NASA TECHNICAL NOTE

NASA TN D-5529

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NASA TN D-5529

SURVEY OF A SECOND CLASS OF ORBITS IN THE EARTH-MOON FIELD: $X_R = 1.00$

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • NOVEMBER 1969

1. Report No. NASA TN D-5529	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle Survey of a Second Class of Orbits in the Earth-Moon Field: $X_R = 1.00$		5. Report Date November 1969
		6. Performing Organization Code C-86
7. Author(s) R. F. Hoelker		8. Performing Organization Report No.
9. Performing Organization Name and Address Electronics Research Center Cambridge, Mass. 02169		10. Work Unit No. 129-04-04-06
		11. Contract or Grant No.
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546		13. Type of Report and Period Covered Technical Note
		14. Sponsoring Agency Code
15. Supplementary Notes		

16. Abstract

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The system model is that of the restricted problem of three bodies, with circular motion of the masses and the mass parameter $\mu = 1/80$. Earth and Moon are located on the X_R-axis at -0.0125 and +0.9875, respectively.

The objective of the study is, first, a graphical display of the class of orbits in steps small enough to allow interpolation and, secondly, through the exploit of appropriately composed survey graphs, the exposition of the class' main phase of development as well as their critical transitions.

For comparison a suitably chosen class of Kepler orbits in rotating coordinates is carried in parallel.

17. Key Words	18. Distribution Statement
Earth-Moon Field Mass parameter Graphical Display Kepler Orbits	Unclassified - Unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	75	$3.00

*For sale by the Clearinghouse for Federal Scientific and Technical Information
Springfield, Virginia 22151

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SUMMARY

A class of orbits in the Earth-Moon (E-M) field is discussed based on a sequence of illustrations. The class is defined by the common starting point of all orbits at the normalized coordinate $X_R = 1.00$ on the rotating X_R-axis and by the orthogonality to this axis of the initial velocities of all its orbits. All motion is in a plane.

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For comparison a suitably chosen class of Kepler orbits in rotating coordinates is carried in parallel.

INTRODUCTION

This report is the second in a series of reports intended to depict classes (or series) of orbits of the E-M field as represented by the restricted three-body problem, and to compare these orbits with rotating Kepler orbits.

While the first report of this series (ref. 1) deals with a class of orbits that start far behind the Moon and thereby can show a relatively large degree of similarity to Kepler orbits, the present report differs greatly in this respect.

Its orbits start at the point $X_R = 1.00$; $Y_R = 0.00$, while the Moon is located at $X_R = 0.9875$; $Y_R = 0.00$. The close proximity of the initial point to the Moon's location causes this class not only to contain a large continuous group of orbits that stay in the Moon's field, but also to show the Moon's effect in a larger variety of orbital developments than was the case with the group of orbits of the earlier report.

Thus, on one hand, there is a much reduced chance to set the E-M orbit in correspondence to Kepler orbits. On the other hand, the larger variety of events poses real problems when it comes to presenting the whole series of orbits on a few survey-graphs as was done within five graphs for the earlier series. The problem is met here by providing the reader a choice between a "four-part synopsis" and a group of "twenty survey graphs." A comparison between the two testifies to the amount of interesting material that was deleted in designing the shorter of the two surveys.

Since reference 1 gives an extended introduction into the representation and classification of Kepler orbits in rotating coordinates, the present report merely refers to it.

The main body of the report is arranged so its first part presents the two surveys starting with the shorter one. The twenty graph survey is accompanied by an extended commentary, and is to be looked upon as the core of the report. Here the major phases of the series and their developments, as well as transitions, are discussed. This is paralleled by a survey of Kepler orbits contained on six graphs.

The remainder of the report presents the large number of individual orbits, each one graphed to a time length corresponding to the significance of the development shown. Comments are made at places where it seems necessary for clarity or emphasis.

In parallel to this, individual Kepler orbits are displayed to facilitate comparisons where these are possible. At other places, the Kepler series is continued merely for completeness. (Kepler orbits and surveys of these are placed at the bottom of pages.)

The rich contents of the present E-M series is attested to by the very large number of figures supplied here. The cost of producing them, however, is greatly reduced from those of reference 1 by the use of an automatic plotter (Stromberg-Carlson 4020). Also, only by this means is it possible to draw those long-period and sometimes rather complicated graphs, which the reader encounters in this report.

PREPARATORY INFORMATION

Model Data and Method of Computation

The problem under study is the restricted problem of three bodies, the masses of two of which revolve about each other in circular paths. The third, massless, body is assumed to move coplanar with the two masses. The mass ratio between the smaller mass (Moon "M") and the larger mass (Earth "E") is chosen 1/79, resulting in the value of 1/80 for the mass parameter μ.

In computation as well as when referring to collisions, the masses are consistently considered strictly point-masses. Notwithstanding, some of the graphs show the physical size of the Moon in true relationship to the scale. This serves just to bring the size of the orbits, shown thereon, into the right perspective.

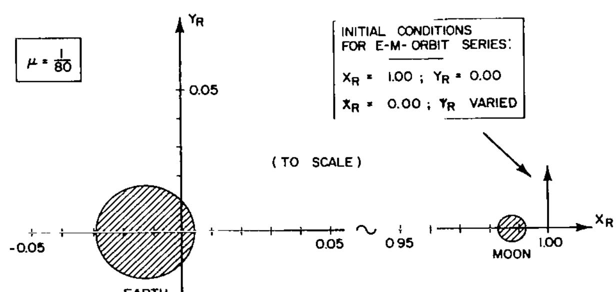
The set of equations of motion is normalized and regularized. Normalization reduces certain parameters to unity, such as the gravitational constant, the sum of the two masses, the distance between them, and their rate of revolution about each other. It gives rise to the designation of μ to the smaller mass and 1 - μ to the larger one. Normalization is a convenient means of reducing to a unified scale problems that belong to the same type but have various magnitudes involved, as the Earth-Moon problem, sun-planet problem, etc., but also the Kepler-problem.

Regularization of the equations of motion is applied to facilitate the calculation of the orbital paths through the singularities that are represented by collisions with either mass. The method of regularization used is that of Arenstorf (ref. 2).

All orbit computation then is carried through in a stepwise integration method of series type. For a description of the deck, see reference 1.

All displays of orbits are given in the rotating (or "synodic") coordinate system X_R, Y_R where the X_R-axis contains the locations of the two masses, the Earth's being at $X_R = -0.0125$ and the Moon's at $X_R = +0.9875$.

Figure 1 depicts the geometry of the system indicating the relative size and location of Earth and Moon on the rotating X_R-



INITIAL CONDITIONS AND GEOMETRIC RELATIONS FOR EARTH-MOON-FIELD ORBITS

Figure 1

axis and giving the initial conditions of the E-M orbits as $X_R = 1.00$, $Y_R = 0.00$ and $\dot{X}_R = 0.00$ and the axis-orthogonal component $\dot{Y}_R$ varied from the negative near-escape case to the positive one.

KEPLER ORBITS

Since reference 1 furnishes a rather elaborate introduction to the characteristics of Kepler orbits as seen from a rotating coordinate system, no information of a general nature is included here.

The series of Kepler orbits included here and serving as comparison to the main-series is chosen so that its initial point is at $X_R = 1.0125$, $Y_R = 0.0$, which is the same distance from the (only) mass as is the initial point of the E-M-series from the larger mass. Its initial velocity is orthogonal to the X_R-axis.

Normalization applied to the equations of the Kepler problem brings this problem in comparative scale with the main problem.

For reasons of unification of procedures, the Kepler orbits are computed by using the same regularized deck that serves for the E-M-orbit computation. This means that the Kepler orbits are derived by stepwise integration. This streamlining of procedures offers the bonus of easy tests for computational accuracy.

The value of "n" listed with most of the Kepler orbits refers to the mean-angular motion in respect to an inertial (or "sidereal") system, "n" carrying a negative sign if the mean angular motion is retrograde; positive if it is direct.

E-M-ORBITS AND SYMMETRY PROPERTIES

Since all orbits start with orthogonal departures from the X_R-axis, a second orthogonal crossing of this axis insures that the orbit is periodic in the rotating reference frame. This follows from the symmetry properties of orbits of the given problem (see e.g., Szebehely, ref. 3, page 426). For clarity of the illustration, periodic orbits are frequently depicted to only half of the periods. There, the marking of the crossing by means of the "right-angle sign" manifests the orbit's periodic character.

Also, because of the symmetry properties by reflection on the X_R-axis, all orbits can be continued backwards (or "upstream") of the starting points, reflecting the downstream path with reversed velocity after reflection. This is particularly important in regard to a search for so-called "free return trajectories," i.e., orbits that start near Earth, circumnavigate the Moon, and return to Earth without requiring a thrust impulse. Thus, all orbits of the present E-M-series that approach the Earth can indeed be considered to be the return phase of free-return trajectories.

Some E-M-orbits are marked by a "n*-value". This serves to point toward the structural similarity of the orbit to that Kepler-orbit that is annotated by the same value for its n-value.

APPLICATIONS AND DIMENSIONING

In cases where the information displayed here is used for determining flight modes of real E-M flights, due consideration must be given to the many simplifications imposed on the real world in representing the environment of these flights by the model of the restricted three-body problem. The most influential departures from the real world are brought about by ignoring the true shape of the orbits of the two masses about each other, the effects of Sun and other planets, and, for the orbital behavior near the masses, the effects of the body shape and inhomogeniety (masscons) of the two masses.

Due to these and other model deviations, the flights in a more realistic model deviate in many details from those represented here, particularly involving matters of periodicity. What is retained, however, is the general topology and the course of structural developments from orbit to orbit. Since the exhibit of these is the primary concern of the survey graphs, especially the group of twenty (Figures 6-25), reference to these is most helpful for real flight orbit selection and design.

For this application, the data listed here are to be transformed from normalized to dimensioned data. This is carried out by multiplying the given data of distances, velocities, and time periods by the following factors:

for distance, multiply by distance E-M (roughly 400 megameter or 240,000 st. miles)

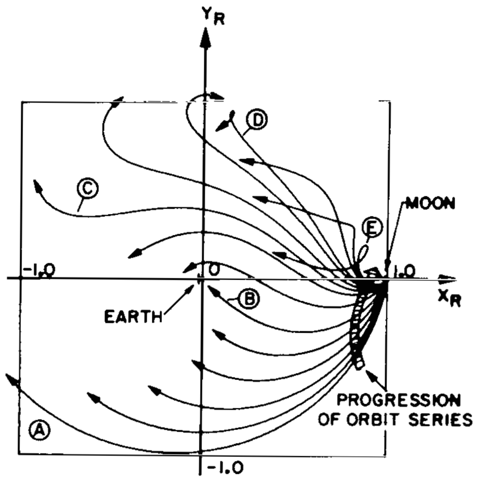
for velocities, multiply by velocity of the moon (roughly 1.0 km/sec or 2400 st. mph)

for time, multiply by ratio of days in one lunar revolution about Earth to 2π (roughly 4.3 to give the time in days).

EARTH-MOON FIELD ORBITS: SYNOPSIS IN FOUR PARTS

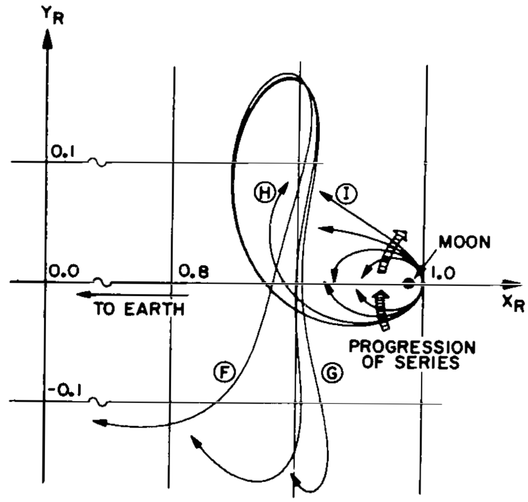
Figures 2-5 represent a very condensed synoptical view of the series of orbits described in the body of this report. This synopsis gives a broad-brush representation of the major features encountered in the run of the series and enables a first orientation in case some particular phase development is searched for.

EARTH-MOON FIELD ORBITS



FIRST OF FOUR-PART SYNOPSIS
OF EARTH MOON FIELD ORBITS
$-2.750 \le \dot{Y}_R\,(T=0) \le -1.366$

Figure 2



SECOND OF FOUR PART SYNOPSIS
OF EARTH MOON FIELD ORBITS
$-1.3645 \le \dot{Y}_R\,(T=0) \le +1.3900$

Figure 3

The progression of the series within each figure is indicated by the heavy arrow pointing in the direction of the increasing initial velocity component Y_R, and also by the progress in alphabetical labeling of some of the orbits.

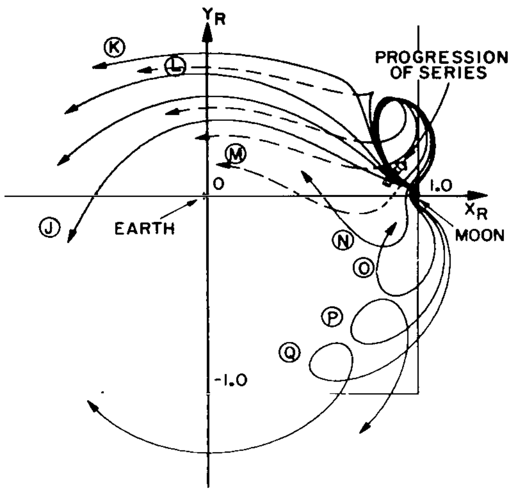
The series starts with orbit "A" which represents an orbit in the neighborhood of the border between escape orbits and orbits "generally bounded." (The term in quotes is to indicate that the region of bounded orbits is interspersed with orbits that escape the system through a close lunar flyby.) The orbits following orbit "A" in the progression move consistently closer to the Earth. Orbit "B" is the Earth-collision orbit. It is the only one in the present class that reaches the Earth in a "simple" flight mode, i.e., without a prior close lunar flyby.

The collision orbit "B" separates orbits whose sense of mean-orbital motion about the Earth is retrograde from those of direct motion about the Earth.

With orbit "E" of this figure, a particular development is starting that is signified by the formation of a loop in the neighborhood of the Moon. The development is carried through on the following graph.

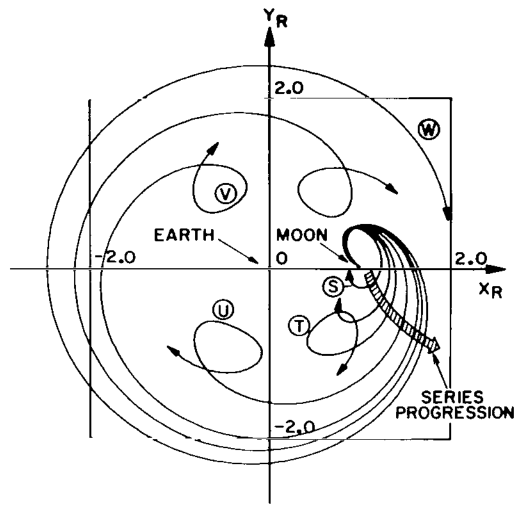
Figure 3 shows the increasing retention of orbits by the Moon's gravity leading finally to those orbits that are confined to the Moon's neighborhood. All orbits that are of the type of lunar satellite orbits are found between the orbits marked "H" and "I".

EARTH-MOON FIELD ORBITS



THIRD OF FOUR-PART SYNOPSIS OF EARTH-MOON-FIELD ORBITS
$1.34510 \leq Y_R\,(T=0) \leq 1.40605$

Figure 4



FOURTH OF FOUR-PART SYNOPSIS OF EARTH-MOON-FIELD ORBITS
$1.414 \leq Y_R\,(T=0) \leq 1.449$

Figure 5

The third graph of this group, Figure 4, shows the resumption of the motion about the Earth starting with orbit "J" and progressing to orbit "K". The move away from the Earth which is noted with these orbits is reversed with the orbits following as orbit "L" to orbit "M".

In the sequence of orbits "N", "O" and "P" the return to retrograde mean motion about the Earth can be recognized. All orbits of this graph pass in front of the Moon.

With the group of orbits on Figure 5 then, the initial velocities are large enough that the orbits approach patterns of Kepler orbits again. Apogees show increasing radial distances and the orbital periods increase as manifested by the forward movement of the perigee loops "T", "U", "V". The last orbit ("W") is close to the upper boundary of bounded trajectories.

With this orbit, the four-part synopsis is concluded.

TWENTY SURVEY GRAPHS OF THE E-M-SERIES WITH SIX SURVEY GRAPHS OF THE KEPLER SERIES

The orbits of the twenty survey graphs of the E-M-series are selected and grouped with the purpose of giving insight into the development of the various formations encountered in the course of progression.

Hence, a reader primarily interested in the study of developments is better served by these survey graphs than by the individual orbits of the succeeding chapter. However, the emphasis on lucidity here excludes the display of orbits to any considerable length of orbital period. This is why many interesting orbits will not be recognized in the survey graphs.

There are, on the other hand, those orbits that contribute border cases between classes of orbits, as between direct and retrograde orbits or between orbits about the Moon and orbits about both Earth and Moon. As far as these bordercase orbits are concerned in the progression of the series, their place and significance are given due mention in this chapter.

Concurrently with the twenty survey graphs of the E-M-series, survey graphs of Kepler orbits are presented. The relatively smaller number of development phases existing in the Kepler series allows rendering of the Kepler series in six graphs. The primary purpose of its inclusion is to provide a ready means of comparing the two series and thereby accord explanations for various features observable in the E-M-series.

For completeness, however, the Kepler series is also carried through where a semblance of the two series is lacking.

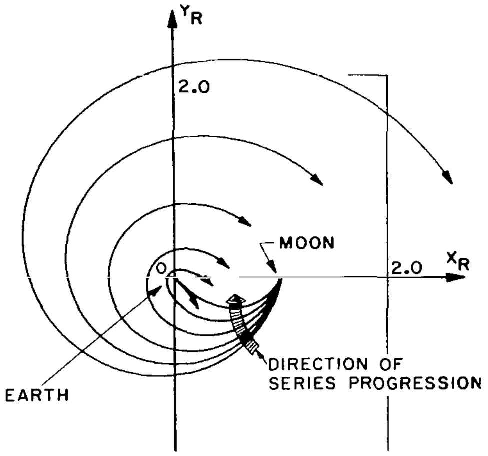
The limits of the Kepler series presentation are the limits of elliptical motion, i.e., the retrograde and the direct parabolic orbit, the sense of motion being understood in reference to an inertial frame.

The discussion of the E-M series, then, is limited to the range that can be set in parallel to the range of elliptical Kepler orbits. This extension proves adequate since the semblance of the two series outside the chosen boundaries is quite good.

Direction of both series progressions is from the lowest negative initial velocity Y_R upwards.

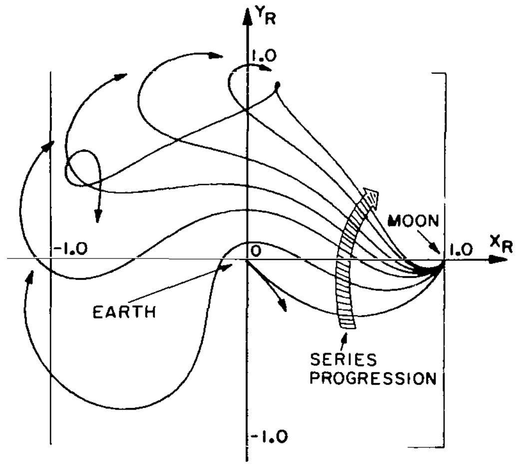
The first graph of the E-M-series (Figure 6) shows the group of orbits that covers the range from the lower limit of the series to the orbit that collides with the Earth. The development of this group is quite regular insofar as distances from the Earth

EARTH-MOON FIELD ORBITS
SURVEY GRAPHS



EARTH MOON FIELD ORBITS
SURVEY GRAPH #1
INITIAL VELOCITY Y_R FROM -2.75 TO -1.795

Figure 6



EARTH MOON FIELD ORBITS
SURVEY GRAPH #2
INITIAL VELOCITY Y_R FROM -1.795 TO -1.399

Figure 7

decrease consistently until the collision orbit is reached. Initial velocity of the collision orbit is -1.795, expressed in normalized units.

A point of significance attached to this collision orbit is its role as central orbit for a region of flyby orbits that may serve the practical interests of E-M flights. By virtue of the orthogonal crossings of the X_R-axis behind the Moon, all flyby orbits in the current series are return branches of symmetrical free-return orbits.

The series is continued on Figure 7 with the collision orbit repeated. This orbit represents the transition from retrograde to direct mean-angular motion about the Earth. This transition holds for the sidereal reference frame as well as the synodic. The development within this group in Figure 7 may be characterized by the increase in perigee distance and the decrease of the apogee loops.

Figures 8 and 9, are the first two graphs of the Kepler series, evidencing a good similarity to their E-M-counterparts. Some orbits here are labelled for easy reference. Orbit "A" is the exact parabolic orbit of inertially retrograde direction. The orbit "C", that collides with the mass, represents in the inertial frame the rectilinear orbit which after collision bounces back to its origin. The transition between opposite directions of mean-orbital motion occurs only once if the orbits are viewed in the inertial system. For the viewer rotating with the synodic system, a very interesting return to the retrograde motion unfolds later in the Kepler series. With the E-M-series, the mean-motion behavior is more complicated.

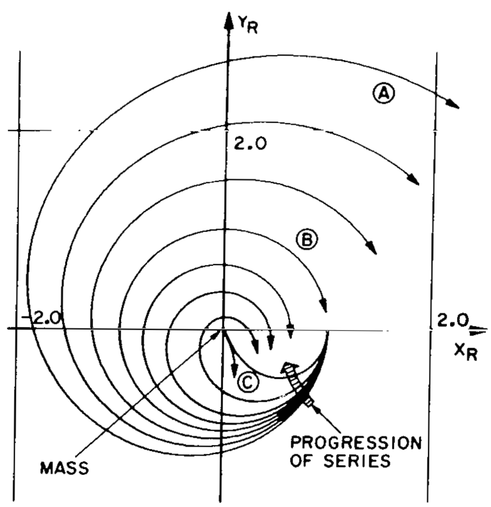
Before leaving the graphs of the Kepler orbits, it may be observed for later reference that the loops caused by the slow-down of orbital speed around the apogee points are decreasing in extension with progression of the series.

From here on, comparability between the two series does not exist for a larger portion of the series.

The two subsequent E-M-series graphs, Figures 10 and 11, in conjunction with the former E-M graph, Figure 7, reveal a significant motion as far as the perigee points of orbits are concerned. Perigees successively move away, advance toward, and again move away from the Earth. This oscillating perigee travel shows up again later in the series.

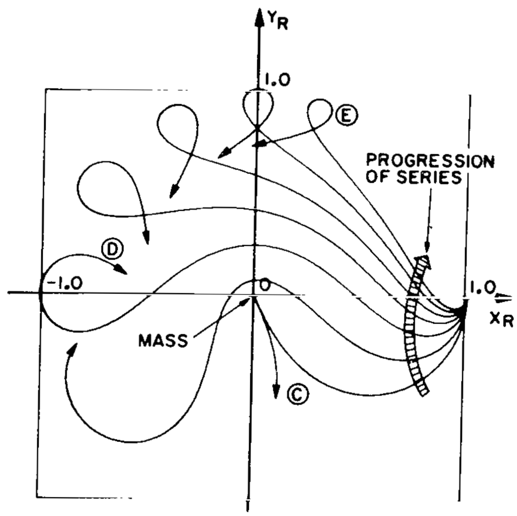
When advancing toward the Earth, the orbits do not approach the Earth closer than shown on Figure 10. At this point, it may be worthwhile to mention that there is no second "simple" Earth collision case in the E-M-series, but quite a number of orbits which show Earth collisions on the second or later "run", i.e.,

KEPLER ORBITS
SURVEY GRAPHS



KEPLER ORBITS
SURVEY GRAPH # 1
$-2.41796 \leq \dot{Y}_R \leq -1.01250$

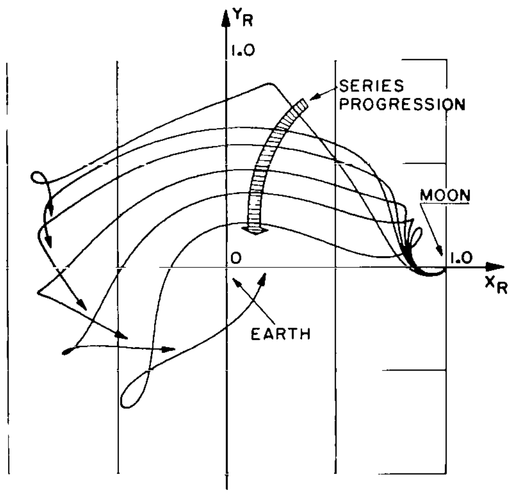
Figure 8



KEPLER ORBITS
SURVEY GRAPH #2
$-1.01250 \leq \dot{Y}_R \leq -0.10978$

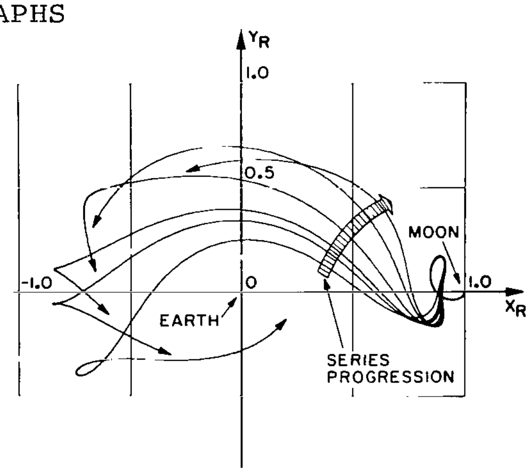
Figure 9

EARTH-MOON FIELD ORBITS
SURVEY GRAPHS



EARTH MOON FIELD ORBITS
SURVEY GRAPH #3
INITIAL VELOCITY Y_R FROM -1.394 TO -1.366

Figure 10



EARTH MOON FIELD ORBITS
SURVEY GRAPH #4
INITIAL VELOCITY Y_R FROM -1.3645 TO -1.3641

Figure 11

after the orbits first had a close lunar flyby.

Returning to the current graphs (Figures 10 and 11), it is important to notice that the oscillatory perigee behavior is coinciding with the formation and growth of a loop structure in the field of the Moon. On Figure 11 this loop structure is expanding into the lower half of the plane.

The significance of this formation is becoming clear on Figure 12. Within the progression represented on this graph, the orbits undergo the transition from those that move in the E-M-field to those that move in the Moon-field only. (The reference to the "Moon-field" is rather loose here. Its extent is determined preferably from the orbital motion about the Moon.)

The transition stretches out over a finite velocity range and produces a number of interesting and significant periodic orbits that are depicted in the chapter of individual orbits. (See Figures 180, 184-187, 195 and others.)

On the following three graphs (Figures 13-15), concern is given to the presentation of the orbits whose motion is confined to the Moon's field. Essentially the development consists of a steady shrinking of the orbits to a minimum and a subsequent expansion. In the course of the contracting phase, the orbits assume more and more the shapes of ellipses. This is particularly true after the development has passed through the near-

EARTH-MOON FIELD ORBITS
SURVEY GRAPHS

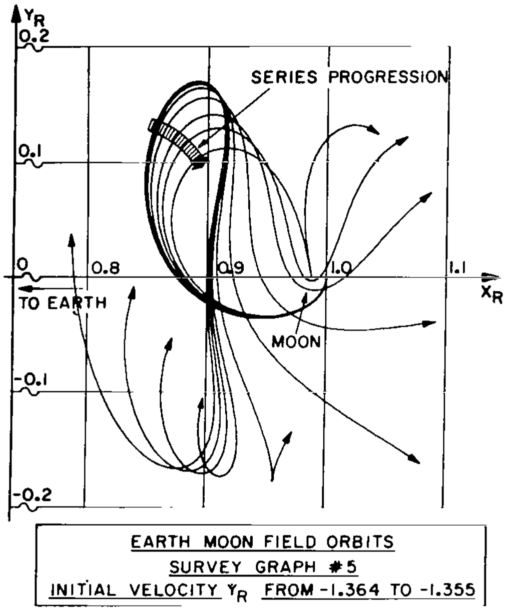


Figure 12

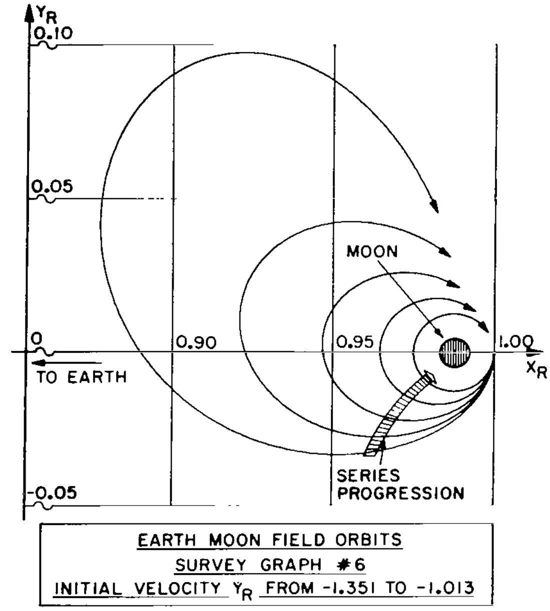


Figure 13

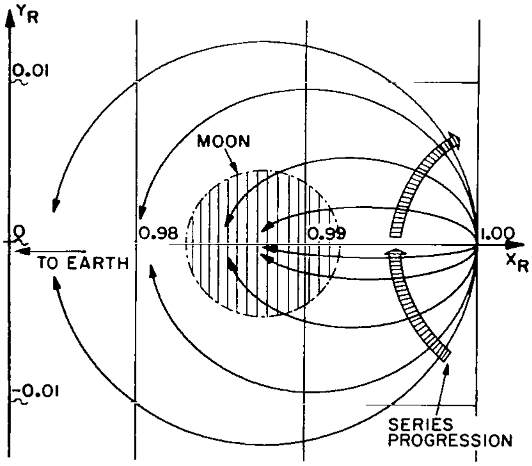
circular orbit shown on Figure 14 as the first one of the group. In the succeeding chapter where these orbits are shown for longer time periods, it is demonstrated that the orbits indeed follow closely Keplerian laws with respect to an inertial system and with the Moon as the central body. The motion of the synodic coordinate system then makes itself apparent in a rotational progression of the orbits. This motion does not come to light within the short periods for which the orbits are shown on Figure 14.

It is relevant to notice on Figure 14 that the orbit of zero initial velocity is very near to the orbit that collides with the Moon, this body, of course, considered as a point-mass. This coincidence does not hold if the orbits are started from larger initial X_R-values (See reference 1).

Furthermore, attention is drawn to the fact that the lunar collision orbit represents the dividing orbit between retrograde and direct angular motion about the Moon. The mean-angular motion about the Earth, however, does not alter at this point in the series.

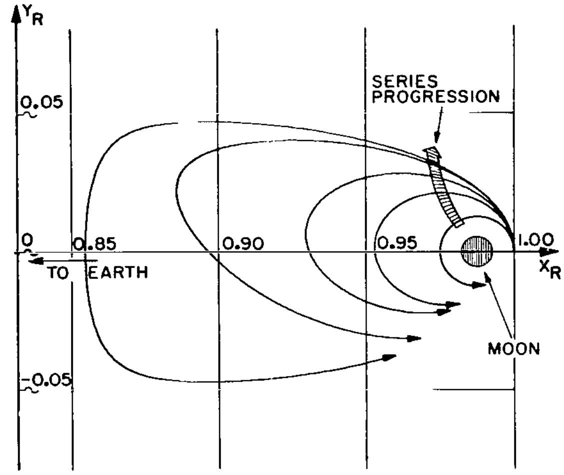
The last orbit on Figure 14 corresponds again to a near-circular path about the Moon, this time with positive mean-angular motion. The development of the series past this circular case is illustrated on Figure 15 which shows that the orbits reach out increasingly toward the Earth. The last orbit here is a periodic

EARTH-MOON FIELD ORBITS
SURVEY GRAPHS



EARTH MOON FIELD ORBITS
SURVEY GRAPH #7
INITIAL VELOCITY Y_R FROM -1.013 TO +0.988

Figure 14



EARTH MOON FIELD ORBITS
SURVEY GRAPH #8
INITIAL VELOCITY Y_R FROM 0.988 TO 1.3279

Figure 15

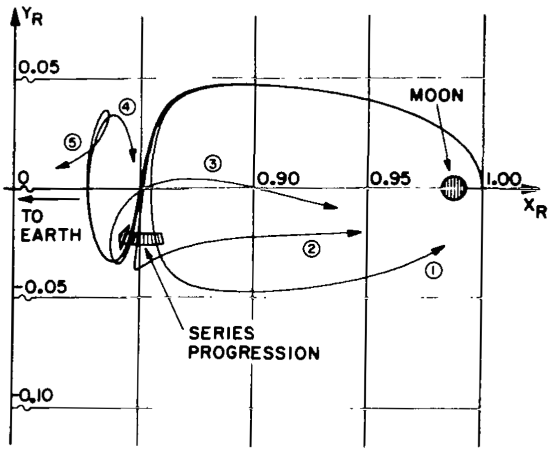
and somewhat triangular one.

Looking back at the velocity range passed through, during the phase of the lunar satellite orbits, one recognizes that from Figure 13 to Figure 15, nearly two units of the normalized velocity coordinate are involved which is approximately half of the full velocity range covered in the series.

What follows on Figure 16 is a short and tenuous phase of transition between the lunar satellite orbits and the orbits in the general E-M field. The exact borderline is manifested in an orbit that lies quite close to the orbit numbered "4" and corresponds to an asymptotic periodic orbit about the cis-lunar equilibrium point.

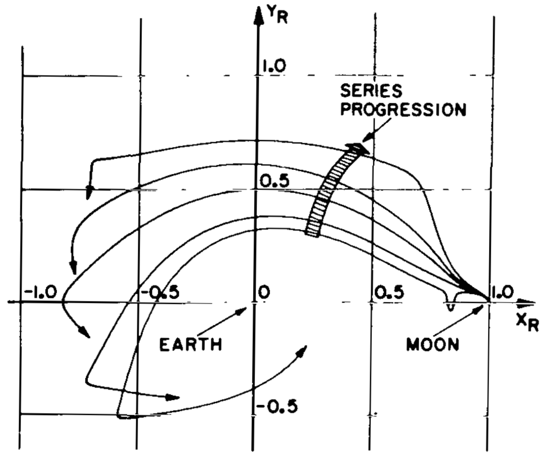
The three subsequent survey graphs (Figures 17-19) are best studied as a unit since together they reveal a second occurrence of the vacillatory perigee behavior observed before on Figures 7, 10, and 11. Here again the orbits stay away from the Earth by a finite minimum distance. Also, the development phase is again connected with a special loop formation near the Moon. The transition that is foreboded by the loop formation here is the change from the direct mean-angular motion about the Earth to the retrograde direction.

EARTH-MOON FIELD ORBITS
SURVEY GRAPHS



EARTH MOON FIELD ORBITS
SURVEY GRAPH #9
INITIAL VELOCITY Y_R FROM 1.3279 TO 1.3283

Figure 16



EARTH MOON FIELD ORBITS
SURVEY GRAPH #10
INITIAL VELOCITY Y_R FROM 1.328 TO 1.390

Figure 17

This transition is taking place within the phase displayed on Figure 20. The threshold orbit is a periodic orbit about the Moon which is characterized by the fact that it exhibits two big "ear lobes" leading and trailing the Moon. (For the threshold orbit, see Figure 310 in the succeeding chapter!)

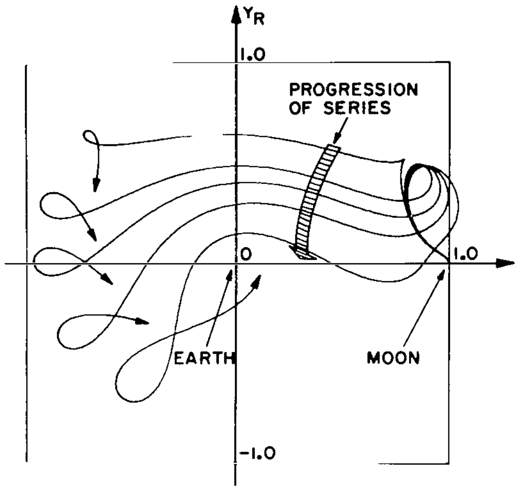
On Figure 21, a consistent group of orbits of retrograde motion about the Earth is displayed.

At this point in the progression we might resume the reference to Kepler orbits and show the continuation of the progression interrupted with Figure 9. This point is an opportune one, since the Kepler series also undergoes a change in the direction of the mean angular motion. This can be studied on Figures 22 and 23.

The continuity of the first orbit on Figure 22, i.e., orbit "F" with the group of Kepler orbits on Figure 9, is well recognizable. The development from here on involves orbits with loops that decrease in size and succeed each other in shorter intervals.

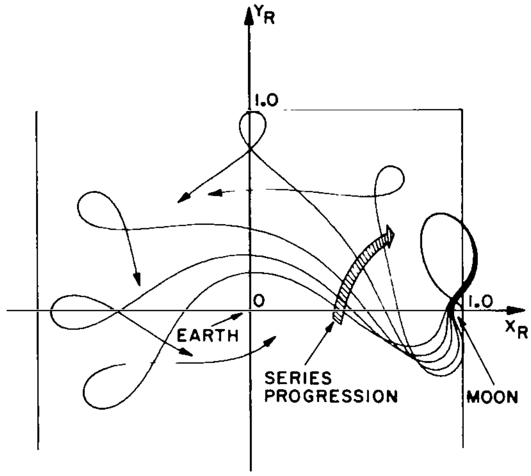
An enlarged view of what follows in the serial progression is given on the next graph (Figure 23). Orbit "I" is the last example selected of the group of orbits with direct mean motion relative to the synodic coordinate system. Orbit "K" is an orbit with retrograde mean motion. In between is the unique orbit "J" which is stationary in the synodic system and periodic with its

EARTH-MOON FIELD ORBITS
SURVEY GRAPHS



EARTH MOON FIELD ORBITS
SURVEY GRAPH #11
INITIAL VELOCITY Y_R FROM 1.395 TO 1.404

Figure 18



EARTH MOON FIELD ORBITS
SURVEY GRAPH #12
INITIAL VELOCITY Y_R FROM 1.4049 TO 1.4056

Figure 19

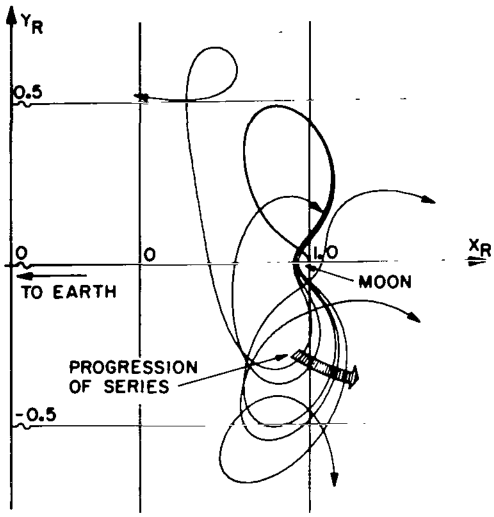
period 2π equal to that of the synodic system relative to the sidereal one. This orbit is the limit between forward and retrograde mean motion.

The evolution of the Kepler orbits shortly after passage to the retrograde direction is pointed out by the three orbit examples "K", "L", and "M" on Figure 23. Distances between successive loops increase again. On the other hand, the shrinkage of the loops continues. This latter process is completed with the arrival of the orbit labeled "L", which is a cusping orbit. Between this orbit and orbit "M" of this figure, all secondary oscillations fade away. Orbit "M" is an orbit of circular motion about the mass, the second one of the Kepler series. With respect to an inertial coordinate system, this circular orbit is of positive mean angular motion.

The reader is referred back now to the E-M-series which is continued in Figures 24 and 25. The process that unfolds within the progression of orbits on these two figures is that of further extrication of the orbits from the lunar attraction. With the last orbit on Figure 25 the process is completed.

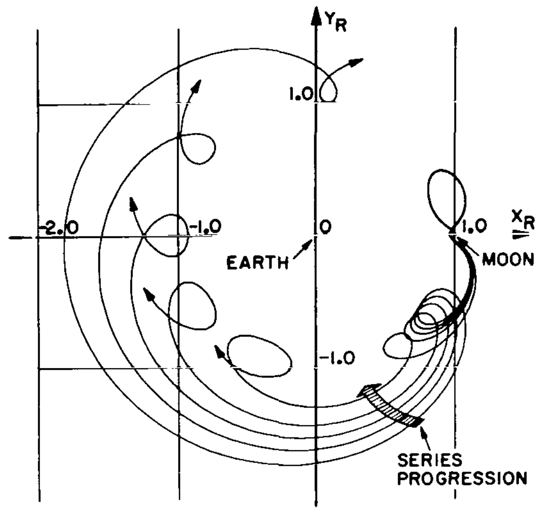
What is not shown here, is a short-lived return of the orbits to direct mean motion about the Earth. This phase and its two periodic bordering orbits may be studied in the next chapter on Figures 319 through 328.

EARTH-MOON FIELD ORBITS
SURVEY GRAPHS



EARTH MOON FIELD ORBITS
SURVEY GRAPH #13
INITIAL VELOCITY Y_R FROM 1.4056 TO 1.4059

Figure 20

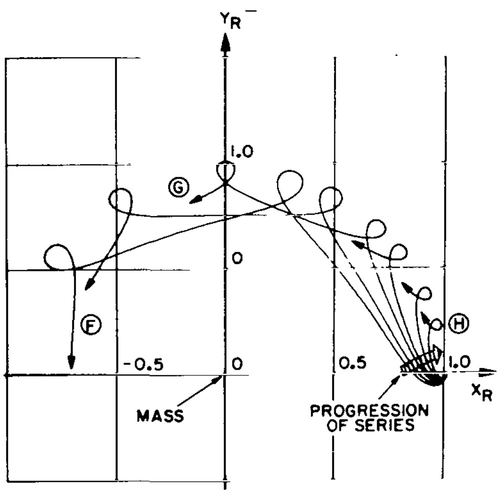


EARTH MOON FIELD ORBITS
SURVEY GRAPH #14
INITIAL VELOCITY Y_R FROM 1.40595 TO 1.40605

Figure 21

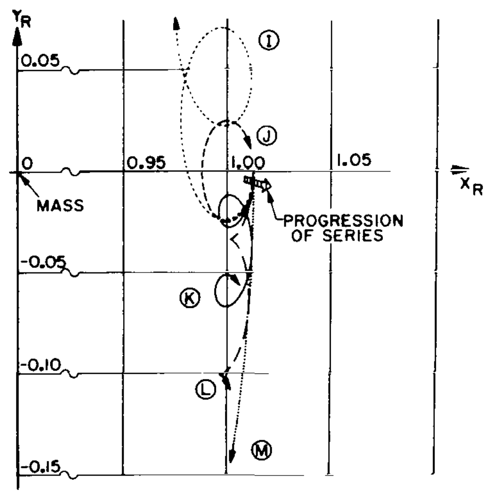
The succeeding orbit group (on Figure 26) reveals a consistent progress in a fashion that is expected. The increasing radial extension implies the growing apogee radius, and the successively later appearing perigee loops point toward the increasing orbit period.

KEPLER ORBITS
SURVEY GRAPHS



KEPLER ORBITS
SURVEY GRAPH #3
$-0.10978 \leq Y_R \leq -0.03800$

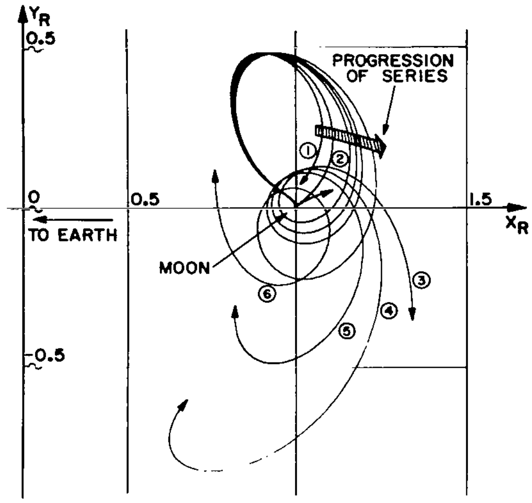
Figure 22



KEPLER ORBITS
SURVEY GRAPH #4
$-0.02744 \leq Y_R \leq -0.01869$

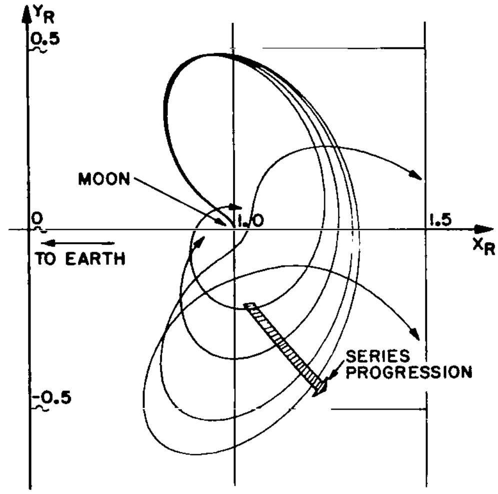
Figure 23

EARTH-MOON FIELD ORBITS
SURVEY GRAPHS



EARTH MOON FIELD ORBITS
SURVEY GRAPH #15
INITIAL VELOCITY Y_R FROM 1.406 TO 1.414

Figure 24



EARTH MOON FIELD ORBITS
SURVEY GRAPH #16
INITIAL VELOCITY Y_R FROM 1.414 TO 1.419

Figure 25

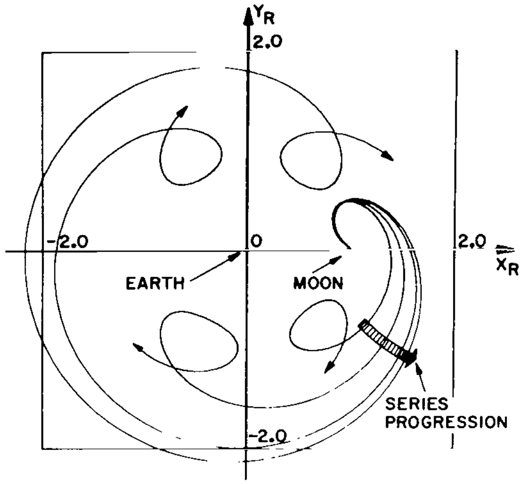
The group of orbits on Figure 27 may be looked upon rather as an interlude than a typical step in the progression. There is a minute velocity progression connected with the total group. Here, the orbits are again "caught" by the Moon and their course after flyby is deflected to generate the pattern in the Earth's field as demonstrated on this figure.

On Figure 28, the consistent pattern of growth started on Figure 26 is resumed. The development "in the large" from here on is rather predictable, though specific "interludes" of the nature shown on Figure 27 are interspersed.

The expanding pattern of spirals is exemplified by a number of orbits on the last graph of the E-M-series. (Figure 29) The last of these orbits is in the neighborhood of an escape orbit.

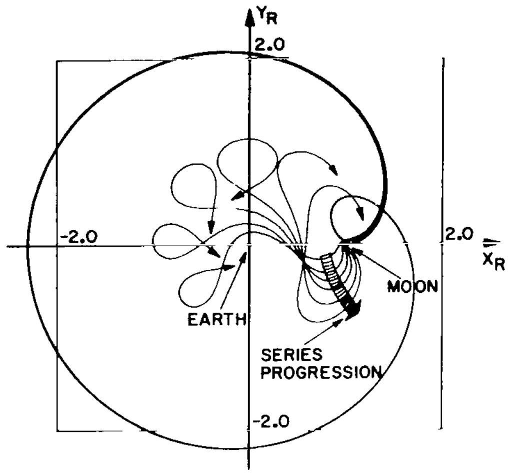
Figures 30 and 31 conclude the series of Kepler orbits. The group on Figure 30 starts out with the circular orbit "N" which is identical to orbit "M" on Figure 23. Orbits beyond the circular one in a sense reverse the pattern sequence that was followed in approaching the circular, since now orbits are again building-up the "secondary" formations as: indentations, cusp, and then increasing loops. Also, the angular spacing of the loops, i.e., of perigee occurrences, increases with the series progression as noticeable on the sequence labelled "O" to "S".

EARTH-MOON FIELD ORBITS
SURVEY GRAPHS



EARTH MOON FIELD ORBITS
SURVEY GRAPH #17
INITIAL VELOCITY Y_R FROM 1.423 TO 1.444

Figure 26

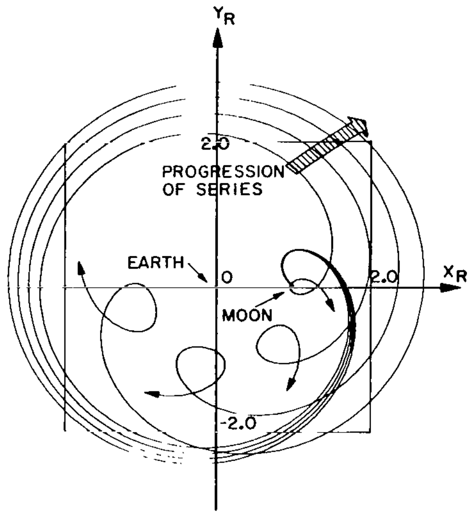


EARTH MOON FIELD ORBITS
SURVEY GRAPH #18
INITIAL VELOCITY Y_R FROM 1.4459 TO 1.4461

Figure 27

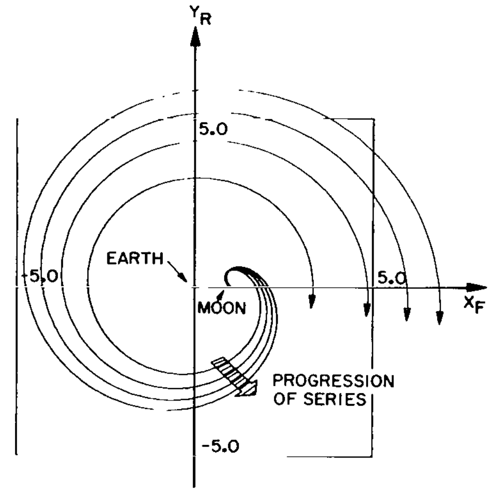
By virtue of this behavior, the trend of patterns of the Kepler series is toward growing semblance to those of the E-M-series. Comparability – at least as far as the earlier parts of the orbits are concerned – between the two series is revealing itself well in the last graph of each series (Figures 31 and 29). The orbits that terminate the E-M-series (on Figure 29) were selected to demonstrate a behavior similar to the last one of the Kepler series (Figure 31), which is, in fact, the inertially-positive branch of the parabolic orbit.

EARTH-MOON FIELD ORBITS
SURVEY GRAPHS



EARTH MOON FIELD ORBITS
SURVEY GRAPH # 19
INITIAL VELOCITY $\dot{Y}_R$ FROM 1.446 TO 1.454

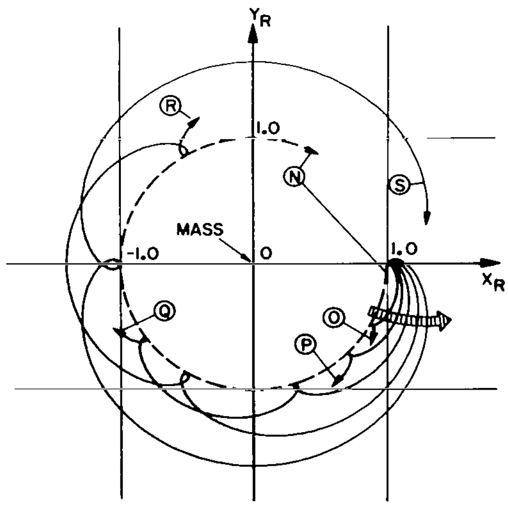
Figure 28



EARTH MOON FIELD ORBITS
SURVEY GRAPH # 20
INITIAL VELOCITY $\dot{Y}_R$ FROM 1.46 TO 1.52

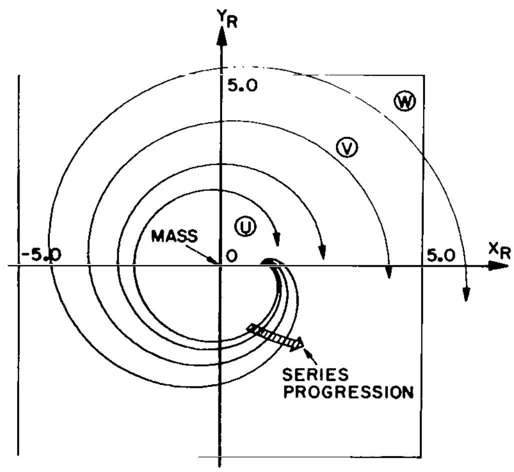
Figure 29

KEPLER ORBITS
SURVEY GRAPHS



KEPLER ORBITS
SURVEY GRAPH # 5
$-0.01869 \leq \dot{Y}_R \leq +0.08848$

Figure 30



KEPLER ORBITS
SURVEY GRAPH # 6
$0.14739 \leq \dot{Y}_R \leq 0.39296$

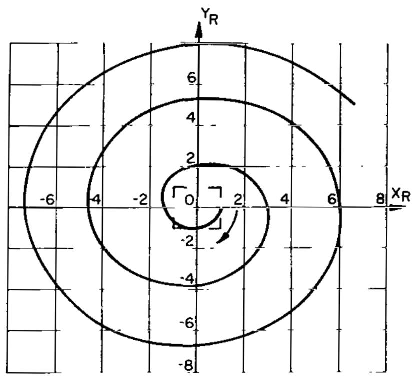
Figure 31

PRESENTATION OF INDIVIDUAL ORBITS OF THE
E-M-SERIES AND THE KEPLER SERIES

The density of sequencing the orbits of the two series in this chapter is, in general, sufficient to let the developments of orbital structures be recognized without difficulty.

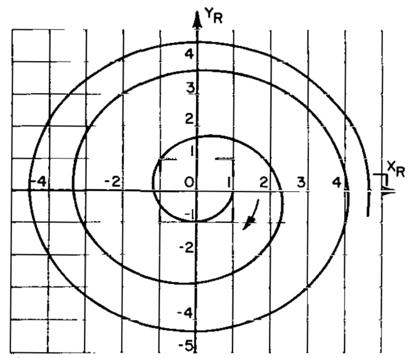
Comments to the figures, therefore, are made only at places where either the orbital behavior seems to require clarification or the significance of the orbital event calls for emphasis.

EARTH-MOON FIELD ORBITS



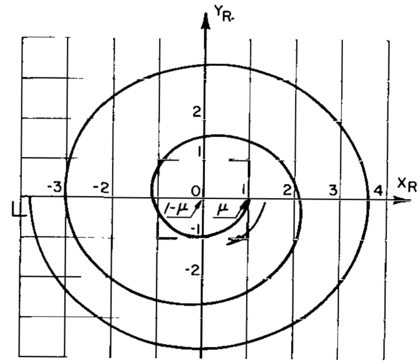
E.M. ORBIT; REFERENCE ORBIT
$\dot{Y}_R = -2.7500$

Figure 32



E.M. ORBIT; $n_* = -1/5$
$\dot{Y}_R = -2.6564$; $P/2 = 15.00$

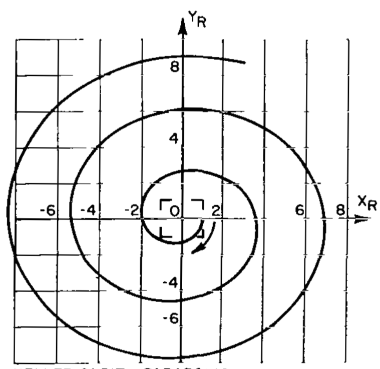
Figure 33



E.M. ORBIT; $n_* = -1/4$
$\dot{Y}_R = -2.6333$; $P/2 = 11.79$

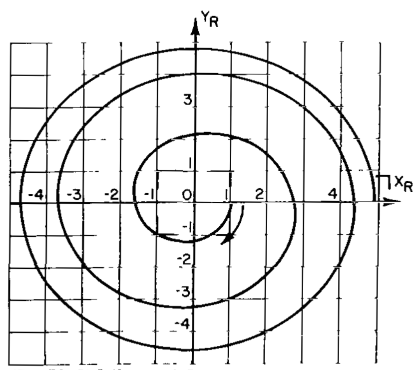
Figure 34

KEPLER ORBITS



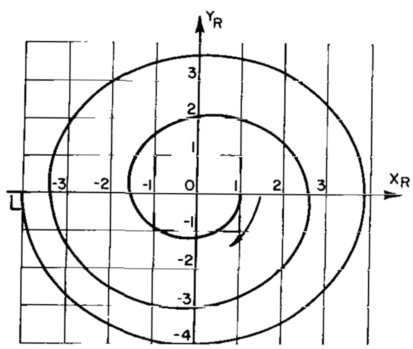
KEPLER ORBIT ; PARABOLIC
$\dot{Y}_R = -2.4180$

Figure 35



KEPLER ORBIT; $n = -1/5$
$\dot{Y}_R = -2.2905$; $P/2 = 15.71$

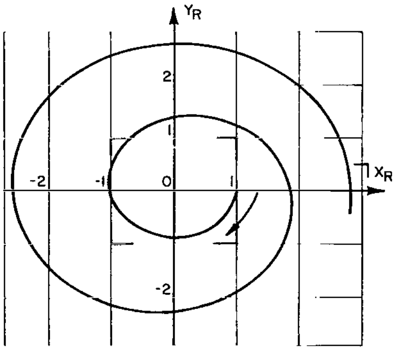
Figure 36



KEPLER ORBIT; $n = -1/4$
$\dot{Y}_R = -2.2688$; $P/2 = 12.57$

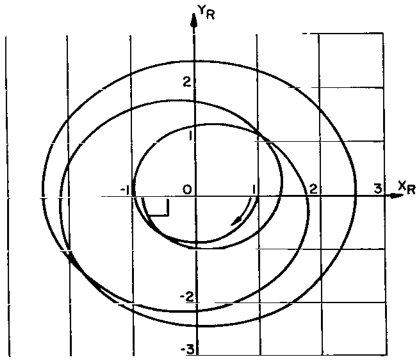
Figure 37

EARTH-MOON FIELD ORBITS



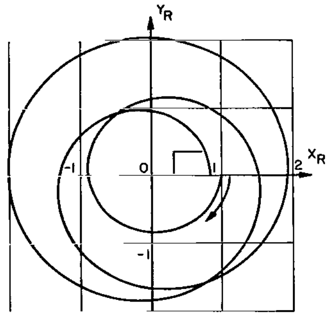
E.M. ORBIT; $n^* = -1/3$
$\dot{Y}_R = -2.5924$; $P/2 = 8.517$

Figure 38



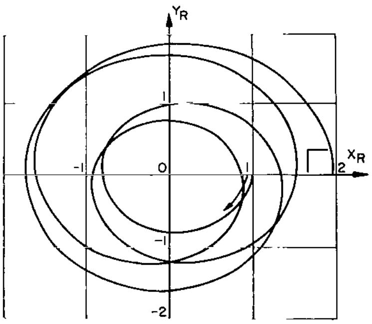
E.M. ORBIT; $n^* = -2/5$
$\dot{Y}_R = -2.5768$; $P/2 = 14.71$

Figure 39



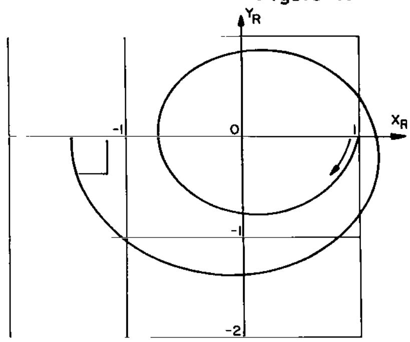
E.M. ORBIT; PERIODIC
$\dot{Y}_R = -2.5355$; $P/2 = 11.53$

Figure 40



E.M. ORBIT; PERIODIC
$\dot{Y}_R = -2.5000$; $P/2 = 14.59$

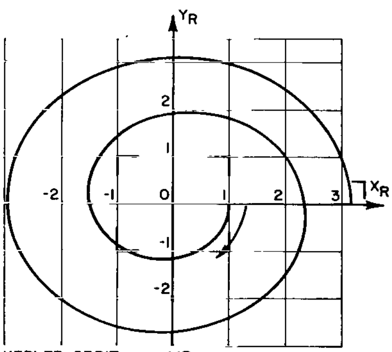
Figure 41



E.M. ORBIT; $n^* = -1/2$
$\dot{Y}_R = -2.4558$; $P/2 = 4.618$

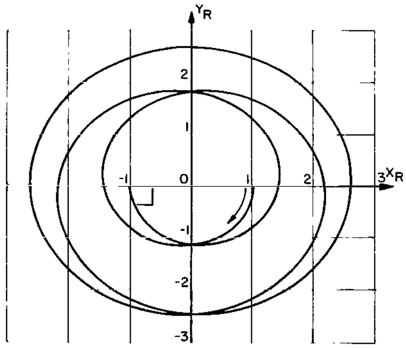
Figure 42

KEPLER ORBITS



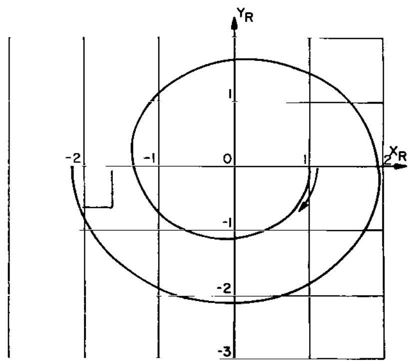
KEPLER ORBIT; $n = -1/3$
$\dot{Y}_R = -2.2350$; $P/2 = 9.43$

Figure 43



KEPLER ORBIT; $n = -2/5$
$\dot{Y}_R = -2.2093$; $P/2 = 15.71$

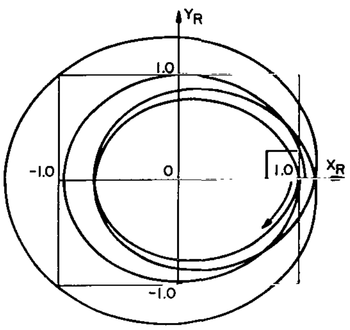
Figure 44



KEPLER ORBIT; $n = -1/2$
$\dot{Y}_R = -2.1723$; $P/2 = 6.283$

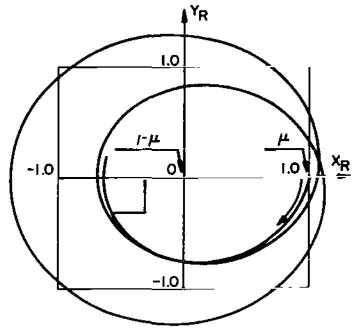
Figure 45

EARTH-MOON FIELD ORBITS



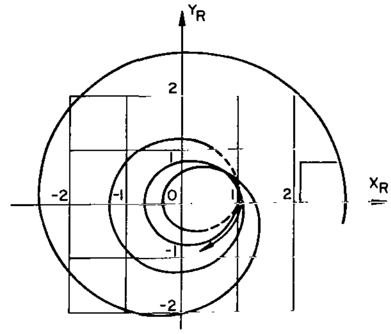
E.M. ORBIT; $n^* = -3/5$
$\dot{Y}_R = -2.4534$; $P/2 = 12.24$

Figure 46



E.M. ORBIT; $n^* = -2/3$
$\dot{Y}_R = -2.4441$; $P/2 = 7.656$

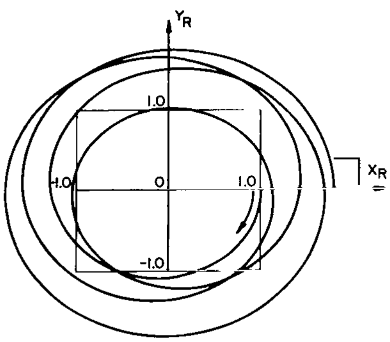
Figure 47



E.M. ORBIT; PERIODIC
$\dot{Y}_R = -2.4360$; $P/2 = 13.68$

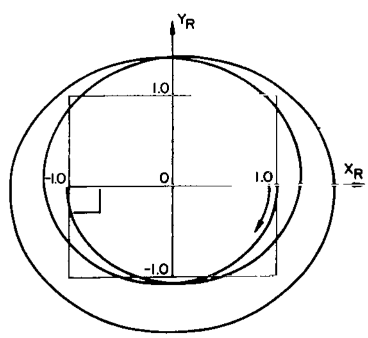
Figure 48

KEPLER ORBITS



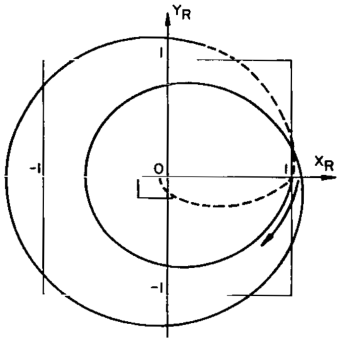
KEPLER ORBIT; $n = -3/5$
$\dot{Y}_R = -2.1367$; $P/2 = 15.71$

Figure 49



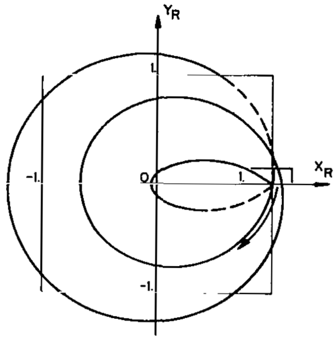
KEPLER ORBIT; $n = -2/3$
$\dot{Y}_R = -2.1135$; $P/2 = 9.425$

Figure 50



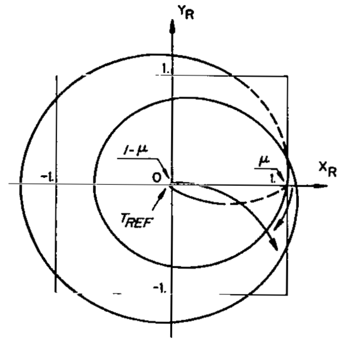
E.M. ORBIT; PERIODIC
$\dot{Y}_R = -2.434739$; $P/2 = 6.704$

Figure 51



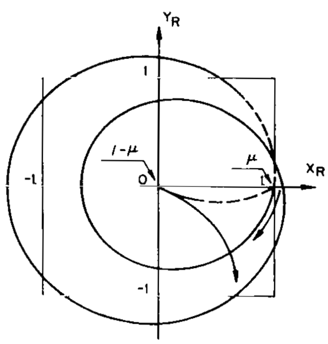
E M. ORBIT; PERIODIC
$\dot{Y}_R = -2\,434735$; $P/2 = 7.176$

Figure 52



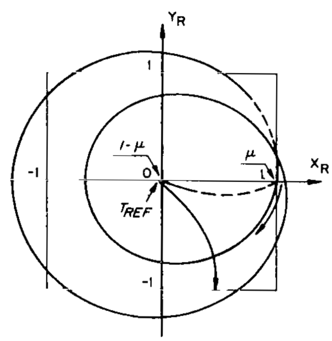
E.M ORBIT
$\dot{Y}_R = -2\,43455$; $T_{REF} = 6.642$

Figure 53



E.M ORBIT; COLLISION WITH EARTH
$\dot{Y}_R = -2\,43448$, $T_{COLL} = 6\,626$

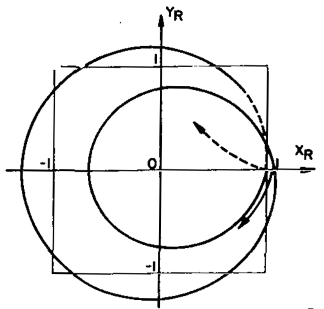
Figure 54



E M ORBIT
$\dot{Y}_R = -2\,43444$; $T_{REF} = 6\,617$

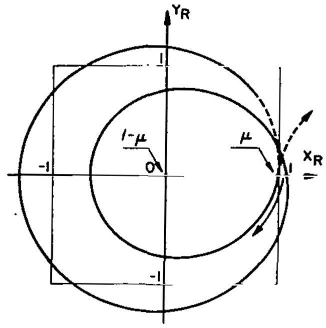
Figure 55

The development on Figures 51 to 59 reappears several times in the progression, modified or reversed. The essential features are: approach to the Moon as shown in the dashed orbital portion of Figure 51 with deflection of the downstream path toward the Earth; periodic orbits in Figures 51 and 52, the half-period marked by the orthogonal crossings of the X_R-axis; development of Earth collision shown on Figure 54, with two neighboring close Earth-flybys in opposing directions (Figures 53 and 55), development of collision with the Moon, occurring on Figure 57 with close



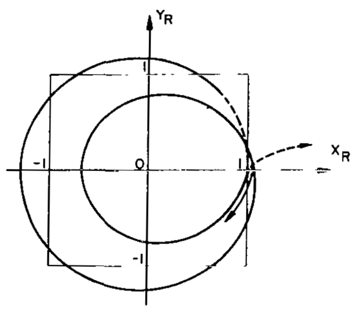
E.M. ORBIT; FLYBY DISTANCE: 1×10^{-3}
$\dot{Y}_R$=-2.4340; T_{FLYBY}=6.199

Figure 56



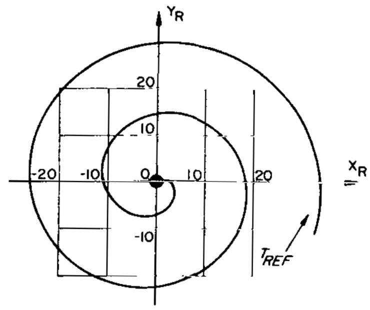
E.M. ORBIT; COLLISION WITH MOON
$\dot{Y}_R$=-2.43366; T_{COLL}=6.189

Figure 57



E.M ORBIT; FLYBY DISTANCE 2×10^{-4}
$\dot{Y}_R$=-2.4335; T_{FLYBY}=6.185

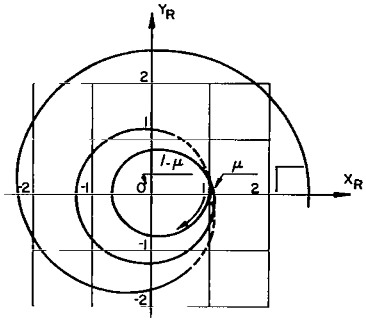
Figure 58



E M. ORBIT
$\dot{Y}_R$=-2.4335; T_{REF}=20.00

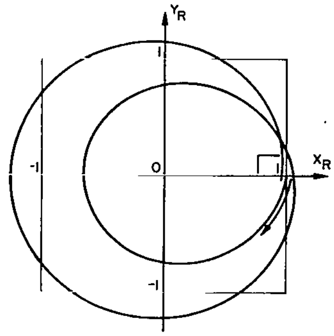
Figure 59

neighboring orbits circumnavigating the Moon in opposite directions (Figures 56 and 58). Figure 59 shows the same orbit conditions as Figure 58, but the period of the orbit is extended to 20 units compared to about 6 units on the former orbit. (The history of the initial movement is blotted out here.) Figure 59 shows the effect of the close lunar flyby. The development, encountered here the first time, is illustrated on Figures 76 through 84 in its clearest form.



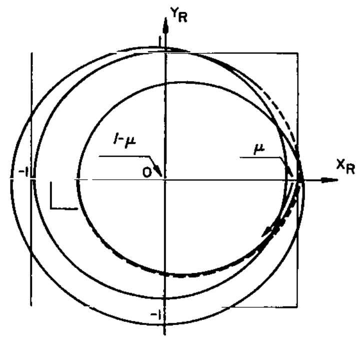
E.M. ORBIT; PERIODIC
$\dot{Y}_R = -2.4322$; $P/2 = 11.30$

Figure 60



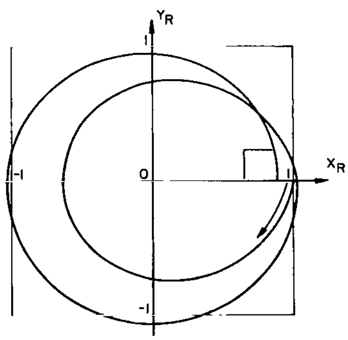
E M ORBIT; PERIODIC
$\dot{Y}_R = -2.4301$; $P/2 = 6.106$

Figure 61



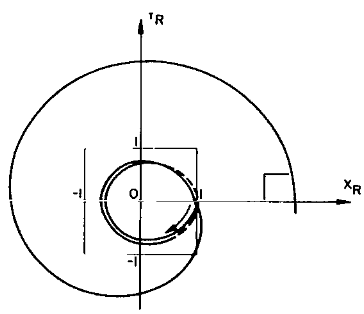
E.M. ORBIT; PERIODIC
$\dot{Y}_R = -2\ 4232$; $P/2 = 10.15$

Figure 62



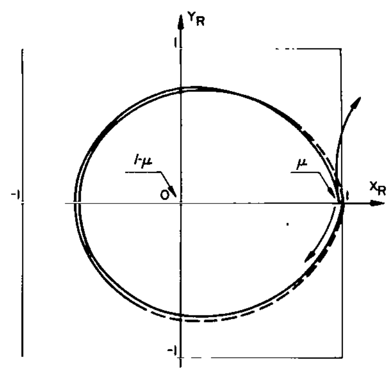
E M. ORBIT; PERIODIC
$\dot{Y}_R = -2.4187$; $P/2 = 5.576$

Figure 63



E.M. ORBIT; PERIODIC
$\dot{Y}_R = -2.41203$; $P/2 = 10.16$

Figure 64



E.M. ORBIT; COLLISION WITH MOON
$Y_R = -2.41161$; $T_{COLL} = 4\ 771$

Figure 65

EARTH-MOON FIELD ORBITS

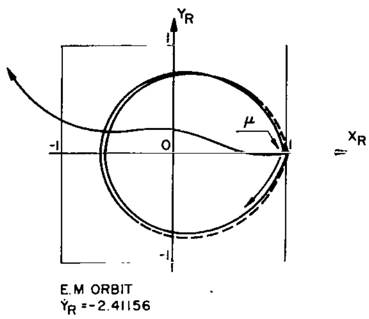


Figure 66

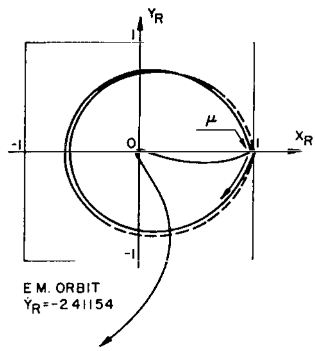


Figure 67

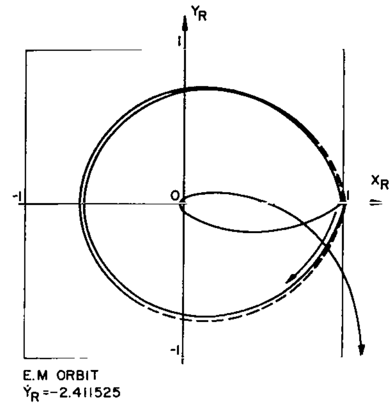


Figure 68

KEPLER ORBITS

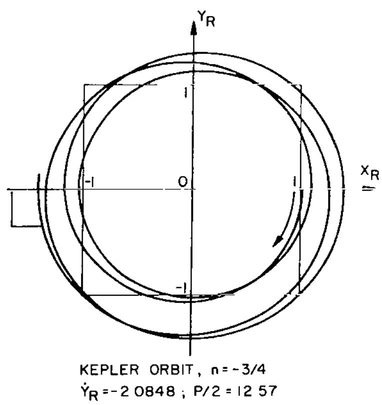


Figure 69

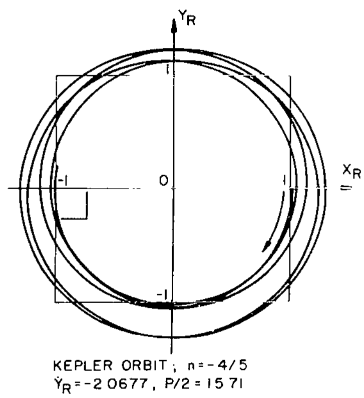
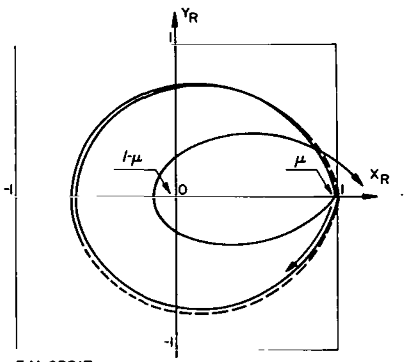
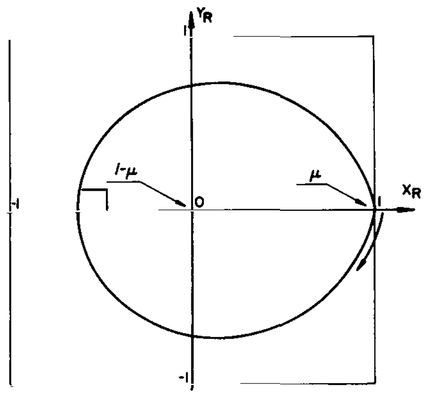


Figure 70



E.M. ORBIT
$\dot{Y}_R = -2.41150$

Figure 71



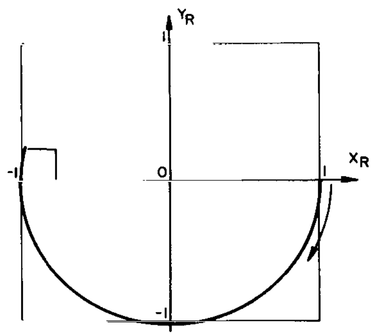
E.M. ORBIT ; PERIODIC
$\dot{Y}_R = -2.41136$; $P/2 = 1.181$

Figure 72

Comments to Figures 71 to 73

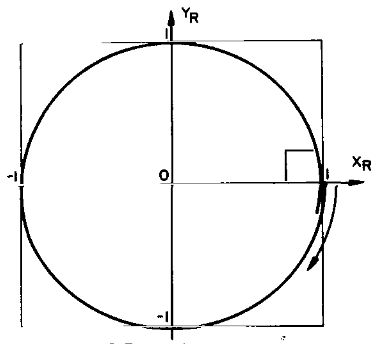
The smaller of the three loops of the orbit on Figure 71 expands with progression of the series and all three loops then collapse into a single loop, which is depicted on Figure 72. This is the only figure in the E-M-series that can be set opposite to the (inertially negative) circular orbit of the Kepler series (Figure 73).

KEPLER ORBITS



KEPLER ORBIT ; CIRCULAR (NEGATIVE)
$\dot{Y}_R = -2.0063$; $P/2 = 1.585$

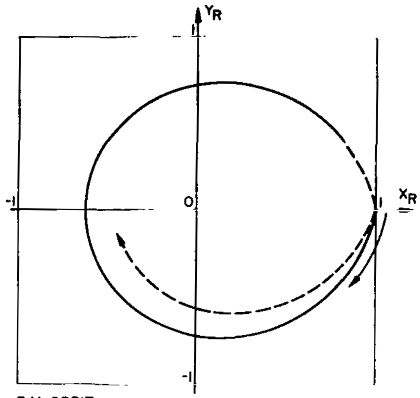
Figure 73



KEPLER ORBIT; $n = -1$
$\dot{Y}_R = -2.0001$; $P/2 = 3.142$

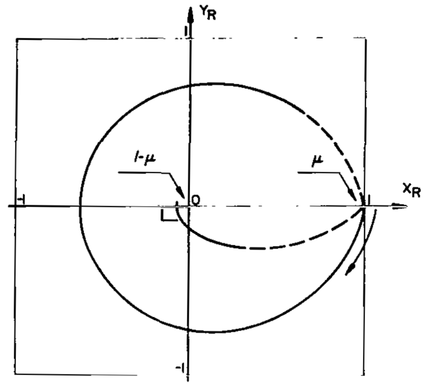
Figure 74

EARTH-MOON FIELD ORBITS



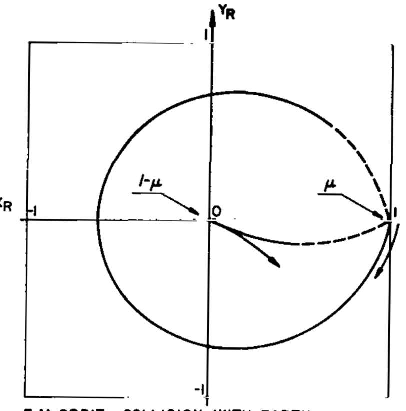
E.M. ORBIT
$\dot{Y}_R = -2.4100$

Figure 75



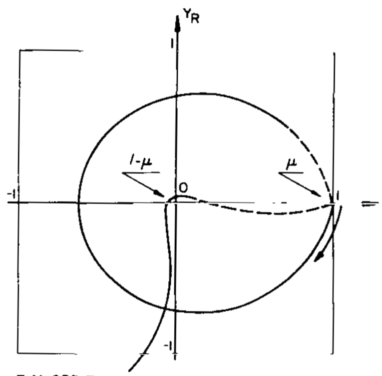
E.M ORBIT; PERIODIC
$\dot{Y}_R = -2.4080$; $P/2 = 2.846$

Figure 76



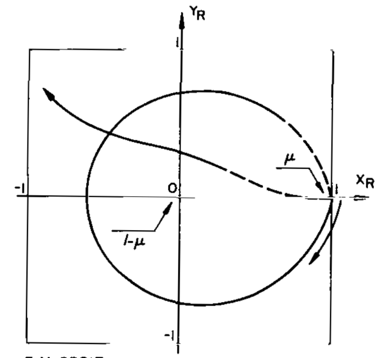
E.M. ORBIT ; COLLISION WITH EARTH
$\dot{Y}_R = -2.40762$; $T_{COLL} = 2.762$

Figure 77



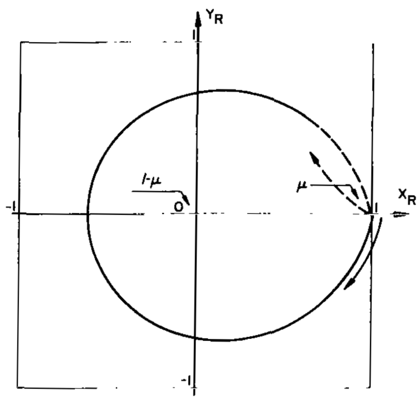
E M ORBIT
$\dot{Y}_R = -2.40740$

Figure 78



E M ORBIT
$\dot{Y}_R = -2.4070$

Figure 79

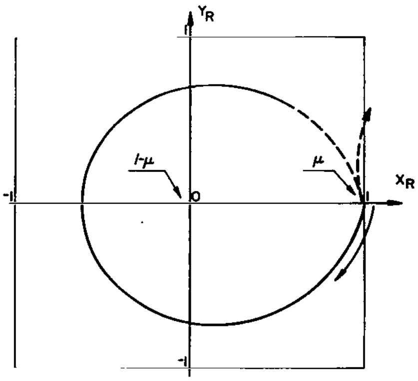


E M. ORBIT
$\dot{Y}_R = -2.4065$

Figure 80

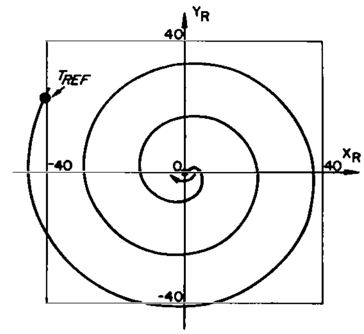
Comments to Figures 76 to 84

On Figures 76 to 84 the development (discussed earlier) involving collisions with Earth and Moon within a close interval of initial conditions is rendered in its clearest form. Observe that the orbit of collision with the Moon (Figure 81) is repeated with much longer time period on Figure 82. While this orbit is not investigated to determine whether it is of escaping type or bounded, the orbit on Figure 84 is included to show the boundedness.



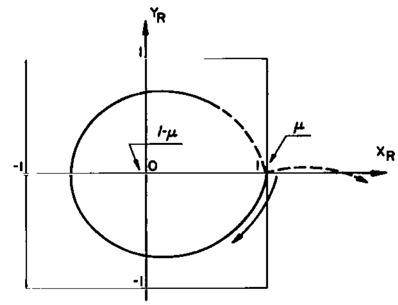
E.M. ORBIT; COLLISION WITH MOON
$\dot{Y}_R = -2.40604$; $T_{COLL} = 2.3346$

Figure 81



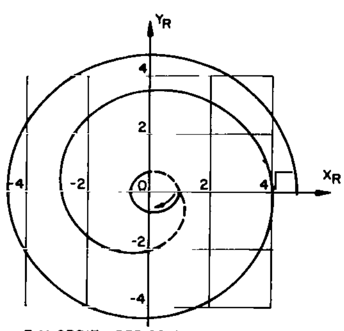
E.M. ORBIT; COLLISION WITH MOON
$\dot{Y}_R = -2.40604$; $T_{REF} = 20.00$

Figure 82



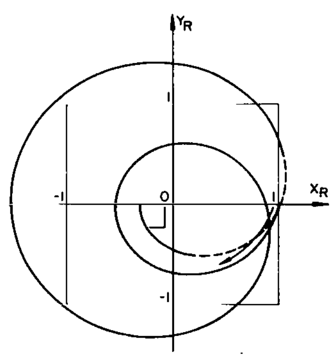
E.M. ORBIT
$\dot{Y}_R = -2.4050$

Figure 83



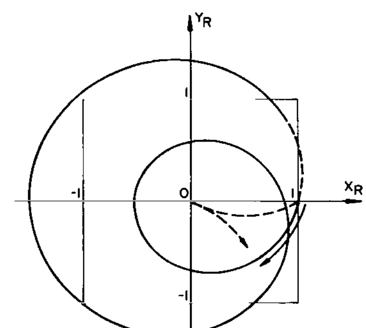
E.M. ORBIT ; PERIODIC
$\dot{Y}_R = -2.400036$; $P/2 = 14.05$

Figure 84



E.M. ORBIT; PERIODIC
$\dot{Y}_R = -2.36314$; $P/2 = 7.185$

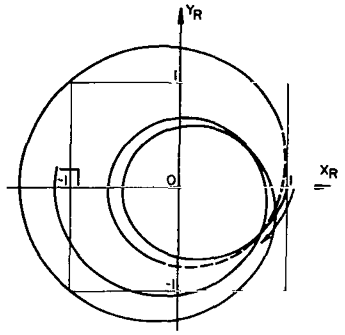
Figure 85



E M ORBIT , COLLISION WITH EARTH
$\dot{Y}_R = -2.35937$, $T_{COLL} = 6.697$

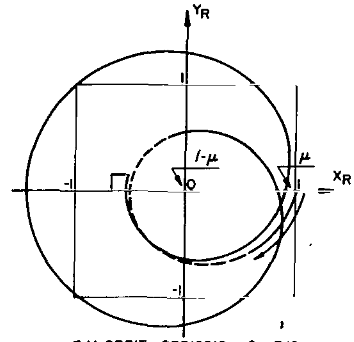
Figure 86

EARTH-MOON FIELD ORBITS



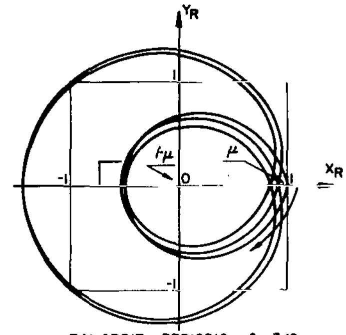
E.M ORBIT; PERIODIC; n*=-4/3
$\dot{Y}_R$=-2.35300; P/2=10.20

Figure 87



E.M. ORBIT; PERIODIC; n*=-3/2
$\dot{Y}_R$=-2.34686; P/2=7.153

Figure 88



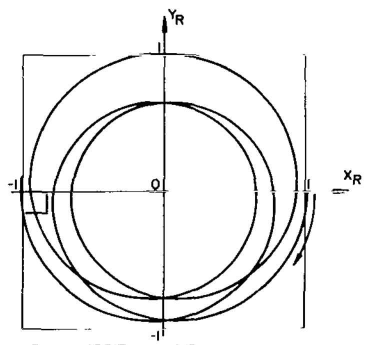
E M ORBIT, PERIODIC; n*=-3/2
$\dot{Y}_R$=-2.34686; P/2=7.153

Figure 89

Comments to Figures 88 to 99

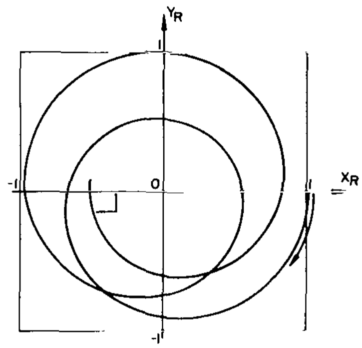
Some of these orbits are shown with half period as well as full period to aid the perception of structural similarity of E-M-orbits with the corresponding Kepler orbits.

KEPLER ORBITS



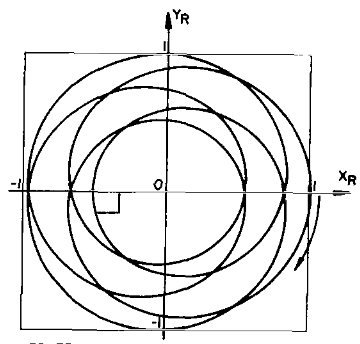
KEPLER ORBIT; n=-4/3
$\dot{Y}_R$=-1.88651; P/2=9.425

Figure 90



KEPLER ORBIT; n=-3/2
$\dot{Y}_R$=-1.82794; P/2=6.283

Figure 91



KEPLER ORBIT; n=-3/2
$\dot{Y}_R$=-1.82794; P/2=6.283

Figure 92

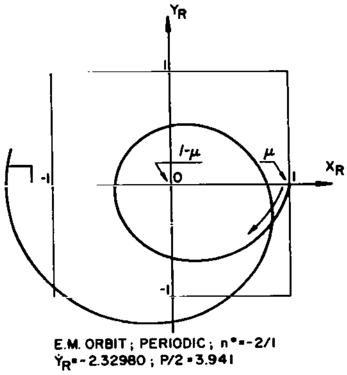


Figure 93

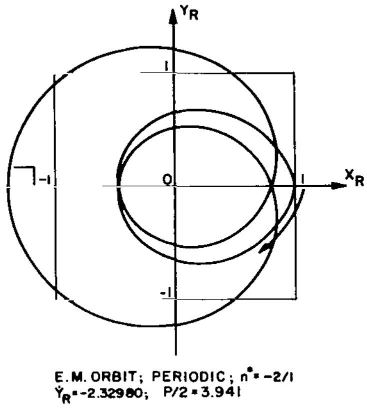


Figure 94

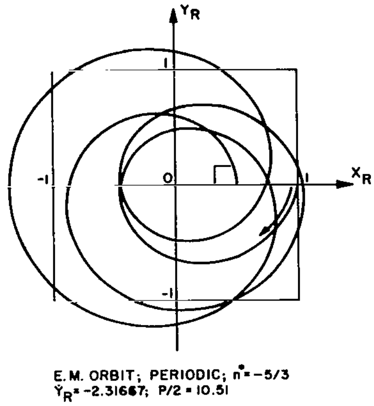


Figure 95

Comments to Figures 93 to 99

Notice that both Figures 95 and 99 show orbits of the structure of $n^* = -5/3$, corresponding to the structure of the Kepler orbit of commensurability $n = -5/3$ as depicted on Figure 96. The sequence of these E-M-orbits with the E-M-orbit of $n^* = -2/1$ on Figure 93 is reversed to that of the corresponding Kepler orbits on Figures 96 and 97.

KEPLER ORBITS

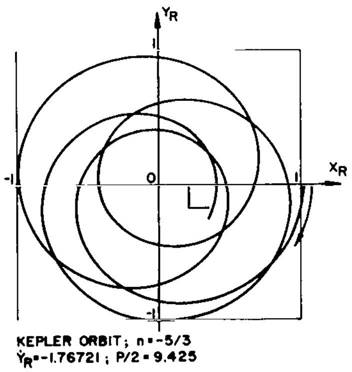


Figure 96

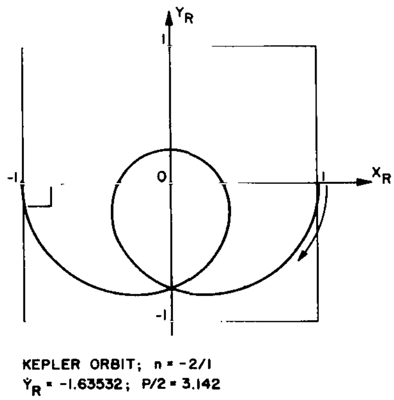


Figure 97

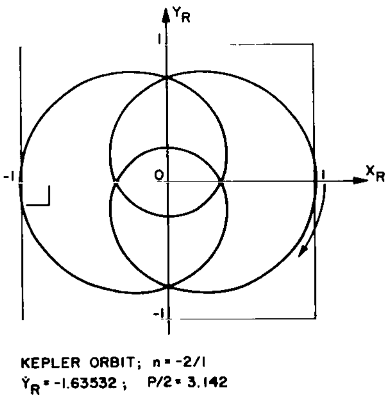
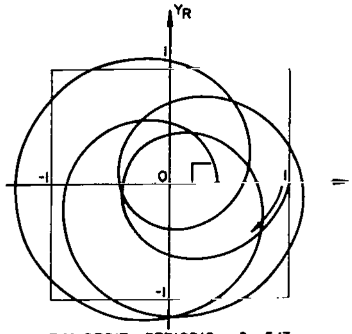


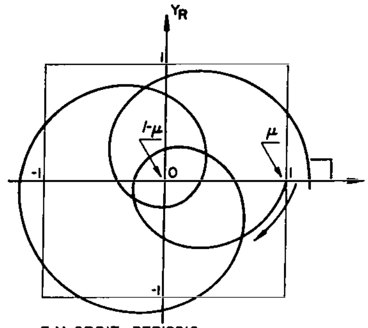
Figure 98

EARTH-MOON FIELD ORBITS



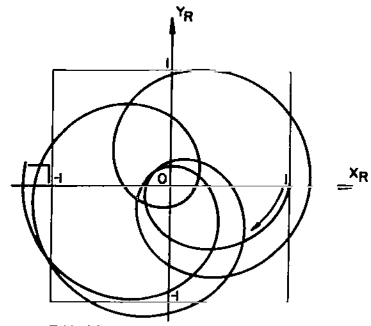
E.M. ORBIT; PERIODIC; $n^* = -5/3$
$\dot{Y}_R = -2.28851$; $P/2 = 10.24$

Figure 99



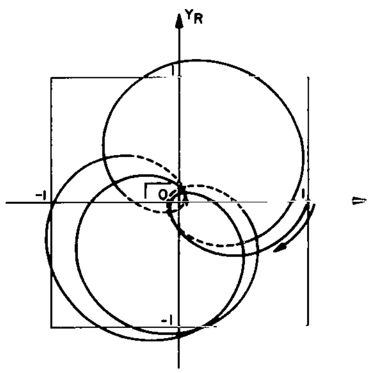
E.M. ORBIT; PERIODIC
$\dot{Y}_R = -2.18550$; $P/2 = 6.73$

Figure 100



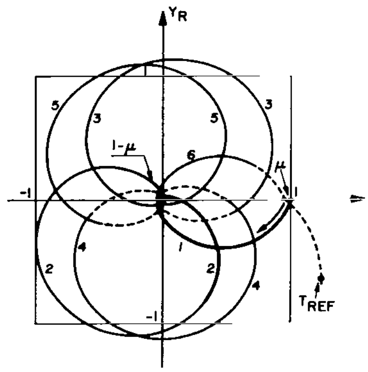
E.M. ORBIT; PERIODIC
$\dot{Y}_R = -2.14009$; $P/2 = 9.828$

Figure 101



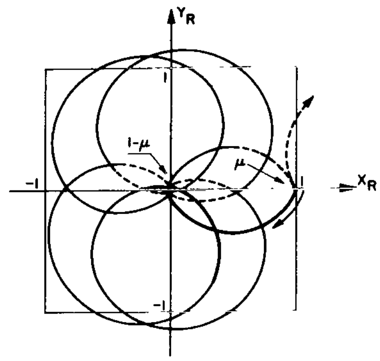
E.M. ORBIT; PERIODIC; $n^* = -7/3$
$\dot{Y}_R = -1.97935$; $P/2 = 9.575$

Figure 102



E. M. ORBIT;
$\dot{Y}_R = -1.9150$; $T_{REF} = 13.35$

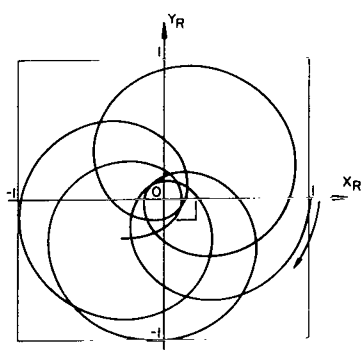
Figure 103



E.M. ORBIT; COLLISION WITH MOON
$\dot{Y}_R = -1.8730$; $T_{COLL} = 12.707$

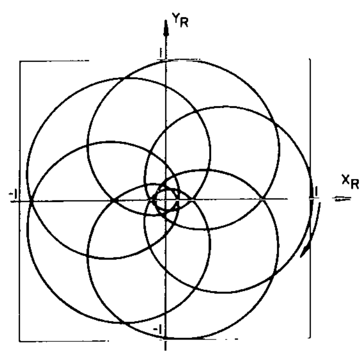
Figure 104

KEPLER ORBITS



KEPLER ORBIT; $n = -7/3$
$\dot{Y}_R = -147736$; $P/2 = 9.425$

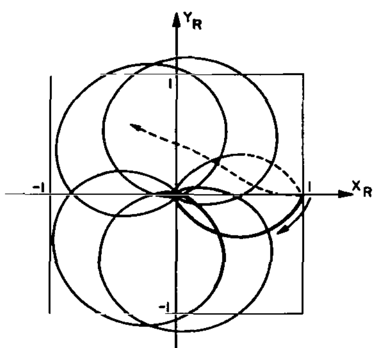
Figure 105



KEPLER ORBIT; $n = -5/2$
$\dot{Y}_R = -1.37759$; $P/2 = 6.283$

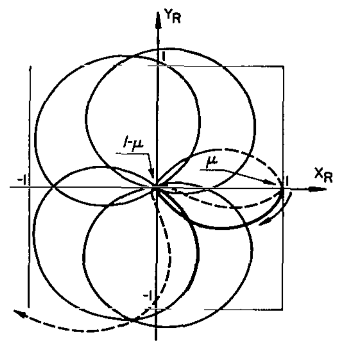
Figure 106

EARTH-MOON FIELD ORBITS



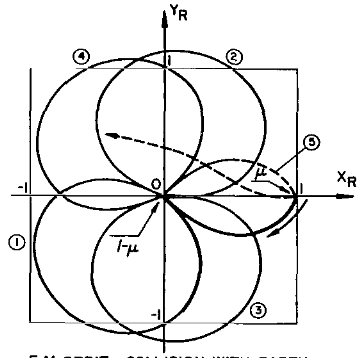
E. M. ORBIT;
$\dot{Y}_R$ = −1.8600

Figure 107



E. M. ORBIT; CLOSEST TO n* = −5/2
$\dot{Y}_R$ = −1.8300; LUNAR PASSAGE TIME: 12.62

Figure 108



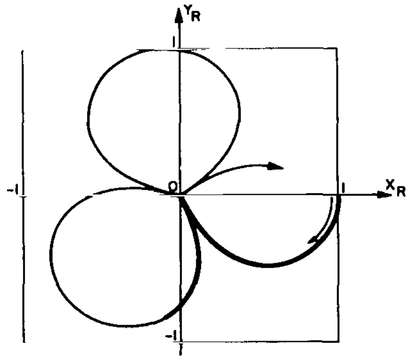
E. M. ORBIT ; COLLISION WITH EARTH
$\dot{Y}_R$ = −1.7950 ; TIME OF FIRST COLLISION: 0.7666

Figure 109

Figure 109 is the case of the orbit colliding with the Earth (this assumed to be a point-mass). After five collisions or near-collisions, the orbit returns to circumnavigate the Moon (dashed curve). The sequence is indicated by the numbers on the loops.

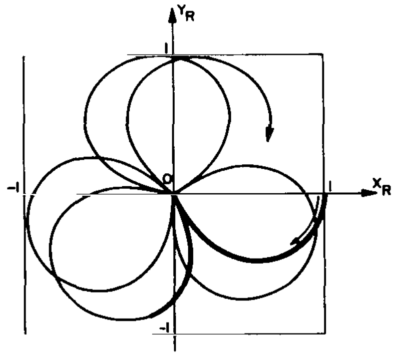
Since lunar circumnavigation does not happen with orthogonal crossing of the X_R-axis, the collision case is not shown to be periodic. The best approach to an orthogonal crossing behind the Moon is obtained with the orbit of Figure 108. On the other hand, a collision with the Moon is encountered with the orbit of Figure 104.

KEPLER ORBITS



KEPLER ORBIT; COLLISION
$\dot{Y}_R$ = −1.0125; TIME OF FIRST COLLISION: 1.132

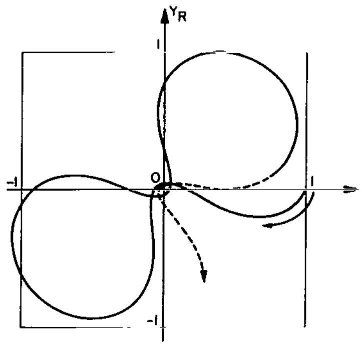
Figure 110



KEPLER ORBIT; COLLISION
$\dot{Y}_R$ = −1.0125; TIME OF FIRST COLLISION: 1.132

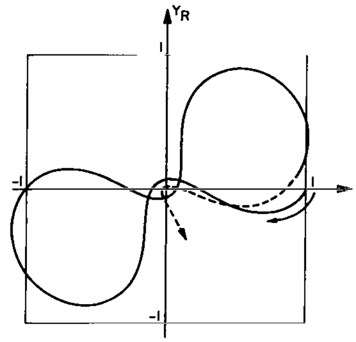
Figure 111

EARTH-MOON FIELD ORBITS



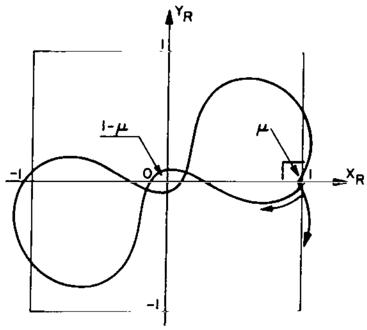
E.M. ORBIT;
$Y_R = -1.650$

Figure 112



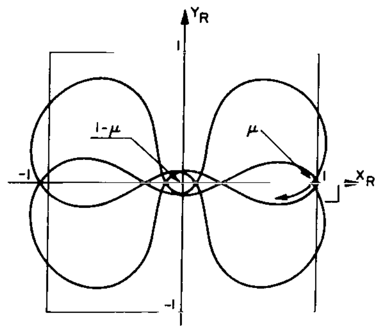
E.M ORBIT;
$Y_R = -1.620$

Figure 113



E.M. ORBIT; PERIODIC
$Y_R = -1.60225$; $P/2 = 6.151$

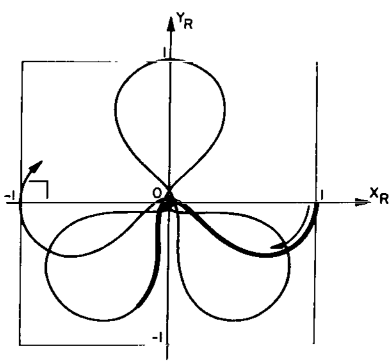
Figure 114



E.M. ORBIT; PERIODIC
$Y_R = -1.60225$; $P/2 = 6.151$

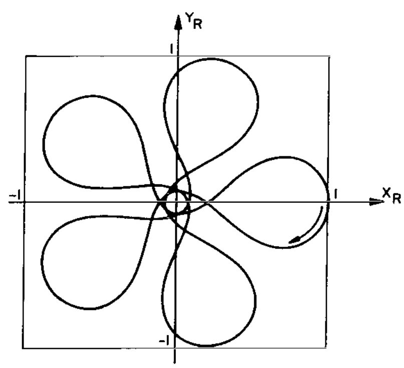
Figure 115

KEPLER ORBITS



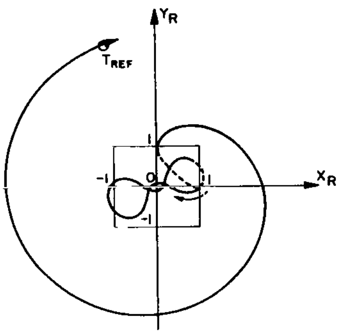
KEPLER ORBIT; $n = +8/3$
$Y_R = -0.783788$; $P/2 = 9.425$

Figure 116



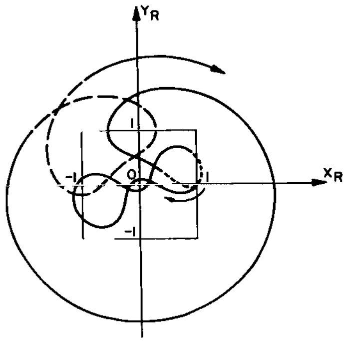
KEPLER ORBIT; $n = +5/2$
$Y_R = -0.647407$; $P/2 = 6.283$

Figure 117



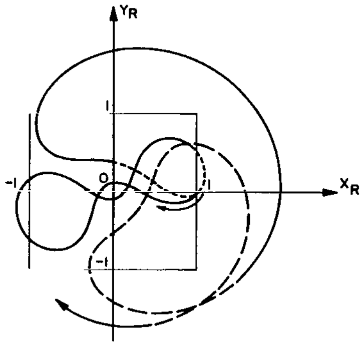
E.M. ORBIT;
$\dot{Y}_R = -1.5900$; $T_{REF} = 15.00$

Figure 118



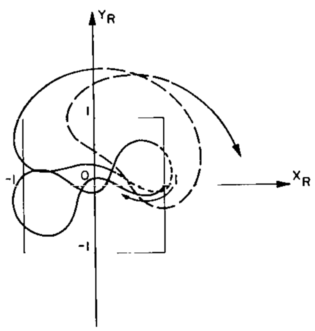
E.M. ORBIT;
$\dot{Y}_R = -1.5880$

Figure 119



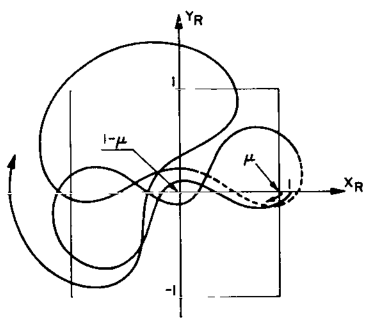
E.M. ORBIT;
$\dot{Y}_R = -1.5860$

Figure 120



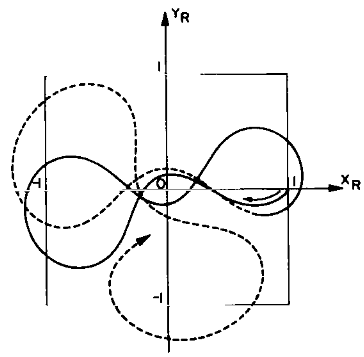
E.M. ORBIT;
$\dot{Y}_R = -1.5840$

Figure 121



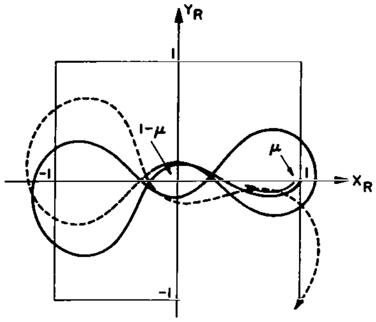
E.M. ORBIT;
$\dot{Y}_R = -1.5800$

Figure 122



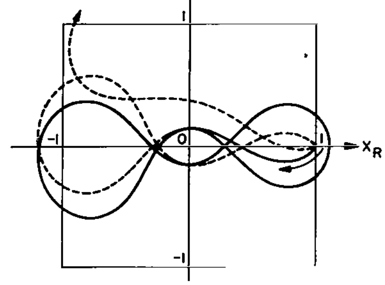
E.M. ORBIT;
$\dot{Y}_R = -1.5700$

Figure 123



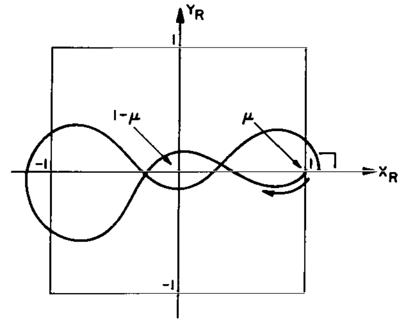
E. M. ORBIT ;
$\dot{Y}_R = -1.5600$

Figure 124



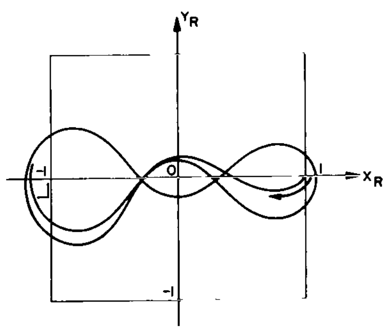
E. M. ORBIT ;
$\dot{Y}_R = -1.5530$

Figure 125



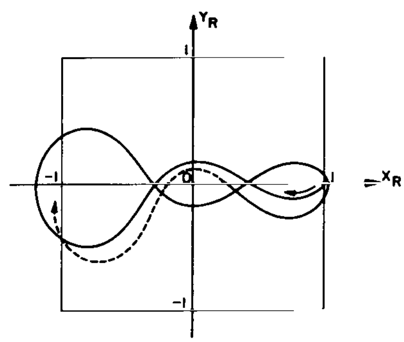
E. M. ORBIT ; PERIODIC
$\dot{Y}_R = -1.55245$; $P/2 = 5.752$

Figure 126



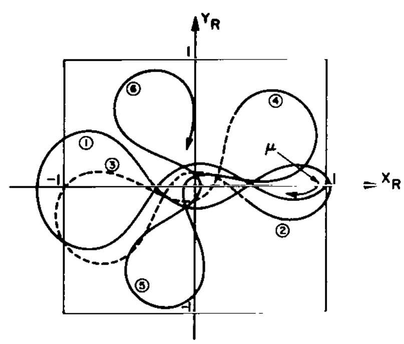
E. M. ORBIT; PERIODIC
$\dot{Y}_R = -1.54681$; $P/2 = 8.637$

Figure 127



E. M. ORBIT ;
$\dot{Y}_R = -1.5400$

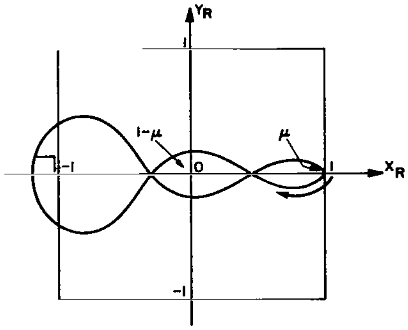
Figure 128



E. M. ORBIT;
$\dot{Y}_R = -1.5400$

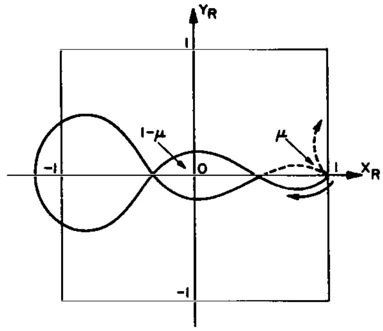
Figure 129

EARTH-MOON FIELD ORBITS



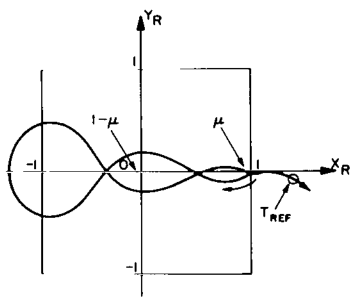
E.M. ORBIT; PERIODIC; $n^* = +2/1$
$Y_R = -1.53667$; $P/2 = 2.714$

Figure 130



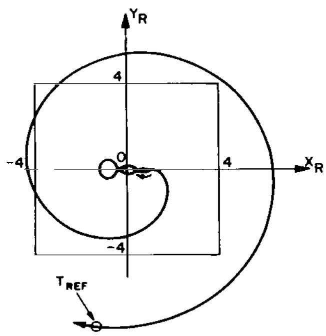
E.M. ORBIT;
$Y_R = -1.5340$

Figure 131



E.M. ORBIT;
$Y_R = -1.5310$; $T_{REF} = 6.0$

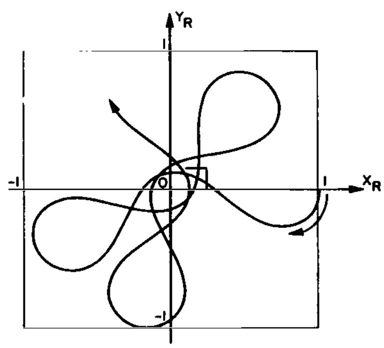
Figure 132



E.M. ORBIT;
$Y_R = -1.5310$; $T_{REF} = 15.00$

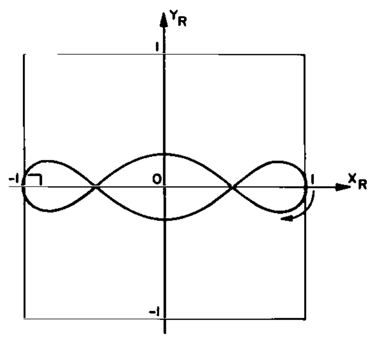
Figure 133

KEPLER ORBITS



KEPLER ORBIT; $n = +7/3$
$Y_R = -0.547637$; $P/2 = 9.425$

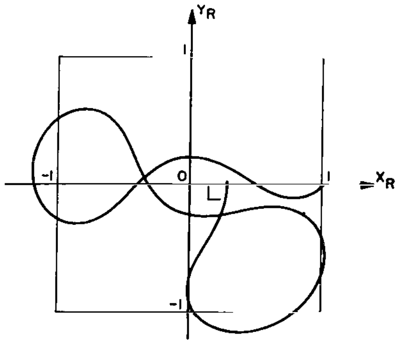
Figure 134



KEPLER ORBIT; $n = +2/1$
$Y_R = -0.389678$; $P/2 = 3.1416$

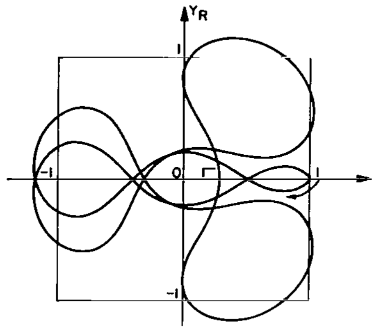
Figure 135

EARTH-MOON FIELD ORBITS



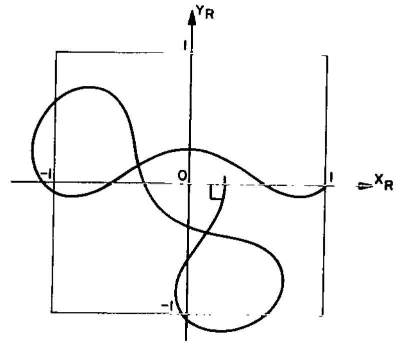
E.M. ORBIT; PERIODIC
$Y_R = -1.51747$; $P/2 = 9.107$

Figure 136



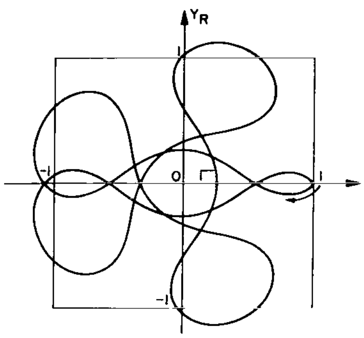
E.M. ORBIT; PERIODIC
$Y_R = -1.51747$; $P/2 = 9.107$

Figure 137



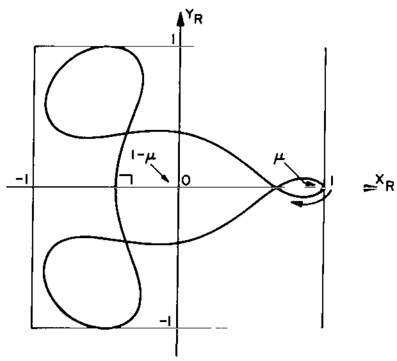
E.M. ORBIT; PERIODIC
$Y_R = -1.49797$; $P/2 = 8.893$

Figure 138



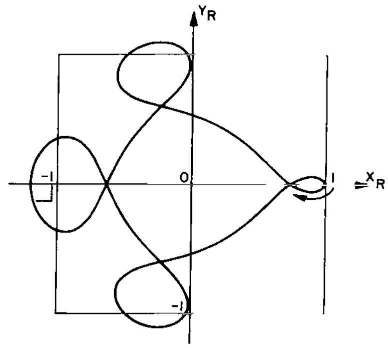
E.M. ORBIT; PERIODIC
$Y_R = -1.49797$; $P/2 = 8.893$

Figure 139



E.M. ORBIT; PERIODIC
$Y_R = -1.45921$; $P/2 = 5.557$

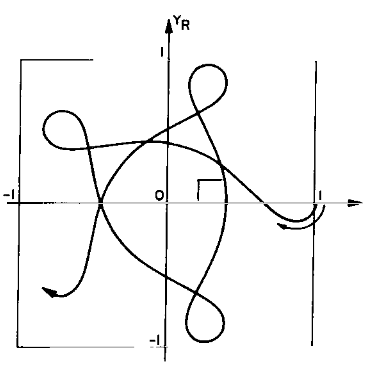
Figure 140



E.M. ORBIT; PERIODIC
$Y_R = -1.43253$; $P/2 = 8.519$

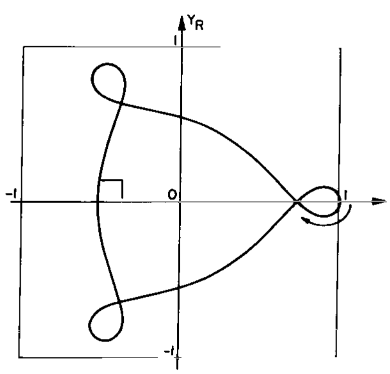
Figure 141

KEPLER ORBITS



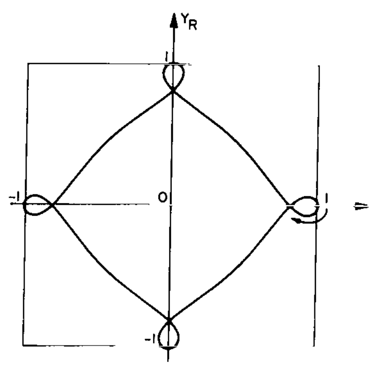
KEPLER ORBIT; $n = +5/3$
$Y_R = -0.257790$; $P/2 = 9.425$

Figure 142



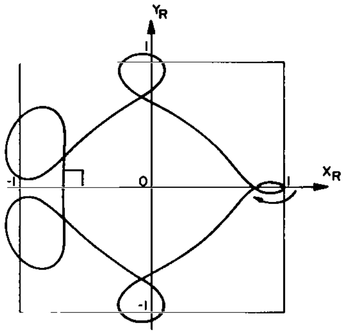
KEPLER ORBIT; $n = +3/2$
$Y_R = -0.19706$; $P/2 = 6.2832$

Figure 143



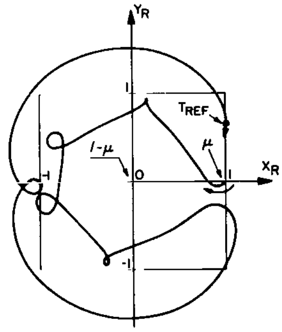
KEPLER ORBIT; $n = +4/3$
$Y_R = -0.13849$; $P/2 = 9.425$

Figure 144



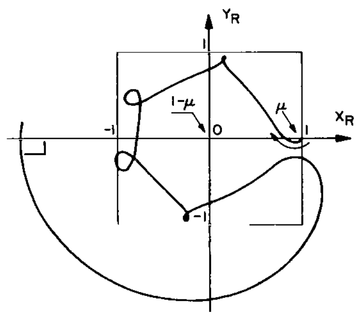
E M ORBIT; PERIODIC
$\dot{Y}_R = -1.41689$; $P/2 = 11.50$

Figure 145



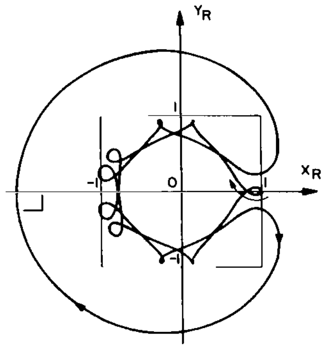
E.M. ORBIT
$\dot{Y}_R = -1.39900$; $T_{REF} = 40.0$

Figure 146



E M.ORBIT; PERIODIC
$Y_R = -1.398481$; $P/2 = 29.05$

Figure 147



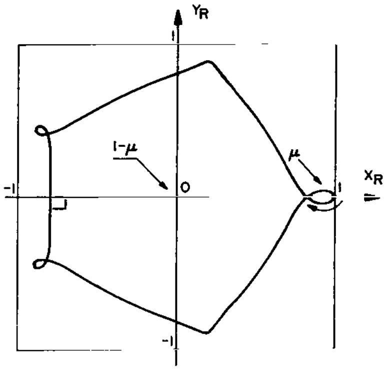
E M ORBIT; PERIODIC
$Y_R = -1.398481$, $P/2 = 29.05$

Figure 148

Comments to Figures 146 to 148

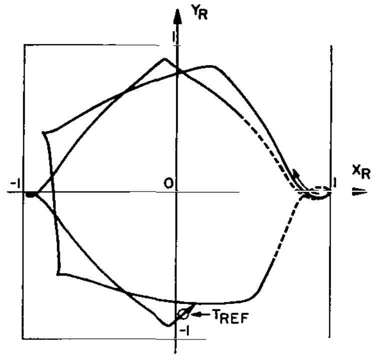
Attention may be directed to the fact that from here on, for a large portion of the series to come, the "breakouts" of the orbits into outside regions materialize through the neck that is formed by the Jacobian zero-velocity curves in the neighborhood of the Moon.

EARTH-MOON FIELD ORBITS



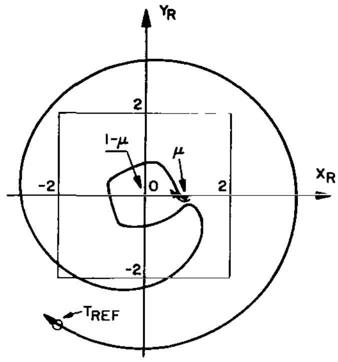
E M. ORBIT; PERIODIC
$\dot{Y}_R = -1.39409$; $P/2 = 11.00$

Figure 149



E.M. ORBIT
$\dot{Y}_R = -1.391530$; $T_{REF} = 35.0$

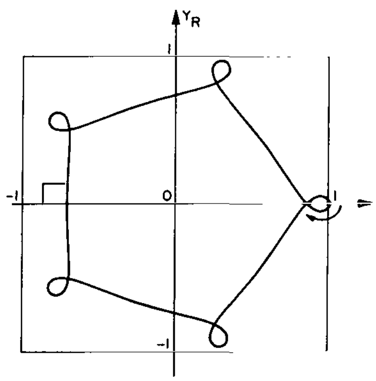
Figure 150



E.M. ORBIT
$\dot{Y}_R = -1.388$; $T_{REF} = 30.0$

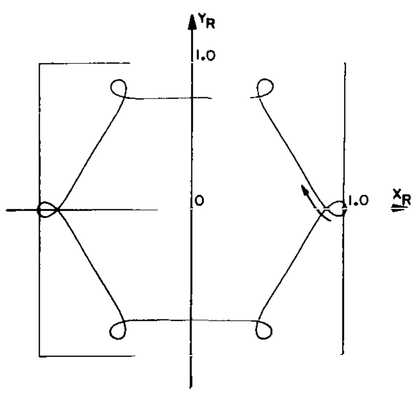
Figure 151

KEPLER ORBITS



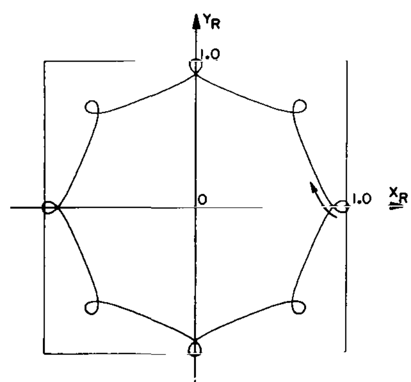
KEPLER ORBIT; $n = +5/4$
$\dot{Y}_R = -0.10978$; $P/2 = 12.57$

Figure 152



KEPLER ORBIT; $n = +6/5$
$\dot{Y}_R = -0.092682$; $P/2 = 15.71$

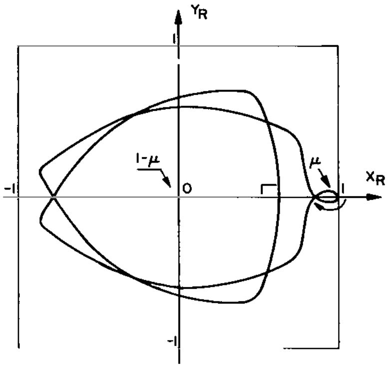
Figure 153



KEPLER ORBIT; $n = +8/7$
$\dot{Y}_R = -0.073242$; $P/2 = 21.99$

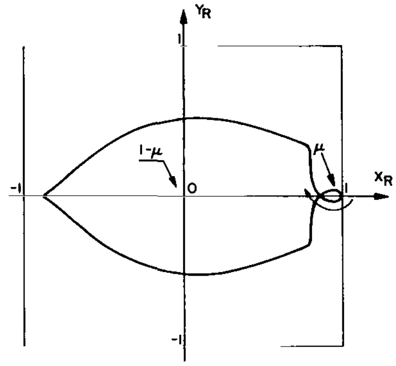
Figure 154

EARTH-MOON FIELD ORBITS



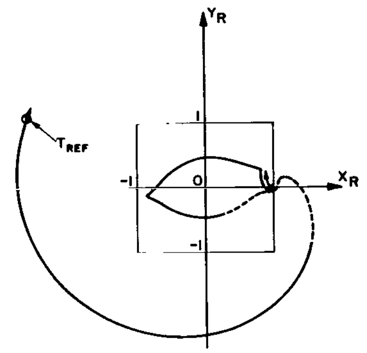
E.M. ORBIT; PERIODIC
$Y_R = -1.37624$; $P/2 = 11.39$

Figure 155



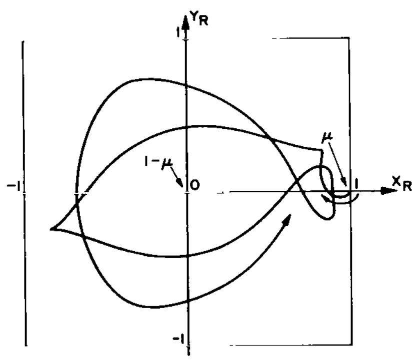
E.M. ORBIT; PERIODIC
$Y_R = -1.37351$; $P/2 = 5.450$

Figure 156



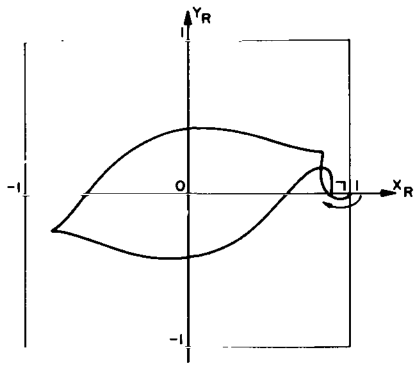
E M. ORBIT
$Y_R = -1.37200$; $T_{REF} = 15.0$

Figure 157



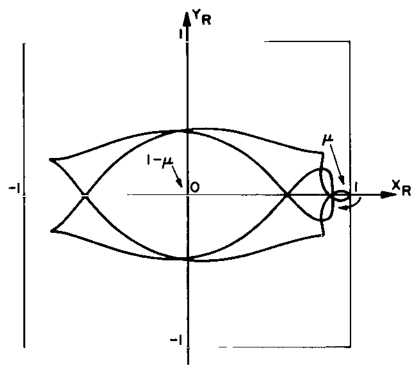
E.M. ORBIT
$Y_R = -1.37080$

Figure 158



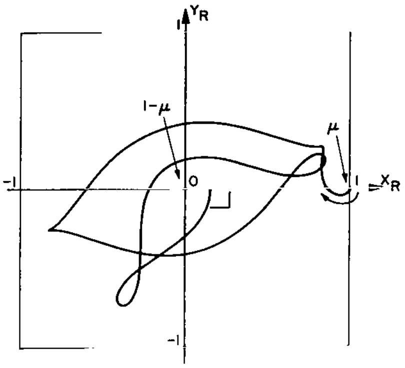
E.M. ORBIT; PERIODIC
$Y_R = -1.370792$; $P/2 = 8.670$

Figure 159



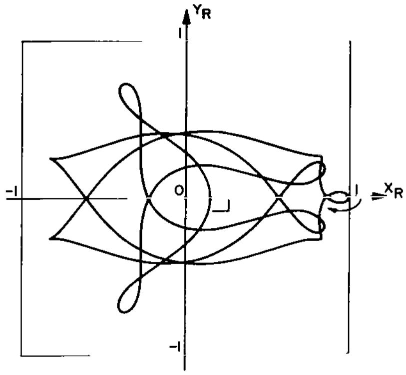
E.M. ORBIT; PERIODIC
$Y_R = -1.370792$; $P/2 = 8.670$

Figure 160



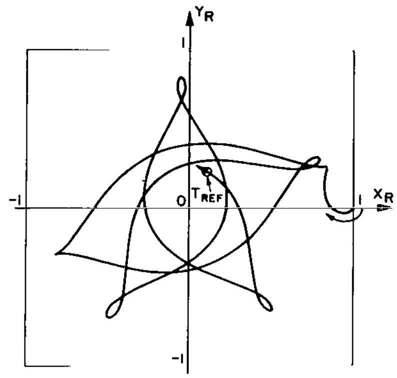
E.M. ORBIT; PERIODIC
$Y_R = -1.370567$; $P/2 = 11.78$

Figure 161



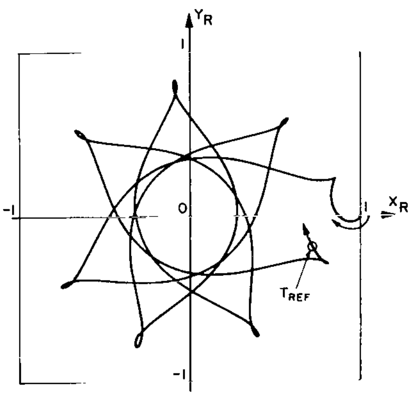
E.M. ORBIT; PERIODIC
$Y_R = -1.370567$; $P/2 = 11.78$

Figure 162



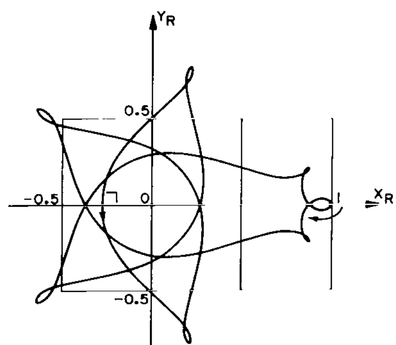
E.M. ORBIT,
$Y_R = -1.37020$; $T_{REF} = 17.0$

Figure 163



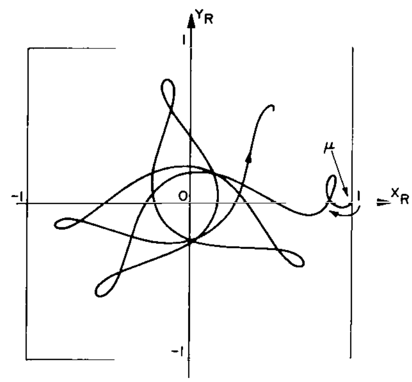
E.M. ORBIT;
$Y_R = -1.36900$

Figure 164



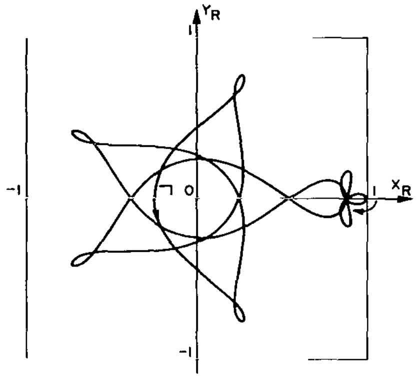
E.M. ORBIT; PERIODIC
$Y_R = -1.367628$; $P/2 = 8.390$

Figure 165



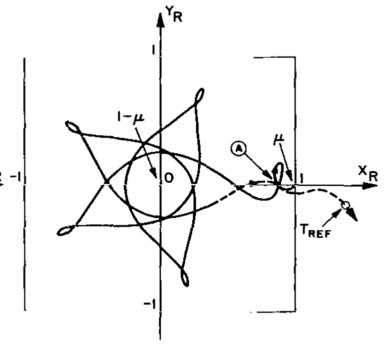
E.M. ORBIT
$Y_R = -1.364800$

Figure 166



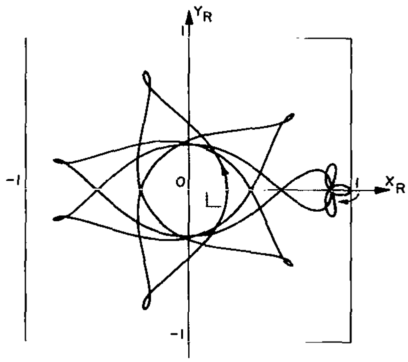
E.M. ORBIT; PERIODIC
Y_R = -1.364521; P/2 = 8.418

Figure 167



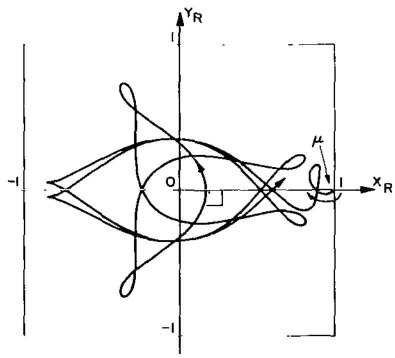
E.M. ORBIT;
Y_R = -1.364500; T_{REF} = 16.0

Figure 168



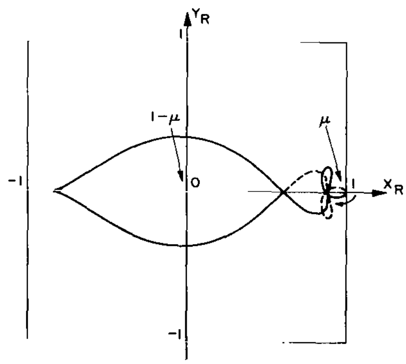
E.M. ORBIT; PERIODIC
Y_R = -1.364383; P/2 = 11.58

Figure 169



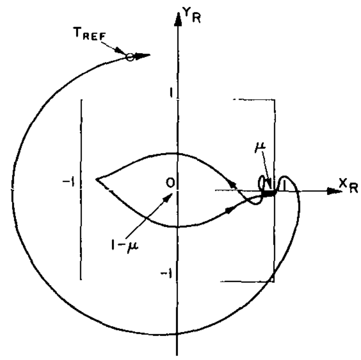
E.M. ORBIT; PERIODIC
Y_R = -1.36433; P/2 = 11.73

Figure 170



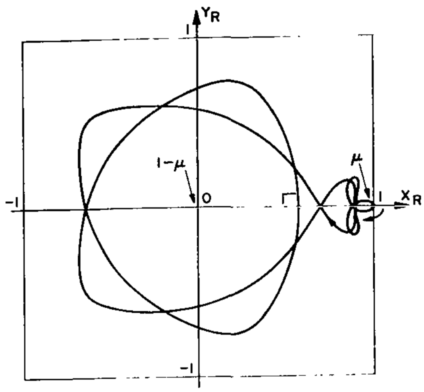
E.M. ORBIT; PERIODIC
Y_R = -1.364312; P/2 = 5.33

Figure 171



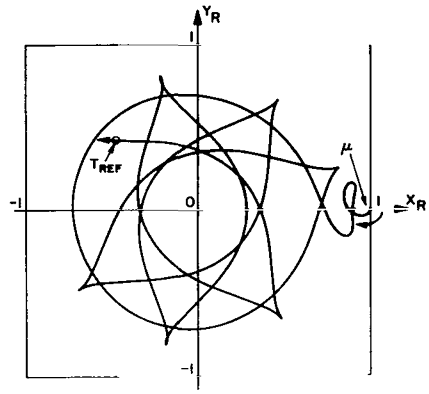
E.M. ORBIT
Y_R = -1.364280; T_{REF} = 20.0

Figure 172



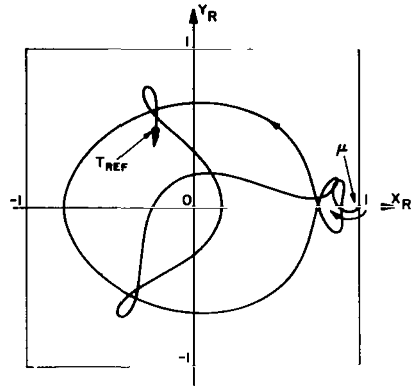
E.M. ORBIT ; PERIODIC
$Y_R = -1.364184$; $P/2 = 11.4$

Figure 173



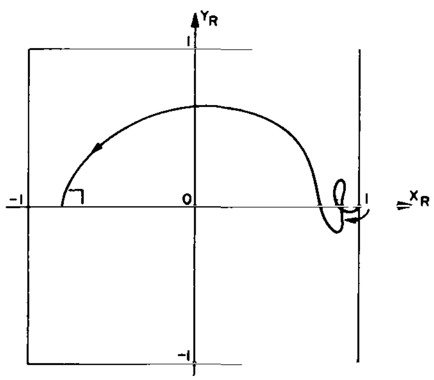
E.M. ORBIT
$Y_R = -1.364145$; $T_{REF} = 30.0$

Figure 174



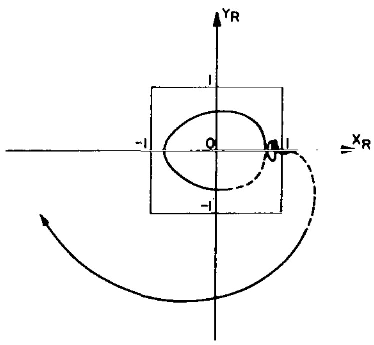
E.M. ORBIT
$Y_R = -1.364130$; $T_{REF} = 20.0$

Figure 175



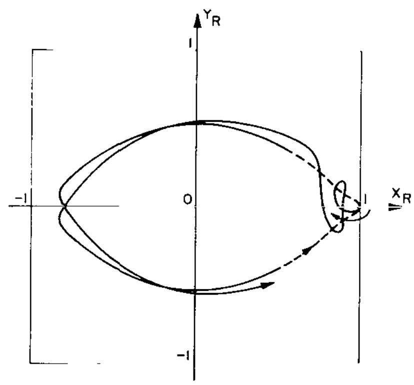
E.M. ORBIT; PERIODIC
$Y_R = -1.364126$; $P/2 = 8.38$

Figure 176



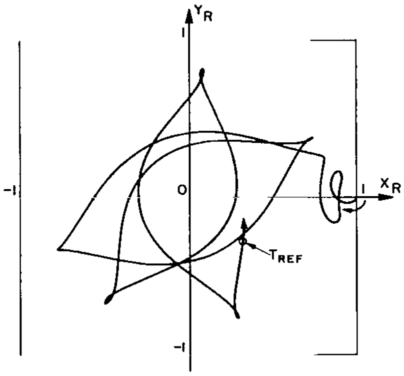
E.M. ORBIT
$Y_R = -1.364125$

Figure 177



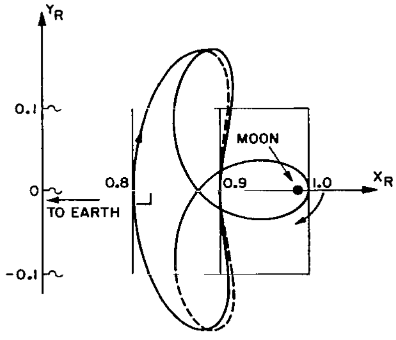
E.M. ORBIT ; PERIODIC
$Y_R = -1.364117$; $P/2 = 12.55$

Figure 178



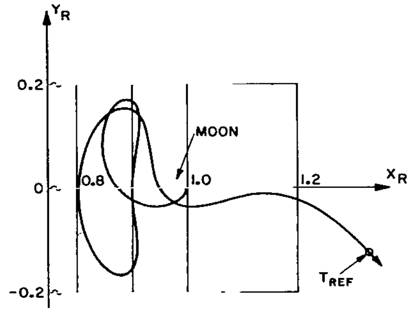
E.M. ORBIT
$Y_R = -1.364110$; $T_{REF} = 20.0$

Figure 179



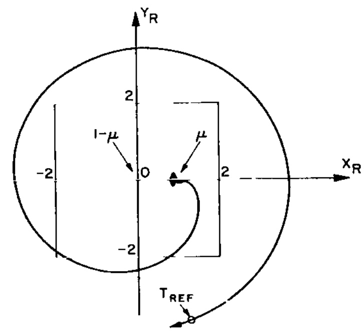
E.M. ORBIT; PERIODIC
$Y_R = -1.364102$; $P/2 = 3.553$

Figure 180



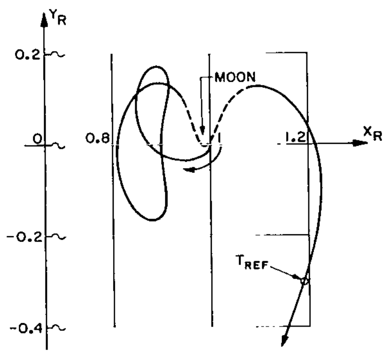
E.M. ORBIT
$Y_R = -1.364100$; $T_{REF} = 6.00$

Figure 181



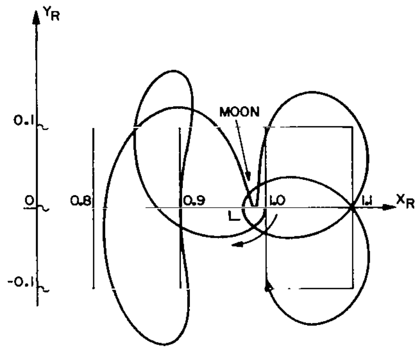
E.M. ORBIT
$Y_R = -1.364100$; $T_{REF} = 15.6$

Figure 182



E.M. ORBIT
$Y_R = -1.364097$; $T_{REF} = 7.00$

Figure 183



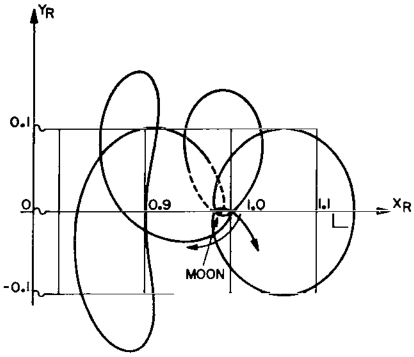
E.M. ORBIT; PERIODIC
$Y_R = -1.364095$; $P/2 = 6.06$

Figure 184

Comments to Figures 180 and 184

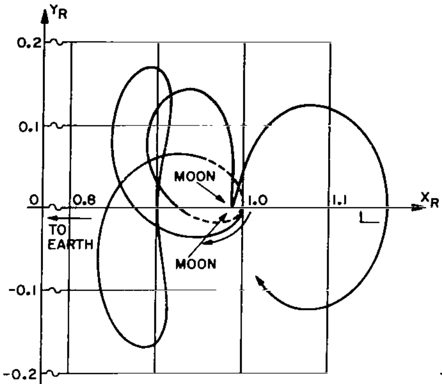
The relatively broad transition area from unconfined orbits toward orbits confined to the Moon's field contains a number of isolated periodic orbits of interesting structure. These start with the orbits on Figure 180 and 184. The reader will recognize others on the following pages.

EARTH-MOON FIELD ORBITS



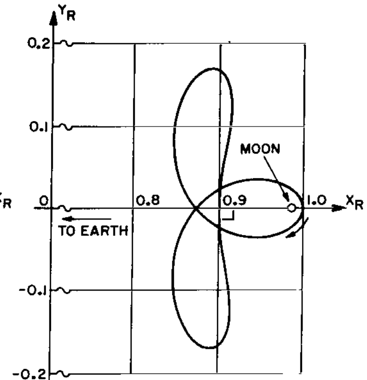
E.M. ORBIT; PERIODIC
Y_R = -1.364091; P/2 = 6.297

Figure 185



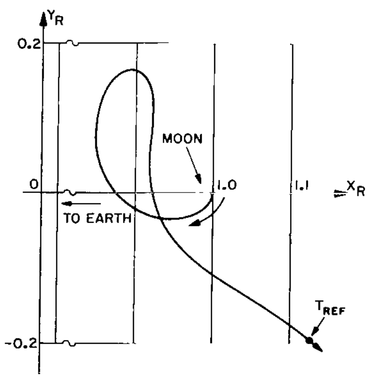
E.M. ORBIT; PERIODIC
Y_R = -1.3640796; P/2 = 6.408

Figure 186



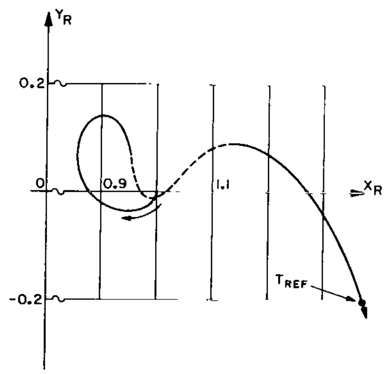
E.M. ORBIT; PERIODIC
Y_R = -1.364065; P/2 = 1.950

Figure 187



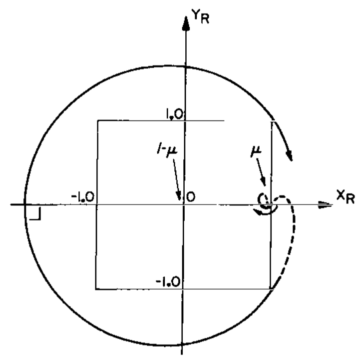
E.M. ORBIT
Y_R = -1.363500; T_{REF} = 3.10

Figure 188



E.M. ORBIT
Y_R = -1.360500; T_{REF} = 3.10

Figure 189



E.M. ORBIT; PERIODIC
Y_R = -1.359620; P/2 = 9.328

Figure 190

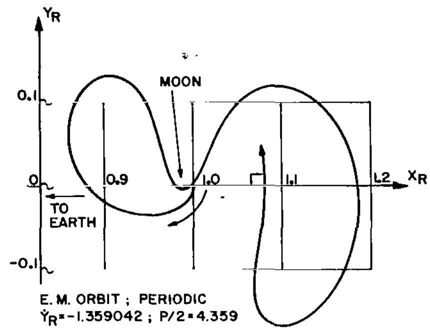


Figure 191

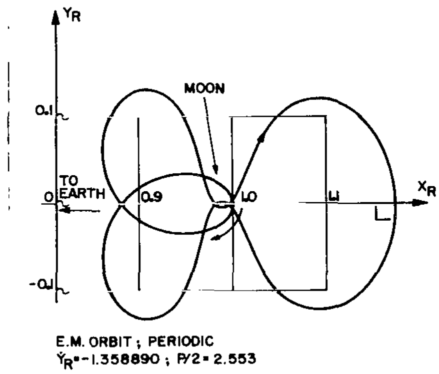


Figure 192

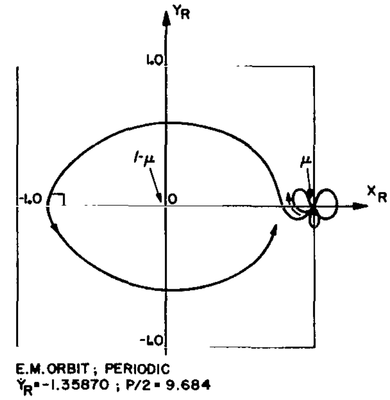


Figure 193

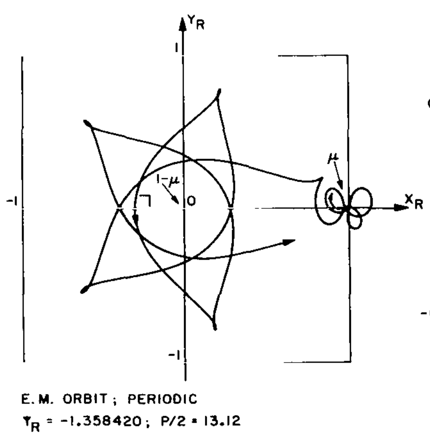


Figure 194

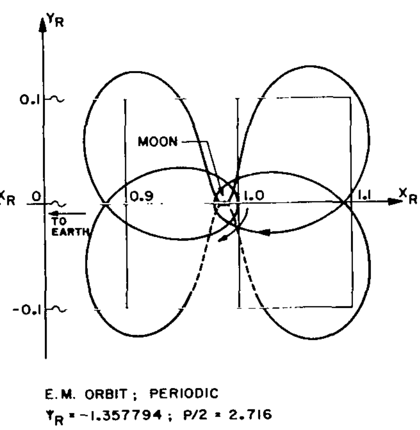


Figure 195

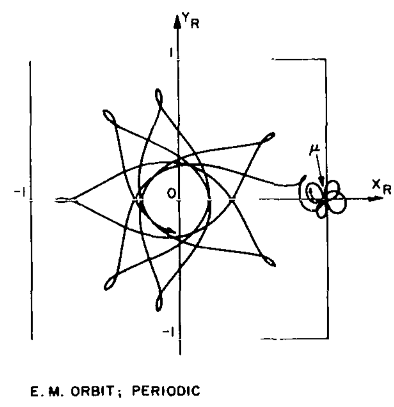
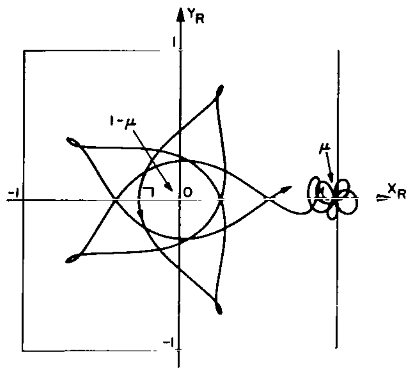
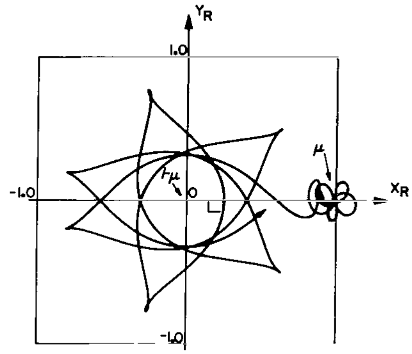


Figure 196



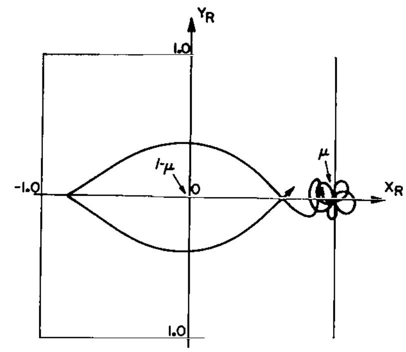
E.M. ORBIT; PERIODIC
$\dot{Y}_R = -1.356354$; $P/2 = 13.56$

Figure 197



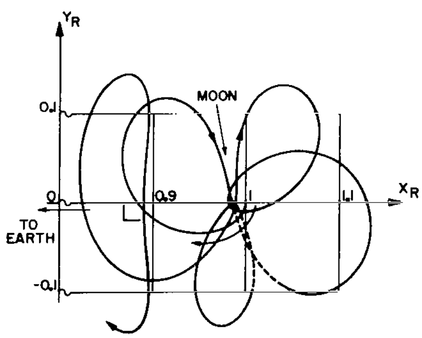
E.M. ORBIT; PERIODIC
$\dot{Y}_R = -1.356345$; $P/2 = 16.74$

Figure 198



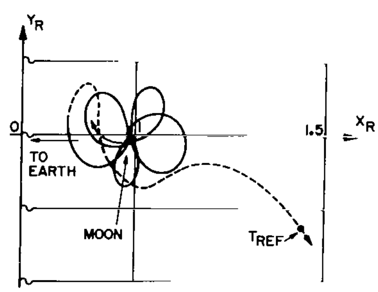
E.M. ORBIT; PERIODIC
$\dot{Y}_R = -1.356339$; $P/2 = 10.55$

Figure 199



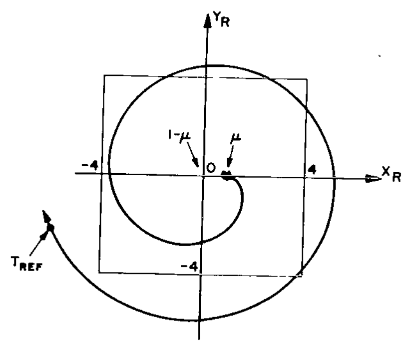
E.M. ORBIT; PERIODIC
$\dot{Y}_R = -1.356327$; $P/2 = 7.076$

Figure 200



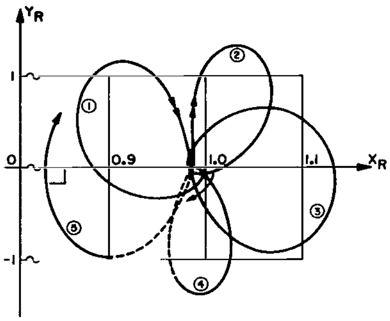
E.M. ORBIT
$\dot{Y}_R = -1.356300$; $T_{REF} = 9.600$

Figure 201



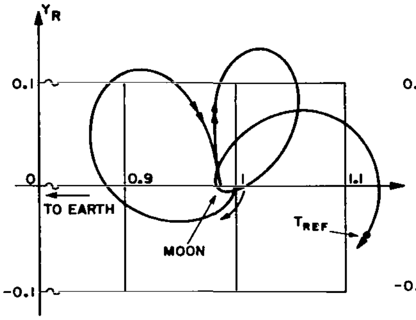
E.M. ORBIT
$\dot{Y}_R = -1.356300$; $T_{REF} = 20.0$

Figure 202



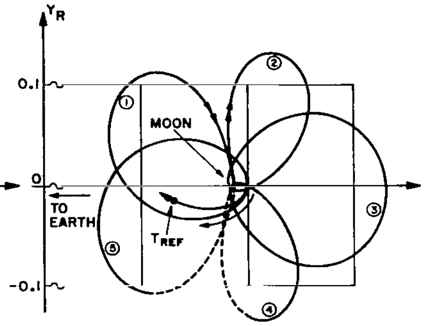
E.M. ORBIT; PERIODIC
$Y_R = -1.35580$; $P/2 = 5.49$

Figure 203



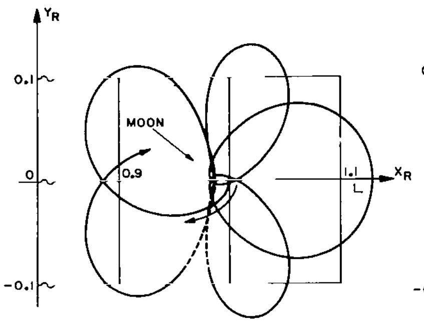
E.M. ORBIT
$Y_R = -1.355100$; $T_{REF} = 3.07$

Figure 204



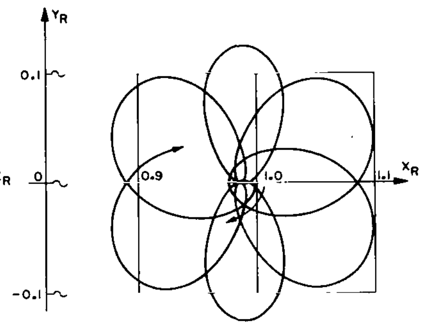
E. M. ORBIT
$Y_R = -1.355100$; $T_{REF} = 5.86$

Figure 205



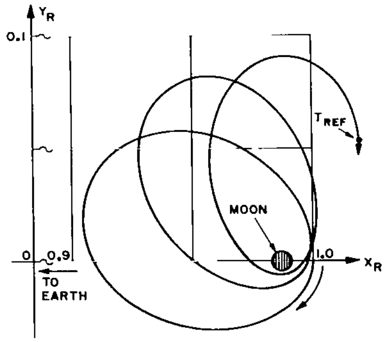
E. M. ORBIT; PERIODIC
$Y_R = -1.35479$, $P/2 = 2.814$

Figure 206



E. M. ORBIT; PERIODIC
$Y_R = -1.35074$; $P/2 = 2.891$

Figure 207



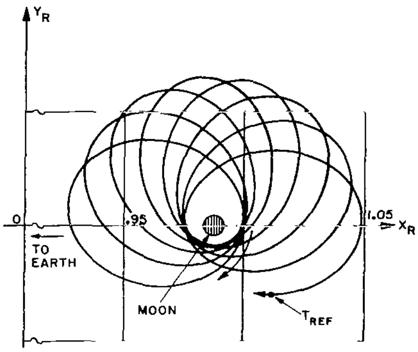
E M. ORBIT
$Y_R = -1.330$; $T_{REF} = 1.75$

Figure 208

Comments to Figures 201 to 206

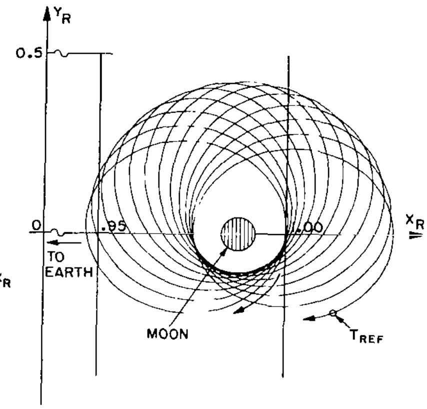
The orbit on Figure 201, shown again in Figure 202 with time extended to T = 20.0, is capable of either escaping or reaching large distances from the masses. With the next four orbits plotted, however, the transition to orbits bounded in the Moon field is realized. The last step toward this is made by the alignment of all loops of the orbit into mean-motion about the Moon in the direct sense of revolution, which is illustrated on Figures 203 to 206. The lunar flyby between the loops numbered "1" and "2" is retrograde on Figure 203 and direct on Figure 206, while Figure 204 and Figure 205 demonstrate the collision case. From here on, the loops form increasingly regular patterns, as shown on the succeeding E-M-graphs.

EARTH-MOON FIELD ORBITS



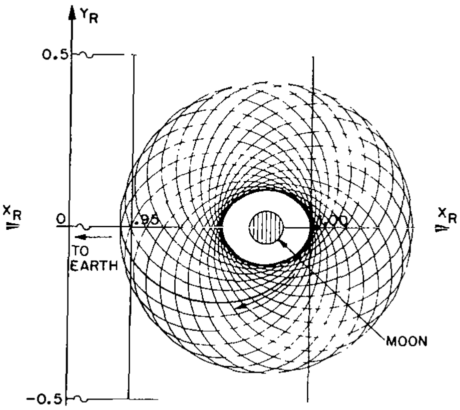
E.M ORBIT
$Y_R = -1.300$; $T_{REF} = 3.14$

Figure 209



E.M. ORBIT
$Y_R = -1.250$; $T_{REF} = 3.14$

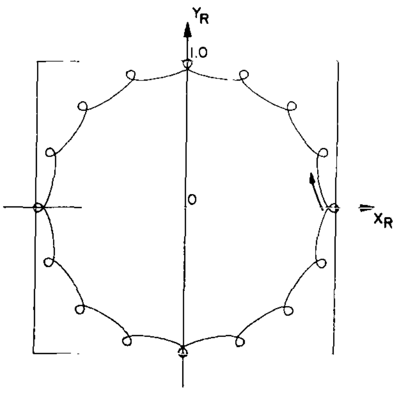
Figure 210



E.M. ORBIT ; PERIODIC
$Y_R = -1.250$; $P/2 = 3.077$

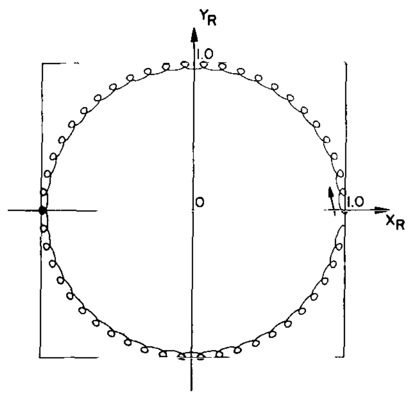
Figure 211

KEPLER ORBITS



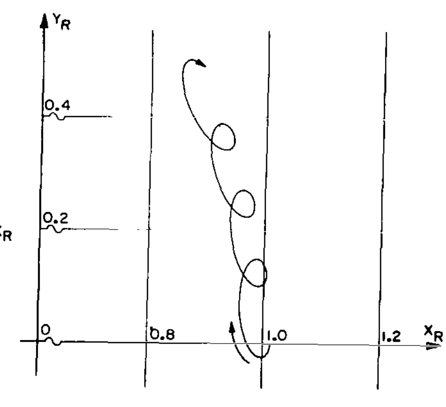
KEPLER ORBIT ; $n = +16/15$
$\dot{Y}_R = -0.047438$; $P/2 = 47.12$

Figure 212



KEPLER ORBIT ; $n = +50/49$
$\dot{Y}_R = -0.031812$; $P/2 = 153.9$

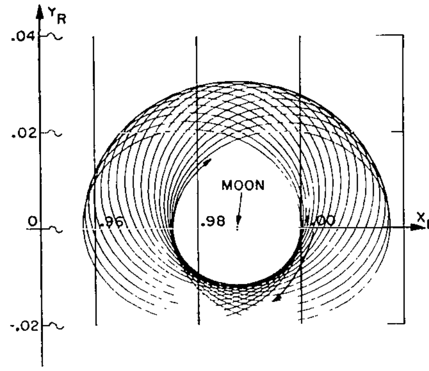
Figure 213



KEPLER ORBIT ; $n = +50/49$
$Y_R = -0.031812$, $P/2 = 153.9$

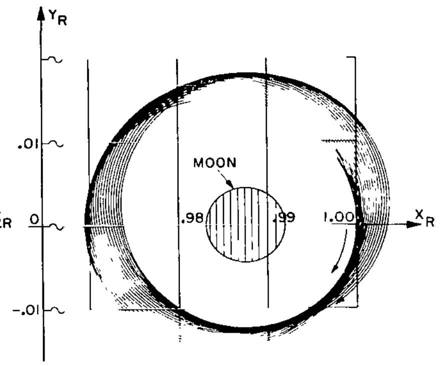
Figure 214

EARTH-MOON FIELD ORBITS



E.M. ORBIT
Y_R = -1.200; TIME SHOWN: 3.14

Figure 215



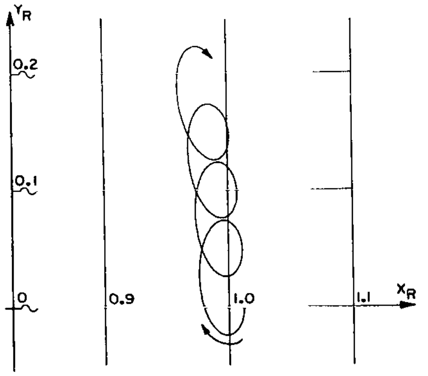
E.M ORBIT
Y_R = -1.100; TIME SHOWN: 2.00

Figure 216

Comments to Figure 216

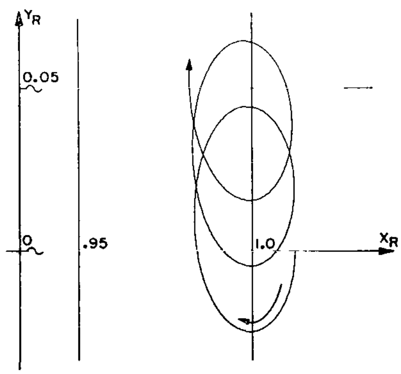
Though the orbit calculation is done with the assumption of the masses being point-masses, the Moon's physical extension is indicated on several of the subsequent figures to convey a feel of the size of the orbits involved.

KEPLER ORBITS



KEPLER ORBIT, n = +125/124
Y_R = -0.027645; P/2 = 389.6

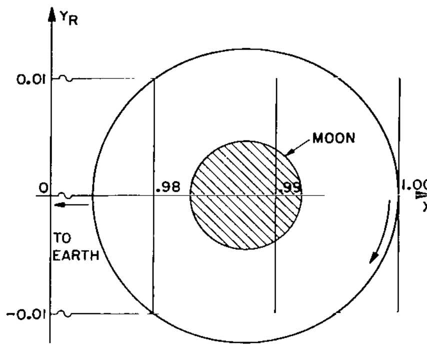
Figure 217



KEPLER ORBIT
Y_R = -0.0260; TIME SHOWN: 15.94

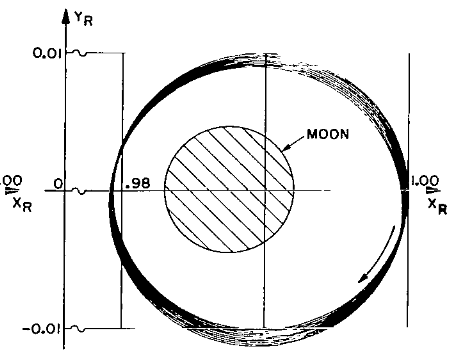
Figure 218

EARTH-MOON FIELD ORBITS



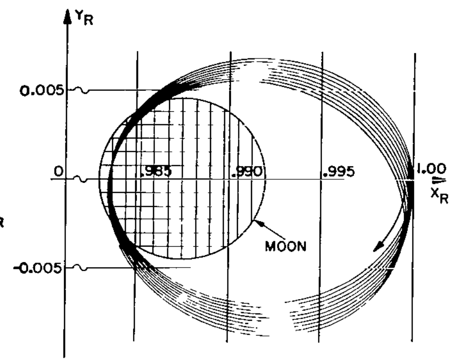
E.M ORBIT; PERIODIC
Y_R = -1.012580; P/2 = 0.03878

Figure 219



E.M. ORBIT
Y_R = -0.900; TIME SHOWN: 0.56

Figure 220



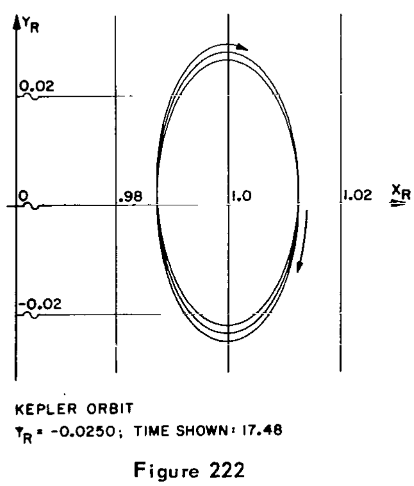
E.M. ORBIT
Y_R = -0.700; TIME SHOWN: 0.39

Figure 221

Comments to Figures 219, 238 and 223

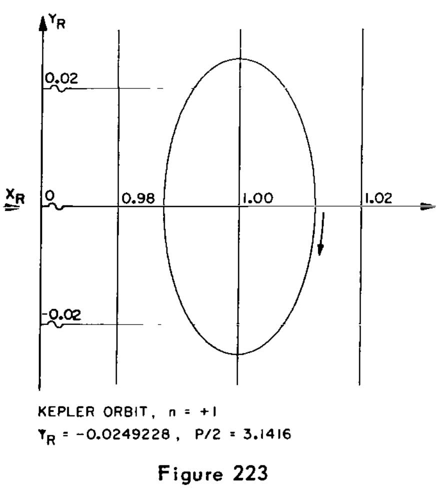
While the Kepler series has only one orbit of the single-loop type about the point (1.0; 0.0), i.e., that reproduced on Figure 223, the E-M-series produces two single-loop type (near-circular) orbits about the Moon. These are shown on Figures 219 and 238.

KEPLER ORBITS



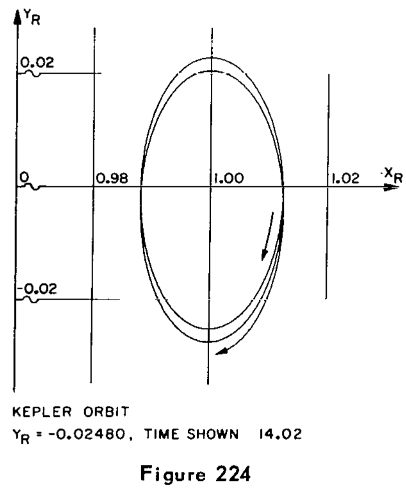
KEPLER ORBIT
Y_R = -0.0250; TIME SHOWN: 17.48

Figure 222



KEPLER ORBIT, n = +1
Y_R = -0.0249228, P/2 = 3.1416

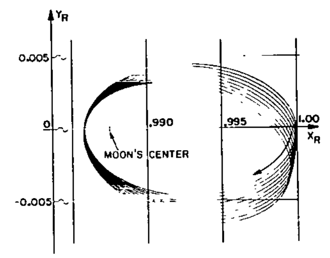
Figure 223



KEPLER ORBIT
Y_R = -0.02480, TIME SHOWN 14.02

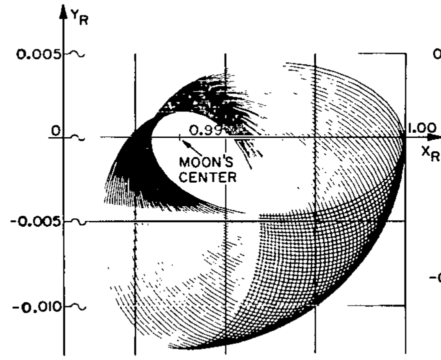
Figure 224

EARTH-MOON FIELD ORBITS



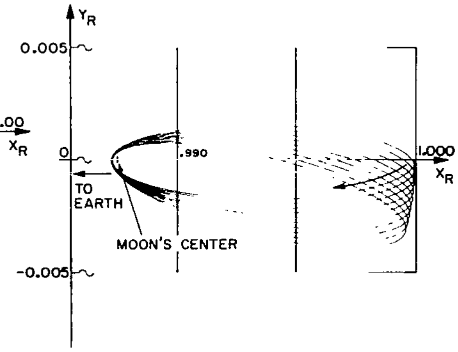
E M ORBIT
Y_R = -0.500; TIME SHOWN: 0.32

Figure 225



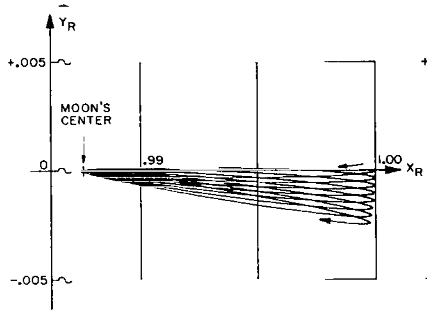
E.M. ORBIT
Y_R = -0.500; TIME SHOWN: 1.575

Figure 226



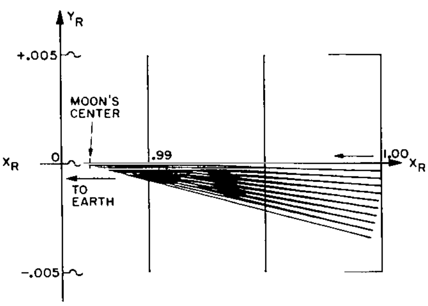
E.M. ORBIT
Y_R = -0.200; TIME SHOWN: 0.27

Figure 227



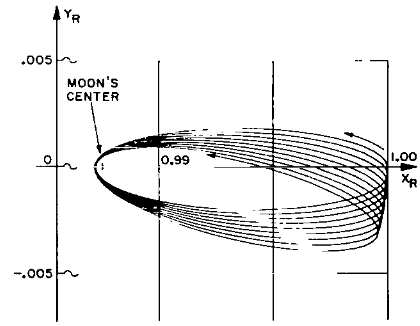
E M ORBIT
Y_R = -0.04; TIME SHOWN: 0.20

Figure 228



E.M. ORBIT
Y_R = 0.000; TIME SHOWN 0.28

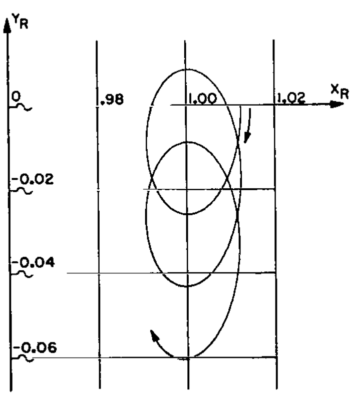
Figure 229



E.M ORBIT
Y_R = +0.200, TIME SHOWN: 0.27

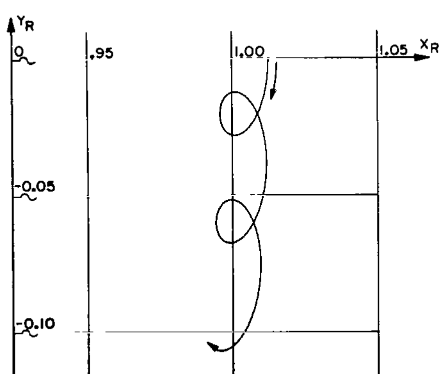
Figure 230

KEPLER ORBITS



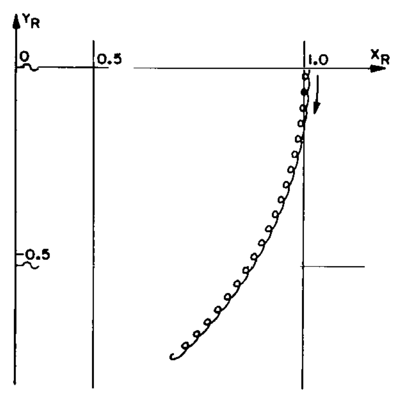
KEPLER ORBIT
Y_R = -0.02400; TIME SHOWN: 15.05

Figure 231



KEPLER ORBIT; n = 159/160
Y_R = -0.022813; P/2 = 502.6

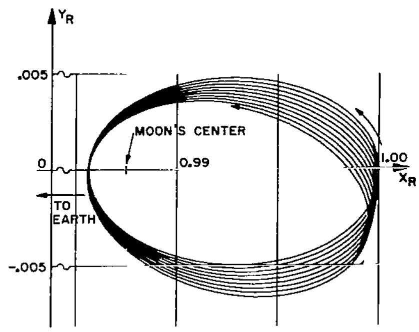
Figure 232



KEPLER ORBIT; n = 159/160
Y_R = -0.022813; P/2 = 502.6

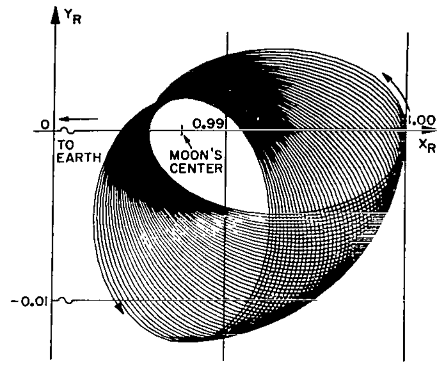
Figure 233

EARTH-MOON FIELD ORBITS



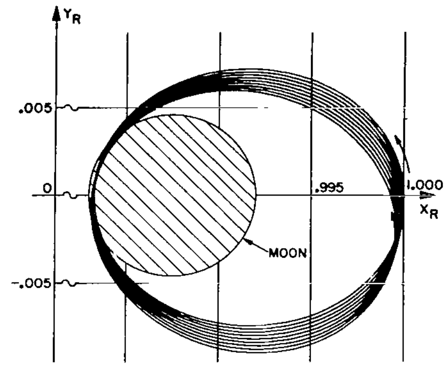
E. M. ORBIT
Y_R = +0.500; TIME SHOWN: 0.32

Figure 234



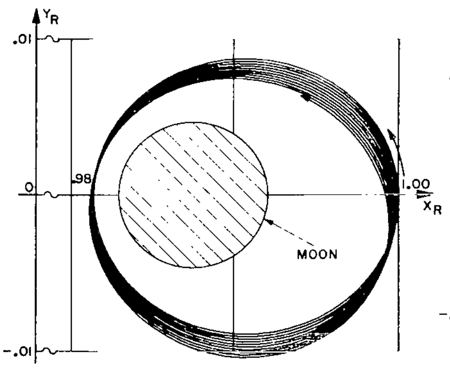
E M. ORBIT
Y_R = +0.500; TIME SHOWN: 1.57

Figure 235



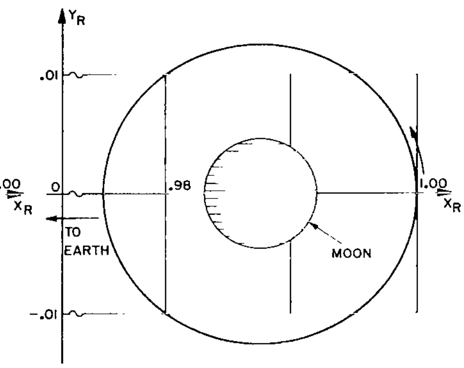
E M ORBIT
Y_R = +0.700; TIME SHOWN 0.39

Figure 236



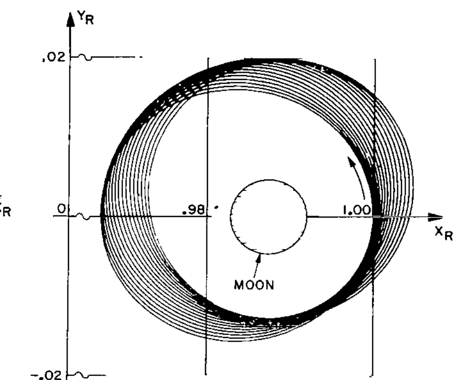
E M ORBIT
Y_R = +0.800, TIME SHOWN 0.52

Figure 237



E M ORBIT; PERIODIC
Y_R = +0.987734; P/2 = 0.03979

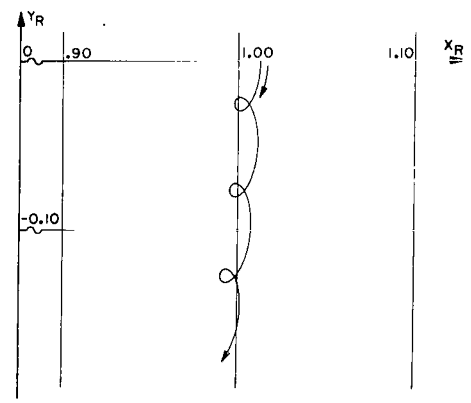
Figure 238



E M. ORBIT
Y_R = +1.100; TIME SHOWN: 2.00

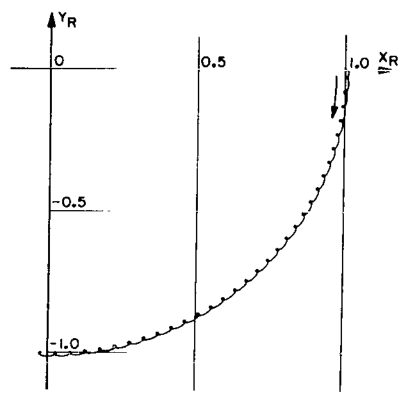
Figure 239

KEPLER ORBITS



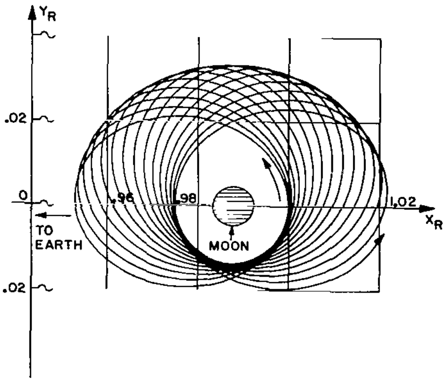
KEPLER ORBIT; n = 124/125
Y_R = -0.022223; P/2 = 392.7

Figure 240



KEPLER ORBIT; n = 124/125
Y_R = -0.022223; P/2 = 392.7

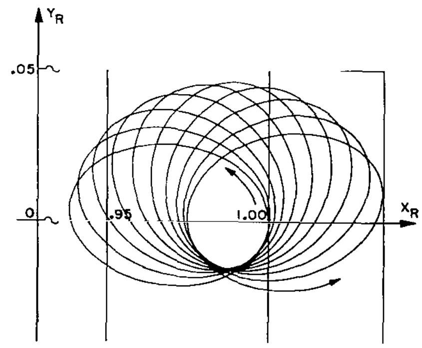
Figure 241



E. M. ORBIT

Y_R = +1.200; TIME SHOWN: 3.15

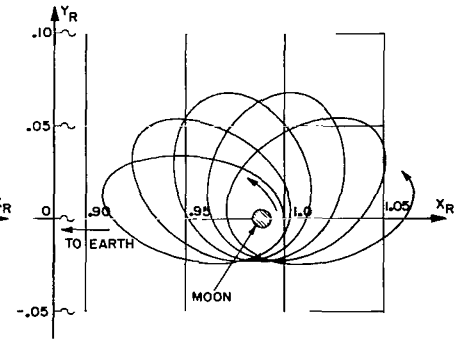
Figure 242



E. M. ORBIT

Y_R = +1.250; TIME SHOWN: 3.15

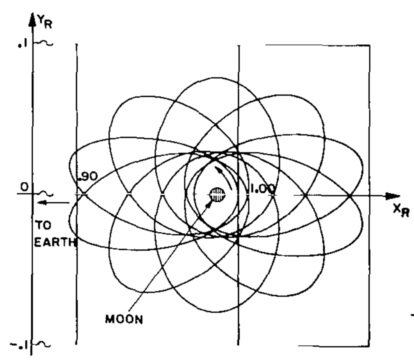
Figure 243



E. M. ORBIT

Y_R = +1.300; TIME SHOWN: 3.22

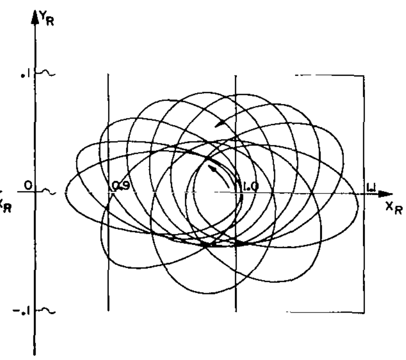
Figure 244



E. M. ORBIT; PERIODIC

Y_R = +1.3115; P/2 = 3.6

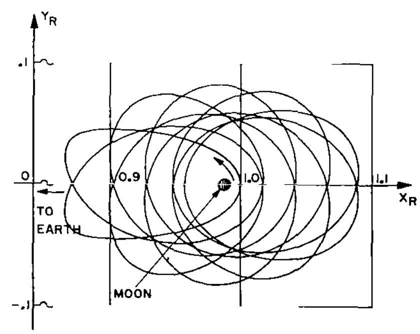
Figure 245



E.M. ORBIT

Y_R = +1.323; TIME SHOWN: 12.0

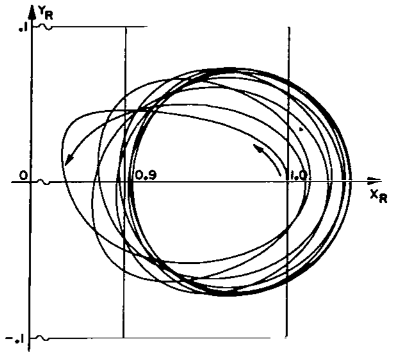
Figure 246



E. M. ORBIT, PERIODIC

Y_R = +1.3260; P/2 = 5.97

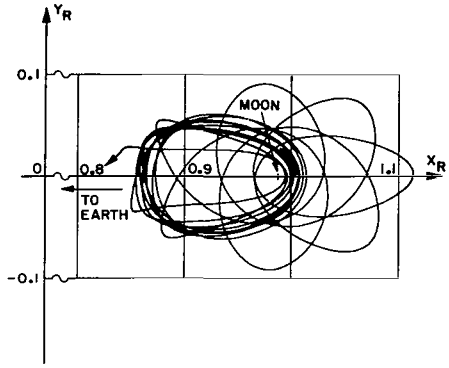
Figure 247



E. M. ORBIT

Y_R = +1.3270; TIME SHOWN : 12.06

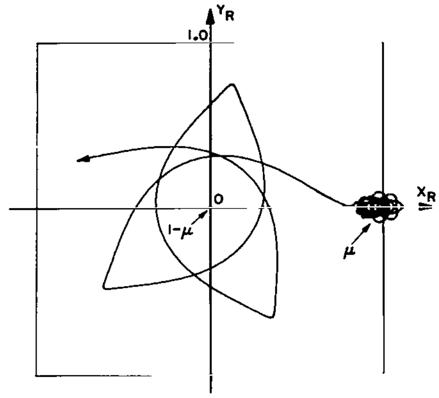
Figure 248



E. M. ORBIT

Y_R = 1.32760; TIME SHOWN : 25.06

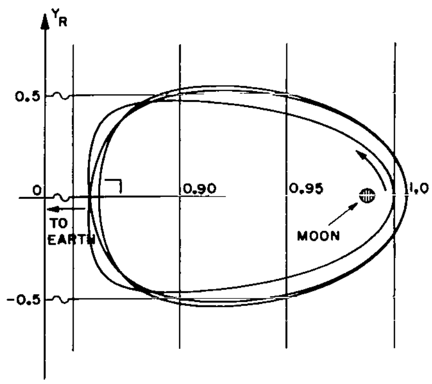
Figure 249



E. M. ORBIT

Y_R = 1.32760 ; TIME SHOWN : 35.02

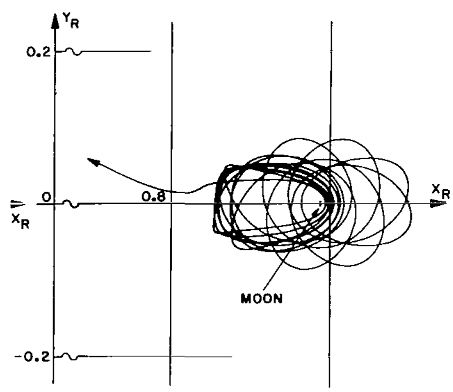
Figure 250



E. M. ORBIT ; PERIODIC

Y_R = 1.327647 ; P/2 = 2.943

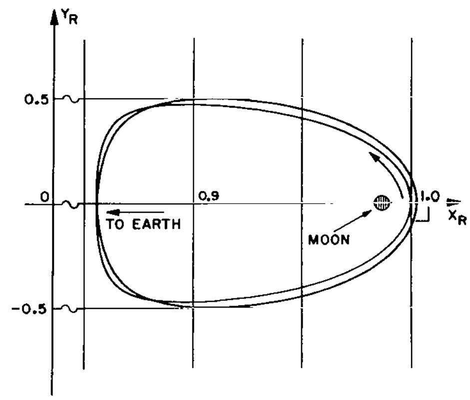
Figure 251



E. M. ORBIT

Y_R = 1.32770; TIME SHOWN : 25.04

Figure 252



E. M. ORBIT ; PERIODIC

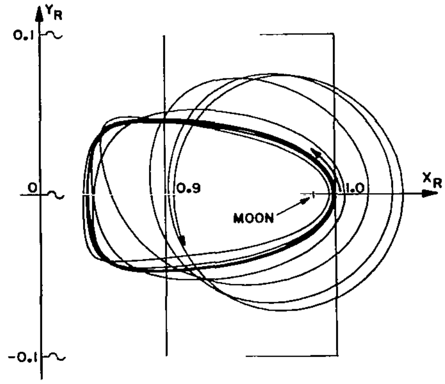
Y_R = 1.32780; P/2 = 2.111

Figure 253

Comments to Figures 251, 253, 255, and 260

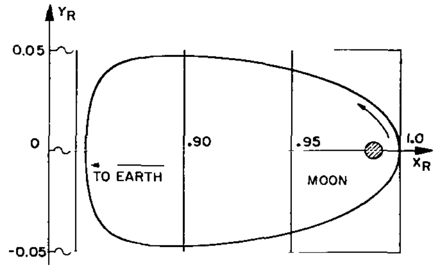
This region of transition from orbits bounded in the Moon's field to general orbits is producing the triangle-shaped periodic orbits of one or more loops as represented on Figures 251, 253, 255, and 260. The last one shows the same number of loops as the first one.

EARTH-MOON FIELD ORBITS



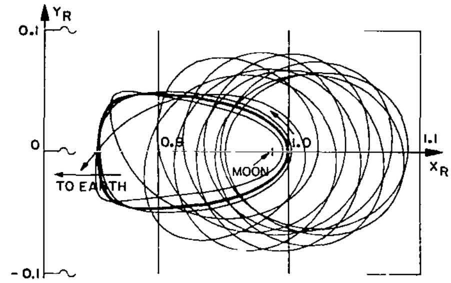
E.M. ORBIT
Y_R = 1.327910; TIME SHOWN: 20.66

Figure 254



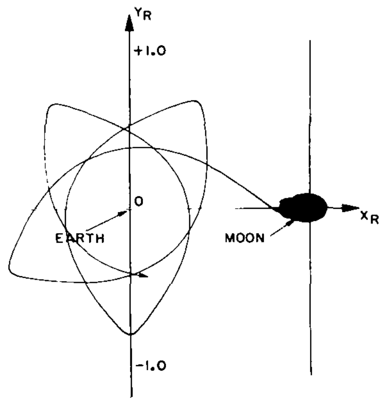
E.M. ORBIT; PERIODIC;
$\dot{Y}_R$=+1.3279118, P/2 = 1.101

Figure 255



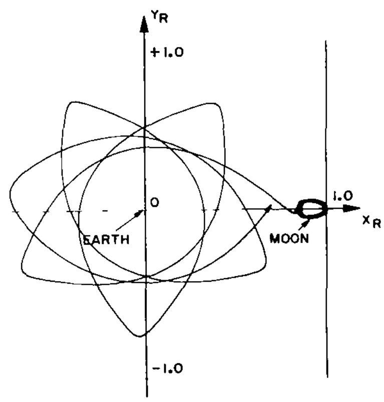
E.M. ORBIT;
$\dot{Y}_R$= +1.3279120; TIME SHOWN: 30.1

Figure 256



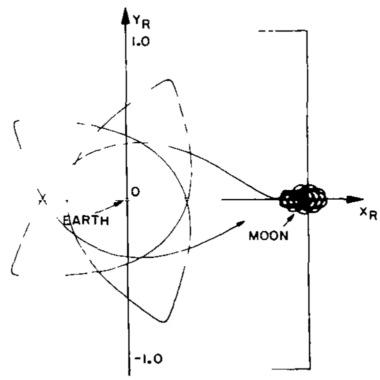
E.M. ORBIT;
$\dot{Y}_R$= +1.3279120; TIME SHOWN: 40.1

Figure 257



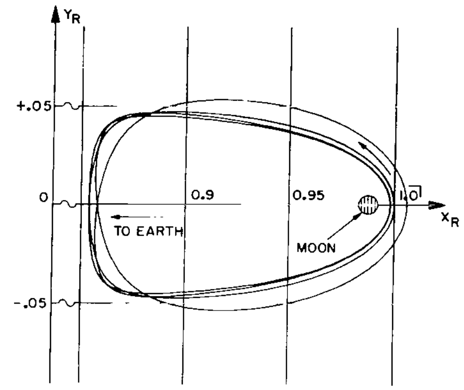
E.M. ORBIT;
$\dot{Y}_R$= +1.327919; TIME SHOWN 31.0

Figure 258



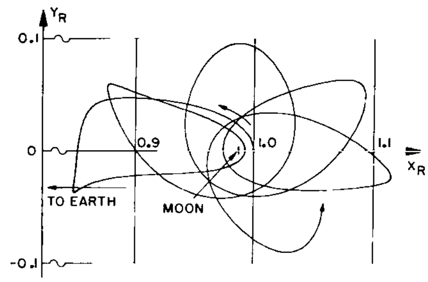
E.M. ORBIT;
$\dot{Y}_R$= +1.327940; TIME SHOWN: 30.0

Figure 259



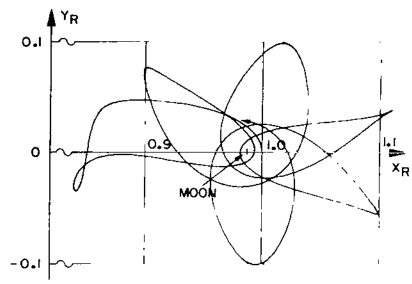
E.M. ORBIT ; PERIODIC
$\dot{Y}_R = +1.3279741$; $P/2 = 4.345$

Figure 260



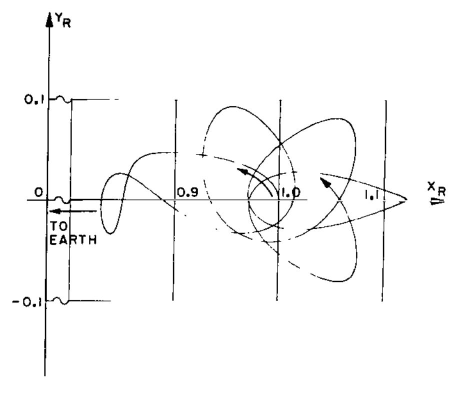
E.M ORBIT ,
$\dot{Y}_R = +1.32816$; TIME SHOWN: 8.04

Figure 261



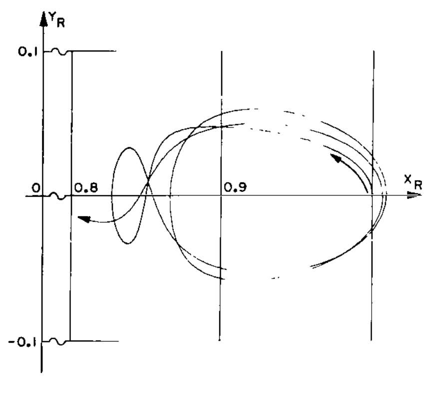
E.M ORBIT ;
$\dot{Y}_R = +1.32822$; TIME SHOWN: 8.0

Figure 262



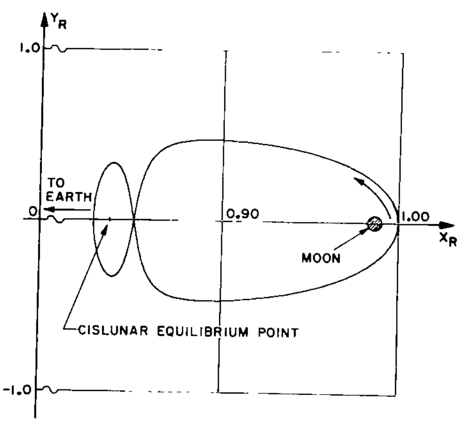
E. M. ORBIT
$\dot{Y}_R = 1.328250$; TIME SHOWN : 8.04

Figure 263



E.M. ORBIT
$\dot{Y}_R = 1.328253$; TIME SHOWN : 8.08

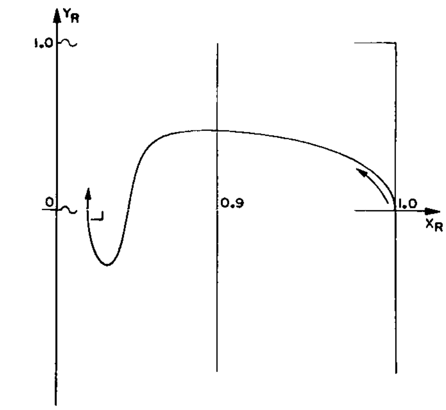
Figure 264



E. M. ORBIT ; PERIODIC
$\dot{Y}_R = 1.32825317$; $P/2 = 2.46$

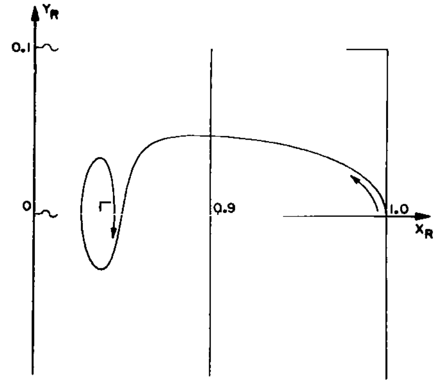
Figure 265

EARTH-MOON FIELD ORBITS



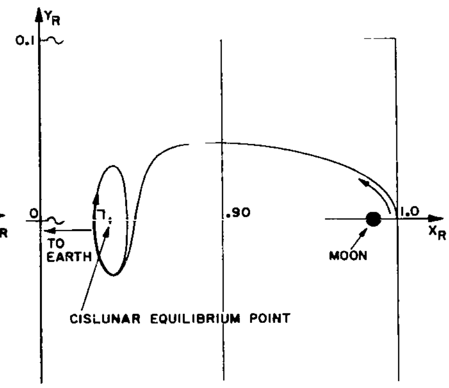
E. M. ORBIT; PERIODIC
Y_R = 1.32825317; P/2 = 2.46

Figure 266



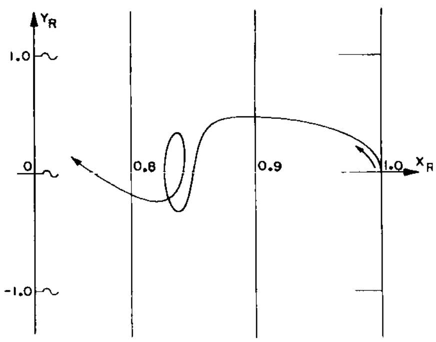
E. M. ORBIT; PERIODIC
Y_R = 1.32825330; P/2 = 3.813

Figure 267



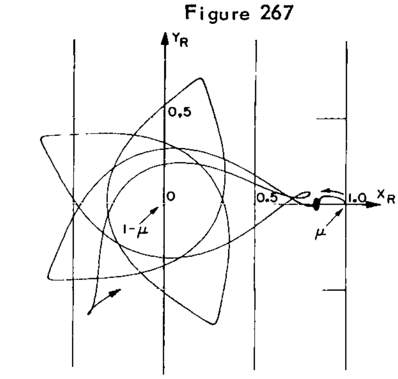
E. M. ORBIT; PERIODIC
Y_R = 1.32825330; P/2 = 5.166

Figure 268



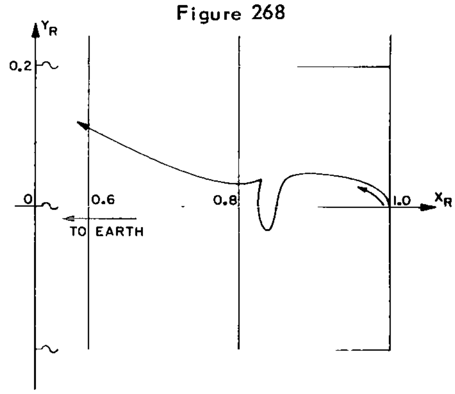
E.M ORBIT
Y_R = 1.3282534, TIME SHOWN: 4.87

Figure 269



E. M. ORBIT
Y_R = 1.3282536, TIME SHOWN: 20.06

Figure 270



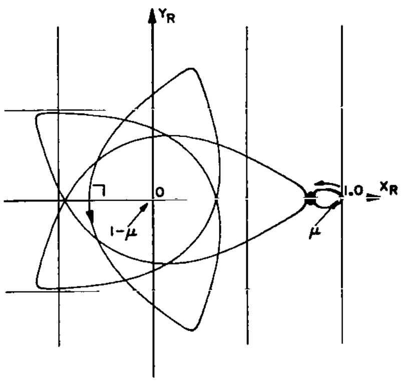
E. M. ORBIT
Y_R = 1.328255; TIME SHOWN: 4.44

Figure 271

Comments to Figures 266, 267, and 268

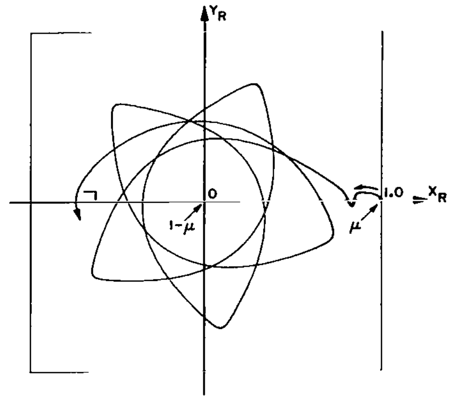
Figure 266, which shows the same orbit as Figure 265, is repeated for the half period in order to have the sequence of 266, 267, 268 demonstrate the argument for the existence of an asymptotic periodic orbit about the cis-lunar libration point. The orthogonal crossing of the X_R-axis is encountered with the second cut of this axis on the first of the three orbits (Figure 266); with the third cut of the X_R-axis on the second orbit (Figure 267); while on Figure 268 the orthogonal crossing is found with the fourth cut of the X_R-axis. The velocity differential between the last two orbits is beyond the digits listed here.

Arguments involving the magnitude of the angles of crossings past the orthogonal crossing on Figure 268 can be made in support of the continuation of the sequence of the three orbits toward the asymptotic periodic one, though no proof of this can be given by the methods of experimental astrodynamics.



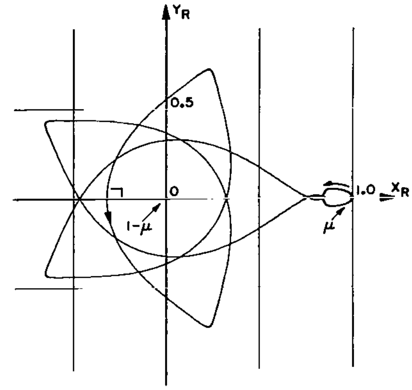
E. M. ORBIT; PERIODIC
Y_R = 1.328265; P/2 = 9.545

Figure 272



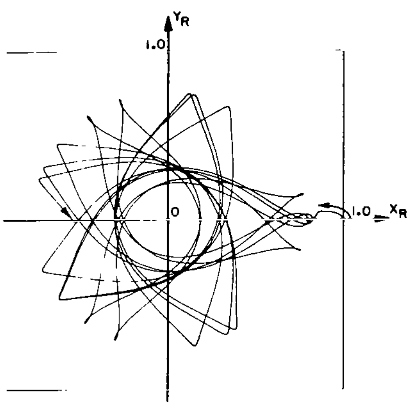
E. M. ORBIT; PERIODIC
Y_R = 1.328273; P/2 = 19.16

Figure 273



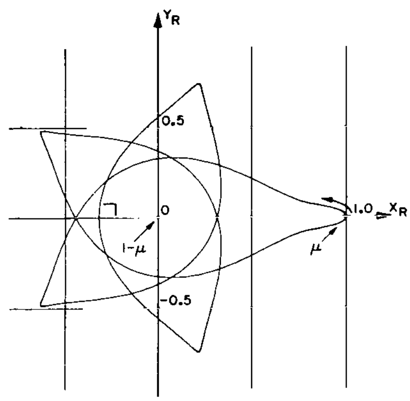
E. M. ORBIT; PERIODIC
Y_R = 1.32891427; P/2 = 7.915

Figure 274



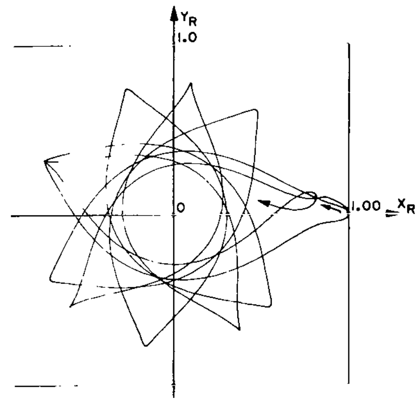
E. M. ORBIT
Y_R = 1.32930; TIME SHOWN: 51.06

Figure 275



E. M. ORBIT; PERIODIC
Y_R = 1.337922; P/2 = 7.285

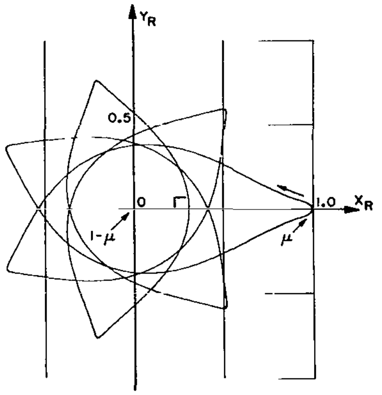
Figure 276



E. M. ORBIT
Y_R = 1.3440; TIME SHOWN: 35.00

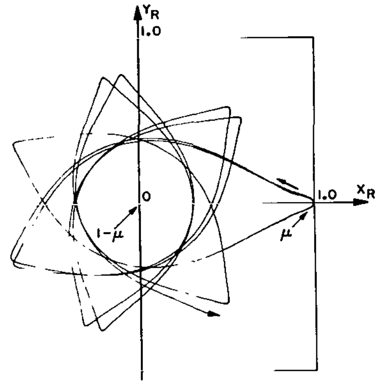
Figure 277

EARTH-MOON FIELD ORBITS



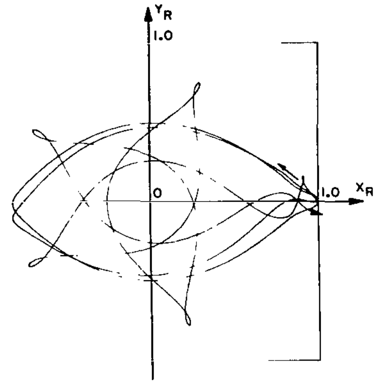
E. M. ORBIT; PERIODIC
Y_R = 1.344819; P/2 = 10.30

Figure 278



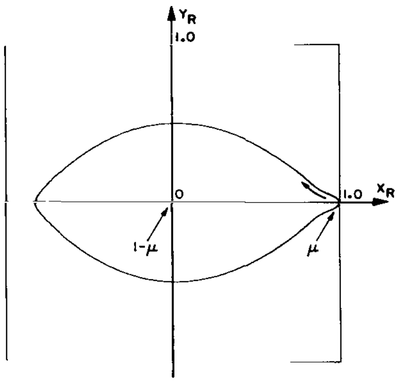
E. M. ORBIT
Y_R = 1.3450; TIME SHOWN : 35.1

Figure 279



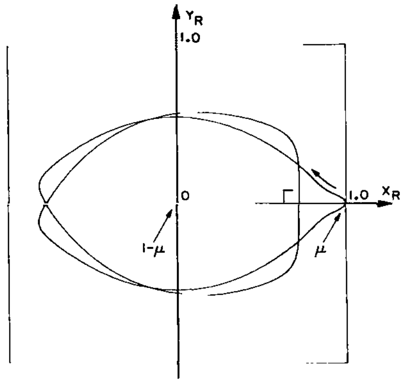
E. M. ORBIT
Y_R = 1.3590; TIME SHOWN : 30.0

Figure 280



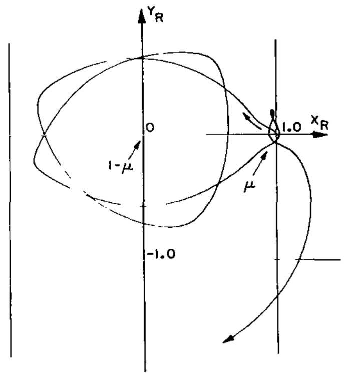
E. M. ORBIT; PERIODIC
Y_R = 1.359414; P/2 = 4.05

Figure 281



E. M. ORBIT; PERIODIC
Y_R = 1.363814; P/2 = 9.008

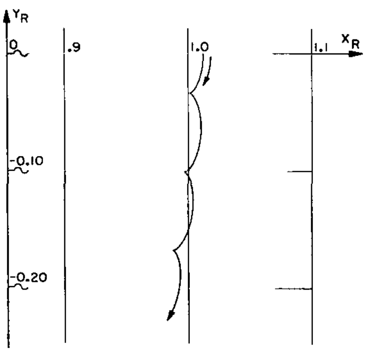
Figure 282



E. M. ORBIT
Y_R = 1.36980; TIME SHOWN : 25.15

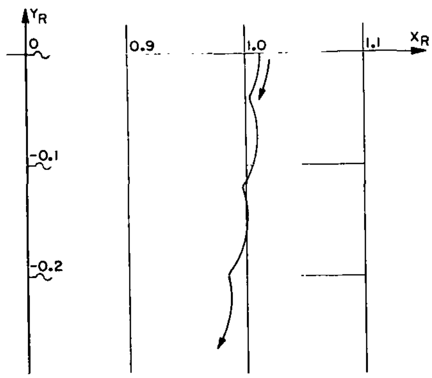
Figure 283

KEPLER ORBITS



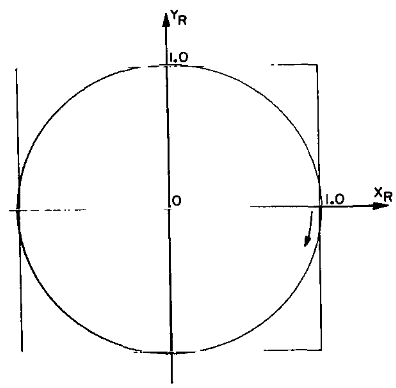
KEPLER ORBIT; CUSPING ORBIT
Y_R = -0.021342; TIME SHOWN: 20.18

Figure 284



KEPLER ORBIT
Y_R = -0.02070; TIME SHOWN: 20.21

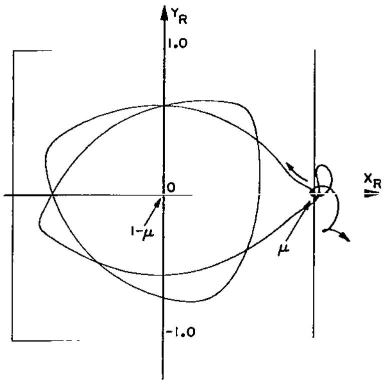
Figure 285



KEPLER ORBIT ; CIRCULAR (POSITIVE)
Y_R = -0.0186920; P/2 = 170.2

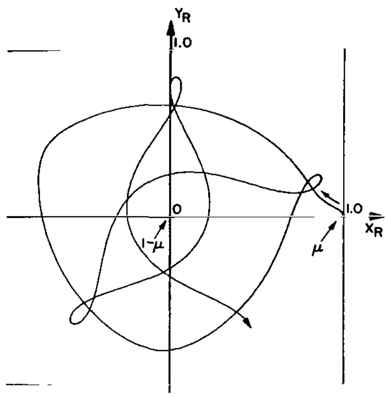
Figure 286

EARTH-MOON FIELD ORBITS



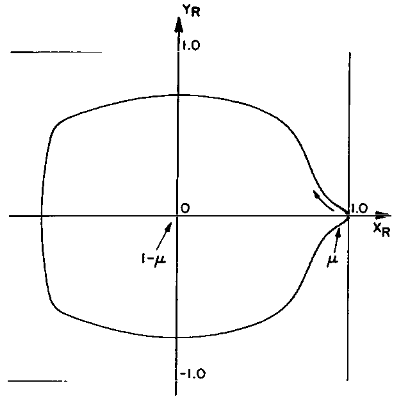
E. M. ORBIT
Y_R = 1.37020; TIME SHOWN: 24.97

Figure 287



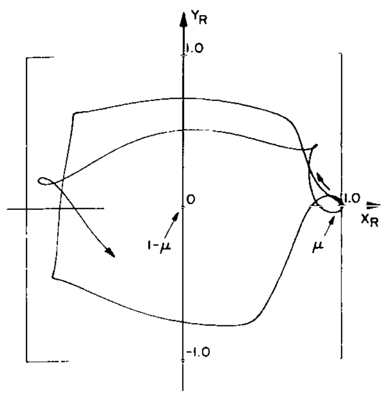
E.M. ORBIT
$\dot{Y}_R$ = 1.3750; TIME SHOWN: 20.14

Figure 288



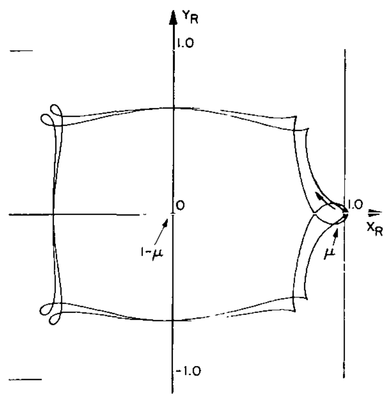
E.M. ORBIT; PERIODIC
$\dot{Y}_R$ = 1.387032; P/2 = 9.263

Figure 289



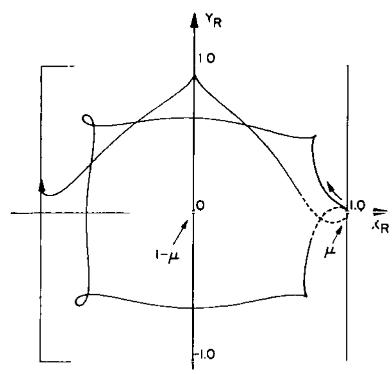
E M ORBIT
$\dot{Y}_R$ = 1 390; TIME SHOWN: 28.03

Figure 290



E M ORBIT; PERIODIC
$\dot{Y}_R$ = 1.395048; P/2 = 19.71

Figure 291



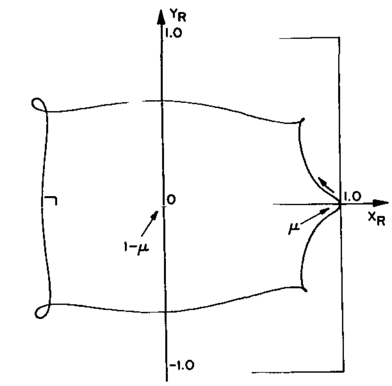
E.M. ORBIT
$\dot{Y}_R$ = 1.39520; TIME SHOWN: 28.01

Figure 292

Comments to Figures 281, 289 and 293

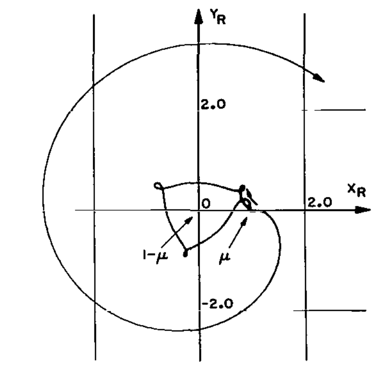
The orbits on Figure 281, 289 and 293 are all examples of "simple" orbits that go around both Earth and Moon.

EARTH-MOON FIELD ORBITS



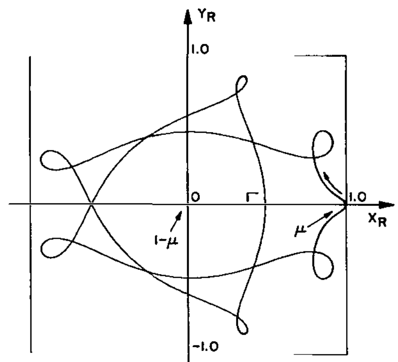
E.M. ORBIT ; PERIODIC
$\dot{Y}_R$ = 1.395543 ; P/2 = 9.766

Figure 293



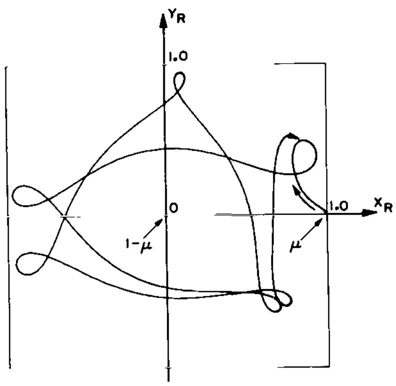
E.M. ORBIT
$\dot{Y}_R$ = 1.398 ; TIME SHOWN : 23.61

Figure 294



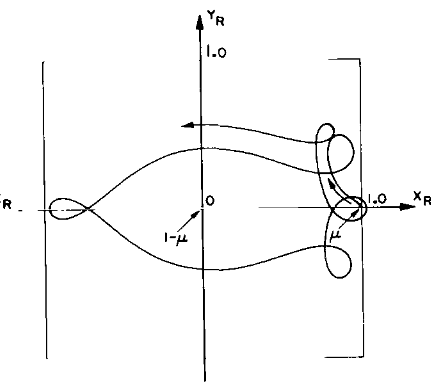
E.M. ORBIT ; PERIODIC
$\dot{Y}_R$ = 1.399745 ; P/2 = 13.00

Figure 295



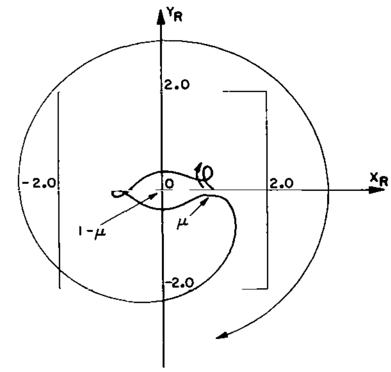
E.M. ORBIT
$\dot{Y}_R$ = 1.4006 ; TIME SHOWN : 27.12

Figure 296



E.M. ORBIT
$\dot{Y}_R$ = 1.4012 ; TIME SHOWN : 1.915

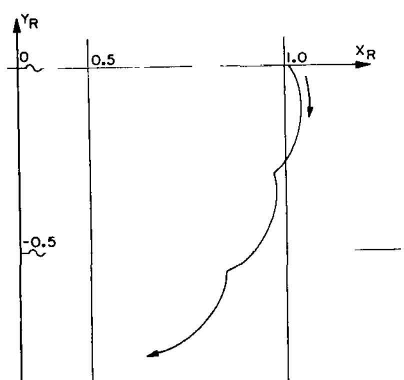
Figure 297



E.M ORBIT
$\dot{Y}_R$ = 1.4015 ; TIME SHOWN : 20.16

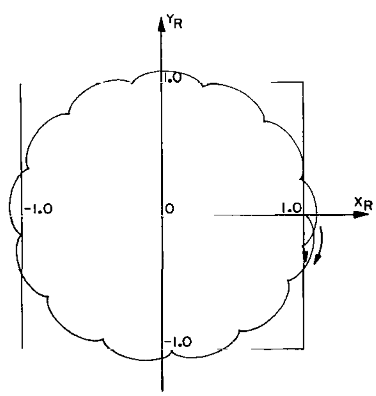
Figure 298

KEPLER ORBITS



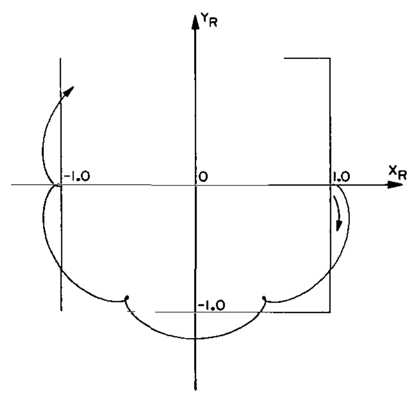
KEPLER ORBIT
$\dot{Y}_R$ = -0.0100 ; TIME SHOWN : 20.20

Figure 299



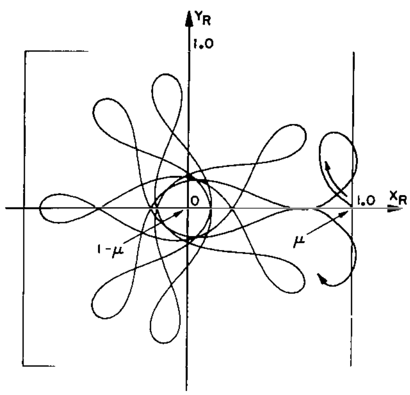
KEPLER ORBIT ; CUSPING ORBIT
$\dot{Y}_R$ = 0.00 ; TIME SHOWN : 90.37

Figure 300



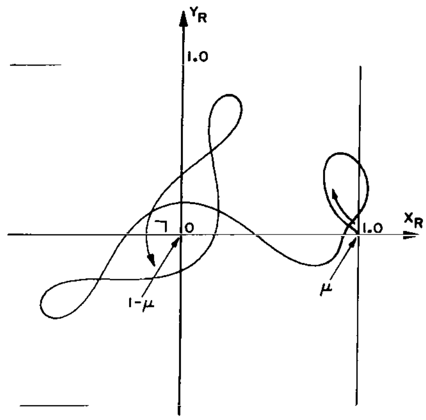
KEPLER ORBIT ; n = +6/7
$\dot{Y}_R$ = +0.023343 ; P/2 = 21.99

Figure 301



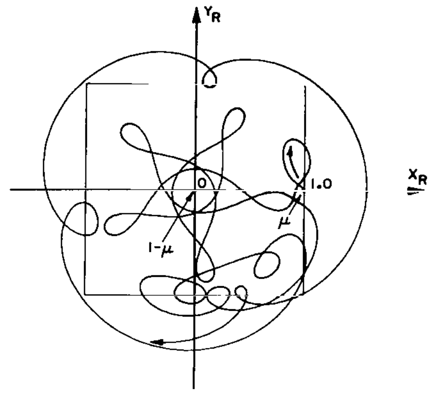
E. M. ORBIT; PERIODIC
Y_R = 1.403960; P/2 = 13.31

Figure 302



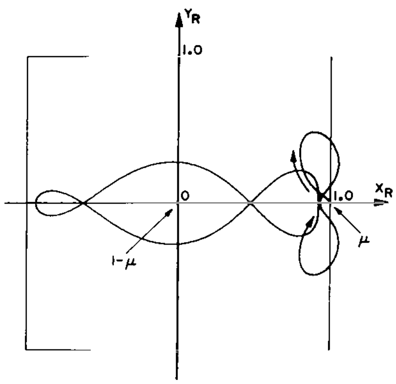
E. M. ORBIT; PERIODIC
Y_R = 1.404951; P/2 = 10.14

Figure 303



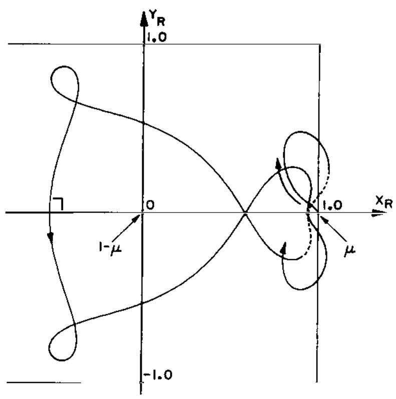
E. M. ORBIT
Y_R = 1.405; TIME SHOWN: 70.12

Figure 304



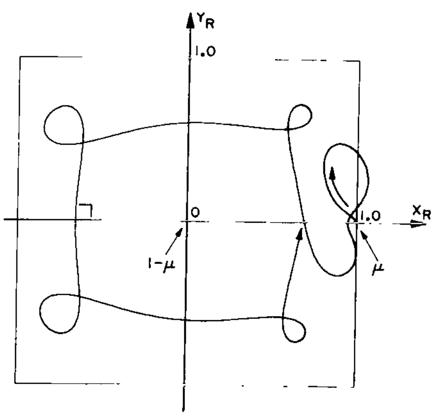
E. M. ORBIT; PERIODIC
Y_R = 1.405183; P/2 = 6.995

Figure 305



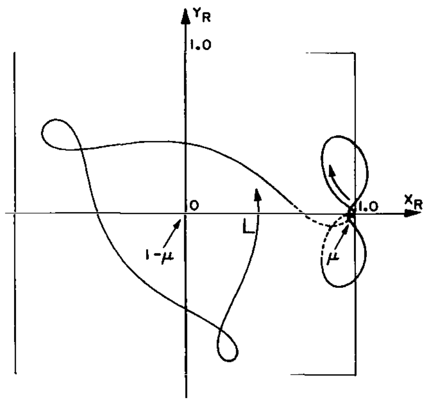
E. M. ORBIT; PERIODIC
Y_R = 1.405420; P/2 = 9.987

Figure 306



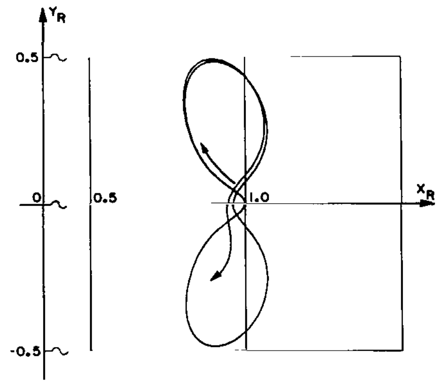
E. M. ORBIT; PERIODIC
Y_R = 1.405595; P/2 = 15.53

Figure 307



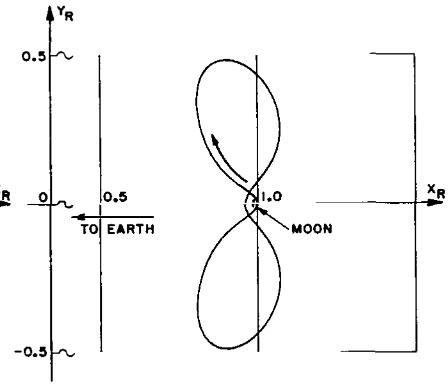
E.M. ORBIT ; PERIODIC
Y_R = 1.405803; P/2 = 16.05

Figure 308



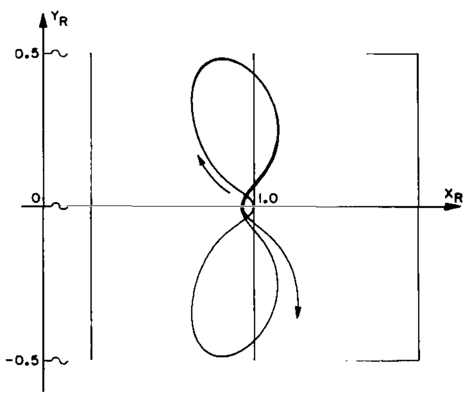
E.M. ORBIT
Y_R = 1.40583380; TIME SHOWN : 12.0

Figure 309



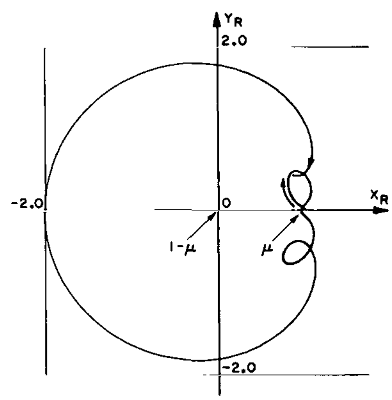
E.M. ORBIT ; PERIODIC
Y_R = 1.4058339; P/2 = 3.744

Figure 310



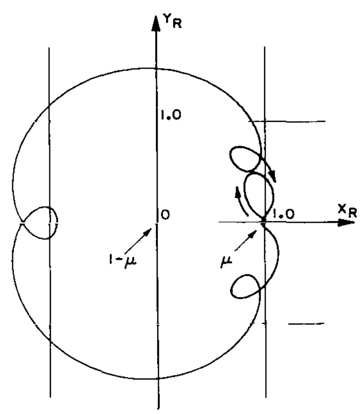
E.M. ORBIT
Y_R = 1.405834 ; TIME SHOWN : 12.11

Figure 311



E.M. ORBIT ; PERIODIC
Y_R = 1.4059395; P/2 = 13.9

Figure 312

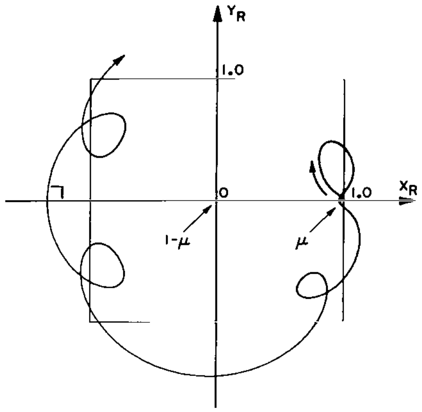


E.M. ORBIT ; PERIODIC
Y_R = 1.4059805; P/2 = 17.16

Figure 313

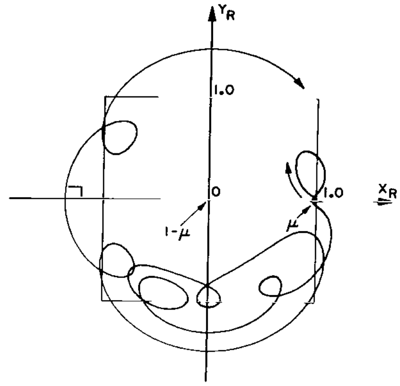
Comments to Figure 310

This orbit separates the orbits of direct motion about Earth from those of retrograde motion.



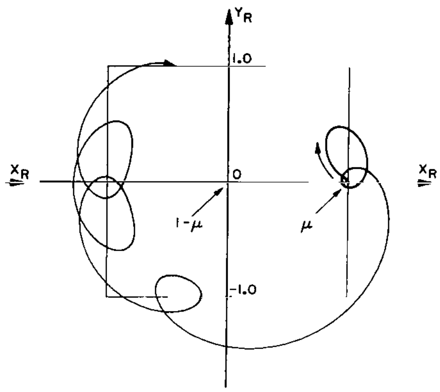
E.M. ORBIT; PERIODIC
Y_R = 1.406001; P/2 = 20.31

Figure 314



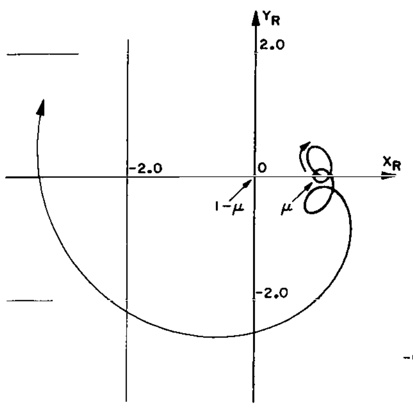
E.M. ORBIT; PERIODIC
Y_R = 1.40605; P/2 = 40.87

Figure 315



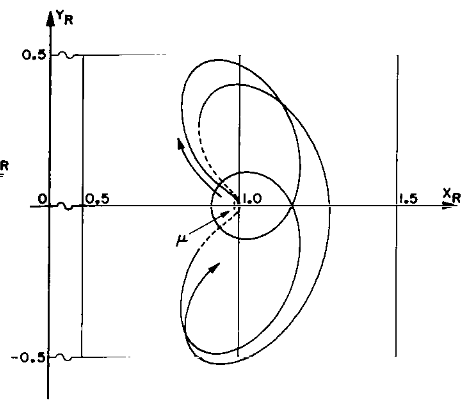
E.M. ORBIT
Y_R = 1.410; TIME SHOWN: 30.04

Figure 316



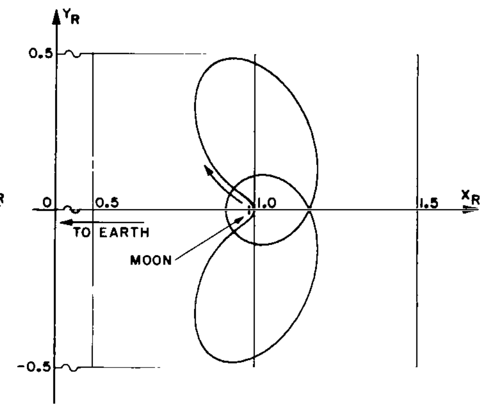
E.M. ORBIT
Y_R = 1.4115; TIME SHOWN: 14.15

Figure 317



E.M. ORBIT
Y_R = 1.411850; TIME SHOWN: 14.08

Figure 318

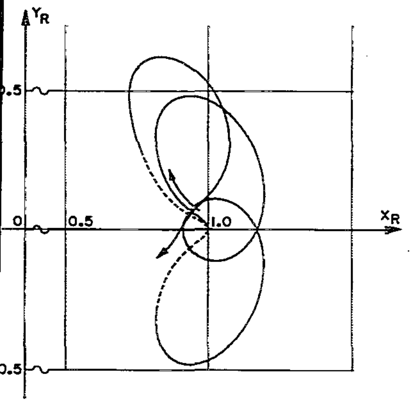


E.M. ORBIT; PERIODIC
Y_R = 1.4118647; P/2 = 4.119

Figure 319

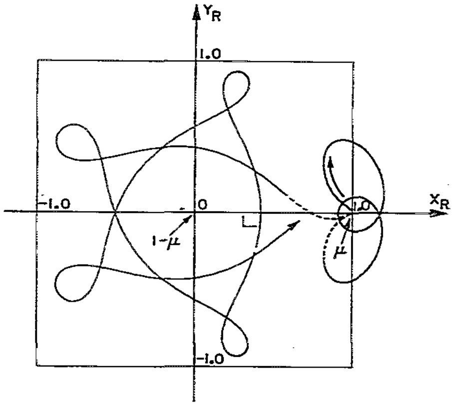
Comments to Figures 319 and 328

With the orbit of Figure 319, the direction of motion about the Earth is returned to the positive one which, however, is maintained for a short step in the progression only. The reassumption of the retrograde motion occurs after passing the periodic orbit about the translunar equilibrium point depicted on Figure 328.



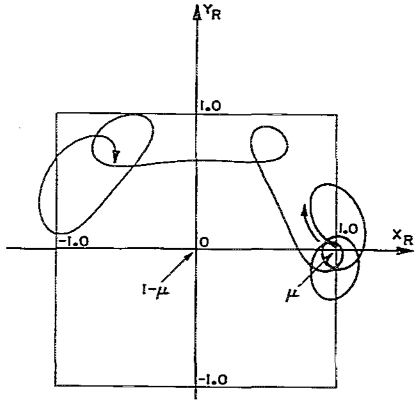
E. M. ORBIT
Y_R = 1.41188 ; TIME SHOWN : 13.04

Figure 320



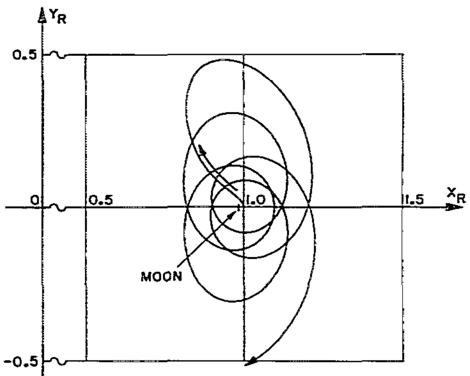
E. M. ORBIT; PERIODIC
Y_R = 1.4120099 ; P/2 = 16.83

Figure 321



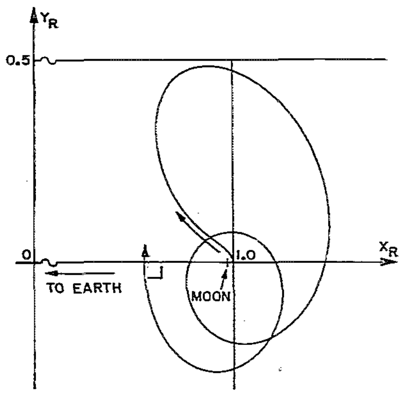
E.M. ORBIT
Y_R = 1.4125; TIME SHOWN : 25.05

Figure 322



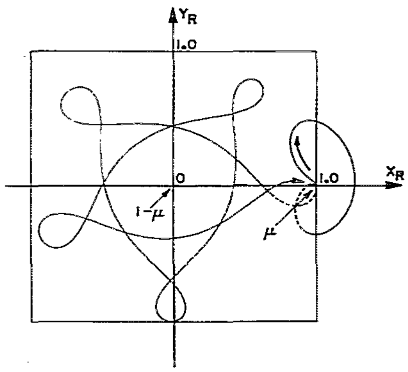
E.M. ORBIT
Y_R = 1.4130 ; TIME SHOWN : 14.07

Figure 323



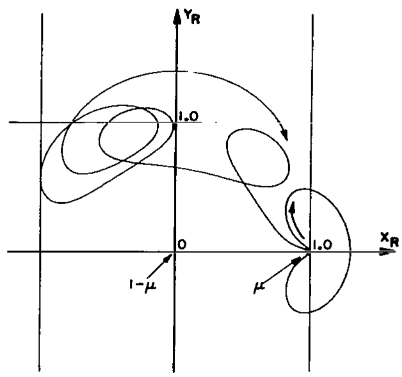
E.M. ORBIT; PERIODIC
Y_R = 1.413573 ; P/2 = 6.838

Figure 324



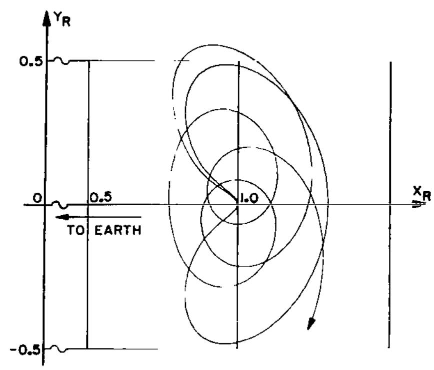
E.M. ORBIT
Y_R = 1.4160 ; TIME SHOWN : 23.22

Figure 325



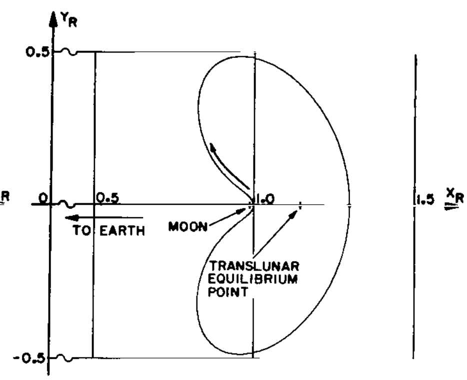
E. M. ORBIT
Y_R = 1.41730 ; TIME SHOWN : 30.09

Figure 326



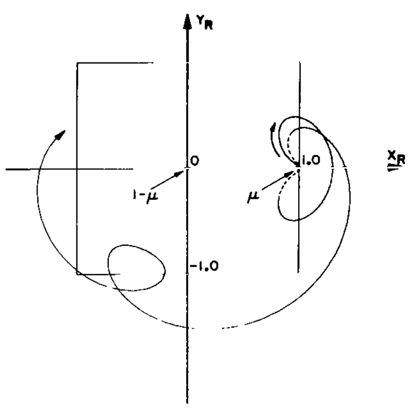
E. M. ORBIT
Y_R = 1.41745 ; TIME SHOWN : 18.25

Figure 327



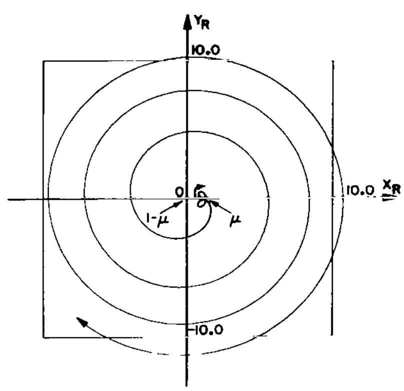
E. M. ORBIT ; PERIODIC
Y_R = 1.417491 ; P/2 = 3.087

Figure 328



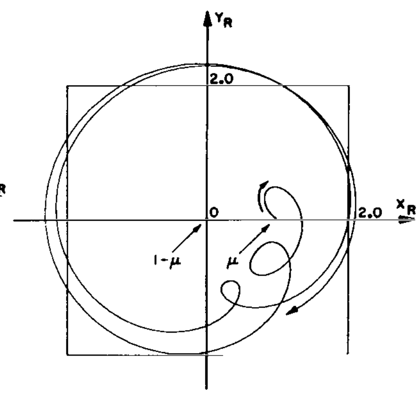
E. M. ORBIT
Y_R = 1.41760 ; TIME SHOWN : 18.07

Figure 329



E. M. ORBIT
Y_R = 1.4190 ; TIME SHOWN : 30.17

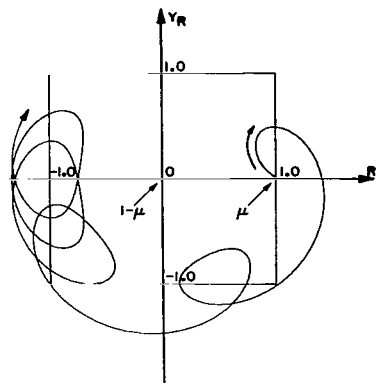
Figure 330



E. M. ORBIT
Y_R = 1.421 ; TIME SHOWN : 30.04

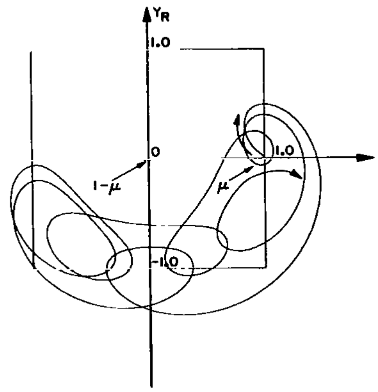
Figure 331

EARTH-MOON FIELD ORBITS



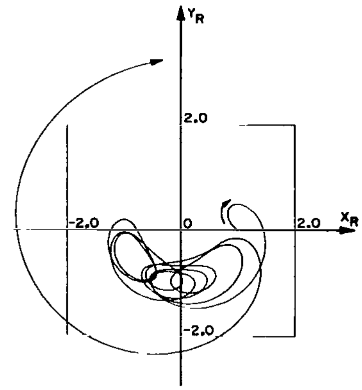
E.M. ORBIT
Y_R = 1.4250 ; TIME SHOWN : 32.02

Figure 332



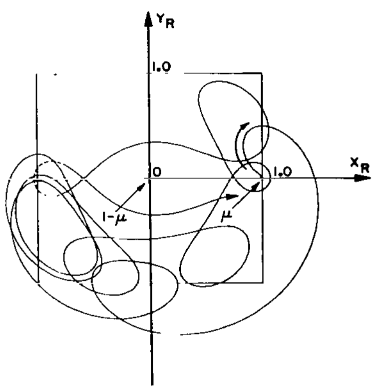
E.M. ORBIT
Y_R = 1.42820 ; TIME SHOWN : 42.02

Figure 333



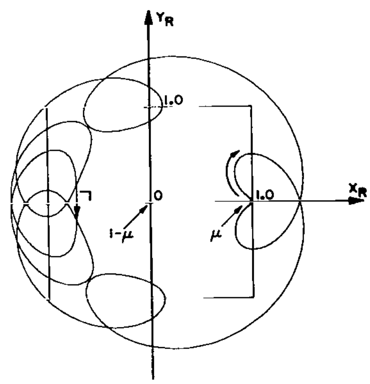
E.M. ORBIT
Y_R = 1.4290 ; TIME SHOWN 60.08

Figure 334



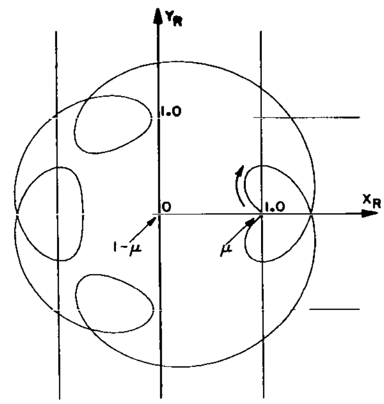
E.M. ORBIT
Y_R = 1.4291 ; TIME SHOWN: 51.06

Figure 335



E.M. ORBIT ; PERIODIC
Y_R = 1.429742 ; P/2 = 22.16

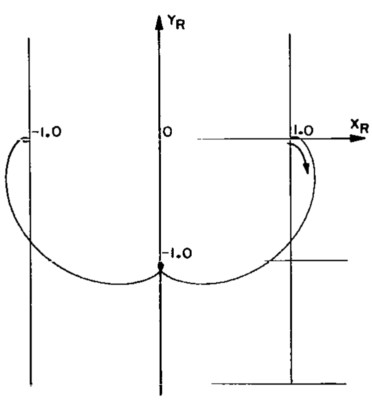
Figure 336



E.M. ORBIT ; PERIODIC
Y_R = 1.430896 ; P/2 = 15.95

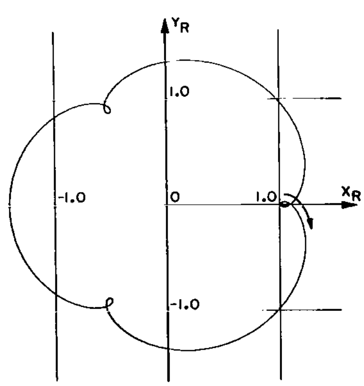
Figure 337

KEPLER ORBITS



KEPLER ORBIT ; n = + 4/5
Y_R = +0.042742 ; P/2 = 15.71

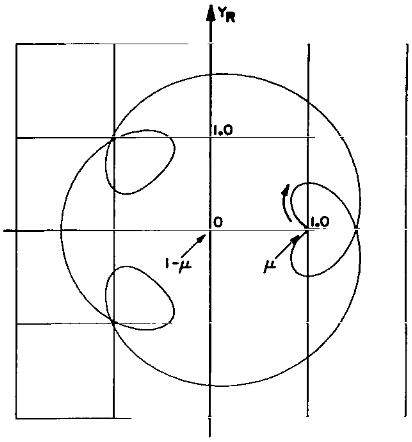
Figure 338



KEPLER ORBIT ; n = + 3/4
Y_R = +0.059800 ; P/2 = 12.57

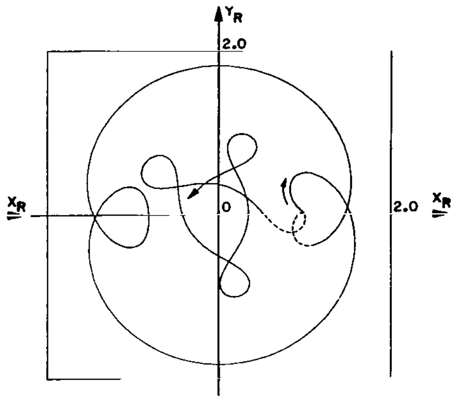
Figure 339

EARTH-MOON FIELD ORBITS



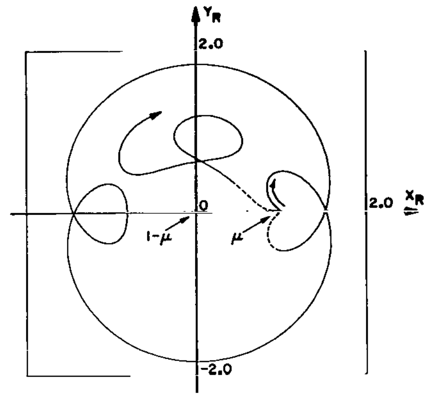
E.M. ORBIT; PERIODIC
$Y_R = 1.432550$; $P/2 = 12.85$

Figure 340



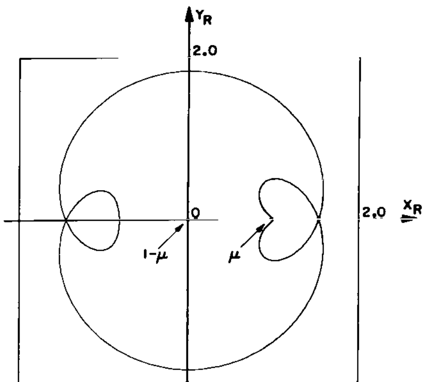
E.M. ORBIT
$Y_R = 1.4360$; TIME SHOWN: 32.03

Figure 341



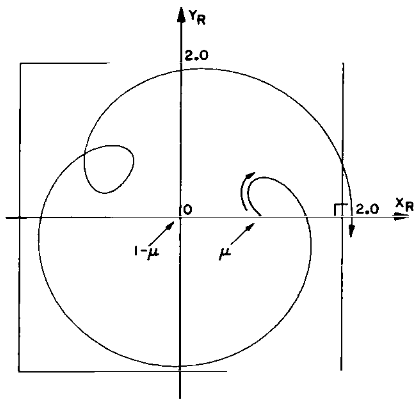
E.M. ORBIT
$Y_R = 1.43610$; TIME SHOWN: 29.03

Figure 342



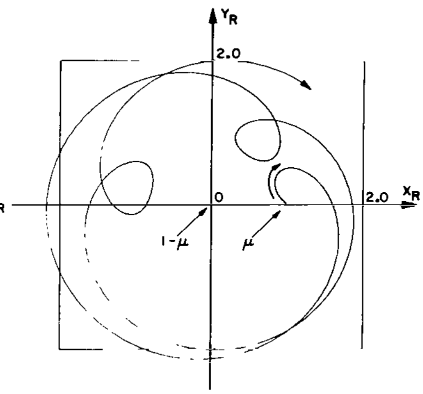
E.M. ORBIT; PERIODIC
$Y_R = 1.436128$; $P/2 = 9.78$

Figure 343



E.M. ORBIT; PERIODIC
$Y_R = 1.438569$; $P/2 = 16.09$

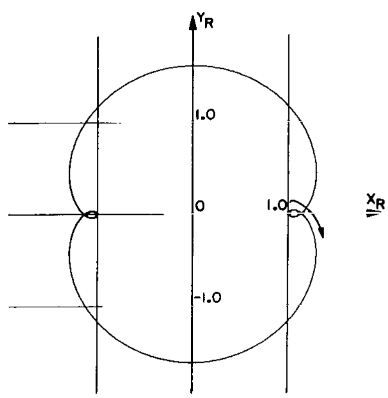
Figure 344



E.M. ORBIT
$Y_R = 1.4440$; TIME SHOWN: 27.61

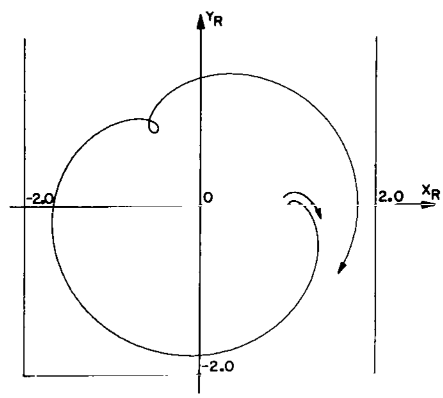
Figure 345

KEPLER ORBITS



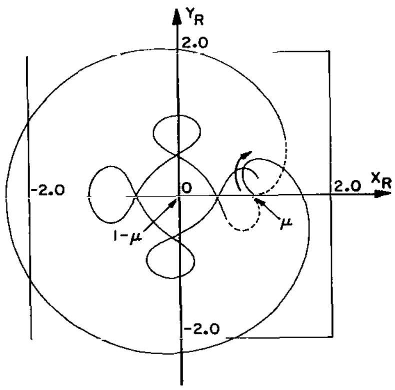
KEPLER ORBIT; $n = +2/3$
$Y_R = 0.088484$; $P/2 = 9.425$

Figure 346



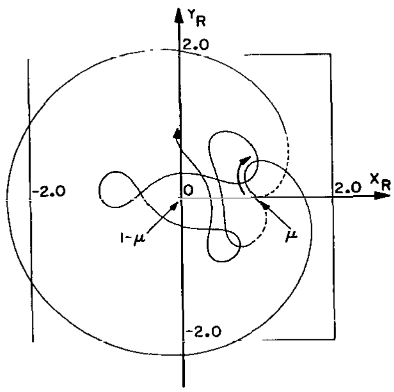
KEPLER ORBIT; $n = +3/5$
$Y_R = 0.11175$; $P/2 = 15.71$

Figure 347



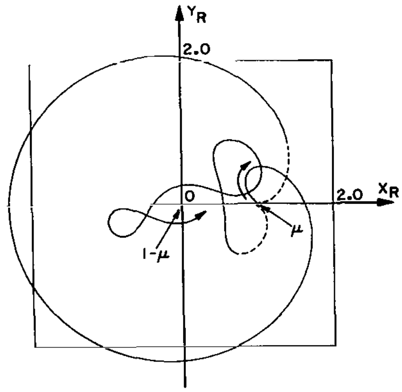
E.M. ORBIT
Y_R = 1.446020; TIME SHOWN: 30.07

Figure 348



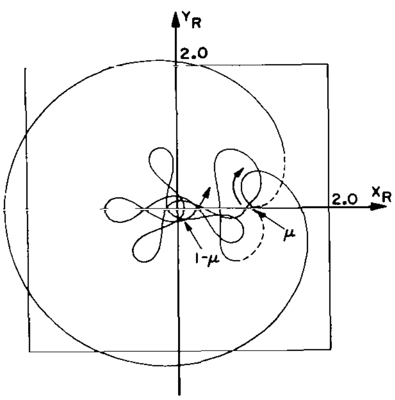
E.M. ORBIT
Y_R = 1.446043; TIME SHOWN: 29.10

Figure 349



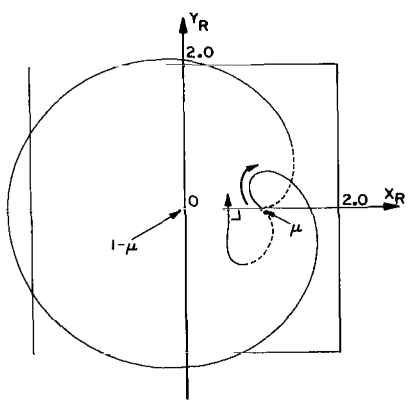
E.M. ORBIT
Y_R = 1.446045; TIME SHOWN: 23.54

Figure 350



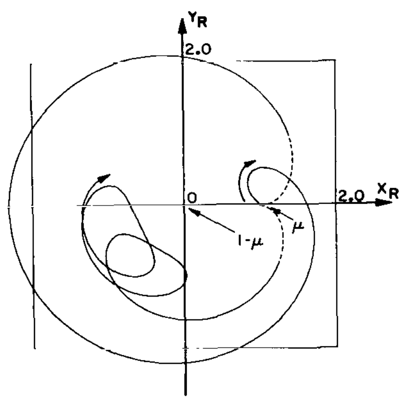
E.M. ORBIT
Y_R = 1.446048; TIME SHOWN: 30.03

Figure 351



E.M. ORBIT; PERIODIC
Y_R = 1.4460503; P/2 = 15.26

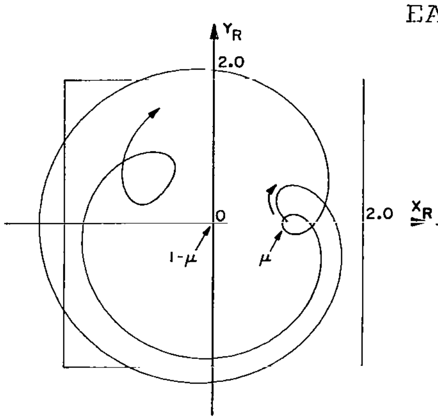
Figure 352



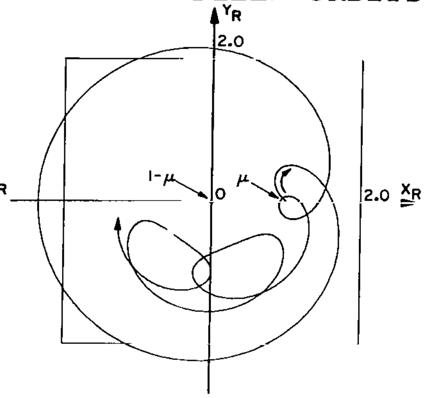
E.M. ORBIT
Y_R = 1.4461; TIME SHOWN: 30.05

Figure 353

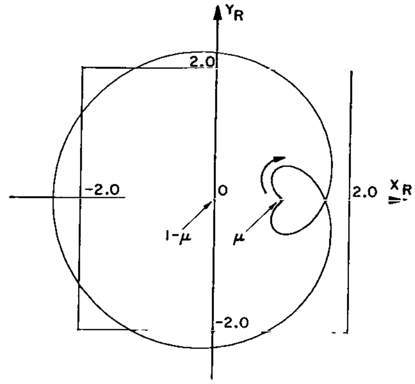
EARTH-MOON FIELD ORBITS



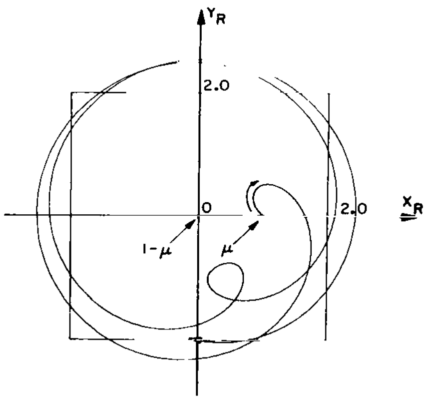
E. M. ORBIT
Y_R = 1.4466; TIME SHOWN: 24.06
Figure 354



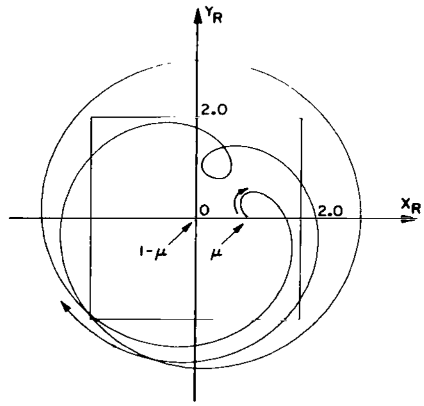
E. M. ORBIT
Y_R = 1.4468; TIME SHOWN: 30.12
Figure 355



E. M. ORBIT; PERIODIC
Y_R = 1.447547; P/2 = 6.707
Figure 356

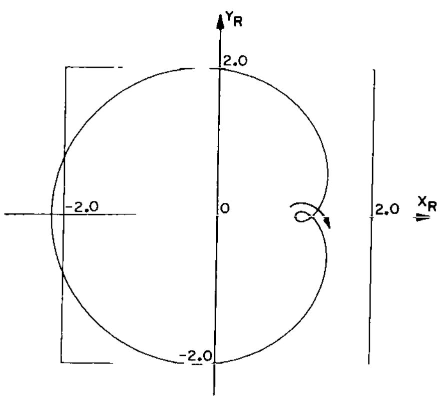


E. M. ORBIT
Y_R = 1.450; TIME SHOWN: 24.75
Figure 357



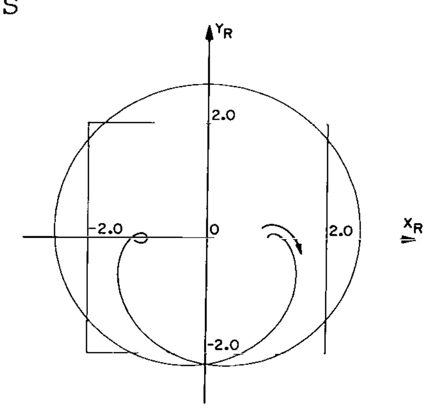
E. M. ORBIT
Y_R = 1.4580; TIME SHOWN: 24.62
Figure 358

KEPLER ORBITS



KEPLER ORBIT; n = +1/2
Y_R = 0.14739; P/2 = 6.283

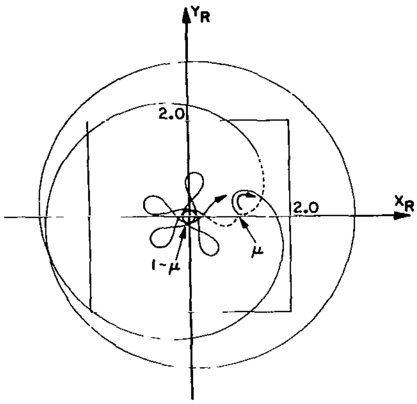
Figure 359



KEPLER ORBIT; n = +2/5
Y_R = 0.18434; P/2 = 15.71

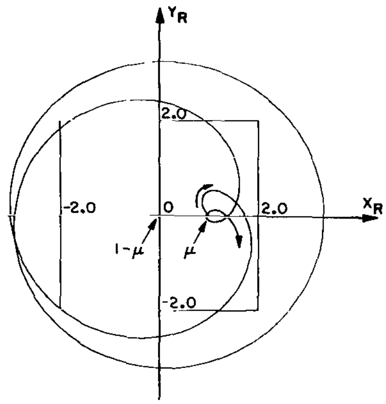
Figure 360

EARTH-MOON FIELD ORBITS



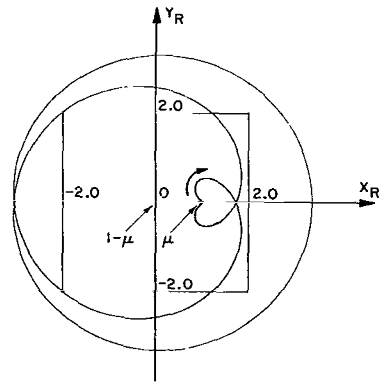
E.M. ORBIT
Y_R = 1.45950; TIME SHOWN: 30.02

Figure 361



E.M. ORBIT; PERIODIC
Y_R = 1.4597613; P/2 = 18.55

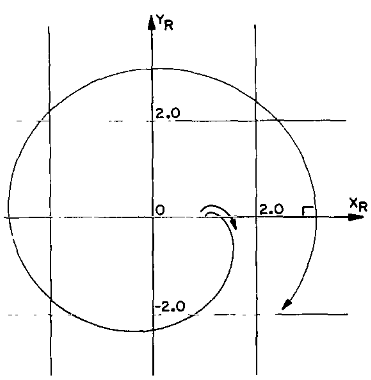
Figure 362



E.M. ORBIT; PERIODIC
Y_R = 1.460332; P/2 = 9.891

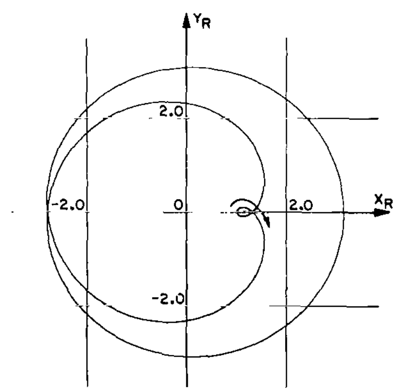
Figure 363

KEPLER ORBITS



KEPLER ORBIT; n = +1/3
Y_R = 0.21002; P/2 = 9.425

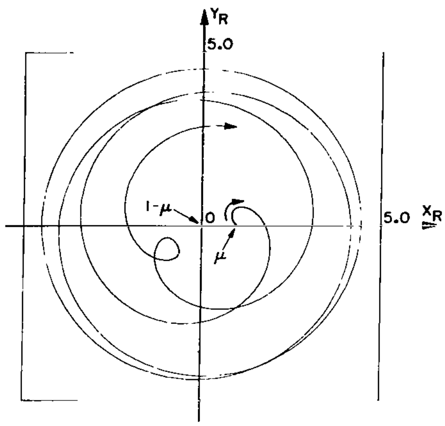
Figure 364



KEPLER ORBIT; n = +1/3
Y_R = 0.21002; P/2 = 9.425

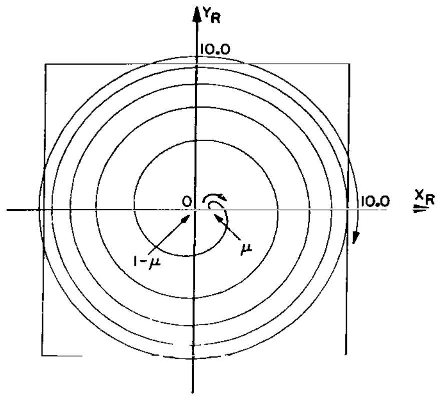
Figure 365

EARTH-MOON FIELD ORBITS



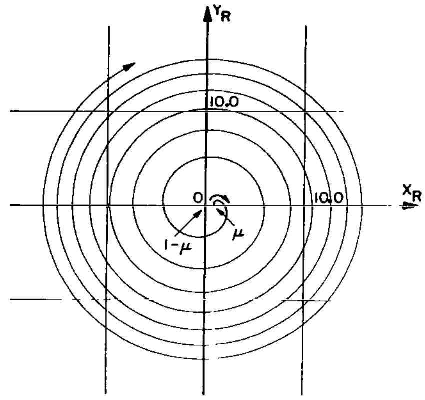
E. M. ORBIT
Y_R = 1.470; TIME SHOWN: 33.15

Figure 366



E M ORBIT
Y_R = 1.490; TIME SHOWN: 35.09

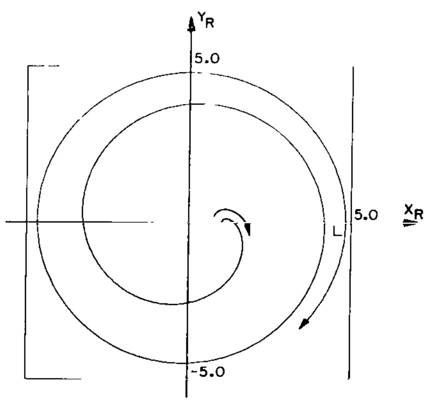
Figure 367



E. M. ORBIT
Y_R = 1.500; TIME SHOWN: 45.22

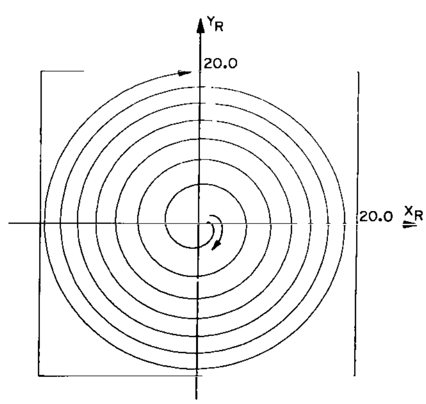
Figure 368

KEPLER ORBITS



KEPLER ORBIT; n = +1/5
Y_R = 0.26551; P/2 = 15.71

Figure 369



KEPLER ORBIT; (PARABOLIC)
Y_R = 0.39296; TIME SHOWN: 45.02

Figure 370

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