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DETERMINING VISIBILITY INTERVALS BETWEEN AN EARTH TRACKING STATION AND A PLANETARY SATELLITE

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SUMMARY

The problem of finding visibility intervals between an Earth tracking station and a planetary satellite is formulated based on the assumption of an infinite speed of light. A modification to account for the finite speed of light is then added to the formulation. An input-output description and listing of a Fortran computer program which calculates visibility intervals is provided in Appendix 1. The program was used to obtain visibility intervals between the Goldstone deep space tracking station and a satellite in a typical elliptic orbit around Mars.

DETERMINING VISIBILITY INTERVALS BETWEEN AN EARTH TRACKING STATION AND A PLANETARY SATELLITE

INTRODUCTION

The purpose of this paper is to provide an efficient technique for determining intervals in which a planetary satellite is visible to a given earth tracking station. This problem is similar to the one treated by Stern (1) who determines communication times between an earth satellite and a planetary satellite. There is also a similarity to the problem of finding shadow times of a satellite—Yeremenko (2) Karytevn (3), and others. But the problem treated in the present paper is not a special case of the ones mentioned above and hence it requires its own separate formulation.

Clearly the visibility between an earth tracking station and a planetary satellite can be obstructed by both the earth and the planet. The approach taken will be to first derive an earth occultation function, S_E, of time from epoch which is positive for times at which the earth fails to occult visibility and negative for times at which the earth does occult visibility. A similar function S_P for planetary occultation is also derived. A visibility function V of time from epoch can then be defined which is equal to one when S_E and S_P are positive, and is equal to minus one otherwise. The discontinuities of V can be easily and accurately determined by numerical methods. The times associated with these discontinuities are either visibility rise or visibility set times depending on the nature of the discontinuity. By this technique, visibility intervals or "windows" can be constructed.

The earth and planetary occultation functions will be derived assuming an infinite speed of light. This permits the application of direct line of sight geometry in the derivations. Later a slight modification of the occultation functions will substantially correct for the error introduced by this assumption.

In Appendix I is an input and output description and listing of a Fortran routine called "OCCULT" which determines visibility windows between an earth tracking station and a planetary satellite. The program is based on the analysis outlined above. This program will be utilized in a later section to determine visibility windows between an earth tracking station and an artificial satellite in a typical two body orbit around Mars.

DERIVATION OF EARTH OCCULTATION FUNCTION

The object of this section is to define a function S_E of time which assumes negative values for times when the Earth occults visibility between an Earth tracking station and a satellite in orbit around a planet, and which assumes positive values otherwise. The derivation will be carried out under the following assumptions:

(1) The speed of light is infinite.

(2) The satellite is in a two body elliptic orbit about the planet.

(3) Earth nutation can be ignored.

(4) The Earth's angular velocity is constant.

(5) The Earth and the planet are perfect spheres.

(6) The interval in which occultation information is desired is small enough so that the position vector of the planet relative to the Earth can be considered independent of time.

(7) All vector quantities are given with reference to a mean equinox of date, mean equator of date coordinate set where the epoch date is a given time of perifocal passage of the planetary satellite.

(8) Along with latitude and longitude, another parameter associated with a given tracking station is its elevation angle δ. Visibility between a tracking station and a satellite will be considered occulted when the angle between the tangent plane at the tracking station and the line of sight to the satellite is less than δ. The angle δ is typically around 5 degrees.

Figure 1 displays the geometric situation at the exact instant of Earth occultation. The position vector from Earth center to tracking station is represented by $\overline{R}_E$. The vector $\overline{R}$ is the position vector of the planet relative to the Earth center. $\overline{R}_p$ is the position vector of the satellite relative to the planet center and $\overline{R}_s$ is the position vector of the satellite relative to the Earth center. The dotted line shown in Figure 1 is the line of sight from the satellite to the tracking station. The angle between this line and the tangent plane at the tracking station is the elevation angle δ. The symbol ψ represents the angle between the position vectors of the tracking station and satellite relative to Earth center and at all times is given by

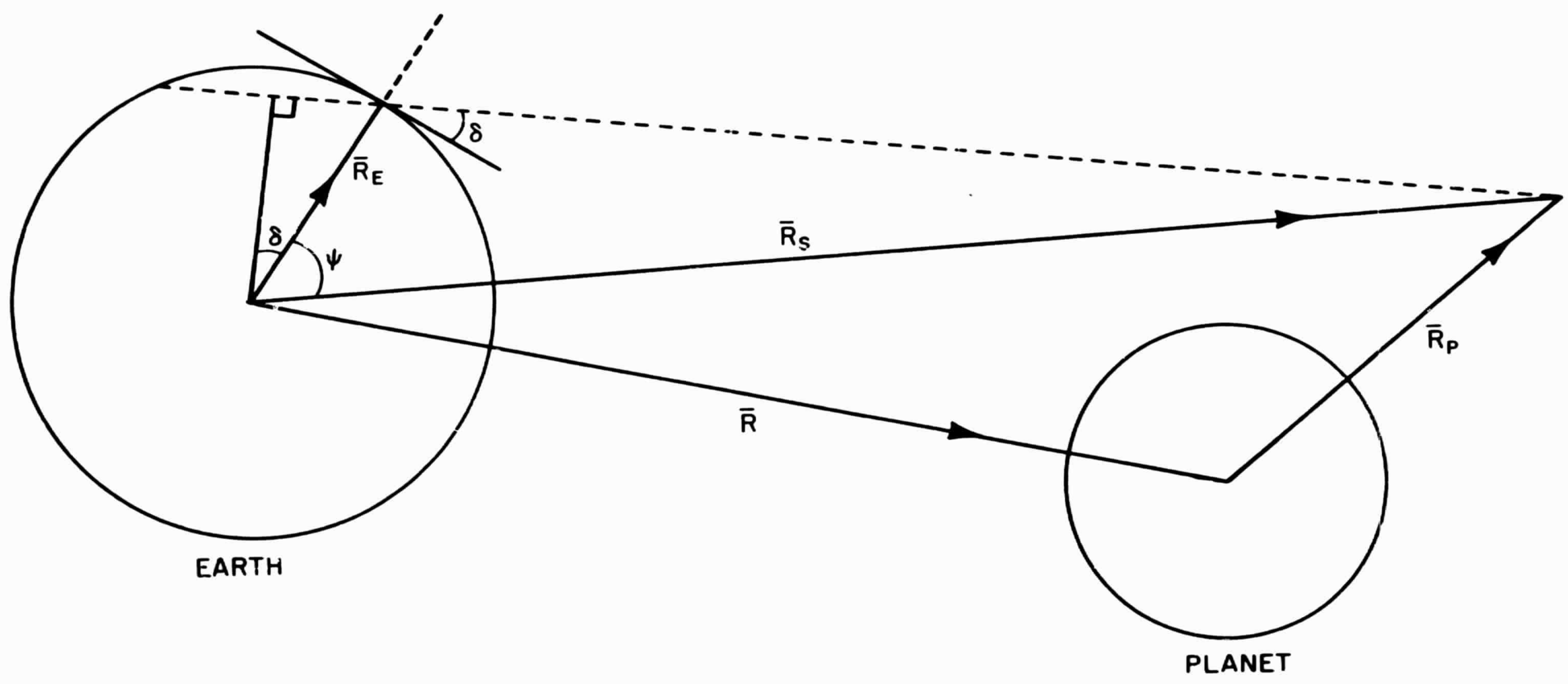

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Figure 1—Geometry for Earth Occultation Function

$$\cos \Psi = \bar{R}_E \cdot \bar{R}_S / |\bar{R}_E| \, |\bar{R}_S| \tag{1}$$

where

$$\bar{R}_S = \bar{R} + \bar{R}_P \tag{2}$$

At the instant of Earth occultation Ψ satisfies the relation

$$\cos(\delta + \Psi) = |\bar{R}_E| \cos \delta \, / \, |\bar{R}_S| \tag{3}$$

With the aid of some trigonometric identities, equation 3 can be written as

$$\cos \delta \cos \Psi - \sin \delta \sqrt{1 - \cos^2\Psi} - |\bar{R}_E| \cos \delta \, / \, |\bar{R}_S| = 0 \tag{4}$$

By substituting the right side of 1 for cos ψ in 4, the Earth occultation function is obtained

$$\frac{\cos \delta \, (\bar{R}_E \cdot \bar{R}_S) - \sin \delta \sqrt{|\bar{R}_E|^2 \, |\bar{R}_S|^2 - (\bar{R}_E \cdot \bar{R}_S)^2} - |\bar{R}_E|^2 \cos \delta}{|\bar{R}_E| \, |\bar{R}_S|} = S_E \tag{5}$$

S_E is clearly zero at Earth occultation. When the satellite is visible to the tracking station, S_E is positive as can be seen by examining 5 when $\Psi = 0$, a condition which clearly implies visibility. During Earth occultation S_E is negative as can be seen by examining Equation (5) when $\Psi = \Pi/2$.

Substantial computing time can be saved by noticing that when ψ is between $\Pi/2$ and $3\Pi/2$, visibility is occulted by the Earth. Hence when $\bar{R}_E \cdot \bar{R}_S < 0$, can be assigned some arbitrary negative value without altering the desired characteristics of S_E. The adjusted definition of the Earth occultation function S_E is

$$S_E = \frac{\cos \delta \, (\bar{R}_E \cdot \bar{R}_S) - \sin \delta \sqrt{|\bar{R}_E|^2 |\bar{R}_S|^2 - (\bar{R}_E \, \bar{R}_S)^2} - |R_E|^2 \cos \delta}{|\bar{R}_E| \, |\bar{R}_S|} \tag{6}$$

when $\bar{R}_E \cdot \bar{R}_S > 0$ and $S_E = -1$ otherwise.

Equations (1) through (6) recursively define S_E as a function of $\bar{R}_E$ and $\bar{R}_P$. In order to define S_E as a function only of time from an epoch date it is necessary to define $\bar{R}_E$ and $\bar{R}_P$ as functions of time from an epoch date.

Let $\mho$ be the epoch date, where $\mho$ is a time of periapsis passage of the planetary satellite. Let T be days from epoch. Let LAT and LONG be the latitude and longitude of the tracking station in radians. Also let H be the right ascension of Greenwich meridian from mean equinox of epoch time $\mho$. Then $\bar{R}_E$ is given by

$$R_E(1) = R_0 \cos(\mathrm{LAT}) \cos(\mathrm{LONG} + H + 2\Pi T)$$

$$R_E(2) = R_0 \cos(\mathrm{LAT}) \sin(\mathrm{LONG} + H + 2\Pi T)$$

$$R_E(3) = R_0 \sin(\mathrm{LAT}) \tag{7}$$

where R_0 is the radius of the earth.

To obtain $\bar{R}_P$ as a function of time from epoch, the orbital elements of the satellite must be available. Let ω = argument of perigee of satellite orbit, Ω = longitude of ascending node of satellite orbit, i = inclination of satellite orbit, a = semi-major axis of satellite orbit, e = eccentricity of satellite orbit. These orbital elements are assumed to be given relative to a mean equinox of epoch mean equator of epoch coordinate set. The unit vectors $\bar{P}$ and $\bar{Q}$ are defined as

$$P(1) = \cos\omega \cos\Omega - \sin\omega \sin\Omega \cos i$$

$$P(2) = \cos\omega \sin\Omega + \sin\omega \cos\Omega \cos i$$

$$P(3) = \sin\omega \sin i$$

$$Q(1) = -\sin\omega \cos\Omega - \cos\omega \sin\Omega \cos i$$

$$Q(2) = -\sin\omega \sin\Omega + \cos\omega \cos\Omega \cos i$$

$$Q(3) = -\cos\omega \sin i \tag{8}$$

In terms of eccentric anomaly E of the satellite, $\bar{R}_P$ can be represented as

$$\bar{R}_p = a(\cos E + e)\bar{P} + a(1 - e^2)^{1/2} \sin E \bar{Q} \tag{9}$$

The functional relationship between eccentric anomaly E and time from epoch T is given by

$$nT = E - e \sin E \tag{10}$$

where n is the mean motion of the satellite orbit. Equation (10) transendentally defines E in terms of T. Hence to obtain the corresponding value of E for a given value of T requires a numerical process.

Equations (8), (9), and (10) recursively define $\bar{R}_p$ as a function of days from epoch T. Thus Equations (1) through (10) recursively define S_E as a function of T where T represents days from an epoch time T, a time of satellite periapsis passage. The function $S_E(T)$ is zero at the instant of initiation or termination of Earth occultation. $S_E(T)$ is negative for times in which Earth occultation occurs and is positive for all other times.

DERIVATION OF PLANET OCCULTATION FUNCTION

In this section a function S_P of time from epoch will be derived which is negative when the line of sight between a planetary satellite and an Earth tracking station is occulted by the planet and is positive otherwise. The same assumptions listed in the derivation of the Earth occultation function will be in force here.

Figure 2 displays the geometric situation at the instant of initiation or termination of planet occultation. The dotted line is the direct line of sight between the satellite and the tracking station. $\bar{R}_E$ is the position vector of the tracking station relative to Earth center. $\bar{R}$ represents the position vector of the planet relative to Earth center. $\bar{R}_t$ is the position vector of the tracking station relative to the planet center. $\bar{R}_P$ is the position vector of the satellite relative to the planet. The symbol Ψ represents the angle between $\bar{R}_t$ and $\bar{R}_P$. $\theta\_$ is the acute angle between $\bar{R}_t$ and the line of sight. ϕ is the angle between $\bar{R}_P$ and the line of sight. At all times Ψ is given by

$$\cos \Psi = \bar{R}_t \cdot \bar{R}_p / |\bar{R}_t| \; |\bar{R}_p| \tag{11}$$

where

$$\bar{R}_t = \bar{R}_E - \bar{R} \tag{12}$$

At an instant of initiation or termination of planetary occultation, Ψ satisfies the relation

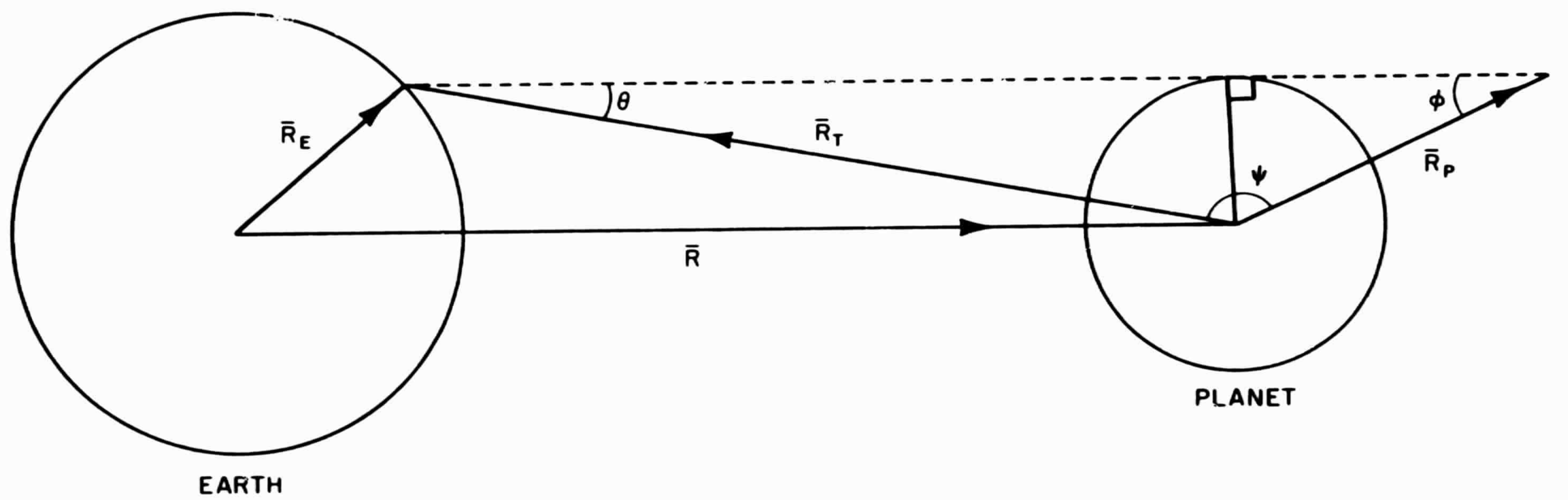

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Figure 2–Geometry for Planetary Occultation Function

$$\Psi = \Pi - \theta - \phi \tag{13}$$

where $\theta < \Pi/2$, $\phi < \Pi/2$ and

$$\cos\theta = X_p / |\bar{R}_t| \ , \ \cos\phi = X_p / |\bar{R}_p| \tag{14}$$

where X_p is the radius of the planet. These facts together with certain trigonometric relations yield

$$\cos\Psi = \frac{\sqrt{(|\bar{R}_t|^2 - X_p^2)(|\bar{R}_p| - X_p^2)} - X_p^2}{|\bar{R}_t|\ |\bar{R}_p|} \tag{15}$$

The difference between the right sides of Equations (11) and (15) is the planetary occultation function

$$S_p = \frac{\bar{R}_t \cdot \bar{R}_p + X_p^2 - \sqrt{(|\bar{R}_t|^2 - X_p^2)(|\bar{R}_p|^2 - X_p^2)}}{|\bar{R}_t|\ |\bar{R}_p|} \tag{16}$$

At an instant of initiation or termination of planetary occultation S_p is zero. When the planet is not occulting visibility, S_p is positive as can be seen by examining Equation (16) when $\Psi = 0$. When the planet occults visibility S_p is negative as can be seen by examining Equation (16) when $\Psi = \Pi$.

Computational time can be saved by noticing that when $-\Pi/2 \leq \Psi \leq \Pi/2$, planetary occultation cannot occur. Thus the planetary occultation function can be redefined as

$$S_p = \frac{\bar{R}_t \cdot \bar{R}_p + X_p^2 - \sqrt{(|\bar{R}_t|^2 - X_p^2)(|\bar{R}_p|^2 - X_p^2)}}{|\bar{R}_t|\ |\bar{R}_p|} \tag{17}$$

when $\bar{R}_t \cdot \bar{R}_p < 0$ and $S_p = 1$ otherwise.

Equation (17) along with Equation (12) recursively define S_p as a function of $\bar{R}_p$ and $\bar{R}_E$. But $\bar{R}_E$ and $\bar{R}_p$ can be expressed as functions of time from epoch as was shown in the previous section. Hence the derivation is complete.

ON THE CALCULATION OF VISIBILITY INTERVALS

The possession of Earth occultation and planetary occultation functions makes the calculation of visibility intervals an easy matter. A function V can be defined in the following way

$$V(T) = 1 \quad \text{if } S_E(T) > 0 \quad \text{and} \quad S_p(T) > 0$$

$$V(T) = -1 \text{ otherwise}$$

The intervals in which V equals one correspond to visibility intervals. The calculation of these intervals is just a question of discovering the location and type of the discontinuities of V. This can be done to any desired accuracy by the application of simple numerical procedures.

The independent variable in the visibility function V is time in days from an epoch where the epoch time is a time of periapsis passage of the planetary satellite. The visibility function is not defined for negative values of T. A uniformly effective procedure for the calculation of visibility intervals within a given time interval is now evident. The time interval in which occultation information is requested must start with a time of periapsis passage $\mathfrak{Z}$. The interval ends at some later time $\mathfrak{Z} + T_0$. To obtain visibility intervals during this period it is necessary to obtain the discontinuities of the visibility function V in the interval $[0, T_0]$. A straightforward numerical procedure incorporated in a computer program can determine these discontinuities and also determine their type. If values immediately to the left of a discontinuity are negative, then the time associated with the discontinuity is the beginning of a visibility interval. Otherwise a time associated with a discontinuity represents the end of a visibility interval.

Some of the assumptions made in the derivation of the Earth and planetary occultation functions place restrictions on the accuracy and effectiveness of the procedure outlined above. Notice that no provisions are made for updating the position vector of the planet relative to the Earth during the interval in which visibility information is requested. Thus the longer this interval is, the more serious the error becomes. Obviously this presents no serious difficulty since any interval in which visibility information is required can be divided into intervals small enough so that this error can be ignored. The position vector of the planet relative to the Earth would then be updated at the beginning of each of these intervals.

Another source of inaccuracy is the assumption of an infinite speed of light. If the distance between the Earth and the body about which the satellite is orbiting is great then the error caused by this assumption can be significant. But a slight alteration in the definitions of the Earth and planetary occultation functions can substantially correct for this error. This is the subject of the next section.

A MODIFICATION TO ACCOUNT FOR THE FINITE SPEED OF LIGHT

The assumption of an infinite speed of light in the derivation of the occultation functions permitted the application of line of sight geometry. But it introduces an error which can be significant when the Earth and the planet in question are separated by a great distance. Stern (1) in determining communication times between an Earth satellite and a planetary satellite effectively deals with this problem.

The treatment provided here is simpler than that given by Stern. It rests on the assumption that the distance between the satellite and the tracking station can be well approximated by the distance between the Earth and the planet. Thus the time of travel in days of the signal from the satellite to the tracking station is $\Delta T = |\bar{R}|/C$ where again $\bar{R}$ is the position vector of the planet relative to the Earth and C is the speed of light in kilometers per day. To determine if occultation occurs at a time T, line of sight geometry can again be used provided the line of sight is understood to be between the tracking station at time T and the satellite at time $T - \Delta T$. Hence in order to correct the occultation functions so as to account for the finite speed of light, all that is necessary is to compute the $\bar{R}_p$ vector of Equations (2) and (17) at time $T - \Delta T$ instead of time T where T is the input variable of the occultation functions.

AN EXAMPLE

In Appendix 1 is an input and output description and listing of a Fortran computer program called "OCCULT." This program calculates visibility intervals between an Earth tracking station and a planetary satellite. The calculation is performed by obtaining the visibility function described in previous sections and locating its discontinuities. These discontinuities are then properly interpreted as either initiation or termination points of visibility. From this information, visibility intervals are then constructed and provided as output.

This program was used to obtain visibility intervals between the Goldstone deep space tracking station and a satellite in a typical Mars orbit. In order to make the geometric situation as clear as possible, Mars was assumed to lie on the principal axis of the mean equinox mean equator of epoch coordinate set. This, of course, makes the example an artificial one, but it permits the reader to grasp the relationship between the Earth-Mars line and the orbit of the satellite since the orbital elements are also given with respect to the mean equinox mean equator of epoch coordinate set. Mars was assumed to be two hundred million kilometers from Earth and the satellite orbit was chosen as to insure that planetary occultation would occur. The parameters involved in the calculation are given below.

Planet – Mars

Radius = 3393.4 KM
Gravitational constant = 42977.8 KM^3/sec^2
distance from Earth = 200,000,000 KM

Tracking Station – Goldstone

longitude = 243.11 degrees
latitude = 35.24 degrees
occultation angle = 5 degrees

Satellite Orbit – Mars Elliptic

Semimajor axis = 5 Mars radii
eccentricity = .5
inclination = 5 degrees
longitude of ascending node = 5 degrees
argument of perigee = 45 degrees

Interval in which occultation information is requested = 3 days.

The results obtained by the OCCULT program are displayed graphically in Figure 3. The solid rectangles on the bottom scale illustrate the time intervals in which visibility is permitted between the Goldstone tracking station and the Mars satellite. The solid rectangles shown on the middle scale indicate the intervals in which the Earth does not interfere with visibility between tracking station and satellite. The solid rectangles on the top scale indicate the intervals in which Mars permits visibility between tracking station and satellite. As one would expect, Earth occultation accounts for the major portion of lost visibility time between tracking station and satellite. In this particular case, during the three day period under investigation Earth occultation accounted for over 93% of lost visibility time. It also should be mentioned that the OCCULT program which performed these calculations contains a modification to account for the finite speed of light.

CONCLUSION

An effective procedure for determining visibility intervals between an Earth tracking station and a planetary satellite has been given in this report. The procedure has been incorporated into a computer program, and input and output description and listing of which is given in Appendix 1. The assumption of an infinite speed of light is a convenience but it can in some situations lead to a significant error. A modification to account for the finite speed of light is easily performed.

The program listed in Appendix 1 was used to calculate visibility intervals between the Goldstone deep space tracking station and a satellite in a typical two body orbit around Mars. The results are displayed graphically in Figure 3. In this particular case, Earth occultation accounted for over 93% of lost visibility time during the three day period under investigation.

ACKNOWLEDGMENTS

I want to thank Mr. Alan W. Ritter of the Bendix Corporation for his programming support during the course of this project.

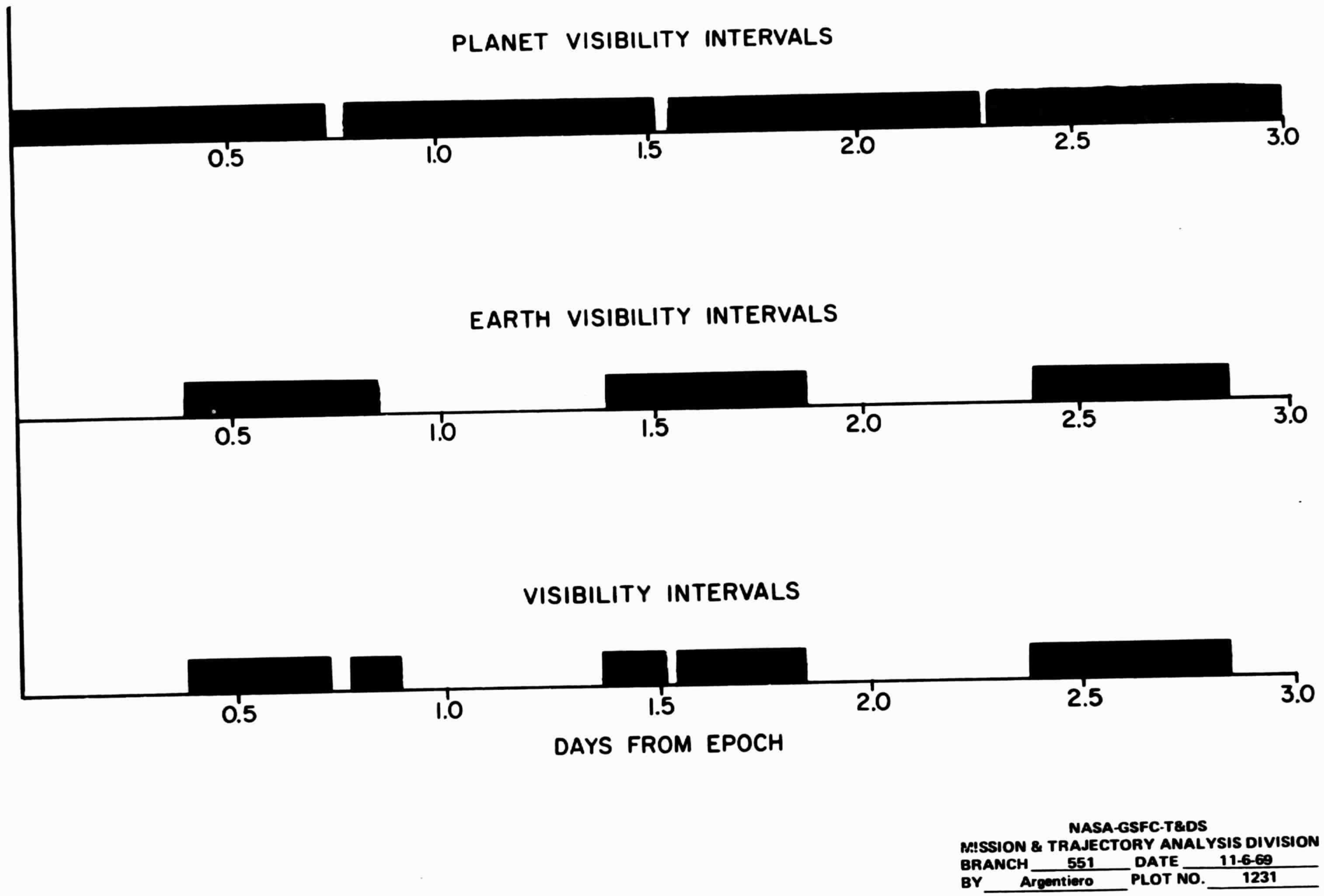

Figure 3—Visibility Intervals Between Tracking Station and Mars Satellite

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APPENDIX 1

Input-Output Description and Listing of "OCCULT"

PROGRAM DESCRIPTION OF "OCCULT"

OCCULT is a double precision Fortran program written for the IBM 360 model 91 or 95 computer. Its purpose is to compute visibility windows between an earth tracking station and a probe executing a closed two body orbit around another body. The program assumed a finite speed of light. An interpretation of the arguments in the calling sequence of OCCULT follows.

CALL OCCULT (EPS, A, XI, SIGMA, OMEGA, U, XP, RP,
XIT, TI, TEND, LAT, LØNG, DELTA, YT, T, N, OPEN)

INPUT

EPS – Eccentricity of satellite orbit about planet

A – Semimajor axis of satellite orbit about planet (in KM)

XI – Inclination of satellite orbit about planet (in radians)

SIGMA – Longitude of the ascending node of satellite orbit about planet (in radians)

OMEGA – Argument of Perigee of satellite orbit about planet (in radians)

U – Gravitational constant of planet in (KM^3/sec^2)

XP – Radius of planet (in KM)

RP – A three dimensional array giving the rectangular coordinates of the planet relative to a mean equinox mean equator of epoch coordinate set (in KM)

XIT – Integer number of days from January 1, 1950 to a time of perifocal passage

TI – Fractional number of days from January 1, 1950 to a time of perifocal passage

TEND – Number of days after perifocal passage in which occultation ifnormation is requested

LAT – Latitude of tracking station (in radians)

LONG – Longitude of tracking station (in radians)

DELTA – Elevation angle of tracking station (in radians)

YT – Time interval, in days, smaller than any visibility window of interest

T – Accuracy desired, in days, in visibility window times

OUTPUT

$$N \begin{cases} = -1 \text{ if there are no visibility windows in interval} \\ = 0 \text{ if there is no occultation in interval} \\ = \text{Number of visibility windows in interval otherwise} \end{cases}$$

OPEN – An N by two array:

OPEN (I, 1) is rise time in days from perifocal passage of I^{th} visibility window

OPEN (I, 2) is set time in days from perifocal passage of I^{th} visibility window

If N = 0 or N = - 1, the OPEN array is undefined.

The OPEN array lists the visibility windows in chronological order.

```
      SUBROUTINE OCCULT(EPS,A,XI,SIGMA,OMEGA,U,RP,XIT,TI,LAT,
     1  LONG,DELTA,TEND,YT,T,XP,N,OPEN)
      IMPLICIT REAL * 8 (A-H,O-Z)
      REAL * 8 LONG,LAT
      COMMON  GHAI,RX,XLONG,XLAT,XN,XEPS,XA,P,Q,XDELTA,XRP,XXP,YTIME
     1  ,XT
      EXTERNAL  VIZ
      DIMENSION P(3),Q(3),RP(3),WAY(102),ZERO(102),YP(2),OPEN(60,2)
      DIMENSION  XRP(3)
      XLONG=LONG
      XLAT=LAT
      XEPS=EPS
      XA=A
      XDELTA=DELTA
      V=(29979250.0D0)*(36.0D0)*(24.0D0)
      ARP=DSQRT(RP(1)*RP(1)+RP(2)*RP(2)+RP(3)*RP(3))
      XT=ARP/V
      DO 60 I=1,3
   60  XRP(I)=RP(I)
      XXP=XP
      RX=6378.165D0
       XN=(DSQRT(U)/(A**1.5D0))*3600.0D0*24.0D0
      CALL EHA(XIT,TI,0.0D0,RA,W)
      GHAI=RA
      P(1)=(DCOS(OMEGA)*DCOS(SIGMA))-(DSIN(OMEGA)*DSIN(SIGMA))
     1 *DCOS(XI)
      P(2)=(DCOS(OMEGA)*DSIN(SIGMA))+(DSIN(OMEGA)*DCOS(SIGMA)
     1 *DCOS(XI))
      P(3)=DSIN(OMEGA)*DSIN(XI)
      Q(1)=-(DSIN(OMEGA)*DCOS(SIGMA))-(DCOS(OMEGA)*DSIN(SIGMA)*
     1 DCOS(XI))
      Q(2)=-(DSIN(OMEGA)*DSIN(SIGMA))+(DCOS(OMEGA)*DCOS(SIGMA)*
     1 DCOS(XI))
      Q(3)=DCOS(OMEGA)*DSIN(XI)
      YP(1)=0.0D0
      YP(2)=TEND
      M=TEND/YT
      DUM= ROOT(M,YP,T,IOUT,VIZ,ZERO,WAY)
      IF(IOUT  .GT. 0)  GO TO 10
      Z=TEND/2.0D0
      Y=VIZ(Z)
      IF(Y  .LT.  0.0D0)  GO TO 5
      N=0
      GO TO 100
    5 N=-1
      GO TO 100
   10 IF(WAY(1)  .GT.  0.0D0)  GO TO 1
      IF(WAY(IOUT)  .LT.  0.0D0)  GO TO 4
      GO TO 50
```

```
    4 IOUT=IOUT+1
      ZERO(IOUT)=TEND
      GO TO 50
    1 DO 20 I=1,IOUT
      J=IOUT+1-I
      L=J+1
   20 ZERO(L)=ZERO(J)
      ZERO(1)=0.0D0
      IOUT=IOUT+1
      IF(WAY(IOUT-1)  .LT.  0.0D0)  GO TO 12
      GO TO 50
   12 IOUT=IOUT+1
      ZERO(IOUT)=TEND
   50 N=IOUT/2
      DO 201 I=1,IOUT
      IF(MOD(I,2)  .EQ.  0)  GO TO 202
      J=(I/2)+1
      OPEN(J,1)=ZERO(I)
      GO TO 201
  202 J=I/2
      OPEN(J,2)=ZERO(I)
  201 CONTINUE
  100 RETURN
      END
      FUNCTION  S1(TIME)
      IMPLICIT REAL * 8 (A-H,O-Z)
      REAL * 8 LONG,LAT
      COMMON  GHAI,RX,LONG,LAT,XN,EPS,A,P,Q,DELTA,RP,XP  ,YTIME
     1  ,XT
      EXTERNAL  EA
      DIMENSION  XX(2)
      DIMENSION  R(3),RPS(3),RS(3),P(3),Q(3),RP(3)
      DIMENSION  ZERO(10),WAY(10)
C     CACULATE HOUR ANGLE
      GHA=GHAI+(2.0D0*3.14159D0*TIME)
C     CACULATE VECTOR FROM EARTH CENTER TO TRACKING STATION
      R(1)=RX*DCOS(LAT)*DCOS(LONG+GHA)
      R(2)=RX*DCOS(LAT)*DSIN(LONG+GHA)
      R(3)=RX*DSIN(LAT)
C     CACULATE ECCENTRIC ANOMALY
      XX(1)=(XN*YTIME)-1.0D0
      XX(2)=XX(1)+2.0D0
      M=3
      XL=.0001D0
      DUM= YROOT(M,XX,XL,IOUT,EA,ZERO,WAY)
      E=ZERO(1)
      X=A*(DCOS(E)-EPS)
      Y=A*DSIN(E)*((1.0D0-(EPS*EPS))**.5D0)
C       CACULATE VECTOR FROM EARTH CENTER TO SATILITE
      DO 2 I=1,3
```

```
    2 RS(I)=(X*P(I))+(Y*Q(I))+RP(I)
C     CACULATE EARTH OCCULTATION FUNCTION
      AR=RX
      ARS=DSQRT((RS(1)*RS(1))+(RS(2)*RS(2))+(RS(3)*RS(3)))
      RDOT=(R(1)*RS(1))+(R(2)*RS(2))+(R(3)*RS(3))
      IF(RDOT  .LT.  0.0D0)  GO TO 4
      GO TO 5
    4 S1=-1.0D0
      GO TO 10
    5 S1=(RDOT*DCOS(DELTA))-(DSQRT((AR*AR*ARS*ARS)-(RDOT*RDOT))
     1 *DSIN(DELTA))-(AR*AR*DCOS(DELTA))
      S1=S1/(AR*ARS)
   10 RETURN
      END
      FUNCTION  S2(TIME)
      IMPLICIT REAL * 8 (A-H,O-Z)
      REAL * 8 LONG,LAT
      COMMON  GHAI,RX,LONG,LAT,XN,EPS,A,P,Q,DELTA,RP,XP ,YTIME
     1  ,XT
      EXTERNAL  EA
      DIMENSION  XX(2)
      DIMENSION   ZERO(10),WAY(10)
      DIMENSION  R(3),RPS(3),RS(3),P(3),Q(3),RP(3)
C     CALCULATE HOUR ANGLE
      GHA=GHAI+(2.0D0*3.14159D0*TIME)
C     CACULATE VECTOR FROM EARTH CENTER TO TRACKING STATION
      R(1)=RX*(DCOS(LAT)*DCOS(LONG+GHA))
      R(2)=RX*DCOS(LAT)*DSIN(LONG+GHA)
      R(3)=RX*DSIN(LAT)
C     CACULATE VECTOR FROM PLANET CENTER TO TRACKING STATION
      DO 1 I=1,3
    1 RS(I)=R(I)-RP(I)
      ARS=(RS(1)*RS(1))+(RS(2)*RS(2))+(RS(3)*RS(3))
      ARS=DSQRT(ARS)
C     CACULATE ECCENTRIC ANOMALY
      XX(1)=(XN*YTIME)-1.0D0
      XX(2)=XX(1)+2.0D0
      M=3
      XL=.0001D0
      DUM= YROOT(M,XX,XL,IOUT,EA,ZERO,WAY)
      E=ZERO(1)
      X=A*(DCOS(E)-EPS)
         Y=A*DSIN(E)*((1.0D0-(EPS*EPS))**.5D0)
C     CACULATE VECTOR FROM PLANET CENTER TO SATILITE
      DO 3 I=1,3
```

```
    3 RPS(I)=(X*P(I))+(Y*Q(I))
      ARPS=(RPS(1)*RPS(1))+(RPS(2)*RPS(2))+(RPS(3)*RPS(3))
      ARPS=DSQRT(ARPS)
C     CACULATE PLANET OCCULTATION FUNCTION
      RDOT=RS(1)*RPS(1)+RS(2)*RPS(2)+RS(3)*RPS(3)
      IF(RDOT  .GT.  0.0D0)  GO TO 5
      GO TO 6
    5 S2=1.0D0
      GO TO 10
    6 Z=DSQRT((ARS*ARS-XP*XP)*(ARPS*ARPS-XP*XP))
      S2=(RDOT+Z-(XP*XP))/(ARS*ARPS)
   10 RETURN
      END
      REAL FUNCTION YROOT*8 (N,P,T,IOUT,FUNC,ZERO,WAY)
C----------INPUT VARIABLES  N=NUMBER OF STEPS  P=TWO PLACE ARRAY
C----------WITH START & STOP OF INTERVAL TO BE EXAMINED
C----------T=ACCURACY DESIRED, FUNC= FUNCTION NAME TO DEFINE
C--------------          CURVE(PASSED W/ EXTERNAL STATEMENT IN CALLING PROG)
C----------OUTPUT VARIABLES  IOUT=NUMBER OF ROOTS FOUND (MAX=100),
C----------ZERO=ARRAY OF ROOTS(MAX=100),WAY=ARRAY OF ROOT RISE/SET
C----------INDICATORS WHERE +1.0D0=SET AND -1.0D0=RISE ROOT
      IMPLICIT REAL * 8 (A-H,O-Z)
      DIMENSION ZERO (100), WAY (100), P(2)
      DUMMY=99.0D0
      XN=N
C------------ M = MAX NUMBER OF ITERATIONS AND IS A FUNC OF P,N,T
      XP= P(2) - P(1)
       IF(T.GE.XP) GO TO 777
      M=(DLOG(XP)  -DLOG(XN)-DLOG(T))/DLOG(2.0D0)
       IF(M.LE.0) GO TO 777
      VALUE = P(1)
      IOUT=0
      RIGHT=+1.0D0
      LEFT=RIGHT
      TEST=FUNC(VALUE)
C------DCOS FUNC WAS USED  FOR CHECK-OUT PURPOSES
       IF(TEST.GE.0.0D0) GO TO 9
       LEFT=-1.0D0
       RIGHT=LEFT
    9  DELTA=(P(2)-P(1))/XN
      VALUE=P(1)-DELTA
       DO 10 ISTEP=1,N
      VALUE=VALUE + DELTA
       KEEP=LEFT
      TEST=FUNC (VALUE)
       RIGHT=+1.0D0
       IF(TEST.LT.0.0D0) RIGHT=-1.0D0
       IF(LEFT.NE.RIGHT) GO TO 11
C---------------------- GO TO 11 . . . CURVE HAS CROSSED AXIS
       GO TO 10
```

```
11      XMARK=VALUE - DELTA
        STOP=VALUE
        CENTER=(VALUE-XMARK)/2.0D0+ XMARK
        DO 14 ITER=1,M
        TEST=FUNC (CENTER)
        SIGN=+1.0D0
        IF(TEST.LT.0.0D0) SIGN=-1.0D0
        IF(SIGN.EQ.LEFT) GO TO 12
C---------------------- GO TO 12 . . . ROOT TO RIGHT OF CENTER
        VALUE=CENTER
        CENTER=( CENTER - XMARK)/2.0D0 + XMARK
        GO TO 14
12      XMARK=CENTER
        CENTER = ( VALUE - CENTER) / 2.0D0 + CENTER
14      C O N T I N U E
100     IOUT=IOUT + 1
        ZERO(IOUT)=CENTER
        WAY(IOUT)=KEEP
        LEFT=+1.0D0
        IF(WAY(IOUT).GT.0.0D0) LEFT=-1.0D0
        RIGHT=LEFT
        VALUE=STOP
10      C O N T I N U E
        YROOT=DUMMY
        GO TO 778
777     ZERO(1)=(P(2)-P(1))/2.0D0
        IOUT=-1
        YROOT=DUMMY
778     RETURN
        END
        FUNCTION  EA(E)
        IMPLICIT REAL * 8 (A-H,O-Z)
        REAL * 8 LONG,LAT
        COMMON  GHAI,RX,LONG,LAT,XN,EPS,A,P,Q,DELTA,RP,XP   ,YTIME
     1  ,XT
        DIMENSION P(3),Q(3),RP(3)
        EA=E-(XN*YTIME)-(EPS*DSIN(E))
        RETURN
        END
        SUBROUTINE EHA ( TW,TF,DA,RA,OMEGA)
        IMPLICIT REAL * 8 (A-H,O-Z)
        DOUBLE PRECISION D
        D=TW
        T=TF*86400.0D0
        OMEGA=0.729211507D-4 - 0.38D-16 * D
        RA=DMOD(DA+OMEGA*T+1.7466477191D0+D*(.0172027914500+D*
     X      0.50641D-14),6.283185307179586D0)
        RETURN
        END
```

```
      FUNCTION  VIZ(TIME)
      IMPLICIT REAL * 8 (A-H,O-Z)
      REAL * 8 LONG,LAT
      COMMON  GHAI,RX,LONG,LAT,XN,EPS,A,P,Q,DELTA,RP,XP    ,YTIME
     1  ,XT
      DIMENSION  R(3),RPS(3),RS(3),P(3),Q(3),RP(3)
      YTIME=TIME-XT
      IF(S1(TIME)  .LT.  0.0D0)  GO TO 10
      IF(S2(TIME)  .LT.  0.0D0)  GO TO 10
      VIZ=1.0D0
      GO TO 15
   10 VIZ=-1.0D0
   15 RETURN
      END
      REAL FUNCTION ROOT*8 (N,P, T, IOUT, FUNC, ZERO, WAY )
C---------- FUNCTION S/R TO COMPUTE THE ZERO CROSSINGS OF A
C-----------CONTINUOUS CURVE ON CART COORDINATES
C-----------CAN BE USED AS A FUNCTION OR A S/R
C-----------INPUT VARIABLES  N=NUMBER OF STEPS  P=TWO PLACE ARRAY
C-----------WITH START & STOP OF INTERVAL TO BE EXAMINED
C-----------T=ACCURACY DESIRED, FUNC= FUNCTION NAME TO DEFINE
C-------------       CURVE(PASSED W/ EXTERNAL STATEMENT IN CALLING PROG)
C-----------OUTPUT VARIABLES  IOUT=NUMBER OF ROOTS FOUND (MAX=100),
C-----------ZERO=ARRAY OF ROOTS(MAX=100),WAY=ARRAY OF ROOT RISE/SET
C-----------INDICATORS WHERE +1.0D0=SET AND -1.0D0=RISE ROOT
      IMPLICIT REAL * 8 (A-H,O-Z)
      DIMENSION ZERO (100), WAY (100), P(2)
      DUMMY=99.0D0
      XN=N
C------------ M = MAX NUMBER OF ITERATIONS AND IS A FUNC OF P,N,T
      XP= P(2) - P(1)
       IF(T.GE.XP) GO TO 777
      M=(DLOG(XP)  -DLOG(XN)-DLOG(T))/DLOG(2.0D0)
       IF(M.LE.0) GO TO 777
      VALUE = P(1)
      IOUT=0
      RIGHT=+1.0D0
      LEFT=RIGHT
      TEST=FUNC(VALUE)
C-------DCOS FUNC WAS USED  FOR CHECK-OUT PURPOSES
       IF(TEST.GE.0.0D0) GO TO 9
       LEFT=-1.0D0
       RIGHT=LEFT
```

```
 9      DELTA=(P(2)-P(1))/XN
        VALUE=P(1)-DELTA
        DO 10 ISTEP=1,N
        VALUE=VALUE + DELTA
        KEEP=LEFT
        TEST=FUNC (VALUE)
        RIGHT=+1.0D0
        IF(TEST.LT.0.0D0) RIGHT=-1.0D0
        IF(LEFT.NE.RIGHT) GO TO 11
C----------------------- GO TO 11 . . . CURVE HAS CROSSED AXIS
        GO TO 10
 11     XMARK=VALUE - DELTA
        STOP=VALUE
        CENTER=(VALUE-XMARK)/2.0D0+ XMARK
        DO 14 ITER=1,M
        TEST=FUNC (CENTER)
        SIGN=+1.0D0
        IF(TEST.LT.0.0D0) SIGN=-1.0D0
        IF(SIGN.EQ.LEFT) GO TO 12
C--------------------- GO TO 12 . . . ROOT TO RIGHT OF CENTER
        VALUE=CENTER
        CENTER=( CENTER - XMARK)/2.0D0 + XMARK
        GO TO 14
 12     XMARK=CENTER
        CENTER = ( VALUE - CENTER) / 2.0D0 + CENTER
 14     C O N T I N U E
 100    IOUT=IOUT + 1
        ZERO(IOUT)=CENTER
        WAY(IOUT)=KEEP
        LEFT=+1.0D0
        IF(WAY(IOUT).GT.0.0D0) LEFT=-1.0D0
        RIGHT=LEFT
        VALUE=STOP
 10     C O N T I N U E
        ROOT=DUMMY
        GO TO 778
 777    ZERO(1)=(P(2)-P(1))/2.0D0
        IOUT=-1
        ROOT=DUMMY
 778    RETURN
        END
```