A Progress Report

on the study of

Effects on the Motion of a Body Attracted

by a Rotating Source

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PROGRESS REPORT

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(Part II)

An approach different from that in Part I* has been used here: the equation of motion (the symbols are identical with those in Part I) may be written

$$(1) \qquad \ddot{\overline{X}} + \frac{\mu}{r^3}\,\overline{X} = \alpha\,\overline{F}$$

where $\alpha\,\overline{F}$ is any small perturbing force. In the case which is to be investigated here $\overline{F}$ is the gradient of a time dependent potential $\phi\,(x, y, z, t)$. The components of $\overline{F}$ are homogeneous functions of x, y, z of degree-4, in other words

$$F_i\,(\lambda x, \lambda y, \lambda z, t) = \lambda^{-4}\,F_i(x, y, z, t)$$

This fact leads to the following treatment of the perturbation of a circular orbit. If r_o is the radius of a circular orbit whose local vector $\overline{X}_o$ satifies (1) with $\alpha = 0$ and if we substitute in (1)

$$\tau = \sqrt{\mu}\,\frac{t}{r_o}\,, \quad \overline{X} = r_o\,\overline{a}\,, \quad \overline{a} = \text{vector}\,(a_{11}\ a_{21},\ a_{31})$$

$$(2) \qquad r = r_o\,\rho, \quad \beta = \frac{\alpha}{\mu r_o^2}\,,$$

equation (1) assumes the dimenionless form

$$(3) \qquad \overline{a}'' + \frac{\overline{a}}{\rho^3} = \beta\,\text{grad}\,\overline{\phi}\,,$$

the prime denoting differentiation with respect to τ where

*Part I refers to Progress Report dated June 14, 1968.

$$\text{(4)} \qquad \text{grad } \phi = \text{vector } \frac{\partial}{\partial a_i} \bar{\phi} \, (a_{11} a_{21} a_{31} t) \, .$$

In order to solve equation (3) we assume $\bar{a}$ in the form of power series

$$\bar{a} = \bar{a}_o + \beta \bar{a}_1 + \beta^2 \quad + \ldots$$

where

$$\bar{a}_o^{\,2} = 1$$

The vector $\bar{a}_i$ yields the i-th variation of the orbit.

It is shown that $\bar{a}_i$ satisfies an equation

$$\text{(5)} \qquad \bar{a}_i'' + \bar{a}_i + \text{const.} \, (\bar{a}_o \, \bar{a}_i) \, \bar{a}_o = \bar{F}_i \, (\bar{a}_o, \, \bar{a}_1, \, \ldots \bar{a}_{i-1}) \, .$$

The equation (5) can be solved explicitly, i.c. can be expressed in terms of functions of the kind $\tau^{\ell} \cos {}^{m}\tau$, $\tau^{\ell} \sin {}^{m}\tau$. Equation (5) represents a linear ordinary inhomogeneous differential equation for $\bar{a}_i$ with nonconstant coefficients.

Equation (5) is solved applying the method of variation of constants. The homogeneous equation

$$\text{(6)} \qquad \bar{a}_i'' + \bar{a}_i + \text{const.} \, (\bar{a}_o \, \bar{a}_i) \, \bar{a}_o = 0$$

has the property: If $a_1^{(j)}, a_2^{(j)}, a_3^{(j)}$, j=1,2,3,4,5,6 are the 6 linearly independent solutions of (6) the determinant of the matrix M whose jth column has the elements $a_1^{(j)}, a_2^{(j)}, a_3^{(j)}, a_1^{(j)'}, a_2^{(j)'}, a_3^{(j)'}$ is independent of τ In case of the first and second variation the solutions can easily be chosen so that the determinant is 1. The elements of the inverse matrix M^{-1} are of a strikingly simple form. All quadratures of (5) can be solved explicitly. The components of $\bar{a}_i$ are polynomials of τ, $\cos\tau$, $\sin\tau$, $\cos \Omega\tau \sin \Omega\tau$ where $\Omega = \frac{r_o}{\sqrt{\mu}} \omega$, ω= angular velocity of the rotating attracting source. Professor Robertson calculated various orbits for various perturbing potentials. His results show that the fundamental formulas (28a), (28b), (29a), (29b) of Part I are of such an accuracy that their deviations from the exact results be in the error range of the computer calculations.