TM-54/30-172
LMSC/HREC A791043

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DESCRIPTION OF A GENERAL LAUNCH VEHICLE SIMULATION PROGRAM AND ITS ASSOCIATED DATA PROGRAM

December 1967

Contract NAS8-20082

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CR-52661

FOREWORD

This report presents the results of work performed by Lockheed's Huntsville Research & Engineering Center while under subcontract to Northrop Space Laboratories (NSL PO 5-09287) in support of the Aero-Astrodynamics Laboratory of Marshall Space Flight Center (MSFC) Mission Support Contract NAS8-20082. This task was conducted in response to the requirement of Appendix E-1, Schedule Order No. 50.

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Section 1
INTRODUCTION

This document describes the Launch Vehicle Simulation Program and the Launch Vehicle Data Program. The system of equations is complete to the extent required to perform load relief analysis and performance evaluation for the Saturn V class of launch vehicles. The prime reference for these equations is Reference 1. Other sources which were used in addition to Reference 1-appear in the Reference section.

General descriptions of the background material of the two digital programs are presented in Appendixes B and C. The Nomenclature comprises Appendix A.

Section 2
SYSTEM EQUATIONS FOR LAUNCH VEHICLE

The equations describe a flexible, sloshing vehicle model in the pitch plane. The following features are included:

- First three vehicle bending nodes
- Propellant sloshing models for the two S-IC stage tanks and the S-II LOX tank
- Bending moment at a particular station along the longitudinal axis of the vehicle
- Control engines dynamics.

The pitch plane relationships are shown in Figures 1 and 2. Following is a summary of all equations-of-motion.

Translational:

$$\ddot{Y} = U_t\phi + A_t\alpha + B_t\beta + G_{t\ell_1}\eta_1 + G_{t\ell_2}\eta_2 + G_{t\ell_3}\eta_3 - \bar{N}_{t\ell_1}\ddot{Y}_{s1} - \bar{N}_{t\ell_2}\ddot{Y}_{s2} - \bar{N}_{t\ell_3}\ddot{Y}_{s3} + B_E\ddot{\beta}$$

where

$$U_t = (F - D)/m$$

$$A_t = q\,S_A\,C_{N_\alpha}/m$$

$$B_t = F_c/m$$

$$G_{t\ell_i} = (F/m)\,Y_i'(x_E) + (q\,S_A/m)\,C_{Nb\ell_i}$$

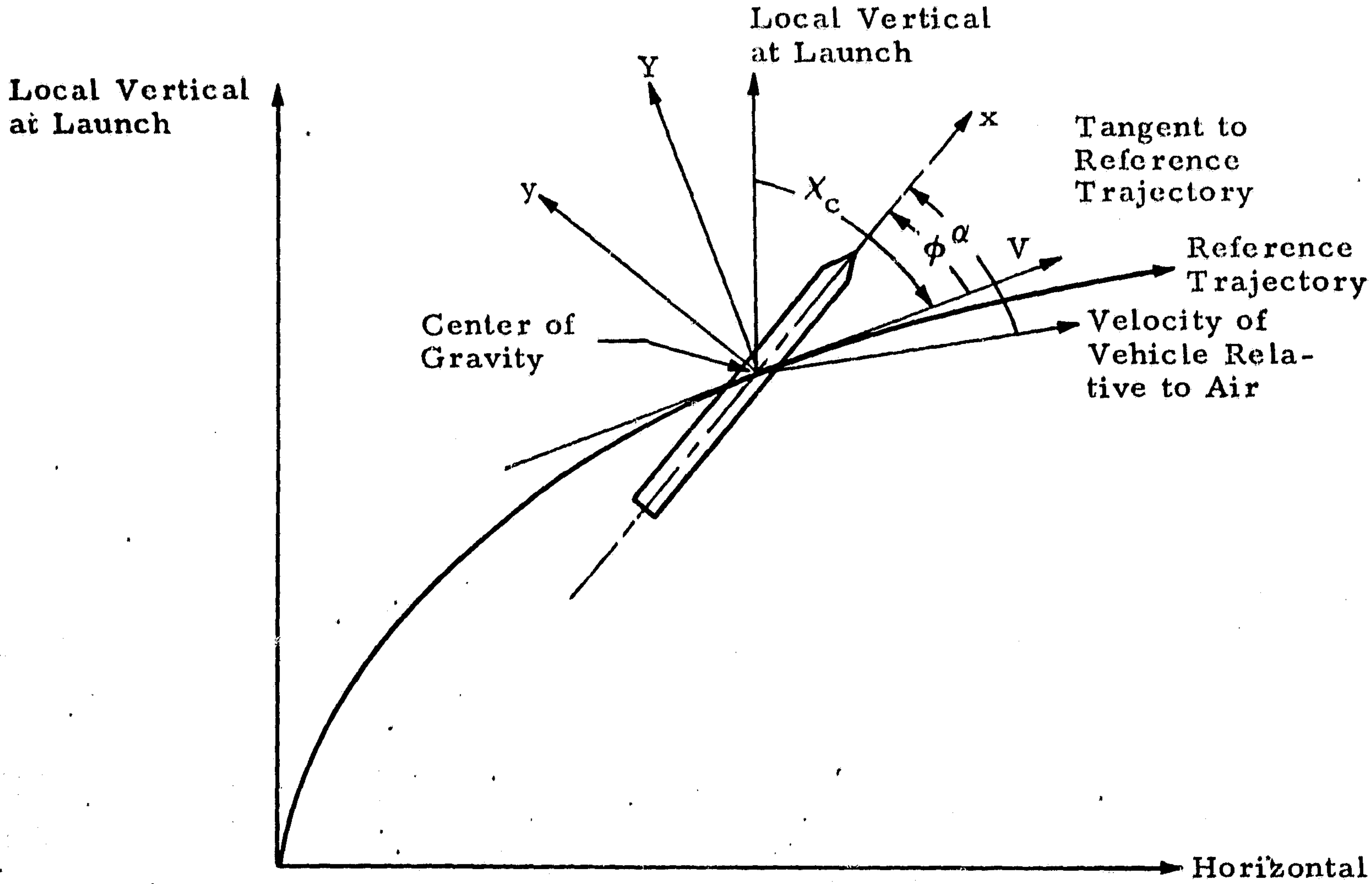

Figure 1 - Pitch Plane Geometry

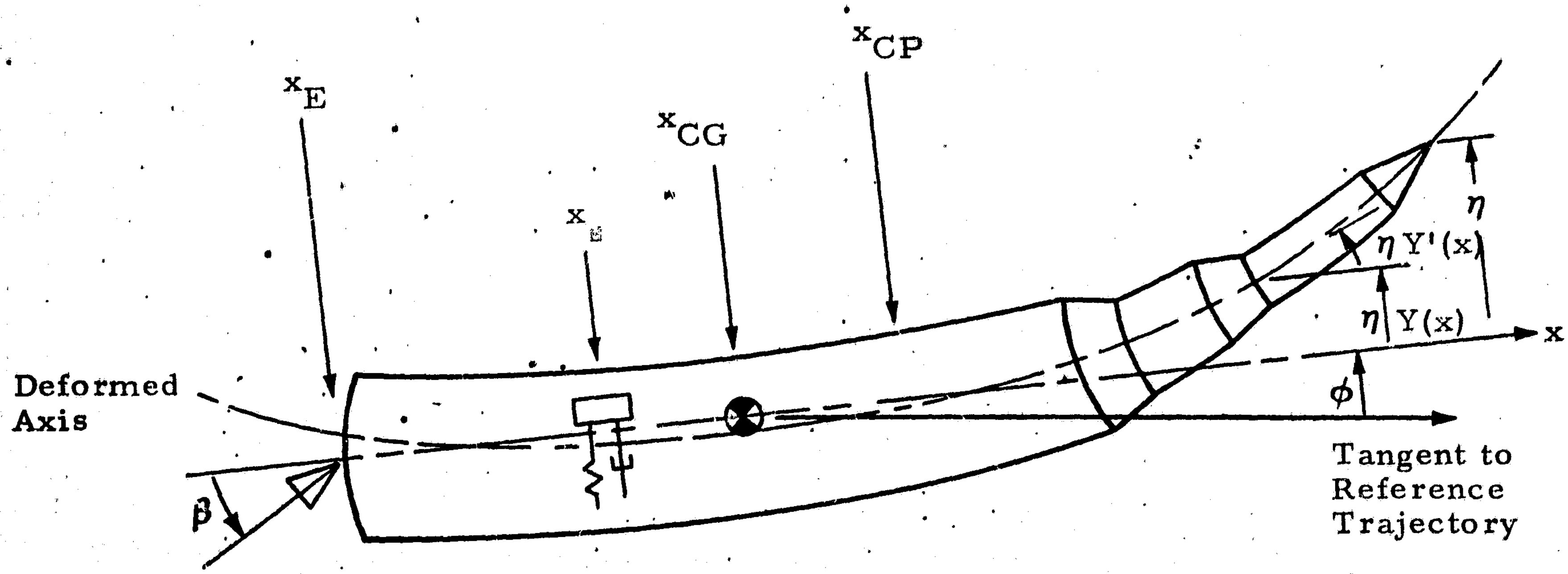

Figure 2 - Pitch Plane Flexibility

$$\bar{N}_{t\ell_i} = m_{s_i}/m$$

$$B_E = 4\, m_E\, \ell_E/m$$

$$C_{Nb\ell_i} = \int_L \frac{\partial C_{N_\alpha}}{\partial x}\, Y_i'(x)\, dx$$

Rotational:

$$\ddot{\phi} = -A_r\,\alpha - B_r\,\beta + \bar{N}_{r\ell_1}\,\ddot{Y}_{s1} + \bar{N}_{r\ell_2}\,\ddot{Y}_{s2} + \bar{N}_{r\ell_3}\,\ddot{Y}_{s3} + N_{r\ell_1}\,Y_{s1}$$

$$+ N_{r\ell_2}\,Y_{s2} + N_{r\ell_3}\,Y_{s3} + G_{r\ell_1}\,\eta_1 + G_{r\ell_2}\,\eta_2 + G_{r\ell_3}\,\eta_3 + C_E\,\ddot{\beta}$$

where

$$A_r = q\, S_A\, C_{N_\alpha}\,(x_{CG} - x_{CP})/I$$

$$B_r = F_c\,(x_{CG} - x_E)/I$$

$$\bar{N}_{r\ell_i} = (x_{CG} - x_{S_i})\, m_{s_i}/I$$

$$N_{r\ell_i} = U_t\, m_{s_i}/I$$

$$G_{r\ell_i} = -F\left[Y_i(x_E) + (x_{CG} - x_E)\,Y_i'(x_E)\right]/I + (q\,S_A/I)\,C_{mb\ell_i}$$

$$C_E = -4\left[I_E + m_E\,\ell_E\,(x_{CG} - x_E)\right]/I$$

$$C_{mb\ell_i} = \int_L \frac{\partial C_{N_\alpha}}{\partial x}\,(x - x_{CG})\, Y_i'(x)\, dx$$

Angle of Attack:

$$\alpha = \phi - \dot{Y}/V + \alpha_w$$

where

$$\alpha_w = \tan^{-1}(V_w \cos\chi_c)/(V - V_w \sin\chi_c)$$

Propellant Sloshing:

$$\ddot{Y}_{s1} = - \bar{N}_{s\ell_1} \dot{Y}_{s1} - N_{s\ell_1} Y_{s1} - \ddot{Y} + \bar{U}_{s\ell_1} \ddot{\phi} + U_t \phi$$

$$- \bar{G}_{s\ell_{11}} \ddot{\eta}_1 - \bar{G}_{s\ell_{12}} \ddot{\eta}_2 - \bar{G}_{s\ell_{13}} \ddot{\eta}_3$$

$$+ G_{s\ell_{11}} \eta_1 + G_{s\ell_{12}} \eta_2 + G_{s\ell_{13}} \eta_3 \; .$$

$$\ddot{Y}_{s2} = - \bar{N}_{s\ell_2} \dot{Y}_{s2} - N_{s\ell_2} Y_{s2} - \ddot{Y} + \bar{U}_{s\ell_2} \ddot{\phi} + U_t \phi$$

$$- \bar{G}_{s\ell_{21}} \ddot{\eta}_1 - \bar{G}_{s\ell_{22}} \ddot{\eta}_2 - \bar{G}_{s\ell_{23}} \ddot{\eta}_3$$

$$+ G_{s\ell_{21}} \eta_1 + G_{s\ell_{22}} \eta_2 + G_{s\ell_{23}} \eta_3$$

$$\ddot{Y}_{s3} = - \bar{N}_{s\ell_3} \dot{Y}_{s3} - N_{s\ell_3} Y_{s3} - \ddot{Y} + \bar{U}_{s\ell_3} \ddot{\phi} + U_t \phi$$

$$- \bar{G}_{s\ell_{31}} \ddot{\eta}_2 - \bar{G}_{s\ell_{32}} \ddot{\eta}_2 - \bar{G}_{s\ell_{33}} \ddot{\eta}_3$$

$$+ G_{s\ell_{31}} \eta_1 + G_{s\ell_{32}} \eta_2 + G_{s\ell_{33}} \eta_3$$

where

$$\bar{N}_{s\ell_i} = 2 \zeta_{s_i} \omega_{s_i}$$

$$N_{s\ell_i} = \omega_{s_i}^{2}$$

$$\bar{U}_{s\ell_i} = x_{CG} - x_{s_i} \; .$$

$$\bar{G}_{s\ell_{ik}} = Y_k(x_{s_i})$$

$$G_{s\ell_{ik}} = U_t Y'_k(x_{s_i})$$

Vehicle Bending:

$$\ddot{\eta}_1 = -\bar{G}_{b\ell_1}\dot{\eta}_1 - G_{b\ell_1}\eta_1 + B_{b\ell_1}\beta + D_{E_1}\ddot{\beta}$$
$$+ G_{E1}\ddot{\phi} - \bar{N}_{b\ell_{11}}\ddot{Y}_{s1} - \bar{N}_{b\ell_{12}}\ddot{Y}_{s2} - \bar{N}_{b\ell_{13}}\ddot{Y}_{s3}$$
$$+ N_{b\ell_{11}}Y_{s1} + N_{b\ell_{12}}Y_{s2} + N_{b\ell_{13}}Y_{s3}$$
$$+ N_{\dot{\eta}_1}\alpha + N_{\eta_{12}}\eta_2 + N_{\eta_{13}}\eta_3 - N_{\dot{\eta}_{12}}\dot{\eta}_2 - N_{\dot{\eta}_{13}}\dot{\eta}_3$$

$$\ddot{\eta}_2 = -\bar{G}_{b\ell_2}\dot{\eta}_2 - G_{b\ell_2}\eta_2 + B_{b\ell_2}\beta + D_{E_2}\ddot{\beta}$$
$$+ G_{E2}\ddot{\phi} - \bar{N}_{b\ell_{21}}\ddot{Y}_{s1} - \bar{N}_{b\ell_{22}}\ddot{Y}_{s2} - \bar{N}_{b\ell_{23}}\ddot{Y}_{s3}$$
$$+ N_{b\ell_{21}}Y_{s1} + N_{b\ell_{22}}Y_{s2} + N_{b\ell_{23}}Y_{s3}$$
$$+ N_{\dot{\eta}_2}\alpha + N_{\eta_{21}}\eta_1 + N_{\eta_{23}}\eta_3 - N_{\dot{\eta}_{21}}\dot{\eta}_1 - N_{\dot{\eta}_{23}}\dot{\eta}_3$$

$$\ddot{\eta}_3 = -\bar{G}_{b\ell_3}\dot{\eta}_3 - G_{b\ell_3}\eta_3 + B_{b\ell_3}\beta + D_{E_3}\ddot{\beta}$$
$$+ G_{E3}\ddot{\phi} - \bar{N}_{b\ell_{31}}\ddot{Y}_{s1} - \bar{N}_{b\ell_{32}}\ddot{Y}_{s2} - \bar{N}_{b\ell_{33}}\ddot{Y}_{s3}$$
$$+ N_{b\ell_{31}}Y_{s1} + N_{b\ell_{32}}Y_{s2} + N_{b\ell_{33}}Y_{s3}$$
$$+ N_{\dot{\eta}_3}\alpha + N_{\eta_{31}}\eta_1 + N_{\eta_{32}}\eta_2 - N_{\dot{\eta}_{31}}\dot{\eta}_1 - N_{\dot{\eta}_{32}}\dot{\eta}_2$$

where

$$\bar{G}_{b\ell_i} = 2\,\zeta_{b_i}\,\omega_{b_i} + (q\,S_A/T_i\,V)\,C_{db\ell_{ik}}$$

$$G_{b\ell_i} = \omega_{b_i}^{\,2} - (q\,S_A/T_i)\,C_{b\ell_{ik}}$$

$$B_{b\ell_i} = F_c\,Y_i\,(x_E)/T_i$$

$$D_{E_i} = 4\,[m_E\,\ell_E\,Y_i\,(x_E) - I_E\,Y_i'\,(x_E)]/T_i$$

$$G_{E_i} = 4\,[m_E\,(\ell_E + x_{CG} - x_E)\,Y_i\,(x_E) - m_E\,\ell_E\,(x_{CG} - x_E)\,Y_i'\,(x_E)]/T_i$$

$$\bar{N}_{b\ell_{ik}} = m_{s_k}\,Y_i\,(x_{s_k})/T_i$$

$$N_{b\ell_{ik}} = U_t\,m_{s_k}\,Y_i'\,(x_{s_k})/T_i$$

$$N_{\dot{\eta}_i} = (q\,S_A/T_i)\,C_{Ab\ell_i}$$

$$N_{\eta_{ik}} = (q\,S_A/T_i)\,C_{b\ell_{ik}}$$

$$N_{\dot{\eta}_{ik}} = (q\,S_A/T_i\,V)\,C_{db\ell_{ik}}$$

$$C_{Ab\ell_i} = \int_L \frac{\partial C_{N_\alpha}}{\partial x}\,Y_i(x)\,dx$$

$$C_{b\ell_{ik}} = \int_L \frac{\partial C_{N_\alpha}}{\partial x}\,Y_i(x)\,Y_k'(x)\,dx$$

$$C_{db\ell_{ik}} = \int_L \frac{\partial C_{N_\alpha}}{\partial x}\,Y_i(x)\,Y_k(x)\,dx$$

Gimbaled Engines:

$$\ddot{\beta} = -\bar{N}_E \dot{\beta} - N_E \beta + N_E \beta_\Lambda - J_E \ddot{\phi} + K_{E_1} \ddot{\eta}_1$$

$$+ K_{E_2} \ddot{\eta}_2 + K_{E_3} \ddot{\eta}_3 + L_E Y - P_E \phi$$

when $|\beta| > 5.15^\circ$, $\beta = (\text{sign}\beta)\ 5.15^\circ$, $\dot{\beta} = \ddot{\beta} = 0$, where

$$\bar{N}_E = 2\,\zeta_E\,\omega_E$$

$$N_E = \omega_E^2$$

$$J_E = [m_E\,\ell_E\,(x_{CG} - x_E) - I_E]/I_E$$

$$K_{E_i} = [m_E\,\ell_E\,Y_i\,(x_E) - I_E\,Y_i'\,(x_E)]/\,I_E$$

$$L_E = m_E\,\ell_E/I_E$$

$$P_E = U_t\,L_E$$

Bending Moment Equation:

$$BM_x = M'_\alpha\,\alpha + M'_\beta\,\beta + M'_{\ddot{\eta}_1}\,\ddot{\eta}_1 + M'_{\ddot{\eta}_2}\,\ddot{\eta}_2 + M'_{\ddot{\eta}_3}\,\ddot{\eta}_3$$

Lateral Acceleration Normal to Vehicle Longitudinal Axis at Sensor Station, x_a:

$$\ddot{y}(x_a) = \ddot{Y} - \ell_a\,\ddot{\phi} + Y_1(x_a)\,\ddot{\eta}_1 + Y_2(x_a)\,\ddot{\eta}_2$$

$$+ Y_3(x_a)\,\ddot{\eta}_3 - U_t\,[\dot{\phi} + Y_1'(x_a)\,\dot{\eta}_1$$

$$+ Y_2'(x_a)\,\eta_2 + Y_3'(x_a)\,\eta_3]$$

where $\ell_a = x_{CG} - x_a$.

Position Gyro:

$$\phi_i = \phi + Y'_1(x_G)\,\eta_1 + Y'_2(x_G)\,\eta_2 + Y'_3(x_G)\,\eta_3$$

Rate Gyro:

$$\theta_i = \dot{\phi} + Y'_1(x_{RG})\,\dot{\eta}_1 + Y'_2(x_{RG})\,\dot{\eta}_2 + Y'_3(x_{RG})\,\dot{\eta}_3$$

Control:

$$\beta_c = \text{Option 1 or Option 2}$$

$$\text{Option 1} = K_N \Big\{ F_N[(a_o + a_2/s)\,\phi_i + a_1 F_{rg}\,\theta_i]$$

$$+ g_2 F_a \ddot{y}(x_a) + g_3 Y + g_4 \dot{Y} \Big\}$$

where

$$F_{rg} = \frac{(1 + s/\omega_{r1})}{(1 + s/\omega_{r2})}$$

$$F_a = \frac{1}{1 + s/\omega_a}$$

F_N = Option A, Option B or Option C

$$\text{Option A} = \frac{1 + (2\zeta_1/\omega_1)\,s + (1/\omega_1^2)\,s^2}{[1 + (2\zeta_2/\omega_2)\,s + (1/\omega_2^2)\,s^2]\,[1 + (2\zeta_3/\omega_3)\,s + (1/\omega_3^2)\,s^2]}$$

Option B =

$$\frac{[1 + (2\zeta_1/\omega_1)\,s + (1/\omega_1^2)\,s^2]\,[1 + (2\zeta_2/\omega_2)\,s + (1/\omega_2^2)\,s^2]}{[1 + (2\zeta_3/\omega_3)\,s + (1/\omega_3^2)\,s^2]\,[1 + (2\zeta_4/\omega_4)\,s + (1/\omega_4^2)\,s^2]\,[1 + s/\omega_5]}$$

Option C =

$$\frac{[1 + (2\zeta_1/\omega_1)\, s + (1/\omega_1^2)\, s^2][1 + (2\zeta_2/\omega_2)\, s + (1/\omega_2^2)\, s^2]}{[1 + (2\zeta_3/\omega_3)\, s + (1/\omega_3^2)\, s^2][1 + (2\zeta_4/\omega_4)\, s + (1/\omega_4^2)\, s^2][1 + (2\zeta_5/\omega_5)\, s + (1/\omega_5^2)\, s^2]}$$

$$\text{Option 2} = (a_o + a_2/s)\, F_{pg}\, \phi_i + a_1\, F_{rg}\, \theta_i + g_2\, F_a\, \ddot{y}(x_a) + g_3\, Y + g_4\, \dot{Y}$$

where

$$F_a = \frac{1}{1 + s/\omega_a}$$

$$F_{pg} = \frac{[1 + (2\zeta_1/\omega_1)\, s + (1/\omega_1^2)\, s^2][1 + s/\omega_2]}{[1 + (2\zeta_3/\omega_3)\, s + (1/\omega_3^2)\, s^2][1 + (2\zeta_4/\omega_4)\, s + (1/\omega_4^2)\, s^2]}$$

$$F_{rg} = \frac{[1 + (2\zeta_5/\omega_5)\, s + (1/\omega_5^2)\, s^2][1 + (2\zeta_6/\omega_6)\, s + (1/\omega_6^2)\, s^2]}{[1 + (2\zeta_7/\omega_7)\, s + (1/\omega_7^2)\, s^2][1 + (2\zeta_8/\omega_8)\, s + (1/\omega_8^2) s^2][1 + s/\omega_9]}$$

Actuator Transfer Function:

$$A(s) = \frac{(3.3)\,(10^3)\,(s^2 + 5.35\, s + 2547)}{(s^2 + 29.9\, s + 1188)(s^2 + 135\, s + 7090)}$$

$$\beta_A = A(s)\, \beta_c$$

Section 3
CONTROL LOOP OPTIONS

The control feedback options are illustrated in Figures 3 and 4. As can be seen, the main difference is that Option 2 restricts all compensation to the individual instrument loops. Gain scheduling is included in Appendix B.

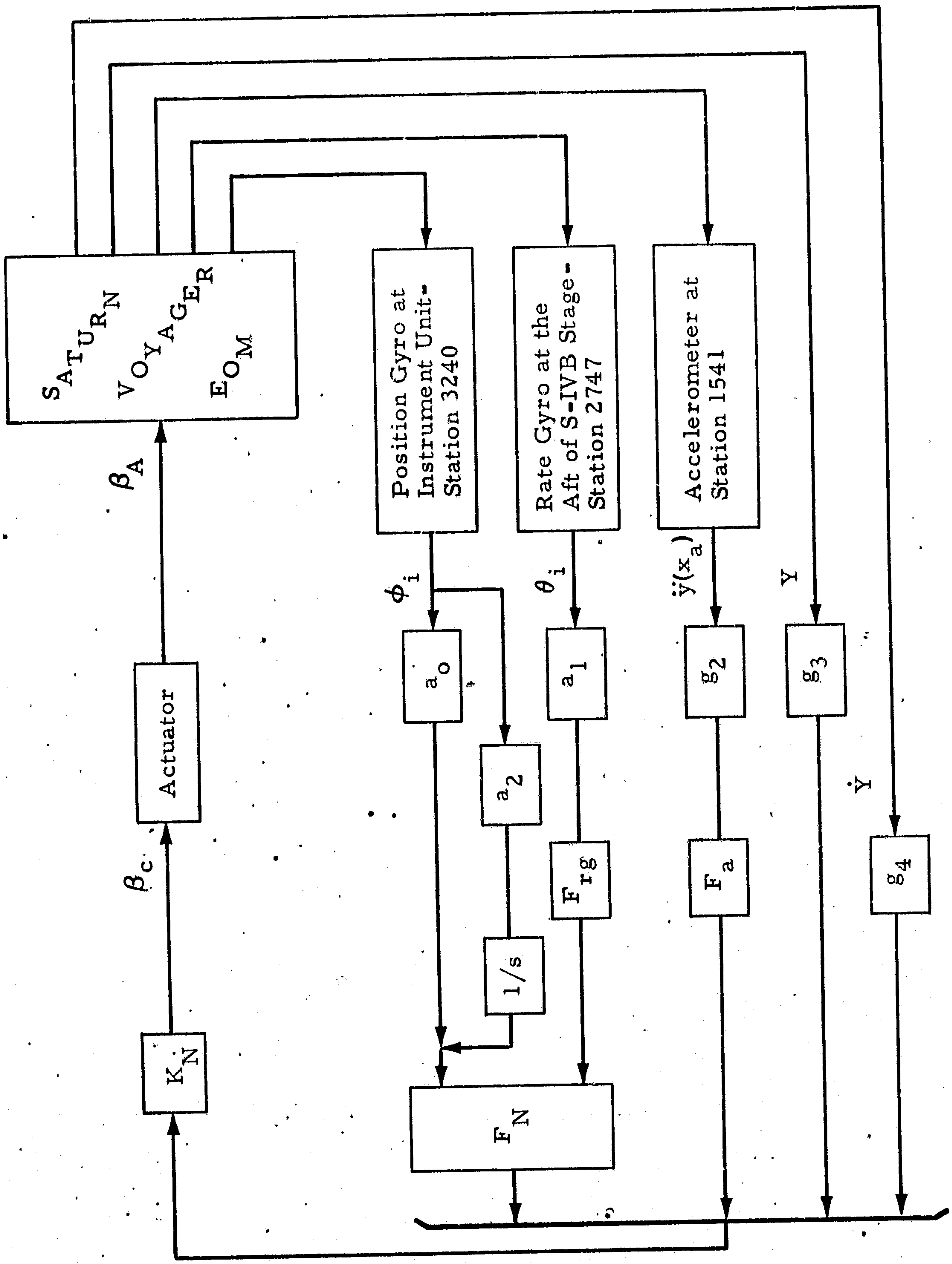

Figure 3 - General Control System — Option 1

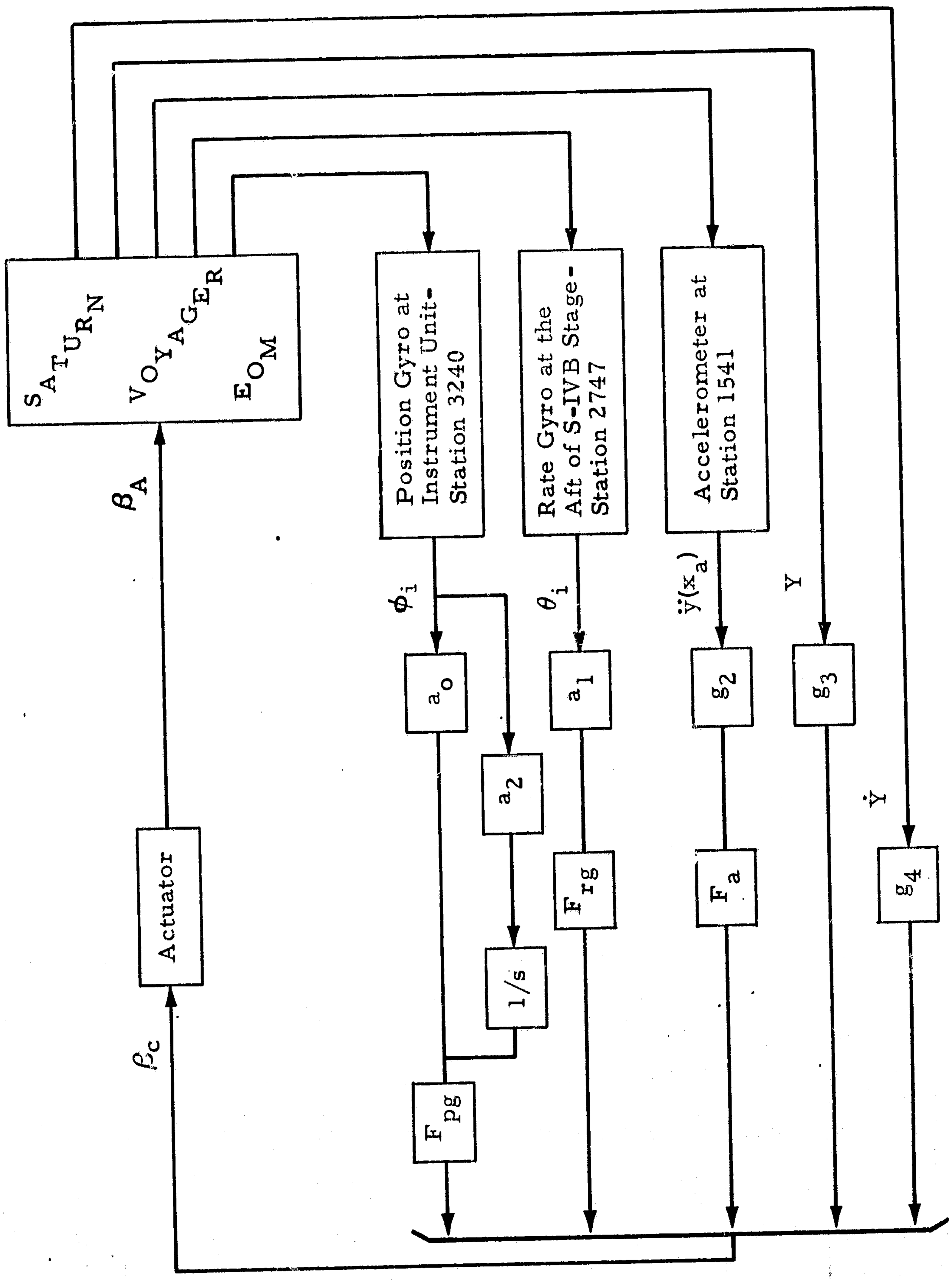

Figure 4 - General Control System – Option 2

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Appendix A

NOMENCLATURE

Appendix A
NOMENCLATURE

$A(s)$	actuator transfer function
a_o	attitude feedback control gain
a_1	attitude rate feedback control gain
a_2	control gain for integral of attitude feedback; set equal to a_o when used
BM_x	bending moment at station x; kg-m
C_{N_α}	normal force coefficient gradient; 1/rad
D	aerodynamic drag; newtons
F	total thrust of booster engines; newtons
F_a	accelerometer feedback filter transfer function
F_c	total thrust of control engines; newtons
F_N	notch and roll off filter transfer function
F_{pg}	position feedback filter transfer function
F_{rg}	attitude rate feedback filter transfer function
g_2	accelerometer feedback gain; rad-sec^2/m
g_3	drift feedback gain; rad/m
g_4	drift rate feedback gain; rad-sec/m
I	vehicle pitch mass moment of inertia; newton-m-sec^2
I_E	engine mass moment of inertia about gimbal point; newton-m-sec^2
K_N	loop gain term

ℓ_E	difference between engine gimbal coordinate and engine mass center of gravity $(x_E - x_{EG})$; m
m	total vehicle mass; kg
m_E	engine mass; kg
m_s	modal slosh mass; kg
M'_α	aerodynamic bending moment coefficient; kg-m/rad
M'_β	bending moment coefficient due to control engines deflection; kg-m/rad
$M'_{\ddot{\eta}}$	bending moment coefficient due to bending mode acceleration; kg-sec^2
q	aerodynamic pressure; newtons/m^2
s	Laplacian operator
S_A	aerodynamic reference area; m^2
T	bending mode generalized mass; kg
V	vehicle velocity; m/sec
V_w	wind velocity; m/sec
x_a	accelerometer longitudinal coordinate; m
x_{CG}	vehicle center of gravity longitudinal coordinate; m
x_{CP}	vehicle center of pressure longitudinal coordinate; m
x_E	control engines gimbal plane longitudinal coordinate; m
x_{EG}	control engine center of gravity longitudinal coordinate; m
x_G	position gyro longitudinal coordinate; m
x_{RG}	rate gyro longitudinal coordinate; m
x_s	modal slosh mass longitudinal coordinate; m
$Y, \dot{Y}, \ddot{Y}$	drift, drift rate, drift acceleration, respectively, normal to the reference trajectory; m, m/sec, m/sec^2
$Y(x)$	bending mode normalized deflection at station x

$Y'(x)$	slope of bending mode normalized deflection curve at station x; 1/m
$Y_s, \dot{Y}_s, \ddot{Y}_s$	slosh mass displacement, velocity, acceleration, normal to reference trajectory, m, m/sec, m/sec^2
$\ddot{y}(x_a)$	lateral acceleration normal to vehicle deformed centerline at station x_a; m/sec^2

<u>Greek</u>

α	vehicle angle of attack; radians
α_w	angle between wind velocity vector and vehicle velocity vector; radians
β	control engines gimbal angle; radians
β_A	actuator output; radians
β_c	commanded control engines gimbal angle; radians
ζ_b	bending mode damping ratio
ζ_E	damping ratio of control engine
ζ_s	slosh mode damping ratio
$\zeta_1 - \zeta_8$	control filters damping ratios
$\eta, \dot{\eta}, \ddot{\eta}$	amplitude, amplitude rate, amplitude acceleration, respectively, of bending mode at nose of vehicle; m, m/sec, m/sec^2
θ_i	rate gyro output; rad/sec
ϕ	vehicle pitch attitude error; radians
ϕ_i	position gyro output; radians
χ_c	commanded pitch attitude relative to vertical at time of launch; radians
ω_a	accelerometer feedback filter break frequency, rad/sec
ω_b	bending mode undamped natural frequency; rad/sec
ω_E	undamped natural frequency of control engine; rad/sec

ω_{r1}, ω_{r2}	rate gyro feedback filter break frequencies, rad/sec
ω_s	slosh mode undamped natural frequency; rad/sec
$\omega_1 - \omega_9$	control filters undamped natural frequencies, rad/sec

Numerical subscripts when affixed to bending mode terms denote the particular vehicle bending mode. Numerical subscripts affixed to slosh terms denote a particular tank, where tank 1 is the S-IC fuel tank, tank 2 is the S-IC LOX tank, tank 3 is the S-II LOX tank.

Appendix B

LAUNCH VEHICLE SIMULATION PROGRAM

Appendix B
LAUNCH VEHICLE SIMULATION PROGRAM

B.1 GENERAL

A description of background material to the Launch Vehicle Simulation Program is given below. The program utilization report, Reference 5, contains sufficient information for machine use of the program.

B.2 EQUATIONS OF MOTION

The equations as shown in the main body of the report are undesirable for digital simulation since they are implicit in form and would require an excessively large amount of machine time to solve. In general the implicit equations may be represented by the following:

$$\begin{bmatrix} \ddot{\beta} \\ \ddot{Y} \\ \ddot{\phi} \\ \ddot{\eta}_1 \\ \ddot{\eta}_2 \\ \ddot{\eta}_3 \\ \ddot{Y}_{s1} \\ \ddot{Y}_{s2} \\ \ddot{Y}_{s3} \end{bmatrix} = \begin{bmatrix} T_1 \end{bmatrix} \begin{bmatrix} \ddot{\beta} \\ \ddot{Y} \\ \ddot{\phi} \\ \ddot{\eta}_1 \\ \ddot{\eta}_2 \\ \ddot{\eta}_3 \\ \ddot{Y}_{s1} \\ \ddot{Y}_{s2} \\ \ddot{Y}_{s3} \end{bmatrix} + \begin{bmatrix} T_2 \end{bmatrix} \begin{bmatrix} \dot{x}, & x \end{bmatrix} \tag{B.1}$$

where

$\begin{bmatrix} \dot{x}, x \end{bmatrix}$ represents all lower derivatives of the state variables

$\begin{bmatrix} T_1 \end{bmatrix}$, $\begin{bmatrix} T_2 \end{bmatrix}$ represent matrices of time-varying coefficients.

In order to express the system equations in explicit form it is only necessary to solve for the highest order derivatives as shown below.

$$[\ddot{x}] = [T]\,[\ddot{x}] + [F] \quad \text{, where } [F] \text{ denotes all terms of order lower than second} \tag{B.2}$$

$$[\ddot{x}] = \left\{[I] - [T]\right\}^{-1} [F] \quad \text{, where } [I] \text{ is a unit matrix} \tag{B.3}$$

The system of equations is now in explicit form and may be simulated digitally. Since the time varying coefficients in Equation (B.1) are known for a particular vehicle and trajectory, the inverse matrix in Equation (B.3) can be evaluated off-line in a separate digital program which has been termed the "Launch Vehicle Data Program" (see Appendix C).

The general flow of the indicated operation in Equations (B.1) through (B.3) is shown in Figure B-1. Note that a straightforward inverse is not performed since it is not necessary to obtain an equation for each second order state variable in terms of all lower order state variables. The equations are arranged in cascade beginning with the slosh terms and ending with the $\ddot{\beta}$ equation.

The complete system of equations is written for the simulation in the form shown below. These equations constitute the Launch Vehicle Simulation Program version of the equations of motion. The angle of attack equation is first order and was therefore not included in the rearrangement of the system equations.

$$\alpha = \phi - \dot{Y}/V + \alpha_w \tag{B.4}$$

$$F_1 = U_t \phi + A_t \alpha + B_t \beta + G_{t\ell_1} \eta_1 + G_{t\ell_2} \eta_2 + G_{t\ell_3} \eta_3 - K_{86} \dot{\beta} - K_{87} (\beta - \beta_A) - K_{88} \phi \tag{B.5}$$

Equation (B.1) → $\ddot{\beta} = f(\ddot{Y}, \ddot{\phi}, \ddot{\eta}_1, \ddot{\eta}_2, \ddot{\eta}_3, F)$

↓

$\ddot{\beta}$ equation substituted into $\ddot{Y}$ through $\ddot{Y}_{s3}$ equations

↓

Resultant $\ddot{Y}$ equation solved for $\ddot{Y}$ and substituted into $\ddot{\phi}$ through $\ddot{Y}_{s3}$ equations → $\ddot{Y} = f(\ddot{\phi}, \ddot{\eta}_1, \ddot{\eta}_2, \ddot{\eta}_3, \ddot{Y}_{s1}, \ddot{Y}_{s2}, \ddot{Y}_{s3}, F)$

↓

Resultant $\ddot{\phi}$ equation solved for $\ddot{\phi}$ and substituted into $\ddot{\eta}_1$ through $\ddot{Y}_{s3}$ equations → $\ddot{\phi} = f(\ddot{\eta}_1, \ddot{\eta}_2, \ddot{\eta}_3, \ddot{Y}_{s1}, \ddot{Y}_{s2}, \ddot{Y}_{s3}, F)$

↓

Resultant $\ddot{\eta}_1$ through $\ddot{Y}_{s3}$ equations regrouped and arranged in matrix format

↓

$\ddot{Y}_{s1}$, $\ddot{Y}_{s2}$, $\ddot{Y}_{s3}$ solved for individually in terms of lower order state variables →

$\ddot{Y}_{s1} = f(F)$
$\ddot{Y}_{s2} = f(F)$
$\ddot{Y}_{s3} = f(F)$

↓

$\ddot{\eta}_1$, $\ddot{\eta}_2$, $\ddot{\eta}_3$ solved for individually in terms of lower order state variables and $\ddot{Y}_{s1}$, $\ddot{Y}_{s2}$, $\ddot{Y}_{s3}$. →

$\ddot{\eta}_1 = f(\ddot{Y}_{s1}, \ddot{Y}_{s2}, \ddot{Y}_{s3}, F)$
$\ddot{\eta}_2 = f(\ddot{Y}_{s1}, \ddot{Y}_{s2}, \ddot{Y}_{s3}, F)$
$\ddot{\eta}_3 = f(\ddot{Y}_{s1}, \ddot{Y}_{s2}, \ddot{Y}_{s3}, F)$

Figure B-1 - General Derivation Flow

$$F_2 = - A_r \alpha - B_r \beta + N_{r\ell_1} Y_{s1} + N_{r\ell_2} Y_{s2} + N_{r\ell_3} Y_{s3} + G_{r\ell_1} \eta_1 + G_{r\ell_2} \eta_2 + G_{r\ell_3} \eta_3 - K_{89} \dot{\beta} - K_{90} (\beta - \beta_A) - K_{91} \phi \tag{B.6}$$

$$F_3 = - \bar{G}_{b\ell_1} \dot{\eta}_1 - G_{b\ell_1} \eta_1 + B_{b\ell_1} \beta - K_{92} \dot{\beta} - K_{93} (\beta - \beta_A) - K_{94} \phi + N_{b\ell_{11}} Y_{s1} + N_{b\ell_{12}} Y_{s2} + N_{b\ell_{13}} Y_{s3} + K_{149} \alpha + K_{152} \eta_2 + K_{153} \eta_3 - K_{158} \dot{\eta}_2 - K_{159} \dot{\eta}_3 \tag{B.7}$$

$$F_4 = - \bar{G}_{b\ell_2} \dot{\eta}_2 - G_{b\ell_2} \eta_2 + B_{b\ell_2} \beta - K_{95} \dot{\beta} - K_{96} (\beta - \beta_A) - K_{97} \phi + N_{b\ell_{21}} Y_{s1} + N_{b\ell_{22}} Y_{s2} + N_{b\ell_{23}} Y_{s3} + K_{150} \alpha + K_{154} \eta_1 + K_{155} \eta_3 - K_{160} \dot{\eta}_1 - K_{161} \dot{\eta}_3 \tag{B.8}$$

$$F_5 = - \bar{G}_{b\ell_3} \dot{\eta}_3 - G_{b\ell_3} \eta_3 + B_{b\ell_3} \beta - K_{98} \dot{\beta} - K_{99} (\beta - \beta_A) - K_{100} \phi + N_{b\ell_{31}} Y_{s1} + N_{b\ell_{32}} Y_{s2} + N_{b\ell_{33}} Y_{s3} + K_{151} \alpha + K_{156} \eta_1 + K_{157} \eta_2 - K_{162} \dot{\eta}_1 - K_{163} \dot{\eta}_2 \tag{B.9}$$

$$F_6 = - \bar{N}_{s\ell_1} \dot{Y}_{s1} - N_{s\ell_1} Y_{s1} + U_t \phi + G_{s\ell_{11}} \eta_1 + G_{s\ell_{12}} \eta_2 + G_{s\ell_{13}} \eta_3 \tag{B.10}$$

$$F_7 = - \bar{N}_{s\ell_2} \dot{Y}_{s2} - N_{s\ell_2} Y_{s2} + U_t \phi + G_{s\ell_{21}} \eta_1 + G_{s\ell_{22}} \eta_2 + G_{s\ell_{23}} \eta_3 \tag{B.11}$$

$$F_8 = - \bar{N}_{s\ell_3} \dot{Y}_{s3} - N_{s\ell_3} Y_{s3} + U_t \phi + G_{s\ell_{31}} \eta_1 + G_{s\ell_{32}} \eta_2 + G_{s\ell_{33}} \eta_3 \tag{B.12}$$

$$F_9 = F_2 + K_{101} F_1 \tag{B.13}$$

$$F_{10} = F_3 + K_{102} F_1 \tag{B.14}$$

$$F_{11} = F_4 + K_{103} F_1 \tag{B.15}$$

$$F_{12} = F_5 + K_{104} F_1 \tag{B.16}$$

$$F_{13} = F_6 - K_{105} F_1 \tag{B.17}$$

$$F_{14} = F_7 - K_{105} F_1 \tag{B.18}$$

$$F_{15} = F_8 - K_{105} F_1 \tag{B.19}$$

$$F_{16} = F_{10} + K_{108} F_9 \tag{B.20}$$

$$F_{17} = F_{11} + K_{109} F_9 \tag{B.21}$$

$$F_{18} = F_{12} + K_{110} F_9 \tag{B.22}$$

$$F_{19} = F_{13} + K_{111} F_9 \tag{B.23}$$

$$F_{20} = F_{14} + K_{112} F_9 \tag{B.24}$$

$$F_{21} = F_{15} + K_{113} F_9 \tag{B.25}$$

$$F_{22} = K_{114} F_{19} + K_{115} F_{20} + K_{116} F_{21} - F_{16} \tag{B.26}$$

$$F_{23} = K_{117} F_{19} + K_{118} F_{20} + K_{119} F_{21} - F_{17} \tag{B.27}$$

$$F_{24} = K_{120} F_{19} + K_{121} F_{20} + K_{122} F_{21} - F_{18} \tag{B.28}$$

$$\ddot{Y}_{s1} = Q_1 F_{22} + Q_4 F_{23} + Q_7 F_{24} \tag{B.29a}$$

$$\ddot{Y}_{s2} = Q_2 F_{22} + Q_5 F_{23} + Q_8 F_{24} \tag{B.29b}$$

$$\ddot{Y}_{s3} = Q_3 F_{22} + Q_6 F_{23} + Q_9 F_{24} \tag{B.29c}$$

$$\ddot{\eta}_1 = K_{106} \ddot{Y}_{s1} + K_{107} \ddot{Y}_{s2} + K_{67} \ddot{Y}_{s3} - C_{11} F_{19} - C_{21} F_{20} - C_{31} F_{21} \tag{B.30a}$$

$$\ddot{\eta}_2 = K_{68} \ddot{Y}_{s1} + K_{69} \ddot{Y}_{s2} + K_{70} \ddot{Y}_{s3} - C_{12} F_{19} - C_{22} F_{20} - C_{32} F_{21} \tag{B.30b}$$

$$\ddot{\eta}_3 = K_{71} \ddot{Y}_{s1} + K_{72} \ddot{Y}_{s2} + K_{73} \ddot{Y}_{s3} - C_{13} F_{19} - C_{23} F_{20} - C_{33} F_{21} \tag{B.30c}$$

$$\ddot{\phi} = K_{123} F_9 + K_{124} \ddot{\eta}_1 + K_{125} \ddot{\eta}_2 + K_{126} \ddot{\eta}_3 + K_{127} \ddot{Y}_{s1} + K_{128} \ddot{Y}_{s2} + K_{129} \ddot{Y}_{s3} \tag{B.31}$$

$$\ddot{Y} = K_{105} F_1 - K_{130} \ddot{\phi} + K_{131} \ddot{\eta}_1 + K_{132} \ddot{\eta}_2 + K_{133} \ddot{\eta}_3 - K_{134} \ddot{Y}_{s1} - K_{135} \ddot{Y}_{s2} - K_{136} \ddot{Y}_{s3} \tag{B.32}$$

$$\ddot{\beta} = - \bar{N}_E \dot{\beta} - N_E (\beta - \beta_A) - J_E \ddot{\phi} + K_{E_1} \ddot{\eta}_1 + K_{E_2} \ddot{\eta}_2 + K_{E_3} \ddot{\eta}_3 + L_E \ddot{Y} - P_E \phi; \ (\beta \text{ limited}). \tag{B.33}$$

The coefficients in the preceding equations are consistent with those used in the original equations of motion. The "K" coefficients have resulted from the manipulations indicated in Figure B-1 and are consistent with the output quantities of the Launch Vehicle Data Program.

The bending moment equation remains unchanged since it is an "open-loop" equation. Equations (B.34) through (B.36) give the unfiltered instrument loop signals in the form used in the simulation.

$$\ddot{y}(x_a) = \ddot{Y} - \ell_a \ddot{\phi} + K_{137}\,\ddot{\eta}_1 + K_{138}\,\ddot{\eta}_2 + K_{139}\,\ddot{\eta}_3 - U_t\,\dot{\phi}$$

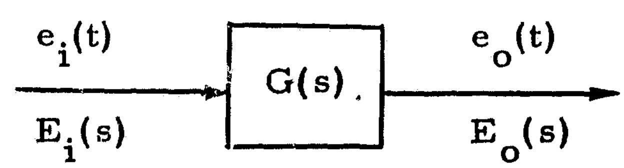
$$- K_{140}\,\eta_1 - K_{141}\,\eta_2 - K_{142}\,\eta_3 \qquad \text{(B.34)}$$

$$\phi_i = \phi + K_{143}\,\eta_1 + K_{144}\,\eta_2 + K_{145}\,\eta_3 \qquad \text{(B.35)}$$

$$\theta_i = \dot{\phi} + K_{146}\,\dot{\eta}_1 + K_{147}\,\dot{\eta}_2 + K_{148}\,\dot{\eta}_3 \qquad \text{(B.36)}$$

B.3 FILTERS

The actuator transfer function and all filters are simulated by the following general procedure. Given that:



$$\frac{E_o}{E_i} = G(s) = \frac{N(s)}{D(s)} \qquad \text{(B.37)}$$

where

$$N(s) = 1 + A_1\, s + A_2\, s^2 + \ldots + A_n\, s^n \qquad \text{(B.38a)}$$

$$D(s) = 1 + B_1\, s + B_2\, s^2 + \ldots + B_m\, s^m \qquad \text{(B.38b)}$$

Let

$$\frac{E_o^*}{E_i} = \frac{1}{D(s)} \qquad \text{(B.39)}$$

Then

$$\frac{E_o}{E_o^*} = \frac{E_i}{E_o^*}\ \frac{E_o}{E_i} = D(s)\ \frac{N(s)}{D(s)} = N(s) \qquad \text{(B.40)}$$

Substitution of Equation (B.38a) into Equation (B.40) gives

$$E_o = E_o^* + A_1\, s\, E_o^* + A_2\, s^2\, E_o^* + \ldots + A_n\, s^n\, E_o^* \qquad \text{(B.41)}$$

In the time domain after rearranging terms

$$e_o^{n*} = \frac{1}{A_n}\, e_o - \frac{1}{A_n}\, e_o^* - \frac{A_1}{A_n}\, e_o^{I*} - \frac{A_2}{A_n}\, e_o^{II*} \ldots \qquad \text{(B.42)}$$

where "I" is the first derivative, "II" is the second derivative, and "n" is the highest derivative.

Substitution of Equation (B.38b) into Equation (B.39) yields

$$E_i = E_o^* + B_1\, s\, E_o^* + B_2\, s^2\, E_o^* + \ldots + B_m\, s^m\, E_o^* \qquad \text{(B.43)}$$

In time notation with rearrangement of terms

$$e_o^{m*} = \frac{1}{B_m}\, e_i - \frac{1}{B_m}\, e_o^* - \frac{B_1}{B_m}\, e_o^{I*} - \frac{B_2}{B_m}\, e_o^{II*}$$

$$\ldots(-)\ \frac{B_n}{B_m}\, e_o^{n*} \ldots \qquad \text{(B.44)}$$

where "m" is the highest derivative.

Substitution of Equation (B.42) into Equation (B.44) gives

$$e_o^{m*} = \frac{1}{B_m} e_i - \frac{1}{B_m} e_o^{*} - \frac{B_1}{B_m} e_o^{I*} - \frac{B_2}{B_m} e_o^{II*} - \frac{B_n}{B_m A_n} e_o$$

$$+ \frac{B_n}{B_m A_n} e_o^{*} + \frac{B_n A_1}{B_m A_n} e_o^{I*} + \frac{B_n A_2}{B_m A_n} e_o^{II*} + \ldots \qquad (B.45)$$

After terms are grouped and when e_o is solved for, Equation (B.45) becomes

$$e_o = \frac{A_n}{B_n} e_i + \left(1 - \frac{A_n}{B_n}\right) e_o^{*} + \left(A_1 - \frac{A_n B_1}{B_n}\right) e_o^{*I} + \left(A_2 - \frac{A_n B_2}{B_n}\right) e_o^{*II}$$

$$+ \ldots (-) \frac{B_m A_n}{B_n} e_o^{m*} \qquad (B.46)$$

Equation (B.46) expresses the output of the filter as a function of the input, e_i, and e_o^{*} with its derivatives. Equation (B.44) is the expression for the highest derivative of e_o^{*} as a function of the input and lower derivatives of e_o^{*}. Therefore, these equations, with a high-accuracy integrator such as Runge-Kutta-Gill, can be used to calculate the transfer of the filter, G(s). These computations are now compatible with the remainder of the simulation since a differentiation routine is not required.

B.4 GAIN SCHEDULING

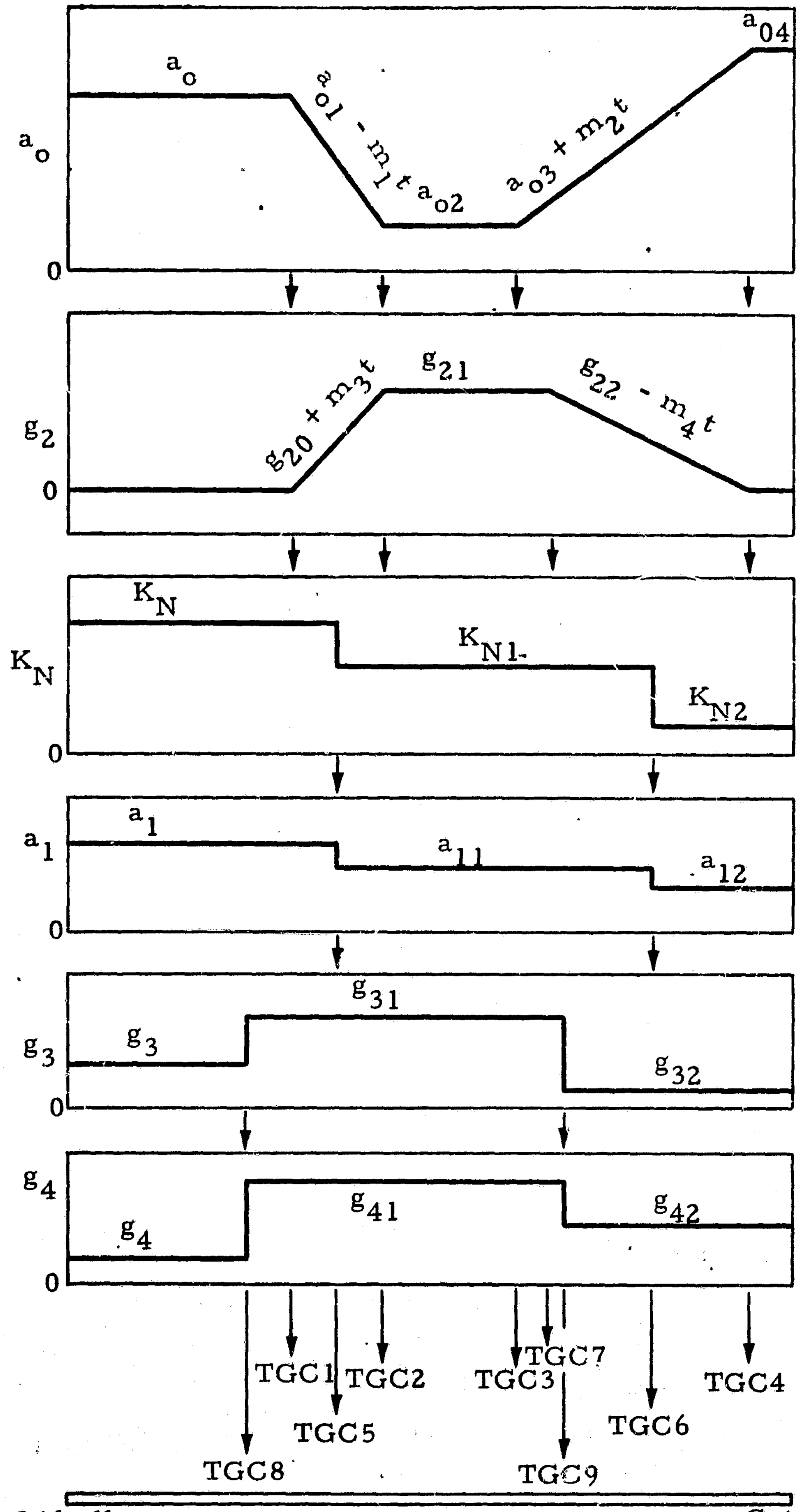
The gain terms as illustrated in Figures 3 and 4 have been programmed with the following general gain change schedule.

$a_o = a_o$	$0 \le t < TGC1$
$a_o = a_{o1} - m_1 t$	$TGC1 \le t < TGC2$
$a_o = a_{o2}$	$TGC2 \le t < TGC3$
$a_o = a_{o3} + m_2 t$	$TGC3 \le t < TGC4$
$a_o = a_{o4}$	$TGC4 \le t$
$K_N = K_N$	$0 \le t < TGC5$
$K_N = K_{N1}$	$TGC5 \le t < TGC6$
$K_N = K_{N2}$	$TGC6 \le t$
$a_1 = a_1$	$0 \le t < TGC5$
$a_1 = a_{11}$	$TGC5 \le t < TGC6$
$a_1 = a_{12}$	$TGC6 \le t$
$g_2 = 0$	$0 \le t < TGC1,\ t \ge TGC4$
$g_2 = g_{20} + m_3 t$	$TGC1 \le t < TGC2$
$g_2 = g_{21}$	$TGC2 \le t < TGC7$
$g_2 = g_{22} - m_4 t$	$TGC7 \le t < TGC4$
$g_3 = g_3$	$0 \le t < TGC8$
$g_3 = g_{31}$	$TGC8 \le t < TGC9$
$g_3 = g_{32}$	$TGC9 \le t$
$g_4 = g_4$	$0 \le t < TGC8$
$g_4 = g_{41}$	$TGC8 \le t < TGC9$
$g_4 = g_{42}$	$TGC9 \le t$

An example relationship among schedules is illustrated in Figure B-2.

B.5 DATA

All time varying coefficients are compiled into the program in the form of block data. These coefficients number over 160 and are generated in a



Liftoff

Cutoff

S-IC Stage Flight Time

Figure B-2 - General Gain Schedule

compatible format by the Launch Vehicle Data Program. The block data presently in the program source deck is for the Saturn/Voyager, 45-foot cylindrical payload, and the 1973 mission trajectory. All block data routines, however, are in the form of sub-programs. Data for any desired vehicle or trajectory may be used by simply recompiling the program with the new generated data from the Launch Vehicle Data Program.

The external excitation to the system equations is the wind bias angle, α_w. Several tables of α_w versus flight time have been computed and compiled into the program with the option to select any one of the cases. The vehicle is perpendicular to the wind shear in the yaw plane and experiences greater loads than in the pitch plane for the same wind profile. Therefore, yaw plane wind angle has been included as an option in addition to pitch plane wind angle. The cases included are

- MSFC measured wind profiles 1639 and 515-06
- MSFC reversed shear wind profile with maximum May-December envelope or maximum annual envelope
- MSFC 95% synthetic wind profile with superimposed gust at Mach 1, maximum $q\alpha$, maximum $\hat{q}$, or Mach 2 with maximum May-December envelope or maximum annual envelope.

Under extreme wind shear conditions vehicle angle of attack, α, considerably exceeds the linearized region of the aerodynamic data. Consequently rigid body nonlinear aerodynamics have been included in the program as an option. When used, the program performs a table look-up on the rigid body aero coefficients as a function of flight time and α every computational pass. When α falls below two degrees, the program automatically uses linearized aero coefficients.

Appendix C

LAUNCH VEHICLE DATA PROGRAM

Appendix C
LAUNCH VEHICLE DATA PROGRAM

The Launch Vehicle Data Program is a companion program to the Launch Vehicle Simulation Program described in Appendix B. The purpose of the program is to eliminate any computation from the simulation program which is a function only of the launch vehicle characteristics and reference trajectory. The output of the program is a listing in table form of the coefficients versus flight time. Also, the output tables are automatically card-punched in the formats required by the Launch Vehicle Simulation Program.

The program essentially mechanizes the indicated operation in Figure B-1 and outputs time tables of the coefficients contained in Equations (B.4) through (B.36) of Appendix B. The required inputs are the vehicle and trajectory parameters such as aerodynamic pressure, vehicle mass, normalized bending mode deflections and slopes, slosh modal mass, etc. A complete description of input requirements and output characteristics is presented in the program utilization report, Reference 5.