AM-BASEBAND TELEMETRY SYSTEMS

Volume 4: Problems Relating to AM-Baseband Systems

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COMMUNICATION SYSTEMS GROUP

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UNIVERSITY OF ALABAMA UNIVERSITY, ALABAMA

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ABSTRACT

An analysis is performed to determine the effect of channel carrier phases on the crest factor of an AM-baseband signal for dc and random modulation. Also, the manner in which a DSB signal is distorted when it drifts within the passband of a bandpass filter is investigated for a sinusoidal modulating signal. The effect of demodulation phase errors is studied for SSB, DSB, and quadrature DSB (QDSB) systems having random modulation. Finally, the effect of intermodulation distortion is studied for an AM-baseband, and the carrier phases which minimize the effect of intermodulation distortion are determined.

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LIST OF SYMBOLS

$e_{DSB}(t)$	DSB signal
$e_{SSB}(t)$	SSB signal
$e_Q(t)$	quadrature DSB signal
$m(t)$	modulating signal
$m(t)$	Hilbert transform of m(t)
$E(t)$	error signal
$E_N(t)$	normalized mean-square error
$\phi(t)$	demodulation carrier phase error
$e_c(t)$	demodulation carrier
$e_d(t)$	demodulated output
$e_{in}(t)$	input signal
ω_n	carrier frequency
ω_x	filter center frequency
ω_m	modulating frequency
ω_ε	error frequency
$\phi(\omega)$	phase characteristic
$\phi_L(\omega)$	linear portion of phase characteristic
$\phi_N(\omega)$	nonlinear portion of phase characteristic
S	slope of phase characteristic
M_U	magnitude of upper sideband component
M_L	magnitude of lower sideband component
b_n	amplitude of Channel n carrier

ϕ_k^n	phase of the k^{th} component on the Channel n data signal
ψ_n	phase of Channel n carrier
f_n	Channel n frequency
$M^n(t)$	Channel n modulating signal
$S(t)$	baseband signal
$S'(t)$	scaled baseband signal
$x(t)$	general input
$y(t)$	general output
ω_A, ω_B	modulating frequencies
a_k	coefficient of $x^k(t)$ in $y(t)$
α, β	carrier phases
$d(t)$	intermodulation distortion
$A(t)$	envelope signal
$\phi(t)$	phase perturbation
$e'_n(t)$	Channel n signal
k	index of summation
$\Phi_M(f)$	power spectrum of modulation
$\Phi_{BB}(f)$	baseband power density spectrum
z	crest factor
$e_n(t)$	Channel n carrier
$e_{sc}(t)$	synthesized demodulation carrier
n	channel number
$e_{out}(t)$	output signal
$m_1(t)$, $m_2(t)$	modulating signals in QDSB system

$\varepsilon(t)$	phase error of quadrature demodulation carrier
$e_{id}(t)$	intermodulation distortion signal on a channel
γ	reference phase of intermodulation carrier

I. INTRODUCTION

Previous reports[1,2,3] have dealt with carrier synthesis from a common pilot, carrier synthesis from AM modulated carriers and automatic gain control. There are problems of interest in AM-baseband systems which have not been investigated in these reports. These are the crest factor of a multiplex of DSB signals, carrier drift, dynamic phase errors of the demodulation carrier, and intermodulation distortion. These four topics are investigated in this report.

When the demodulation carriers in an AM-baseband system are derived from a common pilot, the carriers must be harmonically related. The manner in which this affects the crest factor of a multiplex of DSB channels is studied for random and dc modulation, for a condition with and without pre-emphasis, and for several carrier phases.

The problem of carrier drift is extremely difficult to solve analytically. Passing a DSB signal through a filter with an amplitude and phase characteristic,which is non-symmetrical with respect to the DSB signal, results in an output which cannot generally be written in closed form. The analysis is performed for a sinusoidal modulating signal to illustrate the manner in which the signal is distorted by the filter. If results are required for a particular filter, the derived expression may be programmed with the parameters of the filter of interest.

The effect of dynamic phase errors in the demodulation carrier is investigated for random signals. The analysis is performed for DSB, SSB, and quadrature DSB (QDSB) systems.

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[1] Superscripts refer to numbered references.

The effect of intermodulation distortion is determined for DSB systems having sinusoidal modulation on harmonically related carriers. The results are generalized to random modulation, and the carrier phases which minimize the effect of intermodulation on carrier synthesis are determined.

II. CREST FACTOR OF A MULTIPLEX OF DSB SIGNALS

In a DSB/FM telemetry system, the crest factor (ratio of the peak value to the rms value) of the signal used to deviate the FM transmitter, i.e., the baseband signal, is an important parameter. Good performance of the telemetry link requires that the baseband crest factor be low. This follows from the fact that the signal-to-noise ratio at the FM receiver output is proportional to the mean-square value of transmitter deviation.[4] If the crest factor is high the transmitter sensitivity must be sufficiently low so that the signal peaks do not deviate the transmitter beyond its linear region. This causes the deviation to be relatively low in the interval between peaks with a resulting loss in signal-to-noise ratio.

The value of crest factor is determined for several values of carrier phase in this section. The peak value of the baseband signal is defined as that value which the absolute value of the baseband exceeds 1% of the time. Pre-emphasis and the case of dc modulation are also considered.

A. Baseband Structure

The assumed baseband is generated by translating 15 different data spectra to the appropriate baseband locations using 15 harmonically related carriers. The frequency of the Channel n carrier is denoted by f_n, where

$$f_n = nf_1 \quad . \tag{2.1}$$

The Channel n carrier is thus represented by

$$e_n(t) = b_n \cos (2\pi f_n t + \psi_n) \; , \tag{2.2}$$

or

$$e_n(t) = b_n \cos(2\pi n f_1 t + \psi_n) , \tag{2.3}$$

where b_n is the amplitude and ψ_n is the initial phase of the Channel n carrier.

The modulation will be represented by 10 sinusoidal components, each having a random initial phase. The assumed modulation bandwidth is $\frac{1}{4} f_1$. Therefore, the modulation on Channel n, $M^n(t)$, can be represented as

$$M^n(t) = \sum_{k=1}^{10} \sin\left[\frac{2\pi}{10} (\frac{1}{4} f_1) kt + \phi_k^n\right] \tag{2.4}$$

or

$$M^n(t) = \sum_{k=1}^{10} \sin\left[(0.025) 2\pi f_1 kt + \phi_k^n\right] , \tag{2.5}$$

where ϕ_k^n represents random initial phases. Thus, by choosing a different set of random initial phases for each n, the modulating signals for different channels can be made independent.

Multiplying $e_n(t)$ and $M^n(t)$, yields the Channel n signal, $e_n'(t)$.. Thus

$$e_n'(t) = \sum_{k=1}^{10} \sin\left[(0.025) 2\pi f_1 kt + \phi_k^n\right] b_n \cos\left[2\pi f_1 nt + \psi_n\right] . \tag{2.6}$$

The composite baseband signal, S(t), is the sum of all such signals or

$$S(t) = \sum_{n=1}^{15} e_n'(t) ,$$

which is

$$S(t) = \sum_{n=1}^{15} \sum_{k=1}^{10} \sin\left[(0.025)2\pi f_1 kt + \phi_k^n\right] b_n \cos\left[2\pi f_1 nt + \psi_n\right] \quad .(2.7)$$

This is the signal for which the crest factor will be determined.

Any desired pre-emphasis taper can be considered by appropriately adjusting the values of the b_n's. The pre-emphasis taper used is one in which a breakpoint is located between Channels 6 and 7. This is illustrated in Figure 2-1 for f_1 = 4kHz. The corresponding values of b_n are given in Table 2-1.

Channel No. (n)	Value of b_n
1	1.0
2	1.0
3	1.0
4	1.0
5	1.0
6	1.0
7	1.0116
8	1.288
9	1.445
10	1.604
11	1.756
12	1.926
13	2.09
14	2.24
15	2.40

Table 2-1. Values of b_n Used for Pre-emphasis

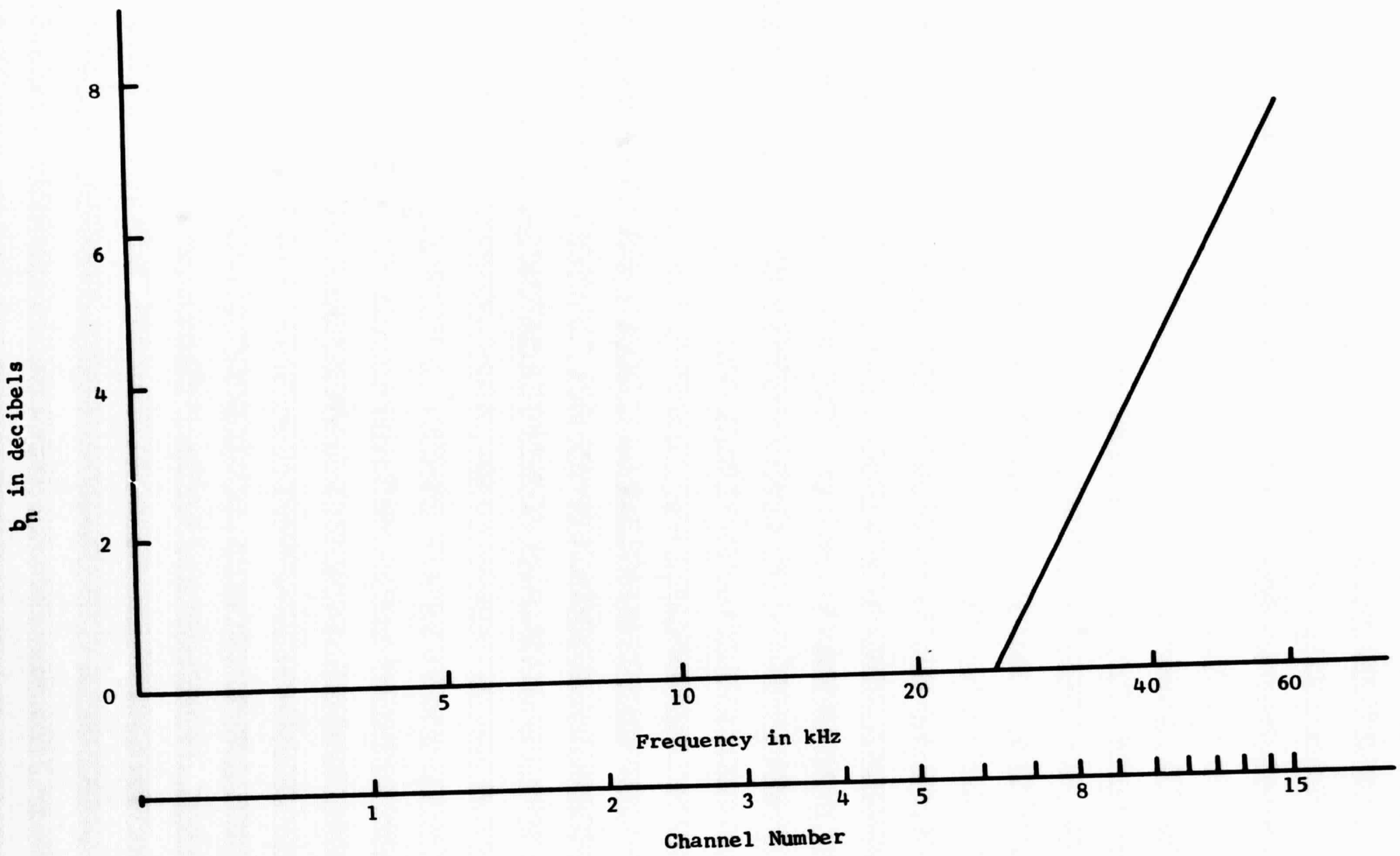

Figure 2-1 Pre-emphasis Taper

B. Determination of the Crest Factor

The easiest method of determining the crest factor of the baseband signal, S(t), as given by (2.7) is by using a digital computer. The function S(t) may be sampled by evaluating it for particular values of t and from the samples the rms value of S(t) can be determined. This value is divided into the peak value to give the crest factor. In order to do this easily, Equation (2.7) is first scaled by letting f_1 = 1 Hz. This yields

$$S'(t) = \sum_{n=1}^{15} \sum_{k=1}^{10} \sin\left[(0.025)2\pi kt + \phi_k^n\right] b_n \cos\left[2\pi nt + \psi_n\right]. \qquad (2.8)$$

The modulation spectrum for each scaled channel is shown in Figure 2-2 and the resulting normalized baseband spectrum in Figure 2-3. From Figure 2-3, it is evident that the highest frequency in the baseband signal, S'(t) is 15.25 Hz. Also, every frequency in the baseband is a multiple of 0.025 Hz. Thus, S'(t) is periodic with a period of 40 sec. Since the highest frequency in the baseband is the 610th harmonic of the fundamental frequency, 0.025 Hz, taking 6100 equally distributed samples over the 40 sec period allows S'(t) to be sampled at 10 times the highest frequency.

The purpose of the investigation is to determine the effect of the carrier initial phases, ψ_n. Therefore, three cases will be considered, These are:

Case 1	$\psi_n = 0$	all n
Case 2	$\psi_n = 0$	n odd
	$\psi_n = \pi$	n even
Case 3	ψ_n randomly distributed with a uniform distribution function between 0 and 2π.	

$$M^n(t) = \text{Channel } n \text{ Modulating Signal}$$

$$= \sum_{k=1}^{10} \sin\left[2\pi(0.025)k\,t + \phi_k^n\right]$$

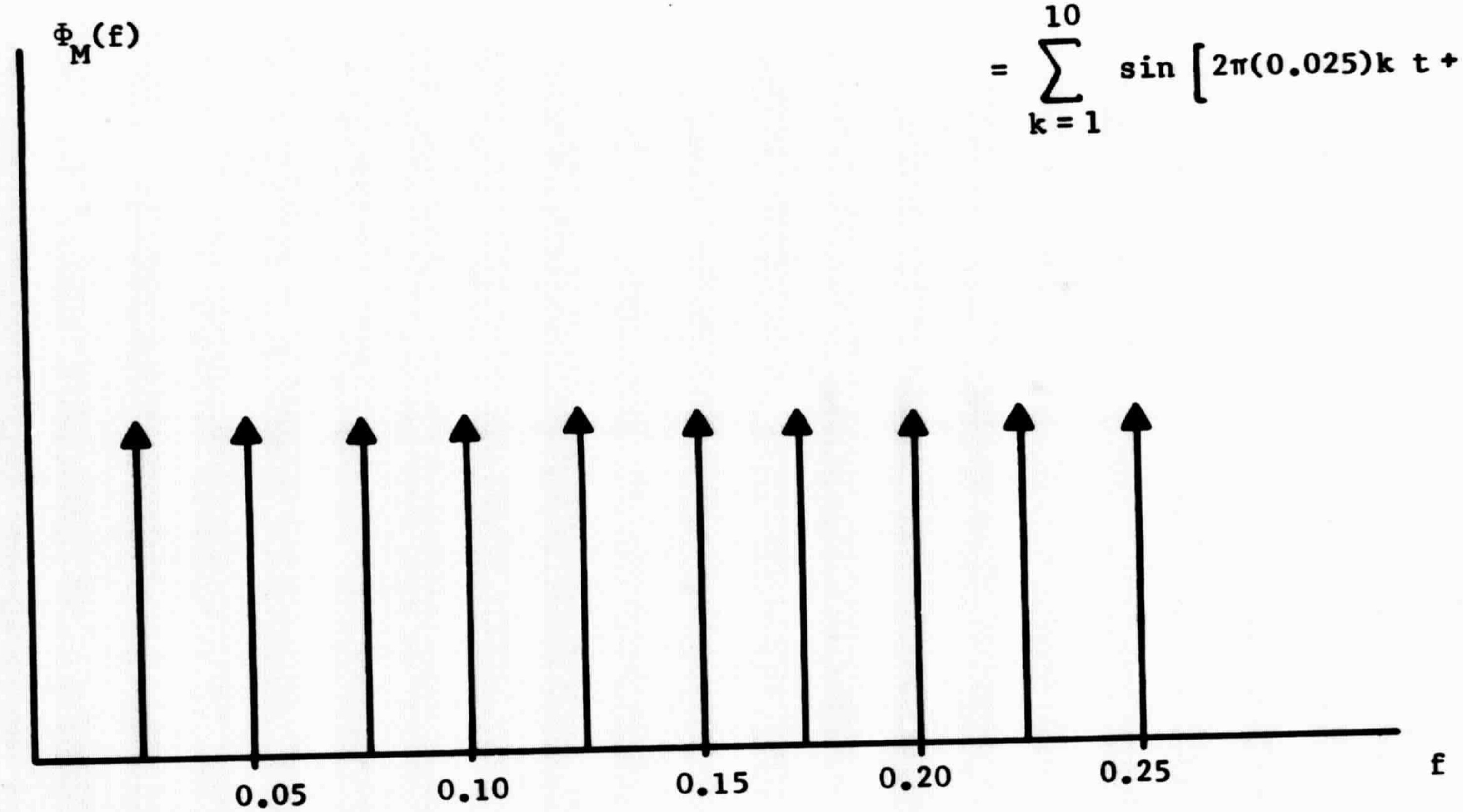

Figure 2-2 Normalized Modulation Spectrum

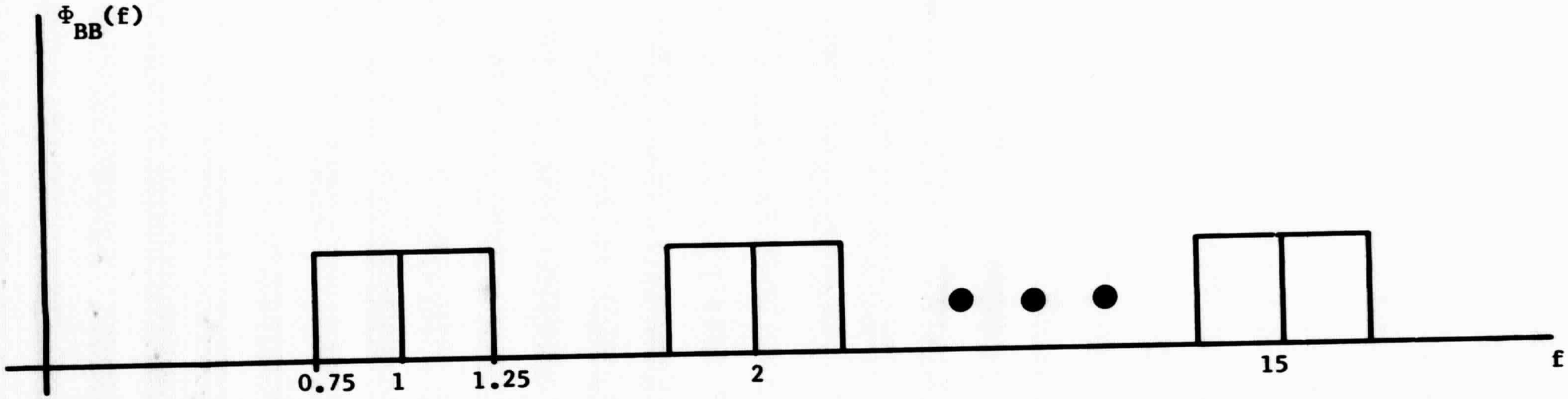

Figure 2-3 Normalized Baseband Spectrum

The effect of pre-emphasis and dc modulation are also considered. To determine the effect of pre-emphasis, data is run for the values of b_n given in Table 1 and then for $b_n = 1$ (all n). The effect of dc modulation on all channels can be determined by letting $M^n(t)$ be unity for all n. This yields

$$S'(t) = \sum_{n=1}^{15} b_n \cos(2\pi nt + \psi_n) \tag{2.9}$$

for the baseband signal.

C. Results

The results of this investigation are given in Table 2-2. The values given in the table represent an average determined from six computer runs. This was done to compensate round-off errors and initial phase effects. The conclusion is reached that where random modulation is present, the carrier phase is unimportant. This is expected since in a suppressed carrier system, the carrier phase is masked by the modulation, i.e., there is no component in the baseband spectrum having identically the channel carrier phase. It is also clear from the results that pre-emphasis has little effect on the crest factor.

For the random modulation case without pre-emphasis, the theoretical value of the crest factor is approximately 2.58. This follows from the fact that S(t) will have a distribution very close to a Gaussian one since S(t) will be the sum of a large number of statistically independent sinusoids. With the crest factor defined as that value exceeded by S(t) 1% of the time, assuming a Gaussian distribution yields

$$\frac{1}{\sqrt{2\pi}} \int_{-z}^{z} \exp\left[-\frac{t^2}{2}\right] dt = 0.99$$

Channel Modulation	Case	Ratio of Peak to RMS	
		With Pre-Emphasis	Without Pre-Emphasis
Random Modulation on all Channels	$\psi_n = 0$ all n	2.62	2.58
	$\psi_n = \begin{cases} 0 & n \text{ odd} \\ \pi & n \text{ even} \end{cases}$	2.65	2.63
	ψ_n Random	2.65	2.58
Dc Modulation on all Channels	$\psi_n = 0$ all n	4.72	5.10
	$\psi_n = \begin{cases} 0 & n \text{ odd} \\ \pi & n \text{ even} \end{cases}$	4.35	4.72
	ψ_n Random	2.40	2.55

Peak = value which $|S'(t)|$ exceeds 1% of the time.

Table 2-2 Table of Results

or

$$\frac{1}{\sqrt{2\pi}} \int_0^z \exp\left[-\frac{t^2}{2}\right] dt = 0.495 ,$$

where z is the crest factor. The value of z is found from a normal probability table to be 2.58, a value agreeing well with the experimental results.

For the case of dc modulation, the modulation was assumed to be 1 unit in amplitude and, as expected, the crest factor is a strong function of carrier phase.

III. EFFECT OF CARRIER DRIFT

When a DSB demodulator is preceded by a bandpass filter, distortion results if the filter characteristic is not symmetrical, in both amplitude and phase, with respect to the channel carrier frequency. Non-symmetry can result if the channel carrier frequency should drift due to a frequency drift in the airborne oscillator from which the channel carrier is derived. Such a drift, even though small, can be significant. For example, if a 60 kHz carrier should drift only 0.5%, the resulting carrier shift is 300 Hz, which is approximately 10% of the channel bandwidth for a typical wideband channel. The effect of this carrier drift in terms of waveform distortion can be determined analytically for the case of sinusoidal modulation.

In determining the effect of carrier drift, the filter phase characteristic will first be considered independent of the amplitude characteristic. The result will then be modified to account for the effect of amplitude.

A. Carrier Centered in Filter

Consider the input to the filter to be the DSB signal given by

$$e_{in}(t) = \cos(\omega_n + \omega_m)t + \cos(\omega_n - \omega_m)t \ . \qquad (3.1)$$

Figure 3-1 illustrates the assumed filter phase characteristic with the carrier frequency equal to the center frequency of the filter, ω_x. This condition, $\omega_n = \omega_x$, results in the phase shift of the upper sideband, $\phi(\omega_n + \omega_m)$, being equal in magnitude and opposite in sign to the phase shift of the lower sideband, $\phi(\omega_n - \omega_m)$. This means that the sideband

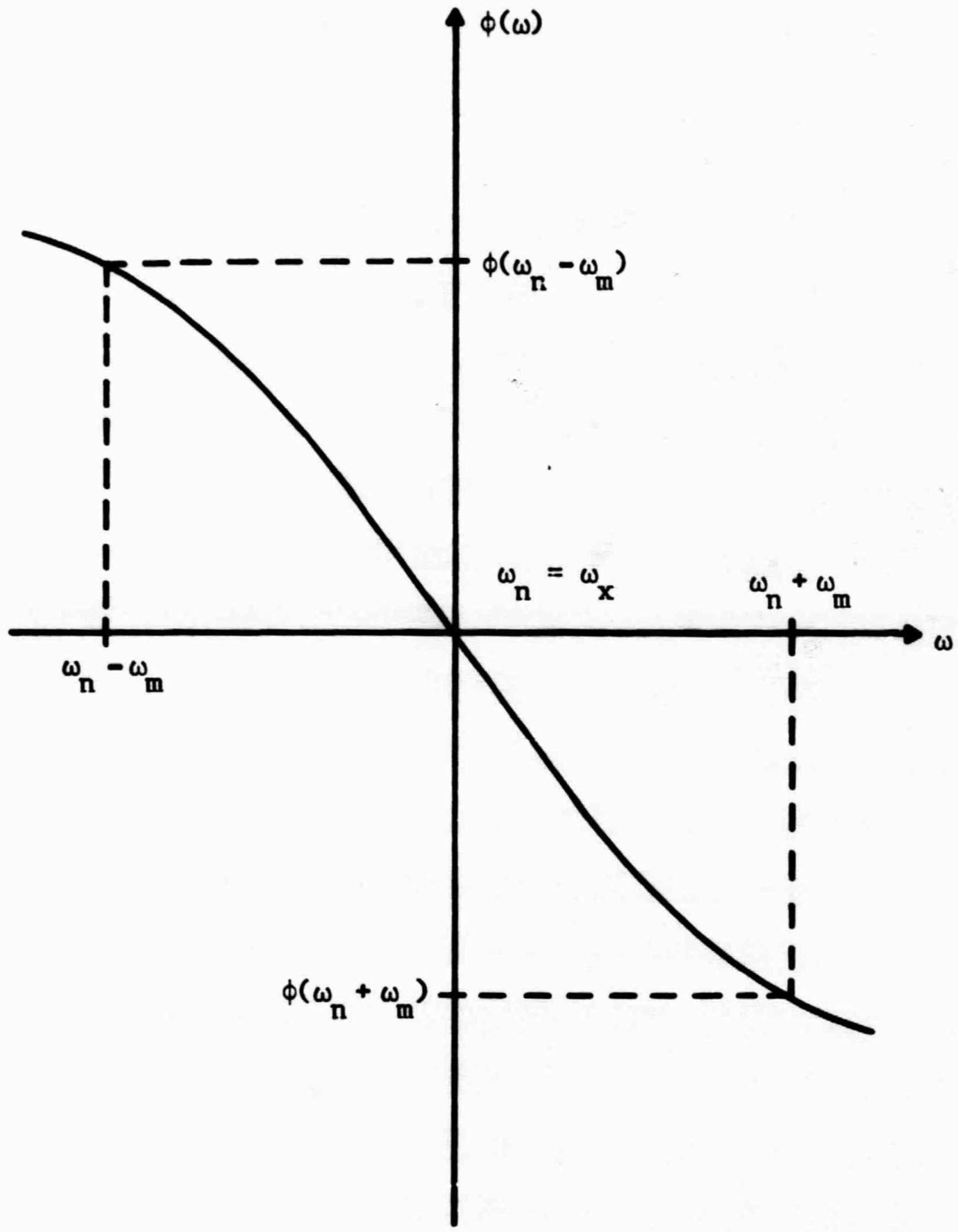

Figure 3-1 General Phase Characteristic with Carrier Centered

components, when demodulated, will be coherent. Figure 3-2 shows the situation which exists when the carrier frequency has drifted an amount ω_ε so that the DSB signal is no longer centered in the filter. The carrier frequency is now

$$\omega_n = \omega_x + \omega_\varepsilon \quad . \tag{3.2}$$

Under this condition $\phi(\omega_n + \omega_m)$ is not equal to $-\phi(\omega_n - \omega_m)$ and the sidebands will not add coherently when demodulated.

In order to look at the problem mathematically, it is convenient to express the phase characteristic as the sum of a linear phase component, $\phi_L(\omega)$, and a nonlinear component, $\phi_N(\omega)$. Such a decomposition is illustrated in Figure 3-3. The linear portion of the characteristic is given by

$$\phi_L(\omega) = S(\omega - \omega_x) \quad , \tag{3.3}$$

where S represents the slope of the phase characteristic at ω_x and is thus a negative number. The total phase shift at a frequency ω is then

$$\phi(\omega) = S(\omega - \omega_x) + \phi_N(\omega) \quad . \tag{3.4}$$

The filter output is obtained from the input by shifting the phase of each component by an appropriate amount. Thus, the output is given by

$$\begin{aligned} e_{out}(t) &= \cos\left[(\omega_n + \omega_m)t + \phi(\omega_n + \omega_m)\right] \\ &\quad + \cos\left[(\omega_n - \omega_m)t + \phi(\omega_n - \omega_m)\right] . \end{aligned} \tag{3.5}$$

For this case where $\omega_n = \omega_x$, (3.4) yields

$$\phi(\omega_n + \omega_m) = S\omega_m + \phi_N(\omega_n + \omega_m) \tag{3.6}$$

and

$$\phi(\omega_n - \omega_m) = -S\omega_m + \phi_N(\omega_n - \omega_m) \quad . \tag{3.7}$$

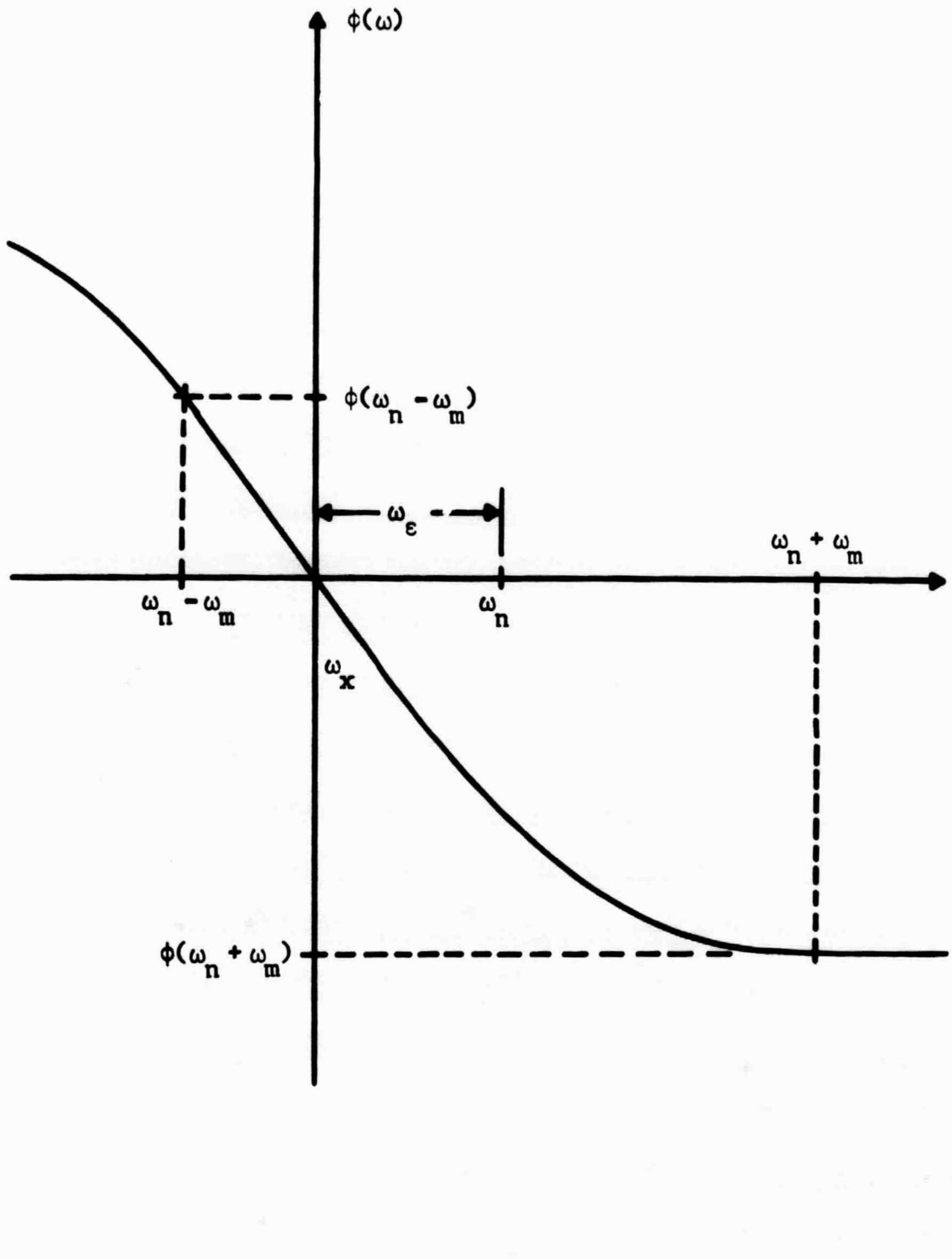

Figure 3-2 General Phase Characteristic with Carrier not Centered

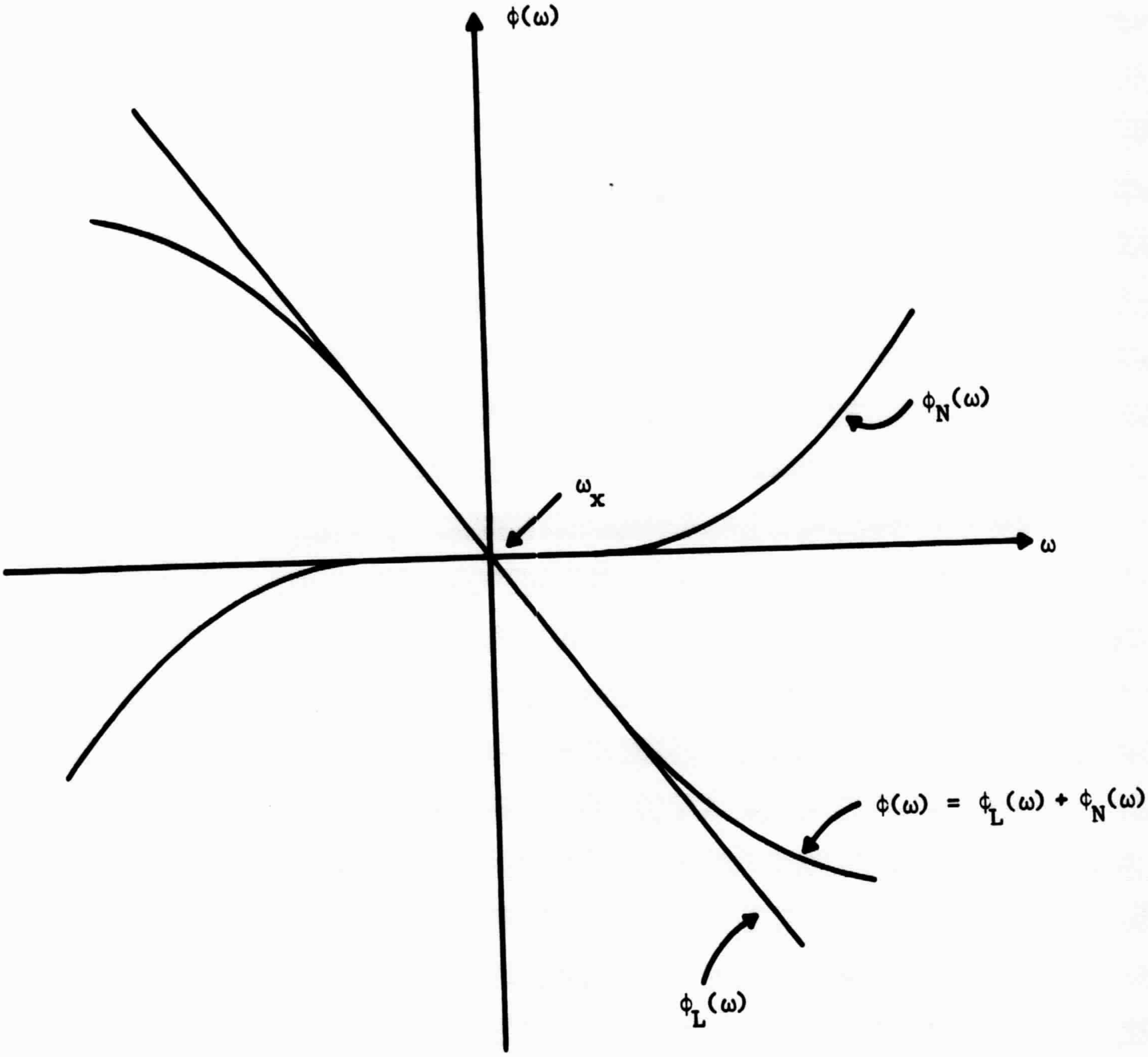

Figure 3-3 Decomposition of the Phase Characteristic

In order to simplify notation, let

$$\phi_N(\omega_n + \omega_m) = \Phi_N , \tag{3.8}$$

and

$$\phi_N(\omega_n - \omega_m) = -\Phi_N , \tag{3.9}$$

since the phase characteristic is odd about ω_x and $\omega_x = \omega_n$. Thus,

$$\begin{aligned} e_{out} &= \cos\left[(\omega_n + \omega_m)t + S\omega_m + \Phi_N\right] \\ &+ \cos\left[(\omega_n - \omega_m)t - S\omega_m - \Phi_N\right] \end{aligned} \tag{3.10}$$

which after demodulation, i.e., multiplying by cos $\omega_n t$ and filtering the components centered around $2\omega_n$, becomes

$$e_d(t) = \cos\left[\omega_m t + S\omega_m + \Phi_N\right] . \tag{3.11}$$

The term $S\omega_m$ is a linear phase shift proportional to frequency and it yields simply a time delay of the signal, even if the signal spectrum is complex. The term Φ_N is not proportional to frequency and thus in the case of a complex spectrum, it yields distortion. This yields the familiar result that for complex signals, distortion results if the phase is non-linear.

B. Phase Effects for Carrier Not Centered in Filter

If the carrier is not centered in the filter, the phase shifts imparted to the upper and lower sideband components can be written as

$$\phi(\omega_n + \omega_m) = S(\omega_n + \omega_m - \omega_x) + \phi_N(\omega_n + \omega_m) \tag{3.12}$$

and

$$\phi(\omega_n - \omega_m) = S(\omega_n - \omega_m - \omega_x) + \phi_N(\omega_n - \omega_m) \tag{3.13}$$

respectively. Using (3.2) this can be written

$$\phi(\omega_n + \omega_m) = S(\omega_m + \omega_\varepsilon) + \phi_N(\omega_n + \omega_m) \tag{3.14}$$

and

$$\phi(\omega_n - \omega_m) = S(-\omega_m + \omega_\varepsilon) + \phi_N(\omega_n - \omega_m) . \tag{3.15}$$

Substitution of these values into (3.5) yields

$$\begin{aligned} e_o(t) &= \cos\left[(\omega_n + \omega_m)t + S\omega_m + S\omega_\varepsilon + \phi_N(\omega_n + \omega_m)\right] \\ &+ \cos\left[(\omega_n - \omega_m)t - S\omega_m + S\omega_\varepsilon + \phi_N(\omega_n - \omega_m)\right] \end{aligned} \tag{3.16}$$

for the filter output.

In order to determine the expression for the demodulated output, it is necessary to multiply (3.16) by the synthesized carrier. In order to derive the expression for the synthesized carrier, the assumption will be made that the frequency of the drift is low so that it can be tracked by the carrier synthesis loop, and is not affected by the dynamic response of the carrier synthesis loop. This is a reasonable assumption since the frequency of the drift will be slow, due mainly to temperature variations in the airborne oscillator, while the bandwidth of the carrier synthesis loop is wide enough to pass higher frequency variations, such as baseband recorder flutter. If this condition holds, the synthesized carrier will be given by

$$e_{sc}(t) = \cos\left[\omega_n t + \phi(\omega_n)\right] \tag{3.17}$$

or

$$e_{sc}(t) = \cos\left[\omega_n t + S\omega_\varepsilon + \phi_N(\omega_n)\right] . \tag{3.18}$$

Multiplying (3.16) and (3.18) and filtering the $2\omega_n$ components yields

$$\begin{aligned} e_d(t) &= \frac{1}{2} \cos\left[\omega_m t + S\omega_m + \phi_N(\omega_n + \omega_m) - \phi_N(\omega_n)\right] \\ &+ \frac{1}{2} \cos\left[-\omega_m t - S\omega_m + \phi_N(\omega_n - \omega_m) - \phi_N(\omega_n)\right] \end{aligned} \tag{3.19}$$

for the demodulated output. In general the assumption can be made that

$\phi_N(\omega_n) \approx 0$. This follows from the fact that within the passband of a reasonable filter $\phi_N(\omega) \approx 0$. Figure 3-4 illustrates this for a Butterworth filter. It can be seen that ω_n would have to lie in the region of the filter bandedge before $\phi_N(\omega_n)$ becomes appreciable. Thus, $e_d(t)$ can be written as

$$e_d(t) = \frac{1}{2}\cos\left[\omega_m t + S\omega_m + \phi_N(\omega_n + \omega_m)\right] + \frac{1}{2}\cos\left[-\omega_m t - S\omega_m + \phi_N(\omega_n - \omega_m)\right] \quad . \tag{3.20}$$

Since $\phi_N(\omega_n + \omega_m)$ and $\phi_N(\omega_n - \omega_m)$ are not equal in magnitude and opposite in sign, it is necessary to obtain a symmetrical form for these functions in order for the two terms in (3.20) to be added to yield an equation of the form

$$e_d(t) = A(\omega)\cos\left[\omega_m t + S\omega_m + \psi\right] \quad . \tag{3.21}$$

This operation is illustrated in Figure 3-5. Let

$$\phi_N(\omega_n + \omega_m) = B \tag{3.22}$$

and

$$\phi_N(\omega_n - \omega_m) = -A \; . \tag{3.23}$$

Then

$$\phi_N(\omega_n + \omega_m) = \frac{A+B}{2} - \frac{A-B}{2} = B \tag{3.24}$$

and

$$\phi_N(\omega_n - \omega_m) = -\frac{A+B}{2} - \frac{A-B}{2} = -A \; . \tag{3.25}$$

Thus (3.20) can be written as

$$e_d(t) = \frac{1}{2}\cos\left[\omega_m t + S\omega_m + \frac{A+B}{2} - \frac{A-B}{2}\right] + \frac{1}{2}\cos\left[\omega_m t + S\omega_m + \frac{A+B}{2} + \frac{A-B}{2}\right], \tag{3.26}$$

which, upon application of the trigonometric identity

$$\cos x \cos y = \frac{1}{2}\cos(x+y) + \frac{1}{2}\cos(x-y) \; , \tag{3.27}$$

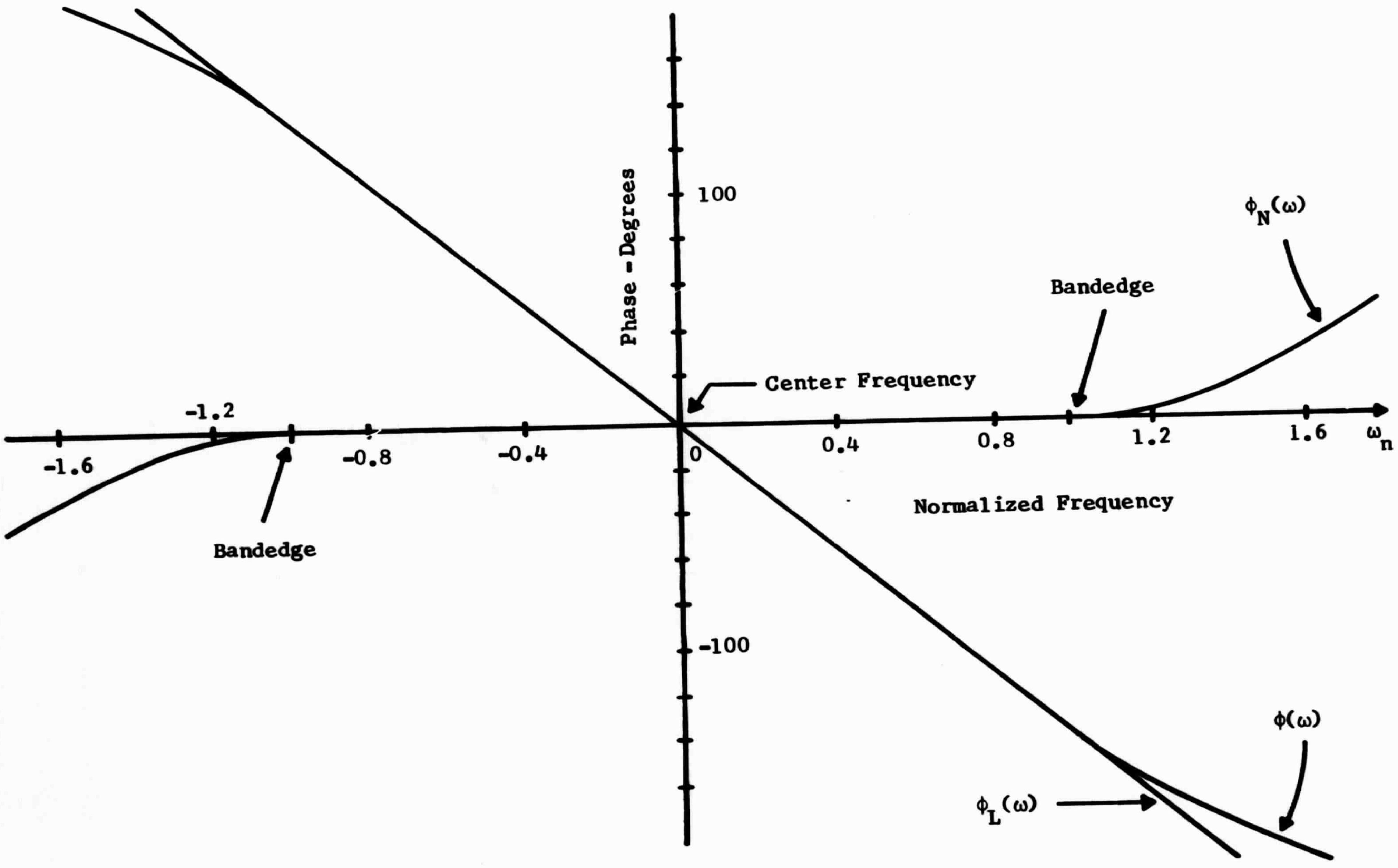

Figure 3-4 Phase Characteristic of Third-Order Butterworth Filter

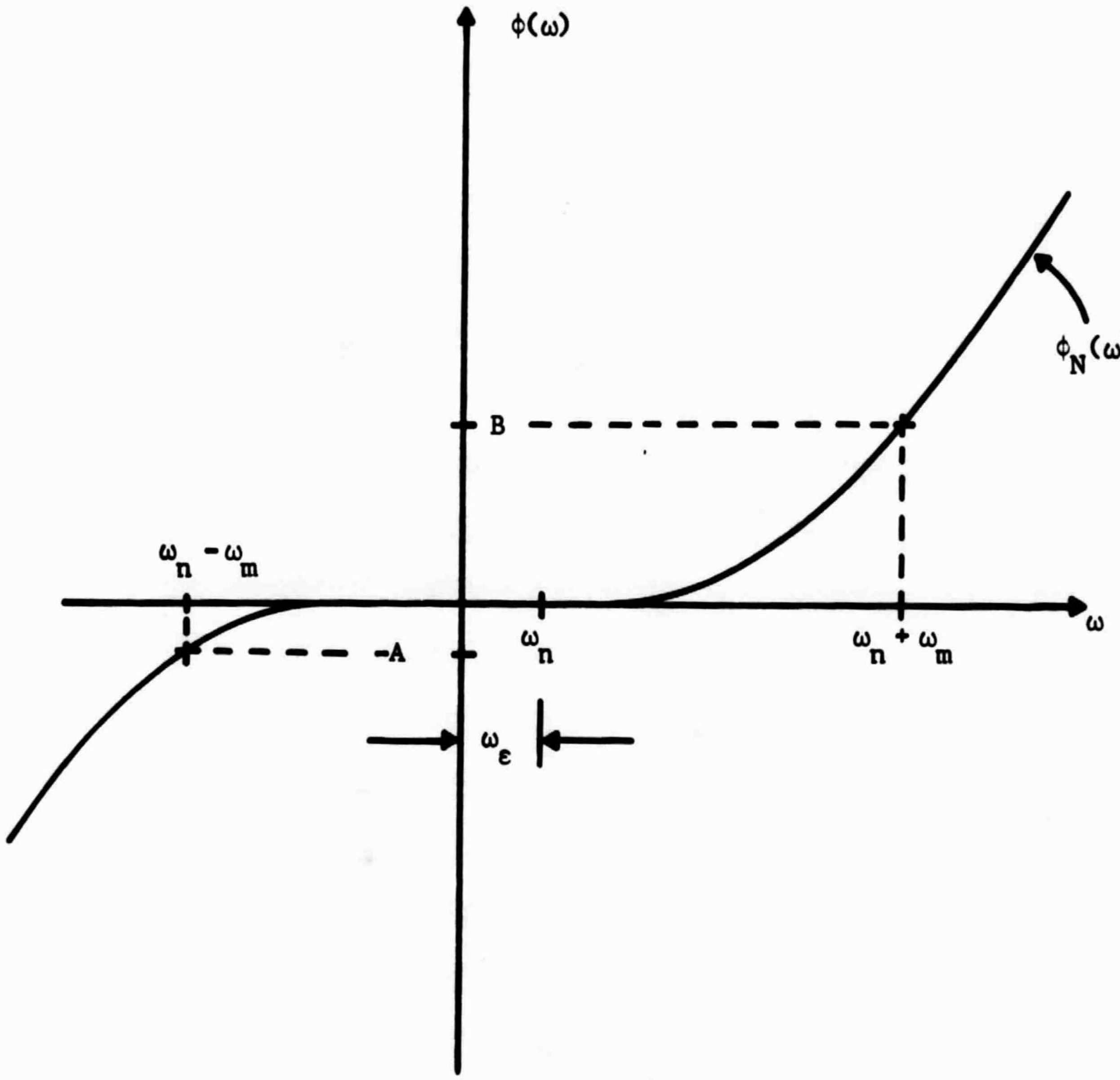

Figure 3-5 Definition of A and B

yields

$$e_d(t) = \cos\left(\frac{A-B}{2}\right)\cos\left[\omega_m t + S\omega_m + \frac{A+B}{2}\right]. \tag{3.28}$$

The preceding equation shows that if phase is linear, the output is unaffected by the carrier drift. This follows from the fact that if phase is linear, $A = B = 0$, and (3.28) becomes (3.11) with $\Phi_N = 0$. Of course, this result assumes that the drift frequencies are sufficiently low so that the synthesized carrier is that given by (3.18).

Note that the effect of the nonlinear phase characteristic is both a dynamic attenuation and a dynamic phase perturbation. The attenuation exceeds 1% if $\frac{A-B}{2}$ exceeds approximately 10°. A 1% error between $e_d(t)$ as given in (3.28) and the ideal output, $e_d'(t)$, where

$$e_d'(t) = \cos\left[\omega_m t + S\omega_m\right] \tag{3.29}$$

occurs for $\frac{A+B}{2}$ much less than 10°. The parameters A and B are defined such that both are positive and therefore $\frac{A+B}{2}$ is greater than $\frac{A-B}{2}$. The conclusion is reached that the attenuation term is unimportant in most cases.

C. Amplitude Effects for Carrier Not Centered in Filter

In order to include the effects of amplitude, assume that the magnitude of the filter transfer function at $(\omega_n + \omega_m)$ is M_U and that the magnitude of the transfer function at $(\omega_n - \omega_m)$ is M_L. Equation (3.16), the expression for the filter output, then becomes

$$\begin{aligned} e_{out}(t) = {} & M_U \cos\left[(\omega_n + \omega_m)t + S\omega_m + S\omega_\varepsilon + \phi_N(\omega_n + \omega_m)\right] \\ & + M_L \cos\left[(\omega_n - \omega_m)t - S\omega_m + S\omega_\varepsilon + \phi_N(\omega_n - \omega_m)\right]. \end{aligned} \tag{3.30}$$

This yields

$$e_d(t) = \frac{M_U}{2} \cos\left[\omega_m t + S\omega_m + \frac{A+B}{2} - \frac{A-B}{2}\right]$$
$$+ \frac{M_L}{2} \cos\left[\omega_m t + S\omega_m + \frac{A+B}{2} + \frac{A-B}{2}\right] \tag{3.31}$$

for the demodulated output. Let

$$x = \omega_m t + S\omega_m + \frac{A+B}{2} \tag{3.32}$$

and

$$y = -\frac{A-B}{2} \quad . \tag{3.33}$$

The demodulated output can then be written

$$e_d(t) = \left[\frac{M_U}{4} + \frac{M_L}{4}\right] \cos (x+y) + \left[\frac{M_U}{4} - \frac{M_L}{4}\right] \cos (x+y)$$
$$+ \left[\frac{M_U}{4} + \frac{M_L}{4}\right] \cos (x-y) - \left[\frac{M_U}{4} - \frac{M_L}{4}\right] \cos (x-y) \tag{3.34}$$

or

$$e_d(t) = \frac{1}{4} (M_U + M_L)\left[\cos (x+y) + \cos (x-y)\right]$$
$$+ \frac{1}{4} (M_U - M_L)\left[\cos (x+y) - \cos (x-y)\right] \quad , \tag{3.35}$$

which is

$$e_d(t) = \frac{1}{2} (M_U + M_L) \cos y \cos x$$
$$- \frac{1}{2} (M_U - M_L) \sin y \sin x \quad . \tag{3.36}$$

The preceding expression can be placed in the compact form

$$e_d(t) = M' \cos (x + \tan^{-1} \psi') \ , \tag{3.37}$$

where

$$M' = \left[\frac{1}{4} (M_U + M_L)^2 \cos^2 y + \frac{1}{4} (M_U - M_L)^2 \sin^2 y\right]^{1/2} . \tag{3.38}$$

and

$$\psi' = \tan^{-1} \frac{(M_U - M_L) \sin y}{(M_U + M_L) \cos y} \quad . \tag{3.39}$$

From the above expressions, the filter output can be computed for any particular set of values. It is convenient to note that if $M_U \approx M_L$ and $A \approx B$, then

$$e_d(t) = \frac{1}{2} (M_U + M_L) \cos y \cos x \ . \tag{3.40}$$

Using the preceding expressions, the output of any particular filter, with a DSB input, can be found if the modulation is sinusoidal.

IV. EFFECT OF DEMODULATION PHASE ERRORS

In suppressed carrier systems errors result in the demodulated waveform if a phase error exists in the synthesized carrier used for demodulation. The manner in which this phase error affects the demodulated output is of considerable practical importance. The sensitivity of SSB and DSB to demodulation phase errors has been previously investigated for the case of sinusoidal modulation[5]. The following investigation is more general in that the modulating signal is assumed random and also quadrature DSB (QDSB) is considered.

A. Phase Errors in a DSB System

A general DSB signal, prior to demodulation, can be expressed as

$$e_{DSB}(t) = m(t)\cos \omega_n t \;, \tag{4.1}$$

where ω_n is the carrier frequency and $m(t)$ is the information bearing signal. The demodulation carrier, $e_c(t)$, can be expressed as

$$e_c(t) = 2 \cos\left[\omega_n t + \phi(t)\right], \tag{4.2}$$

where $\phi(t)$ represents the phase error. Multiplying (4.1) and (4.2) and filtering out the components centered about $2\omega_n$ yields

$$e_d(t) = m(t)\cos \phi(t) \tag{4.3}$$

for the demodulated output.

In order to investigate the error introduced by a nonzero $\phi(t)$, an error function, $E(t)$, is defined as the difference between $e_d(t)$, the actual demodulated output, and $m(t)$, the ideal demodulated output. For the DSB case the error function, $E_{DSB}(t)$, is

$$E_{DSB}(t) = \left[1 - \cos \phi(t)\right] m(t) . \tag{4.4}$$

In practical cases, $\phi(t)$ will be small since this is a requirement for good system performance. Thus, $\phi(t)$ will be assumed sufficiently small to make $\phi^n(t)$ negligible for $n > 2$. This assumption allows $\cos \phi(t)$ to be replaced by the first two terms of its series expansion so that

$$E_{DSB}(t) \approx \frac{1}{2} \phi^2(t) m(t) . \tag{4.5}$$

The mean-square error, $\overline{E_{DSB}^{\,2}(t)}$, can be written as

$$\overline{E_{DSB}^{\,2}(t)} = \frac{1}{4} \overline{\phi^4(t)\, m^2(t)} . \tag{4.6}$$

In general $\phi(t)$ and $m(t)$ will be statistically independent so that

$$\overline{E_{DSB}^{\,2}(t)} \approx \frac{1}{4} \overline{\phi^4(t)}\; \overline{m^2(t)} . \tag{4.7}$$

Both the modulating signal, $m(t)$, and the phase error, $\phi(t)$, are assumed to be zero mean Gaussian random variables with variances $\sigma_m^{\,2}$ and $\sigma_\phi^{\,2}$, respectively. Thus

$$\overline{m^2(t)} = \sigma_m^{\,2} . \tag{4.8}$$

It has been shown in a previous report[6] that the mean-square value of $\phi^2(t)$, $\overline{\phi^4(t)}$, is given by

$$\overline{\phi^4(t)} = 3\sigma_\phi^{\,4} . \tag{4.9}$$

so that

$$\overline{E_{DSB}^{\,2}(t)} \approx \frac{3}{4} \sigma_\phi^{\,4} \sigma_m^{\,2} \tag{4.10}$$

for the system mean-square error, or

$$E_{N(DSB)} \approx \frac{3}{4} \sigma_\phi^{\,4} , \tag{4.11}$$

where $E_{N(DSB)}$ is the system mean-square error normalized with respect to the mean-square value of m(t). The function

$$\sqrt{\overline{E_{N(DSB)}}} \approx \sqrt{\frac{3}{4}} \quad \sigma_{\phi}^{2} \tag{4.12}$$

is the normalized rms error and is the usual quantity of interest.

B. Phase Errors in a QDSB System

A general QDSB signal, prior to demodulation, can be expressed as

$$e_Q(t) = m_1(t) \cos \omega_n t + m_2(t) \sin \omega_n t \ , \tag{4.13}$$

where $m_1(t)$ and $m_2(t)$ represent the two information bearing signals on the two quadrature carriers. Multiplying (4.13) and (4.2) will demodulate $m_1(t)$, and in like manner, a demodulation carrier of

$$2 \sin\left[\omega_n t + \varepsilon(t)\right]$$

will demodulate $m_2(t)$. Because of the symmetry involved, it is necessary only to consider one of these processes. If the mean-square values of $m_1(t)$ and $m_2(t)$ are equal, the error expression will be the same for both cases.

Multiplying (4.13) and (4.2) yields

$$e_d(t) = m_1(t) \cos \phi(t) + m_2(t) \sin \phi(t) \tag{4.14}$$

after the $2\omega_n$ terms are filtered. Using the approximation that $\phi^n(t)$ is negligible for $n > 2$ yields

$$e_d(t) \approx m_1(t)\left[1 - \frac{1}{2}\, \phi^2(t)\right] + m_2(t)\ \phi(t) \tag{4.15}$$

for the demodulated output. Thus, the error function for QDSB is

$$E_Q(t) = \frac{1}{2}\, \phi^2(t) m_1(t) - \phi(t) m_2(t) \tag{4.16}$$

and the mean-square error is

$$\overline{E_Q^{\,2}(t)} = \frac{1}{4}\, \overline{\phi^4(t) m_1^{\,2}(t)} - \overline{\phi^3(t) m_1(t) m_2(t)} + \overline{\phi^2(t) m_2^{\,2}(t)} \ . \tag{4.17}$$

The functions $\phi(t)$, $m_1(t)$, and $m_2(t)$ are all assumed statistically independent random variables with zero means. From (4.17) and (4.9) we then have

$$\overline{E_Q^{\,2}(t)} = \frac{3}{4}\ \sigma_\phi^{\,4}\sigma_{m_1}^{\ \,2} + \sigma_\phi^{\,2}\sigma_{m_2}^{\ \,2} . \tag{4.18}$$

In a typical situation the signals $m_1(t)$ and $m_2(t)$ will have equal variances. If this is the case

$$\overline{E_Q^{\,2}(t)} = (\frac{3}{4}\ \sigma_\phi^{\,4} + \sigma_\phi^{\,2})\sigma_m^{\,2} \tag{4.19}$$

where

$$\sigma_m^{\,2} = \sigma_{m_1}^{\ \,2} = \sigma_{m_2}^{\ \,2} . \tag{4.20}$$

This yields a normalized rms error of

$$\sqrt{E_{N(Q)}} = \sigma_\phi \sqrt{\frac{3}{4}\ \sigma_\phi^{\,2} + 1} . \tag{4.21}$$

C. Phase Errors in an SSB System

If in (4.14) $m_1(t)$ is set equal to $m(t)$ and $m_2(t)$ is set equal to $\hat{m}(t)$, the Hilbert transform of $m(t)$, the result is the SSB signal[7]

$$e_{SSB}(t) = m(t)\cos\,\omega_n t + \hat{m}(t)\,\sin\,\omega_n t . \tag{4.22}$$

Comparison of (4.22) with (4.17) shows that

$$\overline{E^2_{\ SSB}(t)} = \frac{1}{4}\ \overline{\phi^4(t)}\ \overline{m^2(t)}$$

$$- \overline{\phi^3(t)}\ \overline{m(t)\hat{m}(t)} + \overline{\phi^2(t)}\ \overline{\hat{m}^2(t)} \tag{4.23}$$

since $\phi(t)$ and $m(t)$ are assumed statistically independent. It is easily shown that[8]

$$\overline{m^2(t)} = \overline{\hat{m}^2(t)} \tag{4.24}$$

so that (4.23) can be written

$$\overline{E^2_{SSB}(t)} = \frac{3}{4}\ \sigma_\phi^{\ 4}\sigma_m^{\ 2} + \sigma_\phi^{\ 2}\sigma_m^{\ 2}\ . \tag{4.25}$$

Comparison of (4.25) with (4.19) shows that the mean-square error for SSB is exactly the same as the mean-square error in a QDSB system if all modulating signals have the same mean-square value.

In Figure 4-1 the normalized rms error is illustrated for the three cases of interest. The curve illustrates that DSB is much less sensitive to a phase error in the demodulation carrier than either SSB or QDSB.

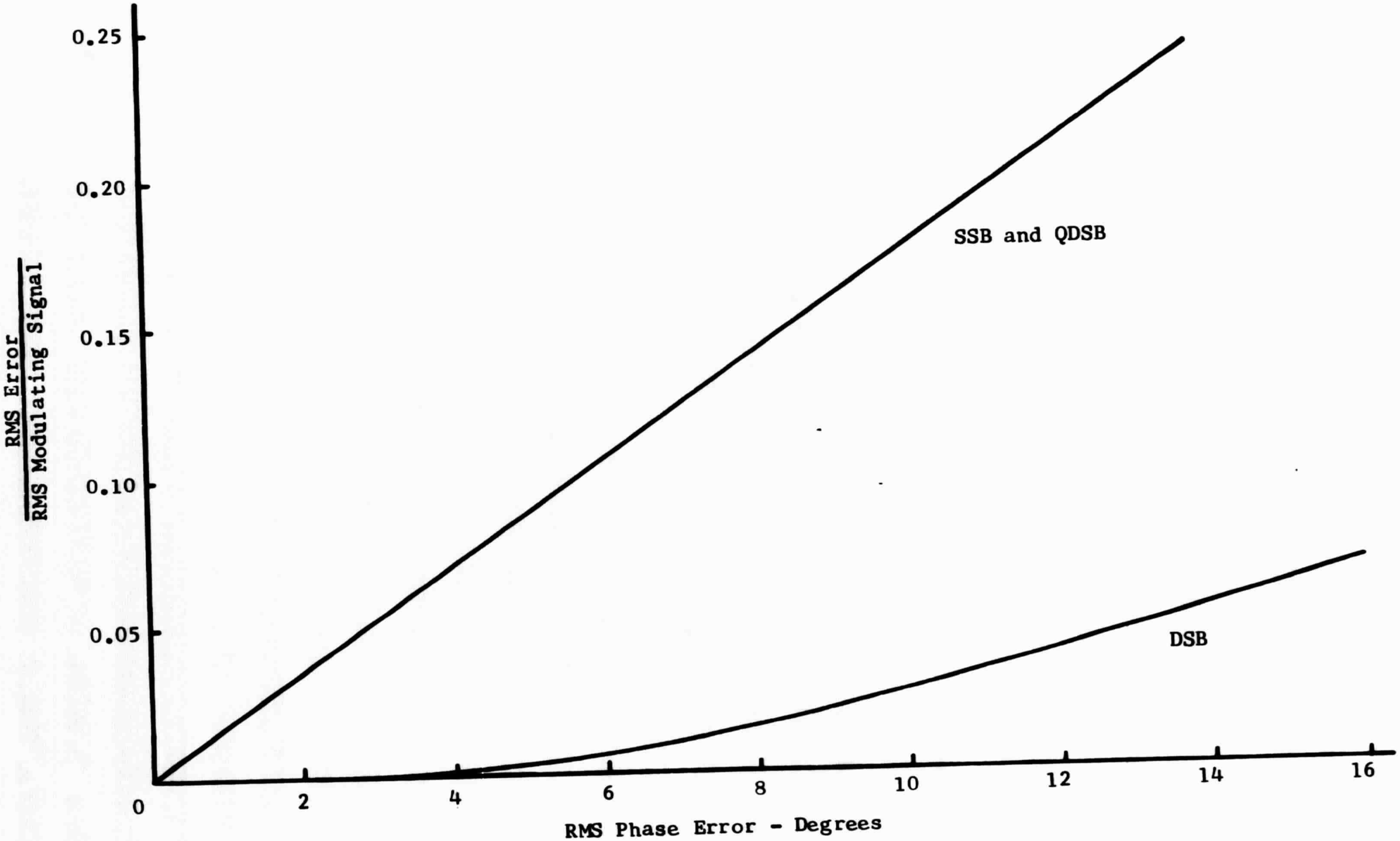

Figure 4-1 Normalized RMS Error

V. INTERMODULATION IN AM-BASEBAND SYSTEMS

Intermodulation results in distortion in AM-baseband systems. In this section the effect of intermodulation will be investigated for the case where the channel carriers are harmonically related to determine the effect of the carrier phase.

In order to simplify the analysis, the baseband will be approximated by two channels, each having sinusoidal modulation, as illustrated in Figure 5-1. The harmonically related carriers have frequencies ω_1 and $2\omega_1$ and initial phases α and β, respectively. The sinusoidal modulating frequencies are ω_A and ω_B.

Intermodulation distortion results from nonlinearities in the system. Therefore, if the output, y(t), is related to the input, x(t), by the equation

$$y(t) = \sum_{k=1}^{\infty} a_k \left[x(t)\right]^k , \tag{5.1}$$

where at least one a_k for $k > 1$ is nonzero, intermodulation distortion results. The analysis will assume that $a_k = 0$ for $k > 2$ so that

$$y(t) = a_1 x(t) + a_2 x^2(t) . \tag{5.2}$$

From Figure 5-1, the baseband signal can be written as

$$\begin{aligned} x(t) = \cos(\omega_1 t - \omega_A t + \alpha) + \cos(\omega_1 t + \omega_A t + \alpha) \\ + \cos(2\omega_1 t - \omega_B t + \beta) + \cos(2\omega_1 t + \omega_B t + \beta) . \end{aligned} \tag{5.3}$$

The intermodulation distortion component of y(t) is $a_2 x^2(t)$. Substituting x(t) yields, after some manipulation,

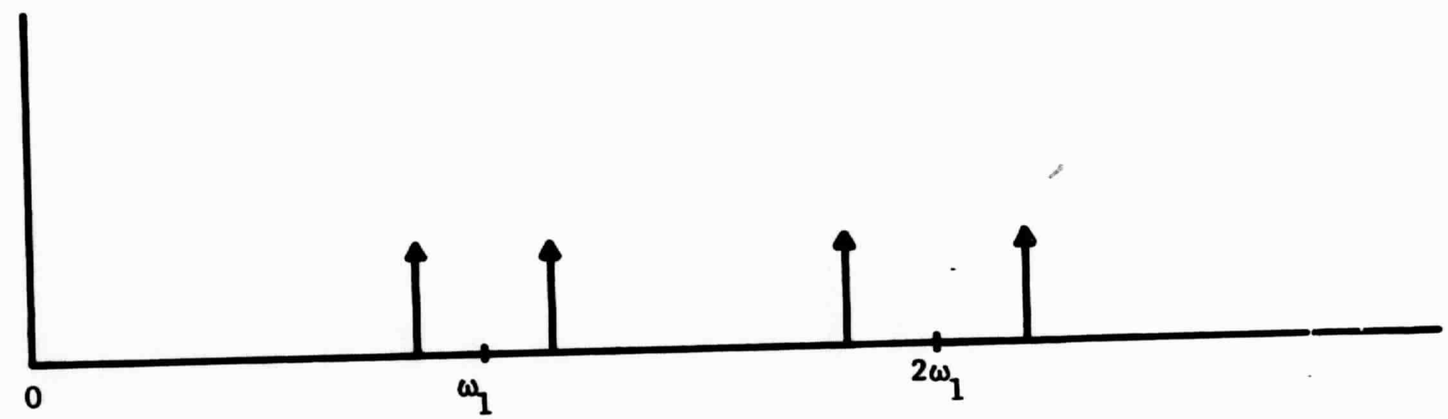

Figure 5-1 Baseband Spectrum

$$
\begin{aligned}
x^2(t) = {} & 2 + \cos 2\omega_A t + \cos 2\omega_B t \\
& + \cos(\omega_1 t - \omega_A t - \omega_B t - \alpha + \beta) + \cos(\omega_1 t - \omega_A t + \omega_B t - \alpha + \beta) \\
& + \cos(\omega_1 t + \omega_A t - \omega_B t - \alpha + \beta) + \cos(\omega_1 t + \omega_A t + \omega_B t - \alpha + \beta) \\
& + \cos(2\omega_1 t + 2\alpha) \\
& + \frac{1}{2}\cos(2\omega_1 t - 2\omega_A t + 2\alpha) + \frac{1}{2}\cos(2\omega_1 t + 2\omega_A t + 2\alpha) \\
& + \cos(3\omega_1 t - \omega_A t - \omega_B t + \alpha + \beta) + \cos(3\omega_1 t - \omega_A t + \omega_B t + \alpha + \beta) \\
& + \cos(3\omega_1 t + \omega_A t - \omega_B t + \alpha + \beta) + \cos(3\omega_1 t + \omega_A t + \omega_B t + \alpha + \beta) \\
& + \cos(4\omega_1 t + 2\beta) \\
& + \frac{1}{2}\cos(4\omega_1 t - 2\omega_B t + 2\beta) + \frac{1}{2}\cos(4\omega_1 t + 2\omega_B t + 2\beta) \; .
\end{aligned} \tag{5.4}
$$

The spectrum of $x^2(t)$ is illustrated in Figure 5-2. From this it can be seen that if the channel carriers are harmonically related, intermodulation distortion consists of DSB spectra having carrier frequencies equal to harmonics of the original channel carrier frequencies and a lowpass spectrum exists below the lowest DSB spectrum. Additionally, the phases of the carriers in the intermodulation distortion spectrum are simple harmonics of the original baseband carriers.

Since the intermodulation distortion on a given channel is a DSB signal, it can be represented by a signal of the form

$$
e_{id}(t) = a_2 d(t)\cos(\omega_n t + \gamma) \; , \tag{5.5}
$$

where $d(t)$ represents the intermodulation distortion, ω_n is the carrier frequency of the channel of interest, γ is the reference phase of intermodulation carrier, and a_2 is the scale factor from (5.2). Thus, the complete Channel n signal, the original signal plus the intermodulation component, can be represented as

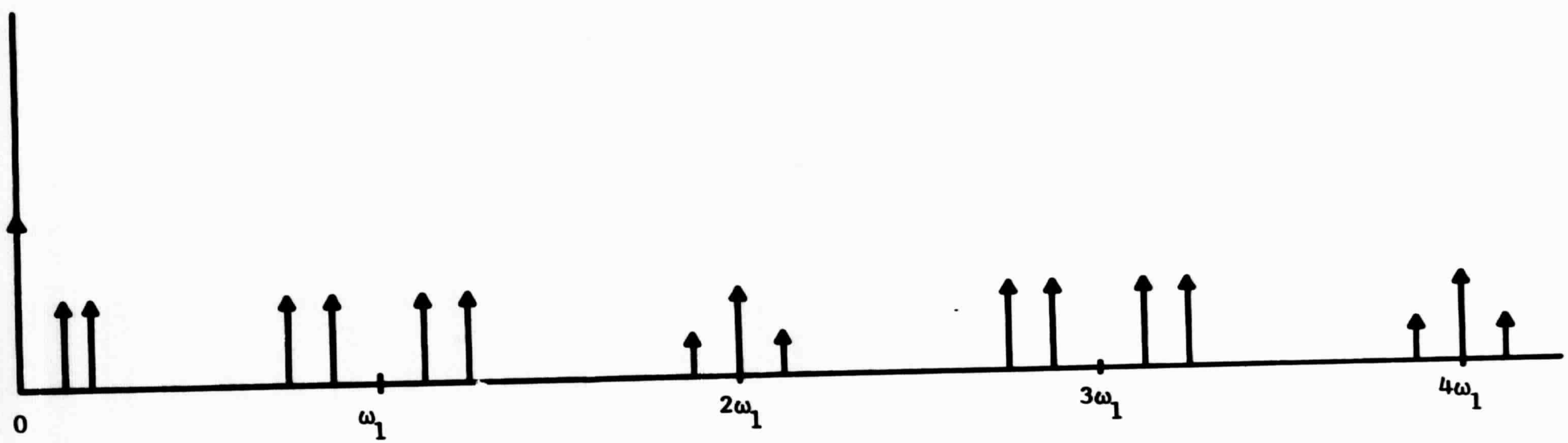

Figure 5-2 Intermodulation Spectrum

$$e'_n(t) = a_1 m(t)\cos \omega_n t + a_2 d(t)\cos(\omega_n t + \gamma) \quad , \tag{5.6}$$

and an appropriate value of γ must be found.

Equation (5.6) may be expanded as

$$\begin{aligned} e'_n(t) &= a_1 m(t)\cos \omega_n t \\ &\quad + a_2 d(t)\left[\cos \omega_n t \cos \gamma - \sin \omega_n t \sin \gamma\right] , \end{aligned} \tag{5.7}$$

which is

$$\begin{aligned} e'_n(t) &= \left[a_1 m(t) + a_2 d(t)\cos \gamma\right]\cos \omega_n t \\ &\quad + a_2 d(t)\sin \gamma \sin \omega_n t \end{aligned} \tag{5.8}$$

This can be written as

$$e'_n(t) = A(t)\cos\left[\omega_n t + \phi(t)\right] \quad , \tag{5.9}$$

where

$$A(t) = \sqrt{\left[a_1 m(t) + a_2 d(t)\cos \gamma\right]^2 + \left[a_2 d(t)\sin \gamma\right]^2} \tag{5.10}$$

and

$$\phi(t) = \tan^{-1} \frac{a_2 d(t)\sin \gamma}{a_1 m(t) + a_1 d(t)\cos \gamma} \tag{5.11}$$

If γ is either 0 or π radians, the zeros of (5.9) will correspond to the zeros of

$$m(t)\cos \omega_n t \ .$$

Thus, if carrier synthesis is to be performed by operating any one of the channel signals, the effect of intermodulation distortion can be removed, as far as carrier synthesis is concerned, by making the channel carriers harmonically related and making the initial phases 0 or π radians. It should be noted that the effect of A(t) can be removed by using a limiter since the carrier synthesis process is not sensitive to amplitude information.

The preceding results have value when the demodulation carrier is derived from a pilot as well as when it is derived from the channel signal. The information gained here is that the initial phases should be 0 or π radians so that the effect of intermodulation components can be eliminated when the channel carriers are harmonically related.

VI. SUMMARY

The crest factor of a multiplex of DSB signals, having harmonically related carriers, is not a function of the carrier phases if random data is being telemetered and all data signals are statistically independent. If dc modulation is present on two or more channels, the crest factor is a function of the carrier phases.

One consideration which does affect the choice of carrier phase is intermodulation. When the carriers are harmonically related, intermodulation distortion takes the form of DSB signals added to the individual channel signals. When carrier synthesis is performed using the channel signal, the effect of intermodulation is minimized by making the carrier phases 0 or π radians.

Carrier drift of a DSB signal can cause distortion when the DSB signal is filtered by a bandpass filter. This effect results because at the filter output, the sidebands do not add coherently if the filter is unsymmetrical with respect to the channel carrier. One solution to this problem is to utilize one highly stable oscillator and derive all carriers from this oscillator.

A dynamic phase error in the demodulation carrier results in a dynamic error in the demodulated output. This effect is much more pronounced for SSB and QDSB than for DSB. A one percent system requires that the rms phase error be less than approximately seven degrees if the system is DSB, while less than a one degree error is required if the system is SSB or QDSB.

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