UNIVERSITY OF SOUTHERN CALIFORNIA

THE STUDY OF SYNCHRONIZATION TECHNIQUES FOR OPTICAL COMMUNICATION SYSTEMS

Third Quarterly Report
Covering Period of 1 June 1969 to 1 October 1969

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1.1 Introduction

This study program is devoted to an investigation of problems associated with synchronizing an optical communication system. The objective of the effort is to indicate design procedures, assess system performance, and predict future areas of needed study in synthesizing and improving system operation. The organization and guideline of the study, the particular problem areas, and their applications are covered in the first quarterly report, March, 1969.

The research effort is primarily analytical in nature, and is divided into two categories. The first involves tasks with direct application to the synchronization problem, while the second involves related areas also being studied under the grant. This document reports technical progress during the third quarter, but detailed results will be published in separate interim reports, and are omitted here.

1.2 Pure Synchronization with Direct Detection

This task is devoted to the general problem of phase locking to a shot noise process. This basically represents the problem one encounteres when a phaselock loop follow's a narrow band photodetector in an optical communication system. The above system is used when pure timing information is transmitted over an optical link by intensity modulating a CW laser source with a periodic signal and using direct detection at the receiver. The phase error resulting from this tracking procedure therefore represents the timing error in the overall system.

In the analysis completed to date, the differential equation of the steady state probability density of the loop tracking error has been derived via the Kolgomorov Theory. This was found to be an infinite order partial differential equation whose n^{th} coefficient decreases as $1/A^n$, where A is the signal to noise ratio of the shot noise (more specifically, A is related to the average number of photons in the tracking loop bandwidth). During the past quarter the first perturbation to the solution of the truncated differential equation has been studied. By applying perturbation methods, it can be established that the first correction term to the truncated solution is obtained by consideration of a differential equation composed of the truncated portion, with the addition of the next term of the infinite equation. This latter equation had been previously solved by a series solution using a digital computer, and the perturbation methods have established some degree of meaning to the resulting solution.

1.3 Synchronization with Heterodyne Detection

The problem of phase locking after a heterodyne detector is being considered although no new results are reported for this period. The problem is different from that in Section 1-2 in that the statistics of the shot noise process are not known in general, and the assumption of poisson counts may not be valid. The problem of determining these count distributions is basically that of analyzing count statistics at the output of a photo-detector when the input intensity is a sum of two sinusoids (plus background noise). Although the moment generating function is known, the inversion to obtain the density is extremely difficult. For design purposes, one can assume the intensity levels are sufficiently high so that the detector output signal can be viewed as a continuous signal, rather than shot noise. In this case analysis would proceed using "wave functions"--the input to the tracking loop being simply the synchronizing signal plus additive gaussian noise. One objective of this study task is to determine the range of validity of this procedure.

1.4 Synchronization with Pulsed Lasers

When the synchronizing signal is transmitted as narrow periodic pulses (e.g., with pulsed laser operation) the tracking is often accomplished by a split gate tracker. This type of system is currently being studied, using the techniques of Section 1.2. The differential equation of the phase error is being examined, with hopes of determining the probability density and the variance of the tracking error. Such a result would be useful for a tradeoff study utilizing narrow and wide gates. The differential equation is similar to that for phase lock loops, except the correlation function of the split gate trackers enters in, rather than the sinusoidal error terms. Methods to solve the equation are being studied. The results would also have application to laser radar systems.

1.5 Effect of Tracking Error on Error Rates in PPM Systems

The probability of error in a binary PPM system is known for the case of perfect synchronization (i.e., the system counts photo-electrons exactly over the signalling intervals). When timing errors occur, the error probabilities are a function of this timing uncertainty.

Consider a binary PPM system in which the transmitter sends a known signal intensity in one of two adjacent time intervals, representing a binary one or zero. The receiver effectively counts photo-electrons in the first and second intervals independently and decides one or zero depending upon which has the largest count. Thus if k_i is the count in the i^{th} interval, $i = 1, 2$, the receiver decides a binary one was sent if $k_1 \geq k_2$, and zero otherwise. The operation implicitly assumes perfect synchronization (timing) between the transmitter and receiver, so that counting occurs exactly over the proper interval. If a one were sent, an error occurs if $k_2 > k_1$. Under a narrow band assumption the probability density of k_i is given by

$$p(k_i) = Pos(k, x_i) \qquad i = 1, 2 \tag{1}$$

where $Pos(k, x_i)$ represents a poisson density in the integer variable k with intensity x_i. Thus, if a binary one was sent over a 2T bit interval with signal power N, in the presence of background power M, the above becomes

$$p(k_1) = \mathrm{Pos}[\,k, (N+M)T] \tag{2a}$$

$$p(k_2) = \mathrm{Pos}[\,k, MT] \tag{2b}$$

Now assume an error of Δ sec occurs in the timing of the counting intervals. That is, the receiver begins counting for 2T sec (T sec for the first interval and T sec for the second interval) with an error of Δ sec in the starting time. In this case the receiver counts over portions of the adjacent bit intervals instead of the proper intervals. Assuming there is equal probability that adjacent bits are ones or zeros, the counting statistics in (2) are modified to

$$\left.\begin{aligned} p(k_1) &= \mathrm{Pos}[\,k, N(T-\Delta) + MT] \\ p(k_2) &= \mathrm{Pos}[\,k, MT + \frac{N\Delta}{2}\,] \end{aligned}\right\} \quad \text{for } \Delta > 0 \tag{3a}$$

and

$$\left.\begin{aligned} p(k_1) &= \mathrm{Pos}[k, N(T- \frac{\Delta}{2}) + MT] \\ p(k_2) &= \mathrm{Pos}[\,k, MT + N\Delta] \end{aligned}\right\} \quad \text{for } \Delta < 0\,. \tag{3b}$$

If we define $K_s = NT$, $K_n = MT$ as the signal and noise energy in the counting interval T, then the probability of error when a one is transmitted is

$$PE/\Delta > 0 = \sum_{k_2=0}^{\infty} \mathrm{Pos}[k_2, k_s(\frac{\Delta}{2T}) + k_n] \sum_{k_1=0}^{k_2} \mathrm{Pos}[k_1, ks(1- \frac{\Delta}{T}) + k_n] \tag{4a}$$

and

$$PE/\Delta < 0 \;=\; \sum_{0}^{\infty} Pos[k_2, K_s(\frac{|\Delta|}{T}) + k_n] \sum_{k_1=0}^{k_2} Pos[k_1, K_s(1-\frac{|\Delta|}{2T}) + K_n] \tag{4b}$$

depending upon whether Δ is positive or negative. (If a zero was sent, the error probabilities for Δ positive and negative would be reversed. Hence, the above equations can be interpreted as an average error probability, assuming zeros and ones are equally likely.) Note that P_E depends only upon the summation in brackets, and the effect of Δ is to alter the value of K_s and K_n. That is

$$PE/\Delta > 0 \;=\; PE(K_s', K_n' \mid \Delta = 0) \tag{5a}$$

$$PE/\Delta < 0 \;=\; PE(K_s', K_n'' \mid \Delta = 0) \tag{5b}$$

where

$$K_s' = K_s(1 - \frac{3|\Delta|}{2T})$$

$$K_n' = K_n + (\frac{\Delta}{2T})$$

$$K_n'' = K_n + K_s(\frac{|\Delta|}{T})$$

and the functions on the right are error probabilities for perfect timing ($\Delta = 0$) perviously derived by Gagliardi and Karp. That is, the timing error can be accounted for by merely reinterpreting the effective signal and noise energies. Note that timing errors act to reduce the effective signal energy and increase the effective noise, the overall result degrading the error probability.

The effect of timing errors can be assessed by computing an average (over Δ) error probability, $\bar{P}E$. Thus,

$$\overline{PE} = \int_0^{\infty} PE(K_s', K_n' | \Delta = 0)\, p(\Delta)\, d\Delta + \int_{\infty}^{0} PE(K_s', K_n'' | \Delta = 0)\, p(\Delta)\, d\Delta \qquad (6)$$

where $p(\Delta)$ is the probability density of the timing error Δ. It appears these integrals are difficult to evaluate in closed form for the types of $p(\Delta)$ derived in Section 1-1. Present study is devoted to consideration of these integrals.

1.6 Synchronization as an Estimation Problem

The problem of synchronization as an optical system is also being studied as a theoretical problem in estimating the phase variable within a shot noise process. Using a simplified model for the estimating system, (one that counts photo-electrons over intervals and uses these as inputs to the estimator) optimal procedures for deriving maximum likelihood estimates of phase variables have been derived for certain a priori densities. The unrealizable (non-causal) system takes the form of a feedback signal generator (tracking loop) in which the signal has the form of a cross-correlation signal, normalized by the synchronizing signal. This can be interpreted as the "typical" feedback signal, normalized by the variance of the shot noise. When the non-stationary shot effect is negligible (high average intensity level) the feedback signal approaches that for the additive gaussian noise case.

Study in Related Areas

2.1 Properties of Shot Noise Processes

Study is continuing on the investigation of the true statistics of the photo-electron count at the detector output. This requires averaging a poisson density, whose intensity is

$$u(t, T) = \int_{t}^{t+T} |e(x)|^2 \, dx \tag{7}$$

where

T = photodetector bandwidth

$e(T_t)$ = received radiation field

The averaging must be carried out over the statistics of U(t, T). A method is being studied to derive the probability density of U(t, T) as a continuous function of t, using diffusion theory. This would allow specification of the probability density for the electron counts as a diffusion process.

2.2 Optimal Detection and Filtering

The optimal mean square estimator of the signal portion of the received intensity is presently being studied. Such a device would be placed after the photodetector, and would be used to recover the stochastic signal intensity from the shot noise process. The system resembles the form of an optimal Wiener filter, with terms due to both shot noise and signal and noise cross products. In addition, it can be shown that the overall estimator is never linear, since it must contain an additive bias in order to minimize the estimation error. (This is primarily due to the presence of a d. c. term in the shot noise intensity.) The addition of such a bias always renders a system nonlinear. The optimal estimator which accounts for spatial effects is presently being examined.

2.3 Information Rate of an Optical Link

An attempt is being made to derive the information rate of an optical system utilizing photodetectors at the receiver. The information rate is defined to be the entropy of the photo-electron counts at the detector output minus the equivocation of the detector. That is, it is the information content at the output minus the information at the output due to ambiguities in reception. The study is motivated by the fact that there exists some confusion in the literature concerning the information rate of the transmitted field and the information rate observed at the detector output. In the effort to date attention has been confined to band limited white gaussian fields. It has been shown by others that the probability of a count k of photo-electrons at the detector output, when receiving a one dimensional gaussian field, is given by

$$P(k) = \binom{2BT+k}{k} \left(\frac{1}{1+N_0}\right)^{2BT+1} \left(\frac{N_0}{1+N_0}\right)^{k} \qquad (8)$$

where B is the field bandwidth, T is the detector integration time, and N_0 is field spectral level. This can be viewed as the 2BT convolution of

Bose-Einstein (BI) probabilities, implying that the detector can be considered as the sum of 2BT +1 independent "mode" counts, each BI distributed. Thus, if we denote each such mode count as k_i, then the received field inherently contains the count vector $\underline{k} = \{k_1, k_2, \cdots, k_{2BT+1}\}$, but the output count is

$$k = k_1 + k_2 + \cdots, k_{2BT+1} \tag{9}$$

The maximum entropy in the field, as computed by Gordon, Bowen, and others, is generally taken as the entropy per mode summed over all modes. The entropy per mode is known to be

$$H(N_0) = (N_0 + 1) \log(N_0 + 1) - N_0 \log N_0 \tag{10}$$

and the total entropy is then

$$H_{Total} = (2BT+1)\, H(N_0) \tag{11}$$

This is generally taken as the Gordon bound for the information content of the optical field, but note that it assumes that each mode count can be individually detected. The receiver, whose count is described by (9), does not observe individual mode counts of $2BT \geq 1$, and information is destroyed. Thus, the output information rate is

$$R = H_{out} - H_{un} \tag{12}$$

where

$$H_{out} = \sum_k P(k) \log P(k)$$

$$H_{un} = E_x \left\{ \sum_k P(k|x) \log P(k|x) \right\}$$

$$p(k|x) = x^k e^{-x} | k!$$

and x is the field intensity integrated over T. For the case where $2BT >> 1$ and $N_0 << 1$, P(k) in (8) approaches a poisson density. The total entropy in (11) increases, but the rate R in (12) approaches zero. Physically, this means the detector integrates the field over a "long" time and the information in the field is destroyed. When $2BT << 1$, (11) still holds, but the rate is altereed. In particular, if $N_0 >> 1$, H_{un} in (12) approaches $\frac{1}{2} H_{Total}$ and therefore, $R \rightarrow \frac{1}{2} H(N_0)$. That is, half the information of the field is lost in detection. Further study on this problem is in progress.