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ADVANCED STUDY OF VIDEO SIGNAL PROCESSING IN LOW SIGNAL TO NOISE ENVIRONMENTS

By

Frank Carden
George Davis

A Quarterly Progress Report

Submitted to

NATIONAL AERONAUTICAL SPACE ADMINISTRATION
WASHINGTON D. C.

NASA RESEARCH GRANT NGR-32-003-037

Electrical Engineering Department
Communication Research Group

June - September 1969

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AN FM CLICK MODEL IN ADDITIVE NORMAL NOISE AND SOME STATISTICAL PROPERTIES OF THE CLICK EVENT

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Abstract

An FM click model is developed from statistical inferences made on the constituent components of the quadrature noise model.

Calculations based on the developed model predicted the occurrence of the "multi-modal" click. This event was subsequently verified experimentally.

The mathematical model indicates that there exists a minimum peak phase rate during the click which is exhibited by all click events, and indicates an optimal threshold level for click suppressing FM threshold extending schemes.

I. Introduction

Temporal properties of the click event, in particular the distribution of click amplitude, are desirable for the development of FM detectors which suppress the effects of the click phenomenon [1]. A click model predicted on this basis does not exist. Rice's [2] Dirac impulse model yields no results for the click amplitude. Hess's [3] model is a particular model prescribing the temporal characteristics and has been applied to the first order phase-lock loop, but does not contain probablistic information available from Rice's [2] proposed explanation of the click event. The model developed here is quite general and permits particular model studies subject to the distribution of the initial phase rate.

The input to the discriminator is

$$\begin{aligned} e_1(t) &= A \cos \omega_c t + n(t) \\ &= (A + x(t)) \cos \omega_c t - y(t) \sin \omega_c t \end{aligned} \tag{1}$$

where n(t) is assumed to be a stationary normal process with zero mean, variance σ^2, and power spectral density (one sided) $G_n(f)$. The power spectral density is taken to be symmetrical about f_c, yielding the noise processes x(t) and y(t) independent stationary normal processes with zero means and variances σ^2. The power spectra $G_X(f)$ and $G_Y(f)$ are one sided and are given by

$$G_X(f) = G_Y(f) = 2G_n(f + f_c) \qquad 0<f<\infty \tag{2}$$

The power spectral density $G_n(f)$ is constrained to satisfy

$$\int_{-\infty}^{\infty} \omega^2 G_n(f)df<\infty \tag{3}$$

in order that the process $\dot{x}(t)$ exists in the mean squared sense. This restriction yields a finite variance for $\dot{x}(t)$.

II. Density of the Initial Phase Rate Given Click Occurrence.

The click event is conditioned by the following[1]:

a) $x(t)<-A$

b) $y(t)=0$ (4)

c) $\dot{y}(t)>0$

The click is assumed to occur if equations (4) are satisfied. The polar-rectangular transformation gives the functional relationship between $\theta(t)$ and the processes $x(t)$ and $y(t)$ (the formula is general and must be specialized to the quadrant of interest).

$$\theta(t) = \tan^{-1} \frac{y(t)}{A + x(t)} \tag{5}$$

Differential yields

$$\dot{\theta}(t) = \frac{(A + x(t))\dot{y}(t) - y(t)\dot{x}(t)}{(A + x(t))^2 + (y(t))^2} \tag{6}$$

The initial phase rate is defined here to be the value of $\dot{\theta}(t)$ when $\theta(t) = \pi$ (when $\theta(t) = \pi$, $y(t) = 0$). Equation 6 simplifies to

$$\dot{\theta}(0) = \frac{\dot{y}(0)}{A + x(0)} \tag{7}$$

where t has been set equal to 0 when $\theta(t) = \pi$.

Utilizing formula (7) as the form of $\dot{\theta}(0)$, the conditioning equations (4) can be incorporated to determine the conditional random variable $\dot{\theta}(0)|$click. Let $Z = \dot{\theta}(0)|$click occurred. Then

$$Z = \frac{\dot{Y}(0)}{A + X(0)} \Bigg| \; X(0)<-A, \; \dot{Y}(0)>0 \tag{8}$$

where Z depends on Y(t) only through time. That is, at some time during the click, $y(t) = 0$ initializing time ($t = 0$ when $y(t) = 0$). Thus equation (8) specifies the conditional random variable $\theta(0)|$click. The designations

[1]These are due to Rice and condition the negative click. The positive click occurs if $\dot{y}(t)<0$.

$\dot{Y}(0)$ and $\dot{X}(0)$ represent the random variables characterizing the processes $\dot{y}(t)$ and $x(t)$ at $t = 0$. Equation (8) can be written

$$Z = \frac{\dot{Y}(0)|\dot{Y}(0)>0}{A + X(0)|X(0)<-A} \tag{9}$$

since the random variables $\dot{Y}(0)$ and $X(0)$ are independent ($G_n(f)$ is symmetric about f_c).

Thus, the random variable Z is a transformation of the conditional random variables $\dot{Y}(0)|\dot{Y}(0)>0$ and $X(0)|X(0)<-A$. In the following development, the simplification of notation below is adopted:

$$\dot{Y} = \dot{Y}(0)|\dot{Y}(0)>0 \qquad\qquad X = X(0)|X(0)<-A \tag{10}$$

The density function of Z can be determined once the joint density of $\dot{Y}$ and X is known. This joint density is the product of the respective densities of $\dot{Y}$ and X. These density functions are evaluated directly and only the results are included here.

$$f_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma_1 K_1}\, e^{-1/2\left(\frac{x}{\sigma_1}\right)^2} \qquad -\infty<x<-A \tag{11}$$

$$f_{\dot{Y}}(y) = \frac{1}{\sqrt{2\pi}\,\sigma_2 K_2}\, e^{-1/2\left(\frac{y}{\sigma_2}\right)^2} \qquad 0<y<\infty \tag{12}$$

and

$$\begin{aligned} \sigma_1{}^2 &= E[X^2(t)] \\ \sigma_2{}^2 &= E[\dot{Y}^2(t)] \\ K_1 &= F_X(-A) \\ K_2 &= F_{\dot{Y}}(0) \end{aligned} \tag{13}$$

The relations in equation 13 are for the original processes $x(t)$ and $\dot{y}(t)$.

Under the transformation

$$Z = \frac{\dot{Y}}{A + X} \qquad\qquad W = X \tag{14}$$

The joint density of Z and W is obtained. The marginal of Z is then obtained by

$$g_Z(z) = \int_{\Omega(W)} f_{Z,W}(z,\omega)\,d\omega \tag{15}$$

where $\Omega(W)$ is the range of the random variable W. The density obtained in this form is readily normalized and in order to reduce the notation a further change of variable is made. Let

$$T = \frac{Z}{2\pi r} \qquad r = \frac{1}{2\pi}\ \frac{\sigma_2}{\sigma_1} \tag{16}$$

be made; r is the radius of gyration of the noise spectral density. The resulting density of the normalized initial phase rate is

$$g_T(t) = \frac{e^{-\left(\frac{\rho t^2}{1 + t^2}\right)}}{2\pi K_1 K_2 (1 + t^2)} \left[e^{-\left(\frac{\rho}{1 + t^2}\right)} - \frac{\sqrt{\pi\rho}}{1 + t^2}\ \mathrm{erfc} \sqrt{\frac{\rho}{1 + t^2}} \right]$$

$$-\infty<t<0 \tag{17}$$

and ρ is the carrier to noise ratio $\left(\frac{A^2}{2\sigma_1{}^2}\right)$.

Comparing $tg_T(t)$ with the function $\frac{t}{1 + t^2}$ and utilizing the limit comparison test for the integral

$$E[T] = \int_{-\infty}^{0} tg_T(t)dt \tag{18}$$

results in the mean of T being non-existant. All higher moments also diverge. As a result, an expected or average initial phase rate does not exist. Therefore, a value of $\dot{\theta}(0)|$click must be chosen in some other way. Once a value is chosen, say $T = t_1$, a probability statement may be made to gauge the value's usefulness. That is,

$$F_T(t) = P(T < t_1) = \int_{-\infty}^{t_1} g_T(t)dt \tag{19}$$

which is plotted for various signal to noise ratios in Figure 1.

With the density function $g_T(t)$ known, the conditional density of $X|T = t_1$ can be determined. This density is employed to assign an initial value to the process $x(t)$ so that the future states $X(t_i)$ can be predicted. With the conditional density of $X|T = t_1$ known, the initial state $x(0)$ is assigned the following value:

$$x(0) = E\,[X|T = t_1] \tag{20}$$

This value determines the value of $\mathring{y}(0)$ also.

$$\begin{aligned}\mathring{y}(0) &= E[\mathring{Y}|T = t_1] = E[t_1(A + X)|T = t_1] = t_1(A + E[X|T = t_1]) \\ &= t_1(A + x(0)\end{aligned} \tag{21}$$

The expected values of these conditional random variables are chosen so that an average click given $T = t_1$ is obtained. In the same manner, the expected value of $\mathring{X}(0)$, required later, is determined

$$E[\mathring{X}|T = t_1] = E[\mathring{X}] = 0 \tag{22}$$

State Prediction.

The future states of the processes $X(t)$ and $Y(t)$ are to be predicted based upon the assigned values $x(0)$, $\mathring{x}(0)$, $y(0)$, and $\mathring{y}(0)$. The predictor is the linear one which minimizes the expected squared prediction error. The predictor is

$$\hat{X}(t) = \alpha X(0) + \beta\mathring{X}(0) \tag{23}$$

with expected squared prediction error

$$\varepsilon = E[X(t) - \hat{X}(t)]^2$$

The parameters α and β are determined

$$\begin{aligned}\alpha &= \rho_{XX}(t) \\ &= \frac{\rho_{\mathring{X}X}(t)}{(2\pi r)} - \frac{\mathring{\rho}_{XX(t)}}{(2\pi r)}\end{aligned} \tag{24}$$

and $\rho_{XX}(t)$ is the correlation coefficient for lag t, determined by the power spectral density $G_X(f)$. It can be shown that the linear predictor of the form of equation (23), the expected squared prediction error for $\dot{X}(t)$ is $\frac{d\hat{X}(t)}{dt}$.

Click Model.

The click model is obtained by substitution of the respective predictions for $x(t)$, $\dot{x}(t)$, $y(t)$, and $\dot{y}(t)$ into equation (6) yielding the click model.

$$\dot{\theta}_m(t) = (\frac{1}{2\pi r})^2 \frac{\dot{\rho}^2_{XX}(t)x(0)\dot{y}(0) - (A + \rho_{XX}(t)x(0))\rho_{\dot{X}X}(t)\dot{y}(0)}{(A + \rho_{XX}(t)x(0))^2 + ((\frac{1}{2\pi r})^2\dot{\rho}_{XX}(t)\dot{y}(0))^2} \tag{25}$$

The envelope $R(t)$ is specified when $x(t)$ and $y(t)$ are specified and $R_m(t)$ and $\dot{\theta}m(t)$ are not functionally independent. In fact

$$R_m(t) = \sqrt{(A + \rho_{XX}(t)x(0))^2 + ((\frac{1}{2\pi r})^2\dot{\rho}_{XX}(t)\dot{y}(0))^2} \tag{26}$$

so that the denominator of (25) is $R^2{}_m(t)$. This result clearly indicates the dependence existing between the envelope and phase rate.

Results

Calculations based on equations (25) and (26) yield results of considerable interest. The interaction of $\dot{\theta}_m(t)$ and $R_m(t)$ was investigated and an explanation of a predicted phenomenon is forwarded.

The initial result, that was subsequently verified experimentally, was the multi-modal click. Equation (25) is used to generate click wave shapes once the power spectral density of the noise, the carrier to noise ratio, and the initial value $\dot{\theta}(0)$ are specified. As the initial value decreases, a peculiar event, named the multi-modal click, begins to occur. The wave shape is illustrated in Figures 2 and 3, along with clicks not exhibiting this property. The click model only generates even clicks (about the mid-event time) but can be generalized to generate asymmetrical clicks.

In order to determine the existence of multi-modal clicks a large number of photographs of clicks were studied.[2] Figure 4a illustrates three clicks. The second one from the left exhibits an indention at the peak. On an expanded scale figure 4b illustrates the bi-modal nature of this type of click. Figure 4c shows a negative going bi-modal click while figure 4d shows a uni-modal. Occasionally tri-modal clicks occur as predicted. In order to observe such multi-modal clicks it is necessary to observe the waveform before the low-pass post-detection filter because this filter smooths the clicks and makes the waveforms appear similar.

Analyses of the calculations of $\dot{\theta}_m(t)$ and $R_m(t)$ indicate that these multi-modal clicks occur when the envelope $R_m(t)$ decreases from its initial (mid-event value. Equation (25) then indicates that for small initial phase rates, the numerator of $\dot{\theta}_m(t)$ is a relative constant compared to $R^{-2}{}_m(t)$ and that $R^{-2}{}_m(t)$ dominates the behavior of $\dot{\theta}_m(t)$. This observation is valid for initial phase rates less than that required for the uni-modal click. Above this value, the numerator becomes more significant in determining the click shape.

Calculations of $\theta_m(t)$ for wide ranges of initial phase rates have presented another significant result. There exists a minimum peak value of $\dot{\theta}_m(t)$ which is exceeded by all clicks generated. The variation of peak phase rate (maximum phase rate during click event) versus the initial phase rate is shown in Figure 5. The minimum peak value is located at the bottom of the disk-shaped curve. The curves all intersect the line $|\dot{\theta}_m(t)|_{max} = |\theta_m(0)|$ at a value which is the first uni-modal click initial phase rate for a particular signal to noise ratio. This result would indicate the existence of an optimum threshold level for click suppressing devices of the threshold class.

[2]Photographs supplied by G. Dickey Arndt, Information Systems Division, NASA Manned Spacecraft Center, Houston, Texas.

Summary

In order to extend FM threshold click elimination is essential. A probabilistic mathematical click model was developed that predicts the time contour. ~~and~~ hence, the model may be used to enchance click elimination schemes. Further, the model predicts a phenomenon heretofore unreported and unobserved. Experimental results verify such a multi-modal event.

REFERENCES

[1] F. J. Loch, "An FM Threshold Extension Technique for the Apollo Unified S-Band Communications System". IEEE National Telemetry Conference, 1968, Houston, Texas.

[2] S. O. Rice, "Noise In FM Receivers", Proceedings Symposium on Time Series Analysis, ed. M. Rosenblatt, Chapter 25, pp 395-422, John Wiley & Sons, Inc., New York, 1962.

[3] Donald T. Hess, "Cycle Slipping in a First-Order Phase-Locked Loop," IEEE Transaction Communication Technology, vol. COM-16. No. 2, pp 255-260, April 1968.

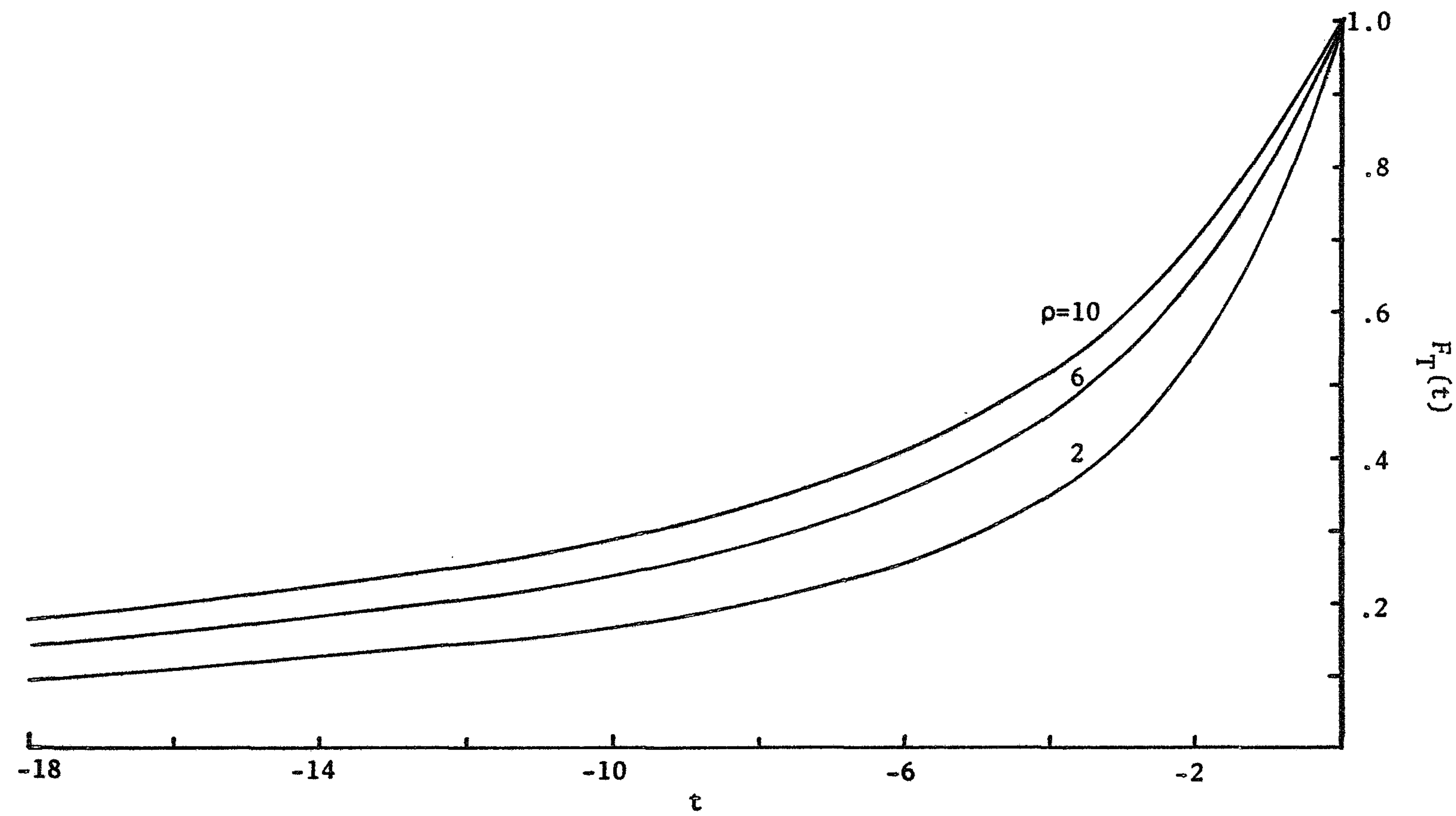

FIGURE 1. Distribution Function of Normalized Initial Phase Rate

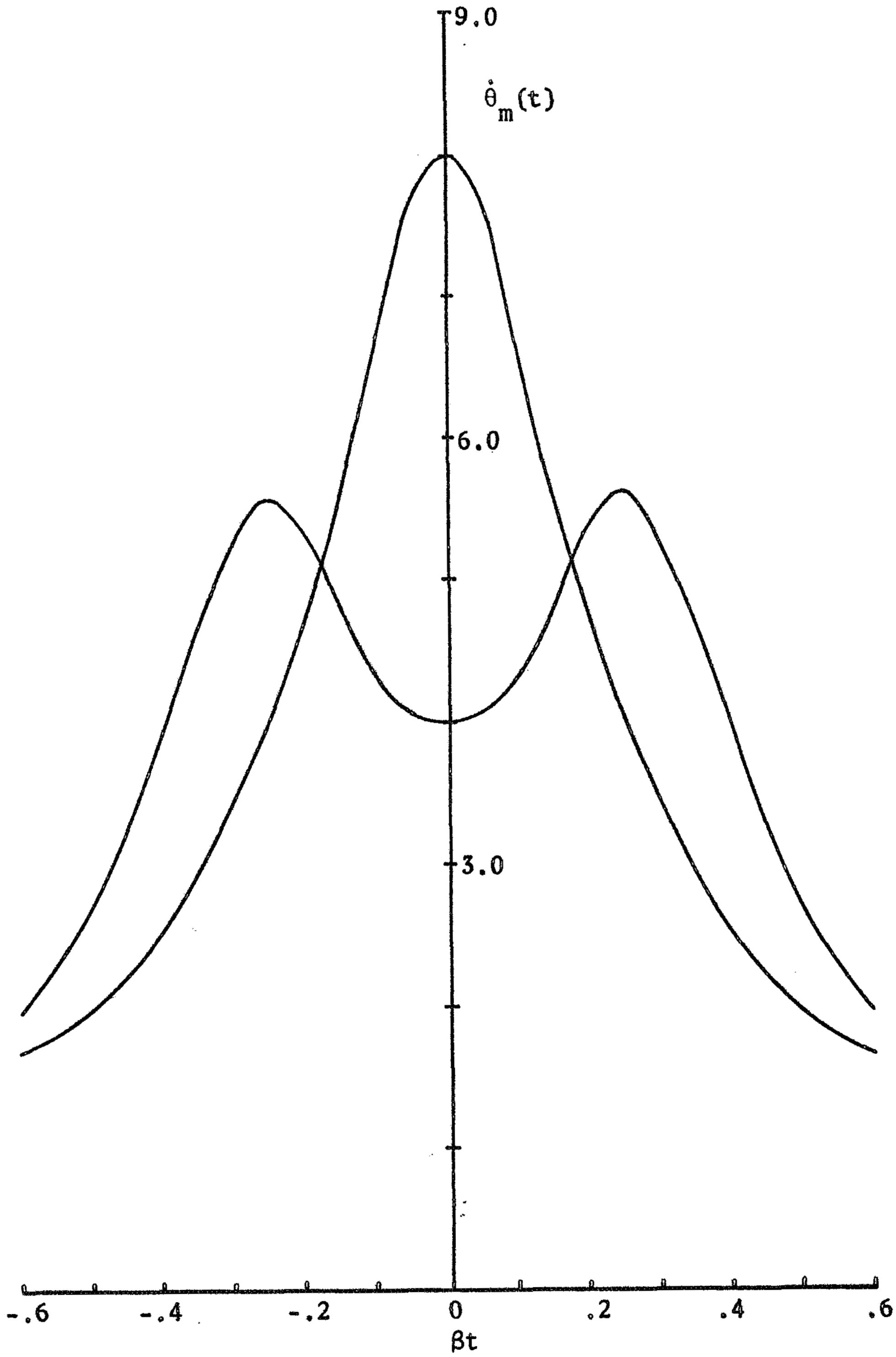

FIGURE 2. Click Model Results, Ideal Filter, ρ=2

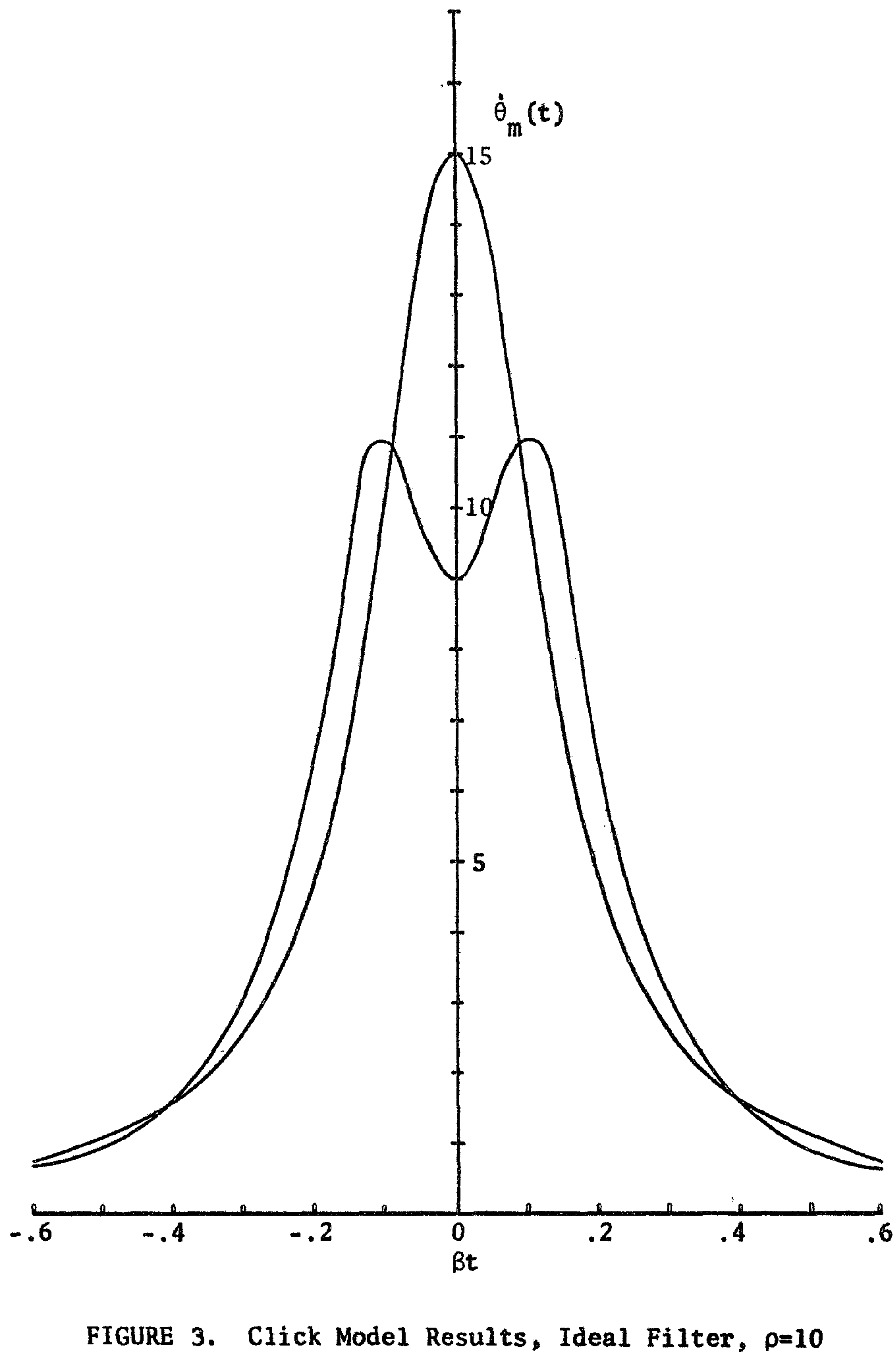

FIGURE 3. Click Model Results, Ideal Filter, ρ=10

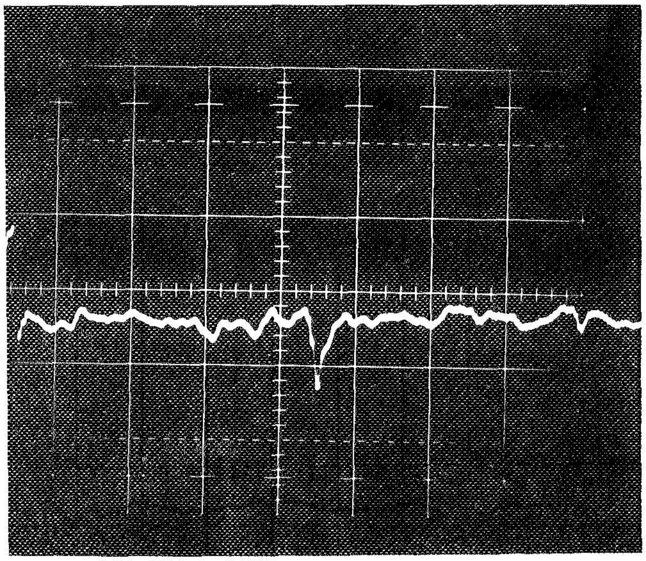

Fig. 4a 2 μsec = 1 cm

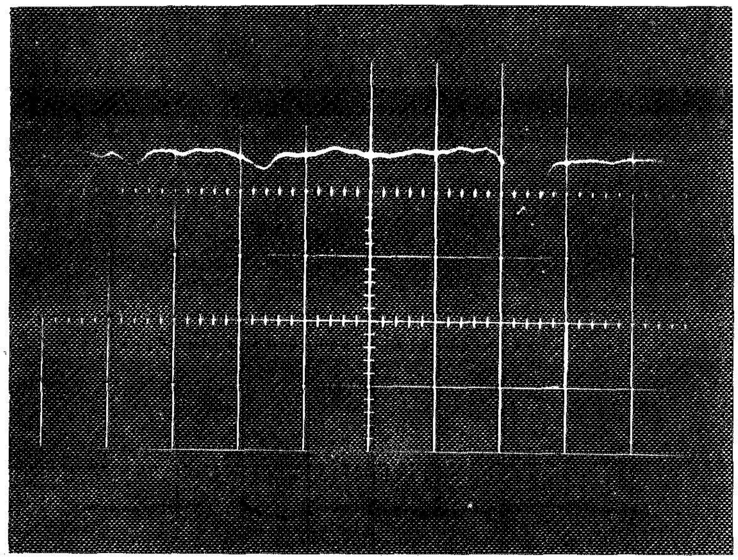

Fig. 4b .5 μsec = 1 cm

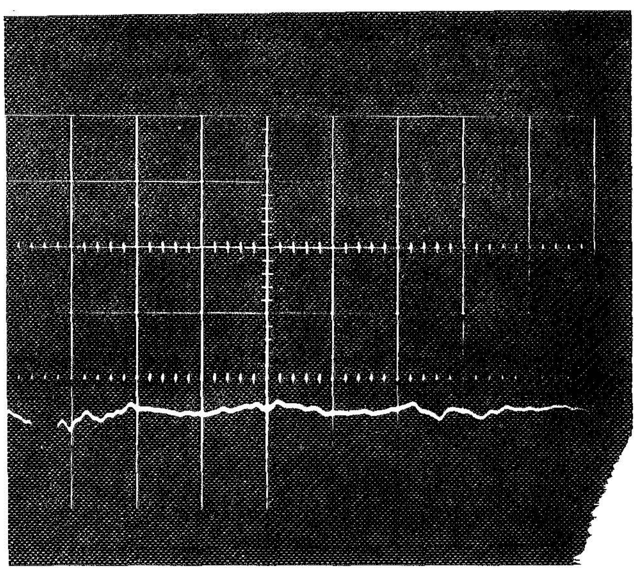

Fig. 4c .5 μsec = 1 cm

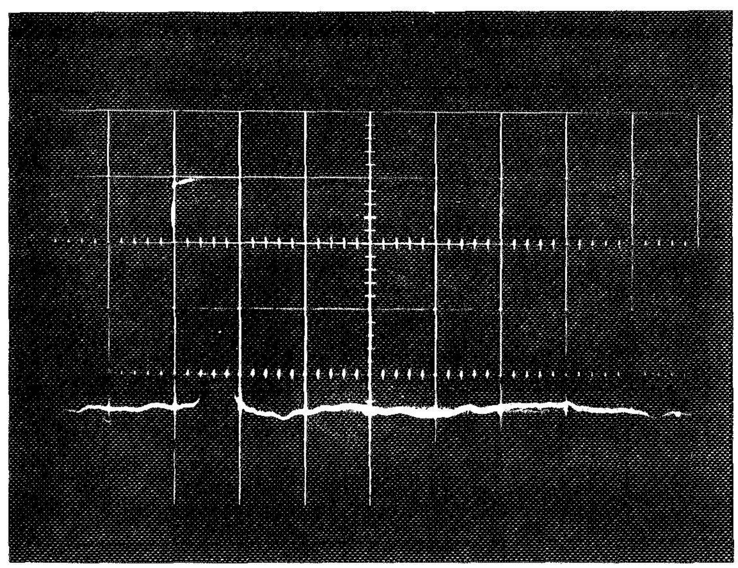

Fig. 4d .5 μsec = 1 cm

Fig. 4 Various model clicks. B_{IF} = 1 mHz

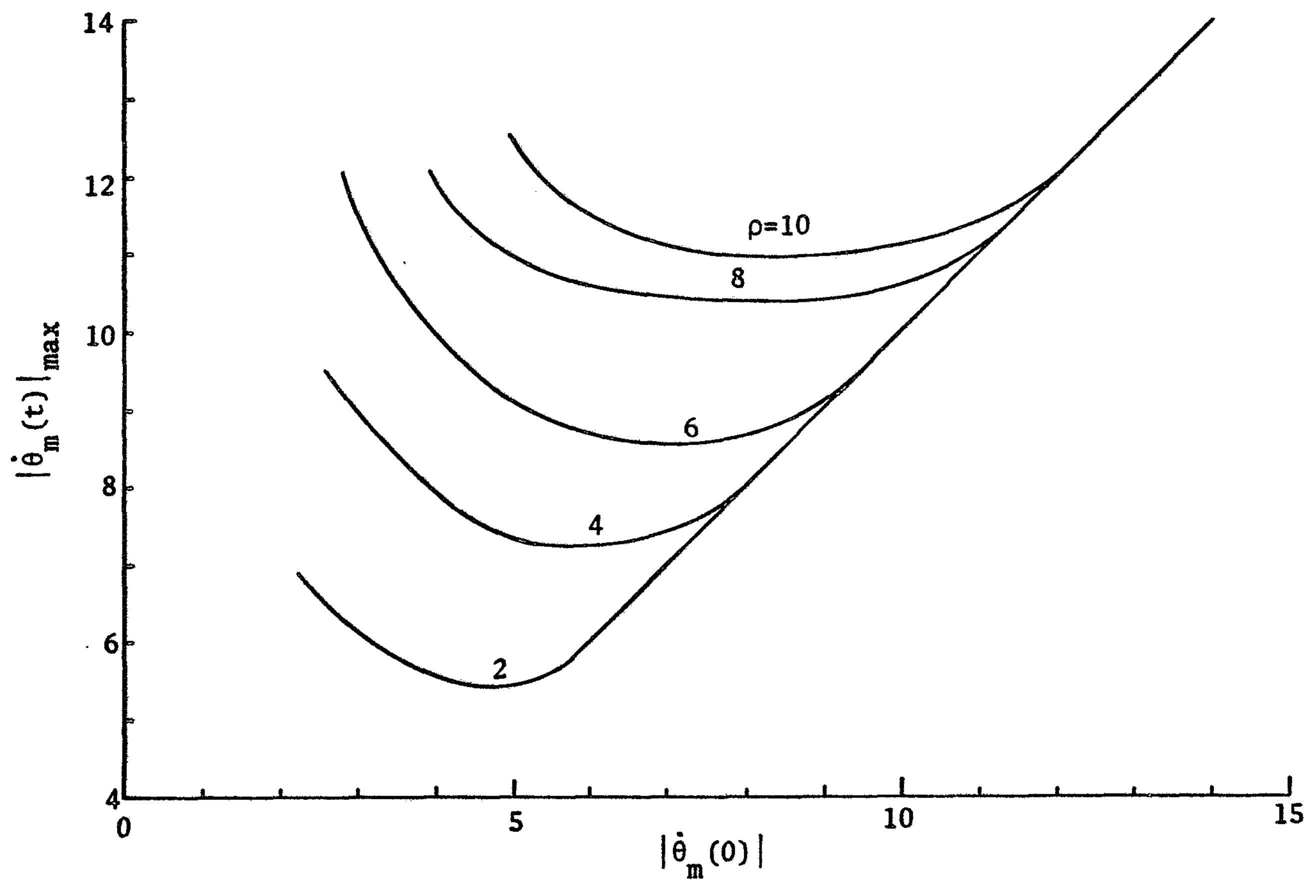

FIGURE 5. Maximum Phase Rate Versus Initial Phase Rate, Ideal Filter