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ACCELERATIONS ON 24-HOUR SATELLITES AND LOW ORDER LONGITUDE TERMS IN THE GEOPOTENTIAL

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ABSTRACT

The tracking record of six 24-hour satellites (Syncom 2,3; Early Bird; ATS 3,5 and Intelsat 2F3) has been analysed and reduced to a set of 35 well determined accelerations due to the anomalous geopotential. The record covers the years 1963-1969 and has a fair distribution in longitude. From this record two "best" resonant geopotential fields to fourth order are derived as well as new locations of the four east-west equilibrium points for the geostationary satellite.

There is considerable improvement in the low order field of other recent (1969) gravity models, in contrast to the "best" models in 1966, as judged by this 24-hour acceleration data.

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ACCELERATIONS ON 24-HOUR SATELLITES AND LOW ORDER LONGITUDE TERMS IN THE GEOPOTENTIAL

INTRODUCTION

Since 1963, there have been literally scores of satellites put into synchronous orbit, mostly for communications purposes. An examination of the maneuver free tracking record of these nearly geostationary objects has revealed long term accelerations which are almost entirely due to the anomalous geopotential of low order.[1] The record of the objects which are closest to being geostationary, and which therefore suffer, generally, the largest such accelerations for the longest times, has permitted some very sensitive tests to be made of solutions for the low order geopotential field.[2] It has also permitted an excellent value of the 2,2 harmonic (associated with equatorial ellipticity) to be derived. Because the past record has been sparse in longitude distribution, values of the harmonics 3,3 and 3,1 which also influence this data significantly, have been less well determined. These third order harmonics (and to a lesser extent, the fourth order harmonics), while not dominant on the accelerated drift (east-west) of the 24 hour satellite, do affect the east-west equilibrium points for a geostationary satellite by as much as 4 degrees. Therefore, in order to determine all the equilibrium longitudes for these satellites to high accuracy ($<1°$) we need reasonably good values of 3,3 and 3,1 or what amounts to the same thing, a good distribution of longitudes in the 24 hour record.

In 1967, the available data (of high quality) on nearly geostationary satellites was over a fairly limited longitude range on only three synchronous satellites.[2] In this report, data from three more 24-hour satellites are added to the previous record; Intelsat 2F3, ATS 3 and ATS 5. The three new accelerations, at well separated longitudes, in combination with the old data, provide estimates of the 4 equatorial equilibrium points for geostationary satellites that are now felt to be good to better than 3/4° of their true locations. In addition, the solution for the 3,3 harmonic is now felt to be quite realistic from the total acceleration record.

NEW TRACKING DATA AND REDUCTION TO GEOPOTENTIAL ACCELERATIONS

ATS 3 (1967 111A)

The tracking data used consisted of 3010 smoothed range and range rate observations from the Goddard Space Flight Center's Tracking Stations at Rosman North Carolina and Mohave, California.[3] The data period was from Feb. 12 to April 26, 1969 when the satellite was slowly drifting near 72° west. It's orbit inclination at this time was 0.43°. The eccentricity was 0.00008 and the semimajor axis was 6.6103 earth radii. This data was processed by the Goddard Orbit and Gravity Determination Program known as GEOSTAR[4] (GEOpotential and STAtion Recovery). This program solved (in a least-squares sense through this data) for a "best" set of six orbit elements as well as the 2,2 gravity harmonic. The reference trajectory of the solution included the effects of solar radiation pressure, the gravity fields of the sun and moon and a geopotential field complete through 8,8.

The 2,2 coefficient of this field was corrected in order to estimate as closely as possible the geopotential acceleration in this free drift arc. Since the arc was not over precisely one longitude, two runs of GEOSTAR were made. The first corrected for both C_{22} and S_{22} and served (through the correlation between them) to determine merely at what geographic longitude the acceleration was best determined. For this purpose it was interesting that only the ratio of the standard errors in the determination of C_{22} and S_{22} was necessary. Since the absolute correlation between these coefficients seemed too high to be trusted, a second run was made fixing S_{22} and solving for only C_{22} to calculate more precisely the actual geopotential acceleration and it's error at this "best" longitude.

The geopotential convention and notation employed is as follows:

$$V_e = \frac{\mu}{r} \left[1 + \sum_{\ell=2}^{\infty} \sum_{m=0}^{\ell} \left(\frac{r_e}{r} \right)^{\ell} P_{\ell m} (\sin \varphi) \right.$$

$$\left. \{ C_{\ell m} \cos m\lambda + S_{\ell m} \sin m\lambda \} \right],$$

where μ is the earth's Gaussian gravity constant (3.98601×10^5 km^3/sec^2), r is the radius from the earth's center of mass to the satellite's, r_e is the mean equatorial radius of the earth (6378.16 km), φ is the satellite's geocentric latitude and λ is its geocentric (geographic) longitude. The $P_{\ell m}$ are associated Legendre functions. The gravity coefficients $C_{\ell m}$ and $S_{\ell m}$ represent longitude dependent harmonic terms ($H_{\ell m}$) when $m \neq 0$. These are the terms which concern us here, and in particular, for the high altitude 24 hour satellites, only the lowest orders (ℓ) of these terms (approximately for $\ell \leq 4$), because of the distance damping factor $(r_e/r)^{\ell}$ in the potential function. The more easily visualized amplitudes $J_{\ell m}$ and phase angles $\lambda_{\ell m}$ of the harmonic terms $(H_{\ell m})$ are related to the $C_{\ell m}$, $S_{\ell m}$ coefficients of these terms by the formulas:

$$J_{\ell m} = [C_{\ell m}^2 + S_{\ell m}^2]^{1/2}$$

$$\lambda_{\ell m} = \frac{1}{m} \tan^{-1} (S_{\ell m}/C_{\ell m}).$$

The resonant (long-term) geopotential (east-west) acceleration on the 24 hour satellite is given by:[2]

$$\ddot{\lambda} = 12\,\pi^2 \sum_{\ell - m \text{ EVEN}} F_{\ell m} J_{\ell m} \sin m\,(\lambda - \lambda_{\ell m}) \frac{\text{radians}}{\text{sidereal day}^2}.$$

$$= 12\pi^2 \sum F_{\ell m} \, [C_{\ell m} \sin m\lambda - S_{\ell m} \cos m\lambda]. \qquad (1)$$

The relevant coefficients $F_{\ell m}$, to fourth order, $\ell \leq 4$, are given as:

$$F_{22} = \frac{6}{a^2} [(\cos i + 1)/2]^2$$

$$F_{31} = \frac{-3}{2a^3} \left\{ \frac{(1 + \cos i)}{2} - \frac{5 \sin^2 i \, (1 + 3 \cos i)}{8} \right\}$$

$$F_{33} = \frac{45}{a^3} [(\cos i + 1)/2]^3$$

$$F_{22} = \frac{-15}{a^4}\left\{\frac{(1+\cos i)^2}{4} - \frac{7\sin^2 i \cos i\,(1+\cos i)}{4}\right\}$$

$$F_{44} = \frac{420}{a^4}\,[(\cos i + 1)/2]^4.$$

In these coefficients, a is the orbit semimajor axis in earth radii ($\sim$6.61), and i is it's inclination.

In order to estimate the error i . $\ddot{\lambda}$ from a set of solved-for gravity coefficients G_i with it's associated covariance matrix cov G_{ij} determined in an experiment from data whose standard error is S, ($\ddot{\lambda}$ being a linear function of the G_i 's) I use the familiar error propagation formula:

$$S(\ddot{\lambda}) = S\left[\sum_{i,j} A_i A_j \operatorname{cov} G_{ij}\right]^{1/2}, \tag{2}$$

where

$$\ddot{\lambda} = \sum_i A_i G_i .$$

The normalized variance of $\ddot{\lambda}$ with respect to two determined gravity coefficients is:

$$\left[\frac{S^2(\ddot{\lambda})}{S^2}\right] = A_1^2 \operatorname{cov} G_{11} + A_2^2 \operatorname{cov} G_{22} + 2A_1A_2 \operatorname{cov} G_{12}.$$

Optimizing this with respect to a single independent variable of the A's gives the equation:

$$0 = 2 A_1 A_1' \operatorname{cov} G_{11} + 2 A_2 A_2' \operatorname{cov} G_{22} + 2 \operatorname{cov} G_{12} [A_1' A_2 + A_2' A_1] .$$

For

$G_1 = C_{22}$ $G_2 = S_{22}$, and the independent variable being λ;

equation (1) gives:

$$A_1 = 12 \pi^2 F_{22} \sin 2\lambda, \quad A_2 = -12 \pi^2 F_{22} \cos 2\lambda,$$

$$A_1' = 2 (12 \pi^2) F_{22} \cos 2\lambda, \quad A_2' = 2 (12 \pi^2) F_{22} \sin 2\lambda .$$

The optimized λ then, is determined from the equation:

$$0 = \sin 2\lambda \cos 2\lambda \operatorname{cov} C_{22} - \sin 2\lambda \cos 2\lambda \operatorname{cov} S_{22}$$

$$+ [- \cos^2 2\lambda + \sin^2 2\lambda] \operatorname{cov} (C_{22} S_{22}).$$

Using the double angle formulas this equation reduces to:

$$\tan 4\lambda = \frac{2 \operatorname{cov} (C_{22} S_{22})}{\operatorname{cov} C_{22} - \operatorname{cov} S_{22}} .$$

The standard deviations of the gravity coefficients and the correlation coefficient (r) have as their definition:

$$\sigma(C_{22}) = [S^2 \operatorname{cov} C_{22}]^{1/2}$$

$$\sigma(S_{22}) = [S^2 \operatorname{cov} S_{22}]^{1/2},$$

and

$$r = \frac{\operatorname{cov} (C_{22} S_{22})}{[\operatorname{cov} C_{22} \operatorname{cov} S_{22}]^{1/2}} .$$

With these definitions, the λ's where the variance of $\ddot{\lambda}$ is minimum (and maximum) are given by:

$$\lambda = \frac{1}{4} \tan^{-1} \left[\frac{2r}{\frac{\sigma(C_{22})}{\sigma(S_{22})} - \frac{\sigma(S_{22})}{\sigma(C_{22})}} \right] . \tag{3}$$

Thus, when the two determined coefficients of $H_{2,2}$ are highly correlated ($r \doteq \pm 1$), the optimized λ's are essentially determined only from the ratio of the standard deviations of these coefficients.

In the first GEOSTAR solution through the ATS 3 data, the standard deviations were:

$$\sigma(C_{22}) = 4.24 \times 10^{-7}$$

$$\sigma(S_{22}) = 2.99 \times 10^{-7},$$

and

$$r \doteq 1 .$$

From Equation (3):

$$\lambda \text{ [for optimum } \ddot{\lambda}] = \frac{1}{4} \tan^{-1} (2.82)$$

$$= n\,(45^\circ) + 17.6^\circ; \quad n = 0, 1, 2, \cdots 7 .$$

A minimum $S(\ddot{\lambda})$ occurs when $n = 7$; at $\lambda = 287.6^\circ$

$$= -72.4^\circ .$$

The gravity results of the second GEOSTAR run through the tracking data, solving only for C_{22} and the six orbit parameters were:

$$10^6\, C_{22} = 1.6388 \pm .0025,$$

using the Smithsonian Astrophysical Observatory 1966 M1 Geopotential Coefficients[5] except for C_{22} in the trajectory. It is interesting that the M1 value of 10^6 C_{22} is quite close to this, being equal to 1.536. This was expected since the M1 field has predicted 24 hour accelerations rather well at many other longitudes.[2] The M1 coefficients through 4,4 with the ATS 3 value of C_{22} are:

Units of 10^{-6}

2,2		3,1		3,3		4,2		4,4	
C	S	C	S	C	S	C	S	C	S
1.6388	-.8721	2.091	.287	.0782	.226	.0738	.148	-.0011	.0049

Using these values in Equation (1) and the $\sigma(C_{22})$ value in Equation (2), I calculate that for this ATS 3 arc:

$$10^5 \ddot{\lambda} \ (\lambda = -72.4°) = -2.163 \pm 0.002 \frac{\text{rad}}{\text{day}^2} .$$

For this study, effects of the higher order geopotential have not been calculated. But from the decline of maximum effects[1] of 2nd, 3rd and 4th order, I estimate a very conservative contribution to $\ddot{\lambda}$ due to the geopotential beyond 4th order would be $\pm 0.010 \times 10^{-5}$ rad/day^2. Other unmodeled forces and biases may produce an additional uncertainty of $\pm 0.01 \times 10^{-5}$ R/D^2 (which seems justified from previous studies of these satellites[1,2]). Therefore, I use a root mean square error of $(.002^2 + .010^2 + .010^2)^{1/2} = .015 \times 10^{-5}$ R/D^2 as the absolute uncertainty of the ATS 3 acceleration for this study.

ATS 5 (1969-69A)

The tracking data used consisted of 850 smoothed range and range rate observations from the stations at Rosman and Mohave[3] during the period Sept. 11

to Oct. 4, 1969 when the spacecraft was in a slow free drift near 107° west. It's orbit at this time had an inclination of 2.78°, an eccentricity of 0.0019 and a semimajor axis of 6.6105 earth radii. The data was processed in two reductions by GEOSTAR exactly as described for ATS 3.

In the first GEOSTAR solution through the data, the standard deviations for C_{22} and S_{22} were:

$$\sigma(C_{22}) = 24.98 \times 10^{-7}$$

$$\sigma(S_{22}) = 17.02 \times 10^{-7}$$

$$r \doteq -1$$

From Equation (3):

$$\lambda \text{ [for optimum } \ddot{\lambda}] = \frac{1}{4} \tan^{-1} (-2.56)$$

$$= n(45°) - 17.2°$$

A minimum $S(\ddot{\lambda})$ occurs when n = 6; at λ = 252.8° = −107.2°.

The gravity results of the second GEOSTAR run through the tracking data, solving only for C_{22} and the six orbit parameters were:

$$10^6 \, C_{22} = 1.6070 \pm .0512 \, .$$

Using the M1 geopotential constants (with the C_{22} above) as in the converged trajectory, in Equation (1); and the $\sigma(C_{22})$ value directly above in Equation (2), I calculate that for this ATS 5 arc:

$$10^5 \ddot{\lambda} \; [\lambda = -107.2°] = 0.232 \pm .047 \; R/D^2 .$$

Considering other likely bias's and model errors, the absolute error in this measurement is estimated as 0.050×10^{-5} R/D^2.

Intelsat 2F3 (1967 26A)

Between April 27, 1967 and (at least) Feb. 19, 1968, the Intelsat 2F3 spacecraft was in free drift between 11.4° and 7.6° west longitude. During this period, the Communications Satellite Corporation reported 12 sets of orbital elements,[6] updated approximately every 4 weeks from new sets of range, azimuth and elevation data taken primarily at the Andover, Maine station. This set of osculating elements is shown in Table 1. The program which derived these elements is believed to be a Cowell numerically integrating program which includes the effects of the sun and moon's gravity. I have converted these osculating elements to Mean Brouwer Elements by removing the short period effects of J_{20} in the geopotential.[7] Such elements still have in them the short period effects of the sun and moon's gravity field. However, Frick and Garber have calculated[8] that the greatest amplitude from these terms is 0.005 degrees in the geographic longitude. This is generally well below the "noise" level in long arc data reductions with 24 hour satellites.

The set of Brouwer mean elements was then examined by the Rapid Orbit Analysis and Determination Program[9,10] (ROAD) which found a best set of 2,2 constants for this long arc by using the mean longitude of the ascending node ($\lambda = M + \omega + \Omega - \theta_g$, where θ_g is the Greenwich Hour Angle) as the principal data type. The program included only the long term effects of 2,2 J_{20}, J_{30} and the sun and moon's gravity in the reference trajectory. The 2,2 determination for this arc was:

Table 1

Osculating Kepler Elements for Intelsat 2F3 in 1967-1968

Time Yr-mo-day-hr-min	a (earth radii)	e	I (deg.)	Node Ω (deg.)	Arg. of per. ω (deg.)	Mean Anomaly M (deg.)
67 04 27 00 35	6.611023	.000283	1.327	287.046	286.461	358.200
05 11 14 05	1153	299	289	7.669	311.907	189.095
06 07 16 00	1051	257	235	8.158	276.258	279.802
06 07 16 00	0859	287	219	7.939	272.959	283.276
07 13 02 10	0969	269	100	8.058	279.361	104.508
08 17 00 00	1069	401	016	7.473	269.164	117.388
10 18 04 00	0771	541	0.894	7.143	287.947	220.827
11 20 01 00	0968	517	826	9.990	291.448	202.327
12 21 00 00	0765	534	729	292.772	301.046	206.000
68 01 09 00 00	0772	462	673	3.405	310.758	214.758
01 26 00 00	0777	300	626	4.537	302.918	238.606
02 19 00 00	0564	310	557	4.147	313.795	252.403

$$10^6\, C_{22} = 1.2556 \pm .2238$$

$$10^6\, S_{22} = -0.5484 \pm .0831$$

$$S(\lambda) = 0.029 \text{ degrees}$$

$$r(C_{22}, S_{22}) = -.99931.$$

Using Equations (1), (2) and (3) these results (with a = 6.611 and i = 1.0°) give the best determined acceleration in this arc as:

$$10^5\, \ddot{\lambda} = .124 \pm .005 \quad R/D^2 \quad \text{at} \quad \lambda = -10.2°.$$

For reasons similar to those given for the ATS 3 reduction, a very conservative estimation of 0.015×10^{-5} R/D^2 was actually used as the measurement error in this arc for this study.

DETERMINATION OF LOW ORDER RESONANT GRAVITY COEFFICIENTS

Adjustments of Measurement Weights for Unestimated Higher Order Effects

The ATS and Intelsat reductions to accelerations have included in the measurement errors an estimation of the effect of terms beyond 4th order which will not be solved for in the inversion of the acceleration data to gravity constants. As long as such terms are neglected in the inverstion, the previous acceleration record[2] should also be degraded similarly. Kozai also discusses this kind of data degradation in Reference 11. Two kinds of inversions of Equation (1) will be presented here; one with and one without 4th order terms. The data in the reductions without fourth order harmonics are degraded by $(0.010^2 + 0.016^2 + 0.018^2)^{1/2} \times 10^{-5}$ R/D^2 = 0.026×10^{-5} R/D^2 to account for unestimated harmonics beyond fourth order (0.010), H_{42} (0.018) [See Reference 1] and H_{44} (0.016) [See Reference 1]. Reductions with 4th order harmonics have the data degraded by 0.010×10^{-5} R/D^2 except for the ATS 3,5 and Intelsat measurements. The new errors are estimated as the root sum of squares of the old errors plus the new degraded values. A list of the observations and weights used in this study is given in Table 2. The measurements for all arcs except ATS 3,5 and Intelsat are taken from Table 3 in Reference 1.

Least Squares Inversions of the Condition Equations

Equation (1) was solved for sets of coefficients of 2nd, 3rd, and 4th order by the usual weighted least squares process, according to the conditioning acceleration

Table 2

Measured Accelerations in 24 Hour Satellite Arcs (1963-1969)

Arc	λ (Degrees East of Greenwich)	a Semimajor Axis (Earth Radii)	I Inclination (Degrees)	$\ddot{\lambda}$ $\left(10^{-5} \frac{\text{Radians}}{\text{Day}^2}\right)$	$\sigma(\ddot{\lambda})$ for Solutions Including 4th Order Terms (10^{-5} R/D^2)	$\sigma(\ddot{\lambda})$ for Solutions Excluding 4th Order Terms (10^{-5} R/D^2)
Syncom 2 (Feb-June '65)	65.3000	6.6109	31.7800	1.0000	0.0540	.059
Syncom 2 (Feb-June '65)	65.3100	6.6109	31.7900	1.0170	0.0540	.059
Syncom 2 (Feb-June '65)	66.1150	6.6112	31.8700	0.9680	0.0649	.068
Syncom 2 (Feb-June '65)	66.3900	6.6112	31.8900	0.8450	0.0860	.090
Syncom 2 (Jul-Mar '65-'66)	69.0000	6.6100	31.4000	0.6760	0.0480	.054
Syncom 2 (Jul-Mar '65-'66)	73.2500	6.6100	31.4000	0.2350	0.0390	.046
Syncom 2 (Jul-Mar '65-'66)	77.6000	6.6100	31.4000	-0.1520	0.0420	.049
Syncom 2 (Jul-Feb '64-'65)	104.5000	6.6176	32.1500	-2.3190	0.0680	.072
Syncom 2 (Jul-Feb '64-'65)	130.0000	6.6170	32.3000	-2.5500	0.0930	.097
Syncom 3 (Sep-Jan '66-'67)	160.7400	6.6100	1.3000	-0.0900	0.0370	.044
Syncom 2 (Jul-Feb '64-'65)	161.0000	6.6165	32.4000	-0.1940	0.0680	.072
Syncom 3 (Nov-May '65-'66)	168.2900	6.6100	0.5300	0.6990	0.0410	.048
Syncom 3 (Nov-May '65-'66)	169.1000	6.6100	0.5300	0.8310	0.0400	.047
Syncom 3 (Mar-May '65)	172.7500	6.6110	0.0	1.0720	0.0990	.102
Syncom 3 (Jan-Mar '65)	176.8000	6.6123	0.0270	1.5770	0.1750	.177
Syncom 3 (Oct-Jan '64-'65)	178.7100	6.6115	0.1100	1.6830	0.0610	.065
Syncom 2 (Apr-Jul '64)	-140.0000	6.6200	32.5800	2.3660	0.1599	.162
Syncom 2 (Apr-Jul '64)	-140.0000	6.6204	32.5800	2.1530	0.0860	.089
ATS 5 (Sep-Oct '69)	-107.2000	6.6105	2.7700	0.2320	0.0500	.056
ATS 3 (Feb-Apr '69)	-72.4000	6.6100	0.4300	-2.1630	0.0150	.030
Syncom 2 (Dec-Mar '63-'64)	-61.0000	6.6120	32.8300	-2.2880	0.1220	.125
Syncom 2 (Dec-Mar '63-'64)	-60.9400	6.6116	32.8250	-2.3200	0.0590	.064
Syncom 2 (Aug-Dec '63)	-55.2350	6.6110	33.0200	-2.2550	0.0860	.090
Syncom 2 (Aug-Dec '63)	-55.2200	6.6111	33.0200	-2.2380	0.0360	.044
Early Bird (Dec-Jan '65-'66)	-35.9800	6.6094	0.7400	-1.9760	0.0250	.035
Early Bird (Oct-Nov '65)	-34.5700	6.6123	0.6400	-1.8490	0.0370	.044
Early Bird (Sep-Oct '65)	-31.8600	6.6118	0.5500	-1.6820	0.0390	.046
Early Bird (Jan-Mar '66)	-31.4600	6.6101	0.8500	-1.6330	0.0170	.030
Early Bird (Jan-Mar '66)	-29.2900	6.6103	0.9000	-1.6440	0.0160	.029
Early Bird (Mar-Apr '66)	-29.8300	6.6101	0.8500	-1.5270	0.0180	.030
Early Bird (Apr-May '66)	-28.8300	6.6103	0.9000	-1.4940	0.0160	.029
Early Bird (Apr-June '65)	-28.7000	6.6105	0.2000	-1.4720	0.0170	.030
Early Bird (June-Aug '65)	-28.5400	6.6112	0.4300	-1.4530	0.0390	.046
Early Bird (June-Aug '65)	-28.3700	6.6112	0.4300	-1.4480	0.0170	.030
Intelsat 2F3 (Apr-Feb '67-'68)	-10.2000	6.6110	1.0000	0.1240	0.0150	.030

data of Table 2. The solutions reported here are the significant new ones for comparison with the previous results[2] without the ATS 3,5 and Intelsat data.

In Table 3 are given these significant new solutions for 2,2, 3,1, and 3,3 without and with fixed 3rd and 4th order coefficients from non-resonant satellite data. The resonant coefficients through 4,4 derived from non-resonant satellite data are also given in Table 3 for comparison purposes. In an addition in Table 3 is a comparison field derived from more extensive data on the 24-hour satellites by a numerically integrating orbit and gravity determination program described in References 9 and 10.

The important conclusion from these solutions is that, except for the SAO '66 M1 field, there is only a mild discrimination by this data between the given low order models. The first solution (without 4th order terms) certainly gives a superior fit to the data but the 3,1 coefficients are still not sufficiently realistic. However, they are considerably closer to the non-resonant models than they were previously from solutions with more limited data.[2] In the previous data set there were only two longitude zones (28°-35° west and 160°-178° east) for the equatorial satellites (Syncom 3 and Early Bird) which could condition the 3,1 harmonic. The effect of 3,1 on the 32° inclined Syncom 2 is almost negligible. As a result, the "best" solution, for 2,2, 3,1, and 3,3, in Reference (2) contained 4 correlation coefficients above $|0.95|$ and 10 (of a total of 15) above $|0.7|$. This new solution (line 1, Table 3) for 2,2, 3,1, and 3,3 contains no correlation coefficients above $|0.9|$ and only two $[(C_{22}, S_{22}), (C_{33}, S_{31})]$ above $|0.7|$. For these reasons, and also because of it's simplicity (not having 4th order terms) I would like to prefer this solution to the others I have made with the data in

Table 3

Solutions for Low Order Gravity Coefficients from 35 Accelerations of 24-Hour Satellites

Solution	Unnormalized Coefficients (Units: 10^{-6}) 2,2 C	2,2 S	3,1 C	3,1 S	3,3 C	3,3 S	4,2 C	4,2 S	4,4 C	4,4 S	Weighted Standard Deviation of Fit to Data: S
1. No fourth order coefficients	1.561	-.898	2.74	-1.19	.062	.171	–		–		0.86
2. For 2,2 and 3,3 with SAO '66 M. 3,1, 4,2, 4,4[5]	1.570	-.900	2.09	0.29	.100	.187	.074	.148	-.0011	.0049	1.12
3. For 2,2, 3,1, 3,3 with SAO '66 M1 4,4, 4,4[5]	1.568	-.896	2.87	-.80	.080	.174	.074	.148	-.0011	.0049	1.05
4. With the SAO '66 M1 Model Only[5]	1.536	-.872	2.09	0.29	.078	.226	.074	.148	-.0011	.0049	2.68
5. With the SAO '69 B13.1 Model Only*	1.553	-.911	2.12	0.26	.111	.180	.068	.152	-.0021	.0075	1.27
6. With 2,2, 3,1, 3,3 derived from more extensive 24-hour data with SAO '66 M1 4,2, 4,4[5]	1.578	-.905	2.11	0.32	.100	.198	.074	.148	-.0011	.0049	1.10

*E. M. Gaposchkin (Smithsonian Astrophysical Observatory, Camb., Mass.), Private Communication, Oct. 1969.

Table 2. However, I recognize that 3,1 is still not sufficiently well represented by this data and must therefore give solution 2. In Table 3 equal rank with solution 1. The somewhat greater standard deviation of solution 2 is in large part due to the smaller measurements errors used for it. The addition of more data on ATS 3 & 5 (at 107°W and 40°-70°W) and the first use of ATS 1 data (at 151°W), now being processed, should considerably improve the overall solution for the 3rd order harmonics.

<u>Equilibrium Points for Geostationary Satellites</u>

A more thorough study of the east-west equilibrium points (where Equation (1) is zero) will be made in the future when the new ATS data is processed. However, even at this time, much can be learned from a tabulation of the points calculated from various recent non-resonant data models as well as from the 24-hour data directly (see Table 4).

Table 4 reveals a good deal of convergence among all the models, as could be anticipated from the agreement of their coefficients. The conclusions from Table 4 as well as from additional calculations; propagating the error in the 24-hour satellite solutions, is that:

$$\lambda_E = 75 \pm 3/4° \text{ (stable)}$$

$$\lambda_E = 162 \pm 1/4° \text{ (unstable)}$$

$$\lambda_E = 255 \pm 1/2° \text{ (stable)}$$

$$\lambda_E = 348.5 \pm 1/2° \text{ (stable)}$$

The bounds on all of these points overlap (at less than 1σ) those of the previous study with 24-hour accelerations[2] except for the Pacific stable point (which

Table 4

East-West Equilibrium Points for Geostationary Satellites

Field	Indian Ocean (Stable)	Pacific (Unstable)	Pacific (Stable)	Atlantic (Unstable)
Table 3 Solution 1	74.7	162.1	255.5	348.6
Table 3 Solution 2	74.9	162.1	254.9	348.4
SAO '66 M1	75.6	162.0	254.1	349.0
SAO '69 B13.1	74.4	161.7	254.9	348.1
From non-resonant satellites, all Doppler Observations (1968)	74.9	161.7	254.6	348.3

overlaps at less than 2σ). The principal reason for the error in the Pacific stable point is the lack of good data close to it. The error bounds on this point from the previous study (253.3 ± 0.9°) were the greatest for all the equilibrium positions. But this estimate should have been considered not weak enough because of the large number of high correlations in the solution. This can easily introduce significant bias into an error extrapolation when observable higher order terms are neglected, as they were in the 2,2 3,1 and 3,3 solution which gave this point. For example, a much less correlated solution for only 2,2 and 3,3 from the previous data, using more realistic 3,1 4,2 and 4,4 values and giving the same fit as the truncated 3rd order solution, showed a pacific stable point at 254.3°. The new preferred solutions (Lines 1 and 2 of Table 3) show much better agreement for this and the other equilibrium points than solutions from the previous study with more limited data.

CONCLUSIONS

The addition of ATS 3,5 and Intelsat 2F3 accelerations has produced a much better representation of resonant geopotential coefficients through 3rd order from the full 24-hour satellite record. It has also produced significantly better estimates of the east-west equilibrium points for geostationary satellites (in accuracy and precision).

Tests of the full 24-hour acceleration record with recent Smithsonian Astrophysical Observatory (SAO) gravity fields (determined without synchronous satellites) shows a significant improvement in 1969 solutions (for low order terms) than those obtained in 1966. In fact there is little power in the 24-hour acceleration data (at present) to discriminate between solutions directly from this data and the 1969 SAO fields. However, it is expected that a more extensive 24-hour record (including ATS 1 and a finer analysis of the Syncom 2 drift since 1965) will reveal significant differences in the 1969 SAO fields as well as a superior 3rd order resonance field.[10]

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