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FINAL REPORT

CONTRACT NAS8-21019

AN INVESTIGATION STUDY ON MODEL REFERENCE ADAPTIVE TECHNIQUES AS APPLIED TO ATTITUDE CONTROL SYSTEM FOR LAUNCH VEHICLES

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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SECTION I

INTRODUCTION

The purpose of this report is to present the results of a study conducted for the National Aeronautics and Space Administration, George C. Marshall Space Flight Center under Contract NAS8-21019 entitled "An Investigation Study on Model Reference Adaptive Techniques as Applied to Attitude Control System for Launch Vehicles."

Conventional methods for designing an attitude control system for a large flexible booster are highly dependent on specific parameters of the elastic modes and the aerodynamic parameters. The bending mode gain and phase margins of these systems are very sensitive to variations in the mode frequencies and mode slopes at the sensor locations. The greater this sensitivity the more difficult it is to design a control system which will maintain adequate stability margins for all flight times. A conventional control system must be designed so that the nominal system is as near optimum as possible based on the available design data and the limited stability margins. Changes in design data due to improved modelling of the vehicle or changes in payload, mission and trajectory may then necessitate a redesign of the attitude control system. It would be most desirable to have a control system with sufficient stability margins so that moderate changes in design data could be made without redesign of the system.

The purpose of this study is to investigate the feasibility of and to define future areas of investigation for the development of a prototype model reference adaptive control system applicable to launch vehicle of the SATURN V Class. The specific objectives of the study are to conduct a comprehensive survey on model reference adaptive techniques that are applicable to attitude control systems including specification of the significant characteristics of each design configuration, and, subsequent to this survey, to choose the most promising of these techniques and, using data representative of a SATURN V vehicle, to conduct an analysis to establish the feasibility of using these techniques to design a practical adaptive attitude control system.

The report is divided into five sections, including this introductory section, and two appendices. Section II presents the results of the survey of model reference adaptive control techniques. Section III presents the justification for choosing two of these techniques for further analysis to determine their feasibility for the design of an adaptive control system for a SATURN V Class launch vehicle. Sections IV and V present the results of these studies. Section IV is based on the M.S. thesis of Mr. R. Kramer and Section V is based on the Ph.D. thesis of Dr. M. Currie. These documents were submitted to the Department of Engineering, University of California, Los Angeles in December, 1968 and March, 1969,

respectively, and are available on request. Appendices A and B list the equations of motion, the notation, and the vehicle parameters used in the detailed studies discussed in Sections IV and V.

It should be noted that in each of the major sections (Roman numerals I - V) equations, figures, etc. are numbered independent of the other sections.

SECTION II

SURVEY OF APPLICABLE MRAC TECHNIQUES

During the last ten years model reference adaptive control (MRAC) was born and thoroughly developed in many different directions. A great deal of the pioneer development work has been carried out at two universities: U.C.L.A. and M.I.T. Presently, many industrial organizations and other universities are also contributing to MRAC technology.

The following paragraphs discuss several of the MRAC developments which appear to have merit for application to the control of flexible launch vehicles. However, since this is a survey of MRAC, we have limited the discussion to those details that are important for a general understanding of the basic concepts and the overall approaches relevant to particular techniques.

1. U.C.L.A. Developments in MRAC

The first MRAC development at U.C.L.A. took place in the 1959-1960 time period, with the work of Margolis and Leondes [1, 2, 3, 4]. Theirs was one of the first adaptive control techniques to make use of a model. In their approach the model and the plant are driven by the same input, and the outputs of the model and plant are differenced to form an error signal. The error is then used to adjust parameters in the model in order to force the model to have the same dynamics, and hence the same response, as the plant. Once the model (and hence, plant parameters are known), system compensation can be changed appropriately. In a strict sense, some may not consider this technique to be MRAC, but the technique does utilize an explicit model.

In 1961-1963 Donalson and Leondes [5,6,7] developed their MRAC technique. The model, in their development, represents the desired response of a closed-loop system composed of an unknown plant and variable compensation. Both the model and the system are driven by the command input. Then the difference between the outputs of the model and the system is used to vary the system compensation. The objective here is to force the system to have the same dynamics as the model. This is the more common approach to MRAC.

From 1963 to the present Hsieh [8], Kishi [9], Sworder [10], and Watson [11] have all furthered the development of adaptive control at U.C.L.A., although none

of their efforts were concerned particularly with MRAC. In 1965, Hatcher [12] conducted a study of the effects of external disturbances on the performance of the MRAC technique of Donalson and Leondes.

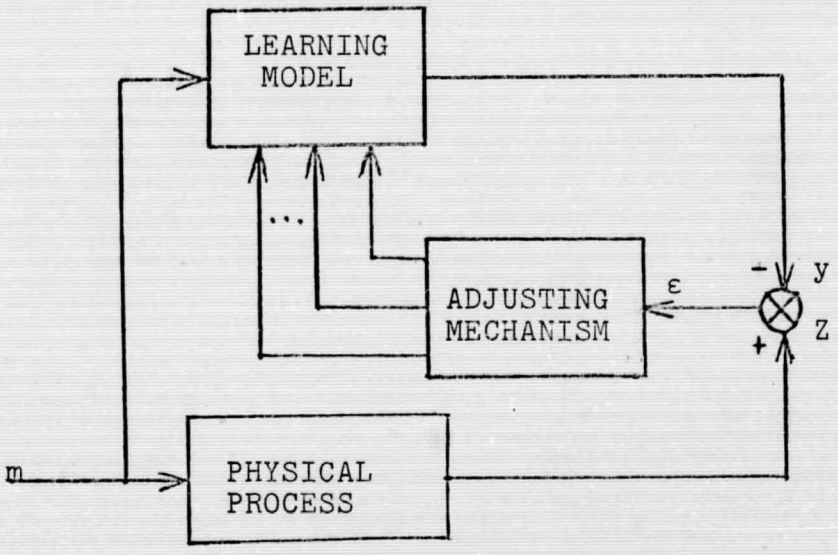
An entirely new technique is presently being developed by Currie and Stear. This technique is of the Margolis and Leondes type in that the parameters of a model are changed. However, the model in this case is altogether different. It is an optimal state estimation filter of the Kalman type. To adapt, the parameters of this Kalman filter are changed in order to minimize the difference between the system and filter outputs.

The next two sections provide a more detailed discussion of the MRAC techniques of Margolis and Leondes and Donalson and Leondes (including the results of Hatcher). The technique of Currie and Stear is discussed in detail in a later section of this report.

Margolis and Leondes Technique

A general block diagram of Margolis and Leondes' technique is given in Figure 1-1. The adjusting mechanism operates on the difference, ϵ, between the plant and system outputs. The object is to alter the parameters of the model in order to make the model and plant dynamics identical.

In MRAC, the measure of how closely the model and



THE LEARNING MODEL WITH ADJUSTING MECHANISM

FIGURE 1-1

plant dynamics match is always taken to be some non-negative function or functional of the error, ε, and possibly several of its derivatives. This measure may be designated $f(\varepsilon)$. For most of their work Margolis and Leondes used the simple quadratic function

$$f_o(\varepsilon) = \varepsilon^2. \tag{1.01}$$

as their performance measure. However, other measures of error were also considered, although only in relation to tracking a simple first order process. For example,

$$f_1(\varepsilon) = \varepsilon^2 + k_1\dot{\varepsilon}^2 \tag{1.02}$$

was found to bring about slightly improved response over the use of $f_o(\varepsilon) = \varepsilon^2$. A significant improvement in traking performance was obtained by using

$$f_2(\varepsilon) = (\varepsilon + k_2\dot{\varepsilon})^2. \tag{1.03}$$

This form of $f_2(\varepsilon)$ is particularly useful because it controls to some extent the dynamic response of ε. That is, if an adjusting mechanism can be found that will maintain $f_2(\varepsilon) = 0$, then ε is forced to decay exponentially with a time constant equal to k_2. The last error measure studied was

$$f_3(\varepsilon) = \int_o^t \varepsilon^2(\tau)d\tau \tag{1.04}$$

The tracking scheme employing this error measure was found to be completely inadequate. In fact it led to instabilities while tracking a first order process driven by only

a step input.

In order to design an adjusting mechanism some means must be established for making $f(\varepsilon)$ zero, or at least minimizing $f(\varepsilon)$. Margolis and Leondes selected the method of steepest descent as the basis for designing the adjusting mechanism. In their development, the process dynamics were of the following form:

$$\sum_{i=0}^{n} a_i(t) \frac{d^i z}{dt^i} = m(t) \tag{1.05}$$

To simplify the development, the model dynamics were chosen to be of the same order:

$$\sum_{i=0}^{n} \alpha_i(t) \frac{d^i y}{dt^i} = m(t) \tag{1.06}$$

The error between system and model is

$$\epsilon = z - y \tag{1.07}$$

The object of the adjusting mechanism is to force each $\alpha_i(t)$ toward $a_i(t)$ such that ϵ approaches zero. The steepest descent approach to this problem is to control the rate of change of each $\alpha_i(t)$ according to the relation

$$\frac{d\alpha_i}{dt} = -k \frac{\partial f}{\partial \alpha_i} \tag{1.08}$$

where k is a constant. Substituting f_o for f and $z - y$ for ϵ in this equation yields

$$\frac{d\alpha_i}{dt} = -2k\epsilon \frac{\partial y}{\partial \alpha_i} \qquad (1.09)$$

There are certain inherent restrictions on the rates-of-change of the α_i's based on minimizing ϵ^2. First, as Margolis points out, even if the model parameters were to instantaneously adjust to the system parameters the model output would not instantaneously change to equal the system output because of the stored energy in the model. The two outputs would be close only after the stored energy had been dissipated. This might be two to three times the lowest system time constant. (Of course this is true only for a stable process. For an unstable process, the stored energy, and hence the error, ϵ, would increase without limit.) Therefore, the responses of the α_i's, must be at least two to three times slower than the system response.

In addition to the inherent limitation on the response speeds of the α_i's, some restrictions are placed on the rates-of-change of the process parameters (i.e. the a_i's) in order to simplify the design of the adjusting mechanism. They are assumed to vary slowly in comparison to the adjustment speed of the model parameters (i.e. the α_i's). Clearly then, there is a hierarchy of variables and parameters, established by their required or desired response speeds. The process variables are the fastest, the model parameters are intermediate in response speed, and the process parameters are the slowest.

With this hierarchy in mind the development of the Margolis-Leondes adjusting mechanism can now be continued. Let u_i be defined by the expression

$$u_i = \frac{\partial y}{\partial \alpha_i} \tag{1.10}$$

Then the steepest descent adjustment for f_o is described by the expression

$$\frac{d\alpha_i}{dt} = -2k\epsilon u_i \tag{1.11}$$

Taking the partial derivative of the model equation with respect to α_j gives

$$\sum_{i=o}^{n} \alpha_{i(t)} \frac{\partial}{\partial \alpha_j} \frac{d^i y}{dt^i} + \frac{d_j y}{dt^j} = 0 \quad . \tag{1.12}$$

Considering y to be an explicit function of t and the $\alpha_i(t)$ and using the chain rule for differentiation leads to

$$\frac{d^i y}{dt^i} = \frac{\partial^i y}{\partial t^i} + \begin{bmatrix} \text{terms involving time} \\ \text{derivatives of the } \alpha_i(t) \end{bmatrix} \tag{1.13}$$

Margolis and Leondes make the assumption that, because of the previously discussed hierarchy, the terms in brackets can be ignored and hence

$$\frac{d^i y}{dt^i} = \frac{\partial^i y}{\partial t^i} \tag{1.14}$$

Substitution of Eq.(1.14) into Eq.(1.12) finally gives the following desired equations to be solved for obtaining

the $u_j(t)$.

$$\sum_{i=0}^{n} \alpha_i(t) \frac{d^i u_j}{dt^i} = - \frac{d^j y}{dt^j} \qquad (1.15)$$

Equations(1.11) and (1.15) together define the adjusting mechanism. Note that $u_j(t)$ is just the model response to an input equal to $-d^j y/dt^j$. The constant k should be selected so that the adjustment speeds of the model parameters fall in the middle between the response speeds of the process variables and the process parameters.

The above approach to adaptive parameter tracking has been studied in great detail by Margolis and Leondes [1,2,3, 4] for first order processes with one variable parameter. In addition, they also studied first order processes with two variable parameters, and second order processes in great detail. In deriving the equations of the adjusting mechanism for the second order process, the following performance criterion was used:

$$f(\epsilon) = [\epsilon + q_1 \dot{\epsilon} + q_2 \ddot{\epsilon}]^2.$$

In general, for n'th order processes of the form of equation (1.05), the recommended performance criterion is

$$f(\epsilon) = [\epsilon + q_1 \epsilon^{(1)} + q_2 \epsilon^{(2)} + \ldots + q_n \epsilon^{(n)}]^2 \qquad (1.16)$$

Now consider the possibility of applying the Margolis and Leondes adaptive technique to the control of a flexible launch vehicle. At the outset one is confronted with the problem that the form of the process differential equation that describes the vehicle dynamics is

$$\sum_{i=0}^{N} a_i(t)\ \theta^{(i)} = \sum_{j=0}^{N-1} b_j(t) m^{(j)} \qquad (1.17)$$

where θ is the vehicle attitude measured by an attitude sensor, and m is the thrust vector command. Unfortunately this equation is different from the one used above (equation (1.05)). There is also concern over the fact that the model, in reality, would not be of the same order as the process, a fact that was not considered in the Margolis-Leondes development. A further significant drawback is the inability of the adjusting mechanism to track unstable processes--the launch vehicle is unstable. This difficulty could be overcome by making the model track the closed loop system, but then the model order would be increased significantly. There is also a practical problem associated with using the recommended performance criterion equation (1.16). Its minimization would require the measurement of N derivatives of both the model and the process outputs. Whenever N is greater than two, obtaining the required process output derivatives is not generally considered feasible.

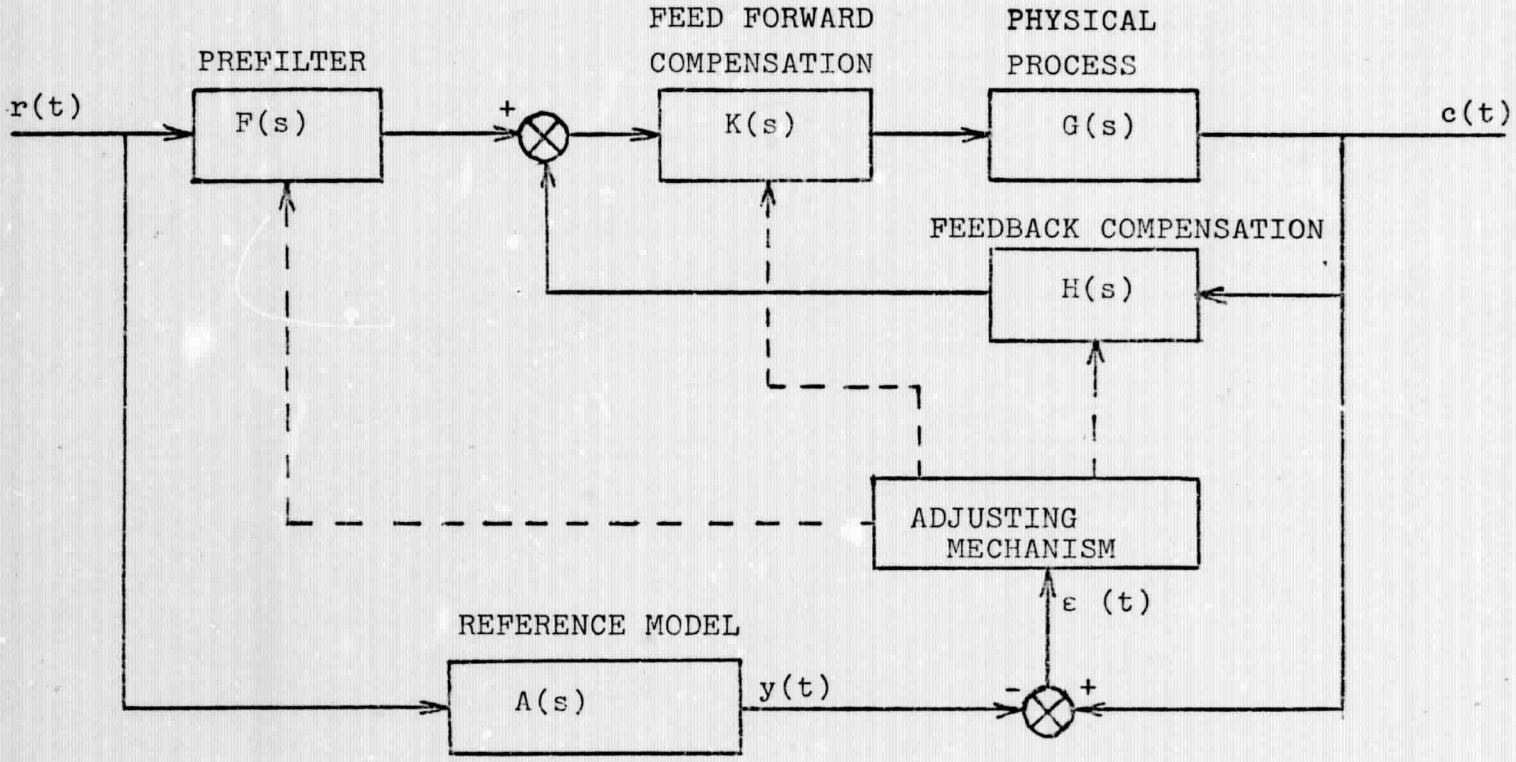
In addition to the above problems, disturbance and noise effects were not investigated by Margolis and Leondes and it is not apparent that their inclusion will not result in significant misadjustments of the process parameters. Finally there was no systematic study of global stability of the overall system; only linearized analysis with step inputs was completed.

From the above discussion it must be concluded that the Margolis and Leondes technique does not appear to be directly suited to adaptive control of a launch vehicle. However, their work has provided insight into the development of performance criteria, the use of steepest descent, and the relationships between the relative speeds of the various system parameters.

A MRAC technique which is somewhat more applicable to the control of a launch vehicle is described below.

Donalson and Leondes Technique

The MRAC technique of Donalson and Leondes is shown in Figure 1-2. In their technique, a reference model is selected which has the same dynamics as is desired for the closed loop system. The error between the model and system outputs is then used as a basis for establishing the dynamics of the three compensation blocks. The development of the adjusting mechanism is very similar to the



GENERALIZED MODEL REFERENCED ADAPTIVE
CONTROL SYSTEM
FIGURE 1-2

Margolis and Leondes development. However, Donalson and Leondes begin with more general process dynamics. They assume the process may be represented by the following transfer function:

$$G(s) = \frac{\sum_{i=0}^{M} g_{in} s^i}{\sum_{i=0}^{N} g_{id} s^i}, N \geq M \qquad (1.18)$$

In fact, all blocks of Fig. 1-2 are represented by similar transfer functions.

The assumptions that are required for the development of the Donalson-Leondes adjusting mechanism are also similar to the ones used by Margolis and Leondes:

Assumption 1: The coefficients in G(s) vary slowly as compared with the basic time constants of the physical process and the reference model.

Assumption 2: The coefficients in G(s) vary slowly as compared with the rate at which the adjusting mechanism, which is to be designed, adjusts the parameters in F(s), K(s), and H(s).

Assumption 3: The adjusting mechanism will be designed so that it adjusts the parameters in F(s), K(s), and H(s) rapidly as compared with the rate at which $f(\epsilon)$ is caused to change because of the effect of input r(t)

Two performance criteria are considered in developing the adjusting mechanism. They both have as many derivatives of $\epsilon(t)$ as there are terms in the denominator of the closed-loop transfer function.

$$f_1(\epsilon) = \frac{1}{2} \sum_{i=0}^{N} q_i \left(\frac{d^i \epsilon}{dt^i} \right)^2 \tag{1.19}$$

$$f_2(\epsilon) = \frac{1}{2} \left(\sum_{i=0}^{N} q_i \frac{d^i \epsilon}{dt^i} \right)^2 \tag{1.20}$$

$f_2(\epsilon)$ is identical with the general performance function used by Margolis and Leondes. The development of the adjusting mechanism based on $f_2(\epsilon)$ is described in the following paragraphs.

The transfer functions of the closed-loop system and the reference model are given as

$$\frac{F(s)K(s)G(s)}{1 + K(s)G(s)H(s)} = \frac{\sum_{i=0}^{M} \alpha_{in} s^i}{\sum_{i=0}^{N} \alpha_{id} s^i} \tag{1.21}$$

and

$$A(s) = \frac{\sum_{i=0}^{M} a_{in} s^i}{\sum_{i=0}^{N} a_{id} s^i} \tag{1.22}$$

The steepest descent approach to minimizing $f_2(\epsilon)$, as before, provides the equations for adjusting the $\alpha_i(t)$. That is,

$$\frac{d\alpha_i}{dt} = - \frac{\partial f_2}{\partial \alpha_i} \tag{1.23}$$

In this equation, unlike before, there is no constant k on the right. This is due to the fact that it is now assumed to be part of the q_i's (i.e., the constants in $f_2(\epsilon)$).

Donalson and Leondes observed that it may be undesirable to require explicit knowledge of the α_i's. Apparently their reasoning for this was based on the fact that, in a particular application, the parameters of the compensation transfer functions are to be adjusted--not the parameters of the closed-loop system. Furthermore, if the α_i's are not known explicitly, then $\frac{\partial f_2}{\partial \alpha_i}$ cannot be computed. They, in effect, compute $\frac{\partial f_2}{\partial \alpha_i}$ in the following way: The error between system and model is given as

$$\epsilon(t) = c(t) - y(t). \tag{1.24}$$

The partial derivatives of $\epsilon(t)$ with respect to α_i and a_i are

$$\frac{\partial \epsilon}{\partial \alpha_i} = \frac{\partial c}{\partial \alpha_i} \tag{1.25}$$

and

$$\frac{\partial \epsilon}{\partial a_i} = - \frac{\partial y}{\partial a_i} \tag{1.26}$$

From assumption two, we know that the α_i's are going to closely track the a_i's. This implies, to a good approximation, that

$$\frac{\partial c}{\partial \alpha} = \frac{\partial y}{\partial a_i} \tag{1.27}$$

Then, from this comes

$$\frac{\partial f_2}{\partial \alpha_i} = - \frac{\partial f_2}{\partial a_i} \tag{1.28}$$

and the steepest descent equation becomes

$$\frac{d\alpha_i}{dt} = \frac{\partial f_2}{\partial a_i} \tag{1.29}$$

From here on, the development is almost identical with the one of Margolis and Leondes. It leads to an adjusting mechanism which is described by the following differential equations:

$$\frac{d\alpha_{id}}{dt} = - \left[\sum_{j=0}^{N} q_j \frac{d^j \epsilon}{dt^j} \right] \left[\sum_{j=0}^{N} q_j \frac{d^j u_{in}}{dt^j} \right] \tag{1.30}$$

$$\frac{d\alpha_{in}}{dt} = - \left[\sum_{j=0}^{N} q_j \frac{d^j \epsilon}{dt^j} \right] \left[\sum_{j=0}^{N} q_j \frac{d^j u_{in}}{dt^j} \right] \tag{1.31}$$

and

$$\sum_{i=0}^{N} a_{id} \frac{d^i u_{jd}}{dt} = - \frac{d^j y}{dt^j} \tag{1.32}$$

$$\sum_{i=0}^{N} a_{in} \frac{d^i u_{jn}}{dt^i} = \frac{d^j r}{dt^j} \qquad (1.33)$$

Donalson and Leondes did not carry the general development beyond this point. They indicated that the α_i's are functions of the parameters of the compensation transfer functions F(s), K(s) and H(s); but that these functions are not unique. Therefore, a general functional relationship doesn't exist. In applying this technique to the control of a flexible launch vehicle, one of the significant design problems would be the determination of the structure of the compensation transfer functions, and the time derivative of the compensation parameters from the steepest computations for the α_i's.

Because of the similarity of the two techniques, the comments relating to the Margolis and Leondes MRAC application to the flexible launch vehicle are valid for the Donalson and Leondes application. The present technique is slightly more general--due to the inclusion of dynamics in the numerator of the model transfer function. However, the advantage of a more general model is somewhat lost because its use introduces the need for the measurement of derivatives of the input signal. This may not always be practical.

Donalson and Leondes were one of the first to apply Lyapunov theory to the stability analysis of adaptive

control systems and they did achieve some interesting results using this theory.

2. M.I.T. Developments in M.R.A.C.

One of the earliest MRAC systems was developed at M.I.T., in 1958, by Whitaker, Yamron and Kezer [13]. This technique made use of an explicit model, and the object in adapting was to vary compensation parameters so that the closed loop system dynamics would match the model dynamics. In 1961, Osburn [14] presented a detailed analysis of the MRAC technique developed by Whitaker, et. al.

The basic approach used in the MRAC developments at M.I.T. is as follows:

If the error between the plant and model output is e(t), then the objective of the adaptive controller is to minimize F(e) by making use of a method suggested by steepest descent where

$$F(e) = \frac{1}{2}\int_0^t e^2(t)dt \qquad (2.01)$$

If one of the system adaptive parameters is ρ, then the steepest descent method suggests changing ρ in accordance with the expression

$$\Delta\rho = -k\frac{\partial F}{\partial \rho} = -k\int_0^t \frac{\partial e}{\partial \rho}\, e\,dt \qquad (2.02)$$

where k is a constant. It can be shown that the quantity $(\partial e/\partial \rho)$ is obtained by passing e(t) through a filter which, in the frequency domain, represents the reciprocal of the transfer function between ρ and e. The filter that is

generally used is a fixed approximation to the exact one. From the above equation, it follows that the time rate-of-change of the adaptive parameter satisfies the relation

$$\dot{\rho} = -k \frac{\partial e}{\partial \rho} e \qquad (2.03)$$

Of course there is a different mechanization of this equation for each parameter.

Also in 1961, Kezer, Hoffman and Engel [15] developed a modification of the MIT technique for the control of a flexible missile. This work is the most germaine to the objectives of the current study and it is discussed in the following paragraphs.

The Implicit Model Reference Technique of Kezer, Hoffman and Engel

With the model reference technique developed by Whitaker, et al. [13], the desired response is obtained from an explicit model which is forced by the same control signal that forces the plant. With the implicit model reference technique [15] system parameters are again adapted to minimize a functional of the response error. However, in this case the response error is not generated by subtracting the response of an explicit model from that of the plant. Instead, the response error in the implicit model reference technique is obtained from the plant itself as described below.

Applying the model reference technique to a flexible missile (plant) and assuming that the model is an exact representation of the rigid body missile, the response error will consist only of body bending and will have the form

$$\varepsilon = \sum_{i=1}^{\infty} q_i \eta_i \tag{2.04}$$

By locating rate gyros at different positions within the vehicle, a response of the form

$$r(t) = \sum_{i=1}^{\infty} (Y'_{\phi i} - Y'_{\dot{\phi} i})\eta_i \tag{2.05}$$

may be generated. This is an implicit model reference technique which eliminates the necessity for the rigid body model, and hence for the requirement that the model be an <u>exact</u> representation of the rigid body vehicle.

Assuming that the gains associated with bending modes of order greater than the first are negligible, the response error may be approximated by

$$\varepsilon = \eta_1 .$$

If the controllable parameters, P_j, are adjusted to minimize the functional

$$J = \int_0^t \varepsilon^2(t)\,dt \tag{2.06}$$

then the first variation of J vanishes at the minimum; i.e.,

$$\delta J = \sum_{j=I}^{R} \frac{\partial}{\partial P_j}\left\{\int_o^t \varepsilon^2 dt\right\}\delta P_j = 0 \qquad (2.07)$$

For the case where there is a single controllable parameter P and where the limits of integration are independent of parameter variations, equation 2.07 reduces to

$$\delta J = \int_o^t \frac{\partial}{\partial P}\left\{\varepsilon^2\right\}\delta P dt \qquad (2.08)$$

$$= 2\int_o^t \left(\frac{\partial \varepsilon}{\partial P}\right) \varepsilon \, \delta P dt = 0 \qquad (2.09)$$

which implies that

$$\int_o^t \frac{\partial \varepsilon}{\partial P} \varepsilon \, dt = 0 \qquad (2.10)$$

at the minimum. Consideration of the notion of steepest descent leads immediately to a parameter adjustment mechanism in which the rate of parameter variation is made proportional to the integrand of the first variation; i.e., to

$$\frac{dP}{dt} = -k \frac{\partial \varepsilon}{\partial P} \varepsilon \qquad (2.11)$$

Note that this is the same parameter adjustment formula that was obtained by Donalson and Leondes (6) using a modified steepest descent development (See equation 1.23).

Figure (2-1) is a block diagram representation of an implicit model reference autopilot with a single controllable parameter, λ_e, for bending suppression. Assuming that higher order bending modes are negligible, the response error becomes

$$\epsilon = (Y'_{\phi i} - Y'_{\dot{\phi} i})\eta_1 \tag{2.12}$$

and ϵ can be obtained by integrating the difference of the signals obtained from the rate gyros. It can be shown that the weighting function W(t), where

$$W = \partial\eta_1/\partial\lambda_e \tag{2.13}$$

can be obtained as the response of a weighting function filter with input

$$= (Y'_{\phi i} - Y'_{\dot{\phi} i})\eta_1 \tag{2.14}$$

The rate of change of the parameter is given by Equation (2.11) which may be rewritten in the form

$$\frac{d\lambda_e}{dt} = -kW\eta_1 \tag{2.15}$$

When vehicles with higher order bending modes must be considered, the previous analysis still applies with the following changes. If n bending modes must be considered, the response error may be approximated by

$$\epsilon = \sum_{i=1}^{n} q_i\eta_i \tag{2.16}$$

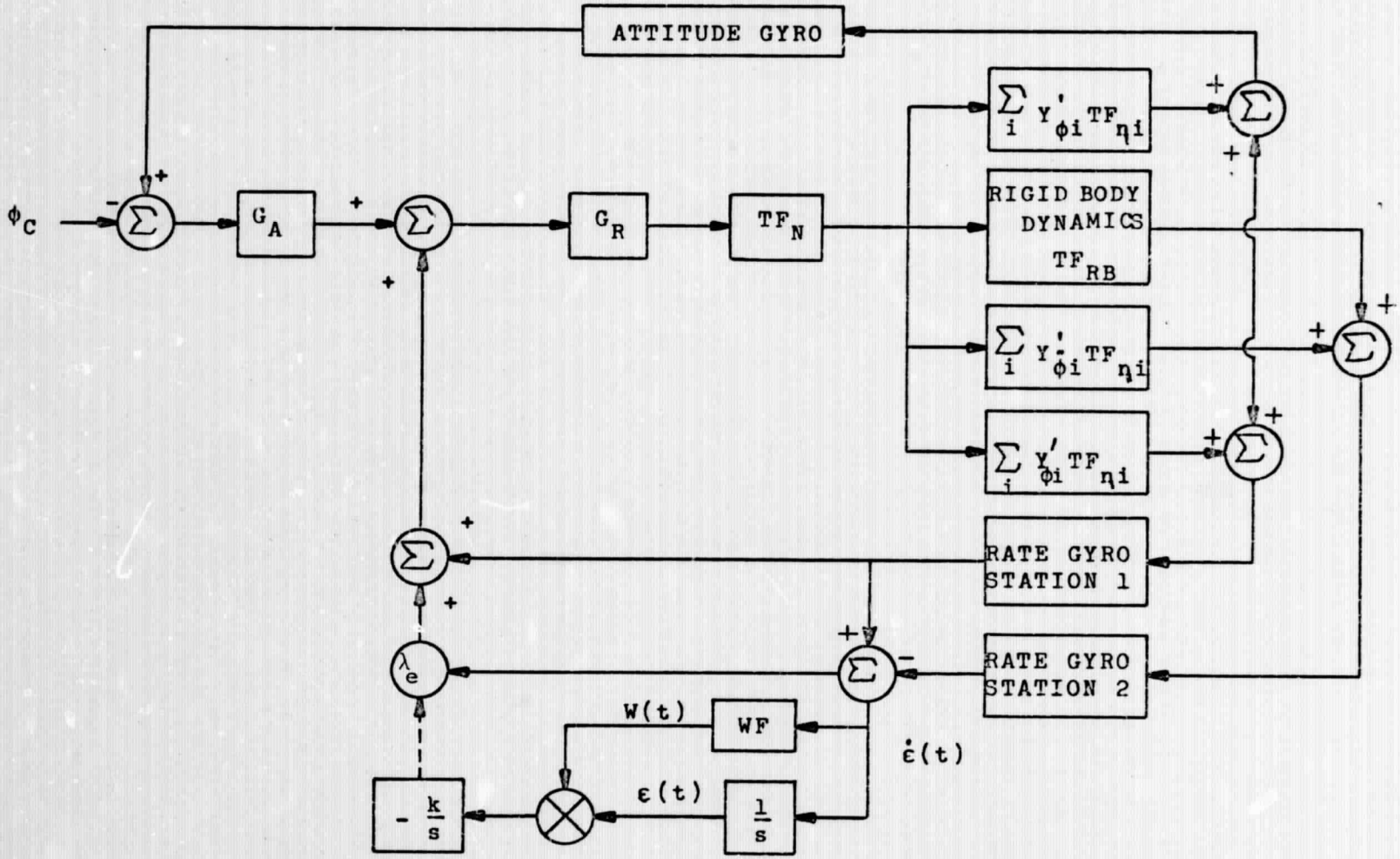


FIGURE 2-1 VEHICLE WITH ADAPTIVE AUTOPILOT

Minimization of

$$J = \int_0^t \varepsilon^2 dt \tag{2.17}$$

by variation of λ_e then requires that

$$\delta J = \frac{\partial}{\partial \lambda} \left\{ \left[\int_0^t \varepsilon^2 dt \right] \delta \lambda_e \right. \tag{2.18}$$

$$= 2 \int_0^t \frac{\partial \varepsilon}{\partial \lambda_e} \varepsilon \delta \lambda_e \, dt = 0 \tag{2.19}$$

Using the same logic as before, a corresponding expression for the time rate-of-change of the adaptive parameter is obtained; i.e.,

$$\frac{d\lambda_e}{dt} = -k \, w'/\varepsilon \tag{2.20}$$

where the weighting function, w' is given by

$$w' = \frac{\partial}{\partial \lambda_e} \left\{ \sum_{i=1}^{n} q_i r_i \right\} \tag{2.21}$$

It may be shown analytically that for the case of a single bending mode, the transfer function WF of the weighting function filter W' is given by

$$WF = TF_{[\beta, y_1]} (Y'_{\phi_i} - Y'_{\dot{\phi}_i}) TF_{\eta_1} \tag{2.22}$$

Equation (2.22) implies that the weighting function filter (for the case of a single bending mode) has order m+2, where m is the order of the rigid body autopilot. For the case where N bending modes must be considered the weighting function filter is given by

$$WF = TF_{[\beta, y_1]} \sum_{i=1}^{N} (Y'_{\phi_i} - Y'_{\dot{\phi}_i}) TF_{\eta_i} \tag{2.23}$$

which is of order m+2N. The inclusion of additional bending modes necessitates the use of higher order weighting function filters whose realization may exceed the capacity of the flight computer.

The previous analysis is based on the assumption that a signal

$$\varepsilon = \sum_{i=1}^{M} q_i \eta_i \tag{2.24}$$

may be obtained by integrating the difference signal obtained from the rate gyros. The signal actually obtained is of the form

$$\epsilon = \phi_1 - \phi_2 + \sum_{i=1}^{n} Y'_{\phi_i} \eta_i - \sum_{i=1}^{n} Y'_{\dot{\phi}_i} \eta_i \tag{2.25}$$

where ϕ_1 and ϕ_2 are the integrated rigid body rates from the two rate gyros. Equation (2.25) reduces to

$$\epsilon = \sum_{i=1}^{n} (Y'_{\phi_i} - Y'_{\dot{\phi}_i})\eta_i = \sum_{i=1}^{n} q_i \eta_i \qquad (2.26)$$

only if the dynamics of the rate gyros are identical in the region of dynamic response of interest.

3. Adaptive Control Techniques Based on Use of a Kalman Filter

A much more recently developed technique which makes use of a model of the plant consists of the use of a Kalman filter for combined estimation of the state and unknown parameters of a plant and use of the resulting estimates of the plant parameters to vary the control law in an appropriate way. Because the variation of the control law is straightforward, only the combined estimation of states and parameters will be reviewed here.

Combined State and Parameter Estimation via a Kalman Filter

The use of a Kalman filter to estimate an augmented state vector is discussed in this section. The augmented state vector consists of the actual linear plant states plus the parameters arranged as a vector. This idea was originally proposed in 1963 by Kopp and Orford [16] where they reported results for a second order continuous system with two parameters. The general equations for discrete and continuous systems were stated (although with conceptual errors) by Kumar in 1964[17].

This technique was extended and then applied to a launch vehicle by McDonnell-Douglas for NASA-MSFC in 1966[18]. The extension was to allow the parameter vector to consist only of selected plant parameters instead of all of them. Even before the technique was used in the latter application, it was realized that the approach was still impractical

in this modified form. This is because a matrix riccati differential equation had to be solved on-line, which causes the order of the resulting system to be $N(N + 3)/2$, where N is the number of plant states and unknown parameters. For example, to estimate two parameters of a third order plant requires the real-time solution of 20 nonlinear differential equations. Adding one more state raises the number of equations to 27, which illustrates the rapid growth of complexity for such a method.

A statement of the continuous equations will be given to clarify the above statements and for reference of other sections to follow.

Equations of Combined Estimation

A linear system in state space form is written as

$$\dot{x} = Ax + Bu + w$$
$$y = Hx + v \qquad (3.01)$$

where,

x = state (n_x-dimensional)

u = control (n_u-dimensional)

w = mean zero white gaussian plant noise (n_w-dimensional)

y = observation (n_y-dimensional)

v = mean zero white gaussian observation noise (n_v-dimensional) and, A, B, and H are appropriately dimensioned matrices.

Assume that there are unknown parameters that are denoted by a vector a, which is n_a-dimensional, whose rate varies according to a zero mean white gaussian parameter noise, w_a, i.e.,

$$\dot{a} = w_a \tag{3.02}$$

Then, define an augmented state vector, $\underline{x}$, by

$$\underline{x} = \begin{pmatrix} x \\ a \end{pmatrix} \tag{3.03}$$

and using Equations (3.01) and (3.02) we have

$$\dot{\underline{x}} = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix} \underline{x} + \begin{pmatrix} B \\ 0 \end{pmatrix} u + \begin{pmatrix} w \\ w_a \end{pmatrix} \tag{3.04}$$

which may be written concisely as

$$\dot{\underline{x}} = \underline{A}\,\underline{x} + \underline{B}u + \underline{w} \tag{3.05}$$

$$= f(\underline{x},u) + \underline{w} \tag{3.06}$$

We will denote the estimate (conditional expected value) of a variable $(\cdot)$ by $(\hat{\cdot})$ and the estimation error, $(\cdot) - (\hat{\cdot})$, by $(\bar{\cdot})$. It then turns out that, neglecting products of estimation errors, the form of the estimation equation for $\underline{x}$ is that of a Kalman filter. That is

$$\dot{\hat{\underline{x}}} = \hat{\underline{A}}\,\hat{\underline{x}} + \hat{\underline{B}}u + \underline{K}(y - \hat{y}) \tag{3.07}$$

where

$$\hat{y} = \underline{H}\,\hat{\underline{x}} \tag{3.08}$$

and

$$\underline{K} = \underline{P}\,\underline{H}'\,R^{-1} \tag{3.09}$$

and where in this last equation,

R = observation noise covariance coefficient matrix, i.e.,
$\mathrm{Cov}(v(t), v(\tau)) = R(t)\delta(t - \tau)$,

H = [H,0], 0 being a matrix, and

$\underline{P}$ = estimation error covariance of the (linearized) augmented state.

The latter matrix is the solution of the following matrix riccati equation:

$$\dot{\underline{P}} = \underline{A}^{*}\underline{P} + \underline{P}\,\underline{A}^{*\prime} - \underline{P}\,\underline{H}'\,R^{-1}\,\underline{H}\,\underline{P} + \underline{Q} \tag{3.10}$$

$$A^{*} = \left[\frac{\partial f_i(\underline{x}, u)}{\partial \underline{x}_j}\right]_{\underline{x}=\hat{\underline{x}}} \tag{3.11}$$

and

$\underline{Q}$ = covariance coefficient of $\underline{w}$.

Note that it turns out in this formulation that the "A-matrix" in the riccati equation is not the same "A-matrix" which appears explicitly in the augmented state equation. This is different than the usual Kalman filter. For example, consider the first order system

$$\dot{x} = a_1 x + bu \tag{3.12}$$

with a_1 being the unknown parameter. Then

$$\underline{A} = \begin{bmatrix} a_1 & 0 \\ 0 & 0 \end{bmatrix} \tag{3.13}$$

whereas

$$\underline{A}^* = \begin{bmatrix} a_1 & \hat{x} \\ 0 & 0 \end{bmatrix} \tag{3.14}$$

Since $\hat{x}$ appears in $\underline{A}^*$, the riccati equation will not generally settle to an equilibrium value.

Also note that the present problem is really a nonlinear problem which has been linearized to obtain a result like a Kalman filter.

Some Observations of the Combined Estimation Equations

We will now make some observations of the previous equations which will allow a better understanding of them and will lead to the modified approach to combined estimation described in Section V. In addition, in their present form a general computer program for their solution involves many needless multiplications and additions with zero.

First, by inspection, equation (3.08) can be written as

$$\hat{y} = H\hat{x}$$

Next, writing P as

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \tag{3.15}$$

We can write (3.09) as

$$\underline{K} = \begin{bmatrix} P_{11} & P_{21}' \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} H' \\ 0 \end{bmatrix} R^{-1} = \begin{bmatrix} P_{11} & H' \\ P_{21} & H' \end{bmatrix} R^{-1} \tag{3.09'}$$

Now, inspection of Equation (3.09') shows that we can easily distinguish two gains, one associated with the state estimation and one associated with parameter estimation. They are, respectively,

$$K^x = P_{11} H' R^{-1} \tag{3.09'a}$$

and

$$K^a = P_{21} H' R^{-1} \tag{3.09'b}$$

Then, equation (3.07) may be rewritten as

$$\dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}u + K^x(y - \hat{y}) \tag{3.07a}$$

and

$$\dot{\hat{a}} = K^a(y - \hat{y}) \ . \tag{3.07b}$$

With the exception of the equation for the Kalman gain and the fact that estimates of the plant matrices are used instead of their actual values (which are unknown in this case), the first equation is an ordinary Kalman state estimator. Further discussion of these equations will be given in the next section. The form inspires a practical approach to combined estimation.

The form of $f(\underline{x},u)$ is such that $\underline{A}^*$ can always be written as

$$\underline{A}^* = \begin{bmatrix} \hat{A} & \hat{M} \\ 0 & 0 \end{bmatrix} \tag{3.16}$$

where

$$M = \left[\frac{\partial f_i}{\partial a_j}\right] \tag{3.17}$$

Hence, a little algebra shows that the riccati equation in (3.10) may be written equivalently as (dropping circumflex notation on A and M)

$$\dot{P}_{11} = AP_{11} + P_{11}A' - P_{11}NP_{11} + Q_{11} + MP_{21} + P_{21}'M' \tag{3.10a}$$

$$\dot{P}_{21} = P_{21}(A' - NP_{11}) + P_{22}M' \tag{3.10b}$$

$$\dot{P}_{22} = -P_{21}NP_{21}' + Q_{22} \tag{3.10c}$$

where

$$N = H'R^{-1}H$$

The first of these equations looks like the usual riccati equation for the Kalman state estimator with an added forcing function (the last two terms) which causes coupling with the second equation. The second equation is linear. The equilibrium solution for fixed M was obtained for these equations for a first order plant with one parameter. However, it was not without some subtle limit

arguments. Computer solution of equilibrium values for higher order systems could not be obtained for the equations in this form. The reason for attempting to obtain equilibrium solutions will be explained in a later section, where it will also be explained how the situation was remedied.

4. Other MRAC Techniques

Since the work of Whitaker and Margolis and Leondes, there have been a great number of new MRAC developments. These techniques are based on essentially the same objective. That is, to adapt or control an adjusting mechanism based on some sort of error between the model and system responses. The various MRAC techniques differ because they are based on very different approaches for designing the adjusting mechanisms. Some of these techniques will be discussed in the next few sections.

Parks Approach

In a good many of the MRAC developments (starting with Donalson and Leondes) Liapunov's second method was used to analyze system stability. This method, in general, requires a certain amount of ingenuity to arrive at an appropriate Liapunov function. A Liapunov function is a positive definite scalar function, $V(x)$, of the system state x, which has the property that its time derivative, $\dot{V}(x)$, along trajectories of the state is negative definite. $V(x)$ is somewhat analagous to the energy contained in the system; if the energy builds up with time (i.e. $\dot{V}(x) > 0$) then the system is unstable.

Unlike the majority of previous developments, Parks [19] considers the problem of stability from the outset.

He defines a particular Liapunov function, V(x), for each system under consideration, and then determines an adjusting mechanism such that $\dot{V}(x) < 0$ for a given input type (usually a constant).

Parks develops adaptive controllers for first and second order systems. In each case, he compares the design with an adaptive controller that is based on the "MIT" rule. He shows that, by using Liapunov theory as a basis of design, an adaptive controller can be developed which is always stable for a constant input. This is in contrast to the adaptive controllers developed by the MIT rule which have a significant region of instability.

In general, Park's approach leads to the requirement to measure derivatives of the errors between the model and system responses. However, he shows that if the plant transfer function is positive real, then the adjustment can be made stable for a constant input without measuring any of the error derivatives.

A disadvantage of Park's approach, with respect to its application to the control of a launch vehicle, is that it does not consider the effects of disturbances. Possibly the approach could be extended to eliminate this shortcoming. A possible major problem is associated with finding a suitable Liapunov function for a realistic set of process dynamics.

Dressler's Technique

A general approach to the synthesis of adaptive controllers has been developed by Dressler [20]. The approach has an advantage in that it is extremely simple to implement Its development is carried out in state variable notation and stays strictly in the time domain. The adaptive control system (i.e. plant plus adaptive controller) is described by

$$\dot{x}(t) = F_s(t)x(t) + D_s(t)u(t) \qquad (4.01)$$

$$c(t) = M(t)x(t), \qquad (4.02)$$

where x is the n-dimensional state vector, u is the q-dimensional control, and c is a scalar output. The matrices F_s, D_s and M are of appropriate dimensions. The reference model is described by a differential equation of the same order as the system:

$$\dot{y}(t) = F_D y(t) + D_D u(t) \qquad (4.03)$$

$$c_D(t) = My(t) \qquad (4.04)$$

The adaptive controller acts to minimize the error

$$e(t) = c(t) - c_D(t) \qquad (4.05)$$

by forcing the following relationship to hold:

$$[e(t + \Delta t) - e(t)]\, e(t) \leq 0. \qquad (4.06)$$

Here, Δt is some small increment of time. This is equivalent to

$$\dot{e}(t)e(t) \leq 0, \tag{4.07}$$

which would also be obtained from Park's approach by taking $\frac{1}{2} e^2(t)$ as a Liapunov function.

In the adaptive controller development, Dressler makes two basic assumptions:

Assumption 1: The model and plant matrices are sufficiently close together (i.e. $F_s \doteq F_D$ and $D_s \doteq D_D$), so that the error difference $[e(t + \Delta t) - e(t)]$ is a linear function of the differences between the corresponding elements of the model and plant matrices.

Assumption 2: The rates of change of the plant parameters, the system input and the model output are negligible compared with those of the adaptive parameters.

The development of the equations for the adaptive controller is rather long. However, the final result is not. The equations which describe the adaptive controller are:

$$f_{ij}(t) = - \alpha_{ij} y_j(t) e(t) \tag{4.08}$$

$$\dot{d}_{m\ell}(t) = - \beta_{m\ell} u_\ell(t) e(t) \tag{4.09}$$

Here $f_{ij}(t)$ and $d_{m\ell}(t)$ represent the controller gains in $F_s(t)$ and $D_s(t)$. The α_{ij} and $\beta_{m\ell}$ are constants, and $y_j(t)$ and $u_\ell(t)$ are elements of the model state vector and the input vector.

If we interpret the Laplace operator as a differential operator (i.e. $s \triangleq \frac{d}{dt}$), then the plant differential equation may be written as

$$\left(s^n + \sum_{k=0}^{n-1} a_k s^k\right) y(t) = \left(\sum_{j=0}^{m} b_j s^j\right) x(t). \quad (m < n) \tag{4.10}$$

As with most of the other developments, the model and plant have the same order. The differential equation for the model is

$$\left(s^n + \sum_{k=0}^{n-1} \alpha_k s^k\right) y_m(t) = \left(\sum_{j=0}^{m} \beta_j s^j\right) x(t). \tag{4.11}$$

In the equation error approach $y_m(t)$ is not observable. So, $y(t)$ is substituted for $y_m(t)$ in the model equation in order to develop the equation error,

$$\epsilon_E = \left(s^n + \sum_{k=0}^{n-1} \alpha_i s^k\right) y(t) - \left(\sum_{j=0}^{m} \beta_j s^j\right) x(t). \tag{4.12}$$

In most practical systems it is either not desirable or not possible to measure the n derivatives of y and the m derivatives of x. However, these measurements are necessary to compute the equation error ϵ_E. To circumvent this problem the following variables are defined:

$$y_k = L^{-1}\{M_k(s)\ L[y(t)]\} \tag{4.13}$$

$$x_j = L^{-1}\{M_j(s)\ L[x(t)]\} \tag{4.14}$$

where L is the Laplace operator. The estimating $M_k(s)$ is certainly not unique. One possible mechanization of the filter is

$$M_k(s) = \frac{s^k}{(\tau s+1)^n} \qquad (4.15)$$

If the equation error, ϵ_E, is divided by $(\tau s+1)^n$, a quantity results which Lion calls the "generalized" equation error, ϵ, and which is in terms of the variables defined above.

Dressler's technique has a distinct advantage over some of the earlier adaptive controllers in that the only additional dynamics required are those of the model. (That is, a separate weighting function for each adaptive parameter is not necessary.)

As with many of the other techniques, Liapunov's second method has been used to determine the stability boundaries for some example systems. Dressler does not consider disturbance effects, but their inclusion would not be too difficult. It would only be necessary to augment the state equations with the disturbance dynamics. This would lead naturally into the application of Kalman filter theory for the development of an estimator for the non-measurable states.

The Equation Error Technique by Lion

The MRAC techniques described above fall into the general category of "response error" systems. That is, the adaptive mechanization is based on an error signal which is the difference between the plant output and the model output,

when both the plant and model are driven by the same input.

"Equation error" systems operate on a different error signal. Figure 4-1 shows the two types of error systems for the situation in which the plant and model dynamics are represented by ratios of polynomials in the Laplace operator, s. The equation error approach appears to have certain advantages over the more classical approach in terms of enlarged stability regions and increased parameter tracking speed.

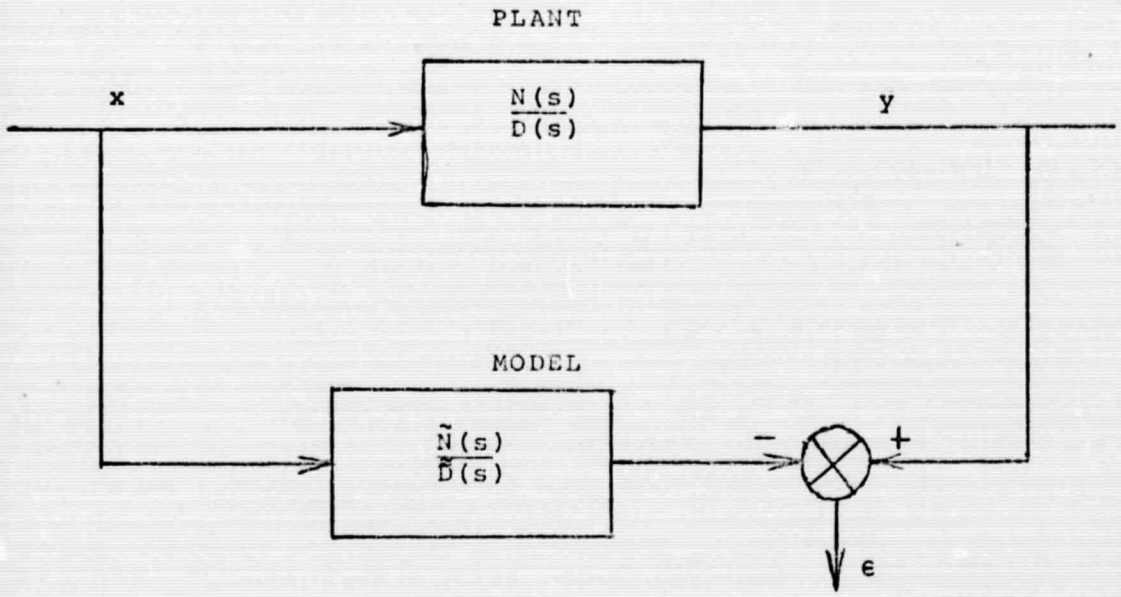
The equation error technique described by Lion [21] is presented in the next few paragraphs. Earlier work by Shipley, Engel and Hung [22] contained much of Lion's development, although it was on a somewhat ad hoc basis. Consider the equation

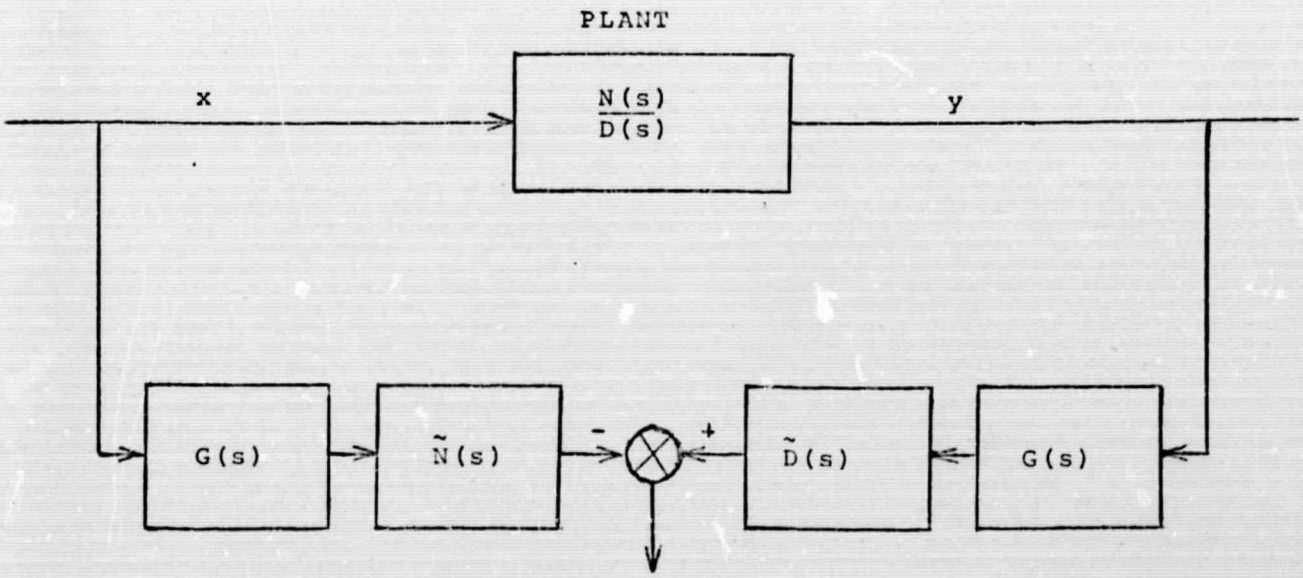
$$\epsilon = y_n + \sum_{k=0}^{n-1} \alpha_k y_k - \sum_{j=0}^{m} \beta_j x_m \qquad (4.16)$$

Having developed the expression for the equation error, it is now necessary to develop the adjusting mechanism that will drive the error to zero. Lion as well as Shipley, Engel and Hung took the simple error-squared performance criteria

$$F(\epsilon) = \frac{1}{2} \epsilon^2, \qquad (4.17)$$

and applied the technique of steepest descent to arrive at the parameter adjusting mechanization. The steepest descent approach leads to the following parameter adjustment rates





TWO TYPES OF "EQUATION ERROR" SYSTEMS
FIGURE 4-1

$$\dot{\alpha}_i = - k \frac{\partial F}{\partial \alpha_j} = -k\epsilon y_i \qquad (4.18)$$

$$\dot{\beta}_j = - k \frac{\partial F}{\partial \beta_j} = -k\epsilon x_j \qquad (4.19)$$

where k is the constant loop gain

Lion proved that the above parameter estimation scheme is globally, asymptotically stable for a wide class of system inputs including inputs composed of sinusoids of different frequency, equal in number to at least one-half the total number of parameters to be estimated. However, he did assume that none of the sinusoidal signals may have a phase shift of $k\pi$ radians (k an integer) through the plant. This is to insure enough independent conditions so that the model parameters will not only converge, but will converge to the plant parameters.

To maximize the speed of convergence it is necessary to optimize the loop gain k. A value that is too low will result in sluggish performance, while a high value will result in poorly damped performance. However, Lion has described an approach that effectively removes the upper limit on the convergence speed. To do this he defines $p = m + n - 1$ independent equation errors

$$\epsilon_1 = y_{n+i} + \sum_{k=0}^{n-1} \alpha_k y_{k+i} - \sum_{j=0}^{m} \beta_j x_{j+i} \quad 0 \leq i \leq p-1 \qquad (4.20)$$

Of course, for the y_i and x_i to be available, the order of the denominator pole in the estimating filter must be changed to give

$$M_k(s) = \frac{s^k}{(\tau s+1)^{n+p-1}} \tag{4.21}$$

Then, application of steepest descent to the performance criterion

$$F = \frac{1}{2} \sum_{i=0}^{p-1} \epsilon_i^2 \tag{4.22}$$

produces the parameter adjustment rates

$$\dot{\alpha}_i = - k \sum_{l=0}^{p-1} \epsilon_l y_{l+i} \tag{4.23}$$

$$\dot{\beta}_j = k \sum_{l=0}^{p-1} \epsilon_l x_{l+j} \tag{4.24}$$

Lion shows that the above approach can be made to converge arbitrarily fast simply by making the loop gain k high enough. Also, he shows that the same global, asymptotic stability holds for this more general development.

The application of the equation error technique for estimating the parameters of plants of any order is straight forward. However, the practical limitations on plant order are not presently known. For example, it is not certain what effects disturbances and sensor noise have on parameter

identification by this method in general. Shipley, Engel and Hung have indicated that disturbances for which some kind of measurement may be made available (such as an angle-of-attack measurement in the presence of disturbing winds for a flight control system) can be treated as additional system inputs by simply adding terms to the plant and model equations. In addition, Lion has noted that if some of the noise properties are known, then the estimating filter, $M_k(s)$, may be designed to minimize the effects of the noise.

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SECTION III

The choice of the two techniques of model reference adaptive control to be developed and evaluated in detail in the course of the present study was based on an extensive literature review, the results of which are summarized in Section II. This literature review indicated the striking similarity between the UCLA and the MIT work in this area and the eventual evolution of the UCLA and MIT approaches to an essentially common approach to the problem. Moreover, except for a small amount of work on response error techniques, this common approach came to be predominant among the approaches based on concepts from classical (pre-state space) control theory. In this predominant position, it was applied to a fairly wide class of control problems with relatively good success as far as could be determined from from the level of detail with which these application studies were carried out, and one of these applications was to the control of a flexible booster. The application to control of a flexible booster was so successful to the depth carried out that some knowledgeable people in the field of adaptive control felt that no further evaluation of it was required. Nevertheless, some important questions remained unanswered and it was felt that an in-depth evaluation was required to see if this approach really lived up to its promise as the best of the known classical approaches when applied to a

realistic model of a real booster such as SATURN V. Thus the technique developed at UCLA and MIT and applied in a general but insufficiently detailed way to flexible boosters by Whitaker and his associates was selected as one of the two techniques to be evaluated in depth for the SATURN V application during this study.

More recently, the general adaptive control problem was attacked from a more modern point of view using theory based on state-space methods and use of the Kalman filter in the identification process. Early experience with this approach indicated much promise but the mechanization problem was too difficult due to complexity of the resulting system. Since experience was beginning to verify the general feeling that the form of the Kalman filter was more important than the exact mechanization of the optimal gains, it was felt that exploration of such a technique using a simplified mechanization of the gains might very well lead to quite good overall performance. In any case, review of the literature indicated that this was the most promising of the approaches based on modern control concepts and its exploration in depth for the SATURN V should go a long way toward evaluating the potential of currently available modern control techniques for solving very difficult feedback control problems of the type encountered in the SATURN

V application.

Considering the experience to date there was good reason to expect this approach to do a good job on the tough SATURN V problem.

SECTION IV

1. INTRODUCTION

The problem of providing adequate stability margins for the flexible modes of booster vehicles has become more acute as a result of trends in vehicle structural design. As vehicle size increases, the flexible mode frequencies approach the frequency range of the control modes; this coupled with the uncertainties associated with vehicle parameters and the variation of parameters with flight conditions, tends to exceed the capability of conventional autopilots to provide adequate suppression of the flexible response.

Self-adaptive techniques have been shown to be effective in providing improved stability, and are not as susceptible to failure, in the face of parameter variations and uncertainties, as are conventional autopilots. Several self-adaptive techniques have been investigated to determine their effectiveness in suppressing the flexible response in booster vehicles and aircraft. The techniques investigated include: rate gyro blending, tracking notch filters, multiple sensors, model reference techniques, and rigid body separation.[1,2,7,8,11,12]

This section examines the application of one such self-adaptive technique, the model reference technique, to the suppression of the flexible response in vehicles of the Saturn V AS-501 class.

2. THEORY OF MODEL REFERENCE ADAPTIVE CONTROL

2.1 Theory of Operation

Figure 2-1 is a functional representation of the basic model reference adaptive system.

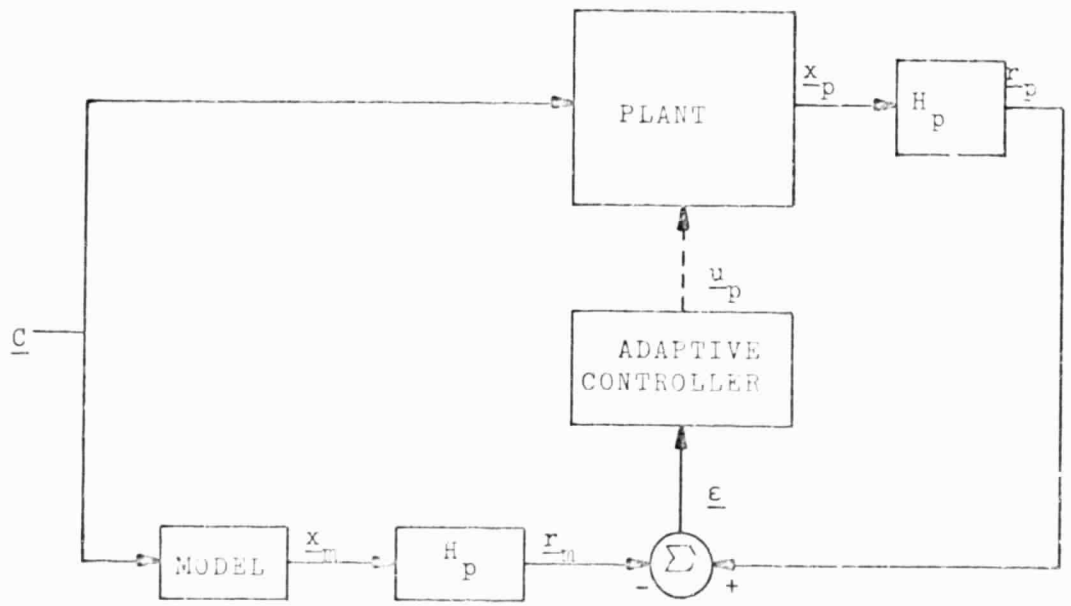


FIGURE 2-1 MODEL REFERENCE ADAPTIVE SYSTEM

The equations governing the plant and model are:

Plant Equations:

$$\dot{\underline{x}}_p = \underline{f}_p(\underline{x}_p, \underline{C}, \underline{P}, t) \tag{2.01}$$

Model Equations:

$$\dot{\underline{x}}_m = \underline{f}_m(\underline{x}_m, \underline{C}, t). \tag{2.02}$$

The system is comprised of a plant (or process to be controlled), a reference model which incorporates the desired plant characteristics, and an adaptive controller. The state vector of the plant $\underline{x}_p$ is dependent upon a set of controllable parameters

$$\underline{P} = \left[P_1 \ \ldots \ P_T \right]$$

as indicated by the plant equations (Eq.2.01). The objective of the adaptive process is to force $\underline{x}_p$ to conform to the equations set forth in the model; this is accomplished by adjusting the controllable parameters within the plant $\underline{P}$, so that the plant equations approximate those of the model (Eq. 2.02). To determine deviations of the plant from the nominal (as characterized by the model) the plant response $\underline{r}_p$ is compared to the response of the reference model $\underline{r}_m$ and an error response $\underline{\varepsilon}$, defined by

$$\underline{\varepsilon} \triangleq \underline{r}_p - \underline{r}_m \tag{2.03}$$

is obtained. The adaptive controller defines an adaptive policy

$$\underline{u}_P = \underline{u}_P(\underline{\varepsilon})$$

which determines the adjustment of the controllable parameters, and causes a predefined performance index

$$J(\underline{\varepsilon}) = \int_{-\infty}^{t} ||\underline{\varepsilon}||^2 \, dt \tag{2.04}$$

to be minimized (where $||\underline{\varepsilon}||$ denotes the Euclidean norm of $\underline{\varepsilon}$).

2.2 The Adaptive Policy

An adaptive policy which minimizes the performance index may be derived by use of steepest descent techniques. Consider a Euclidean space E_P, which forms the domain of the vector $\underline{P}$, and let s denote arclength in E_P, then

$$ds^2 = \langle d\underline{P}, d\underline{P} \rangle$$

and

$$1 = \left\langle \frac{d\underline{P}}{ds}, \frac{d\underline{P}}{ds} \right\rangle \tag{2.05}$$

(where $\langle \underline{n}, \underline{L} \rangle$ denotes the inner product of $\underline{n}$ and $\underline{L}$). The performance index as given by equation 2.04 may be regarded as a functional defined on E_P and hence

$$\frac{dJ}{ds} = \left\langle \nabla_{\underline{P}} J, \frac{d\underline{P}}{ds} \right\rangle \tag{2.06}$$

(where

$$\nabla_{\underline{P}} J \triangleq \text{grad}_{\underline{P}} J).$$

Introducing a Lagrange multiplier and combining eqs. 2.05 and 2.06

$$\frac{dJ}{ds} = \left\langle \nabla_{\underline{P}} J, \frac{d\underline{P}}{ds} \right\rangle + \lambda \left[1 - \left\langle \frac{d\underline{P}}{ds}, \frac{d\underline{P}}{ds} \right\rangle \right]. \qquad (2.07)$$

A necessary condition for dJ/ds to be maximized with respect to the directional cosines $d\underline{P}/ds$, is that

$$\nabla_{\frac{d\underline{P}}{ds}} \left\{ \frac{dJ}{ds} \right\} = 0 . \qquad (2.08)$$

Substitution of Eq. 2.07 into Eq. 2.08 yields

$$\frac{1}{\partial\lambda} \nabla_{\underline{P}} J = \frac{d\underline{P}}{ds} , \qquad (2.09)$$

premultiplying by $(d\underline{P}/ds)^T$

$$\frac{1}{\partial\lambda} \left\langle \frac{d\underline{P}}{ds} , \nabla_{\underline{P}} J \right\rangle = \left\langle \frac{d\underline{P}}{ds} , \frac{d\underline{P}}{ds} \right\rangle$$

$$= 1 \qquad (2.10)$$

and from Eq. 2.09

$$\frac{1}{\partial\lambda} \left\langle \frac{1}{\partial\lambda} \nabla_{\underline{P}} J, \nabla_{\underline{P}} J \right\rangle = 1 \qquad (2.11)$$

Solving for the Lagrange multiplier λ, and substituting into Eq. 2.09, the necessary condition for maximum (or steepest) descent becomes

$$\frac{d\underline{P}}{ds} = - \nabla_{\underline{P}} J \left[\left\langle \nabla_{\underline{P}} J, \nabla_{\underline{P}} J \right\rangle \right]^{-1/2} . \tag{2.12}$$

Regarding $\underline{P}$ as a vector function of time t,

$$\dot{\underline{P}} = \frac{d\underline{P}}{ds} \frac{ds}{dt} \tag{2.13}$$

where the velocity is given by

$$\frac{ds}{dt} = \left[\left\langle \dot{\underline{P}} , \dot{\underline{P}} \right\rangle \right]^{1/2} , \tag{2.14}$$

now $\dot{\underline{P}}$ may be expressed as

$$\dot{\underline{P}} = - \left[\frac{\left\langle \dot{\underline{P}} , \dot{\underline{P}} \right\rangle}{\left\langle \nabla_{\underline{P}} J, \nabla_{\underline{P}} J \right\rangle} \right]^{1/2} \nabla_{\underline{P}} J. \tag{2.15}$$

If the velocity magnitude is assumed to be proportional to the magnitude of the gradient, Eq. 2.15 becomes

$$\dot{\underline{P}} = - k \nabla_{\underline{P}} J. \tag{2.16}$$

In terms of the performance index

$$J = \int_{\infty}^{t} ||\underline{\varepsilon}||^2 dt$$

$$= \int_{\infty}^{t} \underline{\varepsilon}^T . \underline{\varepsilon} \, dt$$

equation 2.16 becomes

$$\dot{\underline{P}} = - k \nabla_{\underline{P}} \left\{ \int_{\infty}^{t} \underline{\varepsilon}^T . \underline{\varepsilon} \, dt \right\} \tag{2.17}$$

as the limits of integration are not functions of $\underline{P}$

$$\dot{\underline{P}} = - k \int_{-\infty}^{t} \nabla_{\underline{P}} \left\{ \underline{\varepsilon}^{T} \cdot \underline{\varepsilon} \right\} dt \tag{2.18}$$

$$\dot{\underline{P}} = -2k \int_{-\infty}^{t} W(t) \underline{\varepsilon}(t) dt \tag{2.19}$$

where W(t) is a weighting function matrix:

$$W(t) = \left[w_{ij}(t) \right]$$

$$= \left[\frac{\partial \varepsilon_j(t)}{\partial P_i(t)} \right] \quad . \tag{2.20}$$

The elements of the weighting function matrix

$$w_{ij}(t) = \partial \varepsilon_j(t) \Big/ \partial P_i$$

are determined by taking the partial derivatives of the differential equations for response error as a function of the input quantities.

If the parameter adjustment mechanism is implemented by equating the rate of parameter adjustment to the adaptive signal

$$\dot{\underline{P}} = \underline{u}_{\underline{P}} \tag{2.21}$$

the required adaptive policy is:

$$\underline{u}_{\underline{P}} = -2k \int_{-\infty}^{t} W(t) \, \underline{\varepsilon}(t) dt \quad . \tag{2.22}$$

For the case of a system with a single controllable parameter P, and a scalar response error ε, the adaptive policy reduces to

$$\dot{u}_p = -2k \int_{-\infty}^{t} W(t)\varepsilon(t)dt \qquad (2.23)$$

with

$$W(t) = \frac{\partial \varepsilon(t)}{\partial P} \quad . \qquad (2.24)$$

3. REDUCTION OF FLEXIBLE RESPONSE IN BOOSTER VEHICLES

3.1 Application of Model Reference Techniques

The adaptive policy defined by Eqs. 2.23 and 2.24 (repeated below: Eqs. 3.01 and 3.02) acts to minimize the integral of the response error squared

$$u_p = -2k \int_{-\infty}^{t} W(t)\varepsilon(t)dt \qquad (3.01)$$

$$W(t) = \partial\varepsilon(t)/\partial P \quad . \qquad (3.02)$$

If the response error were a measure of the response of the flexible modes, represented by the function

$$\varepsilon = F(\eta_1, \eta_2 \ldots),$$

then u_p would act to minimize

$$J(\varepsilon) = \int_{-\infty}^{t} F^2 dt \; .$$

If the system response error is defined as a weighted sum of the responses of the flexible modes

$$\varepsilon \overset{\Delta}{=} \sum_i \alpha_i \eta_i \qquad (3.03)$$

the adaptive policy becomes

$$u_p = -2k \int_{-\infty}^{t} W(t) \sum_i \alpha_i \eta_i dt \qquad (3.04)$$

with weighting function

$$W(t) = \frac{\partial}{\partial P} \left\{ \sum_i \alpha_i \eta_i dt \right\} \; . \qquad (3.05)$$

Eqs. 3.04 and 3.05 completely define the adaptive policy.

By locating the vehicle rate gyro in the instrument unit station, the plant response rate, as sensed by the rate gyro, is

$$\dot{r}_P = \dot{\phi} + \sum_i y'_{\phi i}\, \dot{\eta}_i \,.$$

If the model incorporates the vehicle dynamics with the rate gyro located in the rate gyro station, the model response rate is

$$\dot{r}_m = \dot{\phi} + \sum_i y'_{\dot{\phi} i}\, \dot{\eta}_i \quad ,$$

and the response error is then

$$\varepsilon(t) = \sum_i (y'_{\phi i} - y'_{\dot{\phi} i})\eta_i \,.$$

However in applying the model reference adaptive technique to the flexible booster problem it is possible to generate the response error without including a model in the system.

By locating two rate gyros at separate stations within the vehicle, and differencing their outputs, the rigid body information is cancelled and the resulting flexible response rate signal is

$$\gamma = \sum_i (y'_{\phi i} - y'_{\dot{\phi} i})\dot{\eta}_i \,. \qquad (3.06)$$

Assuming that parameter variations are slow relative to fluctuations in system response, the response error may be generated by integrating the flexible response rate signal γ, and equating the α_i of Eq. 3.03 to the weighting

terms of Eq. 3.06,

$$\alpha_i = y'_{\phi i} - y'_{\dot{\phi} i} \tag{3.07}$$

$$\varepsilon = \sum_i (y'_{\phi i} - y'_{\dot{\phi} i})\eta_i \ . \tag{3.08}$$

The weighting function

$$W(t) = \partial\varepsilon(t)/\partial P \tag{3.09}$$

is obtained by taking the partial derivative of the differential equations for $\varepsilon(t)$ as a function of the input quantities. If the resulting system of differential equations is linear, the weighting function may be expressed in terms of a convolution integral

$$W(t) = \int_{-\infty}^{t} WF_{\phi C}(t,\tau)\phi_C(\tau)d\tau. \tag{3.10}$$

Expressing the input quantities in terms of $\dot{\varepsilon}(t)$, Eq. 3.10 becomes

$$W(t) = \int_{-\infty}^{t} WF(t,\tau)\dot{\varepsilon}(\tau)d\tau \tag{3.11}$$

which implies that the weighting function may be obtained as the response of a weighting filter with input $\dot{\varepsilon}(t)$.

Since the weighting function and response error may be generated by operating on signals obtained from the plant (vehicle) itself, it is not necessary to include a reference model in the system. Figure 3-1 is a functional representation of the application of the aforementioned

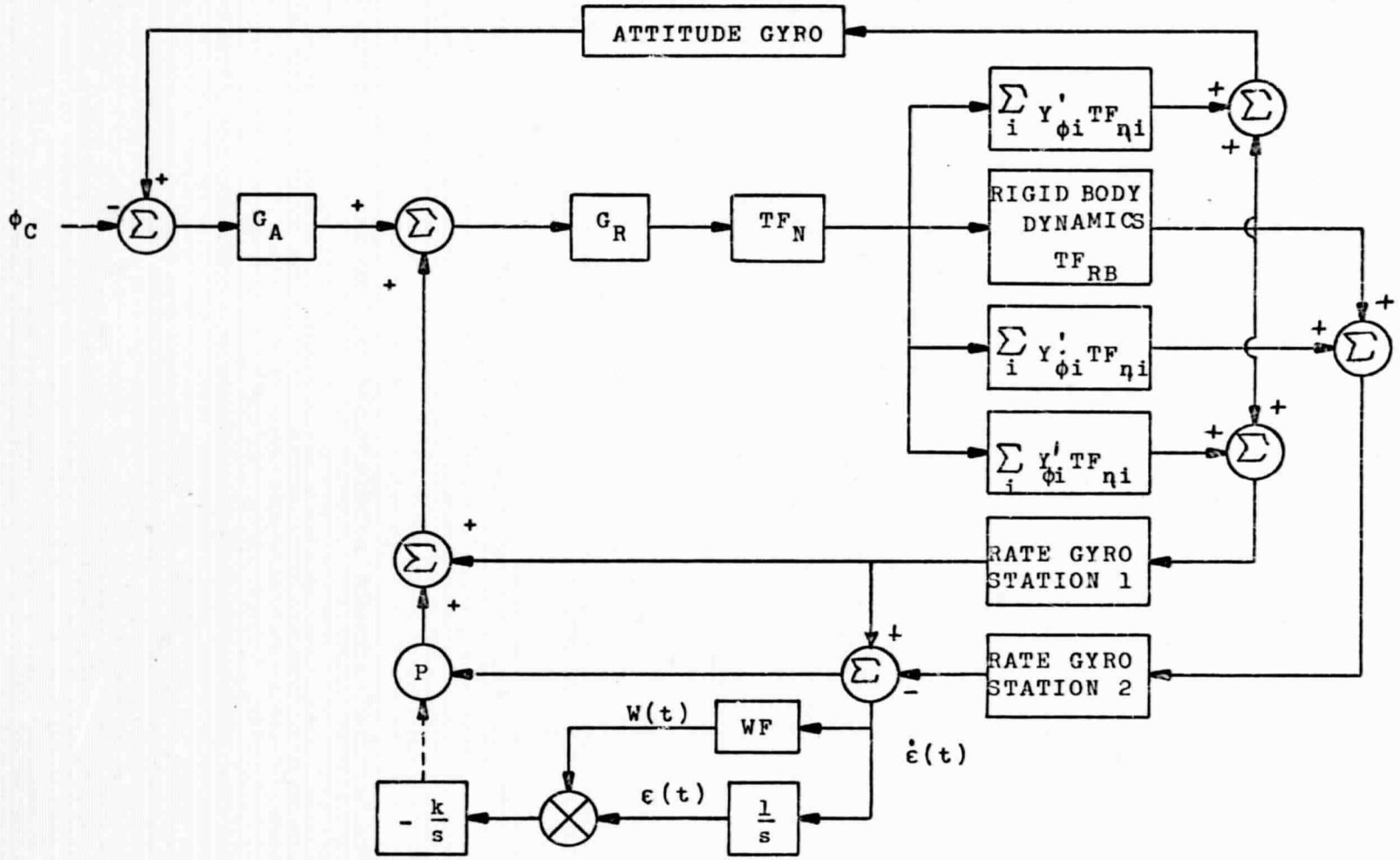


FIGURE 3-1 VEHICLE WITH ADAPTIVE AUTOPILOT

model reference technique to a flexible booster vehicle.

3.2 Derivation of the Weighting Filter

The differential equations relating response error as a function of the input quantities may be obtained by analysis of the functional representation of the system. In terms of the transfer functions of Figure 3-1,

$$\varepsilon(s) = - G_A\, G_R\, TF_N \Big(\sum_i (y'_{\phi i} - y'_{\dot{\phi} i}) TF_{\eta i} \Big) \phi_C(s) \Big/ \Big(1 - G_R\, TF_N \Big[(S + G_A)(TF_{RB} + \sum_i y'_{\phi i}\, TF_{\eta i}) + sP \sum_i (y'_{\phi i} - y'_{\dot{\phi} i}) TF_{\eta i} \Big] \Big) . \qquad (3.12)$$

The weighting function is obtained by taking the partial derivative of the response error with respect to P.

$$W(s) = \partial \varepsilon(s) / \partial P \qquad (3.13)$$

$$= - sG_A \Big(G_R\, TF_N \sum_i (y'_{\phi i} - y'_{\dot{\phi} i}) TF_{\eta i} \Big)^2 \phi_C(s) \Big/ \Big(1 - G_R\, TF_N \Big[(s + G_A)(TF_{RB} + \sum_i y'_{\phi i} TF_{\eta i}) + sP \sum_i (y'_{\phi i} - y'_{\dot{\phi} i}) TF_{\eta i} \Big] \Big)^2 \qquad (3.14)$$

Expressed in terms of the response error, the weighting function becomes

$$W(s) = G_R\ TF_N \sum_i (y'_{\phi i} - y'_{\dot{\phi} i}) TF_{\eta i} \varepsilon(s) /$$

$$\left(1 - G_R\ TF_N \left[(s + G_A)(TF_{RB} + \sum_i y'_{\phi i} TF_{\eta i}\right)\right.$$

$$\left. + sP \sum_i (y'_{\phi i} - y'_{\dot{\phi} i}) TF_{\eta i}\right]\Big) \qquad (3.15)$$

which implies that the weighting filter of Eq. 3.11 may be expressed in terms of the transfer functions as

$$WF(s) = G_R\ TF_N \sum_i (y'_{\phi i} - y'_{\dot{\phi} i}) TF_{\eta i} /$$

$$\left(1 - G_R\ TF_N \left[(s + G_A)(TF_{RB} + \sum_i y'_{\phi i} TF_{\eta i}\right.\right.$$

$$\left.\left. + sP \sum_i (y'_{\phi i} - y'_{\dot{\phi} i}) TF_{\eta i}\right]\right). \qquad (3.16)$$

4. APPLICATION TO THE SATURN V AS-501

4.1 Purpose and Scope

In order to evaluate the effectiveness of the model reference technique in reducing flexible response, the technique was applied to the Saturn V AS-501 launch vehicle. For purposes of the study, it was assumed that the vehicle was characterized by two flexible modes, third order actuator dynamics, and third order rigid body dynamics. The Saturn V AS-501 vehicle equations and data as supplied by Marshall Space Flight Center are listed in appendices A and B.

The system comprised of the vehicle, conventional compensation networks, and the adaptive mechanism was subjected to analysis under flight conditions corresponding to: lift-off, maximum dynamic pressure, and burnout. The analysis and digital computer simulations were used to evaluate the effects of parameter adjustment on the stability margins and on flexible response.

4.2 System Design

The design objective was to provide compensation, both conventional and adaptive, which would guarantee adequate stability margins for all modes of the system and minimize the response of the flexible modes in the face of parameter uncertainties and variations.

To accomplish the design objective the autopilot was designed according to the following design principles:

1. The conventional compensation should provide adequate stability margins for the rigid body and flexible modes under conditions of lift-off, maximum dynamic pressure, and burnout.[10]
2. The adaptive compensation should act to maintain or improve the stability margins associated with the flexible modes, and suppress the flexible response.

4.3 Conventional Compensation

To assure the stability of the vehicle under all flight conditions, conventional compensation was designed to provide a minimum of 6db. rigid body gain margin, 30° phase margin, 6db. gain margin of the peak amplitude for those flexible modes that are gain stabilized, and 45° phase margins for those flexible modes that are phase stabilized.[4] Figure 4-1 is a functional representation of the vehicle with autopilot, incorporating both conventional and adaptive compensation. Attitude and attitude rate are sensed by rate-integrating and rate gyros located in the vehicle instrument unit.

In the rate loop the autopilot employs phase lag compensation to stabilize the rigid body and a notch filter to stabilize the flexible response. Phase lead

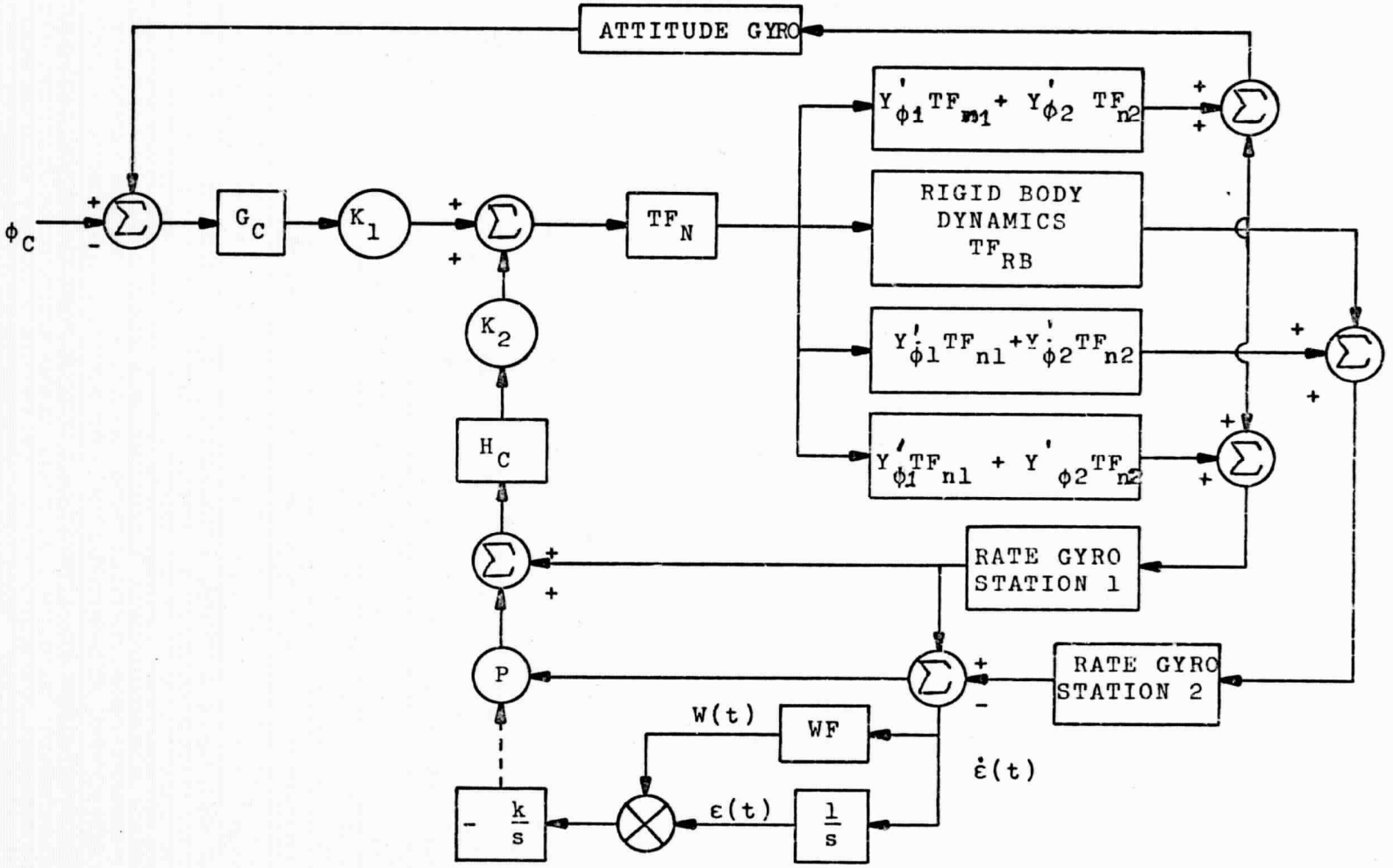


FIGURE 4-1 SATURN V AS-501 WITH ADAPTIVE AUTOPILOT

compensation is included in the attitude loop to assure the appropriate gain and phase margins. Figure 4-2 is a functional representation of the conventional compensation network. The phase vs gain characteristics of the conventionally compensated system are shown in Figures 4-3 thru 4-5.

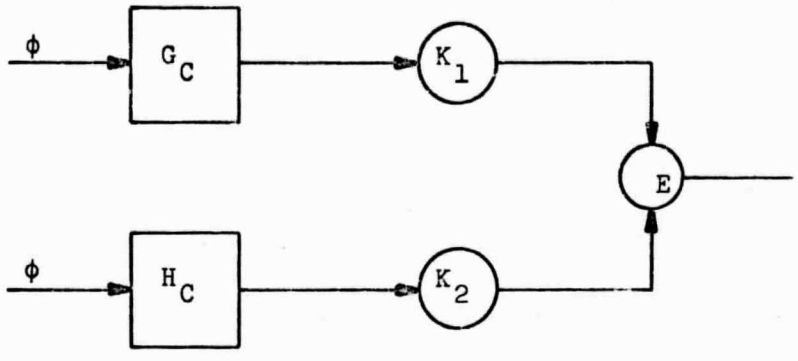
4.4 The Weighting Filter

In terms of the transfer functions of Figure 4-1 the weighting filter may be expressed as

$$WF(s) = K_2 H_C TF_N\left[(y'_{\phi i} - y'_{\dot{\phi}1})TF_{\eta 1} + (y'_{\phi 2} + y'_{\dot{\phi}2})TF_{\eta 2}\right] \Big/ \Big(1 - TF_N\left[(K_1 G_C + K_2 sH_C)(TF_{RB} + y'_{\phi 1}TF_{\eta 1} + y'_{\dot{\phi}2}TF_{\eta 2})\right.$$

$$\left. + PK_2 sH_C\left[(y'_{\phi 1} - y'_{\dot{\phi}1})TF_{\eta 1} + (y'_{\phi 2} - y'_{\dot{\phi}2})TF_{\eta 2}\right]\right). \qquad (4.01)$$

Figures 4-6, 4-7, and 4-8 are pole-zero plots of an approximation to the weighting filter, (with P = 0.0) at times corresponding to lift-off, maximum dynamic pressure, and burnout, respectively. The approximate weighting filter contains the dominant poles and zeros of the weighting filter and in addition contains all poles of the weighting filter whose variations are large with respect to variations in P.



$$G_C = \frac{s + 1.1}{s + 80.0}$$

$$H_C = \frac{(s + 3)(s^2 + s + 49.25)}{(s + .022)(s + 40)(s + 60)}$$

$$K_1 = .125$$

$$K_2 = 5.0$$

FIGURE 4-2

SATURN V AS-501 CONVENTIONAL COMPENSATION

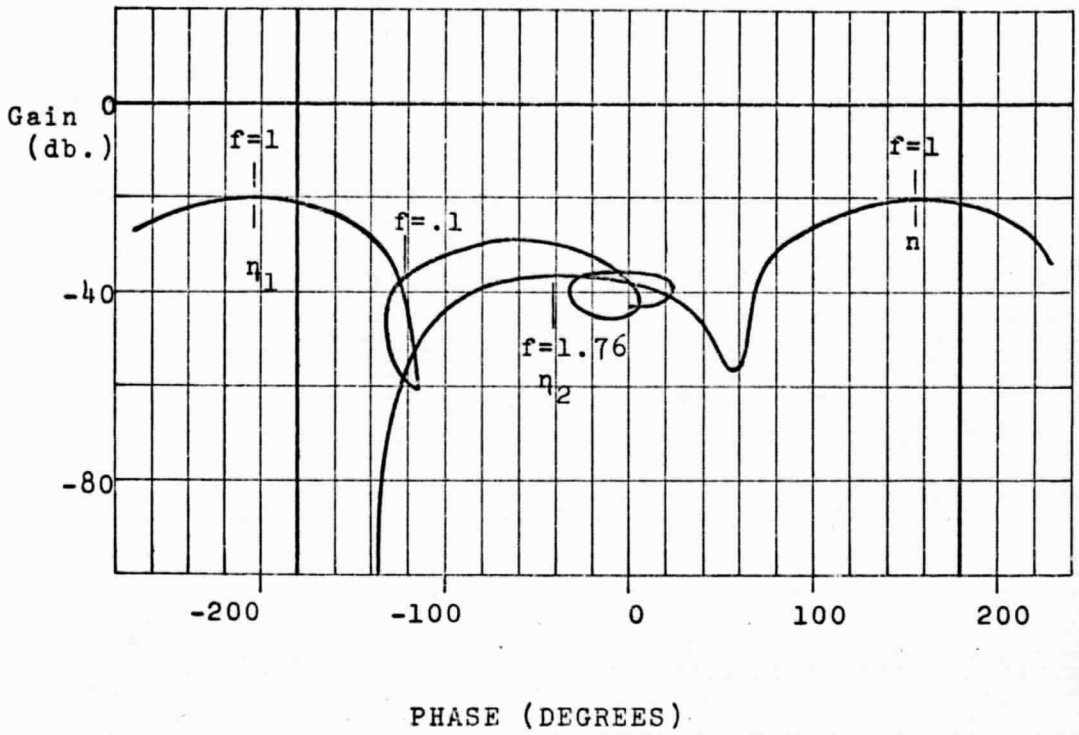


FIGURE 4-3

PHASE VS GAIN CHARACTERISTICS
OF CONVENTIONALLY COMPENSATED SATURN V AS-501 AT LIFT-OFF

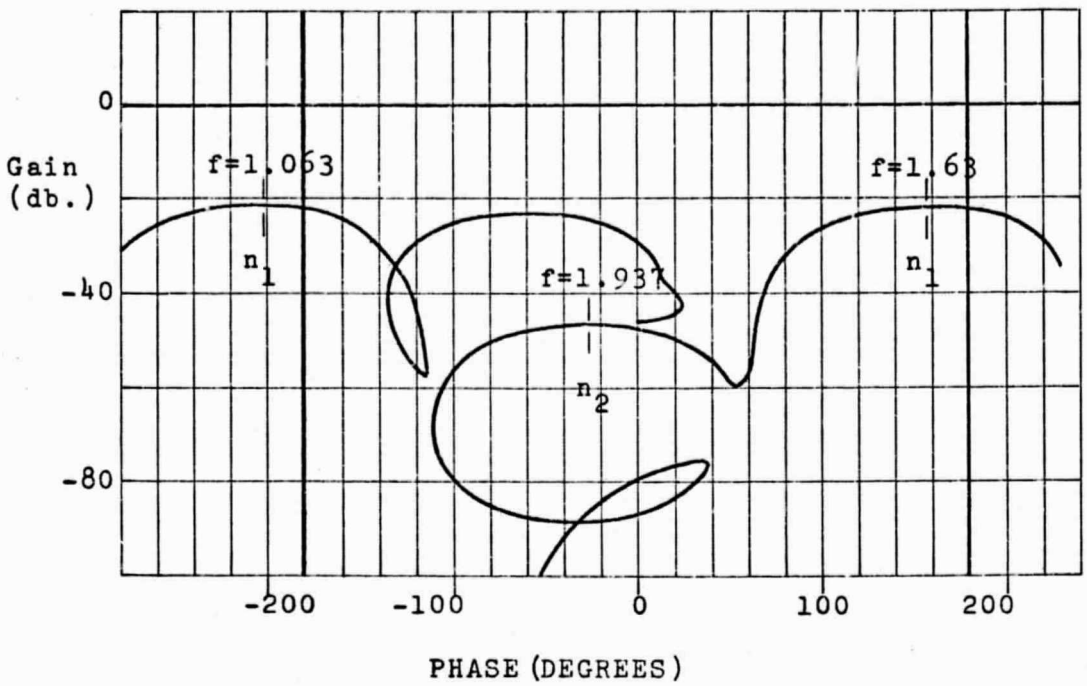


FIGURE 4-4

PHASE VS GAIN CHARACTERISTICS OF
CONVENTIONALLY COMPENSATED SATURN V AS-501
AT MAXIMUM DYNAMIC PRESSURE

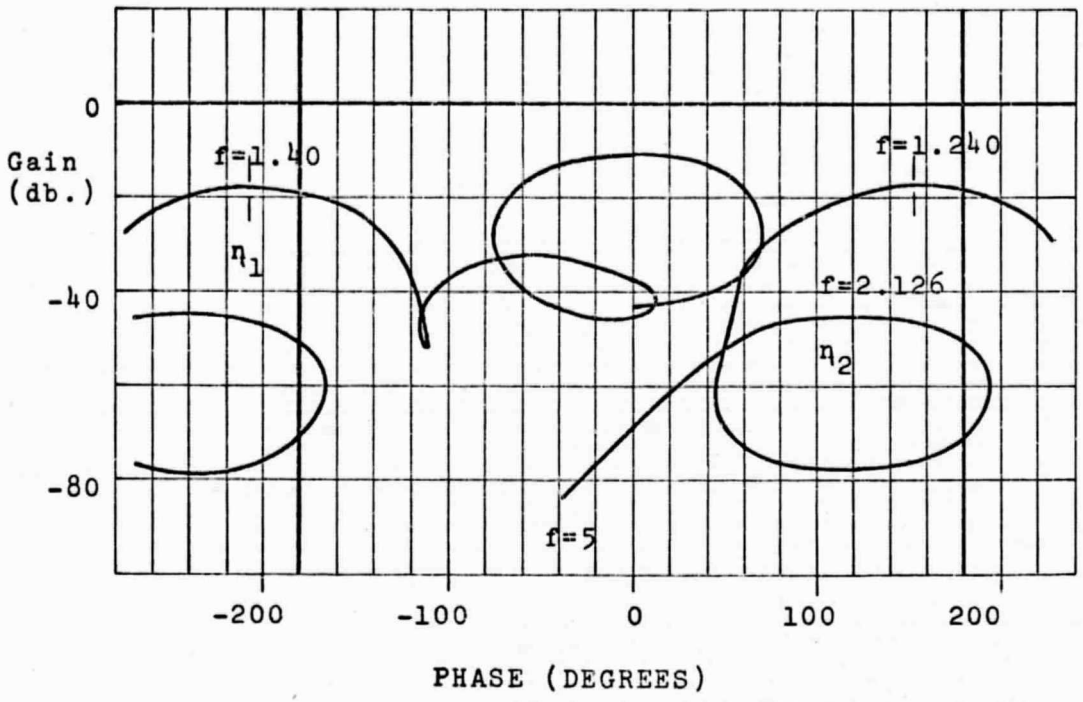


FIGURE 4-5
PHASE VS GAIN CHARACTERISTICS OF
CONVENTIONALLY COMPENSATED SATURN V AS-501 AT BURNOUT

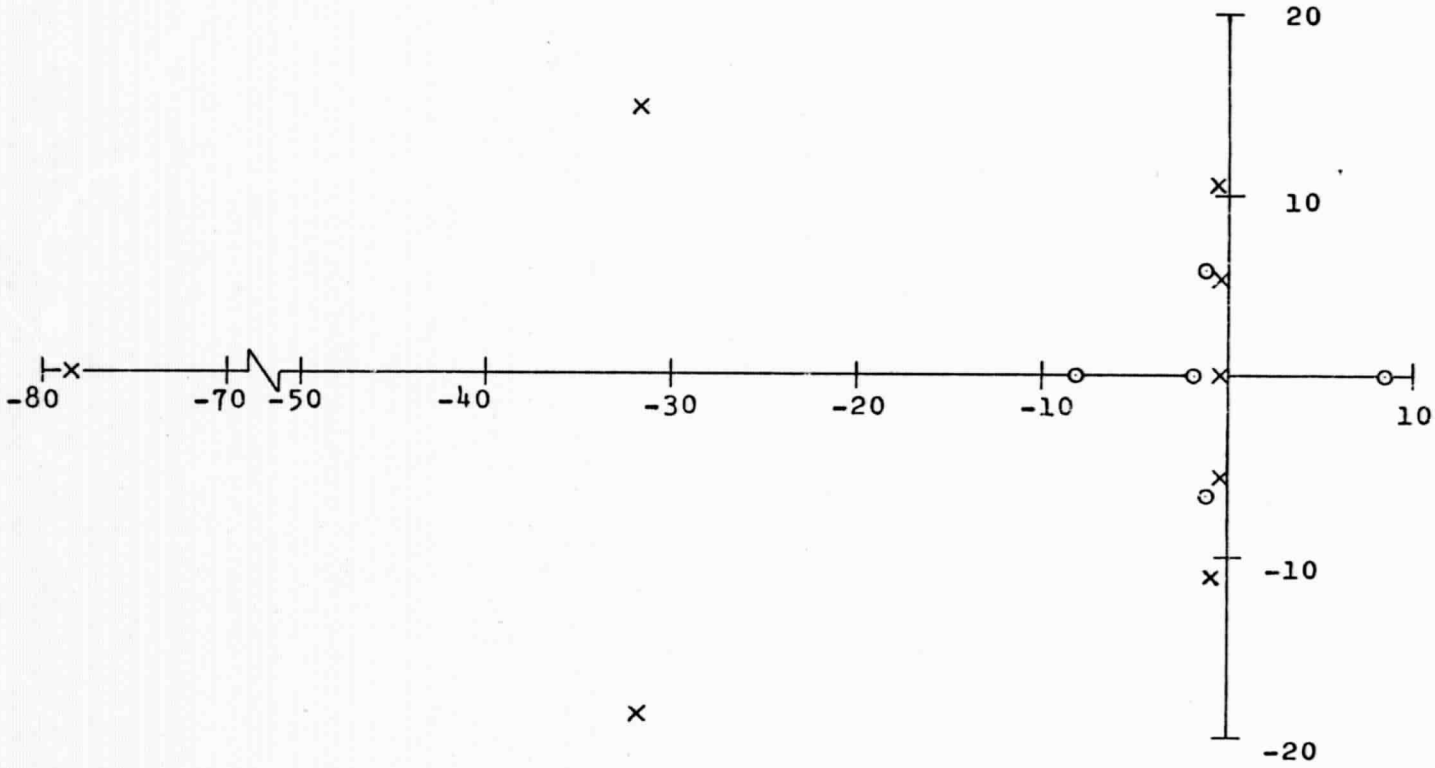


FIGURE 4-6
WEIGHTING FILTER POLE-ZERO PLOT AT LIFT-OFF; P = 0

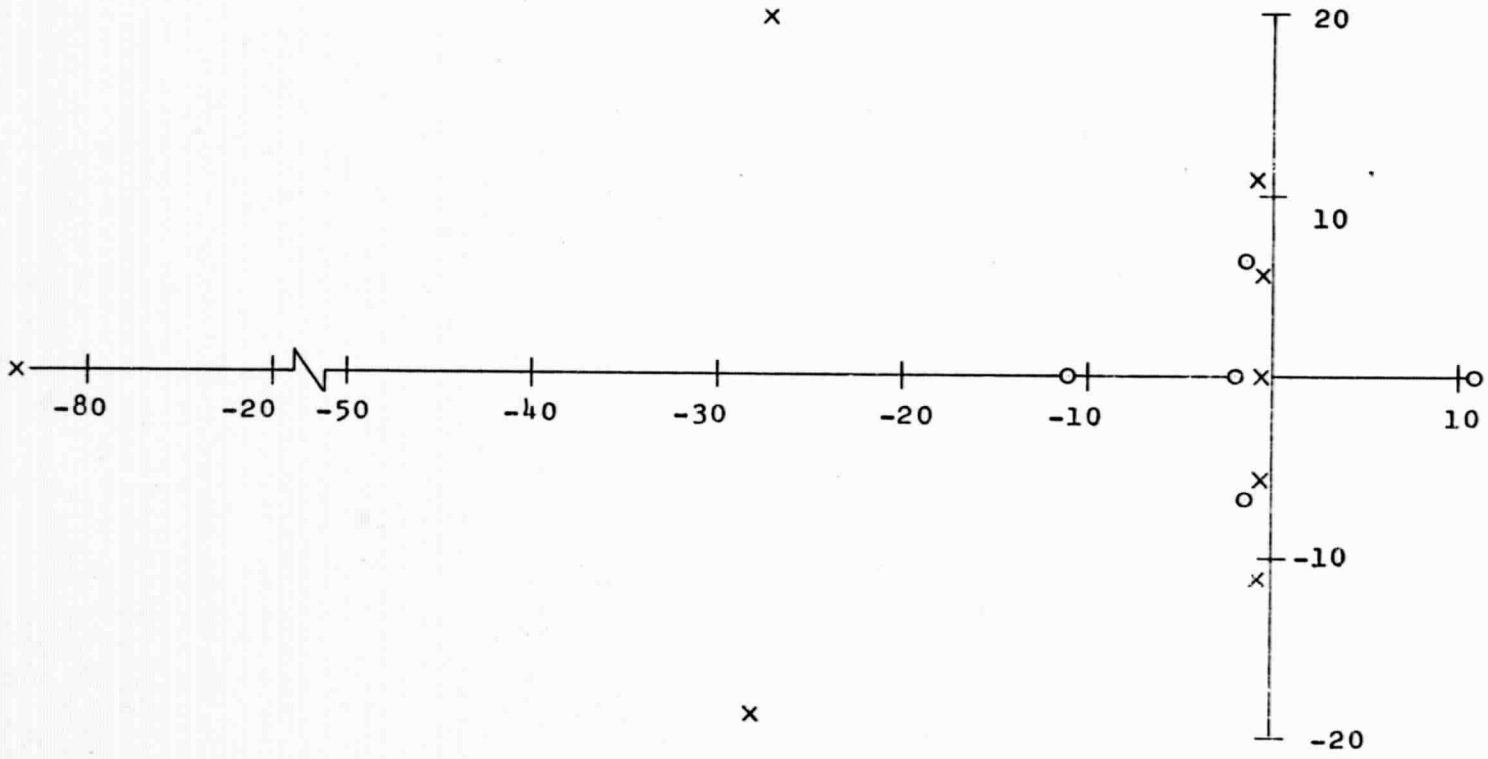


FIGURE 4-7
WEIGHTING FILTER POLE-ZERO PLOT
AT MAXIMUM DYNAMIC PRESSURE; P = 0

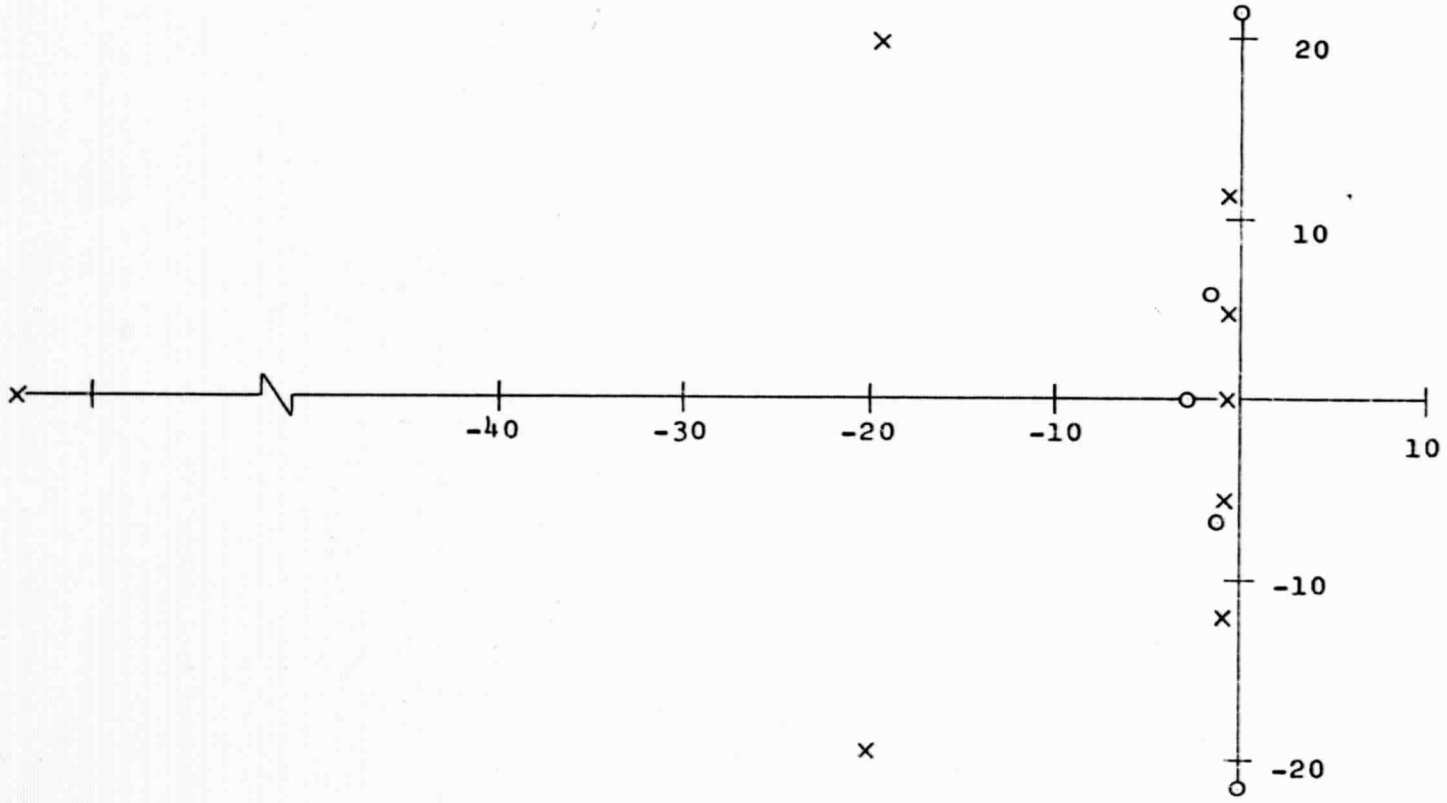


FIGURE 4-8

WEIGHTING FILTER POLE-ZERO PLOT AT BURNOUT ; P = 0

5. RESULTS

5.1 Effects of Adaptation on Flexible Mode Response

Fixed point trajectory studies were carried out to determine the effects of parameter adaptation on flexible response and stability margins. Fixed point simulations were run under fixed point trajectory conditions corresponding to lift-off, maximum dynamic pressure, and burnout, with system excitation being provided by an impulse function as shown in Figure 5-1. The flexible mode response, with adaptive loop open (P=0.0), is shown in Figures 5-2 thru 5-4. With the adaptive loop closed and the controllable parameter constrained to remain within the region of stability, the flexible responses pictured in Figures 5-5 thru 5-7 were obtained. Comparison of the response of the nonadaptive system (P=0.0) to the response of the adaptive system indicates that for conditions of lift-off and maximum dynamic pressure, adaptation of P produces no significant reduction in the residues associated with the flexible response. Under these conditions (lift-off and maximum dynamic pressure) adaptation of P does result in a reduction of the damping associated with the flexible response.

At burnout adaptation increases the initial amplitude of the first mode response, but the damping ratio is improved by a factor of approximately 3.5. Adaptation

results in a significant reduction in the initial amplitude of the second mode response, however, the damping ratio is reduced by a factor of approximately 5.0, and the amplitude of the adapted response eventually exceeds that of the nonadaptive response.

Figures 5-8 and 5-9 show the response of the flexible modes with the adaptive loop closed and with no constraints on the range of the controllable parameter. The responses pictured indicate that removal of the constraints on P results in an unstable system.

5.2 Effects of Adaptation on Stability Margins

Figures 5-10, 5-11, and 5-12 are phase vs gain plots for the system with the adaptive loop closed. The range of the controllable parameter was restricted to insure that the system would remain stable and phase-gain data was taken with the controllable parameter fixed at its steady state value. The effects of adaptation on the stability margins may be determined by comparing Figures 5-10, 5-11, and 5-12 to Figures 4-3, 4-4, and 4-5, respectively.

Under conditions of lift-off, adaptation resulted in an 8db. reduction in gain margin for both the first and second flexible modes. For conditions of maximum dynamic pressure adaptation resulted in a 6db. reduction in gain margin for the first flexible mode and a 12db. reduction

in gain margin for the second mode. At burnout, adaptation resulted in an 18db. increase in gain margin for the first flexible mode and a 13db. reduction in gain margin for the second flexible mode. Variations in the controllable parameter had no significant effect on the stability margins associated with the rigid body.

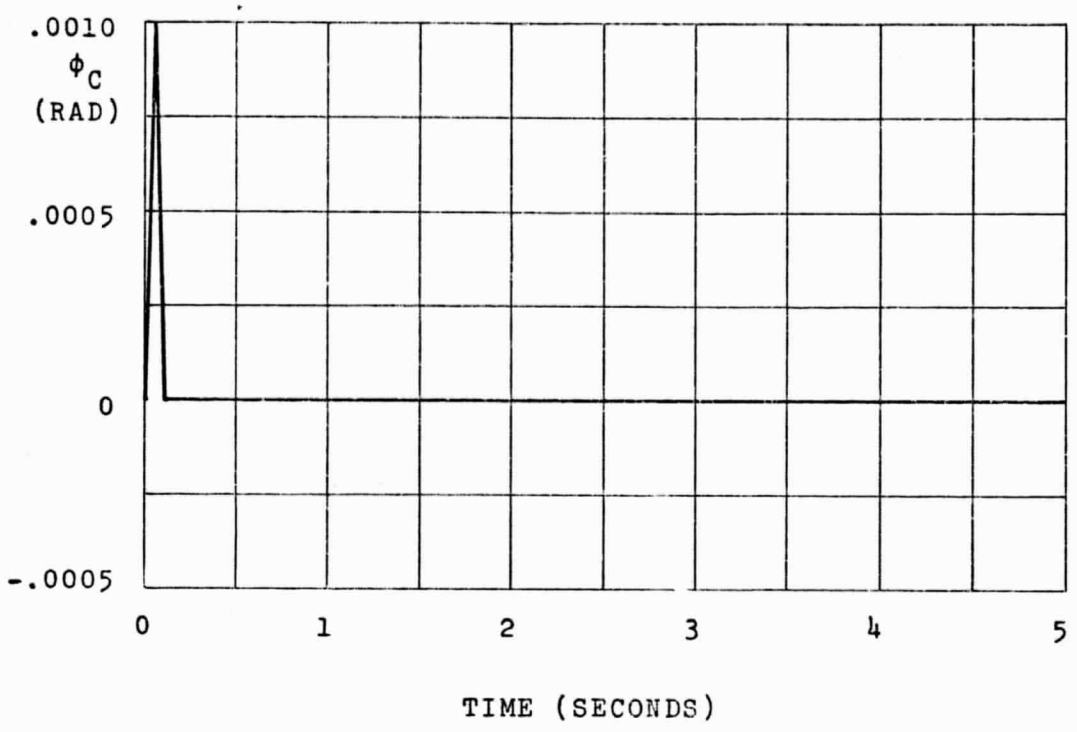


FIGURE 5-1
SYSTEM EXCITATION

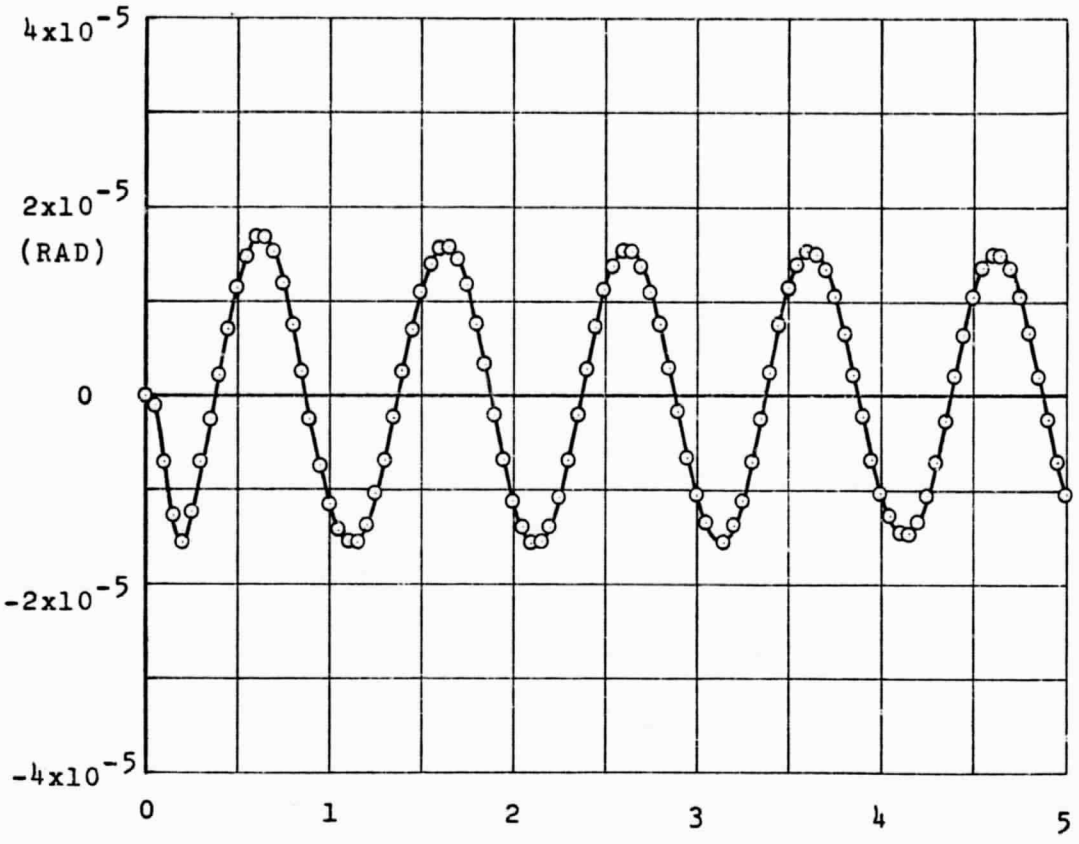


FIGURE 5-2a

FLEXIBLE RESPONSE: η_1 at LIFT-OFF,

P = 0

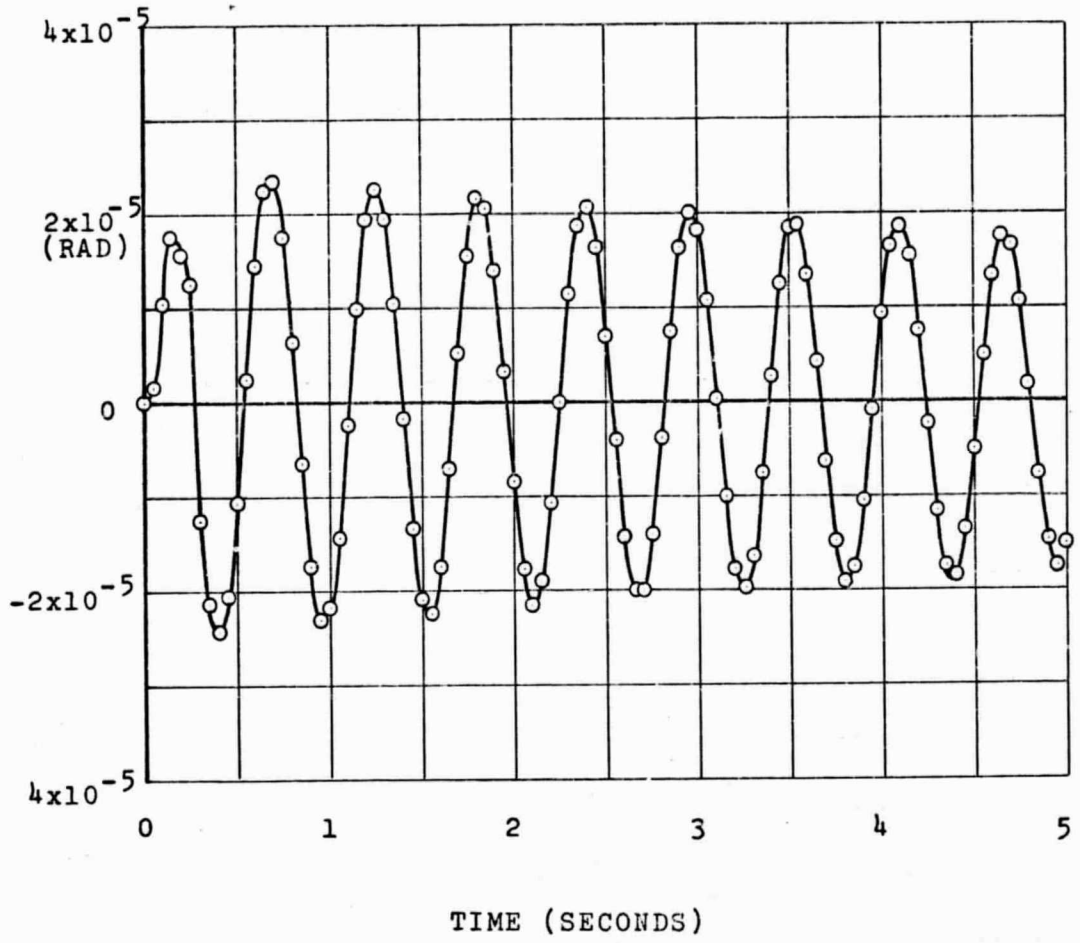


FIGURE 5-2b

FLEXIBLE RESPONSE: n_2 AT LIFT-OFF,

P = 0

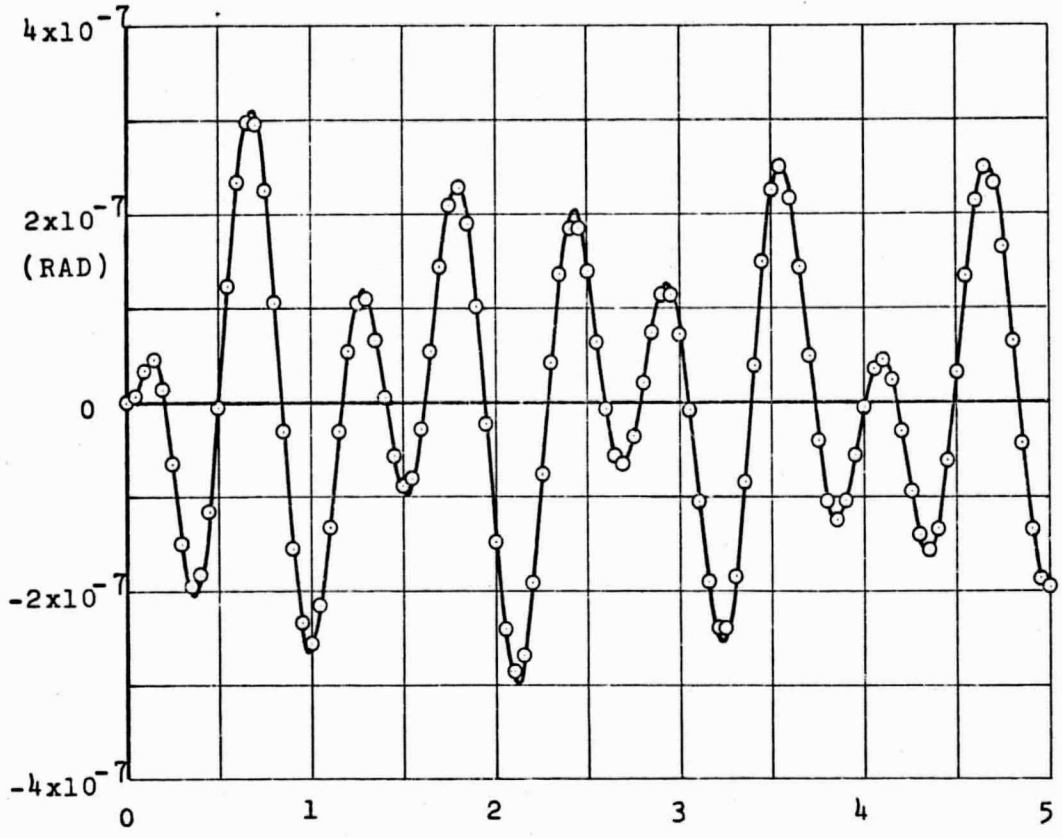


FIGURE 5-2c
RESPONSE ERROR ϵ AT LIFT-OFF, P = 0

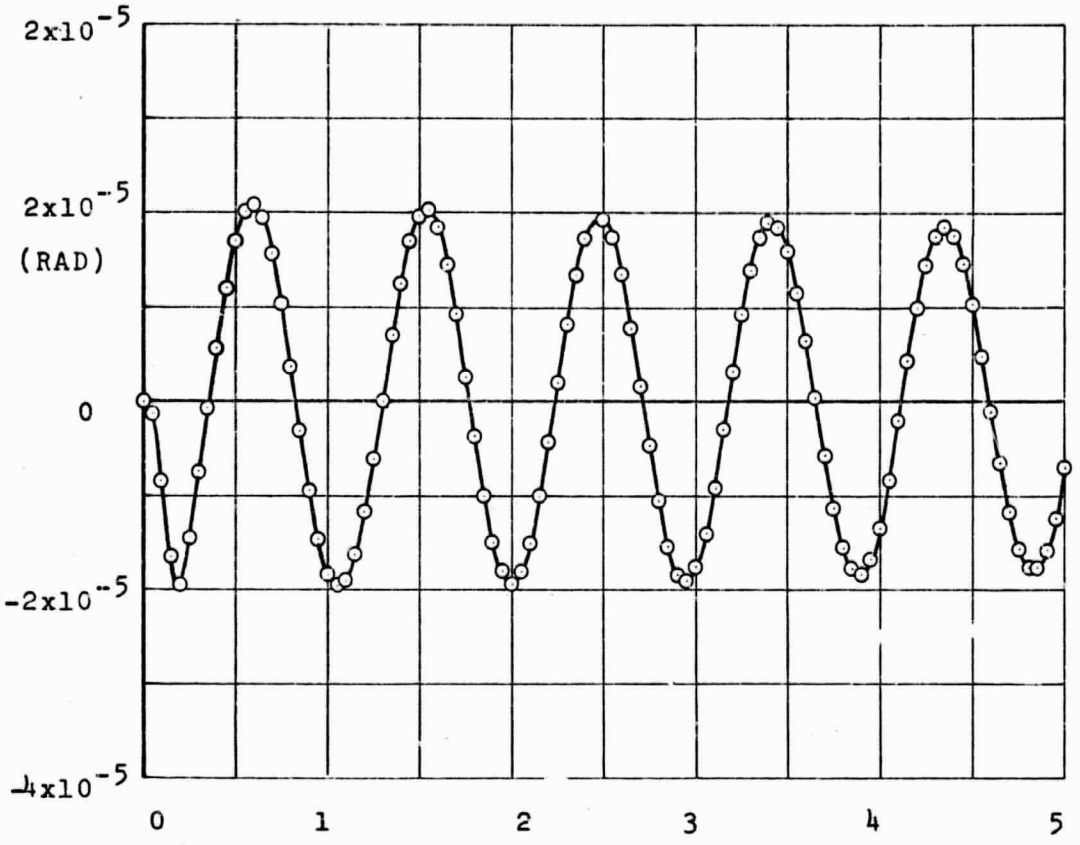


FIGURE 5-3a

FLEXIBLE RESPONSE: η_1 AT MAXIMUM DYNAMIC PRESSURE, P = 0

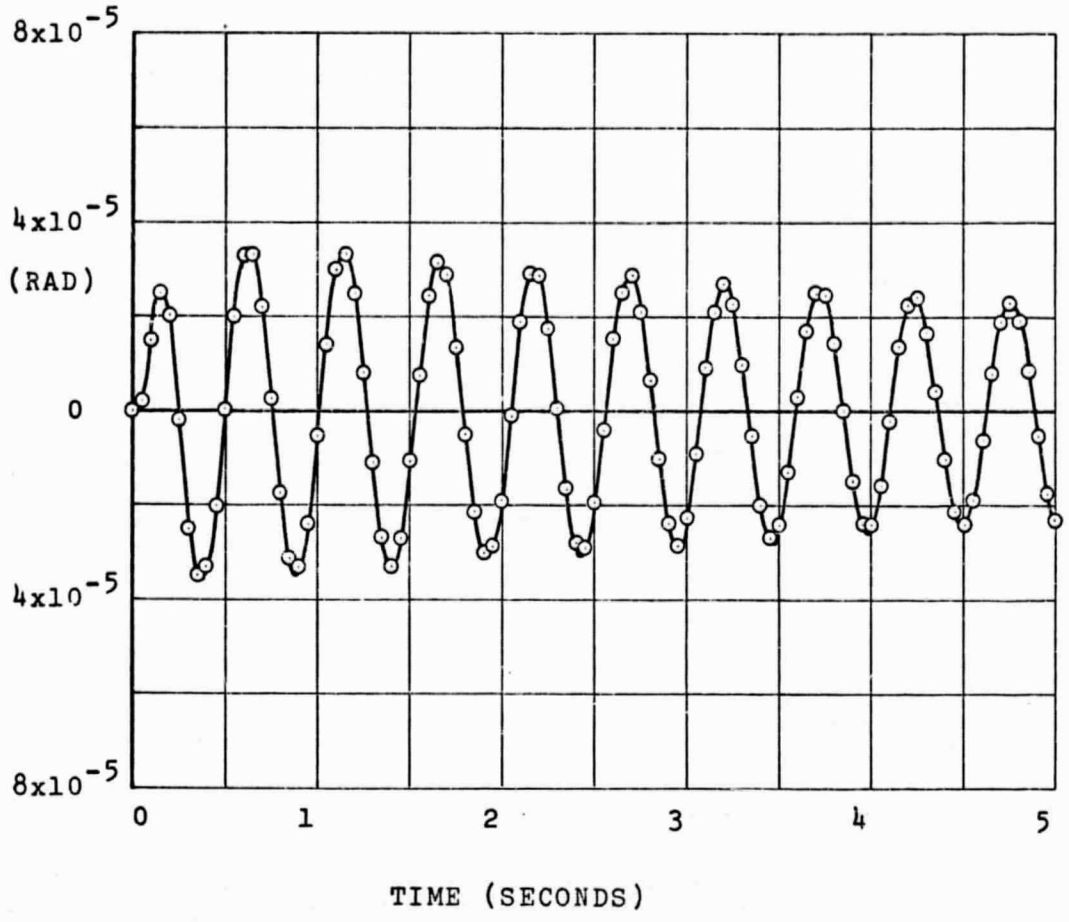


FIGURE 5-3b

FLEXIBLE RESPONSE: η_2 AT MAXIMUM DYNAMIC PRESSURE, P = 0

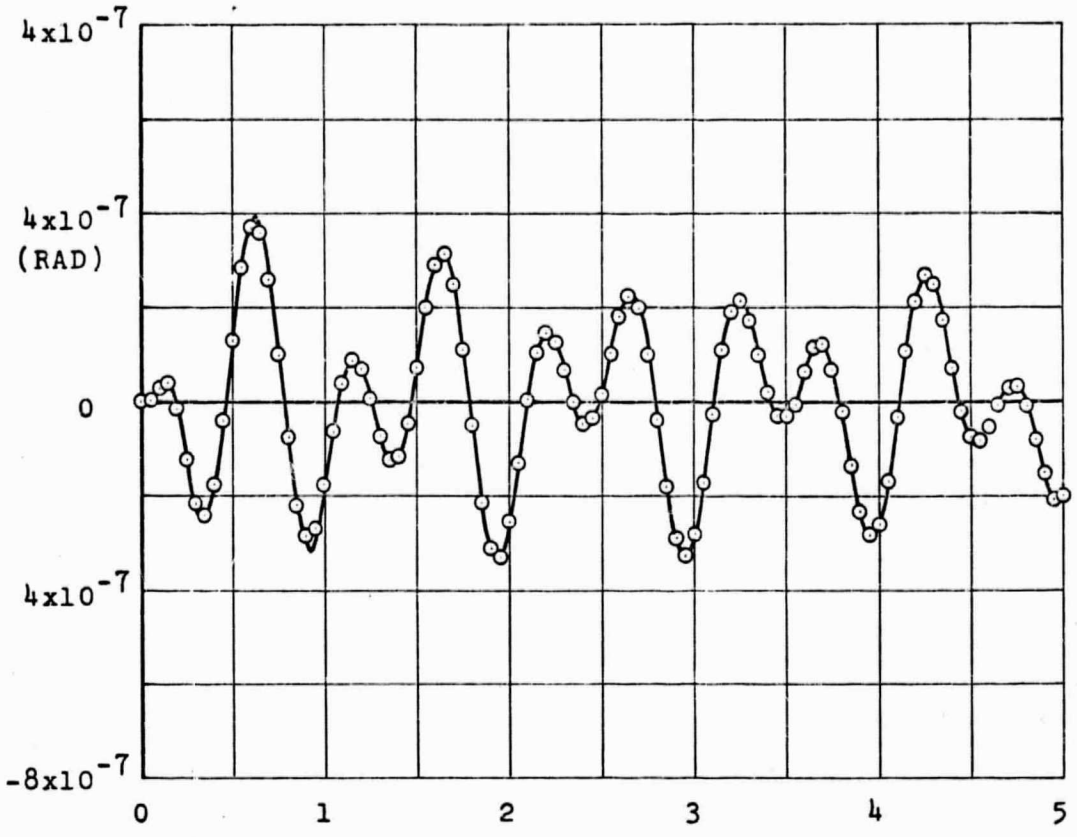


FIGURE 5-3c

RESPONSE ERROR ϵ AT MAXIMUM DYNAMIC PRESSURE,

P = 0

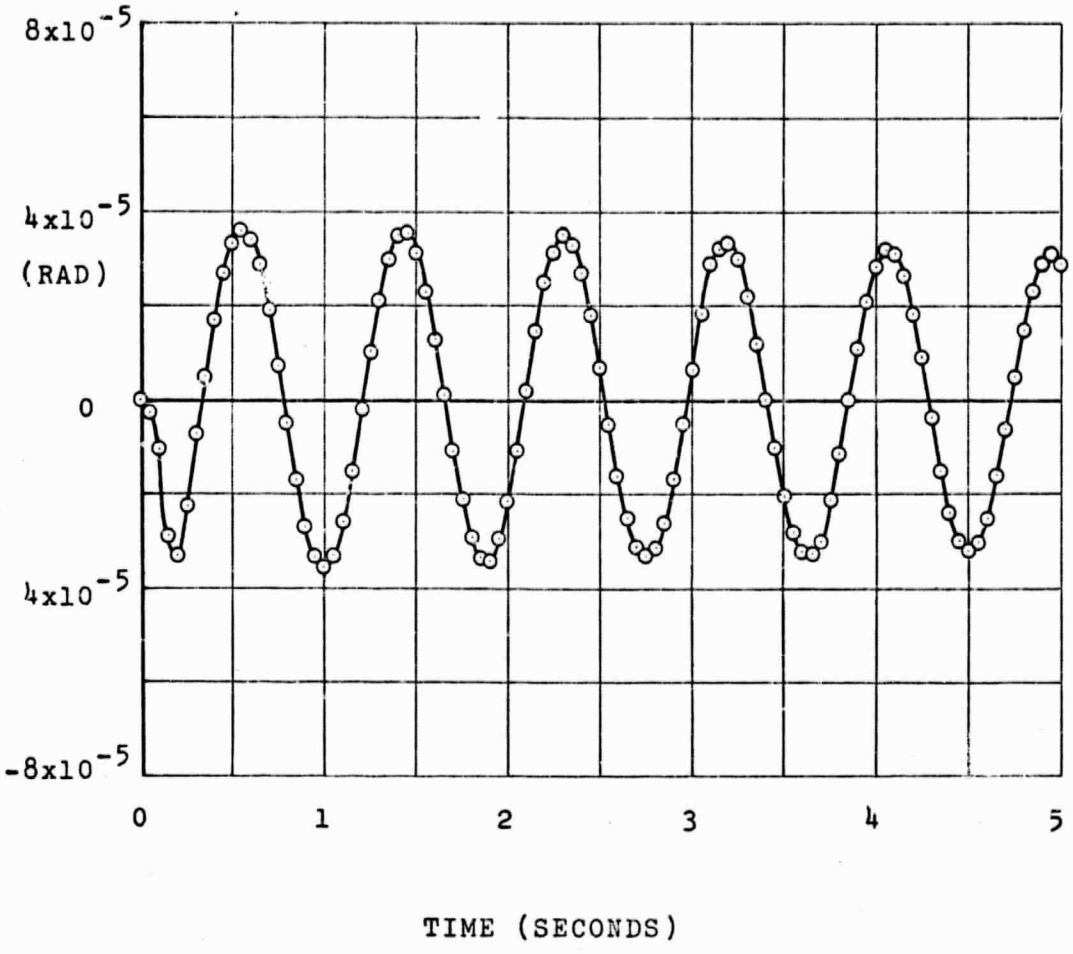


FIGURE 5-4a

FLEXIBLE RESPONSE: η_1 AT BURN OUT, P = 0

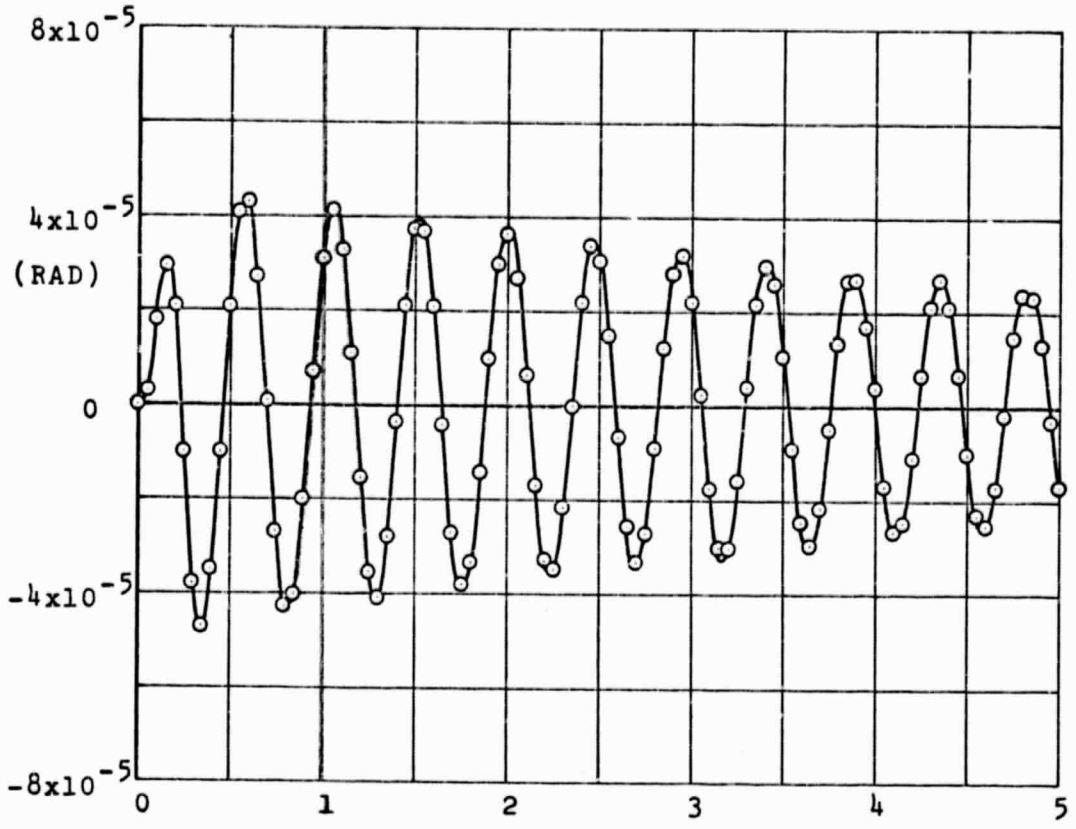


FIGURE 5-4b

FLEXIBLE RESPONSE: n_2 AT BURNOUT, P = 0

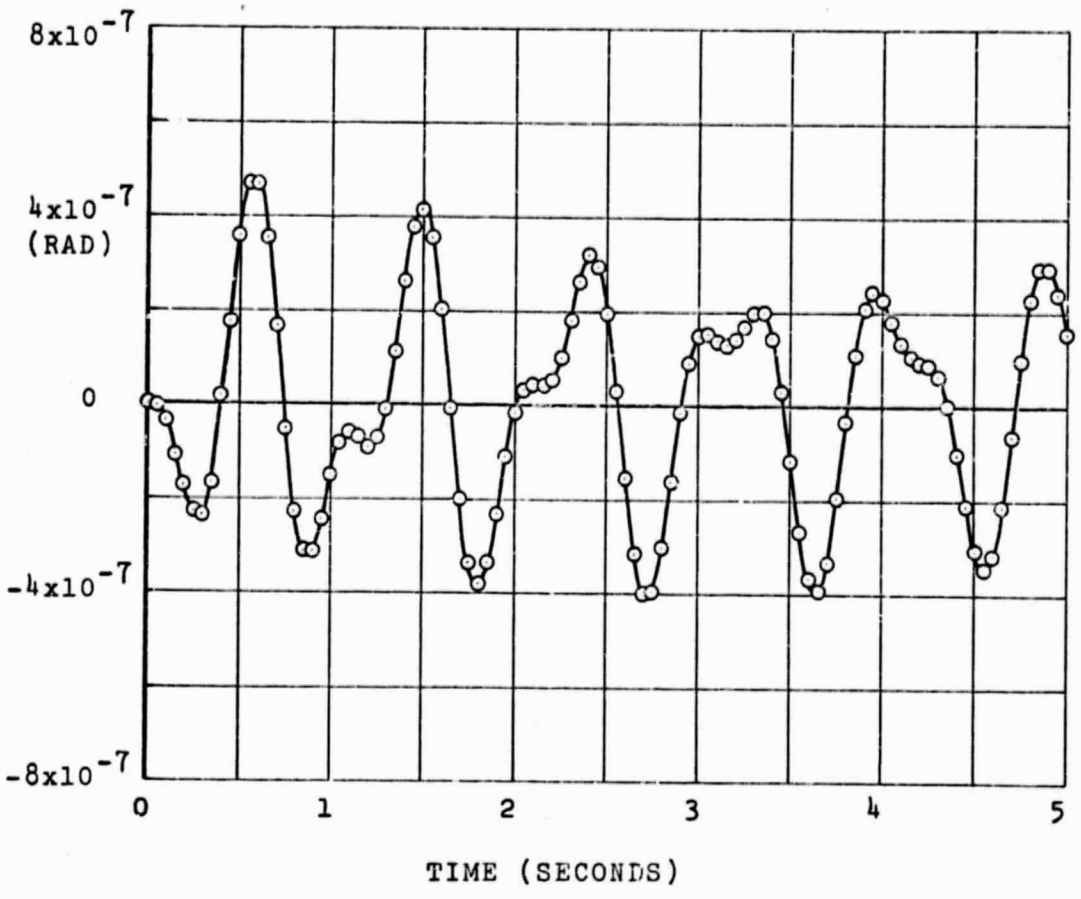


FIGURE 5-4c

RESPONSE ERROR ε AT BURNOUT, P = 0

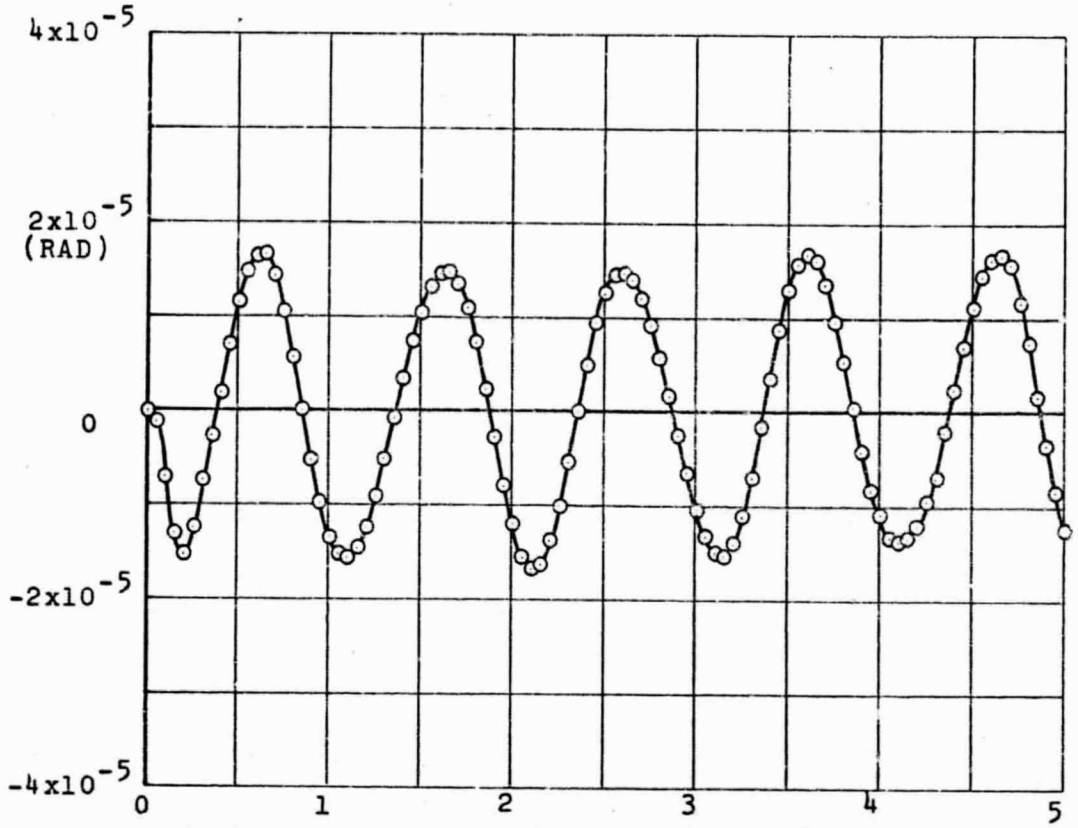


FIGURE 5-5a

FLEXIBLE RESPONSE: η_1 AT LIFT-OFF, P-ADAPTIVE

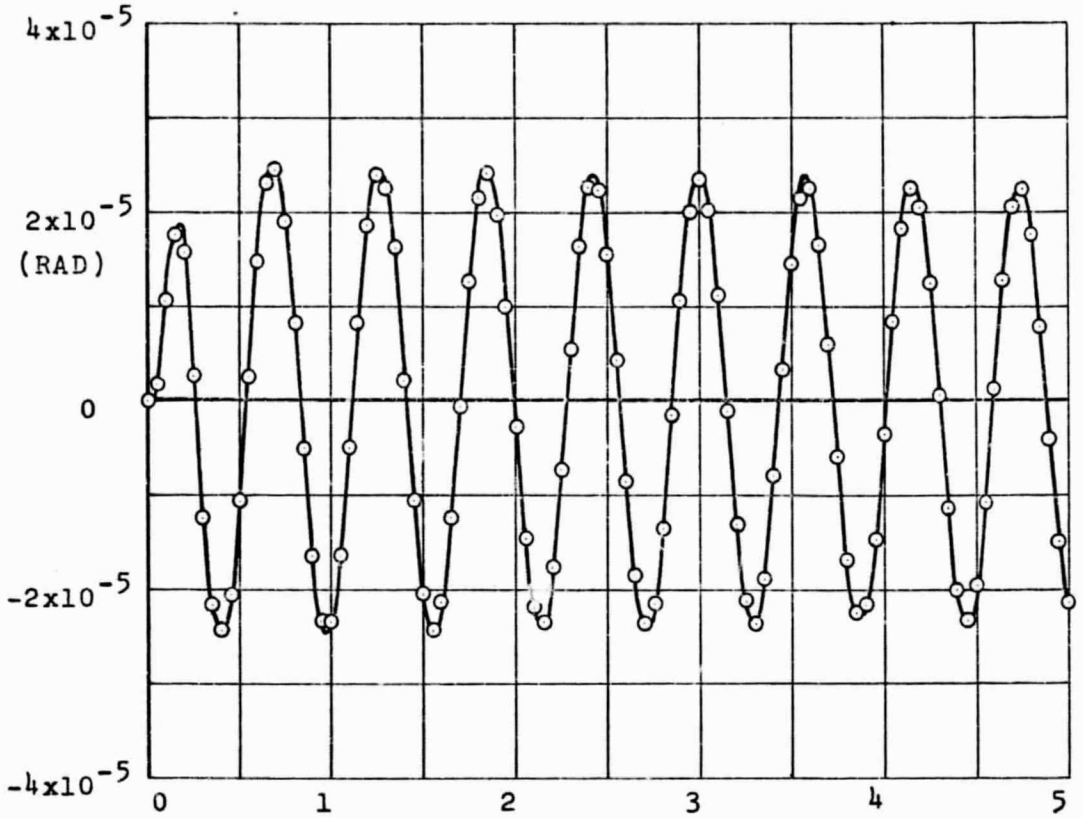


FIGURE 5-5b
FLEXIBLE RESPONSE: η_2 AT LIFT-OFF
P-ADAPTIVE

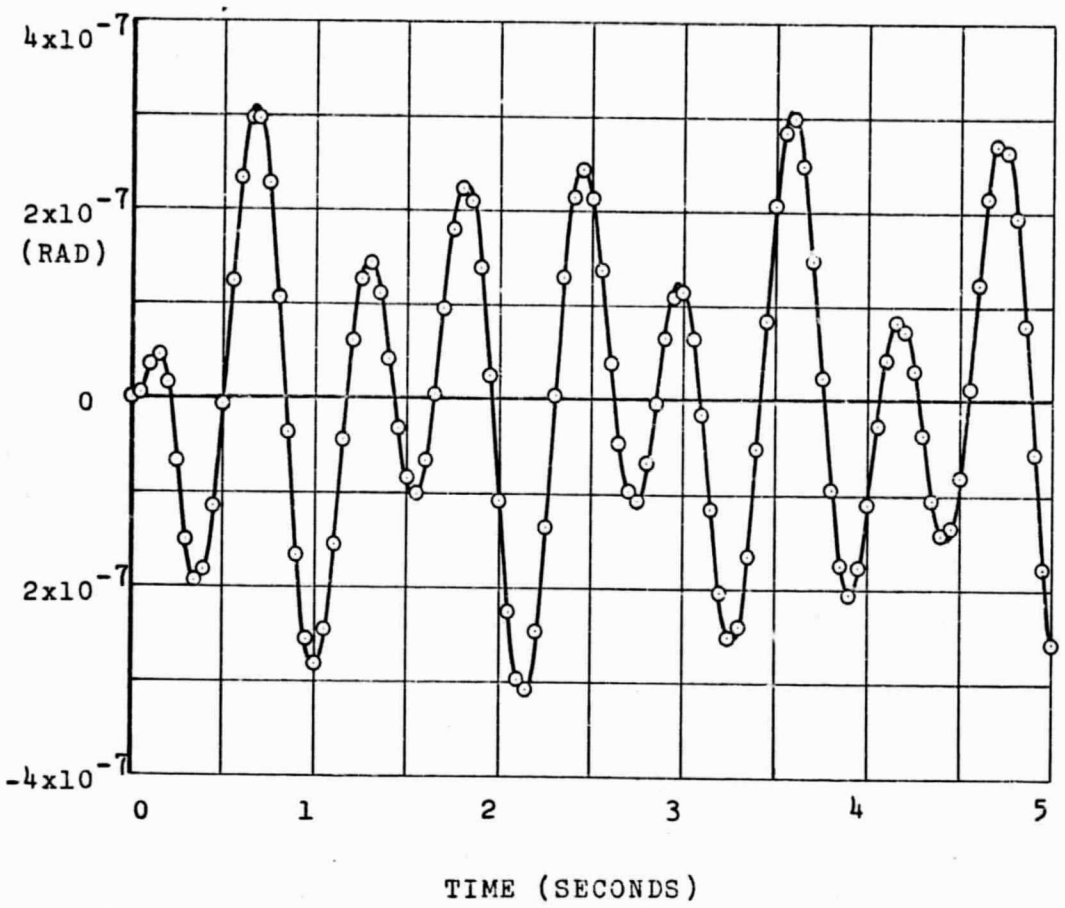


FIGURE 5-5c
RESPONSE ERROR ε AT LIFT-OFF,
P-ADAPTIVE

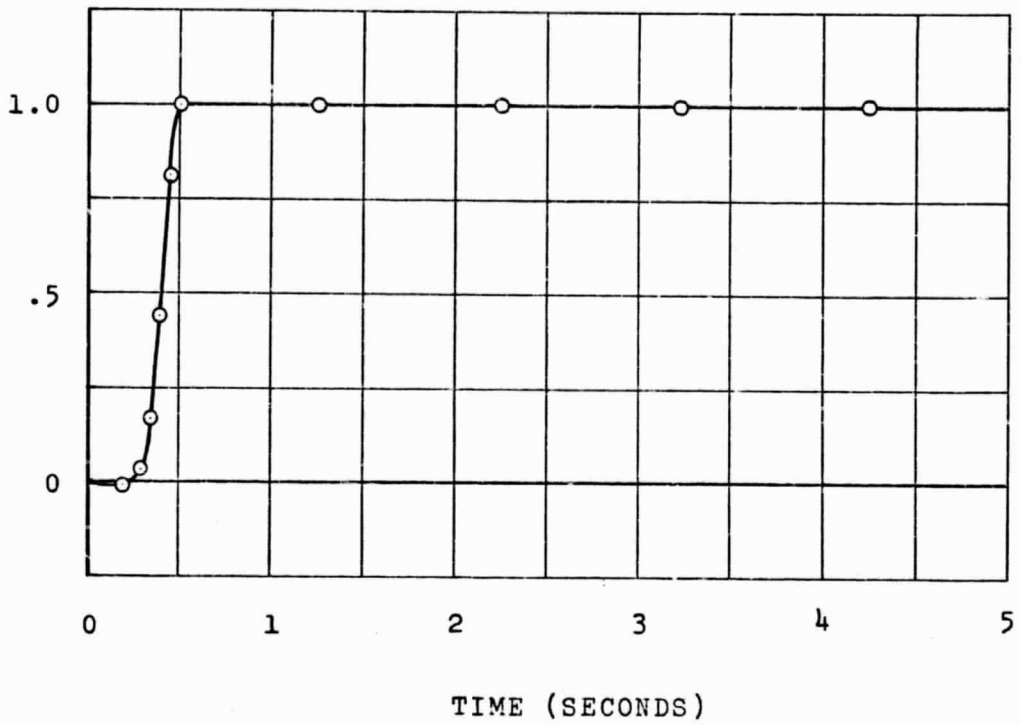


FIGURE 5-5d
CONTROLLABLE PARAMETER P, AT LIFT-OFF

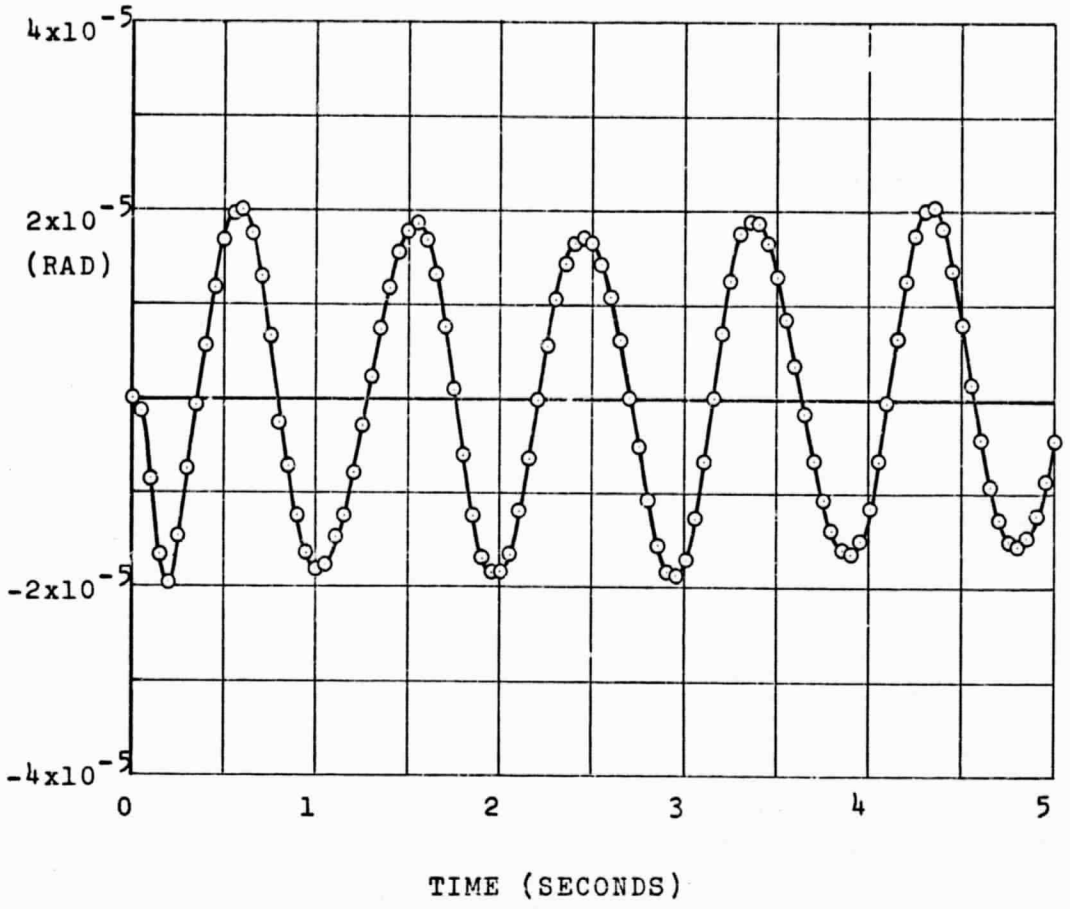


FIGURE 5-6a

FLEXIBLE RESPONSE: η_1 AT MAXIMUM DYNAMIC PRESSURE

P-ADAPTIVE

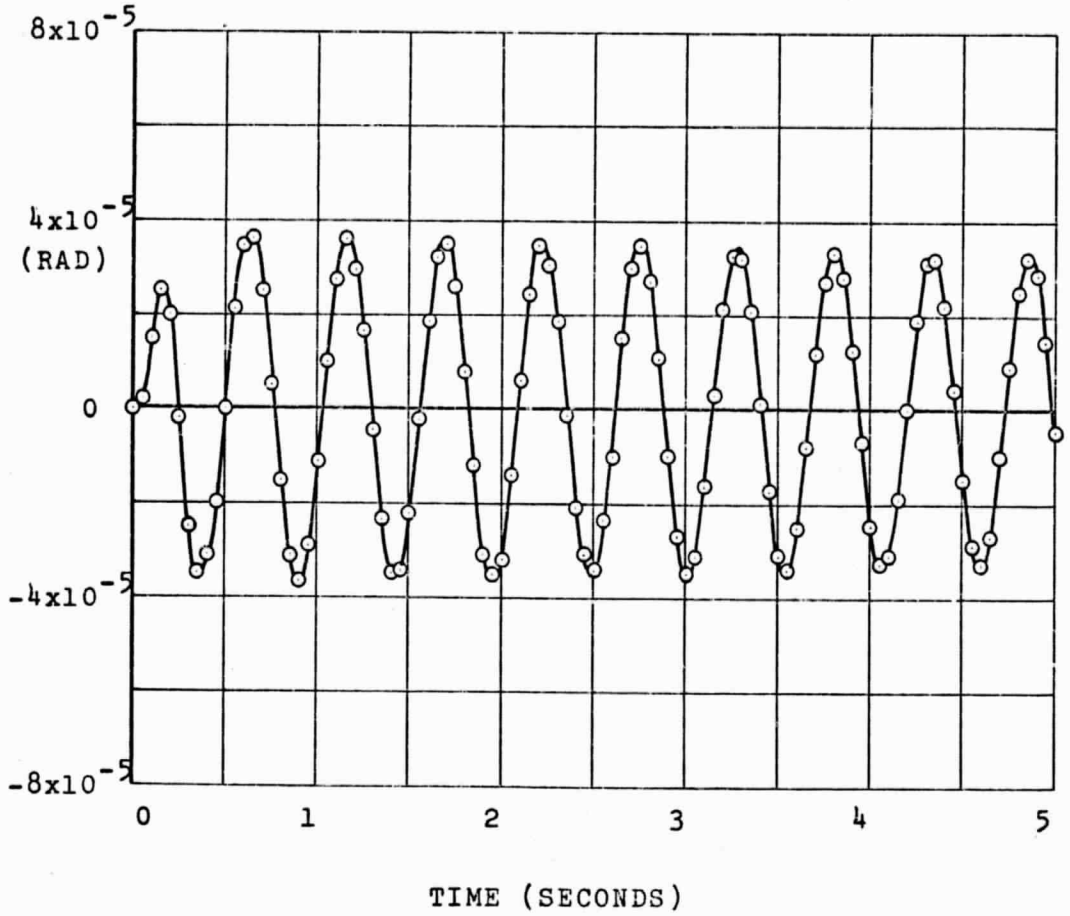


FIGURE 5-6b

FLEXIBLE RESPONSE: η_2 AT MAXIMUM DYNAMIC PRESSURE, P-ADAPTIVE

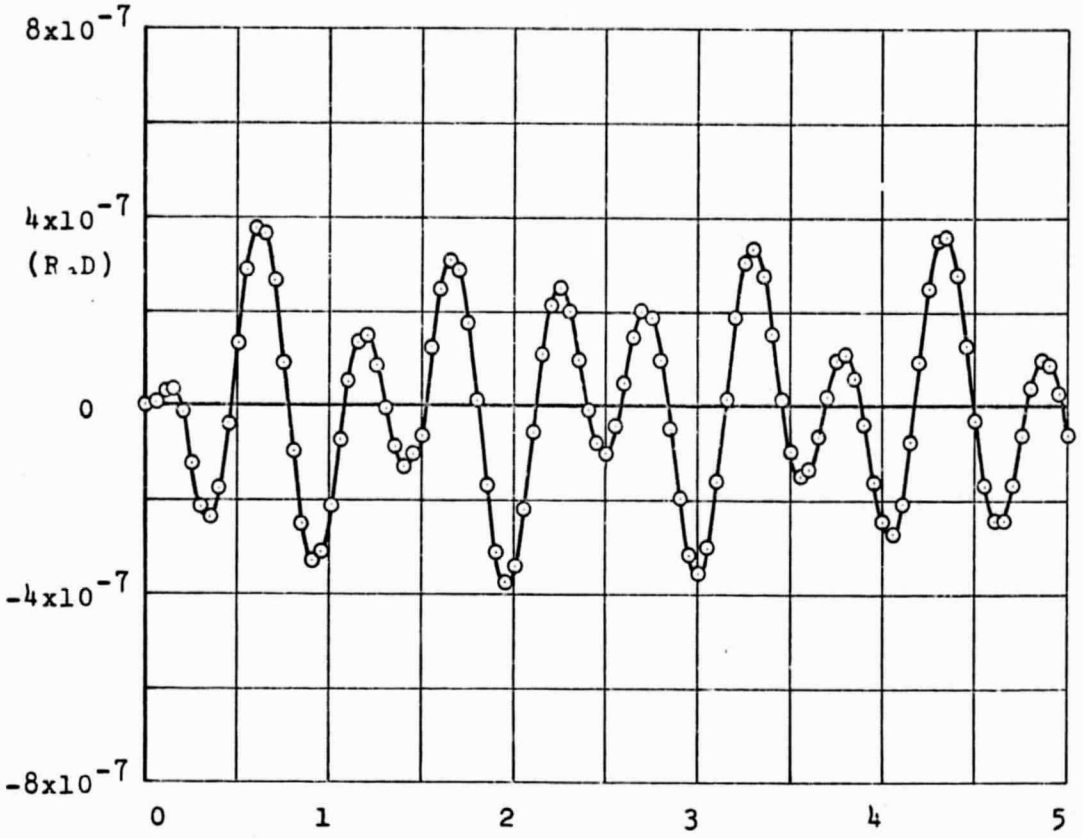


FIGURE 5-6c
RESPONSE ERROR ε AT MAXIMUM DYNAMIC PRESSURE,
P-ADAPTIVE

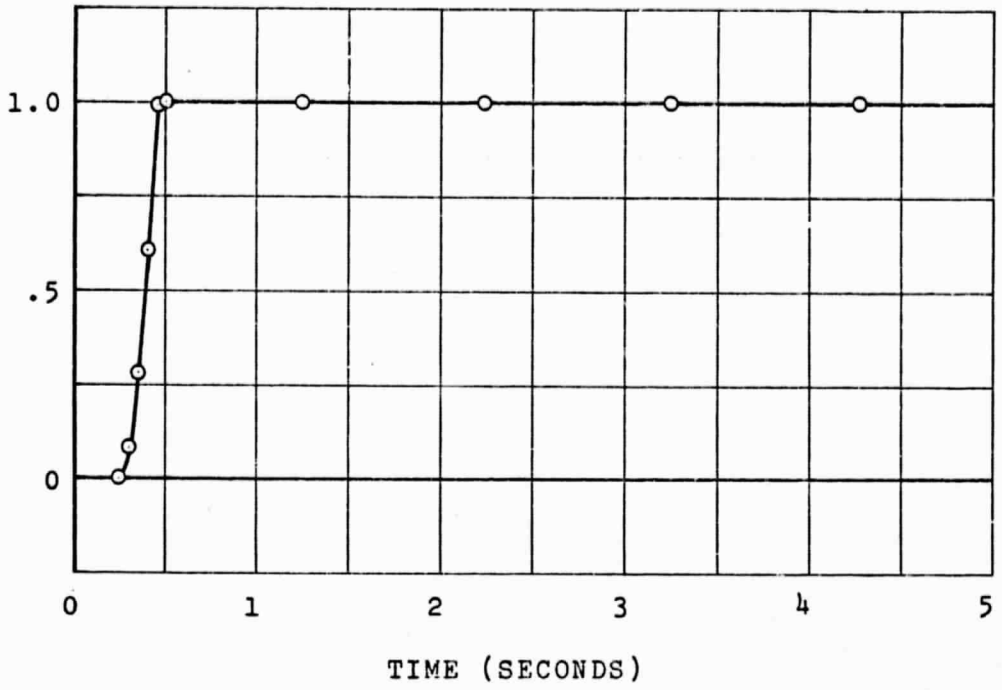


FIGURE 5-6d

CONTROLLABLE PARAMETER P, AT MAXIMUM DYNAMIC PRESSURE

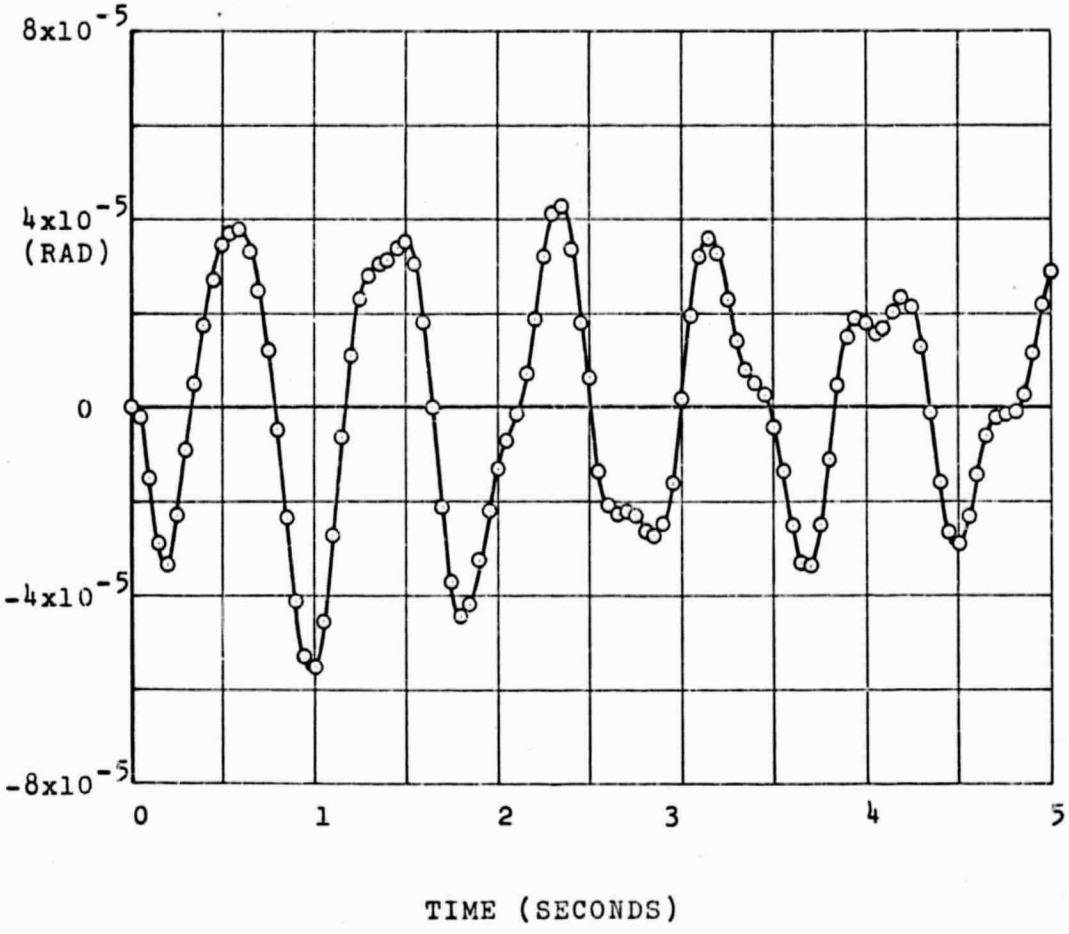


FIGURE 5-7a
FLEXIBLE RESPONSE: η_1 AT BURNOUT,
P-ADAPTIVE

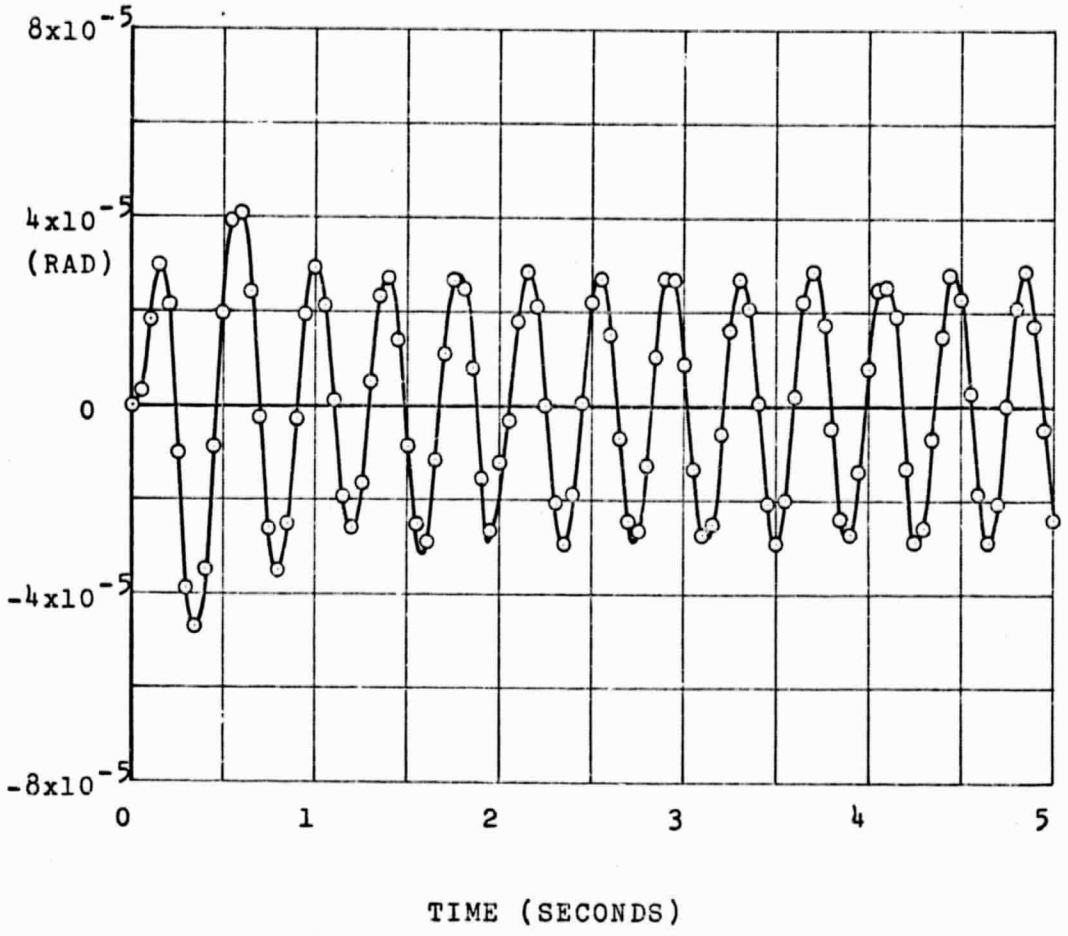


FIGURE 5-7b
FLEXIBLE RESPONSE: n_2 AT BURNOUT,
P-ADAPTIVE

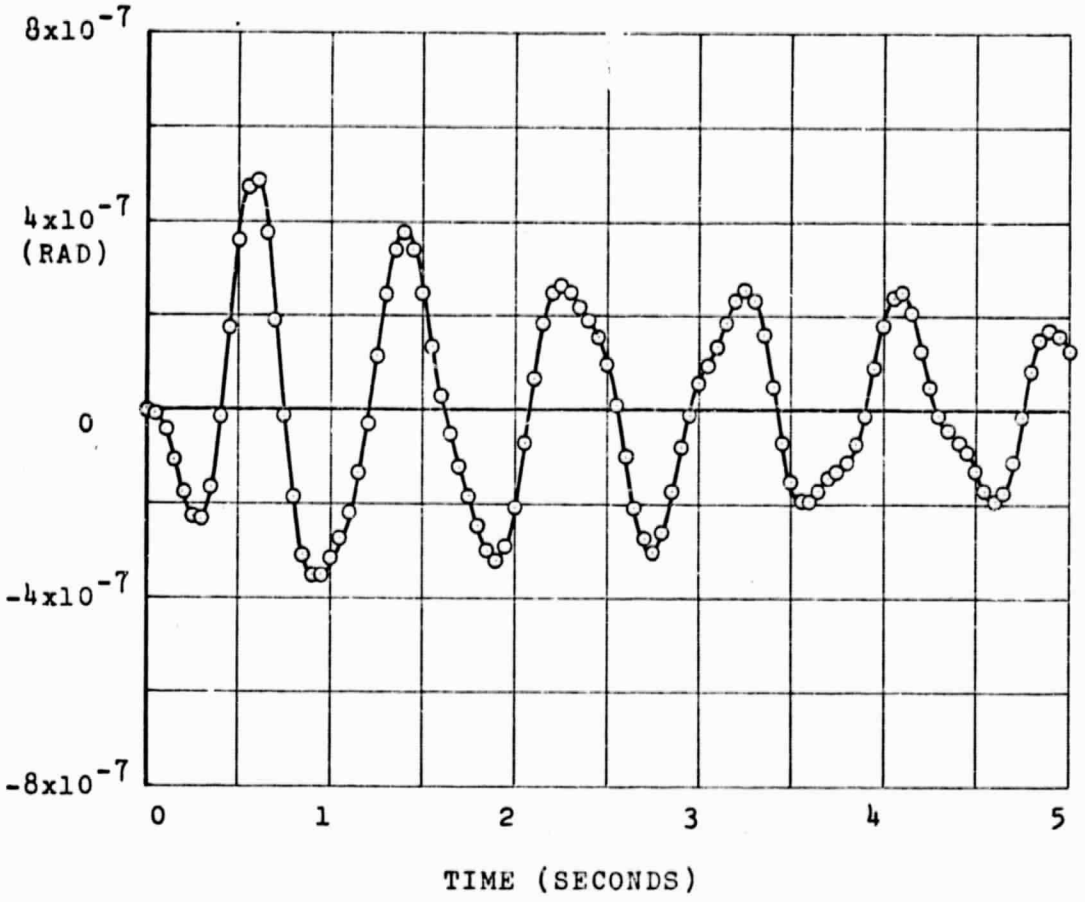


FIGURE 5-7c
RESPONSE ERROR ε AT BURNOUT, P-ADAPTIVE



FIGURE 5-7d
CONTROLLABLE PARAMETER P, AT BURNOUT

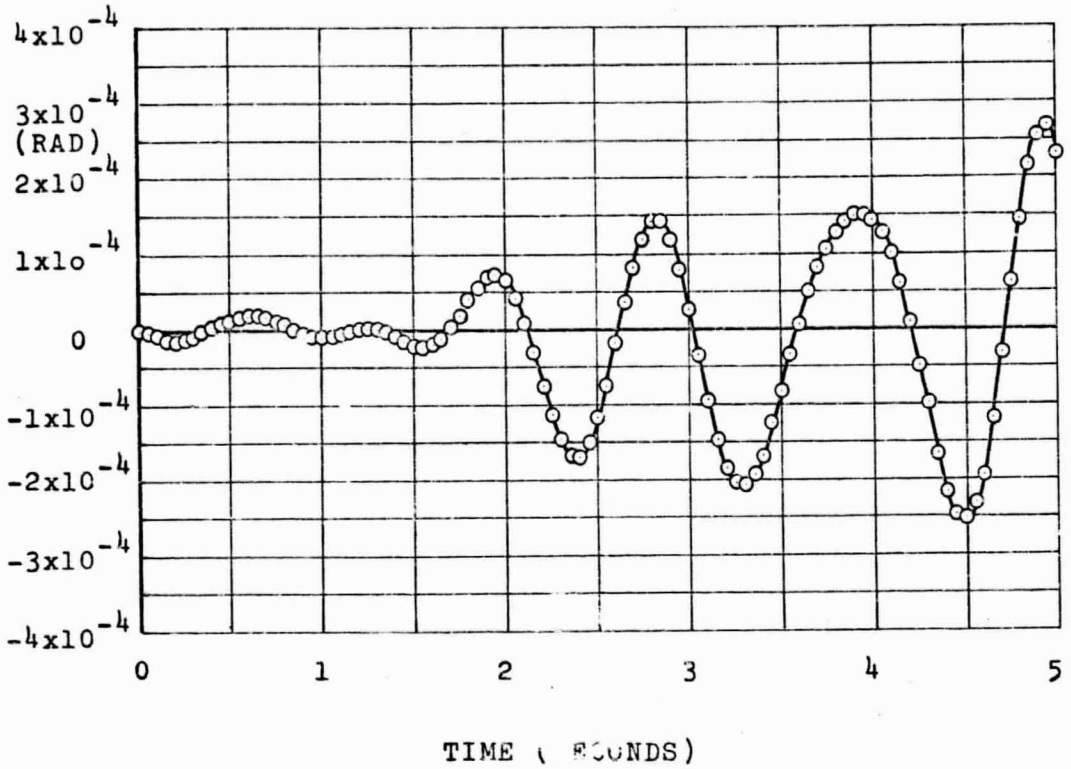


FIGURE 5-8a

FLEXIBLE RESPONSE: η_1 AT LIFT-OFF

P-ADAPTIVE AND UNCONSTRAINED

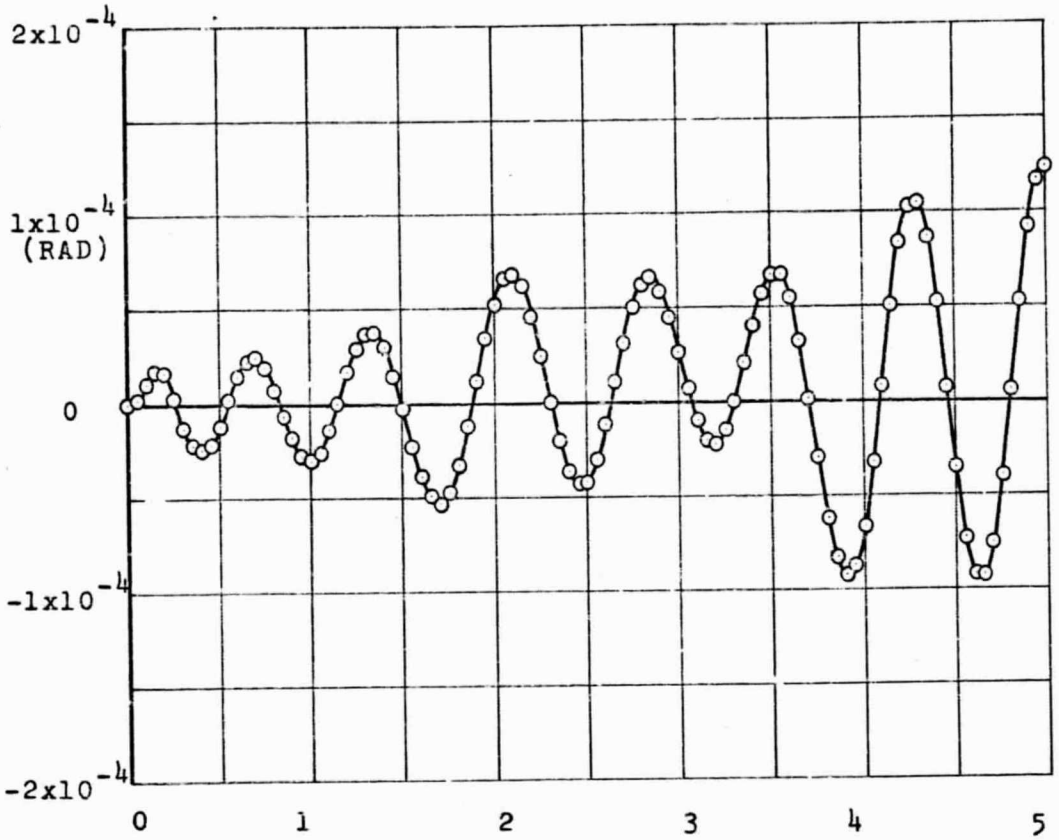


FIGURE 5-8b

FLEXIBLE RESPONSE: η_2 AT LIFT-OFF,

P-ADAPTIVE AND UNCONSTRAINED

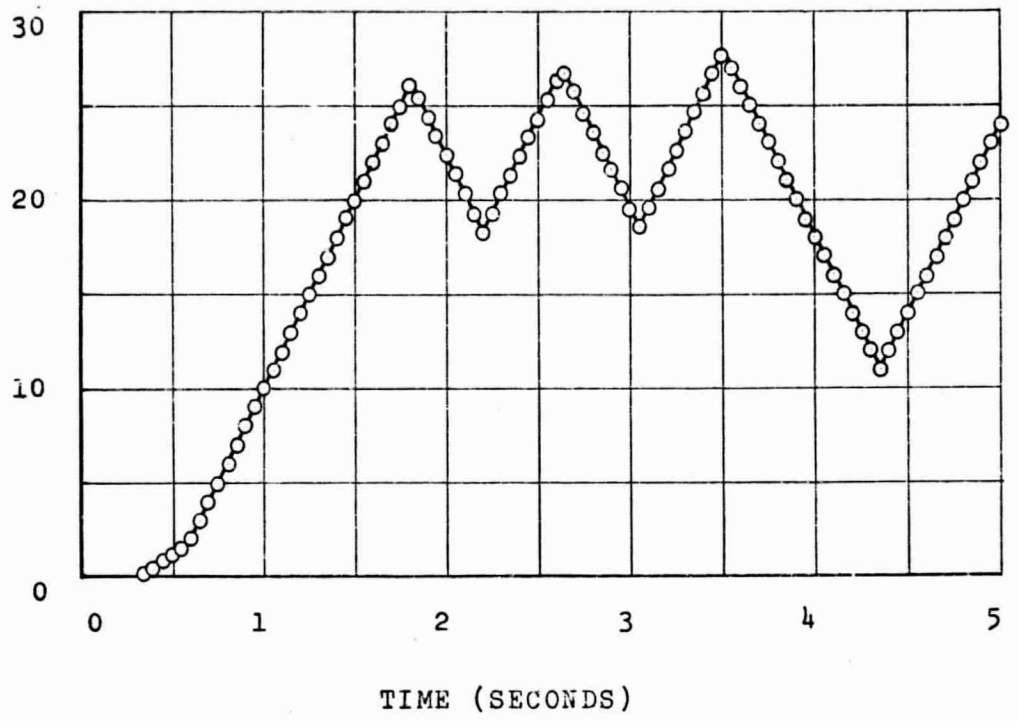


FIGURE 5-8c
CONTROLLABLE PARAMETER P, AT LIFT-OFF P
UNCONSTRAINED

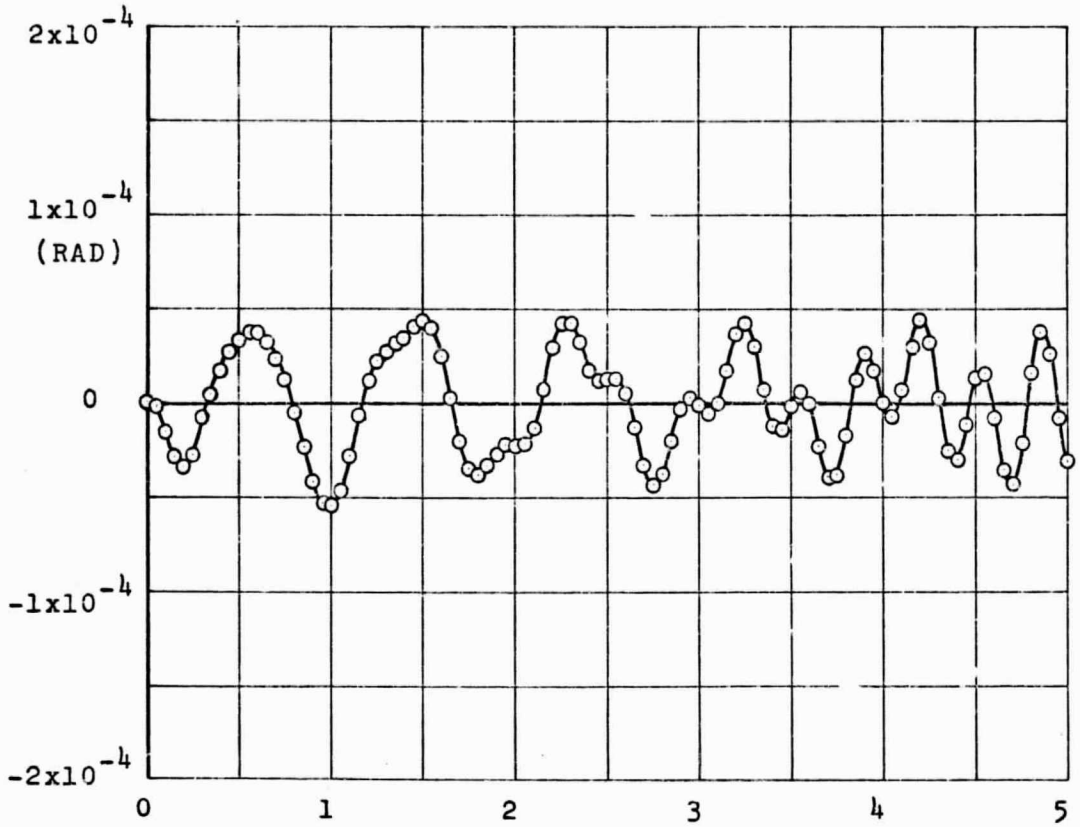


FIGURE 5-9a

FLEXIBLE RESP NSE: η_1 AT BURNOUT,

P-ADAPTIVE AND UNCONSTRAINED

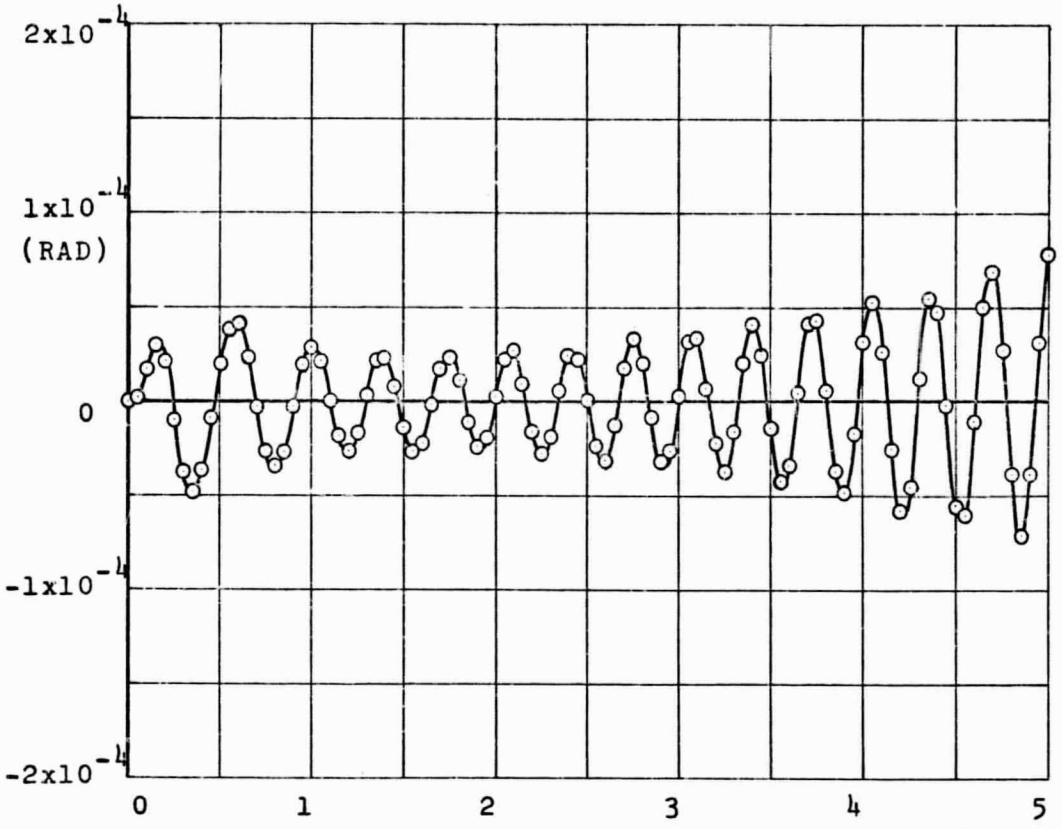


FIGURE 5-9b

FLEXIBLE RESPONSE: η_2 AT BURNOUT,

P-ADAPTIVE AND UNCONSTRAINED

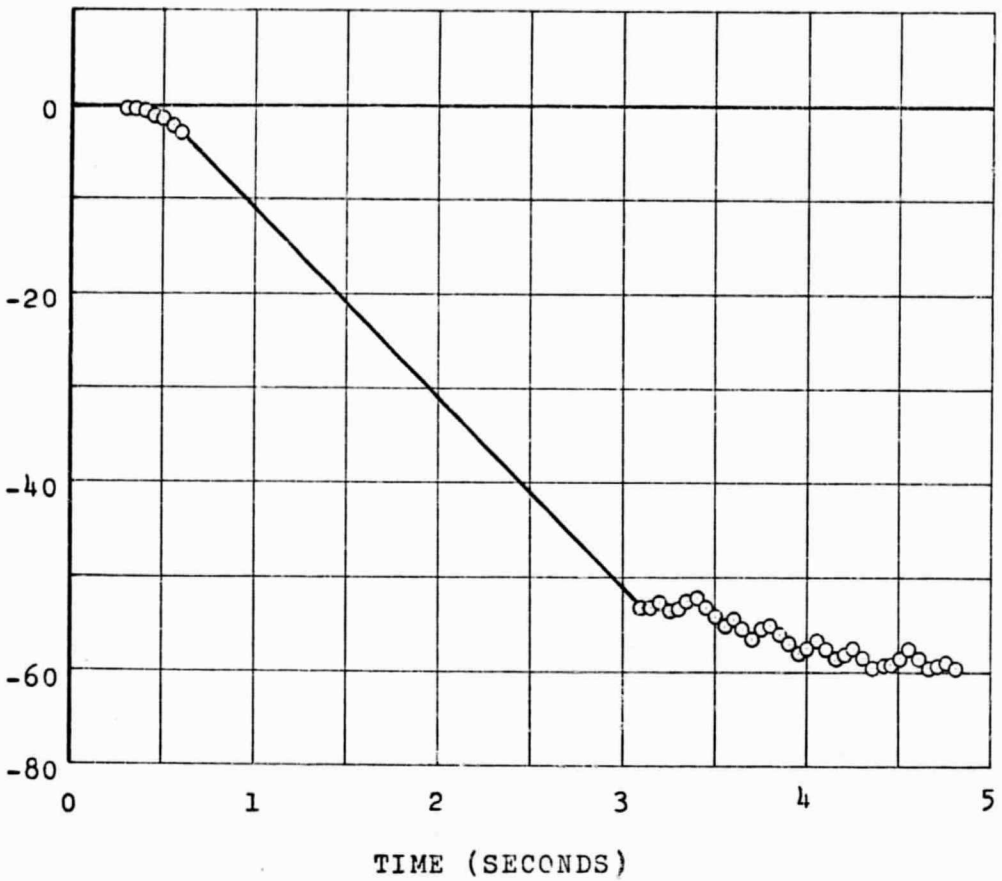


FIGURE 5-9c
CONTROLLABLE PARAMETER P, AT BURNOUT, P
UNCONSTRAINED

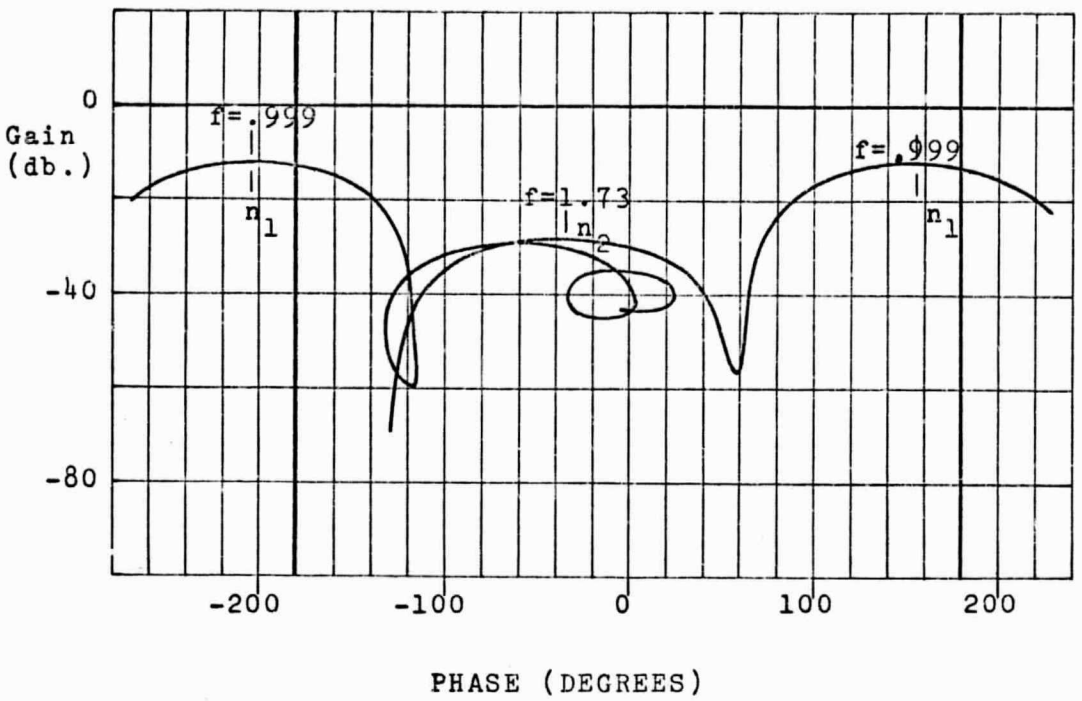


FIGURE 5-10

PHASE VS GAIN CHARACTERISTICS OF ADAPTIVE SATURN V AS-501 AUTOPILOT AT LIFT-OFF, P=1.0

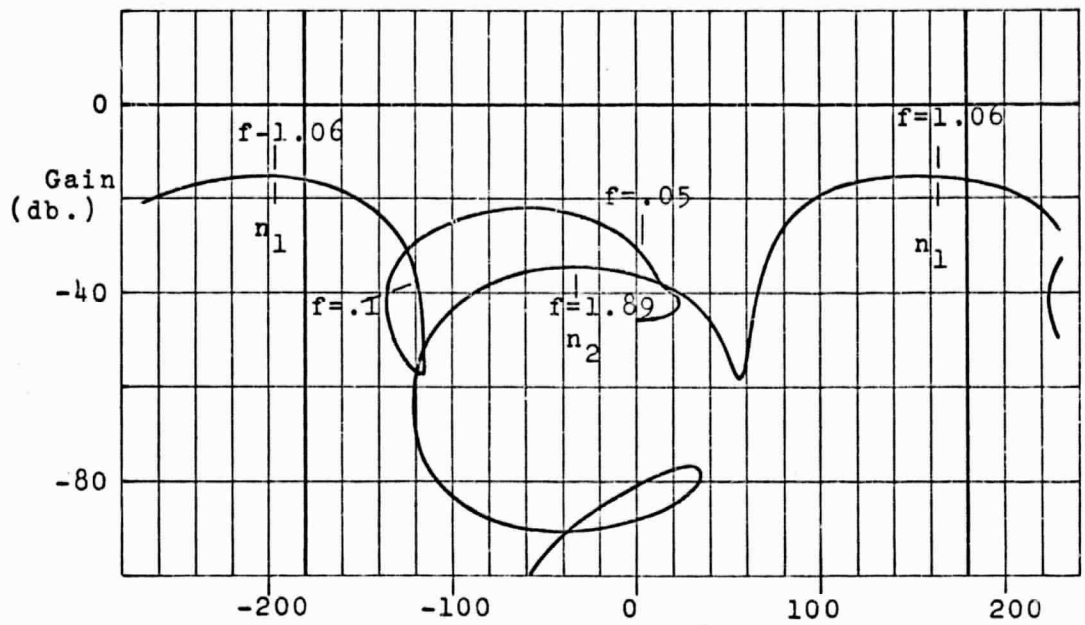


FIGURE 5-11

PHASE VS GAIN CHARACTERISTICS OF ADAPTIVE SATURN V AS-501 AUTOPILOT AT MAXIMUM DYNAMIC PRESSURE, P=1.0

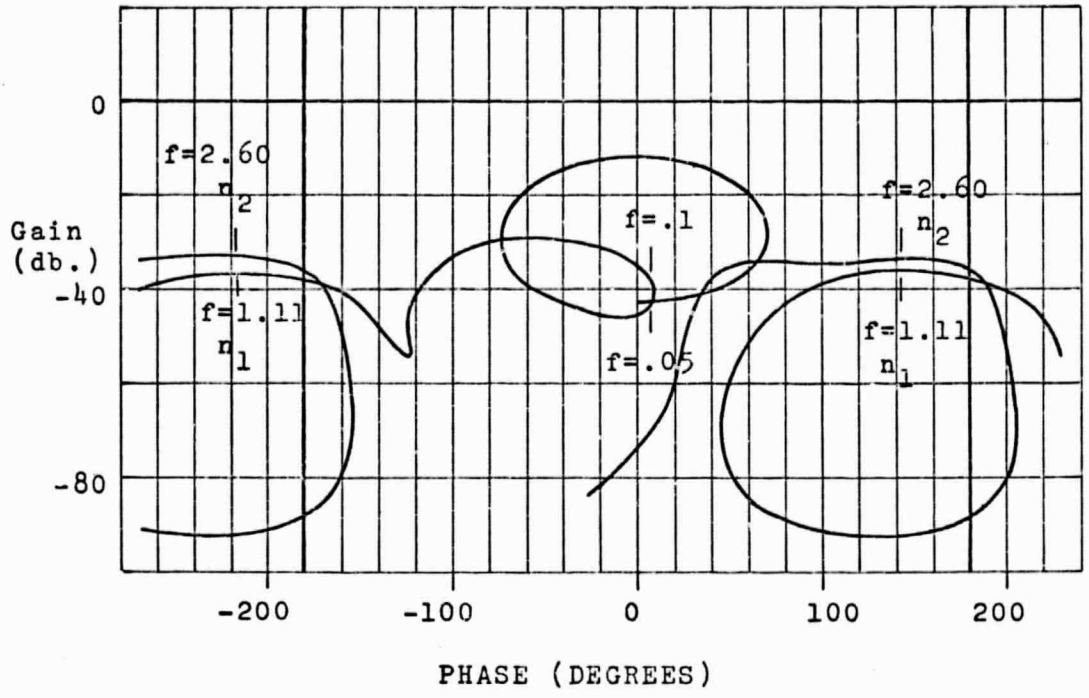


FIGURE 5-12

PHASE VS GAIN CHARACTERISTIC OF ADAPTIVE SATURN V AS-501 AUTOPILOT AT BURNOUT, P=-12.0

6. ANALYSIS

The responses of the flexible modes of the system may be expressed as

$$\eta_1 = k_1 e^{s_1 t} \tag{6.01}$$

and

$$\eta_2 = k_2 e^{s_2 t} \quad . \tag{6.02}$$

For the adaptive system the residues k_1 and k_2 and the complex frequencies s_1 and s_2 are functions of the controllable parameter

$$k_1 = k_1(P) \tag{6.03}$$

$$k_2 = k_2(P) \tag{6.04}$$

$$s_1 = s_1(P) \tag{6.05}$$

and

$$s_2 = s_2(P) \quad . \tag{6.06}$$

The adaptive policy

$$\dot{P} = u_P \tag{6.07}$$

$$u_P = -2k \int_{-\infty}^{t} W(t)\varepsilon(t)dt \tag{6.08}$$

is constrained to operate with a "knowledge" of the history and current state of the system; subject to this constraint the system will adapt to minimize the residues

associated with the response error, ε.

Figures 6-1 and 6-2 are root contour plots for variations in P at lift-off and at the point of maximum dynamic pressure. Analysis of Figures 6-1 and 6-2 indicates that the residues k_1 and k_2 will be minimized by positive adaptation of P, this adaptation will however, result in a reduction of the real components of the complex frequencies s_1 and s_2 and hence in a reduction in damping of the flexible modes. Figure 6-3 is a root contour plot for variations in P at burnout. Analysis of Figure 6-3 indicates that negative adaptation of P results in a reduction in residues and an increase in flexible mode damping for

$$-4.0 < P < 0.0 \ . \qquad (6.09)$$

For values of P less than -4.0 the damping of the second flexible mode is decreased.

In all three cases: lift-off, maximum dynamic pressure, and burnout, poles associated with the control modes showed the greatest variation with variations in P, and hence determined the direction of adaptation. This implies that the choice of compensation networks and control gains was critical in determining adaptive policy.

The results of this study demonstrate a major fault of the model reference technique. Variations in the controllable parameters manifest themselves in two ways: first by effecting the residues associated with the response error, and second by effecting the complex frequencies associated with the response error. The model reference technique employing the performance index

$$J(\underline{\varepsilon}) = \int_{-\infty}^{t} ||\underline{\varepsilon}||^2 dt$$

adapts the controllable parameters so as to minimize the residues, without regard to the effect on the complex frequencies. As a result, in attempting to minimize residues the adaptive mechanism may seriously reduce the stability margins or possibly force the system to become unstable.

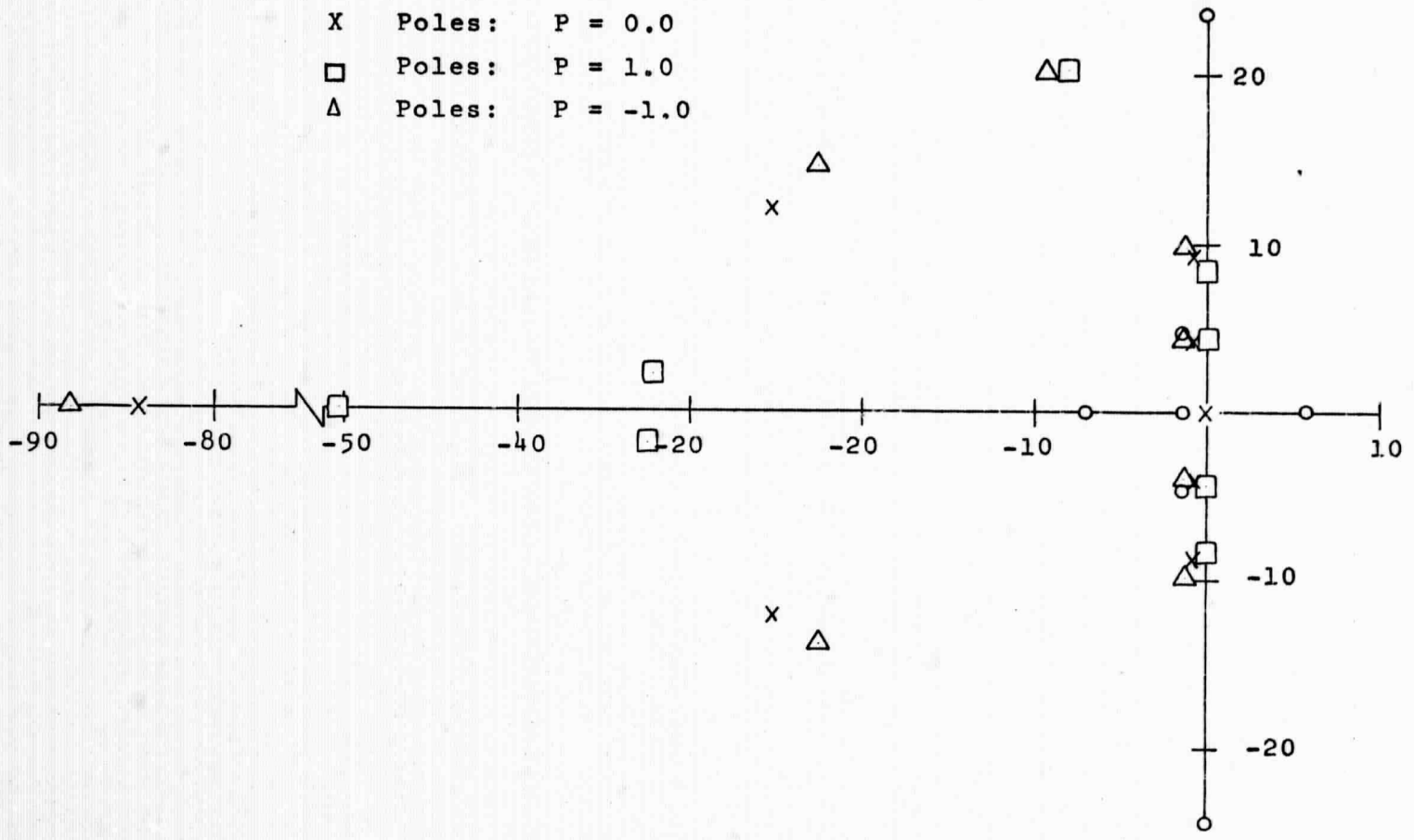


FIGURE 6-1

WEIGHTING FILTER ROOT CONTOUR FOR VARIATIONS IN P AT LIFT-OFF

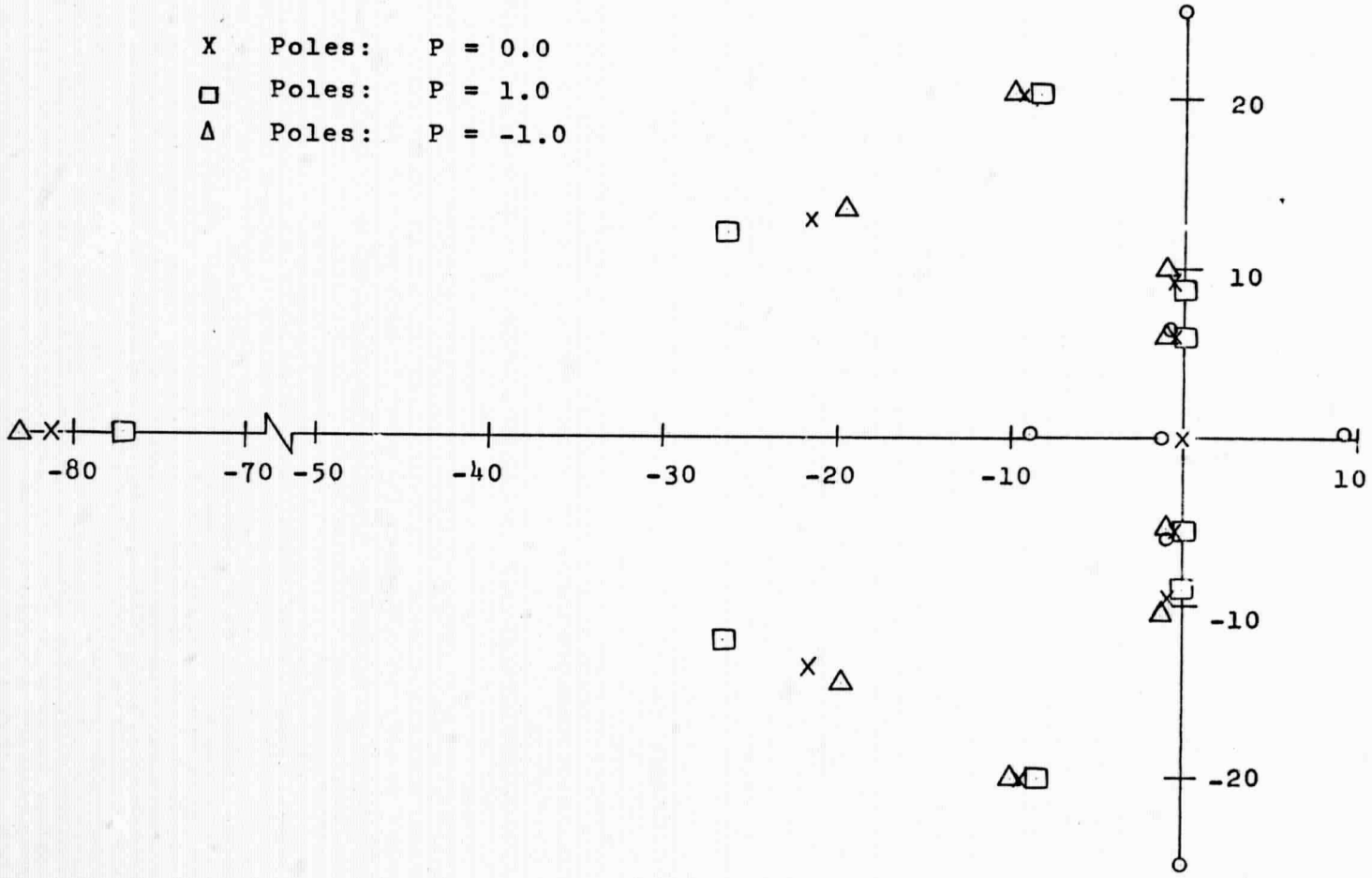


FIGURE 6-2
WEIGHTING FILTER ROOT CONTOUR FOR VARIATIONS IN P
AT MAXIMUM DYNAMIC PRESSURE

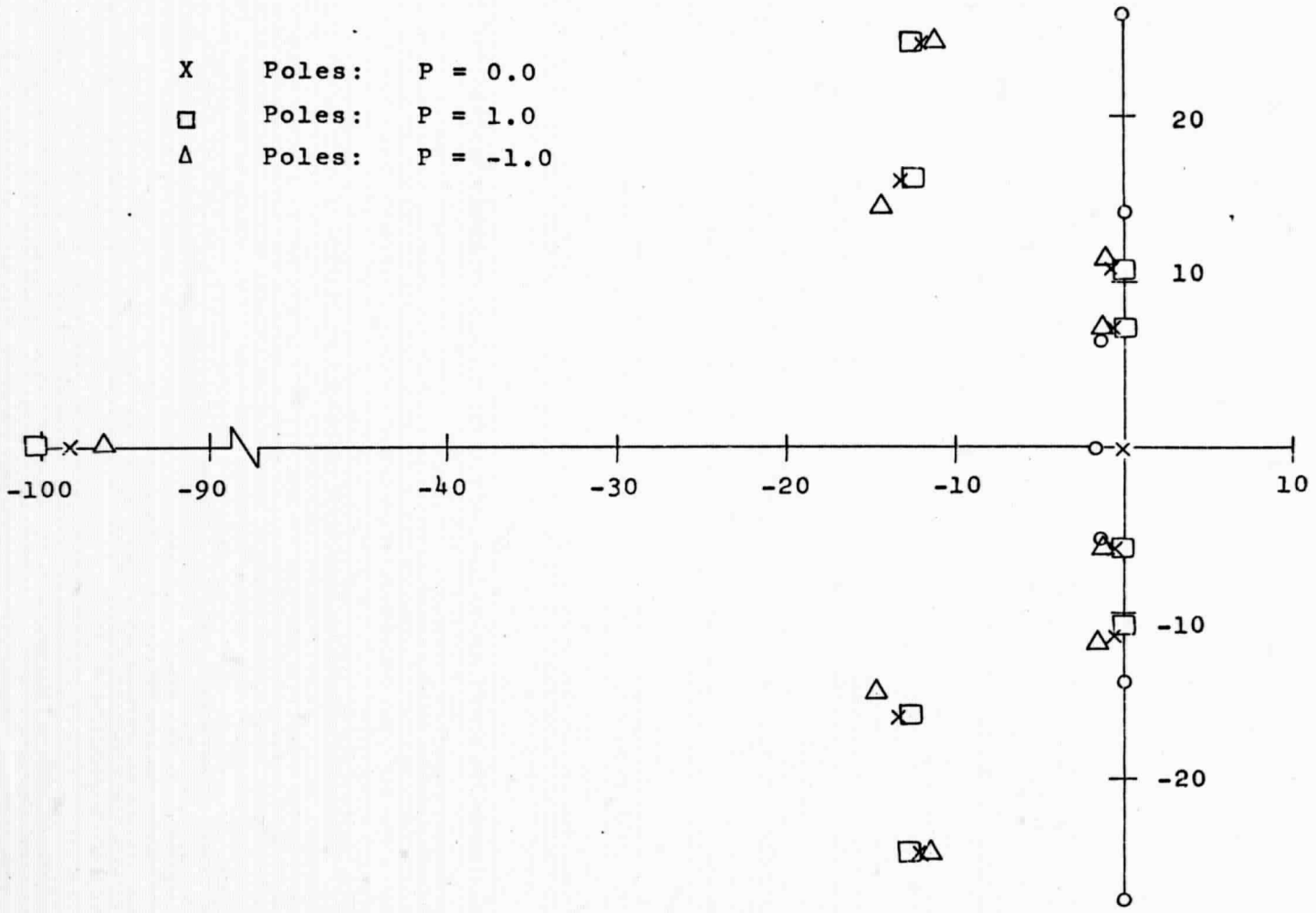


FIGURE 6-3
WEIGHTING FILTER ROOT CONTOUR FOR VARIATIONS IN P AT BURNOUT

7. CONCLUSIONS AND RECOMMENDATIONS

7.1 Conclusions

Based on the results of this study it is not recommended that the model reference adaptive technique discussed in this paper be employed to suppress the flexible response in booster vehicles. The study results clearly indicate that under certain flight conditions, adaptation may reduce the stability margins for the flexible modes and effectively excites flexible response. If no limitations are placed on the range of the adaptive parameter, adaptation may cause the flexible modes to become unstable.

It is apparent from the analysis of the system that the response of the adaptive mechanism is highly sensitive to the control mode response, and hence to the choice of compensation networks and control gains. With this in mind two approaches may be taken to develop a workable model reference system.

7.2 Recommendations for Further Study

An alternate approach to suppressing flexible response is to select an alternate design for the conventional compensation networks and control gains. Although no guidelines were established for making an alternate selection, the possibility always exists that some

alternate design will yield a system for which adaptation will provide improved stability margins.

A second approach is to choose an alternate performance index. Since the effects of parameter variations on damping do not manifest themselves immediately a choice of performance index which incorporated some form of prediction might provide a measure of the effects of parameter variations on damping. For sufficiently small τ

$$\varepsilon(t + \tau) \cong \varepsilon(t) + \tau \cdot \dot{\varepsilon}(t) \ .$$

Since $\varepsilon(t)$ and $\dot{\varepsilon}(t)$ may both be obtained by differencing the rate gyro signals, a more promising performance index might be

$$J(\varepsilon) = \int_{-\infty}^{t} (\varepsilon + \tau\dot{\varepsilon})^2 dt \ .$$

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SECTION V

V. AN APPROACH TO ON-LINE PARAMETER ESTIMATION AND ITS APPLICATION TO SATURN V

1. INTRODUCTION

In previous sections there has been described the combined estimation method via a Kalman filter of implementing the parameter adjustment. This combined estimation approach is not practical for on-line estimation because of the large number of simultaneous nonlinear differential equations required to be solved. However, inspection of the structures of the discussed combined estimation and the general adaptive controller has indicated that the state gains may not vary a great deal and may possibly be approximated by constants or linearly corrected values. A method of implementing such an approach is discussed in this section. It involves the use of a priori gains determined from equilibrium solutions to the riccati equation of the Kalman filter approach to combined estimation.

2. Equilibrium Solutions to the Riccati Equation

The determination of the equilibrium solution to the matrix riccati equation is often used for quadratic optimal control and for KALMAN (state) estimation. It has not been previously proposed as a way to get solutions to the combined estimation problem via Kalman filtering (CE/KF) theory. In fact, it is sure that one equilibrium solution

will not be satisfactory at all. As an alternative, it is proposed that a set of equilibrium solutions be used to determine a set of gains for a set of parameter perturbations. Then, in place of gains determined from a dynamic solution of the riccati equation, approximations to these gains may be used as a function of the parameter estimates. Note that parameters of the riccati equation may be both parameters and states of the plant. The explicit values of solutions to the riccati equation are not necessary. Instead only the gains determined from those equilibrium solutions are necessary. This reduces the number of terms that must be reviewed by the designer and, also, simplifies the design implementation.

There are many methods available for the equilibrium solution to the riccati equation. One that was found very useful was that proposed by O'Donnell (Reference 5). This was implemented by McDonnell-Douglas Corporation (MDC) in the form of a FORTRAN IV computer subroutine called R1CSS6 (Reference 2). In using this subroutine for an application, it was found that no equilibrium solution could be found when the $\underline{A}^*$ matrix (A^* was defined in part 3.2 of Section II) was used. In running out a hand solution to a first order example with one parameter, it was found that no equilibrium solution could be found directly. Instead some subtle limit argument had to be used to get a solution. Some terms were than added to $\underline{A}^*$

in the places representing time constants for the parameters, i.e., on the bottom part of the main diagonal. The terms were of a magnitude such that the time constants made no practical effect on the problem. The equilibrium solutions were then obtained via the RICSS6 computer subroutine. The physical meaning to the problem of obtaining the solution for the original $\underline{A}^*$ is that the parameter estimation error covariance was unbounded. Adding a term representing a time constant remedies that problem.

3.0 Discussion of Approach

The riccati equation for CE/KF does not, in general, settle to an equilibrium. This is why it has not been previously proposed to use the equilibrium solution for this problem. But, the "Newton is more important than Gauss" philosophy indicates that the use of the true values of the Kalman gains in state estimation is not essential to a Kalman filter. In fact, there has surely never been, and there never will be, a case where a Kalman filter was used in an environment where the plant and measurement covariances were really those used in the design of the filter. So, one should not be too surprised when the state estimator works satisfactorily when the gains are determined from equilibrium solutions of the riccati equation for CE/KF.

There remains the question of the parameter gains. There are several ways to implement the parameter gains. Indeed, the whole body of literature on parameter identification and MRAC is essentially addressed to this problem. With this background, it should not be surprising that the equilibrium solutions for the CE/KF approach should be able to be used to obtain these gains. One satisfying aspect of this is that, except for the lead or lag that would be introduced by the dynamic solution to the riccati equation, the gains will be of the right sign in this approach. The stability of a Kalman filter also shows that the overall approach will yield stable operation within a region where the linear approximation is sufficiently valid. This will then allow the adaptive loop to be stable for step inputs for any a priori specified parameter mismatch. This is the best that one can find for current MRAC systems design (Reference 6), and these systems assume perfect measurements and no plant noise.

In summary, it appears that the above approach will allow an adaptive controller to be designed in a procedural manner analogous to the manner in which quadratic optimal control and Kalman filtering are used to design for a known parameter linear system.

In the remainder of this section, the application of this approach to the Saturn V and the results achieved are presented and discussed.

4. Launch Vehicle Data

The following is a discussion of the plant, filter, and controller determination.

The aerodynamic and structural parameters are those of Appendix A and B at the end of this report. The state space matrices were computed from this data with the SPLAV computer program described in Reference 2. The A, B, and H matrices for a fifth order system are shown in Figure 4-1) at 80 seconds after lift-off which is the Max. q time. The 5 states were $\dot{z}$, ϕ, $\dot{\phi}$, η_1, and $\dot{\eta}_1$, i.e., rigid body and first bending mode states. The H matrix resulted from one attitude sensor and one rate sensor located between the third stage and the spacecraft, while the second rate sensor was between the second and third stages.

In some work recently conducted at MDC (Reference 2) on a Saturn V with a more detailed plant description, it was found that bending modes were relatively unimportant in the feedback. Hence, in this study we have used only rigid body feedback. Further, since the numerical values of the plant used here and in the cited study were nearly the same, the numerical values of a set of feedback gains on the rigid body which were determined in that study were used here. They are: .00132, .54401, and .96499 which multiply $\dot{z}$, ϕ, and $\dot{\phi}$, respectively.

The filter used to estimate states was fifth order to give estimates of the three rigid body states and the first bending mode. The estimation of the bending mode was used in order to get a better estimate of the rigid body states. The bending mode estimate was not used in the feedback.

The filter gains were determined from consideration of an adjoined system which included two unknown parameters. The parameters were the coefficients of the bending mode which are uncertain because of the incomplete knowledge of the bending mode natural frequency and damping ratio. Specifically, the two parameters were taken to be

$$a_{54} = -\omega_1^2$$

$$a_{55} = -2\xi_1\omega_1 \quad .$$

The equilibrium solution to a 7 x 7 riccati equation was solved using a subroutine designated as RICSS6 (Reference 2). Perturbations on some elements of the $\underline{A}^*$ matrix were conducted, which in this case is

$$\underline{A}^* = \left[\begin{array}{cccc|cc} & & & 0 & 0 & 0 \\ & \hat{A} & & & 0 & 0 \\ & & & & 0 & 0 \\ & & & & \hat{n}_1 & \hat{n}_1 \\ \hline 0 & 0 & 0 & 0 & \varepsilon_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \varepsilon_1 \end{array}\right]$$

Figure 4-1. Plant Matrices for Rigid Body and First Bending At Max. q Flight Time

$$A = \begin{bmatrix} -.015007 & 28.807 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ -.000371 & .18955 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -44.622 & -.13360 \end{bmatrix}$$

$$B^T = (17.687 \ , \ 0 \ , \ -1.1515 \ , \ 0 \ , \ 253.389)$$

$$H = \begin{bmatrix} 0 & 1 & 0 & .01463 & 0 \\ 0 & 0 & 1 & 0 & .00701 \\ 0 & 0 & 1 & 0 & .01463 \end{bmatrix}$$

It was found that perturbations on $\hat{a}_{54}$ and $\hat{a}_{55}$ made very little difference in the solution for the equilibrium P in this problem. Perturbations in $\dot{\hat{\eta}}_1$ had even less effect. The major effect came from perturbations in $\hat{\eta}_1$, but this changed, essentially, only the gains associated with the parameters, i.e., those corresponding to the parameter identifier. Hence, constant gains were used in the filter to obtain state estimates. The "Newton is more important than the Gauss" philosophy is a further justification for not accounting for the changes in the state estimator gains due to parameter changes.

The gains in the parameter estimator changed signs with a sign change in $\hat{\eta}_1$, as might be expected from past MRAC work. This is because this gain is analogous to the output of a "weighting function filter" as originally discussed by Osborn in Reference 3. The presently discussed approach is a generalization of this to multiple output systems. However, here we have not resorted to a steepest descent analysis of a deterministic system, but, instead, we use a relinearization of the error covariance equation. The qualitative results of this approach are a gain which can be fairly well approximated by a linear function of $\hat{\eta}_1$ for small values of $\hat{\eta}_1$, and a constant for larger values. It is interesting that the direct application of Osborn's approach yielded a gain ("weighting function filter' output) proportional to

to the system input, and experimentally it was determined that a limiter on this gain gave better results. The present approach yields the same qualitative result analytically, and in addition, specifies the values for the limiter settings.

The simulation was done with discrete equations. This caused a problem with the filter gains because they are close to the values obtained from the Riccati equation only if the time interval is made very small. That is, it is not sufficient to merely compute the transition matrix to good accuracy for a reasonably small time interval, as is the case for simulation of a continuous linear system by discrete equations. This will become evident by considering the following first order example.

Suppose the (scalar) plant equation is

$$\dot{n} = an + w \tag{4.01}$$

and the measurements are

$$y = hx + v \quad . \tag{4.02}$$

The riccati equation for the estimation error covariance is then

$$\dot{p} = 2a\,p - p^2\,\frac{h^2}{r} + q \tag{4.03}$$

from which the equilibrium solution (taking h = 1 with no loss of generality) must satisfy

$$\frac{p^2}{r} - 2a\,p - q = 0 \qquad (4.04)$$

This equation will be compared to the analogous equation for the discrete error covariance.

Let C* denote the covariance of the discrete plant noise, and let C be the covariance of the discrete measurement noise. These are related to the continuous system noise parameters within $O(T)$ by

$$C^* = q\,T \qquad (4.05)$$

and

$$C = \frac{r}{T} \qquad (4.06)$$

The (scalar) discrete covariance equations in equilibrium are

$$k = \frac{ph}{C} \qquad (4.07)$$

$$p = pb - k\,hpb \qquad (4.08)$$

$$pb = \phi^2 p + C^* \qquad (4.09)$$

where, pb and p are the apriori and posteriori error covariances, respectively. Eliminating k and pb from these equations, we have (for h = 1)

$$p^2\,\frac{\phi^2}{r} - p\left(\frac{2 - 1}{T} + \frac{q}{r}\,T\right) - q = 0 \qquad 4.10)$$

Now, within $O(T^2)$, $\phi^2 = 1 + 2aT$ so that this becomes

$$\frac{p^2}{r}(1 + 2aT) - p\left(2a + \frac{q}{r}T\right) - q = 0 \qquad (4.11)$$

Hence, the latter approximates the continuous covariance equation only if

$$|2aT| << 1 \tag{4.12}$$

and

$$\frac{q}{r} T << |2a| \tag{4.13}$$

Putting a in terms of a time constant, we may write a = 1/T.C. If T is chosen as T = k(T.C.) for k << 1/2, then $|2aT|$ is 2k, and equation (4.12) is satisfied. But, T q/r is k(T.C.) q/r whereas $|2a|$ is 2/T.C. For many nonpathelogical choices of T.C and q/r, we see that Equation (4.13) may not be satisfied. Hence, the discrete equation may not approximate the continuous equation even when T is chosen to be much smaller than the system time constant.

This problem was encountered in the launch vehicle simulation. The Kalman continuous gains for attitude measurement errors were very different values (even in sign) from those computed from the discrete equation when a time step of T = .01 seconds was used. Making T = .001 seconds made the discrete values somewhat more like the continuous values (they were at least the same sign). Since a value of T = .001 is impractical for both a digital simulation and for an on-line control computer, the Kalman gains for the simulation were determined from equilibrium solutions to the discrete equations.

5. Covariance Matrices

The only parameters that are required to specify the state and parameter estimators at the max. q condition are the covariance matrices for the plant disturbance, measurement noise, and parameter changes. These parameters completely specify the filters because only steady solutions to the riccati equations are used, thereby not requiring initial error covariances. There are two ways in which to view the plant being disturbed. One is through wind disturbance and another is through considering engine gimbal angle as a random process. Both of these were considered and it was determined that the latter was more critical. The details will not be shown here, but the basis for the comparison will be explained.

The measurement noise was considered to be the second bending mode response. Consider, for a moment, the state estimation problem. For inputs of wind or engine gimbal angles, the response of the second bending mode will be proportional to the input, as will be the rigid body and first bending mode. Hence, as the input magnitude increases, both the Q and R (in the riccati equation) will also increase in proportion. It is easy to show that this will cause P to increase in proportion, but the Kalman gain, K, will remain constant. To put it another way, if $QH'R^{-1}$ is a constant, then so is K. Further, we would not want the

state estimates to be changed too fast in response to the measurement noise. This leads to choosing the disturbance for which $QH' R^{-1}$ is the least of all possible values. This was the basis on which engine excitation was chosen as the plant noise source.

Now consider the combined estimation problem, instead of only state estimation considered above. The relative sizes of Q and R are not now important because the covariances associated with parameter changes are not functions of the input noise magnitude. This makes the combined estimation problem more difficult than state estimation, since the noise magnitude remains as a filter parameter. However, we can still normalize R and the state portions of Q, and then carry the normalizing factor along in the parameter part of Q. This is what will be done.

Measurement Noise Covariance

In this application, the second bending mode was assumed to be the source of the measurement noise, Roundoff errors are small by comparison. We take the variance coefficient, r, of a measurement as being the magnitude of the sensed second bending mode dc response to an rms engine defection. The value of 1° was chosen because Reference 2 states that a standard deviation engine deflection, S_{β}, of 0.65° appears possible whereas 1.3° was obtained with the currently used flight control system. Writing

$$\ddot{\eta}_2 = 2\zeta\omega_2 \dot{\eta}_2 + \omega_2^2 \eta_2 + B_7\beta \tag{5.01}$$

we have

$$dc_2 \triangleq \frac{(\eta_2)ds}{\beta} = \frac{B_7}{\omega_2^2} \tag{5.02}$$

$$= \frac{477.2}{150.} = \underline{3.18} \tag{5.03}$$

The actuator is well approximated by a second order transfer function with a break at 5.5 hertz and a first order transfer function with a break at 7 hertz. It is roughly approximated with a first order transfer function with a break at 6 hertz, which will be used in the following calculations.

The spectral density of the input to the actuator is assumed to be white noise with a spectral density of G_β. The output variance for a first order filter with a break at ω_o will be $G_\beta\ \omega_o/2$, so that

$$G_\beta = \frac{2\ s_\beta^2}{\omega_o} = \frac{2(1/57.3)^2}{2\pi 6} = \frac{2(1/57.3)^2}{2\pi 6} = \underline{.16 \times 10^{-6}}(\text{sec}) \tag{5.04}$$

The normalized value of r_{11} is

$$\overline{r_{11}} = \frac{r_{11}}{G_\beta} = (Y'_{\phi_2}\ dc_2)^2 \tag{5.05}$$

$$= (3.18)^2 (1.27) \times 10^{-3})^2$$

$$= \underline{.163 \times 10^{-4}} \qquad (5.06)$$

The normalized spectral density of the rate measurement noises are similarly computed. For the first rate sensor

$$\bar{r}_{22} = \left(Y'_{\phi 2} \left|\frac{\dot{\eta}_2}{\beta}\right|\right)^2 \qquad (5.07)$$

$$= (Y'_{\phi 2}\ \omega\ dc_2)^2$$

$$= (.00513(3.18)\omega)^2$$

$$= \underline{.262\ \omega^2 \times 10^{-3}} \ . \qquad (5.08)$$

and for the second rate sensor,

$$\bar{r}_{33} = \left(Y'_{\phi 2} \left|\frac{\dot{\eta}_2}{\beta}\right|\right)^2 \qquad (5.09)$$

$$= \bar{r}_{11}\ \omega^2$$

$$= .163\ \omega^2 \times 10^{-4} \qquad (5.10)$$

Since we are concerned with state and parameter estimation in the range of 1 cps, we choose ω as 6.28 so that $\bar{r}_{22}$ and $\bar{r}_{33}$ $.104 \times 10^{-1}$ and $.644 \times 10^{-3}$, respectively.

Plant Disturbance Covariances

Since the engine gimbal angle acts through B, we have for the state covariance coefficient

$$Q = B\, B'\, G_\beta \tag{5.11}$$

so that the normalized state covariance coefficient is

$$\bar{Q} = B\, B' \tag{5.12}$$

Parameter Change Covariances

We will now derive the relation between the change in a parameter and the covariance coefficient of the white noise process assumed to govern its rate of change. The parameter rate of change is assumed to be given by

$$\dot{a} = w$$

where, w is a mean zero white noise with a covariance coefficient of q. The variance in the change of a in the time interval T is (assuming the initial time to be zero with no loss of generality)

$$\text{Var}\,(a(T)) = E\left(\int_0^T w\,dt_1 \int_0^T w\,dt_2\right) \tag{5.13}$$

$$= \int_0^T q\,dt = qT \tag{5.14}$$

The parameters of interest in the present study are

$$x_6 = a_{54} = -\omega_1^2 \tag{5.15}$$

and

$$x_7 = a_{55} = -2\zeta\,\omega_1 \tag{5.16}$$

It is noted that if we take ζ as a constant and ω_1 as having a gaussian rate of change, then a_{55} has a gaussian rate of change. But, a_{54} will not change in this manner, and neither will the cross distribution. However, it was found that the variance for changes of a_{54} was not much different when computed for changes in ω_1 or equivalent changes in a_{54}, so the computation of q_{66} will be shown only per equation (5.15). On the other hand, the cross covariance will be computed subsequently, and not just assumed to be zero. This approach is not strictly to theory, but the original assumption that the parameter rates are governed by gaussian white noise processes should not be taken that literally. The gaussian noise assumption could be carried through strictly with ω_1 and ζ as the unknown parameters, but that approach is not pursued here.

If we write Var (a(T)) as $s_a^2(T)$, then $s_a(T)$ is a standard deviation of a change in the parameter a after a time period T. The covariance coefficient is then

$$q = \frac{s_a^2(T)}{T} \tag{5.17}$$

The assumed $s_a(T)$ for certain T's are shown in Figure 5.1, along with the resulting q's, for a_{54} and a_{55}. Each is shown for two conditions; lift-off and maximum dynamic pressure (max. q). Liftoff is a parameter mismatch condition, whereas max. q is a parameter tracking situation. That is why the T's are much smaller for the lift-off cases because this is

Figure 5-1 Normalized Disturbance Covariance Coefficient Matrix for the Engine Gimbal Angle on the Rigid Body and First Bending Mode

	Max. q			Lift-Off		
	$T_{(sec)}$	$s_a(T)$	$q(T)$	$T_{(sec)}$	$s_a(T)$	$q(T)$
a_{54}	140	20.5	3.	1	20.5	420.
a_{55}	140	.0267	4.85×10^{-6}	1	.0267	6.78×10^{-4}
Cross-Correlation	140	1.4	$.383 \times 10^{-2}$	1	1.4	.538

really an adaptation time rather than a time taken for a vehicle parameter to change. Also shown in the figure are the cross covariances which were computed in the following described manner.

If we write

$$\Delta a(t) = a(t) - a(0) \tag{5.18}$$

then, denoting the initial value of ω_1 by ω_o,

$$\begin{aligned} -\Delta a_{54}(T) &= \omega_1^2 - \omega_o^2 \\ &= (\omega_o + \Delta\omega)^2 + \omega_o^2 \\ &= 2\omega_o \Delta\omega - \Delta\omega^2 \end{aligned} \tag{5.19}$$

$$-\Delta a_{55}(T) = 2\xi \Delta\omega \quad . \tag{5.20}$$

Then, the covariance is

$$s_{67}^2(T) = E(\Delta a_{54}(T) \quad a_{55}(T)) \tag{5.21}$$

$$= E(4\xi\omega_o \Delta\omega^2 + 2\xi \Delta\omega^3)$$

$$= 4\xi\omega_o E(\Delta\omega^2)$$

$$= -2 a_{55}(0) s_\omega^2(T) \tag{5.22}$$

Thus, the cross covariance coefficient is

$$q_{67}(T) = - \frac{2 a_{55}(0) s_\omega^2(T)}{T} \tag{5.23}$$

Each of these coefficients were normalized when used in computations, i.e., they were divided by G_β.

6. Results of Saturn V Launch Vehicle Design

The Saturn V launch vehicle design was analyzed in the frequency domain to check its stability, gain and phase margins, and parameter sensitivity. This was, of course, for a filter with state gains only, i.e., a nonadaptive system.

A simulation which included the time dependent solution to the riccati equation for the estimation covariance was used for two purposes. One purpose was to obtain a reference performance by which to judge the a priori gain system. The second purpose was to determine a time at chich to switch from "lift-off" gains to max. q gains. It turned out to serve another purpose, i.e., to reveal that some of the discrete system gains were not close to being the same as the continuous system gains. This problem has already been discussed in a previous section. Hence, equilibrium solutions to the discrete equations for fixed values of the bending mode states were used to determine gains for the max. q simulation. Further, selected transient solution values for these fixed states were used to determine "lift-off" gains, which were found to be in about the same relation to the max. q gains as was the relation for those found from the continuous equation. A simulation of the a priori gain system was also conducted to show final verification of the approach applied to the launch

vehicle. It turned out that finding the equilibrium solution to the riccati equation showed the reasons for some problems in the on-line gain system.

Frequency Domain Analysis

An updated version of the SYANAL computer program (Reference 2) was used to obtain, among other things, the closed loop poles and the open loop frequency response of the system.

The results show the system to be very stable and nearly (bending mode) parameter insensitive. The latter results from the value of R that was used in the filter. A larger value for R would cause the system to be more sensitive to bending mode parameters because values of the Kalman gain would be reduced. That is, the system could have withstood more second bending mode signal in the sensor signals, or less sensors. The parameter insensitivity does, however, preclude the real need for adaptive control of the bending mode parameters, but it will still be pursued here.

The high degree of stability of the design is caused by the control gains that were used. As mentioned, these were taken from a previously designed controller. However, the Nyquist diagrams in that report were upside down due to a sign error, and when turned over show that the design has a (too) high degree of stability.

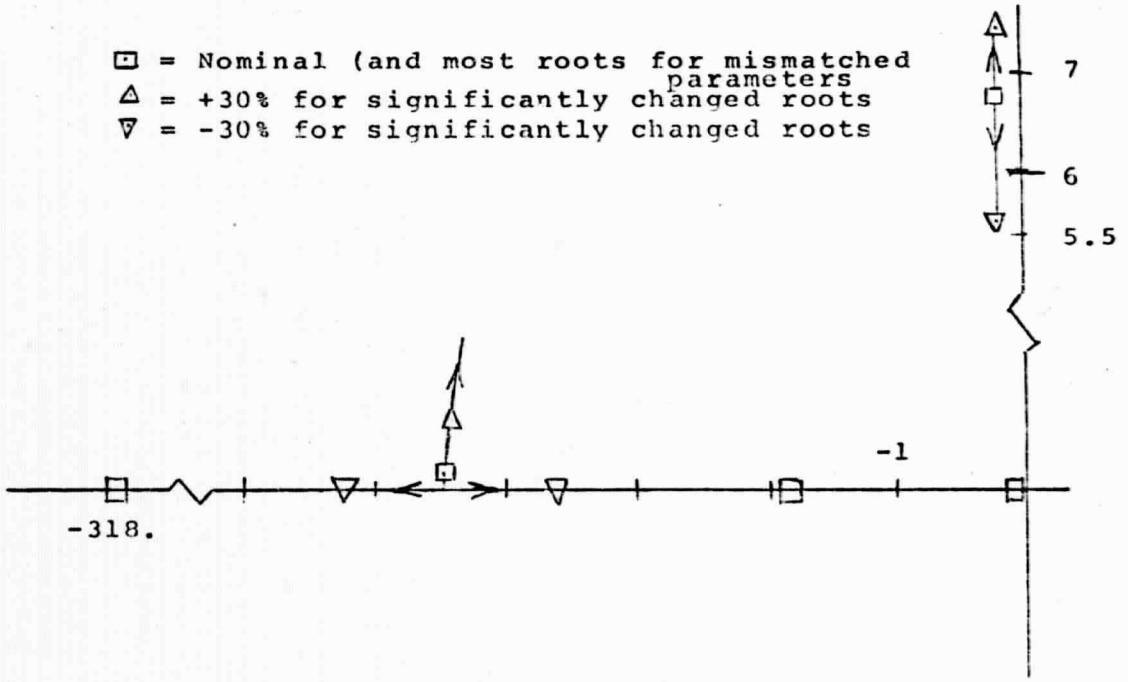
In spite of these shortcomings, the system design has gain stabilized the first bending mode and provided a stable rigid body. This will be adequate to show the validity of the adaptive method which is the prime concern of the discussion.

The closed loop roots were obtained for nominal parameters being studied, individually and together. The results are shown in Figure 6-1. These are for max. q filter gains, but the use of lift-off gains gave essentially the same results, lending credence to the "Newton is more important than Gauss" philosophy of filter design.

The figure shows that most of the roots are (nearly) invariant to changes in the first bending mode parameter $a_{54} = -\omega_1^2$. The first bending mode roots do change, of course, and the corresponding roots caused by the filter also are moved. But, for $\pm$ 30% change in the parameter, no indication of instability occurs. Perturbations of $a_{55} = -2\zeta\omega_1$ (not shown) resulted in much less change to the roots.

The open loop frequency response (opened at engine command angle) is shown in Figure 6.2 for the system at max. q conditions.

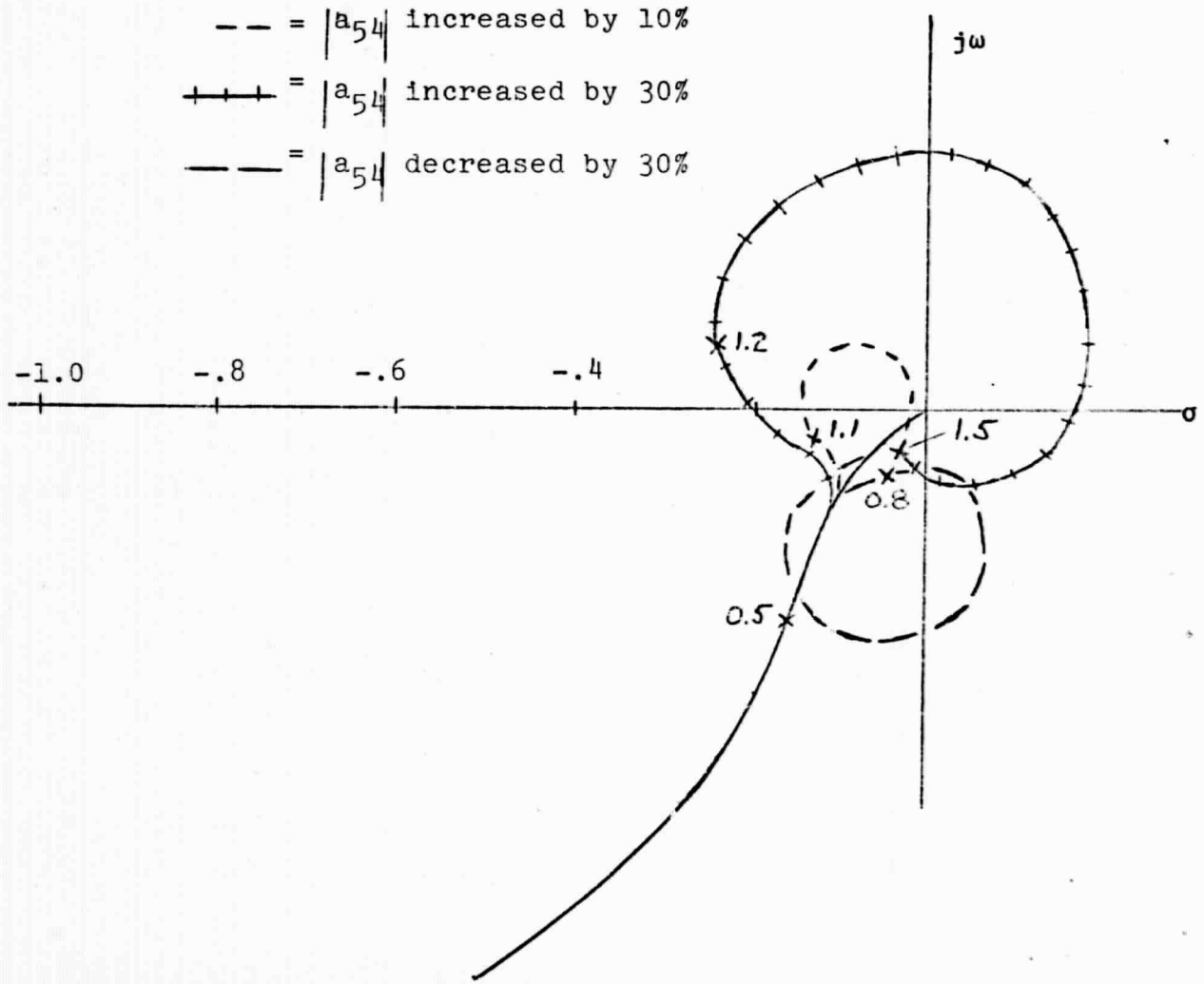
The results show the system to have about 50° of phase margin, which is virtually invariant to the bending mode parameter perturbations. The gain margin is reduced by



(roots with negative imaginary parts not shown)

SYSTEM CLOSED LOOP ROOTS (REAL, IMAGINARY)
FOR NOMINAL AND +30% MISMATCHED PARAMETERS

FIGURE 6-1



FREQUENCY RESPONSE OF OPEN LOOP SYSTEM

FIGURE 6.2

positive changes in the magnitude of a_{54}, but a 30% change still leaves a 14 db. margin.

The lag introduced by a computing interval of .01 seconds adds only .54° at .15 hertz, which is an insignificant reduction in phase margins. A 4° lag at 1.1 hertz actually increases the gain margin of the perturbed system.

Simulation Results

The simulation of the combined estimation problem via the on-line gain system was initially performed to determine a switching time for the gains in the a priori-gain system, and for a comparison of adaptive performance. However, it was found that the state gains reached nearly constant values after about 2 seconds, and that those corresponding to the attitude position measurements were much different from those obtained from the riccati equation equilibrium solution. The reason for this was traced to the fact that very small computing intervals may be required to get the discrete covariance equation to yield the same values as the continuous equation. This has been shown in a previous subsection via a first order example. It was verified in the larger order problem by rerunning the discrete covariance equation computer program for a smaller computing interval. But it is too costly in computing time to use a computing interval less than .01 seconds, sc the discrete gain equations were then used to compute Kalman gains (for states and parameters) for the discrete simulation.

(Implementation of such a system via a digital controller would require the same procedure.) In addition, as mentioned previously, the transient solution of the discrete gain equations for fixed bending states were used to determine the "lift-off" gains for the adaptive loop. Simulation running times were for 5 or 10 seconds of flight time.

Computer time is used rapidly in these simulations, especially for the on-line gain system, so this phase was used only when absolutely necessary. Of course, an analog simulation of the system would be useful for full flight time design verification, fine tuning and failure mode checking.

The on-line gain system solves the riccati differential equation as a function of time and system variables to determine the filter gain. This is an impractical approach because of the order of the differential equations that result, i.e., $n(n+3)/2$ for an n-th order system (including parameters). A simulation of the analogous discrete system was conducted in this study to determine a point (in time) at which to switch gains in the a priori gain system. The simulation also offers a result by which to compare the adapting qualities of the a priori-gain system.

In the a priori gain system we determine the filter gains off-line by using the equilibrium solution to the riccati equation. These gains are, in general, functions of the system parameters and states. However, the "Newton is more important than Gauss" philosophy indicates that using the exact values for the filter gains is not critical. Hence, we use approximations to the gains which are obtained from the equilibrium solution to the riccati equation.

In the present study, it was found that nearly all of the state-gains were very insensitive to parameter changes and all were very insensitive to the states which occurred in the riccati equation. Those state-gains which were sensitive varied in a hyperbolic manner, and thus did not change sign. Because of this, they were approximated by constants corresponding to nominal parameter gains. It should be realized that the more sensitive gains will be the first place the designer should look if a system turns out to have too much parameter sensitivity.

The parameter-gains were all very insensitive to parameter changes and to the first bending mode rate variable. Conversely, they were very sensitive to the bending mode position variable. In fact, their signs were determined by that variable, all parameter-gains having the same sign as the first bending mode position variable.

It was found that the on-line gain system could adapt a 105 mismatch in a_{54} in about 1.9 second, and increasing this mismatch up to 505 made hardly any increase in the adapting time. This simulation was done with two sinusoidal dither signals, one at .5 hertz and one at 1.5 hertz, each with a 1° amplitude. Another simulation with a (nearly) gaussian random input with a 1° standard deviation cut this adapting time approximately in half. A larger amplitude sinusoid was also found to decrease the adaption time, of course.

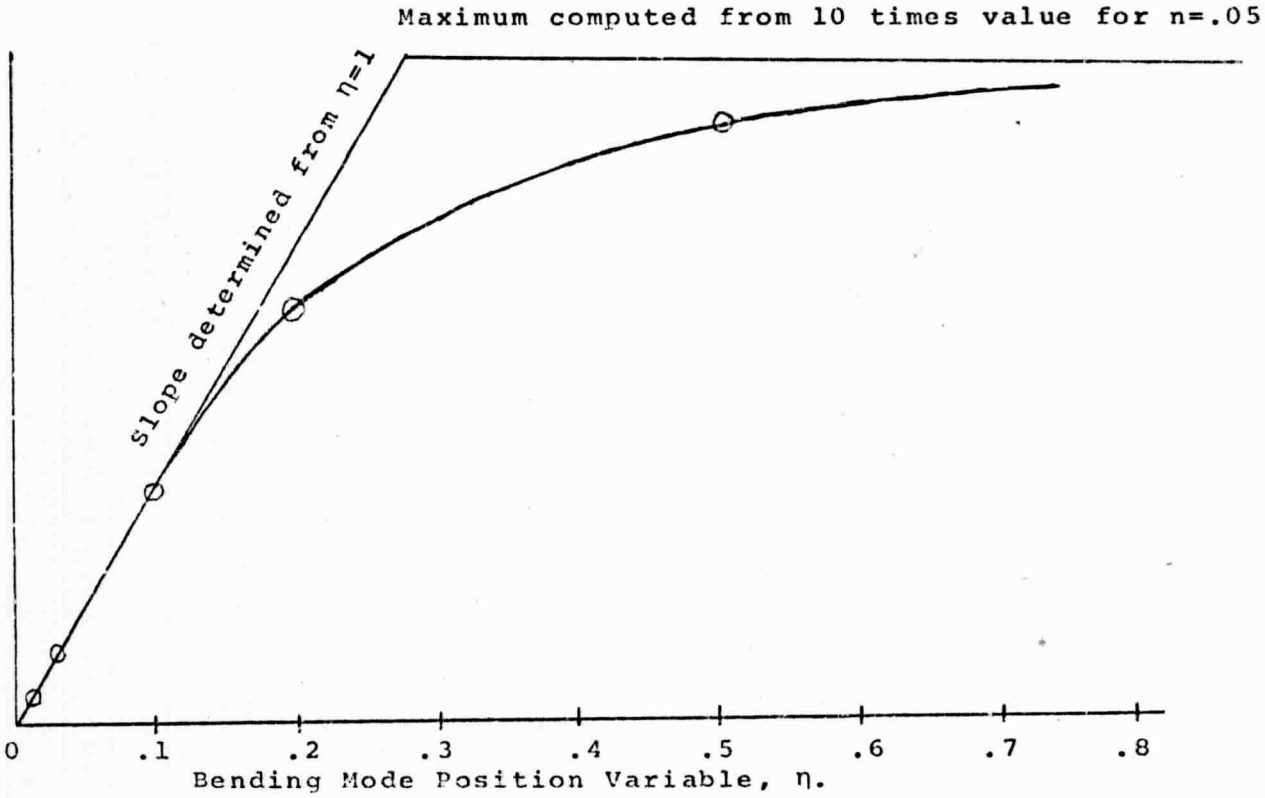
In both of the cases, the adaption of the a_{55} parameter resulted in about a 55 bias. The probable reason for this is the lack of system sensitivity to the estimate of a_{55}. This was exhibited in the root locus and Nyquist results. Also, inspection of a standard Bode plot for a second order system shows the high sensitivity to natural frequency and the low sensitivity to damping ratios when the damping ratio is less than 0.1 (the value of 0.01 was being used in the problem).

From these results and some contemplation about the physical problem, a switching time of about 2 seconds is a reasonable specification, provided that enough signal strength is available in the 1 hertz range in that initial time interval. There should be no practical problem in providing dither signals of the type described above during the first two seconds of launch of a large launch vehicle.

A representative plot of a max. q parameter gain is shown in Figure G-3. It is seen that a linear gain is a good approximation for the value of the bending mode position variable less than about 0.3, and that a maximum gain is attained by a value of about one. The maximum gain was found to be about 10 times the .05 value of bending mode position. The same kind of functional relationship was found for gains obtained from the discrete equations. Gain slopes were determined from gain data corresponding to the bending mode position value of 0.1, as shown on the figure.

The determination of lift-off gains was not as simple as were the max. q gains. A plot like the above figure was made for each .05 seconds of flight time for the first second or so. Then the range of the bending mode position variable was considered for that period of time. From this a very approximate slope and maximum value were determined. This was done graphically for a few gains, and then a criterion of selection was decided upon. It was to use the t = 0.4 data for a bending mode position value of 0.1, the maximum gain being taken to be the gain corresponding to that position value, and the slope being 10 times that gain value.

Simulation results for a "lift-off" gains showed about 1.2 second adaptation time for 10%, 30%, and 50% mismatch of a_{54} when the above described 1^{o} amplitude sinusoidal dither



REPRESENTATIVE PLOT OF PARAMETER GAINS
FOR VARYING FIRST BENDING MODE POSITION VARIABLE

FIGURE 6-3

was used. The a_{55} parameter adapted equally rapidly, but to a value which had about a 5% bias (as in the on-line system).

For max. q gains, the a_{54} adapter followed a 1% per second parameter change with about a 6% lag, after 10 seconds but a_{54} was only designed to track a standard deviation of about .3% per second. The max. q gains could zero out a 10% mismatch of a_{54} in less than 10 seconds. The a_{55} adapter did not (for practical purpose) respond to the 1% per second ramp until about a 3% error was present. It lagged the ramp by about 8% after 10 seconds. A 10% mismatch was reduced to less than 7% in 5 seconds. The a_{55} adapter was designed for about a .15% per second standard deviation.

The effect of the actuator on system stability is very small, causing less than 5° of phase lag at frequencies below 0.6 hertz. In a simulation of the above system with the actuator added to the plant, the results were, for practical purposes, identical to the system without the actuator. For this reason, the actuator dynamics were not included in any subsequent simulations.

A series of simulations were made with a seventh order plant; consisting of three rigid body states and two states each for first and second bending modes. A single sinusoid with a 1° amplitude and a frequency of 0.6 hertz was used for excitation.

First, the model was kept at fifth order and maximum q gains were used in the filter. It was found that the parameter estimates had large nearly sinusoidal oscillations with

amplitudes corresponding to about 20% error in the estimate of ω_1, and a frequency of about 3.3 hertz. The control far exceeded the limits of $\pm 5^{\circ}$ as a result. This was an unexceptable design structure because lower gains on the parameter adjustment to decrease the oscillations would cause the adapting time to be too small. The reason for this difficulty is that it was assumed in the design that the second bending mode had a flat frequency response at its dc value, when in fact it increases more than 30 db at about 2 hertz. Also, it is not an independent white noise, as assumed, but a correlated colored noise. The simulation then showed that the assumptions were too strongly violated to get a satisfactory result with this design.

If the second bending mode was added to the filter with parameters the same as the plant, there would occur a perfect cancellation of second bending in the measurements and the results would be for practical purposes the same as when second bending had not been included at all in the simulation. This is not practical, however, because the second bending mode parameters are not shown exactly. But, knowledge of the approximate values of the second bending mode parameters can be used to filter out the second bending mode signals which are in the measurements. The most logical approach for this is to increase the dimension of the filter to include the best estimates of the second bending mode dynamics. In the subsequent simulations it was assumed that

the estimate of ω_2 was taken to be 10% larger than its actual value. The generalization of this approach will be discussed in the sequel.

Some non-zero values had to be assigned to the Kalman gains which drive the second bending mode estimates. Two approaches to gain setting were tried to see just how critical the Kalman gain values might be. The steady-state solution to the discrete covariance equation was used to determine the ordinary Kalman filter state gains using the same R as was used previously, and with Q computed in the same way as before, i.e., BB', but with a corresponding increase in dimension because of the inclusion of the second bending mode. The first Kalman gain that was used in the simulation was one which was the same as before except for the addition of some values for the second bending mode, the latter being those obtained from the above computation. The second set of gains were for all of the state gain values obtained in this previous calculation, and parameter gains unchanged from before. It turned out that, even though these state gains differed by factors of about 4 to 100, the simulation results were usually about the same, with the second set of gains yielding the better parameter identification (having the smaller state gains). These results are described in the following:

For max. q gains on the parameters and no initial parameters and no initial parameter mismatch, the estimated parameter values deviated from their actual values by less than 1%. A mismatch condition was not simulated for this case.

For L.O. gains on the parameters and no initial parameter mismatch, the estimated parameter values deviated from their actual values by an amount corresponding to about a 10% error in the estimate of ω_1 (being a few percent smaller for the second set of state gains). When an initial mismatch of 10% of ω_1 was used, the parameter estimation errors crossed zero in less than 1.4 seconds for the first set of state gains, and in less than one second for the second set.

A final experiment was conducted, which yielded some rather pleasing implications. It was reasoned that the estimate of the first bending mode amplitude is more likely to be the same sign as the actual variable when the estimate is large. Therefore, if all of the L.O. values of slopes (with respect to η_1) on the parameter gains were reduced, then the sensitivity to bad bending state estimates of the parameter adjuster could be reduced. An increase in adaptation time would also result, but not in the same proportion. This was verified by reducing all slopes by a factor of 10, the effect being much more pronounced with the second set of gains and will be the only case discussed in detail.

For the second set of state gains and 0.1 times the L.O. gains, the following results were observed. When there was no initial parameter mismatch, the parameter estimates remained within 1% of their actual values. When the error in the estimate of the first bending mode frequency was initially at 10%, it was adjusted to within 3% in one second, and remained less than 1% after 2.75 seconds. It is of interest that the gain slopes in this experiment were 2 to 10 times the corresponding values for the maximum q values.

It is noted that previous experiments have shown (and theory predicts) that even faster adaptation times will occur when the excitation has more frequency content.

The above experiments indicate that a further simplification in the determination of system gains is possible, i.e. the usual Kalman filter gains work well in the adaptive system. Further, one must include in the filter the best estimate of the dynamics of states whose bandwith will not allow a white noise approximation to be used relative to states which are being adopted. This leads to a dichotomy of the frequency domain as follows. States with dynamics which cut off below a certain frequency are important for feedback and these primary states must be adapted if parameter values are too uncertain a priori. Secondary states whose dynamics cut off in a frequency band up to a second frequency must still be included as filter states in order to obtain sufficiently accurate estimates of the primary states.

These states may also have to be adapted if their parameter values are too uncertain. Next, there are third order states which may not have been needed in the filter if some other states did not need to be adaptive. And there are, of course, higher order effects which can be totally neglected in the filter. The new dichotomy is that the third order states may have to be included in an adaptive system, where they would not have been required when the parameters of other states were not so uncertain as to require adaptation.

These results clearly show the operation of the a priori gain system is a practical approach to adaptive control, since it not only matches the performance of the on-line adaptive system, but can even exceed it. Moreover, after the design is completed, it looks like a design that might have been designed more by art, but it has a theoretical backing for the values used for all gains.

References - Section V

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APPENDIX A

Equations of Motion

1. Attitude

$$\ddot{\phi} = -C_1\alpha - C_2\beta$$

2. Normal Acceleration

$$\ddot{z} = \frac{F - X}{m}\phi + \frac{R'}{m}\beta + \frac{N'}{m}\alpha$$

3. Angle of Attack

$$\alpha = \phi - \frac{\dot{z}}{V}$$

4. Actuator Dynamics

$$\frac{\beta}{\beta_{in}} = \frac{W_b W_n^2}{(s + W_b)(s^2 + 2\xi_n W_n + W_n^2)}$$

5. Flexible Mode Dynamics

$$\ddot{\eta}_i + 2\xi_i W_i \dot{\eta}_i + W_i^2 \eta_i = \left(\frac{R' Y_{\beta L}}{M_i}\right)\beta + \frac{(s_E Y_{\beta i} - I_E Y'_{\beta i})}{M_i}\ddot{\beta}$$

6. Miscellaneous Equations

$$\dot{\theta}_{RG} = \dot{\phi} + \sum_{i=1}^{2} Y'_{\phi i}\dot{\eta}_i \quad \text{Rate gyro response}$$

$$\theta_A = \phi + \sum_{i=1}^{2} Y'_{\phi i}\eta_i \quad \text{Attitude gyro response}$$

$$C_1 = - L_\alpha qS \; C_{N\alpha}$$

$$C_2 = \frac{R' L_{cg}}{I_{XX}}$$

$$R' = \frac{4}{5}F$$

$$N_\alpha = Q \; S \; C_{N\alpha}$$

$$N' = N_\alpha + X$$

APPENDIX B

Vehicle Parameters

Parameter (Units)	Lift-Off	Maximum Dynamic Pressure	Burn-Out
F,(KN)	33962.0	38134.8	38898.1
X,(N)	93516.0	1610200.0	27243.0
θ,(N/m^2)	669.8	37613.0	1819.7
V,(m/sec)	33.94	540.22	2142.0
m,(Kg)	2601502.0	1691715.0	817604.0
X_{cg}(m)	27.42	29.66	43.23
I_{XX}(Kg-m^2)	844118979.0	775421160.0	438127471.0
$C_{N\alpha}$	4.62	4.38	4.4
X_{cp}/D	3.56	4.05	5.1
f_1,(Hz)	0.9932	1.0637	1.1653
M_1,(Kg)	13430.	10603.0	8780.0
$Y_{\beta 1}$	0.09881	0.08849	0.15645
$Y'_{\beta 1}$,(m^{-1})	-0.00481	-0.00448	-0.00551
$Y'_{\phi 1}$(m^{-1})	0.00795	0.00701	0.00516
$Y'_{\dot{\phi} 1}$(m^{-1})	0.01505	0.01463	0.01344
f_2,(Hz)	1.7614	1.9496	2.1238
M_2,(Kg)	5230.0	4055.0	3029.0
$Y_{\beta 1}$	-0.06256	-0.06361	-0.06516
$Y'_{\beta 1}$,(m^{-1})	0.00353	0.00393	0.0033
$Y'_{\dot{\phi} 2}$(m^{-1})	-0.00336	-0.00513	-0.00594
$Y'_{\phi 2}$(m^{-1})	0.00469	0.00127	-0.00181

$\xi_1 = 0.01$

$\xi_2 = 0.01$

$S_E = 13500.0$ Kg - m

$I_E = 47195$ Kg - m^2

$S = 79.4$ m^2

$f_b = 7.0$ Hz

$f_n = 5.5$ Hz

$\xi_n = 0.33$