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A Progress Report
on the study of

Effects on the Motion of a Body Attracted
by a Rotating Source

(Grant NGR 33-007-034 - Sup. 1)

to

National Aeronautics and Space Administration

Prepared by:

Herbert Knothe
Professor of Mathematics

FACILITY FORM 502

N70-12315	
(ACCESSION NUMBER)	(THRU)
4	1
(PAGES)	(CODE)
Cr-106986	19
(NASA CR OR TMX OR AD NUMBER)	(CATEGORY)

Division of Research
Clarkson College of Technology
Potsdam, New York 13676

October 15, 1969

PROGRESS REPORT

Effect on the Motion of a Body Attracted by a Rotating Source (Part II)

An approach different from that in Part I^{*} has been used here: the equation of motion (the symbols are identical with those in Part I) may be written

$$(1) \quad \ddot{\bar{X}} + \frac{\mu}{r^3} \bar{X} = \alpha \bar{F}$$

where $\alpha \bar{F}$ is any small perturbing force. In the case which is to be investigated here $\bar{F}$ is the gradient of a time dependent potential $\phi(x, y, z, t)$. The components of $\bar{F}$ are homogeneous functions of x, y, z of degree -4, in other words

$$F_i(\lambda x, \lambda y, \lambda z, t) = \lambda^{-4} F_i(x, y, z, t)$$

This fact leads to the following treatment of the perturbation of a circular orbit. If r_0 is the radius of a circular orbit whose local vector $\bar{X}_0$ satisfies (1) with $\alpha = 0$ and if we substitute in (1)

$$\tau = \sqrt{\mu} \frac{t}{r_0}, \quad \bar{X} = r_0 \bar{a}, \quad \bar{a} = \text{vector } (a_{11}, a_{21}, a_{31})$$

$$(2) \quad r = r_0 \rho, \quad \beta = \frac{\alpha}{\mu r_0^2},$$

equation (1) assumes the dimensionless form

$$(3) \quad \bar{a}'' + \frac{\bar{a}}{\rho^3} = \beta \text{ grad } \bar{\phi},$$

the prime denoting differentiation with respect to τ where

^{*}Part I refers to Progress Report dated June 14, 1968.



$$(4) \quad \text{grad } \phi = \text{vector } \frac{\partial}{\partial a_i} \bar{\phi} (a_{11} a_{21} a_{31} t) \cdot$$

In order to solve equation (3) we assume $\bar{a}$ in the form of power series

$$\bar{a} = \bar{a}_0 + \beta \bar{a}_1 + \beta^2 + \dots$$

where

$$\bar{a}_0^2 = 1$$

The vector $\bar{a}_i$ yields the i -th variation of the orbit.

It is shown that $\bar{a}_i$ satisfies an equation

$$(5) \quad \bar{a}_i'' + \bar{a}_i + \text{const.} (\bar{a}_0 \bar{a}_1) \bar{a}_0 = \bar{F}_i (\bar{a}_0, \bar{a}_1, \dots, \bar{a}_{i-1}) \cdot$$

The equation (5) can be solved explicitly, i.e. can be expressed in terms of functions of the kind $\tau^l \cos m\tau$, $\tau^l \sin m\tau$. Equation (5) represents a linear ordinary inhomogeneous differential equation for $\bar{a}_i$ with nonconstant coefficients.

Equation (5) is solved applying the method of variation of constants. The homogeneous equation

$$(6) \quad \bar{a}_i'' + \bar{a}_i + \text{const.} (\bar{a}_0 \bar{a}_1) \bar{a}_0 = 0$$

has the property: If $a_1^{(j)}$, $a_2^{(j)}$, $a_3^{(j)}$, $j=1,2,3,4,5,6$ are the 6 linearly independent solutions of (6) the determinant of the matrix M whose j th column has the elements $a_1^{(j)}$, $a_2^{(j)}$, $a_3^{(j)}$, $a_1^{(j)'}$, $a_2^{(j)'}$, $a_3^{(j)'}$ is independent of τ . In case of the first and second variation the solutions can easily be chosen so that the determinant is 1. The elements of the inverse matrix M^{-1} are of a strikingly simple form. All quadratures of (5) can be solved explicitly. The components of $\bar{a}_i$ are polynomials of τ , $\cos \tau$, $\sin \tau$, $\cos \Omega \tau$, $\sin \Omega \tau$ where $\Omega = \frac{r_0}{\sqrt{\mu}} \omega$, ω = angular velocity of the rotating attracting

source. Professor Robertson calculated various orbits for various perturbing potentials. His results show that the fundamental formulas (28a), (28b), (29a), (29b) of Part I are of such an accuracy that their deviations from the exact results be in the error range of the computer calculations.