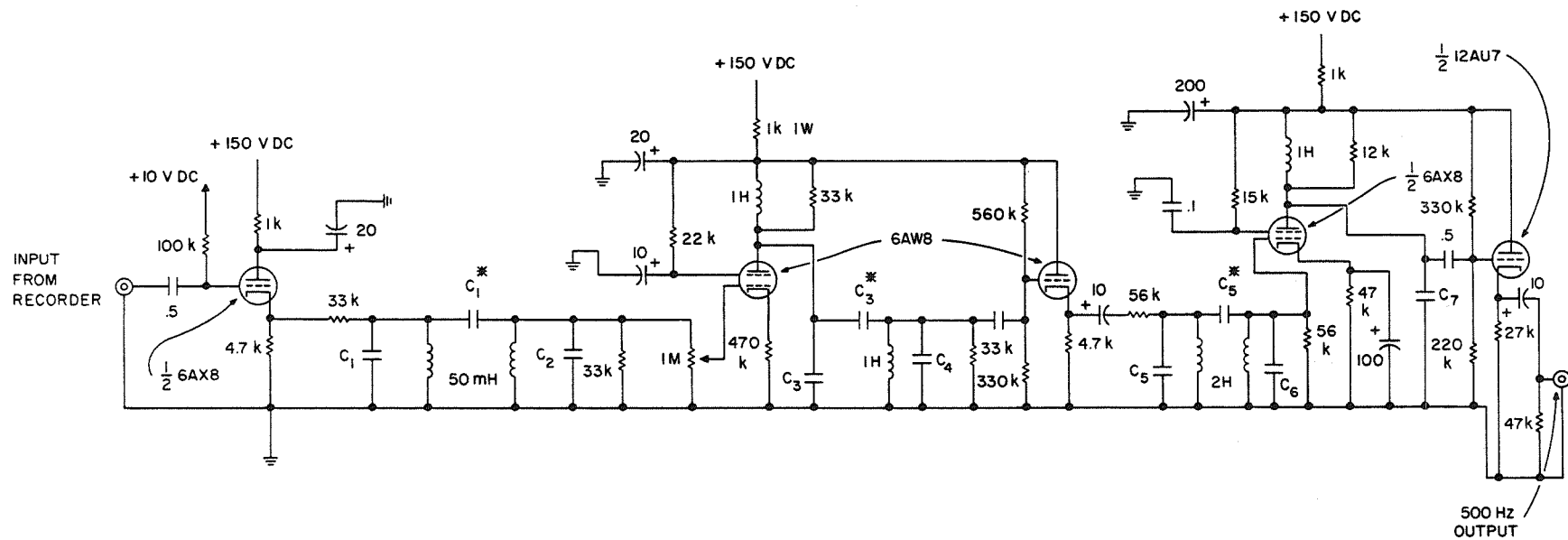
Figure 5.7. Block diagram of spaced station recording system.

## 5.7 Spaced Station Playback Instrumentation

On playback, the amplitude data was reconstructed by the filters and discriminators in the output section of the Model 3 recording adapter. The outputs of the recording adapter were then filtered so that the total system response fell off at a rate of 60 dB per decade above a frequency of approximately 10 Hz.

Before a phase comparison between any two stations could be performed, the original two 500 Hz signals must be recovered from the sum of the three frequencies recorded on the tape. The necessary filtering and heterodyning circuitry is shown in Figures 5.8 through 5.10. The large number of filtering stages employed was necessary to keep the phase response of any one stage reasonably flat over the passband involved so that each of the resulting 500 Hz signals would suffer the same time delay in traveling through the system.

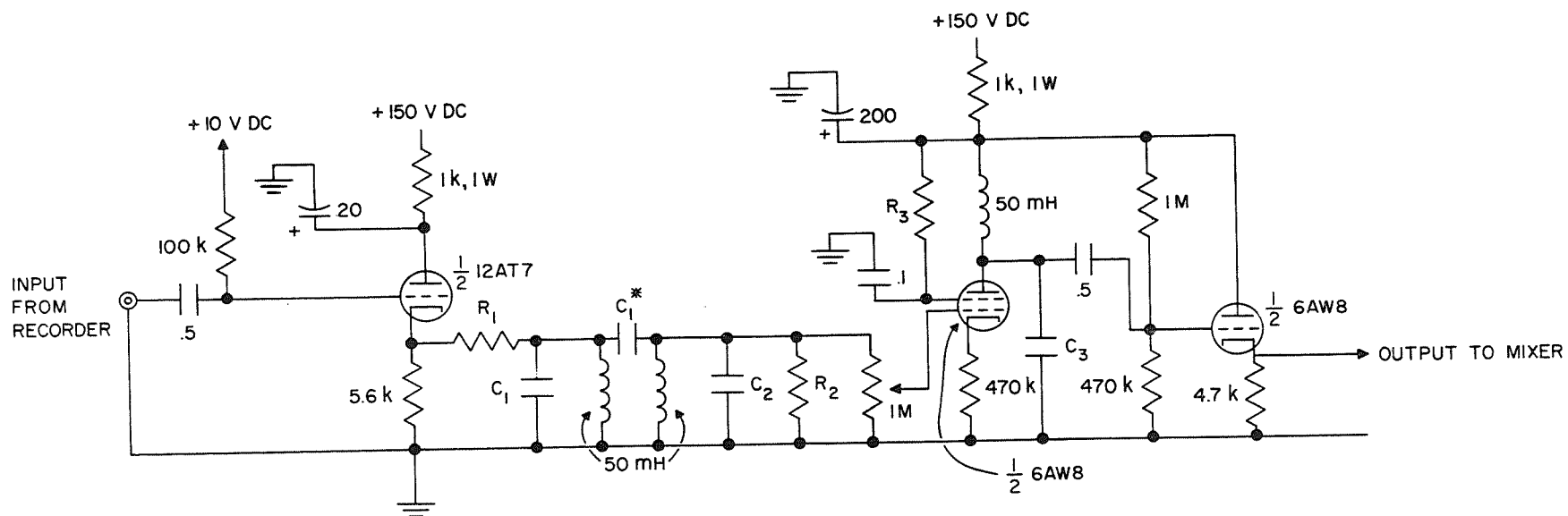
The phase difference between the two 500 Hz signals finally emerging from the filters can then be obtained by employing the circuitry shown in Figure 5.11, which essentially consists of a pair of zero crossing detectors, a flip flop, and an R.C. integrator arranged so that the integrated output is proportional to the difference in the times of occurrence of the zero crossings of each signal. Unfortunately, the free space distance between the satellite and each station changes as the satellite moves, and the resulting output on a phase comparator which recycles every



$C_1, C_2, C_3, C_4, C_5, C_6, C_7$  ADJUSTED FOR RESONANCE AT 500 Hz

$C_1^*, C_3^*, C_5^*$  ADJUSTED FOR CRITICAL COUPLING AT RESONANCE

Figure 5.8. 500 Hz output filter.



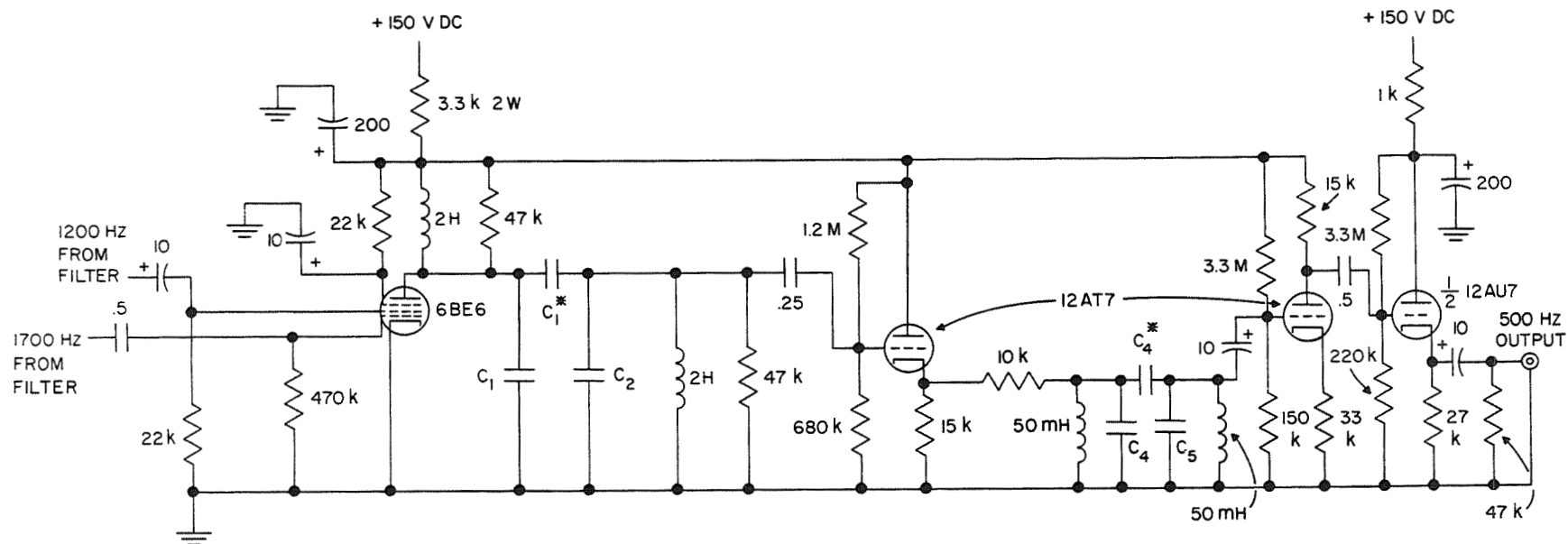
FOR 1200 HZ FILTER;  $R_1 = 10\text{ k}$ ,  $R_2 = 12\text{ k}$ ,  $R_3 = 18\text{ k}$

FOR 1700 HZ FILTER;  $R_1 = R_2 = 18\text{ k}$

$C_1$ ,  $C_2$ ,  $C_3$  ADJUSTED FOR RESONANCE AT FILTER FREQUENCY

$C_1^*$  ADJUSTED FOR CRITICAL COUPLING AT RESONANCE

Figure 5.9. 1200 Hz and 1700 Hz output filters.



$C_1, C_2, C_4, C_5$  ADJUSTED FOR RESONANCE AT 500 HZ  
 $C_1^*, C_4^*$  ADJUSTED FOR CRITICAL COUPLING AT RESONANCE

Figure 5.10. Output mixer-filter.

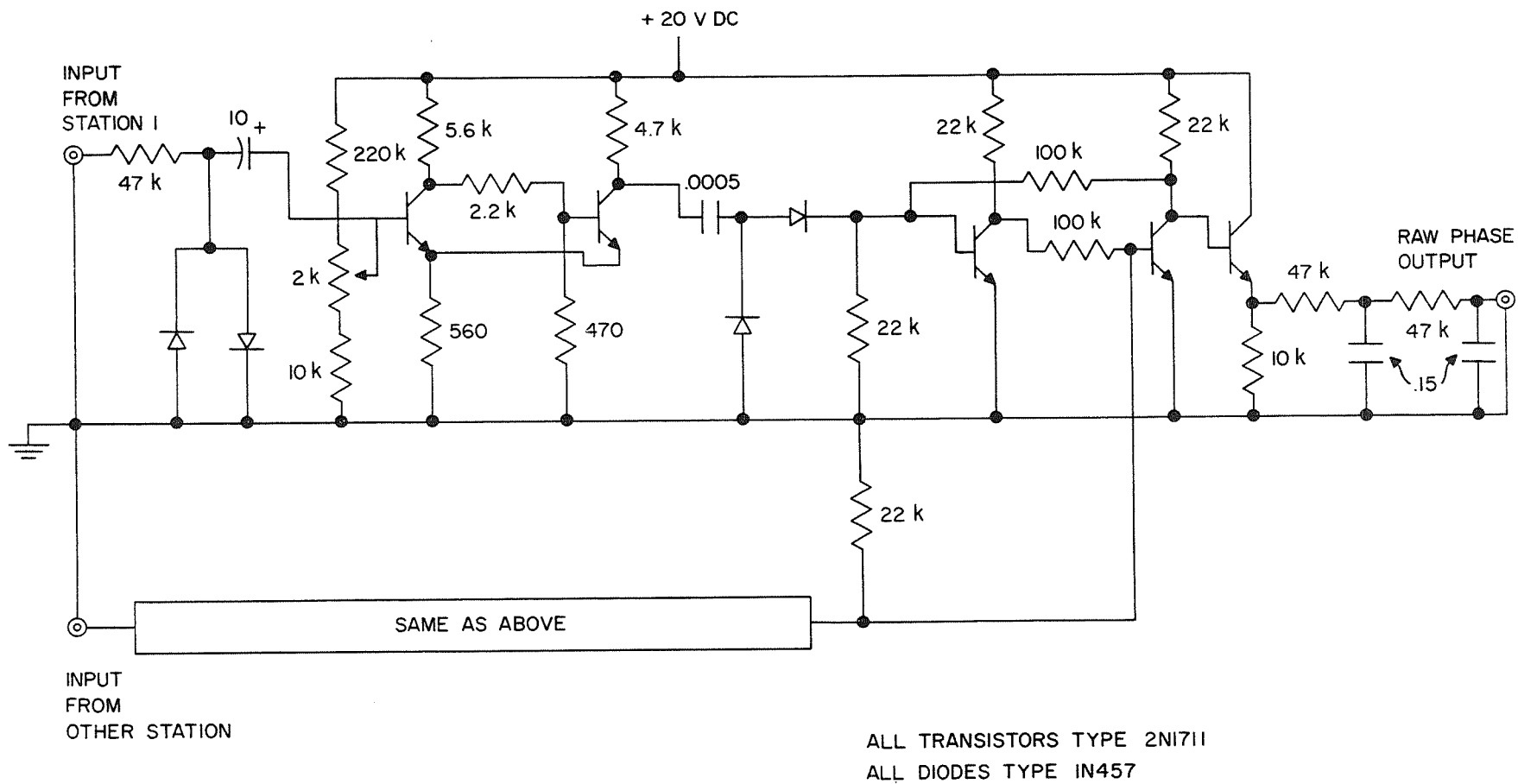


Figure 5.11. Raw phase comparator.

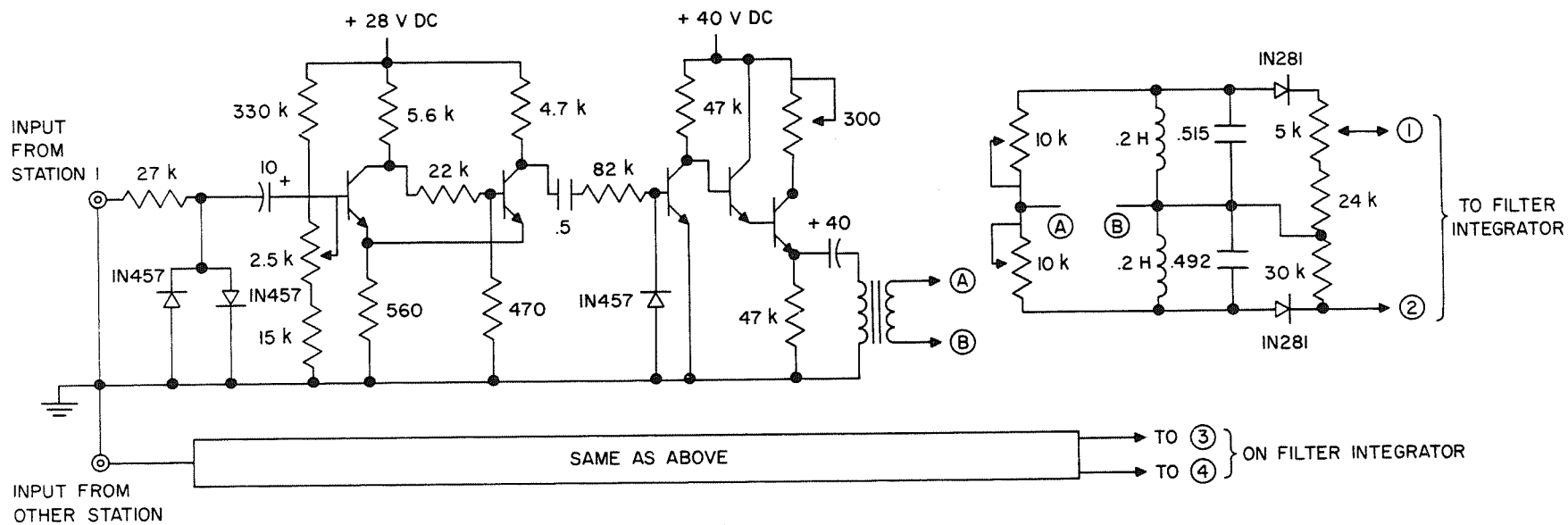
360 degrees is a sawtooth wave whose frequency and sense depends on the manner in which the satellite orbit intersects the hyperboloids of constant phase of the two antennas involved. Any scintillation present would be superimposed upon this sawtooth waveform, and the analysis of the scintillation becomes difficult, especially when the free space phase shift is near 360 degrees, for then any scintillation present will tend to cause the comparator to recyle and the resulting waveshape will be very hard to interpret. If the sawtooth rate is rapid compared to any slow scintillation, it will tend to obscure the slower waveform and also make scintillation interpretation difficult.

If the rate of change of the free space phase difference is examined it is seen to change only slowly with time, the change in fact being linear for the satellite near the point of closest approach. Since a linear change of phase difference with time corresponds to a constant frequency difference between the two signals, a slowly changing rate of phase difference corresponds to a frequency difference which changes only slowly with time. If each of the two original 500 Hz signals whose phase difference is desired is then passed through an identical frequency discriminator, the difference in the outputs of the discriminators will vary slowly with time in the absence of scintillation. If this difference is then passed through a high pass filter, and the resulting signal is integrated, the integrated output will

correspond to the phase difference between the two original signals with the slowly varying term due to the movement of the satellite removed.

The necessary circuitry for accomplishing the discrimination, subtraction, filtering, and integration is shown in Figures 5.12 and 5.13. The high pass filter had a cutoff frequency of about 1/12 Hz and was adapted from a design in the Burr Brown Handbook of Operational Amplifier Active RC Networks (1966). The integrated phase output was further filtered through a passive low pass filter with a 3 dB cutoff frequency of 12 Hz, and an attenuation of at least 30 dB by frequencies of 30 Hz or higher. The total phase system had a sensitivity of approximately 1/3 volt per radian.

The three amplitude outputs, the output of the simple flip flop phase comparator, and the output of the discriminator system were displayed along with time information on a Model 350 Sanborn oscillograph. A block diagram of the entire playback system is shown in Figure 5.14. A section of a typical quiet record is shown in Figure 5.15. From top to bottom, the traces are: one second time marker from oscillograph, raw phase, corrected phase, stations two, one and three amplitudes, and WWV time signals. The oscillograph speed is five millimeters per second, and the sensitivity of the corrected phase output is five millimeters per radian. The sudden decreases in amplitude evident on the three amplitude traces are due to a zeroing circuit which reduced the input amplitude to zero once a minute for calibration purposes.



ALL TRANSISTORS: 2N2219  
TRANSFORMERS: TRIAD A-89 A

Figure 5.12. Discriminator circuitry.

# OPERATIONAL AMPLIFIER BIAS

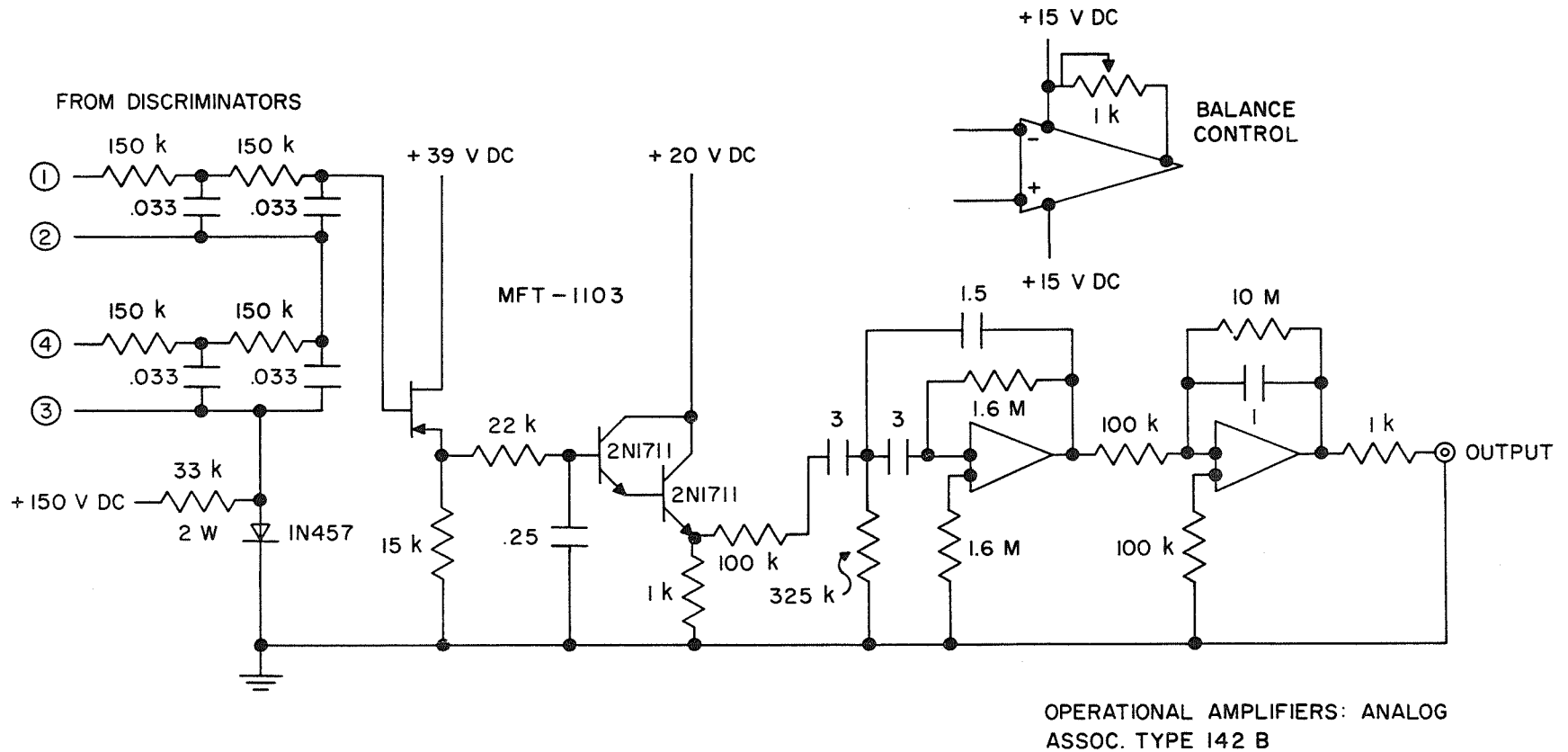


Figure 5.13. Phase filter-integrator.

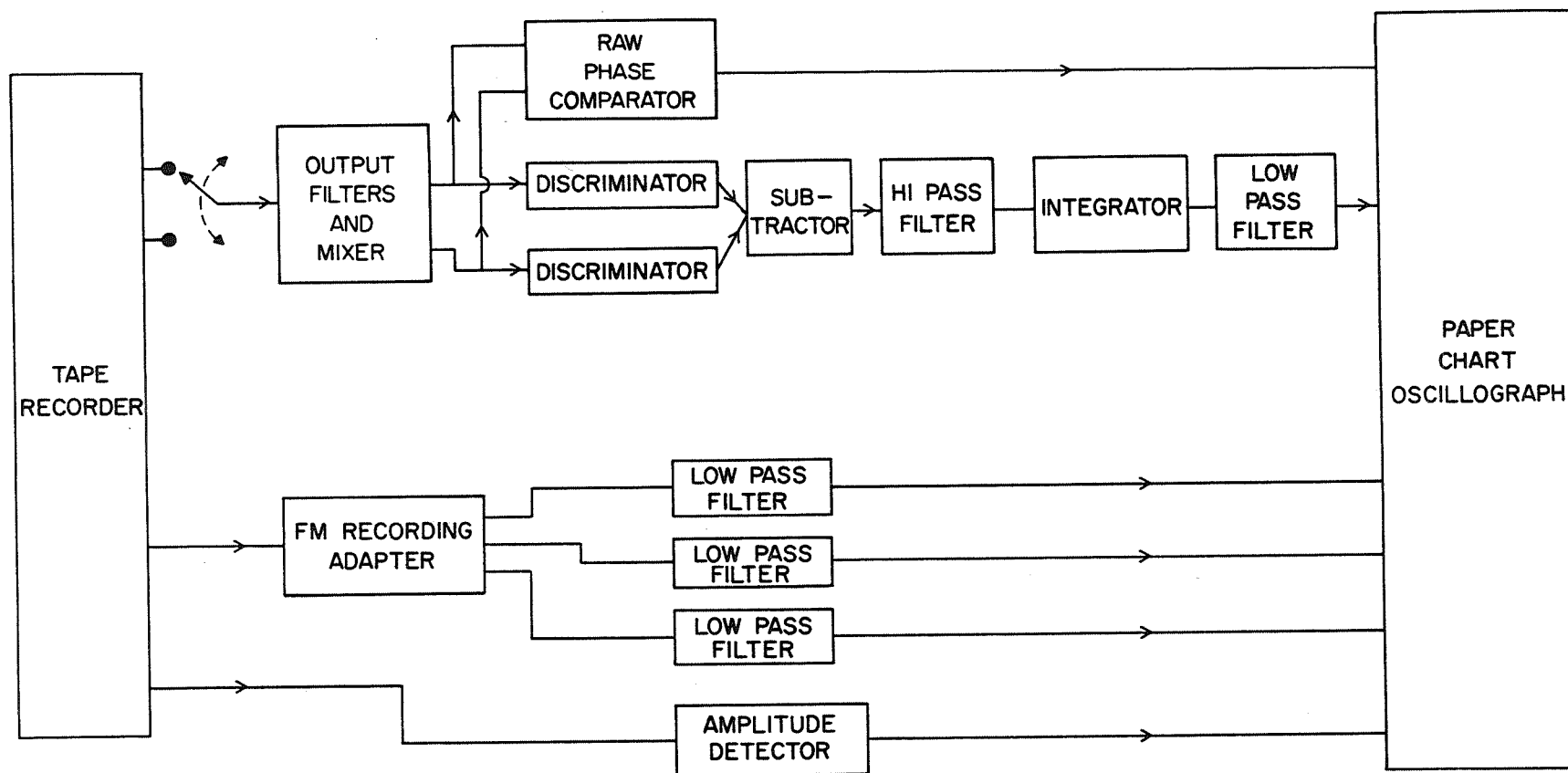


Figure 5.14. Spaced receiver playback system block diagram.

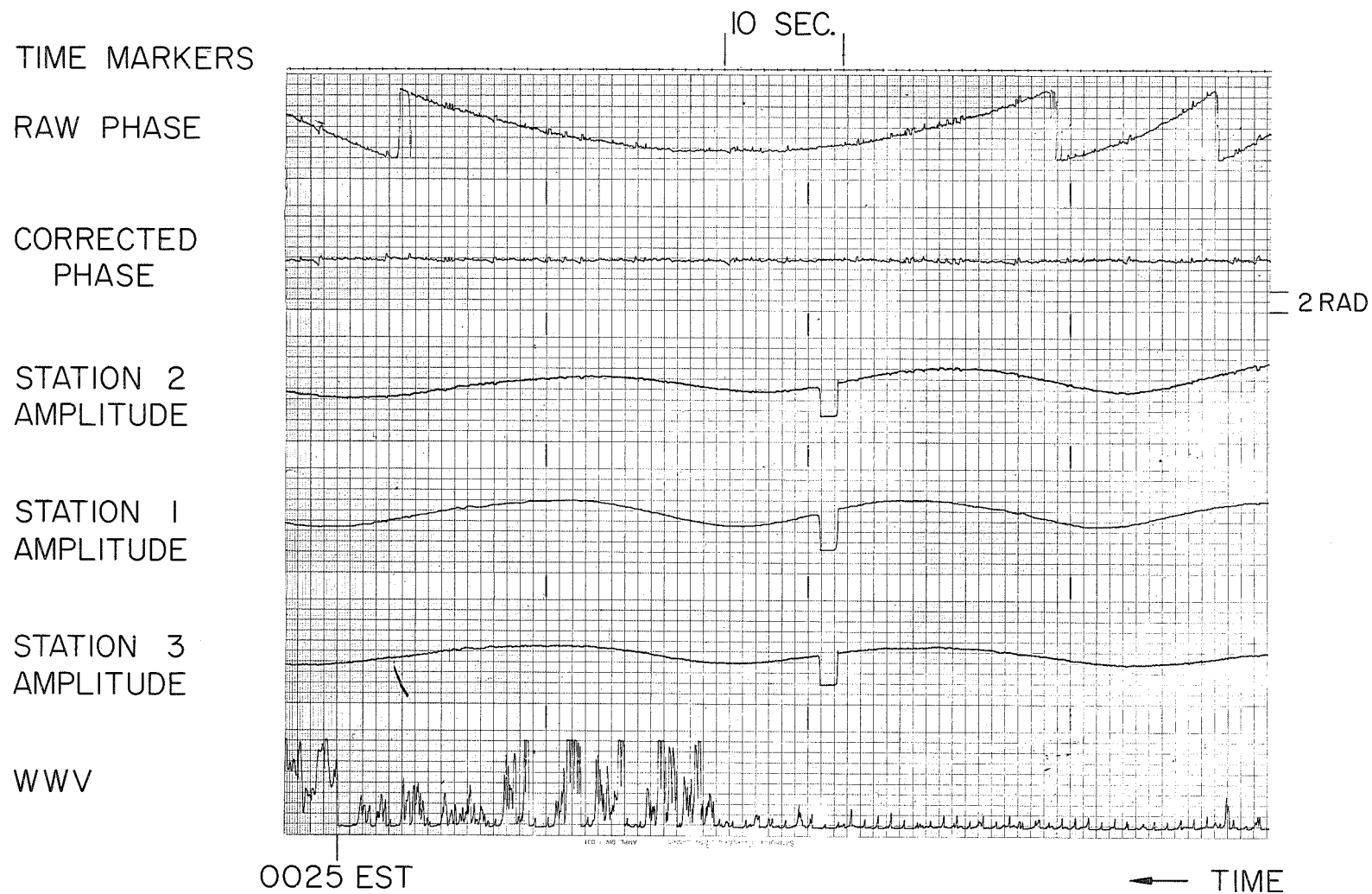


Figure 5.16. Spaced station record for pass 18660, 29 June, 1968, with phase measured between stations 1 and 3.

The quasiperiodic fading visible in the amplitude traces is due to Faraday rotation and the fact that the patterns of the receiving antennas do not have unity axial ratio except for small satellite zenith angles. In this figure, the phase being displayed is measured between receiving locations one and three.

A section of a typical scintillating pass is shown in Figures 5.16 and 5.17. The phase difference between stations one and two is displayed in Figure 5.16, and the phase difference between stations one and three in Figure 5.17. The system playback rate and phase sensitivity are the same as in Figure 5.15. The difference in the amplitude of the phase fluctuations evident in a comparison of the phase outputs of the two figures is due to the fact that the pattern elongation is predominately along the 1-3 baseline at this point in the orbit, and the magnitude of the phase difference as seen by the 1-3 pair is reduced in accordance with equation 4.36.

The rather sudden cessation of the scintillation activity visible at the left of the record is characteristic of many of the passes observed. Similar cessations have been observed by Yeh and Swenson (1964). It should also be noted that the amplitude and phase scintillation stop at the same time implying a common source.

## 5.8 Differential Doppler Instrumentation

To verify the spaced receiver results, a few differential Doppler measurements were performed with the 20 and 40 MHz signals

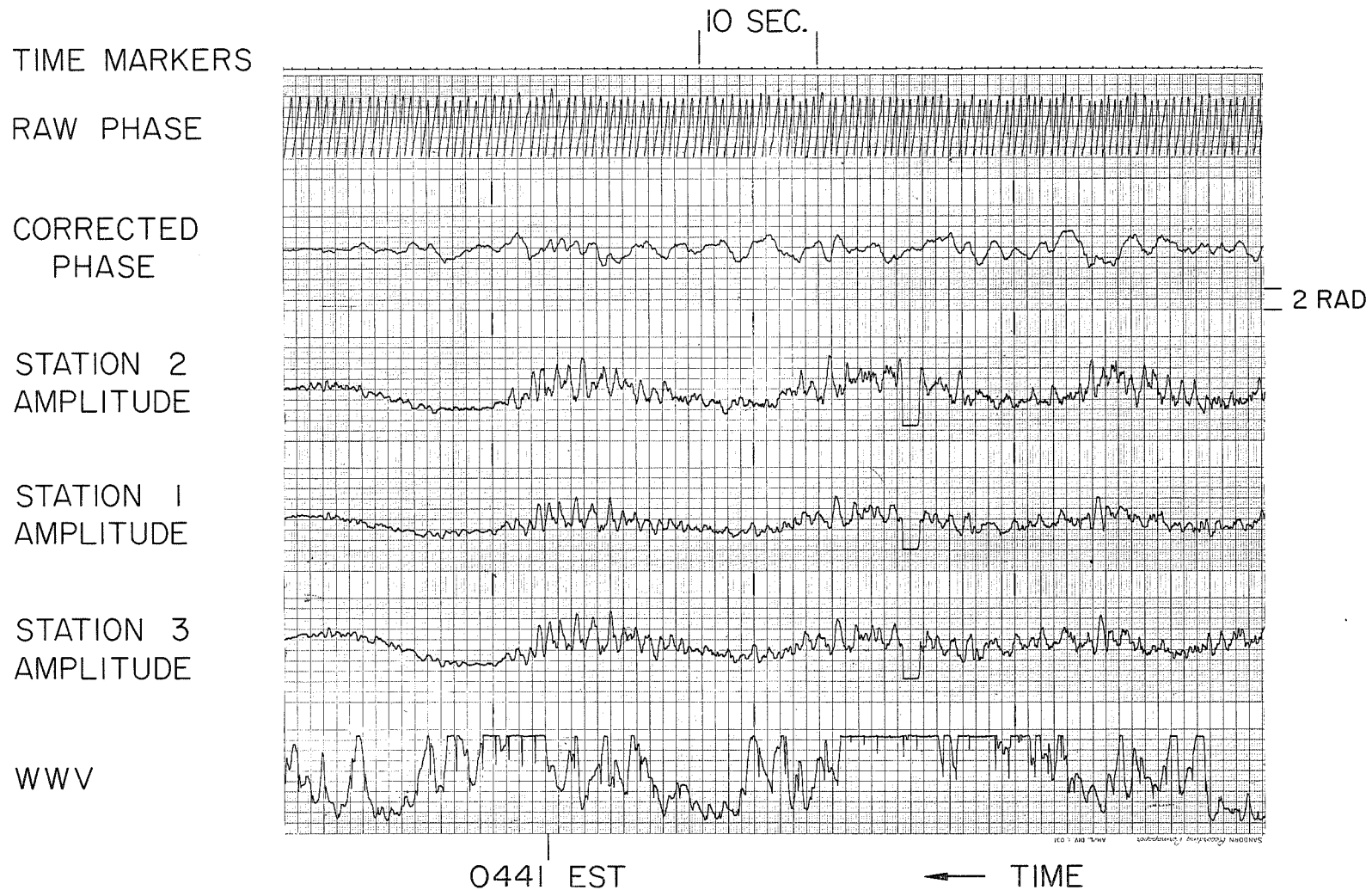


Figure 5.17. Spaced station record for pass 18154, 23 May, 1968, with phase measured between stations 1 and 2.

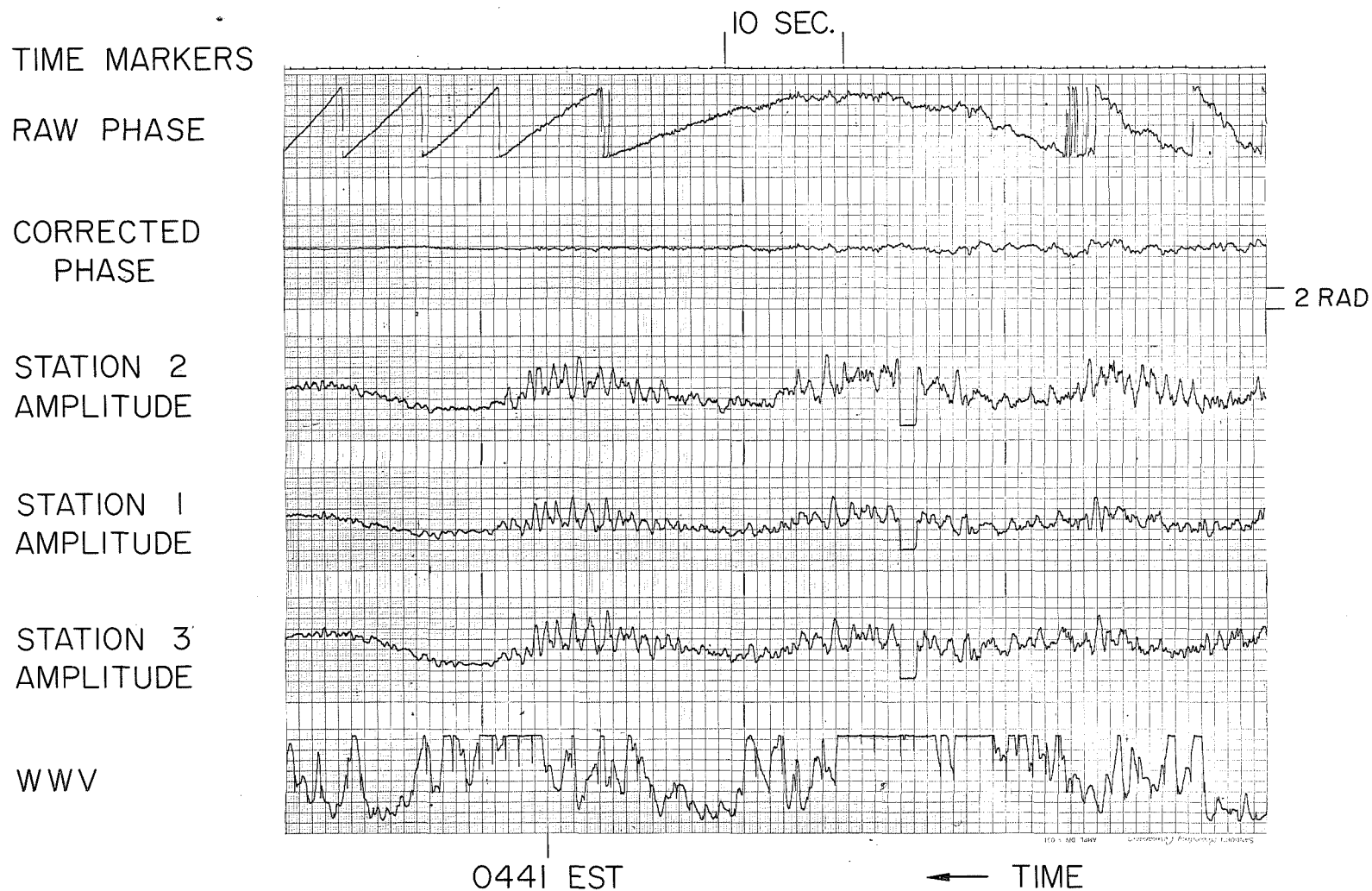


Figure 5.18. Spaced station record for pass 18154, 23 May, 1968, with phase measured between stations 1 and 3.

received at location one. The 20 MHz antenna employed was a horizontal turnstile constructed of aluminium tubing and fed with a matching network identical to that shown in Figure 5.2. The capacitors at the ends of the folded dipoles were not employed as the antenna dielectric consisted of air.

The receiving apparatus consisted of a Tapetone TC-20 converter, and a Specific Products SR-7 receiver similar to the spaced station instrumentation. The audio load in the 20 MHz receiver was tuned at 250 Hz. A 20 MHz signal derived from the 1.000260 MHz voltage controlled oscillator was injected into the converter and heterodyned with the satellite signal to produce an audio output at 250 Hz, which was sent over a leased telephone line to the main processing site. On arrival, the signal was filtered, limited, filtered further, and added to a similarly limited and filtered 500 Hz signal derived from the 40 MHz satellite transmission. The resulting signal was recorded on one track of the magnetic tape.

On playback, the individual signals were recovered by the proper filtering. The 250 Hz signal was doubled in frequency and phase compared to the 500 Hz signal using the phase comparator shown in Figure 5.18. The Schmitt triggers in the input circuitry were designed to function with input signals of low amplitude and were adapted from a circuit developed by Robinson (1964). An examination of the comparator circuitry will show that the output voltage decreases linearly from a maximum to a minimum value as

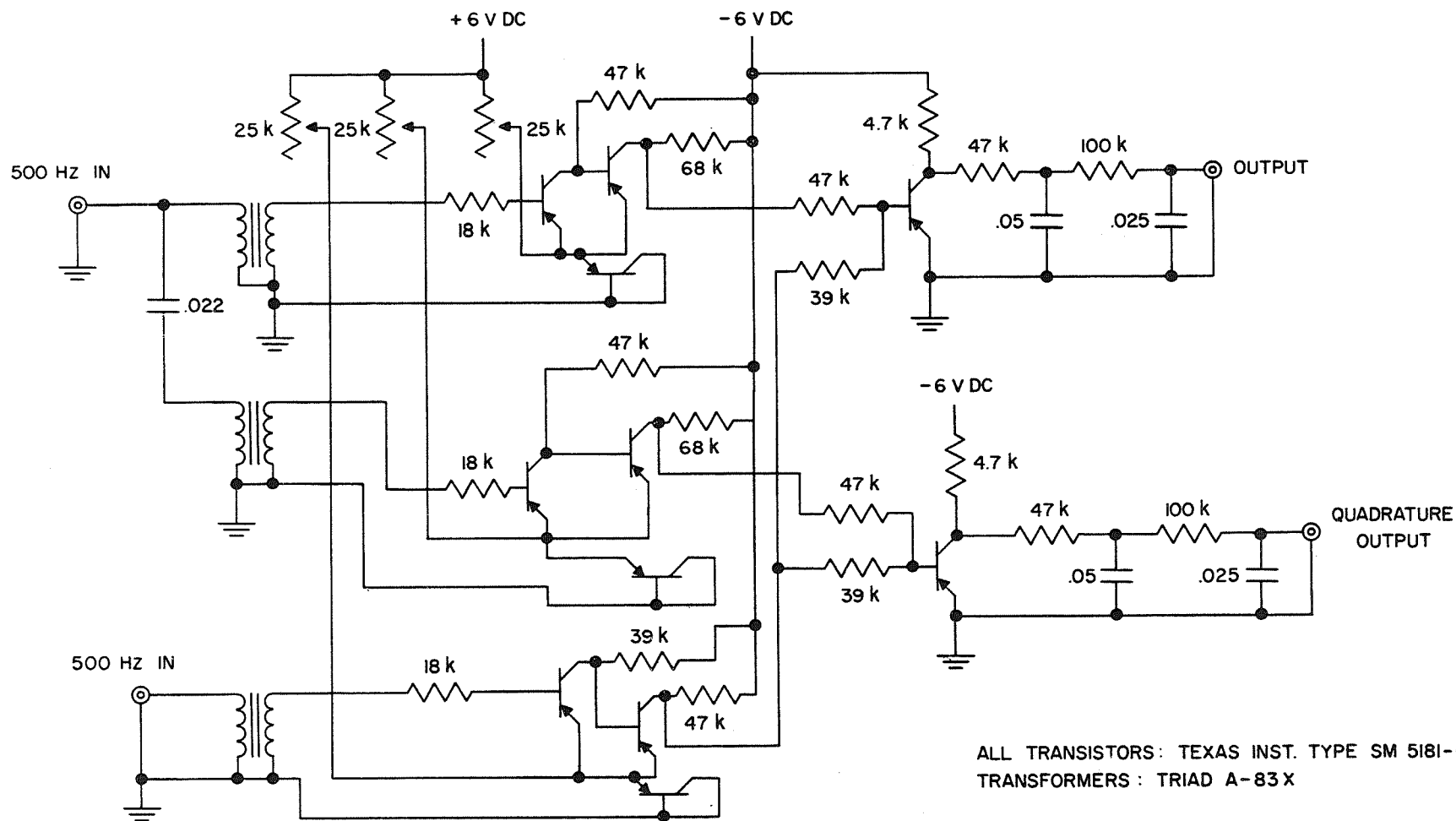


Figure 5.18. Differential doppler phase comparator.

the phase difference of the inputs varies from  $0^\circ$  to  $180^\circ$ . The output then increases toward the maximum value as the phase difference goes from  $180^\circ$  to  $360^\circ$ . Thus, each complete output cycle corresponds to a  $360^\circ$  phase difference. The effects of scintillation may be obtained by noting the difference between the actual rate of change of the phase difference with time and the rate expected from an ionosphere with no irregularities. This is done by counting the number of cycles of phase change with time starting from some arbitrary point, and noting the difference between this function and a smooth curve. To facilitate the identification of the sense of the phase difference when the scintillation causes a reversal in the steady phase variation due to the changing satellite position, a second phase comparison is made between one of the original input signals, and a second signal derived by shifting the phase of the other input signal by  $90^\circ$ . This output is referred to as the quadrature output in Figure 5.18. A block diagram of the entire Doppler instrumentation system is shown in Figure 5.19.

A section of a typical scintillating record is shown in Figure 5.20. The topmost trace is the time marker from the Sanborn oscillograph. The next two traces contain the two outputs from the phase comparator. The next trace contains the 40MHz amplitude as derived from the spaced receiver instrumentation. The next two traces contain the inputs to the phase comparator, the signal derived from the 40 MHz transmission being topmost. The bottommost trace contains WWV time signals. It is evident from the second

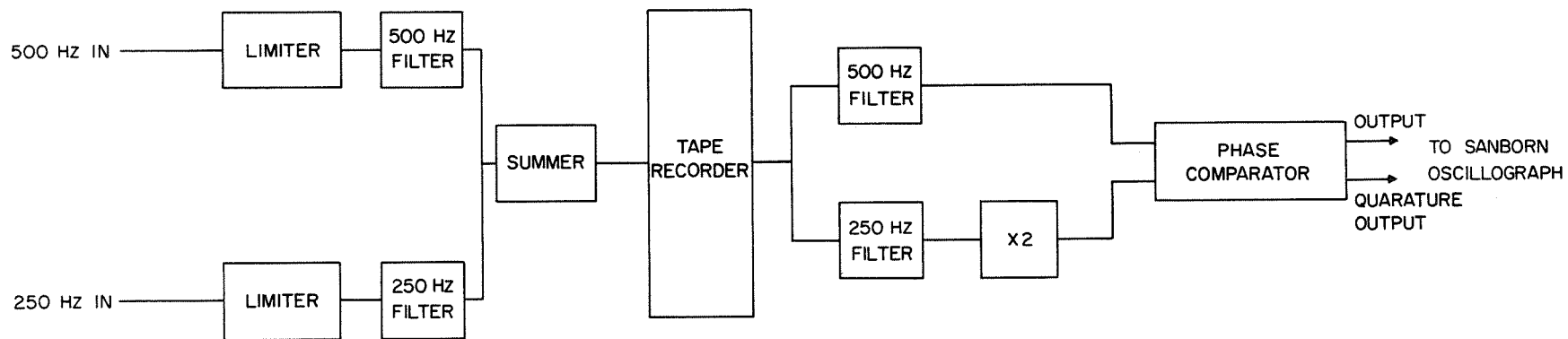


Figure 5.19. Differential doppler instrumentation.

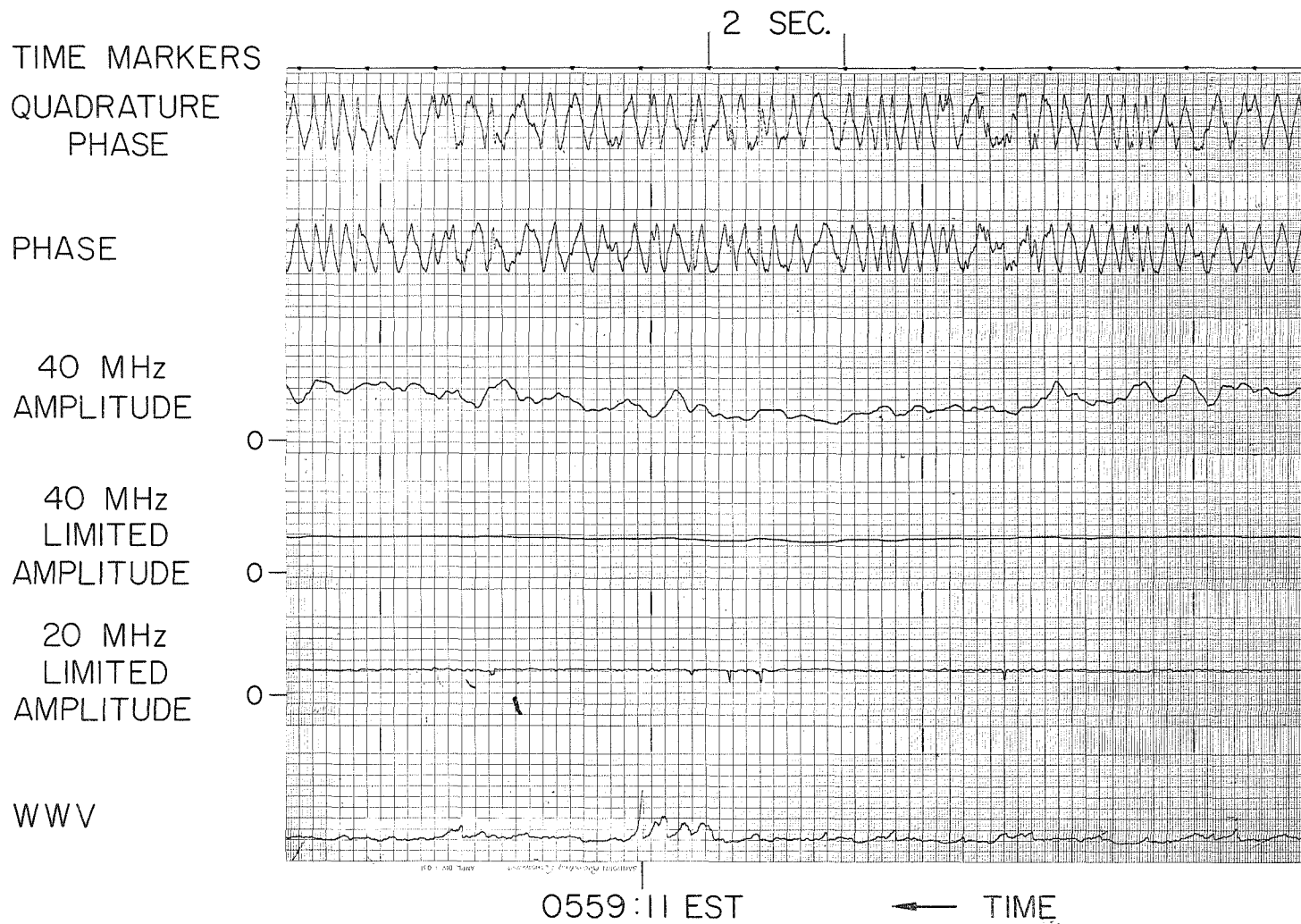


Figure 5.20. Differential Doppler record for pass 18141, 22 May, 1968.

and third traces from the bottom that limiting of the signals is virtually complete even before the signals are limited by the Schmitt triggers in the phase comparator. The chart speed in this Figure is 25 millimeters per second.

## VI. METHOD OF ANALYSIS

### 6.1 Introduction

The data obtained with the experimental apparatus can be analyzed in accordance with the equations developed in Chapter IV. However, some details of the actual analysis are involved enough to warrant further elaboration.

### 6.2 Spaced Station Correlation Analysis

Oscillographs of selected sections of the spaced receiver recordings were obtained at playback speeds of 25 and 50 millimeters per second. These oscillographs were sampled at a rate of 25 samples per second, and the sampled values were transferred to IBM cards with the aid of a Gerber model GDDRS-3 Scaling Machine. In view of the low pass filtering employed prior to oscillographic playback, this sampling rate was sufficient to prevent aliasing.

Before further processing, it was necessary to remove the effects of Faraday rotation from the amplitude data. For this purpose a second degree polynomial was fitted in the least mean square sense to the first 160 data samples. The value of the data at the 80th sample was then replaced by the difference between the original data value and the value of the polynomial at this point. Similarly, the 81st sample point was replaced by the difference between its value and that of a polynomial fitted over the 2nd to

the 161st sample values. This process was continued until each point more than 79 samples from the ends of the data had been redefined. Since this process effectively throws away 158 sample values, and since the Faraday rotation causes only slow changes in the average value of the received amplitude, it was deemed sufficient to define the first and last 79 sample values by obtaining the differences between the actual data values and the values of the polynomials fitted over the first and last 160 points respectively. All new data values were then normalized by dividing them by the value of the appropriate polynomial from which they were obtained.

This process yielded a series of normalized differences from the average values of received amplitude, the averaging having been done over a 6.4 second range. Tests made with a similar averaging technique over ranges of 5 and 8 seconds showed that the 6.4 second range was long enough to effectively remove the effects of Faraday rotation without significantly affecting the scintillation data. The auto and cross correlations of the corrected data were then computed using regular IBM supplied Fortran subroutines on an IBM 360 digital computer. These correlation functions were then fitted in the vicinity of their origins with Gaussian curves, the fitting being accomplished in the least mean square sense.

Once the equivalent Gaussian coefficients had been obtained, the values of  $V_g$ ,  $V_c$ ,  $Ax_1$ ,  $Ax_2$ ,  $\Omega_a$ , and  $\xi_g$  can be obtained by applying the equations of section 4.3. In practice, the assumption that the pattern can be represented by an ellipsoid of revolution

can be expected to hold only approximately. Furthermore, errors are introduced in the scaling and curve fitting processes. Therefore, it may be expected that the coefficients of the ellipsoid, A, B, C, F, G, and H defined in equation 4.5 will be known only approximately. If the elongation of the ellipse is large, then an examination of equations 4.11 and 4.12 will show that the product AB will be very nearly equal to H squared. As a result if A, B, and H are only approximately known, the resulting values of major axis size, pattern drift velocity magnitude and direction, and the value of characteristic random velocity will be highly uncertain depending as they do on the quantity  $AB-H^2$ . It will, therefore, be more accurate to obtain  $\xi_g$  from the satellite orbital data, rather than from the correlation analysis. It will also be found that the magnitude of the drift velocity is best determined by first obtaining the component in the direction of the minor axis of the diffraction pattern by using equation 4.18. Similarly  $V_c$  is best estimated by the use of the approximate relation given in equation 4.20. Unfortunately, no similar approximate relation may be derived for the size of the major axis, and equation 4.14 must be used even though it may be numerically inaccurate. Physically, this is a reflection of the fact that it is not possible to obtain highly accurate measures of pattern elongation with this method, when the long dimension of the pattern is much larger than the spacing between receiving stations.

To obtain results from the phase data, the oscillograph trace of the phase difference corrected for the effects of satellite motion was scaled along with the amplitude data. The best mean square straight line over the whole data sample was obtained and subtracted from the data, so that the data used for calculation had zero mean value. The reduced data was then autocorrelated, fitted in the vicinity of the origin with a function of the form given in equation 4.37, and the corresponding values of  $L_{cp12}$  and  $\phi_0$  were obtained.

### 6.3 The Average Height of the Scintillating Region and the Orientation of the Ground Pattern

The average height of the scintillation region may be found from a knowledge of the pattern drift velocity on the ground, and a knowledge of the satellite position as a function of time by employing equations 4.3 and 4.4. The satellite position as a function of time was obtained from Goddard Space Flight Center orbital data which gives the satellite position for every minute of time to a precision better than one km in height, longitude and latitude. Due to the large inclination of the satellite orbit, linear interpolation was permissible for determining the satellite position for smaller time increments.

Once the height of the scintillating region is determined, the theoretical orientation of the resulting ground pattern can be

calculated by the use of equation 3.42. This requires a knowledge of the Earth's magnetic field as a function of height. The magnetic field was represented by a Legendre polynomial expansion with terms up to the tenth degree. The coefficients of the model were the Goddard Space Flight Center coefficients for epoch 1965.0.

#### 6.4 The Estimation of Power Spectral Densities

In the interpretation of the Doppler measurements, it is of interest to obtain estimates of the power spectra of the Doppler phase variations and compare these to the power spectra of the amplitude variations obtained from the spaced receiver experiment. The desired spectral estimates were obtained by the method of Blackman and Tukey (1958), which is outlined below.

Any physically realizable data will be of limited time extent. The power spectrum computed from this sample may differ from the actual power spectrum of the sample population from which the sample can be imagined to have been drawn. Due to the Wiener Khintchine theorem, the power spectrum may be computed by first obtaining the autocovariance function of the data sample, and then Fourier transforming it to yield the computed spectral estimates. If the autocovariance function computed as a function of time is then multiplied by a function of time  $D(t)$ , and the result is Fourier transformed, Blackman and Tukey have shown that the

resulting spectral estimate  $P_e(f)$  is related to the spectrum density of the parent population by

$$P_e(f) = \int P(f_1) Q(f-f_1) df_1 \quad (6.1)$$

where  $Q(f)$  is the Fourier transform of  $D(t)$  and is known as the spectral window corresponding to  $D(t)$ .

From the theory of Fourier transforms, it is evident that as the "width" of  $D(t)$  becomes large in time, the "width" of  $Q(f)$  will decrease and  $P_e(f)$  will become a better estimate of  $P(f)$ . However, it can be shown that the statistical error associated with a computed value of autocovariance increases as the time lag at which the autocovariance was obtained increases. Therefore, it is desirable to have  $D(t)$  span a relatively small length of time for high statistical accuracy.

It is thus seen that high statistical accuracy and high resolution of the shape of the observed spectrum are mutually exclusive requirements, and some compromise between these requirements is necessary in practice. Blackman and Tukey recommend a  $D(t)$  of the form

$$\begin{aligned} D(t) &= .5 (1 + \cos \pi t/T_m) & t \leq T_m \\ &= 0 & t > T_m \end{aligned} \quad (6.2)$$

where  $T_m$  is a length of time less than 5 or 10 percent of the total sample length. This choice of  $D(t)$  results in a  $Q(f)$  which has a main lobe roughly triangular in shape with base width  $2/T_m$  Hz and with sidelobes of height less than 3 percent of the main lobe.

In this study resolution of the features of the spectra was judged somewhat more important than agreement between the observed spectra and those of some imaginary sample population, and so  $T_m$  was chosen at approximately 20 percent of the total sample length. In practice, it was found necessary to numerically prewhiten the data to minimize the effects of the sidelobes of  $Q(f)$ . The exact amount of prewhitening necessary depends on the exact shape of the spectrum being estimated. By trying different numerical filtering of the input data, and different values for  $T_m$ , it was found that a prewhitening of the highest frequencies by 20 to 26 dB in power over the lowest frequencies was adequate for consistent results. The details of the prewhitening method employed, and the corrections necessary to remove the effects of the prewhitening from the final results, as well as the modifications necessary for computation with data in digital form are given in Blackman and Tukey.

## VII. EXPERIMENTAL RESULTS

### 7.1 Introduction

Experimental data was collected during the Spring of 1968 and processed as discussed in Chapter VI. Since the majority of scintillation observed occurred during the hours of darkness, and since that which did sometimes occur during the daylight hours was usually found at the extreme ends of the pass, attention was given only to the nighttime records. Due to the lack of an automated data reduction system only a few passes could be analyzed in detail. Six passes were chosen which displayed to the eye varying degrees of scintillation activity. It is felt that these passes are varied enough to indicate the types of results to be expected from a spaced receiver experiment. The orbital characteristics and times of closest approach are given in Table I.

### 7.2 Height of the Scintillating Region and Orientation of the Ground Pattern Using the Method of Similar Fades

The one dimensional method described in section 4.6 provides a simple way of obtaining approximate values for the height of the scintillating region without the need of a full correlation analysis. This technique, in fact, does not even require that the correlogram on the ground be Gaussian, but simply that the pattern elongation be large, and the station spacings be close enough so that similar features are recognizable to the eye at the three stations. Since

Table I. Satellite Orbital Data.

| Orbit<br>Number | Date    | Direction<br>of Pass | Time of Closest<br>Approach (EST) |
|-----------------|---------|----------------------|-----------------------------------|
| 17616           | 4-14-68 | South-North          | 0136,44"                          |
| 18141           | 5-22-68 | North-South          | 0558,28"                          |
| 18154           | 5-23-68 | North-South          | 0439,58"                          |
| 18168           | 5-24-68 | North-South          | 0507,26"                          |
| 18359           | 6-07-68 | North-South          | 0241,51"                          |
| 18523           | 6-19-68 | North-South          | 0107,40"                          |

the results obtained with the full correlation analysis do show that the elongation was indeed large, the selected passes were played back at oscillograph speeds of 10 and 25 millimeters per second, and the time delays between similar features in the amplitude patterns at the three ground stations were estimated by eye at 10 to 20 second intervals for all portions of the passes in which similar scintillating features were recognizable.

The major error in this method is the time uncertainty in estimating the time of maximum crosscorrelation by eye since a scintillating region of appreciable thickness will cause the shape of the features to be slightly different at each station. To minimize this effect as much as possible, the time delays corresponding to several closely spaced features were obtained, and the average value of these readings was used in the calculations. The time uncertainty introduced by this technique is estimated at approximately  $\pm .35$  seconds. The resulting geocentric heights were calculated from equations 4.3, 4.4, and 4.33. These heights, together with error bars corresponding to the estimated error in obtaining time delays, are shown in Figures 7.1 through 7.6. The horizontal scale in these figures is the latitude of the ionospheric point intersected by the straight line from the satellite to receiving station one. Continuing scintillation for which no heights could be obtained due to the weakness of the satellite signal, or the lack of similar fades at all three ground stations, is indicated by an arrow. A cross indicates the sudden onset or cessation of scintillation.

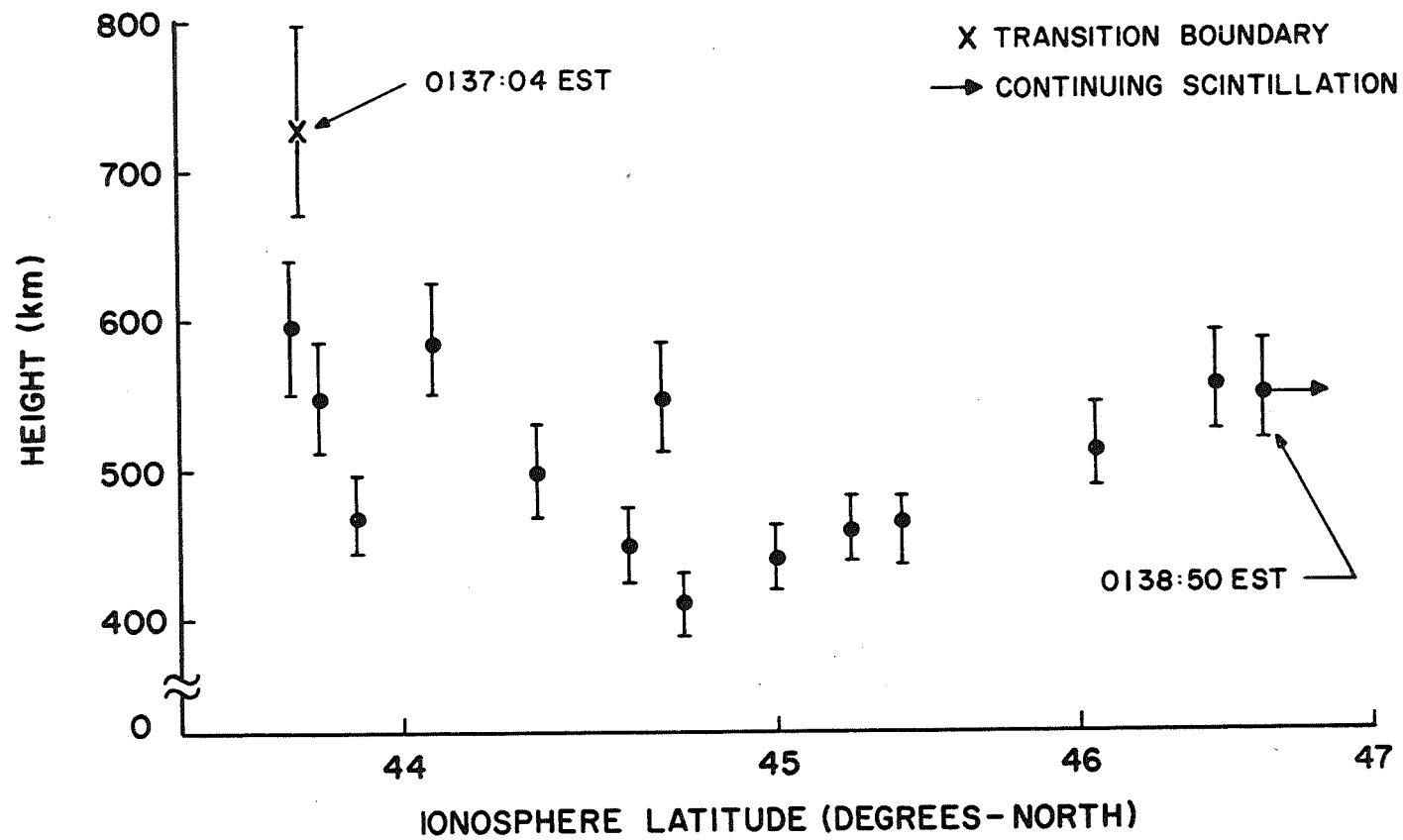


Figure 7.1. Irregularity heights for pass 17616, 14 April 1968.

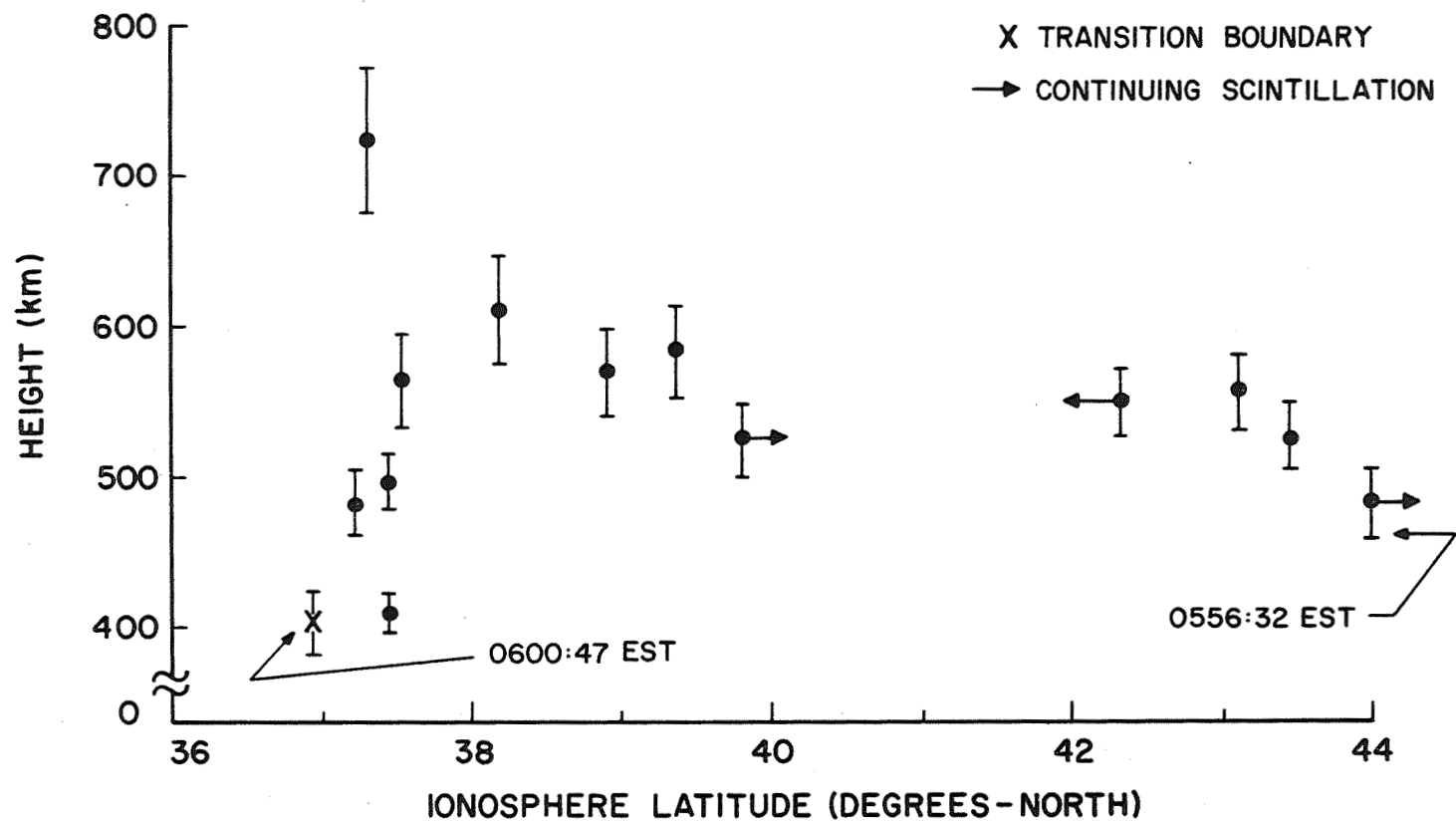


Figure 7.2. Irregularity heights for pass 18141, 22 May 1968.

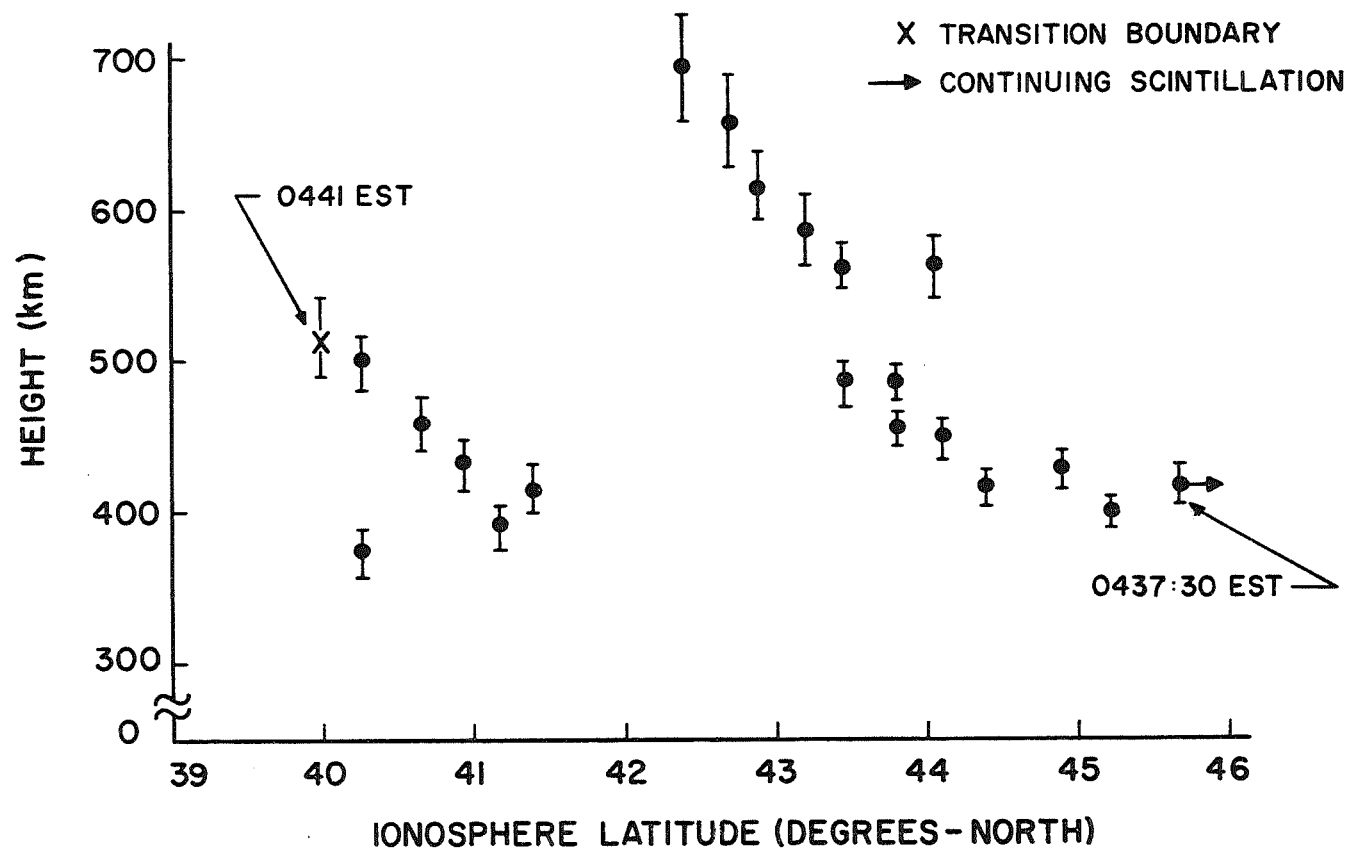


Figure 7.3. Irregularity heights for pass 18154, 23 May 1968.

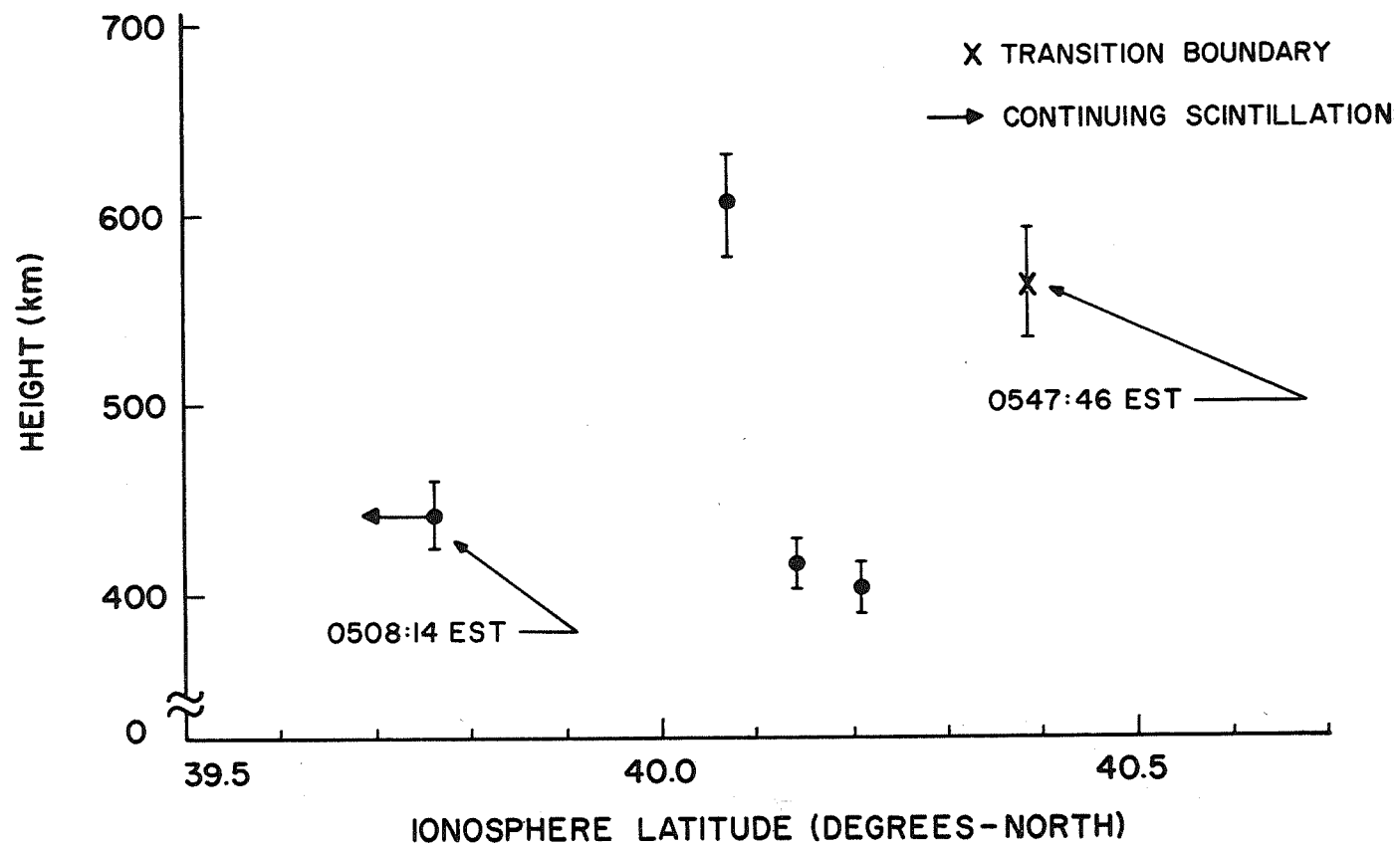


Figure 7.4. Irregularity heights for pass 18168, 24 April 1968.

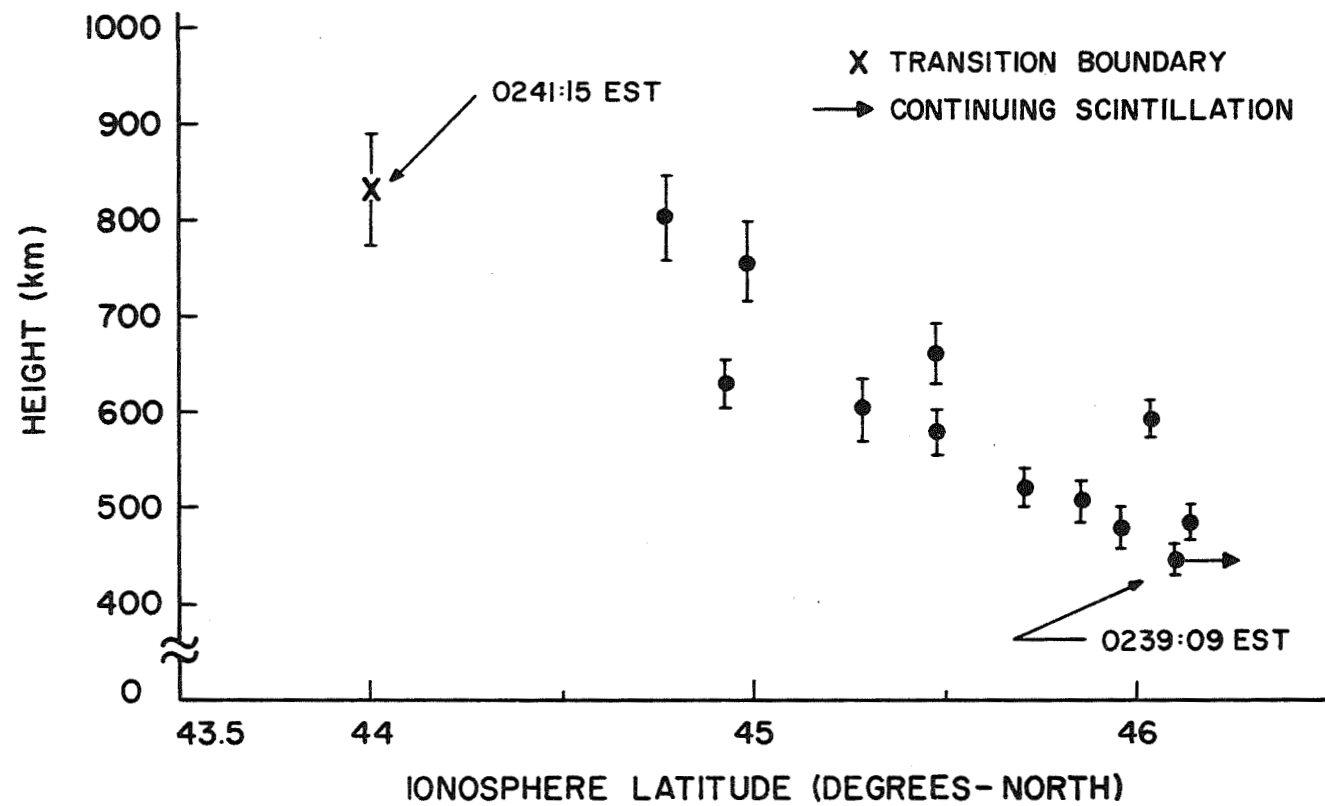


Figure 7.5. Irregularity heights for pass 18359, 7 June 1968.

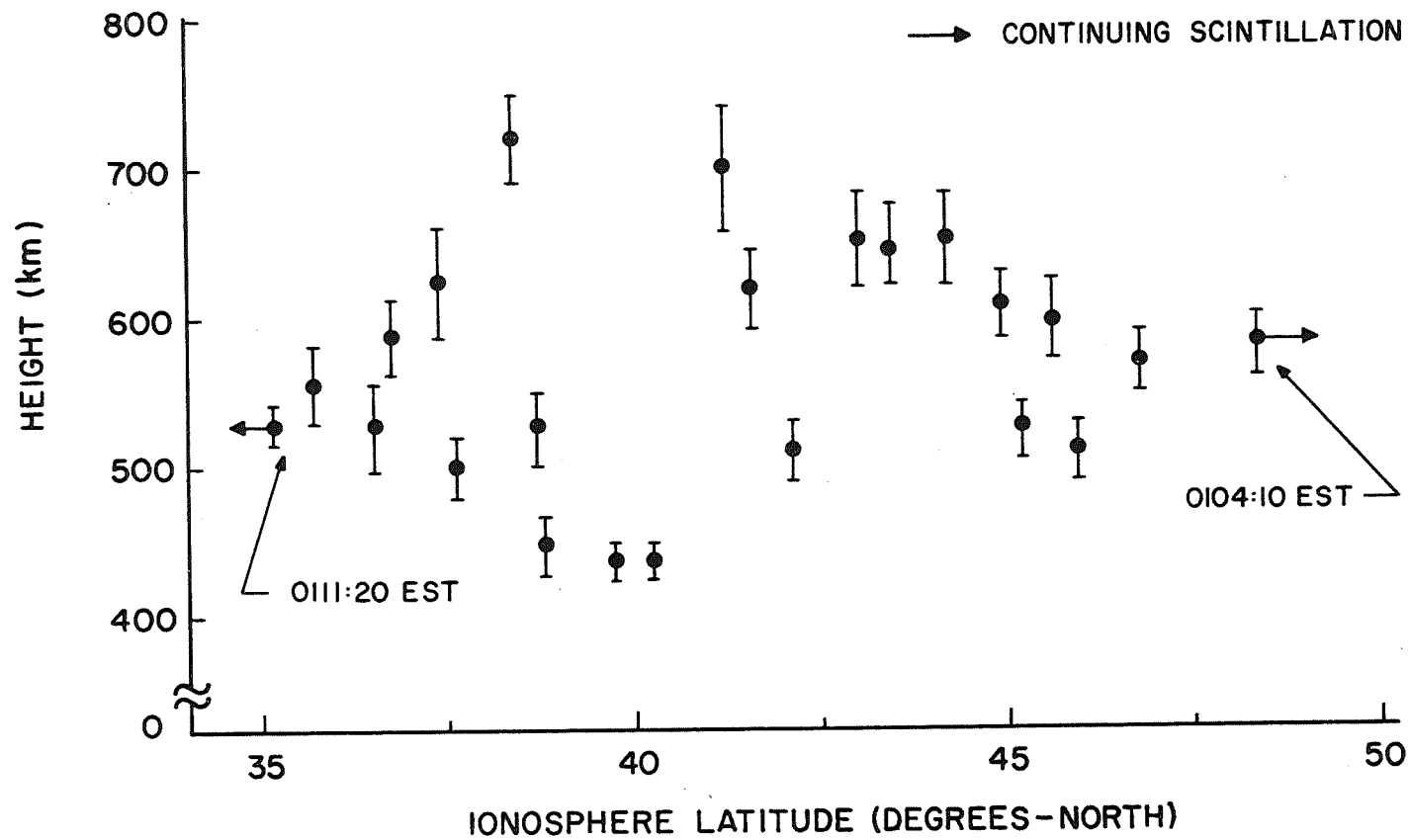


Figure 7.6. Irregularity heights for pass 18523, 19 June 1968.

No evident relation between the occurrence of scintillation, or the height of the scintillation region, and the position of the satellite could be found although it was clear that it was more usual for scintillation to be seen to the north, rather than to the south of the station. Undoubtedly some of the scatter of the heights obtained was due to errors in obtaining time delays which exceeded the estimated error limits. However, it is apparent that a complex height structure still existed and that substantial variations in the height of the scintillating region occurred during a single pass.

The apparent existence of more than one value of height for a single ionospheric latitude was undoubtedly due to a nonuniform irregularity distribution with height as discussed by Jespersen and Kamas (1964). Figure 7.7, which is taken after Jespersen and Kamas, illustrates this effect. It should be pointed out, however, that the absence of such multiple valued structure does not necessarily indicate a region of small height extent. It may simply indicate a region of comparatively uniform height distribution.

The orientations of the lines of maximums of the amplitude patterns on the ground were also calculated from the measured time delays with equation 4.32. The theoretical orientations of the patterns on the ground caused by ellipsoidal irregularities elongated by a factor of 10 to 1 along the Earth's magnetic field were obtained from the ground contours given in equation 3.42. For these calculations, the theoretical value of the Earth's magnetic field was computed at the height given by each set of time delays. In all

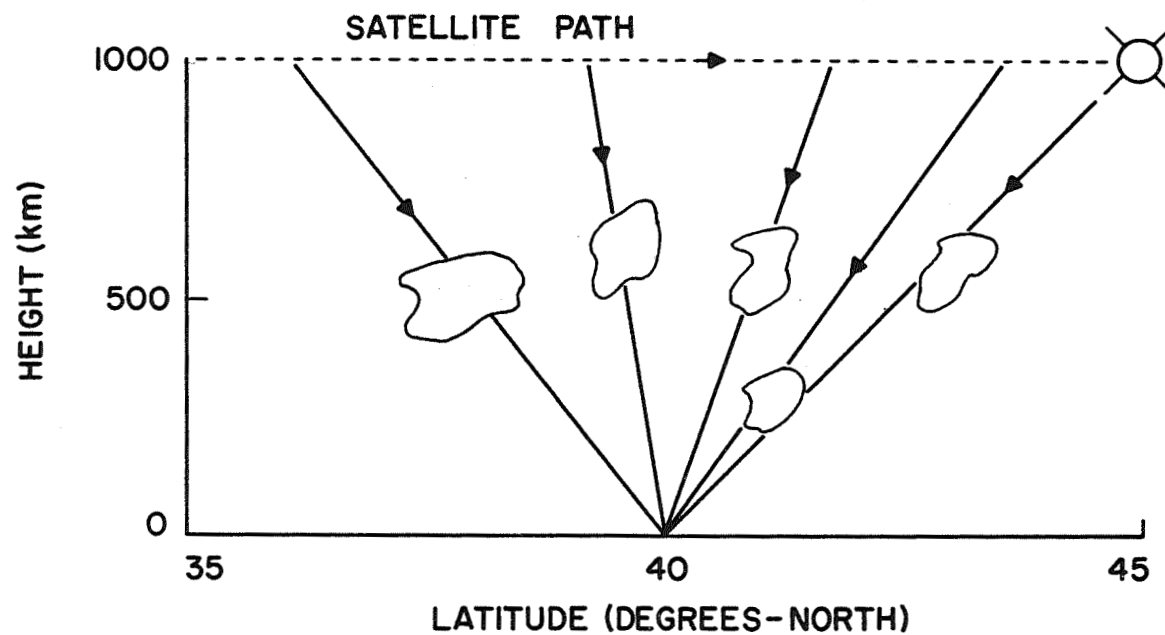


Figure 7.7. Diagrammatic representation of irregularity height distribution which can cause multiple values of irregularity height at a single value of ionospheric latitude.

cases the theoretical and experimental orientations were very close, the r.m.s. deviation between the two orientations being less than seven degrees, which is not considered to be significant.

### 7.3 Correlation Analysis Results

#### 7.3.1 Correlogram Characteristics

Since the irregularity heights discussed in section 7.2 changed so greatly with small changes in latitude, and since the amplitude scintillation index was often observed by eye to change substantially during a short section of a satellite pass, the patterns on the ground could not be expected to remain statistically stationary for an extended period of time. The manual method of data sampling which was employed also made the transferral of large amounts of data onto IBM cards impractical. Therefore, the length of a data sample for the correlation analysis was chosen as 10 to 12 seconds. Ten sections of oscillograph records were chosen which appeared to the eye to display varying degrees of scintillation. It was felt that these samples showed sufficient diversity to indicate the types of ionospheric effects which were occurring.

On analysis, one of the data sections yielded a maximum crosscorrelation in amplitude between any pair of stations of less than .5. Such low values of crosscorrelation were probably due to a very thick irregularity layer. Since the parameters of such a layer could not be expected to be obtained from the analysis

which was being employed, this data section was discarded. The remaining data sections are identified in Table II, where the times given refer to the time at the middle of the data section. In this Table,  $\rho_{\max 1, 2}$  and  $\rho_{\max 1, 3}$  are the maximum values of the crosscorrelations in amplitude between stations one and two and one and three respectively.  $S_a$  is the amplitude scintillation index defined in equation 2.16.

The experimental amplitude and phase correlation functions were fitted with theoretical curves as described in Chapter VI. The amplitude autocorrelation and crosscorrelations were fitted with Gaussian forms with zero and nonzero mean values respectively. The phase difference autocorrelations,  $\rho_{\phi d}$  were fitted with that theoretical form of  $\rho_{\phi d}$  given in equation 4.37 which was pertinent to a Gaussian form for  $\rho_{\phi}$ . In general, it was found that the fits for the amplitude correlations were good for values of experimental correlation higher than about .3. The fits to the phase difference autocorrelations were not quite so good, but were satisfactory for values of phase difference autocorrelation greater than about .5. Typical examples of the experimental and fitted theoretical curves are shown in Figures 7.8 through 7.10.

### 7.3.2 Ground Pattern Characteristics

Once the equivalent parameters of the ground patterns had been obtained, the sizes and orientations of the ground patterns deduced from equations 4.13 through 4.15, 4.38, and 4.42. The values of mean square phase deviation,  $\phi_0$ , could be obtained from

Table II. Amplitude Crosscorrelations and Scintillation Indexes.

| Data<br>Section<br>Number | Pass<br>Number | Time<br>(EST) | $\rho$ max 1-2 | $\rho$ max 1-3 | $S_a$ |
|---------------------------|----------------|---------------|----------------|----------------|-------|
| 1                         | 17616          | 0137,18"      | .96            | .95            | .312  |
| 2                         | 18141          | 0559,04"      | .48            | .90            | .145  |
| 3                         | 18141          | 0600,03"      | .84            | .96            | .495  |
| 4                         | 18154          | 0438,47"      | .86            | .84            | .485  |
| 5                         | 18154          | 0440,20"      | .74            | .88            | .215  |
| 6                         | 18168          | 0507,55"      | .77            | .61            | .217  |
| 7                         | 18359          | 0240,29"      | .92            | .90            | .123  |
| 8                         | 18523          | 0105,43"      | .90            | .73            | .283  |
| 9                         | 18523          | 0108,50"      | .58            | .72            | .226  |

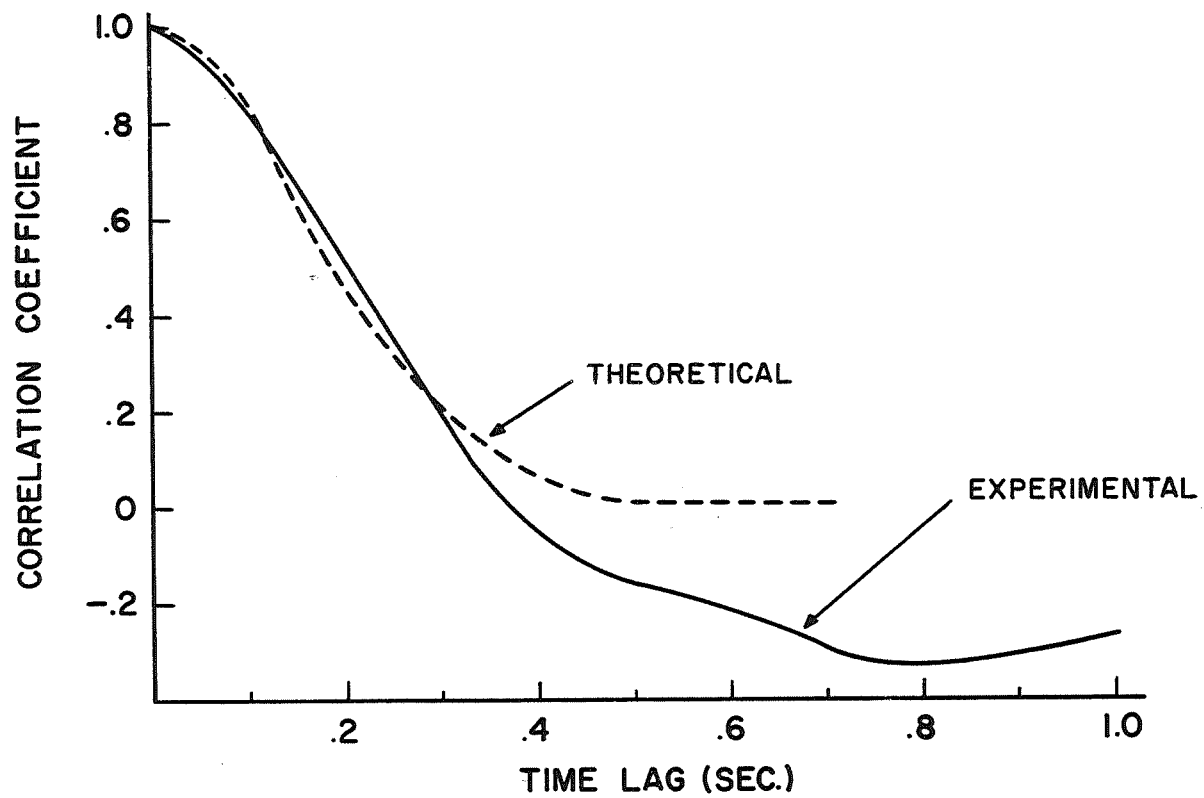


Figure 7.8. Autocorrelation in amplitude at station 1 for pass 18141, 22 May 1968, 0600 EST.

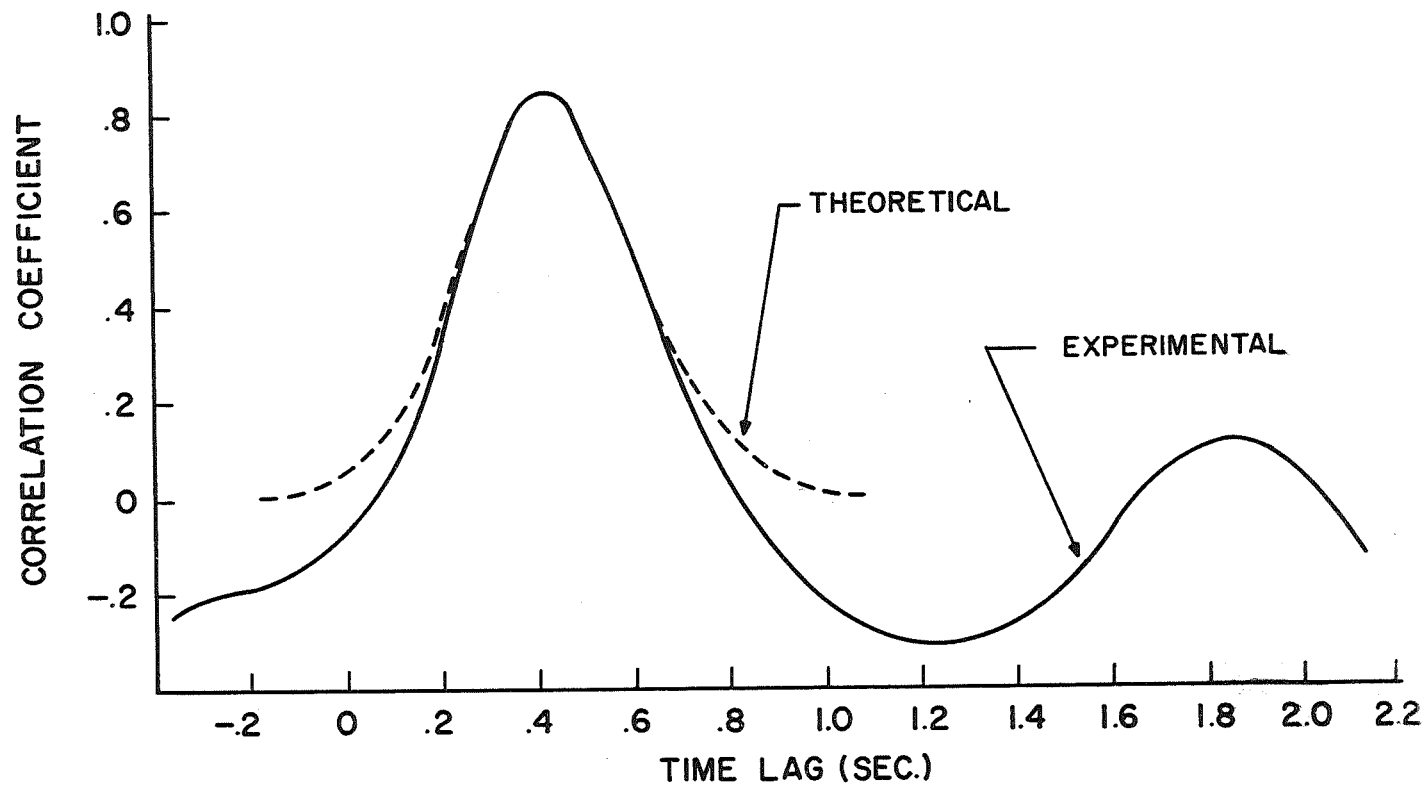


Figure 7.9. Crosscorrelation in amplitude between stations 1 and 2 for pass 18141, 22 May 1968, 0600 EST.

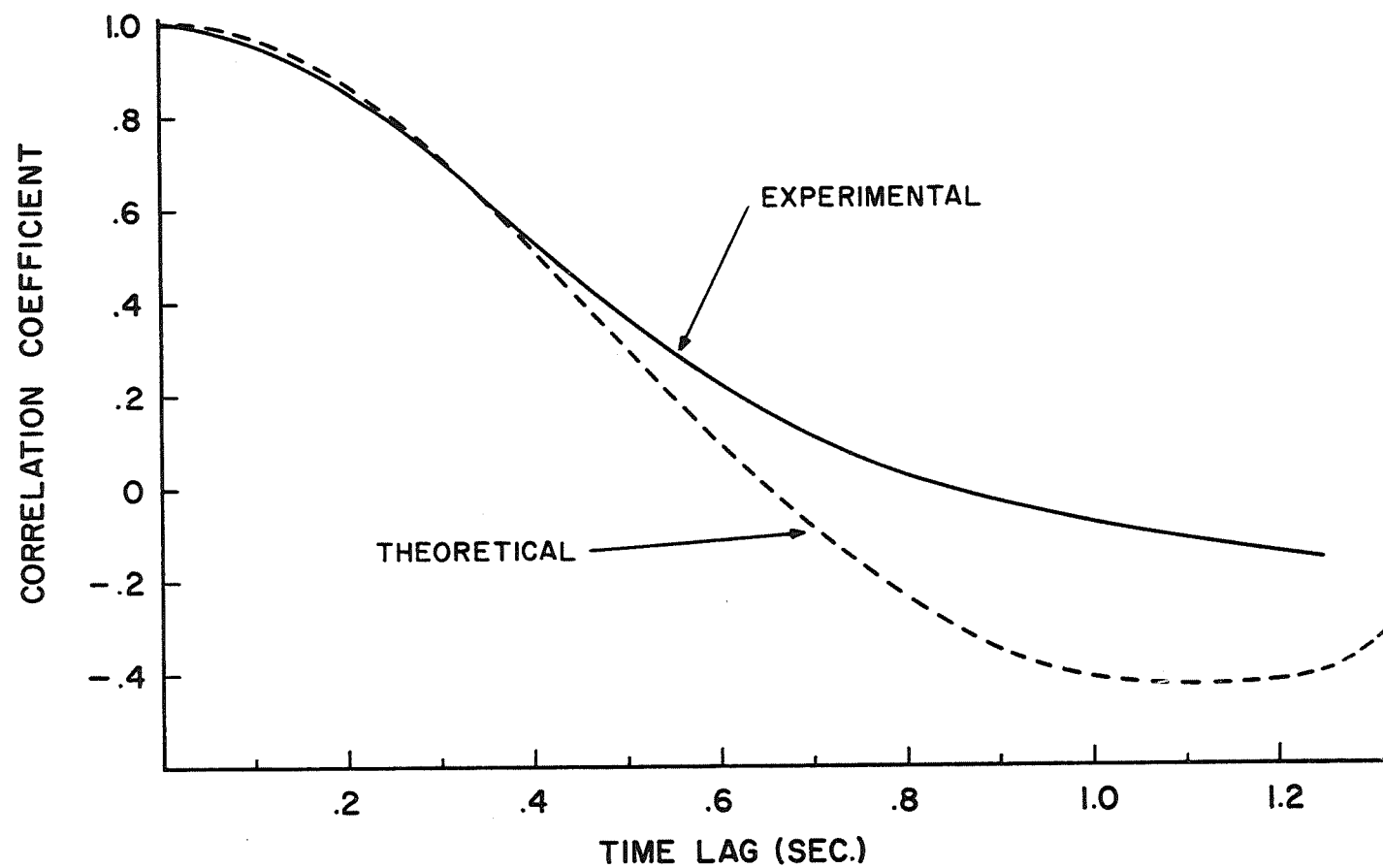


Figure 7.10. Autocorrelation of phase difference between stations 1 and 2 for pass 18141, 22 May 1968, 0600 EST.

equation 4.36. Since the phase analysis depended on a one dimensional approximation to the ground phase pattern, it is of interest to obtain the amplitude pattern orientations and minor axis scales from the one dimensional similar fades analysis.

For this purpose, several sets of time lags between similar features were obtained from each 10 to 12 second section of record to which the correlation analysis had been applied. A value of  $V_g$  was obtained from each set of time lags with equation 4.33. These values were then averaged to yield average values of pattern drift velocity for each 10 to 12 second period. The scale sizes in time of the autocorrelations obtained at station one from the correlation analysis were then converted into scale sizes in distance by multiplying by the average values of drift velocity discussed above. Similarly, average values of pattern orientation were computed from the observed time lags and equation 4.32. These values were used to convert the scales obtained above into scales along the direction transverse to that of pattern elongation.

The results of these calculations are given in Tables III and IV. All angles in Table IV are measured from local east. The theoretical orientations in Table IV were obtained by determining the orientations of the major axes of the ellipses determined by equation 3.42 for an irregularity elongated by a factor of 10 along the direction of the Earth's magnetic field. The good agreement obtained between the amplitude pattern parameters deduced from the two dimensional full correlation analyses and the parameters

Table III. Ground Scale Sizes and Phase Deviations

| Data<br>Section<br>Number | <u>Amplitude Measurements</u> |                           |                    | <u>Phase Measurements</u> |           |
|---------------------------|-------------------------------|---------------------------|--------------------|---------------------------|-----------|
|                           | Major Axis                    | Minor Axis                | Minor Axis         | Minor Axis                | RMS       |
|                           | Scale                         | Scale                     | Scale              | Scale                     | Phase     |
|                           | (Correlation<br>Analysis)     | (Correlation<br>Analysis) | (Similar<br>Fades) |                           | Deviation |
|                           | km                            | km                        | km                 | km                        | radians   |
| 1                         | 13.63                         | 1.34                      | 1.39               | 5.27                      | 3.34      |
| 2                         | 5.96                          | .73                       | .73                | 3.66                      | .72       |
| 3                         | $\infty$                      | .81                       | .88                | 3.56                      | 2.75      |
| 4                         | $\infty$                      | .57                       | .51                | 1.84                      | 1.77      |
| 5                         | $\infty$                      | .54                       | .54                | 3.93                      | 1.73      |
| 6                         | 2.13                          | .61                       | .65                | 1.39                      | .28       |
| 7                         | 5.74                          | .83                       | .87                | 5.57                      | .84       |
| 8                         | 2.77                          | .42                       | .43                | 1.46*                     | .29       |
| 9                         | 2.68                          | .57                       | .57                | 6.32                      | 2.99      |

\* Size along 1-2 baseline.

Table IV. Pattern Orientations

| Data<br>Section<br>Number | Theoretical<br>Orientation<br><br>degrees | <u>Amplitude Pattern</u> |                  | Phase<br>Pattern<br><br>degrees |
|---------------------------|-------------------------------------------|--------------------------|------------------|---------------------------------|
|                           |                                           | Correlation<br>Analysis  | Similar<br>Fades |                                 |
|                           |                                           | degrees                  | degrees          |                                 |
| 1                         | -41.2                                     | -42.3                    | -41.1            | -42.0                           |
| 2                         | - 3.0                                     | - 4.1                    | - 2.1            | - 6.5                           |
| 3                         | 24.5                                      | 29.3                     | 19.7             | 28.4                            |
| 4                         | 57.1                                      | 61.1                     | 58.5             | 64.3                            |
| 5                         | 26.8                                      | 30.0                     | 27.6             | 34.5                            |
| 6                         | 83.0                                      | 79.3                     | 83.0             | 59.7                            |
| 7                         | 64.4                                      | 72.0                     | 64.4             | 58.6                            |
| 8                         | 71.8                                      | 77.4                     | 74.3             | *                               |
| 9                         | 3.5                                       | 6.0                      | 7.0              | 0.0                             |

\* Not available.

obtained from the one dimensional similar fades approach provides justification for the analysis used for the phase pattern. Except for data sections 6 and 8, the agreement between the orientations of the amplitude and phase patterns shows that the amplitude and phase effects are both due to field aligned irregularities.

Unfortunately, no real value of pattern orientation could be obtained from the data of section 8, and so the phase scale size given for this data section is measured along the 1-2 baseline. It will be noted that in both sections 6 and 8, the theoretical pattern orientation is nearly parallel to the 1-2 baseline. Under these circumstances the mean square values of phase difference obtained on this baseline will be very small, and any noise in the system would tend to have a large effect. A reduction of signal coherence, and consequently, scale size, along the 1-2 baseline coupled with a large value of mean square phase difference along the 1-3 baseline would yield imaginary values of field orientation as actually occurred for data section 6.

If the irregularities responsible for the phase effects were at a considerably different height than those causing the amplitude effects, the ground velocity used to convert the phase measurements from units of time to units of distance would be incorrect and could cause discrepancies in the results to occur. However, when the effect of the point source magnification is considered, this effect is minimal and a height difference of more than 200 kilometers would be necessary to yield field alignment from the data obtained

from data section 8. Horizontal gradients in the ambient ionization or non field aligned irregularities, possibly at D region heights, could also be responsible for discrepancies in the orientation of the phase pattern, and yet not effect the amplitude pattern appreciably.

### 7.2.3 Ionospheric Parameters

To convert the ground scale sizes into ionospheric sizes, the average height of the irregularity layer must be known. Although this can be obtained from the similar fades analysis, it is of interest to obtain the layer height directly from values of  $V_g$  obtained from the correlation analysis since these values may differ somewhat from values of  $V_g$  obtained from the similar fades analysis when the layer is very thick. As discussed in Chapter VI, equation 4.16 is highly inaccurate for an elongated ground pattern, and  $V_g$  is obtained instead from equation 4.18 and a knowledge of the pattern orientation. With the average layer height known, the corrections for the oblique incidence of the incident wave and the finite distance of the source from the layer which were developed in section 3.42 of Chapter III may be applied to yield apparent ionospheric scale sizes.

The layer thickness as obtained by the James (1962) method can be obtained from a knowledge of the characteristic random velocity,  $V_c$ , through equation 4.21. Since the 1-2 baseline is almost parallel with the satellite track, equation 4.20 provides a more accurate value for  $V_c$  than does equation 4.19.

The results of these calculations are given in Table V. The layer thickness is defined as  $4\Delta_t$ ,  $\Delta_t$  being .707 standard deviations of the irregularity height distribution which is assumed to be Gaussian as defined in connection with equation 4.21. Since the true pattern drift velocity as measured by the correlation analysis is equal to the difference of the squares of the drift velocity as measured by the similar fades method and the characteristic random velocity (Briggs, 1968), a 300 km thick layer with average height of 500 km would yield a difference in height of only 1 kilometer between the full correlation and similar fades methods. The differences actually observed are therefore probably due to the fact that the correlation technique provides a better representation of the average behavior of the pattern throughout a long time interval than does the similar fades technique, which essentially looks at only a few of the larger pattern features.

It should be again pointed out that the values of ionospheric scale size obtained from the amplitude pattern may not correspond to the actual size of any irregularity due to the effects of the spatial filtering of the diffraction process in the near field, and the large values of ionospheric phase deviation which exist. If the ray optical approximation holds, the phase scale sizes would be expected to be representative of actual ionospheric parameters. However, the large values of scintillation index observed in data sections 3 and 4, and the comparatively small change in the scale sizes observed on the ground as compared to the large difference in phase deviations evident between these sections and

Table V. Irregularity Parameters

| Data<br>Section<br>Number | <u>Amplitude Measurements</u>                               |                                                      |                              | Irregularity<br>Elongation | Minor<br>Axis<br>Scale | Phase<br>Measure-<br>ment<br>Minor<br>Scale |
|---------------------------|-------------------------------------------------------------|------------------------------------------------------|------------------------------|----------------------------|------------------------|---------------------------------------------|
|                           | Geocentric<br>Height<br>(Correlation<br>Analysis)<br><br>km | Geocentric<br>Height<br>(Similar<br>Fades)<br><br>km | Layer<br>Thickness<br><br>km |                            |                        |                                             |
| 1                         | 569                                                         | 589                                                  | 36                           | 8.2                        | .59                    | 2.39                                        |
| 2                         | 547                                                         | 528                                                  | 295                          | 13.4                       | .33                    | 1.74                                        |
| 3                         | 474                                                         | 500                                                  | 77                           | $\infty$                   | .40                    | 2.02                                        |
| 4                         | 479                                                         | 469                                                  | 74                           | $\infty$                   | .31                    | 1.01                                        |
| 5                         | 400                                                         | 406                                                  | 88                           | $\infty$                   | .34                    | 2.50                                        |
| 6                         | 477                                                         | 516                                                  | 224                          | 17.0                       | .32                    | .73                                         |
| 7                         | 654                                                         | 644                                                  | 83                           | 5.7                        | .35                    | 2.34                                        |
| 8                         | 607                                                         | 591                                                  | 68                           | 5.7                        | .19                    | .66*                                        |
| 9                         | 448                                                         | 466                                                  | 170                          | 7.4                        | .25                    | 2.93                                        |

\* Size along 1-2 baseline direction.

the corresponding sections 2 and 5 obtained later in the two respective passes indicates that the ray optical approximation probably did not hold during these two periods, and the resulting scale sizes and phase deviations must be considered as minimum values only.

The relatively small values of  $S_a$ ,  $\phi_0$ , and minor axis scale size for data section 6 suggest that this data section may be amenable to exact analysis with the small phase deviation theory. If the theoretical, rather than the experimental direction of orientation is employed, the scale size of the ground phase pattern along the minor axis direction becomes .69 km. If the Gaussian form of ionospheric phase autocorrelation is assumed to hold and the two dimensional small phase deviation analysis of Bowhill (1961) is applied, it is found that an irregularity characterized by a minor axis scale of .39 km and a major axis scale of 8.8 km gives closest agreement with the observed data. Such an irregularity would produce an amplitude pattern on the ground of scale .62 by 2.6 km after correction for diffractive point source, and zenith angle effects. The amplitude scintillation index computed from the observed value of  $\phi_0$  would be .22. The phase pattern would have a minor axis scale on the ground of .67 km.

When these values are compared to the values actually obtained of .61 by 2.1 km for the scale size in amplitude, .22 for  $S_a$ , and .69 km for the scale size in phase, it is seen that the agreement is good and that if the discrepancy in orientation is ignored,

it does appear that in this case the small phase deviation theory adequately describes the diffraction patterns of both amplitude and phase.

However, the large variations between the experimental sizes of the amplitude and phase patterns observed for the other data samples implies that the usual theory of a uniform layer of irregularities with Gaussian correlation of ionospheric phase may be somewhat oversimplified. The picture which emerges from the results of this experiment is that of an irregularity layer of varying thickness, often thin locally, but of large height extent when measured over hundreds of kilometers horizontally. The actual relation between the amplitude and phase scale sizes observed depends on the actual ionospheric phase deviation, and on the actual form of the distribution of irregularity sizes. Especially important are the tails of the size distribution where the irregularity sizes most likely to influence the amplitude pattern are seen. Unfortunately, the form of the distribution is unlikely to be truly Gaussian over the whole range of scale sizes, although the patterns produced by the irregularities appear to have Gaussian correlograms for large values of correlation.

#### 7.4 Differential Doppler Results

Since the results of the correlation analysis imply that large irregularities are important in producing the phase pattern, and that the total phase deviation due to the irregularities

reaches high values, it is of interest to look at differential Doppler data available for the same time periods as those used for the correlation analysis. To ascertain the effect of low frequency terms in the spectrum 40 seconds was chosen as a suitable data length, even though it was unlikely that the phase statistics were completely stationary throughout this time interval. Unfortunately, it is only possible to obtain accurate values of phase difference from the records produced by the high speed phase comparator if the 20 MHz signals are of sufficient strength, and if the scintillation is not so severe as to obliterate the quasi-periodic phase changes due to the effects of the ambient ionosphere. It is also desirable that the period of the phase changes due to the ambient ionosphere be fairly slow.

It was found that the 40 second intervals encompassing data sections 2 and 8 best met these requirements. The data was passed through the high speed phase comparator, and the output sampled at a rate of 10 samples per second. To minimize the variation in phase difference due to the varying position of the satellite throughout the data interval, a second degree polynomial was fitted to, and subtracted from the data. This had the effect of filtering out all phase variations of less than  $1/40$  Hz. The resulting data were expressed in terms of radians at 40 MHz and spectrally analyzed as described in Chapter VI.

The resulting spectra are shown in Figures 7.11 and 7.12. For a comparison, the amplitude spectra of the 12 second data

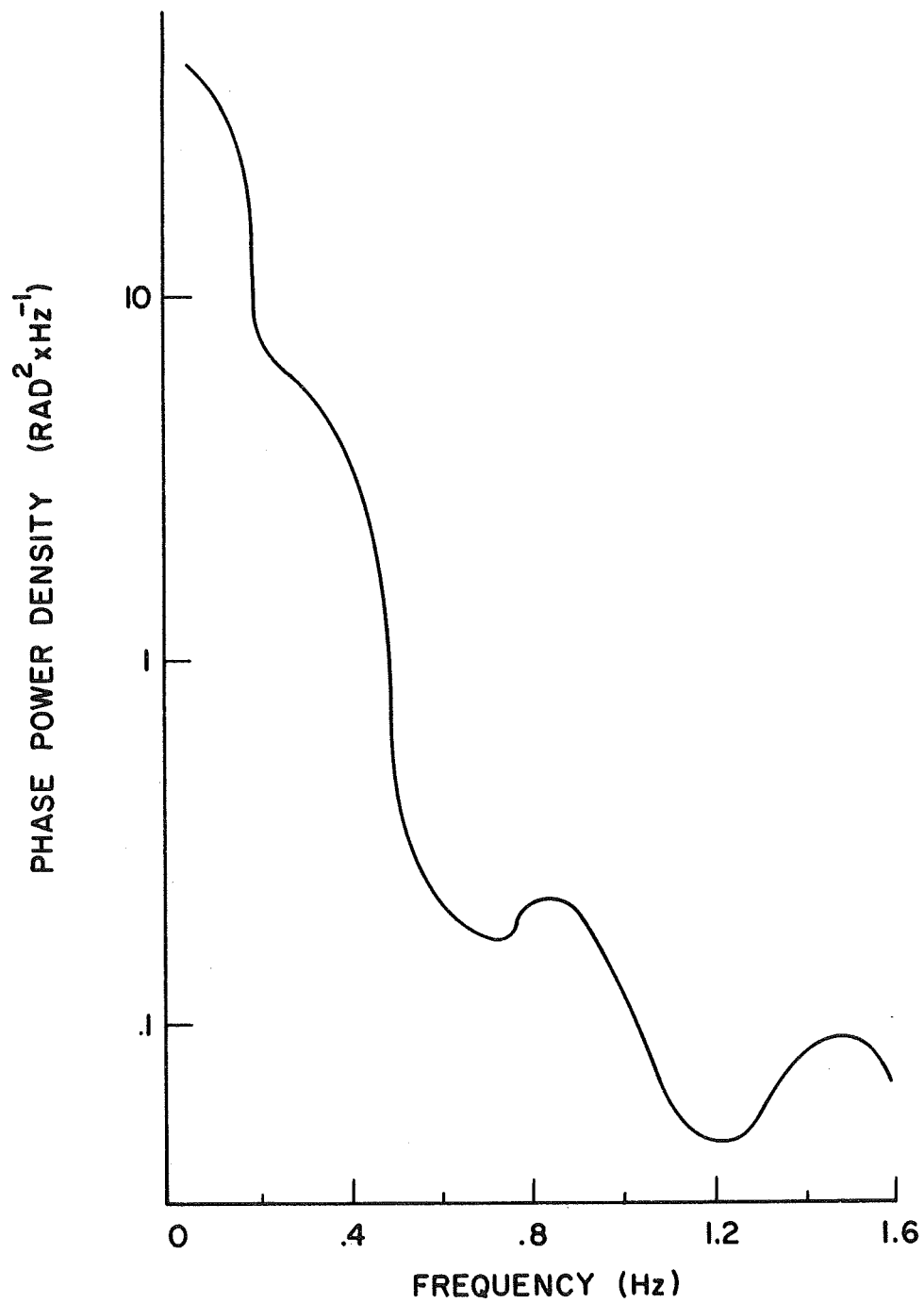


Figure 7.11. Phase power spectrum from Doppler measurements for pass 18141, 22 May 1968, 0559 EST.

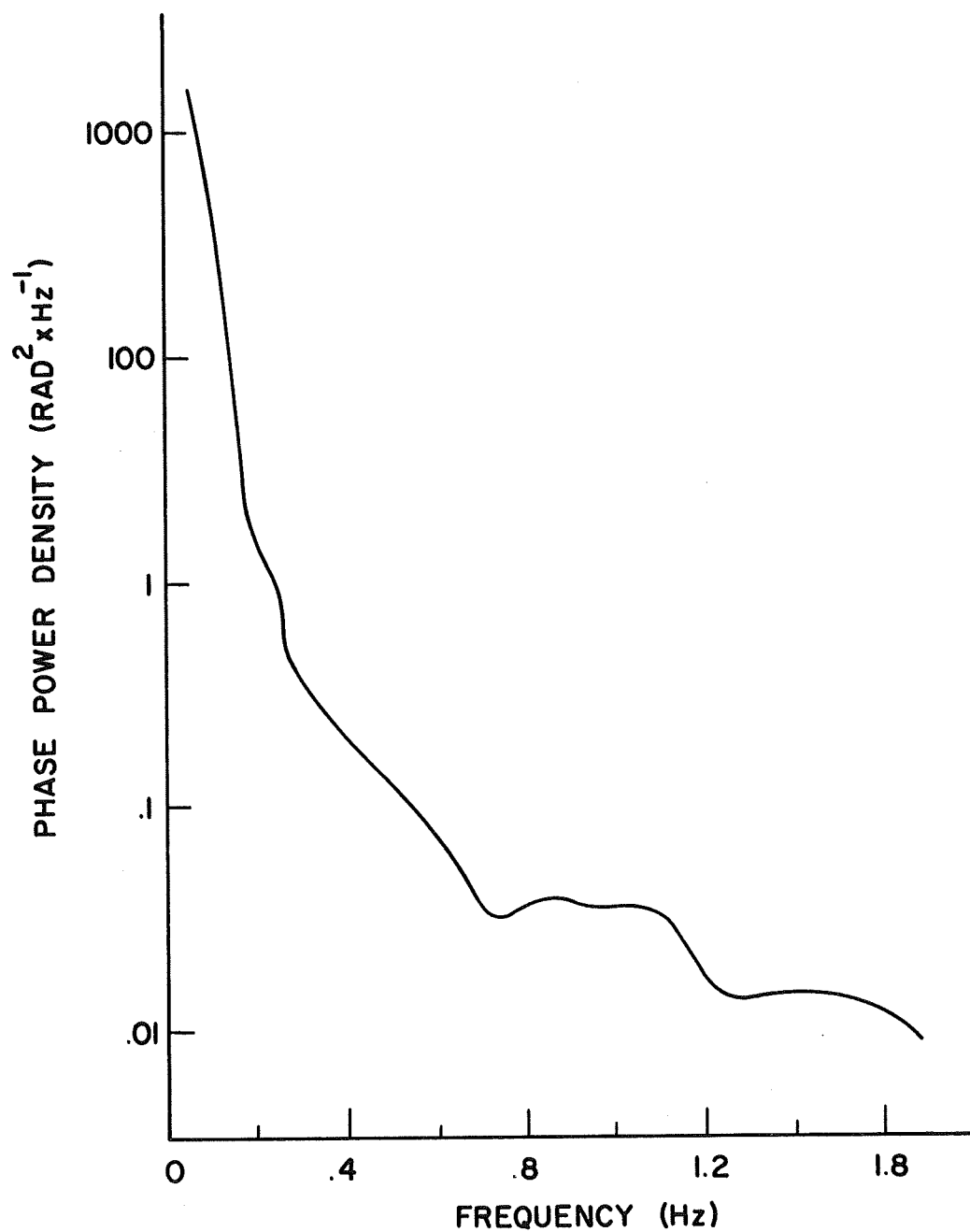


Figure 7.12. Phase power spectrum for pass 18523, 19 June 1968, 0108:50 EST.

sections taken from the same portion of the passes are shown in Figures 7.13 and 7.14. To make the Doppler results more directly comparable to the correlation results, a linear term of 10 second length was "run" through the 40 second data sections in the manner described in section 6.2 of Chapter VI so as to filter out frequencies lower than 1/10 Hz. The resulting data was autocorrelated and the resulting autocorrelation functions are compared in Figures 7.15 and 7.16 to the amplitude autocorrelation functions derived in the correlation analysis. These results clearly show that the phase patterns contain considerably more low frequency components than do the amplitude patterns.

The autocorrelograms of phase may be converted from correlations versus time to those versus distance by multiplying the time scales by the appropriate ground velocities. If this is done, and the autocorrelograms are fitted with equivalent Gaussian functions, the resulting scale sizes may be compared to those deduced from the spaced receiver experiment. After correction for the finite distance of the source, the Doppler results yielded ionospheric minor axis sizes of 2.74 and 3.47 km compared to the spaced receiver results of 1.74 and 2.93 km. The r.m.s. values of phase deviation,  $\phi_0$ , obtained from the Doppler results were .53 and 3.62 radians compared to the values .72 and 2.98 radians of the spaced receiver analysis.

The larger values obtained from the Doppler data are probably due to the imperfect removal of zenith angle and other

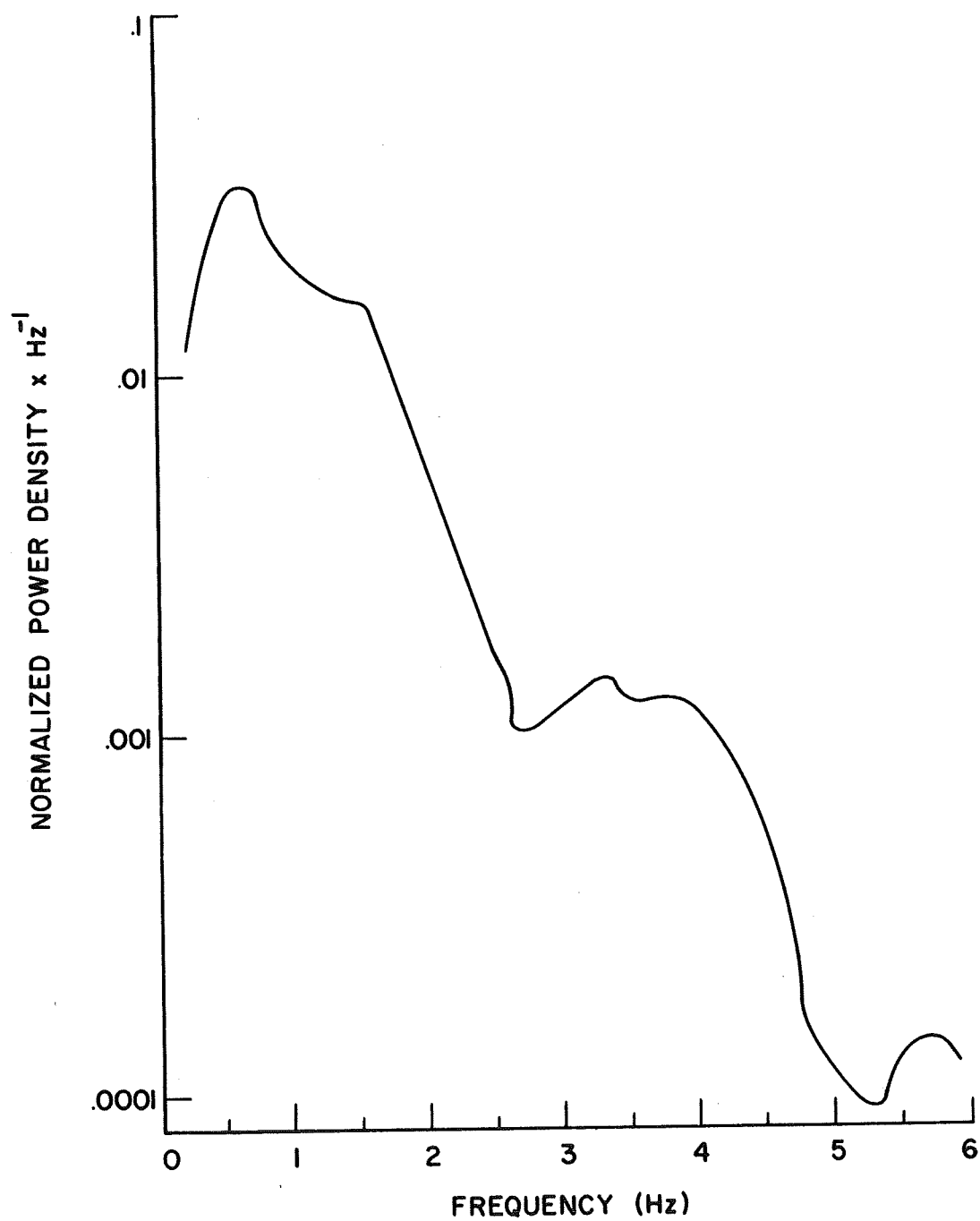


Figure 7.13. Amplitude power spectrum for pass 18141, 22 May 1968, 0559 EST.

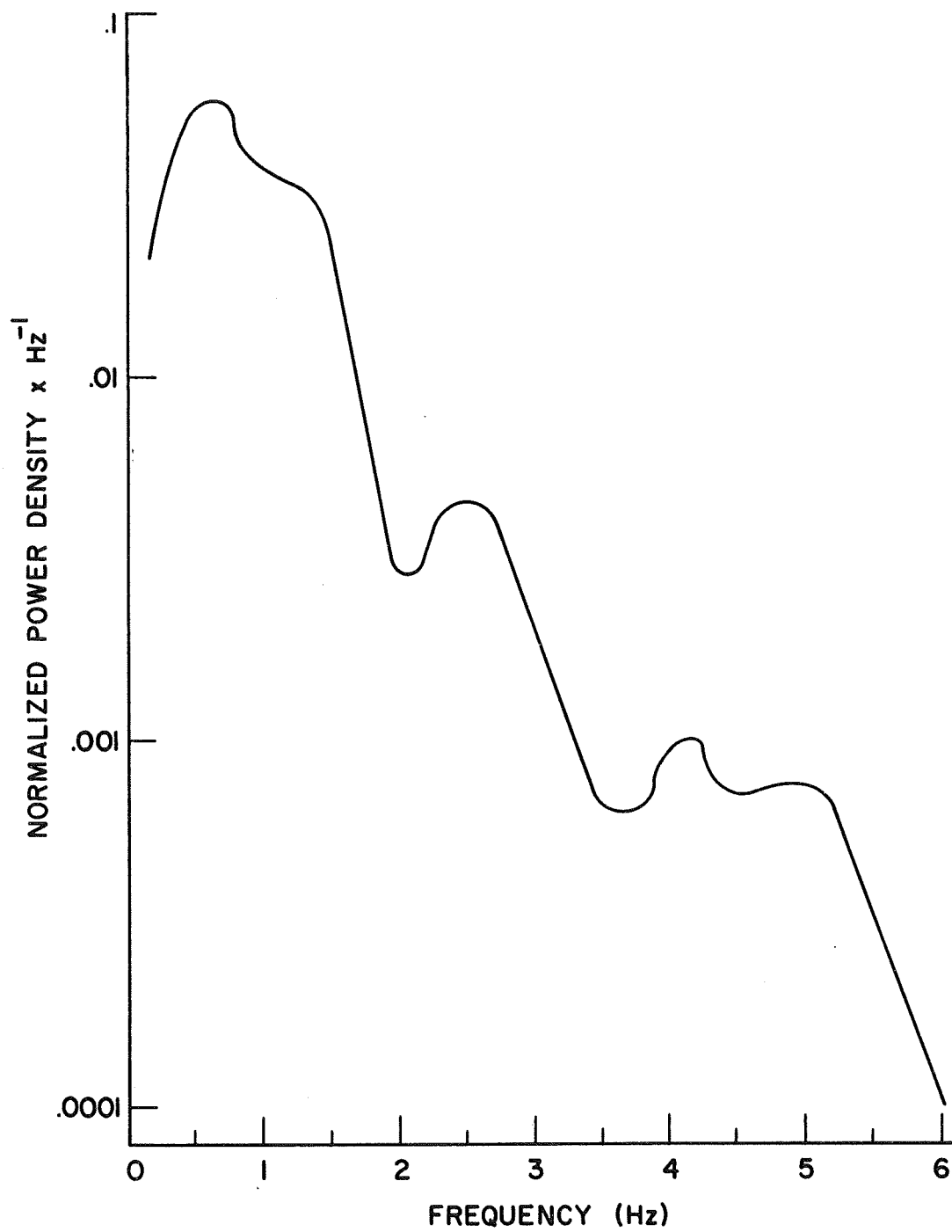


Figure 7.14. Amplitude power spectrum for pass 18523, 19 June 1968, 0108:50 EST.

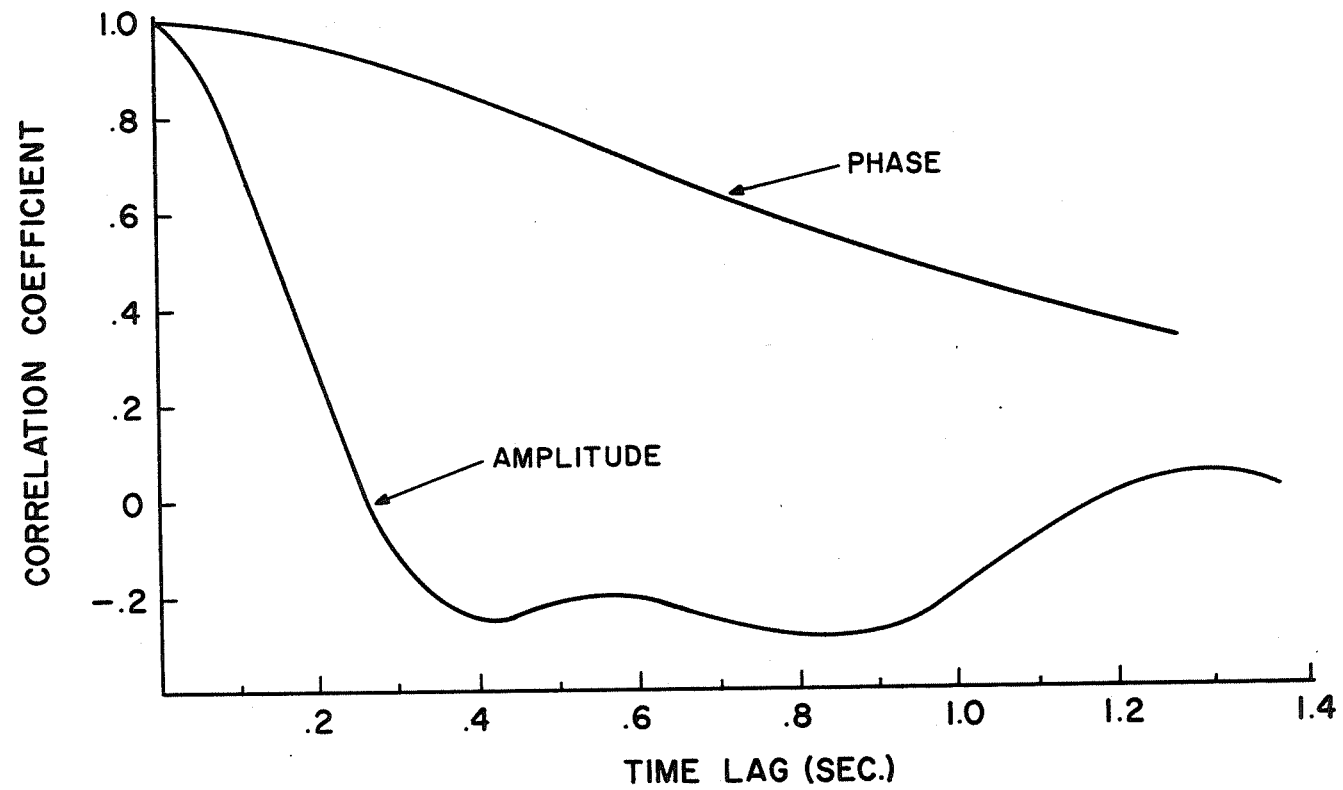


Figure 7.15. Amplitude and phase autocorrelegrams for pass 18141, 22 May 1968, 0559 EST.

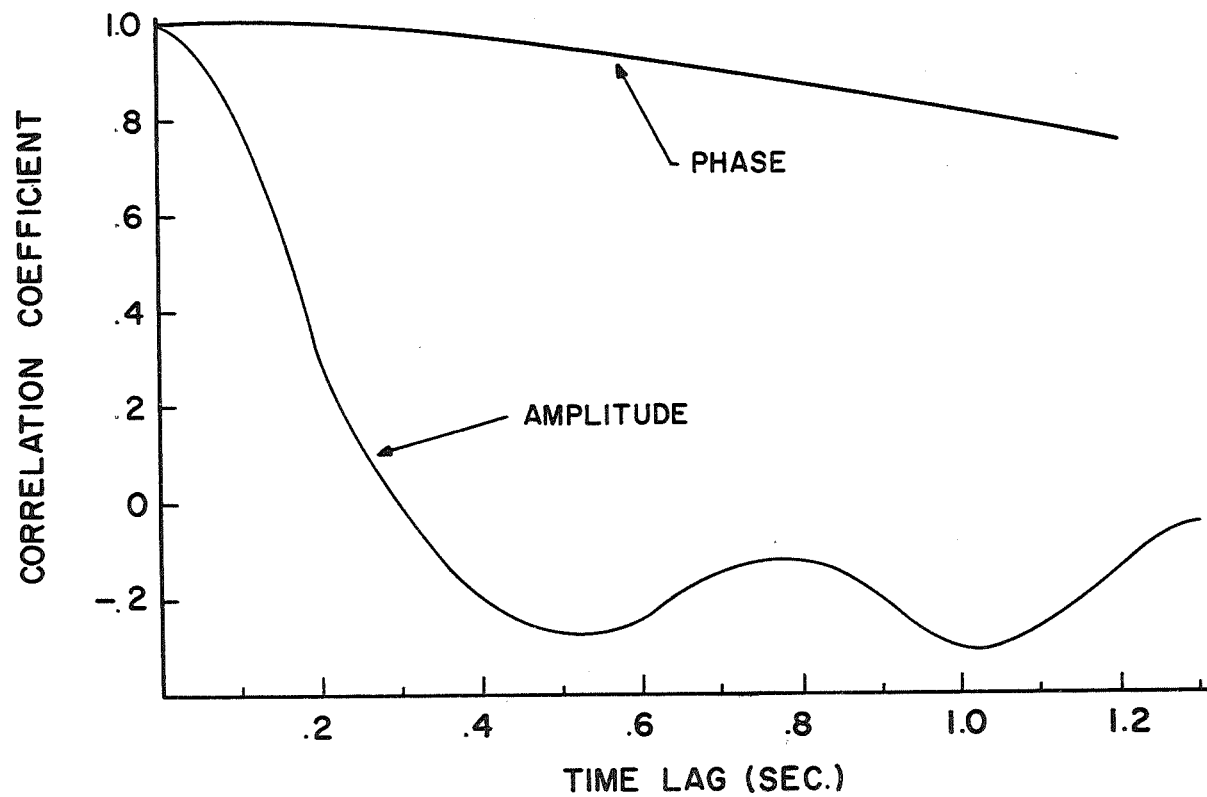


Figure 7.16. Amplitude and phase autocorrelogram for pass 18523, 19 June 1968, 0108:50 EST.

large scale variations which have little effect on the spaced receiver experiment due to the filtering action of the relatively small baseline employed. The nonstationarity of the data may also be involved. Nevertheless it is clear that the phase spectrums are predominately concentrated at the very low frequencies and reach high values there. If the phase spectra of the intervals including data sections 2 and 9 are compared, it is seen that the spectrum of section 9 has a larger ratio of low to high frequencies than in section 2, and these low frequencies represent a much higher phase deviation. Therefore, if the difference between the amplitude and phase scale sizes obtained from the correlation analysis is due to a high value of phase deviation, and a differing spectral distribution of irregularity sizes, it would be expected that the ratio of sizes would be much larger for section 9 than for section 2. In fact such is seen to be the case with the ratios being 11.7 and 4.2 respectively.

## VIII. DISCUSSION AND CONCLUSIONS

### 8.1 Introduction

In this chapter, a comparison is made between the results obtained in this study and those obtained by others. Some sources of error which have not been considered previously are discussed. The findings of the study are summarized, and the conclusions of the study are discussed. Suggestions for further work are presented.

### 8.2 Comparisons with Other Work

#### 8.2.1 Amplitude Pattern Measurements

Hewish (1952) at Cambridge, England in late 1951 and early 1952 estimated the average irregularity height at 400 km, and the ionospheric scale sizes from both amplitude and phase measurements as slightly under 1 km. Kent (1959) obtained minimum heights of 275 to 400 km and amplitude scale sizes of approximately .5 km at Cambridge in 1959. In 1961 DeBarber and Ross (1966) obtained average heights of 350 to 540 km at State College, Pa. They estimated the layer thickness as computed by the James (1962) method at approximately 90 km and obtained ionospheric scale sizes of approximately .5 by 10 km when their correction for the diffractive spatial filtering effects is ignored.

In 1963, Jespersen and Kamas (1964) obtained heights between 250 and 400 km at Boulder, Colorado. The transverse scale sizes

were found to be on the order of 1 km. The average irregularity heights were found to vary appreciably during a single pass in about one third of the passes studied. The layer thickness as determined by the James method was found to vary from 120 to 470 km, with the average thickness being 184 km.

At Urbana, Illinois, McClure and Swenson (1964) obtained essentially the same results during 1963 for the irregularity heights and height variations. They obtained a somewhat smaller spread of layer thicknesses, however, the range of values being 50 to 185 km. They pointed out that these estimates should be more reliable than those of Jespersen and Kamas since their station spacing was a factor of 20 greater. In May and June of 1967 Lyszka (1968) reported heights of approximately 600 km at Sagamore Hill, Massachusetts and at Athens, Greece. The data exhibited considerable scatter and indicated that the height structure was very complicated and could not be represented as a layer of uniform average height. Lyszka also reported that similar measurements at the same locations obtained in 1965 yielded average heights 100 to 200 km lower.

In the auroral zone, Basler and Dewitt (1962) obtained heights ranging from 250 to 1000 km in 1959. In 1960 Frihagen (1963) obtained heights ranging from 200 to 600 km and height spreads of over 200 km in a single pass. The pertinent irregularities were .6 by 1.8 km, but the size estimates were very approximate. Lyszka (1963) obtained heights from 200 to over 800 km in 1962 with height variations in a single pass of over 400 km not being uncommon. He estimated the ionospheric scales at .7 by 10 km.

### 8.2.2 Magnitude of Phase Deviations

Since most of the measurements of irregularity characteristics have concentrated on measurements of the amplitude diffraction patterns, the magnitudes of the total phase deviation experienced by the wave in propagating through the irregularity layers are only obtained indirectly. A large value of  $S_a$  or a Rayleigh probability distribution of received amplitude can be taken as evidence of high phase deviation, but as shown in Chapter II, the absence of such effects does not necessarily mean that the associated phase deviation is low unless it is known that the observer is in the far field region of the diffraction pattern. Attempts to determine the degree of irregularity phase deviation by multifrequency studies are complicated by the fact that the effective distance of the ground from the irregularity region is a function of frequency. The size of the pattern on the ground would then be expected to change as the frequency was varied even in the case of small total ionospheric phase deviation, unless the observer can be assumed to be wholly in the near or far field regions of the diffraction process for all frequencies being employed.

Nevertheless, the small phase effects observed by DeBarber, Chisholm, and Ross (1963), the similar forms of power spectra obtained at 54 and 150 MHz by Jespersen and Kamas (1964), and the scintillation rate studies of Yeh and Swenson (1964), all indicate that during periods of low sunspot number, large phase deviations

are rarely observed at frequencies of 50 MHz and above at midlatitude sites. In the aurora zone, however, it appears that larger phase deviations are much more common. (Little, Reid, Stiltner, and Merritt, 1962; Yeh and Swenson, 1964)

### 8.2.3 Spectrum of Irregularity Sizes

Little work has been done on the direct measurement of the spectrum of irregularity sizes responsible for night time scintillation. In 1957 Gruber (1961) at Lincoln Laboratories employed a phase switched interferometer to deduce amplitude and phase spectra from radio star measurements at 50 and 200 MHz. He found generally good agreement between the amplitude and phase spectra. However, the phase switched interferometer responds to both the amplitude and phase of the received signal, and the separation into amplitude and phase components is not direct. Furthermore, Gruber estimated the pertinent irregularities were only 100 to 150 km above the Earth's surface, and so it is possible that he was observing E region, rather than F region irregularities.

In 1962, Moorcroft and Forsyth (1963) obtained results at Saskatoon which revealed discrepancies between measurements made on a pair of phase switched interferometers of different antenna spacings. They interpreted these effects as being due to large ionospheric structures which would cause phase scintillation but whose amplitude patterns would not be fully developed due to the large size of the irregularities involved. In late 1964 and

early 1965, Lansinger (1966) obtained amplitude and phase spectra at College, Alaska, using a phase swept interferometer of improved design. This instrument gave direct readings of amplitude and phase difference rather than the indirect readings of the phase switched type. He obtained amplitude and phase results similar to those reported in this study in that the autocorrelograms of phase always took longer to decay than did those of amplitude. The resulting phase power spectra were dominated by low frequency components. Since only two antennas were employed, it was not possible to test for field alignment of the irregularities producing the phase pattern, and Lansinger therefore concluded that the phase effects were due to ionospheric tilts or wedges of a few kilometers in size, while the amplitude effects were due to smaller irregularities coexisting with the larger structures.

In late 1965, Rao (1967) obtained at Urbana, Illinois, a linear variation of irregularity phase power density versus irregularity spatial wavelength over the range of spatial wavelengths of 8 to 400 km. These results were obtained from an analysis of Faraday rotation data. By an analysis of differential Doppler data, Porcello and Hughes (1969) determined that in the auroral zone, the irregularity phase power density was proportional to the spatial wavelength raised to the 2.8 power over the range of wavelengths of .08 to 8 km. By comparison, the differential Doppler results obtained in this study indicate that the phase power density was approximately proportional to the spatial wavelength raised to the 3.2 power over the range of spatial wavelengths of 3 to 30 km.

Wagner (1962) has performed a theoretical study of the zenith angle dependence of the amplitude scintillation index under the assumptions of small ionospheric phase deviation, and a statistically homogeneous irregularity region. By comparing his results to records from Cambridge, England obtained in 1952, and from College, Alaska in 1962, he deduced that the irregularity spatial power spectrum was nearly Gaussian at Cambridge, but that it contained more high frequency components than would a Gaussian distribution at College. His derivation seems questionable in that it appears to assume random phasing of the components of the angular spectrum even at small distances from the irregularity layer, but it is still interesting in that it suggests that at least in auroral zones, the Gaussian form of size spectrum may not be valid.

#### 8.2.4 Discussion

It is interesting to note that the irregularity heights obtained in this study compare well to the values obtained in 1967 at Sagamore Hill and Athens, and that all these heights are somewhat higher than those obtained at mid latitude stations earlier in the solar cycle. The complex height structure, the large values of irregularity phase deviation, and the nature of the phase power spectra, seem to be more suggestive of auroral zone results, than of those observed at midlatitude stations during solar cycle minimum.

The implication is therefore present that at solar cycle maximum, the F region irregularities observed at midlatitudes may

be aurorally related, and therefore caused by particle precipitation. This view is upheld by the work of Forsyth and Paulson (1961) who suggested, as a result of radio star measurements, "that the scintillations arise most strongly in regions of the atmosphere closely associated with the auroral zone and that these regions migrate southward during years of intense sunspot activity" (Forsyth and Paulson, p. 502). Supporting evidence is also found in the work of Yeh, Simonich, Mawdsley, and Preddey (1968) who found a significant crosscorrelation between the occurrence of night time scintillation at two geomagnetically conjugate stations. Aarons, Mullen, and Whitney (1969) have also obtained agreement between diurnal changes in the extent of the region of the ionosphere which causes the scintillation, and the diurnal extent in the region over which soft particle precipitation may be expected. It should be noted that the irregularity scale sizes as obtained from the amplitude pattern seem to be in good agreement with sizes obtained at this location in 1961 by DeBarber and Ross (1966), and that these scale sizes are also representative of auroral values. This implies that many features of the amplitude pattern are relatively insensitive to solar activity. This is probably due to the fact that the amplitude pattern is not as directly related to the ionospheric parameters as is the phase pattern. The inadequacy of the amplitude pattern size as an indicator of ionospheric conditions has also been discussed by Moorcroft and Forsyth (1963), who found little correlation between amplitude pattern measurements and

auroral activity, but good agreement between phase fluctuations and auroral and magnetic activity.

### 8.3 Sources of Error

A major source of error in this work was the use of the ray optical approximation necessitated by the large phase deviations which appeared to be present. The magnitude of the error introduced by this approximation is hard to estimate, but the exact solutions obtained for the cosinusoidal screen indicate that for ionospheric scales greater than about 1 km, and values  $\phi_0$  less than 2.5 radians, the errors in measured scale size should be less than 20 percent, and in the measured value of  $\phi_0$ , less than 10 percent. If the ray optical approximation does not hold, the size of the phase ground pattern will be less than the true irregularity size and so the measured values of ionospheric scale size and phase deviation must be treated as minimum values.

Another source of error which must be considered is the comparatively small lengths of data used in the correlation and spectral analyses. These may not be typical of the random phenomenon being studied, and in this sense may be expected to yield irregularity parameters which are somewhat different from those of the hypothetical parent population of which they may be considered to be members. The errors involved in the correlation analyses are hard to estimate from purely experimental data, but numerical calculations using the methods suggested by Awe (1964a)

indicate that the standard error may be as large as +50 percent, -35 percent, for the scales of the largest irregularities. Blackman and Tukey (1958) have shown that the statistical error associated with a power spectral estimate depends on the ratio of the maximum time lag for which the autocorrelation is computed to the total sample length. For the ratio of 20% used in this study, the probable error associated with the spectral estimates is approximately  $\pm 1.4$  dB.

It should be pointed out that the results obtained from the experimental measurements were measures of ionospheric conditions which actually did exist. If they were atypical in a statistical sense, they nevertheless pointed out different extremes in ionospheric behavior which could be expected to occur. Since the statistics of the diffraction pattern at the ground were non stationary, measurements over longer and longer time intervals would not yield results which approached any limiting values, and so the parent population existed only conceptually, and had no physical significance.

As the satellite moved, the orientation of the ground pattern changed so that the ground pattern statistics were non stationary even if the statistics of the irregularity distribution were stationary. For the time intervals used in this study, this change in ground pattern orientation was always less than 8 degrees and therefore should have little effect on the results. The longer time intervals involved in the Doppler studies involved pattern

rotations of as much as 17 degrees, but fortunately the pattern orientations were such that angular changes of this amount represented changes of less than 5 percent in the pattern sizes along the direction of pattern motion.

The simple theory used to describe the Doppler measurements assumed that 20 and 40 MHz ray paths were coincident straight lines through the ionosphere. Since the ionospheric index of refraction is a function of frequency such as assumption is incorrect. If the effects of the Earth's magnetic field are ignored, and a plane Earth geometry is assumed, it may be shown that a parabolic electron density profile with half thickness 150 km, and peak plasma frequency 4 MHz will produce a maximum ray path separation of approximately 3 km for a satellite zenith angle of 45 degrees. In practice, the irregularities in data sample 2 probably were above the height of maximum electron density, and the thickness of the irregularity layer of data sample 9 was less than 300 km as indicated from experimental results, so the actual ray separation for the two data sections which were analyzed may have been closer to 1.5 km. However, since the 20 MHz signal contributed 9/10 of the total power in the spectrum, any ray separation which was present would not be expected to effect the nature of the results, at least qualitatively.

#### 8.4 Summary

An experiment was performed to obtain the characteristics of the amplitude and phase of 40 MHz signals received on the ground

after propagation through a layer of inhomogeneities of ionospheric electron density. Equipment was designed and constructed to record on magnetic tape the amplitude and phase of the signals received from three ground stations spaced approximately 2.5 km apart. Specialized playback equipment was built to remove from the records of phase difference between pairs of stations on the ground the effect of the varying length of the free space distance from the moving satellite to the ground stations.

Measures of the size, orientation, and velocity of the amplitude patterns on the ground were obtained by the method developed by Kent and Koester (1966). Calculations performed with a simpler one dimensional analysis yielded essentially the same results. On the assumption that the phase pattern was moving over the ground at the same velocity as the amplitude pattern, characteristic sizes of the phase pattern in the direction transverse to its elongation were obtained, and the alignments of the phase patterns on the ground were found. These measurements of phase pattern characteristics appear to be the first to include the anisotropic shape of the phase pattern, and appear to be the first which indicate that the phase ground pattern is frequently aligned in the same direction as is the amplitude pattern.

To interpret these measurements in terms of ionospheric parameters, the amplitude and phase diffraction patterns due to a sinusoidal phase screen of arbitrary phase deviation were computed. The effects of additional spatial components were

considered, and it was seen that close to the screen, the amplitude pattern would have a more rapid spatial variation than would the phase pattern. It was seen that this would be the case even for a screen of small phase deviation due to the filtering effect of the diffraction process. Although this situation had been previously recognized, it does not previously appear to have been sufficiently appreciated.

The relationship between the patterns produced by a periodic screen and those produced by a screen whose phase fluctuations were a random variable of position was considered, and it was seen that close to the screen, the conclusions of the periodic analysis should be applicable to the random case since only a limited portion of the screen would be contributing to the ground pattern. It was also seen that if the phase deviations of the incident wave were produced not by an infinitely thin screen, but by a thick region of inhomogeneities, the thin screen analysis could be used, at least to describe the phase pattern, if the thickness of the screen was small enough so that little diffraction occurred within the inhomogeneity layer. The effects on the ground pattern of oblique incidence and finite satellite distance from the scattering layer were derived in a slightly more general manner than had previously appeared in the literature.

Data collected during the Spring of 1968 were analyzed in light of the theoretical results. Measures of the height and thickness of the irregularity region were deduced from the amplitude

measurements, and apparent ionospheric scale sizes were deduced from both the amplitude and phase measurements.

The complicated nature of the irregularity height structure, and the high values of the phase deviations observed were somewhat unexpected in light of previous midlatitude measurements. The observation of field aligned structures by measurements of the phase patterns appears to be the first to be reported in the literature and the large scale sizes indicated by the phase measurements, as compared to the scales indicated by the amplitude measurements, indicates that amplitude measurements alone are not, as commonly supposed, adequate indicators of ionospheric structure.

#### 8.5 Conclusions

It was concluded that the irregularities responsible for night time scintillation during the Spring of 1968 were located primarily at heights of 400 to 700 km above the Earth's surface. The height structure was very complex. The amplitude and phase patterns on the ground were both elongated in the direction which would be expected if the causative irregularities were elongated along the Earth's magnetic field. The apparent scale of the irregularities transverse to the direction of elongation was typically 2 to 3 km. The transverse scales deduced from the amplitude measurements were typically a factor of four to seven times smaller than the corresponding phase scales. The total phase deviation imposed on the incident wave by the irregularity layer frequently exceeded one radian.

## 8.6 Suggestions for Further Work

Due to the small amount of data employed in this study, the results may be regarded as being of a preliminary nature. Further data should be obtained and analyzed to detect any diurnal, seasonal, and solar cycle trends which may exist. Due to the large amount of time necessary for manual data scaling, it would appear that an automated data handling system would be mandatory for such extended data collection and analysis.

It would be desirable to have a diffraction theory which would give physically meaningful results for all values of phase deviation. Since the mathematical difficulties involved made the development of such a theory doubtful, a spaced receiver experiment utilizing higher satellite frequencies is advisable, since at a sufficiently high frequency the theory developed for small  $\phi_0$  could be employed exactly.

Since the limited baseline length employed in this experiment may have suppressed large irregularities, spaced receiver experiments with even longer baselines are desirable. However, as the baseline length is increased, the problem of maintaining a phase reference between all stations becomes severe. This problem might be overcome by employing dispersive Doppler measurements at receiving sites spaced some distance apart on the ground. This technique would seem to have considerable promise if one of the two frequencies being compared was sufficiently high so that it could be assumed to traverse the ionosphere unaffected. The

resulting phase differences would then be in terms of phase at a single point on the ground and could be analyzed to obtain irregularity heights and layer thicknesses in a way exactly analogous to the amplitude measurements.

A basic problem encountered in attempts to describe the properties of a medium by means of measurements of its effects on signals propagating through it, is the question of exactly what types of results are realistically obtainable from measurements of integrated effects at a few points on the ground. Model studies perhaps employing laser beams and dust particles, or acoustical waves and hot gases may be of help in clarifying this problem. Studies with large numbers of ground stations; such as those of Briggs (1968) are also necessary to insure that three ground stations are sufficient to adequately specify the ground pattern characteristics.

BIBLIOGRAPHY

- Aarons, J., J.P. Mullen, and H.E. Whitney, "The Scintillation Boundary," Jour. Geophys. Research, 74, 884, 1969.
- American Radio Relay League, A.R.R.L. Antenna Book, Rumford Press, Concord, New Hampshire, 5th edition, 1949.
- Awe, Olumuyiwa, "Errors in Correlation between Time Series," Jour. Atmosph. Terr. Phys., 26, 1239, 1964a.
- Awe, Olumuyiwa, "Effects of Errors in Correlation on the Analysis of the Fading of Radio Waves," Jour. Atmosph. Terr. Phys., 26, 1257, 1964b.
- Basler, R.P., and R.N. DeWitt, "The Heights of Ionospheric Irregularities in the Auroral Zone," Jour. Geophys. Research, 67, 587, 1962.
- Beckmann, P., "Signal Degeneration in Laser Beams Propagated through a Turbulent Atmosphere," Radio Science, 69D, 629, 1965.
- Blackman, R.B., and J.W. Tukey, The Measurement of Power Spectra From the Point of View of Communications Engineering, Dover Publications Inc. New York, 1958.
- Bowhill, S.A., "Statistics of a Radio Wave Diffracted by a Random Ionosphere," Jour. Research NBS, 65D, 275, 1961.
- Bramley, E.N., "Diversity Effects in Spaced Aerial Reception of Ionospheric Waves," Proc. Instn. Elect. Engrs. 98 Part III, 19, 1951.
- Bramley, E.N., "The Diffraction of Waves by an Irregular Refracting Medium," Proc. Royal Soc. A225, 515, 1954.
- Bramley, E.N., "Diffraction of an Angular Spectrum of Waves by a Phase Changing Screen," Jour. Atmosph. Terr. Phys. 89, 1, 1967.
- Brennan, D.G., "The Extrapolation and Interpolation of Spatial Correlation Functions," Statistical Methods in Radio Wave Propagation, edited by W.C. Hoffman, Pergamon Press, New York, 1960.

- Briggs, B.H., G.J. Phillips, and D.H. Shinn, "Analysis of Fading at Spaced Receivers," Proc. Phys. Soc. B63, 106, 1950.
- Briggs, B.H., and I.A. Parkin, "On the Variation of Radio Star and Satellite Scintillation with Zenith Angle," Jour. Atmosph. Terr. Phys. 25, 339, 1963.
- Briggs, B.H., "On the Analysis of Moving Patterns in Geophysics-I. Correlation Analysis," Jour. Atmosph. Terr. Phys., 30, 1777, 1968.
- Burr Brown Research Corporation, Handbook of Operational Amplifier Active RC Circuits, Burr Brown Research Corp., Tucson, Ariz., 1966.
- Crosby, M.G., "Carrier and Side Frequency Relations with Multi-tone Frequency of Phase Modulation," RCA Review, 3, 103, 1938.
- DeBarber, J.P., "The Diffraction of High Frequency Radio Waves by Irregularities in the Ionosphere with Emphasis on Waves Emanating from Artificial Earth Satellites," Ionosphere Research Laboratory Report No. 169, The Pennsylvania State University, 1962.
- DeBarber, J.P., G.E. Chisholm, W.J. Ross, "The Nature of the Irregularities in Electron Density causing Scintillations in Satellite Signals," Proceedings of the International Conference on the Ionosphere Held at Imperial College, London, July 1962, edited by A.C. Strickland, The Institute of Physics and the Physical Society, London, 1963.
- DeBarber, J.P., and W.J. Ross, "The Diffraction of H.F. Radio Waves by Ionospheric Irregularities," Spread F and its Effects upon Radiowave Propagation and Communication, Agardograph 95, edited by P. Newman, Technivision, London, 1966.
- Feinstein, J., "Some Stochastic Problems in Wave Propagation-Part II," Trans. Inst. Radio Engrs. AP-2, 63, 1954.
- Fejer, J.A., "The Diffraction of Waves in passing through an Irregular Medium," Proc. Royal Soc. A220, 455, 1953.
- Fooks, G.L., "Ionospheric Drift Measurements using Correlation Analysis; Methods of Computation and Interpretation of Results," Jour. Atmosph. Terr. Phys. 27, 979, 1965.
- Forsyth, P.A., and K.V. Paulson, "Radio Star Scintillations and the Auroral Zone," Canadian Jour. Phys., 39, 502, 1961.

- Frihagen, J., "Study of Fine Scale Ionospheric Structure by Satellite Techniques," Radio Astronomical and Satellite Studies of the Atmosphere, edited by J. Aarons, North Holland Publishing Co., Amsterdam, 1963.
- Gruber, S., "Statistical Analysis of Radio Star Scintillation," Jour. Atmosph. Terr. Phys., 20, 59, 1961.
- Hewish, A., "The Diffraction of Radio Waves in Passing Through a Phase Changing Ionosphere," Proc. Royal Soc. A209, 81, 1951.
- Hewish, A., "The diffraction of Galactic Radio Waves as a Method of Investigating the Irregular Structure of the Ionosphere," Proc. Royal Soc. A214, 494, 1952.
- James, P.W., "The Correlation Analysis of the Fading of Radio Signals Received from Satellites," Jour. Atmosph. Terr. Phys., 24, 237, 1962.
- Jespersen, J.L., and G. Kamas, "Satellite Scintillation Observations at Boulder, Colorado," Jour. Atmosph. Terr. Phys., 26, 457, 1964.
- Jones, R.E., G.H. Millman, and R.J. Nertney, "The Heights of Ionospheric Winds as Measured at Long Radio Wavelengths," Jour. Atmosph. Terr. Phys., 3, 91, 1953.
- Jones, I.L., "Further Observations of Radio Star Scintillation," Jour. Atmosph. Terr. Phys., 10, 26, 1960.
- Kelleher, R.F., "Some Statistical Properties of the Ground Diffraction Patterns of Vertically Reflected Radio Waves," Jour. Atmosph. Terr. Phys., 28, 213, 1966.
- Kent, G.S., "High Frequency Fading Observed on the 40 mc/s Wave Radiated from Artificial Satellite 1957a," Jour. Atmosph. Terr. Phys., 16, 10, 1959.
- Kent, G.S., and J.R. Koster, "Some Studies of Night-Time F-Layer Irregularities at the Equator using Very High Frequency Signals Radiated from Earth Satellites," Spread F and its Effects upon Radiowave Propagation and Communication, Agardograph 95, edited by P. Newman, Technivision, London, 1966.
- Lansinger, J.M., "High-Latitude Radio Star Scintillation Measurements at 68 Mc/s Made With a Phase-Sweep Interferometer System," Radio Science, 1 (new series), 1176, 1966.

- Liszka, L., "A Study of Ionospheric Irregularities using Satellite Transmissions at 54 Mc/s," Arkiv For Geofysik, 4, No. 9, 1963.
- Liszka, L., "JSSG-Spaced Receiver Measurements," Kiruna Geophysical Observatory Report, Kiruna, Sweden, 1968.
- Little, C.G., and A.C.B. Lovell, "Origin of the Fluctuations in the Intensity of Radio Waves from Galactic Sources," Nature, 165, 423, 1950.
- Little, C.G., G.C. Reid, E. Stiltner, and R.P. Merritt, "An Experimental Investigation of the Scintillation of Radio Stars Observed at Frequencies of 223 and 456 Megacycles per Second from a Location Close to the Auroral Zone," Jour. Geophys. Research, 67, 1763, 1962.
- McClure, J.P., and G.W. Swenson, "Beacon Satellite Studies of Small Scale Ionospheric Inhomogeneities," report from Dept. of Elect. Eng., Engineering Experiment Station, University of Illinois, 1964.
- Mercier, R.P., "Diffraction by a Screen Causing Large Random Phase Fluctuations," Proc. Camb. Phil. Soc. Math. Phys. Sci., 58, 382, 1962.
- Mitra, S.N., "A Radio Method of Measuring Winds in the Ionosphere," Proc. Inst. Elect. Engrs., 96 Part III, 441, 1949.
- Moorcroft, D.R., and P.A. Forsyth, "On the Relation between Radio Star Scintillations and Auroral and Magnetic Activity," Jour. Geophys. Research, 68, 117, 1963.
- Phillips, G.J., and M. Spencer, "The Effects of Anisometric Patterns in the Measurement of Ionospheric Drifts," Proc. Physical Soc. B, LXVIII, 481, 1955.
- Porcello, L.J., and L.R. Hughes, "Observed Fine Structure of a Phase Perturbation Induced during Transauroral Propagation," Jour. Geophys. Research, 73, 6337, 1968.
- Rao, N.N., "Ionospheric Electron Content and Irregularities Deduced from BE-C Satellite Transmissions," Jour. Geophys. Research, 72, 2929, 1967.
- Ratcliffe, J.A., and J.L. Pawsey, "A Study of the Intensity Variations of Downcoming Wireless Waves," Proc. Cambridge Phil. Soc., 29, 301, 1933.

- Ratcliffe, J.A., "Some Aspects of Diffraction Theory and their Application to the Ionosphere," Reports on Progress in Physics, XIX, edited by A.C. Stickland, The Physical Society, London, 1956.
- Robinson, D.D., "Diode Coupled Schmitt Trigger," Electronics, 37, 50, Dec. 14, 1964.
- Sales, G.S., "A Summarization of Turbulence and Ionospheric 'Wind' Measurement Theories with Application of the Later to Preliminary Data at 75 KC/S," Ionosphere Research Laboratory Report No. 88, The Pennsylvania State Univ., 1956.
- Spencer, M., "The Shape of Irregularities in the Upper Atmosphere," Proc. Phys. Soc., 68B, 493, 1955.
- Wagner, L.S., "Zenith Angle Dependence of Radio Star Scintillation," Jour. Geophys. Research, 67, 4187, 1962.
- Yeh, K.C., and G.W. Swenson, "F-Region Irregularities Studied by Scintillation of Signals From Satellites," Radio Science, 68D, 881, 1964.
- Yeh, K.C., D. Simonich, J. Mawdsley, and G.F. Preddy, "Scintillation Observations at Medium Latitude Geomagnetically Conjugate Stations," Radio Science, 3, (New Series), 690, 1968.