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THE ROLE OE STAGED SPACE PROPULSION SYSTEMS IN INTERPLANETARY MISSIONS

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16. Abstract

Conditions for obtaining maximum performance of tandem stages of space propulsion systems are examined by means of theoretical analyses and with a numerical simulation of the trajectories flown. The often-used assumption that the inert fraction is constant is shown to yield unduly optimistic predictions compared with the numerical results or with results of an analytic method based on more realistic scaling laws for inert masses. The computerized numerical method is used to assess advantages of staging chemical and nuclear propulsion systems in several high-energy interplanetary missions for which single-stage performance is inadequate.

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THE ROLE OF STAGED SPACE PROPULSION SYSTEMS IN INTERPLANETARY MISSIONS

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SUMMARY

Conditions for obtaining maximum performance from two or more stages of space propulsion are examined both by theoretical means and by a procedure which utilizes numerical integration of the equations of motion in planetary gravitational fields and which includes empirically derived scaling laws for inert masses. Comparison of results indicates that the assumption of a constant value for the inert fraction in an often-used theoretical model leads to optimistic predictions of advantages in performance to be gained by staging space propulsion systems, even though gravity losses are taken into account.

In general, results of the numerical method indicate that the performance of two stages of space propulsion (cryogenic chemical, solid-core nuclear systems or their combinations) does not significantly surpass that of a single stage unless the required energies are so high that the payload fractions are small (about 0.10 or less). Overall performance of two stages used to escape from Earth orbit is found to be insensitive to the ratio of the thrust of the second stage to that of the first from about 0.10 to 1.0. Performance of a combination of nuclear and chemical stages is better when the chemical system serves as the second rather than the first stage of a two-stage system. Results of the study indicate that adding a third stage does little or nothing to increase performance over that of two stages.

The use of two stages of space propulsion rather than one is applied in several high-energy interplanetary missions. Results suggest that although markedly better performance is achieved, the actual performance of two-stage systems in most of the missions studied is still so poor that other means for accomplishing such missions should be considered. Possible alternate means include the use of planetary gravitational fields to add energy to spacecraft trajectories, and the development and use of advanced propulsion systems such as low-thrust electric systems (solar or nuclear power), or liquid- or gaseous-core nuclear engines of high specific impulse.

INTRODUCTION

The energy requirements of certain interplanetary missions are so large that using a single-stage chemical or solid-core nuclear propulsion system for Earth escape or target capture or both would result in excessively large initial masses in Earth orbit for the payloads delivered (ref. 1). These high-energy missions include solar probes to within 0.05 of the Sun, synchronous solar orbiters at a heliocentric distance of 0.176 AU (for a 27-day period), solar probes inclined to the plane of the ecliptic at angles greater than about 45°, missions to orbit the outer planets for trip times shorter than those required for Hohmann transfers, and orbital missions to Mercury. The purpose of this paper is to examine whether using two tandem stages rather than one stage during Earth escape or at the target or both will improve the capability of chemical or solid-core nuclear upper-stage propulsion systems sufficiently to justify the added operational and systems complexity involved.

Although many analyses and studies pertaining to staging of rockets and propulsion systems have been made, it appears that attention has been confined to surface launch vehicles or to vehicles in gravity-free space (see, e.g., refs. 2-4). Results of such studies have either limited or no application to the use of multiple stages of propulsion intended for escaping from or effecting capture into relatively low orbits about planets with appreciable gravitational fields. Consequently, other approaches are developed and applied in the present study.

The first part of the present study deals with the conditions for optimum staging of two or more propulsion systems to obtain the largest payload fraction. First, the usual theoretical model in which the inert masses scale linearly with propellant mass is adopted in order to determine the relationships among the various systems parameters which lead to maximum overall performance. A second model that includes the effect of "fixed" inert masses is similarly analyzed. In a different approach the payload fraction of two stages in tandem is optimized on the basis of numerical integration of the equations of motion. This latter method takes gravity losses into account and enables the selection of the thrust levels of the two stages that result in the maximum payload. Results of the theoretical and numerical methods are compared for propulsion systems having the same specific impulse.

The second part of the paper assesses the advantages in performance resulting from the use of two stages of propulsion, rather than one, to achieve the high energies typical of several of the aforementioned interplanetary missions.

NOTATION AND DEFINITIONS

$a(t)$	instantaneous acceleration
a_i	initial acceleration
c	exhaust velocity of propellants
I_{sp}	specific impulse
k	ratio of inert mass to propellant mass

M_i initial gross mass of first stage

M_F mass of inerts which do not vary appreciably with propellant mass

M_{IN} total mass of stage inerts

M_P mass of stage propellants

M_S mass of propellant module

R ratio of stage payload to stage gross mass

R(n) ratio of payload of last of n stages to initial gross mass of first stage

T thrust

t time

t_s optimum time for separation of first stage

V_∞ hyperbolic excess speed (speed relative to a massive body at an infinite distance)

ΔV total velocity increment, including gravity losses

σ $\frac{dM_S}{dM_P}$, slope of curve of inert mass of propellant module versus propellant mass

Subscripts

1,2,. . .n first, second, . . . nth stage

opt values found to yield optimum results (e.g., maximum performance)

Definitions

emos A unit of speed equal to Earth mean orbital speed about the Sun, 29.7848 km/sec

gravity loss A loss in performance of a propulsion system using finite thrust to accelerate a stage to a final velocity and altitude in a gravitational field as compared with the hypothetical performance of the same system using an infinitely large impulsive thrust to attain the same final orbital energy. The loss is associated with the change in gravitational potential imparted to those portions of usable propellants yet unconsumed during the thrusting period. It is usually expressed as an equivalent velocity increment which if added to the impulsive velocity increment and the sum used in the rocket equation $M_i/(M_i - M_p) = e^{\Delta V/c}$ would yield the same performance as in the case of the finite thrusting. In general, numerical methods are required to evaluate the performance of propulsion stages having finite thrust.

CONDITIONS FOR OPTIMAL STAGING

The rationale for using more than one stage of propulsion to impart velocity to a payload is that dropping spent propulsion stages reduces the total mass that undergoes acceleration and thereby enables a higher final velocity to be attained with a given payload, or allows a heavier payload to be accommodated for a given terminal velocity, or permits a lower initial mass for a given payload and terminal velocity. The viewpoint adopted here is that given either payload or initial mass, the purpose of staging upper-stage propulsion systems used for escape from or capture into planetary orbits is to maximize the ratio of payload to initial mass. Terminal velocities required for interplanetary missions can, of course, be determined independently from the energy requirements (hyperbolic excess speeds).

Theoretical Analysis

The payload M_L of a given propulsion stage is defined by the equation

$$M_L = M_i - M_P - M_{IN}$$

where M_i is the initial gross mass of the stage, M_P is the mass of propellants expended in achieving a velocity change ΔV, and M_{IN} denotes the mass of inerts (those of the rocket engine, propellant tank and other structures, and reserve and ullage propellants). The payload fraction can be expressed by

$$R = \frac{M_L}{M_i} = 1 - \frac{M_P\left[1 + \left(M_{IN}/M_P\right)\right]}{M_i}$$

Two methods of treating the ratio of the mass of inerts to the propellant mass will be considered. For one, the ratio will be assumed to have a constant value σ for a given stage. This assumption has often been adopted in propulsion and mission-analysis studies because of the simplicity it affords. In the other method, the inert masses will be considered to consist partly of certain "fixed" masses that do not change significantly with propellant mass, and partly of other masses that vary linearly with propellant mass. This view results in an inert fraction that varies in a nonlinear fashion with M_P.

Linear inert scaling law.- If the ratio of M_{IN}/M_P is assumed to be equal to a constant σ, the equation for the payload fraction of a propulsion stage can be written

$$R = (1 + \sigma)e^{-\Delta V/c} - \sigma$$

where c is the exhaust velocity of propellants relative to the spacecraft (equal to the product of g_o, the standard gravitational acceleration of Earth ($9.80665 \ m/sec^2$), and I_{sp} (specific impulse) of the stage). In the case of two tandem stages, the payload of the first stage becomes the initial gross mass of the second; the payload of the second stage is the mission payload. The ratio of the latter to the initial gross mass of the first stage is therefore given by

$$R(2) = R_1 R_2 \tag{1}$$

where $R(2)$ denotes the payload fraction obtained with two stages of propulsion, and the subscripts 1 and 2 refer to the first and second stages, respectively. The payload fractions of each stage are given by

$$R_1 = (1 + \sigma_1)e^{-\Delta V_1/c_1} - \sigma_1 \tag{2a}$$

and

$$R_2 = (1 + \sigma_2)e^{-\Delta V_2/c_2} - \sigma_2 \tag{2b}$$

Use is made of the condition that the sum of the velocity increments ΔV_1 and ΔV_2 is equal to the total ΔV required in a given application, that is,

$$\Delta V = \Delta V_1 + \Delta V_2 \tag{3}$$

Setting equal to zero the derivative of $R(2)$ with respect to either ΔV_1 or ΔV_2 yields conditions for maximum $R(2)$:

$$\frac{R_1 c_1 e^{\Delta V_1/c_1}}{1 + \sigma_1} = \frac{R_2 c_2 e^{\Delta V_2/c_2}}{1 + \sigma_2} \tag{4}$$

Even if a constant value is assumed for the inert fraction, an explicit expression for ΔV_1 or ΔV_2 is not obtained for two stages having different specific impulses and different inert fractions. The expression obtained, namely

$$\Delta V_1 = c_1 \, ln\left\{\frac{1 + \sigma_1}{\sigma_1}\left[1 - \frac{c_2}{c_1} + \frac{c_2}{c_1}\frac{\sigma_2}{1 + \sigma_2} e^{(\Delta V - \Delta V_1)/c_2}\right]\right\}$$

could be used to find ΔV_1 and ΔV_2 for given values of ΔV, c_1, c_2, σ_1, and σ_2 by graphical means, iteration, or other devices.

If the two stages have the same specific impulse ($c_1 = c_2 = c$), some simplication results:

$$\Delta V_1 = 0.5\ \Delta V + 0.5c\ \ln\left(\frac{\sigma_2}{\sigma_1}\ \frac{1 + \sigma_1}{1 + \sigma_2}\right)$$

An equation often used to express the conditions for optimum staging of two similar stages is obtained when it is assumed in addition that $\sigma_1 = \sigma_2 = \sigma$ Thus, we have

$$\Delta V_1 = \Delta V_2 = 0.5\ \Delta V$$

$$R_1 = R_2 = R$$

and

$$R(2) = R^2 = \left[(1 + \sigma)e^{-\Delta V/2c} - \sigma\right]^2 \tag{5}$$

This result can be extended easily to any number n of similar stages, that is,

$$R(n) = \left[(1 + \sigma)e^{-\Delta V/nc} - \sigma\right]^n \tag{6}$$

In reference 2, essentially the same results as shown in the foregoing were obtained by seeking the conditions for achieving maximum final velocity for a given payload with two stages.

Nonlinear scaling law for inerts.- In practical propulsion systems, the mass of inerts does not scale linearly with the mass of usable propellants. In the case of the propellant module (propulsion module less the engine), masses of certain items tend to remain relatively constant for rather wide variations in the propellant capacity of the tanks. One useful scaling law for calculating the mass of a propellant module (empty, except for reserve propellants and pressurization devices) is (ref. 1)

$$M_S = \frac{A}{\rho^{0.533}} M_P{}^{0.9} + B \tag{7}$$

in which A and B are constants, ρ is the specific gravity of the propellant, and M_S is the inert mass of the tank and associated structures and components. This equation was used for the plots in figure 1 for modules designed to contain liquid hydrogen and liquid hydrogen/liquid oxygen; the figure also shows the ratio of M_S to M_P in each case. It is evident that the ratio M_S/M_P increases as the mass of required propellant decreases.

The mass of a given rocket engine depends chiefly upon its design thrust level, which, in practice, is not varied with propellant mass but is selected on the basis of stage acceleration levels (i.e., initial gross weight). Also, in the interests of economy, a given engine is likely to be used in several applications involving a rather wide range of propellant requirements.

An additional more-or-less "fixed" inert mass is that of interstaging structures used to interconnect the payload with the propulsion system, and one stage with another.

If the inert mass of a stage is considered to include both "fixed" masses and those that vary linearly with propellant mass, the inert fraction k_1 of the first stage would be

$$k_1 = \frac{\sigma_1 M_{P_1} + M_{F_1}}{M_{P_1}} = \sigma_1 + \frac{M_{F_1}}{M_i\left(1 - e^{-\Delta V_1/c_1}\right)}$$

where σ is dM_S/dM_P, a constant, and M_{F_1} is the fixed mass of the first stage. The payload fraction R_1 then becomes

$$R_1 = \left(1 + \sigma_1\right)e^{-\Delta V_1/c_1} - \sigma_1 - \frac{M_{F_1}}{M_i}$$

For a second stage, however, the inert fraction k_2 involves the payload fraction of the first stage, that is,

$$k_2 = \frac{\sigma_2 M_{P_2} + M_{F_2}}{M_{P_2}} = \sigma_2 + \frac{M_{F_2}}{R_1 M_i\left(1 - e^{-\Delta V_2/c_2}\right)}$$

Likewise, the second-stage payload fraction is now

$$R_2 = \left(1 + \sigma_2\right)e^{-\Delta V_2/c_2} - \sigma_2 - \frac{M_{F_2}}{R_1 M_i}$$

The overall payload fraction of the two-stage system can be expressed by

$$R(2) = \left[\left(1 + \sigma_1\right)e^{-\Delta V_1/c_1} - \sigma_1\left(1 + \frac{M_{F_1}}{\sigma_1 M_i}\right)\right]\left[\left(1 + \sigma_2\right)e^{-\Delta V_2/c_2} - \sigma_2\right] - \frac{M_{F_2}}{M_i} \tag{8}$$

Conditions for obtaining maximum performance of two stages in this case are found to be

$$\frac{R_1 c_1 e^{\Delta V_1/c_1}}{1 + \sigma_1} = \frac{\left[R_2 + \left(M_{F_2}/R_1 M_i\right)\right]c_2}{R_2 + \sigma_2 + \left(M_{F_2}/R_1 M_i\right)}$$

or, in terms of velocity increments,

$$\Delta V_1 = c_1 \; ln\left\{\frac{1 + \sigma_1}{\sigma_1}\left[1 - \frac{c_2}{c_1} + \frac{c_2}{c_1}\frac{\sigma_2}{1 + \sigma_2} e^{\left(\Delta V - \Delta V_1\right)/c_1}\right]\left(1 + \frac{M_{F_1}}{\sigma_1 M_i}\right)^{-1}\right\}$$

As in the previous example based on a linear scaling law, graphical or iteration methods or other schemes are required to obtain values of ΔV_1 or ΔV_2 in the general case.

If both stages have the same specific impulse and utilize identical propellants (i.e., $c_1 = c_2 = c$, and $\sigma_1 = \sigma_2 = \sigma$), fairly simple expressions are obtained for the stage velocity increments:

$$\Delta V_1 = 0.5 \; \Delta V - 0.5c \; ln\left(1 + \frac{M_{F_1}}{\sigma M_i}\right) \tag{9a}$$

$$\Delta V_2 = 0.5 \; \Delta V + 0.5c \; ln\left(1 + \frac{M_{F_1}}{\sigma M_i}\right) \tag{9b}$$

The equation for the maximum payload fraction of the two similar stages can be written as

$$R(2) = \left[(1 + \sigma)e^{-\Delta V/2c} - \sigma\left(1 + \frac{M_{F_1}}{\sigma M_i}\right)^{1/2}\right]^2 - \frac{M_{F_2}}{M_i} \tag{10}$$

This expression can be compared with equation (5) which was derived earlier on the basis of a linear scaling law for inerts. It is clear that a simple definition for the inert fraction leads to a more optimistic view of the efficacy of staging than does the more practical and realistic definition used to obtain equation (10).

Calculations for more than two stages become increasingly involved and lead to inconvenient solutions. The case of three stages having the same specific impulse and using identical propellants is treated here. The payload fraction $R(3)$ can be expressed as

$$R(3) = R_1 R_2 R_3$$

$$= \left\{R_1\left[(1 + \sigma)e^{-\Delta V_2/c} - \sigma\right] - \frac{M_{F_2}}{M_i}\right\}\left[(1 + \sigma)e^{-\Delta V_3/c} - \sigma\right] - \frac{M_{F_3}}{M_i} \tag{11}$$

If ΔV_3 is considered fixed, one finds by solving the expression

$$\frac{\partial R(3)}{\partial \; \Delta V_1} = - \frac{\partial R(3)}{\partial \; \Delta V_2} = 0$$

that

$$\Delta V_1 = \Delta V_2 - c\, ln\left(1 + \frac{M_{F_1}}{\sigma M_i}\right)$$

or, with the condition that the sum of the three-stage velocity increments is equal to the total change ΔV required,

$$\Delta V_1 = 0.5\left(\Delta V - \Delta V_3\right) - 0.5c\, ln\left(1 + \frac{M_{F_1}}{\sigma M_i}\right) \tag{12a}$$

and

$$\Delta V_2 = 0.5\left(\Delta V - \Delta V_3\right) + 0.5c\, ln\left(1 + \frac{M_{F_1}}{\sigma M_i}\right) \tag{12b}$$

as might be expected from the results obtained in the case of two similar stages (cf. eq. (9)). Substituting these values of ΔV_1 and ΔV_2 into equation (11) gives an expression for $R(3)$ in terms of ΔV_3, ΔV, c, σ and the ratios of the fixed masses to the initial mass. If we set

$$A = 1 + \frac{M_{F_1}}{\sigma M_i} \qquad C = 1 + \sigma$$

$$B = \frac{M_{F_2}}{\sigma M_i} \qquad D = e^{\Delta V/2c}$$

the expression for $R(3)$ becomes

$$R(3) = \left[\left(\frac{C\sqrt{A}}{D}\, e^{\Delta V_3/2c} - \sigma A\right)\left(\frac{C}{D\sqrt{A}}\, e^{\Delta V_3/2c} - \sigma\right) - \sigma B\right]\left(Ce^{-\Delta V_3/c} - \sigma\right) - \frac{M_{F_3}}{M_i} \tag{13}$$

From the procedure used in its derivation, equation (13) represents the locus of first-maximum values of $R(3)$ for all values of ΔV_3 in the range of ΔV. To find the value of ΔV_3 corresponding to the overall maximum value of $R(3)$, we equate to zero the result of differentiating the right-hand member of equation (13) with respect to ΔV_3 and solve the resultant equation for the appropriate root. In terms of $x = e^{\Delta V_3/2c}$, the resultant equation can be written as

$$x^4 - \frac{\sigma D}{C} A^{1/2} x^3 - DA^{1/2} x - \left(\frac{B - \sigma A}{C}\right) D^2 = 0 \tag{14}$$

In general, only one root of this equation will yield a maximum value of $R(3)$. Analysis shows that typically only one real positive root of equation (14) exists when the coefficients are related to propulsion systems. Once this root x_1 is found, it can be used in equation (13) to find the maximum performance of three similar stages directly, or, alternatively, the three ΔV can be calculated from

$$\Delta V_3 = 2c \ln x_1$$

and from equation (12); these ΔV can be used to calculate the payload fractions R_1, R_2, and R_3, and then $R(3) = R_1 R_2 R_3$.

Numerical Method

Although the equations derived in the foregoing may appear entirely applicable to assessing performance of two or perhaps more stages of space propulsion systems, they all suffer from a common defect. This defect arises from the lack of exact definitions of the total velocity change required, ΔV, and of the ΔV supplied by each stage. The equations apply actually to systems of infinitely high thrust (impulsive ΔV). However, in practice, the stage ΔV will generally include "gravity" losses which depend on the accelerations, speeds, flight-path angles, and distances from planets at thrust initiation of each stage, as well as on the specific impulse. It is not known *a priori* what the total ΔV, including stage gravity losses, will be for a given required hyperbolic excess speed. Nor do the equations derived contain information relating to the thrust levels and, hence, engine masses appropriate to each stage. To be truly representative of actual staging performance, the theoretical equations should be supplemented by additional information from other sources.

In addition, it was noted in the theoretical approach that solutions, in the case of dissimilar stages, are not mathematically convenient.

In view of the foregoing, a numerical approach was developed to include gravity losses and realistic scaling laws for inerts in the simulation of the staging of two propulsion systems having either similar or unlike characteristics. In brief, equations of motion of a given initial mass undergoing tangential thrust are first integrated numerically from an initial velocity and planetary distance to some fraction f of a given hyperbolic excess speed V_∞ $(0 \leq f \leq 1)$. Tangential thrust has been shown to give results within a fraction of 1 percent of those based on an optimal steering program in similar applications and involves guidance requirements less complex than those of the optimal method (e.g., ref. 5). The payload at termination of the first-stage thrusting is then calculated and used as the initial mass of the second stage. Integration is resumed with this mass and with an input thrust level of the second stage until the given hyperbolic excess velocity is reached. The payload of the second stage is then calculated, as is the product of the two payload fractions, $R(2)$. The fraction f is decreased incrementally from unity (which represents a single stage only) toward zero. The fraction f that produces the largest overall payload fraction is then found, as well as the largest value of $R(2)$ and its components R_1 and R_2. The largest value of $R(2)$ obtained in this way will not always necessarily be the maximum ratio possible; inasmuch as f has a lower limit of zero, the first stage will contribute at least parabolic velocity, and it is conceivable that in some instances dropping of the first stage at lower velocity could yield a larger payload. However, for hyperbolic excess speeds as large as 0.1 emos (earth mean orbital speed, 29.7848 km/sec), analysis of the detailed data from computer runs shows that the slopes of the curves of payload fraction versus f for both stages reach equal absolute values at $f \geq 0$ satisfying the conditions for the maximum value of the product of the fractions. Other quantities of interest are calculated for each value of f. These include the impulsive ΔV, gravity losses, inert fractions, and thrusting times of each of the two stages. For departure from a planetary orbit (circular or elliptic) the integration proceeds normally with the acceleration due to thrust increasing with time since the mass decreases. For capture into a planetary orbit, however, the integration commences at the final conditions of payload mass, velocity, and position in the orbit and proceeds toward the mostly unknown initial conditions, with mass increasing with time (thrust acceleration decreasing). An automated iteration scheme is employed in this case. The foregoing procedure is incorporated into a program for a high-speed digital computer.

With such a computer program, the effects on two-stage performance can be found readily for a number of parameters such as initial acceleration of the first stage, thrust level of the second stage, hyperbolic excess speed, and magnitude of stage inerts. These effects are discussed in the following sections.

To assist the reader in relating performance capabilities of propulsion systems discussed subsequently to mission energy requirements, hyperbolic excess speeds for typical high-energy missions are listed here.

Mission	Hyperbolic excess speeds, emos (km/sec)		
	Earth depart	Planet arrival	Planet depart
Solar probe to 0.05 AU	0.680(20.254)		
45° inclination to ecliptic	0.695(20.700)		
Synchronous solar orbit	0.445(13.254)	0.7249(21.590)	
Jupiter orbiter (550 days)	0.353(10.514)	0.400(11.914)	
Saturn orbiter (3 yr)	0.395(11.765)	0.395(11.765)	
Uranus orbiter (6 yr)	0.432(12.867)	0.413(12.301)	
Mercury manned orbiter (370 days)	0.323(9.623)	0.2819(8.396)	0.3643(10.851)

<u>Initial acceleration of first stage</u>.- In order to gain an insight into the factors that influence the selection of the initial acceleration of the first stage of a two-stage propulsion system, the performance of a single stage in achieving hyperbolic excess speed in the gravity field of a planet is first examined. Two approaches to the problem of determining the variation of performance with initial acceleration are possible. For example, in the case of injection from a parking orbit about Earth into some heliocenter trajectory, one can assume that the initial mass is fixed by the orbital payload capability of a given launch vehicle, or, conversely, that a rocket engine of given

thrust is available, to be used either singly or clustered to give quantum steps in thrust level. Either viewpoint may be applicable depending upon circumstances. The two approaches can be expected to produce somewhat different results since the variation of inert fractions with acceleration will not be the same in each case. Results are given here for both assumptions in the cases of chemical and nuclear stages. Data are obtained with the computer program described, utilizing an option which bypasses calculations for a second stage. Inert masses include, for the present purpose, only those of the propellant modules and of the rocket engines as given in appendix A or figure 2.

Figure 3 shows the variations of payload fraction with initial accelerations for a chemical propulsion system. In part (a) of figure 3, an initial mass of 113,100 kg (250,000 lbm) is assumed and the engine thrust is varied. The effect of doubling the mass of the propellant module is also shown. In part (b), the engine thrust is held fixed and the initial mass is varied. In both cases, the figure shows that the effect of the initial acceleration upon payload fraction is small as long as the acceleration is at least one-third of an Earth g.[1] The difference between the optimum initial accelerations obtained by the two foregoing methods is shown in figure 4. Although varying the thrust to suit the initial mass appears to result in somewhat larger optimum initial accelerations than does adjusting the mass to a given engine thrust level, figure 4 also shows that the optimum payload fraction is essentially unaffected. Figure 5 indicates that an increase in propellant-module inert mass tends to require slightly larger accelerations to obtain the maximum payload fractions. The payload fractions differ by about 0.05 to 0.065 for a given hyperbolic excess velocity; for equal values of payload fraction, the extra inerts decrease the hyperbolic excess velocity attainable by about 0.048 to 0.10 emos (1.4 to 3.0 km/sec).

A similar analysis is made for a nuclear propulsion system. Results are shown in figures 6, 7, and 8. It is noted in figure 6 that the performance of the nuclear stage is more sensitive to initial acceleration than that of the chemical system, particularly when the data are obtained by varying the initial mass while holding the thrust level constant. The more rapid falling off of payload fraction with increasing initial acceleration observed in figure 6(b) is largely owing to the fact that the inert masses (e.g., the engine mass) constitute an increasingly larger fraction of the total mass as the initial acceleration is increased. As noted in figure 7, the optimum payload fraction is larger when thrust is constant than when initial mass is constant. This difference is due to the initial masses also being larger and hence the inert fractions smaller for a fixed thrust level. For the chemical system discussed previously, the effects just noted are scarcely discernible chiefly because the engine mass represents only a small fraction of the inert mass, in contrast to the nuclear system, and the initial masses are more nearly the same. The effect of doubling the inert masses of the propellant module has little effect on the values of optimum initial acceleration when the nuclear

[1]This is true only for initial masses and thrust levels as large as those considered here. For masses or thrust levels considerably smaller, the sensitivity of payload fraction to initial acceleration is greater and the optimum acceleration is lower.

engine has a fixed thrust level of 444,822 N (100,000 lbf) (fig. 8). The effect on optimum payload fraction is significant, however.

From the results just presented, the variation of the optimum initial acceleration with hyperbolic excess speed is not so large that the use of an average value would result in noticeable loss in performance over a wide range of energy requirements. This fact has a bearing on the choice of the initial acceleration for the first stage of a two-stage system. The hyperbolic excess speed attained at the end of optimum first-stage thrusting will vary from near-parabolic values to generally not more than about half the total speed required. Hence, the initial acceleration found to be optimum for a single stage at a given hyperbolic excess speed can be considered essentially optimum for the first stage of a two-stage system designed to attain the same energy starting from the same parking orbit and same initial mass as the single stage.

Effect of engine lifetime on selection of initial acceleration. - Although it is desirable to match engine thrust and initial gross mass to achieve the maximum payload fraction as closely as is practical, there may be another consideration, at least for the nuclear engine. Although current plans call for several hours (perhaps as many as 10) of total operating time for a multiple-restart nuclear engine, a "single-use" time may, for one reason or another, be limited to some definite period. The effects of limiting single-use time on the minimum permissible initial acceleration of the nuclear engine is examined here. The time required to achieve a given velocity increment can be expressed in terms of the initial acceleration a_i and specific impulse of the propulsion system as follows:

$$t = \left(1 - e^{-\Delta V/g_o I_{sp}}\right) \frac{g_o I_{sp}}{a_i}$$

For a given allowable time of engine operation, the minimum initial acceleration is

$$(a_i)_{min} = \left(1 - e^{-\Delta V/g_o I_{sp}}\right) \frac{g_o I_{sp}}{t_{allow}}$$

This relationship is plotted in figure 9 against ΔV for a nuclear engine having a specific impulse of 820 sec and for allowable operating times of 1800, 2700, and 3600 sec. An auxiliary scale shows typical hyperbolic excess speeds attained with these ΔV in departing Earth from a circular orbit (altitude 485 km) with an initial acceleration of 0.25 Earth g. (The scale would be shifted somewhat for other altitudes and other values of a_i.) A comparison of figure 9 with figure 6(b) shows that an allowable operation time of 1800 sec imposes possible penalties in payload fraction ranging from zero to about 18 percent for hyperbolic excess speeds up to 0.5 emos in the case of Earth departure with a thrust level of 444,822 newtons (100,000 lbf). Increasing the allowable time to 2700 sec reduces the largest penalty to about 2 percent at a V_∞ of about 0.5 emos. With an allowable time of 3600 sec, the minimum thrust-to-initial-weight ratio is smaller than desirable for maximum payload fraction except for hyperbolic excess speeds as large as 0.6 emos when a nuclear engine of the above thrust is used (fig. 6(b)). In this instance (V_∞ = 0.6), the payload fraction reduces to a mere 0.10 percent or so if the engine is not operated beyond 3600 sec. Staging in this case could improve the payload fraction and reduce the operating time required of either stage to considerably less than 3600 sec.

Effect of second-stage thrust level.- In a previous section ("Initial acceleration of first stage") it was found that the initial acceleration which would result in maximum performance of a single stage during escape from Earth to a given hyperbolic excess speed could also be considered as essentially optimum for the first stage of a two-stage space propulsion system required to attain the same energy under similar initial conditions. For the present, the foregoing result will be used to fix the initial acceleration of the first stage in an investigation of the effect on performance of varying the thrust level of the second stage of a two-stage vehicle.

Some feeling for the factors involved in selecting a thrust level for the second stage may be obtained from applying the results of previous theoretical analyses in combination with certain assumptions. For example, the adoption of the assumption that the single-stage optimal initial acceleration is also essentially optimum for a first stage leads to another valid assumption according to the following heuristic reasoning. The thrust-acceleration history of both the single stage and the first stage of the two-stage vehicle is given by

$$a(t) = \frac{a_i}{1 - (a_i/c)t}$$

where $a(t)$ is the acceleration due to thrust at a time t following thrust initiation. Associated with this acceleration profile is a trajectory which, with a given thrusting program (tangential steering is assumed here) and an optimum initial acceleration, makes the best compromise between gravity losses on the one hand and propellant consumption and inert masses on the other in attaining a given hyperbolic excess velocity. Such a trajectory results in maximum performance of a single stage. Under the present assumption, the two-stage vehicle has the same acceleration history and trajectory up to the optimum time t_s for separation of the first stage and start of the second-stage engine (assumed to occur simultaneously with separation). It therefore appears reasonable to assume that the second stage would attain essentially maximum performance if it continued the optimum single-stage trajectory until the remaining energy were provided.

If the foregoing assumption is adopted, it follows that the acceleration history of the single stage should remain unbroken. Then at time t_s

$$a(t_s) = \frac{T_1}{M_i e^{-\Delta V_1/c_1}} = \frac{T_2}{R_1 M_i}$$

Hence, the optimum ratio of second-stage thrust to that of the first stage is

$$\left(\frac{T_2}{T_1}\right)_{opt} = R_1 e^{\Delta V_1/c_1} = 1 + \sigma - \sigma\left(1 + \frac{M_{F_1}}{\sigma M_i}\right)e^{\Delta V_1/c_1}$$

where the definition of the payload fraction R_1 of the first stage is that used in the analytic method based on nonlinear inert fractions. The same theory gives, for similar stages (cf. eq. (9))

$$e^{\Delta V_1/c} = \left(1 + \frac{M_{F_1}}{\sigma M_i}\right)^{-1/2} e^{\Delta V/2c} \tag{15}$$

Thus, in terms of the total required velocity increment ΔV, the ratio can be expressed as

$$\left(\frac{T_2}{T_1}\right)_{opt} = 1 + \sigma - \sigma\left(1 + \frac{M_{F_1}}{\sigma M_i}\right)^{1/2} e^{\Delta V/2c}$$

Equation (15) indicates that T_2 should always be less than T_1, and decrease relative to T_1 as energy requirements (V_∞) increase.

If a good estimate for ΔV (including gravity losses) is available for a given hyperbolic excess speed, equation (15) may be useful. However, the sensitivity of performance to second-stage thrust level is not given by the theory. To investigate this sensitivity, and, incidentally, to assess the validity of equation (15) and the underlying assumption, the numerical procedure is applied to obtain the variation of performance of two-stage propulsion systems with the ratio T_2/T_1. A combination of an initial mass and a first-stage thrust level is selected to give an initial acceleration found earlier to yield essentially maximum payload fractions for a single stage over a range of hyperbolic excess speeds. The thrust level of the second stage is then varied in the simulation of departure from an Earth parking orbit. Results are presented in figure 10 for similar chemical and nuclear stages and for combinations of chemical and nuclear systems. It is apparent that when it is advantageous to use two stages rather than one, the thrust level of the second stage is generally not critical from the standpoint of performance as long as it is at least about one-tenth that of the nearly optimum first stage. This insensitivity is potentially advantageous in that a rocket engine suitable for a single-stage space propulsion system could be used in both stages of a two-stage space propulsion system with little or no degradation in the performance attainable with optimum engine sizing.

In view of the flatness of the curves in figures 10(a) and (b), comparison of the theoretical predictions of $(T_2/T_1)_{opt}$ with the numerical results is somewhat academic. The trend of the analytic predictions, however, is borne out by the computer-obtained results, namely a decrease in the optimum second-stage thrust level with increasing V_∞.

Effects of using nonoptimum first-stage acceleration.- Since it is not economically practical to design, develop, and produce a wide variety of rocket engines having thrust levels ideally suited to each of many interplanetary missions, the use of a given engine either singly or in clustered arrangements for a number of missions will result in nonoptimum initial accelerations in both single-stage and multiple-stage applications. Figures 3 and 6 indicate that a given initial mass in Earth orbit could be matched without appreciable loss in performance with single-stage propulsion systems with thrust levels ranging from about 0.3 to 0.8 of the initial mass for chemical stages, and from about 0.25 to 0.50 of the initial mass for nuclear stages. From another viewpoint, it is clear that a single stage of given thrust level could be used with initial masses from about 1.5 to 4.0 times the thrust level of chemical engines or from approximately 4.0 to 6.0 times the thrust level of nuclear engines without significant loss in performance.[2] (Engine operating time limitations may prohibit using certain otherwise acceptable low initial accelerations.) In the discussion of two-stage space propulsion systems in the previous section, it was found that for a given initial spacecraft mass and a first-stage thrust level which gave essentially optimum first-stage initial acceleration, the thrust level of the second stage could be varied over nearly an order of magnitude without appreciable change in the overall payload fraction obtained.

Although the foregoing remarks tend to justify developing relatively few space propulsion engines of different thrust levels, it remains to complete the analysis by assessing the effects on two-stage performance of using off-optimum first-stage initial accelerations. Accordingly, additional data similar to those presented in figure 10 were obtained with the same first-stage thrust levels but with different initial masses. Examples of results obtained by varying initial accelerations as much as a factor of 8 are shown in figure 11.

In the case of two stages of chemical propulsion (fig. 11(a)), the optimum first-stage initial acceleration may be taken as 0.4 g. The variation of performance with second-stage thrust tends to increase somewhat as the first-stage acceleration departs from the optimum value. This larger variation is due to the fact that with off-optimum first-stage performance the overall performance can be affected more by second-stage characteristics such as thrust level. Even so, the range of second-stage thrust levels which could be used without serious penalty in overall performance is large even in the extreme cases shown for off-optimum first-stage acceleration. If the thrust level of the second stage is selected near its optimum value, the loss in overall performance due to off-optimum first-stage initial acceleration is proportionately less than that correspondingly incurred in the case of a single stage (represented by intercepts on the vertical axis $T_2/T_1 = 0$).

In the case of nuclear propulsion, a study of figure 11(b) leads to essentially the same conclusions as those in the preceding paragraph. One

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[2]These figures are representative for only initial masses and thrust levels as large as those considered here. For smaller values, the ranges cited would be reduced.

difference can be noted, however. At hyperbolic excess velocities as large as 0.5 emos, the data indicate that the overall performance of two stages may remain unchanged or slightly improved if the initial acceleration of the first stage is reduced to about one-half that found to give maximum performance of a single stage. The difference between the effect of initial acceleration on chemical and nuclear two-stage performance when the first-stage thrust level is fixed is one of degree only; it is chiefly due to the larger fixed inert masses (particularly that of the engine) of the nuclear stages. As the initial mass is increased to produce a smaller initial acceleration, the inert fractions of both nuclear stages decrease more in proportion than do those of the chemical stages; hence, the payload fraction of the former has a more pronounced tendency to increase despite the incurrence of larger gravity losses. Advantages of using such a low acceleration, however, could be prohibited if limitations on single-use operating times were required in practice as discussed earlier. Figure 12 displays maximum-performance thrusting times of each stage as a function of first-stage initial acceleration and of a second-stage thrust for a hyperbolic excess speed of 0.55 emos (16.5 km/sec). For an initial acceleration as small as 0.1 of an Earth g, the first-stage nuclear engine would have to operate more than about 30 percent longer than an assumed 3600-sec capability. At $a_i = 0.15$ g, a single stage would thrust about 4500 sec; with the addition of a second stage having optimally half the thrust of the first stage, the first stage would require only about 3000 sec and the second nearly 2000 sec. Figure 12 also indicates in this case that although reducing the second-stage thrust from 0.5 to, say, 0.2 that of the first stage diminishes the overall performance by only 1 or 2 percent (cf. fig. 11(b)), the operating time of the second-stage engine correspondingly increases to beyond an assumed 3600-sec limit. It may be concluded that, under assumptions made here, the first stage of a two-stage nuclear space propulsion system designed for high-energy Earth-escape maneuvers should have an initial acceleration in the neighborhood of that found optimum for a single stage for similar purposes, and the second-stage thrust level should be from one-half to the same as that of the first stage.

In general, results of this section indicate that the effect of using nonoptimum thrust levels of first-stage units of two-stage systems is not so large from the standpoint of performance as it is for single-stage space propulsion systems.

COMPARISON OF RESULTS OF THEORETICAL AND NUMERICAL METHODS OF PROPULSION STAGING

In order to make meaningful comparisons among the predictions of the theoretical analyses and of the numerical method, certain ground rules are observed here. One such rule is that the magnitude of the total velocity increment ΔV required for a given hyperbolic excess speed in the theoretical formulas is to be that calculated by the numerical method for a single stage having a near-optimal initial acceleration. In this way, gravity losses will be taken into account in both methods, and comparison between the performances of single and multiple stages will be meaningful. Another rule is that inert masses such as those of the engines and propellant modules shall be calculated in essentially the same way in both approaches. For example, in using equations (5) and (6), the value of the constant inert fraction σ is calculated by dividing the sum of M_S (eq. (7)) and the mass of the engine (appendix A, or fig. 2) by the mass of the propellant M_P. The latter is calculated from

$$M_P = M_i\left(1 - e^{\Delta V/nc}\right)$$

where M_i is the initial gross mass of the spacecraft and n is the number of stages. In the use of equation (10), the value of σ is taken to be the slope of the straight line that best fits the appropriate curve of M_S versus M_P in figure 1. The scaling law for propellant modules shown in figure 1 can be closely approximated over a rather wide range of M_P by

$$M_S = \sigma M_P + M_{S_o}$$

The fixed mass M_F of each stage is computed as the sum of M_{S_o} and the mass of the stage engine. For the present comparison, other inert masses such as those of interstaging structures, meteoroid and thermal protection of tanks, and payload adapters are not included. In subsequent examples of the application of staging to interplanetary missions, these other inerts are taken into account. A simple computer program was written to solve equation (14) and to use the appropriate root for calculating ΔV_1, ΔV_2, ΔV_3, R_1, R_2, R_3, and $R(3)$ in the case of three similar stages. The solution of equation (9) and calculations for R_1, R_2, and $R(2)$ for two similar stages were also included as options.

Similar Propulsion Stages

Figure 13 shows the performance of single- and multiple-stage propulsion systems over a range of hyperbolic excess speeds. An initial parking orbit (altitude = 485 km) about Earth is assumed. Two types of propulsion systems are considered, chemical stages using liquid hydrogen/liquid oxygen with an I_{sp} of 450 sec, and solid-core nuclear stages with an I_{sp} of 820 sec.

Chemical stages.- Figure 13(a) shows the predictions of the theoretical method based on the nonlinear inert scaling law to be in excellent agreement with those of the numerical-integration method in the case of two chemical stages. As anticipated earlier, the method derived from the assumption that the inert fraction is constant and the same for all stages tends to overestimate the performance advantage of two stages over a single stage. From the comparisons shown for two- and three-stage performance, the addition of a third stage of propulsion for Earth departure does not appear to be warranted.

Nuclear stages.- Because of the larger inert mass (particularly that of the engine) of nuclear stages, predictions of the theory based on a constant inert fraction are shown in figure 13(b) to exaggerate the performance advantages of two nuclear stages over a single stage much more than in the previous example of chemical staging. Theoretical results based upon more realistic inert scaling laws again agree well with the numerical calculations. As in the case of chemical space propulsion systems, a third stage does not appear to be warranted here. According to theoretical results (not shown here), if the same sized nuclear engine (thrust level in the neighborhood of 100,000 lbf) were used in each of the three stages, the performance would be poorer than that of two stages having identical engines.

From the foregoing comparisons, the theoretical method which was derived on the basis of a nonlinear scaling law for the inert fraction may appear to be a convenient and adequately accurate means of assessing performance of two- or three-stage space propulsion systems. However, it will be recalled that the theoretical results were based on the assumption that the total velocity increment required was that calculated by integration of the equations of motion for a single stage having the same initial acceleration as the first stage of a two-stage system. As a matter of fact, examination of the output of the computer program shows that the sum of ΔV_1 and ΔV_2 is within about 1 percent of the ΔV for a single stage (gravity losses included in all stages). The question arises whether the use of the readily computed single-stage impulsive velocity increment could be substituted for the total ΔV in the theoretical method without significant change in results. Figure 14 shows the effects of neglecting gravity losses in the theoretical calculations for performance of the two-stage systems just discussed. In the case of two chemical stages, figure 14(a) indicates that differences in predicted performance range from about 10 percent at $V_\infty = 0.4$ to 25 percent at $V_\infty = 0.6$ emos (18 km/sec). For two nuclear stages (fig. 14(b)), corresponding discrepancies are about 25 and 100 percent. The relatively larger errors noted for nuclear stages are associated with the considerably greater gravity losses (10 to 12 percent of impulsive ΔV) which in the examples are about three times as large as those of the chemical stages for corresponding V_∞.

The foregoing analysis leads to the conclusion that for applications such as Earth escape, failure to take gravity losses into account in assessing performance of two-stage space propulsion systems results in unacceptably optimistic expectations. It follows that some such approach as the numerical method adopted here is required to provide at least the proper total ΔV requirements and appropriate engine sizes in order to obtain meaningful results with the analytic expressions derived in this report. Also, since no closed form was obtained for staging two dissimilar propulsion systems, an algorithm would need to be devised and probably be programmed for a computer in order to analyze the effectiveness of staging in a number of applications. The present computer program based upon numerical integration of the equations of motion solves the general staging problem and provides a convenient method for evaluating the advantages or disadvantages of using two similar or dissimilar stages rather than a single stage of propulsion in diverse situations typical of interplanetary missions.

In view of the foregoing, staging and staging applications will be studied further on the basis of data obtained with the computer program described earlier.

All the data presented thus far have been obtained with departure from an Earth parking orbit as a frame of reference. Results and conclusions derived from presentation of such data would not be materially different should another planet be used as a basis for calculations. The chief difference noted would be changes in the performance curves of payload fraction versus hyperbolic excess speed V_∞. The shape of these curves would differ because the performance of given propulsion systems in attaining the energy associated with V_∞ depends upon both the impulsive velocity increment and the magnitude of the gravity loss, and both of these vary from one planet to another. For example, for the small planet Mercury, the impulsive velocity increments are smaller than those at Earth for V_∞ less than about 0.23 emos, but become increasingly larger beyond this value. Likewise, the gravity losses incurred in escaping Mercury with optimum initial accelerations are comparable with, or in some cases larger than, those incurred in escaping Earth to the same V_∞. In view of the disparities in mass and radius of the two planets (mass and radius of Mercury are about 0.054 and 0.38 that of Earth, respectively), this latter fact may appear paradoxical. However, an analysis of the trajectories followed in departing both planets and of the optimum initial accelerations lends credence to the observation. It is found that for the same absolute initial acceleration and same impulsive velocity increment, the flight path is steeper and the radial distance (in planetary radii) is greater for departure from a parking orbit about Mercury than about Earth. This means that the overall propellant load receives a larger gain in Mercurial gravitation potential than it does in the terrestrial potential. The work done in changing the potential level of the unused portion of the usable propellant, of course, constitutes a "gravity loss," expressed as an equivalent velocity increment additional to the impulsive velocity increment. Hence, although the strength of the gravitational field of Mercury is only about 0.376 that of Earth, the gravity losses under similar conditions at the two planets tend to become more alike. In the case of chemical propulsion, analysis shows that maximum performance is obtained with essentially the same absolute initial acceleration (0.4 Earth g, or 1.064 Mercury g) for V_∞ to about 0.4. Gravity losses are found to be generally larger by as much as 15 percent for Mercury than for Earth for corresponding V_∞. At a hyperbolic excess speed of 0.5 emos, the optimum initial acceleration has decreased to 0.15 Earth g and the maximum payload fraction has decreased to zero in the case of escape from Mercury. At this V_∞, the impulsive ΔV and gravity loss are nearly 17 and 200 percent larger, respectively, for Mercury than for Earth, resulting in a total velocity increment 20 percent or 2.256 km/sec greater. In the case of nuclear propulsion, the optimum initial acceleration for escape from Mercury to a given V_∞ is about one-half that correspondingly found for Earth; gravity losses would generally be about the same at Mercury as at Earth under these conditions. However, such low accelerations could not be used for V_∞ much larger than 0.2 emos without requiring engine operating times longer than 3600 sec; consequently, some sacrifice in performance would be required in such instances.

In the following examples of the use of two stages of space propulsion, rather than one, in missions to other planets, effects of planet mass and size on energy requirements are automatically taken into account in the computer program based upon numerical integration of the equations of motion.

OTHER STAGING CHARACTERISTICS

Before presenting results of applying two-stage propulsion systems to specific missions, two other characteristics of staging are examined here. For one, the effect of varying the inert masses of the stages is assessed. The other is the comparison of performances of two dissimilar stages (e.g., nuclear and chemical) with those of similar stages.

Effects of Inert Mass

Although masses of chemical and nuclear engines may be considered to be known or predictable within limits sufficiently accurate for preliminary mission analysis, estimates of the masses of propellant modules (propellant tanks and associated equipment and structures) are subject to more uncertainty at present. The nominal value of the constant A cited in appendix A for the empirically derived scaling law for propellant modules is about one-half that found to be representative of a number of launch-vehicle stages on which the law was based (cf. ref. 1). These stages, however, included masses of thermal insulation and of meteoroid shielding which are accounted for separately in the inert scaling laws adopted here for space propulsion systems. Also, the use of current materials and structural technologies in the design and fabrication of future space propulsion stages should logically result in stage inert fractions lower than those of stages designed some years ago for launch vehicles. Preliminary calculations indicate that a value of 0.10 for the constant A should not be unduly optimistic for future propellant modules incorporating tanks with common bulkheads for containing liquid oxygen and liquid hydrogen, or modules containing liquid hydrogen only. In view of the demonstrated dependence of staging effectiveness on the inert scaling law employed in the theoretical approaches, it is pertinent to examine the effect of varying the inert fraction of the propellant module in the numerical simulation of staging. Figure 15 shows the effect of doubling the constant A in the formula for calculating the mass of the propellant module. The only other inert mass included in this comparison is that of the engine. One conclusion from figure 15 is that although increasing the inert mass decreases the payload capacity of both single-stage and two-stage propulsion systems, the performance of the two-stage system is relatively less affected than that of the single stage. As a result, advantages of staging become significant at lower energy requirements and are also relatively larger at a given V_∞ as inert fractions increase. Another noteworthy observation is that the data indicate that reducing the structural mass of the propellant module significantly can yield gains in performance comparable with those obtained by adding a second stage. This is particularly true for the nuclear system shown in figure 15(b).

Performance of Dissimilar Stages

Although data for combinations of chemical and nuclear stages were calculated and presented earlier (figs. 10(c), 10(d)), the purpose there was to assess the effect of the magnitude of the second-stage thrust on performance. Here the objective is to compare the performance of two-stage propulsion systems consisting of a nuclear first stage and a chemical second stage (N/C), of a chemical first stage and a nuclear second stage (C/N), of two chemical stages (C/C), and of two nuclear stages (N/N). Figure 16 shows performance curves for such space propulsion systems as applied to escape from an Earth parking orbit (altitude = 485 km) to various hyperbolic excess speeds. Included for reference are the performances of single-stage nuclear (N) and chemical (C) systems.

From a study of figure 16, it is clear that if both chemical and nuclear propulsion systems were available, the chemical system should be used as the second, rather than the first, stage of a two-stage space propulsion system for injecting paylods into high-energy heliocentric trajectories from an Earth parking orbit. The nuclear/chemical combination yields higher performance than the combination of two nuclear stages only at such large velocity requirements that the payload is at best a few percent of the initial mass in Earth orbit. This situation is analogous to the demarcation in performance that can be delineated for chemical and nuclear single stages in terms of ΔV and of either initial mass in Earth orbit or payload. Figure 17 shows curves of equal performance for chemical and nuclear space propulsion systems similar to those considered as component stages here. In the case of the subject two-stage vehicles with an initial mass of 226,800 kg, at sufficiently large V_∞ the mass remaining after separation of the nuclear first stage will be less than some such initial-mass boundary value, with the result that a chemical system with its relatively low inert masses will perform better as a second stage than the heavier nuclear system despite the advantage of the higher I_{sp} of the latter. The same line of reasoning can be followed in terms of a payload boundary curve and the second-stage performance. Likewise, these same factors help to explain why the performance of two chemical stages becomes equal to or better than that of two nuclear stages as the payload fraction tends toward zero, as noted in figure 16.

From the comparisons between the performances of two-stage and single-stage space propulsion systems here in figure 16 and earlier in figure 13, one noteworthy general observation can be made: advantages in performance resulting from the use of two stages rather than one stage of propulsion become significant only at such large energy requirements that the payload fractions are usually no larger than about 0.10.

APPLICATIONS OF STAGING IN HIGH-ENERGY MISSIONS

The use of two tandem stages to achieve the large velocity changes inherent in several high-energy solar-exploration missions is examined in the following sections. Arbitrarily ruled out for consideration here are trajectories which utilize the gravitational field of Jupiter or of other planets to reduce ΔV requirements.

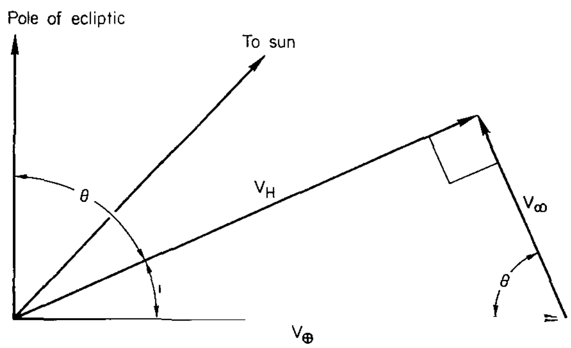
Solar Probe to 0.05 AU

Hyperbolic excess speeds required to inject a payload into heliocentric orbits having an aphelion distance equal to the distance of the Earth from the Sun at the time of launch and various perihelion distances are shown in figure 18 for a summer launch date (July 7, when Earth is at aphelion and velocity requirements for solar probes are least). For a perihelion distance of 0.05 AU, the probe must have a residual velocity of 0.68 emos (20.254 km/sec) after escaping from Earth.

Figure 19 shows the payload capabilities of an Earth-departure propulsion system consisting of a nuclear first stage and a chemical second stage. Data for initial masses in Earth orbit of 45,360 kg (100,000 lbm), 113,400 kg (250,000 lb), and 181,440 kg (400,000 lb) are shown. Interstaging masses for connecting the two propulsion stages (diameters assumed to be 10 m), as well as for connecting the payload (diameter of 1.5 m) to the second stage are included in the calculations (see appendix A). Performance of a single nuclear stage also is shown for several initial masses. Figure 19 indicates that the single stage with an initial mass as large as 453,600 kg cannot deliver any payload closer than about 0.1 AU from the Sun. With an initial mass of 113,400 kg, the two-stage system cannot attain the objective perihelion distance. If the initial mass is increased to 181,400 kg, a payload of about 1450 kg (payload fraction of 0.008) could, under the assumptions used in the calculations, be sent to 0.05 AU by the two-stage system considered. From these results, it is obvious that the mass required in Earth orbit for even a small solar probe of only a few hundred kilograms is representative of the capability of two current Saturn V launch vehicles. Studies at the Mission Analysis Division indicate that the use of a nuclear stage in a suborbital mode with the Saturn V launch vehicle could increase the payload by perhaps 20 to 30 percent compared with its use in an orbit-start mode. In this case, one standard Saturn V and a nuclear/chemical system might be sufficient for small but perhaps adequate payloads to 0.1 AU. On the other hand, analyses (e.g., ref. 6) have indicated that the subject solar-probe mission could be accomplished with a smaller launch vehicle (e.g., a Saturn I-B) and a solar electric or nuclear electric upper-stage propulsion system, although the mission time required may exceed one year. Hence, although staging of nuclear and chemical rocket engines does provide much better performance than a single nuclear stage at large hyperbolic excess speeds, the initial mass requirements for a solar probe to within 0.05 AU or closer are still of such magnitude that consideration of more advanced types of upper-stage propulsion for this mission may well be warranted.

Probes in Heliocentric Orbits Inclined to Ecliptic

From the viewpoint of scientists concerned with solar and space physics, instrumented probes orbiting the Sun in planes inclined to the solar equator at relatively large angles (45° to 90°) would be highly desirable to extend our knowledge and understanding of solar phenomena and of the space environment above and below the ecliptic. Since the solar equator is inclined by only 7.25° to the ecliptic, the inclinations of the probe orbits with respect to the ecliptic would be almost equal to those with respect to the solar equator. If departure from Earth coincides with the passage of Earth through the descending node of the solar equator on the ecliptic (heliocentric longitude = 255°.356) the solar latitude reached by the probe will be the largest possible for a given inclination of the probe trajectory with respect to the ecliptic, and ΔV requirements will be nearly minimum, since Earth is approaching its aphelion (lowest orbital velocity). To achieve the largest inclination of the probe trajectory for a given hyperbolic excess velocity, the latter is made to lie in a direction normal to the plane of the trajectory as shown in sketch (a) below.



Sketch (a)

In this method, it will be noted that the resulting heliocentric velocity of the probe V_H is initially less than the orbital velocity $V_\oplus$ of the Earth. The probe will therefore have a perihelion distance less than 1 AU which decreases with increasing angle of inclination. For observation of solar phenomena at high solar latitudes, such probe orbits could provide relatively high resolution at the highest latitude attained. If an orbit at 1 AU is stipulated, ΔV requirements grow much more rapidly with inclination angle than in the method described.

Figure 20 shows the hyperbolic excess speeds required to attain probe-trajectory inclinations with respect to the ecliptic up to 90°. Figure 19 indicates that a single nuclear upper-stage propulsion system is incapable of placing any payload into heliocentric trajectories inclined by as much as about 40°. For an initial mass of 113,400 kg in Earth orbit, the nuclear-chemical two-stage system could achieve an inclination of only about 41° or 42° for a small payload of 100 or 200 kg. Increasing the initial weight to 181,437 kg would enable an inclination of about 47° or 48° to be attained with a small payload and two stages. Here again, the use of a suborbital start of the nuclear stage (with a Saturn V launch vehicle) could improve overall performance somewhat.

From the foregoing results, it appears that achieving out-of-ecliptic probe misions with angles of inclination as large as 45° or larger is costly, in terms of initial mass required in Earth orbit, with nuclear or chemical upper-stage propulsion despite the significant increases in performance over a single stage afforded by a two-stage system. As in the case of the solar probe discussed previously, advanced propulsion systems such as low-thrust electric propulsion are worthy of consideration for such high-energy missions. As indicated in reference 7, the use of electric propulsion (powerplant specific mass of 30 kg/kW) in combination with relatively small launch vehicles (e.g., Atlas/Centaur, Titan III-D/Agena D) could place payloads of a few hundred kilograms into solar orbits at 1 AU with inclinations extending to perhaps 60°. A somewhat larger launch vehicle, with a capability similar to that of a Saturn I/Centaur, would enable angles up to about 70° to be attained. The times involved are of the order of 500 days with low-thrust systems.

Synchronous Solar Orbiter

Establishing an instrumented spacecraft into an orbit having a period equal to that of the solar photosphere at some latitude (e.g., 30°) appears to be highly desirable from a scientific viewpoint. Such an orbit would uniquely permit observation of the complete history of certain transient solar phenomena (e.g., sunspots, flares) from first appearance to final demise. However, energy requirements for this mission are enormous. At Earth departure, a velocity increment of about 9.4 km/sec is required to reach the requisite perihelion distance of 0.176 AU. To capture into a solar orbit at this distance requires a decrease in speed of more than twice that, 21.6 km/sec. No single chemical or nuclear system presently under development can provide this latter velocity change. When thermal protection and propellant boiloff, shielding against micrometeorite puncture, and other inerts are included in the mass of the stages, calculations indicate that a combination of a nuclear first stage and a chemical second stage requires a gross mass of several million kilograms at perihelion arrival in order to place an instrumented probe weighing only a few hundred kilograms into the circular solar orbit. Clearly, missions having such energy requirements as are inherent in the solar synchronous orbiter must await development of more advanced propulsion devices.

Outer Planet Orbiter Missions

Figure 21 shows typical variations with trip time of optimum hyperbolic excess speeds for both Earth departure and planet arrival in the case of ballistic trajectories to Jupiter, Saturn, and Uranus. Hohmann transfer times, for which total energy requirements are minimum, are indicated in the figure. The arrival speeds at the planets are seen to be particularly sensitive to variations in trip time.

The velocity changes required for capture depend not only upon the arrival speeds but also upon the mass of the planet and upon the radius of the capture orbit. Examples of the variation of capture ΔV with arrival speed for selected circular capture orbits about Jupiter, Saturn, and Uranus are given in figure 22. Included for comparison is the variation of impulsive ΔV with V_∞ for Earth departure. It is interesting to note that although the capture requirements at Saturn and Uranus are smaller than those at Jupiter for hyperbolic excess speeds less than about 0.3 to 0.4 emos (9 to 12 km/sec), they are larger at greater arrival speeds. Figure 21 indicates that the arrival speed is about 0.4 emos for a 3-year trip to Saturn and for a 6-year trip to Uranus. An arrival speed of 0.4 emos corresponds to a 550-day trip time to Jupiter. Hence, a study of orbiter missions to Jupiter involving trip times as short as 550 days will provide an upper limit on capture performance at Saturn and Uranus for much longer trip times. Impulsive ΔV serves as a valid criterion here since, at the orbital altitudes considered, gravity losses are negligible.

Simulation of capture into an orbit about Jupiter (the orbit of the Jovian satellite Ganymede at 15 planet radii) was carried out for payloads in orbit of 4536 kg (10,000 lb) and 907 kg (2000 lb) and for various capture propulsion systems. Inert masses included those required by propellant boil-off and tank thermal insulation, by meteoroid shielding, and for interstaging between propulsion stages, as well as those associated with tank structure and engines. Results are shown as the variation of payload fraction with trip time in figure 23. (No nuclear performance is shown for the smaller payload since the assumed nuclear engine (thrust of 444,800 N) proved to be too heavy to compete with the chemical stage even at the shorter trip times.) Results indicate that staging of two chemical systems gives better performance than does a single chemical stage for all trip times shown if payloads as large as 4536 kg are placed in orbit. For trip times less than 500 days, a combination of a nuclear first stage with a chemical second stage provides the best performance. Even at a trip time of 500 days, however, the payload fraction is less than 0.04 in the case of the 4536 kg payload. As can be seen in the figure, the payload fraction decreases significantly with decreasing payload mass. Hence, the gross mass required at arrival can be expected to be large for a reasonable size payload for trip times as short as 500 days.

Figure 24 shows the variation of the overall payload fraction with trip time for the total mission including planet capture and Earth departure. Note the scale change between figures 23 and 24. For the range of trip times shown, a single nuclear stage gives better performance at Earth departure except at trip times as short as 400 days for which a nuclear/chemical two-stage system is slightly superior (see figs. 21 and 19). The relative overall performances of the various combinations of propulsion systems used for Earth departure and planet capture are thus much the same as for the capture phase shown in the previous figure. What this means in terms of requirements for initial mass in Earth orbit is indicated in figure 25. For payloads as large as 4536 kg, part (a) of the figure indicates that reductions of trip times become significantly more costly in terms of the mass required in Earth orbit for trip times less than about 750-800 days regardless of the propulsion systems used. Shown on the figure for comparison purposes is the nominal value of the Earth-orbital payload capability of the current two-stage Saturn V launch vehicle. Figure 25(a) indicates that the use of two chemical stages rather than either a nuclear/chemical combination or a single chemical stage for the orbit capture maneuver is to be preferred from a performance standpoint. However, the figure also shows that, for a given mass in Earth orbit, replacing a single chemical capture stage by two chemical stages will reduce the trip time by only a few weeks. The effect of reducing the required payload is shown in part (b) of figure 25. It is noteworthy that a fivefold reduction in payload results in considerably less than a twofold reduction in the weight required in Earth orbit for missions of the same trip time. The figure shows that for a fixed initial mass the use of two stages rather than one during the capture phase reduces trip time by about 50 days at best.

As remarked earlier, the analysis of the role of staging in the Jupiter orbiter mission can serve to gage the effectiveness of staging in orbiter missions to Saturn and Uranus. It was noted that if a trip time of three years is stipulated for a mission to Saturn, figure 21 indicates that the hyperbolic excess speed at arrival (0.4 emos) is the same as that for a 550-day trip to Jupiter. As shown in figure 22, the ΔV required for capture at this arrival velocity is essentially the same for the two planets. Hence, the performance of the capture stage(s) would be essentially the same for both planets under these conditions for identical payloads delivered provided that the inert masses were equivalent. Chiefly because of the much longer trip time of the Saturn mission, the inert masses represented by propellant boiloff, tank thermal insulation and micrometeoroid protection would be larger than those for the 550-day Jupiter mission. Hence the capture phase at Saturn for a 3-year trip time would require a larger gross mass at arrival than the similar maneuver for a 550-day trip to Jupiter for a given delivered payload. Likewise, figure 21 shows that the hyperbolic excess speed required at Earth departure is larger by about 0.04 emos (1.2 km/sec) for the 3-year Saturn trip than for the 550-day Jupiter mission. It can be expected, therefore, that the masses required in Earth orbit would be considerably larger than those shown in figure 22 for the 550-day Jupiter trip time for comparable delivered payloads. The same conclusion can be drawn for a Uranus orbiter mission which takes 6 years of travel time.

More advanced space propulsion systems may be desirable to reduce the trip times to planets beyond Jupiter. Reference 8 contains examples of orbiter missions to outer planets by means of combined high- and low-thrust propulsion systems. A 3-year mission to Uranus, for example, is shown there to require in the neighborhood of 90,000 kg (200,000 lbm) in Earth orbit for a delivered payload in Uranus orbit (18.367 radii) of 4536 kg (10,000 lbm) if a chemical propulsion system is used in combination with a low-thrust electric system (propulsion system specific mass = 10 kg/kW) during Earth departure and if capture is accomplished by low-thrust spiraling. Use of a nuclear high-thrust stage rather than the chemical stage would correspondingly require about 57,000 kg (125,000 lb) in Earth orbit. Calculations with larger values of specific mass which may be more typical of first-generation nuclear-electric propulsion systems indicate that the masses in Earth orbit cited above might be doubled, or the payloads halved. Even so, the mission appears to be within the capability of one launch vehicle of the standard Saturn V class if nuclear-electric systems were available.

Manned Orbiter Mission to Mercury

As an example of a mission for which initial mass required in Earth orbit is unreasonably large when single stages of solid-core nuclear propulsion are used for each of three required major velocity changes, the manned orbiter mission to Mercury is selected to determine if the use of two stages will reduce the initial mass required to more acceptable values.

In any one year, there are three opportunities for accomplishing the Mercury round-trip mission. One of these periods generally requires much lower total velocity changes than the other two. Associated with each of these opportunities is an optimum period (staytime) between arrival and departure. One of the most favorable opportunities is selected for the present purpose. Hyperbolic excess speeds required in this case are, in emos, 0.3231, 0.2819, and 0.3643 for Earth departure, Mercury arrival, and Mercury departure, respectively. Atmospheric braking upon return to Earth is assumed. A staytime of 80 days is optimum for minimizing energy requirements. Trip times of 90 days, Earth to Mercury, and 200 days, Mercury to Earth, are required. An orbit close to the planet surface (altitude = 93 km (50 n. mi.)) is chosen for the mission.

A mass of 45,360 kg (100,000 lb) is assumed for the spacecraft on the return leg. This mass includes those of a mission module and of an Earth entry vehicle. Compared in figure 26 are the masses required at departure and arrival at Mercury, and the initial masses in Earth orbit for the use of a single stage in each phase with those required when two stages of propulsion are used. Also indicated in the figure are the number of propulsion modules required in each stage either to maximize the payload fraction or to constrain the nuclear engine operating time to no more than 2700 sec. The thrust level of the nuclear engines is assumed to be 444,822 newtons (100,000 lbf).

Although the advantages of staging are quite modest in terms of payload fractions at any one of the three phases, the overall reduction in the requirement for mass in Earth orbit is significant, nearly 40 percent. Even so, however, undertaking this mission with current launch vehicles and with solid-core nuclear spacecraft propulsion systems still does not appear practical. Results of reference 1 indicate that the mass in Earth orbit for this mission could be reduced to about 400 metric tons if liquid- or gaseous-core nuclear rockets with specific impulses of 2000 sec were available. In reference 9, it was shown that use of combined nuclear and low-thrust electric propulsion for the subject mission might reduce the mass in Earth orbit to about one-fourth of that required by high-thrust nuclear systems. It must be concluded that manned missions having energy requirements comparable with those of the Mercury mission examined here must await development of propulsion systems having much higher performance than that possible with solid-core nuclear or chemical engines, whether staged or not.

CONCLUDING REMARKS

An assessment has been made of performance advantages that might be obtained by the use of more than one stage of space propulsion to provide the large velocity changes inherent in space missions of particularly high energy.

In analyzing the conditions under which multiple stages of propulsion would achieve maximum performance, it was found that a commonly employed simplified theory based upon the concept of a constant value for the inert fraction gives unduly optimistic predictions for advantages of using more than one stage for a given velocity change. The basis for comparison was a method of solution that employs an empirically derived, nonlinear scaling law for inert masses and depends upon numerical integration of the equations of motion of vehicles undergoing tangential thrust acceleration within the gravitational field of a planet. A second theoretical analysis based on an inert scaling law nearly equivalent to that used in the numerical method was found to give results almost identical to those of the numerical method in the case of performance of two similar stages. However, both of the theoretical solutions require that equivalent "gravity-loss" velocities be known in order that meaningful results will be obtained in applications to maneuvers close to planets. Gravity losses due to finite thrust are calculated and taken into account in the method based upon numerical integration. Furthermore, analytic solutions in the general case of dissimilar stages are not explicit and therefore require indirect methods of approximation to yield answers. The method based on numerical simulation of a two-stage vehicle operating in the near vicinity of planets was designed to accommodate combinations of stages having different specific impulses, different inert characteristics, and different thrust levels. This method also gives individual stage operating times more realistic than can be obtained from the analytic solution. For such reasons, the numerical method is employed to obtain the data required in the study.

Other results and conclusions obtained from the present study are briefly summarized as follows.

(a) In general, for assumed nominal estimates of inert masses, advantages of staging become significant only at such large velocity requirements that the ratios of payload mass to initial mass are about 10 percent or less. If inert masses are considered to be larger than the assumed nominal values, results indicate that staging becomes advantageous at velocity requirements somewhat less than those found in the case of nominal inert values, although performance at corresponding energies is reduced.

(b) The use of more than two stages to provide large velocity change does not appear to be warranted.

(c) If chemical and nuclear stages are to be combined for a two-stage propulsion system, better performance will result if the chemical system is used as the second, rather than the first stage.

(d) The optimum initial acceleration (thrust level) of the first stage of a two-stage space propulsion system is not essentially different from that found for a single stage similar to the first stage. Performance of a two-stage system is less sensitive to the initial acceleration than that of a single stage.

(e) The thrust level of the second stage of a two-stage propulsion system can vary by about an order of magnitude (from about 0.1 to the same as that of the near-optimum first stage) without seriously degrading overall performance. Allowable engine operating time may impose a lower limit on thrust level under certain conditions (e.g., very high mission-energy requirements).

(f) Although staging of chemical and nuclear systems makes possible the accomplishment of certain high-energy missions that are beyond the capability of single stages (e.g., a solar probe to 0.05 AU or closer, and probes at heliocentric latitudes of 45° or higher), the ratio of the mass required in Earth orbit to the payload mass is so unfavorable that consideration of development of more advanced propulsion systems (e.g., solar-electric or nuclear-electric) for such missions may be warranted.

(g) Staging of chemical or solid-core nuclear propulsion systems to reduce trip times for orbiter missions to planets beyond Mars does not appear sufficiently rewarding to justify the added complexity involved.

(h) In the case of the extremely difficult manned orbiter mission to the planet Mercury, the use of two, rather than one, nuclear stages for Earth escape, planet capture, and planet escape reduces the single-stage requirement for mass in Earth orbit by about 40 percent. Even so, however, the initial mass required is several million kilograms. To cope with the high energy requirements of such missions and to bring the initial mass requirements down to more practical values, development of more advanced propulsion systems such as a liquid- or gaseous-core nuclear engine or a nuclear-electric low-thrust system would be necessary.

The foregoing results indicate that although using more than one stage of space propulsion for high-energy interplanetary missions does make many such missions feasible, the cost in terms of Earth launch vehicle requirements yet remains so high with currently envisaged chemical and nuclear systems that consideration of more advanced space propulsion systems for such missions may be warranted.

National Aeronautics and Space Administration
Moffett Field, Calif., 94035, Aug. 5, 1969

APPENDIX

INERT SCALING LAWS

Scaling laws for inert masses and formulas for interstaging weight used in this study are summarized below.

Mass of Chemical Engines
(Ref. 1)

The scaling law for chemical engines was empirically derived from characteristics of liquid-rocket engines ranging in thrust from 15,000 lbf to over 10^6 lbf.

$$M_E = \tau T_E + 45 \text{ kg}$$

where

τ 0.0125

T_E engine thrust, kg (force)

Mass of Nuclear Engines

The variation of the mass of the solid graphite-core nuclear engine with thrust level is given in figure 2. The curve is based on data obtained from the NASA Space Nuclear Propulsion Office.

Mass of Propellant Module
(ref. 1)
(Total mass of propulsion module excluding engine and thermal and meteoroid protection)

$$M_S = N^{0.1}\left(A/\sigma^{0.533}\right)M_P{}^{0.9} + 500 \text{ N kg}$$

A 0.10 for nominal value

σ specific gravity of propellant

M_P mass of propellant

N number of propulsion modules

Mass of Propellant Boiloff and Thermal Insulation (ref. 1)

The following scaling law is based upon minimizing the sum of the masses of boiloff and tank insulation, and upon the use of cylindrical tanks with hemispherical ends and a length-to-diameter ratio of 3.0.

$$M_{BO} + M_{INS} = B\left(M_P/\sigma\right)^{2/3}(t\ \Delta T/L)^{1/2}$$

B 0.034 for nominal value

t exposure time, days

ΔT temperature difference across insulation

L latent heat of vaporization of propellant, kcal/kg

The insulation material is assumed to be of the high-performance type with a conductivity of 4.44×10^{-5} kcal/hr-m/K and a density of 80 kg/m^3.

Meteoroid Shielding (ref. 1)

The following scaling law is based upon the 1963a Whipple distribution of near-Earth cometary particles, upon the Nysmith-Summers penetration theory, a particle density of 0.5 kg/km^3 and an average particle velocity of 20 km/sec, upon the use of aluminum as shield material, and upon the same tank geometry described above. For single-sheet protection, the mass M_m of the shield is given by

$$M_m = C\left(M_P/\sigma\right)^{5/6}t^{1/4}$$

C 0.06 for nominal value for heliocentric distances less than that of the asteroid belt

C 0.10 for trips through the asteroid belt

The use of a double-sheet shield ("Whipple bumper") is assumed when warranted. The following conditions determine the mass of shielding material which is required in addition to the mass of the propellant tank.

$$\Delta M_{met} = 0; \qquad M_m \leq M_S'$$

$$\Delta M_{met} = M_m - M_S'; \qquad M_S' < M_m \leq 4/3\ M_S'$$

$$\Delta M_{met} = 1/3\ M_S'; \qquad 4/3\ M_S' < M_m < 16/3\ M_S'$$

$$\Delta M_{met} = 1/4\ M_m - M_S'; \qquad M_m > 16/3\ M_S'$$

where

$$M_S' = 0.8 M_S$$

Interstaging and Clustering Structures

The masses of various interstaging and clustering structures are derived from results of a contract study, NAS8-20007 (ref. 9).

For interstage structures used to interconnect propulsion modules, the following scaling laws are used.

For nuclear systems

$$M_{INT} = 142(D_1 + D_2)$$

For chemical systems

$$M_{INT} = 65(D_1 + D_2)$$

where D_1 and D_2 are the diameters in meters.

For clustering two or more propulsion systems in parallel, the mass required is

$$M_{CLUST} = 970(N - 1)$$

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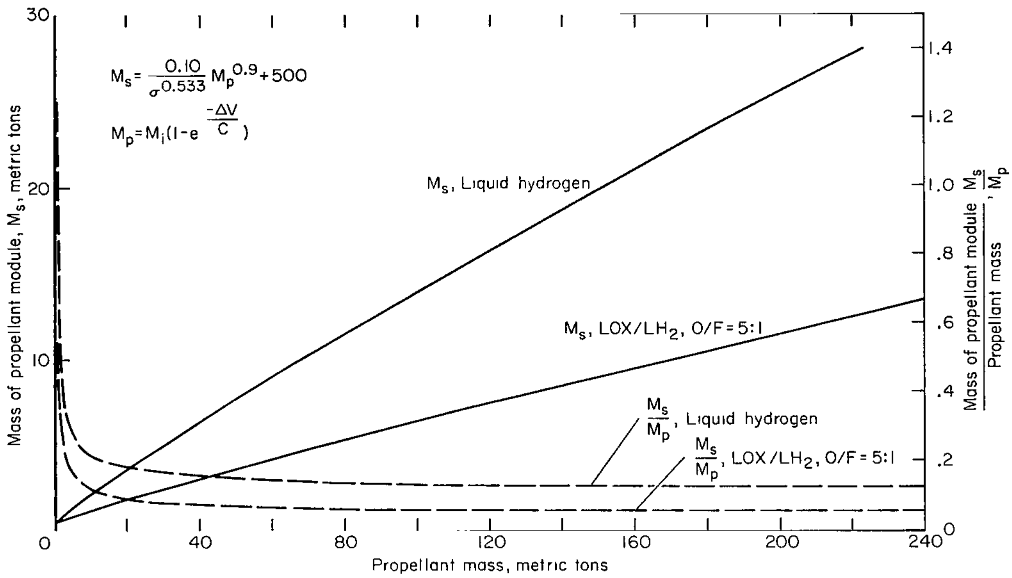


Figure 1.- Plots of scaling law for propellant-module inert mass and inert fraction.

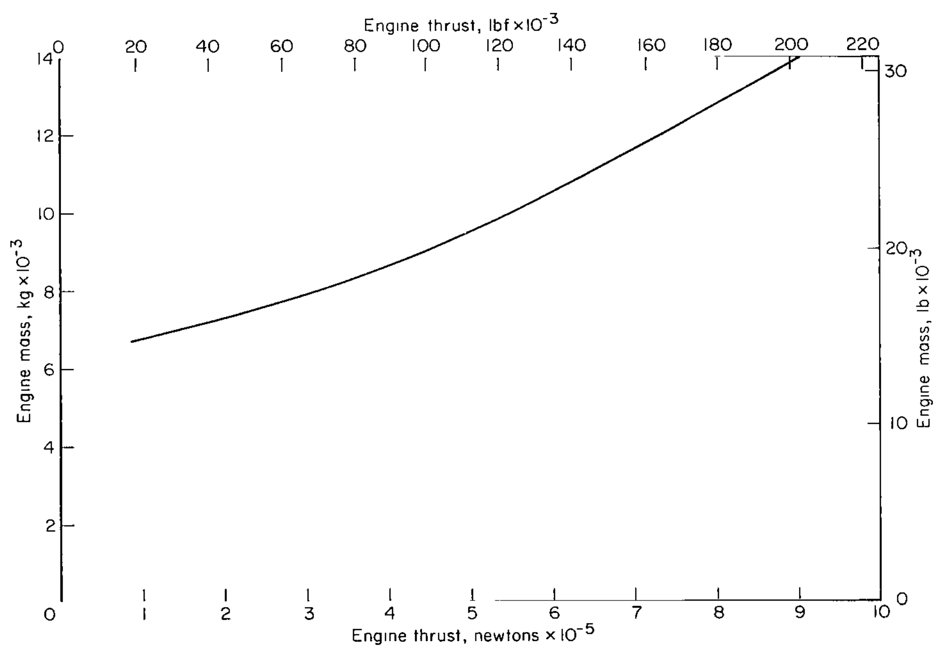
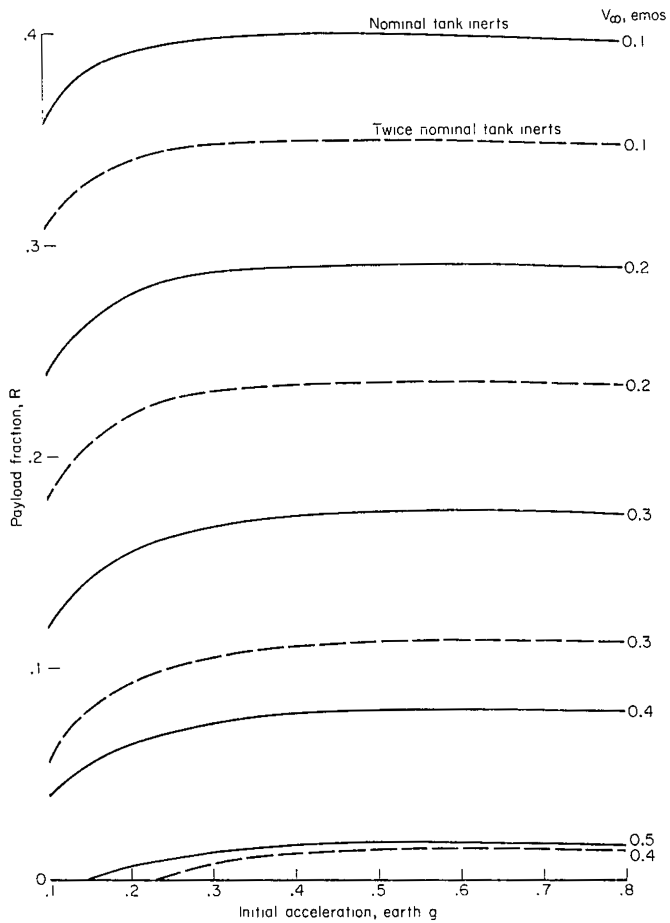
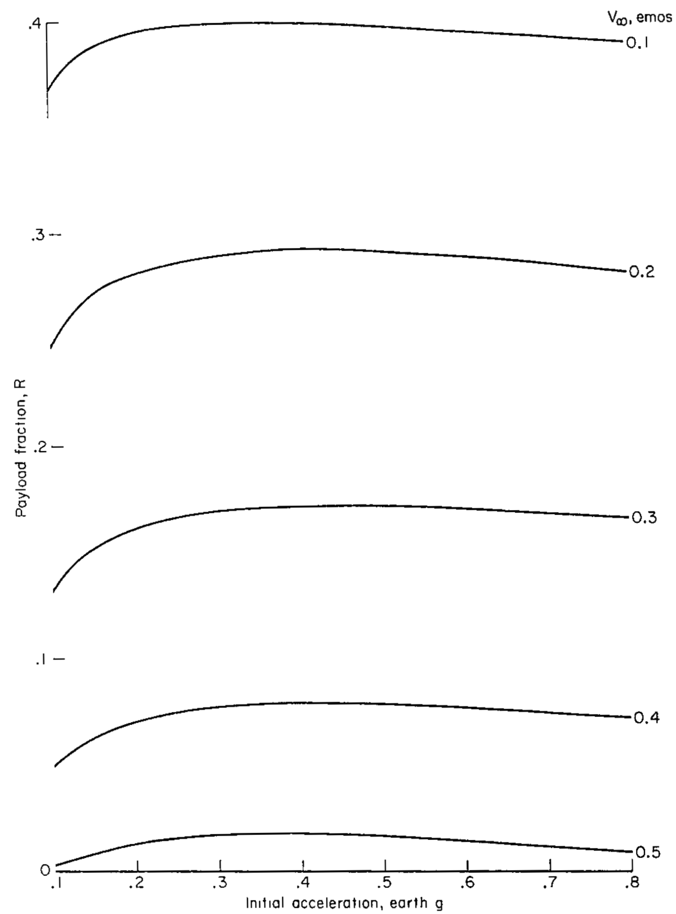


Figure 2.- Variation of nuclear engine mass with engine thrust.



(a) Initial mass = 113,400 kg;
thrust variable.

Figure 3.- Variation of payload fraction with initial acceleration; chemical propulsion system, I_{sp} = 450 sec; Earth departure.



(b) Thrust constant (444,822 N); initial mass
variable.

Figure 3.- Concluded.

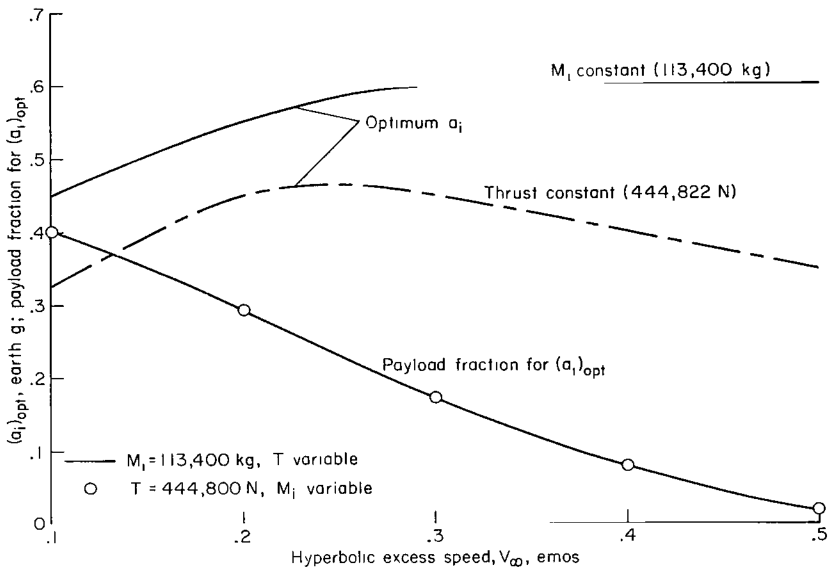


Figure 4.- Variations of optimum initial accelerations and associated payload fractions with hyperbolic excess speed for chemical upper-stage propulsion system; Earth departure.

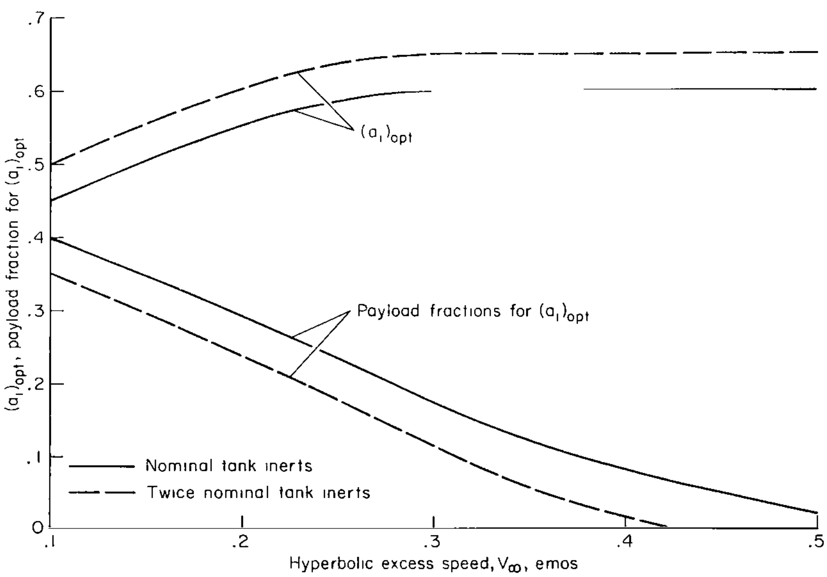
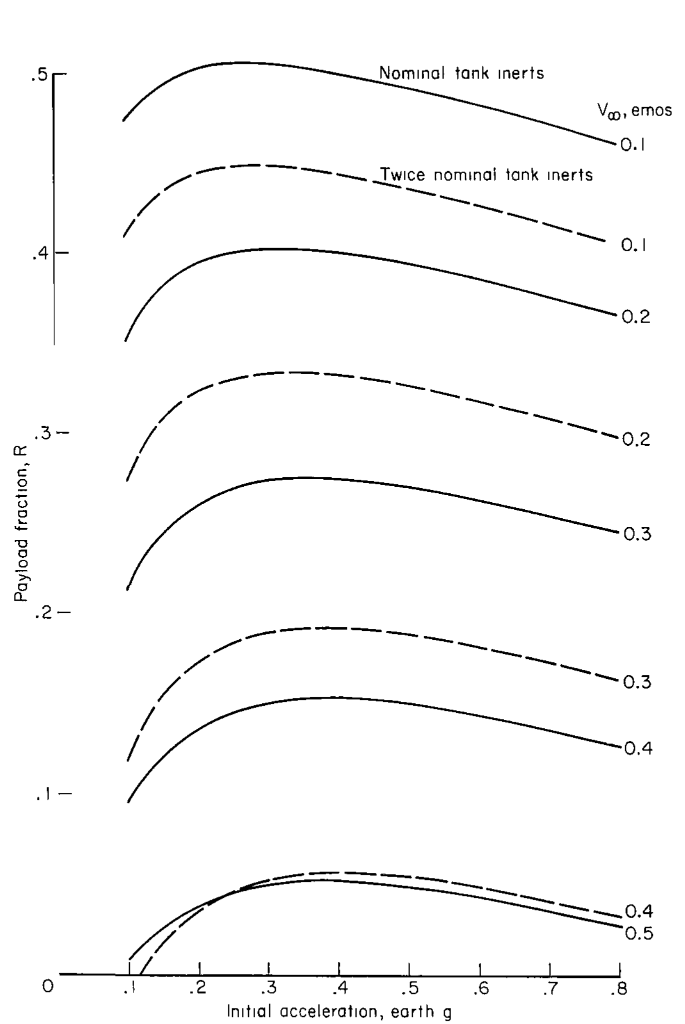
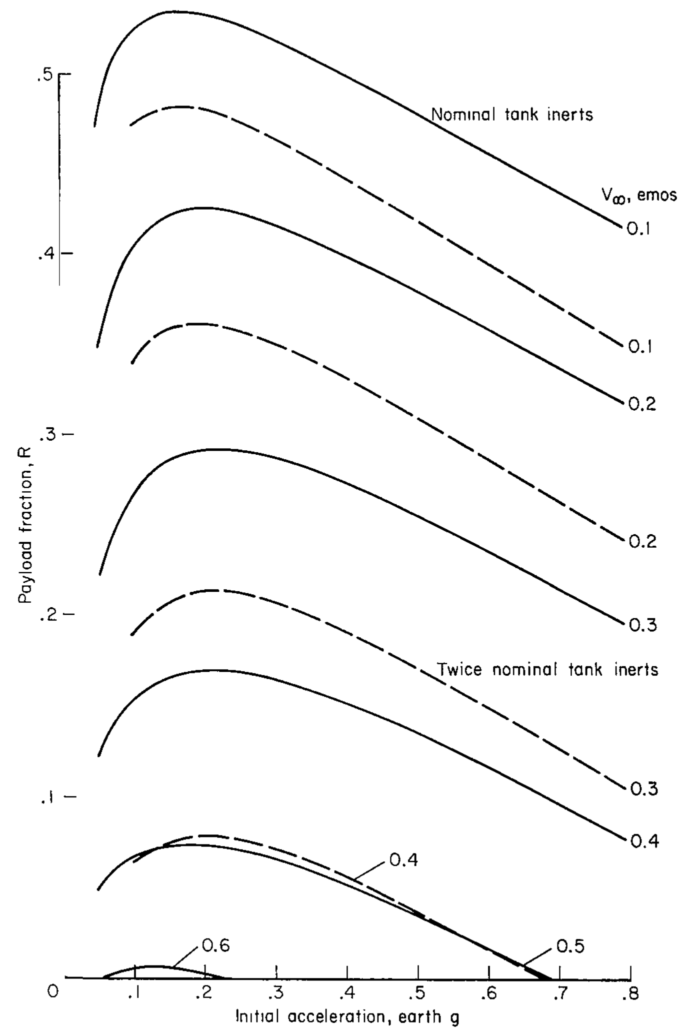


Figure 5.- Effect of inert mass on optimum initial acceleration and on optimum payload fraction during Earth departure with chemical upper stage; initial mass = 113 400 kg (250,000 lbm).



(a) Initial mass constant (113,400 kg); thrust variable.



(b) Thrust constant (444,822 N); initial mass variable.

Figure 6.- Variation of payload fraction with initial acceleration; nuclear propulsion system, I_{sp} = 820 sec; departure from circular Earth orbit, 485 km altitude.

Figure 6.- Concluded.

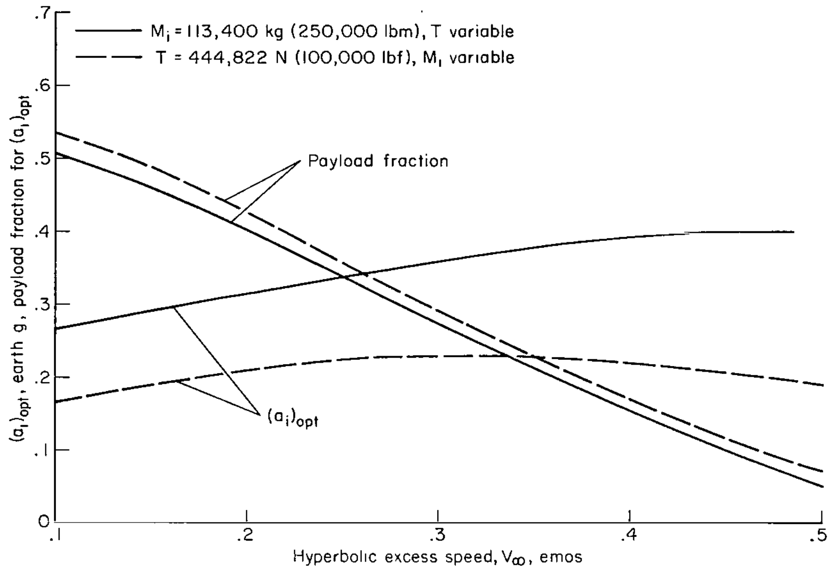


Figure 7.- Variations of optimum initial accelerations and associated payload fractions with hyperbolic excess speed for nuclear upper-stage propulsion system; Earth departure.

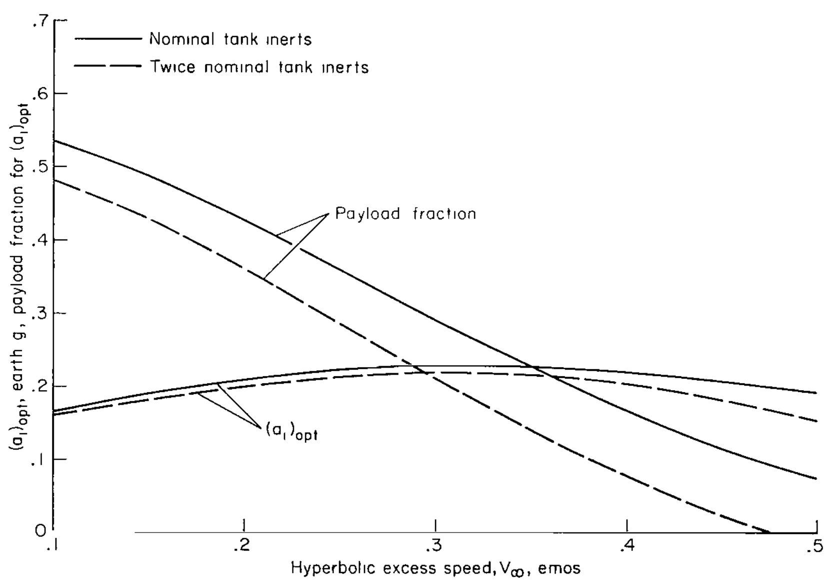


Figure 8.- Effect of inert mass on optimum initial acceleration and on optimum payload fraction during Earth departure with nuclear single stage; thrust = 444,822 N (100,000 lbf).

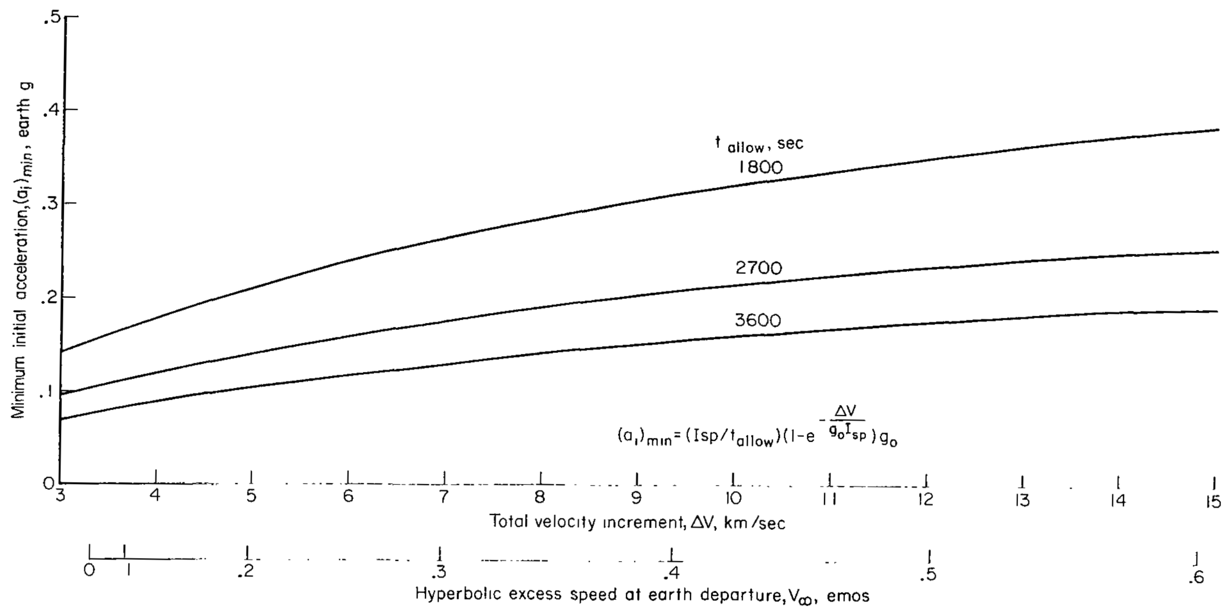
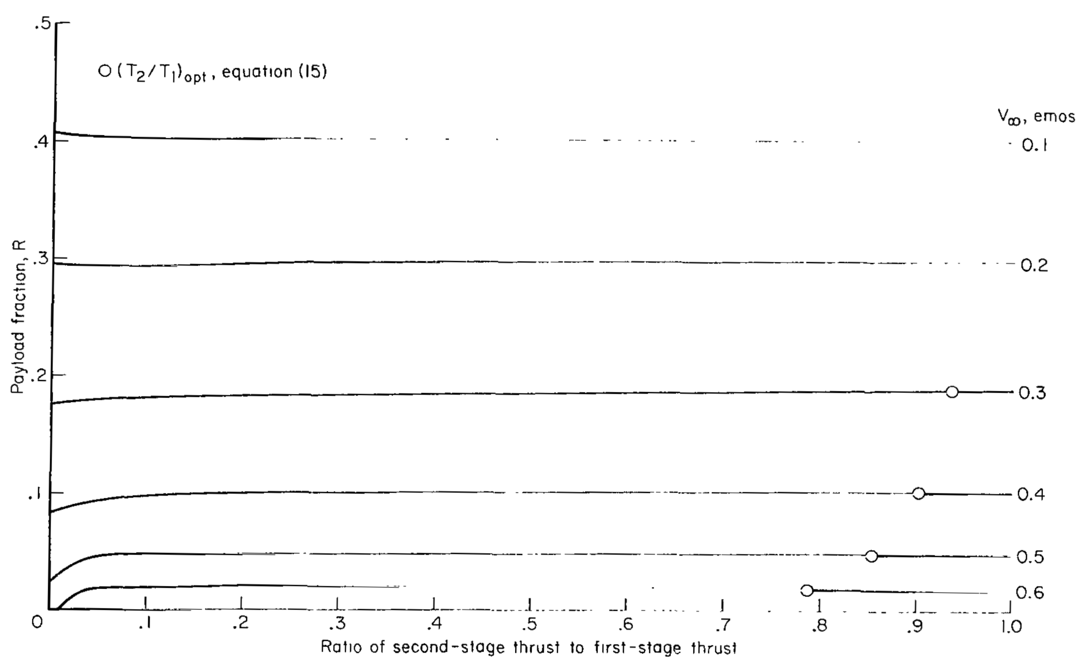
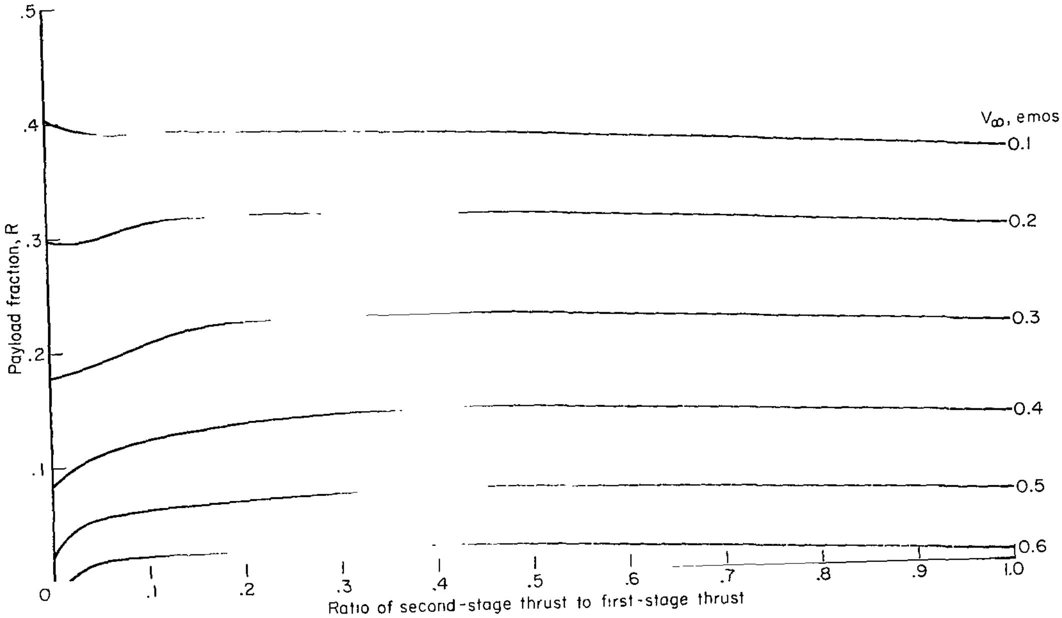


Figure 9.- Variation of minimum initial acceleration with ΔV for given allowable operating times of nuclear rocket engines; I_{sp} = 820 sec.



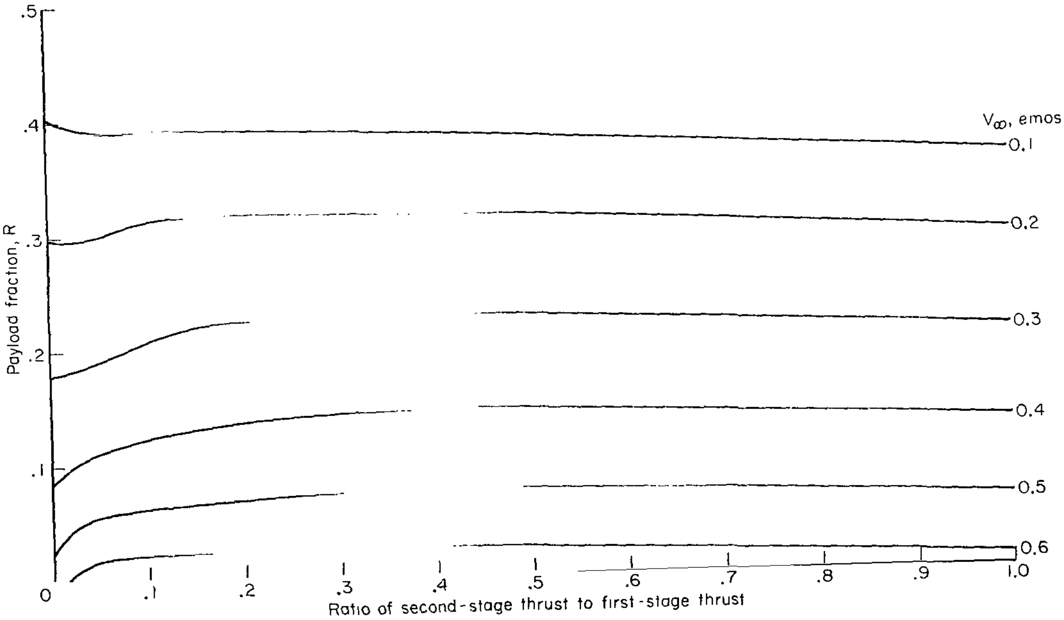
(a) Chemical stages, M_i = 226,795 kg, T_1 = 889,643 N.

Figure 10.- Effect of second-stage thrust level on performance of two-stage vehicle; Earth departure.



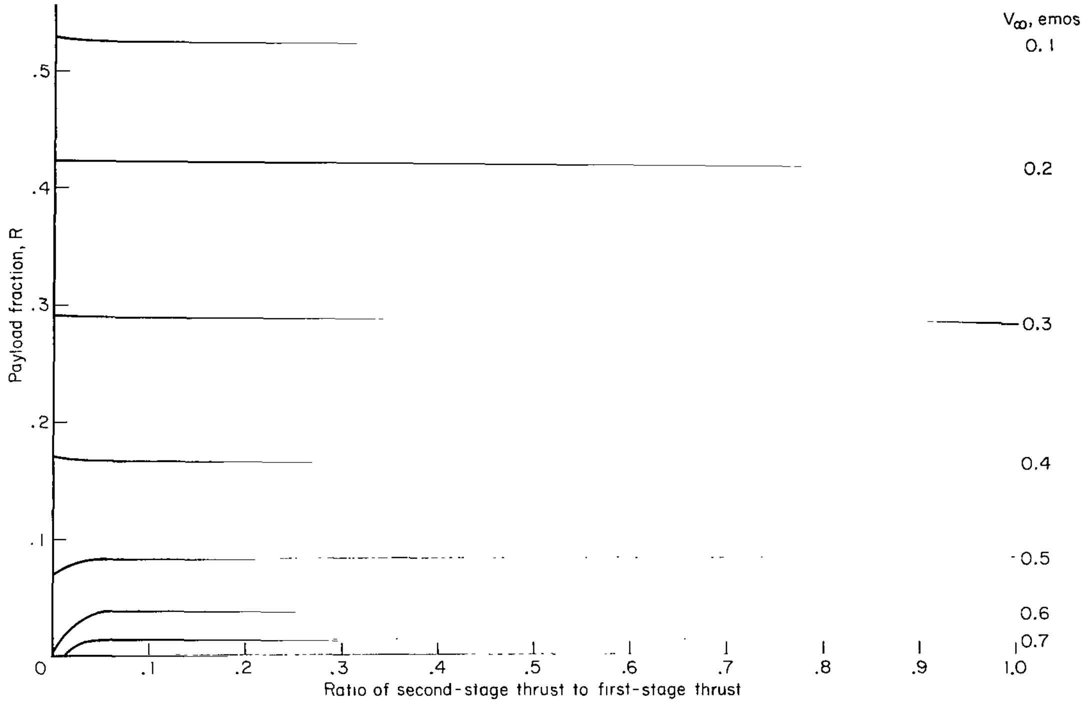
(b) Nuclear stages, M_i = 181,437 kg, T_1 = 444,822 N.

Figure 10.- Continued.



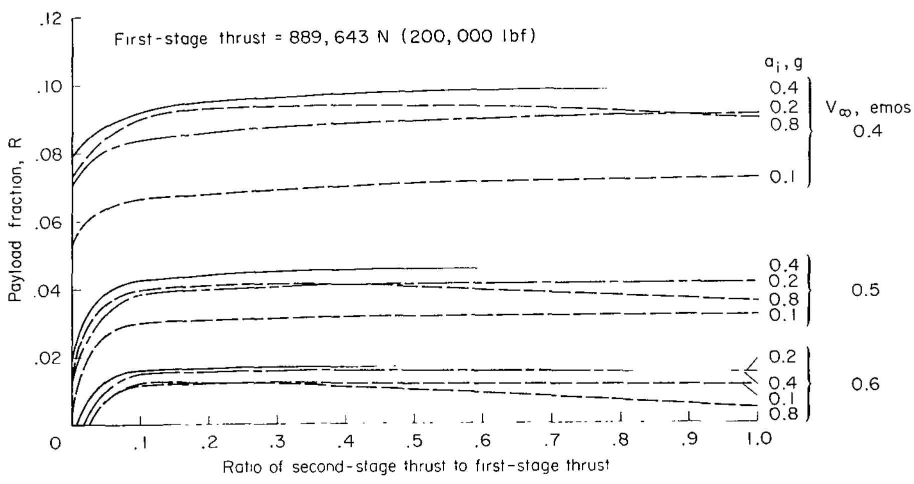
(c) Stage 1: Chemical, T_1 = 889,643 N; M_i = 226,796 kg.
Stage 2: Nuclear.

Figure 10.- Continued.



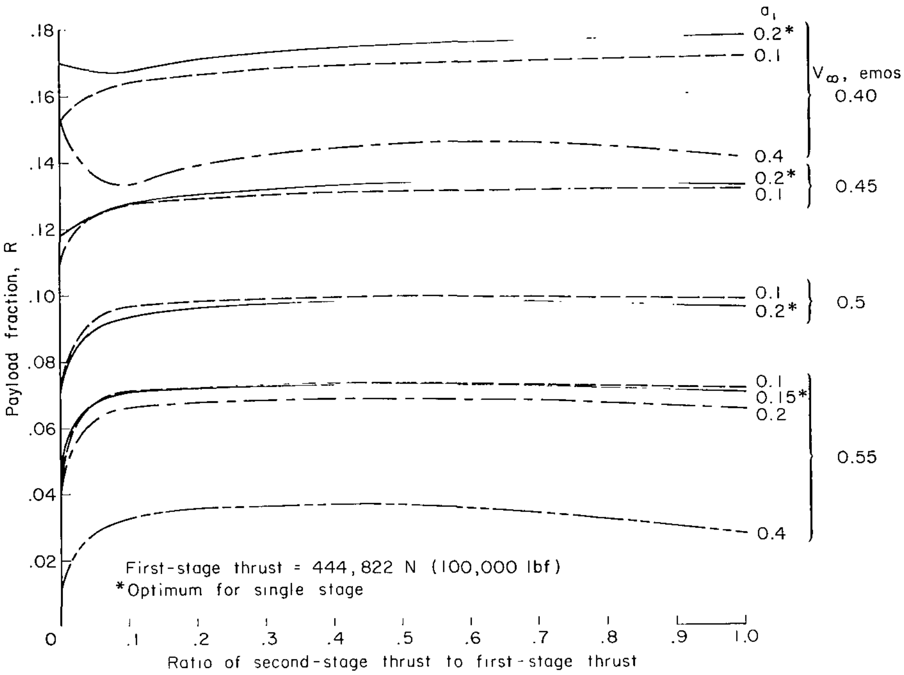
(d) Stage 1: Nuclear, T_1 = 444,822 N, M_i = 181,437 kg.
Stage 2: Chemical.

Figure 10.- Concluded.



(a) Chemical propulsion, I_{sp} = 450 sec.

Figure 11.- Effects of first-stage acceleration on overall performance of two stages during Earth escape.



(b) Nuclear propulsion, I_{sp} = 820 sec.

Figure 11.- Concluded.

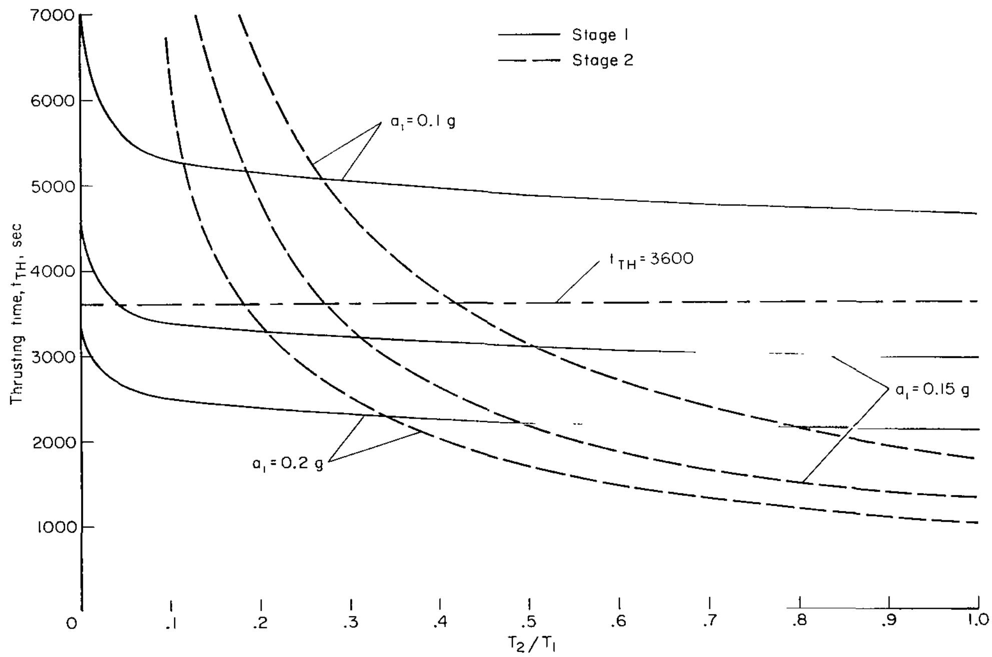
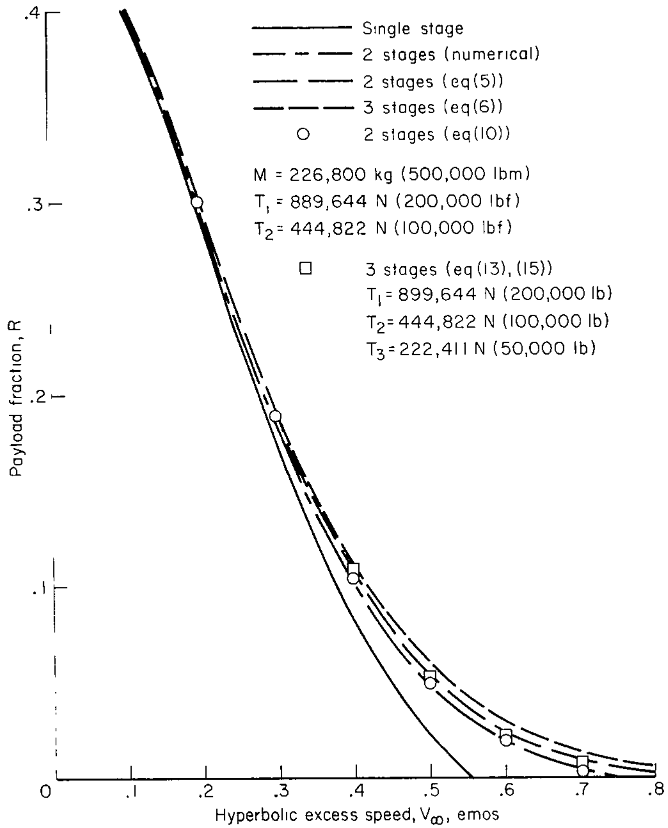
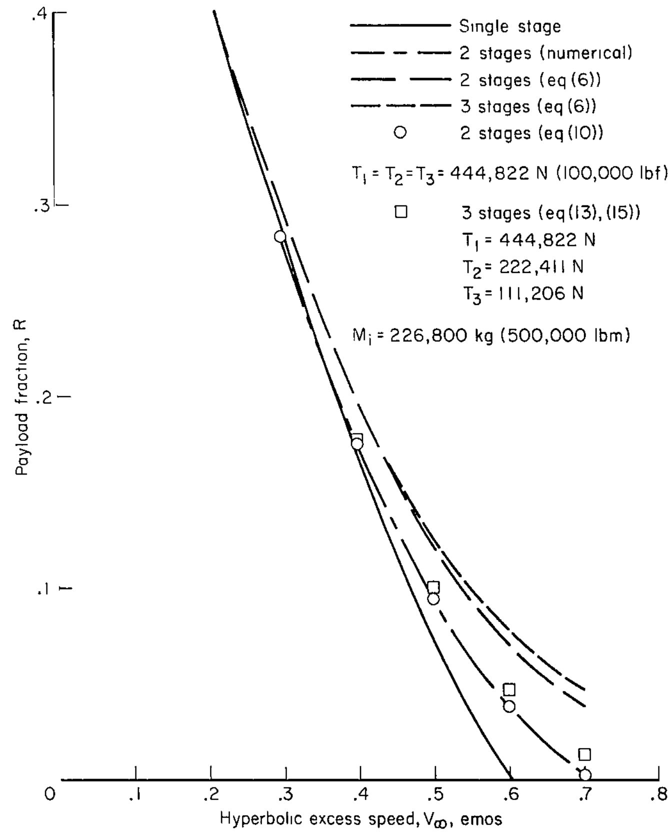


Figure 12.- Comparison of stage thrusting times for different first-stage initial accelerations; V_∞ = 0.55, T_1 = 444,822 N.



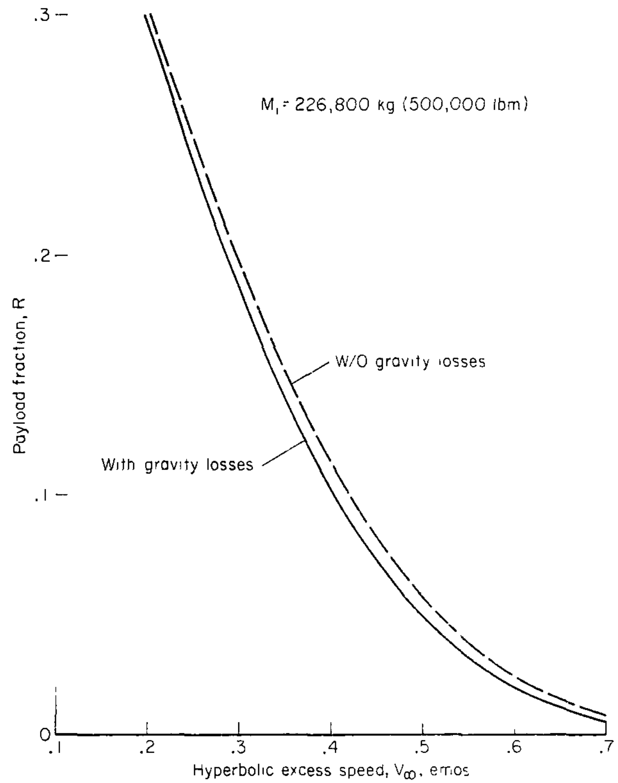
(a) Chemical stages, I_{sp} = 450 sec.

Figure 13.- Comparison of theoretical and numerical predictions of performance for similar propulsion stages.

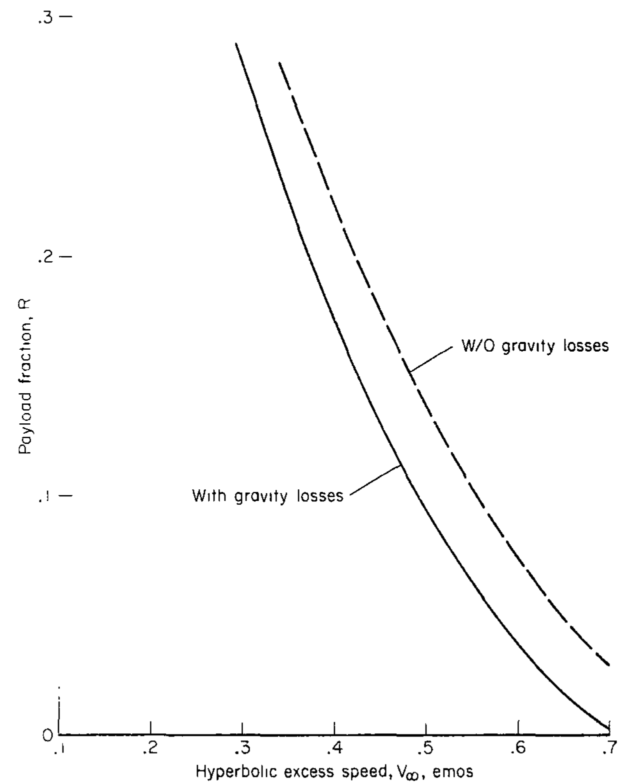


(b) Nuclear stages, I_{sp} = 820 sec.

Figure 13.- Concluded.



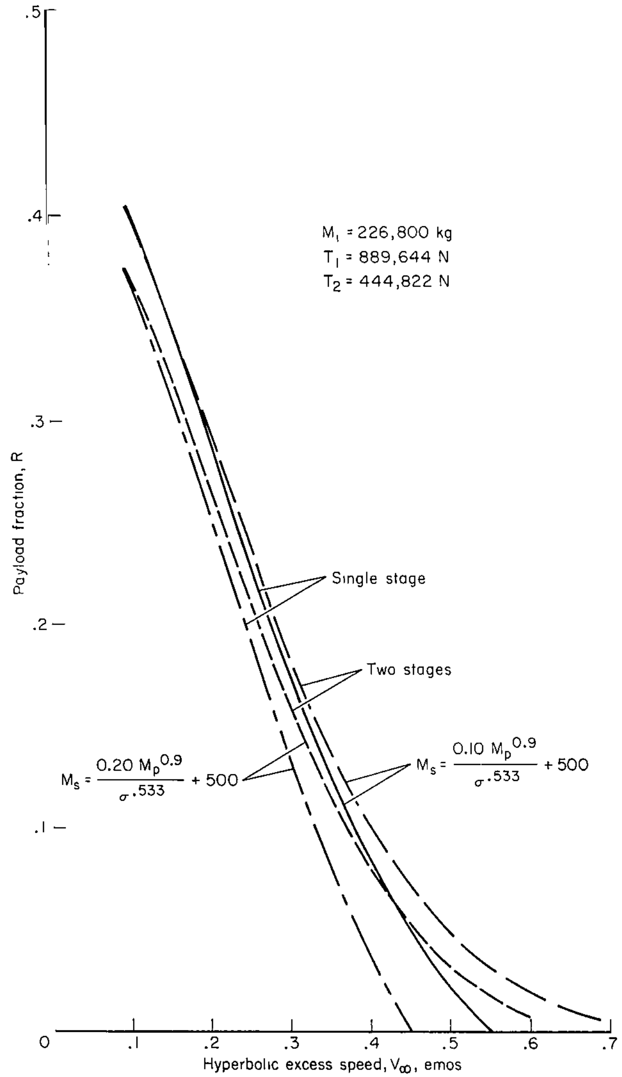
(a) Chemical stages, I_{sp} = 450 sec.



(b) Nuclear stages, I_{sp} = 820 sec.

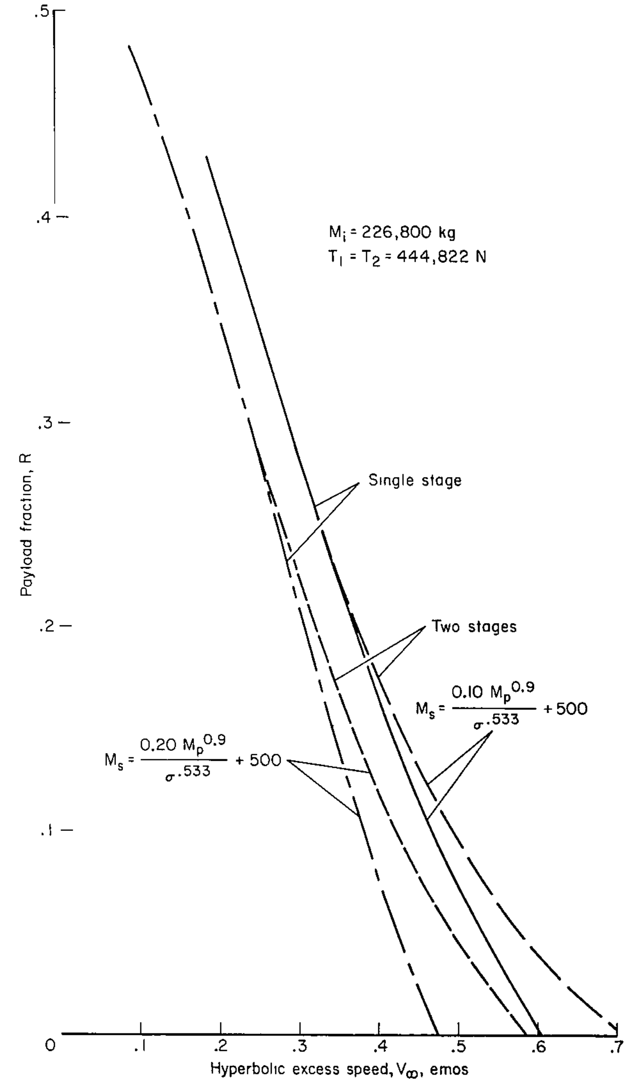
Figure 14.- Effect of gravity losses on theoretical prediction of performance of two stages; Earth escape.

Figure 14.- Concluded.



(a) Chemical propulsion.

Figure 15.- Effect of magnitude of propellant module mass on effectiveness of staging; Earth departure.



(b) Nuclear propulsion.

Figure 15.- Concluded.

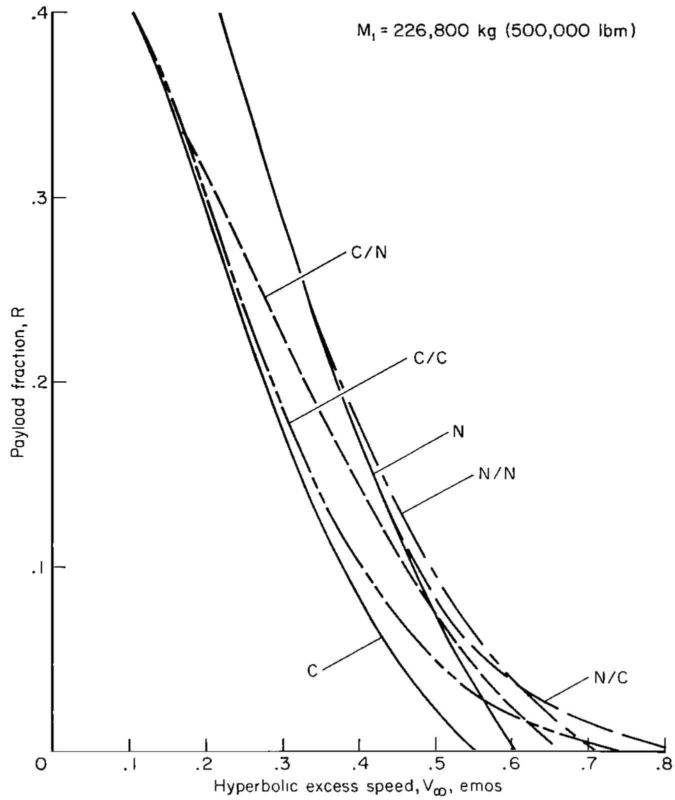


Figure 16.- Comparison of performances of similar and dissimilar two-stage propulsion systems duing Earth departure.

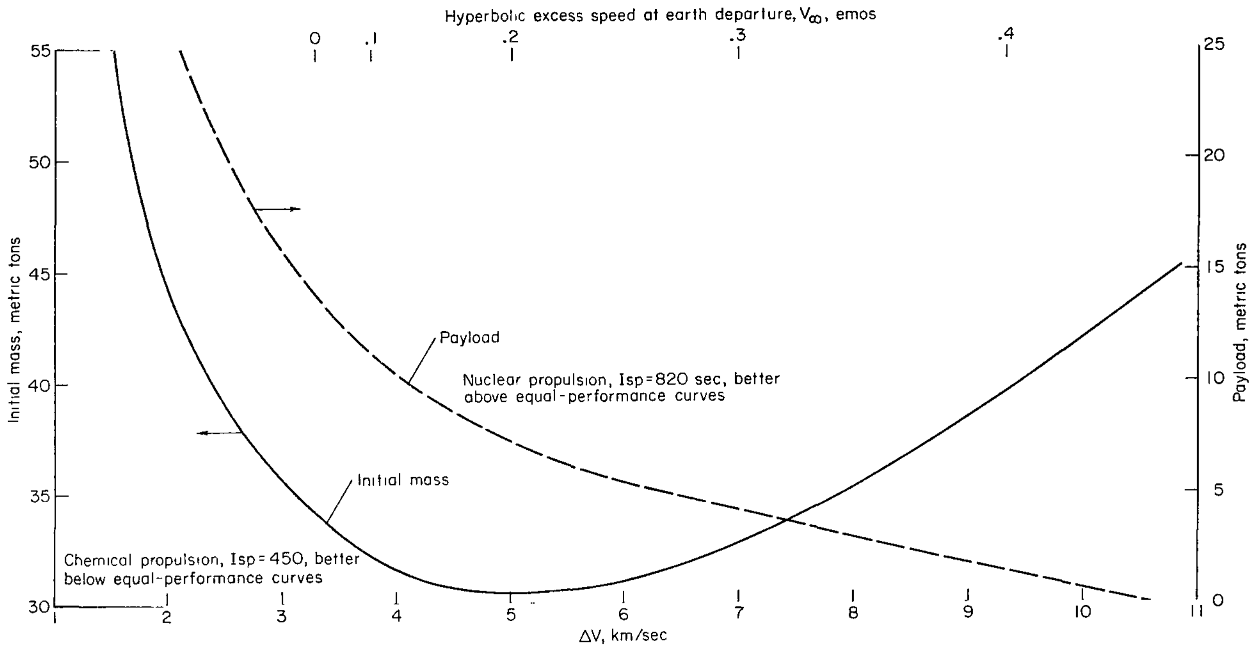


Figure 17.- Curves of equal performance of chemical and nuclear propulsion systems as a function of velocity increment ΔV and of initial mass in Earth orbit or payload mass.

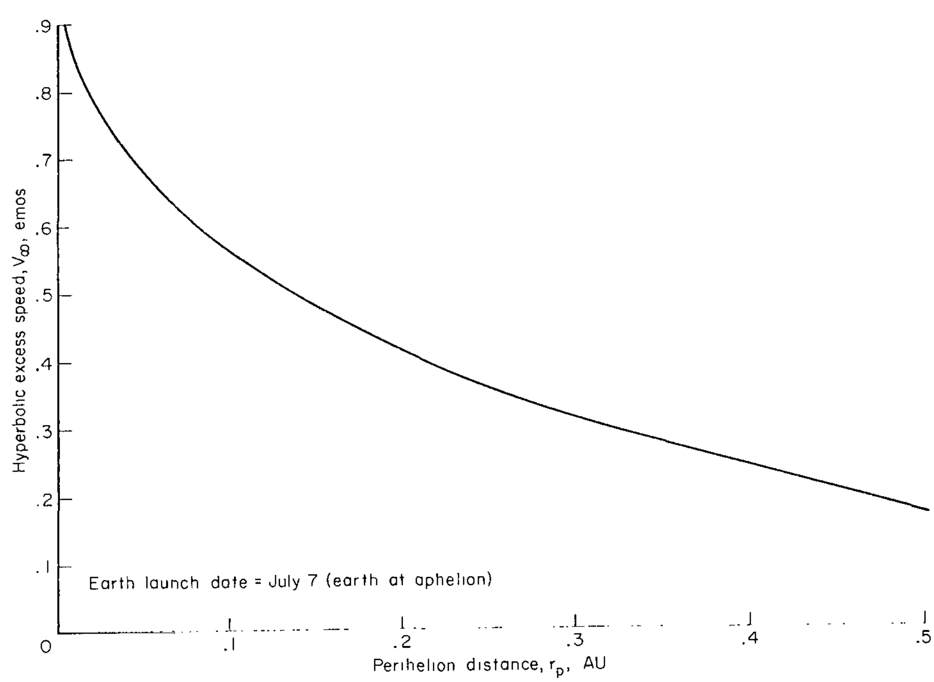


Figure 18.- Variation of hyperbolic excess speed with perihelion distance of solar probes.

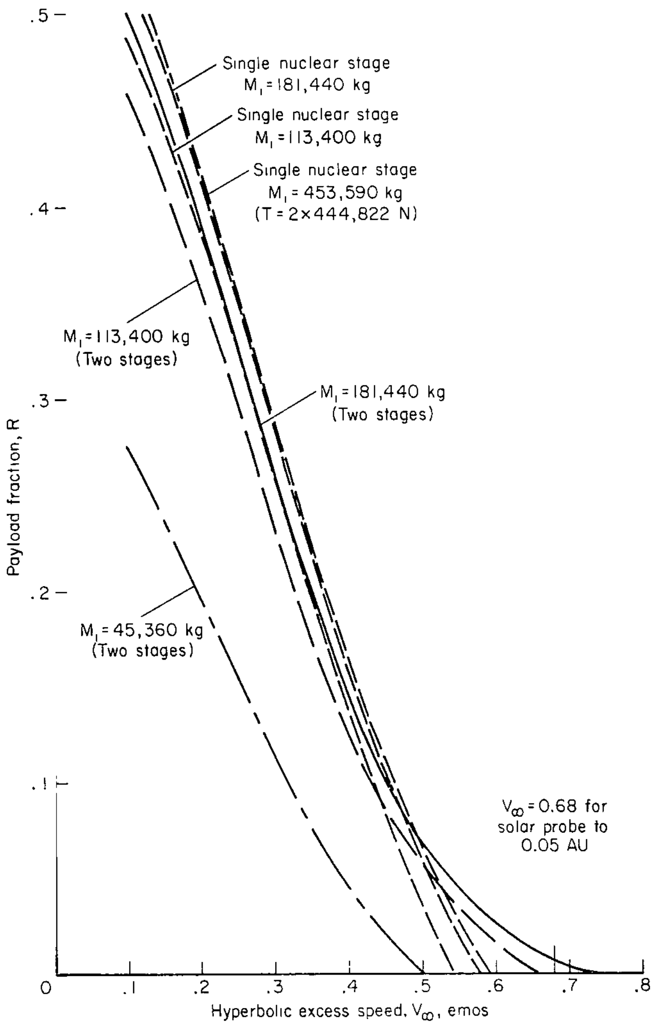


Figure 19.- Performance of nuclear first stage and chemical second stage in departing from an Earth orbit (altitude = 485 km).

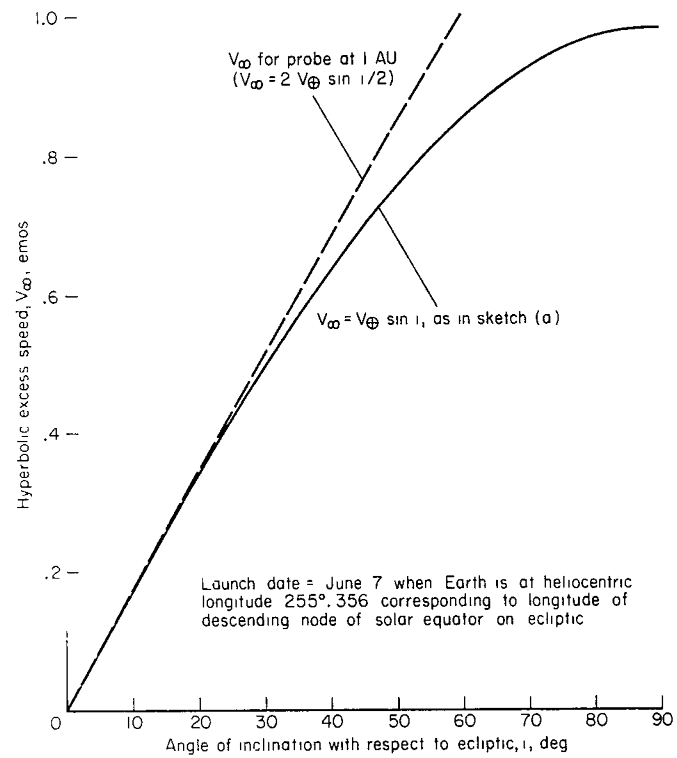
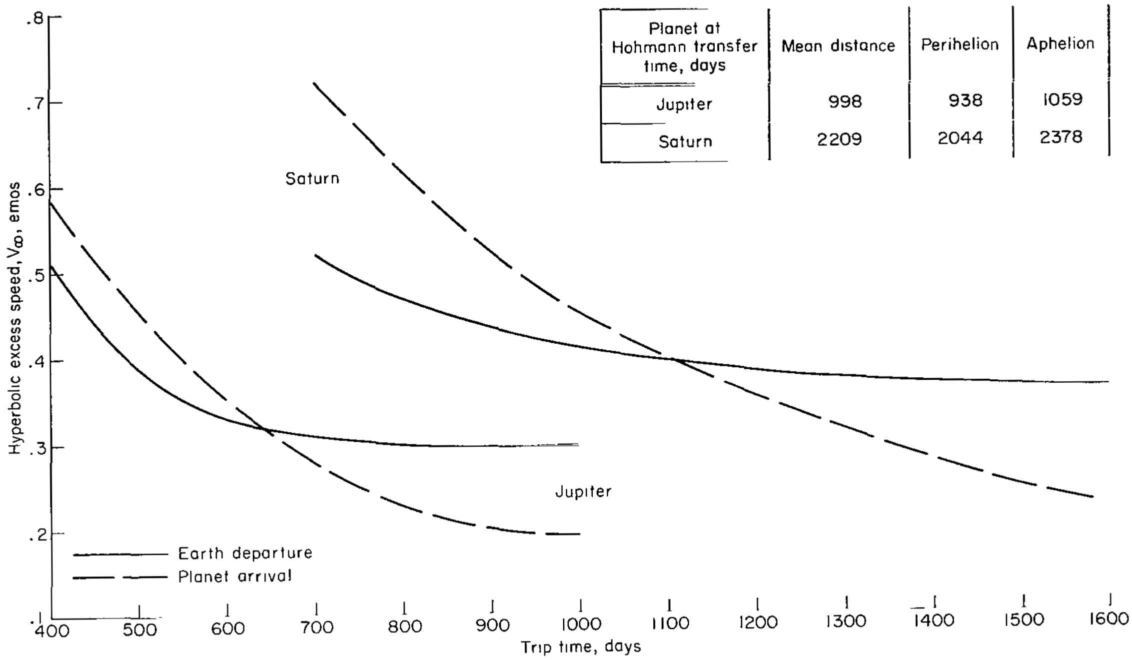


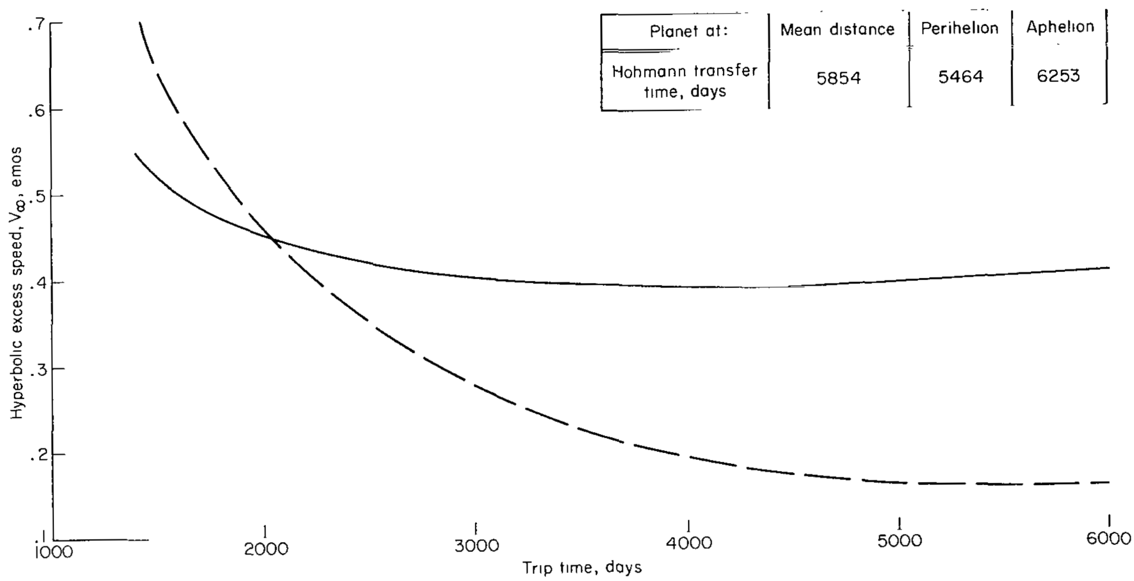
Figure 20.- Variation of hyperbolic excess speed with angle of inclination of heliocentric orbit with respect to ecliptic.



Planet at Hohmann transfer time, days	Mean distance	Perihelion	Aphelion
Jupiter	998	938	1059
Saturn	2209	2044	2378

(a) Missions to Jupiter and Saturn.

Figure 21.- Variation of hyperbolic excess speeds with trip time for orbiter missions.



Planet at:	Mean distance	Perihelion	Aphelion
Hohmann transfer time, days	5854	5464	6253

(b) Mission to Uranus.

Figure 21.- Concluded.

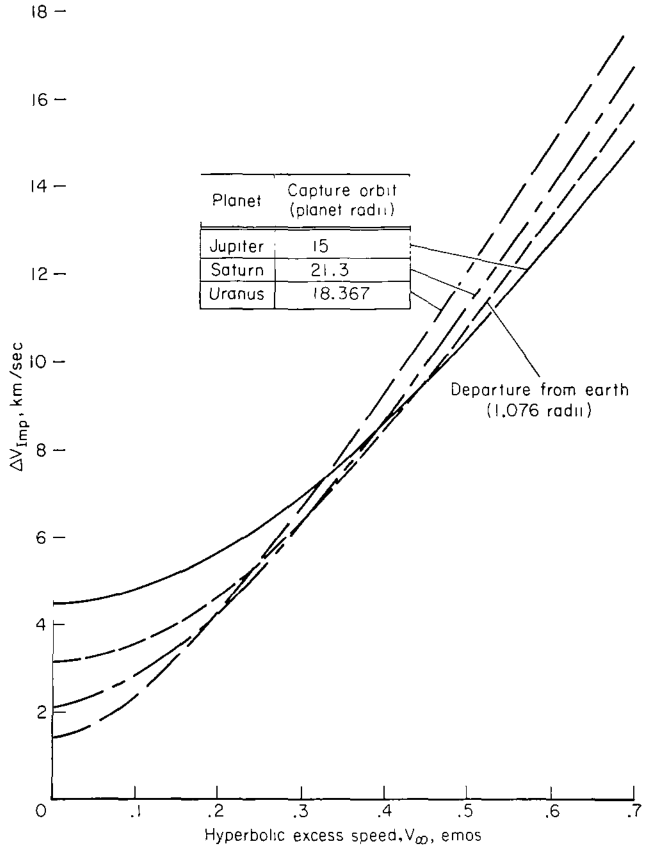


Figure 22.- Variation of impulsive ΔV with hyperbolic excess speed at arrival at outer planets.

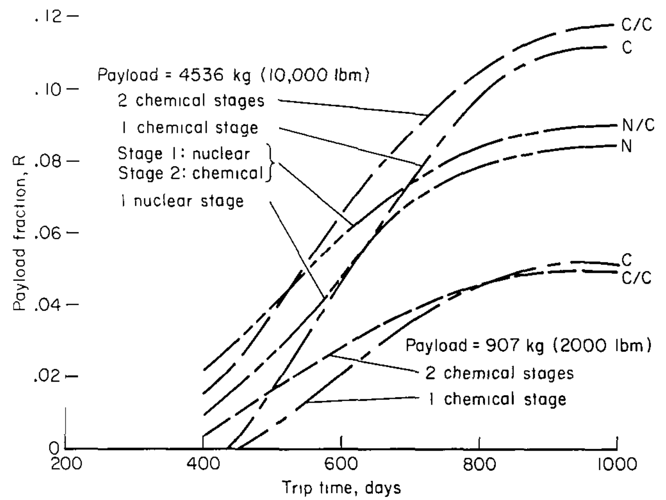


Figure 23.- Performance of various propulsion systems in achieving capture into circular orbit about Jupiter.

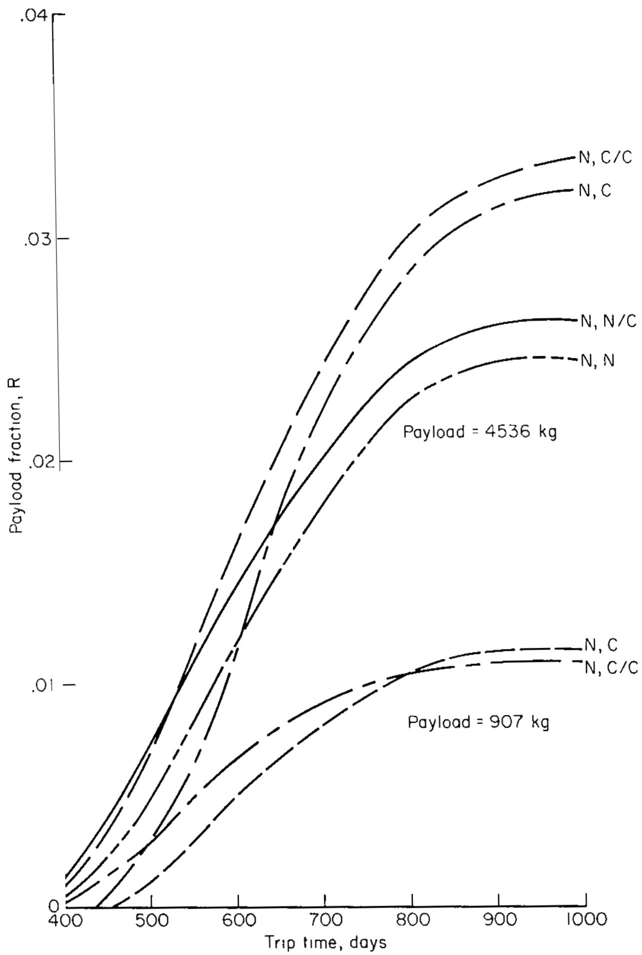
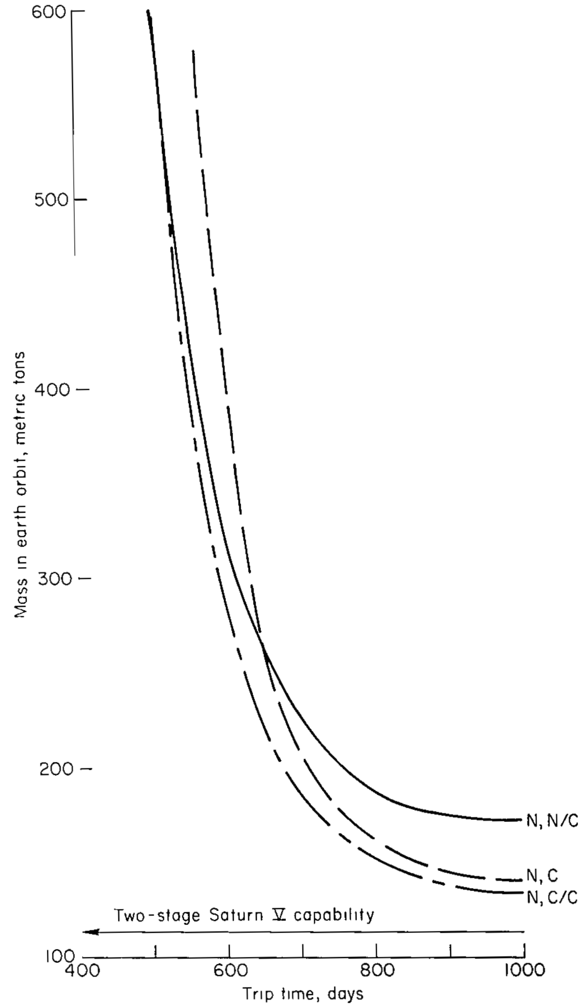
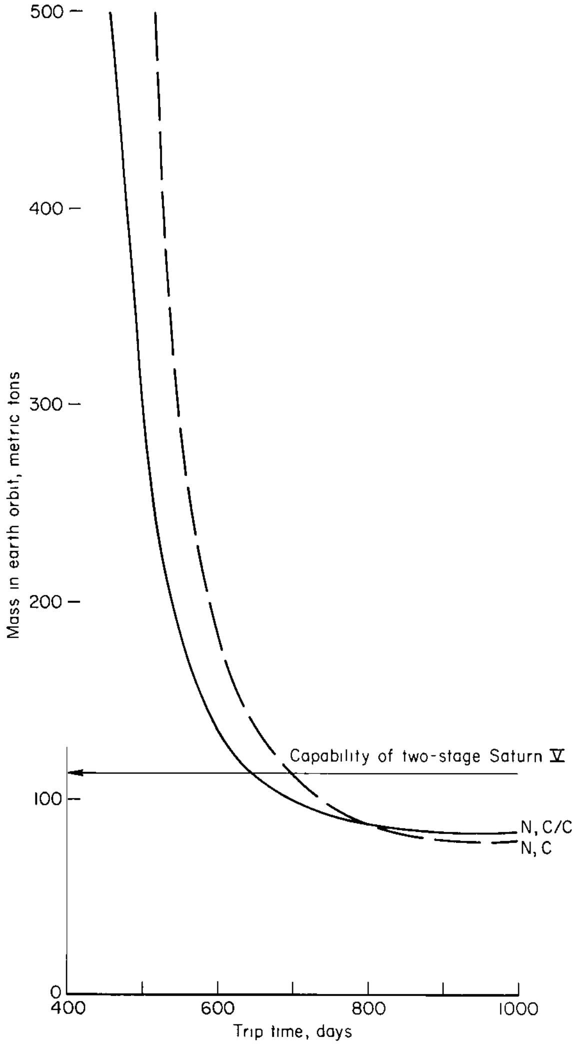


Figure 24.- Performance of various propulsion systems used to deliver unmanned payloads to orbit about Jupiter from an orbit about Earth.



(a) Payload = 4,536 kg (10,000 lbm).

Figure 25.- Requirements for mass in Earth orbit for Jupiter orbiter mission.



(b) Payload = 907 kg (2000 lbm).

Figure 25.- Concluded.

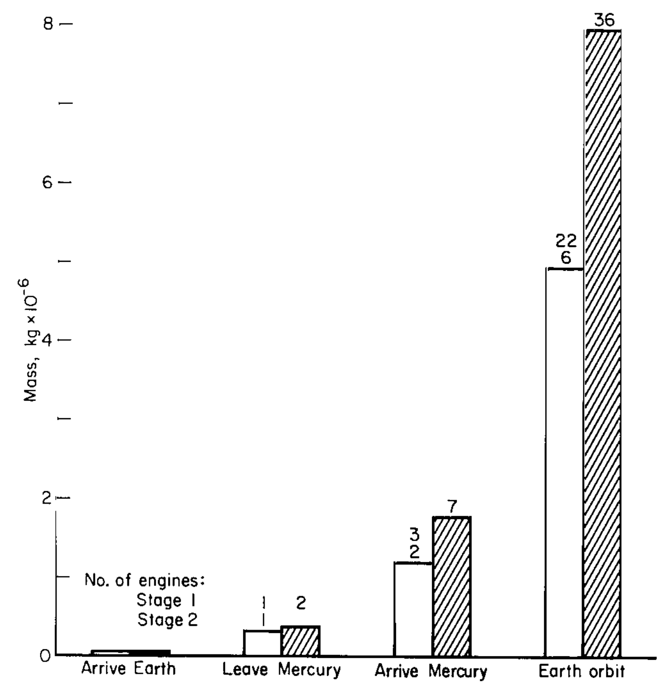


Figure 26.- Effectiveness of staging in a manned orbiter mission to Mercury with nuclear propulsion.

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