FINAL REPORT

Contract No. NAS 12-2041

DESIGNERS MANUAL FOR ELECTRICAL AND ELECTRONIC FILTERS

Department of Electrical Engineering
University of Missouri - Columbia
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Project Director:
Dr. George W. Zobrist

Group Leader:
Mr. Joseph Barbay

Contributors:
Mr. Ron Becker
Mr. William Boyer
Dr. G. V. Lago
Mr. Cal Marvin
Mr. Jim Pratt
Mr. Larry Robertson
Mr. Jim Rodgers
Mr. Lee Sorenson

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Department of Electrical Engineering
University of Missouri - Columbia
Columbia, Missouri

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CHAPTER ONE

Normalization and Frequency Transformation

1.1 Introduction. The intent of this handbook is to provide the NASA engineer with a reference text in which he can find design procedures applicable to many of his filter design problems. These design procedures will serve as a background in the techniques of filter design, a background which will enable the engineer to solve many practical filter problems. The design techniques presented in this handbook are all standard, well documented techniques. A bibliography is included at the end of each chapter so that the reader may pursue the techniques included in the chapter in greater detail if desired. The computer program, NASAP-69, is an integral part of each design procedure.

NASAP-69 is utilized in the analysis of filters which result from the application of the various design techniques. This essential step in each design procedure insures that the final filter fulfills the original specifications, as the filter specifications associated with a design problem are checked against the final filter. NASAP-69 is used to obtain the transfer functions of a filter. The transfer function is found as a rational function of a ratio of polynomials in terms of s. This can be done for both passive and active filters. Other network functions, such as input impedance can be found by use of NASAP-69. Knowledge of the sensitivity of these transfer functions and network functions to changes in filter element values is also obtained by the use of NASAP-69. For example, the sensitivity of a voltage ratio function of a filter to changes in element values could be displayed in the form of a sensitivity coefficient matrix. This sensitivity coefficient matrix could be used by the designer to spot elements of the filter which have particularly critical values. The sensitivity coefficient matrix is used by the Missouri version of NASAP-69 to perform a worst-case analysis of the particular function under consideration. The worst-case variation of the function is exhibited in the form of the maximum variation in the Bode plot of the magnitude and phase of the function.

1.2 Normalization. The technique of network normalization is a useful practice which can be an invaluable tool when designing filters. Normalization is a powerful tool, as a normalized filter can be used to represent many similar filters. The element values of the normalized filter are of convenient size, a benefit which is best appreciated when doing slide rule calculations involving the element values of the filter. Normalization of element values will also minimize the possibility of exponent overflow when using NASAP-69. The attenuation and phase characteristics for a normalized filter can be found using the convenient element values. The attenuation and phase characteristics of other filters based on the normalized filter can be found by performing simple transformations upon the attenuation and phase characteristics of the normalized filter.

There are two quantities used in the process of normalizing a network for steady-state analysis. These two normalizing quantities are:

1. ω_0, the normalization frequency,
2. R_0, the normalization impedance.

The normalization quantities are selected to yield convenient normalized element values in the resulting normalized network. In many filter applications the normalizing quantities have been selected so that a particularly critical frequency of the normalized filter is one radian and the design resistance of the normalized filter is one ohm. The filter designer must select suitable values for the normalizing quantities when the process of normalization is reversed in order to produce the desired filter. By proper selection of ω_0 and R_0, a normalized low-pass filter may be transformed to a low-pass filter, or even some other type of filter such as a band-pass filter. In this section the process of filter normalization is considered. In the next section a more general transformation, by which the normalized low-pass filter can be transformed to high-pass, band-pass, band-elimination and other filters, is discussed.

To normalize a given network first select values for the normalizing quantities ω_0 and R_0. These quantities are selected so that the element values appearing in the normalized network are of convenient size. Then transform all element values of the network to element values of the normalized network by applying the following rules:

1. Replace each resistor R by its normalized value R/R_0.
2. Replace each capacitor by its normalized value $\omega_0 C R_0$.
3. Replace each inductance by its normalized value $\omega_0 L/R_0$.
4. Replace the radian frequency ω by its normalized value ω/ω_0.

Table 1.1 is a summary of the above rules. The prime quantities in table 1.1 denote unnormalized network

	Network quantity	Normalized quantity	Network quantity in terms of normalized quantity
Frequency	$s' = j\omega'$	$s = j\omega/\omega_0$	$s' = \omega_0 s$
Resistance	R'	$R = R'/R_0$	$R' = R_0 R$
Inductance	L'	$L = \omega_0 L'/R_0$	$L' = R_0 L/\omega_0$
Capacitance	C'	$C = \omega_0 C' R_0$	$C' = C/\omega_0 R$
Inductive impedance	XL'	$XL = sL$	$XL' = R_0 XL$
Capacitive impedance	XC'	$XC = 1/sC$	$XC' = R_0 XC$
Impedance	Z'	$Z = Z'/R_0$	$Z' = R_0 Z$

Table 1.1 Network normalization summary

parameters, the unprime quantities are the normalized network parameters. The reverse process, transforming the normalized network back to the unnormalized network is also indicated in the table.

The network shown in figure 1.1-a will be used to demonstrate the process of network normalization. After the network is normalized, the voltage transfer functions for the normalized network and for the original network are compared. Then to complete the example, the normalized network is transformed to a new network. The voltage transfer function V2'/V1' for the network of figure 1.1-a is given by

$$\frac{V2'}{V1'} = \frac{\dfrac{R2'/C1's'}{R2' + 1/C1's'}}{\dfrac{R2'/C1's'}{R2' + 1/C1's'} + R1'}$$

$$= \frac{R2'/(R1' + R2')}{1 + \left(\dfrac{R1'R2'}{R1' + R2'}\right) C1's'}$$

$$\frac{V2'}{V1'}(j\omega) = \frac{2/3}{1 + j(2\text{x}10^{-3})\omega'} .$$

This voltage ratio will be compared to a similar transfer function for the normalized network.

In order to normalize the network, a value for R_0 and ω_0 must be selected. In this case $R_0 = 1\text{x}10^3$ and $\omega_0 = 500$ will result in a normalized network with convenient element values. Choosing R_0 to be $1\text{x}10^3$ eliminates the 10^3 factor in the element values R1 and R2. The value selected for ω_0 forces the breakpoint radian frequency of the voltage transfer function to one radian, as can be seen from the plot of the voltage transfer function for the normalized network figure 1.2-a. These values will also force the normalized value of C1 to be of convenient size.

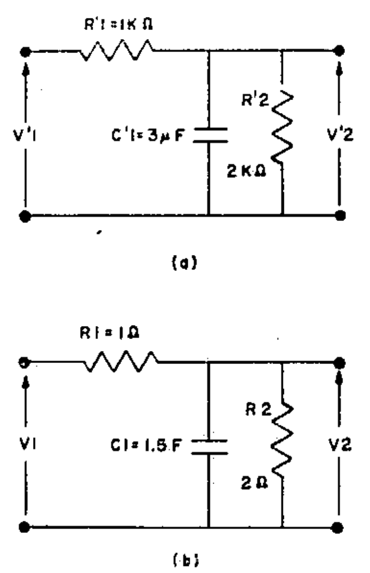

Figure 1.1 Example of network normalization. (a) Original network. (b) Normalized network.

Using table 1.1 the values of the normalized network elements are given by

$$R1 = \frac{R1'}{R_o} = \frac{1x10^3}{1x10^3} = 1\Omega$$

$$R2 = \frac{R2'}{R_o} = \frac{2x10^3}{1x10^3} = 2\Omega$$

$$C1 = \omega_o C1' R_o = (500)(3x10^{-6})(1x10^3) = 1.5f.$$

The normalized network is shown in figure 1.1-b.

The voltage ratio for this network is given by

$$G(j\omega) = \frac{V2}{V1}(j\omega)$$

$$= \frac{R2/(R1 + R2)}{1 + j \frac{R1\ R2}{R1 + R2} C1\omega}$$

$$= \frac{2/3}{1 + j(2/3)(1.5)}$$

$$= \frac{2/3}{1 + j\omega_{\circ}}$$

The magnitude and phase of the voltage transfer function V2'/V1' and V2/V1 for the network and normalized network are shown in figures 1.2-a and 1.2-b. Comparison of the response curves for the network and the normalized network shows that they are quite similar. In fact, the curves for the network response can be obtained from the normalized network response by "sliding" it a distance of $\omega_0 = 500$ to the right.

Now we can use the normalized network to find a new network with a similar frequency response, but with a breakpoint of $\omega = 2000$ and with a C1' equal to 2μf. This frequency response can be obtained by shifting the frequency response curve of the normalized network to the right (figure 1.2-a) by a factor of 2000. The breakpoint of the new network frequency response curve is given by

$$\omega_c' = 2000\omega_c = 2000.$$

This is the ω_0 to use to find the element values of the new network when performing the reverse of the normalization process as given in the last column of table 1.1.

The value of R_0 can be found by considering that

$$C1 = \omega_0 C1' R_0$$

and that C1′ is specified to be 2µf. Thus

$$R_o = \frac{C1}{\omega_o C1'} = \frac{1.5}{(2x10^3)(2x10^{-6})} = 375\Omega.$$

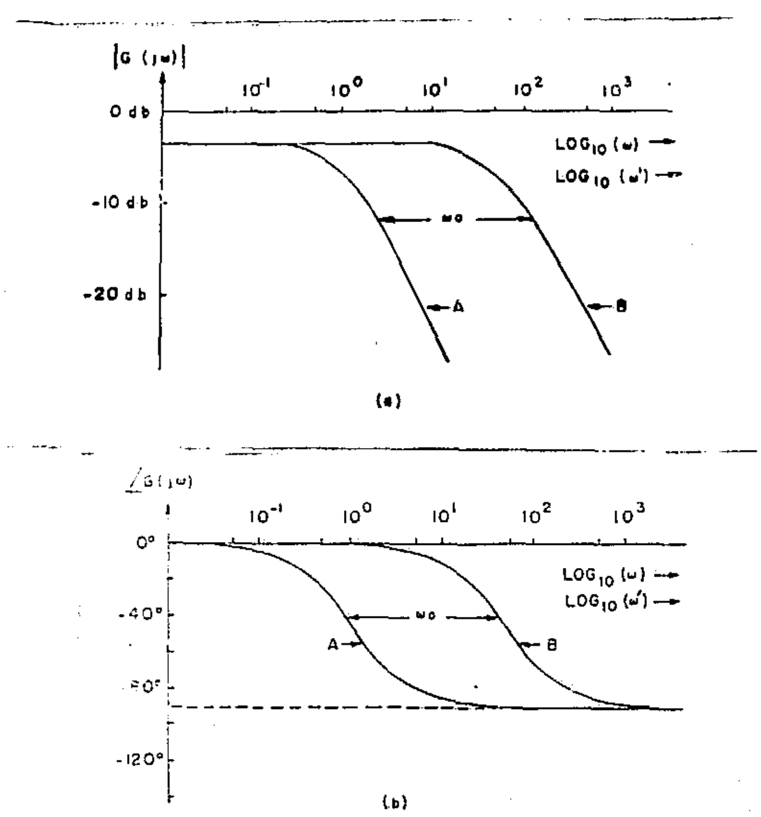

Figure 1.2 Magnitude and phase of voltage transfer function G(jω) for network (B) and normalized network (A) shown in figure 1.1-a and 1.1-b. (a) Magnitude. (b) Phase.

Now using the relationship found in table 1.1, the physical parameters of the new network are

$$R1' = R1R_o = (1)(375) = 375\Omega$$

$$R2' = R2R_o = (2)(375) = 750\Omega$$

$$C1' = C1/\omega_o R_o = 1.5/(2x10^3)(375) = 2.0\mu f.$$

For these values of R1′, R2′ and C1′ the voltage transfer function G(jω) is given by

$$G(j\omega) = \frac{\frac{750}{1125}}{1 + j\left(\frac{281250}{1125}\right)(2x10^{-6})\omega}$$

$$= \frac{2/3}{1 + j5x10^{-4}\omega.}$$

The breakpoint for this function is given by

$$\omega_c' = \frac{1}{5x10^{-4}} = 2000 \text{ radian/sec.}$$

as desired.

This example then has shown how a network is normalized using table 1.1. The effect upon the voltage transfer function V2'/V1' of the original network was to shift it to the left by a factor of ω_0 as shown in figure 1.2.

1.3 Frequency transformation. In the example presented in section 1.2, a low-pass filter was obtained by transforming a normalized low-pass filter to a new filter with the desired frequency response and impedance level. Similar transformations can be used to obtain high-pass, band-pass, multiple-bandpass, and other types of filters from a normalized low-pass filter. The normalized filter typically has a 1 ohm design resistance and a 1 radian cutoff frequency. A step-by-step description of the following four transformations will be given:

1. Normalized low-pass to a low-pass filter.
2. Normalized low-pass to a high-pass filter.
3. Normalized low-pass to a band-pass filter.
4. Normalized low-pass to a band-elimination.

The first transformation has already been used in section 1.2. Transformation (1) is stated in two parts, an impedance level shift and then a frequency transformation. The impedance level change is the same for all four transformations. For transformation (2) the impedance level shift and frequency transformation are combined in one step. In transformations (3) and (4) the impedance level shift is omitted for simplicity.

<u>Normalized low-pass filter to low-pass filter.</u>

Description: This transformation converts a normalized low-pass filter (1 ohm, 1 radian) to a low-pass filter with a design resistance of R_0 ohms and a cutoff frequency of ω_c radians. This transformation is accomplished in two steps, the first is an impedance level change, the second is a shift of the frequency response of the normalized low-pass filter. This transformation is an application of table 1.1.

Transformation:

1. Shift of impedance level, from 1 ohm to R_0 ohms.
   a. Substitute a resistance $R_0 R$ for each resistance R.
   b. Substitute a capacitance C/R_0 for each capacitance C.
   c. Substitute an inductance $R_0 L$ for each inductance L.
2. Shift of cutoff frequency, from $\omega = 1$ radian to $\omega = \omega_c$ radians.
   d. Substitute an inductance L/ω_c for each inductance L.
   e. Substitute a capacitance C/ω_c for each capacitance C.

Using this transformation a single normalized low-pass filter provides a basis from which many low-pass filters can be obtained.

Normalized low-pass filter to high-pass filter

Description: This transformation converts a normalized low-pass filter (1 ohm, 1 radian) to a high-pass filter with a design resistance of R_0 ohms and a cutoff frequency of ω_c radians. The impedance level change and the frequency transformation have been combined in this transformation. The magnitude of the frequency response $G(j\omega)$ for the resulting high-pass filter is described by

1. a region of passband for $|\omega| > \omega_c$ and
2. a region of stopband for $|\omega| < \omega_c$.

Transformation:

a. Substitute a resistance $R_0 R$ for each resistance R.

b. Substitute for each inductance L a capacitance $1/R_0 L\omega_c$.

c. Substitute for each capacitance C an inductance $R_0/\omega_c C$.

Normalized low-pass filter to band-pass filter

Description: This transformation converts a normalized low-pass filter (1 ohm, 1 radian) to a band-pass filter with a design resistance of 1 ohm and a band-pass region between the frequencies ω_1 and ω_2. An impedance level shift can be performed, if desired, in the same manner as was done in the normalized low-pass filter to low-pass filter transformation. The magnitude of the frequency response $|G(j\omega)|$ for the band-pass filter is described by

1. a region of pass band for $\omega_1 < \omega < \omega_2$ and
2. a region of stop band for $\begin{pmatrix} |\omega| < \omega_1 \\ |\omega| > \omega_2 \end{pmatrix}$.

In this transformation inductors are replaced by LC series resonant circuits and capacitors are replaced by LC tank circuits. The resonant frequency of these circuits is denoted by ω_r. The bandwidth of the filter is denoted by B and is found from

$$B = \omega_2 - \omega_1 .$$

The resonant frequency ω_r of the series and parallel resonant LC circuits is given by

$$\omega_r = \sqrt{\omega_1 \omega_2} .$$

Transformation:

a. Substitute for each inductance L a series resonant circuit with an inductance of L/B and capacitance of $B/L\omega_r^2$. The resonant frequency of this circuit is

$$\frac{1}{\sqrt{(L/B)(B/L\omega_r^2)}} = \omega_r = \sqrt{\omega_1 \omega_2} .$$

b. Substitute for each capacitance C a tank circuit with an inductance of $B/C\omega_r^2$ and capacitance of C/B. The resonant frequency of this circuit is

$$\frac{1}{\sqrt{(B/C\omega_r^2)(C/B)}} = \omega_r = \sqrt{\omega_1 \omega_2} .$$

Normalized low-pass filter to band-elimination filter

Description: This transformation converts a normalized low-pass filter (1 ohm, 1 radian) to a band-pass filter with a design resistance of 1 ohm and a band-elimination region between the frequencies ω_1 and ω_2. An impedance level shift can be performed, if desired, in the same manner as was done in the normalized low-pass filter to low-pass filter transformation. The magnitude of the frequency response $|G(j\omega)|$ for the band-elimination filter is described by

1. a region of pass band for $\begin{pmatrix} |\omega| < \omega_1 \\ |\omega| > \omega_2 \end{pmatrix}$ and

2 a region of stop band for $\omega_1 < \omega < \omega_2$.

In this transformation inductors are replaced by LC tank circuits and capacitors are replaced by LC series resonant circuits. The resonant frequency of these circuits is denoted by ω_r. The band width of the filter is denoted by B and is found from

$$B = \omega_2 - \omega_1 .$$

The resonant frequency ω_r of the series and parallel resonant LC circuits is given by

$$\omega_r = \sqrt{\omega_1 \omega_2} .$$

Transformation:

a. Substitute for each inductance L a tank circuit with an inductance 1/BL and a capacitance BL/ω_r^2. The resonant frequency of this circuit is

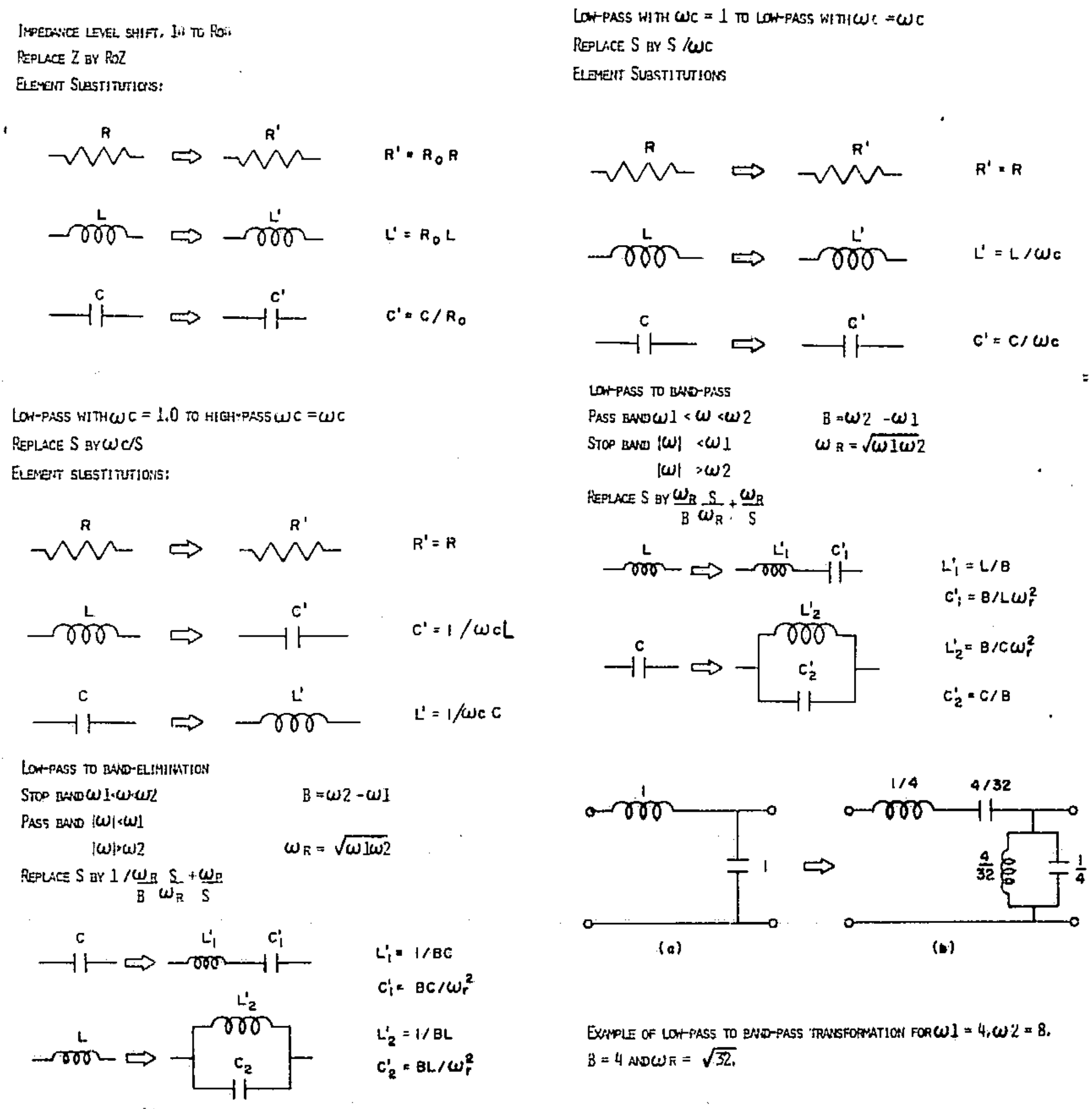
IMPEDANCE LEVEL SHIFT, 1Ω TO R0Ω
REPLACE Z BY R0Z
ELEMENT SUBSTITUTIONS:
R' = R0 R
L' = R0 L
C' = C / R0
LOW-PASS WITH ωc = 1.0 TO HIGH-PASS ωc = ωc
REPLACE S BY ωc/S
ELEMENT SUBSTITUTIONS:
R' = R
C' = 1 / ωcL
L' = 1/ωc C
LOW-PASS TO BAND-ELIMINATION
STOP BAND ω1<ω<ω2
B = ω2 - ω1
PASS BAND |ω|<ω1
|ω|>ω2
ωR = √ω1ω2
REPLACE S BY 1 / ωR/B (S/ωR + ωR/S)
L'1 = 1/BC
C'1 = BC/ωr²
L'2 = 1/BL
C'2 = BL/ωr²
LOW-PASS WITH ωc = 1 TO LOW-PASS WITH ωc = ωc
REPLACE S BY S /ωc
ELEMENT SUBSTITUTIONS
R' = R
L' = L / ωc
C' = C/ ωc
LOW-PASS TO BAND-PASS
PASS BAND ω1 < ω < ω2
B = ω2 - ω1
STOP BAND |ω| < ω1
ωR = √ω1ω2
|ω| > ω2
REPLACE S BY ωR/B (S/ωR + ωR/S)
L'1 = L/B
C'1 = B/Lωr²
L'2 = B/Cωr²
C'2 = C/B
1/4
4/32
4/32
1/4
(a)
(b)
EXAMPLE OF LOW-PASS TO BAND-PASS TRANSFORMATION FOR ω1 = 4, ω2 = 8.
B = 4 AND ωR = √32.

Table 1.2

$$\frac{1}{\sqrt{(1/BL)(BL/\omega_r^2)}} = \omega_r = \sqrt{\omega_1 \omega_2}$$

b. Substitute for each capacitance C a series resonant circuit with an inductance 1/BC and a capacitance of BC/ω_r^2. The resonance frequency of this circuit is

$$\frac{1}{\sqrt{(1/BC)(BC/\omega_r^2)}} = \omega_r = \sqrt{\omega_1 \omega_2}.$$

Table 1.2 is a summary of these transformations. The impedance level shift is treated as a separate transformation in table 1.2. Also included in table 1.2 is an example of normalized low-pass filter to band-pass filter for $\omega_1 = 4$ and $\omega_2 = 8$. In this case $B = 4$ and $\omega_r = \sqrt{32}$.

1.4 Generalized frequency transformation. The foregoing frequency transformations are all specific examples of a general frequency transformation. If $A(\omega)$ is the attenuation characteristic of a normalized low-pass filter where $A(\omega)$ has a passband for $|\omega| < 1$ and a stopband for $|\omega| > 1$, the filter may be transformed to a new filter with attenuation characteristics $A[X(\omega)]$ such that $A[X(\omega)]$ has a passband for $|X(\omega)| < 1$ and a stopband for $|X(\omega)| > 1$. In order to realize such a filter each L or C element of the normalized filter whose impedances are a function of ω must be modified so that their impedances are a function of $X(\omega)$. For example, if the impedance function $ZL = j\omega L$ appears in the normalized filter, it must be replaced by an impedance $jX(\omega)L$. The impedance $ZC = 1/j\omega C$ must be replaced by an impedance $1/jX(\omega)C$. For the transformation to be realizable, it is sufficient that the quantities $jX(\omega)L$ and $1/jX(\omega)C$ are realizable reactances.

To transform a normalized low-pass filter with an attenuation characteristic $A(\omega)$ to $A[X(\omega)]$, proceed as follows:

a. Replace each inductive reactance $j\omega L$ by the reactance $jX(\omega)L$.

b. Replace each capacitive reactance $1/j\omega C$ by the reactance $1/jX(\omega)C$.

For an example, the low-pass to band-pass filter transformation will be developed using the general transformation. A general form for $X(\omega)$ from which realizable filters will result is

$$X(\omega) = H\omega \frac{(\omega_i^2 - \omega^2)(\omega_j^2 - \omega^2)....}{(\omega_m^2 - \omega^2)(\omega_n^2 - \omega^2)....}$$

where $0 \leq \omega_m < \omega_i < \omega_n < \omega_j......$

In order to develop the band-pass filter transformation found in table 1.2, let

$$X(\omega) = H \frac{(\omega^2 - \omega_i^2)}{\omega}.$$

The nature of the attenuation characteristic of the filter that will be produced by the transformation can be ascertained by remembering that it will have a region of passband for ω such that $|X(\omega)| < 1$ and a stop band for ω such that $|X(\omega)| > 1$. For the $X(\omega)$ that has been selected

$$|X(0)| = \infty > 1$$
$$|X(\omega_i)| = 0 < 1$$
$$|X(\infty)| = \infty > 1.$$

Thus the attenuation characteristic of the new filter must have a region of stopband, a region of passband, and then a region of stopband as ω goes from 0 to ∞. This description of the attenuation characteristics fits that of a band-pass filter. In order to investigate the new characteristics more fully the frequencies where $X(\omega) = \pm 1$ will now be considered.

The cutoff frequencies are the ω's where $X(\omega) = \pm 1$. Let ω_1 designate the lower cutoff frequency and ω_2 the upper cutoff frequency. Then the following four equations may be written:

$$X(\omega_1) = H \frac{\omega_1^2 - \omega_r^2}{\omega_1} = -1 \qquad (1.1)$$

$$X(-\omega_1) = H \frac{\omega_1^2 - \omega_r^2}{\omega_1} = +1 \qquad (1.2)$$

$$X(\omega_2) = H \frac{\omega_2^2 - \omega_r^2}{\omega_2} = +1 \qquad (1.3)$$

$$X(-\omega_2) = H \frac{\omega_2^2 - \omega_r^2}{-\omega_2} = -1 \qquad (1.4)$$

as $X(\omega) = \pm 1$ for the cutoff frequencies, ω_1 and ω_2. The plus or minus sign is assigned by assuming that $\omega_1 < \omega_r < \omega_2$. In these equations ω_r has been used in place of ω_i for convenience in notation in a later step. Now using equation (1.1) and equation (1.4) we find that

$$\frac{\omega_1^2 - \omega_r^2}{\omega_1} = \frac{\omega_2^2 - \omega_r^2}{\omega_2}$$

which can be solved for ω_r as

$$\omega_r = \sqrt{\omega_1 \omega_2}\,. \tag{1.5}$$

Now substituting equation (1.5) into equation (1.1) we find that

$$H\,\frac{\omega_1^2 - \omega_1\omega_2}{\omega_1} = -1, \text{ or}$$

$$H = \frac{1}{\omega_2 - \omega_1} = \frac{1}{B}$$

where $B = \omega_2 - \omega_1$.

Thus the general transformation $X(\omega)$ to be used to transform a normalized low-pass filter to a band-pass filter is given by

$$X(\omega) = \frac{1}{B}\left(\frac{\omega^2 - \omega_r^2}{\omega}\right) \tag{1.6}$$

$$= \frac{\omega_r}{B}\left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right)$$

The first step of the general transformation is to replace the inductive reactance $j\omega L$ by the reactance $jX(\omega)L$. Now consider a typical inductance L of the normalized low-pass filter. After the replacement is made its reactance is given by

$$j\,\frac{\omega_r}{B}\left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right)L = \frac{\omega_r}{B}\left(\frac{j\omega}{\omega_r} + \frac{\omega_r}{j\omega}\right)L$$

Now replace $j\omega$ by s to get

$$\frac{\omega_r}{B}\left(\frac{s}{\omega_r} + \frac{\omega_r}{s}\right)L = \frac{sL}{B} + \frac{\omega_r^2 L}{sB}$$

which can be recognized as an inductor $L' = L/B$ and a capacitor $C' = B/\omega_r^2 L$ in series. The resonant frequency of the series circuit is

$$\frac{1}{\sqrt{(L/B)(B/\omega_r^2 L)}} = \omega_r .$$

The second step of the general transformation is to replace the capacitive reactance $1/j\omega C$ by the reactance $1/jX(\omega)C$,

$$\frac{1}{\frac{j\omega_r}{B}\left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right)C} = \frac{1}{\frac{j\omega C}{B} + \frac{\omega_r^2}{j\omega B}} .$$

Now replace $j\omega$ by s to get

$$\frac{1}{\frac{sC}{B} + \frac{\omega_r^2 C}{sB}}$$

which is recognized as a tank circuit with an inductance of $B/\omega_r^2 C$ and a capacitance of C/B. The resonant frequency of this tank circuit is found from

$$\frac{1}{\sqrt{(B/\omega_r^2 C)(C/B)}} = \omega_r .$$

1.5 Example: Use of a frequency transformation. The previous transformations can be very useful in filter design. For example, a band-pass filter may be designed by the following method:

1. Transform the specifications of a desired filter to specifications of a normalized low-pass filter.
2. Using the specifications of the normalized low-pass filter, select an appropriate low-pass filter and obtain element values for the selected filter.
3. Transform the normalized low-pass filter to the desired filter.

In this example, the first step of the above method is illustrated. The specifications of a normalized low-pass filter which will transform into a band-pass filter with a desired frequency response is found. The desired band-pass attenuation characteristic is specified by table 1.3.

Frequency range in hertz	Attenuation in db
0 - 3,350	50db
3,350 - 4,000	Unspecified
4,000 - 8,000	0 - 3db
8,000 - 8,150	Unspecified
8,150 - ∞	30db

Table 1.3 Specification of a band-pass filter

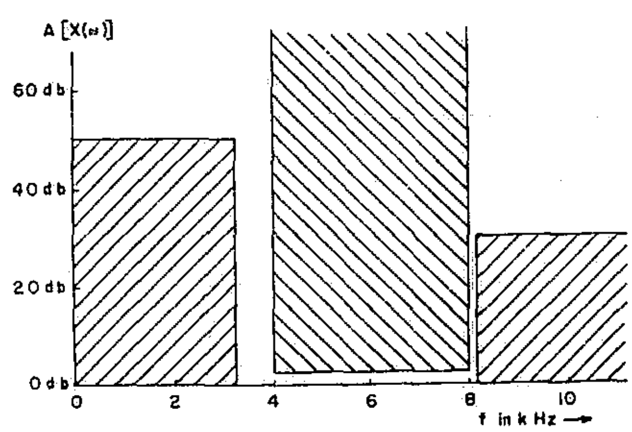

Figure 1.3 Specification of a band-pass filter

Figure 1.3 is a graphical representation of these specifications for a band-pass filter. The attenuation curve of the filter must not enter any portion of the shaded area of figure 1.3. These specifications for the desired band-pass filter now are stated as specifications for a normalized low-pass network. This is accomplished by application of the normalized low-pass filter to band-pass filter transformation in a reverse manner. To do this the lower and upper cutoff frequencies of the band-pass filter must be selected. By inspection of figure 1.3 it is seen that selection of the upper cutoff frequency presents the most critical choice to the designer since the gap of unspecified attenuation in this region is more narrow than that of the gap in the region of unspecified attenuation near the lower frequency cutoff. Let the upper cutoff frequency be in the middle of the narrow gap

$$f_2 = \frac{8000 + 8,150}{2} = 8,075 \text{ hertz.}$$

The lower cutoff frequency is selected, somewhat arbitrarily to be

$$f_1 = 3,800 \text{ hertz}$$

which results in a bandwidth B, in hertz, of

$$B = f_2 - f_1 = 4,275 \text{ hertz.}$$

The frequency transformation given by equation (1.6), restated here for convenience,

$$X(\omega) = \frac{1}{B} \frac{\omega^2 - \omega_r^2}{\omega} \qquad (1.6)$$

will now be used to find the attenuation specifications of the normalized low-pass filter from those of the band-pass filter. The specifications of table 1.3 are constraints upon the attenuation $A[X(\omega)]$, the attenuation characteristics of the band-pass filter. A radian frequency ω of the band-pass filter corresponds to a frequency $X(\omega)$ of the normalized low-pass filter. Thus, the ranges of ω, given in table 1.3 can be directly transformed into ranges of the normalized frequency for the normalized network by use of equation (1.6). This will then give the attenuation specifications for $A(\omega)$, the attenuation characteristic of the normalized network. For example, equation (1.6) can be written as

$$X(\omega) = \frac{\omega^2 - \omega_1\omega_2}{\omega(\omega_2 - \omega_1)} = \frac{\omega - \frac{\omega_1\omega_2}{\omega}}{(\omega_2 - \omega_1)}$$

from which it can be seen that for $\omega = 0$, $X(\omega) = -\infty$. Also, for $\omega = \omega_2$ we find $X(\omega) = 1$ and for $\omega = \omega_1$, $X(\omega) = -1$. For convenience in calculating the constraints upon $A(\omega)$, rewrite equation (1.6) as

$$X(2\pi f) = \frac{4\pi^2 (f - f_1 f_2)}{4\pi^2 f(f_2 - f_1)} \quad . \qquad (1.7)$$

Now using equation (1.7) the attenuation specifications for the low-pass filter are found and recorded in table 1.4.

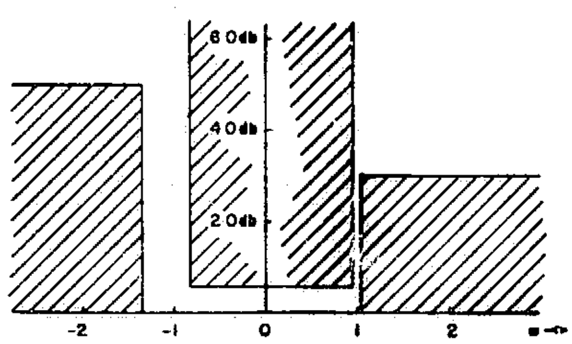

Figure 1.4 Graphical representation of attenuation specifications of table 1.4 - asymmetric.

Band-pass filter frequency, hertz	Attenuation specifications in db	Low-pass filter frequency, radians
0 to 3,350	50	-∞ to -1.356
3,350 to 3,800	Unrestricted	-1.356 to -1.0
3,800 to 4,000	Unrestricted	-1.0 to -0.8588
4,000 to 8,000	0-3	-0.8588 to 0.974
8,000 to 8,075	Unrestricted	0.974 to 1.0
8,075 to 8,150	Unrestricted	1.0 to 1.0254
8,150 to ∞	30	1.0254 to ∞

Table 1.4 Asymmetric specifications

The attenuation characteristic of the normalized low-pass filter $A(\omega)$ must be symmetric with respect to $\omega = 0$. Inspection of figure 1.4 shows that these specifications are not symmetrical. This situation is remedied by selecting, from table 1.4, the most stringent specifications. The most stringent specifications become apparent if figure 1.4 is drawn on a piece of tracing paper, folded back upon itself along the vertical axis, and viewed by holding a light behind the paper. Such a set of specifications is given in table 1.5.

Figure 1.5 is a graphical representation of the symmetric specifications given in table 1.5 for $A(\omega)$, the attenuation characteristic of the required normalized low-pass filter. The next step in the design process is to obtain a normalized low-pass filter that meets these attenuation specifications. This is done in later chapters of this book.

Low-pass filter frequency, radian	Attenuation specifications in db
- ∞	50 db
-1.356 to -1.0254	30 db
-1.0254 to -0.974	Unrestricted
-0.974 to 0.974	0 - 3 db
0.974 to 1.0245	Unrestricted
1.0254 to 1.356	30 db
1.356 to ∞	50 db
±1.00	Cutoff frequency

Table 1.5 Symmetric specifications

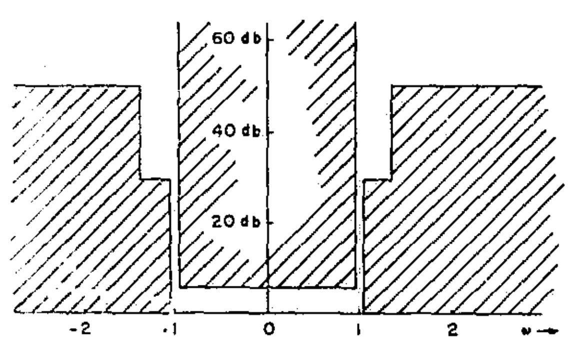

Figure 1.5 Graphical representation of attenuation specifications of table 1.5 - symmetric.

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CHAPTER TWO

Analysis of Filters Using Flowgraph Techniques

2.1 Introduction. This chapter is presented to familiarize the user of this handbook with the basic computer program NASAP-69, which is primarily an analysis tool and if used with some discretion, an aid in filter synthesis. At this point the reader may ask, "Why use NASAP-69? Won't ECAP or SCEPTRE, for example, handle filter problems with ease?" The answers to these and other questions should become apparent to the reader as he progresses through this chapter. Other computer programs do handle filters very well and perhaps NASAP-69 is not the optimum filter program, if one exists, but at this time NASAP-69 does offer substantial advantages in the frequency domain over other programs and therefore is helpful in the analysis and synthesis of filters.

What is NASAP-69? NASAP-69 stands for the 1969 version: Network Analysis for Systems Application Program. Specifically, it is a digital computer program developed and maintained by the Electronics Research Center of NASA. NASAP-69 has been developed for the circuit designer, offering him a number of computational packages available as one program. The program and algorithms within are not merely silhouettes of existing programs but are an alternate approach, as will become evident in this chapter.

2.2 Signal Flowgraphs. A brief but detailed review of linear graph theory is needed since NASAP-69 uses flowgraph techniques and algorithms as the backbone of the program.

Claude Shannon discovered the topological gain formula for open flowgraphs during World War II, but his work was never published.[1] In 1952, Samuel Mason[2] rediscovered the same formula.

Coates in the late 50's developed a slightly different variation of the Mason formula. W.W. Happ generalized the Shannon formula to include flowgraphs and went on further to develop flowgraph sensitivity algorithms.

Flowgraphs are perhaps best described as a collection of nodes and directed line segments called transmittances constructed to satisfy Kirchhoff's Voltage Law (KVL), Kirchhoff's Current Law (KCL), and Voltage-Current (V-I) relationships for a particular circuit under study.

Consider the following simple filter, Figure 2.1.

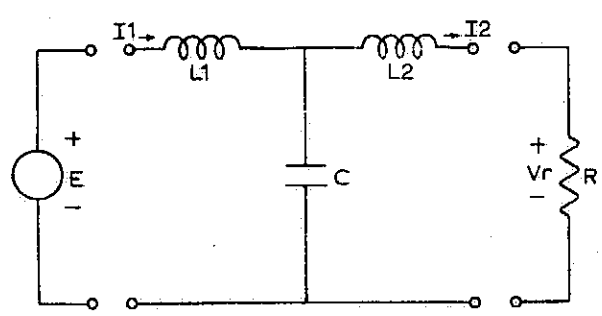

Figure 2.1. Simple LC filter.

Writing the Laplace transformed KVL, KCL, and V-I relationships results in:

$$E = I1(L1s + 1/Cs) + I2(-1/Cs)$$

$$0 = I1(-1/Cs) + I2(1/Cs + R + L2s).$$

Figure 2.2 is the Mason graph for the above system of equations which may be rewritten as:

$$I1 = I2(1/Cs + R + L2s)/(1/Cs)$$

$$I2 = (E - I1(L1s + 1/Cs))/(-1/Cs)$$

$$Vr = RI2.$$

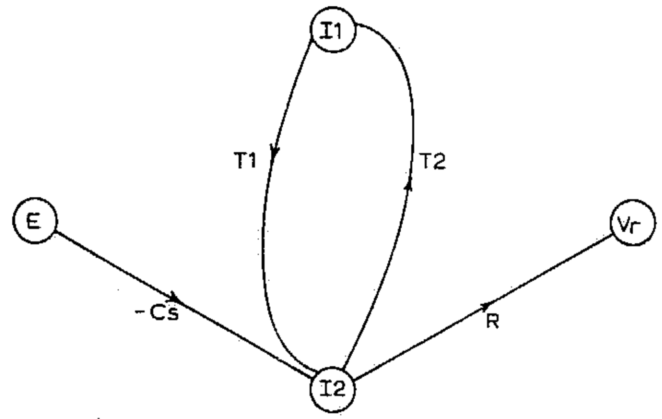

Figure 2.2 Mason flowgraph of figure 2.1.
$T1 = Cs(L1s + 1/Cs)$, $T2 = Cs(L2s + R + 1/Cs)$.

The Mason flowgraph of figure 2.2 is termed an open flowgraph and the relationship between E and Vr can be found by removing the loop between nodes I1 and I2, as shown in figure 2.3.

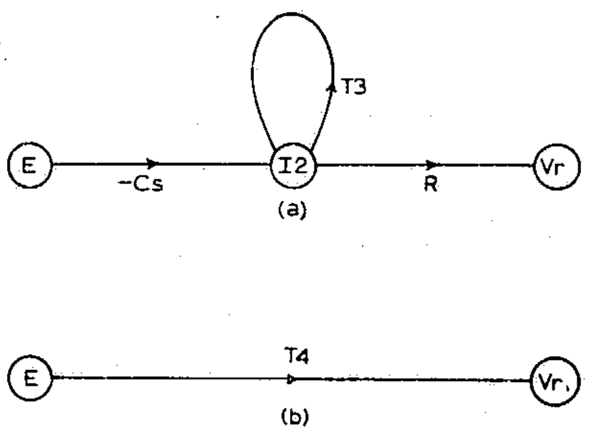

Figure 2.3. (a) Self loop at node I2. (b) Removal of node I2.
$T3 = (Cs)^2(L1s + 1/Cs)(L2s + R + 1/Cs)$, $T4 = (-RCs)/(1-T3)$.

The final gain then by Mason's method is:

$$\frac{Vr}{E} = \frac{R}{L1L2Cs^3 + L1CRs^2 + (L1 + L2)s + R}.$$

One can solve simple circuits such as this example readily, but a better method is needed, one that is more adaptable to computer usage. Happ developed an algorithm to close the flowgraph, thereby solving the flowgraph problem in terms of feedback loops.[3] The preceding example was presented and solved heuristically and now a review of linear graph theory will be discussed prior to the flowgraph algorithm.

2.3 Linear Graph Theory and Flowgraphs. It is assumed that the reader is familiar with linear graph theory, namely the following terms: nodes, edges, cut-sets, trees, and loops of linear graphs. A junction of two or more elements in a circuit corresponds to a node of the linear graph. Each element is represented by an edge between two nodes of the graph and an oriented edge generally denotes the assumed current direction through the corresponding element. A tree is a collection of edges such that all nodes of the graph are connected, yet there are no loops formed by the edges. If one selects the tree branches to be voltage elements, it then follows that every cotree link voltage can be expressed in terms of the voltages using KVL. Usually the tree branch voltages are referred to as an independent set of voltages from which any voltage of the graph can be calculated. The cotree links, on the other hand, form an independent set of currents (if the cotree links are considered to be current elements). NASAP-69 has an algorithm which formulates the voltage equations in terms of the independent set of voltages and the current equations in terms of the independent set of currents. Some versions of NASAP have a routine which picks an acceptable tree to form these equations, other versions require the designer to pick a tree, and still other versions find an optimum tree (to be defined later).

Before a flowgraph can be constructed the voltage current relationships must be provided for each element. A resistor, for example, would be either a voltage controlled current element or a current controlled voltage element.

An element could also be a transconductance of a dependent current source in a F.E.T. or perhaps the voltage gain of an operational amplifier. In the first case current would be dependent on another element's voltage and in the latter case the element would have a voltage to voltage relationship.

Before proceeding to the flowgraph gain algorithm, it is necessary to define the basic concept of a flowgraph. Each element of a circuit is transformed into two nodes in the flowgraph. A resistor, R, for example, is represented by a current node and a voltage node in the flowgraph. Between the nodes is a directed path, called a transmittance.

The value of transmittance is R if the resistor is a voltage element or 1/R if the resistor is a current element (i.e., if the resistor is in the tree it is a current controlled voltage source with a voltage of IR, or if the resistor is in the cotree it is a voltage controlled current source with a current of V/R). The node at which the directed path begins is called the origin node and the other node is then the target node. If there are N elements in a circuit the flowgraph then contains N origin nodes and N target nodes. By applying the concepts just discussed it is easy to show that: (1) the set of voltage origin nodes of a flowgraph form an independent set of tree voltages, and (2) the set of current origin nodes of a flowgraph similarly form an independent set of cotree currents. It can be shown that KVL can be represented by directed paths (with values of +1 or -1) from the voltage origin nodes to the voltage target nodes. Note that for a passive tree element a voltage is a target node for its own current node but that it becomes an origin node to the other voltage nodes. KCL may be represented in an analogous fashion using current target and current origin nodes. It is important to note that all transmittances leaving an origin node are directed away from the node, however, they may carry a weight of minus one.

A directed loop is defined as a closed path consisting of a sequence of transmittances taken in the direction of the arrow. No node may be traversed more than once for each closed path. The value of the loop is the product of the transmittances' weights of the directed loop. There are three types of loops pertinent to the flowgraph algorithm. First, there are the first order loops defined above as simple directed loops. Secondly, there are higher order loops defined as node disjointed first order loops. Node disjointed loops have no nodes in common and their value is the product of the values of the first loops of which they are composed. Third, and last, is the zero order loop defined to have a value of one, but with no flowgraph physical significance.

As an example of the terminology, consider again figure 2.1. The flowgraph for this circuit is shown in figure 2.4. Nodes 1,2,3,4, and 5 correspond to elements E, L1, C, L2, and R, respectively. The tree was selected to be the elements E, C, and R. As a result, the independent set of voltage nodes is nodes 1, 3, and 5 and the independent set of current nodes is 2 and 4.

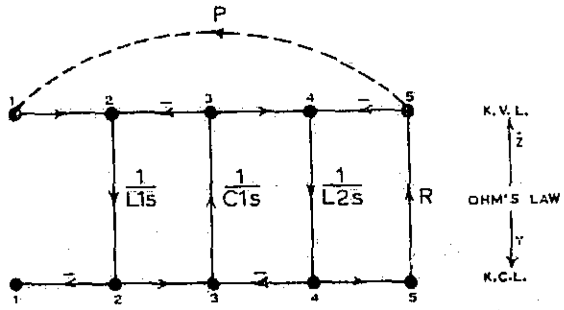

Figure 2.4 Flowgraph of figure 2.1.

The top row of nodes is considered to be voltage nodes and therefore the interconnections between the nodes must satisfy KVL. Nodes 1,3, and 5 are the independent set of voltage nodes and will be called the voltage origin nodes. Nodes 2 and 4 are the dependent set of voltage nodes and will be called the voltage target nodes. The lower row of nodes of the flowgraph is the dual of the upper row of nodes, that is, the lower nodes must satisfy KCL. The reader should be able to verify this concept and the representation of KVL and KCL in the flowgraph. It should be clear that the voltage origin and target nodes of the top row become the current target and origin nodes of the bottom row, respectively. The interconnections between the upper and lower rows are just a graphical representation of Ohm's Law.

In figure 2.4 there are three first order loops. One such loop is from node 2 to node 3 and back to node 2. This loop has a weight of $-1/(L1Cs^2)$. Another first order loop would be from node 4 to node 5 and back to node 4 with a weight of -R/L2s. The two loops just mentioned have no common element (i.e., no common node in the flowgraph) and therefore they are called node disjointed loops of the second order with a combined weight of $R/(L1L2Cs^3)$.

Flowgraphs may be of two types: the open flowgraph and the closed flowgraph. Consider once again a flowgraph and impose an additional constraint on the flowgraph. This constraint is a transmittance with a weight that is the reciprocal of the ratio of the two nodes which it connects. For example, if the transmittance denoted by P connects two current nodes, say I_i and I_j such that $I_i = PI_j$ then we would say P is the value of the current gain between nodes i and j. This transmittance, as the example implies, is the cause-effect relationship for which we are solving. Generally this transmittance is called a "dummy" transmittance and is denoted as P. Any flowgraph modified in this manner is called a closed flowgraph and any flowgraph not modified in this manner is called an open flowgraph.[4] The solid lines of figure 2.4 obviously form the open flowgraph of figure 2.1. If we wish to solve for an input-output voltage gain, we would close the flowgraph in figure 2.4 by connecting node 1 to node 5 by the dashed line, also shown in figure 2.4. Figure 2.4 with P included is now a closed flowgraph.

Flowgraphs as a rule have no common topological structure. However, for non-active, ladder type filters the general closed flowgraph appears in figure 2.5. The structure of the flowgraph becomes more involved as the circuit departs from the pure ladder network. In most cases dealing with filters the dummy transmittance represents a voltage gain since the attenuation and phase shift of the total response are usually the prime objectives.

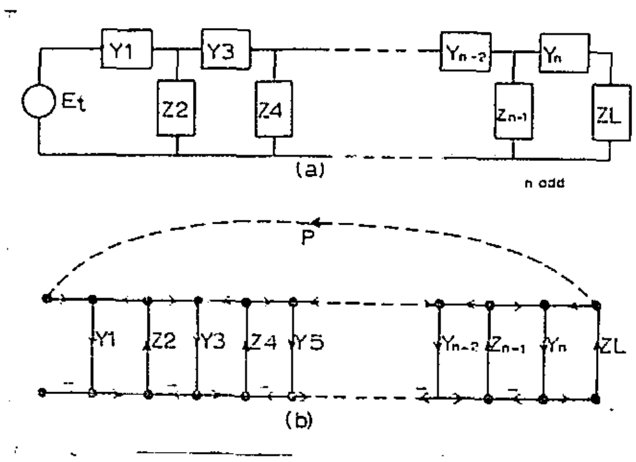

Figure 2.5 (a) Ladder filter. (b) General flowgraph.

Perhaps at this point one should note that the flowgraph of figure 2.5 is not the only possible flowgraph, but is a flowgraph which contains the minimum number of flowgraph loops.

2.4 Topology Equation. NASAP-69 is a topological program which solves for the transfer function in terms of flowgraph loops. We have already seen one topological solution illustrated previously which was solved by Mason's method. This method however is not as adaptable to computer usage as the topological algorithm used in NASAP-69.

The NASAP-69 topology algorithm was developed by Happ as an extension of work done previously by Shannon, and Mason. In recent years the topology equation has become known simply as H, where H is defined as:

$$H = \sum_{\text{over all orders}} (-1)^N L(N). \qquad (2.1)$$

L(N) is defined as the sum of the values of all the loops with order N. L(0) is defined as the zeroth order loop with a value always equal to one. The summation is from zero to K where K is the highest order of flowgraph loops. Note also that the sign depends on the order of the loop so that in effect:

$$H = 1 - \Sigma \text{ first order loops} + \Sigma \text{ second order loops} - \ldots\ldots\ldots\ldots + \Sigma \text{ nth order loops.}$$

Happ has shown that if the flowgraph is closed, the topology equation becomes singular and solvable for the unknown dummy transmittance.[5] That is, if H = 0, then we can solve for P, the dummy transmittance, whose value is the transfer function between the two nodes which P connects.

H is composed to two types of loops, those that contain the dummy transmittance, P, and those that do not. Since the value of P is included in the value of the loops which contain P and no loops contain any multiple powers of P, we may write:

$$H = PH(P') + H(\overline{P}) \tag{2.2}$$

where $PH(P')$ denotes those loops that contained P and $H(\overline{P})$ denotes those loops that do not contain P. Since H = 0, we may solve for P:

$$P = -H(\overline{P})/H(P'). \tag{2.3}$$

P is then the negative ratio of the loops which do not contain P over the loops which did contain P but are now deprived of P. NASAP-69 finds all the loops of a flowgraph and merely sorts the loops into two types and sums their values, thereby producing the transfer function.

Recall the example presented earlier. The loops and their values are from the closed flowgraph of figure 2.4:

	Nodes	
Loop 1	123451	P(1/L1s)(1/Cs)(1/L2s)R
Loop 2	232	-(1/L1s)(1/Cs)
Loop 3	343	-(1/Cs)(1/L2s)
Loop 4	454	-R(1/L2s)
Loop 5	232-454	(1/L1s)(1/Cs)(-R)(1/L2s)

Solving for Vr/E:

$$\frac{Vr}{E} = \frac{1}{P} = \frac{-R/(L1L2Cs^3)}{1 + 1/(L1Cs^2) + 1/(L2Cs^2) + R/L2s + R/(L1L2Cs^3)}$$

$$= \frac{R}{L1L2Cs^3 + L1CRs^2 + (L1 + L2)s + R.}$$

Note at this point that the zeroth order loop was included in the denominator and that the sign changes for some of the terms. The results agree with the results found previously by a type of Mason's method.

2.5 Sensitivity Analysis. Algorithms have also been developed and incorporated into NASAP-69 which relate various types of sensitivity coefficients to the topology structure of the flowgraph. For the interested reader the development of these algorithms is presented in various papers but only a brief description of an algorithm will be discussed here.[6]

If P is the transfer function and Q is an arbitrary circuit component parameter then the three types of sensitivities may be written:

$$G_Q^P = Q/P, \text{ the large signal sensitivity,}$$

$$B_Q^P = \frac{dQ}{dP}, \text{ the small signal sensitivity,}$$

and $$S_Q^P = \frac{d(\log P)}{d(\log Q)} = \frac{dP}{P} - \frac{Q}{dQ}, \text{ the classical Bode sensitivity.}$$

In terms of loops S_Q^P may be written:

$$S_Q^P = \frac{H(\overline{Q},P')}{H(P')} - \frac{H(\overline{Q},\overline{P})}{H(\overline{P})}$$

where $H(\overline{Q},P')$ are the loops devoid of Q, containing P, but deprived of P, and $H(\overline{Q},\overline{P})$ are the loops devoid of both P and Q.

The various versions of NASAP differ in their ability to deal with sensitivity functions. The standard program allows only the sensitivity of the transfer function with respect to one element. While the NASAP-MU version calculates the sensitivity function for each element, if desired. The NASAP-MU version also finds a worst case analysis for both magnitude and phase by using a sensitivity matrix.[7] In addition, gradient and tolerance matrices of the magnitude and phase are also calculated and printed out.[8] The worst case analysis, tolerance matrix, and gradient matrices permit the NASAP-MU user to go beyond analysis and initiate a design algorithm. Appendix D explains the NASAP-MU version and its capability to design by analysis.

2.6 Capabilities of NASAP-69. NASAP-69 has both inherent advantages and disadvantages and perhaps should be classified as a special purpose program. Certainly the capability to solve for a transfer function, explicitly as a function of s, is one advantage. On the other hand, NASAP-69 can only solve for one transfer function per problem. Further, if more than one input is desired the program user must use superposition to find the total response. Quite obviously, the NASAP-69 program was not written to obtain the current and voltages of every element of a circuit.

At the present time the available version of NASAP-69 handles only small signal linear circuits. As a result the models for transistors and other active devices, can be accurate in only a certain frequency range. Filters are generally linear (the exception is active filters) and usually the filter designer is interested in only an input-output type of result. Consequently NASAP-69 is ideally suited for filters.

The NASAP-69 program will handle at a maximum, (b-1) elements were b is defined as the word size of the particular machine being used. An IBM 360 for example, can handle 31 elements. This may seem like a severe restriction but in actuality a more stringent limitation exists, namely, the number of flowgraph loops. The maximum number of flowgraph loops is limited to the core size of the machine being used. The number of flowgraph loops also determines to a large degree the amount of computer time needed to solve a problem. Usually the computer time factor, along with loop storage, is the feasibility factor and not the maximum number of elements.

Since the number of feedback loops is dependent upon the tree chosen it is very important to choose an optimum tree which generates a minimum number of loops. Some versions of NASAP automatically select a tree while other versions permit the user to select a tree. As a general rule, if the user selects a tree it is best to select a star-like tree as compared to a linear tree.[9]

2.7 NASAP-69 Coding. The NASAP program has been a cooperative effort on the part of several universities and consequently there exists several versions of NASAP, each with some advantages over the others. A user's guide and programmer's manual for NASAP-69 has been published which describes in detail the coding and running of NASAP-69 problems.[10] The NASAP-69 program presented in the manual features field free input formats.

Essentially the input encoding has two groups of cards, one group to communicate the circuit information to the computer and the other to instruct the computer in the output request. These two groups of cards are located between three control cards as shown in figure 2.6. Therefore, it can be seen that the major tasks confronting the NASAP-69 user are two-fold: (1) the transformation of the circuit diagram into a form that is acceptable to the computer program, and (2) the formulation of the desired output requests.

1. NASAP PROBLEM

2. Circuit description cards:
   Topology
   Element identification
   Numerical values
   Dependencies

3. OUTPUT

4. Output requests cards:
   Transfer function
   Sensitivity
   Frequency response
   Transient

5. EXECUTE

Figure 2.6 NASAP-69 Input.

The circuit information to be encoded must uniquely determine the circuit topology, the circuit elements, and the dependencies. Consequently, the nodes of the circuit are numbered consecutively starting with the number 1, and the elements are relabeled according to a letter-number format, where each component label is uniquely determined according to type, by one of the letter symbols,

R - resistor
L - inductor
C - capacitor
V - voltage source
I - current source,

immediately followed by a number to further distinguish the elements of the same type. After the nodes are numbered each element must be assigned a current variable direction. The directions are somewhat arbitrary for passive elements, but care must be taken for active devices such as dependent voltage sources.

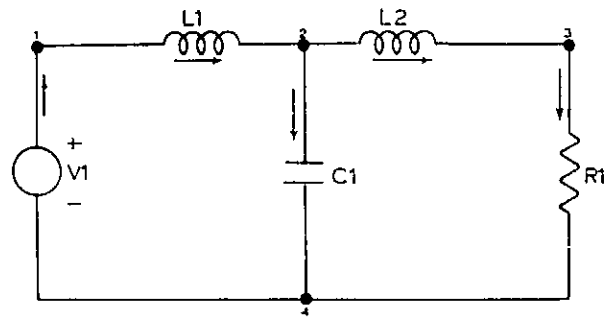

Figure 2.7 Current directions assigned and nodes numbered.

Recall the circuit of figure 2.1. By numbering the nodes, renaming the elements, and assigning current directions, the circuit now appears as in figure 2.7. The current direction associated with either an independent or dependent voltage source should be established in the positive sense of the voltage rise from minus to plus as illustrated in figure 2.7. In dealing with either dependent or independent current sources, the positive voltage sense is taken in the direction of positive current flow as shown in figure 2.8. In addition, figure 2.8 shows the current-voltage relationship for both dependent and independent active devices and passive elements.

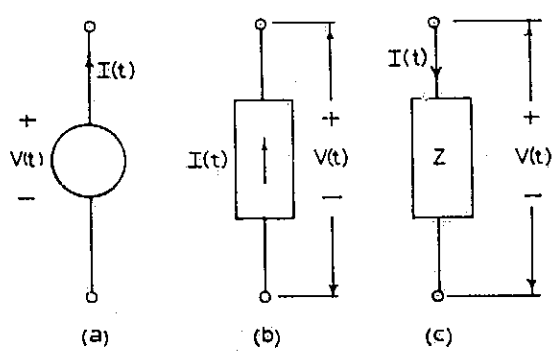

Figure 2.8 Current and voltage assigned directions. (a) voltage source. (b) Current source. (c) Passive element.

The circuit should now be ready for computer input. Each element identifier will require one data card divided into five fields of information. The fields are in field free format form but they must be kept in proper order. Starting with the element identifier in the first field, the second and third fields contain, respectively, the numbers of the origin node and the target node of the assigned current flow through the element. The fourth field accepts the numerical value of the circuit element if the element is not a dependent source. If the element is a dependent source it accepts the dependency parameter value. The fifth field position is employed only if a dependency exists, which is indicated by writing either an I, for current, or V, for voltage, followed by a letter-number symbol of the element upon which the dependency exists. The general form of the circuit data is summarized as shown below (the parentheses are for clarity only).

(identifier)(origin node)(target node)(numerical or dependency value)(dependency)

Scale factors are allowed when specifying resistor, inductor, and capacitor values.

RESISTOR	K	10^3
	M	10^6
CAPACITOR	F	1
	PF	10^{-12}
	UF	10^{-6}

INDUCTOR	H	1
	UH	10^{-6}
	MH	10^{-3}

After the circuit information cards are completed, the output request cards are completed. The primary output is the transfer function and all other outputs are related to it. A transfer function of the form:

$$T = \frac{\text{Output Variable}}{\text{Input Variable}} = \frac{Q_o}{Q_i}$$

can be specified by a user if the input quality Q_i is a driving source variable and Q_o is any current or voltage variable associated with any passive device or any dependent active source. Neither Q_o nor Q_i may be the voltage variable of an independent voltage source or the current variables of an independent current source. Q_o may however, be the current variable of an independent current source. The poles and zeros of the transfer function as well as the function itself are automatically printed out in tabular form.

The card which requests a transfer function and the sensitivity* of this function to some element is of the following format: (voltage or current output variables) (element-number identifier) / (voltage or current input variable) (element-number identifier) / (an optional sensitivity element-number identifier) where, again, the parentheses are not included in the actual program. If sensitivity is

*NASAP-69 increases the component specified by 1% and calculates the percentage increase or decrease of the transfer function.

not desired the second slash and third field are simply deleted.

Since the transfer function is a function of s, NASAP-69 can calculate, upon command, the necessary quantities to produce a Bode diagram. NASAP-69 prints out the quantities: Log_{10} (Frequency) ω, ω (Frequency), the magnitude of the transfer function in decibel units, and the phase angle in degrees, all in tabular form. It will also generate two printer graphic plots, one of the magnitude, in decibels, of the transfer function versus the Log_{10} (Frequency) ω. The other, a plot of the phase angle, in degrees, of the transfer function versus Log_{10} (Frequency).

In the output request section of the NASAP-69 input deck, a user may specify the range and incrementing frequency values for a Bode plot. A card containing the word, FREQ, followed by three numerical values: FREQ(log_{10} of the lower bound frequency) (log_{10} of the upper bound frequency) (log_{10} of the frequency increment) must be inserted after the transfer function request card, where the parentheses are not included.

Another of the NASAP-69 options is the transient response for an impulse excitation to the network. The transient response output consists of three forms:

(1) a convenient mathematical representation,

(2) a table of 100 equally spaced response values computed from time, $t = 0$, to an upper bound value specified by the user, and,

(3) a printer output plot.

The user must provide an output request card with the word TIME on it followed by a numeric that establishes the upper time limit of the time response.

If the impulse response of the network is desired, a card punched with the word TIME followed by a number should be inserted after the Bode output request card, if one is present. Otherwise, the impulse response request card should follow the transfer function request card. The termination card contains only the word EXECUTE.

Recalling the example presented previously in figures 2.1 and 2.8, the following is the NASAP-69 input deck:

```
NASAP PROBLEM
V1      4      1      1.0
L1      1      2      1.0H
L2      2      3     50MH
C1      2      4       .6UF
R1      3      4      1K

OUTPUT
VR1/VV1/L1
TIME .1
EXECUTE
```

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CHAPTER THREE

Image Parameter Design

3.1 Introduction. In this chapter the design of filters by the use of image parameters is studied. The use of image parameters for filter design was introduced by G.A. Campbell, O.J. Zobel and others in the 1920's. The image parameter techniques are now being replaced by modern network synthesis procedures.

3.2 Image parameters. The -terminal-pair, linear, passive, reciprocal network shown in figure 3.1 can be described by means of the open-circuit impedance matrix [Z]. For the indicated direction of terminal current, I2, the terminal voltages and currents are related by the open-circuit impedance matrix as

$$\begin{bmatrix} E1 \\ E2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I1 \\ -I2 \end{bmatrix} \qquad (3.1)$$

where the open-circuit impedance parameters are defined by

$$z_{11} = \left.\frac{E1}{I1}\right|_{I2 = 0} \qquad z_{12} = \left.\frac{E1}{-I2}\right|_{I1 = 0}$$

$$z_{21} = \left.\frac{E2}{I1}\right|_{I2 = 0} \qquad z_{22} = \left.\frac{E2}{-I2}\right|_{I1 = 0}.$$

If all the elements of the network are assumed to be bilateral, then the network is a reciprocal network and

$$z_{12} = z_{21}. \qquad (3.2)$$

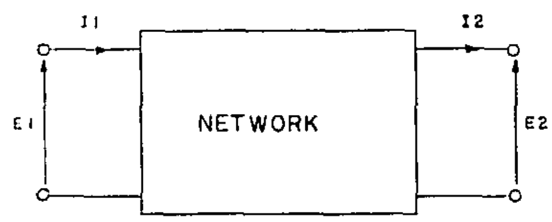

Figure 3.1 A two-terminal-pair, linear, passive, reciprocal network.

The network may also be described by a set of general circuit parameters in the form of the chain matrix when two, two-terminal-pair networks are connected in cascade. An important property of the chain matrix is that the resulting network can be described by a new chain matrix which is obtained from the product of the chain matrices of the original two networks. The chain matrix representation of the network shown in figure 3.1 is given by

$$\begin{bmatrix} E1 \\ I1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} E2 \\ I2 \end{bmatrix} \qquad (3.3)$$

where the chain matrix parameters are defined by

$$A = \left.\frac{E1}{E2}\right|_{I2 = 0} \qquad B = \left.\frac{E1}{I2}\right|_{E2 = 0}$$

$$C = \left.\frac{I1}{E2}\right|_{I2 = 0} \qquad D = \left.\frac{I1}{I2}\right|_{E2 = 0}.$$

If the terminal current I2 of figure 3.1 is constrained to be zero, we find from equation (3.1) that

$$E2 = z_{21} I1$$

or

$$\left.\frac{E2}{I1}\right|_{I2 = 0} = z_{21}$$

and from equation (3.3) that

$$I1 = CE2$$

or

$$\left.\frac{E2}{I1}\right|_{I2 = 0} = \frac{1}{C} .$$

Thus we have

$$\left.\frac{E2}{I1}\right|_{I2 = 0} = z_{21} = \frac{1}{C} . \qquad (3.4)$$

Equation (3.3) can be used to find the terminal quantities, E2 and I2, in terms of the terminal quantities, E1 and I1. This relation is given as

$$\begin{bmatrix} E2 \\ I2 \end{bmatrix} = \frac{1}{AD - BC} \begin{bmatrix} D & -B \\ -C & A \end{bmatrix} \begin{bmatrix} E1 \\ I1 \end{bmatrix} . \qquad (3.5)$$

Now if the terminal current I1 is constrained to be zero we find from equation (3.1) that

$$\left.\frac{E1}{-I2}\right|_{I1 = 0} = z_{12}$$

and from equation (3.5) that

$$\left.\frac{E1}{-I2}\right|_{I1 = 0} = \frac{AD - BC}{C}$$

thus, we have

$$\left.\frac{E1}{-I2}\right|_{I1 = 0} = z_{12} = \frac{AD - BC}{C} . \qquad (3.6)$$

From equations (3.2), (3.4), and (3.6) we see that

$$AD - BC = 1. \qquad (3.7)$$

This result is a consequence of the reciprocal nature of the network, and can be used to rewrite equation (3.5) as

$$\begin{bmatrix} E2 \\ I2 \end{bmatrix} = \begin{bmatrix} D & -B \\ -C & A \end{bmatrix} \begin{bmatrix} E1 \\ I1 \end{bmatrix} . \qquad (3.8)$$

The chain matrix for a network formed by the cascade connection of two networks can readily be found if the chain matrix for each of the smaller networks is known. Figure 3.2 shows a two-port network which has been formed by the cascade connection of two, two-port networks. By inspection of figure 3.2 it can be seen that

$$E1 = E1' \qquad E2' = E1'' \qquad E2'' = E2$$

$$I1 = I1' \qquad I2' = I1'' \qquad I2'' = I2.$$

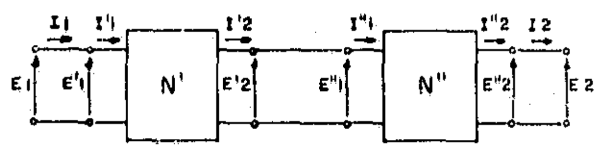

Figure 3.2 A two-port network formed from two, two-port networks connected in cascade.

These relationships can now be used to find the chain matrix of the whole network from the chain matrices of the two smaller networks. As

$$\begin{bmatrix} E1 \\ I1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} E2 \\ I2 \end{bmatrix}, \quad \begin{bmatrix} E1' \\ I1' \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} E2' \\ I2' \end{bmatrix} \text{ and}$$

$$\begin{bmatrix} E1'' \\ I1'' \end{bmatrix} = \begin{bmatrix} A'' & B'' \\ C'' & D'' \end{bmatrix} \begin{bmatrix} E2'' \\ I1'' \end{bmatrix} .$$

Then, using the above relationships we find

$$\begin{bmatrix} E1' \\ I1' \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} E2' \\ I2' \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} E1'' \\ I1'' \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} A'' & B'' \\ C'' & D'' \end{bmatrix} \begin{bmatrix} E2'' \\ I2'' \end{bmatrix}$$

or

$$\begin{bmatrix} E1 \\ I1 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} A'' & B'' \\ C'' & D'' \end{bmatrix} \begin{bmatrix} E2 \\ I2 \end{bmatrix}$$

so that the chain matrix for the composite network is given by

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} A'' & B'' \\ C'' & D'' \end{bmatrix} .$$

Thus the chain matrix of the new network is just the matrix product of the two chain matrices for the two networks that were connected in cascade to form the new network. This result can be generalized for the cascade connection of n networks.

The image parameters for the two-port network can be expressed in terms of the general parameters of the chain matrix. Let Zi1, Zi2 denote the image impedances of a two-port network and γ denote the propagation constant of the network. Figure 3.3 shows a network terminated in its image impedances, Zi1 and Zi2. Refering to Figure 3.3,

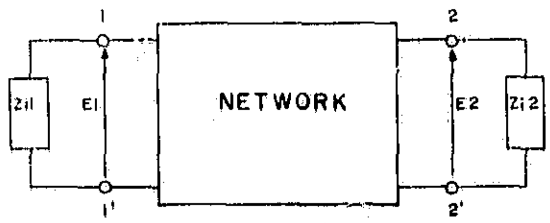

Figure 3.3 Two-port network terminated in its image impedances.

the image impedances are defined as follows: The image impedance Zi2 is the impedance which, when connected to terminals (2,2′) of the network, results in an input impedance of Zi1 when looking into the terminals (1,1′). The image impedance Zi1 is the impedance which, when connected to terminals (1,1′) of the network, results in an input of Zi2 when looking into the terminals (2,2′). For a symmetric network Zi1 and Zi2 are equal. In this case Zi1 and Zi2 will be denoted by Zc. If the ratio of Zi1/Zi2 is denoted by n^2 the propagation constant is defined by

$$\frac{E1}{E2} = n \exp -\gamma \qquad (3.9)$$

when port 2 is terminated in the image impedance Zi2.

The image impedance can be found in terms of the parameter of the chain matrix by considering the network and the termination made to the network as shown in figure 3.4. For the termination of the network shown in figure 3.4-a, using equation (3.3), we can write

$$Zi1 = \frac{E1}{I1} = \frac{AE2 + BI2}{CE2 + DI2}$$

and then E2 = (Zi2)I2 can be used to substitute for E2, so that

$$Zi1 = \frac{E1}{I1} = \frac{AZi2 + B}{CZi2 + D}. \qquad (3.10)$$

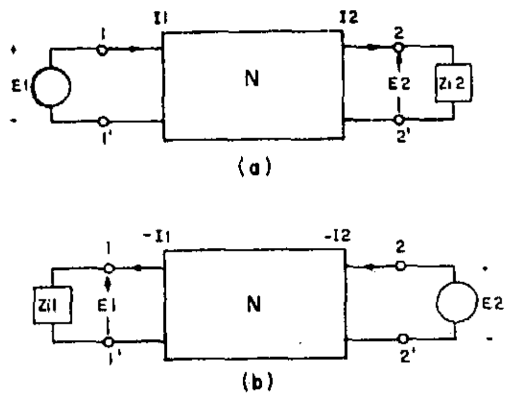

Figure 3.4 Terminated Network.

For the terminations and indicated current directions shown in figure 3.5-b we can write

$$Z11 = \frac{E1}{-I1} = \frac{AE2 + BI2}{-CE2 - DI2}$$

by a second application of equation (3.3). Since terminals (1,1′) are terminated in the their image impedance, the impedance looking into terminals (2,2′) is just Z12. For the indicated direction of current I2 we can write

$$E2 = Z12(-I2)$$

and then use this relationship to substitute for E2 in the previous equation to obtain

$$Z11 = \frac{E1}{-I1} = \frac{-AZ12 + B}{CZ12 - D} \quad . \qquad (3.11)$$

Now equation (3.10) and (3.11) are equated to yield

$$\frac{AZ12 + B}{CZ12 + D} = \frac{-AZ12 + B}{CZ12 - D}$$

$$AC(Z12)^2 - ADZ12 + BCZ12 - BD = -AC(Z12)^2 + BCZ12 - ADZ12 + BD$$

or

$$Z12 = \sqrt{BD/AC} \quad . \qquad (3.12)$$

Similarly

$$Zi1 = \sqrt{AB/CD} \quad . \qquad (3.13)$$

and

$$n^2 = Zi1/Zi2 = \sqrt{AB/CD} \; / \; \sqrt{BD/AC} = A/D \qquad (3.14)$$

The network and terminations as shown in figure 3.4-a can now be used to find an expression for the propagation constant γ in terms of the parameters of the chain matrix. From equation (3.3)

$$\frac{E1}{E2} = A + \frac{BI2}{E2} = A + \frac{B}{Zi2} = A + B\sqrt{AC/BD}$$

or

$$\frac{E1}{E2} = \sqrt{A/D} \; (\sqrt{AD} + \sqrt{BC}) \qquad (3.15)$$

and

$$\frac{I1}{I2} = \frac{CE2}{I2} + D = CZi2 + D = C\sqrt{BD/AC} + D$$

or

$$\frac{I1}{I2} = \sqrt{D/A} \; (\sqrt{AD} + \sqrt{BC}). \qquad (3.16)$$

Comparison of equations (3.15), (3.16) and (3.9) reveals that exp γ must be given by

$$\exp \gamma = \sqrt{AD} + \sqrt{BC} \quad . \qquad (3.17)$$

Equation (3.7) can be used to find an expression for exp-γ as

$$\exp \gamma \; \exp{-\gamma} = 1 = AD - BC = (\sqrt{AD} + \sqrt{BC})(\sqrt{AD} - \sqrt{BC})$$

or

$$\exp{-\gamma} = \sqrt{AD} - \sqrt{BC} \quad . \qquad (3.18)$$

Expressions involving cosh γ and sinh γ are obtained by use of equations (3.17) and (3.18) as

$$\cosh \gamma = \frac{\exp \gamma + \exp -\gamma}{2} = \sqrt{AD} \qquad (3.19)$$

$$\sinh \gamma = \frac{\exp \gamma - \exp -\gamma}{2} = \sqrt{BC} \,. \qquad (3.20)$$

Equations (3.12), (3.14), (3.19) and (3.20) can be solved for the parameter A, B, C, and D of the chain matrix in terms of the image parameters. The results are

$$A = n \cosh \gamma \qquad B = nZi2 \sinh \gamma$$

$$C = \frac{1}{nZi2} \sinh \gamma \qquad D = \frac{1}{n} \cosh \gamma \,. \qquad (3.21)$$

Substituting equation (3.21) into equation (3.3) yields

$$\begin{bmatrix} E1 \\ I1 \end{bmatrix} = \begin{bmatrix} n \cosh \gamma & n\, Zi2 \sinh \gamma \\ \left(\frac{1}{nZi2}\right) \sinh \gamma & \left(\frac{1}{n}\right) \cosh \gamma \end{bmatrix} \begin{bmatrix} E2 \\ E2 \end{bmatrix} \qquad (3.22)$$

$$= \begin{bmatrix} n & 0 \\ 0 & \frac{1}{n} \end{bmatrix} \begin{bmatrix} \cosh \gamma & Zi2 \sinh \gamma \\ \left(\frac{1}{Zi2}\right) \sinh \gamma & \cosh \gamma \end{bmatrix} \begin{bmatrix} E2 \\ I2 \end{bmatrix}$$

which is the chain matrix representation of a network with image impedances Zi1 and Zi2 or the equivalent cascade connection of a network consisting of a transformer with a turns ratio of n (note that n may be complex) and a network with characteristic impedance Zc = Zi2. This equivalence is shown in figure 3.5. For a symmetric network the chain matrix becomes

$$\begin{bmatrix} \cosh \gamma & Zc \sinh \gamma \\ 1/Zc \sinh \gamma & \cosh \gamma \end{bmatrix} \,.$$

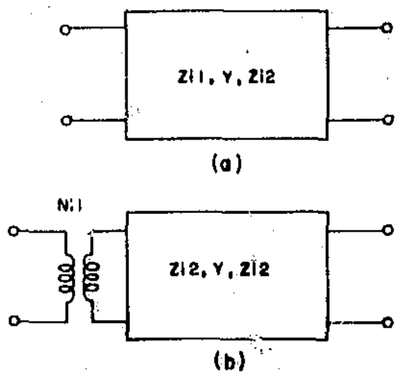

Figure 3.5 Equivalent network for a two-port network. (a) network. (b) equivalent network.

If two such symmetric networks, one with a propagation constant of γ_1, and the other with a propagation of γ_2, are cascaded and both with an image impedance of Zc, then the chain matrix for the cascaded connection can be found by forming the matrix product

$$\begin{bmatrix} \cosh \gamma_1 & Zc \sinh \gamma_1 \\ 1/Zc \sinh \gamma_1 & \cosh \gamma_1 \end{bmatrix} \begin{bmatrix} \cosh \gamma_2 & Zc \sinh \gamma_2 \\ 1/Zc \sinh \gamma_2 & \cosh \gamma_2 \end{bmatrix}$$

$$= \begin{bmatrix} \cosh (\gamma_1 + \gamma_2) & Zc \sinh (\gamma_1 + \gamma_2) \\ 1/Zc \sinh (\gamma_1 + \gamma_2) & \cosh (\gamma_1 + \gamma_2) \end{bmatrix}.$$

The image parameters for a given network are readily determined by use of the following parameters:

Zsc_1 -- the impedance seen when looking into port one with port two short-circuited (E2 = 0)

Zoc_1 -- the impedance seen when looking into port one with port two open-circuited (I2 = 0)

Zsc_2 -- the impedance seen when looking into port two with port one short-circuited (E1 = 0)

Zoc_2 -- the impedance seen when looking into port two with port one open-circuited ($I1 = 0$).

These parameters can be found in terms of A, B, C and D as follows:

$$Zsc_1 = \frac{B}{D} \qquad Zsc_2 = \frac{B}{A}$$

$$Zoc_1 = \frac{A}{C} \qquad Zoc_2 = \frac{D}{C} .$$

From equations (3.12), (3.13), (3.19) and (3.20) it follows that

$$Z11 = \sqrt{Zsc_1 \ Zoc_1} \qquad (3.23)$$

$$Z12 = \sqrt{Zsc_2 \ Zoc_1} \qquad (3.24)$$

and

$$\gamma = \tanh^{-1} \sqrt{\frac{Zsc_1}{Zoc_1}} = \tanh^{-1} \sqrt{\frac{Zsc_2}{Zoc_2}} . \qquad (3.25)$$

Example: NASAP-69, chain parameters and image parameters.

The network of figure 3.6 will be used as an example of the calculation of the chain parameters and image parameters of a network using NASAP. The source shown connected to the network in figure 3.6 is used to determine the chain matrix parameter A which is defined by

$$A = \left. \frac{E1}{E2} \right|_{I2 = 0} .$$

The input table to find A, using NASAP-UMC, is given by Table 3.1. The input information for the standard version of NASAP-69 for the same problem would be much simpler.

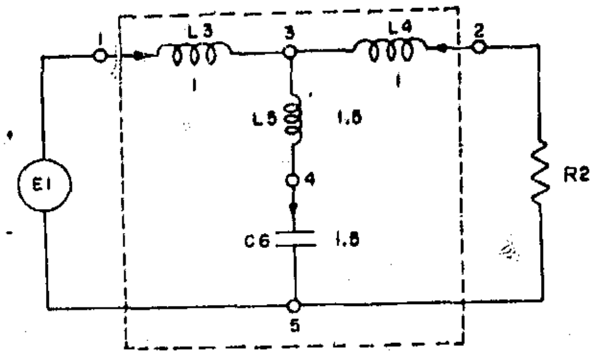

Figure 3.6 Network for calculation of chain parameters and image parameters.

A	B	C	D	E	F	G	H	K	
1	5	0	2	1	0	0	1	0	1.E0
2	5	1	2	2	0	0	0	0	1.E50
1	3	1	3	3	1	0	0	0	1.E0
3	2	0	4	4	-1	1	0	0	1.E0
3	4	1	5	5	1	0	0	0	1.5E0
4	5	0	6	6	1	1	0	0	1.5E0

Table 3.1 Input data for NASAP-UMC to find parameter A .

Using NASAP-UMC, the chain matrix parameters were found to be

$$A = \frac{3.75s^2 + 1}{2.25s^2 + 1} \qquad B = \frac{6s^3 + 2s}{2.25s^2 + 1}$$

$$C = \frac{1.5s}{2.25s^2 + 1} \qquad D = \frac{3.75s^2 + 1}{2.25s^2 + 1}$$

and the open and short circuit parameters were found to be

$$Zoc_1 = Zoc_2 = \frac{3.75s^2 + 1.0}{1.5}$$

$$Zsc_1 = Zsc_2 = \frac{6s^3 + 2s}{3.75s^2 + 1} .$$

Now using the chain matrix parameters and equations (3.12) and (3.13), the image impedances Zi1 and Zi2 are found to be

$$Zi1 = Zi2 = \sqrt{4s^2 + 1.333} .$$

This result can be verified by use of the open and short circuit impedance and equations (3.23) and (3.24).

3.3 m-derived low-pass filters. The image parameters can be used to design m-derived low-pass filters. The synthesis of an m-derived low-pass filter is a simple process. The tolerance requirement for the element values obtained by this syntheses procedure is low, but the number of elements required to realize the filter may be more than if some other synthesis procedures had been used.

The constant-K or prototype filter sections shown in figure 3.7 are basic to the synthesis procedure. The impedances Z1 and Z2 are such that

$$Z1Z2 = R^2. \qquad (3.26)$$

The constant R is termed the design resistance and is equal to one ohm for a network which has been normalized to one ohm.

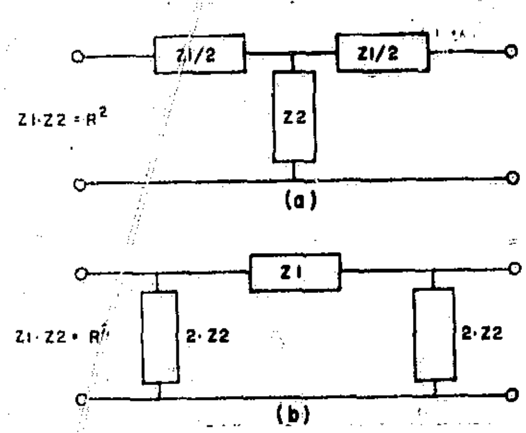

Figure 3.7 Constant-K filter section. (a) T section. (b) π section.

The image parameters for the T and π prototype sections are obtained by use of equations (3.23) and (3.25). For the T section we have

$$Zoc = \tfrac{1}{2}Z1 + Z2$$

$$Zsc = \frac{\tfrac{1}{4}(Z1)^2 + Z1Z2}{\tfrac{1}{2}Z1 + Z2}$$

and thus the image impedance is given by

$$Zi = \sqrt{ZscZoc} = \sqrt{(\tfrac{1}{2}Z1 + Z2)\frac{(\tfrac{1}{4}(Z1)^2 + Z1Z2)}{(\tfrac{1}{2}Z1 + Z2)}} \quad .$$

Since $Z1Z2 = R^2$ we have

$$Zi = \sqrt{R^2 + \frac{(Z1)^2}{4}}$$

or

$$Zi = R\sqrt{1 + \frac{(Z1)^2}{4R^2}} \quad . \qquad (3.27)$$

The propagation constant γ can be found by application of equation (3.21)

$$\cosh \gamma = (1/n)A = A$$

where n is one, since the network is symmetric. For the T and π configuration A is found to be given by

$$A = \left.\frac{E2}{E1}\right|_{I2 = 0} = 1 + Z1/2Z2,$$

Thus, for both the T and the π filter sections we have

$$\cosh \gamma = 1 + \frac{Z1}{2Z2} \qquad (3.28)$$

which is equivalent to

$$\gamma = 2 \sinh^{-1}\left(\sqrt{\frac{Z1}{4Z2}}\right) = 2 \sinh^{-1}\left(\frac{Z1}{2R}\right) . \qquad (3.29)$$

For the π prototype section the short circuit and open circuit impedances are given by

$$Zsc = \frac{2Z2Z1}{Z1 + 2Z2} \qquad Zoc = \frac{2Z2(Z1 + 2Z2)}{Z1 + 4Z2}$$

and the image impedance is found from an application of equation (3.23) to be

$$Zi = \sqrt{ZscZoc} = \sqrt{\left(\frac{2Z2Z1}{Z1 + 2Z2}\right)\left(\frac{2Z2(Z1 + 2Z2)}{Z1 + 4Z2}\right)}$$

or

$$Zi = \frac{R}{\sqrt{1 + (Z1)^2/4R^2}} \,. \qquad (3.30)$$

The propagation constant γ, in general, is a complex number with a real part α and an imaginary part $j\beta$. Thus

$$\gamma = \alpha + j\beta. \qquad (3.31)$$

For a symmetric network terminated in its characteristic impedance equation (3.9) becomes

$$\frac{E1}{E2} = \exp\gamma = \exp(\alpha + j\beta) = \exp\alpha \,\exp j\beta$$

$$= \exp\alpha(\cos\beta + j\sin\beta) = \exp\alpha\angle\beta \,.$$

Thus the magnitude of the ratio E1/E2 is determined by α and the phase angle of the ratio is determined by β. α is the attenuation of the section in nepers, β is the phase shift in radians. The attenuation of the network can be expressed in db as follows

$$Adb = 20\log_{10}(|E1/E2|) = 20\log_{10}(|\exp\alpha|)$$

$$= (20\log_{10} e)\alpha = 8.686\alpha \text{ db.} \qquad (3.32)$$

From equation (3.32) it is clear that 1 neper equals 8.686 db.

Equation (3.31) and equation (3.28) can be combined to yield

$$\cosh(\alpha + j\beta) = 1 + \frac{Z1}{2Z2}$$

and cosh $(\alpha + j\beta)$ can be expanded as

$$\cosh(\alpha + j\beta) = \cosh\alpha \cosh j\beta + \sinh\alpha \sinh j\beta$$

$$= \cosh\alpha \cos\beta + j \sinh\alpha \sin\beta.$$

If the cosh γ is real, i.e. Z1/Z2 real, two cases may be defined:

Case I

Passband: $\alpha = 0, \beta \neq 0$

$$\beta = \cos^{-1}\left(1 + \frac{Z1}{2Z2}\right) \qquad (3.33a)$$

where $-1 \leq \left(1 + \frac{Z1}{2Z2}\right) \leq +1$

Case II

Stopband: $\alpha \neq 0, \beta = n\pi$

$\cos\beta = \pm 1$

for $\left(1 + \frac{Z1}{2Z2}\right) < -1$

$\beta = \pm\pi$

$$\alpha = \cosh^{-1}\left(1 + \frac{Z1}{2Z2}\right) \qquad (3.33b)$$

for $\left(1 + \frac{Z1}{2Z2}\right) > +1$

$\beta = 0$

$$\alpha = \cosh^{-1}\left(1 + \frac{Z1}{2Z2}\right).$$

The frequencies where a transition from bandpass to stopband is made are the frequencies where

$$1 + \frac{Z1}{2Z2} = \pm 1$$

or where

$$\frac{Z1}{4Z2} = -1$$

and

$$\frac{Z1}{4Z2} \doteq 0. \qquad (3.33c)$$

A low-pass filter section will now be studied using the above results.

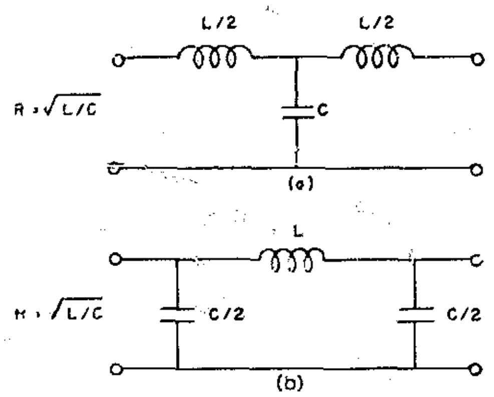

Figure 3.8 Low-pass prototype sections. (a) T sections. (b) π sections.

The low-pass prototype filter sections of figure 3.8 are obtained by letting Z1 be jωL and Z2 be 1/jωC. The characteristic impedance for the T low-pass prototype section is given by

$$Zc = R\sqrt{1 - \omega^2/\omega_c^2}\ . \qquad (3.34)$$

While the characteristic impedance for the π low-pass prototype section is given by

$$Zc = R/\sqrt{1 - \omega^2/\omega_c^2} \qquad (3.35)$$

where

$$\omega_c = 2/\sqrt{LC} \text{ and } R = \sqrt{Z1Z2} = \sqrt{L/C}.$$

The propagation constant for both the T and π prototype sections are found from equation (3.28)

$$\cosh \gamma = 1 + \frac{Z1}{2Z2}$$

by use of the hyperbolic relation

$$\sinh (\tfrac{1}{2}x) = \sqrt{\frac{\cosh x - 1}{2}} .$$

γ is found to be

$$\gamma = 2\sinh^{-1} \left(\frac{Z1}{2R}\right) = 2\sinh^{-1} \left(\frac{j\omega L}{2\sqrt{L/C}}\right) \qquad (3.36)$$

$$= 2\sinh^{-1} (j\omega/\omega_c).$$

The radian frequency for the transition from a region of passband to a region of stopband is found to be

$$|\omega| = \omega_c .$$

by applying equation (3.33-c) . For the passband region $\alpha = 0$ and

$$\beta = \cos^{-1} (1 - 2\omega^2/\omega_c^2) = 2 \sin^{-1}(\omega/\omega_c) \qquad (3.37)$$

for $|\omega| > \omega_c$.

Equations (3.37) and (3.38) apply only when the prototype sections for figure (3.38) are terminated in their characteristic impedance. As seen from equations (3.34) and (3.35) the characteristic impedance for these two prototype sections is not a rational function of frequency, hence, they are not realizable with a finite number of elements.

The low-pass filter indicated in figure 3.9 will be used to demonstrate the effect of terminating a low-pass filter in an impedance other than its characteristic impedance.

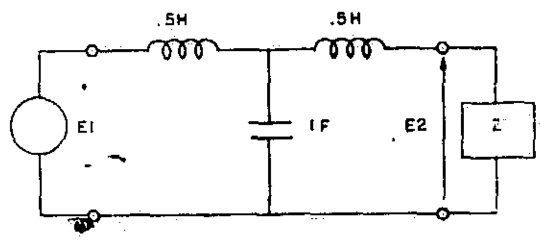

Figure 3.9 Terminated low-pass filter.

For this filter $Z1 = j\omega$, $Z2 = 1/j\omega$ and hence $\omega_c = 1$ and $R = 1$. The characteristic impedance is then

$$Zc = 1\sqrt{1 - \omega^2}$$

and the voltage ratio $E2/E1$ is given by

$$\frac{E2}{E1} = \exp -\gamma = \exp -\alpha \exp -j\beta = \exp -\alpha .$$

NASAP-69 is a convenient means of obtaining the voltage ratio $|E2/E1|$ when the filter is terminated in a resistive load. In this case port two is terminated by a one ohm resistive load, which is the value of the characteristic impedance at $\omega = 0$. Figure 3.10 is a plot of $|E1/E2|$ versus frequency for the improper termination of the network and termination of the network in its characteristic impedance. The m-derived prototype section to be discussed next is used to reduce the problems associated with termination and insufficient attenuation.

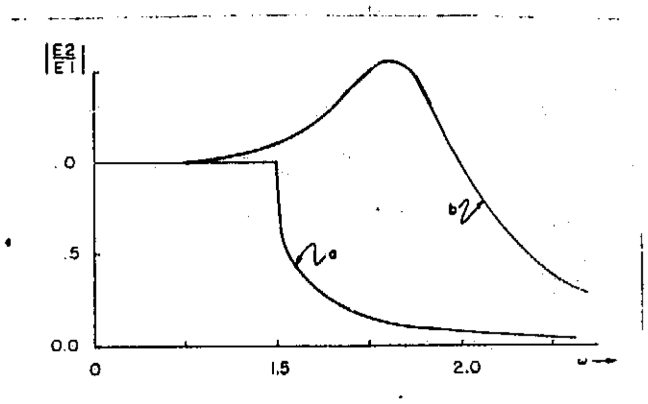

Figure 3.10 Comparison of the voltage transfer function, |E2/E1|. (a) Z = Zc. (b) Z(1,1') = 0 ohm, Z(2,2') = 1 ohm.

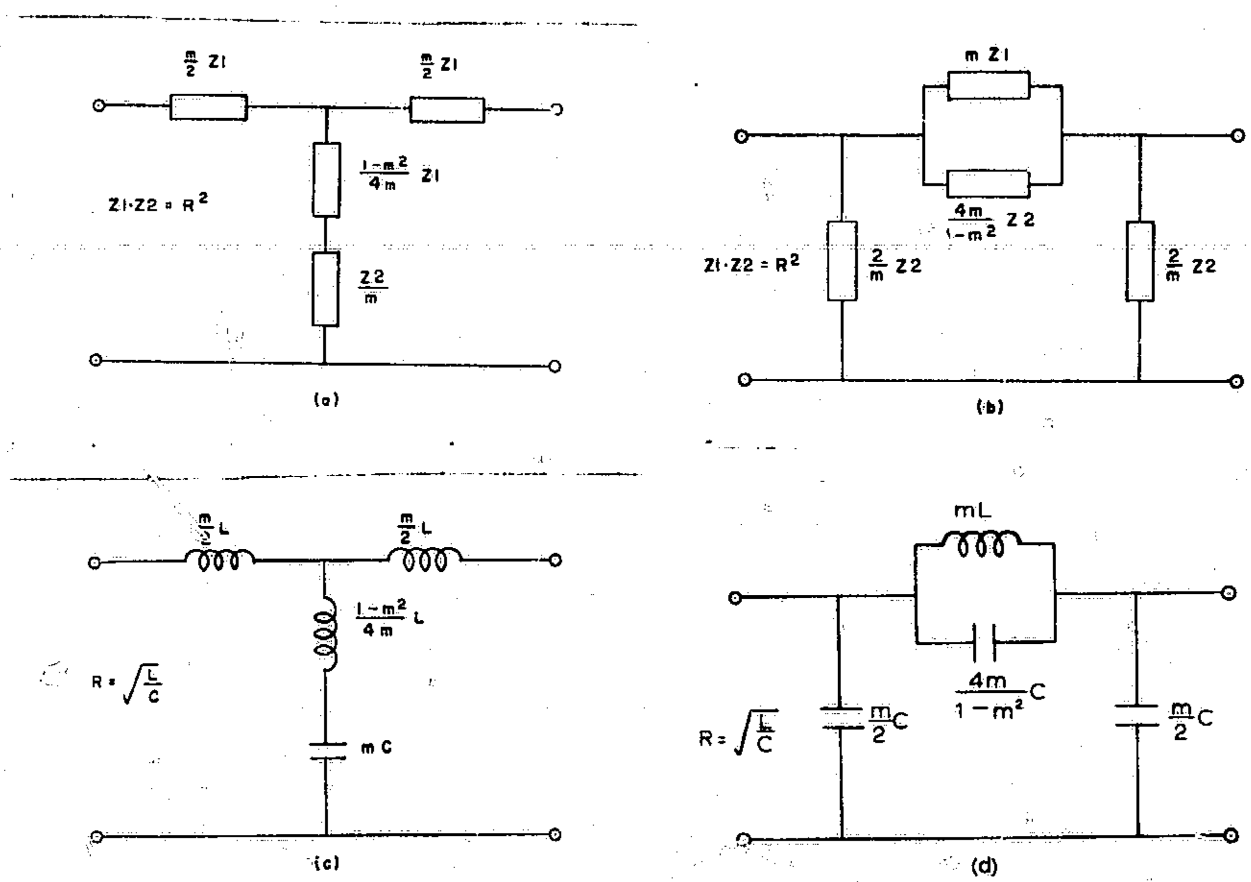

Figure 3.11 m-derived filter sections. (a) m-derived T section. (b) m-derived π section. (c) m-derived low-pass T section. (d) m-derived low-pass π section.

The m-derived T section and π section are shown in figures 3.11-a and 3.11-b. These filter sections are symmetric and their image impedance is denoted by Zc, the characteristic impedance. It will now be shown that characteristic impedances for the m-derived T and π sections are the same as the characteristic impedances for the prototype T and π sections. For the T section we have

$$Zoc = \left(\frac{1 + m^2}{4m}\right) Z1 + \frac{Z2}{m}$$

$$Zsc = \frac{\frac{1}{4}(Z1)^2 + R^2}{Zoc}$$

and Zc for the m-derived T filter section is then given by

$$Zc = \sqrt{ZocZsc} = \sqrt{(Zoc)\left(\frac{\frac{1}{4}(Z1)^2 + R^2}{Zoc}\right)} = R\sqrt{1 + (Z1/4R^2)}.$$

Comparison of this result with equation (3.27) shows that the characteristic impedance of the m-derived T section and the prototype T section are the same. Thus, the m-derived T section and the prototype T section may be connected in cascade to form more complex filters. For the π section we have

$$Zsc = \frac{4mR^2}{(1 + m^2)Z1 + 4Z2}$$

$$Zoc = \frac{Z2[(1 + m^2)Z1 + 4Z2]}{m(Z1 + 4Z2)}$$

and Zc for the m-derived π filter section is then given by

$$Zc = \sqrt{ZscZoc} = \sqrt{\frac{4R^2 Z2}{Z1 + 4Z2}} = \frac{R}{\sqrt{1 + Z_1^2/4R^2}} \quad . \quad (3.39)$$

Thus, the characteristic impedance for the m-derived π section is the same as that for the prototype π section, equation (3.30). The m-derived π section and the prototype π section may be cascaded since their characteristic impedance is the same.

The propagation constant γ for the m-derived T and π section is the same, since the ratio is

$$\left.\frac{E1}{E2}\right|_{I2 = 0} = A = \frac{(1 + m^2)Z1 + 4Z2}{(1 - m^2)Z1 + 4Z2}$$

for both networks. Using equation (3.21) and half-angle relation for a hyperbolic sine function, the propagation constant for the T and π sections is found to be

$$\gamma = 2\sinh^{-1}\left(\frac{m(Z1/2R)}{\sqrt{1 + (1 - m^2)(Z_1^2/4R^2)}}\right). \qquad (3.40)$$

As γ is a function of m, the attenuation characteristic of the m-derived T and π sections is influenced by the value of m selected by the designer.

For the low-pass case $Z1 = j\omega L$, $Z2 = 1/j\omega C$ and $R^2 = L/C$. The m-derived low-pass T and π sections are shown in figures 3.11-c and 3.11-d, respectively. The characteristic impedances are the same as those of the low-pass prototype T and π sections of figure 3.8. The propagation constant for both the T and π m-derived sections is

$$\gamma = 2\sinh^{-1}\left(\frac{jm(\omega/\omega_c)}{\sqrt{1 - (1 - m^2)(\omega/\omega_c)^2}}\right) \qquad (3.41)$$

where $\omega_c = 2/\sqrt{LC}$.

The attenuation and phase characteristics can be found for the passband, $|\omega| > \omega_c$, and the stopband, $|\omega| > \omega_c$, by using the hyperbolic relation

$$\sinh(\tfrac{1}{2}\alpha + j\tfrac{1}{2}\beta) = \sinh\tfrac{1}{2}\alpha\cosh j\tfrac{1}{2}\beta + \cosh\tfrac{1}{2}\alpha\sinh j\tfrac{1}{2}\beta$$
$$= \sinh\tfrac{1}{2}\alpha\cos\tfrac{1}{2}\beta + j\cosh\tfrac{1}{2}\alpha\sin\tfrac{1}{2}\beta.$$

For the passband: $\alpha = 0,\ \beta \neq 0,\ |\omega| < |\omega_c|$

sinh $(\tfrac{1}{2}\gamma)$ is imaginary, therefore

$$\frac{\beta}{j} = 2\sin^{-1}\left(\frac{jm(\omega/\omega_c)}{\sqrt{1 - (1 - m^2)(\omega/\omega)^2}}\right)$$

$$\beta = 2\sinh^{-1}\left(\frac{m(\omega/\omega_c)}{\sqrt{1 - (1 - m^2)(\omega/\omega_c)^2}}\right).$$

For the stopband: $\alpha \neq 0,\ |\omega| > \omega_c$

condition I, $\beta = n\pi$, n odd

sinh$(\tfrac{1}{2}\gamma)$ is imaginary, therefore

$(1 - m^2)(\omega/\omega_c)^2 < 1$

$|\omega| < \omega_c/\sqrt{(1 - m^2)} = \omega_\infty$

or

$\omega_c < |\omega| < \omega_\infty$

and α is given by

$$\alpha = 2\cosh^{-1}\left(\frac{m(\omega/\omega_c)}{\sqrt{1 - (1 - m^2)(\omega/\omega_c)^2}}\right).$$

condition II, $\beta = n\pi$ even

sinh $(\tfrac{1}{2}\gamma)$ is real, therefore

$(1 - m^2)(\omega/\omega_c^2) > 1$

$|\omega| < \omega_c/\sqrt{(1-m^2)} = \omega_\infty$

or $\omega_\infty < |\omega| < \infty$

and α is given by

$$\alpha = 2\sinh^{-1}\left(\frac{m(\omega/\omega_c)}{\sqrt{(1 - m^2)(\omega/\omega_c)^2 - 1}}\right).$$

The attenuation in db is given by

$$\text{Atten.} = 8.686\begin{Bmatrix} 0 & |\omega| < \omega_c \\ 2\cosh^{-1}\dfrac{m(\omega/\omega_c)}{\sqrt{1 - (1 - m^2)(\omega/\omega_c)^2}} & \omega_c < |\omega| < \omega_\infty \\ 2\sinh^{-1}\dfrac{m(\omega/\omega_c)}{\sqrt{(1 - m^2)(\omega/\omega_c)^2 - 1}} & \omega_\infty < |\omega| < \infty \end{Bmatrix}. \tag{3.42}$$

The attenuation in db for several m-derived filter sections as a function of the normalized frequency ω/ω_c is plotted in figure 3.12. As can be seen from this figure the attenuation of an m-derived section becomes infinite. The frequency where this occurs is $\omega_\infty = \omega_c/\sqrt{(1 - m^2)}$. A table of attenuation in db for various values of m is included at the end of this chapter, table 3.1. Table 3.1 and figure 3.12 may be used as a design aid when designing low-pass filters in the following manner:

1. Specify the low-pass filter to be obtained in a manner similar to table 1.5.
2. Use table 3.1 and figure 3.12 to select appropriate filter sections to meet specifications of step one.

The specifications of table 1.5 will be used to illustrate the procedure. The formation of table 1.5 is the first step. Since the pass-band attenuation is 0 db the 0-3 db requirement will be met in the region from 0 to +0.974 radian for any of the m-derived sections selected. The attenuation of 1.0254 radians must be 30 db, so a T section with $\omega_\infty = 1.04$ is selected to produce the desired

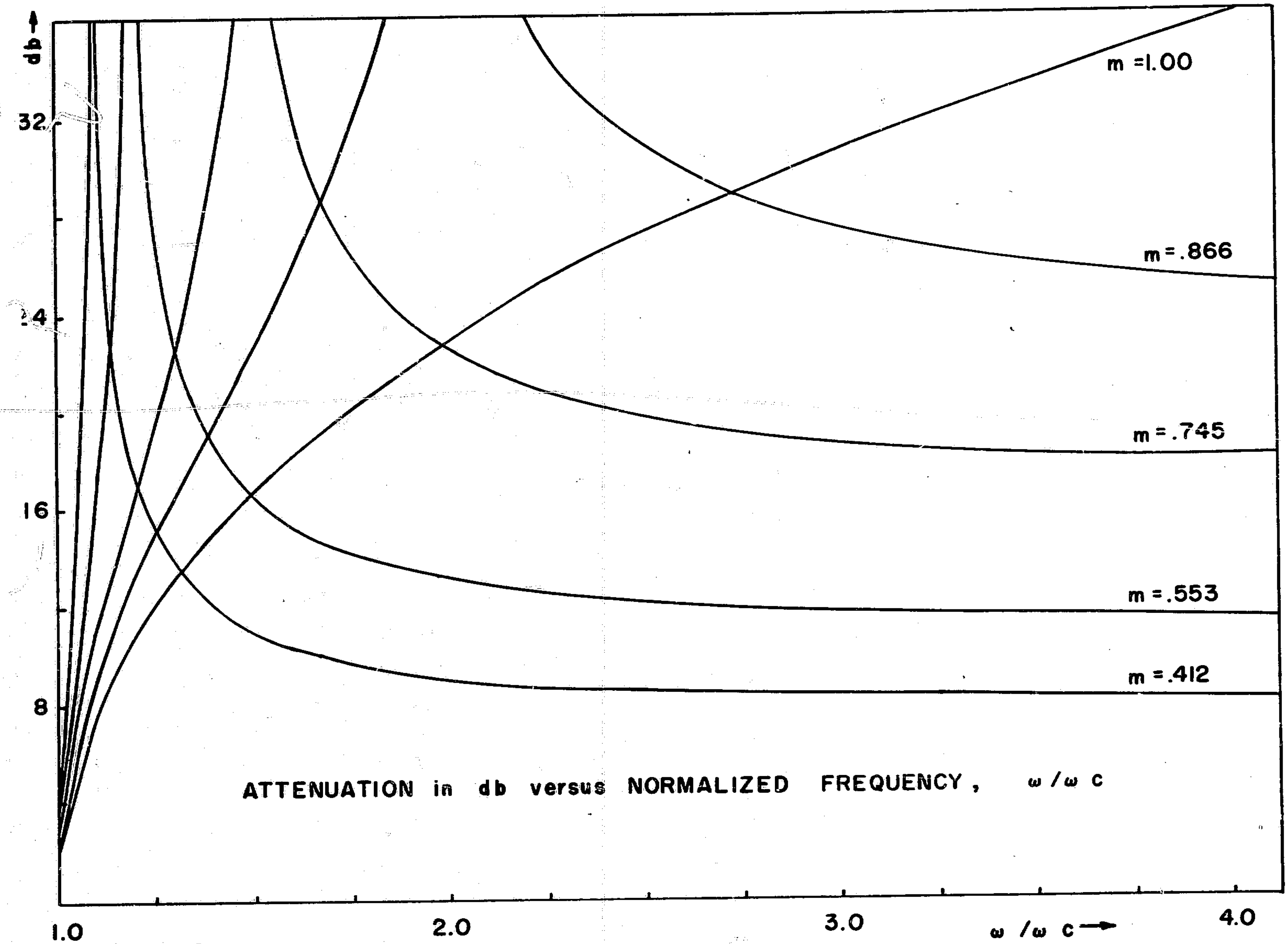
db
32
24
16
8
m =1.00
m = .866
m = .745
m = .553
m = .412
ATTENUATION in db versus NORMALIZED FREQUENCY, ω /ω c
1.0
2.0
3.0
4.0
ω /ω c

FIG. 3.12

attenuation. Similarly, to meet the 50 db attenuation at 1.356 radians, a T section with $\omega_\infty = 1.45$ is chosen for the filter. To insure that the attenuation remains sufficiently high at high frequencies, a prototype T section is also included in the filter. The attenuation curve for the network formed by the cascade of the three sections is found by use of table 3.1. The total attenuation is just the sum of the attenuation of each section. Table 3.2 gives the attenuation for each section and the total attenuation for normalized radian frequency from 1 radian to 8 radians. The attenuation calculation of table 3.2 assumes that the filter is terminated in the characteristic impedance of the section given by equation (3.34). This is not a physically realizable termination impedance. The termination of m-derived filters will be discussed in the next section. Then a filter for this example will be terminated in the indicated manner and then analyzed by NASAP-69 to determine if the specifications for the filter have been met.

3.4 Termination of m-derived filters. In the preceding section the calculation of attenuation for the m-derived filters was based on the assumption that the filter sections were terminated in their characteristic impedances, $Zc = R\sqrt{1 - (\omega/\omega_c)^2}$ for the T section and $Zc = R/\sqrt{1 - (\omega/\omega_c)^2}$ for the π section. It already has been pointed out that

Normalized radian frequency ω/ω_c	M = 0.2747 ω_∞ = 1.04 db	M = 0.7241 ω_∞ = 1.45 db	M = 1 prototype db	Total attenuation db
1.000	0.0	0.0	0.0	0
1.025	19.1	5.4	3.9	28.4
1.04	∞	6.9	4.9	∞
1.06	20.5	8.6	6.0	35.1
1.10	13.8	11.4	7.7	32.9
1.14	11.3	13.9	9.1	34.3
1.18	10.0	16.3	10.3	36.6
1.22	9.1	18.7	11.3	39.1
1.26	8.5	21.2	12.3	42.0
1.30	8.0	24.1	13.1	45.2
1.35	7.5	28.5	14.1	50.1
1.40	7.2	35.4	15.1	57.7
1.50	6.7	36.8	16.7	60.2
1.75	6.1	24.1	20.1	50.3
2.00	5.7	21.0	22.9	49.6
4.00	5.1	16.8	35.1	57.0
8.00	4.9	16.0	48.1	69.0

Table 3.2 Tabulated attenuation of filter designed to meet specifications of table 1.5.

this termination is not possible since both these impedances are an irrational function of frequency. One possible means of termination is to approximate the characteristic impedance by a resistance R as indicated in figure 3.13. The filter's input and output are both terminated in the resistance R. The effect of this mismatch may be minimized by altering the characteristic impedance of the filter section so that it is more nearly approximated by a constant resistance.

The filter sections found in figure 3.14-a and 3.14-b are the m-derived half sections. Two identical half sections may be connected in cascade to form a full m-derived

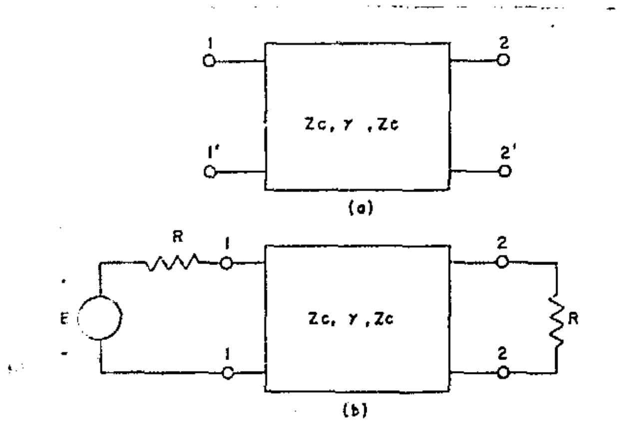

Figure 3.13 Terminated m-derived filter. (a) network. (b) terminated network.

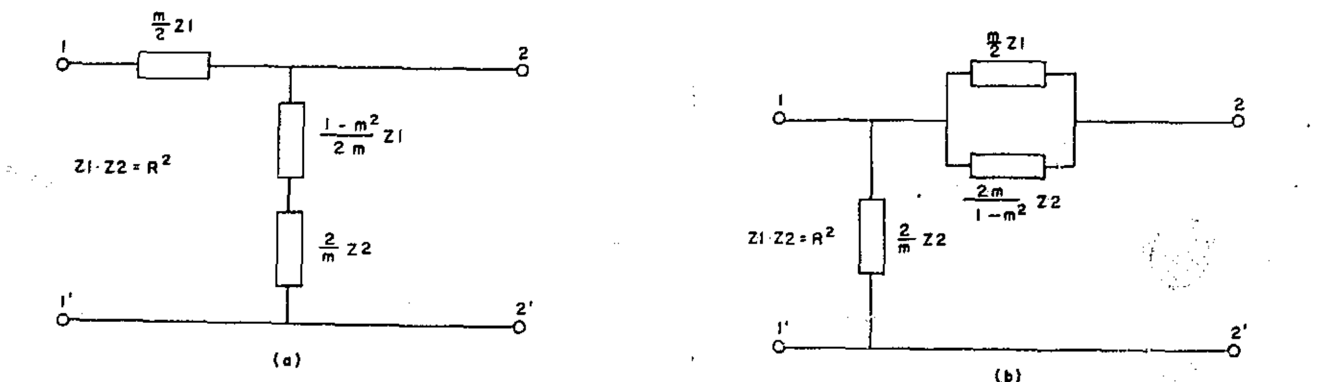

Figure 3.14 m-derived half sections. (a) T m-derived half section. (b) π m-derived half section.

section. For example, two half T sections similar to the section of figure 3.14-a may be cascaded to form an m-derived T section. The half sections are not symmetrical. Thus, their image impedances Zi1 and Zi2 are not the same. For the T half section the open and short circuit impedances looking into terminal pair (1,1′) are

$$Zoc = \frac{Z1 + 4Z2}{2m}$$

$$Zsc = \frac{mZ1}{2}$$

and Zi1 is found to be

$$Zi1 = \sqrt{ZosZsc} = \sqrt{Z_1^2 + 4Z1Z2} = R\sqrt{1 + Z_1^2/4R^2}.$$

Therefore, the image impedance Zi1 for the T m-derived half section is the same as the T m-derived section, equation (3.27). The image impedance for the same section, looking into the terminal pair (2,2′), will now be found. The open and short circuit impedances are

$$Zoc = \frac{[(1-m^2)Z_1^2 + 4Z1Z2]}{2mZ1}$$

$$Zsc = \frac{m[(1 - m^2)Z_1^2 + 4Z1Z2]}{2(Z1 + 4Z2)}$$

and Zi2 is

$$Zi2 = \sqrt{\frac{[(1 - m)^2 Z_1^2 + 4Z1Z2]^2}{4(Z_1^2 + 4Z1Z2)}} = \frac{R[(1 - m^2)(Z_1^2/4R^2)]}{\sqrt{1 + (Z_1^2/4R^2)}} \quad (3.43)$$

The open and short circuit impedance for the π m-derived half section is found to be

$$Zoc\Big|_{\text{port 2 open}} = \frac{2Z2}{m}$$

$$Zsc\Big|_{\text{port 2 short}} = \frac{2mZ_1^2 Z2}{Z_1^2 + 4Z1Z2}$$

$$Zoc\Big|_{\text{port 1 open}} = \frac{2Z2(Z_1^2 + 4Z1Z2)}{m[(1 - m^2)Z_1^2 + 4Z1Z2]}$$

$$Zsc\Big|_{\text{port 2 short}} = \frac{2mZ_1^2 Z2}{(1 - m^2)Z_1^2 + 4Z1Z2}.$$

The image impedances for the π m-derived half section are then

$$Zi1 = \frac{R}{\sqrt{1 + Z_1^2/4R^2}} \quad (3.44)$$

$$Zi2 = \frac{R\sqrt{(1 + Z_1^2/4R^2)}}{1 + (1 - m^2)(Z_1^2/4R^2)}, \quad (3.45)$$

The image impedance Zi1 for the π m-derived half section is the same as the π m-derived section, equation (3.30). As a result, the π m-derived half section and the π m-derived section may be connected in cascade. The propagation constants of the half sections are just one half that of a full section.

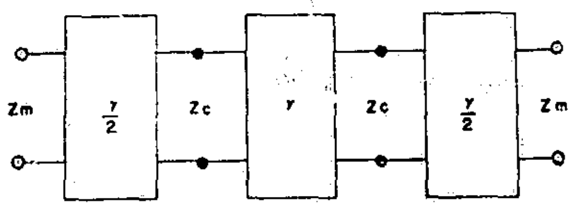

Figure 3.16 m-derived filter terminated in half sections.

The image impedance Zi2 for both the T and π m-derived half sections involved the parameter m and were quite different from the characteristic impedance for the T and π m-derived sections. By proper choice of m, the image impedance Zi2 for the half sections may be made more nearly a constant as compared to the characteristic impedance of the full sections. Figure 3.15 shows the maximum loss in db for several values of m for a filter terminated in an m-derived half section. A value of m = 0.6 reduces the reflection loss over much of the bandpass region. This suggests a method of terminating m-derived filters. A section of the filter which has an m near .6 is selected.

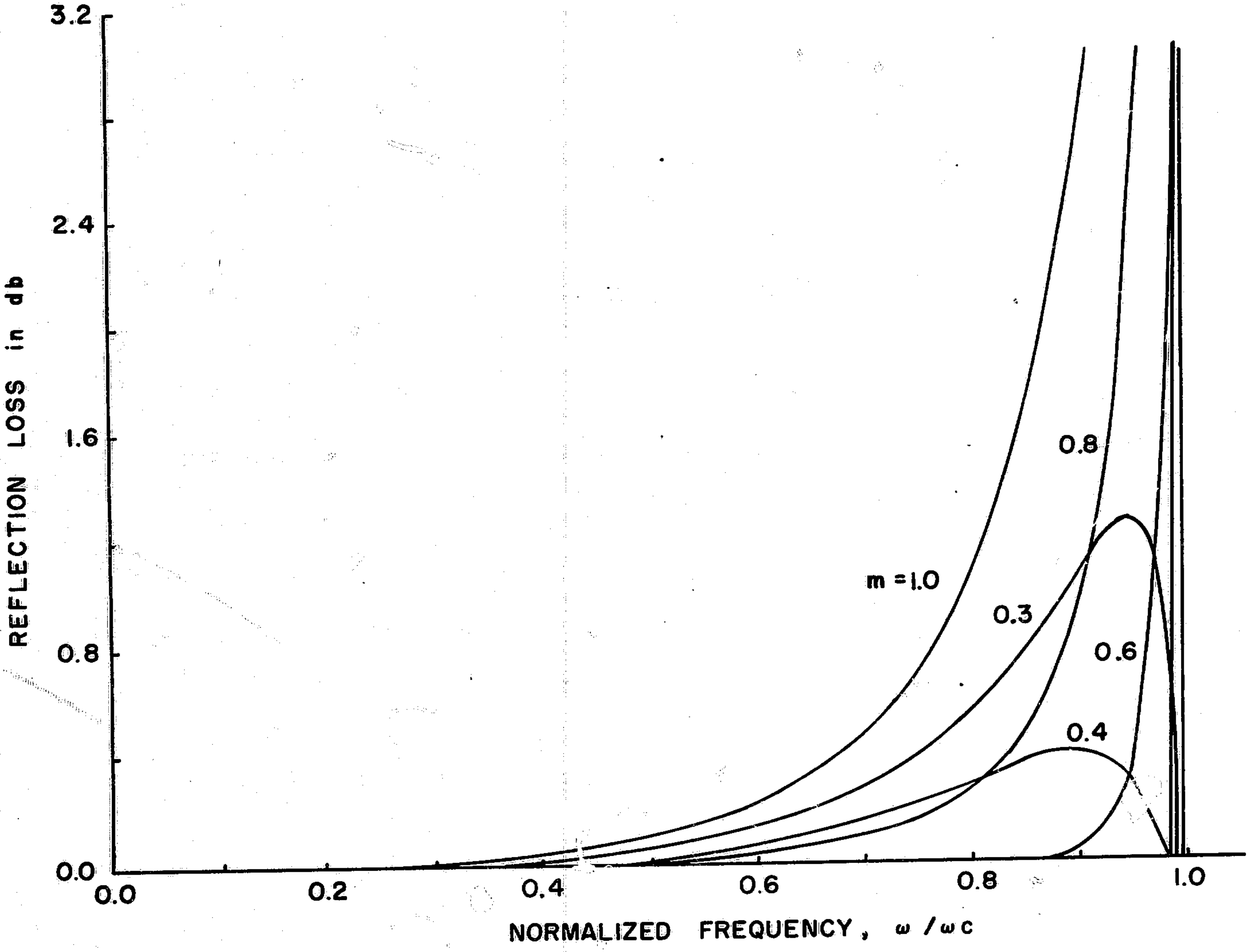
3.2
2.4
1.6
0.8
0.0
REFLECTION LOSS in db
0.0
0.2
0.4
0.6
0.8
1.0
NORMALIZED FREQUENCY, ω / ωc
m = 1.0
0.8
0.3
0.6
0.4

FIG. 3.15

This section is divided into two half sections which are placed at the two ports of the filter. The half sections are connected as shown in figure 3.16. The filter is then to be terminated in the approximate image impedance for the filter, R. The low-pass filter specified in table 1.5 and table 3.2 will now be completed using half sections for termination.

The filter that was designed to meet the specifications of table 1.5 consisted of 3 m-derived sections, m = 1.00, m = 0.7241, and m = 0.2747. The section for m = 0.7241 may be used to obtain the two half sections for termination. For this filter the T m-derived sections were chosen.

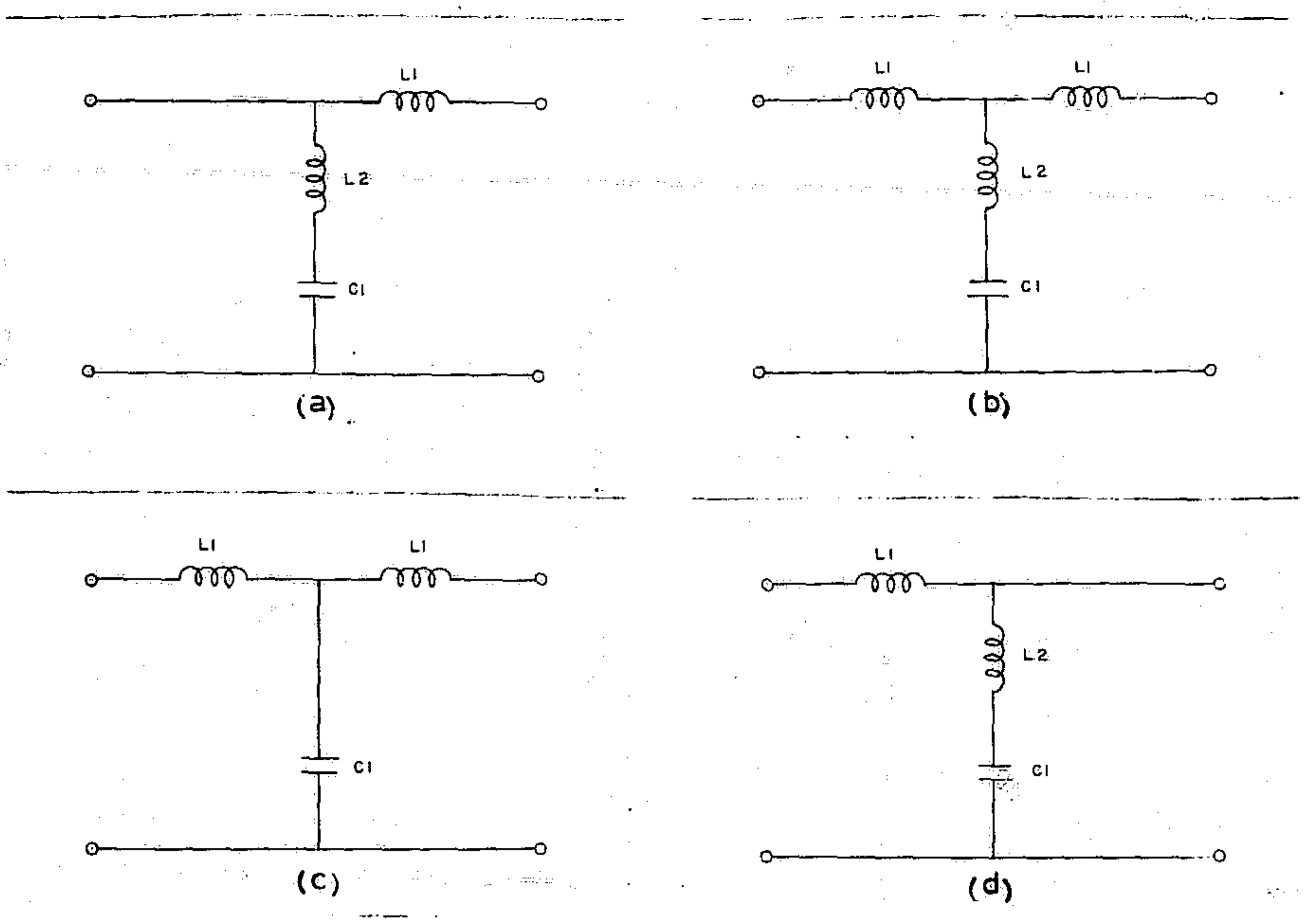

Figure 3.17 Four m-derived sections. (a) first section, m = 0.7241. (b) second section, m = 0.2747. (c) third section, m = 1.00. (d) fourth section, m = 0.7241.

The element values are found as follows:

For the first section, m = 0.7241

$$L1 = \frac{mL}{2} = \frac{(0.7241)(2)}{2} = 0.7241 \text{ h}$$

$$L2 = \frac{(1 - m^2)L}{2m} = \frac{(1.5243)}{.7241} = \frac{.4757}{.7241} = .6569 \text{ h}$$

$$C1 = \frac{mC}{2} = \frac{(0.724)(2)}{2} = 0.7241 \text{ f.}$$

For the second section m = 0.2741

$$L1 = \frac{mL}{2} = \frac{(0.2747)(2)}{2} = 0.2747 \text{ h}$$

$$L2 = \frac{(1 - m^2)L}{4m} = \frac{(1 - .0755)(2)}{(4)(.1378)} = 1.6828 \text{ h}$$

$$C1 = mC = (0.2747)(2) = .5494 \text{ f.}$$

For the third section, m = 1.00

$$L1 = \frac{L}{2} = \frac{2}{2} = 1.00 \text{ h}$$

$$C2 = C = 2.00 \text{ f}$$

For the fourth section, m = 0.7241

$$L1 = \frac{mL}{2} = 0.7241 \text{ h}$$

$$L2 = \frac{(1 - m^2)L}{2m} = .6769 \text{ h}$$

$$C2 = \frac{mL}{2} = 0.7241 \text{ f.}$$

L and C have been selected so that $\omega_c = 1$, and R = 1.0 ohm. NASAP-69 may now be used to insure th t the designed filter meets the specifications; see chapter 8 where this has been done.

Table 3.1

ATTENUATION PER SECTION IN DB FOR A NORMALIZED M-DERIVED LOW-PASS FILTER

OMEGA-0	1.01	1.02	1.03	1.04	1.05	1.06	1.07	1.08	1.09	1.10	1.12	1.14	1.16
M	0.1404	0.1971	0.2396	0.2747	0.3049	0.3317	0.3558	0.3777	0.3979	0.4166	0.4503	0.4801	0.5068
OMEGA													
1.005	15.39	9.67	7.68	6.60	5.89	5.38	5.00	4.69	4.44	4.23	3.91	3.66	3.46
1.010	INF	15.49	11.66	9.79	8.64	7.84	7.24	6.78	6.40	6.09	5.60	5.23	4.94
1.015	20.06	23.09	15.58	12.69	11.03	9.92	9.11	8.49	7.99	7.59	6.95	6.48	6.11
1.020	15.49	INF	20.22	15.67	13.35	11.88	10.83	10.05	9.43	8.92	8.15	7.57	7.13
1.025	13.15	25.35	27.17	19.03	15.76	13.82	12.50	11.53	10.78	10.17	9.25	8.57	9.05
1.030	11.66	20.22	INF	23.31	18.41	15.84	14.19	13.00	12.09	11.37	10.29	9.51	9.02
1.035	10.60	17.47	28.69	30.00	21.54	18.04	15.93	14.48	13.40	12.56	11.31	10.42	9.74
1.040	9.79	15.67	23.31	INF	25.64	20.53	17.80	16.02	14.73	13.74	12.31	11.30	10.54
1.045	9.16	14.36	20.37	31.15	32.18	23.53	19.87	17.65	16.11	14.95	13.30	12.16	11.31
1.050	8.64	13.35	18.41	25.64	INF	27.51	22.26	19.43	17.56	16.20	14.30	13.02	12.08
1.055	8.21	12.54	16.97	22.59	33.11	33.96	25.18	21.42	19.12	17.50	15.32	13.88	12.84
1.060	7.84	11.88	15.84	20.53	27.51	INF	29.09	23.74	20.93	18.89	16.37	14.75	13.60
1.065	7.52	11.32	14.94	19.00	24.39	34.75	35.48	26.60	22.76	20.39	17.46	15.63	14.36
1.070	7.24	10.83	14.19	17.80	22.26	29.09	INF	30.46	25.04	22.06	18.60	16.54	15.13
1.075	7.00	10.42	13.55	16.83	20.68	25.91	36.16	36.80	27.85	23.95	19.82	17.47	15.91
1.080	6.78	10.05	13.00	16.02	19.43	23.74	30.46	INF	31.67	26.19	21.13	18.45	16.71
1.085	6.58	9.72	12.52	15.33	18.41	22.12	27.24	37.41	37.98	28.97	22.56	19.47	17.53
1.090	6.40	9.43	12.09	14.73	17.56	20.83	25.04	31.67	INF	32.76	24.16	20.56	18.39
1.095	6.24	9.16	11.71	14.21	16.83	19.73	23.38	28.42	38.52	39.04	26.00	21.72	19.26
1.100	6.09	8.92	11.37	13.74	16.20	18.89	22.06	26.19	32.76	INF	28.18	22.98	20.19
1.110	5.82	8.50	10.79	12.95	15.14	17.47	20.07	23.16	27.23	33.75	34.66	25.93	22.22
1.120	5.60	8.15	10.29	12.31	14.30	16.37	18.60	21.13	24.16	28.18	INF	29.87	24.57
1.130	5.40	7.84	9.88	11.76	13.60	15.48	17.46	19.62	22.09	25.08	35.50	36.29	27.46
1.140	5.23	7.57	9.51	11.30	13.02	14.75	16.54	18.45	20.56	22.98	29.87	INF	31.25
1.150	5.07	7.34	9.20	10.89	12.52	14.13	15.77	17.50	19.36	21.42	26.72	37.03	37.72
1.160	4.94	7.13	8.92	10.54	12.08	13.60	15.13	16.71	18.38	20.19	24.57	31.35	INF
1.170	4.81	6.94	8.67	10.22	11.70	13.13	14.57	16.04	17.57	19.20	22.97	28.16	39.38
1.180	4.70	6.77	8.44	9.94	11.36	12.72	14.08	15.46	16.87	18.36	21.70	25.98	32.67
1.190	4.60	6.61	8.24	9.69	11.05	12.36	13.65	14.95	16.28	17.65	20.67	24.34	29.45
1.200	4.51	6.47	8.06	9.47	10.78	12.03	13.27	14.50	15.75	17.04	19.81	23.05	27.34

1.220	4.34	6.23	7.74	9.07	10.30	11.49	12.62	13.75	14.88	16.02	18.43	21.10	24.26
1.240	4.20	6.02	7.46	8.73	9.90	11.01	12.08	13.13	14.17	15.21	17.36	19.67	22.26
1.260	4.08	5.83	7.23	8.45	9.56	10.62	11.63	12.61	13.58	14.55	16.51	18.57	20.90
1.280	3.97	5.67	7.02	8.20	9.27	10.28	11.24	12.17	13.09	13.99	15.81	17.63	19.66
1.300	3.88	5.53	6.84	7.98	9.02	9.98	10.91	11.80	12.66	13.52	15.22	16.95	18.75
1.350	3.68	5.25	6.48	7.54	8.50	9.39	10.24	11.04	11.83	12.59	14.09	15.57	17.07
1.400	3.53	5.02	6.19	7.20	8.11	8.94	9.73	10.48	11.20	11.91	13.27	14.59	15.91
1.450	3.41	4.85	5.97	6.93	7.80	8.59	9.34	10.04	10.72	11.38	12.64	13.86	15.05
1.500	3.31	4.70	5.79	6.71	7.54	8.31	9.02	9.69	10.34	10.96	12.15	13.29	14.39
1.550	3.23	4.58	5.63	6.53	7.34	8.07	8.76	9.41	10.02	10.62	11.75	12.83	13.87
1.600	3.16	4.48	5.51	6.38	7.16	7.88	8.54	9.17	9.76	10.34	11.42	12.45	13.44
1.650	3.10	4.39	5.40	6.25	7.01	7.71	8.36	8.96	9.54	10.10	11.14	12.13	13.08
1.700	3.04	4.32	5.30	6.14	6.89	7.57	8.20	8.79	9.35	9.89	10.91	11.87	12.78
1.750	3.00	4.25	5.22	6.04	6.77	7.44	8.06	8.64	9.19	9.71	10.71	11.63	12.52
1.800	2.96	4.19	5.15	5.96	6.68	7.33	7.94	8.51	9.05	9.56	10.53	11.44	12.29
1.850	2.92	4.14	5.08	5.88	6.59	7.24	7.83	8.39	8.92	9.43	10.37	11.26	12.10
1.900	2.89	4.10	5.03	5.82	6.52	7.15	7.74	8.29	8.81	9.31	10.24	11.11	11.93
1.950	2.86	4.06	4.98	5.76	6.45	7.07	7.65	8.20	8.71	9.20	10.12	10.97	11.77
2.000	2.84	4.02	4.93	5.70	6.39	7.01	7.58	8.12	8.62	9.10	10.01	10.85	11.64
2.200	2.76	3.91	4.79	5.53	6.19	6.79	7.34	7.86	8.34	8.80	9.67	10.47	11.22
2.400	2.70	3.82	4.69	5.41	6.06	6.64	7.18	7.68	8.15	8.60	9.43	10.20	10.92
2.600	2.66	3.76	4.61	5.33	5.96	6.53	7.06	7.55	8.01	8.44	9.26	10.01	10.71
2.800	2.63	3.72	4.56	5.26	5.88	6.45	6.96	7.45	7.90	8.33	9.13	9.87	10.55
3.000	2.60	3.68	4.51	5.21	5.82	6.38	6.89	7.37	7.82	8.24	9.03	9.75	10.43
3.500	2.56	3.62	4.43	5.12	5.72	6.27	6.77	7.23	7.67	8.09	8.85	9.55	10.22
4.000	2.53	3.58	4.39	5.06	5.66	6.20	6.69	7.15	7.58	7.99	8.75	9.44	10.09
4.500	2.52	3.56	4.36	5.03	5.62	6.15	6.64	7.10	7.52	7.93	[illegible]	[illegible]	10.00
5.000	2.50	3.54	4.33	5.00	5.59	6.12	6.61	7.06	7.48	7.88	8.62	9.31	9.94
5.500	2.50	3.53	4.32	4.98	5.57	6.09	6.58	7.03	[illegible]	7.85	8.59	9.27	9.90
6.000	2.49	3.52	4.30	4.97	5.55	6.08	6.56	7.01	[illegible]	7.83	8.56	9.24	9.86
7.000	2.48	3.50	4.29	4.95	5.53	6.05	6.53	6.98	7.40	7.79	8.52	9.20	9.82
8.000	2.47	3.49	4.28	4.94	5.51	6.04	6.51	6.96	7.38	7.77	8.50	9.17	9.79
16.000	2.46	3.47	4.25	4.90	5.48	6.00	6.47	6.91	7.33	7.72	8.44	9.10	9.72
50.000	2.45	3.47	4.24	4.90	5.47	5.99	6.46	6.90	7.31	7.70	8.42	9.09	9.70

ATTENUATION PER SECTION IN DB FOR A NORMALIZED M-DERIVED LOW-PASS FILTER

OMEGA-0	1.18	1.20	1.22	1.24	1.26	1.28	1.30	1.35	1.40	1.45	1.50	1.55	1.60
M	0.5309	0.5528	0.5729	0.5913	0.6084	0.6242	0.6390	0.6718	0.6999	0.7241	0.7454	0.7640	0.7806
OMEGA													
1.005	3.30	3.16	3.05	2.95	2.87	2.79	2.73	2.59	2.49	2.40	2.33	2.28	2.23
1.010	4.70	4.51	4.34	4.20	4.08	3.97	3.88	3.68	3.53	3.41	3.31	3.23	3.16
1.015	5.81	5.56	5.36	5.18	5.02	4.89	4.77	4.53	4.34	4.19	4.06	3.96	3.87
1.020	6.77	6.47	6.23	6.02	5.83	5.67	5.53	5.25	5.02	4.85	4.70	4.58	4.48
1.025	7.64	7.30	7.01	6.77	6.56	6.38	6.22	5.89	5.64	5.43	5.27	5.13	5.02
1.030	8.44	8.06	7.74	7.46	7.23	7.02	6.84	6.48	6.19	5.97	5.79	5.63	5.51
1.035	9.21	8.78	8.42	8.11	7.85	7.63	7.43	7.02	6.71	6.47	6.26	6.10	5.96
1.040	9.94	9.47	9.07	8.73	8.45	8.20	7.98	7.54	7.20	6.93	6.71	6.53	6.39
1.045	10.66	10.13	9.69	9.33	9.02	8.75	8.51	8.03	7.66	7.37	7.14	6.94	6.78
1.050	11.36	10.78	10.30	9.90	9.56	9.27	9.02	8.50	8.11	7.80	7.54	7.34	7.16
1.055	12.04	11.41	10.89	10.46	10.10	9.78	9.51	8.95	8.53	8.20	7.93	7.71	7.53
1.060	12.72	12.03	11.48	11.01	10.62	10.28	9.98	9.39	8.94	8.59	8.31	8.07	7.89
1.065	13.40	12.65	12.05	11.55	11.13	10.76	10.45	9.82	9.34	8.97	8.67	8.42	8.21
1.070	14.08	13.27	12.62	12.08	11.63	11.24	10.91	10.24	9.73	9.34	9.02	8.76	8.54
1.075	14.76	13.88	13.18	12.60	12.12	11.71	11.35	10.64	10.11	9.69	9.36	9.09	8.86
1.080	15.46	14.50	13.75	13.13	12.61	12.17	11.80	11.04	10.48	10.04	9.69	9.41	9.17
1.085	16.16	15.12	14.31	13.65	13.10	12.63	12.23	11.44	10.85	10.39	10.02	9.72	9.47
1.090	16.87	15.75	14.88	14.17	13.58	13.09	12.66	11.83	11.20	10.72	10.34	10.02	9.76
1.095	17.61	16.39	15.45	14.69	14.07	13.54	13.09	12.21	11.56	11.05	10.65	10.32	10.05
1.100	18.36	17.04	16.02	15.21	14.55	13.99	13.52	12.59	11.91	11.38	10.96	10.62	10.34
1.110	19.96	18.38	17.20	16.27	15.52	14.90	14.37	13.34	12.59	12.02	11.56	11.19	10.89
1.120	21.70	19.81	18.43	17.36	16.51	15.81	15.22	14.09	13.27	12.64	12.15	11.75	11.42
1.130	23.67	21.35	19.72	18.49	17.52	16.74	16.08	14.83	13.93	13.25	12.72	12.29	11.94
1.140	25.98	23.05	21.10	19.67	18.57	17.68	16.95	15.57	14.59	13.86	13.29	12.83	12.45
1.150	28.82	24.97	22.59	20.92	19.65	18.65	17.84	16.31	15.25	14.46	13.84	13.35	12.95
1.160	32.67	27.24	24.26	22.26	20.80	19.66	18.75	17.07	15.91	15.05	14.39	13.87	13.44
1.170	39.01	30.05	26.15	23.73	22.02	20.72	19.69	17.83	16.57	15.65	14.94	14.38	13.92
1.180	INF	33.87	28.33	25.36	23.33	21.84	20.67	18.61	17.24	16.24	15.48	14.88	14.40
1.190	39.60	40.17	31.17	27.23	24.77	23.03	21.71	19.41	17.91	16.84	16.02	15.39	14.87
1.200	33.87	INF	34.96	29.44	26.38	24.32	22.80	20.24	18.59	17.43	16.56	15.88	15.34

1.220	28.38	34.96	INF	35.97	30.41	27.33	25.25	21.98	20.01	18.65	17.65	16.88	16.27
1.240	25.36	29.44	35.97	INF	36.91	31.33	28.22	23.90	21.49	19.90	18.76	17.88	17.19
1.260	23.33	26.38	30.42	36.91	INF	37.80	32.19	26.99	23.09	21.21	19.89	18.89	18.12
1.280	21.84	24.32	27.33	31.33	37.80	INF	38.63	28.69	24.85	22.59	21.05	19.93	19.06
1.300	20.67	22.80	25.25	28.22	32.19	38.63	INF	32.01	26.83	24.07	22.27	20.99	20.01
1.350	18.61	20.24	21.98	23.90	26.09	28.69	32.01	INF	33.76	28.50	25.68	23.83	22.49
1.400	17.24	18.59	20.01	21.49	23.09	24.85	26.83	33.76	INF	35.34	30.02	27.15	25.25
1.450	16.24	17.43	18.65	19.90	21.21	22.59	24.07	28.50	35.34	INF	36.79	31.41	28.49
1.500	15.48	16.56	17.65	18.76	19.89	21.05	22.27	25.68	30.02	36.79	INF	38.12	32.70
1.550	14.88	15.88	16.88	17.88	18.89	19.93	20.99	23.83	27.15	31.41	38.12	INF	39.36
1.600	14.40	15.34	16.27	17.19	18.12	19.06	20.01	22.49	25.25	28.49	32.70	39.36	INF
1.650	13.99	14.89	15.76	16.63	17.50	18.36	19.24	21.48	23.88	26.56	29.75	33.91	40.52
1.700	13.66	14.51	15.35	16.17	16.98	17.80	18.61	20.67	22.82	25.15	27.79	30.92	35.04
1.750	13.37	14.19	14.99	15.78	16.55	17.32	18.09	20.01	21.99	24.07	26.35	28.93	32.02
1.800	13.12	13.91	14.69	15.44	16.19	16.92	17.65	19.46	21.30	23.21	25.24	27.47	30.00
1.850	12.90	13.67	14.42	15.15	15.87	16.58	17.28	19.00	20.73	22.50	24.35	26.34	28.52
1.900	12.71	13.46	14.19	14.90	15.60	16.28	16.95	18.61	20.25	21.91	23.63	25.43	27.37
1.950	12.54	13.28	13.99	14.68	15.36	16.02	16.67	18.26	19.83	21.41	23.02	24.69	26.45
2.000	12.39	13.11	13.81	14.48	15.14	15.79	16.42	17.96	19.47	20.98	22.50	24.06	25.69
2.200	11.93	12.60	13.26	13.88	14.49	15.09	15.66	17.06	18.41	19.72	21.01	22.31	23.61
2.400	11.61	12.25	12.88	13.47	14.05	14.61	15.16	16.46	17.71	18.91	20.08	21.23	22.37
2.600	11.37	12.00	12.60	13.18	13.74	14.27	14.80	16.04	17.22	18.35	19.44	20.50	21.54
2.800	11.20	11.81	12.40	12.96	13.50	14.02	14.53	15.73	16.86	17.94	18.98	19.98	20.95
3.000	11.06	11.67	12.24	12.79	13.32	13.83	14.32	15.49	16.59	17.63	18.63	19.59	20.52
3.500	10.84	11.42	11.97	12.50	13.01	13.50	13.97	15.09	16.14	17.12	18.05	18.94	19.90
4.000	10.69	11.27	11.81	12.33	12.82	13.30	13.76	14.85	15.86	16.81	17.71	18.56	19.38
4.500	10.60	11.16	11.70	12.21	12.70	13.17	13.62	14.69	15.68	16.61	17.48	18.31	19.11
5.000	10.53	11.09	11.62	12.13	12.61	13.08	13.53	14.58	15.55	16.47	17.33	18.14	18.92
5.500	10.49	11.04	11.57	12.07	12.55	13.01	13.45	14.50	15.46	16.37	17.21	18.02	18.79
6.000	10.45	11.00	11.53	12.03	12.50	12.96	13.40	14.44	15.40	16.29	17.13	17.93	18.68
7.000	10.40	10.95	11.47	11.96	12.44	12.89	13.33	14.35	15.30	16.19	17.02	17.80	18.54
8.000	10.37	10.92	11.43	11.93	12.40	12.85	13.28	14.30	15.24	16.12	16.94	17.72	18.46
16.000	10.29	10.83	11.34	11.83	12.30	12.74	13.17	14.17	15.10	15.96	16.77	17.53	18.25
50.000	10.27	10.81	11.32	11.80	12.27	12.71	13.14	14.14	15.06	15.91	16.72	17.47	18.19

ATTENUATION PER SECTION IN DB FOR A NORMALIZED M-DERIVED LOW-PASS FILTER

OMEGA-0	1.65	1.70	1.75	1.80	1.85	1.90	1.95	2.00	2.50	3.00	3.50	4.00	4.50
M	0.7954	0.8087	0.8207	0.8315	0.8413	0.8503	0.8585	0.8660	0.9165	0.9428	0.9583	0.9682	0.9750
OMEGA													
1.005	2.19	2.15	2.12	2.09	2.07	2.04	2.02	2.01	1.89	1.84	1.81	1.79	1.78
1.010	3.10	3.04	3.00	2.96	2.92	2.89	2.86	2.84	2.68	2.60	2.56	2.53	2.52
1.015	3.80	3.73	3.68	3.63	3.58	3.55	3.51	3.48	3.28	3.19	3.14	3.10	3.08
1.020	4.39	4.32	4.25	4.19	4.14	4.10	4.06	4.02	3.79	3.68	3.62	3.58	3.56
1.025	4.92	4.83	4.76	4.69	4.64	4.59	4.54	4.50	4.24	4.12	4.05	4.01	3.98
1.030	5.40	5.30	5.22	5.15	5.08	5.03	4.98	4.93	4.65	4.51	4.43	4.39	4.36
1.035	5.84	5.74	5.65	5.57	5.50	5.44	5.38	5.33	5.02	4.87	4.79	4.74	4.70
1.040	6.25	6.14	6.04	5.96	5.88	5.82	5.76	5.70	5.37	5.21	5.12	5.06	5.03
1.045	6.64	6.52	6.42	6.33	6.25	6.18	6.11	6.05	5.69	5.53	5.43	5.37	5.33
1.050	7.01	6.89	6.77	6.68	6.59	6.52	6.45	6.39	6.00	5.82	5.72	5.66	5.62
1.055	7.37	7.23	7.12	7.01	6.92	6.84	6.77	6.70	6.30	6.11	6.00	5.93	5.89
1.060	7.71	7.57	7.44	7.33	7.24	7.15	7.07	7.01	6.58	6.38	6.27	6.20	6.15
1.065	8.04	7.89	7.76	7.64	7.54	7.45	7.37	7.30	6.85	6.64	6.52	6.45	6.40
1.070	8.36	8.20	8.06	7.94	7.83	7.74	7.65	7.58	7.11	6.89	6.77	6.69	6.64
1.075	8.66	8.50	8.35	8.23	8.12	8.02	7.93	7.85	7.36	7.13	7.01	[illegible]	6.97
1.080	8.96	8.79	8.64	8.51	8.39	8.29	8.20	8.12	7.61	7.37	7.23	7.15	7.10
1.085	9.26	9.08	8.92	8.78	8.66	8.55	8.46	8.37	7.84	7.60	7.46	7.37	7.31
1.090	9.54	9.35	9.19	9.05	8.92	8.81	8.71	8.62	8.07	7.82	7.67	7.58	7.52
1.095	9.82	9.63	9.46	9.31	9.18	9.06	8.96	8.87	8.30	8.03	7.88	7.79	7.73
1.100	10.10	9.89	9.71	9.56	9.43	9.31	9.20	9.10	8.51	8.24	8.09	7.99	7.93
1.110	10.63	10.41	10.22	10.05	9.91	9.78	9.67	9.56	8.94	8.64	8.49	8.38	8.31
1.120	11.14	10.91	10.71	10.53	10.37	10.24	10.12	10.01	9.34	9.03	8.85	8.75	8.69
1.130	11.64	11.39	11.18	10.99	10.82	10.68	10.55	10.43	9.73	9.40	9.21	9.10	9.03
1.140	12.13	11.87	11.63	11.44	11.26	11.11	10.97	10.85	10.10	9.75	9.56	9.44	9.36
1.150	12.61	12.33	12.08	11.87	11.68	11.52	11.38	11.25	10.46	10.10	9.89	9.77	9.69
1.160	13.08	12.78	12.52	12.29	12.10	11.93	11.77	11.64	10.81	10.43	10.22	10.09	10.00
1.170	13.54	13.22	12.95	12.71	12.50	12.32	12.16	12.02	11.15	10.75	10.53	10.40	10.30
1.180	13.99	13.66	13.37	13.12	12.90	12.71	12.54	12.39	11.48	11.06	10.84	10.69	10.60
1.190	14.44	14.09	13.79	13.52	13.29	13.09	12.91	12.76	11.80	11.37	11.13	10.98	10.89
1.200	14.89	14.51	14.19	13.91	13.67	13.46	13.28	13.11	12.12	11.67	11.42	11.27	11.16

1.220	15.76	15.35	14.99	14.69	14.42	14.19	13.99	13.81	12.73	12.24	11.97	11.81	11.70
1.240	16.63	16.17	15.78	15.44	15.15	14.90	14.68	14.48	13.32	12.79	12.50	12.33	12.21
1.260	17.50	16.98	16.55	16.19	15.87	15.60	15.36	15.14	13.88	13.32	13.01	12.92	12.70
1.280	18.36	17.80	17.32	16.92	16.58	16.28	16.02	15.79	14.43	13.83	13.50	13.30	13.17
1.300	19.24	18.61	18.09	17.65	17.28	16.95	16.67	16.42	14.96	14.32	13.97	13.76	13.62
1.350	21.48	20.67	20.01	19.46	19.00	18.61	18.26	17.96	16.24	15.49	15.09	14.85	14.69
1.400	23.88	22.82	21.99	21.30	20.73	20.25	19.83	19.47	17.45	16.59	16.14	15.86	15.68
1.450	26.56	25.15	24.07	23.21	22.50	21.91	21.41	20.98	18.61	17.63	17.12	16.81	16.61
1.500	29.75	27.78	26.35	25.24	24.35	23.63	23.02	22.50	19.73	18.63	18.05	17.71	17.48
1.550	33.91	30.92	28.93	27.47	26.34	25.43	24.69	24.06	20.83	19.59	18.94	18.55	18.31
1.600	40.53	35.04	32.02	30.00	28.52	27.37	26.45	25.69	21.92	20.52	19.80	19.38	19.11
1.650	INF	41.62	36.11	33.07	31.03	29.52	28.36	27.42	22.99	21.42	20.63	20.16	19.87
1.700	41.62	INF	42.66	37.12	34.06	32.00	30.48	29.29	24.07	22.31	21.43	20.92	20.40
1.750	36.11	42.66	INF	43.65	38.09	35.00	32.92	31.39	25.15	23.18	22.21	21.66	21.30
1.800	33.07	37.12	43.65	INF	44.59	39.01	35.90	33.81	26.24	24.03	22.98	22.37	21.99
1.850	31.03	34.06	38.09	44.59	INF	45.48	39.89	36.77	27.36	24.89	23.73	23.07	22.65
1.900	29.52	32.00	35.00	39.01	45.48	INF	46.34	40.73	28.51	25.73	24.46	23.75	23.30
1.950	28.36	30.48	32.92	35.90	39.89	46.34	INF	47.17	29.70	26.58	25.19	24.41	23.93
2.000	27.42	29.29	31.39	33.81	36.77	40.73	47.17	INF	30.94	27.43	25.99	25.06	24.54
2.200	24.94	26.31	27.74	29.25	30.88	32.66	34.67	37.02	36.89	30.91	28.72	27.58	26.89
2.400	23.51	24.65	25.82	27.00	29.23	29.51	30.85	32.29	47.75	34.77	31.56	30.01	29.10
2.600	22.57	23.59	24.61	25.63	26.67	27.72	28.80	29.91	48.93	39.49	34.53	32.42	31.24
2.800	21.91	22.85	23.78	24.70	25.63	26.55	27.49	28.44	40.45	46.59	37.82	34.80	33.35
3.000	21.42	22.31	23.18	24.03	24.89	25.73	26.58	27.43	36.97	INF	41.75	37.50	35.48
3.500	20.63	21.43	22.21	22.98	23.73	24.46	25.19	25.90	33.02	41.75	INF	45.73	41.26
4.000	20.16	20.92	21.66	22.37	23.07	23.75	24.41	25.06	31.21	37.50	45.77	INF	49.16
4.500	19.87	20.60	21.30	21.99	22.65	23.30	23.93	24.54	30.18	35.48	41.26	49.16	INF
5.000	19.66	20.37	21.06	21.72	22.37	22.99	23.60	24.19	29.52	34.30	39.08	44.52	52.17
5.500	19.51	20.21	20.89	21.54	22.16	22.77	23.36	23.94	29.06	33.52	37.78	42.21	47.29
6.000	19.40	20.09	20.76	21.40	22.01	22.61	23.19	23.76	28.73	32.98	36.91	40.80	44.98
7.000	19.25	19.93	20.58	21.21	21.81	22.40	22.96	23.51	28.29	32.27	35.93	39.17	42.47
8.000	19.16	19.83	20.47	21.09	21.68	22.26	22.81	23.35	28.02	31.85	35.19	38.25	41.16
16.000	18.93	19.59	20.21	20.80	21.38	21.93	22.47	22.98	27.40	30.90	33.93	36.37	38.63
50.000	18.86	19.51	20.13	20.72	21.29	21.84	22.37	22.88	27.22	30.63	33.46	35.88	38.00

ATTENUATION PER SECTION IN DB FOR A NORMALIZED M-DERIVED LOW-PASS FILTER

OMEGA-D	5.00	5.50	6.00	6.50	7.00	7.50	8.00	8.50	9.00	9.50	10.00	12.00	INF
M	0.9798	0.9833	0.9860	0.9881	0.9897	0.9911	0.9922	0.9931	0.9938	0.9944	0.9950	0.9965	1.0000
OMEGA													
1.005	1.77	1.77	1.76	1.76	1.75	1.75	1.75	1.75	1.75	1.75	1.74	1.74	27.33
1.010	2.50	2.50	2.49	2.48	2.48	2.48	2.47	2.47	2.47	2.47	2.47	2.46	27.38
1.015	3.07	3.06	3.05	3.04	3.04	3.03	3.03	3.02	3.02	3.02	3.02	3.01	27.44
1.020	3.54	3.53	3.52	3.51	3.50	3.50	3.49	3.49	3.49	3.49	3.48	3.48	27.49
1.025	3.96	3.94	3.93	3.92	3.92	3.91	3.91	3.90	3.90	3.90	3.89	3.89	27.55
1.030	4.33	4.32	4.30	4.30	4.29	4.28	4.28	4.27	4.27	4.27	4.26	4.26	27.60
1.035	4.68	4.66	4.65	4.64	4.63	4.62	4.62	4.61	4.61	4.61	4.60	4.60	27.65
1.040	5.00	4.98	4.97	4.96	4.95	4.94	4.94	4.93	4.93	4.92	4.92	4.91	27.71
1.045	5.30	5.28	5.27	5.26	5.25	5.24	5.23	5.23	5.22	5.22	5.22	5.21	27.76
1.050	5.59	5.57	5.55	5.54	5.53	5.52	5.51	5.51	5.50	5.50	5.50	5.49	27.81
1.055	5.86	5.84	5.82	5.81	5.80	5.79	5.78	5.78	5.77	5.77	5.76	5.75	27.87
1.060	6.12	6.09	6.08	6.06	6.05	6.04	6.04	6.03	6.02	6.02	6.02	6.01	27.92
1.065	6.37	6.34	6.32	6.31	6.30	6.29	6.28	6.27	6.27	6.26	6.26	6.25	27.97
1.070	6.61	6.58	6.56	6.54	6.53	6.52	6.51	6.51	6.50	6.50	6.49	6.48	28.03
1.075	6.84	6.81	6.79	6.77	6.76	6.75	6.74	6.73	6.73	6.72	6.72	6.71	28.08
1.080	7.06	7.03	7.01	6.99	6.98	6.97	6.96	6.95	6.95	6.94	6.94	6.93	28.13
1.085	7.27	7.24	7.22	7.20	7.19	7.18	7.17	7.16	7.16	7.15	7.15	7.14	28.19
1.090	7.48	7.45	7.43	7.41	7.40	7.39	7.38	7.37	7.36	7.36	7.35	7.34	28.24
1.095	7.68	7.65	7.63	7.61	7.60	7.59	7.58	7.57	7.56	7.56	7.55	7.54	28.29
1.100	7.88	7.85	7.83	7.81	7.79	7.78	7.77	7.76	7.76	7.75	7.75	7.73	28.34
1.110	8.26	8.23	8.20	8.18	8.17	8.15	8.14	8.14	8.13	8.12	8.12	8.10	28.44
1.120	8.63	8.59	8.56	8.54	8.52	8.51	8.50	8.49	8.48	8.48	8.47	8.46	28.54
1.130	8.97	8.94	8.91	8.89	8.87	8.85	8.84	8.83	8.82	8.82	8.81	8.80	28.64
1.140	9.31	9.27	9.24	9.21	9.20	9.18	9.17	9.16	9.15	9.14	9.14	9.12	28.74
1.150	9.63	9.59	9.56	9.53	9.51	9.50	9.48	9.47	9.47	9.46	9.45	9.43	28.84
1.160	9.94	9.90	9.86	9.84	9.82	9.80	9.79	9.78	9.77	9.76	9.75	9.74	28.94
1.170	10.24	10.20	10.16	10.13	10.11	10.10	10.08	10.07	10.06	10.05	10.05	10.03	29.04
1.180	10.53	10.49	10.45	10.42	10.40	10.38	10.37	10.36	10.35	10.34	10.33	10.31	29.14
1.190	10.82	10.77	10.73	10.70	10.68	10.66	10.65	10.63	10.62	10.61	10.61	10.59	29.24
1.200	11.09	11.04	11.00	10.97	10.95	10.93	10.92	10.90	10.89	10.88	10.88	10.85	29.33

1.220	11.62	11.57	11.53	11.49	11.47	11.45	11.43	11.42	11.41	11.40	11.39	11.37	29.53
1.240	12.13	12.07	12.03	11.99	11.96	11.94	11.93	11.91	11.90	11.89	11.89	11.86	29.72
1.260	12.61	12.55	12.50	12.47	12.44	12.42	12.40	12.38	12.37	12.36	12.35	12.32	29.90
1.280	13.08	13.01	12.96	12.92	12.89	12.87	12.85	12.83	12.82	12.81	12.80	12.77	30.09
1.300	13.53	13.45	13.40	13.36	13.33	13.30	13.28	13.26	13.25	13.24	13.23	13.20	30.27
1.350	14.58	14.50	14.44	14.39	14.35	14.33	14.30	14.28	14.27	14.25	14.24	14.21	30.72
1.400	15.55	15.46	15.40	15.34	15.30	15.27	15.24	15.22	15.20	15.19	15.17	15.14	31.15
1.450	16.47	16.37	16.29	16.23	16.19	16.15	16.12	16.09	16.07	16.06	16.04	16.00	31.57
1.500	17.33	17.21	17.13	17.07	17.02	16.97	16.94	16.92	16.89	16.87	16.86	16.81	31.98
1.550	18.14	18.02	17.93	17.85	17.80	17.75	17.72	17.69	17.66	17.64	17.63	17.58	32.38
1.600	18.92	18.78	18.68	18.60	18.54	18.49	18.46	18.42	18.40	18.37	18.35	18.30	32.77
1.650	19.66	19.51	19.40	19.32	19.25	19.20	19.16	19.12	19.09	19.07	19.05	18.99	33.16
1.700	20.37	20.21	20.09	20.00	19.93	19.87	19.83	19.79	19.76	19.73	19.71	19.64	33.53
1.750	21.06	20.89	20.76	20.66	20.58	20.52	20.47	20.43	20.39	20.37	20.34	20.27	33.90
1.800	21.72	21.54	21.40	21.29	21.21	21.14	21.09	21.04	21.01	20.98	20.95	20.88	34.25
1.850	22.37	22.16	22.01	21.90	21.81	21.74	21.68	21.64	21.60	21.56	21.53	21.46	34.59
1.900	22.99	22.77	22.61	22.49	22.40	22.32	22.26	22.21	22.16	22.13	22.10	22.02	34.93
1.950	23.60	23.36	23.19	23.06	22.96	22.88	22.81	22.76	22.71	22.68	22.64	22.56	35.26
2.000	24.19	23.94	23.76	23.62	23.51	23.42	23.35	23.29	23.25	23.21	23.17	23.09	35.58
2.200	26.43	26.11	25.87	25.69	25.56	25.45	25.36	25.29	25.23	25.18	25.13	25.01	[illegible]
2.400	28.51	28.11	27.81	27.59	27.42	27.28	27.17	27.08	27.01	26.95	26.90	26.75	[illegible]
2.600	30.50	29.99	29.62	29.35	29.14	28.97	28.84	28.73	28.64	28.57	28.51	28.33	39.05
2.800	32.41	31.78	31.33	31.00	30.75	30.55	30.39	30.26	30.16	30.07	29.99	29.78	40.05
3.000	34.30	33.52	32.98	32.58	32.27	32.04	31.85	31.70	31.57	31.47	31.38	31.13	41.90
3.500	39.08	37.78	36.91	36.29	35.83	35.47	35.19	34.97	34.79	34.64	34.51	34.16	43.13
4.000	44.52	42.21	40.80	39.85	39.17	38.65	38.25	37.94	37.68	37.47	37.29	36.92	45.02
4.500	52.17	47.39	44.98	43.49	42.47	41.73	41.16	40.72	40.37	40.09	39.85	39.21	46.72
5.000	INF	54.86	49.97	47.47	45.91	44.83	44.03	43.43	42.95	42.57	42.25	41.43	48.25
5.500	54.86	INF	57.29	52.31	49.74	48.11	46.97	46.14	45.49	44.98	44.57	43.50	49.65
6.000	49.97	57.29	INF	59.51	54.45	51.81	50.13	48.95	48.07	47.39	46.95	45.49	51.94
7.000	45.91	49.74	54.45	61.55	INF	63.44	58.26	55.51	53.74	52.49	51.53	49.31	53.24
8.000	44.03	46.97	50.13	53.73	58.26	65.20	INF	66.85	61.57	58.74	56.89	53.14	55.27
16.000	40.68	42.56	44.32	45.98	47.56	49.08	50.55	51.99	53.41	54.83	56.25	62.31	65.05
50.000	39.89	41.59	43.15	44.58	45.90	47.14	48.29	49.38	50.41	51.39	52.32	55.66	84.46

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CHAPTER FOUR

Introduction to Modern Network Synthesis Techniques

4.1 Introduction. In the previous chapter the design of filters by the use of image parameters was outlined. These image parameter design techniques have been replaced by modern network synthesis procedures. This chapter presents an introduction to the methods of network synthesis which can be used to realize a filter.

4.2 Transfer functions. The response of a network at a port, due to the excitation of the network at some other port, is related in a cause and effect manner by a transfer function. For example, the voltage appearing at the output port (the effect) may be compared to the current injected at the input port (the cause) by means of an impedance transfer function. Such a transfer function would have a dimension of impedance. Other transfer functions can have dimensions of admittance or can be dimensionless as are voltage ratio transfer functions. The transfer functions that will be considered here represent the ratio of Laplace transformed network voltages and currents. A general form of this type of transfer function can be given as a ratio of polynomials in s as

$$F(s) = \frac{P(s)}{Q(s)} = \frac{a_m s^m + a_{m-1} s^{m-1} + \ldots + a_1 s + a_0}{b_n s^n + b_{n-1} s^{n-1} + \ldots + b_1 s + b_0}$$

$$= \frac{H(s - z_1)(s - z_2) \ldots (s - z_m)}{(s - p_1)(s - p_2) \ldots (s - p_n)}$$

where $H = \frac{a_n}{b_n}$.

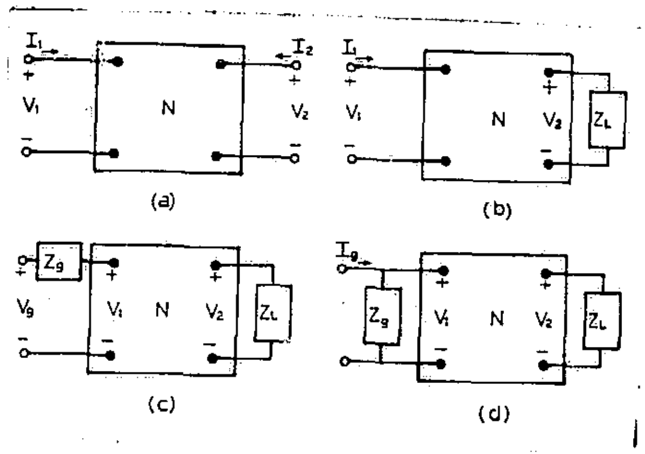

Figure 4.1 Network used to define the transfer functions; G21, Z21, Y21, G2g, Z2g, and Y2g.

The open-circuit impedance parameters for the two-port network of figure 4.1-a are defined as

$$z_{11} = \frac{V1}{I1}\Big|_{I2\,=\,0} \qquad z_{12} = \frac{V1}{I2}\Big|_{I1\,=\,0}$$

$$z_{21} = \frac{V2}{I1}\Big|_{I2\,=\,0} \qquad z_{22} = \frac{V2}{I2}\Big|_{I1\,=\,0}. \qquad (4.1)$$

The short-circuit admittance parameters for the same two-port network are defined as

$$y_{11} = \frac{I1}{V1}\Big|_{V2\,=\,0} \qquad -y_{12} = \frac{I1}{V2}\Big|_{V1\,=\,0}$$

$$-y_{21} = \frac{I2}{V1}\Big|_{B2\,=\,0} \qquad y_{22} = \frac{I2}{V2}\Big|_{V1\,-\,0}. \qquad (4.2)$$

If, for the network shown in figure 4.1-a, the voltage ratio transfer function G21 is defined to be

$$G21 = \frac{V2}{V1} \qquad (4.4)$$

it can be found in terms of the open-circuit impedance parameters and short-circuit admittance parameters as

$$G21 = \frac{V2}{V1} = \frac{z_{21}}{z_{11}} = \frac{-y_{21}}{y_{22}} \quad . \qquad (4.5)$$

For the same network, if we define the transfer impedance Z21 to be

$$Z21 = \frac{V2}{I1} \qquad (4.6)$$

and the transfer admittance to be

$$Y21 = \frac{-I2}{V1} \qquad (4.7)$$

both Z21 and Y21 may be expressed in terms of the open-circuit impedance parameters as

$$Z21 = \frac{V2}{I1} = z_{21} = \frac{-y_{21}}{\Delta y} \qquad (4.8)$$

and

$$-Y21 = \frac{I2}{V1} = \frac{-z_{21}}{\Delta z} = y_{21} \qquad (4.9)$$

where

$$\Delta z = \begin{vmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{vmatrix} \quad \text{and} \quad \Delta y = \begin{vmatrix} y_{11} & -y_{12} \\ -y_{21} & y_{22} \end{vmatrix} \quad .$$

For the network shown in figure 4.1-b the voltage, impedance, and admittance transfer function may be defined and found as

$$G21 = \frac{V2}{V1} = \frac{z_{21}}{\Delta z + z_{11} ZL} = \frac{-y_{21}}{y_{22} + YL} \qquad (4.10)$$

$$Z21 = \frac{V2}{I1} = \frac{z_{21} ZL}{z_{22} + ZL} = \frac{-y_{21}}{\Delta y + y_{11} YL} \qquad (4.11)$$

$$-Y21 = \frac{I2}{V1} = \frac{z_{21}}{\Delta z + z_{11} ZL} = \frac{-y_{21} YL}{y_{22} + YL} \quad . \qquad (4.12)$$

For the network shown in figure 4.1-c the voltage ratio transfer function and admittance transfer function is given by

$$G2g = \frac{V2}{Vg} = \frac{z_{21} ZL}{(z_{11} + Zg)(z_{22} + ZL) - z_{21} z_{12}} \quad (4.13)$$

and

$$Y2g = \frac{I2}{Vg} = \frac{z_{21}}{(z_{11} + Zg)(z_{22} + ZL) - z_{21} z_{12}} \quad . \ (4.14)$$

The voltage ratio transfer function for the network of figure 4.1-d is the same as that for the network in figure 4.1-b, while the impedance transfer function Z2g is

$$Z2g = \frac{V2}{Ig} = \frac{z_{21} ZgZL}{(z_{11} + Zg)(z_{22} + ZL) - z_{12} z_{21}} \quad . \quad (4.15)$$

It is possible to define and express a current ratio transfer function in terms of the open-circuit impedance parameters and the short-circuit admittance parameters.

In order to observe the properties of a transfer function, Z21 for the ladder network of figure 4.2-a and the voltage ratio transfer function for the constant resistance lattice network of figure 4.2-b have been found by the use of NASAP-69. These two transfer functions will be used to illustrate the general properties of transfer functions as listed in Appendix B.

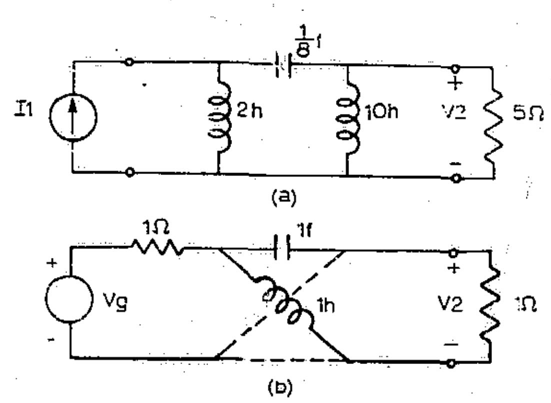

Figure 4.2 (a) Ladder network. (b) Constant resistance lattice network.

For the ladder network of figure 4.2-a the transfer function Z21(s) is

$$Z21(s) = \frac{P(s)}{Q(s)} = \frac{5s^3}{s^3 + 3s^2 + 4s + 2} \quad . \qquad (4.16)$$

Note that when s is real Z12(s) is real. This is a general property of transfer functions of the form P(s)/Q(s) when the polynomials P(s) and Q(s) have real coefficients. Now Q(s) can be tested by means of a continued fraction expansion (Appendix B) to determine if it is a Hurwitz polynomial

$m(s) = \text{even part} = 3s^2 + 2$

$n(s) = \text{odd part} = s^3 + 4s$

$3s^2 + 2 \;\overline{)\, s^3 + 4s}\;(s/3$

$\underline{s^3 + 2s/3}$

$10s/3 \;\overline{)\, 3s^2 + 2}\;(9s/10$

$\underline{3s^2}$

$2 \;\overline{)\, 10s/3}\;(10s/6$

$\underline{10s/3}$.

Since all three quotient terms are positive, Q(s) is Hurwitz and hence, Z12(s) does not have any poles in the right-half plane. A stable transfer function has no poles in the right-half portion of the s plane, and this is the case when the denominator of the transfer function is a Hurwitz polynomial. The zeros of Z12(s) are all located at s = 0, i.e., Z12(s) has a third order zero at the origin. Ladder networks consisting of positive RLC elements do not have any of their poles and zeros located in the right-half s plane.

The amplitude response, $|Z12(j\omega)|$, and the phase response, Arg(Z12(jω)) can be found by separating Z12(s) into its even and odd parts as

$$Z12(s) = \frac{0 + 5s^3}{(3s^2 + 2) + (s^3 + 4s)} = \frac{m_1(s) + n_1(s)}{m_2(s) + n_2(s)} \ . \qquad (4.17)$$

Then Z12(jω) is

$$Z12(j\omega) = \frac{0 + j(-5\omega^3)}{(2 - 3\omega^2) + j(4\omega - \omega^3)} = \frac{m_1(j\omega) + n_1(j\omega)}{m_2(j\omega) + n_2(j\omega)} \qquad (4.18)$$

from which the amplitude response may be found to be

$$\left|Z12(j\omega)\right| = \sqrt{\frac{(0 + 25\omega^6)}{(2 - 3\omega^2)^2 + (4\omega - \omega^3)^2}} = \sqrt{\frac{m_1^2(j\omega) + n_1^2(j\omega)}{m_2^2(j\omega) + n_2^2(j\omega)}} \qquad (4.19)$$

and the phase response is

$$Arg(Z12(j\omega)) = 270^\circ - \arctan\left[\frac{(4\omega - \omega^3)}{(2 - 3\omega^2)}\right]$$

$$= \arctan\left[\frac{n_1(\omega)}{m_1(\omega)}\right] - \arctan\left[\frac{n_2(\omega)}{m_2(\omega)}\right] . \qquad (4.20)$$

The voltage-ratio transfer function G2g for the lattice network of figure 4.2-b is

$$G2g(s) = \frac{1}{2}\,\frac{(s - 1)}{(s + 1)} \quad . \qquad (4.21)$$

The voltage-ratio transfer function of equation (4.21) is an example of a transfer function of a network with a zero in the right-half s plane.

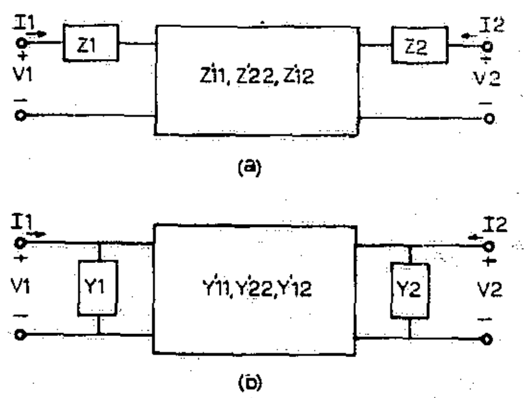

Figure 4.3 Two-port network with series impedance and shunt admittance added.

The open-circuit impedance parameters for a network can be found in terms of short-circuit admittance parameters as

$$z_{11} = \frac{y_{22}}{\Delta y} \qquad z_{12} = \frac{-y_{12}}{\Delta y}$$

$$z_{12} = \frac{-y_{21}}{\Delta y} \qquad z_{22} = \frac{y_{11}}{\Delta y} .$$

If y_{22} and Δy, y_{11} and Δy, and y_{12} and Δy have no common factors, there will be no cancellation between the numerators and denominators of z_{11} and z_{22}, and z_{12}. z_{11}, z_{22}, and z_{12} will then have the same poles. Now consider the network shown in figure 4.3-a, where it is assumed that the open-circuit parameters z'_{11}, z'_{22}, and z'_{12}, for the subnetwork, all have the same poles. The open-circuit impedance parameters for the entire network are given as

$$z_{11} = z'_{11} + Z1$$

$$z_{22} = z'_{22} + Z2$$

$$z_{12} = z'_{12} \qquad .$$

The poles of z_{11} are the poles of z'_{11} and Z1 while the poles of z_{22} are the poles of z'_{22} and Z2. The poles of z_{12} are the poles of z'_{12}. Thus z_{11} and z_{22}, for this network, can have poles which are not poles of z_{12}. All poles of z_{12} are poles of z_{11} and z_{21}, but a pole of z_{11} or z_{22} need not be a pole of z_{12}.

Using figure 4.3-b it can be shown that all poles of y_{12} are poles of y_{11} and y_{22}, but a pole of y_{11} or

y_{22} need not be a pole of y_{12}. A pole of a driving point function which is not a pole of the transfer function is called a "private pole". Private poles of a driving point function may be realized as indicated in figure 4.3-a and 4.3-b.

4.3 Zeros of transmission. A transfer function relates the response of a network at an output port, say port (2,2′) to the excitation at an input port, say port (1,1′). For example, the transfer impedance function Z12(s) relates an output voltage, V2, to an input current, I1. Now if for some finite input of current, there results an output of voltage, there has been a transmission through the network. If a zero output results from a non-zero input of current at some frequency, z_{12} is said to have a zero of transmission at that particular frequency.

A question of some importance in the two-port synthesis procedure to be shortly introduced, is how the zeros of transmission are produced in a ladder network. We see from inspection of the ladder network of figure 4.4-a that there is only one path through the network by which signals from the input may reach the output. Thus a zero of transmission can not result from the cancellation of signals arriving by means of multiple paths. Since there is but one path of transmission, a zero of transmission can only be caused by an open circuit or a short circuit in this single transmission path. A pole in a series arm may cause a break in the path of transmission and result in a zero of transmission. A zero in a shunt arm of the ladder could result in the signal path being shorted out and therefore producing a zero of transmission. Under certain circumstances the effect of a pole in a series arm or the effect of a zero in a shunt arm may be cancelled out by the behavior of the rest of the network. When this happens the series pole or shunt zero does not produce a zero of transmission.

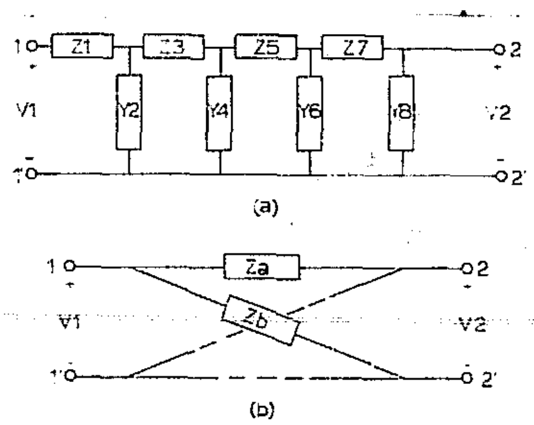

Figure 4.4 (a) Ladder network with possible zero of transmission when Zi = ∞. (b) Lattice network with zero when Za = Zb, balanced bridge.

Any zeros of transmission of a ladder network correspond to poles of a series arm or zeros of a shunt arm. If the series and shunt arm of the ladder network are to be constructed from RLC elements the series poles and shunt zeros will not appear in the right-half s plane. Thus a ladder network consisting of RLC elements must have its zero of transmission confined to the left-half of the s plane and the $j\omega$ axis. If the series and shunt arms of the ladder network are to be constructed entirely of RC elements the zeros of transmission would be restricted to the negative real axis of the s plane. For a ladder network consisting of LC elements only, the zeros of transmission must occur on the imaginary axis in conjugate pairs.

4.4 Two-port synthesis by ladder development. The Cauer ladder development that will now be discussed is a method of synthesizing a two-port network for which open-circuit and short circuit parameters have been specified. The following combination of parameters are possible:

$-y_{12}$ and y_{11}
$-y_{12}$ and y_{22}
z_{12} and z_{11}
z_{12} and z_{22} .

The method is to synthesize the specified driving point function, using a ladder network, in such a manner as to produce the zeros of transmission of the transfer function. The method realizes the poles and zeros of the driving point and the transfer function. The scale factor of the transfer function specified and the scale factor of the transfer function of the network may not be the same because the synthesis procedure has no means of forcing the two to be equal.

If the driving point and transfer function specifications are given in terms of the short-circuit admittance parameters the specifications may take the form of

$$-y_{12} = \frac{a_0 + a_1 s + a_2 s^2 + . \ . \ . + a_{n-1} s^{n-1} + a_n s^n}{q(s)} \qquad (4.22)$$

and

$$y_{11} = \frac{b_0 + b_1 s + b_2 s^2 + . \ . \ . + b_{i-1} s^{i-1} + b_i s^i}{q(s)}$$

or

$$-y_{12} = \frac{a_0 + a_1 s + a_2 s^2 + . \ . \ . + a_{n-1} s^{n-1} + a_n s^n}{q(s)} \qquad (4.24)$$

and

$$y_{22} = \frac{c_0 + c_1 s + c_2 s^2 + . \ . \ . + c_{n-1} s^{n-1} + c_n s^n}{q(s)} \ .$$

The coefficients a_i , b_i , and c_i are restricted to be such that either

$$a_i \geq 0$$
$$b_i \geq a_i \qquad i = 0,1,2. \ . \ . \qquad (4.25)$$

or

$$a_i \geq 0$$
$$c_i \geq a_i \qquad i = 0,1,2. \ . \ . \ . \qquad (4.26)$$

This implies that the degree of the numerator polynomial of the transfer function can not exceed that of the driving point function.

The specified driving point function must have all the properties of a driving point function. For example, it must be a positive real function. The transfer function need not be positive real, and since some of the coefficients

a_1 may be zero without violation of the coefficient condition, the transfer function can have multiple zeros of transmission. Two such cases of multiple zeros of transmission are of particular interest. In the first case all the zeros of transmission are at infinity and in the second case all the zeros are at the origin. There may be no missing coefficients in the denominator polynomial of the transfer function unless all even or odd coefficients are missing as the denominator polynomial must be a Hurwitz polynomial.

For a simple example of the method of synthesis let us realize a two-port network specified by

$$z_{21} = \frac{2}{s^3 + 4s}$$

$$z_{11} = \frac{3s^2 + 2}{s^3 + 4s} \quad . \qquad (4.27)$$

The denominator of z_{21} and z_{11} must have the same poles. The zeros of transmission of z_{21} are all at infinity. In this case z_{11} may be realized by means of a continued fraction expansion of $1/z_{11}$ and the zeros of the resulting network will all be at infinity. For the general case, where all the zeros are not located at infinity or the origin, some effort is required to insure that the zeros of transmission are properly located. The continued fraction expansion of $1/z_{11}$ is

```
3s² + 2 |s³ + 4s  (s/3     <———————————————— Y2(s)
         s³ + 2s/3
         ————————————
              10s/3 |3s² + 2  ( 9s/10   <———— Z3(s)
                     3s²
                    ——————————————
                          2 |10s/3  (5s/3  <—— Y4(s)
                             10s/3
                             =====
```

The network represented by this continued fraction expansion is shown in figure 4.5. The short-circuit

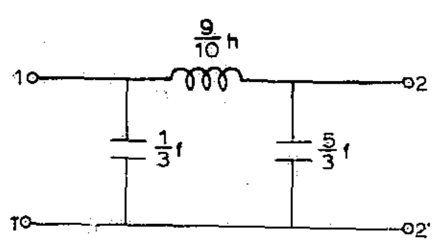

Figure 4.5 Ladder network for specification of equation (4.27).

admittance parameters for the network can be used to show that the specifications have been met. Using NASAP-69 the short circuit admittance parameters are found to be

$$y_{11} = \frac{3s^2 + 10}{9s}$$

$$y_{22} = \frac{15s^2 + 10}{9s}$$

$$y_{21} = y_{12} = \frac{-10}{9s} .$$

Then Δy is found to be

$$\Delta y = \frac{5s^2 + 20}{9}$$

and

$$z_{11} = \frac{y_{22}}{\Delta y} = \frac{15s^2 + 10}{9s} \; \frac{9}{5s^2 + 20} = \frac{3s^2 + 2}{s^3 + 4} \, ,$$

$$z_{12} = \frac{-y_{12}}{\Delta y} = \frac{10}{9s} \; \frac{9}{5s^2 + 20} = \frac{2}{s^3 + 4} \, .$$

NASAP-69 could have been employed to find z_{11} and z_{12} directly.

For the specifications given by equation (4.27) the desired two-port network was realized by forming the continued fraction expansion of $1/z_{11}$. The network resulting from the continued fraction expansion had its zeros located at infinity. For a more general set of specifications, say

$$z_{21} = \frac{(s^2 + 1)(s^2 + 4)}{s(s + 16)}$$

$$z_{11} = \frac{(s^2 + 9)(s^2 + 25)}{s(s^2 + 16)} \, , \qquad (4.28)$$

simple expansion of the driving point function will not result in a network with a transfer function which has the desired zeros. A more complex procedure of expanding the driving point impedance must be employed in order to insure that the desired zeros are obtained in the final result. The more complex expansion makes use of two synthesis procedures. The first is the shifting of a zero by partial removal of a pole and the second is complete removal of a pole. The complete removal of a pole is discussed first.

If Z is an LC driving point impedance function, it may have a pole at the origin, at infinity, or a conjugate pair of poles on the imaginary axis. The pole in question is removed from Z by splitting Z into two simpler networks. With impedance functions Zp and Z1. The driving point impedance function Zp now contains the removed pole. Zp is in a form that can be recognized as a network consisting of LC elements. The form of the function Zp and the form of the resulting network depend on the location of the pole to be removed. The driving point impedance function Z1 is found by subtracting the impedance function Zp from Z. The forms of the impedance function Zp for the three pole locations are given in table 4.1.

The function

$$Z(s) = \frac{90s^3 + 15s^2 + 16s}{45s^3 + 18s^2 + 5s + 2} \tag{4.29}$$

has a factor of $(s^2 + 1/9)$ in its denominator (Appendix B) and hence has conjugate poles at $s = \pm j1/3$. For complete removal of this pole from Z(s), k must be one and 2K is found as

$$2K = \lim_{s^2 \to -1/9} \frac{(s^2 + 1/9)}{s} \frac{90s^3 + 15s^2 + 16s}{(s^2 + 1/9)(45s + 18)}$$

$$= \left. \frac{90s^3 + 15s^2 + 16s}{s(45s + 18)} \right|_{s = j1/3} = 1/3 \quad .$$

The driving point impedance function to be subtracted from Z(s) to find Z1(s) is

$$Zp = \frac{2Ks}{s^2 + \omega^2} = \frac{1/3s}{s^2 + 1/9}$$

Pole location	Zp	Residue	Network
pole at origin	$Zp = \frac{kK_o}{s}$	$K_o = \lim_{s \to 0} sZ(s)$	series capacitor
pole at infinity	$Zp = kK_\infty s$	$K_\infty = \lim_{s \to \infty} \frac{1}{s}Z(s)$	series inductor
pole at $s = \pm j\omega_1$	$Zp = \frac{2kK_1 s}{s^2 + \omega_1^2}$	$2K_1 = \lim_{s^2 \to -\omega_1^2} \frac{(s^2 + \omega_1^2)Z(s)}{s}$	LC tank circuit

Table 4.1 Impedance function Zp for a pole at the origin, infinity and $\pm j\omega_1$. For complete pole removal $k = 1$, for partial pole removal $0 < k < 1$.

which is recognized as an LC tank circuit where

$$C = \frac{1}{2K} = 3 \text{ fd}$$

and

$$L = \frac{1}{\omega^2 C} = 3 \text{ h} \quad .$$

To remove the pole at $s = \pm j1/3$ from the impedance function Z(s) subtract Zp(s) from Z(s) to obtain Z1(s).

$$Z1(s) = \frac{90s^3 + 15s^2 + 16s}{45s^3 + 18s^2 + 5s + 2} - \frac{1/3s}{(s^2 + 1/9)}$$

$$= \frac{90s^3 + 10s}{(s^2 + 1/9)(45s + 18)} = \frac{(s^2 + 1/9)\,90s}{(s^2 + 1/9)(45s + 18)}$$

$$= \frac{10s}{5s + 2} \quad .$$

Examples of the removal of a pole at the origin and infinity can be found in Appendix B.

The partial removal of a pole from a driving point impedance function is accomplished in the same manner, except the constant k in table 4.1 is no longer taken to be one. k is given some value between zero and one. When this is done the poles of the function Z1(s) are the same as those of the function Z(s) as no cancellation between the numerator and denominator occur in Z1(s), but the zeros of Z1(s) are shifted to new locations. The amount the zeros of Z1(s) are shifted depends on the value of k. Figures 4.6, 4.7, and 4.8 show the effect of the partial removal of a pole at the origin, at infinity, and at $s = \pm j\omega$.

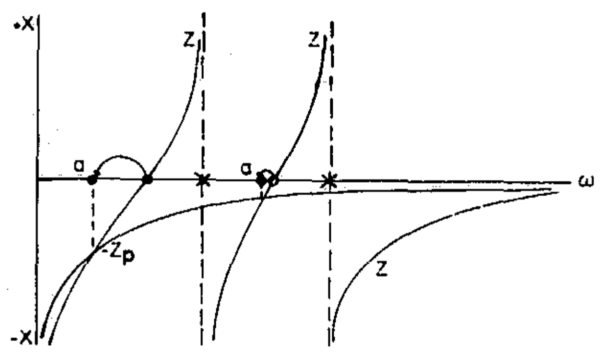

Figure 4.6 Partial removal of a pole at the origin, $Zp = \frac{kK_0}{s}$. "a" denotes a zero of Z1.

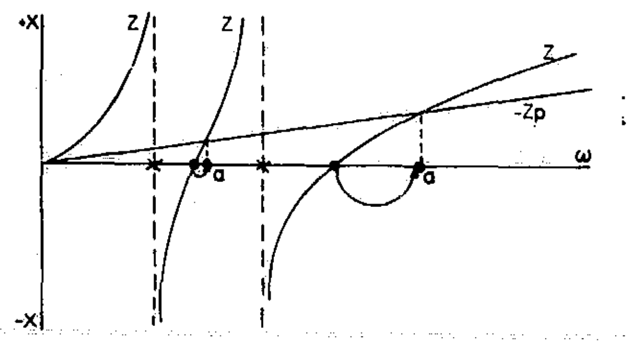

Figure 4.7 Partial removal of a pole at infinity $Zp = kK_\infty s$. "a" denotes a zero of Z1.

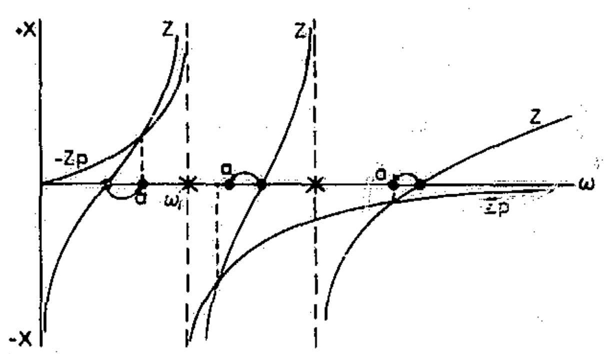

Figure 4.8 Partial removal of conjugate imaginary poles at $s = \pm\omega_1$, $Zp = \frac{kK_1 s}{s^2 + \omega_1^2}$. "a" denotes a zero of Z1.

The following observations were made by M.E. Van Valkenburg in his book, <u>Modern Network Synthesis</u>:

(1) The partial removal of a pole shifts the zero toward that pole, the amount of shift depending on the value of k and the proximity of a zero to that pole.

(2) In no case can a zero be shifted beyond an adjacent pole. Typically, the shift can be only a fraction of that distance.

(3) The complete removal of a pole at the origin shifts the adjacent zero to the origin, and the complete removal of a finite pole shifts an adjacent zero toward the position of the removed pole [the other zero vanishing with the pole to maintain the equality of the number of zeros (including those at zero and infinity) to the number of poles].

(4) The partial or complete removal of a pole at the origin does not affect a zero at infinity, nor does the partial or complete removal of a pole at infinity affect a zero at the origin.

(5) There are limits on the amount a given partial pole removal can shift a given zero. However, by using several steps of zero shifting (by weakening of several poles or by successive weakening of poles of impedance and admittance), some zero can be moved to any location on the imaginary axis of the s plane.

The techniques of zero shifting by partial pole removal and pole removal may be used to obtain a two-port network with the specifications of equation (4.28). The procedure is a repeated application of two steps. First a zero of z_{11} is shifted so that the new function has a zero that corresponds to a zero of z_{12}. Then the pole of the reciprocal function which corresponds to the common zero is removed. As an example of this process a two-port network corresponding to the specifications of equation (4.28) will be found. Equation (4.28) is repeated here for convenience.

$$z_{11} = \frac{(s^2 + 9)(s^2 + 25)}{s(s^2 + 16)}$$

$$z_{21} = \frac{(s^2 + 1)(s^2 + 4)}{s(s^2 + 16)} \quad . \qquad (4.28)$$

Step 1. Zero shifting operation. The first step of the procedure is to shift a zero of z_{11} so that the new impedance function formed and z_{12} have a common zero, see figure 4.9. It was decided to shift the zero at $s = j3$ to $s = j2$ by the partial removal of the pole at the origin. The new driving point impedance function is

$$Z1(s) = z_{11}(s) - \frac{kK_o}{s} \quad .$$

It is desired to force Z1(s) to zero at $s = j2$, or

$$z_{11}(j2) - \frac{kK_o}{j2} = 0$$

$$\frac{kK_o}{j2} = z_{11}(j2)\Big|_{s^2 = -4} = \frac{(-4 + 9)(-4 + 25)}{(j2)(-4 + 16)}$$

$$kK_o = \frac{35}{4} \quad .$$

Now as a check, k_o may be found so that it can be determined if k is between zero and one.

$$K_o = \lim_{s \to 0} sz_{11} = \frac{(s^2 + 9)(s^2 + 25)}{(s^2 + 16)}\Big|_{s = 0} = \frac{225}{16} \quad .$$

k may now be found as

$$k = \frac{kK_o}{K_o} = \frac{35}{4}\frac{16}{225} = \frac{28}{45}$$

hence $0 < k < 1$.

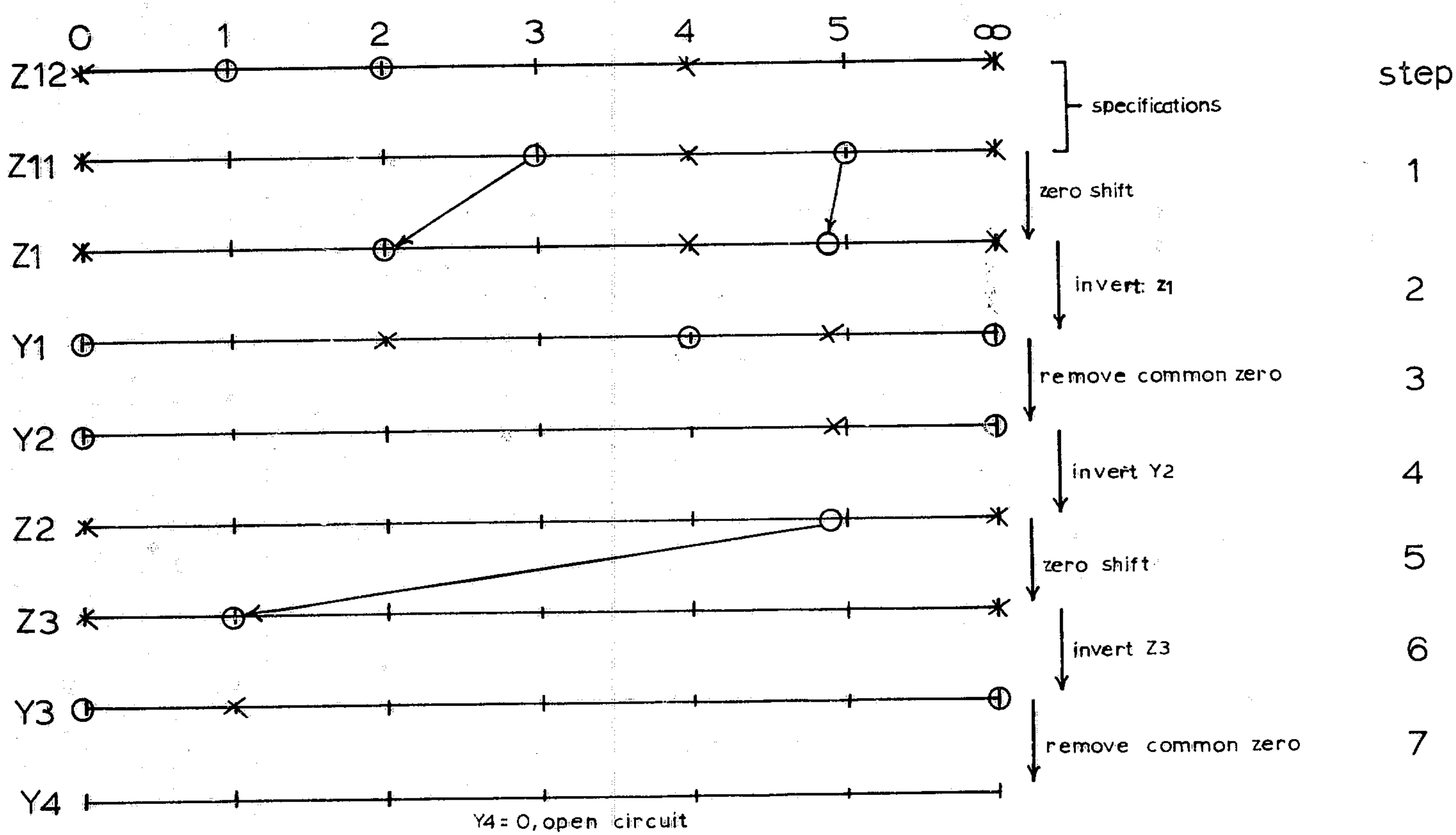

Figure 4.9 Ladder development of specifications for LC two-port network, equation (4.28)

The new driving point impedance can now be found as

$$Z1(s) = z_{11}(s) - \frac{35}{4s} \longleftarrow \text{series capacitor of } 4/35 \text{ f} \approx 0.1143 \text{ f}$$

$$= \frac{(s^2 + 9)(s^2 + 25)}{s(s^2 + 16)} - \frac{35/4}{s}$$

$$= \frac{s^4 + 25.25s^2 + 85}{s(s^2 + 16)} .$$

Now (s + 4) must be a factor of the numerator since the object of the procedure was to force a zero of Z1(s) to be located at s = j2. Thus the numerator can be factored by long division.

$$\begin{array}{r|l} & s^2 + 21.25 \\ s^2 + 4 & s^4 + 25.25s^2 + 85 \\ & s^4 + 4.00s^2 \\ \hline & 21.25s^2 + 85 \\ & 21.25s^2 + 85 \\ \hline \end{array}$$

$$Z1(s) = \frac{(s^2 + 4)(s^2 + 21.25)}{s(s^2 + 16)} .$$

At this point we see that $z_{11}(s)$ consists of a 4/35 f capacitor in series with Z1(s). Z1(s) has a zero at s = j2, this is also a zero of $z_{12}(s)$.

Step 2. Invert Z1 to get Y1.

$$Y1 = \frac{s(s^2 + 16)}{(s^2 + 4)(s^2 + 21.25)} = \frac{2As}{(s^2 + 4)} + \frac{2Bs}{(s^2 + 21.25)} .$$

The residues 2A and 2B are found as

$$2A = \lim_{s^2 \to -4} \left(\frac{s^2 + 4}{s}\right) Y1 = \left.\frac{(s^2 + 16)}{(s^2 + 21.25)}\right|_{s^2 = -4} = \frac{12}{17.25}$$

$$2B = \lim_{s^2 \to -21.25} \left(\frac{s^2 + 21.25}{s}\right) Y1 = \left.\frac{(s^2 + 16)}{(s^2 + 4)}\right|_{s^2 = -21.25}$$

$$= \frac{5.25}{17.25}$$

Thus $Y1 = \frac{(12/17.25)s}{(s^2 + 4)} + \frac{(5.25/17.25)s}{(s^2 + 21.25)}$

or

$$\frac{(5.25/17.25)s}{(s^2 + 21.25)} = Y1 - \frac{(12/17.25)s}{(s^2 + 4)} \quad . \qquad (4.30)$$

Step 3. Remove the zero at s = j2 of Z1 as a pole of Y1.

$$Y2(s) = Y1(s) - Yp_1 \quad .$$

Now by comparison with equation (4.30) we see that

$$Yp_1 = \frac{(12/17.25)s}{(s^2 + 4)}$$

which is recognized as an LC series resonant circuit with

$$L = \frac{17.25}{12} \approx 1.438 \text{ h}$$

$$C = \frac{3}{17.25} \approx 0.1739 \text{ f}$$

as shown in figure 4.10. The admittance Y2(s) is

$$Y2(s) = \frac{(5.25)(17.25)s}{(s^2 + 21.25)} \quad ,$$

Step 4. Invert Y2(s) to get Z2.

$$Z2(s) = \frac{(s^2 + 21.25)}{(5.25/17.25)s} \quad .$$

Step 5. Shift zero of Z2 at $s^2 = -21.25$ to $s^2 = -1$ to form a second zero in common with $z_{12}(s)$. The new driving point impedance function with a zero at $s^2 = -1$ is

$$Z3(s) = Z2(s) - \frac{kK_0}{s} \quad .$$

It is desired to force Z3(j1) to be zero, or

$$Z2(j) = \frac{kK_0}{j}$$

$$kK_0 = jZ2(j) = \frac{(17.25)(20.25)}{(5.25)} \approx 66.5357 \quad .$$

As a check K_0 is found to be

$$K_0 = \lim_{s^2 \to -1} sZ2(s) = \frac{(17.25)(21.25)}{(5.25)}$$

and

$$k = \frac{kK_0}{K_0} = \frac{(17.25)(20.25)}{(5.25)} \quad \frac{(5.25)}{(17.25)(21.25)} = \frac{20.25}{21.25} .$$

Thus k is between 0 and one as required.

The new driving point impedance Z3(s) can now be found as

$$Z3(s) = Z2(s) - \frac{66.5357}{s} \longleftarrow \text{series capacitor}$$

$$C = \frac{1}{66.5357} = 0.01503$$

$$= \frac{s^2 + 21.25}{\left(\frac{5.25}{17.25}\right)s} - \frac{66.5357}{s}$$

$$Z3(s) = \frac{(s^2 + 1)}{\left(\frac{5.25}{17.25}\right)s} .$$

Thus Z2(s) consists of a 0.01503 capacitor in series with Z3(s). Z3(s) has a zero at s = j1, which is also a zero of $z_{12}(s)$.

Step 6. Invert Z3 to get Y3.

$$Y3(s) = \frac{(5.25/17.25)s}{(s^2 + 1)} .$$

Step 7. Remove zero at s = j1 of Z3 as a pole of Y3.

$$Y3(s) = \frac{(5.25/17.25)s}{(s^2 + 1)}$$

Y3(s) is recognized as an LC series resonant circuit with

L = 3.286 h

C = 0.3043 f.

The complete network is shown in figure 4.10-c.

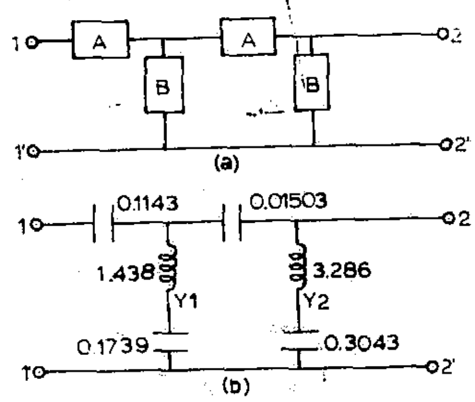

Figure 4.10 (a) Ladder network with zero shifting (A) and zero producing (B) subnetworks indicated.
(b) Final ladder network.

The Cauer method of two-port synthesis can also be used to realize an RC ladder network when an RC driving point and transfer function are specified. There are three techniques which may be used to shift a zero of an RC function as given by M.E. Van Valkenburg:

(1) The removal of a constant, $kZ(\infty)$, from $Z(s)$,
(2) The removal of a constant, $kY(0)$, from $Y(s)$,
(3) The partial removal of a pole from $Z(s)$ or $Y(s)$, like

$$Z1(s) = Z(s) - \frac{kK_1}{s + p_1}$$

where k_1 is the residue at the pole p_1.

Consider for example the driving point and transfer admittance function specifications given by

$$y_{11} = \frac{(s+1)(s+6)}{(s+4)(s+8)}$$
$$-y_{21} = \frac{(s+2)(s+3)}{(s+4)(s+8)} \qquad (4.31)$$

The required two-port network can be realized by the following procedure:

Step 1. Shift zero of y_{11} at s = -1 to s = -2 by partial removal of pole at s = -2.

$$Y1 = \frac{s^2 + 7s + 6}{(s + 4)(s + 8)} - \frac{kK_1 s}{s + 4}$$

$$kK_1 = \frac{s + 4}{s} \left. \frac{(s + 1)(s + 6)}{(s + 4)(s + 8)} \right|_{s = -2} = 1/3$$

$$Y1 = \frac{s^2 + 7s + 6}{(s + 4)(s + 8)} - \frac{1/3s}{(s + 4)} \longleftarrow$$ recognized as R and C in series in a shunt arm. R1 = 3Ω, C = 1/12 f.

$$Y1 = \frac{2}{3} \frac{s^2 + 13s/2 + 9}{(s + 4)(s + 8)} = \frac{2}{3} \frac{(s + 2)(s + 9/2)}{(s + 4)(s + 8)} .$$

Step 2. Invert Y1 to obtain Z1.

$$Z1 = \frac{3}{2} \frac{(s + 4)(s + 8)}{(s + 2)(s + 9/2)}$$

$$= \left[\frac{3}{2} + \frac{36/5}{(s + 2)} + \frac{21/20}{(s + 9/2)}\right]$$

Step 3. Removal of pole from Z1 at s = -2.

$$Z2 = \frac{3}{2} \frac{(s + 4)(s + 8)}{(s + 2)(s + 9/2)} - \frac{36/5}{(s + 2)} \longleftarrow$$ recognize as R and C in parallel in series arm. R4 = 18/5, C2 = 5/36

$$= \frac{3}{2}\left[\frac{10s^2 + 72s + 104}{10(s + 2)(s + 9/2)}\right]$$

$$= \frac{3}{2} \frac{s + 52/10}{s + 9/2}$$

Step 4. Invert Z2 to obtain Y2.

$$Y2 = \frac{2}{3} \frac{s + 9/2}{s + 52/10}$$

Step 5. Shift zero of Y2 at s = -9/2 to s = -3 by partial removal of Y2(0)

$$Y3 = Y2 - kY(0)$$

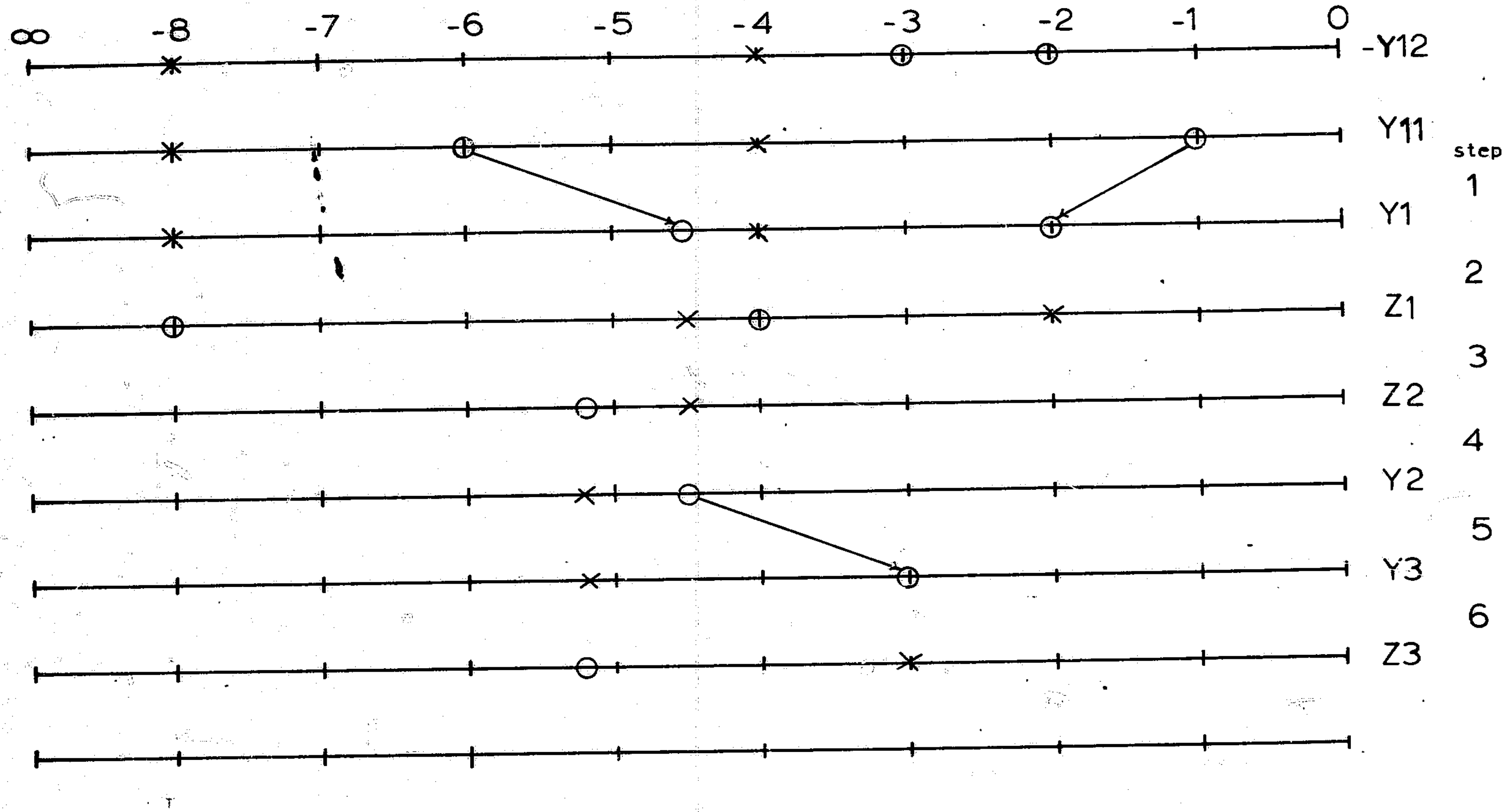

Figure 4.11 Ladder development for specifications of equation (4.31)

$$kY(0) = \frac{2}{3}\left[\frac{s + 9/2}{s + 52/10}\right]\Bigg|_{s = -3} = \frac{30}{66}$$

$$Y3 = \frac{3}{2}\,\frac{s + 9/2}{s + 52/10} - \frac{30}{66}$$ ← recognize as a resistor in a shunt arm R2 = 66/30 Ω.

Step 6. Invert Y3 to obtain Z3.

$$Z3 = \frac{33}{7}\,\frac{(s + 52/10)}{(s + 3)}$$

$$= \left[\frac{33}{7} + \frac{726/70}{s + 3}\right]$$

Z3 is recognized as a 33/7 ohm resistor (R3) in series with an RC tank circuit with R5 = 726/210 Ω and C = 70/726. The complete network is shown in figure 4.12.

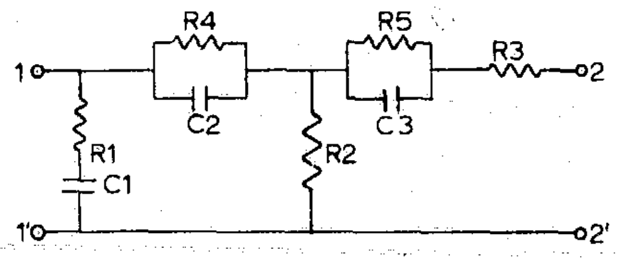

Figure 4.12 Ladder network resulting from specifications of equation 4.31, With R1 = 3 Ω, R2 = 2.2 Ω, R3 = 33/7 Ω, R4 = 18/5 Ω, R5 = 726/210 Ω, C1 = 1/12 f, C2 = 5/36 f, and C3 = 70/726 f.

4.5 Two-port synthesis with one ohm termination. The Cauer ladder development of section 4.3 can be used in the synthesis of two-port LC ladder networks terminated in a one ohm resistance. When a transfer impedance Z21 or transfer admittance Y21 is given as a specification. If the network of figure 4.1-b is taken to be an LC network and if the impedance ZL is one ohm, equation (4.11) becomes

$$Z12 = \frac{V2}{I1} = \frac{z_{21}}{z_{22} + 1} \tag{4.32}$$

and equation (4.12) becomes

$$-Y12 = \frac{I2}{V1} = \frac{-y_{21}}{y_{22} + 1} \; . \tag{4.33}$$

Equations (4.32) and (4.33) will provide a means of obtaining a set of specifications z_{21} and z_{22} or $-y_{21}$ or y_{22} for use with the Cauer ladder development synthesis when a Z12 or Y12 is specified for a two-port network terminated in one ohm.

A property of Z21 or Y21 that is used in obtaining the desired specifications from equations (4.32) or (4.33) is that Z21 and Y21 are odd functions. For example, if the transfer admittance Y21 is given by

$$-Y21 = \frac{P(s)}{Q(s)}$$

and Q(s) is Hurwitz, and can be written as

$$Q(s) = m(s) + n(s)$$

where P(s) is even or odd

m(s) is even

n(s) is odd

then Y21 may be written as

$$-Y21 = \frac{P(s)}{Q(s)} = \frac{P(s)}{m(s) + n(s)} \; . \tag{4.34}$$

Equation (4.34) can be made to comply to the form of equation (4.33) by dividing the numerator and denominator by m(s) or n(s). If P(s) is even divide by n(s) so that

P(s)/n(s) is odd. Y21 would then be given as

$$-Y21 = \frac{P(s)/n(s)}{m(s)/n(s) + 1} \left[\frac{\text{odd}}{\text{even/odd} + 1}\right] .$$

Now using equation (4.33) we can identify

$$-y_{21} = P(s)/n(s) \qquad \text{odd}$$

$$y_{22} = m(s)/n(s) \qquad \text{even/odd} .$$

Since the ratio of the even to odd or odd to even parts of a Hurwitz polynomial (property four, appendix B) can be expanded by means of a continued fraction expansion with all positive quotient terms, y_{22} can be realized as an LC driving point admittance. If P(s) is odd, divide by m(s) so that P(s)/m(s) is odd. Y21 would then be given as

$$-Y21 = \frac{P(s)/m(s)}{n(s)/m(s) + 1} \left[\frac{\text{odd}}{\text{odd/even} + 1}\right] .$$

Once again equation (4.33) can be used to identify y_{21} and y_{22} as

$$-y_{21} = P(s)/m(s) \qquad \text{odd}$$

and

$$y_{22} = n(s)/m(s) \qquad \text{odd/even} .$$

As an example, let us realize the 3rd order admittance function

$$-Y21 = \frac{1}{s^3 + 2s^2 + 2s + 1} = \frac{1}{(2s^2 + 1) + (s^3 + 2s)} \qquad (4.35)$$

as a two port LC network terminated in a one ohm resistance. Since P(s) is even, divide by $n(s) = s^3 + 2s$ to obtain

$$-Y21 = \frac{\frac{1}{s^3 + 2s}}{\frac{(2s^2 + 1)}{(s^3 + 2s)} + 1} = \frac{-y_{21}}{y_{22} + 1}$$

and then identify $-y_{21}$ and y_{22} as

$$-y_{21} = \frac{1}{s^3 + 2s}$$

$$y_{22} = \frac{2s^2 + 1}{(s^3 + 2s)} \;.$$

At this point we have the specifications required for the Cauer ladder development of an LC ladder network. Furthermore, we note that $-y_{21}$ has all its zeros of transmission at infinity and that a ladder with a structure similar to that of the ladder network of figure 4.13-a would have all its zeros of transmission at infinity. Such a ladder network, consisting of inductances in the series arms and capacitances in the shunt arms can be obtained by the continued fraction expansion of y_{22}. This continued fraction expansion is found from

$$Z = 1/y_{22} = \frac{s^3 + 2s}{2s^2 + 1}$$

as

```
2s² + 1 ⟌ s³ + 2s    (s/2                        ←——— Z
          s³ + s/2
          ————————
              3s/2 ⟌ 2s² + 1   (4s/3             ←——— Y
                     2s² + 0
                     ———————
                           1 ⟌ 3s/2   (3s/2      ←——— Z
                               3s/2
                               ════  .
```

The elements of the ladder network, starting at port (2,2′) are identified as a series ½ henry inductor, a shunt 4/3 farad capacitor and a series 3/2 henry inductor. Figure 4.14-a shows the network with the one ohm termination included.

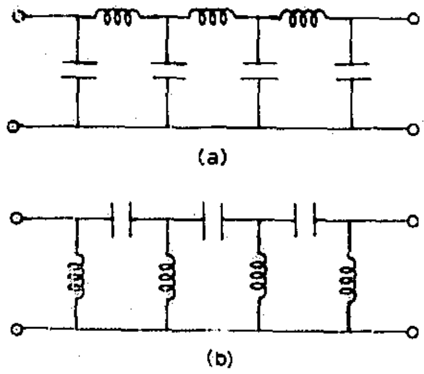

Figure 4.13 (a) Ladder network with all zeros of transmission at infinity (low-pass). (b) Ladder network with all zeros at the origin (high-pass).

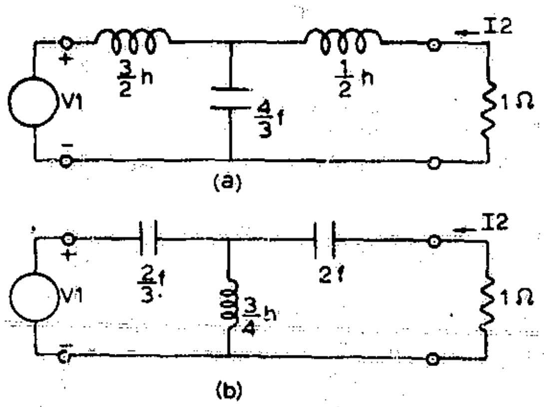

Figure 4.14 (a) Two port network with $-Y12 = \dfrac{1}{s^3+2s^2+2s+1}$.
(b) Two port network with $-Y12 = \dfrac{s^3}{1+2s+2s^2+s^3}$.

The specifications of equation (4.35) for -Y21 can be transformed to

$$-Y21 = \frac{s^3}{1 + 2s + 2s^2 + s^3}$$

by the low-pass to high-pass transformation discussed in Chapter One. In this case s was replaced by 1/s. The short-circuit admittances are then identified as

$$-y_{21} = \frac{s^3}{(2s^2 + 1)}$$

$$y_{22} = \frac{s^3 + 2s}{2s^2 + 1} \quad .$$

The continued fraction expansion of $Z = 1/y_{22}$ is given by

$$
\begin{array}{l}
2s + s^3 \,\overline{)\,1 + 2s^2}\ (1/(2s) \longleftarrow\!\!\!\longrightarrow Z,\ 2\ f \\
\qquad\qquad \underline{1 + \tfrac{1}{2}s^2} \\
\qquad\qquad\quad 3s^2/2 \,\overline{)\,2s + s^3}\ (4/(3s) \longleftarrow Y,\ 3/4\ h \\
\qquad\qquad\qquad\qquad\quad \underline{2s} \\
\qquad\qquad\qquad\qquad\qquad s^3 \,\overline{)\,3s^2/2}\ (3/(2s) \longleftarrow Z,\ 2/3\ f \\
\qquad\qquad\qquad\qquad\qquad\qquad \underline{3s^2/2} \quad .
\end{array}
$$

The high-pass filter is shown in figure 4.14-b. The same results are obtained if the low-pass to high-pass transformation of table 1.2 is applied to the low-pass network of figure 4.14-a.

As a final example of this method of synthesis, consider the transfer impedance specification

$$Z21(s) = \frac{5s^3}{s^3 + 3s^2 + 4s + 2} = \frac{\dfrac{5s^3}{(3s^2 + 2)}}{\dfrac{s^3 + 4s + 1}{3s^2 + 2}} \quad . \qquad (4.36)$$

The driving point and transfer function specification then becomes

$$z_{21} = \frac{5s^3}{3s^2 + 2}$$

$$z_{22} = \frac{s^3 + 4s}{3s^2 + 2} \quad . \qquad (4.37)$$

The continued fraction expansion of z_{22} is

$$
\begin{array}{l}
3s^2 + 2 \,\overline{)\,s^3 + 4s}\ (s/3 \longleftarrow Z,\ 1/3\ h \\
\qquad\quad \underline{s^3 + 2s/3} \\
\qquad\qquad 10s/3 \,\overline{)\,3s^2 + 2}\ (9s/10 \longleftarrow Y,\ 9/10\ f \\
\qquad\qquad\qquad\quad \underline{3s^2} \\
\qquad\qquad\qquad\qquad 2 \,\overline{)\,10s/3}\ (5s/3 \longleftarrow Z,\ 5/3\ h \\
\qquad\qquad\qquad\qquad\quad \underline{10s/3} \quad .
\end{array}
$$

The ladder network which corresponds to this continued fraction expansion is shown in figure 4.15-a. As a check the transfer function Z21(s) can be found by NASAP-69 to be

$$Z12 = \frac{10/3}{s^2 + 3s + 10/3}\,. \qquad (4.38)$$

The transfer impedance of equation (4.37) is not the desired Z12 as specified by equation (4.38). What went wrong? Looking at the network shown in figure 4.15-a we note that the zeros of this network are all at s equals infinity, but the zeros of the open-circuit impedance z_{12} given by equation (4.37) are at the origin. This is the clue to what went wrong. When expanding z_{22} we did not make sure that the resulting network would have the proper zeros of transmission. The proper form of the ladder is that given in figure 4.13-b. In order to expand z_{22} in this manner, invert it to form Y and then reorder the coefficients of the numerator and denominator.

$$Y = \frac{3s^2 + 2}{s^3 + 4s} = \frac{2 + 3s^2}{4s + s^3}$$

$$4s + s^3 \;\overline{)\,2 + 3s^2}\;(1/(2s) \longleftarrow Y,\ 2\text{ h}$$
$$\underline{2 + s^2/2}$$
$$5s^2/2 \;\overline{)\,4s + s^3}\;(8/(5s) \longleftarrow Z,\ 5/8\text{ f}$$
$$\underline{4s}$$
$$s^3 \;\overline{)\,5s^2/2}\;(5/(2s) \longleftarrow Y,\ 2/5\text{ h}$$
$$\underline{5s^2/2}$$

The network which corresponds to this expansion is shown in figure 4.15-b. For this network NASAP-69 found Z12 to be

$$Z21 = \frac{s^3}{s^3 + 3s^2 + 4s + 2}$$

which differs from the desired transfer impedance by a constant multiplier. There is no control over the scale factor when using the Cauer ladder development.

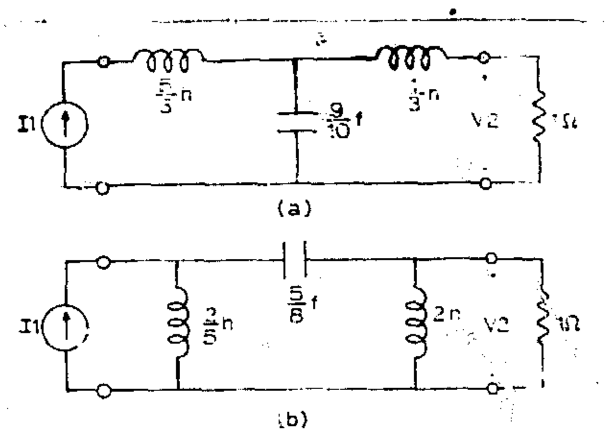

Figure 4.15 Ladder network for specifications of equation 4.37. (a) Incorrect network due to improper expansion of Z_{22}. (b) correct network.

4.6 Constant-Resistance and lattice networks. A constant-resistance two-port network is a two-port network for which the input impedance of port (1,1′) is a constant-resistance R if port (2,2′) is terminated in R ohms and the input impedance of port (2,2′) is R ohms if port (1,1′) is terminated in R ohms. In the terms of Chapter Three, a constant-resistance network has an image impedance of R ohms. Constant-resistance networks can be cascaded, and are readily terminated in their characteristic impedance R. Figure 4.16-a shows a constant-resistance lattice while figure 4.16-b shows a constant-resistance bridged T network. The condition for both networks to be a constant-resistance network is

$$Z_a Z_b = R^2 \ . \tag{4.39}$$

The voltage ratio transfer function G2g for the constant resistance lattice is

$$G2g = \frac{V2}{Vg} = \frac{1}{2}\frac{(Zb - R)}{(Zb + R)} = \frac{1}{2}\frac{(R - Za)}{(R + Za)} \qquad (4.40)$$

while the voltage ratio transfer function G2g for the constant-resistance bridge is

$$G2g = \frac{V2}{Bg} = \frac{R}{R + Za} = \frac{Zb}{Zb + R}\ . \qquad (4.41)$$

These two relationships can be used to realize a desired transfer function.

The lattice network can be used to synthesize a network with the transfer function

$$\frac{V2}{Vg} = \frac{1}{2}\frac{(s - 1)}{(s + 1)}$$

by making the following identification with equation (4.40)

$$Zb = s$$

and then using the constant-resistance requirement

$$ZaZb = R = 1$$

to find Za to be

$$Za = 1/s.$$

Thus we see that Zb is a 1 henry inductor and Za is a 1 farad capacitor. The resulting network is shown in figure (4.17-a).

As a second example of the use of the lattice network to synthesize a given voltage transfer function consider the voltage ratio transfer function G2g given by

$$G2g = \frac{V2}{Vg} = \frac{s^2 - 2s + 2}{s^2 + 2s + 2} = \frac{(s^2 + 2) - 2s}{(s^2 + 2) + 2s} = \frac{\frac{(s^2 + 2)}{2s} - 1}{\frac{(s^2 + 2)}{2s} + 1}\ .$$

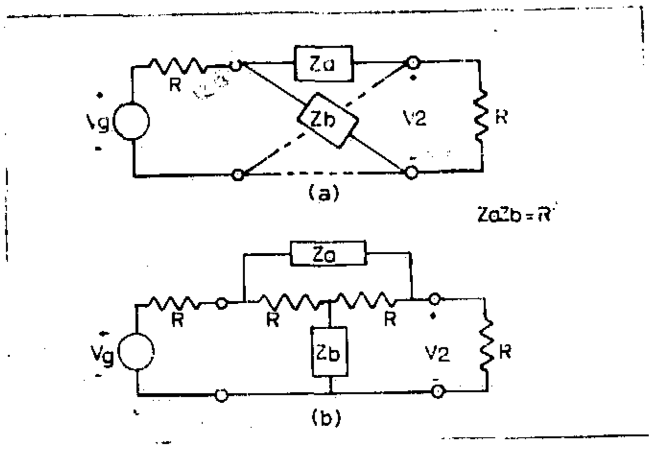

Figure 4.16 Constant-resistance networks. (a) lattice. (b) bridge T.

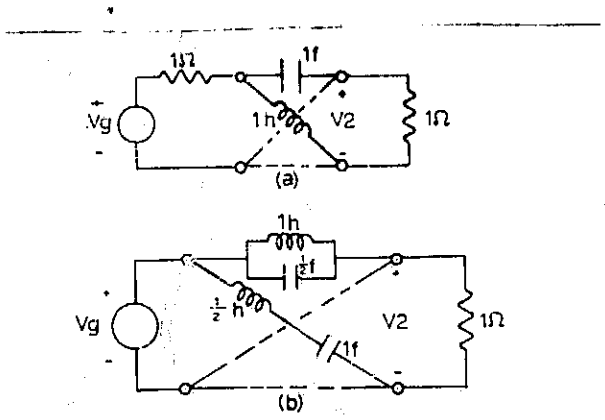

Figure 4.17 Constant-resistance lattice networks.

Using equation (4.40) we can identify Zb as

$$Zb = \frac{s^2 + 2}{2s} = \frac{s}{2} + \frac{1}{s}$$

which is a ½ henry inductor in series with a 1 farad capacitor. Now using the constant resistance requirement

$$ZaZb = 1$$

we find

$$Za = \frac{2s}{s^2 + 2} = \frac{1}{s/2 + 1/s}$$

which is a ½ farad capacitor in parallel with a 1 henry inductor. The network is shown in figure 4.17-b.

As the final example of this section, the voltage ratio transfer function

$$G2g = \frac{s^2 + 1}{s^2 + 2s + 1} = \frac{1}{1 + [2s/(s^2 + 1)]}$$

will be synthesized using the constant-resistance bridge network of figure 4.16-b. Again, R is taken to be one ohm.

Using equation (4.41) we can identify Za as

$$Za = \frac{2s}{s^2 + 1} = \frac{1}{(s/2) + (1/2s)}$$

which is a ½ farad capacitor in parallel with a 2 henry inductor. From the constant-resistance requirement we find

$$Zb = \frac{s^2 + 1}{2s} = \frac{s}{2} + \frac{1}{2s}$$

which is a ½ henry inductor in series with a 2 farad capacitor. The network is shown in figure 4.18.

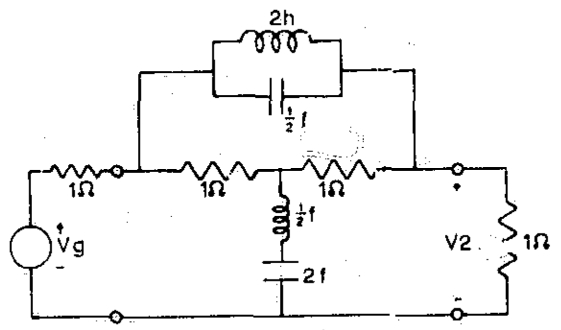

Figure 4.18 Bridge T constant-resistance network, R = 1 ohm.

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CHAPTER FIVE

Butterworth, Chebyshev, and Bessel Filters

5.1 Introduction. The starting point of a filter design problem is usually a set of initial specifications. The initial electrical specifications are usually some restrictions to which an amplitude characteristic, a phase characteristic or a time response must comply. In some instances a combination of the above characteristics may be specified over certain regions of frequency. The first step in the solution of a filter design problem by modern synthesis techniques is to obtain, from the initial electrical specification, a suitable network transfer function H(s). The transfer function H(s) can then be used as the starting point of a synthesis procedure, several of which were presented in Chapter Four.

A convenient form of specifications is a tolerance contour. An amplitude response of a band-pass filter was specified in Chapter One by means of a tolerance contour. For example, see figure 1.3. The phase response of a filter can also be described by a tolerance contour. Such a tolerance contour designates the general region where the amplitude or phase response curve of the desired filter should lie. For example, the amplitude response characteristic for the low-pass filter specified by Table 1.5

must not enter the shaded areas of figure 1.4. Figure 5.1 is a tolerance contour for a low-pass filter. The required cutoff frequency of the passband region, the maximum

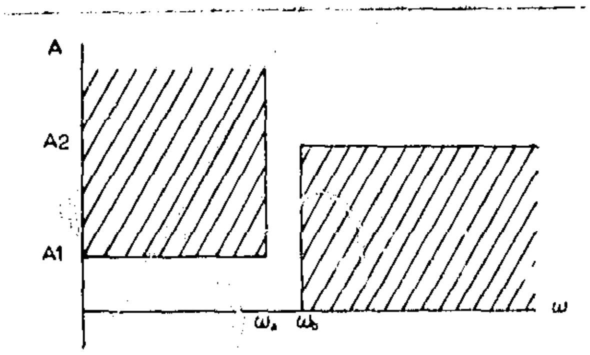

Figure 5.1 Tolerance contour for a low-pass filter.

allowable deviation in attenuation, and the minimum attenuation at a frequency in the stopband can readily be obtained from the tolerance contour by inspection. The Butterworth low-pass filter approximation can be used to find a transfer function H(s) that will have an amplitude response curve that lies in the permissible region of the tolerance contour of a low-pass filter.

5.2 Butterworth Approximation. The ideal normalized amplitude response $|H(j\omega)|$ and phase response $\arg[H(j\omega)]$ for a low-pass filter are shown in figure 5.2-a and b respectively. The ideal low-pass amplitude response is approximated by the magnitude function

$$M(\omega) = \frac{K_0}{\sqrt{1 + f(\omega^2)}} \tag{5.1}$$

where K_o denotes a dc gain constant and $f(\omega^2)$ denotes a polynomial. The magnitude function $M(\omega)$ has all its zeros of transmission at infinity as did the low-pass filter network synthesized in Chapter Four. The polynomial selected for $f(\omega^2)$ determines the significant properties of the magnitude function $M(\omega)$. If $f(\omega^2)$ is selected as

$$f(\omega^2) = \omega^{2n} \tag{5.2}$$

the Butterworth (maximally flat) form of response results. The magnitude function $M(\omega)$ then becomes

$$M(\omega) = \frac{K_o}{\sqrt{1 + \omega^{2n}}} \tag{5.3}$$

Examination of equation (5.3) reveals

1. $M(0) = K_o$,
2. $M(\omega)$ is monotonically decreasing with increasing ω,
3. for $\omega = 1$, M is equal to $K_o/\sqrt{2}$, thus $\omega = 1$ is the 3 db cutoff frequency.

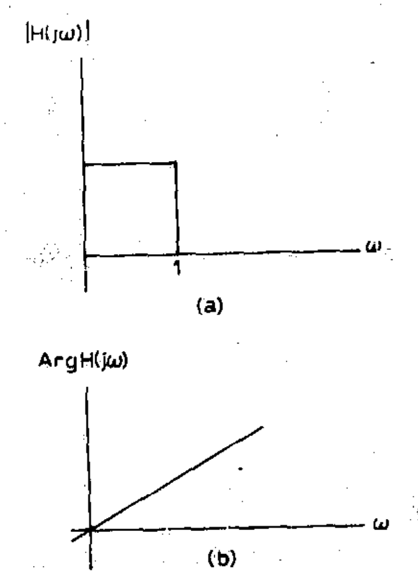

Figure 5.2 Ideal low-pass response. (a) amplitude response. (b) phase response.

The magnitude function $M(\omega)$ for several values of n is plotted in figure 5.3. These curves represent the amplitude response of a Butterworth filter. The ideal low-pass amplitude characteristic is also included in figure 5.3 for comparison with the Butterworth response. Inspection of the figure shows that the passband decreases for large n and that the slope of the attenuation curve in the transition region is steeper for larger n. Hence, the Butterworth response more nearly approximates that of an ideal amplitude response as n is increased.

The Butterworth response is sometimes called the maximally flat response. The reason for this can be seen if the normalized magnitude function

$$\frac{M(\omega)}{K_o} = (1 + \omega^{2n})^{\frac{1}{2}}$$

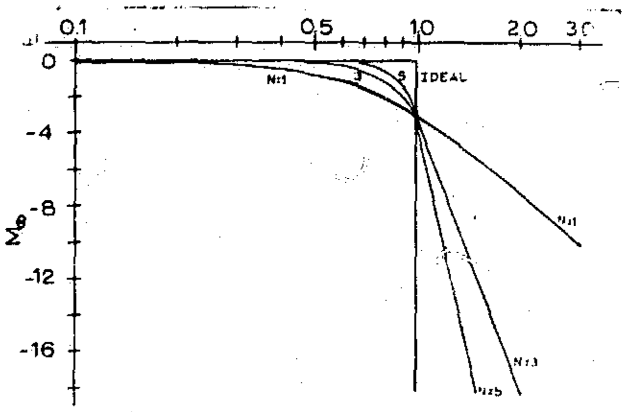

Figure 5.3 Sketch of amplitude response characteristics of a Butterworth low-pass filter for n = 1, 3, and 5.

is expanded by use of the binomial series

$$(1 + x)^{-m} = 1 - mx + \frac{m(m + 1)x^2}{2!} - \frac{m(m + 1)(m + 2)x^3}{3!} + \ldots \qquad (5.4)$$

where

$$x^2 < 1$$

as

$$\frac{M(\omega)}{K_o} = 1 - \frac{\omega^{2n}}{2} + \frac{3\omega^{4n}}{8} - \frac{5\omega^{6n}}{16} + \frac{35\omega^{8n}}{128} + \ldots \qquad (5.5)$$

The first derivative of the magnitude function $M(\omega)/K_o$ is then given by

$$\frac{M'(\omega)}{K_o} = \frac{-2n\omega^{2n-1}}{2} + \frac{12n\omega^{4n-1}}{8} - \frac{30n\omega^{6n}}{16} + \ldots \qquad (5.6)$$

and is zero for $\omega = 0$. Similarily the next 2n-2 derivatives are equal to zero for $\omega = 0$. Thus the Butterworth response has a zero slope at $\omega = 0$, and hence the name maximally flat response.

For large values of ω, $\omega >> 1$, the amplitude of a Butterworth function is approximated by

$$M(\omega) \approx \frac{1}{\sqrt{\omega^{2n}}} = \frac{1}{\omega^n} \qquad (5.7)$$

or

$$Mdb \approx 20\log_{10}(1/\omega^n) = -20n\log_{10}\omega. \qquad (5.8)$$

Thus the Butterworth attenuation function would have an asymptotic slope of 20n db/octave in the stopband.

If a network is to be synthesized using a synthesis procedure given in Chapter Four a network transfer function H(s) must be found that has $M(\omega)$ as its magnitude response. The amplitude function $M(\omega)$ and the transfer function $H(j\omega)$ are related by

$$M^2(\omega) = H(j\omega)H(-j\omega). \qquad (5.9)$$

Let $h(s^2)$ be defined as

$$h(s^2) = H(s)H(-s) \tag{5.10}$$

then

$$M^2(\omega) = h(-\omega^2) = h(s^2) = H(s)H(-s) \tag{5.11}$$

where

$$s^2 = -\omega^2 \text{ and } s = j\omega.$$

To find H(s) from $h(s^2)$ choose the poles of $h(s^2)$ which are in the left hand s plane so that H(s) is Hurwitz. For example, let

$$M^2(\omega) = \frac{1}{1+\omega^6}$$

$$= \frac{1}{1-(-\omega^2)^3}$$

$$= \frac{1}{1-(s^2)^3}. \tag{5.12}$$

where $s^2 = -\omega^2$

$$h(s^2) = \frac{1}{1-(s^2)^3}$$

$$= \frac{1}{(s^3+2s^2+2s+1)(-s^3+2s^2-2s+1)}. \tag{5.13}$$

Thus H(s) is selected as

$$H(s) = \frac{1}{(s^3+2s^2+2s+1)}. \tag{5.14}$$

As a check we see that

$$M^2(\omega) = H(j\omega)H(-j\omega)$$

$$= \frac{1}{(j\omega)^3+2(j\omega)^2+2(j\omega)+1}\ \frac{1}{(-j\omega)^3+2(j\omega)^2-2(j\omega)+1}$$

$$= \frac{1}{(1-2\omega^2)+j(2\omega-\omega^3)}\ \frac{1}{(1-2\omega^2)-j(2\omega-\omega^3)}$$

$$= \frac{1}{1 + \omega^8}$$

as desired.

Table 5.1 gives the factors of H(s) for a Butterworth amplitude function for n = 1 to n = 8. Table 5.2 gives H(s) in the form

$$H(s) = \frac{1}{a_n s^n + a_{n-1} s^{n-1} + . \ . \ . + a_1 s + 1} \ . \tag{5.15}$$

The poles of H(s)H(-s) are located at $s_k = \sigma_k + j\omega_k$ where

$$\sigma_k = \frac{\cos(2k + n - 1)\pi}{2n} = \sin\left(\frac{2k - 1}{n}\right)\frac{\pi}{2} \tag{5.16}$$

$$\omega_k = \frac{\sin(2k + n - 1)\pi}{2n} = \cos\left(\frac{2k - 1}{n}\right)\frac{\pi}{2} \ .$$

The parametric equation of a circle with its center at the origin in the s plane is given by

$$\sigma = a \cos \theta$$

$$\omega = a \sin \theta \ . \tag{5.17}$$

Comparison of equations (5.16) and (5.17) shows that the poles of H(s)H(-s) for the Butterworth response are located at a unit circle with its center at the origin.

n	
1	$(s+1)$
2	$(s^2+\sqrt{2}s+1)$
3	$(s^2+s+1)(s+1)$
4	$(s^2+0.56536s+1)(s^2+1.84776s+1)$
5	$(s+1)(s^2+0.6180s+1)(s^2+1.6180s+1)$
6	$(s^2+0.5176s+1)(s^2+\sqrt{2}s+1)(s^2+1.9318s+1)$
7	$(s+1)(s^2+0.4450s+1)(s^2+1.2465s+1)(s^2+1.8022s+1)$
8	$(s^2+0.3896s+1)(s^2+1.1110s+1)(s^2+1.6630s+1)(s^2+1.9622s+1)$

Table 5.1 H(s) for Butterworth Response

n	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
1	1							
2	$\sqrt{2}$	1						
3	2	2	1					
4	2.613	3.414	2.613	1				
5	3.236	5.236	5.236	3.236	1			
6	3.864	7.464	9.141	7.464	3.864	1		
7	4.494	10.103	14.606	14.606	10.103	4.494	1	
8	5.126	13.138	21.848	25.691	21.848	13.138	5.126	1

Table 5.2 Coefficients of H(s), note $a_0 = 1$ for all n.

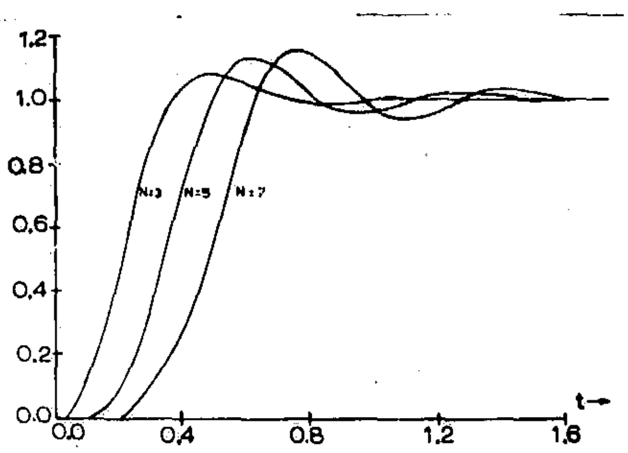

Figure 5.4 Sketch of the step response of a normalized Butterworth filter, for n = 3, 5, and 7.

A sketch of the response of a normalized Butterworth low-pass filter for a unit step input is shown in figure 5.4. Note that the rise time, overshoot, and settling time increases as n increases.

Example: Design of a normalized third order Butterworth low-pass filter. Suppose that a third order Butterworth low-pass filter is desired. Furthermore, port (2,2′) of the network is to be terminated in a resistive load, and the transfer function to be realized is -Y21. From Table 5.1, -Y21 is found to be

$$H(s) = -Y21 = \frac{1}{s^3 + 2s^2 + 2s + 1}\ .$$

The network is realized by the process given in Chapter Four, see equation (4.35). The resulting network is shown in figure 4.14-a. The frequency and impedance transformations of Chapter One can be used to transform the network to a network with the desired cutoff frequency and impedance level. NASAP-69 is used to check the resulting filter to insure that it meets the desired specifications.

5.3 The Chebyshev approximation. A second approximation to the ideal low-pass response will result if we let

$$f(\omega^2) = \epsilon^2 C_n^2(\omega) \tag{5.18}$$

where $0 < \epsilon < 1$ is a real constant and C_n is the nth-order Chebyshev polynomial. The nth-order Chebyshev polynomial is given by

$$\begin{aligned} C_n(\omega) &= \cos(n\cos^{-1}\omega) && \text{for} |\omega| \leq 1 \\ &= \cosh(n\cosh^{-1}\omega) && |\omega| > 1 \ . \end{aligned} \tag{5.19}$$

Chebyshev polynomials can be generated by use of the recursive formula

$$C_n(\omega) = 2\omega C_{n-1}(\omega) - C_{n-2}(\omega), \tag{5.20}$$

Table 5.3 gives the Chebyshev polynomials from C_1 to C_{10}.

The Chebyshev polynomials C_0 through C_5 are plotted versus ω in figure 5.5. The odd order polynomials are in the right hand column while the even order polynomials are in the left hand column. As can be seen by examination of figure 5.5, the Chebyshev polynomials have their zeros for ω between -1 and 1. The maximum value of a Chebyshev polynomial within the interval $-1 \leq \omega \leq -1$ is +1 while the minimum value is -1. For $|\omega| < -1$ and $|\omega| > 1$ the value of the magnitude of a Chebyshev polynomial, $|C_n(\omega)|$, becomes very large as ω goes to ∞. For n even, we note that

n	Chebyshev Polynomial
0	1
1	ω
2	$2\omega^2 - 1$
3	$4\omega^3 - 3\omega$
4	$8\omega^4 - 8\omega^2 + 1$
5	$16\omega^5 - 20\omega^3 + 5\omega$
6	$32\omega^6 - 48\omega^4 + 18\omega^2 - 1$
7	$64\omega^7 - 112\omega^5 + 56\omega^3 - 7\omega$
8	$128\omega^8 - 256\omega^6 + 160\omega^4 - 32\omega^2 + 1$
9	$256\omega^9 - 576\omega^7 + 432\omega^5 - 120\omega^3 + 9\omega$
10	$512\omega^{10} - 1280\omega^8 + 1120\omega^6 - 400\omega^4 + 50\omega^2 - 1$

Table 5.3 Chebyshev polynomials for $n = 0$ to $n = 8$.

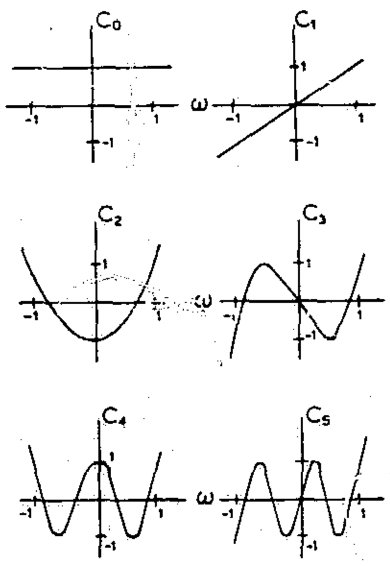

Figure 5.5 Sketch of Chebyshev polynomials C_0 through C_5 versus ω.

$C_n(0) = (-1)^{n/2}$ and $C_n(+1) = 1$. For n odd, $C_n(0) = 0$ and $C_n(\pm 1) = \pm 1$.

The Chebyshev magnitude type of response is given by

$$M(\omega) = \frac{1}{\sqrt{1 + \epsilon^2 C_n^2(\omega)}} . \tag{5.21}$$

The maximum value of $M(\omega)$ occurs at a value of ω such that $C_n(\omega) = 0$. The minimum value of $M(\omega)$ occurs at a value of ω such that $|C_n(\omega)| = 1$ and is a function of ϵ. The maximum value of a Chebyshev magnitude function is unity while the minimum value is $1/\sqrt{1 + \epsilon^2}$. For n odd $M(0) = 1/\sqrt{1 + \epsilon^2}$. Since $|C_n(\omega)| = 1$ for all n, $M(1) = 1/\sqrt{1 + \epsilon^2}$. Figure 5.6 shows a typical Chebyshev magnitude response for both even and odd n. The most distinguishing feature of the Chebyshev magnitude response when compared with the Butterworth magnitude response is the presence of the ripple (the ripple-like variation between 1 and $1/\sqrt{1 + \epsilon^2}$ in

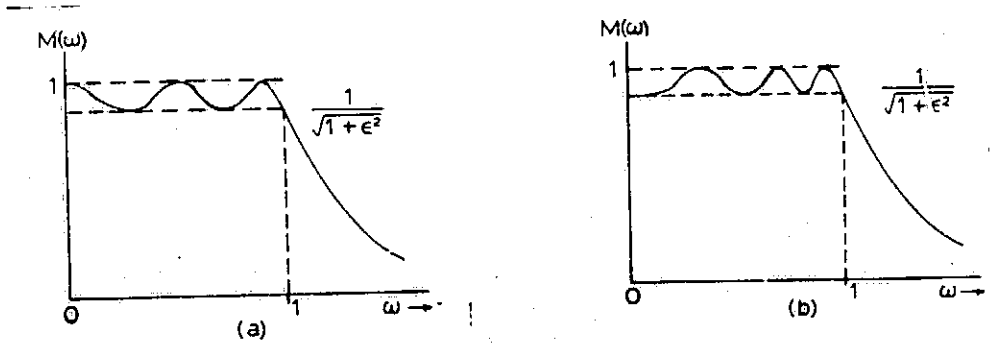

Figure 5.6 Sketch of a typical Chebyshev magnitude response. (a) n odd, (b) n even.

the Chebyshev magnitude response). The minimum value of the Chebyshev magnitude response in the passband can be controlled by the choice of ϵ, $0 < \epsilon < 1$. There is no such control of the minimum value of the Butterworth response in the passband.

The ripple width or the maximum deviation in the magnitude function can be found from the tolerance contours used to specify the low-pass filter. The ripple width for a Chebyshev magnitude response is given as

$$1 - \frac{1}{(1 + \epsilon^2)^{\frac{1}{2}}} \approx \frac{\epsilon^2}{2}, \quad \text{for } \epsilon << 1 \tag{5.22}$$

Hence the value of ϵ^2 can be selected by knowledge of the maximum deviation in the magnitude function as given by a tolerance contour. For large values of ω where $\omega >> 1$ and $\epsilon C_n(\omega) >> 1$ the Chebyshev magnitude response is approximately equal to

$$M(\omega) \approx \frac{1}{\epsilon C_n(\omega)}. \tag{5.23}$$

The attenuation of a Chebyshev response function is given by

$$\begin{aligned} Adb &= -20\log_{10} M(\omega) \\ &\approx 20\log_{10}\epsilon + 20\log_{10} C_n(\omega). \end{aligned} \tag{5.24}$$

Now for large values of ω, $C_n(\omega)$ can be approximated by the nth-order term of the nth-order Chebyshev polynomial resulting in

$$Adb \approx 20\log_{10}\epsilon + 20\log_{10}(2^{n-1}\omega^n) \tag{5.25}$$

or

$$Adb \approx 20\log_{10}\epsilon + 6(n-1) + 20\log_{10}\omega . \tag{5.26}$$

The Chebyshev attenuation response function thus has a slope of approximately 20n db/octave in the stopband, which is the same as the Butterworth attenuation function.

The poles of the Chebyshev magnitude response function occur for

$$C_n(\omega) = \pm \frac{j}{\epsilon} \tag{5.26-a}$$

where

$$C_n(\omega) = \cos n\omega.$$

Now let $\omega = u + jv$ and equation (5.26-a) becomes

$$C_n(\omega) = \cos n\omega = \cos n(u + jv)$$

$$= \cos nu \cosh nv - j \sin nu \sinh nv = \pm \frac{j}{\epsilon}\,. \tag{5.27}$$

Equation (5.27) is satisfied when

$$\cos nu \cosh nv = 0 \tag{5.28-a}$$

and

$$\sin nu \sinh nv = \pm j/\epsilon. \tag{5.28-b}$$

Equation (5.28-a) implies that $\cos nu = 0$ as $\cosh nv \neq 0$, thus

$$u = (1/n)(2k - 1)(\pi/2),\ k = 1,2,\ldots,2n. \tag{5.29}$$

For these values of u, $\sin nu = \pm 1$, so that

$$nv = \sinh^{-1}(1/\epsilon). \tag{5.30}$$

Let "a" be a v such that equation (5.30) is satisfied then

$$a = (1/n)\sinh^{-1}(1/\epsilon). \tag{5.31}$$

The position of the poles of the Chebyshev response are then located at

$$s = j\cos(u + jv) = j\cos(\ (\pi/2n)(2k - 1) + ja). \tag{5.32}$$

If $s_k = \sigma_k + j\omega_k$ the pole locations are given by

$$\sigma_k = \pm\ \sinh a\ \ \sin((2k - 1)(\pi/2n))$$

$$k = 1,2,3,\ldots,2n \tag{5.33}$$

and

$$\omega_k = \cosh a \cos((2k - 1)(\pi/2n)).$$

Equation (5.33) is the parametric equation of an ellips. This will be seen if both expressions are squared and added to obtain

$$\sigma_k^2/\sinh^2 a + \omega_k^2/\cosh^2 a = 1 . \tag{5.34}$$

If equations (5.33) are divided by cosh a they become

$$\frac{\sigma_k}{\cosh a} = \tanh a \sin((2k - 1)(\pi/2n))$$

$$k = 1,2,3,. . .2n \tag{5.35}$$

and

$$\frac{\omega_k}{\cosh a} = \cos((2k - 1)(\pi/2n)). \tag{5.35}$$

If equations (5.35) are compared with equations (5.16) for the Butterworth pole locations we find the imaginary part of the Chebyshev pole s_k divided by cosh a and the imaginary part of the Butterworth pole are the same. The real part of the Chebyshev pole divided by cosh a has an additional factor of tanh a when compared to the real portion of the Butterworth pole. Thus the Chebyshev pole locations can be obtained from the Butterworth pole locations. This technique will be used in the following example.

Example: A transfer function H(s) with a Chebyshev amplitude characteristic is desired. The Chebyshev amplitude characteristic should have a maximum of 1 db ripple in the passband and should have at least 20 db of attenuation at $\omega = 2$.

We know that $|C_n(1)| = 1$ for any n, thus $H(j1) = 1/\sqrt{1 + \epsilon^2}$. The relation can be used to find ϵ as

$$20\log_{10}|H(j1)| = -1 = 20\log_{10} 1/\sqrt{1 + \epsilon^2}$$

or

$$\log_{10} 1/\sqrt{1 + \epsilon^2} = \frac{-1}{20}$$

which means

$$\frac{1}{\sqrt{1 + \epsilon^2}} = 10^{-1/20} = \frac{1}{10^{1/20}} .$$

Now by use of a slide rule's loglog scales we find

$$\frac{1}{\sqrt{1 + \epsilon^2}} = \frac{1}{1.122} = .891 .$$

Thus an $\epsilon = .508$ will insure a 1 db ripple in the passband. Now n, the order of the Chebyshev polynomial, must be found. Equation (5.25) can be used for this purpose, as Adb = 20 for $\omega = 2$.

$$20 \approx 20\log_{10} 0.508 + 20\log_{10}(2^{n-1} 2^n)$$

$$1 = -\log_{10} 1.97 + (2^{n-1})\log_{10} 2$$

$$(2^{n-1}) = \frac{1.294}{\log_{10} 2} = 4.32$$

$$n = \frac{5.32}{2} = 2.66$$

The next largest integer is 3, therefore, n = 3. There are design charts available for selected values of ϵ from which n, for a specified attenuation at a given frequency, can be read directly. For example, see White Electromagnetics' <u>A Handbook on Electrical Filters</u> or Zuerev's <u>Handbook of Filter Synthesis</u>.

The pole locations for $\epsilon = 0.508$ and $n = 3$ can now be found using equations (5.31), (5.35), and Table 5.1.

$$a = (1/3)\ \sinh^{-1}(1/.508)$$
$$= (1/3)\ \sinh^{-1}(1.97) = (1/3)(1.43)$$
$$= 0.477$$
$$\tan a = \tan(0.477) = 0.444.$$

Now the factors of the 3rd-order Butterworth response function from Table 5.1 are $(s + 1)$ and $(s^2 + s + 1)$. Thus the poles are located at

$$s_1 = -1.0,\ s_2 = -0.5 + j0.866,\ s_3 = -0.5 - j0.866.$$

The real and imaginary part of the Chebyshev poles are then found using equations (5.35).

$$\frac{\sigma_k}{\cosh a} = \sigma_k \tanh a$$

$$\frac{\omega_k}{\cosh a} = \omega_k$$

where σ_k and ω_k are the real and imaginary parts of the kth Butterworth pole.

The Chebyshev poles are just

$$\sigma_1 = -1 \cosh(0.477) \tanh(0.477)$$
$$= -0.496$$
$$\sigma_2 = \sigma_3 = (-0.5) \cosh(0.477) \tanh(0.477)$$
$$= -0.248$$
$$\omega_2 = (0.866) \cosh(0.477)$$
$$= 0.9666$$
$$\omega_3 = (-0.866) \cosh(0.477)$$
$$= -0.9666$$

where

$$\cosh(0.477) \approx 1.116.$$

The desired network transfer function is

$$H(s) = \frac{1}{(s+0.496)(s+0.248+j0.996)(s+0.248-j0.9961)}.$$

Poles of a Chebyshev amplitude response function for selected values of ϵ and n from 1 to 10 are listed in Table 11.9 of Weinberg's <u>Network Analysis and Synthesis</u>.

Example: The low-pass filter specified in Table 1.5 will now be considered. The order n of the required Chebyshev polynomial is desired. The complexity of the required filter is indicated by n. Initial estimation of n for the Butterworth case indicates that a Butterworth filter with the required attenuation of 30 db at $\omega = 1.0245$ would be too complex and require too many elements. Using the 3 db ripple requirement, find n for the Chebyshev response function, thus determining the complexity of the required Chebyshev filter.

$$20\log_{10}|H(j1)| = -3 = 20\log_{10} 1/\sqrt{1+\epsilon^2}$$

$$\log_{10} 1/\sqrt{1+\epsilon^2} = -3/20$$

which means

$$\frac{1}{\sqrt{1+\epsilon^2}} = 10^{-3/20}$$

$\sqrt{1+\epsilon^2} = 1.$[illegible]2

$\epsilon = \sqrt{.99}$[illegible]$ = .997$.

A value for n may now be found. In this example, using equation (5.25) for this purpose will be more difficult than in the pr[illegible]vious examples, since the attenuation is not specified for $\omega = 2$, but for $\omega = 1.0245$. In this case equation (5.25) becomes

$$30 = 20\log_{10}1 + 20\log_{10}[(2^{n-1})(1.0245^{n})]$$

$$1.5 = .3(n-1) + \log_{10}1.0245^{n}.$$

Assume $\log_{10}1.0245^{n} << .3(n-1)$, an assumption which would tend to result in too large a value of n, then

$$n - 1 = 5$$
$$n = 6 .$$

Therefore a very conservative value of n would seem to be $n = 7$. Now a Chebyshev response function for $\epsilon = .9907$ and $n = 7$ could be found and then the techniques of Chapter Four could be used to synthesize the required network. The resulting low-pass filter can be checked by NASAP-69 to see if it meets the given specifications. Rather than taking this approach to realize the filter, a much simpler method will be employed. The simpler approach involves the use of special tables introduced in Chapter 6. The filter will be realized by use of these tables and then analyzed by NASAP-69. Since equation (5.25) was developed for $\omega >> 1$, and we have used it for $\omega \approx 1.0$ it is anticipated that analysis by NASAP-69 will show that the Chebyshev filter with $\epsilon = .997$ and $n = 7$ does not meet the required specifications.

5.4 Bessel Approximation. Bessel polynomials are used to approximate transfer functions with a linear phase response.

Consider a system transfer function given by

$$H(s) = k\exp -sT \; . \tag{5.36}$$

The frequency response of the system is given by

$$H(j\omega) = k\exp -j\omega T \tag{5.37}$$

where

$$M(\omega) = |H(j\omega)| = k$$

and the phase response is

$$\phi(\omega) = \text{Arg}H(j\omega) = -\omega T. \tag{5.38}$$

Equation (5.38) is a linear function of ω. The response of a system with the form of system transfer function of equation (5.36) is just the excitation delayed by some time, T. The delay T is obtained by differentiating the phase response.

$$\text{Delay} = \frac{-d\phi(\omega)}{d\omega} = T \tag{5.39}$$

The delay time, T, is sometimes called the envelope delay or the group delay. A circuit with such a delay cannot be constructed by the use of discrete elements, however, it can be approximated in the following manner. Let T = 1 sec. and rewrite equation (5.39) as

$$H(s) = \frac{1}{\exp sT}$$

$$= \frac{1}{\sinh s + \cosh s} \; . \tag{5.40}$$

Now we will approximate H(s). First divide the numerator and denominator of (5.40) by sinh s to get

$$H(s) = \frac{1/\sinh s}{\cosh s/\sinh s + 1} \; . \tag{5.41}$$

The fractions cosh s and sinh s are expanded as

$$\cosh s = 1 + \frac{s^2}{2!} + \frac{s^4}{4!} + \frac{s^6}{6!} + \cdot \cdot \cdot \cdot \qquad (5.42)$$

and

$$\sinh s = s + \frac{s^3}{3!} + \frac{s^5}{5!} + \frac{s^7}{7!} + \cdot \cdot \cdot \cdot \qquad (5.43)$$

Equations (5.42) and (5.43) can be combined to compute coth s = cosh s + cosh s/sinh s in the form of a continued fraction expansion as

$$\coth s = \frac{1}{s} + \cfrac{1}{\frac{3}{s} + \cfrac{1}{\frac{5}{s} + \cfrac{1}{\frac{7}{s} + \cdots}}} \qquad (5.44)$$

The coths is approximated by truncating the continued fraction expansion (2n - 1)/s term. For n = 3, coth s is approximated by including terms 1/s through 5/s as

$$\coth s = \frac{1}{s} + \cfrac{1}{\frac{3}{5} + \cfrac{1}{\frac{5}{s}}}$$

$$= \frac{6s^2 + 15}{s^3 + 15s} = \frac{m}{n} = \frac{\cosh s}{\sinh s} . \qquad (5.45)$$

Note that this is the ratio of an even function to an odd function. Since all the quotients of equation (5.44) are positive, m + n is a Hurwitz polynomial for $m = 6s^2 + 15$ and $n = s^3 + 15s$.

If m is identical with cosh s and n with sinh s, equation (5.40) becomes

$$H(s) = \frac{k_0}{m + n} \tag{5.46}$$

where k_0 is chosen so that $H(0) = 1$. For the m and n of equation (5.45) we would have

$$H(s) \frac{k_0}{15 + 15s + 6s^2 + s^3} = \frac{k_0}{B_3} \tag{5.47}$$

where

$$H(0) = 1 = \frac{k_0}{15}$$

which implies

$$k_0 = 15$$

and

$B_3 = 15 + 15s + 6s^2 + s^3$ is the 3rd-order Bessel polynomial. In general B_n is of the form

$$B_n(s) = b_0 + b_1 s + . . . + b_n s^n \tag{5.48}$$

and is given by the recursion formula

$$B_n = (2n - 1) B_{n-1} + s^2 B_{n-2} \tag{5.49}$$

where $B_0 = 1$ and $B_1 = s + 1$ are used to start the generation of B_n. Table 5.4 gives the coefficients of B_n for $n = 0$ to $n = 7$. Table 5.5 gives the roots of the Bessel functions for $n = 1$ to $n = 7$.

n	b_0	b_1	b_2	b_3	b_4	b_5	b_6	b_7
0	1							
1	1	1						
2	3	3	1					
3	15	15	6	1				
4	105	105	45	10	1			
5	945	945	420	105	15	1		
6	10,395	10,395	4,725	1,260	210	21	1	
7	135,235	135,135	62,370	17,325	3,150	378	28	1

Table 5.4 Coefficients of Bessel Polynomial

n	1	2	3	4
	-1.0	-1.5±j0.866667	-2.32219+j0	-2.89621±j0.867234
			-1.83891±j1.75438	-2.10379±j2.65742

n	5	6	7
	-3.64674+j0	-4.24836±j0.86751	-4.97181+j0
	-3.35196±j1.74266	-3.73571±j2.6267	-4.75827±j1.73928
	-2.32467±j3.57102	-2.5159±j4.49267	-4.07014±j3.71718
			-2.68568±j5.42069

Table 5.5 Roots of Bessel Polynomials

Figure 5.7 compares the amplitude response of a 3rd-order Bessel and a 3rd-order Butterworth filter. Figure 5.8 is a comparison of their phase characteristics.

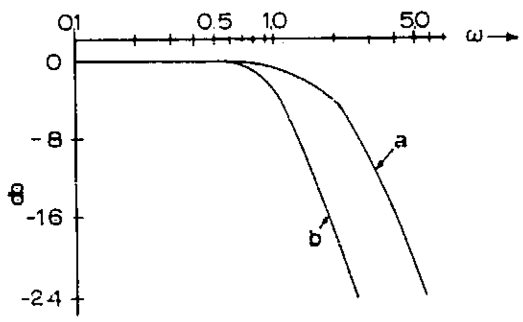

Figure 5.7 Comparison of amplitude response of a Bessel and Butterworth filter, n = 3. (a) Bessel. (b) Butterworth.

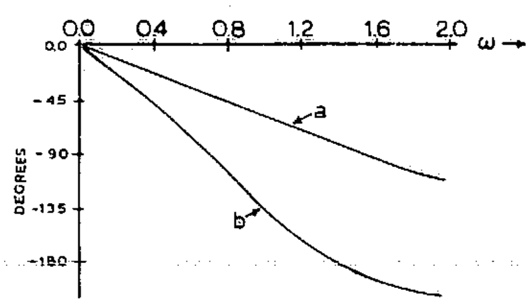

Figure 5.8 Comparison of phase response of a Bessel and Butterworth filter, n = 3. (a) Bessel. (b) Butterworth.

5.5 Concluding remarks. Both the Butterworth and Chebyshev response functions may be used to approximate the amplitude response of a low-pass filter. There are many other possible response functions which also may be used for this purpose. One commonly used function results in a filter with equal ripple in both the pass and stopband. An example of such a filter can be found in Chapter Eight where it is analyzed by use of NASAP-69.

The choice of response function to be used is made by considering the characteristics of each type of filter and selecting the response function that is most suited for the task. For example, the Chebyshev function may be selected instead of the Butterworth function since the maximum attenuation in the passband may be controlled by the choice of ϵ for the Chebyshev function while there is no such control over the passband attenuation with the Butterworth filter. On the other hand the Butterworth response may be selected over the Chebyshev response function if phase distortion is an important consideration. The Bessel response function is useful when one wishes to approximate a linear time delay function.

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CHAPTER SIX

Double Terminated Networks

6.1 Introduction. The synthesis techniques of Chapter Four coupled with the approximations of a transfer function of a low-pass filter discussed in Chapter Five can be used to realize low-pass filters which are terminated in a resistive load. In this chapter a synthesis technique that will realize a low-pass filter that is to be driven by an ideal voltage source with a series resistance or an ideal current source with a shunt resistance and terminated in a resistive load is introduced. The low-pass filters resulting from the techniques of Chapter Four and from the methods of this chapter can be transformed into other types of filters by the use of the transformations given in Chapter One. In the final section of this chapter a table of element values for Butterworth and Chebyshev low-pass filters is presented. This table simplifies the design of double terminated filters. Similar tables for other forms of filters appear in the references listed at the end of this chapter. The program used to generate the element values appears in Appendix D.

The Darlington synthesis procedure is used to realize networks which are to be driven and terminated as shown in figure 6.1. The amplitude of the filter response can be conveniently specified by either the voltage transfer function

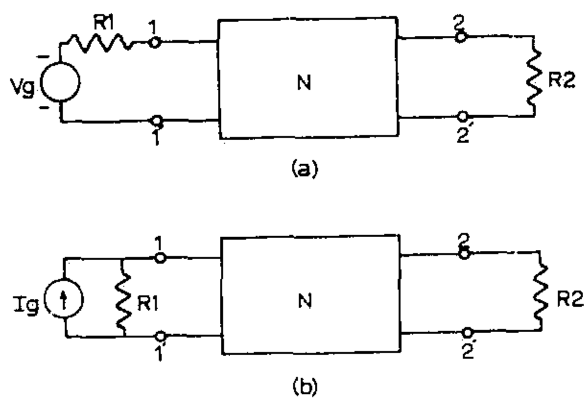


Figure 6.1 Double terminated networks.

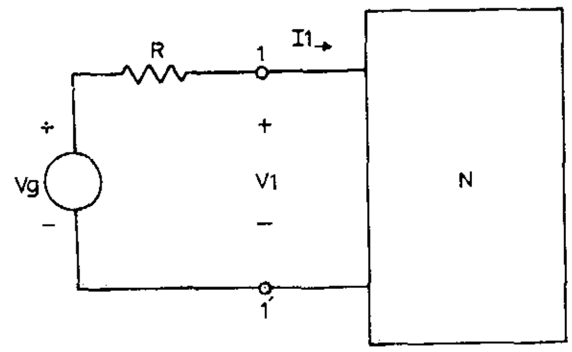


Figure 6.2 Single-port network.

|V2/Vg| or the impedance transfer function |V2/Ig|. From the specification of the magnitude of either the voltage or impedance transfer function the magnitude scattering coefficients $|s_{11}(j\omega)|^2$ and $|s_{12}(j\omega)|^2$ for the LC network are found. Using $|s_{11}(j\omega)|^2$, the driving point impedance $Z_{11}(s)$ is found and then synthesized. The scattering parameter's representation of a two-port network will be reviewed before the details of the Darlington procedure are presented.

<u>6.2 Scattering parameters.</u> The single-port network of figure 6.2 will be used to introduce the concepts involved in the scattering matrix representation of a network. Define the voltage V_i (incident voltage) and V_r (reflected voltage) as

$$V_i = \tfrac{1}{2}\,(V1 + R_{o1} I1) \qquad (6.1\text{-a})$$

$$V_r = \tfrac{1}{2}\,(V1 - R_{o1} I1) \qquad (6.1\text{-b})$$

where V1 and I1 are the terminal voltage and current of the single-port network and the network is characterized by the driving point impedance Z1(s) = V1/I1. The parameter R_{o1} is an arbitrary, dimensionless, positive constant referred to as the reference impedance factor. If equation (6.1-b) is added to (6.1-a), and then equation (6.1-b) is subtracted from (6.1-a) the terminal voltage and current are found in terms of the incident and reflected voltage as

$$V1 = V_i + V_r \qquad (6.2\text{-a})$$

$$I1 = \frac{V_i}{R_{o1}} - \frac{V_r}{R_{o1}} \qquad (6.2\text{-b})$$

It is convenient to define two new parameters, the incident parameter "a_1" and the reflected parameter "b_1" in terms of the incident and reflected voltage. The parameters "a_1" and "b_1" are defined as

$$a_1 = \tfrac{1}{2}\,(V1/\sqrt{R_{o1}} + \sqrt{R_{o1}}\; I1) \qquad (6.3\text{-a})$$

$$b_1 = \tfrac{1}{2}\,(V1/\sqrt{R_{o1}} - \sqrt{R_{o1}}\; I1). \qquad (6.3\text{-b})$$

And by comparing equation (6.1) and (6.3) we see that

$$V_i = a_1\sqrt{R_{o1}}$$

$$V_r = b_1\sqrt{R_{o1}}\ .$$

Thus equation (6.2) expressing V1 and I1 in terms of V_i and V_r can be rewritten

$$V1 = (a_1 + b_1)\sqrt{R_{o1}} \qquad (6.4\text{-a})$$

$$I1 = \frac{(a_1 - b_1)}{\sqrt{R_{o1}}} \qquad (6.4\text{-b})$$

The power consumed by terminals (1,1′) of the single-port network is given by

$$P = \mathrm{Re}(V1I1^*)$$

$$= \mathrm{Re}\left[(a_1 + b_1)\sqrt{R_{o1}}\ \frac{(a_1 - b_1)^*}{\sqrt{R_{o1}}}\right]$$

$$= Re(a_1 a_1^* - b_1 b_1^* + a_1^* b_1 - a_1 b_1^*)$$

$$= (a_1 a_1^* - b_1 b_1^*)$$

$$= (|a_1|^2 - |b_1|^2) \qquad (6.5)$$

where the asterisk denotes the complex conjugate. If the scattering parameter s_{11} is defined as

$$s_{11} = \frac{b_1}{a_1} \qquad (6.6)$$

equation (6.5) becomes

$$P = |a_1|^2 (1 - |s_{11}|^2) \quad . \qquad (6.7)$$

Since the power consumed by the network when it is excited by a sinusoidal source is never negative we see that $|s_{11}(j\omega)| \leq 1$.

The scattering parameter s_{11} can be expressed in terms of V1 and I1 by use of equations (6.3-a) and (6.3-b) as

$$s_{11} = \frac{b_1}{a_1} = \frac{V1/\sqrt{R_{01}} - \sqrt{R_{01}}\,I1}{V1/\sqrt{R_{01}} - \sqrt{R_{01}}\,I1}$$

$$= \frac{V1 - R_{01} I1}{V1 + R_{01} I1}$$

or

$$s_{11} = \frac{Z1 - R_{01}}{Z1 + R_{01}} \qquad (6.8)$$

where Z1 has been used to replace the ratio V1/I1. Note here that if $Z1 = R_{01}$ then s_{11} equals zero and from the definition of s_{11}, equation (6.6) we see that the reflected parameter b_1 must be equal to zero.

Now consider the particular single-port network shown in figure 6.3. In this case the network consists of a single resistance, R, the series resistance of the voltage source,

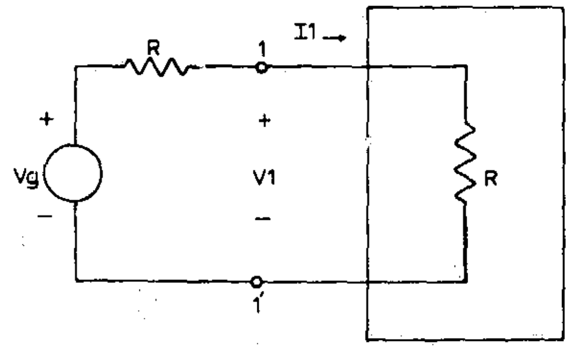


Figure 6.3 Single-port network with Z1 = R, the series resistance of the voltage source.

This is the condition for which the maximum power will be dissipated by the network. The maximum power available from the source is given as

$$Pmax = \frac{|V_g|^2}{2} \frac{1}{R} \qquad (6.9)$$

and must also be equal to

$$Pmax = |a_1|^2 (1 - |s_{11}|^2) \qquad (6.10)$$

by equation (6.7). Since R_{o1} was an arbitrary positive constant it can be choosen in a manner designed to simplify equation (6.10). If s_{11} is zero, equation (6.10) reduces to

$$Pmax = |a_1|^2 \qquad (6.11)$$

and s_{11} is zero if R_{o1} = Z1, or in this case R_{o1} = R the series resistance of the source. Comparing equations (6.9) and (6.11) we find that

$$|a_1|^2 = \frac{|V_g|^2}{2} \frac{1}{R} . \qquad (6.12)$$

Now the network of figure 6.3 will be considered.

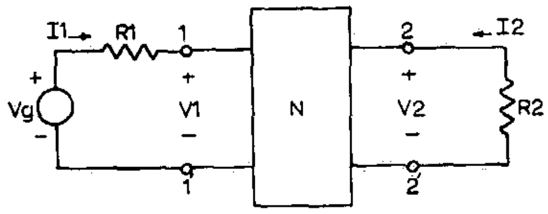


Figure 6.4 Two-port network to be characterized by the scattering matrix [S].

Associated with terminal pair (1,1′) is the arbitrary, dimensionless, positive reference parameter, R_{o1}. Likewise, with terminal pair (2,2′),is associated the arbitrary reference parameter R_{o2}. It is convenient, just as it was in the case of the single-port network, to let R_{o1} equal the series resistance, R1, associated with the voltage source connected to port 1. Similarly, R_{o2} is taken to be equal to R2. Then the parameters a_1, b_1, and a_2, b_2 are defined in a manner similar to the parameters a_1 and b_1 for the single-port network. The defining equations are

$$a_1 = \tfrac{1}{2}(V1/\sqrt{R1} + \sqrt{R1}\ I1) \qquad (6.13)$$

$$b_1 = \tfrac{1}{2}(V1/\sqrt{R1} - \sqrt{R1}\ I1) \qquad (6.14)$$

$$a_2 = \tfrac{1}{2}(V2/\sqrt{R2} + \sqrt{R2}\ I2) \qquad (6.15)$$

$$b_2 = \tfrac{1}{2}(V2/\sqrt{R2} - \sqrt{R2}\ I2). \qquad (6.16)$$

The incident and reflected parameters of port 1 and 2 are related by the scattering matrix [S] in the following manner:

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} . \qquad (6.17)$$

The coefficients of the scattering matrix for the two-port network are given by

$$S_{11} = \frac{b_1}{a_1}\Big|_{a_2 = 0} \qquad S_{12} = \frac{b_1}{a_2}\Big|_{a_1 = 0} \qquad (6.18)$$

$$S_{21} = \frac{b_2}{a_1}\Big|_{a_2 = 0} \qquad S_{22} = \frac{b_2}{a_2}\Big|_{a_1 = 0}.$$

We find from equation (6.15) that the condition $a_2 = 0$ implies that

$$V2/\sqrt{R2} = -\sqrt{R2}\ I2$$

or that

$$\frac{V2}{-I2} = R2$$

which is the case when port 2 is terminated by a resistance R2. The condition $a_1 = 0$ is satisfied when port 1 is terminated by a resistance of R1 ohms. Now using equation (6.12) for S_{11} and equations (6.13) and (6.14) to substitute for a_1 and b_1 we have

$$S_{11} = \frac{[V1/\sqrt{R1} - \sqrt{R1}\ I1]}{[V1/\sqrt{R1} + \sqrt{R1}\ I1]}\Bigg|_{a_2 = 0}$$

$$= \frac{V1/I1 - R1}{V1/I1 + R1}\Bigg|_{a_2 = 0}$$

$$= \frac{Z1 - R1}{Z1 + R1} \qquad (6.19)$$

where Z1 = V1/I1 with port 2 terminated in a resistance R2. Similarly, we have

$$S_{22} = \frac{Z2 - R2}{Z2 - R2} \qquad (6.20)$$

where Z2 = V2/I2 with port 1 terminated in a resistance R1.

The forward transmission coefficient s_{21} is defined by

$$s_{21} = \left.\frac{b_2}{a_1}\right|_{a_2 = 0}$$

where the condition $a_2 = 0$ is satisfied when port 2 is terminated in R2 ohms as the reference impedance is taken to be R2 ohms. This situation is shown in figure 6.5.

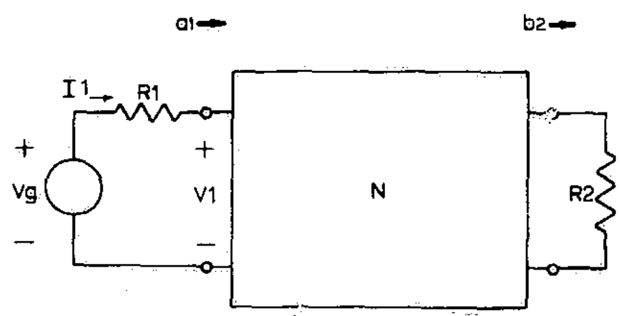


Figure 6.5 Double terminated two-port network with R_{o1} = R1, R_{o2} = R2, and a_2 = 0.

The current I1 flowing into terminals (1,1′) is given by

$$I1 = \frac{V_g}{R1 + Z1} \qquad (6.21)$$

and the terminal voltage V1 is given as

$$V1 = V_g + I1R1. \qquad (6.22)$$

Now equation (6.21) and (6.22) can be combined with equation (6.13), the equation which defines a_1 as follows:

$$a_1 = \tfrac{1}{2}(V1/\sqrt{R1} + \sqrt{R1}\ I1)$$

$$= \frac{1}{2\sqrt{R}}\left[V_g - \frac{R1\ V_g}{R1 + Z1} + \frac{R1 V_g}{R1 + Z} \right]$$

$$a_1 = \frac{V_g}{2\sqrt{R1}} \qquad (6.23)$$

From equation (6.15), the equation which defines a_2, and the fact that $a_2 = 0$ we have

$$a_2 = 0 = \frac{1}{2}(V2/\sqrt{R2} + \sqrt{R2}\ I2)$$

or

$$\sqrt{R2}\ I2 = -\frac{V2}{\sqrt{R2}}$$

and using equation (6.16) we find

$$b_2 = (V2/\sqrt{R2} - \sqrt{R2}\ I2)$$

$$= \frac{1}{2}(V2/\sqrt{R2} + V2/\sqrt{R2}) = \frac{V2}{\sqrt{R2}} . \qquad (6.24)$$

Thus the forward transmission coefficient s_{21} becomes

$$s_{21} = \frac{b_2}{a_1}\bigg|_{a_2 = 0} = \frac{2V2}{Vg}\sqrt{\frac{R1}{R2}} . \qquad (6.25)$$

Equation (6.25) relates the voltage transfer function $G2g = V2/Vg$ to the forward scattering coefficient s_{21} and will be used in the Darlington synthesis procedure. If the ideal voltage source and its series resistance is replaced by its Norton's equivalent circuit, a network similar to figure 6.1-b results where $Ig = Vg/R1$ and equation (6.25) for this configuration becomes.

$$s_{21} = \frac{2V2}{IgR1}\sqrt{\frac{R1}{R2}} = \frac{2}{\sqrt{R1R2}}\ \frac{V2}{Ig} . \qquad (6.26)$$

Equation (6.26) relates the impedance transfer function $Z2g = V2/Ig$ and the forward scattering coefficient s_{21} and is used in the Darlington synthesis procedure when the magnitude response transfer impedance is specified.

Now that the scattering parameters of a two-port network have been found, relationships involving scattering parameters

s_{11} and s_{22} will be found by considering the power flowing into and out of the ports of the network.

The total power delivered to the network is just

$$P = P_1 + P_2 \qquad (6.27)$$

where P_1 is the power delivered to port one and P_2 is the power delivered to port two. The power delivered to port one is

$$P_1 = (a_1 a_1^* - b_1 b_1^*) \qquad (6.28)$$

and P_2 is

$$P_2 = (a_2 a_2^* - b_2 b_2^*) \qquad (6.29)$$

where equations (6.28) and (6.29) are found in a manner similar to the derivation of equation (6.5). From equations (6.27), (6.28), and (6.29) we have

$$\begin{aligned} P &= (a_1 a_1^* - b_1 b_1^*) + (a_2 a_2^* - b_2 b_2^*) \\ &= (a_1 a_1^* + a_2 a_2^* - b_1 b_1^* - b_2 b_2^*) \leq 0 \qquad (6.30) \end{aligned}$$

since the total power delivered to all ports must be positive. Equation (6.30) in matrix form is

$$P = \lfloor a^* \rfloor \; a] - \lfloor b^* \rfloor \; b] \qquad (6.31)$$

where

$$\lfloor a^* \rfloor = \lfloor a_1^* \quad a_2^* \rfloor$$

$$\lfloor b^* \rfloor = \lfloor b_1^* \quad b_2^* \rfloor$$

$$a] = \begin{matrix} a_1 \\ a_2 \end{matrix} \Bigg]$$

$$b] = \begin{matrix} b_1 \\ b_2 \end{matrix} \Bigg] .$$

Using $b] = [S] \; a]$ and $b^* = a^* \; [S^*]^t$, equation (6.31) becomes

$$P = \lfloor a^* \rfloor\, a] - a^* \, [S^*]^t [S][a] \qquad (6.32\text{-a})$$

where

$$[S^*]^t = \begin{bmatrix} S_{11}^* & S_{21}^* \\ S_{12}^* & S_{22}^* \end{bmatrix}$$

$$[S] = \begin{bmatrix} S_{11} & S_{21} \\ S_{12} & S_{22} \end{bmatrix} .$$

For a lossless network the net power delivered to the network is zero; the net power delivered to port one must be equal to the net power flowing from port two. In this case equation (6.32) can be written as

$$P = \lfloor a^* \rfloor\, a] - \lfloor a^* \rfloor\, [S^*]^t [S]\, a]$$

$$= \lfloor a^* \rfloor\, ([I] - [S^*][S])\, a] = 0 \qquad (6.32\text{-b})$$

where

$$[I] = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} .$$

Equation (6.32-b) implies that

$$[I] - [S^*]^t [S] = 0$$

or

$$[S^*][S] = [I] \qquad (6.32\text{-c})$$

which becomes

$$\begin{bmatrix} S_{11}^* \, S_{11} + S_{21}^* \, S_{21} & S_{11}^* \, S_{12} + S_{21}^* \, S_{22} \\ S_{12}^* \, S_{11} + S_{22}^* S_{21} & S_{12}^* \, S_{12} + S_{22}^* \, S_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad (6.33)$$

when $[S^*][S]$ is multiplied out. Thus we have the following four equations:

$$S_{11}^* \, S_{11} + S_{21}^* \, S_{21} = 1 \qquad (6.34)$$

$$S_{11}^* \, S_{12} + S_{21}^* \, S_{22} = 0 \qquad (6.35)$$

$$s_{12}^* \; s_{11} \; + \; s_{22}^* \; s_{21} \; = \; 0 \tag{6.36}$$

$$s_{12}^* \; s_{12} \; + \; s_{22}^* \; s_{22} \; = \; 1 \tag{6.37}$$

Since $s_{11}^* \; s_{11} \; = \; |s_{11}|^2$ equation (6.34) can be written as

$$|s_{11}(j\omega)|^2 = |1 - \; s_{21}(j\omega)|^2 \; . \tag{6.38}$$

This equation is used to find $|s_{11}(j\omega)|$ from $|s_{21}|$ after $|s_{21}(j\omega)|^2$ has been determined. $|s_{21}(j\omega)|^2$ can be found by using equation (6.25) if the voltage ratio G2g is specified, or from equation (6.26) if the current ratio V2/Ig is specified. $|S_{21}(j\omega)|^2$ can be specified in still another manner, by use of a relationship between $|s_{21}(j\omega)|^2$ and the insertion power loss. This relationship will be found after defining insertion power loss.

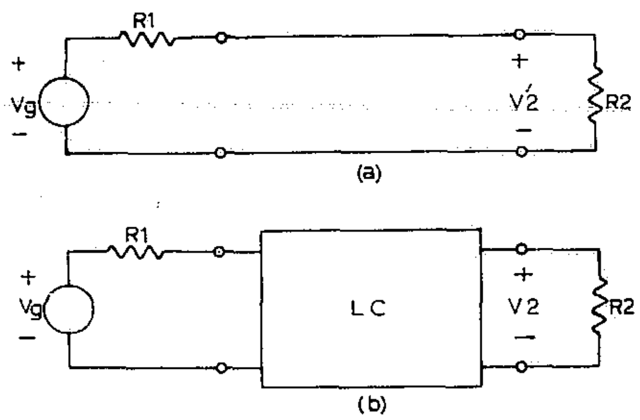


Figure 6.6 Network used to define insertion loss. (a) Circuit without LC network. (b) Circuit with LC network inserted.

The voltage V′2 appearing across the load resistance R2 of the circuit shown in figure 6.6-a is given by

$$V'2 = \frac{R2}{R1 + R2} Vg. \tag{6.39}$$

For the same circuit the power dissipated in the load resistance is

$$P_2' = \left|\frac{Vg}{R1 + R2}\right|^2 R2 \qquad (6.40)$$

while for the circuit shown in figure 6.6-b the power dissipated by the load resistance R2 is

$$P_2 = \frac{|V2|^2}{R2} \quad . \qquad (6.41)$$

The insertion power ratio is defined as the ratio of P'2 to P2

$$\frac{P_2'}{P_2} = e^{2\alpha} = \left(\frac{R2}{R1 + R2}\right)^2 \left|\frac{Vg}{V2}\right|^2 \quad . \qquad (6.42)$$

Now both sides of equation (6.25) are square and equation (6.42) is used to substitute for the ratio $|Vg/V2|^2$ in order that $|s_{21}|^2$ may be found in terms of the insertion power ratio, $e^{2\alpha}$, as

$$|s_{21}|^2 = \frac{4R1\ R2}{(R1 + R2)^2} e^{-2\alpha} \qquad (6.43)$$

6.3 Darlington's synthesis procedure. The basic steps in the Darlington procedure are:

(1) Given any of the specifications $|V2/Vg|^2$, $|V2/Ig|^2$, or $e^{2\alpha}$ use the appropriate relation (equation (6.25), (6.26), or (6.42)) and the desired values of R1 and R2 to find $|s_{12}|^2$. Before equation (6.25), and (6.26) may be used for this purpose they must be rewritten as

$$|s_{21}(j\omega)|^2 = \frac{4R1}{R2} \left|\frac{V2}{Vg}\right| \qquad (6.44)$$

$$|s_{21}(j\omega)|^2 = \frac{4}{R1R2} \left|\frac{V2}{Ig}\right|^2 . \qquad (6.45)$$

For a realizable network $s_{12}(j\omega)^2 \leq 1$.

(2) Using equation (6.38) find $|s_{11}(j\omega)|^2$.

(3) Using $|s_{11}(j\omega)|^2 = [s_{11}(s)\ s_{11}(-s)]\big|_{s\,=\,j\omega}$ find $s_{11}(s)$.

(4) Using $s_{11}(s)$ and equation (6.19) find the driving point impedance Z1(s) for the desired value of R1. Equation (6.19) can be rewritten as

$$Z1(s) = \frac{1 - s_{11}(s)}{1 + s_{11}(s)} \tag{6.46}$$

for this purpose.

(5) Synthesize a network with a driving point impedance Z1(s). The network may or may not be realizable as a ladder network. A ladder realization is possible if all the zeros of $s_{11}(s)$ are at the origin or at infinity. Then the desired network can be realized as a ladder network. This is the case for the low-pass response approximations discussed in Chapter Four.

Example: Synthesize a low-pass filter with a maximally flat magnitude response where the ratio $|V2(j\omega)/Vg(j\omega)|$ is to be given by

$$\left|\frac{V2(j\omega)}{Vg(j\omega)}\right| = \frac{1}{\sqrt{1 + \omega^6}} \tag{6.47}$$

and it is desired that R1 = R2 = 1 ohm. To insure that the function Z1(s), which is found in step 5, will be a positive real function, the magnitude of $|s_{12}(j\omega)|^2$ must be less than or equal to one. In this case the factor 4R1/R2 in equation (6.44) is equal to 4. Therefore, the specification must be scaled to

$$\left|\frac{V2(j\omega)}{Vg(j\omega)}\right| = \frac{1/4}{\sqrt{1 + \omega^6}} \quad . \qquad (6.48)$$

Using equation (6.44), $|s_{12}(j\omega)|^2$ is then found as

$$|s_{12}(j\omega)|^2 = \frac{1}{\sqrt{1 + \omega^6}} \qquad (6.49)$$

completing step 1.

Then $|s_{11}(j\omega)|^2$ is found as

$$|s_{11}(j\omega)|^2 = 1 - \frac{1}{1 + \omega^6} = \frac{\omega^6}{1 + \omega^6} . \qquad (6.50)$$

$s_{11}(s)\ s_{11}(-s)$ is found by replacing ω by s/j as

$$s_{11}(s)\ s_{11}(-s) = \frac{-\ s^6}{1 - s^6} \quad . \qquad (6.51)$$

The poles of this function in the left hand portion of the s plane can be found by using table 5.1. These poles are associated with $s_{11}(s)$ and $s_{11}(s)$ is found as

$$s_{11}(s) = \frac{s^3}{(s^2 + s + 1)(s + 1)} = \frac{s^3}{s^3 + 2s^2 + 2s + 1} \qquad (6.52)$$

where $s_{11}(s)$ has three zeros located at the origin. Now using equation (6.46) Z1 is found as

$$Z1(s) = \frac{2s^2 + 2s + 1}{2s^3 + 2s^2 + 2s + 1} \quad . \qquad (6.53)$$

As the zeros of $s_{11}(s)$ are all at the origin, $Y1 = 1/Z1$ can be realized as a continued fraction expansion of equation (6.53), as follows:

$$\begin{array}{llll}
2s^2 + 2s + 1 & \overline{)\,2s^3 + 2s^2 + 2s + 1} \;(\; s & & \leftarrow Y \\
 & \underline{2s^3 + 2s^2 + \ s} & & \\
 & \qquad s + 1 \ \overline{)\,2s^2 + 2s + 1} \;(\; 2s & & \leftarrow Z \\
 & \qquad \qquad \underline{2s^2 + 2s} & & \\
 & \qquad \qquad \qquad 1 \ \overline{)\,s + 1} \;(\; s & & \leftarrow Y \\
 & \qquad \qquad \qquad \quad \underline{s} & & \\
 & \qquad \qquad \qquad \qquad 1 \ \overline{)\,1} \;(\; 1 & & \leftarrow R2 \\
 & \qquad \qquad \qquad \qquad \quad \underline{1} & &
\end{array}$$

The resulting network is shown in figure 6.7.

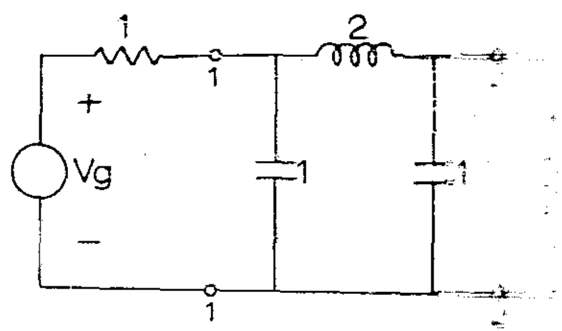
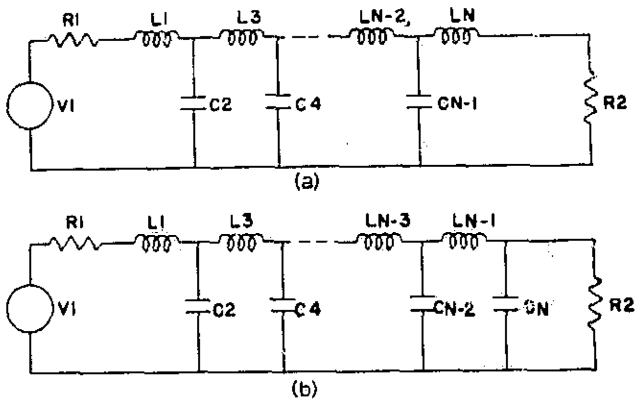


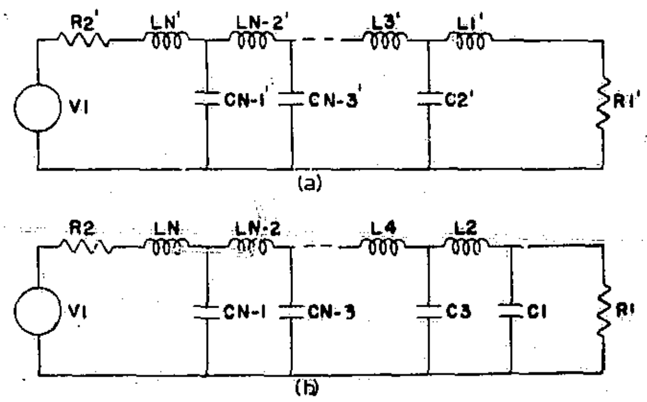
Figure 6.7 Filter synthesized to meet specifications of equation (6.47).

<u>6.4 Tabulated element values of</u> Butterworth and Chebyshev <u>low-pass filters.</u> Explicit formulas for the element values of low-pass filters with Butterworth and Chebyshev response have been found. Weinberg discusses the use of such formulas to find the element values of low-pass filters with Butterworth and Chebyshev response in his book, Modern Network Synthesis. The program found in Appendix D uses these explicit formulas to generate table 6.1 and table 6.2. Table 6.1 and 6.2 are a convenient means of finding the element values of Butterworth and Chebyshev low-pass filters. The general configuration of a Butterworth filter which corresponds to table 6.1 is shown in figure 6.8. Note that there are two configurations shown. Configuration (a) is used if the desired Butterworth filter is an odd order filter (N odd) and configuration (b) is used if an even order filter (N even) is desired.

Configurations for Low-Pass Ladder Networks



Resistance-terminated Butterworth filter. (a) N odd. (b) N even.



Resistance-terminated Chebyshev filter. (a) N odd. (b) N even.

N	L1 OR C1'	C2 OR L2'	L3 OR C3'	C4 OR L4'	L5 OR C5'	C6 OR L6'	L7 OR C7'	C8 OR L8'	L9 OR C9'	C10 OR L10'
R1 = 1 R2 = 1										
1	2.0000									
2	1.4142	1.4142								
3	1.0000	2.0000	1.0000							
4	0.7654	1.8478	1.8478	0.7654						
5	0.6180	1.6180	2.0000	1.6180	0.6180					
6	0.5176	1.4142	1.9319	1.9319	1.4142	0.5176				
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450			
8	0.3902	1.1111	1.6629	1.9616	1.9616	1.6629	1.1111	0.3902		
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	
10	0.3129	0.9080	1.4142	1.7820	1.9754	1.9754	1.7820	1.4142	0.9080	0.3129
R1 = 1 R2 = 2										
1	3.0000									
2	3.3461	0.4483								
3	3.2612	0.7789	1.1811							
4	3.1868	0.8826	2.4524	0.2175						
5	3.1331	0.9237	3.0510	0.4955	0.6857					
6	3.0938	0.9423	3.3687	0.6542	1.6531	0.1412				
7	3.0640	0.9513	3.5532	0.7512	2.2726	0.3536	0.4799			
8	3.0408	0.9558	3.6678	0.8139	2.6863	0.5003	1.2341	0.1042		
9	3.0223	0.9579	3.7426	0.8565	2.9734	0.6046	1.7846	0.2735	0.3685	
10	3.0072	0.9588	3.7934	0.8864	3.1795	0.6808	2.1943	0.4021	0.9818	0.0825
R1 = 1 R2 = 3										
1	4.0000									
2	4.8284	0.2761								
3	4.8473	0.4934	1.6725							
4	4.8105	0.5676	3.5233	0.1386						
5	4.7743	0.5997	4.4239	0.3186	0.9912					
6	4.7446	0.6156	4.9155	0.4233	2.4042	0.0913				
7	4.7206	0.6244	5.2085	0.4882	3.3200	0.2294	0.7006			
8	4.7012	0.6295	5.3950	0.5308	3.9376	0.3258	1.8075	0.0678		
9	4.6853	0.6326	5.5200	0.5601	4.3702	0.3948	2.6209	0.1785	0.5410	
10	4.6720	0.6346	5.6071	0.5809	4.6833	0.4454	3.2293	0.2630	1.4445	0.0540

R1 = 1 R2 = 4

1	5.0000									
2	6.2741	0.1992								
3	6.3870	0.3608	2.1699							
4	6.3840	0.4180	4.6024	0.1018						
5	6.3636	0.4435	5.8036	0.2350	1.2992					
6	6.3425	0.4567	6.4673	0.3130	3.1601	0.0675				
7	6.3238	0.4641	6.8671	0.3618	4.3727	0.1700	0.9225			
8	6.3078	0.4687	7.1244	0.3940	5.1943	0.2417	2.3838	0.0503		
9	6.2941	0.4716	7.2984	0.4162	5.7720	0.2932	3.4607	0.1325	0.7143	
10	6.2825	0.4735	7.4209	0.4321	6.1916	0.3312	4.2683	0.1955	1.9090	0.0401

R1 = 1 R2 = 6

1	7.0000									
2	9.1330	0.1277								
3	9.4254	0.2344	3.1681							
4	9.4868	0.2735	6.7653	0.0665						
5	9.4965	0.2914	8.5666	0.1541	1.9165					
6	9.4918	0.3009	9.5731	0.2058	4.6745	0.0443				
7	9.4830	0.3064	10.1856	0.2384	6.4812	0.1120	1.3672			
8	9.4734	0.3099	10.5832	0.2600	7.7107	0.1594	3.5381	0.0332		
9	9.4643	0.3122	10.8547	0.2750	8.5784	0.1937	5.1424	0.0875	1.0614	
10	9.4558	0.3138	11.0474	0.2858	9.2107	0.2189	6.3484	0.1292	2.8392	0.0265

R1 = 1 R2 = 8

1	9.0000									
2	11.9764	0.0939								
3	12.4442	0.1735	4.1674							
4	12.5685	0.2032	8.9296	0.0493						
5	12.6076	0.2169	11.3305	0.1146	2.5343					
6	12.6190	0.2243	12.6794	0.1533	6.1898	0.0330				
7	12.6199	0.2287	13.5040	0.1778	8.5907	0.0835	1.8121			
8	12.6166	0.2314	14.0417	0.1940	10.2279	0.1190	4.6929	0.0248		
9	12.6117	0.2333	14.4102	0.2053	11.3856	0.1446	6.8248	0.0653	1.4086	
10	12.6064	0.2346	14.6730	0.2135	12.2305	0.1635	8.4293	0.0965	3.7698	0.0198

# ELEMENT VALUES (IN HENRYS AND FARADS) FOR A NORMALIZED CHEBYSHEV FILTER
RIPPLE = 1/10 DB

N	C1 OR L1'	L2 OR C2'	C3 OR L3'	L4 OR C4'	C5 OR L5'	L6 OR C6'	C7 OR L7'	L8 OR C8'	C9 OR L9'	L10 OR C10'
	R1 = 1 R2 = 1									
1	0.3052									
2										
3	1.0316	1.1474	1.0316							
4										
5	1.1468	1.3712	1.9750	1.3712	1.1468					
6										
7	1.1812	1.4228	2.0967	1.5734	2.0967	1.4228	1.1812			
8										
9	1.1957	1.4426	2.1346	1.6167	2.2054	1.6167	2.1346	1.4426	1.1957	
10										
	R1 = 1 R2 = 1/2									
1	0.4579									
2	1.5715	0.2880								
3	2.2746	0.6035	1.3341							
4	2.3545	0.7973	2.6600	0.3626						
5	2.6921	0.8042	3.3882	0.6853	1.4572					
6	2.5561	0.8962	3.3962	0.8761	2.8071	0.3785				
7	2.8260	0.8560	3.7594	0.8685	3.5246	0.7050	1.4932			
8	2.6324	0.9285	3.5762	0.9619	3.5095	0.8950	2.8547	0.3843		
9	2.8839	0.8762	3.8788	0.9121	3.8660	0.8836	3.5703	0.7127	1.5084	
10	2.6688	0.9429	3.6461	0.9887	3.6707	0.9765	3.5472	0.9027	2.8761	0.3870
	R1 = 1 R2 = 1/3									
1	0.6105									
2	2.3497	0.1712								
3	3.4253	0.3914	1.8216							
4	3.7120	0.5161	3.8671	0.2172						
5	4.1422	0.5389	4.9341	0.4442	1.9966					
6	4.0823	0.5906	5.1046	0.5639	4.0826	0.2272				
7	4.3777	0.5785	5.5512	0.5786	5.1294	0.4572	2.0481			
8	4.2247	0.6155	5.4212	0.6284	5.2644	0.5760	4.1536	0.2308		
9	4.4804	0.5942	5.7535	0.6119	5.6978	0.5885	5.1967	0.4623	2.0697	
10	4.2930	0.6266	5.5455	0.6490	5.5485	0.6374	5.3199	0.5810	4.1857	0.2325

R1 = 1 R2 = 1/4

1	0.7631									
2	3.0912	0.1220								
3	4.5446	0.2885	2.3272							
4	5.0046	0.3815	5.0696	0.1559						
5	5.5547	0.4037	6.4880	0.3281	2.5[illegible]7					
6	5.5377	0.4403	6.7916	0.4167	5.3580	0.1632				
7	5.8904	0.4353	7.3489	0.4330	6.7472	0.3379	2.6256			
8	5.7441	0.4601	7.2417	0.4675	7.0032	0.4258	5.4537	0.1659		
9	6.0374	0.4478	7.6338	0.4594	7.5404	0.4404	6.8374	0.3419	2.6542	
10	5.8435	0.4690	7.4193	0.4839	7.4072	0.4742	7.0778	0.4295	5.4970	0.1672

R1 = 1 R2 = 1/6

1	1.0683									
2	4.5464	0.0774								
3	6.7530	0.1889	3.3506							
4	7.5394	0.2506	7.4716	0.0997						
5	8.3428	0.2684	9.5997	0.2153	3.6944					
6	8.3932	0.2916	10.1522	0.2739	7.9091	0.1046				
7	8.8775	0.2906	10.9465	0.2877	9.9902	0.2219	3.7961			
8	8.7261	0.3055	10.8673	0.3094	10.4711	0.2800	8.0550	0.1064		
9	9.1124	0.2995	11.3965	0.3063	11.2317	0.2927	10.1275	0.2246	3.8389	
10	8.8869	0.3119	11.1511	0.3209	11.1133	0.3138	10.5850	0.2826	8.1214	0.1072

R1 = 1 R2 = 1/8

1	1.3736									
2	5.9892	0.0567								
3	8.9466	0.1403	4.3787							
4	10.0512	0.1865	9.8722	0.0733						
5	11.1128	0.2003	12.7123	0.1602	4.8368					
6	11.2235	0.2179	13.5071	0.2041	10.4600	0.0770				
7	11.8455	0.2179	14.5445	0.2153	13.2359	0.1652	4.9726			
8	11.6822	0.2287	14.4866	0.2312	13.9349	0.2087	10.6567	0.0783		
9	12.1681	0.2248	15.1596	0.2297	14.9251	0.2191	13.4207	0.1672	5.0298	
10	11.9040	0.2336	14.8765	0.2401	14.8149	0.2346	14.0887	0.2106	10.7462	0.0790

ELEMENT VALUES (IN HENRYS AND FARADS) FOR A
NORMALIZED CHEBYSHEV FILTER
RIPPLE = 1/4 DB

N	C1 OR L1'	L2 OR C2'	C3 OR L3'	L4 OR C4'	C5 OR L5'	L6 OR C6'	C7 OR L7'	L8 OR C8'	C9 OR L9'	L10 OR C10'
R1 = 1 R2 = 1										
1	0.4868									
2										
3	1.3034	1.1463	1.3034							
4										
5	1.4144	1.3180	2.2414	1.3180	1.4144					
6										
7	1.4468	1.3560	2.3476	1.4689	2.3476	1.3560	1.4468			
8										
9	1.4604	1.3704	2.3800	1.5000	2.4414	1.5000	2.3800	1.3704	1.4604	
10										
R1 = 1 R2 = 1/2										
1	0.7303									
2	1.7288	0.4104								
3	2.5965	0.6465	1.7402							
4	2.2884	0.9039	2.7832	0.4930						
5	2.9282	0.7984	3.7341	0.7177	1.8648					
6	2.4162	0.9771	3.2941	0.9837	2.9094	0.5100				
7	3.0294	0.8341	4.0204	0.8546	3.8585	0.7340	1.9007			
8	2.4631	1.0000	3.4072	1.0463	3.3925	1.0015	2.9490	0.5161		
9	3.0724	0.8478	4.1088	0.8843	4.1199	0.8670	3.8989	0.7404	1.9157	
10	2.4852	1.0100	3.4501	1.0651	3.4927	1.0606	3.4235	1.0085	2.9666	0.5190
R1 = 1 R2 = 1/3										
1	0.9737									
2	2.6983	0.2337								
3	3.8716	0.4279	2.3508							
4	3.7640	0.5729	4.1610	0.2812						
5	4.4396	0.5433	5.3965	0.4742	2.5236					
6	4.0210	0.6299	5.1006	0.6179	4.3438	0.2910				
7	4.6162	0.5713	5.8745	0.5783	5.5708	0.4851	2.5735			
8	4.1167	0.6480	5.3173	0.6668	5.2361	0.6285	4.4023	0.2945		
9	4.6917	0.5822	6.0240	0.6017	6.0086	0.5864	5.6288	0.4893	2.5944	
10	4.1620	0.6561	5.4002	0.6817	5.4294	0.6749	5.2808	0.6328	4.4285	0.2962

R1 = 1 R2 = 1/4

1	1.2171									
2	3.5907	0.1647								
3	5.1234	0.3182	2.9867							
4	5.1282	0.4214	5.4989	0.1990						
5	5.9266	0.4097	7.0768	0.3529	3.2121					
6	5.5080	0.4669	6.8423	0.4540	5.7437	0.2062				
7	6.1786	0.4322	7.7470	0.4354	7.3060	0.3611	3.2774			
8	5.6503	0.4815	7.1583	0.4930	7.0209	0.4618	5.8228	0.2087		
9	6.2867	0.4410	7.9582	0.4543	7.9206	0.4415	7.3830	0.3643	3.3046	
10	5.7179	0.4880	7.2798	0.5050	7.3032	0.4989	7.0810	0.4650	5.8583	0.2099

R1 = 1 R2 = 1/6

1	1.7039									
2	5.3277	0.1036								
3	7.6009	0.2097	4.2769							
4	7.7855	0.2759	8.1555	0.1260						
5	8.8721	0.2738	10.4483	0.2330	4.6108					
6	8.4067	0.3080	10.2931	0.2973	8.5279	0.1307				
7	9.2748	0.2899	11.5036	0.2909	10.7914	0.2385	4.7077			
8	8.6408	0.3184	10.8059	0.3248	10.5620	0.3025	8.6491	0.1324		
9	9.4481	0.2962	11.8387	0.3043	11.7600	0.2950	10.9079	0.2408	4.7482	
10	8.7523	0.3230	11.0038	0.3333	11.0206	0.3287	10.6538	0.3046	8.7036	0.1332

R1 = 1 R2 = 1/8

1	2.1908									
2	7.0446	0.0755								
3	10.0648	0.1562	5.5746							
4	10.4126	0.2051	10.8046	0.0922						
5	11.8024	0.2054	13.8236	0.1738	6.0183					
6	11.2734	0.2298	13.7316	0.2211	11.3063	0.0958				
7	12.3557	0.2179	15.2643	0.2183	14.2825	0.1780	6.1472			
8	11.5987	0.2378	14.4405	0.2423	14.0925	0.2250	11.4700	0.0970		
9	12.5942	0.2228	15.7237	0.2287	15.6054	0.2214	14.4391	0.1797	6.2011	
10	11.7539	0.2414	14.7148	0.2489	14.7268	0.2452	14.2166	0.2267	11.5438	0.0976

ELEMENT VALUES (IN HENRYS AND FARADS) FOR A
NORMALIZED CHEBYSHEV FILTER
RIPPLE = 1/2 DB

N	C1 OR L1'	L2 OR C2'	C3 OR L3'	L4 OR C4'	C5 OR L5'	L6 OR C6'	C7 OR L7'	L8 OR C8'	C9 OR L9'	L10 OR C10'
R1 = 1 R2 = 1										
1	0.6986									
2										
3	1.5963	1.0967	1.5963							
4										
5	1.7058	1.2296	2.5408	1.2296	1.7058					
6										
7	1.7373	1.2582	2.6383	1.3443	2.6383	1.2582	1.7373			
8										
9	1.7504	1.2690	2.6678	1.3673	2.7239	1.3673	2.6678	1.2690	1.7504	
10										
R1 = 1 R2 = 1/2										
1	1.0479									
2	1.5132	0.6538								
3	2.9431	0.6503	2.1903							
4	1.8158	1.1328	2.4881	0.7732						
5	3.2228	0.7645	4.1228	0.7116	2.3197					
6	1.8786	1.1884	2.7589	1.2403	2.5976	0.7976				
7	3.3055	0.7899	4.3575	0.8132	4.2419	0.7252	2.3566			
8	1.9012	1.2053	2.8152	1.2864	2.8479	1.2628	2.6310	0.8063		
9	3.3403	0.7995	4.4283	0.8341	4.4546	0.8235	4.2795	0.7304	2.3719	
10	1.9117	1.2127	2.8366	1.2999	2.8964	1.3054	2.8744	1.2714	2.6456	0.8104
R1 = 1 R2 = 1/3										
1	1.3972									
2	2.8282	0.3109								
3	4.3200	0.4405	2.9371							
4	3.6172	0.6399	4.1985	0.3620						
5	4.7896	0.5293	5.8898	0.4809	3.1130					
6	3.7922	0.6851	4.8770	0.6852	4.3536	0.3722				
7	4.9305	0.5495	6.2770	0.5603	6.0535	0.4901	3.1632			
8	3.8560	0.6990	5.0230	0.7235	4.9937	0.6953	4.4022	0.3759		
9	4.9901	0.5572	6.3947	0.5770	6.4061	0.5671	6.1064	0.4936	3.1841	
10	3.8860	0.7051	5.0780	0.7348	5.1229	0.7314	5.0307	0.6993	4.4238	0.3776

R1 = 1 R2 = 1/4

1	1.7466									
2	3.8432	0.2145								
3	5.6859	0.3308	3.7139							
4	5.0293	0.4646	5.6377	0.2504						
5	6.3476	0.4023	7.6862	0.3613	3.9411					
6	5.2985	0.5009	6.6492	0.4963	5.8461	0.2576				
7	6.5476	0.4187	8.2283	0.4252	7.8993	0.3683	4.0061			
8	5.3972	0.5122	6.8706	0.5272	6.8027	0.5035	5.9120	0.2602		
9	6.6323	0.4250	8.3941	0.4388	8.3940	0.4303	7.9689	0.3710	4.0331	
10	5.4437	0.5172	6.9544	0.5364	6.9993	0.5328	6.8522	0.5064	5.9413	0.2614

R1 = 1 R2 = 1/6

1	2.4452									
2	5.7836	0.1330								
3	8.4006	0.2200	5.2916							
4	7.7362	0.3015	8.4556	0.1560						
5	9.4472	0.2708	11.3001	0.2406	5.6251					
6	8.1894	0.3274	10.1144	0.3220	8.7743	0.1607				
7	9.7660	0.2827	12.1544	0.2861	11.6163	0.2453	5.7207			
8	8.3564	0.3355	10.4840	0.3440	10.3456	0.3267	8.8758	0.1624		
9	9.9014	0.2872	12.4172	0.2959	12.3969	0.2896	11.7206	0.2472	5.7605	
10	8.4353	0.3390	10.6243	0.3506	10.6746	0.3476	10.4216	0.3286	8.9212	0.1632

R1 = 1 R2 = 1/8

1	3.1438									
2	7.6905	0.0965								
3	11.1053	0.1646	6.8796							
4	10.3991	0.2234	11.2532	0.1135						
5	12.5367	0.2039	14.9223	0.1801	7.3211					
6	11.0346	0.2434	13.5532	0.2386	11.6839	0.1170				
7	12.9745	0.2131	16.0900	0.2154	15.3437	0.1837	7.4478			
8	11.2694	0.2497	14.0704	0.2556	13.8641	0.2422	11.8216	0.1182		
9	13.1608	0.2167	16.4502	0.2230	16.4112	0.2180	15.4833	0.1852	7.5006	
10	11.3804	0.2524	14.2670	0.2607	14.3247	0.2583	13.9670	0.2436	11.8832	0.1188

ELEMENT VALUES (IN HENRYS AND FARADS) FOR A
NORMALIZED CHEBYSHEV FILTER
RIPPLE = 1 DB

N	C1 OR L1'	L2 OR C2'	C3 OR L3'	L4 OR C4'	C5 OR L5'	L6 OR C6'	C7 OR L7'	L8 OR C8'	C9 OR L9'	L10 OR C10'
R1 = 1 R2 = 1										
1	1.0177									
2										
3	2.0236	0.9941	2.0236							
4	.									
5	2.1349	1.0911	3.0009	1.0911	2.1349					
6										
7	2.1666	1.1115	3.0936	1.1735	3.0936	1.1115	2.1666			
8										
9	2.1797	1.1192	3.1214	1.1897	3.1746	1.1897	3.1214	1.1192	2.1797	
10										
R1 = 1 R2 = 1/2										
1	1.5265									
2										
3	3.4774	0.6153	2.8540							
4										
5	3.7211	0.6949	4.7448	0.6650	2.9936					
6										
7	3.7916	0.7118	4.9425	0.7348	4.8636	0.6757	3.0331			
8										
9	3.8210	0.7182	5.0013	0.7485	5.0412	0.7429	4.9004	0.6797	3.0495	
10										
R1 = 1 R2 = 1/3										
1	2.0354									
2	2.5721	0.4702								
3	4.9893	0.4286	3.8075							
4	3.0355	0.7929	3.7589	0.5347						
5	5.3830	0.4915	6.6673	0.4622	3.9944					
6	3.1307	0.8287	4.1451	0.8467	3.8812	0.5475				
7	5.4978	0.5050	6.9839	0.5177	6.8280	0.4696	4.0473			
8	3.1647	0.8395	4.2237	0.8764	4.2404	0.8580	3.9186	0.5520		
9	5.5459	0.5101	7.0783	0.5288	7.1141	0.5232	6.8785	0.4724	4.0693	
10	3.1806	0.8442	4.2532	0.8851	4.3088	0.8857	4.2691	0.8623	3.9349	0.5541

R1 = 1 R2 = 1/4

1	2.5442									
2	3.7779	0.3001								
3	6.5048	0.3264	4.7927							
4	4.5699	0.5428	5.3680	0.3406						
5	7.0522	0.3776	8.6301	0.3520	5.0313					
6	4.7366	0.5716	6.0240	0.5764	5.5353	0.3486				
7	7.2126	0.3888	9.0689	0.3973	8.8368	0.3577	5.0989			
8	4.7966	0.5803	6.1592	0.6005	6.1501	0.5836	5.5869	0.3515		
9	7.2800	0.3930	9.2001	0.4064	9.2344	0.4015	8.9024	0.3598	5.1270	
10	4.8247	0.5841	6.2098	0.6076	6.2689	0.6063	6.1890	0.5864	5.6096	0.3528

R1 = 1 R2 = 1/6

1	3.5619									
2	5.8921	0.1796								
3	9.5324	0.2201	6.7917							
4	7.2808	0.3429	8.2956	0.2044						
5	10.3903	0.2570	12.5900	0.2375	7.1380					
6	7.5794	0.3635	9.4434	0.3635	8.5550	0.2093				
7	10.6432	0.2651	13.2768	0.2702	12.8928	0.2413	7.2362			
8	7.6874	0.3697	9.6835	0.3808	9.6350	0.3680	8.6359	0.2110		
9	10.7495	0.2682	13.4827	0.2769	13.5163	0.2731	12.9898	0.2428	7.2770	
10	7.7380	0.3725	9.7735	0.3858	9.8469	0.3843	9.6953	0.3698	8.6717	0.2119

R1 = 1 R2 = 1/8

1	4.5796									
2	7.9318	0.1286								
3	12.5563	0.1657	8.8038							
4	9.9024	0.2517	11.1584	0.1467						
5	13.7259	0.1945	16.5650	0.1789	9.2596					
6	10.3304	0.2677	12.7878	0.2668	11.5115	0.1503				
7	14.0719	0.2009	17.5013	0.2045	16.9660	0.1819	9.3890			
8	10.4856	0.2725	13.1313	0.2802	13.0465	0.2701	11.6220	0.1516		
9	14.2174	0.2033	17.7827	0.2097	17.8168	0.2066	17.0949	0.1830	9.4427	
10	10.5585	0.2746	13.2602	0.2842	13.3503	0.2828	13.1287	0.2715	11.6709	0.1522

ELEMENT VALUES (IN HENRYS AND FARADS) FOR A
NORMALIZED CHEBYSHEV FILTER
RIPPLE = 2 DB

N	C1 OR L1'	L2 OR C2'	C3 OR L3'	L4 OR C4'	C5 OR L5'	L6 OR C6'	C7 OR L7'	L8 OR C8'	C9 OR L9'	L10 OR C10'
R1 = 1 R2 = 1										
1	1.5296									
2										
3	2.7107	0.8327	2.7107							
4										
5	2.8310	0.8985	3.7827	0.8985	2.8310					
6										
7	2.8650	0.9120	3.8774	0.9537	3.8774	0.9120	2.8650			
8										
9	2.8790	0.9171	3.9056	0.9643	3.9597	0.9643	3.9056	0.9171	2.8790	
10										
R1 = 1 R2 = 1/2										
1	2.7943									
2										
3	4.3975	0.5326	3.9184							
4										
5	4.6265	0.5835	5.8503	0.5698	4.0730					
6										
7	4.4917	0.5941	6.0293	0.6136	5.9720	0.6776	4.8742			
8										
9	4.7187	0.5989	6.0921	0.6270	6.1370	0.6195	6.0148	[illegible]	[illegible]	
10										
R1 = 1 R2 = 1/3										
1	3.0591									
2										
3	6.1471	0.3816	5.7161							
4										
5	6.4974	0.4219	8.0681	0.4076	5.4794					
6										
7	6.5974	0.4307	[illegible]	0.4425	[illegible]	0.4131	5.4893			
8										
9	6.6301	0.4334	8.4220	0.4493	8.4834	0.4467	8.7913	0.4152	5.5161	
10										

R1 = 1 R2 = 1/4

1	3.8239									
2										
3	7.9106	0.2955	6.5423							
4										
5	8.3859	0.3285	10.3300	0.3156	6.8118					
6										
7	8.5220	0.3354	10.7009	0.3443	10.5472	0.3199	6.8877			
8										
9	8.5787	0.3380	10.8103	0.3499	10.8798	0.3475	10.6142	0.3215	6.9191	
10										

R1 = 1 R2 = 1/6

1	5.3535									
2										
3	11.4484	0.2028	9.2241							
4										
5	12.1773	0.2268	14.8979	0.2166	9.6101					
6										
7	12.3869	0.2319	15.4674	0.2376	15.2113	0.2196	9.7188			
8										
9	12.4743	0.2338	15.6355	0.2417	15.7234	0.2398	15.3087	0.2207	9.7638	
10										

R1 = 1 R2 = 1/8

1	6.8830									
2	7.2895	0.1875								
3	14.9900	0.1541	11.9205							
4	8.5051	0.3093	9.9546	0.2088						
5	15.9745	0.1729	19.4874	0.1646	12.4250					
6	8.7527	0.3224	10.9256	0.3260	10.2196	0.2130				
7	16.2581	0.1769	20.2574	0.1811	19.8989	0.1669	12.5671			
8	8.8412	0.3263	11.1205	0.3368	11.1251	0.3295	10.3007	0.2144		
9	16.3765	0.1784	20.4848	0.1844	20.5920	0.1828	20.0271	0.1678	12.6259	
10	8.8824	0.3280	11.1932	0.3399	11.2954	0.3396	11.1858	0.3308	10.3363	0.2151

ELEMENT VALUES (IN HENRYS AND FARADS) FOR A
NORMALIZED CHEBYSHEV FILTER
RIPPLE = 3 DB

N	C1 OR L1'	L2 OR C2'	C3 OR L3'	L4 OR C4'	C5 OR L5'	L6 OR C6'	C7 OR L7'	L8 OR C8'	C9 OR L9'	L10 OR C10'
R1 = 1 R2 = 1										
1	1.9953									
2										
3	3.3487	0.7117	3.3487							
4										
5	3.4813	0.7619	4.5375	0.7619	3.4813					
6										
7	3.5185	0.7722	4.6390	0.8038	4.6390	0.7722	3.5185			
8										
9	3.5339	0.7760	4.6691	0.8118	4.7270	0.8118	4.6691	0.7760	3.5339	
10										
R1 = 1 R2 = 1/2										
1	2.9929									
2										
3	5.2910	0.4618	4.8991							
4										
5	5.5259	0.4993	6.9460	0.4917	5.0821					
6										
7	5.5922	0.5069	7.1256	0.5235	7.0869	0.4979	5.1335			
8										
9	5.6197	0.5093	7.1785	0.5296	7.2454	0.5282	7.1292	0.5002	5.1547	
10										
R1 = 1 R2 = 1/3										
1	3.9905									
2										
3	7.2903	0.3758	6.5207							
4										
5	7.6371	0.3652	9.4808	0.3571	6.7635					
6										
7	7.7352	0.3712	9.7463	0.3822	9.6687	0.3615	6.8315			
8										
9	7.7760	0.3734	9.8243	0.3870	9.9043	0.3856	9.7256	0.3632	6.8597	
10										

R1 = 1 R2 = 1/4

1	4.9881								
2									
3	9.3059	0.2625	8.1669						
4									
5	9.7676	0.2866	12.0571	0.2791	8.4724				
6									
7	9.8986	0.2915	12.4111	0.2998	12.2946	0.2826	8.5581		
8									
9	9.9530	0.2934	12.5151	0.3037	12.6097	0.3024	12.3669	0.2839	8.5935
10									

R1 = 1 R2 = 1/6

1	6.9834								
2									
3	13.3528	0.1821	11.4866						
4									
5	14.0481	0.1997	17.2549	0.1936	11.9209				
6									
7	14.2458	0.2033	17.7890	0.2088	17.5946	0.1961	12.0428		
8									
9	14.3279	0.2045	17.9459	0.2117	18.0712	0.2106	17.6984	0.1970	12.0932
10									

R1 = 1 R2 = 1/8

1	8.9787									
2	6.1219	0.2596								
3	17.4070	0.1392	14.8205							
4	6.9104	0.3884	8.2760	0.2861						
5	18.3377	0.1530	22.4760	0.1481	15.3856					
6	7.0661	0.4011	8.8887	0.4087	8.4796	0.2913				
7	18.6027	0.1559	23.1920	0.1600	22.9195	0.1499	15.5441			
8	7.1213	0.4048	9.0096	0.4190	9.0452	0.4128	8.5411	0.2931		
9	18.7129	0.1569	23.4023	0.1623	23.5592	0.1614	23.0554	0.1506	15.6097	
10	7.1470	0.4064	9.0546	0.4219	9.1496	0.4223	9.0917	0.4144	8.5679	0.2939

Figure 6.9 is used in conjunction with table 6.1 when the element values of a Chebyshev filter are to be found. The configuration of figure 6.8-a is used for n odd while the configuration of figure 6.8-b is used for n even. The table of element values for other types of filters may be found in books by Skwirzynski or Zverev as listed in the bibliography for this chapter.

Example: Use table 6.1 to find the element values of a 7th order Chebyshev filter where R1 = R2. The ripple in the passband is to be 3db or less. In this case the element values are found in table 6.1 under the heading, Chebyshev filter, ripple = 3db and R1 = R2. The element values are given as:

L1′ = 3.5185
C2′ = .7722
L3′ = 4.6390
C4′ = 0.8038
L5′ = 4.6390
C6′ = 0.7722
L7′ = 3.5185 .

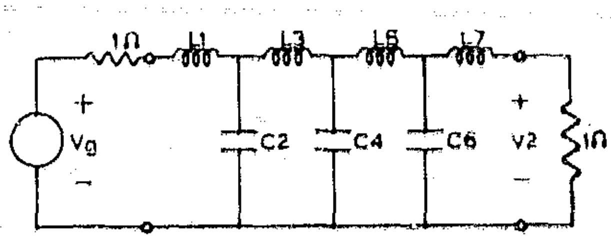


Figure 6.10 Chebyshev filter.

The filter is shown in figure 6.10.

In Chapter Five, using the approximate equation for the attenuation of a Chebyshev filter in the stopband, it was determined that a 7th order Chebyshev filter with a ripple of 3db in the passband should more than meet the specifications given in table 1.5. It was suspected, however, that the results given by equation 5.25 would be in error due to the approximation used in obtaining equation (5.25). NASAP-69 was used to compare the amplitude response of the network to that of the specifications. The amplitude response plotted by NASAP-69 and the tolerance contour specified in table 1.5 are shown in figure 6.11. As suspected, the amplitude response curve of the network enters the forbidden regions of the tolerance contour. The amplitude response curve |Vg/V2| for the network has been shifted down 6db so that it may be compared with the specification |V1/V2| of table 1.5. This is necessary since it is inconvenient to find |V1/V2| using NASAP-69 for the case of a double terminated network.

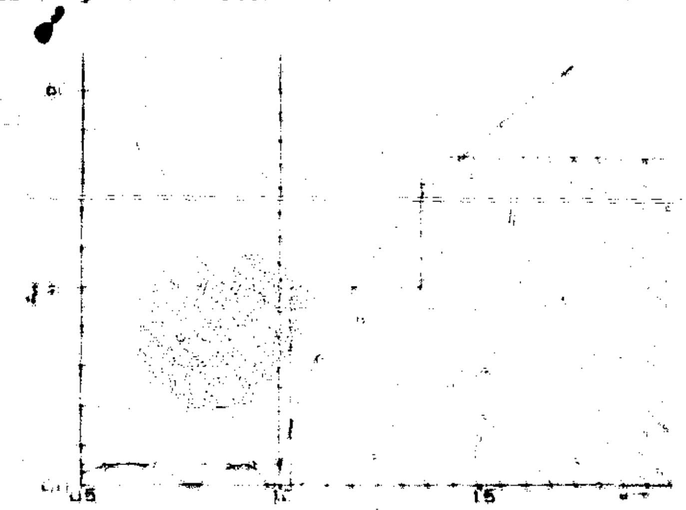


Figure 6.11 Amplitude response for filter of figure 6.10.

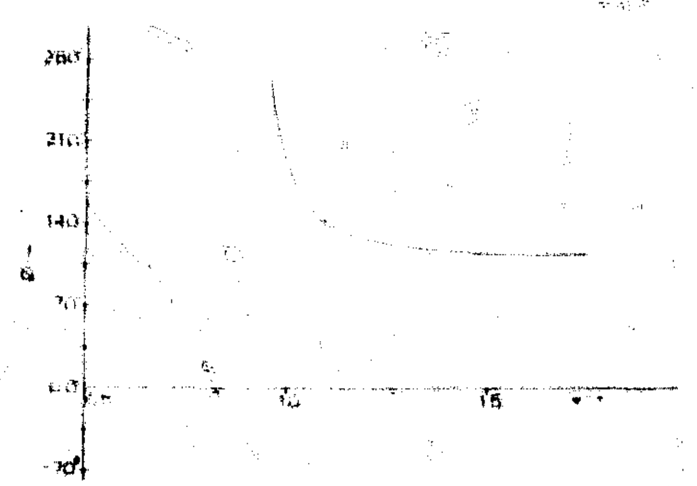


Figure 6.12 Phase response for filter of figure 6.10.

Geffe, P.R., Simplified Modern Filter Design, N.Y., Hayden, 1963.

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CHAPTER SEVEN

Active Filters

7.1 Introduction. This chapter will briefly cover the growing subject of active RC network synthesis which has recently been stimulated by the growth of integrated circuit technology. Using active RC networks and the techniques of this chapter, filters with characteristics unattainable with passive elements can be realized. For example, networks with their poles on the $j\omega$ axis are physically realizable with RC active networks, while they are realizable in theory only with LC elements due to losses associated with physical inductances. The problems arising from the use of lossy inductances are avoided in the synthesis procedures presented in this chapter as the use of inductors in the synthesis procedure is shunned. The elimination of inductors as network elements also reduces any size and weight problems as inductors, especially at low frequencies, are bulky items.

7.2 Active network elements. Controlled sources, operational amplifiers, negative resistances, impedance converters and impedance inverters are all active elements that may be used in an RC synthesis procedure. Four types of ideal controlled source models and their flowgraph representations are shown in figure 7.1. The ideal voltage controlled voltage source is described by the following relations:

$$\begin{bmatrix} I1 \\ V2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \mu & 0 \end{bmatrix} \begin{bmatrix} V1 \\ V2 \end{bmatrix} . \qquad (7.1)$$

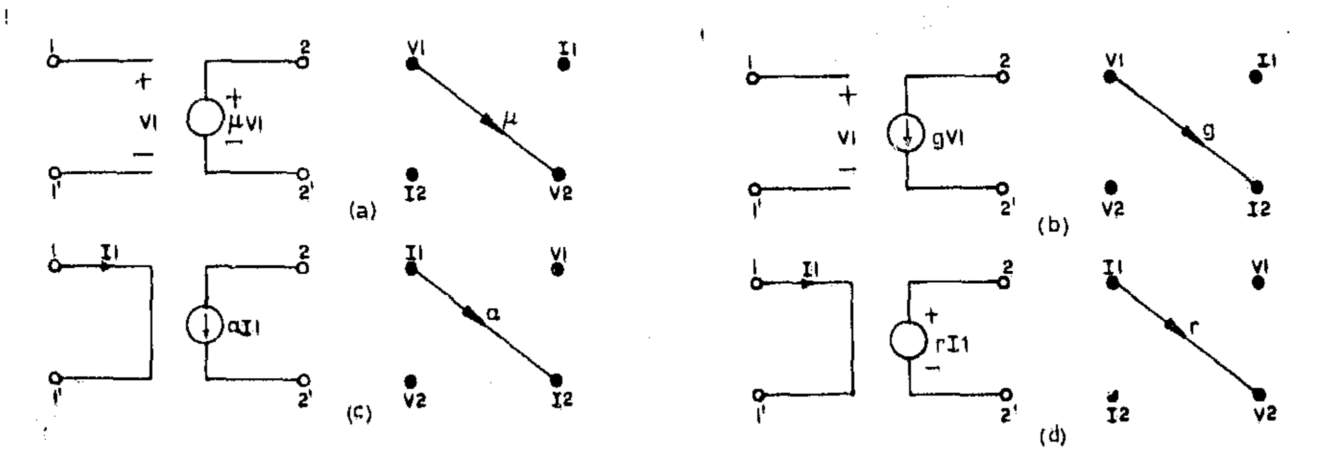

Figure 7.1 Four types of controlled source models.
(a) Voltage controlled voltage source.
(b) Voltage controlled current source.
(c) Current controlled current source.
(d) Current controlled voltage source.

Using equation (7.1) it can be seen that the voltage controlled voltage source has infinite input impedance and zero output impedance. Inspection of the flowgraph representation of the voltage controlled voltage source reveals that there exists a transmission path from the terminal voltage V1 to the terminal voltage V2. Note that there is no transmission from V2 or I2 to V1 or I2, thus the input port, port one, is isolated from the output port, port 2. The power gain is infinite as the input power is zero.

The ideal voltage controlled current source has the matrix relation given by

$$\begin{bmatrix} I1 \\ I2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ g & 0 \end{bmatrix} \begin{bmatrix} V1 \\ V2 \end{bmatrix} . \qquad (7.2)$$

The ideal voltage controlled current source has zero input admittance and zero output admittance. The coupling from the input port and output port is between V1 and I2. The input port is isolated from the output port. The power gain is infinite as the input power is zero when the voltage controlled current source is connected in a network.

The ideal current controlled current source is described by

$$\begin{bmatrix} V1 \\ I2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \alpha & 0 \end{bmatrix} \begin{bmatrix} I1 \\ V2 \end{bmatrix} \qquad (7.3)$$

and has zero input impedance and infinite output impedance. The forward current to current ratio is α. The input is isolated from the output and zero input power is required.

The ideal current controlled voltage source has the following input output relations:

$$\begin{bmatrix} V1 \\ V2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ r & 0 \end{bmatrix} \begin{bmatrix} I1 \\ I2 \end{bmatrix} . \qquad (7.4)$$

The ideal current controlled voltage source has zero input impedance and zero output impedance.

Models for nonideal controlled sources with their flowgraph representations are shown in figure 7.2. These devices have finite input and output impedances which results in finite power gain when the sources are incorporated in networks.

The operational amplifier is a voltage controlled voltage source which has infinite gain, infinite input impedance, and

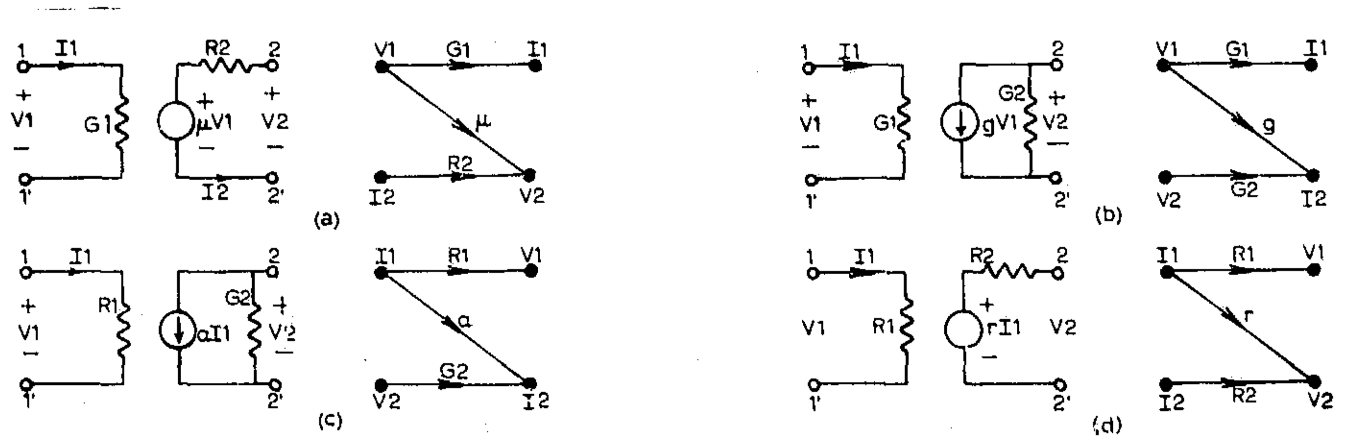

Figure 7.2 Nonideal controlled sources. (a) voltage controlled voltage source. (b) Voltage controlled current source. (c) Current controlled current source. (d) Current controlled voltage source.

zero-output impedance. Ideally the frequency range is from dc to infinite frequency. The ideal operational amplifier model is shown in figure 7.3-a and the symbol for the ideal operational amplifier is shown in figure 7.3-b. The output

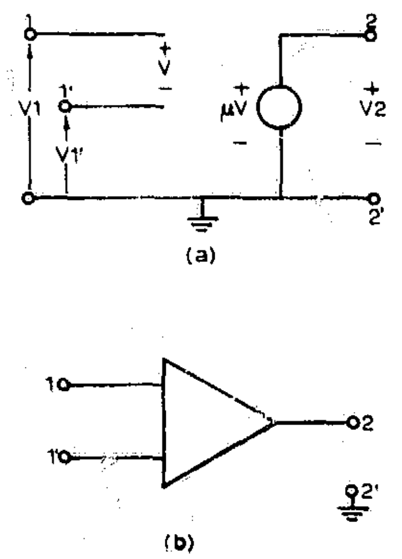

Figure 7.3 Ideal operational amplifier. (a) Model of ideal operational amplifier. (b) symbol for ideal operational amplifier.

voltage, V2, (reference figure 7.3-a) for the ideal operational amplifier is a function of the input voltages V1 and V1'. This relation is

$$V2 = A(V1' - V1) = -AV \qquad (7.4)$$

where the gain A goes to infinity, $A \to \infty$. The output voltage Vo should tend to zero as V goes to zero. Indeed, the ideal operational amplifier has a "zero offset" (V2 is zero when V is zero).

Two basic negative feedback circuits incorporating an ideal operational amplifier are shown in figure 7.4. The configuration of figure 7.4-a introduces a 180° phase shift between the input and output voltage while the configuration of figure 7.4-b does not.

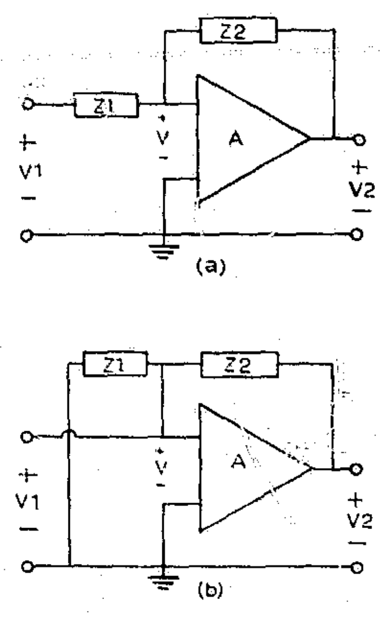

Figure 7.4 Basic negative feedback circuit. (a) Inverting. (b) Non-inverting.

The input impedance of the ideal operational amplifier is infinite; this fact can be used in the analysis of the circuit of figure 7.4-a when writing KCL as the current flowing into the operational amplifier must be zero. Hence

$$\frac{V1 - V}{Z1} + \frac{V2 - V}{Z2} + 0 = 0$$

or

$$\frac{V1 - V}{Z1} = \frac{V - V2}{Z2} \quad . \tag{7.5}$$

Now replace V by - V2/A and obtain

$$\frac{V1 + V2/A}{Z1} + \frac{V2 + V2/A}{Z2} = 0$$

or

$$\frac{V2}{V1} = - \frac{1}{1/A + (Z1/Z2)(1 + 1/A)} \ . \tag{7.6}$$

As A tends to infinity, equation (7.6) becomes

$$\frac{V2}{V1} = - \frac{Z2}{Z1} \ . \tag{7.7}$$

KCL for the currents at the input to the operational amplifier of figure 7.4-b is

$$\frac{-(V + V1)}{Z1} + \frac{V2 - (V + V1)}{Z2} + 0 = 0. \tag{7.8}$$

By replacing V by V2/A and letting A go to infinity the ratio V2/V1 for the circuit of figure 7.4-b may be found to be

$$\frac{V2}{V1} = \frac{Z1 + Z2}{Z1} \quad . \tag{7.9}$$

For an example of the usefulness of the circuit of figure 7.4-a, consider the case where Z1 = 1/Cs and Z2 = R. Then equation 7.7 becomes

$$\frac{V2}{V1} = - \frac{1}{RCS} = \frac{-1}{RC}\,\frac{1}{s} \ , \tag{7.10}$$

In the time domain equation (7.10) becomes

$$V2 = -\frac{1}{RC}\int V1\ dt. \qquad (7.11)$$

The circuit with Z1 = 1/Cs and Z2 = R is called an integrator as the output voltage is just a constant times the integral of the input voltage.

Negative resistances are also a possibility when using active network elements. For example, a negative resistance can be obtained on a portion of the current-voltage characteristic of a tunnel diode. A typical characteristic curve for a tunnel diode is shown in figure 7.5. The negative resistance portion is the section of the curve from a to b. On this

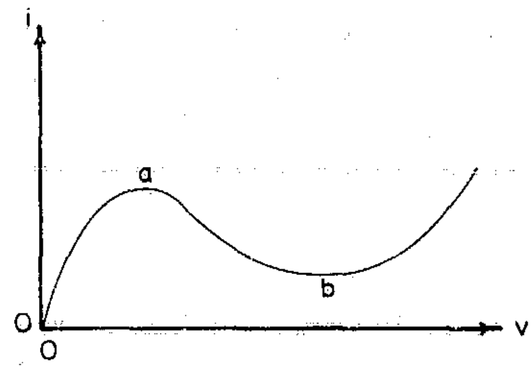

Figure 7.5 Typical characteristic curve of a tunnel diode.

portion of the curve the current-voltage relationship is

$$i(t) = v(t)/-r\ . \qquad (7.12)$$

Impedance converters provide a means of converting some impedance, ZL(s) to a new impedance, Z(s). For example, the negative impedance converter will transform an impedance Z(s) into its negative -Z(s). An ideal impedance converter is a network which will transform a terminating impedance ZL(s) at terminals (2,2′) to a new impedance Z(s) looking into terminals (1,1′) so that

$$Z(s) = K(s)ZL(s) \tag{7.13}$$

where K(s) is a real rational function.

The input impedance looking into terminals (1,1′) of the general network shown in figure 7.6 is

$$Z(s) = \frac{AZL(s) + B}{CZL(s) + D} \tag{7.14}$$

where A, B, C, and D are the parameters of the chain matrix defined in Chapter Three.

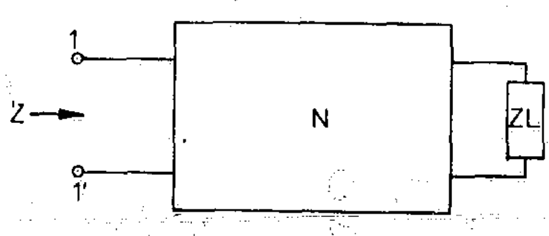

Figure 7.6 Terminated general two-port network.

If the general network with input impedance given by equation (7.14) is to be an impedance converter with an input impedance of the form given in equation (7.13) the following conditions must be met:

(1) $B = C = 0$

(2) $A \neq 0$

(3) $D \neq 0$.

If this is the case, then $K(s) = A/D$. These three conditions may be expressed in terms of the h parameters as:

(1) $h_{11} = h_{22} = 0$

(2) $h_{12}h_{21} = -A/D = -K(s)$.

The h-parameters are defined as

$$h_{11} = \left.\frac{V1}{I1}\right|_{V2 = 0} \qquad h_{12} = \left.\frac{V1}{V2}\right|_{I1 = 0}$$

$$h_{21} = \left.\frac{I2}{I1}\right|_{V2 = 0} \qquad h_{22} = \left.\frac{I2}{V2}\right|_{I1 = 0}$$

and

$$A = -\Delta h/h_{21} \qquad B = -h_{11}/h_{21}$$

$$C = -h_{22}/h_{21} \qquad D = -1/h_{21}$$

where

$$\Delta h = h_{11}h_{22} - h_{12}h_{21} \; .$$

The network represented by the h-parameter matrix

$$\begin{bmatrix} V1 \\ I2 \end{bmatrix} = \begin{bmatrix} 0 & -K_1 \\ -K_2 & 0 \end{bmatrix} \begin{bmatrix} I1 \\ V2 \end{bmatrix} \tag{7.15}$$

where K_1 and K_2 are real positive constants is called a voltage inversion type negative-impedance converter (VNIC). The h parameter matrix for a current inversion type negative-impedance converter (CNIC) is given by

$$\begin{bmatrix} V1 \\ I2 \end{bmatrix} = \begin{bmatrix} 0 & K_1 \\ K_2 & 0 \end{bmatrix} \begin{bmatrix} I1 \\ V2 \end{bmatrix} \tag{7.16}$$

where K_1 and K_2 are real positive constants. The parameters h_{11} and h_{22} are non-zero for an h-parameter matrix representing a nonideal negative-impedance converter, voltage or current.

If the network of figure 7.6 is described by the h parameter matrix of equation (7.15) and port 2 is terminated in an impedance ZL(s) the input impedance looking into port 1 is

$$Z(s) = A\ ZL(s) = -h_{12}h_{21} = -K_1 K_2\ ZL(s)\ . \qquad (7.17)$$

Hence the impedance ZL(s) at port 2 has been converted to its negative at port 1. An ideal transformer is a positive impedance converter.

Impedance converters provide a means of converting an impedance ZL(s) to a new impedance K(s) ZL(s). Impedance inverters are a means of obtaining the inverse of an impedance ZL(s). An ideal impedance converter is a network which will convert an impedance ZL(s) to an impedance Z(s) such that

$$Z(s) = \frac{G(s)}{ZL(s)}\ . \qquad (7.18)$$

Z(s) is the input impedance looking into terminals (1,1′) and G(s) is the inversion factor. The gyrator is an example of an impedance inverter.

The conditions for the general two-port network of figure 7.6 to be an impedance inverter are:

(1) $A = D = 0$

(2) $B \neq 0$

(3) $C \neq 0$.

If this is the case, the inversion factor G(s) of equation (7.18) is just B/C. These conditions for a general two-port network to be an impedance inverter can be stated in terms of the z parameters as

(1) $z_{11} = z_{22} = 0$

(2) $G(s) = -z_{12} z_{21}$

If z_{11} and z_{22} are zero for a network and $z_{12} = -r$ while $z_{21} = r$ the network is characterized by the following open-circuit impedance matrix:

$$[z] = \begin{bmatrix} 0 & -r \\ r & 0 \end{bmatrix} . \qquad (7.19)$$

Such a network is a positive impedance inverter since, if ZL(s) is connected to port 2 the input impedance seen at port 1 is

$$Z(s) = \frac{r^2}{ZL(s)} . \qquad (7.20)$$

The symbolic representation of such a network is shown in figure 7.7-a. The network is known as an ideal gyrator. The nonideal gyrator of figure 7.7-b is characterized by

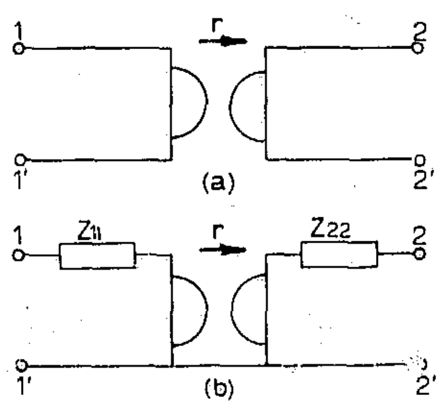

Figure 7.7 Gyrator. (a) Ideal gyrator (b) nonideal gyrator.

the following open-circuit impedance matrix:

$$[Z] = \begin{bmatrix} Z_{11} & -r \\ r & Z_{22} \end{bmatrix} . \qquad (7.21\text{-a})$$

An ideal negative impedance inverter has the following form of open-circuit impedance matrix:

$$[Z] = \begin{bmatrix} 0 & \pm R \\ \pm R & 0 \end{bmatrix} . \qquad (7.21\text{-b})$$

where R is a positive real number. If an impedance ZL(s) is connected to port 2 the input impedance seen at port 1 is

$$Z(s) = \frac{-R^2}{ZL(s)} . \qquad (7.22)$$

For example, if R = 1 and ZL(s) = 1/s, a one farad capacitor, then Z(s) is

$$Z(s) = -s.$$

This is a negative 1 henry inductor. Thus the negative impedance inverter effectively transformed a 1 farad capacitor into a -1 henry inductor. Figure 7.8 shows two realizations of an ideal negative impedance inverter using negative resistance elements.

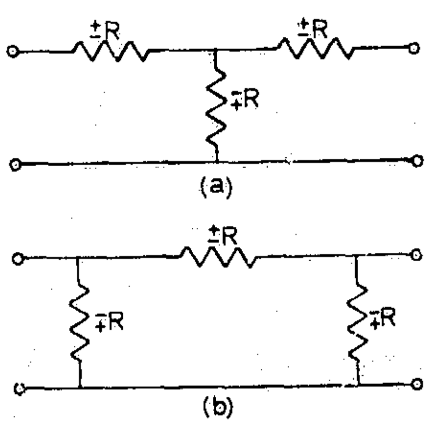

Figure 7.8 Ideal negative impedance inverters. (a) T negative impedance inverter. (b) π negative impedance inverter.

7.3 Synthesis of Transfer Functions Using Negative Resistances. The Cauer method of two-port synthesis introduced in Chapter Four could be used to realize RC ladder networks which had RC type transfer functions with zeros on the negative real axis. If negative resistances are now admitted as usable network elements the transfer functions of the RC and -R networks may have zeros of transmission on the negative real axis and the positive real axis. Just as before, a zero of transmission could be realized in two ways; a pole of impedance in a series arm or a pole of admittance in a shunt arm. The network shown in figure 7.9-a has a pole of impedance on the positive real axis. Such a pole would create a zero of transmission on the positive real axis. The network of figure 7.9-b has a pole of admittance on the positive real axis.

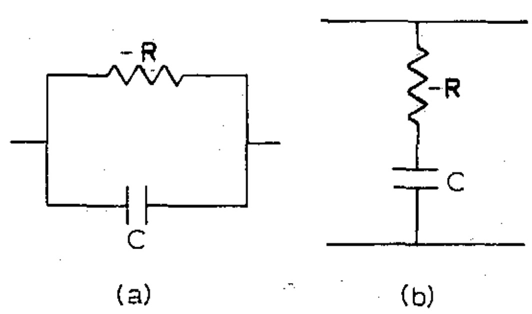

Figure 7.9 Two networks with poles on the positive real axis.
(a) Pole of impedance at $s = +1/RC$.
(b) Pole of admittance at $s = 1/RC$.

Example. Realize a network with the following short-circuit driving point and transfer admittance functions:

$$y_{22} = \frac{(s+2)(s+4)}{(s+3)}$$

$$-y_{21} = \frac{s(s-1)}{(s+3)} \; .$$

Use negative resistance.

Step 1. Shift zero of y_{22} to $s = +1$ by removal of a shunt conductance, to find Y1.

$$Y1 = \frac{(s+2)(s+4)}{(s+3)} - G1$$

$$G1 = y_{22}\Big|_{s=1} = \frac{15}{4}$$

$$Y1 = \frac{s^2+6s+8}{s+3} - \frac{15}{4} = \frac{4s^2+9s-13}{4s+12} = \frac{(s-1)(4s+13)}{4s+12}$$

Note that Y1 has the expected zero at $s = +1$.

Step 2. Invert Y1 to find Z1.

$$Z1 = \frac{4s+12}{4s^2+9s-13} = \frac{1/17}{(s+13/4)} + \frac{16/17}{(s-1)}$$

Step 3. Remove pole of Z1 at $s = +1$ by use of a negative resistance.

$$Z1 = \frac{1/17}{(s+13/4)} + \frac{16/17}{(s-1)}$$

← recognize as -R and C in parallel, in series arm. $R = -16/17\Omega$, $C = 17/16$ f.

$$Z2 = Z1 - \frac{16/17}{(s-1)} = \frac{1/17}{(s+13/4)}.$$

Step 4. Invert Z2 to find Y2.

$$Y2 = \frac{(s+13/4)}{1/17} \; .$$

Step 5. Shift zero of Y2 to $s = 0$ by removal of a shunt conductance, to find Y3.

$$Y3 = \frac{(s+13/4)}{1/17} - G2$$

$$G2 = \frac{(s+13/4)}{1/17}\Big|_{s=0} = \frac{13/4}{1/17} = \frac{221}{4}$$

$= 17s$ ⟵ recognize as a 17f capacitor in a series arm.

The required network is shown in figure 7.9-a.

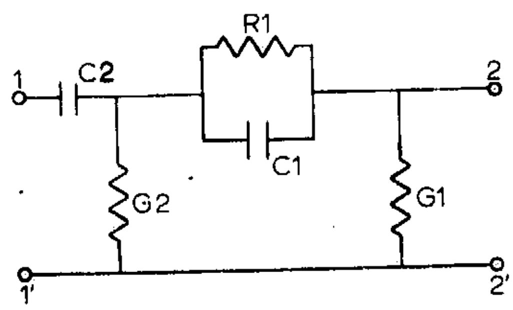

Fig. 7.9-a Network utilizing a negative resistance. G1 = 15/4 mhos, G2 = 221/4 mhos, R1 = -16/17 ohm, C1 = 17/16f, and C2 = 17f.

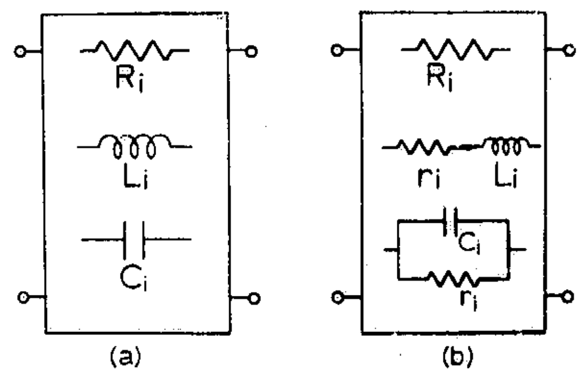

Figure 7.10 (a) Original network with RLC elements only. (b) network with added dissipation.

Using NASAP-69 y_{22} and $-y_{21}$ for the network are found to be

$$y_{22} = \frac{(s+2)(s+4)}{(s+3)}$$

$$-y_{21} = \frac{s(s-1)}{(s+3)} \quad . \qquad (7.39\text{-b})$$

A second method of realizing transfer functions with zeros in the right hand plane is based on the Cauer method and knowledge of the manner in which the poles and zeros of a network shift if dissipative elements are added to a network. Consider the network consisting of RLC elements shown in figure 7.10-a. If to each inductor, L_i, a series resistance r_i is added and to each capacitor, C_i, a parallel conductance g_i is added such that

$$\alpha = \frac{r_i}{L_i} = \frac{g_i}{C_i} \qquad (7.23)$$

then a system function, H, of the original network and the corresponding system function, H_1, of the new network will have the relation

$$H(s) = H_1(s + \alpha). \tag{7.24}$$

This relationship is used to transform a general function into a system function which has poles and zeros in the negative portion of the s plane so that the Cauer method may be used to realize the function. Then the resulting network is transformed to the desired network by addition of appropriate series resistances and shunt conductances. For example, if a network with the driving point and transfer admittance function of

$$y_{11} = \frac{(s - 2)(s + 3)}{(s + 1)(s + 5)} \tag{7.25}$$

$$-y_{21} = \frac{(s - 1)(s)}{(s + 1)(s + 5)}$$

is desired, replacing s by s + 3 gives the driving point and transfer admittance

$$y_{11} = \frac{(s + 1)(s + 6)}{(s + 4)(s + 8)} \tag{7.26}$$

$$-y_{21} = \frac{(s + 2)(s + 3)}{(s + 1)(s + 5)} .$$

An RC network with such a y_{11} and $-y_{21}$ was realized in Chapter Four using the Cauer method (see figure 4.12). Now the desired network may be obtained by adding negative conductances in parallel with capacitors C1, C2, and C3 of figure 4.12. The conductances are found as

$$g1 = -C1\alpha = -3/12 \text{ mhos}$$

$$g2 = -C2\alpha = \frac{-15}{36} \text{ mhos}$$

$$g3 = -C3\alpha = \frac{-210}{726} \text{ mhos.}$$

The resulting network is shown in figure 7.11. Note that

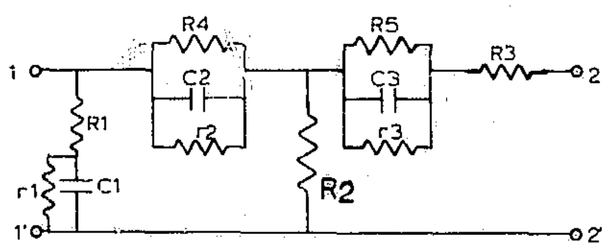

Figure 7.11 Ladder network resulting from specifications of equation 7.25, with R1 - 3Ω, R2 = 2.2Ω, R3 = 33/7Ω, R4 = 33/7Ω, R4 = 18/5Ω, R5 = 726/210Ω, r1 = -4Ω, r2 = -12/5Ω, r3 = -726/210Ω, C1 = 1/12f, C2 = 5/36f, and C3 = 70/726f.

one negative resistance was required for each of the capacitors appearing in the network resulting from the Cauer synthesis procedure.

A third method of synthesis using negative resistances is based on the network configuration shown in figure 7.12. The indicated RC subnetworks are RC ladder networks. The open circuit voltage transfer function G21 is given in terms of the short circuit admittance parameters as

$$G21 = -\frac{y_{21}}{y_{22}} = -\frac{(y'_{21} + y''_{21})}{(y'_{22} + y''_{22})} \qquad (7.27)$$

where the prime parameters refer to the subnetwork N′ and the double prime parameters refer to the subnetwork N″. Let

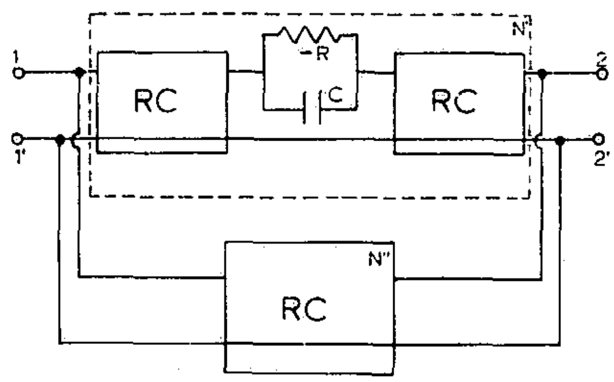

Figure 7.12 RC and -R network with the voltage transfer function given in equation 7.27.

a given voltage transfer ratio be denoted as N(s)/D(s), then this ratio can be put into the form

$$\frac{N(s)}{D(s)} = \frac{\dfrac{a'(s-\alpha)N'(s)}{Q(s)} + \dfrac{a''N''(s)}{Q(s)}}{\dfrac{K'D(s)}{Q(s)} + \dfrac{K''D(s)}{Q(s)}} \qquad (7.28\text{-a})$$

where Q(s) is chosen so that D(s)/Q(s) is an RC driving point admittance and $K' + K'' = 1$. $N'(s)$ and $N''(s)$ are such that

$$N(s) = a'(s-\alpha)N'(s) + a''N''(s) \qquad (7.28\text{-b})$$

and are polynomials with negative real roots which have unity coefficients for the term of highest power of s.

Next realize two RC, two-port networks, N1 and N2, which are specified by

$$\begin{aligned} N1:\quad y^1{}_{22} &= D(s)/Q(s) \\ -y^1{}_{12} &= -(s-\alpha)N'(s)/Q(s) \qquad (7.29) \\ N2:\quad y^2{}_{22} &= D(s)/Q(s) \\ -y^2{}_{12} &= -N''(s)/Q(s)\ . \end{aligned}$$

Let B1 and B2 be the gain constant of the networks N1 and N2 respectively. Using the impedance level transformation of Chapter One, adjust the admittance level of network N1 by a factor of Ka′/B2 where K is yet to be determined. When network N1, just N′, and N2, just N″, are connected in parallel the short-circuit admittance parameters for the entire network become

$$\frac{Ka'}{B1}\, y^1{}_{21} + \frac{Ka''}{B2}\, y^2{}_{21} = K\left(\frac{a'}{B1} + \frac{a''}{B2}\right) y^2{}_{21} \qquad (7.30)$$

$$\frac{Ka'}{B1}\, y^1{}_{22} + \frac{Ka''}{B2}\, y^2{}_{22} = K\left(\frac{a'}{B1} + \frac{a''}{B2}\right) y^2{}_{22}$$

where K is selected so that

$$K\left(\frac{a'}{B1} + \frac{a''}{B2}\right) = 1 \qquad (7.31)$$

Example: Suppose that it is desired to realize a network with the short-circuit admittance parameter

$$y_{22} = \frac{(s + 2)(s + 4)}{(s + 3)} \qquad (7.32)$$

$$-y_{21} = \frac{s^2 - s + 2}{(s + 3)} \quad .$$

Now the numerator of $-y_{21}$ is put into the form of equation 7.28-b as

$$s^2 - s + 2 = s(s - 1) + 2 = a'(s - \alpha)N'(s) + a''N''(s) \qquad (7.33)$$

and the following identifications are made:

$$(s - \alpha) = (s - 1) \qquad (7.34)$$

$$a' = 1,\ a'' = 2$$

$$N'(s) = s,\ N''(s) = 1.$$

The specifications for N1 and N2 are then

N1:

$$y_{22} = \frac{(s + 2)(s + 4)}{(s + 3)} \quad (7.35)$$

$$-y_{21} = \frac{s(s - 1)}{(s + 3)}$$

N2:

$$y_{22} = \frac{(s + 2)(s + 4)}{(s + 3)} \quad (7.36)$$

$$-y_{21} = \frac{1}{s + 3} \quad .$$

A network with the short-circuit admittance specifications of equation (7.35) has already been found in the first example of this section, (see figure 7.9-a). A network with the short-circuit admittance specifications of equation (7.36) may be found by making a continued fraction expansion of y_{22}. This continued fraction expansion is given by

$$y_{22} = 1s + \cfrac{1}{1/3 + \cfrac{1}{9s + \cfrac{1}{1/24}}} \quad . \quad (7.37)$$

The network corresponding to the continued fraction expansion of y_{22}, equation (7.37), is shown in figure 7.13. Using NASAP-69 the short-circuit admittances y_{22} and $-y_{21}$ are found to be

$$y_{22} = \frac{(s + 2)(s + 4)}{(s + 3)} \quad (7.38)$$

$$-y_{21} = \frac{8}{s + 3} \quad . \quad (7.39\text{-a})$$

From equations (7.39-a) and equation (7.39-b) we see that B1 = 1

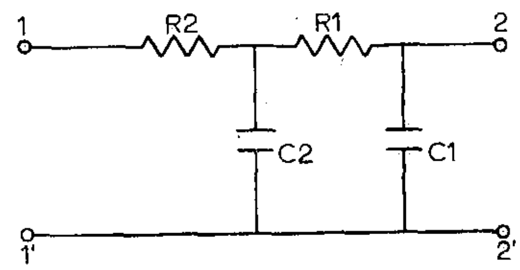

Figure 7.13 Network resulting from short-circuit admittance specifications of equation (7.36). R1 = 1/3Ω, C1 = 1f and C2 = 9f.

and B2 = 8. Using equation (7.31) to find K, we have

$$K = \frac{1}{\frac{a'}{B1} + \frac{C''}{B2}} = \frac{1}{\frac{1}{1} + \frac{2}{8}}$$

$$K = 4/5 .$$

The network N′ is shown in figure 7.14. This network is the same as shown in figure 7.9-a, but the impedance level has been changed by a factor of Ka′/B1 = 4/5. The network N″ is shown in figure 7.15. This network is the same as shown in figure 7.13, but the impedance level has been changed by a factor of Ka″/B2 = 1/5.

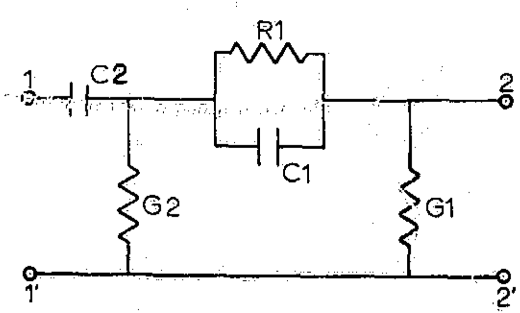

Figure 7.14 Network N′. G1 = 3 mhos, G2 = 221/5 mhos, R1 = -20/17, C1 = 17/20, and C2 = 68/5.

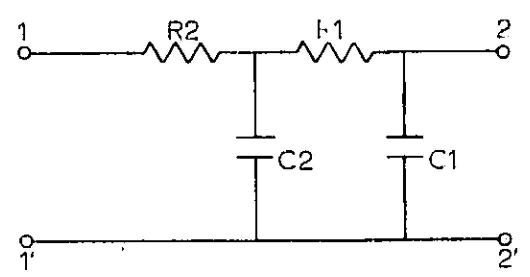

Figure 7.15 Network N″. R1 = 5/3Ω, R2 = 5/24, C1 = 1/5f and C2 = 9/5f.

7.4 Synthesis of Transfer Functions Using Controlled Sources.

The voltage transfer ratio of the circuit shown in figure 7.16 is

$$\frac{V2}{V1} = \frac{-y_{21}}{y_{22} + Y1 - (\mu - 1)Y2} \qquad (7.40)$$

where y_{22} and y_{21} are the short-circuit admittances of the RC two-port network and Y1 and Y2 are RC one-port networks. The gain of the voltage controlled voltage amplifier is μ where μ is greater than one.

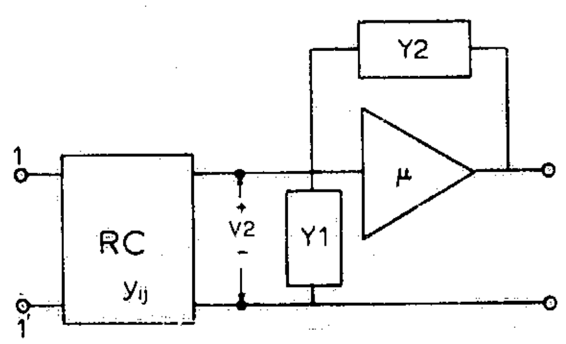

Figure 7.16. Network with voltage transfer ratio given by equation (7.40).

This network configuration is used to synthesize a desired voltage transfer function, $N(s)/D(s)$, by letting $Y1 = 1$ and rearranging the transfer function $N(s)/D(s)$ so that it is of the form of equation (7.40). Let $Q(s)$ denote a polynomial which is chosen so that:

(1) $Q(s)$ has distinct real roots.

(2) The degree of $Q(s)$ is less or equal to that of the maximum degree of $N(s)$ or $D(s)$.

The desired voltage transfer function is now written as

$$\frac{N(s)}{D(s)} = \frac{N(s)/Q(s)}{D(s)/Q(s)} \, . \tag{7.41}$$

Then make a partial fraction expansion of $D(s)/sQ(s)$ and write $D(s)/Q(s)$ in the form

$$D(s) = K_0 + K_\infty s + \sum_i \frac{K_i s}{s + \sigma_i} - \sum_j \frac{h_j s}{s + \delta_j} \tag{7.42}$$

where $K_i \geq 0$ and $h_j \geq 0$.

Using the denominator of equation (7.40) the following identifications are made:

$$y_{22} = K'_0 + K_\infty s + \sum_i \frac{K_i s}{s + \sigma_i} + \sum_j \frac{h'_j s}{s + \delta_j} \tag{7.43}$$

$$(\mu - 1)Y2 = K''_0 + \sum_j \frac{h''_j s}{s + \delta_j}$$

where

$$K_0 = K'_0 - K''_0 + 1 \text{ and } h_j = h''_j - h'_j > 0.$$

From the numerator of equation (7.40) y_{21} is identified as

$$-y_{21} = \frac{N(s)}{Q(s)} \, . \tag{7.44}$$

Now y_{22} and $-y_{21}$ are used as the required specifications for the synthesis of an RC two-port network.

Example: Use the network configuration of figure 7.16 to realize the second order Bessel filter with a voltage transfer function given by

$$\frac{N(s)}{D(s)} = \frac{H}{s^2 + 3s + 3} \qquad (7.45)$$

if the gain of the voltage amplifier is 2.

Let $Q(s) = s + 1$. The partial fraction expansion of $D(s)/sQ(s)$ is

$$\frac{D(s)}{sQ(s)} = \frac{s^2 + 3s + 3}{s(s + 1)} = 1 + \frac{3}{s} - \frac{1}{s + 1}$$

hence

$$\frac{D(s)}{Q(s)} = (s + 3) - \frac{s}{(s + 1)} \quad .$$

Choose $K_0' = 1$ and $h_1' = 1$, then $h_1'' = h_1 + h_1' = 2$ and

$$(u - 1)Y2 = Y2 = 1 + \frac{2s}{s + 1} = \frac{3s + 1}{s + 1}$$

$$y_{22} = s + 3 + \frac{s}{s + 1} = \frac{s^2 + 5s + 3}{s + 1}$$

$$-y_{21} = \frac{H}{s + 1} \quad .$$

The RC two-port network specified by y_{22} and $-y_{21}$ may be found by making a continued fraction expansion of y_{22} as

$$\begin{array}{l} s + 1 \;\overline{)\, s^2 + 5s + 3\,}(s \\ \quad\;\; \underline{s^2 + s} \\ \qquad\quad 4s + 3 \;\overline{)\, s + 4/4\,}(1/4 \\ \qquad\qquad\qquad \underline{s + 3/4} \\ \qquad\qquad\qquad\quad 1/4 \;\overline{)\, 4s + 3\,}(16s \\ \qquad\qquad\qquad\qquad\quad \underline{4s} \\ \qquad\qquad\qquad\qquad\qquad 3 \;\overline{)\, 1/4\,}(1/12. \end{array}$$

The corresponding network is shown in figure 7.17-a.

Y2 is inverted to form Z2, and Z2 is expanded by partial fraction expansion.

$$Z2 = \frac{1}{Y2} = \frac{s + 1}{3s + 1} = \frac{1}{3} + \frac{2/3}{3s + 1} = \frac{1}{3} + \frac{1}{9/2s + 3/2}.$$

Z2 is recognized as a resistance in series with a parallel RC network as shown in figure 7.17-b. Figure 7.18 shows the complete circuit, where the ideal voltage source has been replaced by a nonideal voltage source.

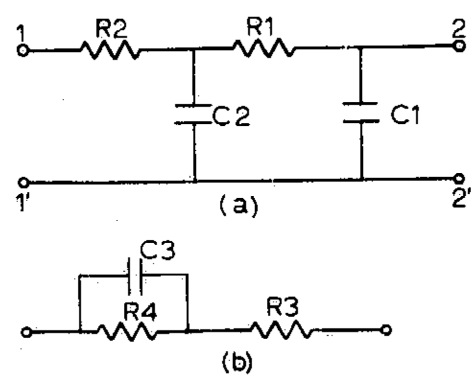

Figure 7.17 R1 = 1/4Ω, R2 = 1/12Ω, R3 = 1/3Ω, R4 = 2/3Ω, C1 = 2f, C2 = 16f, and C3 = 9/2f.

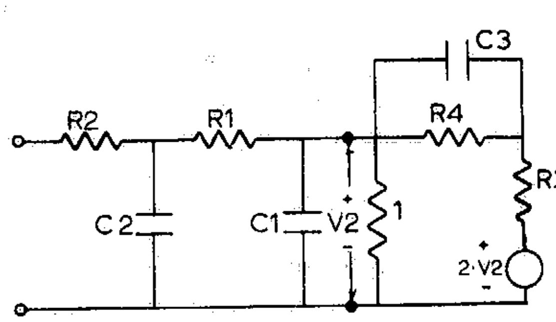

Figure 7.18 Complete circuit. R1 = 1/4Ω, R2 = 1/12Ω, R3 = 1/3Ω, C1 = 1f, C2 = 16f, and C3 = 9/2f.

Filter sections may also be designed using controlled sources by coefficient matching. The circuit configuration of figure 7.19 may be used to design a second-order low-pass filter. The voltage transfer ratio V1/V2 for this circuit is

$$\frac{V2}{V1} = \frac{\dfrac{KG1G2}{C1C2}}{s^2 + \left[\dfrac{G2}{C2}(1 - K) + \dfrac{G1}{C1} + \dfrac{G2}{C1}\right]s + \dfrac{G1G2}{C1C2}}. \qquad (7.46)$$

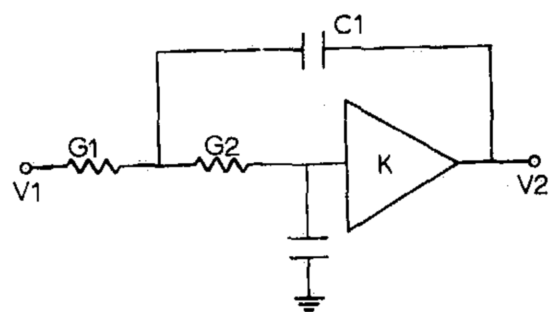

Figure 7.19 Second order low-pass filter section.

If the desired voltage transfer function is given by

$$\frac{V2}{V1} = \frac{H}{s^2 + \beta s + \gamma} \quad \text{then,} \qquad (7.47)$$

by equating like coefficients of equations (7.46) and (7.47) the following design equations are obtained:

$$H = \frac{K\ G1G2}{C1C2}$$

$$\beta = \frac{G2}{C2}(1 - K) + \frac{G1}{C1} + \frac{G2}{C1}$$

$$\gamma = \frac{G1G2}{C1C2}.$$

Note that there are more unknown quantities than there are equations, thus some element values must be chosen arbitrarily so that the remaining equations may be solved. For example, we may select C1 = C2 = 1 and K = 2 then

$$G1 = \beta \qquad (7.48)$$

$$G2 = \frac{\gamma}{\beta}$$

$$H = 2\gamma \; .$$

Example: Use the circuit configuration of figure 7.19 to realize a second-order low-pass Butterworth filter.

The second order Butterworth response is given by

$$\frac{V2}{V1} = \frac{H}{s^2 + \sqrt{2}\, s + 1} \; .$$

Hence $\beta = \sqrt{2}$ and $\gamma = 1$ and using equations (7.48) we find

$C1 = C2 = 1$

$K = 2$

$G1 = \sqrt{2}$

$G2 - 1/\sqrt{2}$

$H = 2$.

The circuit is shown in figure 7.20.

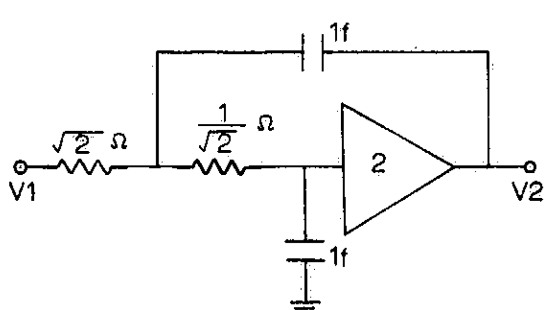

Figure 7.20 Second-order Butterworth filter.

If all the resistances of the circuit shown in figure 7.20 are replaced by capacitors of value $1/R_i$ and all the capacitors are replaced by resistors of value $1/C_i$ the result is the second-order high-pass filter with the voltage transfer ratio of

$$\frac{V2}{V1} = \frac{2s^2}{s^2 + \sqrt{2}s + 1} .$$

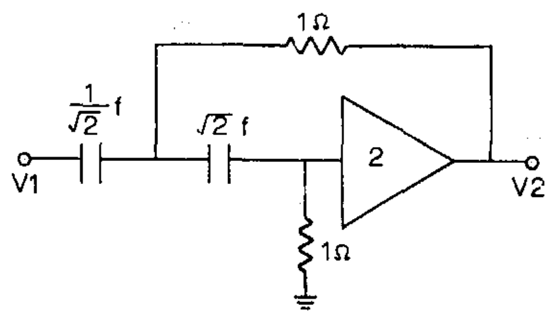

Figure 7.21 High-pass filter.

The resulting filter is shown in figure 7.21.

The second-order filter section shown in figure 7.22 can be used to design elliptic-function active filters with a voltage transfer function of the form

$$\frac{V2}{V1} = \frac{H(s^2 + \alpha)}{s^2 + \beta s + \gamma} . \qquad \gamma > \alpha \qquad (7.49)$$

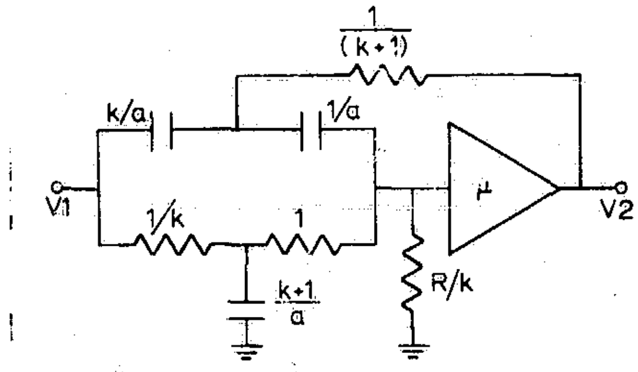

Figure 7.22 Active elliptic-function filter.

The element values for the circuit of figure 7.22 can be found from the relations

$$a = \sqrt{\alpha} \qquad (7.50)$$

$$R = \frac{(K + 1)}{\frac{\gamma}{\alpha} - 1}$$

$$\mu = 2 - K \left[\frac{\beta}{(K + 1)a} - \frac{1}{R}\right]$$

$$H = u \; .$$

where K is an arbitrary positive constant. For K = 1 the circuit shown in figure 7.23 has the transfer function

$$\frac{V2}{V1} = \frac{(3/2)(s^2 + 1/4)}{s^2 + (2/3)s + 1/3} \; . \qquad (7.51)$$

If all the resistors are exchanged for capacitors of value $1/R_i$ and all the capacitors are exchanged for resistors of value $1/C_i$, the resulting circuit is the filter shown in figure 7.24. This circuit has the transfer function

$$\frac{V2}{V1} = \frac{(9/8)(s^3 + 4)}{s^2 + 2s + 3} \; . \qquad (7.52)$$

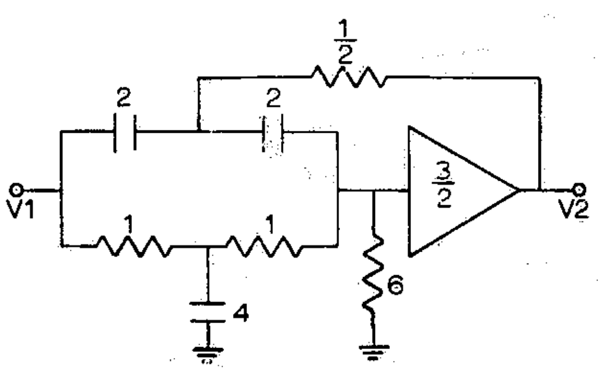

Figure 7.23 Elliptic-function RC filter with transfer function of equation 7.51.

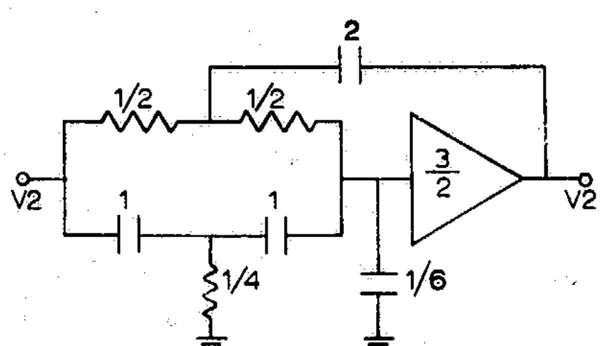

Figure 7.24 Elliptic-function RC filter with transfer functon of equation 7.51.

BIBLIOGRAPHY

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CHAPTER EIGHT

Catalog of Filter Responses.

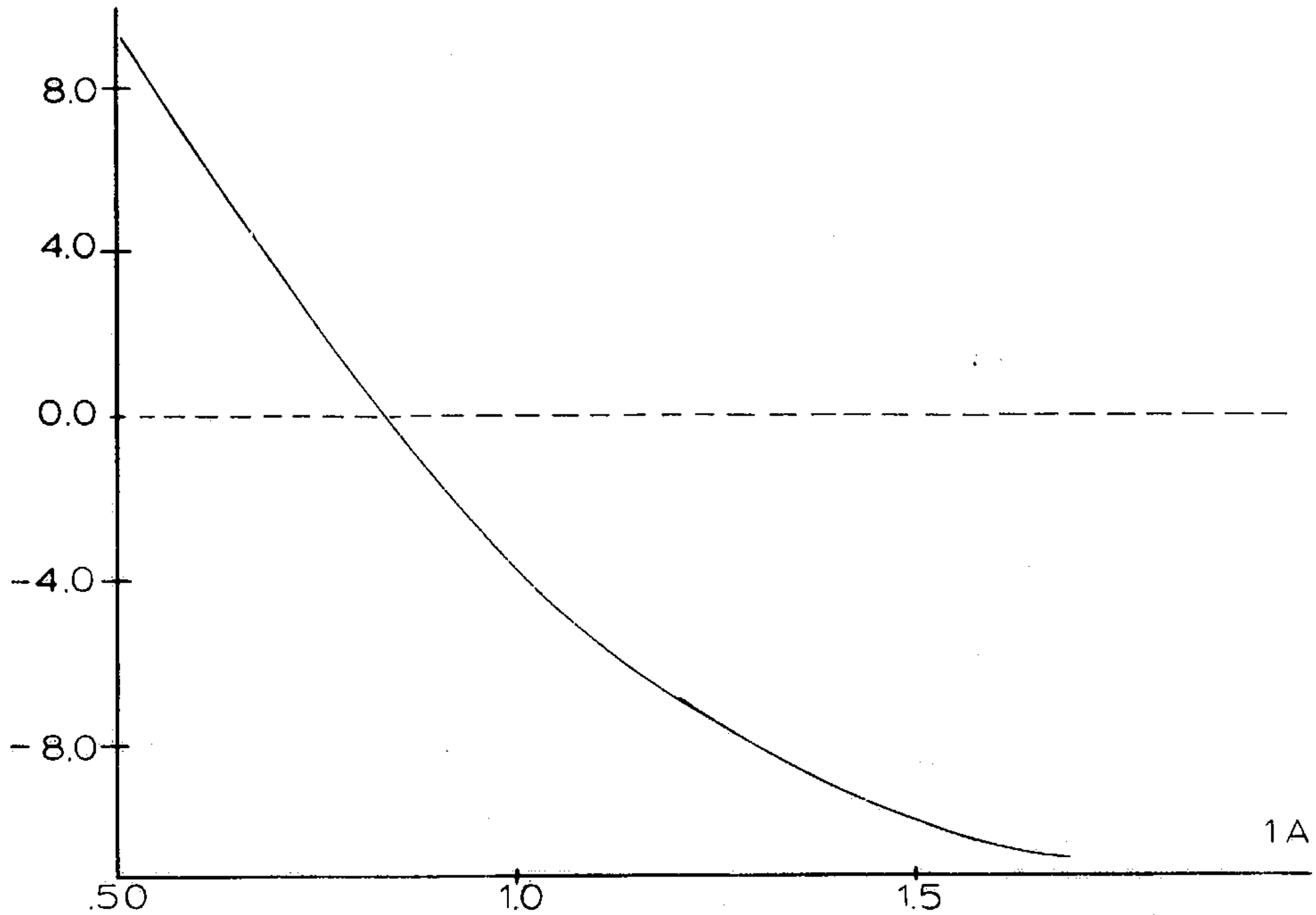
Introduction. In this chapter the magnitude and phase response of some typical filters have been plotted. NASAP-69 was used to obtain the data required for plotting the required responses.

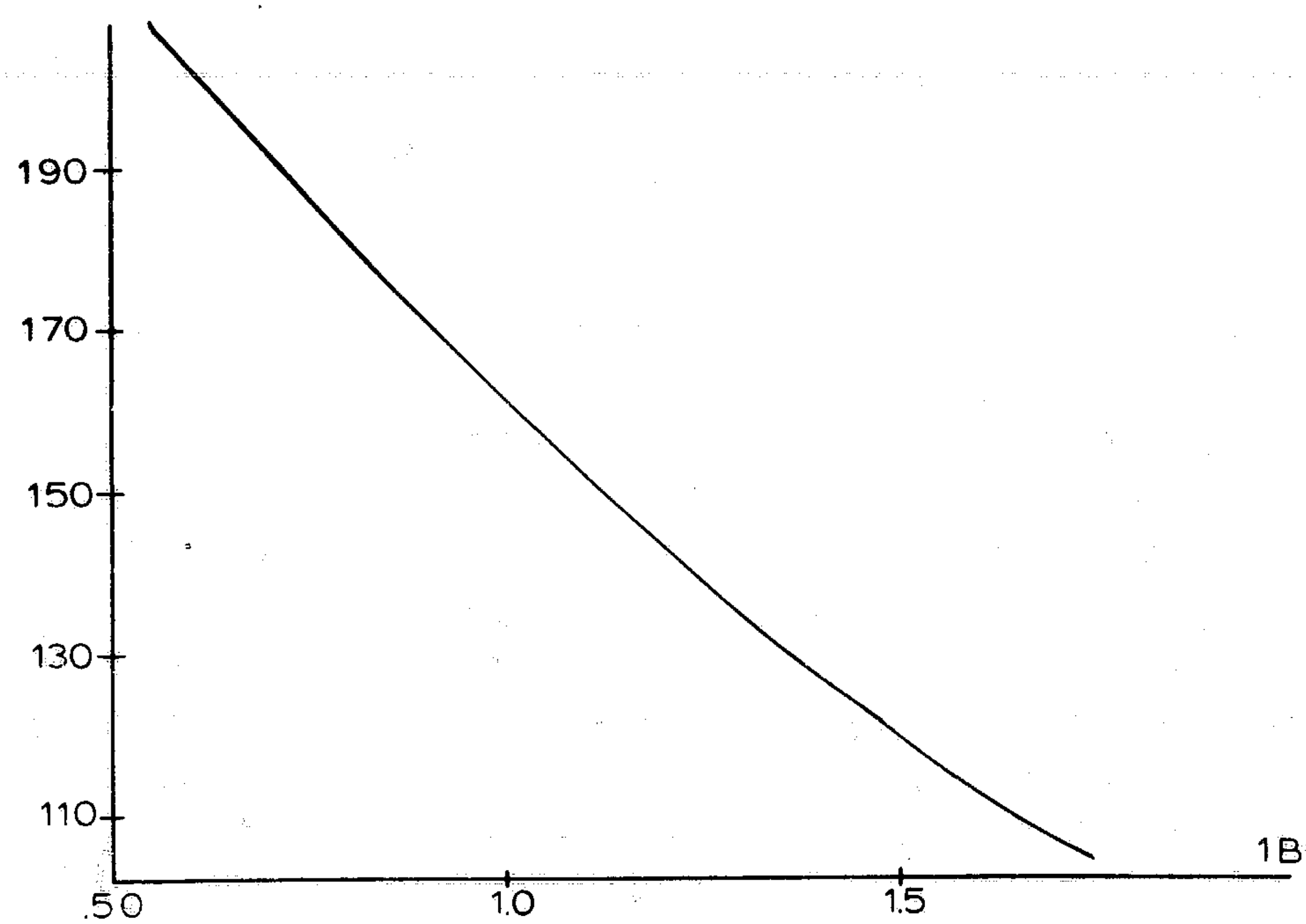
Circuit of figure 4.2-a

1-A $|Z21|$ in db versus ω

$$Z21 = \frac{5s^3}{s^3 + 3s^2 + 4s + 2}$$

1-B phase in degrees versus ω


8.0
4.0
0.0
-4.0
-8.0
1A
.50
1.0
1.5

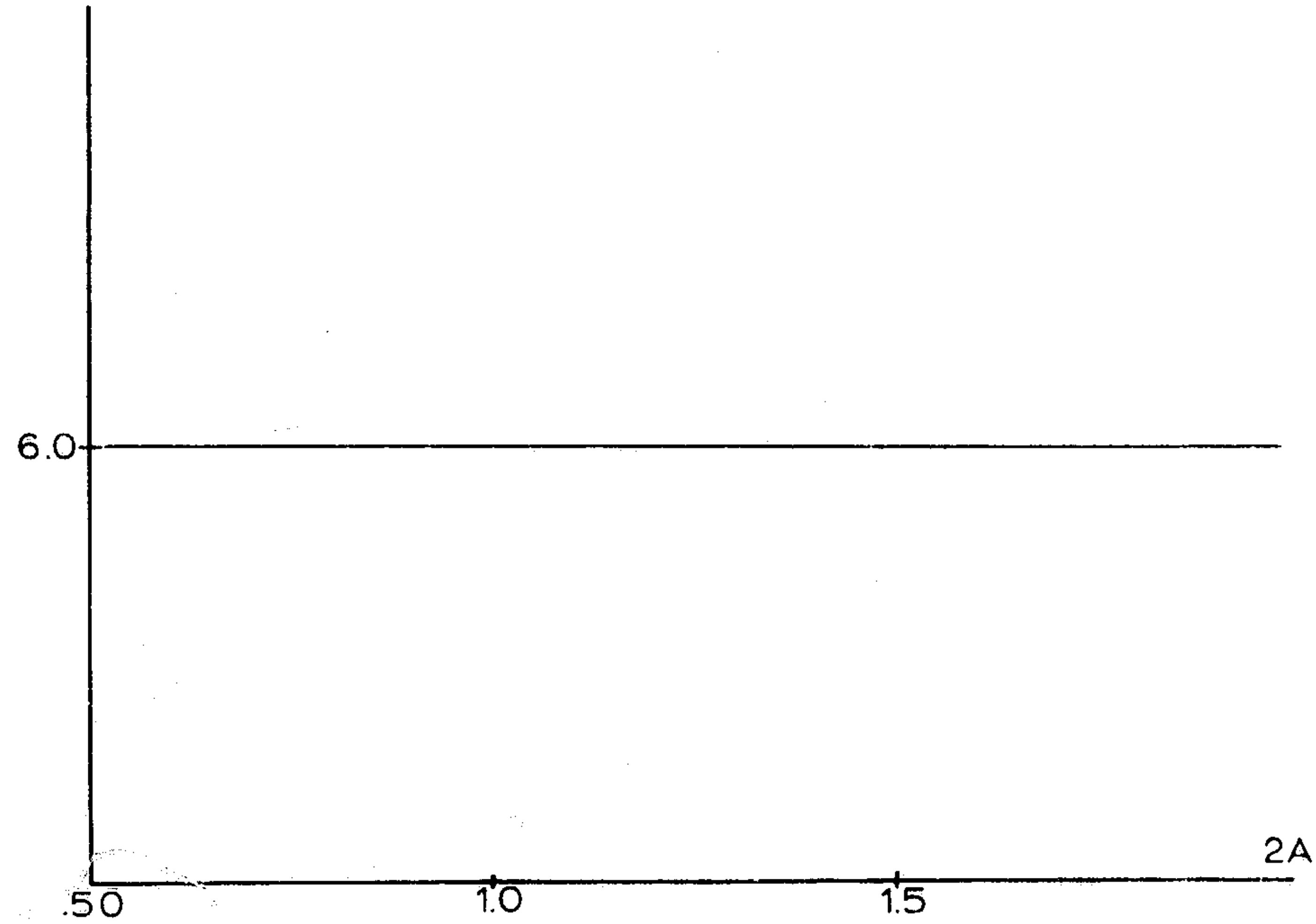

190
170
150
130
110
1B
.50
1.0
1.5

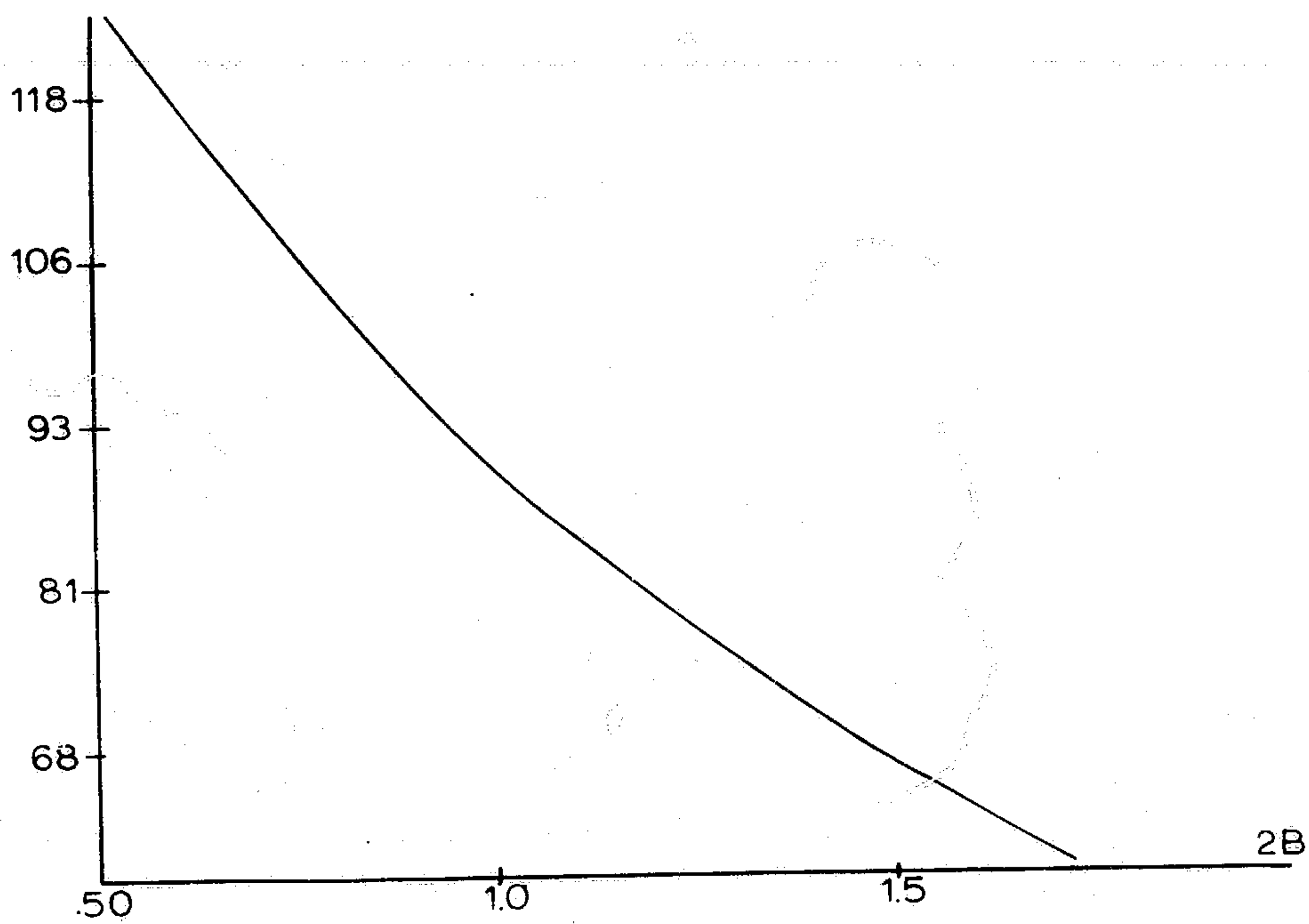
Circuit of figure 4.2-b

2-A $|G2g|$ in db versus ω

$$G2g(s) = \frac{(s - 1)}{2(s + 1)}$$

2-B Phase in degrees versus ω


6.0
2A
.50
1.0
1.5

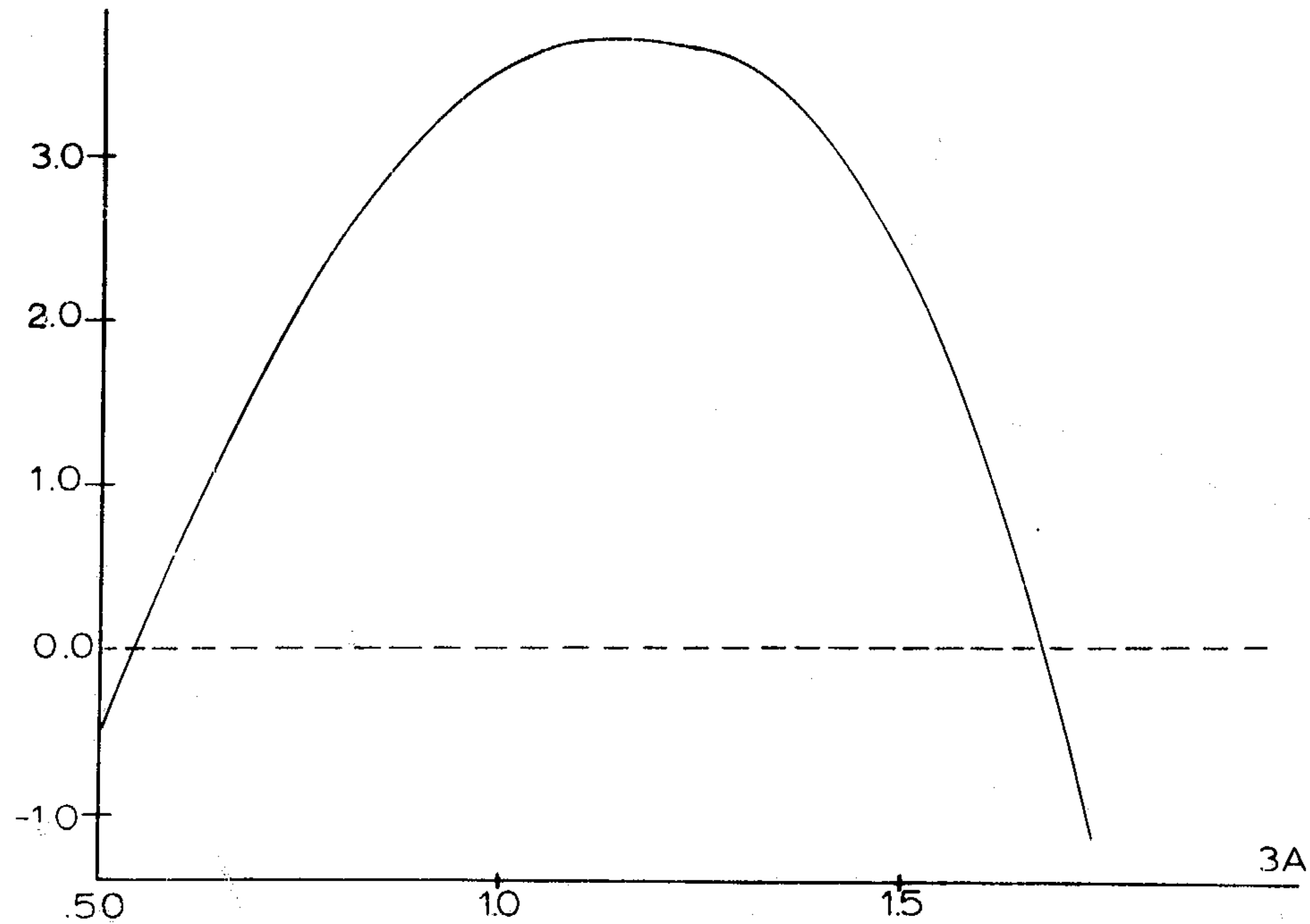

118
106
93
81
68
2B
.50
1.0
1.5

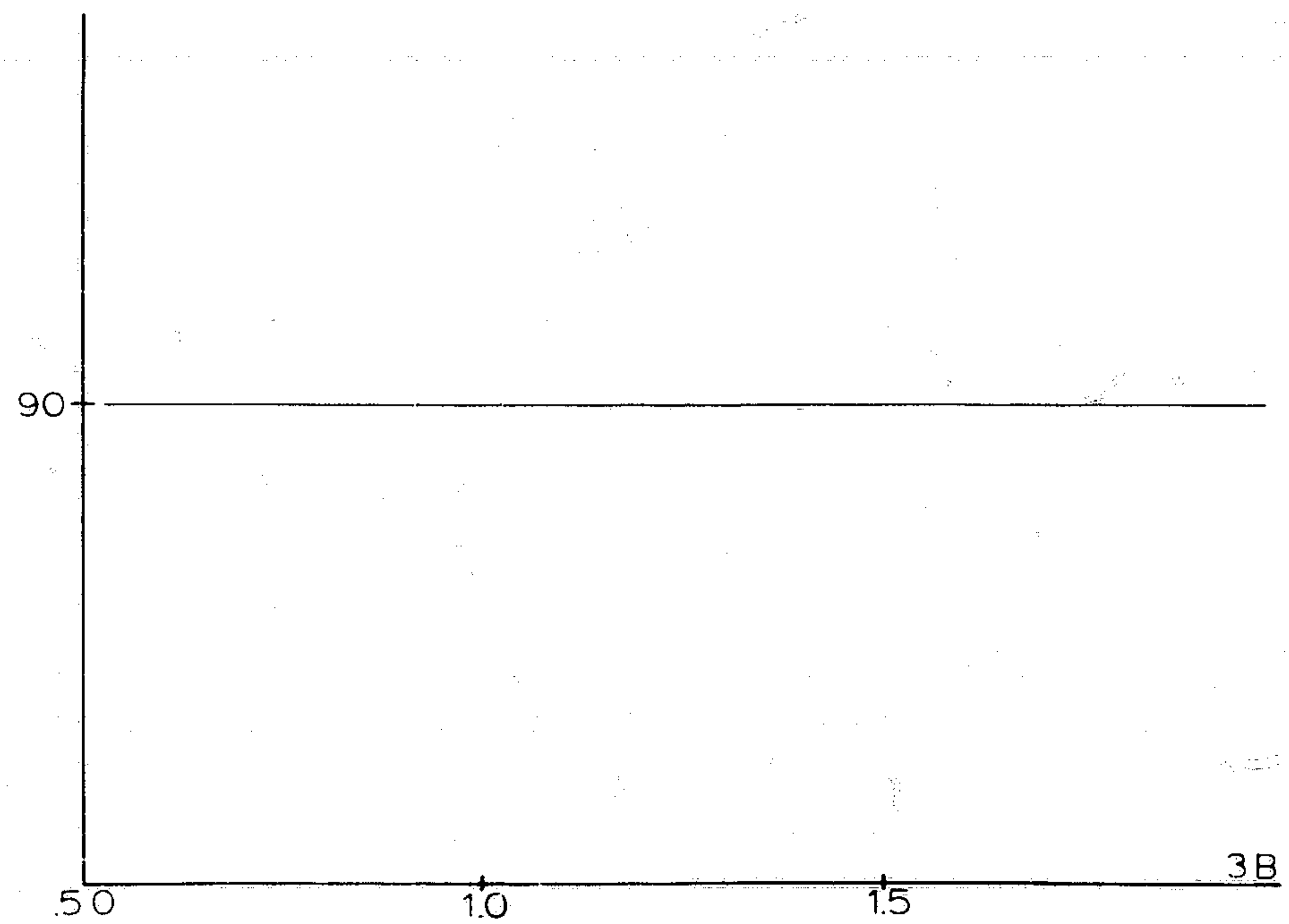
Circuit of figure 4.5

3-A $|Z21(s)|$ in db versus ω

$$Z21 = \frac{-2}{s(s^2 + 4)}$$

3-B phase in degrees versus ω


3.0
2.0
1.0
0.0
-1.0
.50
1.0
1.5
3A

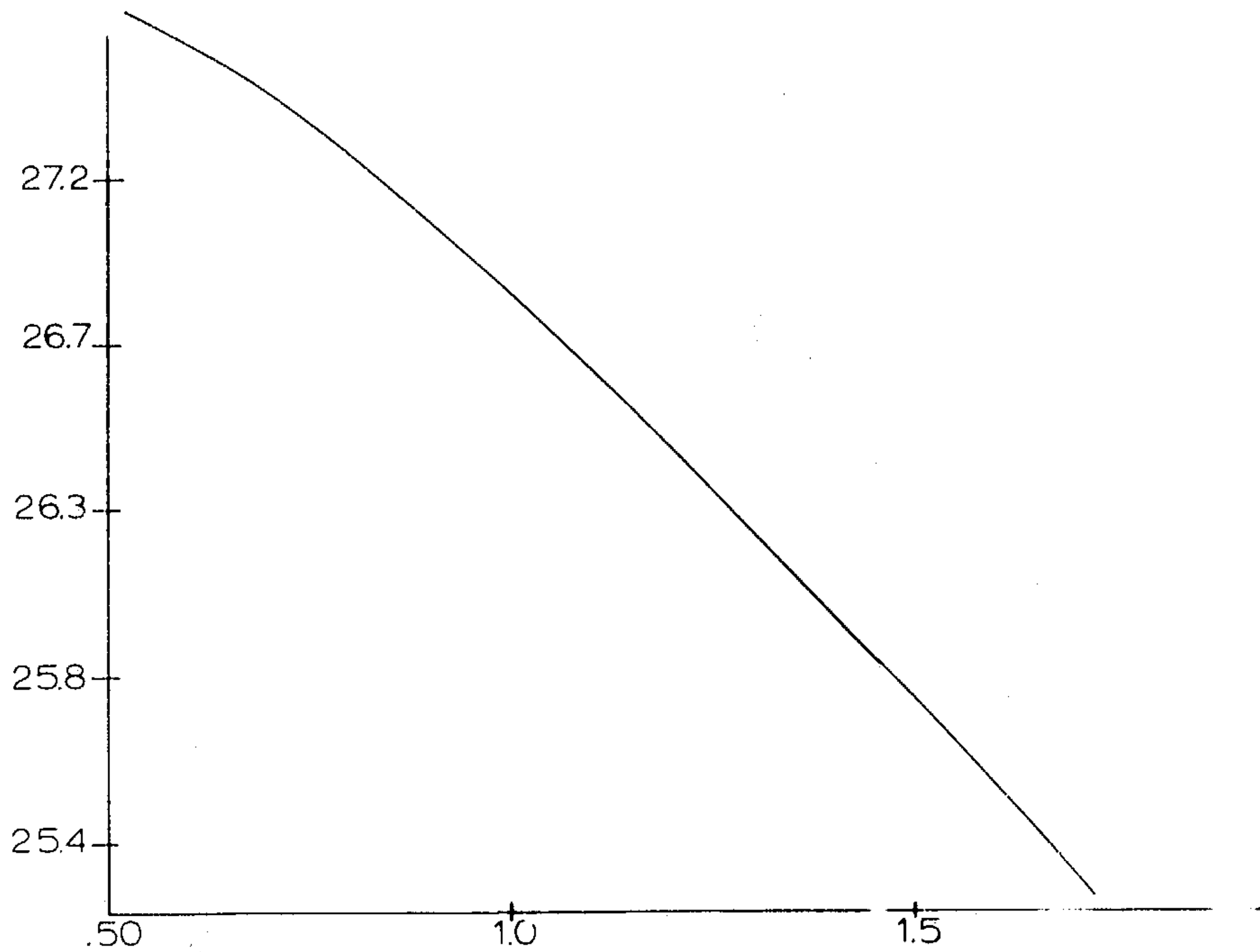

90
.50
1.0
1.5
3B

Circuit of figure 4.12

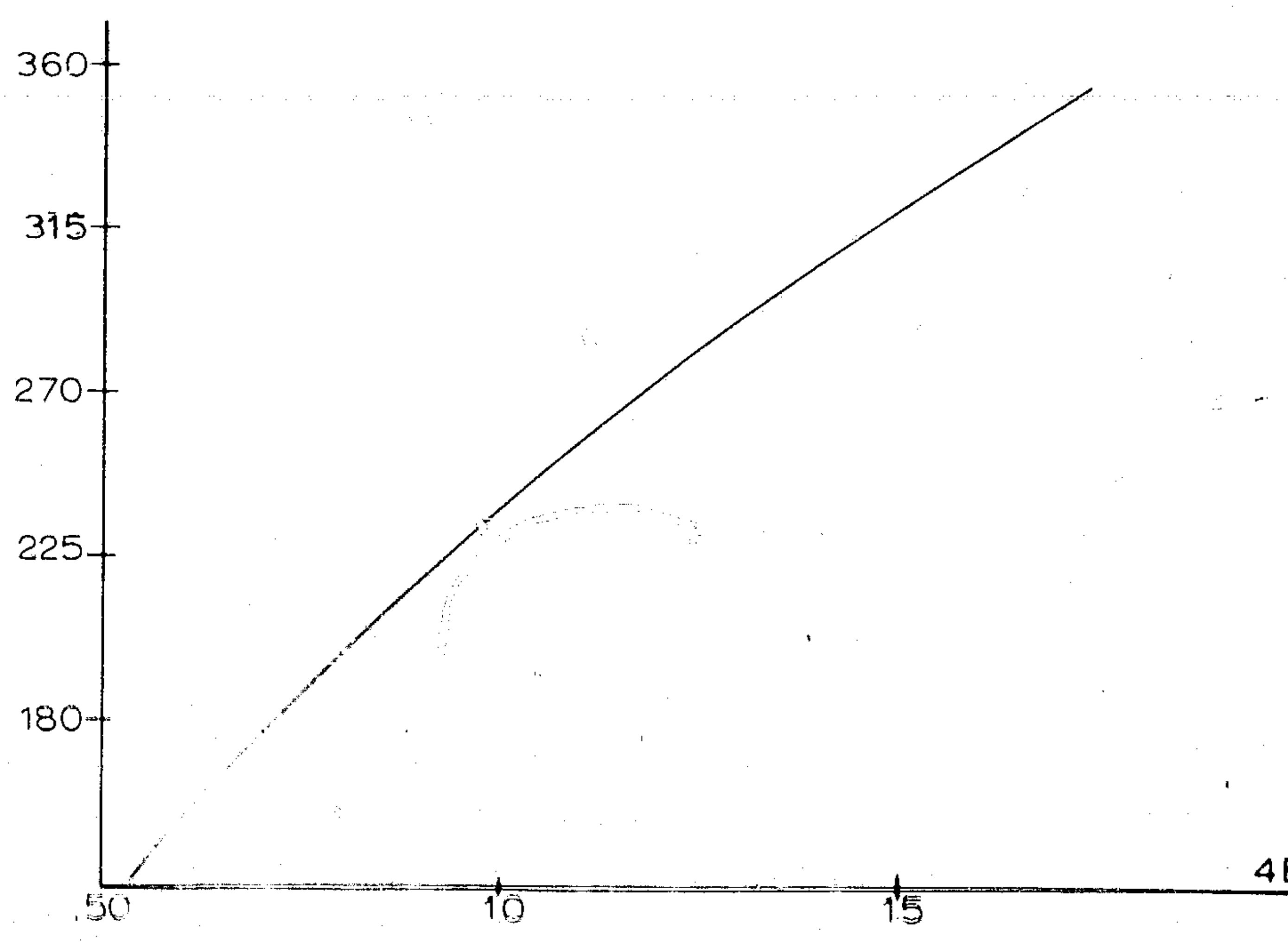
4-A $|y_{21}|$ in db versus ω

$$y_{21} = \frac{.212[(s + 2)(s + 3)]}{(s + 4)(s + 8)}$$

4-B phase in degrees versus ω



4A



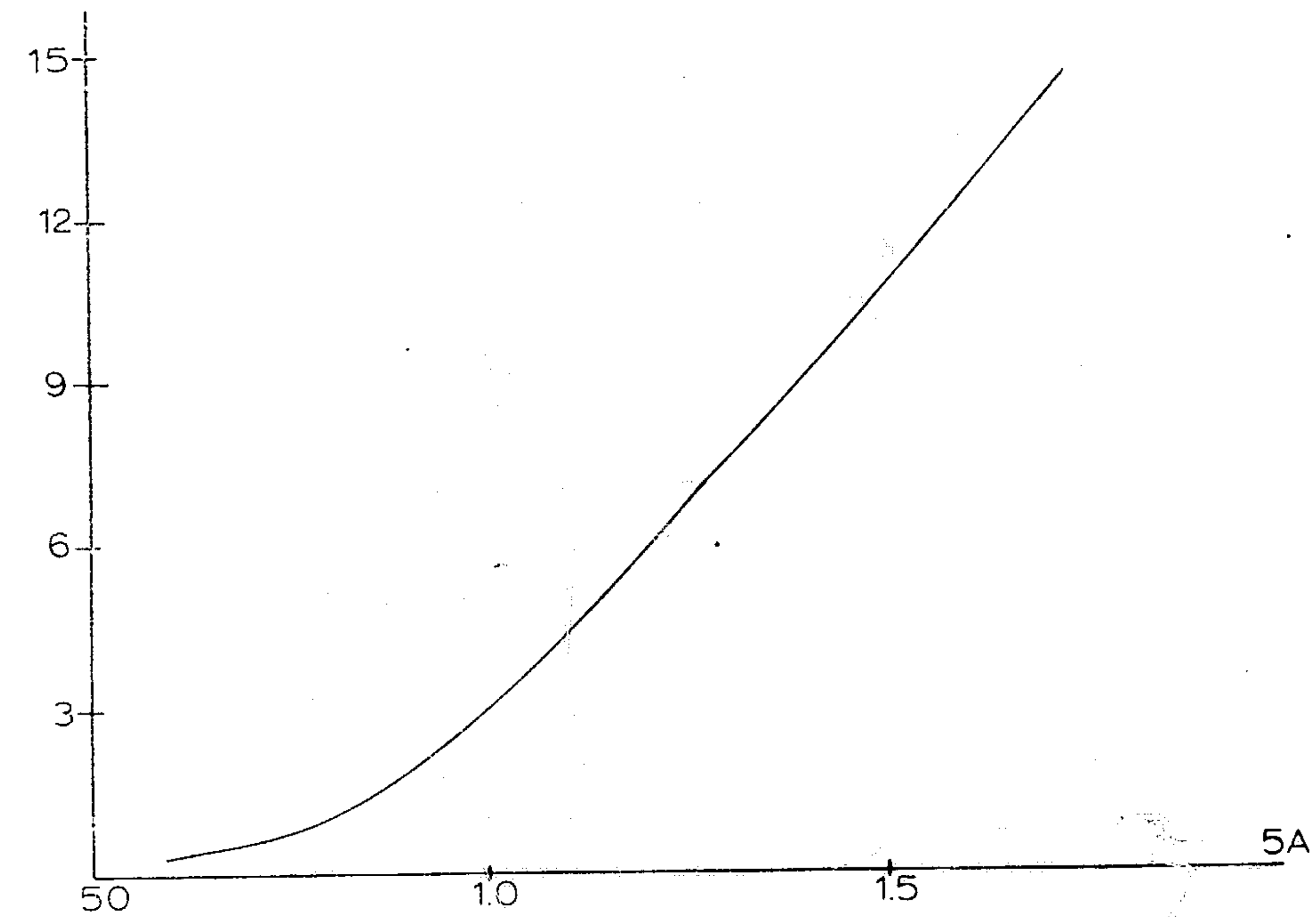
4B

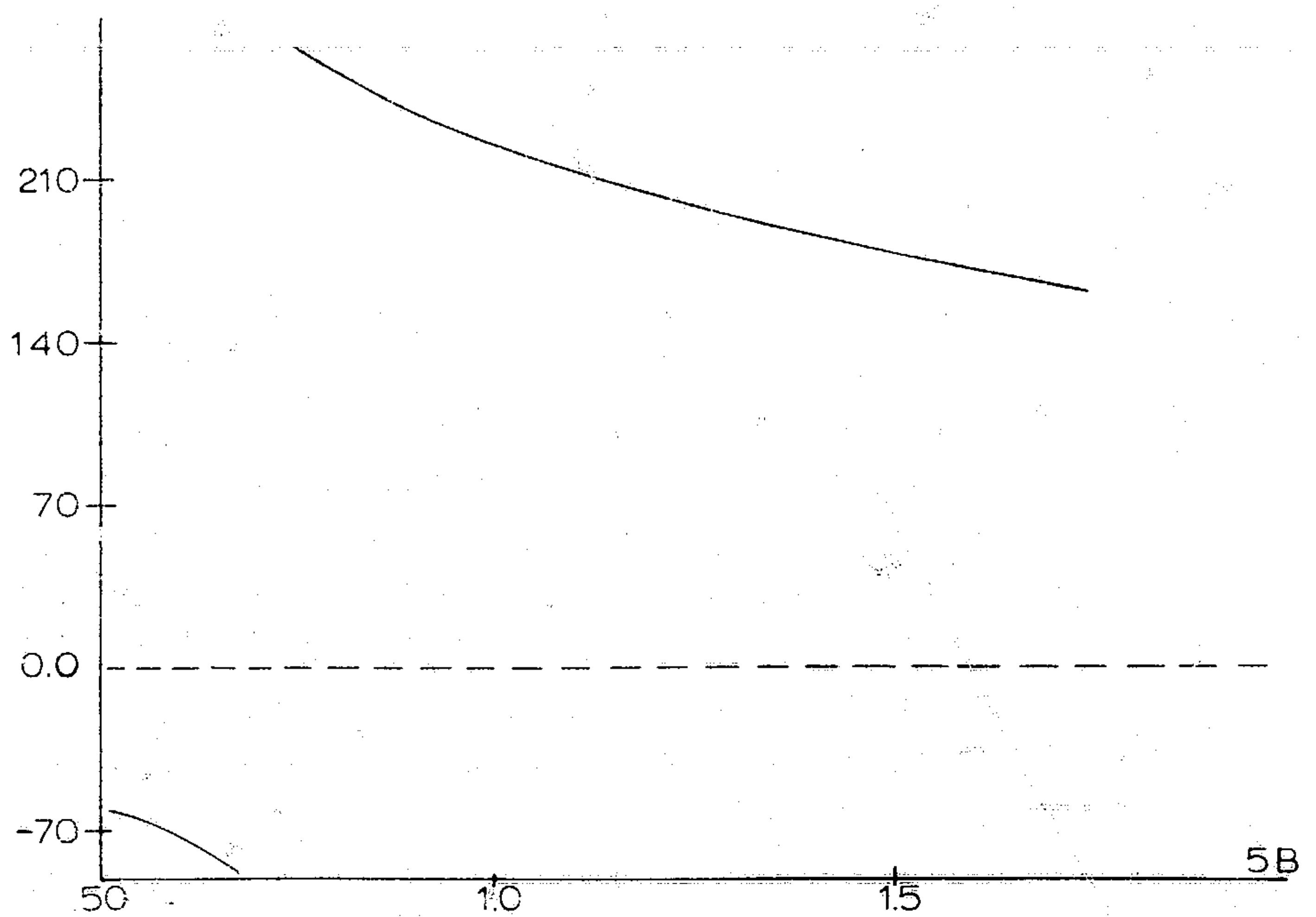
Circuit of figure 4.14-a

5-A $|Y21|$ in db versus ω

$$Y21 = \frac{1}{s^3 + 2s^2 + 2s + 1}$$

5-B phase in degrees versus ω


15
12
9
6
3
50
1.0
1.5
5A

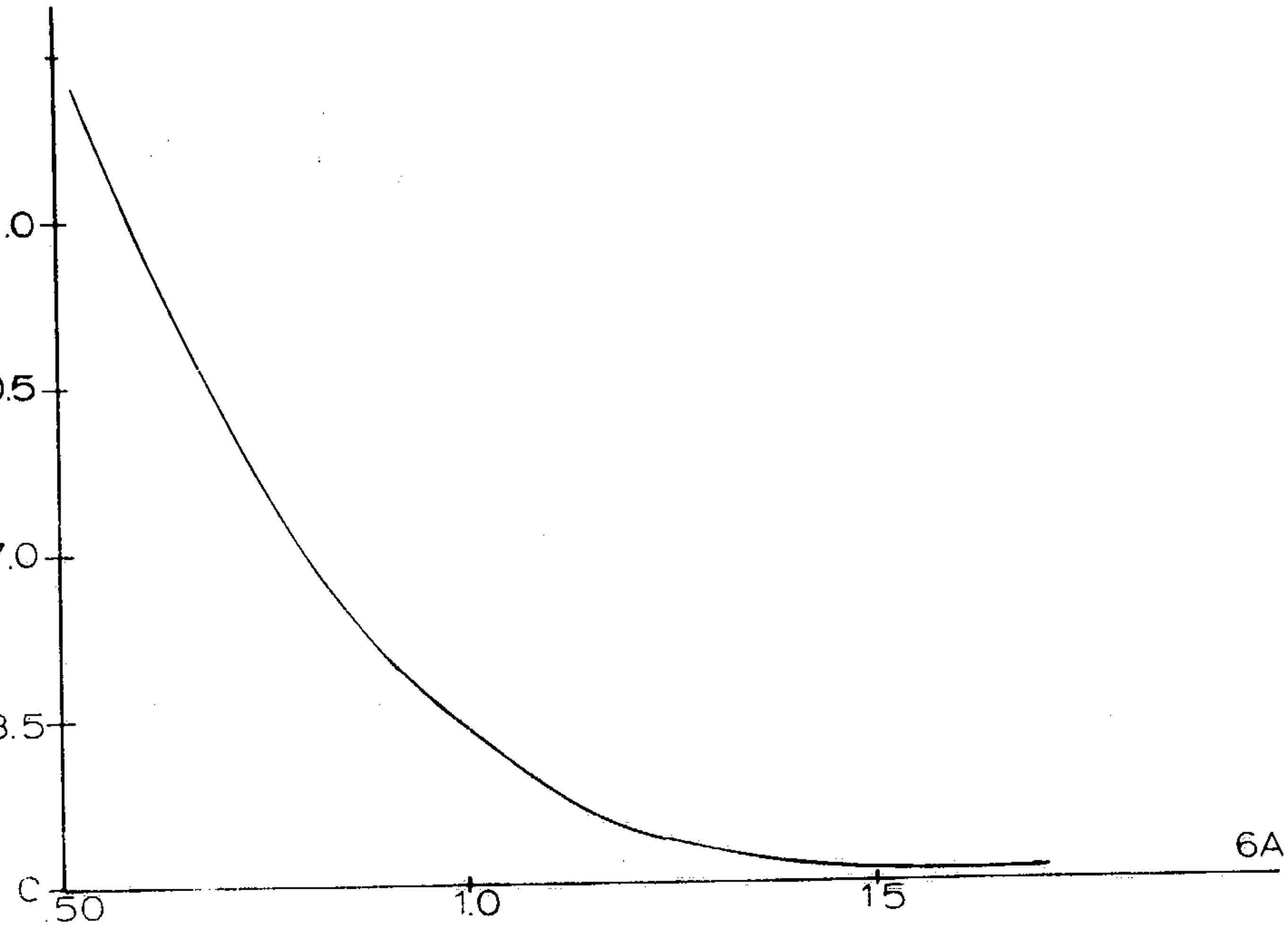

210
140
70
0.0
-70
50
1.0
1.5
5B

Circuit of figure 4.14-b

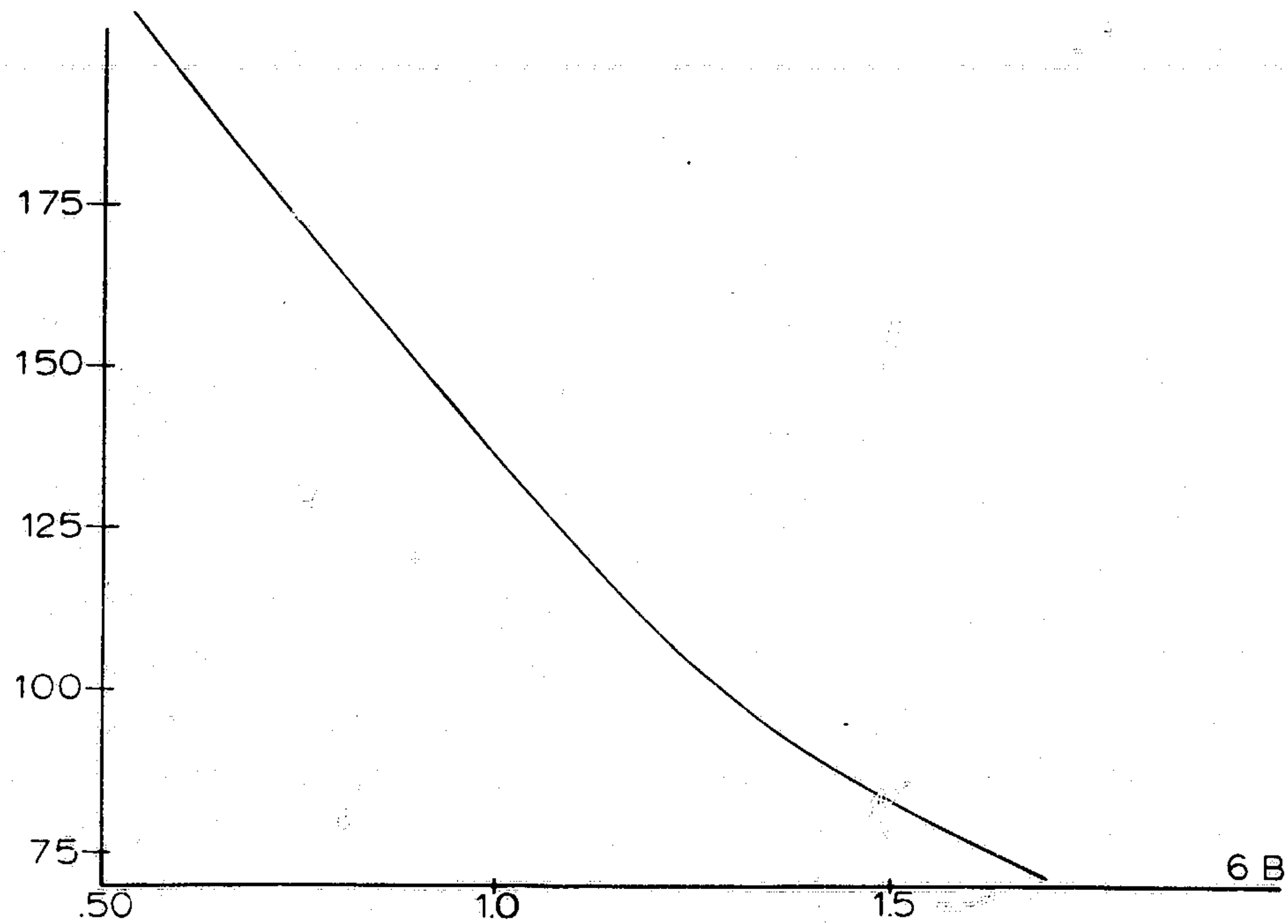
6-A $|Y21|$ in db versus ω

$$Y21 = \frac{s^3}{1 + 2s + 2s^2 + s^3}$$

6-B phase in degrees versus ω



14.0
10.5
7.0
3.5
C
.50
1.0
1.5
6A



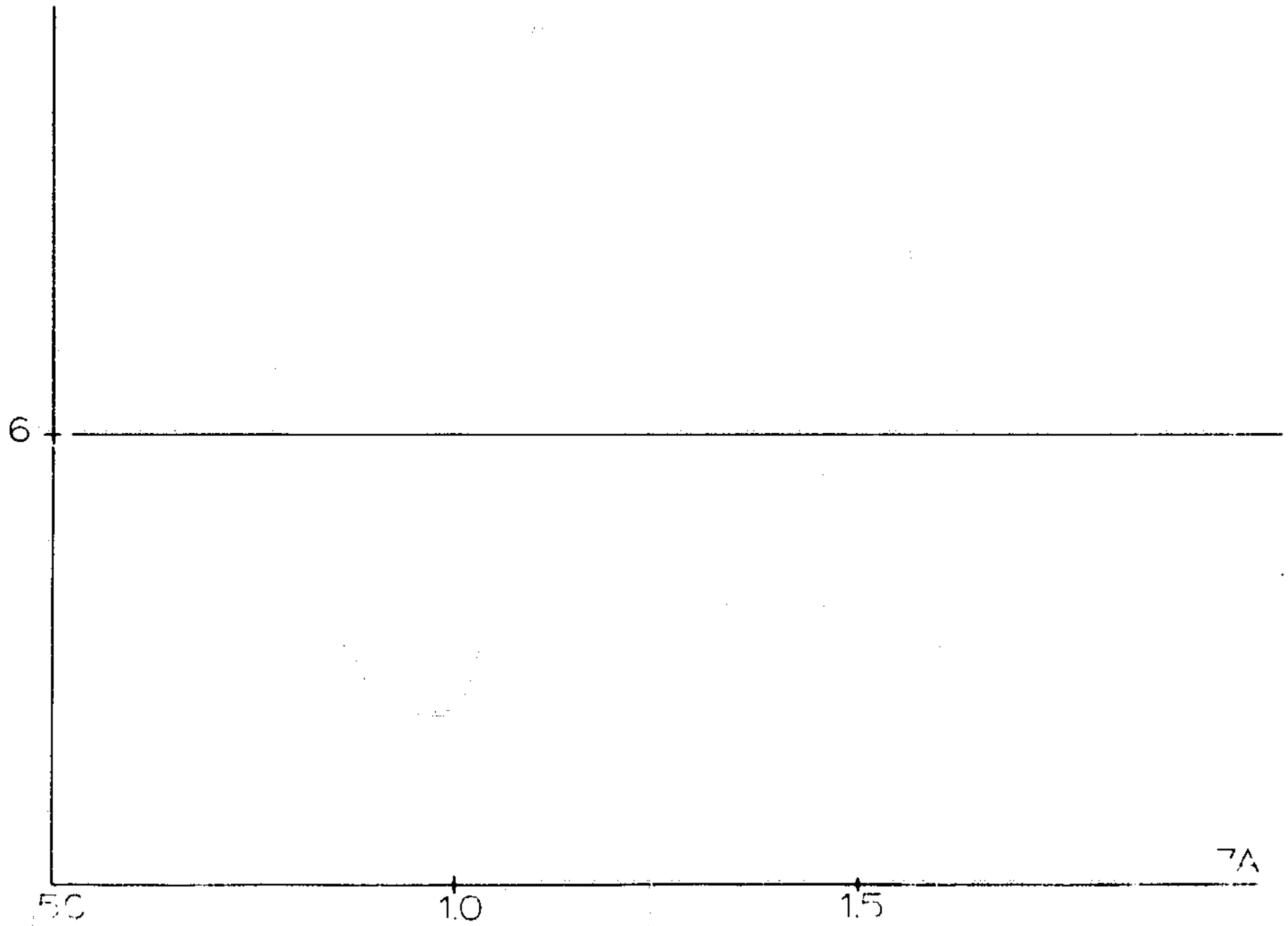
175
150
125
100
75
.50
1.0
1.5
6 B

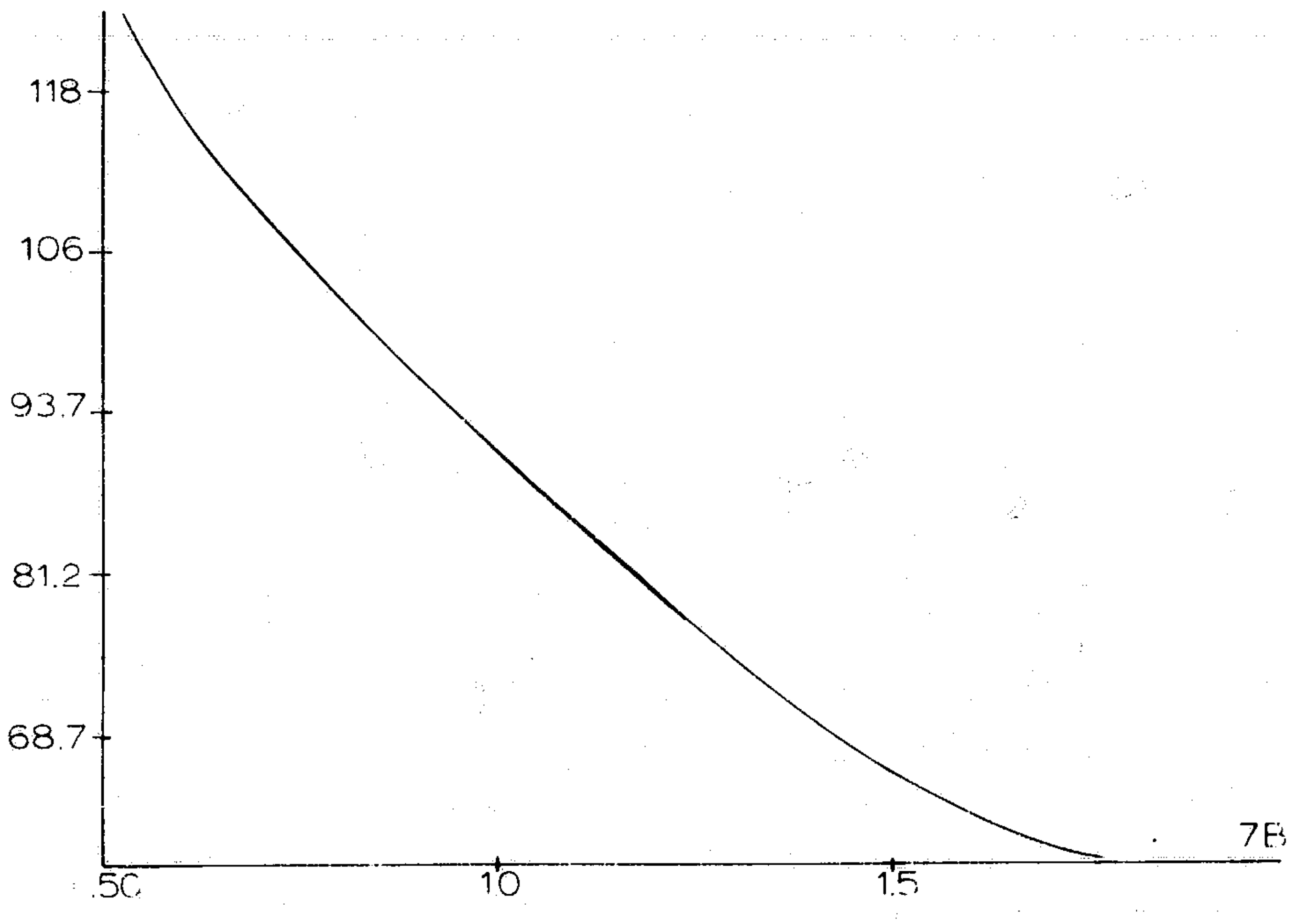
Circuit of figure 4.17-a

7-A $\left|\frac{V2}{Vg}\right|$ in db versus ω

$$\frac{V2}{Vg} = \frac{1}{2}\,\frac{(s - 1)}{(s + 1)}$$

7-B phase in degrees versus ω


6
.50
1.0
1.5
7A

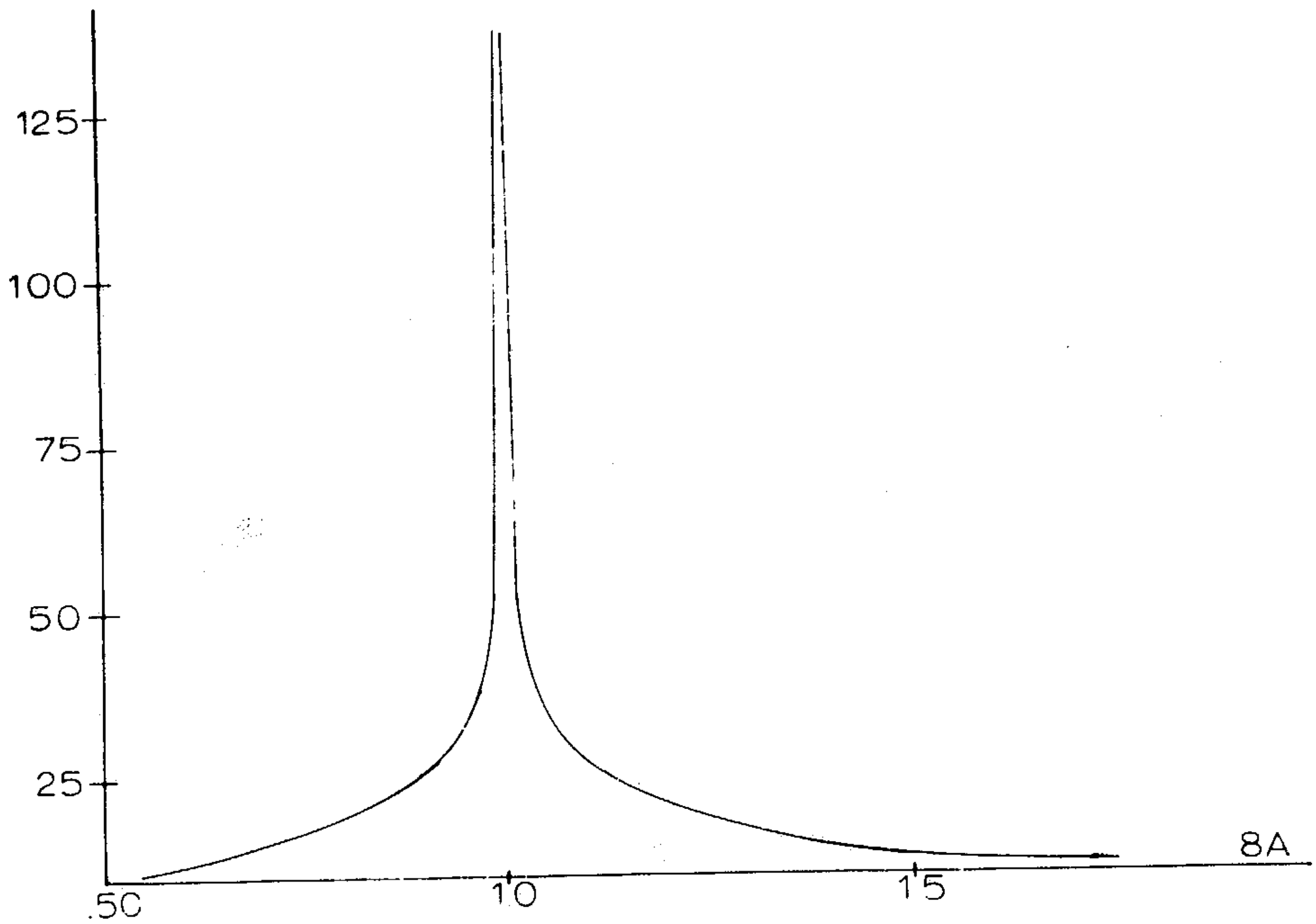

118
106
93.7
81.2
68.7
.50
1.0
1.5
7B

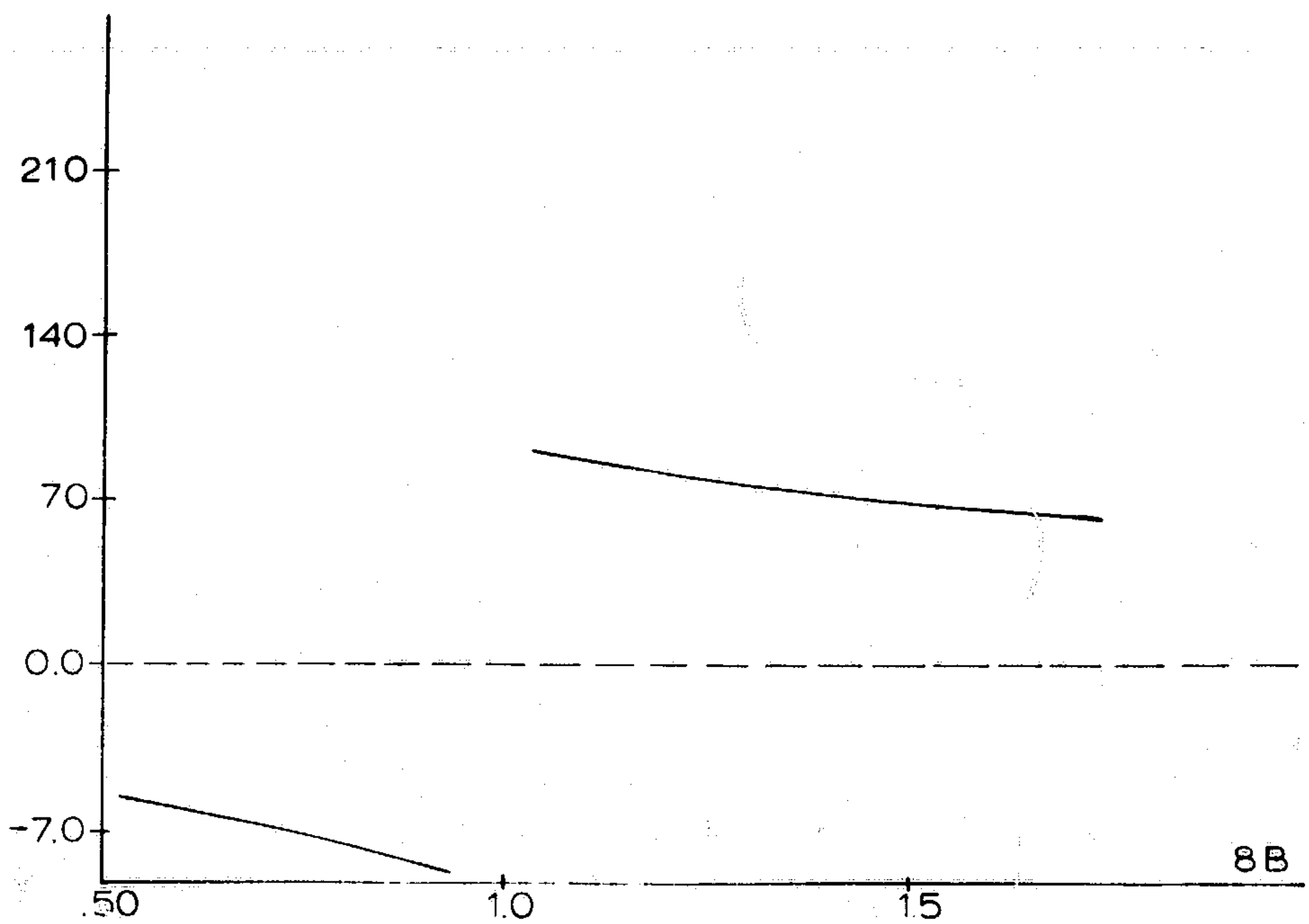
Circuit of figure 4.18

8-A $\left|\frac{V2}{Vg}\right|$ in db versus ω

$$\frac{V2}{Vg} = \frac{s^2 + 1}{2s^2 + 4s + 2}$$

8-B phase in degrees versus ω


125
100
75
50
25
.50
1.0
1.5
8A

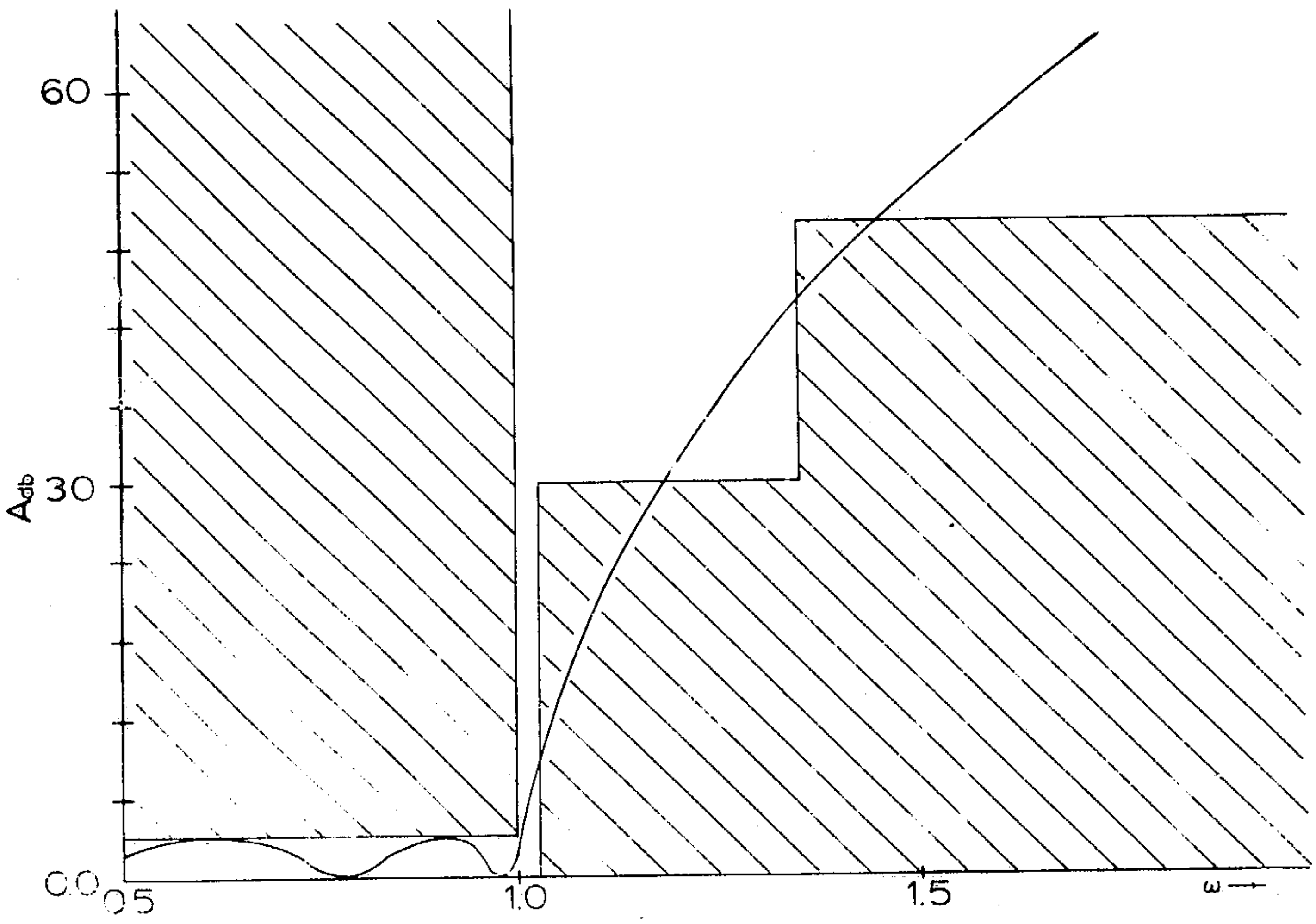

210
140
70
0.0
-7.0
.50
1.0
1.5
8B

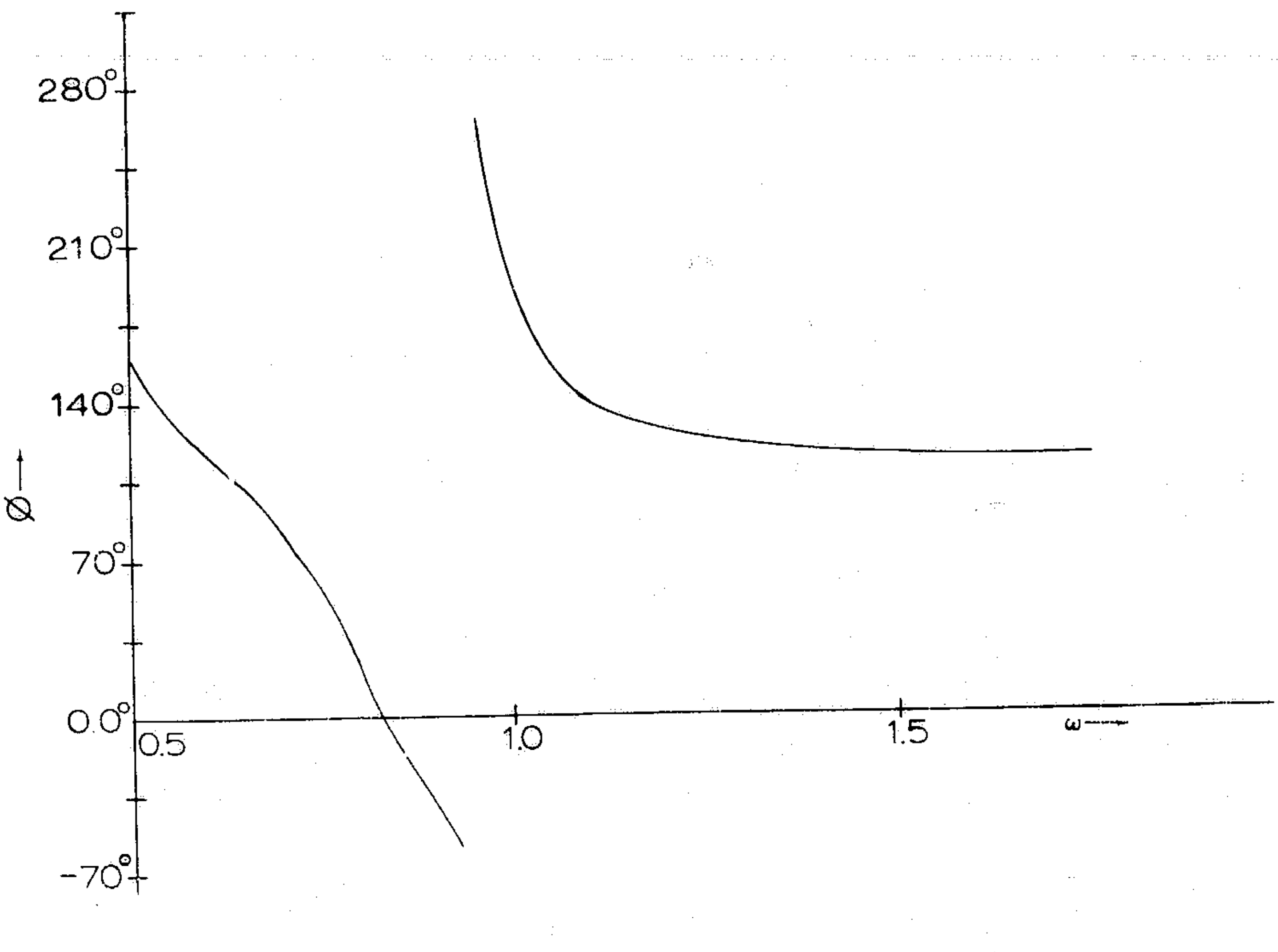
Seventh order Chebyshev filter

with 3db ripple, figure 6.10

9-A $\left|\frac{V2}{Vg}\right|$ in db versus ω

9-B phase in degrees versus ω


60
30
0.0
A_{db}
0.5
1.0
1.5
ω

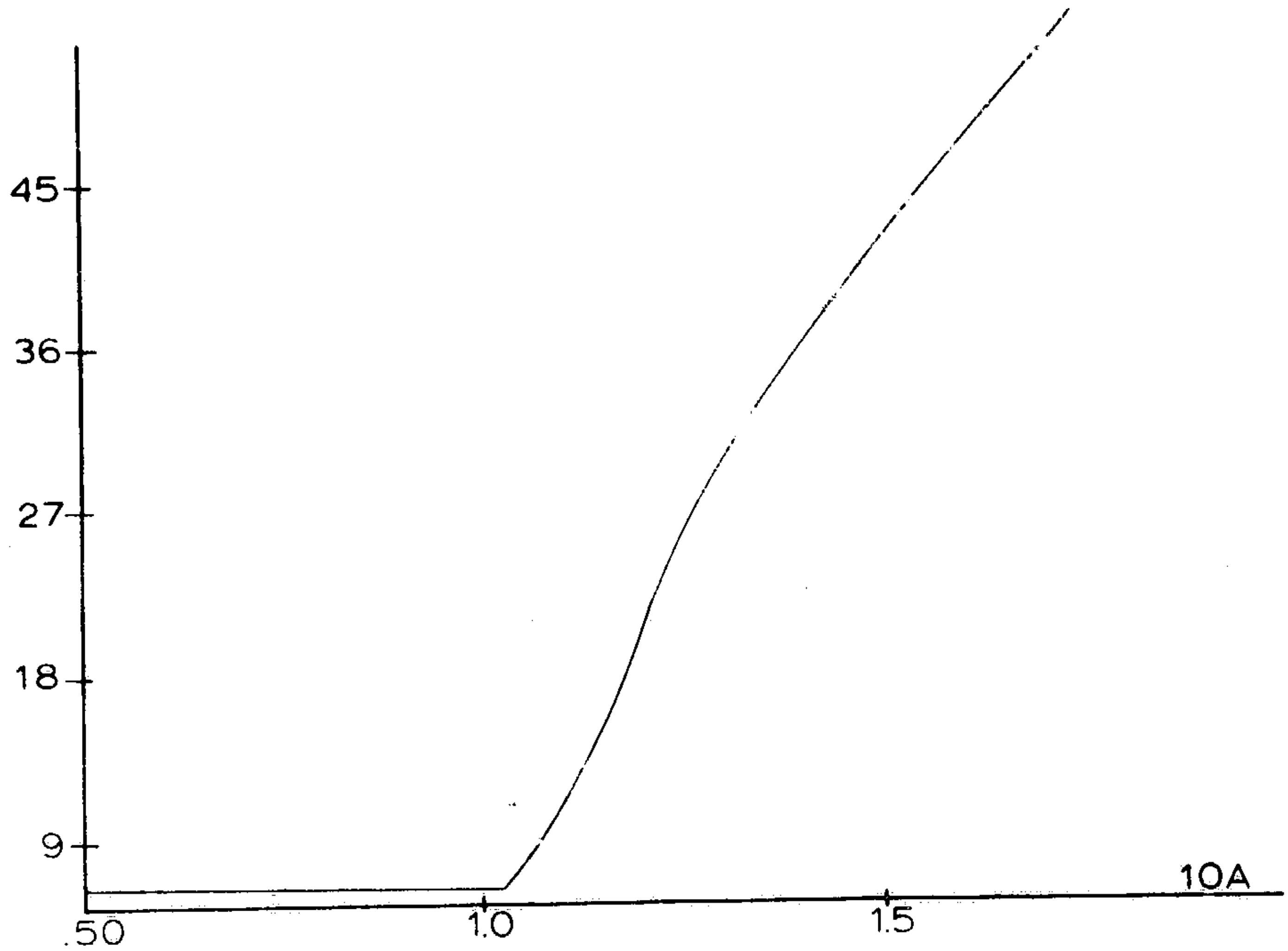

280°
210°
140°
70°
0.0°
-70°
Ø
0.5
1.0
1.5
ω

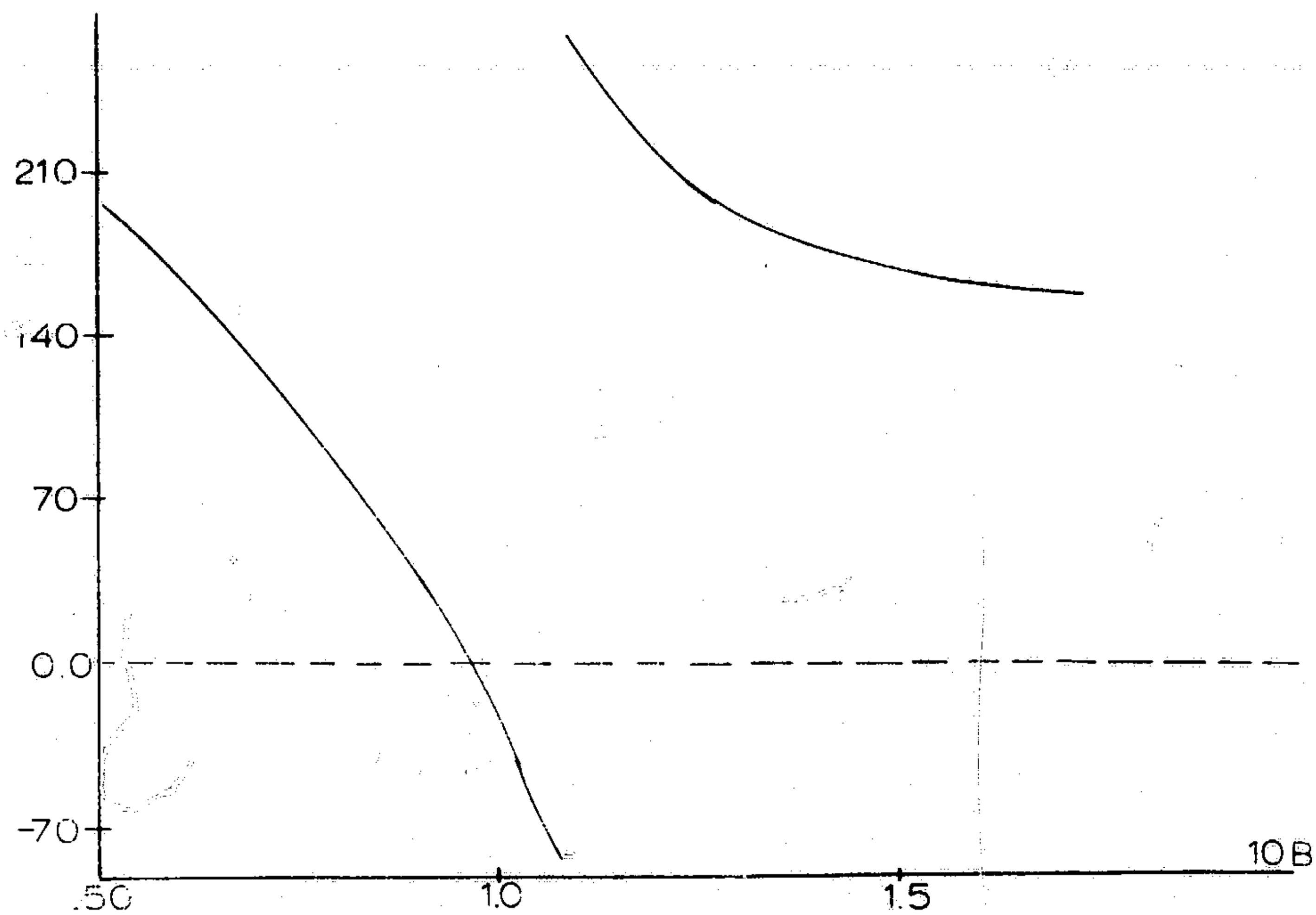
Seventh order Chebyshev filter

with 1/10db ripple

10-A $\left|\frac{V2}{Vg}\right|$ in db versus ω

10-B phase in degrees versus ω

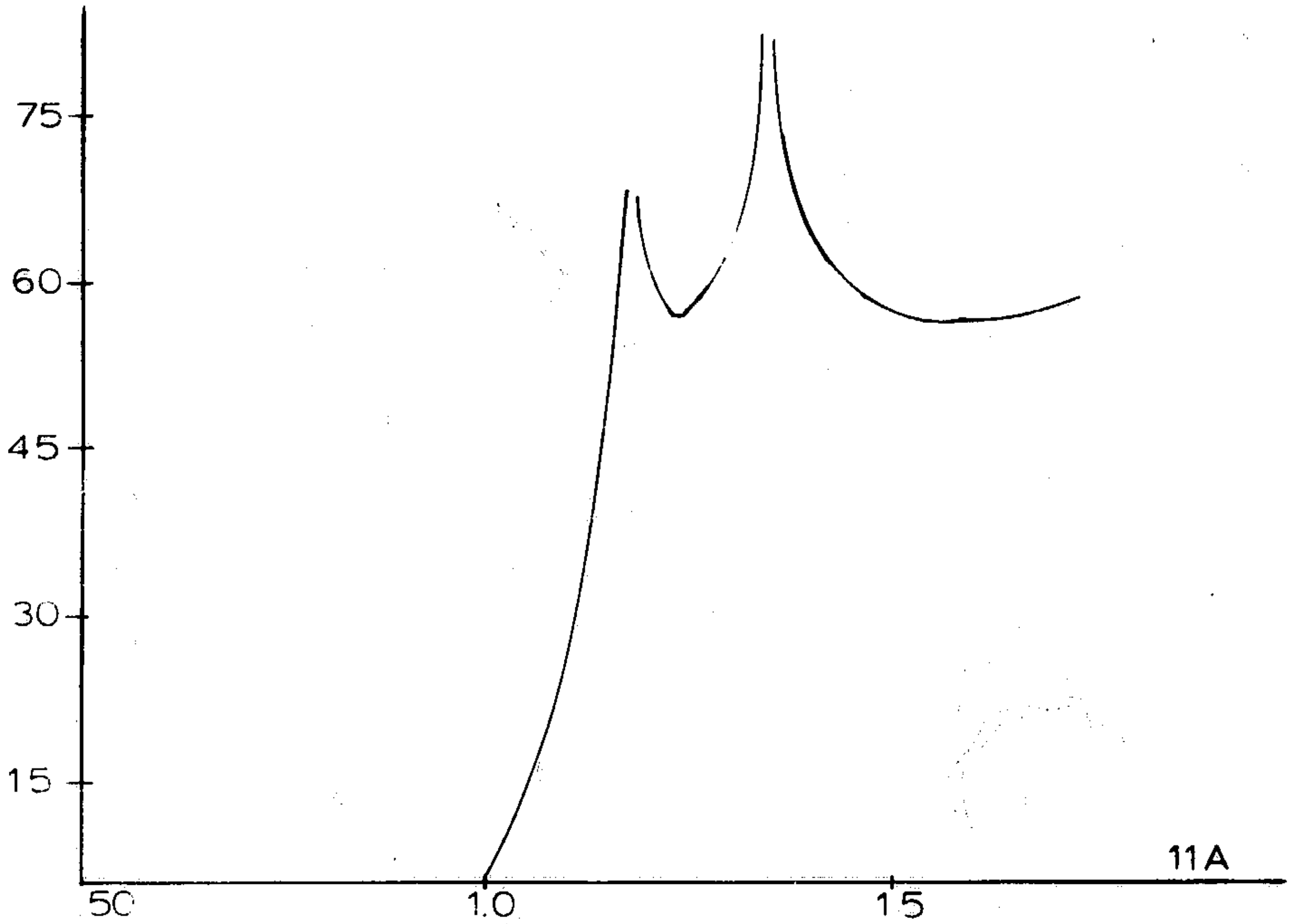

45
36
27
18
9
.50
1.0
1.5
10A


210
140
70
0.0
-70
.50
1.0
1.5
10B

Seventh order elliptical filter

11-A $\left|\frac{V2}{Vg}\right|$ in db versus ω

11-B phase in degrees versus ω


75
60
45
30
15
11A
.50
1.0
1.5

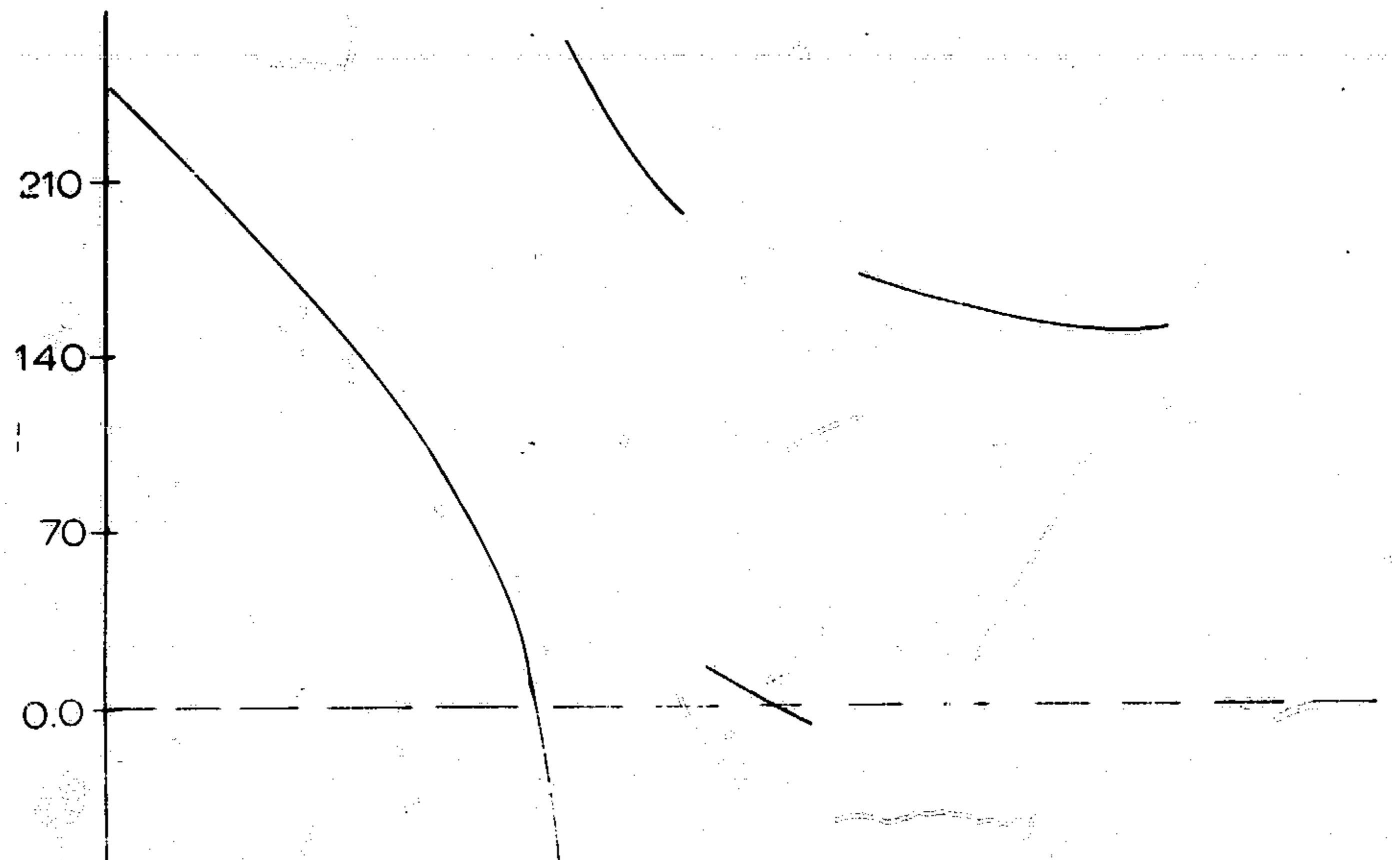
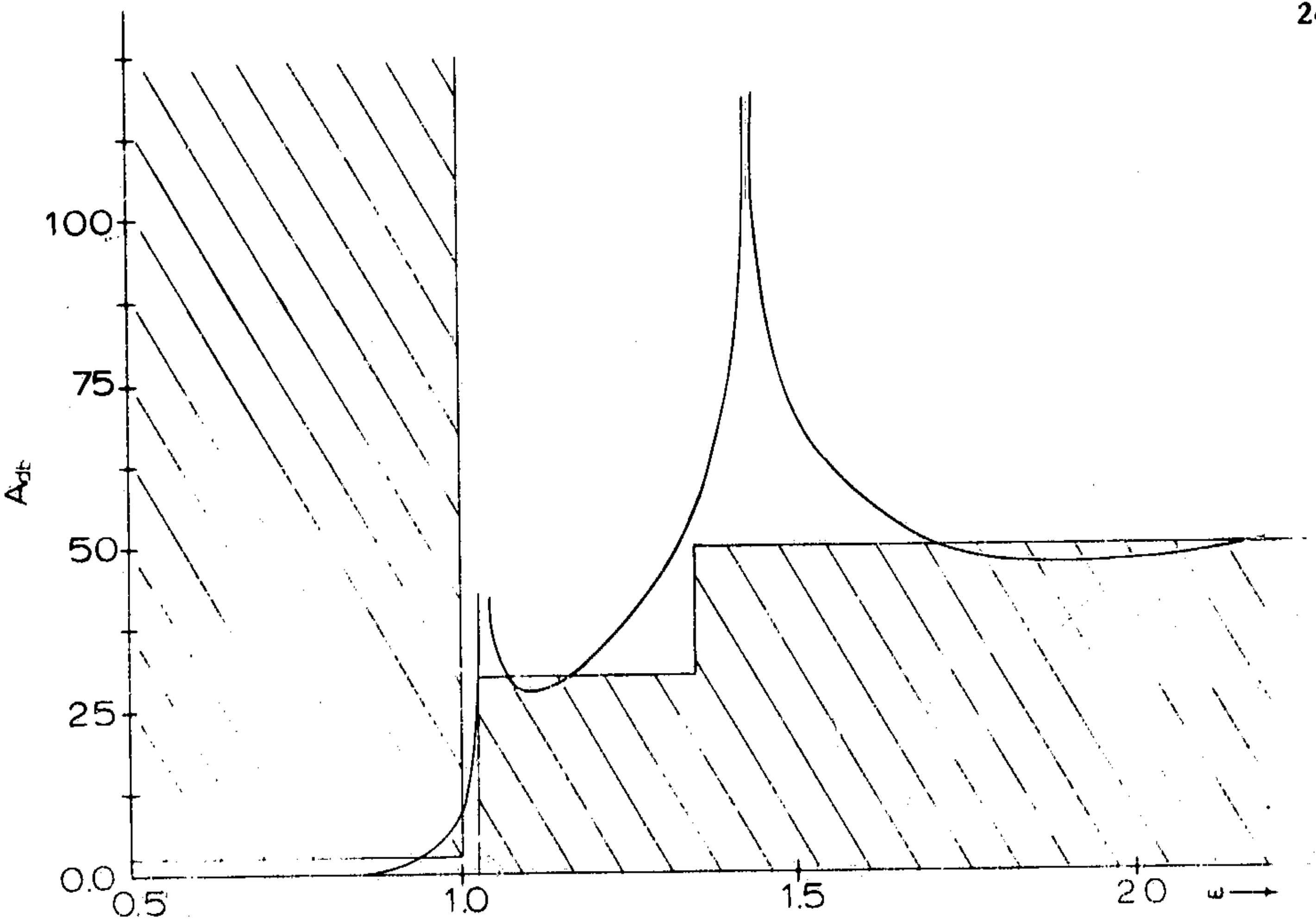
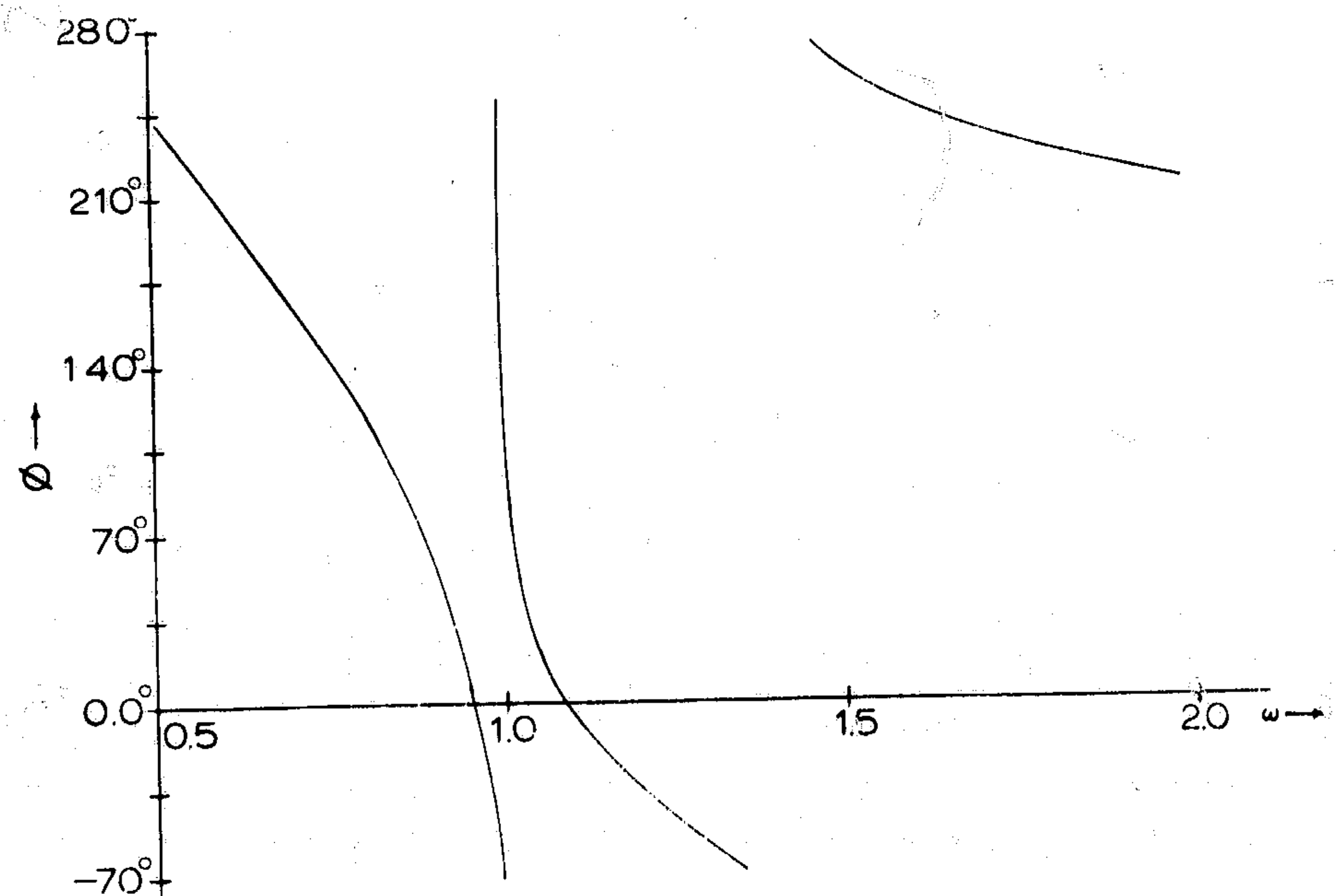
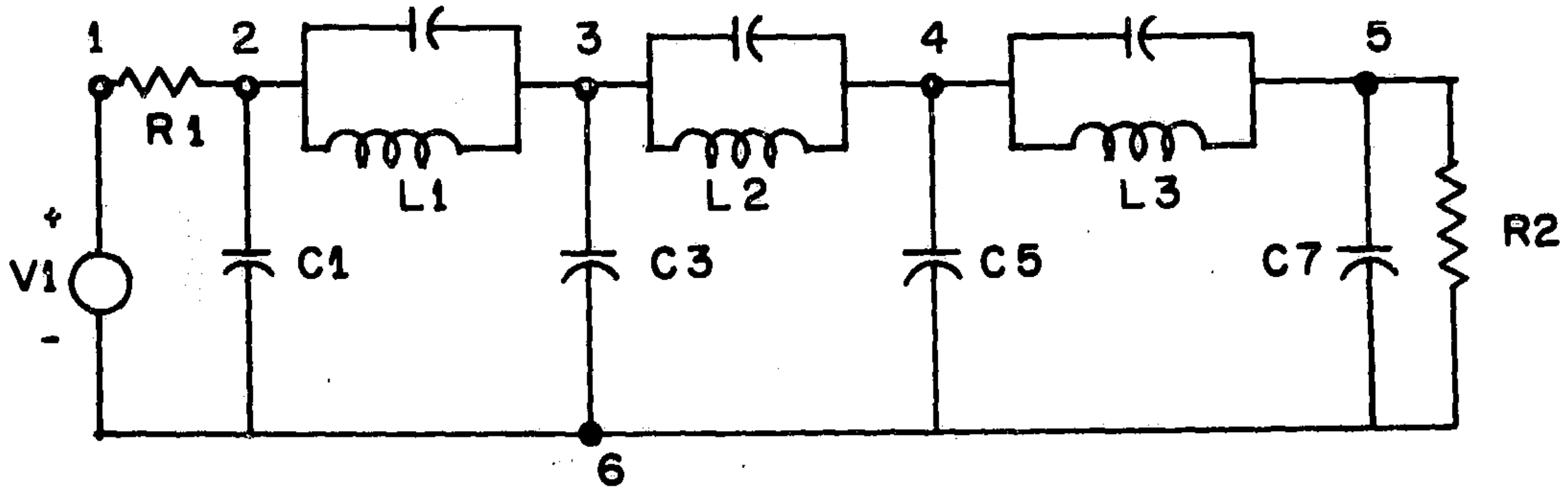

210
140
70
0.0
-70
11B
.50
1.0
1.5

Image parameter filter
designed in Chapter Three
to meet specifications
of Table 1.5.


100
75
50
25
0.0
A_{dt}
0.5
1.0
1.5
2.0
ω →


280°
210°
140°
70°
0.0°
−70°
Ø →
0.5
1.0
1.5
2.0
ω →




```
NASAP PROGRAM ELLIPTICAL FUNCTION FILTER

VL   6   1   1.0
R1   1   2   600.0
R2   5   6   600.0
C1   2   6   .0275UF
C2   2   3   7813.0PF
C3   3   6   .0315UF
C4   3   4   .04685UF
C5   4   6   .02442UF
C6   4   5   .03188UF
C7   5   6   .01558UF
L1   2   3   9.899MH
L2   3   4   4.244MH
L3   4   5   5.232MH
OUTPUT
VR2/VV1
FREQ  -1   6.5   .15
EXECUTE
```

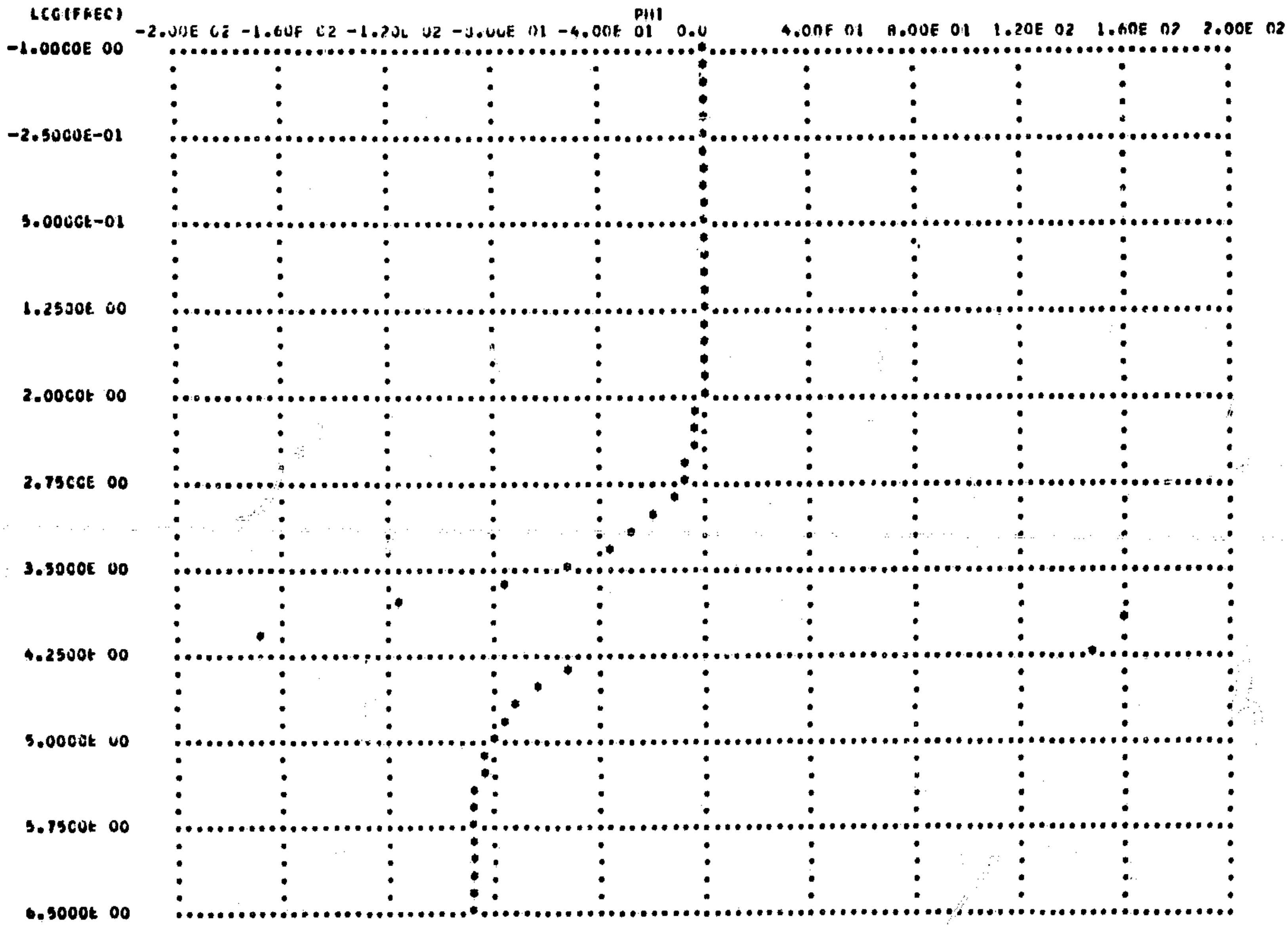
```
NUMBER OF LOOPS PER ORDER

    1=   119
    2=   341
    3=   337
    4=   114
```

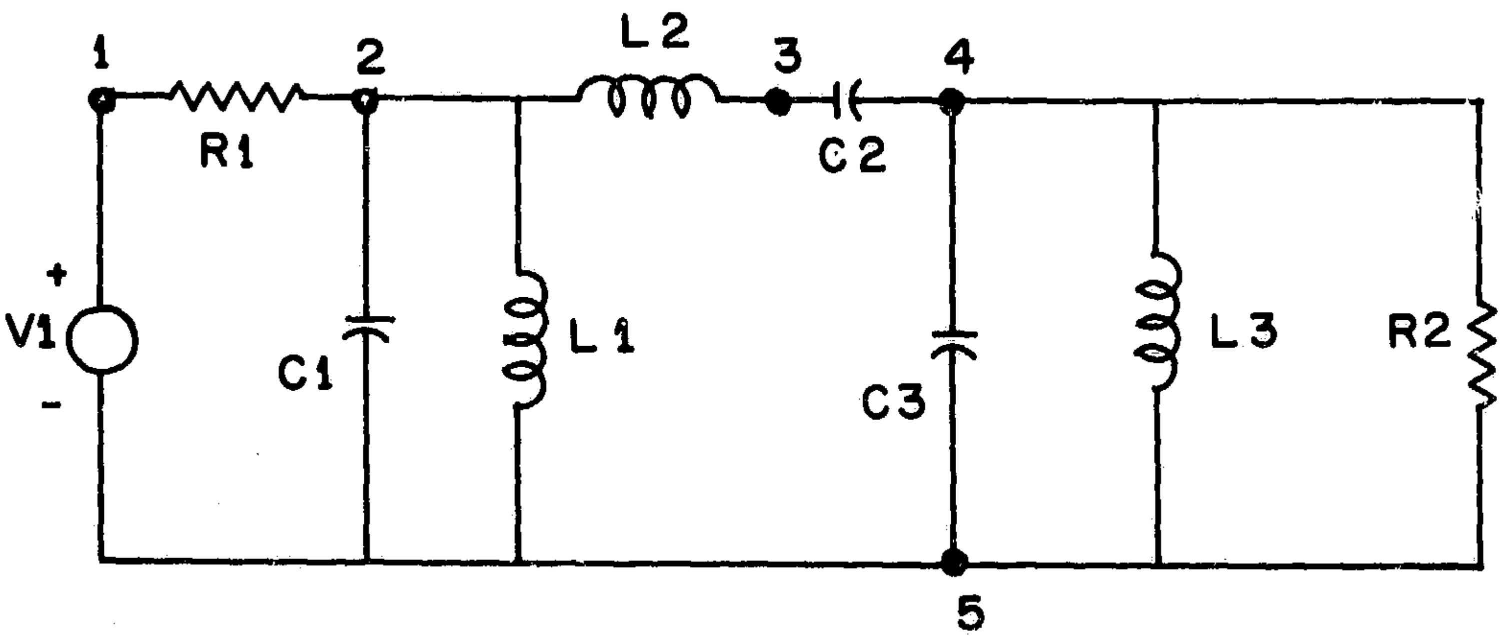
TRANSFER FUNCTION   VR2/VV1

```
        (                                               2             3              4            5             6 )
        (  3.90E 29    +0.0       S   +1.73E 20 S  +0.0        S  +2.40E 10 S  +0.0        S  +1.00E 00 S   )
H(S)= 2.466E 03*------------------------------------------------------------------------------------------------------------
        (                                            2              3              4              5             6             7 )
        (  1.92E 33    +8.81E 28 S   +2.70E 24 S  +6.36E 19 S  +9.43E 14 S  +1.43E 10 S  +9.72E 04 S  +1.00E 00 S   )
```

LOG(FREQ)	FREQ	20.*LOG(ABS(H))	PHI(H)	ABS(H)	LOG(ABS(H))
-0.1000000E 01	0.1000000E 00	-0.6020605E 01	-0.1650447E-02	0.4999998E 00	-0.3010303E 00
-0.8500001E 00	0.1412539E 00	-0.6020605E 01	-0.2331319E-02	0.4999998E 00	-0.3010303E 00
-0.7000002E 00	0.1995264E 00	-0.6020605E 01	-0.3293070E-02	0.4999998E 00	-0.3010303E 00
-0.5500003E 00	0.2818384E 00	-0.6020605E 01	-0.4651587E-02	0.4999998E 00	-0.3010303E 00
-0.4000003E 00	0.3981069E 00	-0.6020605E 01	-0.6570540E-02	0.4999998E 00	-0.3010303E 00
-0.2500004E 00	0.5623409E 00	-0.6020604E 01	-0.9281136E-02	0.4999999E 00	-0.3010302E 00
-0.1000005E 00	0.7943274E 00	-0.6020604E 01	-0.1310995E-01	0.4999998E 00	-0.3010302E 00
0.4999924E-01	0.1122016E 01	-0.6020605E 01	-0.1851825E-01	0.4999998E 00	-0.3010303E 00
0.1999989E 00	0.1584888E 01	-0.6020605E 01	-0.2615770E-01	0.4999998E 00	-0.3010303E 00
0.3499985E 00	0.2238713E 01	-0.6020604E 01	-0.3694872E-01	0.4999998E 00	-0.3010302E 00
0.4999990E 00	0.3162270E 01	-0.6020598E 01	-0.5219151E-01	0.5000000E 00	-0.3010299E 00
0.6499987E 00	0.4466819E 01	-0.6020608E 01	-0.7372236E-01	0.4999996E 00	-0.3010304E 00
0.7999983E 00	0.6309541E 01	-0.6020609E 01	-0.1041355E 00	0.4999995E 00	-0.3010305E 00
0.9499989E 00	0.8912478E 01	-0.6020605E 01	-0.1470956E 00	0.4999998E 00	-0.3010303E 00
0.1099998E 01	0.1258919E 02	-0.6020613E 01	-0.2077777E 00	0.4999993E 00	-0.3010306E 00
0.1249998E 01	0.1778270E 02	-0.6020618E 01	-0.2934934E 00	0.4999990E 00	-0.3010309E 00
0.1399999E 01	0.2511876E 02	-0.6020622E 01	-0.4145711E 00	0.4999988E 00	-0.3010311E 00
0.1549998E 01	0.3549117E 02	-0.6020648E 01	-0.5855967E 00	0.4999973E 00	-0.3010324E 00
0.1699998E 01	0.5011842E 02	-0.6020682E 01	-0.8271745E 00	0.4999953E 00	-0.3010342E 00
0.1849998E 01	0.7079407E 02	-0.6020762E 01	-0.1168412E 01	0.4999907E 00	-0.3010381E 00
0.1999998E 01	0.9999944E 02	-0.6020927E 01	-0.1650422E 01	0.4999812E 00	-0.3010464E 00
0.2149998E 01	0.1412528E 03	-0.6021233E 01	-0.2331262E 01	0.4999636E 00	-0.3010616E 00
0.2299997E 01	0.1995248E 03	-0.6021657E 01	-0.3292946E 01	0.4999277E 00	-0.3010929E 00
0.2449998E 01	0.2818364E 03	-0.6023095E 01	-0.4651259E 01	0.4998564E 00	-0.3011548E 00
0.2599998E 01	0.3981045E 03	-0.6025555E 01	-0.6569674E 01	0.4997149E 00	-0.3012778E 00
0.2749997E 01	0.5623367E 03	-0.6030396E 01	-0.9278811E 01	0.4994365E 00	-0.3015198E 00
0.2899998E 01	0.7943223E 03	-0.6039789E 01	-0.1310383E 02	0.4988967E 00	-0.3019895E 00
0.3049997E 01	0.1122010E 04	-0.6057534E 01	-0.1850296E 02	0.4978785E 00	-0.3028767E 00
0.3199997E 01	0.1584879E 04	-0.6089145E 01	-0.2612439E 02	0.4960698E 00	-0.3044572E 00
0.3349998E 01	0.2238705E 04	-0.6136344E 01	-0.3690729E 02	0.4932680E 00	-0.3069172E 00
0.3499997E 01	0.3162250E 04	-0.6190249E 01	-0.5234583E 02	0.4903290E 00	-0.3095125E 00
0.3649997E 01	0.4466793E 04	-0.6170273E 01	-0.7548904E 02	0.4914980E 00	-0.3085136E 00
0.3799997E 01	0.6309520E 04	-0.6023762E 01	-0.1149146E 03	0.4998181E 00	-0.3011881E 00
0.3949997E 01	0.8912422E 04	-0.6152782E 01	0.1612816E 03	0.4924486E 00	-0.3076391E 00
0.4099997E 01	0.1258913E 05	-0.5099422E 02	-0.1673235E 03	0.2820258E-02	-0.2549711E 01
0.4249996E 01	0.1778259E 05	-0.6433995E 02	0.1474418E 03	0.6067392E-03	-0.3216998E 01
0.4399997E 01	0.2511862E 05	-0.4322092E 02	-0.5237125E 02	0.6901659E-02	-0.2161046E 01
0.4549996E 01	0.3548095E 05	-0.4204590E 02	-0.6423639E 02	0.7901415E-02	-0.2102295E 01
0.4699996E 01	0.5011812E 05	-0.4347673E 02	-0.7204324E 02	0.6701354E-02	-0.2173837E 01
0.4849997E 01	0.7079361E 05	-0.4577791E 02	-0.7758330E 02	0.5141672E-02	-0.2288896E 01
0.4999996E 01	0.9999881E 05	-0.4844624E 02	-0.8110127E 02	0.3781702E-02	-0.2422313E 01
0.5149996E 01	0.1412521E 06	-0.5128435E 02	-0.8371182E 02	0.2727607E-02	-0.2564218E 01
0.5299996E 01	0.1995239E 06	-0.5420427E 02	-0.8555243E 02	0.1948884E-02	-0.2710214E 01
0.5449996E 01	0.2818350E 06	-0.5716438E 02	-0.8689278E 02	0.1386055E-02	-0.2858219E 01
0.5599996E 01	0.3981020E 06	-0.6014445E 02	-0.8777246E 02	0.9835064E-03	-0.3007223E 01
0.5749996E 01	0.5623349E 06	-0.6313449E 02	-0.8842317E 02	0.6970684E-03	-0.3156725E 01
0.5899996E 01	0.7943102E 06	-0.6612949E 02	-0.8888376E 02	0.4937707E-03	-0.3306475E 01
0.6049995E 01	0.1122003E 07	-0.6912698E 02	-0.8920975E 02	0.3496637E-03	-0.3456349E 01
0.6199996E 01	0.1584874E 07	-0.7212575E 02	-0.8944055E 02	0.2475781E-03	-0.3606288E 01
0.6349996E 01	0.2238601E 07	-0.7512509E 02	-0.8960393E 02	0.1752850E-03	-0.3756255E 01
0.6499995E 01	0.3162231E 07	-0.7812477E 02	-0.8971960E 02	0.1240970E-03	-0.3906239E 01

PRECEDING PAGE BLANK NOT FILMED.



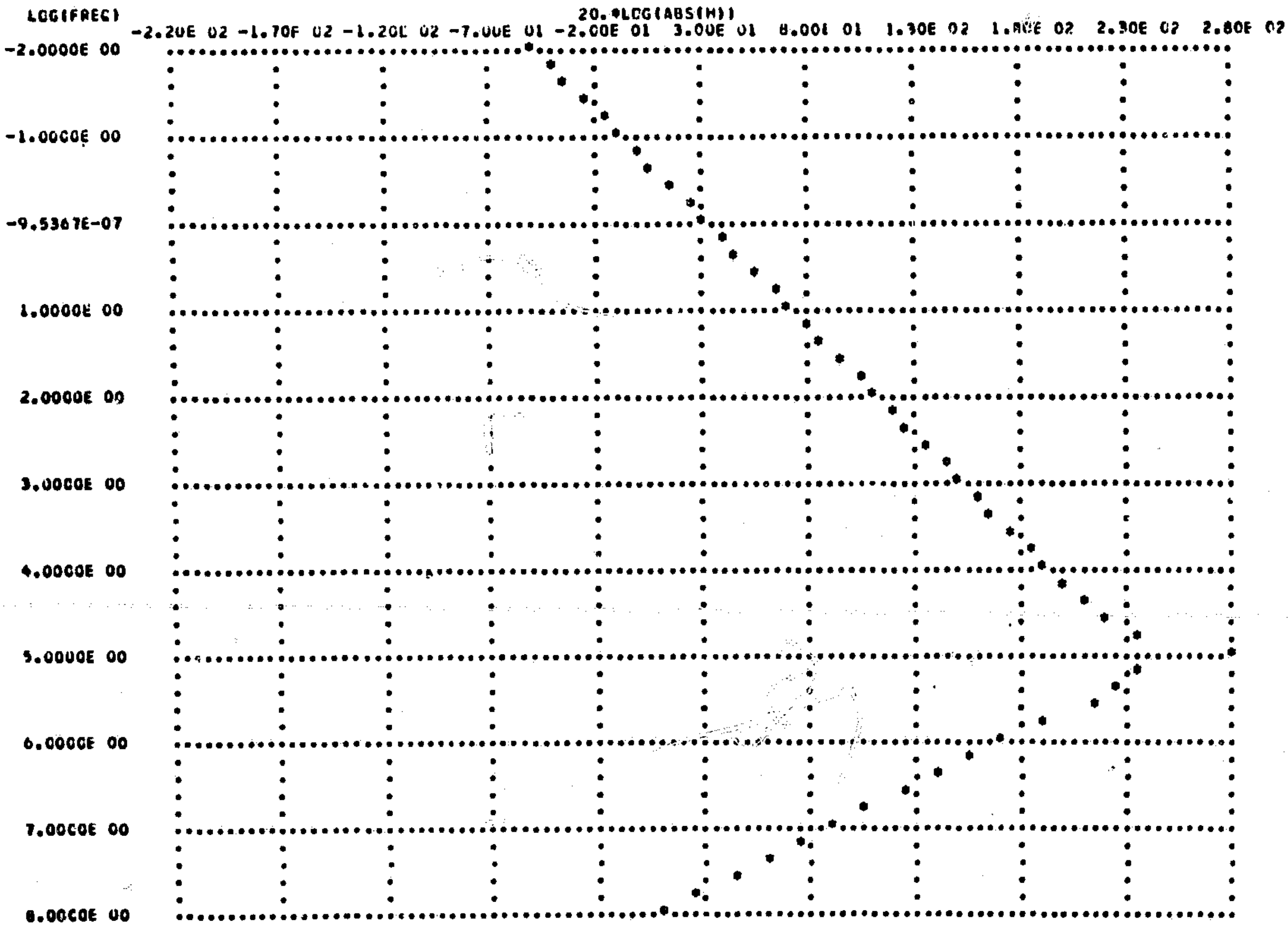


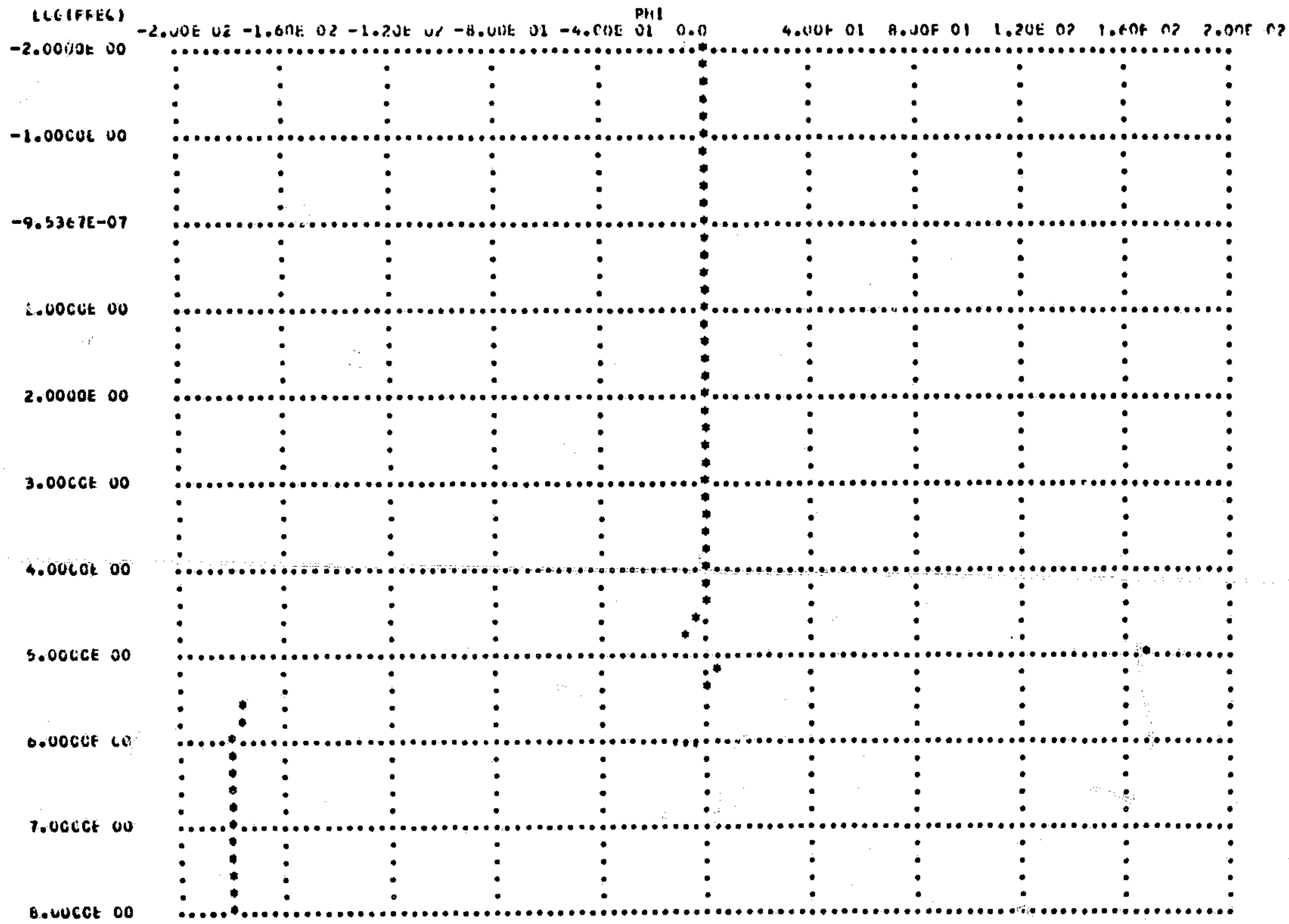

```
NASAP PROGRAM BUTTERWORTH BANDPASS FILTER

V1   5   1   1.0
R1   1   2   100.0
C1   2   5   .16UF
L1   2   5   16.0UH
L2   2   3   3.2MH
C2   3   4   800.0PF
C3   4   5   0.16UF
L3   4   5   1.6UH
R2   4   5   100.0
OUTPUT
VR2/VV1
FREQ   -2    8    .2
EXECUTE



NUMBER OF LOOPS PER ORDER

     1=     8
     2=    12
     3=     4
```

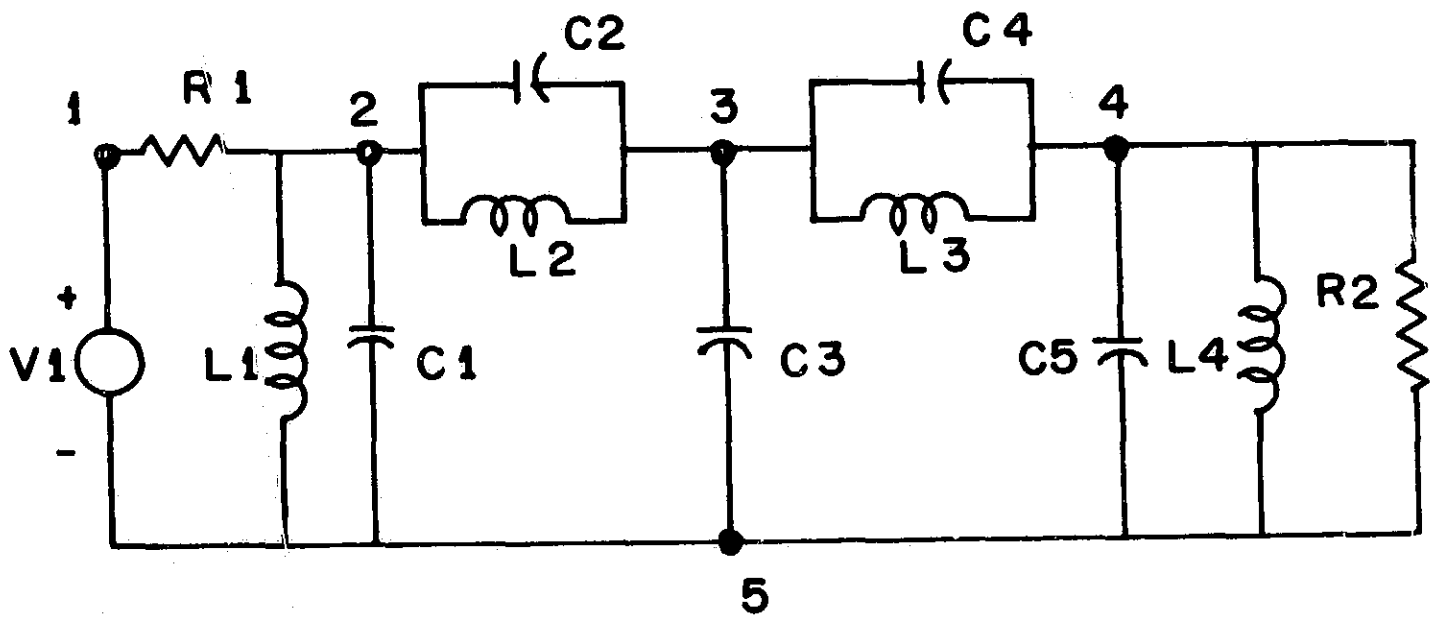
TRANSFER FUNCTION VR2/VV1

$$H(S) = 1.221E\ 14 \cdot \frac{0.0 + 0.0\ S - 4.58E\ 21\ S^{2} + 1.00E\ 00\ S^{3}}{5.56E\ 35 + 1.05E\ 29\ S + 3.21E\ 24\ S^{2} + 3.18E\ 17\ S^{3} + 4.70E\ 12\ S^{4} + 1.25E\ 05\ S^{5} + 1.00E\ 00\ S^{6}}$$

LOG(FREQ)	FREQ	20.*LOG(ABS(H))	PHI(H)	ABS(H)	LOG(ABS(H))
-0.2000000E 01	0.1000001E-01	-0.4863475E 02	-0.6335994E-06	0.3700510E-02	-0.2431738E 01
-0.1799999E 01	0.1584898E-01	-0.4063472E 02	-0.1004189E-05	0.9295296E-02	-0.2031736E 01
-0.1599999E 01	0.2511893E-01	-0.3263474E 02	-0.1591533E-05	0.2334872E-01	-0.1631737E 01
-0.1400000E 01	0.3981078E-01	-0.2463475E 02	-0.2522406E-05	0.5864920E-01	-0.1231738E 01
-0.1200000E 01	0.6309581E-01	-0.1663475E 02	-0.3997741E-05	0.1473200E 00	-0.8317382E 00
-0.1000000E 01	0.1000000E 00	-0.8634775E 01	-0.6335991E-05	0.3700507E 00	-0.4317388E 00
-0.8000011E 00	0.1584890E 00	-0.6348119E 00	-0.1004185E-04	0.9295214E 00	-0.3174060E-01
-0.6000013E 00	0.2511880E 00	0.7365161E 01	-0.1591523E-04	0.2334845E 01	0.3682581E 00
-0.4000006E 00	0.3981067E 00	0.1536519E 02	-0.2522400E-04	0.5864889E 01	0.7682595E 00
-0.2000008E 00	0.6309563E 00	0.2336517E 02	-0.3997733E-04	0.1473192E 02	0.1168259E 01
-0.9536743E-06	0.9999978E 00	0.3136516E 02	-0.6335969E-04	0.3700488E 02	0.1568258E 01
0.1999989E 00	0.1584888E 01	0.3936516E 02	-0.1004183E-03	0.9295192E 02	0.1968258E 01
0.3999987E 00	0.2511878E 01	0.4736514E 02	-0.1591523E-03	0.2334845E 03	0.2368258E 01
0.5999985E 00	0.3981055E 01	0.5536514E 02	-0.2522392E-03	0.5864854E 03	0.2768257E 01
0.7999983E 00	0.6309541E 01	0.6336513E 02	-0.3997718E-03	0.1473182E 04	0.3168257E 01
0.9999981E 00	0.9999948E 01	0.7136513E 02	-0.6335955E-03	0.3700465E 04	0.3568256E 01
0.1199999E 01	0.1584888E 02	0.7936516E 02	-0.1004183E-02	0.9295180E 04	0.3968258E 01
0.1399999E 01	0.2511876E 02	0.8736514E 02	-0.1591521E-02	0.2334842E 05	0.4368258E 01
0.1599998E 01	0.3981053E 02	0.9536514E 02	-0.2522389E-02	0.5864853E 05	0.4768257E 01
0.1799998E 01	0.6309538E 02	0.1033651E 03	-0.3997713E-02	0.1473180E 06	0.5168256E 01
0.1999998E 01	0.9999944E 02	0.1113651E 03	-0.6335959E-02	0.3700470E 06	0.5568257E 01
0.2199998E 01	0.1584883E 03	0.1193651E 03	-0.1004181E-01	0.9295159E 06	0.5968257E 01
0.2399998E 01	0.2511869E 03	0.1273652E 03	-0.1591524E-01	0.2334860E 07	0.6368261E 01
0.2599998E 01	0.3981045E 03	0.1353654E 03	-0.2522421E-01	0.5865025E 07	0.6768270E 01
0.2799998E 01	0.6309536E 03	0.1433658E 03	-0.3997857E-01	0.1473305E 08	0.7168291E 01
0.2999998E 01	0.9999939E 03	0.1513669E 03	-0.6336534E-01	0.3701242E 08	0.7568345E 01
0.3199998E 01	0.1584882E 04	0.1593697E 03	-0.1004414E 00	0.9300062E 08	0.7968485E 01
0.3399998E 01	0.2511868E 04	0.1673767E 03	-0.1592451E 00	0.2337954E 09	0.8368837E 01
0.3599998E 01	0.3981042E 04	0.1753942E 03	-0.2526104E 00	0.5894590E 09	0.8769712E 01
0.3799997E 01	0.6309520E 04	0.1834385E 03	-0.4012556E 00	0.1485699E 10	0.9171926E 01
0.3999997E 01	0.9999914E 04	0.1915503E 03	-0.6395420E 00	0.3780225E 10	0.9577514E 01
0.4199997E 01	0.1584878E 05	0.1996339E 03	-0.1028228E 01	0.9810600E 10	0.9991696E 01
0.4399998E 01	0.2511867E 05	0.2065655E 03	-0.1691178E 01	0.2680878E 11	0.1042828E 02
0.4599998E 01	0.3981041E 05	0.2185385E 03	-0.2964509E 01	0.8451339E 11	0.1092692E 02
0.4799997E 01	0.6309518E 05	0.2326662E 03	-0.6470982E 01	0.4298451E 12	0.1163331E 02
0.4999997E 01	0.9999512E 05	0.2782061E 03	0.1665131E 03	0.8134166E 14	0.1391031E 02
0.5199997E 01	0.1584877E 06	0.2344121E 03	0.4721907E 01	0.5255361E 12	0.1172060E 02
0.5399997E 01	0.2511861E 06	0.2269271E 03	-0.1288132E 01	0.2220029E 12	0.1134635E 02
0.5599997E 01	0.3981027E 06	0.2127017E 03	-0.1746643E 03	0.4316092E 11	0.1063509E 02
0.5799996E 01	0.6309503E 06	0.1900083E 03	-0.1778716E 03	0.3165354E 10	0.9500418E 01
0.5999997E 01	0.9999906E 06	0.1721673E 03	-0.1787918E 03	0.4058499E 09	0.8608363E 01
0.6199997E 01	0.1584877E 07	0.1555072E 03	-0.1792646E 03	0.5961611E 08	0.7775360E 01
0.6399997E 01	0.2511860E 07	0.1392542E 03	-0.1795422E 03	0.9177184E 07	0.6962708E 01
0.6599997E 01	0.3981026E 07	0.1231549E 03	-0.1797126E 03	0.1437959E 07	0.6157746E 01
0.6799996E 01	0.6309500E 07	0.1071156E 03	-0.1798191E 03	0.2268720E 06	0.5355782E 01
0.6999996E 01	0.9999769E 07	0.9110040E 02	-0.1798859E 03	0.3589383E 05	0.4555020E 01
0.7199996E 01	0.1584870E 08	0.7509387E 02	-0.1799280E 03	0.5684527E 04	0.3754694E 01
0.7399996E 01	0.2511832E 08	0.5909166E 02	-0.1799546E 03	0.9007061E 03	0.2954583E 01
0.7599997E 01	0.3981021E 08	0.4309036E 02	-0.1799714E 03	0.1427310E 03	0.2154519E 01
0.7799996E 01	0.6309450E 08	0.2709019E 02	-0.1799819E 03	0.2262096E 02	0.1354510E 01
0.7999996E 01	0.9999736E 08	0.1109030E 02	-0.1799886E 03	0.3595215E 01	0.5545151E 00


LOG(FREQ)
20.*LOG(ABS(H))
-2.20E 02 -1.70F 02 -1.20E 02 -7.00E 01 -2.00E 01 3.00E 01 8.00E 01 1.30E 02 1.80E 02 2.30E 02 2.80E 02
-2.0000E 00
-1.0000E 00
-9.5367E-07
1.0000E 00
2.0000E 00
3.0000E 00
4.0000E 00
5.0000E 00
6.0000E 00
7.0000E 00
8.0000E 00


LOG(FREQ)
PHI
-2.00E 02 -1.60E 02 -1.20E 02 -8.00E 01 -4.00E 01 0.0 4.00E 01 8.00E 01 1.20E 02 1.60E 02 2.00E 02
-2.0000E 00
-1.0000E 00
-9.5367E-07
1.0000E 00
2.0000E 00
3.0000E 00
4.0000E 00
5.0000E 00
6.0000E 00
7.0000E 00
8.0000E 00



NASAP PROGRAM BAND PASS FILTER

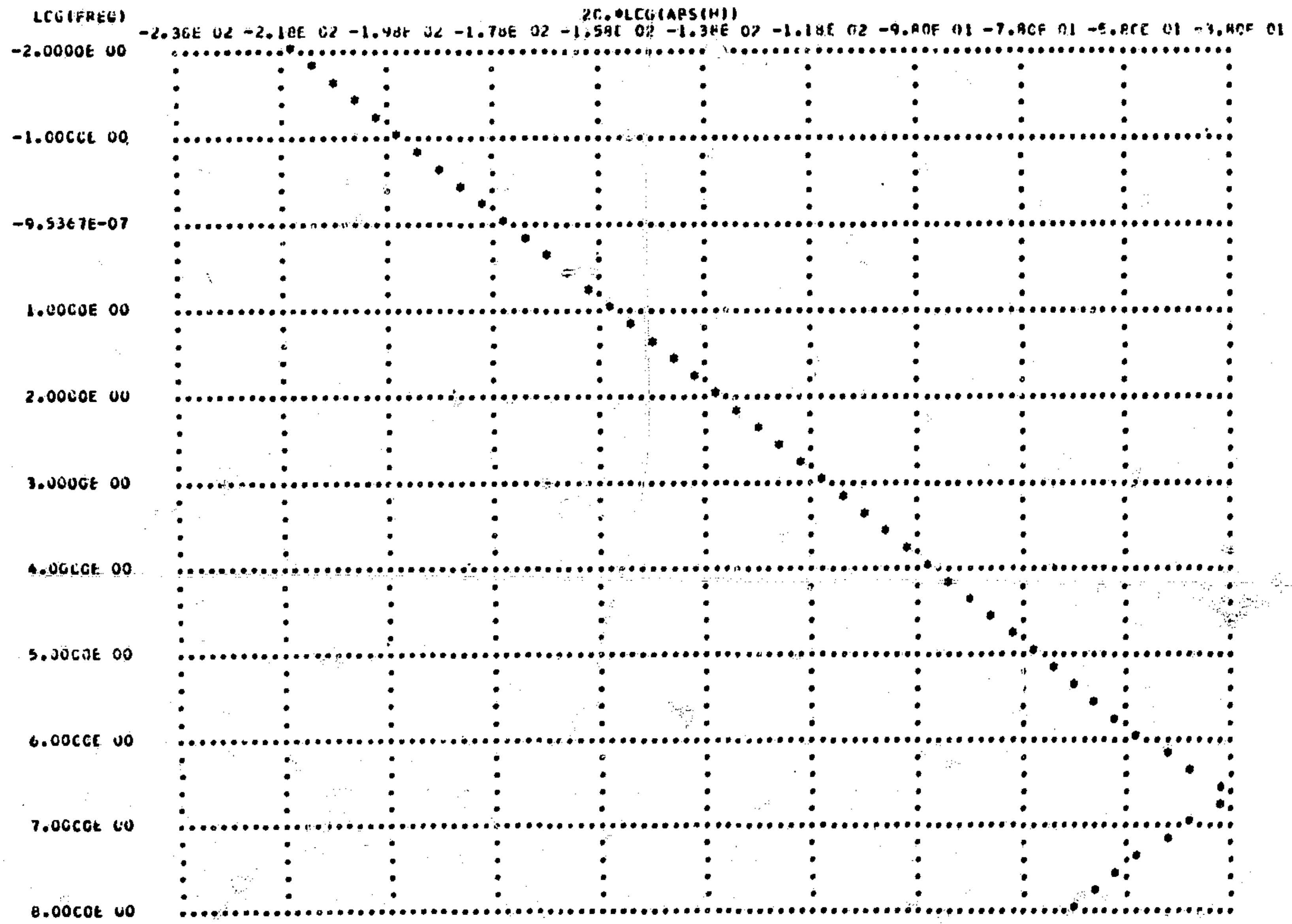
```
V1   5   1   1.0
R1   1   2   150.0
R2   4   5   279.0
C1   2   5   2554.0PF
C2   2   3   319.7PF
C3   3   5   30.0PF
C4   3   4   317.0PF
C5   4   5   1297.0PF
L1   2   5   .3952UH
L2   2   3   2.348UH
L3   3   4   4.328UH
L4   4   5   0.771UH
OUTPUT
VR2/VV1
FREQ   -2    8    .2
EXECUTE
```

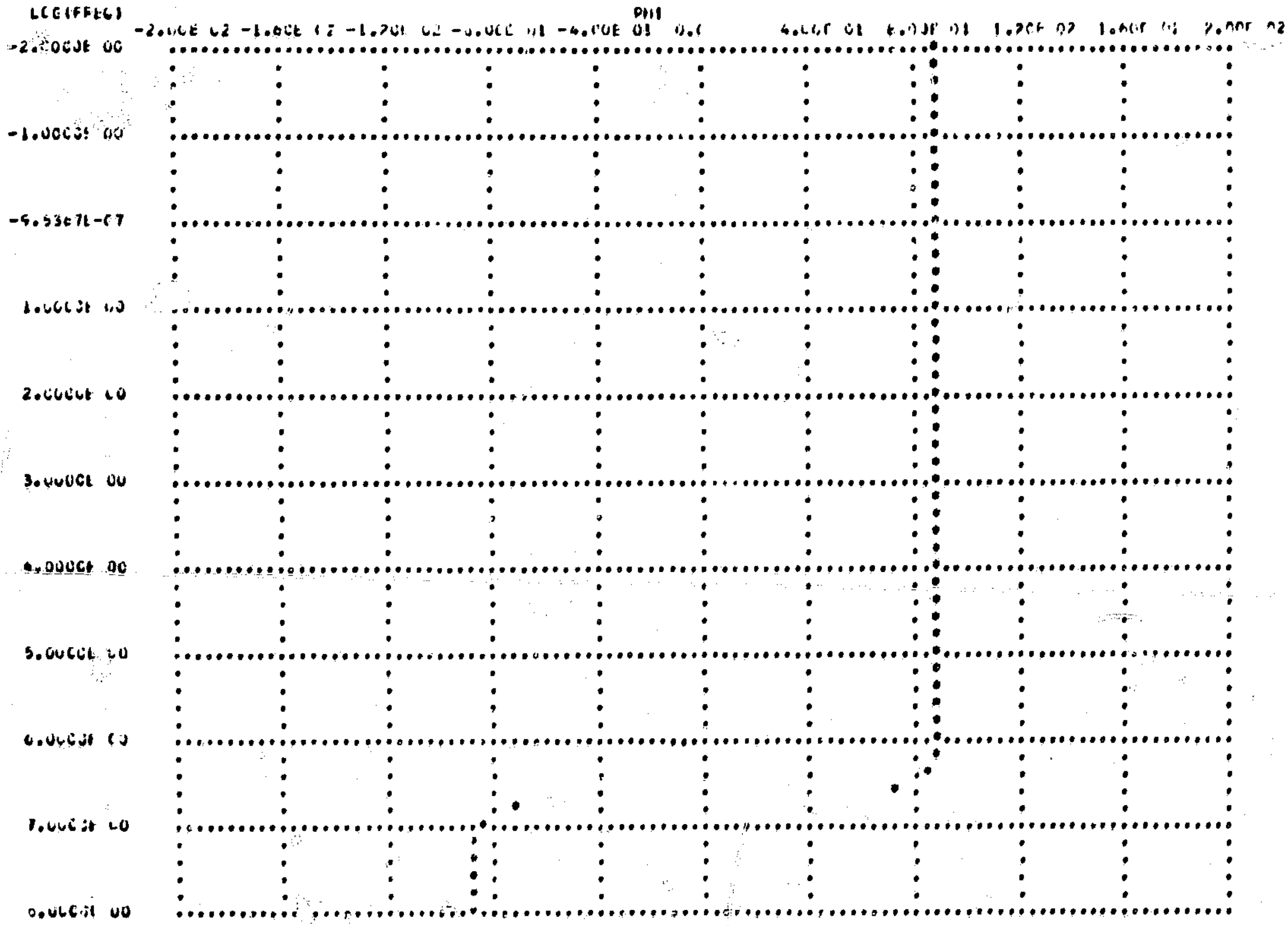
NUMBER OF LOOPS PER ORDER

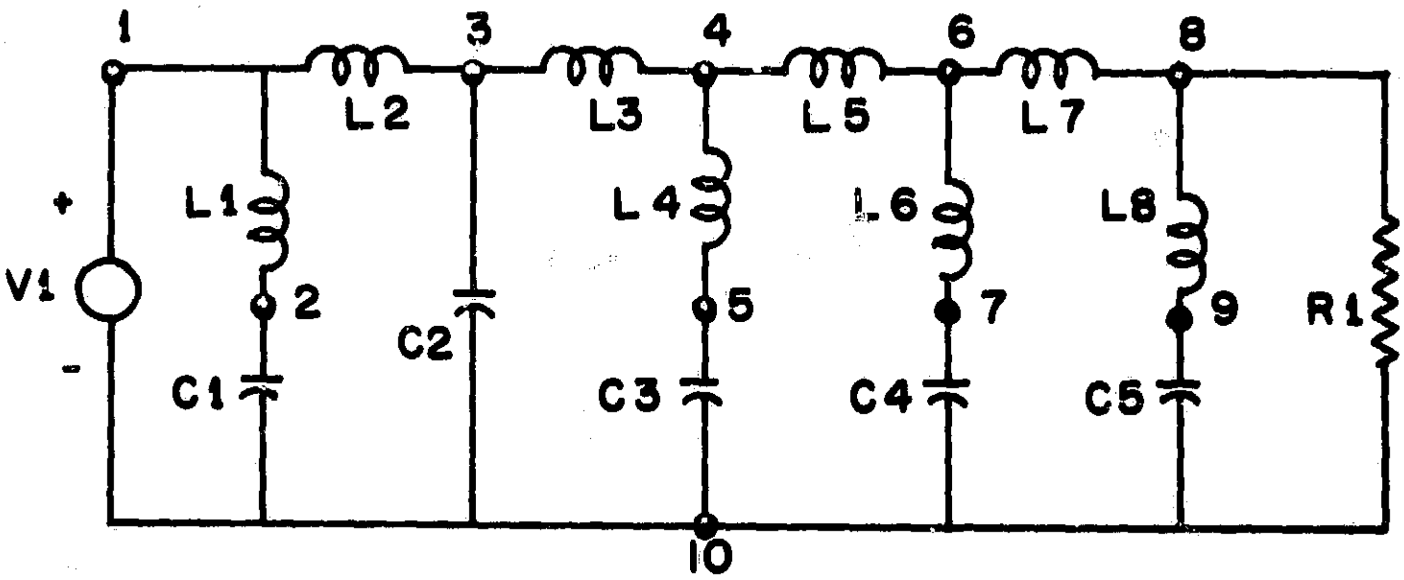
```
1=    33
2=    81
3=    54
```

```
                     (                                                2             3             4             5 )
                     (  0.0          +9.71E 29 S    +0.0          S   +2.00E 15 S   +0.0          S   +1.00E 00 S   )
H(S)= 2.561E 05*----------------------------------------------------------------------------------------------------------
                     (                                                2             3             4             5             6 )
                     (  9.60E 44     +4.79E 36 S    +2.94E 30 S   +9.75E 21 S   +2.98E 15 S   +4.93E 06 S   +1.00E 00 S   )
```

LOG(FREQ)	FREQ	20.*LOG(ABS(H))	PHI(H)	ABS(H)	LOG(ABS(H))
-0.2000000E 01	0.1000001E-01	-0.2157695E 03	0.8999997E 02	0.1627505E-10	-0.1078849E 02
-0.1799999E 01	0.1584698E-01	-0.2117695E 03	0.8999997E 02	0.2579427E-10	-0.1058847E 02
-0.1599999E 01	0.2511893E-01	-0.2077695E 03	0.8999997E 02	0.4088117E-10	-0.1038848E 02
-0.1400000E 01	0.3981078E-01	-0.2037695E 03	0.8999997E 02	0.6479220E-10	-0.1018847E 02
-0.1200000E 01	0.6309581E-01	-0.1997695E 03	0.8999997E 02	0.1026887E-09	-0.9988475E 01
-0.1000000E 01	0.1000000E 00	-0.1957695E 03	0.8999997E 02	0.1627505E-09	-0.9788478E 01
-0.8000011E 00	0.1584890E 00	-0.1917695E 03	0.8999997E 02	0.2579417E-09	-0.9588474E 01
-0.6000013E 00	0.2511880E 00	-0.1877695E 03	0.8999997E 02	0.4088092E-09	-0.9388477E 01
-0.4000006E 00	0.3981067E 00	-0.1837695E 03	0.8999997E 02	0.6479204E-09	-0.9188474E 01
-0.2000008E 00	0.6309563E 00	-0.1797695E 03	0.8999997E 02	0.1026884E-08	-0.8988477E 01
-0.9536743E-06	0.9999578E 00	-0.1757696E 03	0.8999997E 02	0.1627501E-08	-0.8788480E 01
0.1999989E 00	0.1584886E 01	-0.1717695E 03	0.8999997E 02	0.2579413E-08	-0.8588476E 01
0.3999987E 00	0.2511878E 01	-0.1677696E 03	0.8999991E 02	0.4088093E-08	-0.8388479E 01
0.5999985E 00	0.3981055E 01	-0.1637695E 03	0.8999991E 02	0.6479183E-08	-0.8188475E 01
0.7999983E 00	0.6309541E 01	-0.1597695E 03	0.8999991E 02	0.1026880E-07	-0.7988478E 01
0.9999981E 00	0.9999548E 01	-0.1557696E 03	0.8999991E 02	0.1627495E-07	-0.7788481E 01
0.1199999E 01	0.1584888E 02	-0.1517695E 03	0.8999991E 02	0.2579412E-07	-0.7588477E 01
0.1399999E 01	0.2511876E 02	-0.1477696E 03	0.8999991E 02	0.4088089E-07	-0.7388480E 01
0.1599998E 01	0.3981053E 02	-0.1437695E 03	0.8999986E 02	0.6479178E-07	-0.7188477E 01
0.1799998E 01	0.6309538E 02	-0.1397696E 03	0.8999980E 02	0.1026880E-06	-0.6988480E 01
0.1999998E 01	0.9999544E 02	-0.1357696E 03	0.8999976E 02	0.1627495E-06	-0.6788481E 01
0.2199998E 01	0.1584883E 03	-0.1317696E 03	0.8999965E 02	0.2579400E-06	-0.6588482E 01
0.2399998E 01	0.2511869E 03	-0.1277696E 03	0.8999948E 02	0.4088078E-06	-0.6388481E 01
0.2599998E 01	0.3981045E 03	-0.1237696E 03	0.8999921E 02	0.6479167E-06	-0.6188481E 01
0.2799998E 01	0.6309536E 03	-0.1197696E 03	0.8999883E 02	0.1026880E-05	-0.5988481E 01
0.2999998E 01	0.9999539E 03	-0.1157696E 03	0.8999847E 02	0.1627494E-05	-0.5788481E 01
0.3199998E 01	0.1584882E 04	-0.1117696E 03	0.8999707E 02	0.2579402E-05	-0.5588481E 01
0.3399998E 01	0.2511868E 04	-0.1077696E 03	0.8999544E 02	0.4088075E-05	-0.5388481E 01
0.3599998E 01	0.3981042E 04	-0.1037696E 03	0.8999281E 02	0.6479167E-05	-0.5188481E 01
0.3799997E 01	0.6309520E 04	-0.9976962E 02	0.8998860E 02	0.1026879E-04	-0.4988482E 01
0.3999997E 01	0.9999514E 04	-0.9576959E 02	0.8998199E 02	0.1627496E-04	-0.4788480E 01
0.4199997E 01	0.1584878E 05	-0.9176956E 02	0.8997145E 02	0.2579419E-04	-0.4588478E 01
0.4399998E 01	0.2511867E 05	-0.8776942E 02	0.8995479E 02	0.4088168E-04	-0.4388472E 01
0.4599998E 01	0.3981041E 05	-0.8376912E 02	0.8992839E 02	0.6479530E-04	-0.4188457E 01
0.4799997E 01	0.6309518E 05	-0.7976837E 02	0.8988647E 02	0.1027026E-03	-0.3988419E 01
0.4999997E 01	0.9999912E 05	-0.7576645E 02	0.8982008E 02	0.1628085E-03	-0.3788322E 01
0.5199997E 01	0.1584877E 06	-0.7176166E 02	0.8971474E 02	0.2581761E-03	-0.3588083E 01
0.5399997E 01	0.2511861E 06	-0.6774960E 02	0.8954727E 02	0.4097503E-03	-0.3387481E 01
0.5599997E 01	0.3981027E 06	-0.6371925E 02	0.8927968E 02	0.6516846E-03	-0.3185963E 01
0.5799996E 01	0.6309503E 06	-0.5964261E 02	0.8884726E 02	0.1042004E-02	-0.2982131E 01
0.5999997E 01	0.9999906E 06	-0.5544777E 02	0.8812703E 02	0.1688929E-02	-0.2772388E 01
0.6199997E 01	0.1584877E 07	-0.5094376E 02	0.8683104E 02	0.2836692E-02	-0.2547188E 01
0.6399997E 01	0.2511860E 07	-0.4558932E 02	0.8394930E 02	0.5254518E-02	-0.2279467E 01
0.6599997E 01	0.3981026E 07	-0.3976054E 02	0.7006204E 02	0.1027949E-01	-0.1988028E 01
0.6799996E 01	0.6309500E 07	-0.3953716E 02	-0.7049197E 02	0.1054731E-01	-0.1976858E 01
0.6999996E 01	0.9999769E 07	-0.4562672E 02	-0.8399667E 02	0.5231947E-02	-0.2281337E 01
0.7199996E 01	0.1584870E 08	-0.5097508E 02	-0.8684912E 02	0.2826471E-02	-0.2548755E 01
0.7399996E 01	0.2511832E 08	-0.5547632E 02	-0.8813654E 02	0.1693387E-02	-0.2773816E 01
0.7599997E 01	0.3981021E 08	-0.5967014E 02	-0.8885283E 02	0.1038707E-02	-0.2983507E 01
0.7799996E 01	0.6309450E 08	-0.6374628E 02	-0.8928307E 02	0.6496592E-03	-0.3187314E 01
0.7999996E 01	0.9999736E 08	-0.6777643E 02	-0.8954935E 02	0.4084874E-03	-0.3388822E 01


LOG(FREQ)
20.*LOG(ABS(H))
-2.36E 02 -2.18E 02 -1.98E 02 -1.78E 02 -1.58E 02 -1.38E 02 -1.18E 02 -9.80E 01 -7.80E 01 -5.80E 01 -3.80E 01
-2.0000E 00
-1.0000E 00
-9.5367E-07
1.0000E 00
2.0000E 00
3.0000E 00
4.0000E 00
5.0000E 00
6.0000E 00
7.0000E 00
8.0000E 00


PHI
-9.5367E-07
3.0000E 00
4.0000E 00
5.0000E 00

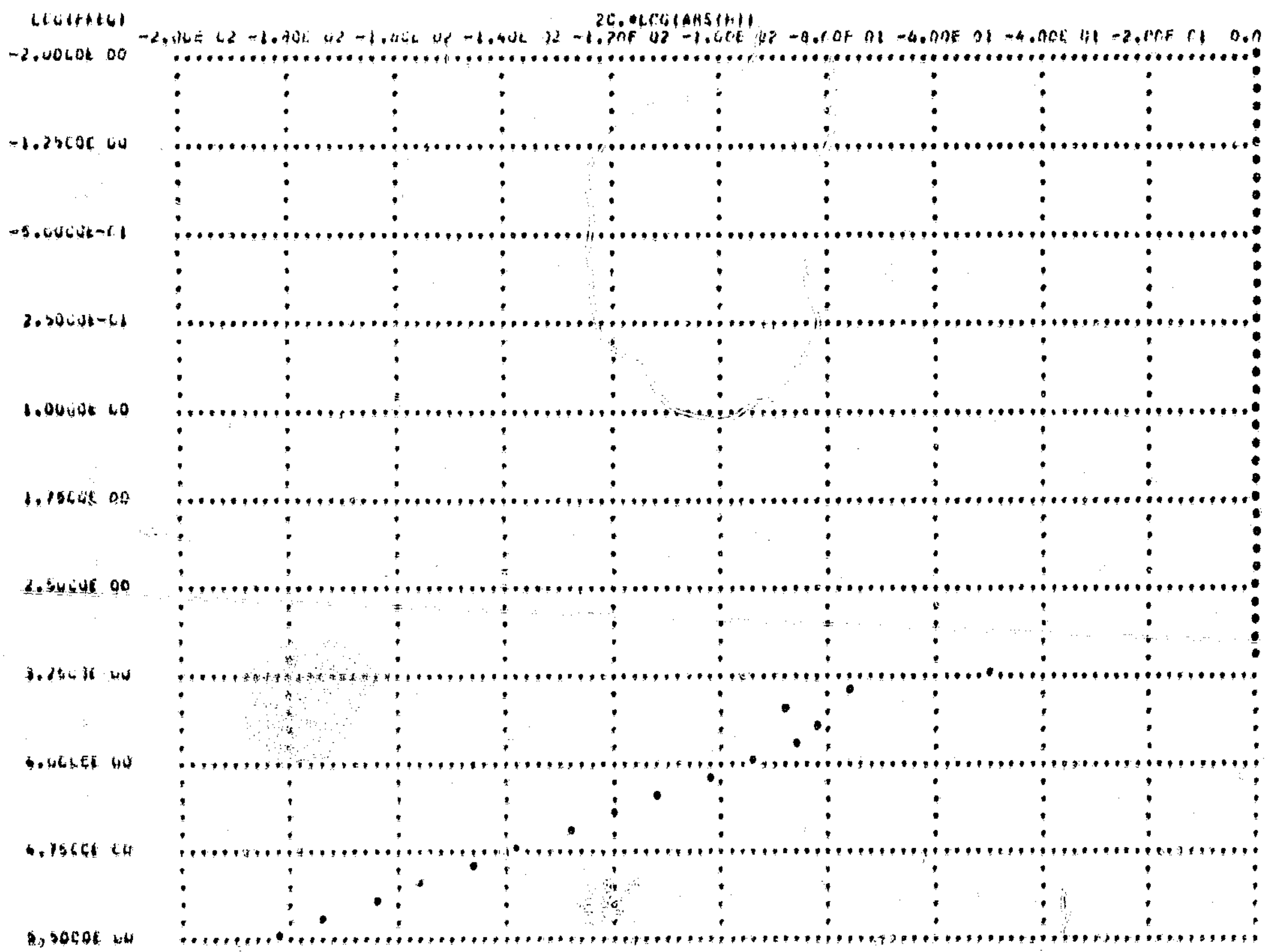


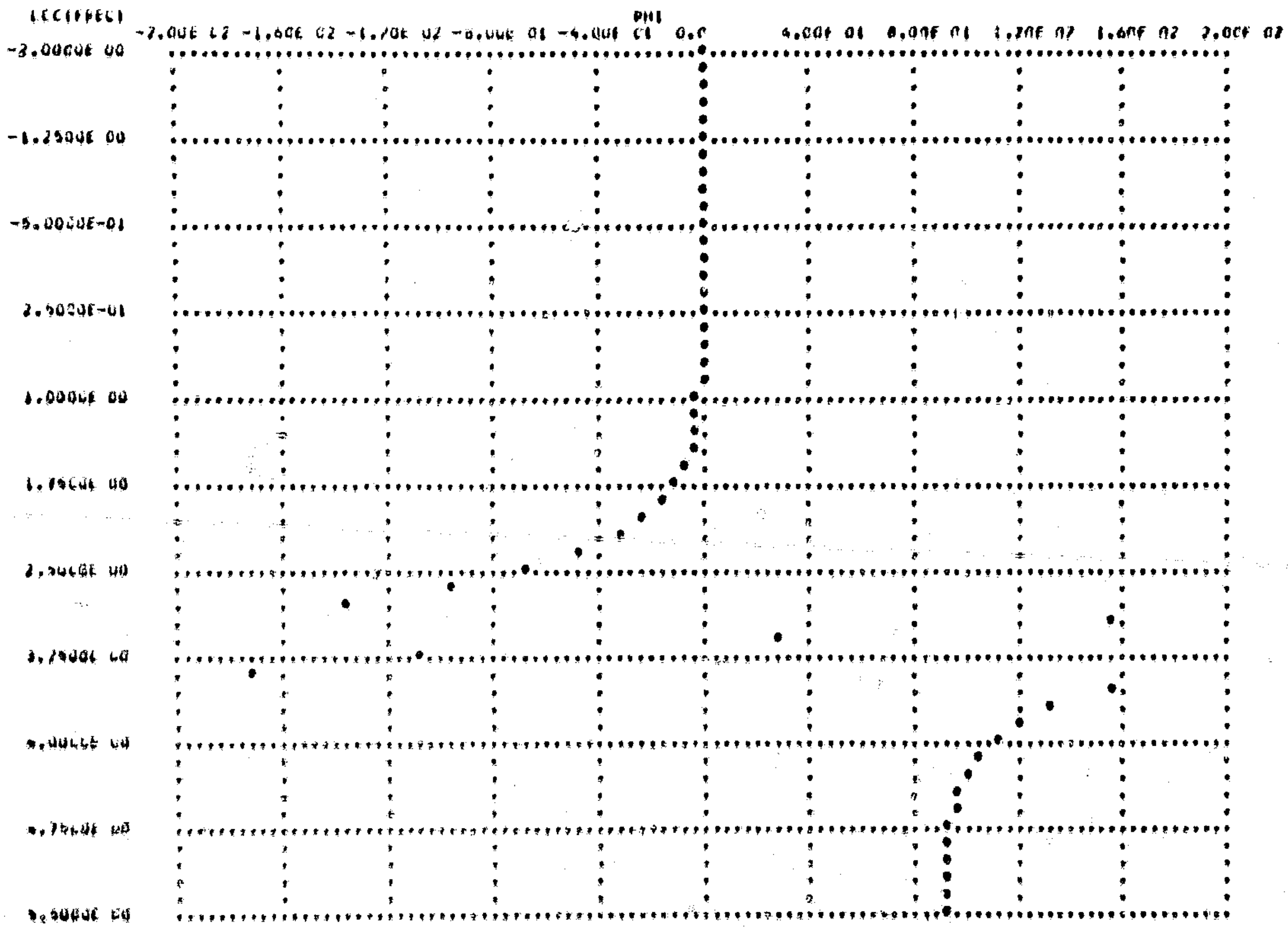

```
NASAP PROGRAM LOW PASS FILTER


V1   10  1   1.0
L1   1   2   35.2MH
C1   2   10  .1UF
L2   1   3   96.0MH
C2   3   10  .33UF
L3   3   4   85.2MH
L4   4   5   59.4MH
C3   5   10  .14UF
L5   4   6   77.5MH
L6   6   7   8.4MH
C4   7   10  .29UF
L7   6   8   88.2MH
L8   8   9   35.2MH
C5   9   10  .1UF
R1   8   10   600.0
OUTPUT
VR1/VV1
FREQ   -2   5.5   .15
EXECUTE



NUMBER OF LOOPS PER ORDER

    1=    34
    2=   160
    3=   300
    4=   265
    5=   107
    6=    15
```

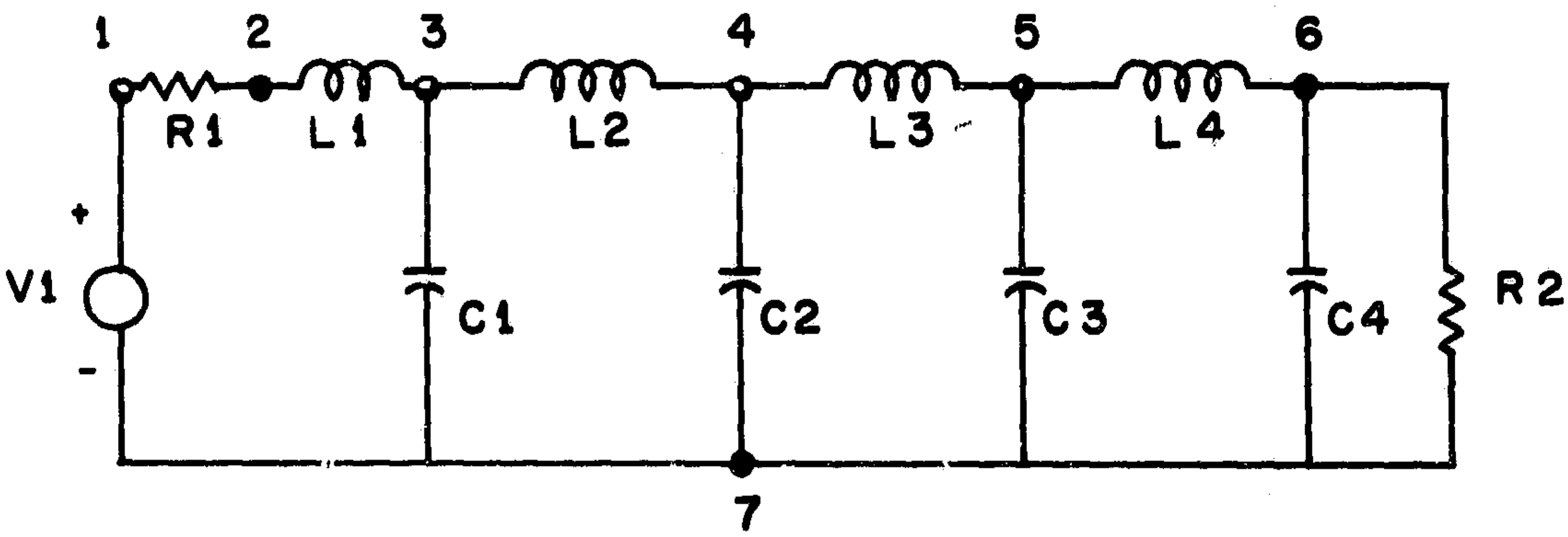
TRANSFER FUNCTION VR1/VV1

```
        ( 3.58E 33  +0.0  S  +7.09E 25 S^2  +0.0  S^3  +4.37E 17 S^4  +0.0  S^5  +1.10E 09 S^6  +0.0  S^7 )
        ( +1.00E 00 S^8 )
H(S)= 5.630E 09*-----------------------------------------------------------------------------------------------
        ( 2.24E 43  +1.30E 40 S  +4.14E 36 S^2  +9.32E 32 S^3  +1.56E 29 S^4  +2.11E 25 S^5  +2.19E 21 S^6  +1.97E 17 S^7 )
        ( +1.25E 13 S^8  +7.55E 08 S^9  +2.33E 04 S^10  +1.00E 00 S^11 )
```

LOG(FREQ)	FREQ	20.*LOG(ABS(H))	PHI(H)	ABS(H)	LOG(ABS(H))
-0.2000000E 01	0.1000001E-01	-0.4659476E-05	-0.2081402E-02	0.9999995E 00	-0.2329738E-06
-0.1849999E 01	0.1412541E-01	-0.4659476E-05	-0.2940062E-02	0.9999995E 00	-0.2329738E-06
-0.1700000E 01	0.1995265E-01	-0.4659476E-05	-0.4152962E-02	0.9999995E 00	-0.2329738E-06
-0.1550000E 01	0.2818386E-01	-0.4659476E-05	-0.5866189E-02	0.9999995E 00	-0.2329738E-06
-0.1400000E 01	0.3981070E-01	-0.4659476E-05	-0.8286219E-02	0.9999995E 00	-0.2329738E-06
-0.1250000E 01	0.5623420E-01	-0.4659476E-05	-0.1170459E-01	0.9999995E 00	-0.2329738E-06
-0.1100000E 01	0.7943285E-01	-0.4659476E-05	-0.1653313E-01	0.9999995E 00	-0.2329738E-06
-0.9500008E 00	0.1122017E 00	0.0	-0.2335365E-01	0.1000000E 01	0.0
-0.8000011E 00	0.1584890E 00	0.0	-0.3298791E-01	0.1000000E 01	0.0
-0.6500015E 00	0.2238715E 00	0.0	-0.4659661E-01	0.1000000E 01	0.0
-0.5000010E 00	0.3162274E 00	-0.6212636E-05	-0.6581956E-01	0.9999993E 00	-0.3106318E-06
-0.3500013E 00	0.4466823E 00	0.0	-0.9297246E-01	0.1000000E 01	0.0
-0.2000017E 00	0.6309549E 00	-0.8801237E-05	-0.1313270E 00	0.9999990E 00	-0.4400619E-06
-0.5000114E-01	0.8912486E 00	0.0	-0.1855046E 00	0.1000000E 01	0.0
0.9999847E-01	0.1258921E 01	0.0	-0.2620316E 00	0.1000000E 01	0.0
0.2499981E 00	0.1778272E 01	-0.1553158E-05	-0.3701297E 00	0.9999998E 00	-0.7765789E-07
0.3999987E 00	0.2511878E 01	-0.4659476E-05	-0.5228221E 00	0.9999995E 00	-0.2329738E-06
0.5499983E 00	0.3548119E 01	-0.4141757E-05	-0.7385049E 00	0.9999995E 00	-0.2070879E-06
0.6999979E 00	0.5011845E 01	-0.1035440E-04	-0.1043166E 01	0.9999988E 00	-0.5177198E-06
0.8499975E 00	0.7079410E 01	-0.4659476E-05	-0.1473513E 01	0.9999995E 00	-0.2329738E-06
0.9999981E 00	0.9999468E 01	-0.1501387E-04	-0.2081601E 01	0.9999983E 00	-0.7506935E-06
0.1149998E 01	0.1412520E 02	-0.2951005E-04	-0.2940066E 01	0.9999966E 00	-0.1475502E-05
0.1299997E 01	0.1995248E 02	-0.5125636E-04	-0.4152987E 01	0.9999941E 00	-0.2562718E-05
0.1449998E 01	0.2818365E 02	-0.1102750E-03	-0.5866166E 01	0.9999873E 00	-0.5513760E-05
0.1599998E 01	0.3981046E 02	-0.2045016E-03	-0.8286780E 01	0.9999765E 00	-0.1022508E-04
0.1749997E 01	0.5623370E 02	-0.3701770E-03	-0.1170631E 02	0.9999574E 00	-0.1850885E-04
0.1899998E 01	0.7943227E 02	-0.6233560E-03	-0.1653835E 02	0.9999282E 00	-0.3116780E-04
0.2049997E 01	0.1122010E 03	-0.8579036E-03	-0.2336909E 02	0.9999012E 00	-0.4289518E-04
0.2199997E 01	0.1584880E 03	-0.2837146E-03	-0.3303910E 02	0.9999673E 00	-0.1418576E-04
0.2349998E 01	0.2238705E 03	0.4380867E-02	-0.4676866E 02	0.1000504E 01	0.2190434E-03
0.2499997E 01	0.3162261E 03	0.2355119E-01	-0.6613817E 02	0.1002715E 01	0.1177560E-02
0.2649997E 01	0.4466794E 03	0.7399517E-01	-0.9681804E 02	0.1008555E 01	0.3699760E-02
0.2799997E 01	0.6309524E 03	0.9965660E-01	-0.1376083E 03	0.1011539E 01	0.4982829E-02
0.2949997E 01	0.8912427E 03	-0.1395190E 00	0.1566920E 03	0.9840660E 00	-0.6975948E-02
0.3099997E 01	0.1258514E 04	0.6736950E 00	0.4932627E 02	0.1080649E 01	0.3368479E-01
0.3249996E 01	0.1778260E 04	-0.4914987E 02	-0.1680189E 03	0.3467416E-02	-0.2459994E 01
0.3399997E 01	0.2511846E 04	-0.7561703E 02	-0.1721113E 03	0.1619945E-03	-0.3780852E 01
0.3549996E 01	0.3548097E 04	-0.8722201E 02	0.1561790E 03	0.4346097E-04	-0.4361001E 01
0.3699996E 01	0.5011812E 04	-0.8180046E 02	0.1339416E 03	0.8127083E-04	-0.4090023E 01
0.3849997E 01	0.7079387E 04	-0.8090906E 02	0.1205507E 03	0.9013851E-04	-0.4045453E 01
0.3999996E 01	0.9999887E 04	-0.9431150E 02	0.1116361E 03	0.1926961E-04	-0.4715575E 01
0.4149996E 01	0.1412521E 05	-0.1025781E 03	0.1051067E 03	0.7431180E-05	-0.5128906E 01
0.4299996E 01	0.1995241E 05	-0.1112558E 03	0.1006700E 03	0.2740130E-05	-0.5562790E 01
0.4449996E 01	0.2818351E 05	-0.1200524E 03	0.9756558E 02	0.9919888E-06	-0.6002620E 01
0.4599996E 01	0.3981022E 05	-0.1289066E 03	0.9533807E 02	0.3561875E-06	-0.6445332E 01
0.4749996E 01	0.5623357E 05	-0.1379236E 03	0.9377853E 02	0.1270046E-06	-0.6896181E 01
0.4899996E 01	0.7943187E 05	-0.1469027E 03	0.9267467E 02	0.4517629E-07	-0.7345108E 01
0.5049995E 01	0.1122036E 06	-0.1559414E 03	0.9189340E 02	0.1600830E-07	-0.7797067E 01
0.5199996E 01	0.1584876E 06	-0.1648801E 03	0.9134036E 02	0.5697660E-08	-0.8244300E 01
0.5349996E 01	0.2238692E 06	-0.1738933E 03	0.9094893E 02	0.2012278E-08	-0.8694167E 01
0.5499996E 01	0.3162232E 06	-0.1828819E 03	0.9067178E 02	0.7174313E-09	-0.9144096E 01


LOG(FREQ)
20.*LOG(ABS(H))
-2.00E 02 -1.80E 02 -1.60E 02 -1.40E 02 -1.20E 02 -1.00E 02 -8.00E 01 -6.00E 01 -4.00E 01 -2.00E 01 0.0
-2.0000E 00
-1.2500E 00
-5.0000E-01
2.5000E-01
1.0000E 00
1.7500E 00
2.5000E 00
3.2500E 00
4.0000E 00
4.7500E 00
5.5000E 00



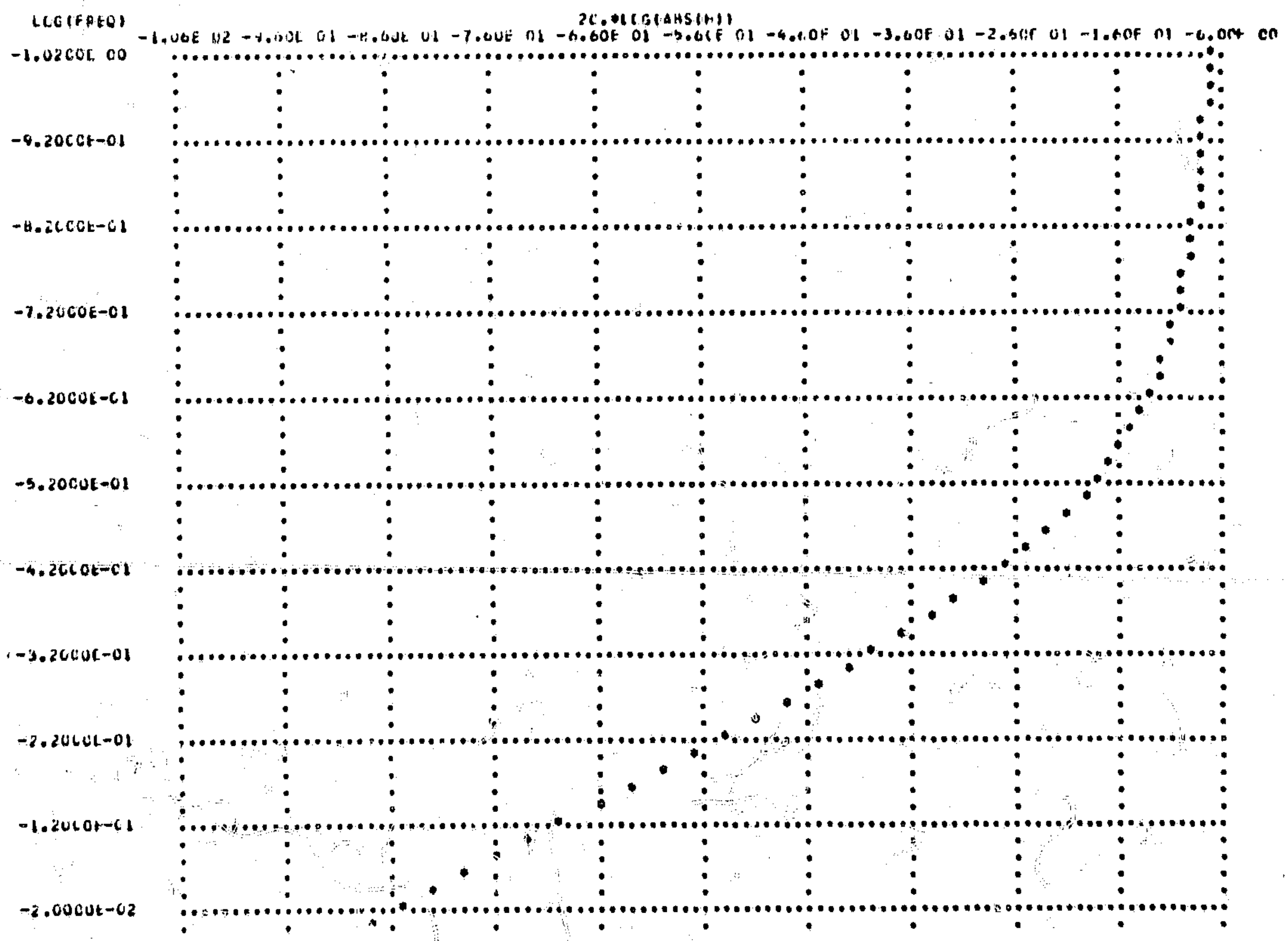


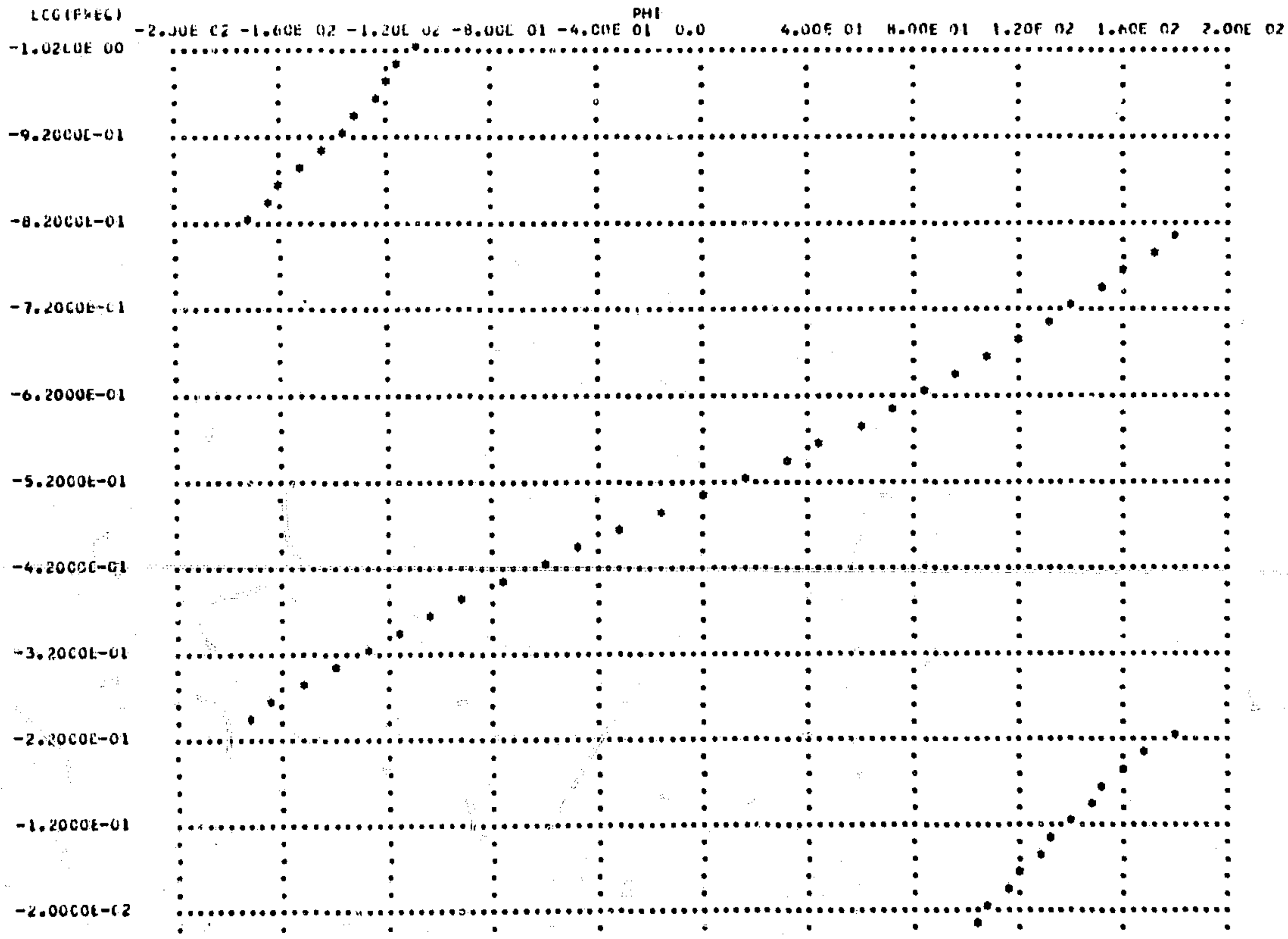

```
NASAP PROGRAM MAXIMALLY FLAT DELAY LOWPASS FILTER

V1    7   1   1
R1    1   2   1
L1    2   3   0.0919H
C1    3   7   0.2719F
L2    3   4   0.4409H
C2    4   7   0.5936F
L3    4   5   0.7303H
C3    5   7   0.8695F
L4    5   6   1.0956H
C4    6   7   2.2056F
R2    6   7   1
OUTPUT
VR2/VV1
FREQ   -1.02    .02   .02
EXECUTE



NUMBER OF LOOPS PER ORDER

     1=    10
     2=    28
     3=    35
     4=    15
     5=     1
```

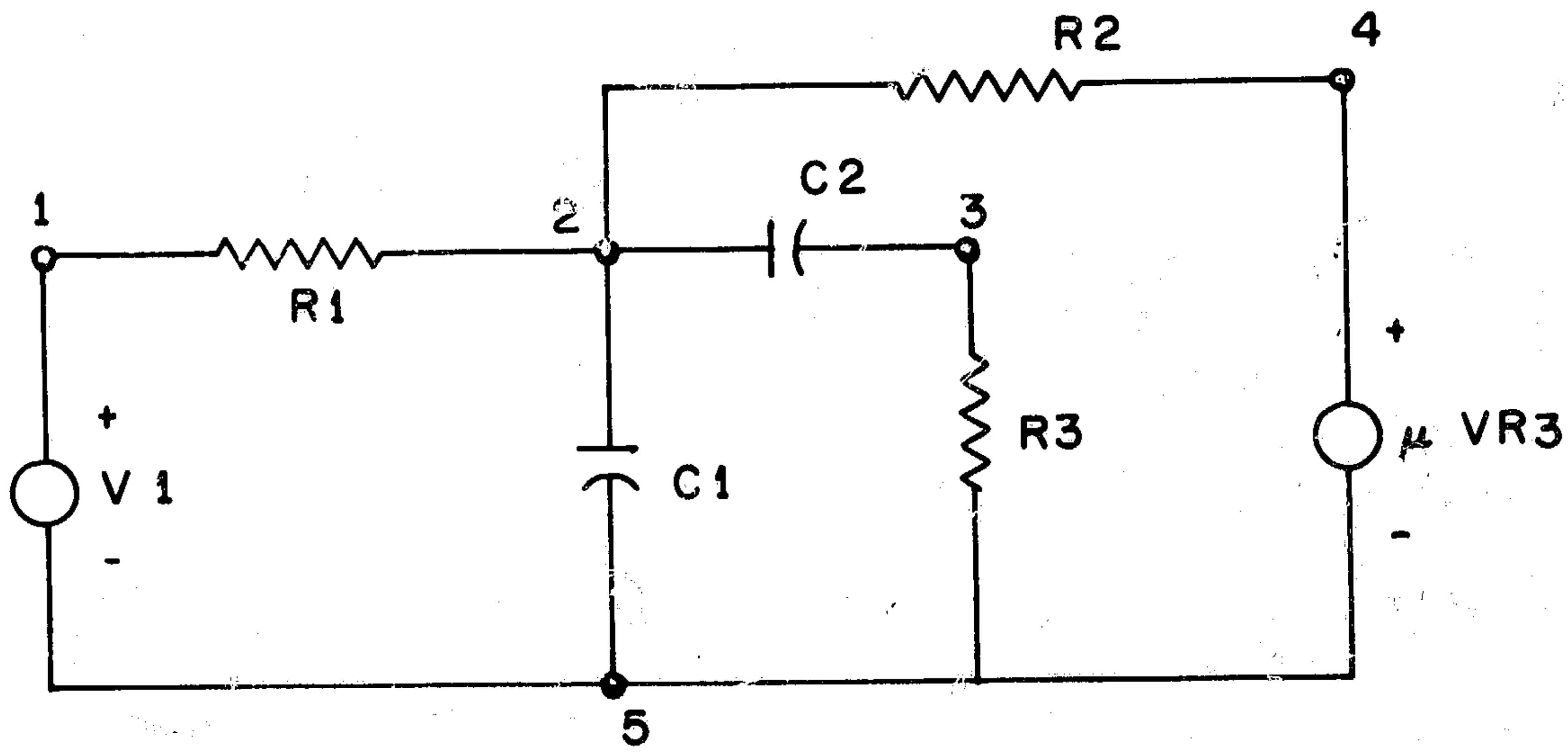
TRANSFER FUNCTION VR2/VV1

```
            (                     )
            (  1.00E 00           )
H(S)= 9.701E 01*--------------------------------------------------------------------------------------------------------------
            (                                  2             3             4             5             6             7 )
            (  1.54E 02    +6.17E 02 S   +9.15E 02 S   +8.32E 02 S   +5.09E 02 S   +2.16E 02 S   +6.73E 01 S   +1.13E 01 S  )
            (                                                                                                               )
            (              8 )
            ( +1.00E 00 S    )
```

LOG(FREQ)	FREQ	20.*LOG(ABS(H))	PHI(H)	ABS(H)	LOG(ABS(H))
-0.1020000E 01	0.9549554E-01	-0.7054623E 01	-0.1093162E 03	0.4423530E 00	-0.3542312E 00
-0.9999995E 00	0.1000003E 00	-0.7186398E 01	-0.1144683E 03	0.4370993E 00	-0.3594199E 00
-0.9799995E 00	0.1047130E 00	-0.7302408E 01	-0.1198629E 03	0.4313995E 00	-0.3651204E 00
-0.9599996E 00	0.1096480E 00	-0.7427712E 01	-0.1255119E 03	0.4252207E 00	-0.3713856E 00
-0.9399995E 00	0.1148156E 00	-0.7565463E 01	-0.1314272E 03	0.4185303E 00	-0.3782732E 00
-0.9199995E 00	0.1202266E 00	-0.7714902E 01	-0.1376212E 03	0.4112964E 00	-0.3858451E 00
-0.8999996E 00	0.1258928E 00	-0.7883488E 01	-0.1441071E 03	0.4034834E 00	-0.3941744E 00
-0.8799996E 00	0.1318258E 00	-0.8066775E 01	-0.1508988E 03	0.3950584E 00	-0.4033388E 00
-0.8599995E 00	0.1380388E 00	-0.8268525E 01	-0.1580105E 03	0.3859880E 00	-0.4134263E 00
-0.8399996E 00	0.1445442E 00	-0.8490672E 01	-0.1654571E 03	0.3762413E 00	-0.4245336E 00
-0.8199996E 00	0.1513564E 00	-0.8735392E 01	-0.1732548E 03	0.3657886E 00	-0.4367696E 00
-0.7999996E 00	0.1584895E 00	-0.9005201E 01	0.1785802E 03	0.3546009E 00	-0.4502601E 00
-0.7799996E 00	0.1659589E 00	-0.9302821E 01	0.1700304E 03	0.3426567E 00	-0.4651411E 00
-0.7599996E 00	0.1737804E 00	-0.9631353E 01	0.1610780E 03	0.3299376E 00	-0.4815677E 00
-0.7399996E 00	0.1819705E 00	-0.9994403E 01	0.1517038E 03	0.3164315E 00	-0.4997202E 00
-0.7199996E 00	0.1905463E 00	-0.1039601E 02	0.1418889E 03	0.3021339E 00	-0.5198006E 00
-0.6999996E 00	0.1995266E 00	-0.1084081E 02	0.1316127E 03	0.2870510E 00	-0.5420406E 00
-0.6799996E 00	0.2089300E 00	-0.1133424E 02	0.1208536E 03	0.2711990E 00	-0.5667118E 00
-0.6599996E 00	0.2187765E 00	-0.1188261E 02	0.1095927E 03	0.2546063E 00	-0.5941306E 00
-0.6399996E 00	0.2290871E 00	-0.1249345E 02	0.9780884E 02	0.2373161E 00	-0.6246728E 00
-0.6199996E 00	0.2398837E 00	-0.1317557E 02	0.8548439E 02	0.2193923E 00	-0.6587784E 00
-0.5999996E 00	0.2511889E 00	-0.1393948E 02	0.7260664E 02	0.2009212E 00	-0.6969742E 00
-0.5799996E 00	0.2630272E 00	-0.1479763E 02	0.5916982E 02	0.1820198E 00	-0.7398815E 00
-0.5599996E 00	0.2754233E 00	-0.1576451E 02	0.4518242E 02	0.1628450E 00	-0.7882253E 00
-0.5399996E 00	0.2884037E 00	-0.1685634E 02	0.3067305E 02	0.1436092E 00	-0.8428174E 00
-0.5199996E 00	0.3019556E 00	-0.1809045E 02	0.1569887E 02	0.1245887E 00	-0.9045233E 00
-0.4999996E 00	0.3162283E 00	-0.1948393E 02	0.3509694E 00	0.1061214E 00	-0.9741967E 00
-0.4799997E 00	0.3311318E 00	-0.2105115E 02	-0.1524054E 02	0.8860159E-01	-0.1052558E 01
-0.4599996E 00	0.3467373E 00	-0.2200151E 02	-0.3091287E 02	0.7243091E-01	-0.1140076E 01
-0.4399996E 00	0.3630786E 00	-0.2473735E 02	-0.4648582E 02	0.5796048E-01	-0.1236868E 01
-0.4199997E 00	0.3801898E 00	-0.2685291E 02	-0.6177844E 02	0.4543119E-01	-0.1342646E 01
-0.3999997E 00	0.3981075E 00	-0.2913545E 02	-0.7663215E 02	0.3493226E-01	-0.1456773E 01
-0.3799996E 00	0.4168698E 00	-0.3156703E 02	-0.9092511E 02	0.2640264E-01	-0.1578352E 01
-0.3599997E 00	0.4365163E 00	-0.3412729E 02	-0.1045749E 03	0.1966234E-01	-0.1706365E 01
-0.3399997E 00	0.4570886E 00	-0.3679588E 02	-0.1175405E 03	0.1446124E-01	-0.1839794E 01
-0.3199997E 00	0.4786305E 00	-0.3955399E 02	-0.1298111E 03	0.1052490E-01	-0.1977699E 01
-0.2999997E 00	0.5011877E 00	-0.4238522E 02	-0.1413988E 03	0.7598680E-02	-0.2119262E 01
-0.2799997E 00	0.5248079E 00	-0.4527011E 02	-0.1523302E 03	0.5447470E-02	-0.2263505E 01
-0.2599997E 00	0.5495416E 00	-0.4821518E 02	-0.1626307E 03	0.3883655E-02	-0.2410759E 01
-0.2399997E 00	0.5754404E 00	-0.5119173E 02	-0.1723625E 03	0.2756214E-02	-0.2559687E 01
-0.2199997E 00	0.6025601E 00	-0.5420445E 02	0.1784609E 03	0.1948841E-02	-0.2710223E 01
-0.1999997E 00	0.6309578E 00	-0.5724167E 02	0.1697947E 03	0.1373737E-02	-0.2862085E 01
-0.1799997E 00	0.6607539E 00	-0.6030092E 02	0.1616018E 03	0.9659492E-03	-0.3015046E 01
-0.1599997E 00	0.6918315E 00	-0.6337852E 02	0.1538560E 03	0.6777570E-03	-0.3168926E 01
-0.1399997E 00	0.7244365E 00	-0.6647160E 02	0.1465215E 03	0.4746995E-03	-0.3323582E 01
-0.1199997E 00	0.7585782E 00	-0.6957776E 02	0.1395730E 03	0.3319795E-03	-0.3478889E 01
-0.9999973E-01	0.7943280E 00	-0.7269538E 02	0.1329855E 03	0.2318707E-03	-0.3634754E 01
-0.7999976E-01	0.8317641E 00	-0.7582199E 02	0.1267361E 03	0.1617710E-03	-0.3791100E 01
-0.5999976E-01	0.8709642E 00	-0.7895714E 02	0.1208038E 03	0.1127569E-03	-0.3947857E 01
-0.3999972E-01	0.9120113E 00	-0.8209944E 02	0.1151605E 03	0.7852876E-04	-0.4104972E 01
-0.1999976E-01	0.9549932E 00	-0.8524779E 02	0.1078153E 03	0.5466125E-04	-0.4262400E 01
0.0	0.1000000E 01	-0.8840149E 02	0.1047252E 03	0.3801013E-04	-0.4420100E 01


LOG(FREQ)
20.*LOG(ABS(H))
-1.06E 02 -9.60E 01 -8.60E 01 -7.60E 01 -6.60E 01 -5.60E 01 -4.60E 01 -3.60E 01 -2.60E 01 -1.60E 01 -6.00E 00
-1.0200E 00
-9.2000E-01
-8.2000E-01
-7.2000E-01
-6.2000E-01
-5.2000E-01
-4.2000E-01
-3.2000E-01
-2.2000E-01
-1.2000E-01
-2.0000E-02


LOG(FREQ)
PHI
-2.00E 02 -1.60E 02 -1.20E 02 -8.00E 01 -4.00E 01 0.0 4.00E 01 8.00E 01 1.20E 02 1.60E 02 2.00E 02
-1.0200E 00
-9.2000E-01
-8.2000E-01
-7.2000E-01
-6.2000E-01
-5.2000E-01
-4.2000E-01
-3.2000E-01
-2.2000E-01
-1.2000E-01
-2.0000E-02

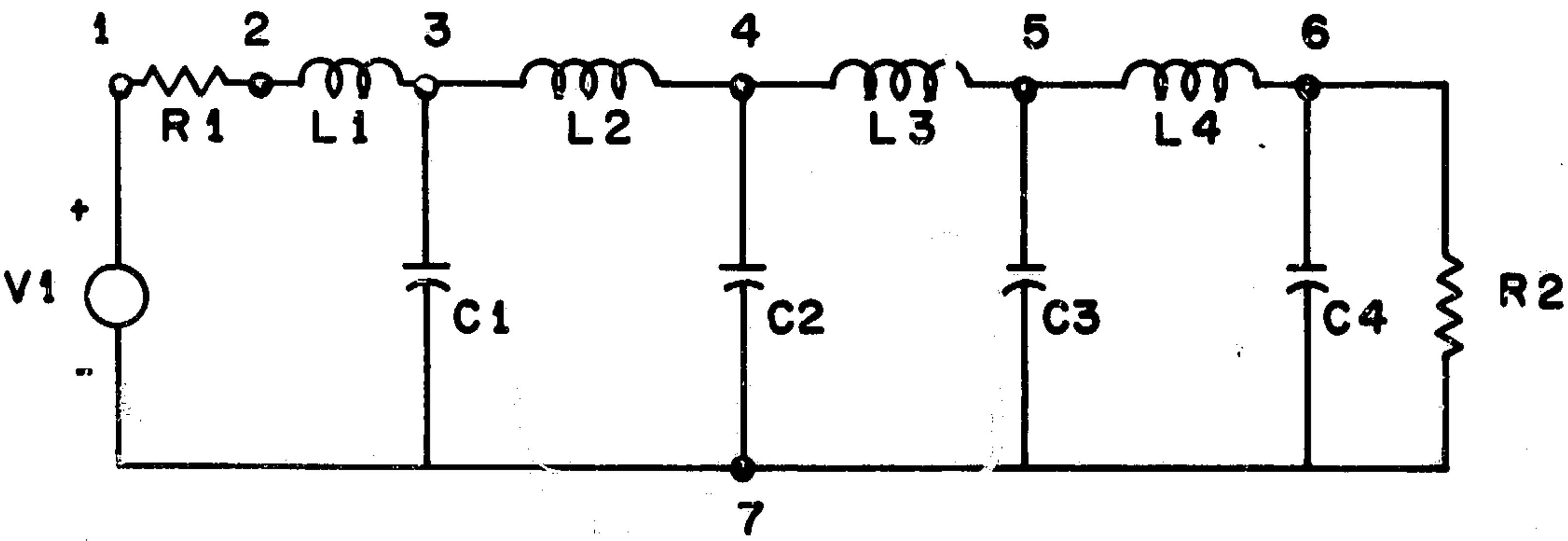


NASAP PROGRAM 2ND ORDER ACTIVE RC BANDPASS FILTER

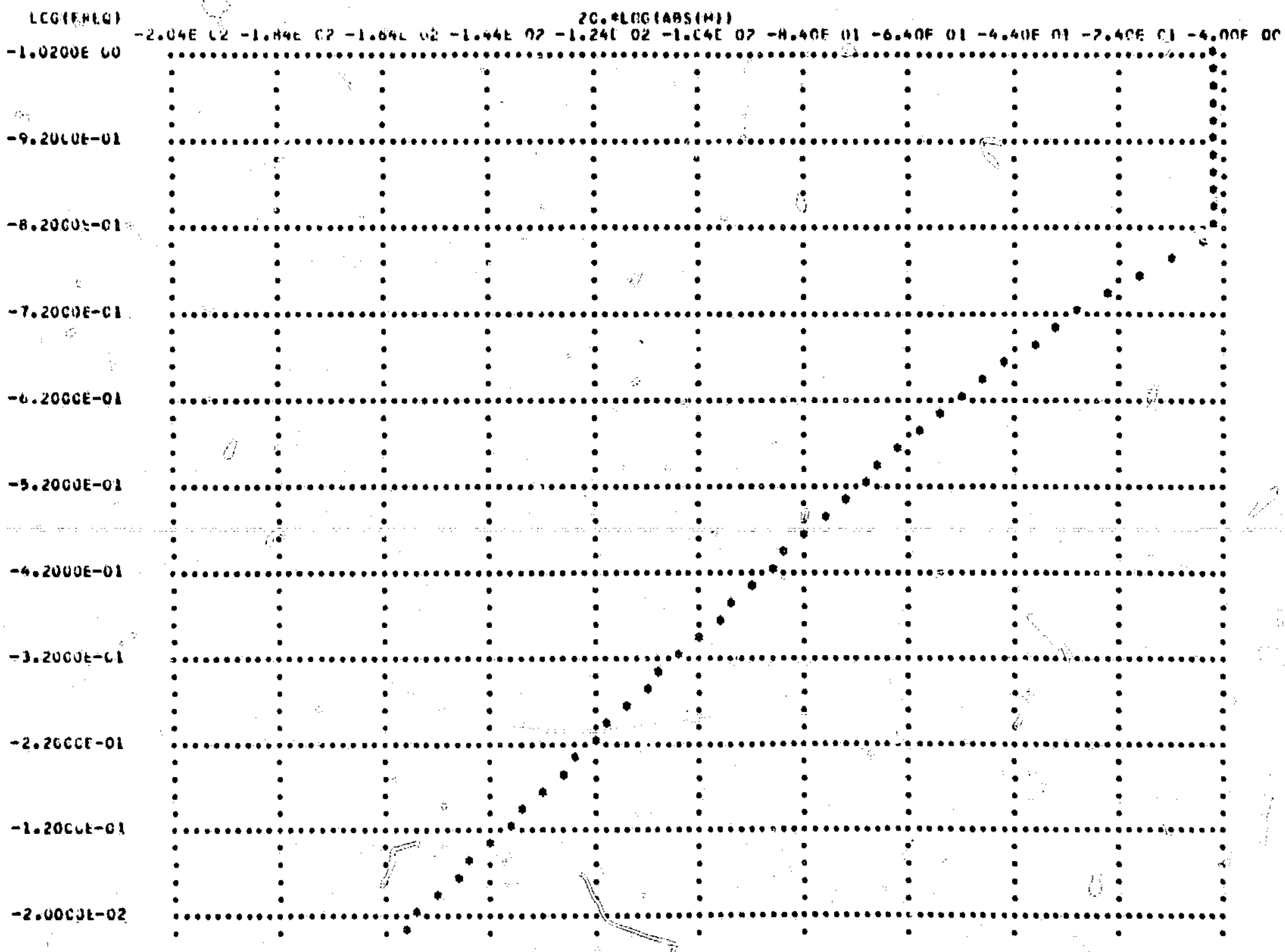
```
V1   5   1   1.0
R1   1   2   1.0
R2   3   5   .2
R3   2   4   1.0
C1   2   5   1.0F
C2   2   3   1.0F
V2   4   5   -11.0   VR2
OUTPUT
VV2/VV1
FREQ   -2   8   .2
EXECUTE
```

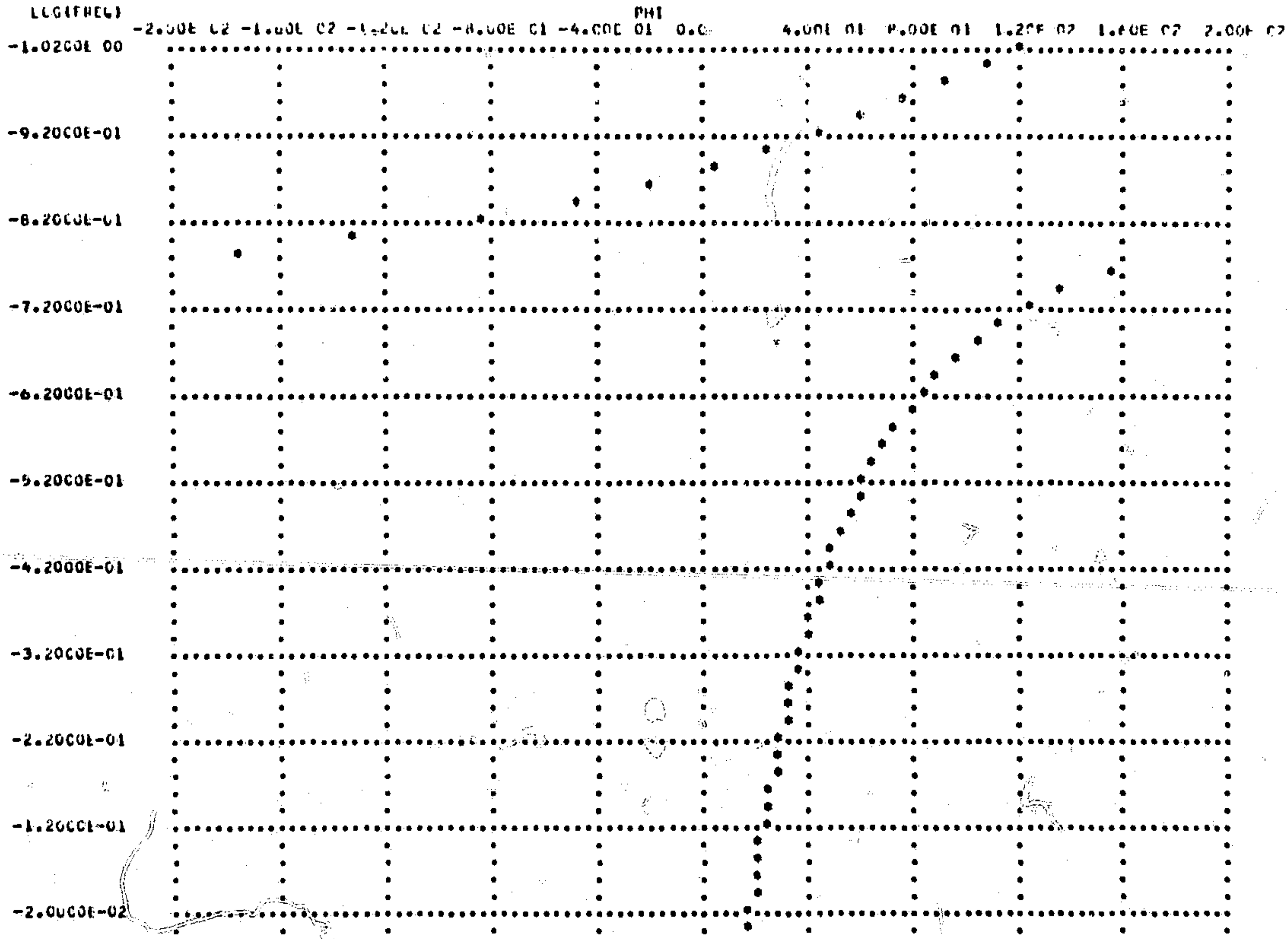
NUMBER OF LOOPS PER ORDER

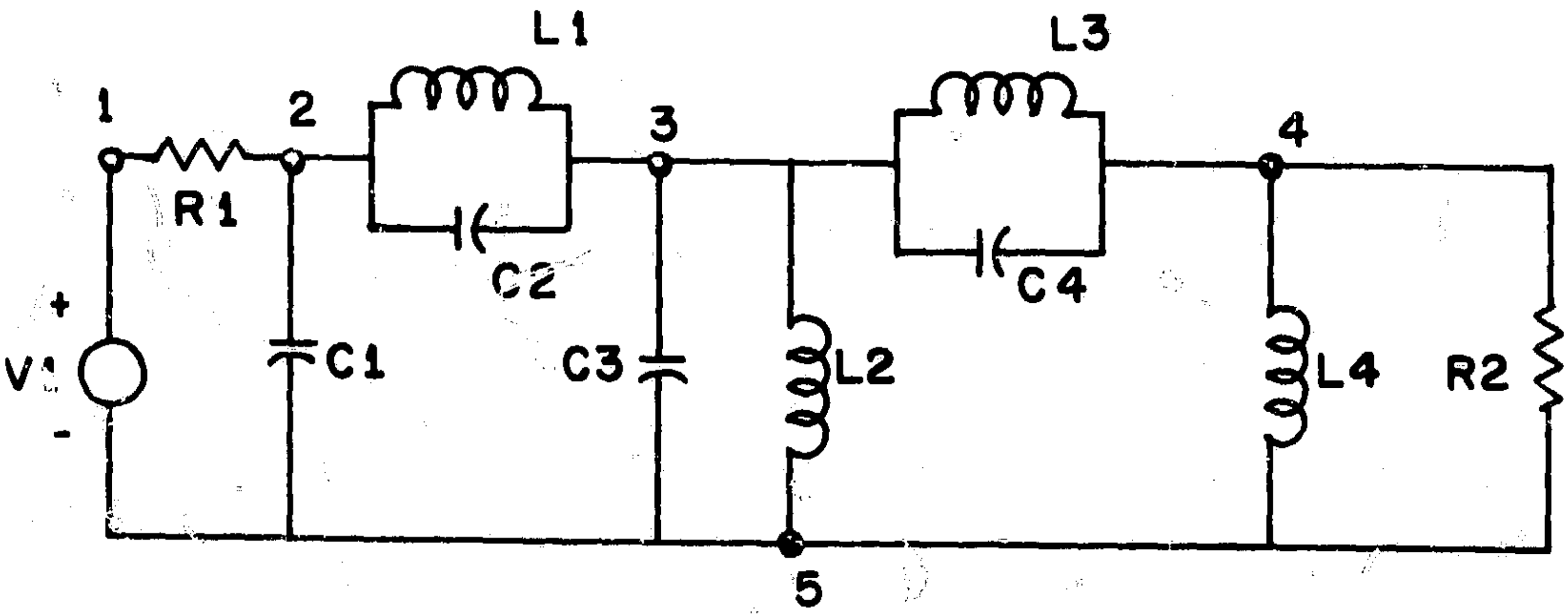
```
1=    6
2=    2
```




```
NASAP PROGRAM LEGENDRE LOWPASS FILTER

VI   7  1  1
R1   1  2  1
L1   2  3   .8205H
C1   3  7  1.4688F
L2   3  4  1.9115H
C2   4  7  1.7672F
L3   4  5  2.0515H
C3   5  7  1.8411F
L4   5  6  1.8501H
C4   6  7  1.5564F
R2   6  7  1
OUTPUT
VR2/VV1
FREQ  -1.02   .02   .02
EXECUTE


  NUMBER OF LOOPS PER ORDER

      1=    10
      2=    28
      3=    35
      4=    15
      5=     1
```


LOG(FREQ)
20.*LOG(ABS(H))
-2.04E 02 -1.84E 02 -1.64E 02 -1.44E 02 -1.24E 02 -1.04E 02 -8.40E 01 -6.40E 01 -4.40E 01 -2.40E 01 -4.00E 00
-1.0200E 00
-9.2000E-01
-8.2000E-01
-7.2000E-01
-6.2000E-01
-5.2000E-01
-4.2000E-01
-3.2000E-01
-2.2000E-01
-1.2000E-01
-2.0000E-02


LOG(FREQ)
PHI
-2.00E 02 -1.60E 02 -1.20E 02 -8.00E 01 -4.00E 01 0.0 4.00E 01 8.00E 01 1.20E 02 1.60E 02 2.00E 02
-1.0200E 00
-9.2000E-01
-8.2000E-01
-7.2000E-01
-6.2000E-01
-5.2000E-01
-4.2000E-01
-3.2000E-01
-2.2000E-01
-1.2000E-01
-2.0000E-02



NASAP PROGRAM 6TH ORDER SYMMETRICAL BANDPASS

```
V1   5   1   1
R1   1   2   1
C1   2   5   2.339F
L1   2   3   .1387H
C2   2   3   4.412F
C3   3   5   51.97F
L2   3   5   .01828H
L3   3   4   .1148H
C4   3   4   14.24F
L4   4   5   .268H
R2   4   5   2.378
OUTPUT
VR2/VV1
FREQ  -1.02   .02   .02
EXECUTE
```

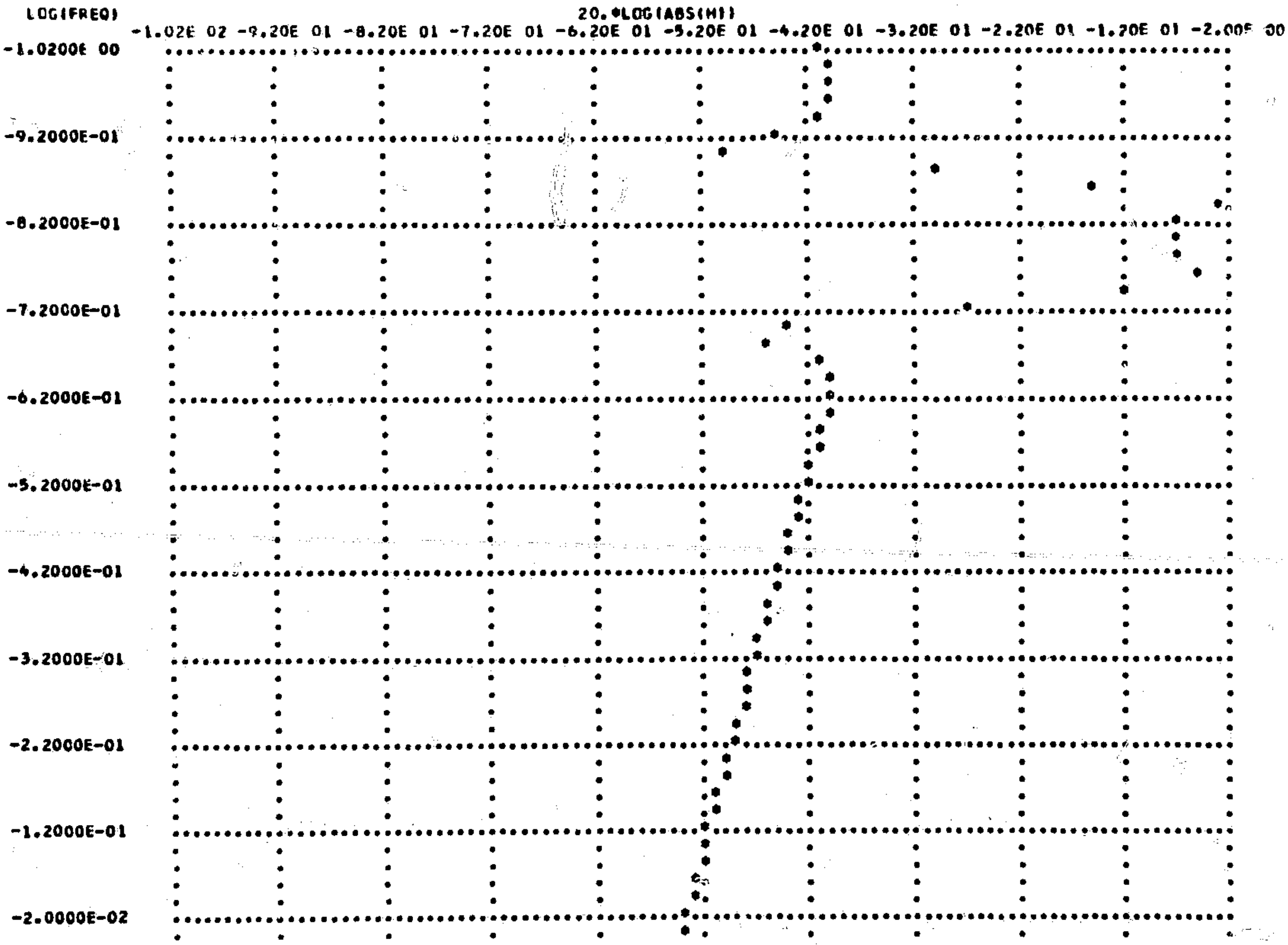
NUMBER OF LOOPS PER ORDER

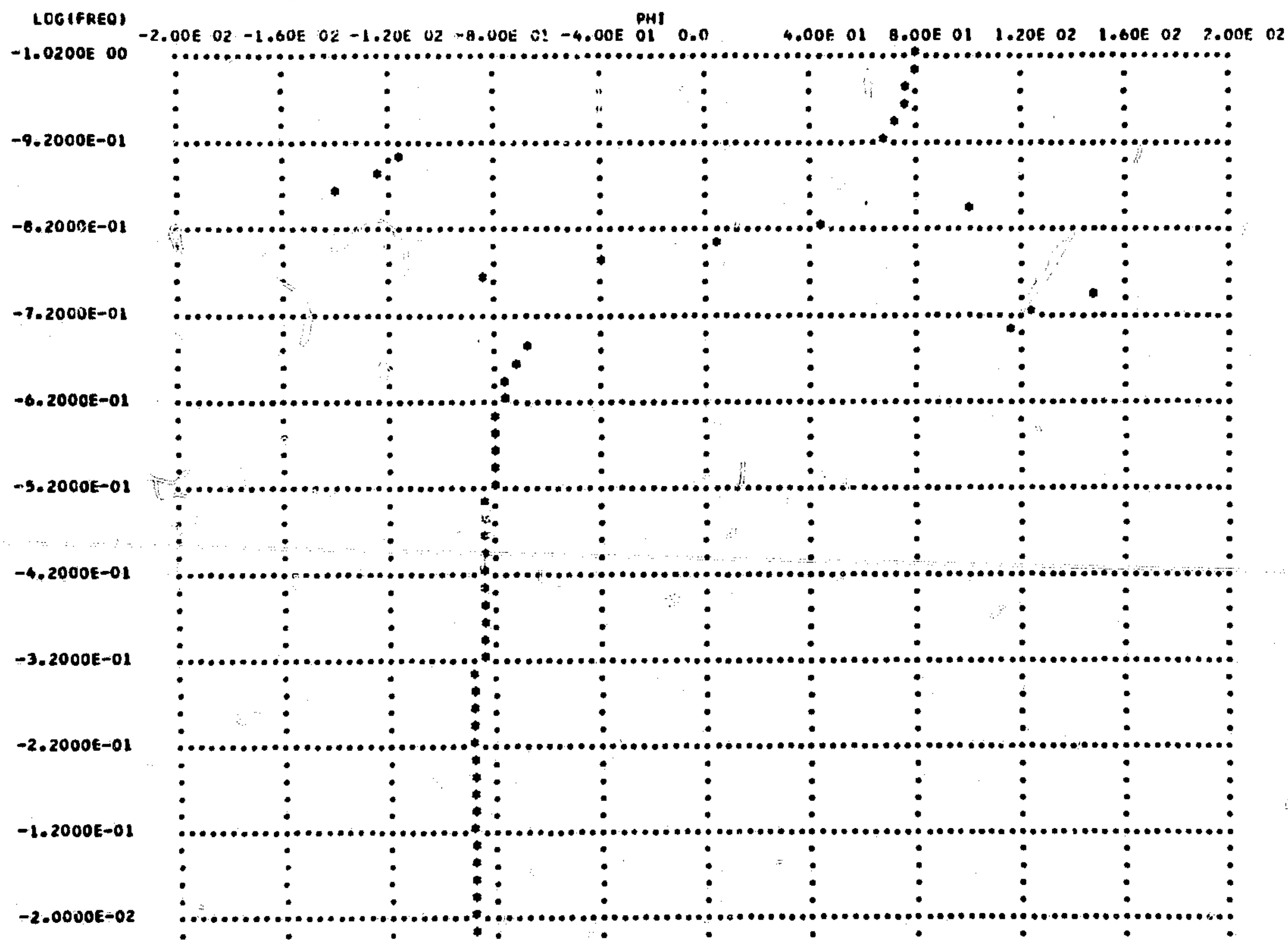
```
1=   45
2=   91
3=   47
```

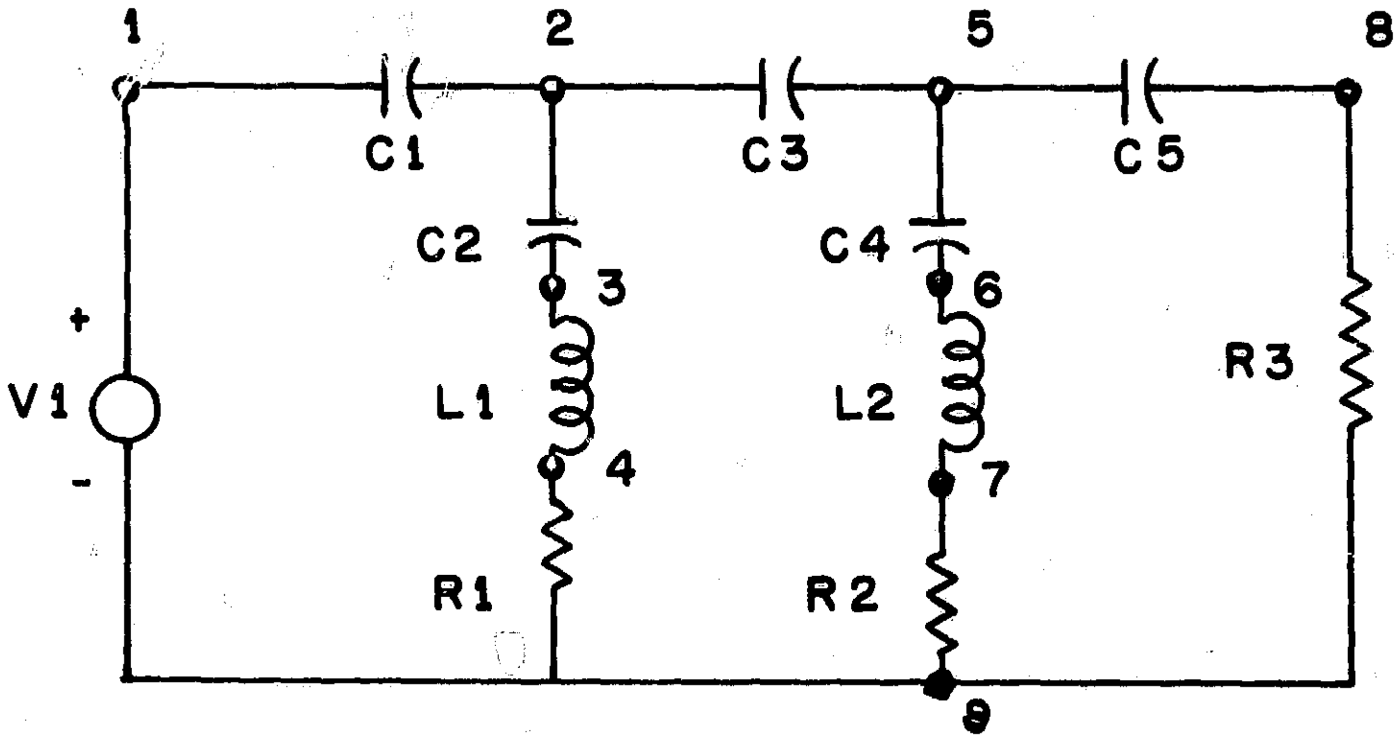
TRANSFER FUNCTION VR2/VV1

```
                   (                                        2                  3                4                  5 )
                   ( 0.0         +1.00E 00 S   +0.0      S   +2.25E 00 S   +0.0       S   +1.00E 00 S    )
H(S)= 1.222E-02*--------------------------------------------------------------------------------------------------
                   (                                        2                  3                4                  5             6 )
                   ( 1.00E 00    +1.93E-01 S   +3.06E 00 S   +3.94E-01 S   +3.06F 00 S   +1.94E-01 S   +1.00E 00 S   )
```

LOG(FREQ)	FREQ	20.*LOG(ABS(H))	PHI(H)	ABS(H)	LOG(ABS(H))
-0.1020000E 01	0.9549946E-01	-0.4064961E 02	0.7950754E 02	0.9279374E-02	-0.2032481E 01
-0.9999995E 00	0.1000003E 00	-0.4025706E 02	0.7835684F 02	0.9708356E-02	-0.2012854E 01
-0.9799995E 00	0.1047130E 00	-0.4001785E 02	0.7694972E 02	0.9979472E-02	-0.2000893E 01
-0.9599996E 00	0.1096480E 00	-0.4010896E 02	0.7518294E 02	0.9875312E-02	-0.2005448E 01
-0.9399995E 00	0.1148156E 00	-0.4101547E 02	0.7288701E 02	0.8896641E-02	-0.2050774E 01
-0.9199995E 00	0.1202266E 00	-0.4475557E 02	0.6975606E 02	0.5783901E-02	-0.2237779E 01
-0.8999996E 00	0.1258928E 00	-0.5002420E 02	-0.1148448E 03	0.3153474E-02	-0.2501210E 01
-0.8799996E 00	0.1318259E 00	-0.2992189E 02	-0.1226098E 03	0.3190841E-01	-0.1496095E 01
-0.8599995E 00	0.1380388E 00	-0.1530292E 02	-0.1416840E 03	0.1717331E 00	-0.7651458E 00
-0.8399996E 00	0.1445442E 00	-0.3194960E 01	0.9949716E 02	0.6922324E 00	-0.1597480E 00
-0.8199996E 00	0.1513564E 00	-0.7097628E 01	0.4482626E 02	0.4416911E 00	-0.3548814E 00
-0.7999996E 00	0.1584895E 00	-0.6811526E 01	0.3851926E 01	0.4564821E 00	-0.3405764E 00
-0.7799996E 00	0.1659589E 00	-0.7168876E 01	-0.3838322E 02	0.4380829E 00	-0.3584438E 00
-0.7599996E 00	0.1737804E 00	-0.4647689E 01	-0.8247665E 02	0.5856196E 00	-0.2323844E 00
-0.7399996E 00	0.1819705E 00	-0.1209361E 02	0.1497837E 03	0.2484959E 00	-0.6046805E 00
-0.7199996E 00	0.1905463E 00	-0.2740001E 02	0.1247059E 03	0.4265786E-01	-0.1370001F 01
-0.6999996E 00	0.1995266E 00	-0.4434769E 02	0.1159257E 03	0.6061994E-02	-0.2217384E 01
-0.6799996E 00	0.2089300E 00	-0.4635945E 02	-0.6906653E 02	0.4808683E-02	-0.2317973E 01
-0.6599996E 00	0.2187765E 00	-0.4134354E 02	-0.7240082E 02	0.8566886E-02	-0.2067177E 01
-0.6399996E 00	0.2290871E 00	-0.4017827E 02	-0.7481848E 02	0.9796821E-02	-0.2008914F 01
-0.6199996E 00	0.2398837E 00	-0.3999454E 02	-0.7666493E 02	0.1000627E-01	-0.1999727E 01
-0.5999996E 00	0.2511889E 00	-0.4019342E 02	-0.7812746F 02	0.9779755E-02	-0.2009671E 01
-0.5799996E 00	0.2630272E 00	-0.4056679E 02	-0.7931821E 02	0.9368293E-02	-0.2028339E 01
-0.5599996E 00	0.2754233E 00	-0.4102501E 02	-0.8030885E 02	0.8886877E-02	-0.2051250E 01
-0.5399996E 00	0.2884037E 00	-0.4152470E 02	-0.8114745E 02	0.8390043E-02	-0.2076236E 01
-0.5199996E 00	0.3019956E 00	-0.4204323E 02	-0.8186768F 02	0.7903837E-02	-0.2102161E 01
-0.4999996E 00	0.3162283E 00	-0.4256831E 02	-0.8249387E 02	0.7440176E-02	-0.2128416E 01
-0.4799997E 00	0.3311318E 00	-0.4309314E 02	-0.8304388E 02	0.7003944E-02	-0.2154657E 01
-0.4599996E 00	0.3467373E 00	-0.4361391E 02	-0.8353145E 02	0.6596360E-02	-0.2180696E 01
-0.4399996E 00	0.3630786E 00	-0.4412862E 02	-0.8396695E 02	0.6216828E-02	-0.2206431E 01
-0.4199997E 00	0.3801898E 00	-0.4463625E 02	-0.8435861E 02	0.5863905E-02	-0.2231812E 01
-0.3999997E 00	0.3981075E 00	-0.4513649E 02	-0.8471303E 02	0.5535737E-02	-0.2256824E 01
-0.3799996E 00	0.4168698E 00	-0.4562927E 02	-0.8503540E 02	0.5230412E-02	-0.2281464E 01
-0.3599997E 00	0.4365163E 00	-0.4611484E 02	-0.8533003E 02	0.4946034E-02	-0.2305742E 01
-0.3399997E 00	0.4570886E 00	-0.4659357E 02	-0.8560040E 02	0.4680816E-02	-0.2329679E 01
-0.3199997E 00	0.4786305E 00	-0.4706577E 02	-0.8584950E 02	0.4433133E-02	-0.2353289E 01
-0.2999997E 00	0.5011877E 00	-0.4753197E 02	-0.8607977F 02	0.4201476E-02	-0.2376598E 01
-0.2799997E 00	0.5248079E 00	-0.4799255E 02	-0.8629326E 02	0.3984489E-02	-0.2399628E 01
-0.2599997E 00	0.5495414E 00	-0.4844791E 02	-0.8649171E 02	0.3780977E-02	-0.2422396E 01
-0.2399997E 00	0.5754404E 00	-0.4889851E 02	-0.8667673E 02	0.3589823F-02	-0.2444926E 01
-0.2199997E 00	0.6025601E 00	-0.4934476E 02	-0.8684956E 02	0.3410056E-02	-0.2467238E 01
-0.1999997E 00	0.6309578E 00	-0.4978696E 02	-0.8701129E 02	0.3240795E-02	-0.2489348E 01
-0.1799997E 00	0.6606939E 00	-0.5022548E 02	-0.8716293E 02	0.3081236F-02	-0.2511274F 01
-0.1599997E 00	0.6918315E 00	-0.5066064E 02	-0.8730537E 02	0.2930677E-02	-0.2533032E 01
-0.1399997E 00	0.7244365E 00	-0.5109271E 02	-0.8743941E 02	0.2788457E-02	-0.2554636E 01
-0.1199997E 00	0.7585782E 00	-0.5152196E 02	-0.8756563E 02	0.2654002E-02	-0.2576098E 01
-0.9999973E-01	0.7943288E 00	-0.5194865E 02	-0.8768469E 02	0.2526774E-02	-0.2597433E 01
-0.7999974E-01	0.8317643E 00	-0.5237300E 02	-0.8779710F 02	0.2406298E-02	-0.2618650E 01
-0.5999970E-01	0.8709642E 00	-0.5279521E 02	-0.8790332E 02	0.2292130E-02	-0.2639761E 01
-0.3999972E-01	0.9120115E 00	-0.5321544E 02	-0.8800380E 02	0.2183872E-02	-0.2660772E 01
-0.1999974E-01	0.9549932E 00	-0.5363390E 02	-0.8809894E 02	0.2081155E-02	-0.2681695E 01
0.0	0.1000000E 01	-0.5405075E 02	-0.8818910E 02	0.1983637E-02	-0.2702538E 01


LOG(FREQ)
20.*LOG(ABS(H))
-1.02E 02 -9.20E 01 -8.20E 01 -7.20E 01 -6.20E 01 -5.20E 01 -4.20E 01 -3.20E 01 -2.20E 01 -1.20E 01 -2.00E 00
-1.0200E 00
-9.2000E-01
-8.2000E-01
-7.2000E-01
-6.2000E-01
-5.2000E-01
-4.2000E-01
-3.2000E-01
-2.2000E-01
-1.2000E-01
-2.0000E-02


LOG(FREQ)
PHI
-2.00E 02 -1.60E 02 -1.20E 02 -8.00E 01 -4.00E 01 0.0 4.00E 01 8.00E 01 1.20E 02 1.60E 02 2.00E 02
-1.0200E 00
-9.2000E-01
-8.2000E-01
-7.2000E-01
-6.2000E-01
-5.2000E-01
-4.2000E-01
-3.2000E-01
-2.2000E-01
-1.2000E-01
-2.0000E-02




```
NASAP PROGRAM  HIGH  PASS

V1   9   1   1
C1   1   2   0.1385UF
C2   2   3   2.43UF
L1   3   4   432.5MH
R1   4   9   53.29
C3   2   5   0.1185UF
C4   5   6   0.5862UF
L2   6   7   754.5MH
R2   7   9   115.6
C5   5   8   0.5274UF
R3   8   9   2K
OUTPUT
VR3/VV1
FREQ  -2   8   .2
EXECUTE
```

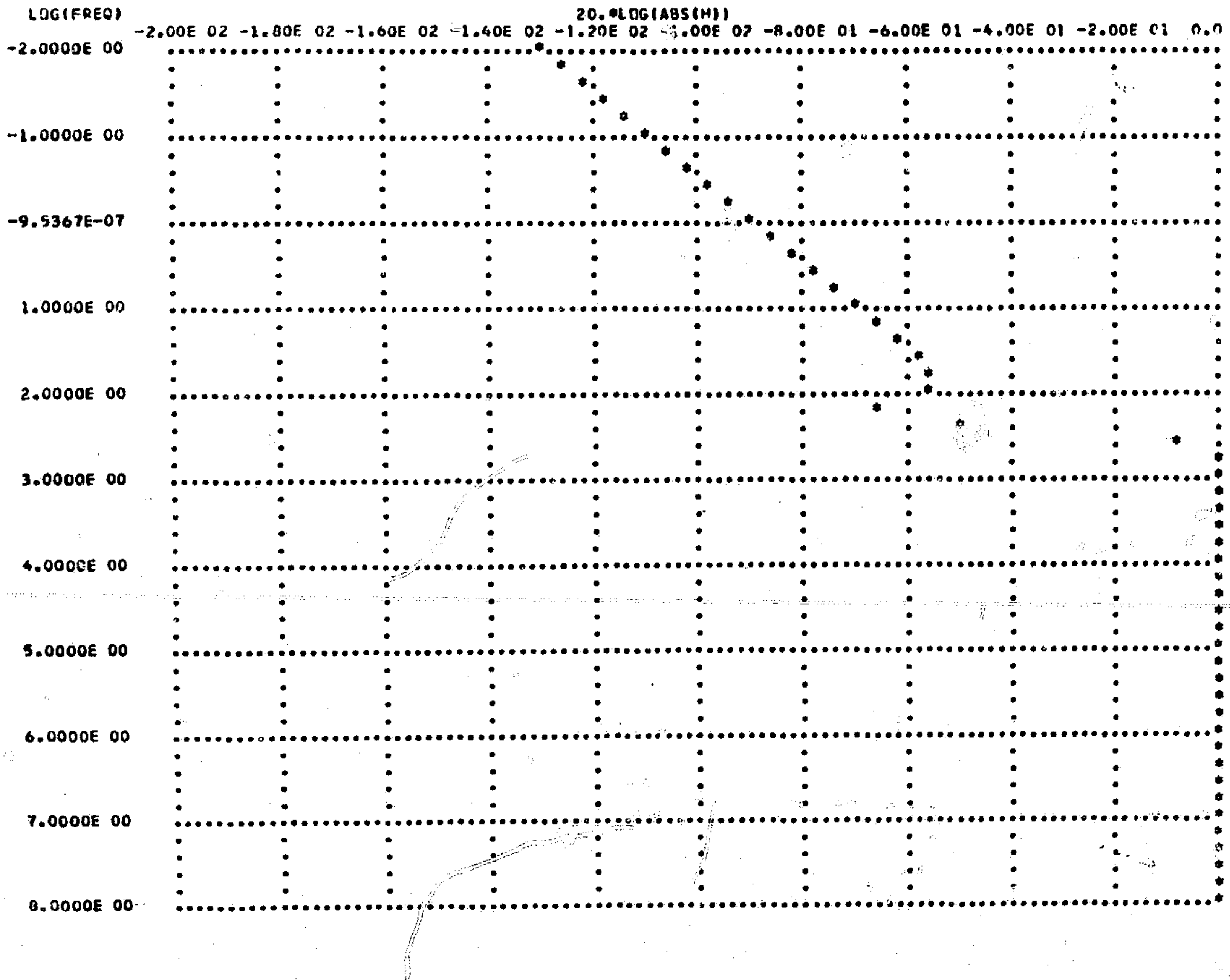
NUMBER OF LOOPS PER ORDER

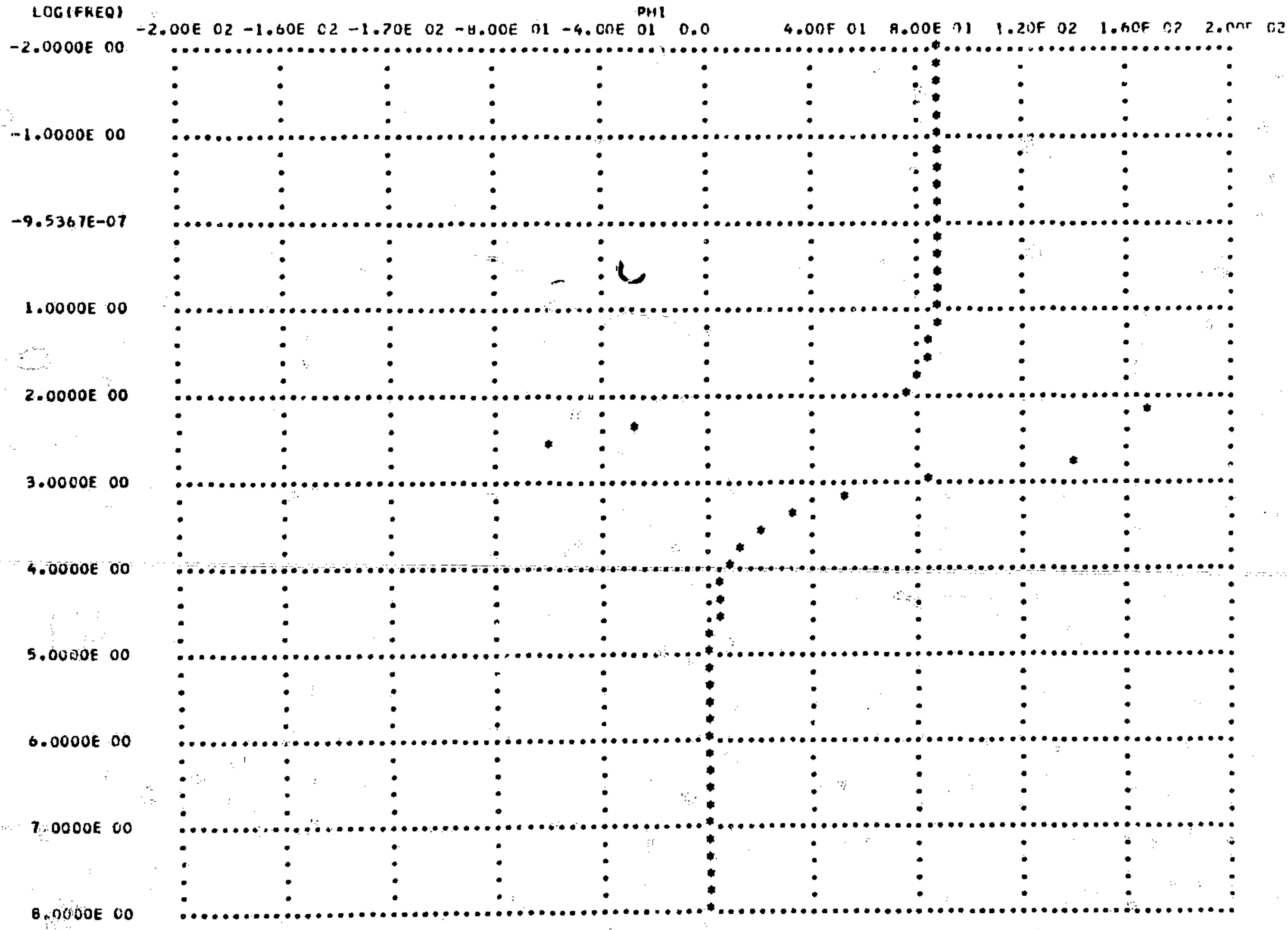
```
1=    17
2=    48
3=    36
```

TRANSFER FUNCTION VR3/VV1

```
                      (                                        2              3              4              5 )
                      ( 0.0        +2.15E 12 S    +4.24E 08 S    +3.23E 06 S    +2.76E 02 S    +1.00E 00 S    )
H(S)= 1.000E 00*------------------------------------------------------------------------------------------------
                      (                                        2              3              4              5 )
                      ( 6.10E 17   +2.66E 14 S    +1.40E 11 S    +4.31E 07 S    +9.05E 03 S    +1.00E 00 S    )
```

LOG(FREQ)	FREQ	20.*LOG(ABS(H))	PHI(H)	ABS(H)	LOG(ABS(H))
-0.2000000E 01	0.1000001E-01	-0.1296311E 03	0.8999834E 02	0.3299488E-06	-0.6481554E 01
-0.1799999E 01	0.1584898E-01	-0.1256311E 03	0.8999734E 02	0.5229346E-06	-0.6281553E 01
-0.1599999E 01	0.2511893E-01	-0.1216311E 03	0.8999586E 02	0.8287951E-06	-0.6081553E 01
-0.1400000E 01	0.3981078E-01	-0.1176311E 03	0.8999347E 02	0.1313550E-05	-0.5881554E 01
-0.1200000E 01	0.6309581E-01	-0.1136311E 03	0.8998970E 02	0.2081836E-05	-0.5681554E 01
-0.1000000E 01	0.1000000E 00	-0.1096311E 03	0.8998369E 02	0.3299484E-05	-0.5481554E 01
-0.8000011E 00	0.1584890E 00	-0.1056311E 03	0.8997418E 02	0.5229316E-05	-0.5281555E 01
-0.6000013E 00	0.2511880E 00	-0.1016311E 03	0.8995915E 02	0.8287877E-05	-0.5081557E 01
-0.4000006E 00	0.3981067E 00	-0.9763116E 02	0.8993527E 02	0.1313536E-04	-0.4881558E 01
-0.2000008E 00	0.6309563E 00	-0.9363129E 02	0.8989740E 02	0.2081784E-04	-0.4681564E 01
-0.9536743E-06	0.9999978E 00	-0.8963155E 02	0.8983746E 02	0.3299300E-04	-0.4481578E 01
0.1999989E 00	0.1584888E 01	-0.8563226E 02	0.8974239E 02	0.5228611E-04	-0.4281613E 01
0.3999987E 00	0.2511878E 01	-0.8163402E 02	0.8959180E 02	0.8285115E-04	-0.4081701E 01
0.5999985E 00	0.3981055E 01	-0.7763844E 02	0.8935318E 02	0.1312432E-03	-0.3881923E 01
0.7999983E 00	0.6309541E 01	-0.7364957E 02	0.8897517E 02	0.2077402E-03	-0.3682479E 01
0.9999981E 00	0.9999948E 01	-0.6967755E 02	0.8837695E 02	0.3281871E-03	-0.3483878E 01
0.1199999E 01	0.1584888E 02	-0.6574806E 02	0.8743253E 02	0.5159371E-03	-0.3287403E 01
0.1399999E 01	0.2511876E 02	-0.6192697E 02	0.8595059E 02	0.8010340E-03	-0.3096349E 01
0.1599998E 01	0.3981053E 02	-0.5838780E 02	0.8366548E 02	0.1203954E-02	-0.2919391E 01
0.1799998E 01	0.6309538E 02	-0.5562512E 02	0.8034810E 02	0.1654792E-02	-0.2781257E 01
0.1999998E 01	0.9999944E 02	-0.5540257E 02	0.7714966E 02	0.1697737E-02	-0.2770129E 01
0.2199998E 01	0.1584883E 03	-0.6675862E 02	0.1664292E 03	0.4592701E-03	-0.3337932E 01
0.2399998E 01	0.2511869E 03	-0.5085362E 02	-0.2930338E 02	0.2866275E-02	-0.2542682E 01
0.2599998E 01	0.3981045E 03	-0.8472850E 01	-0.6037357E 02	0.3770140E 00	-0.4236425E 00
0.2799998E 01	0.6309536E 03	-0.3576095E-01	0.1410596E 03	0.9958913E 00	-0.1788047E-02
0.2999998E 01	0.9999939E 03	-0.1792070E-01	0.8247884E 02	0.9979389E 00	-0.8960348E-03
0.3199998E 01	0.1584882E 04	-0.4348526E-01	0.5087636E 02	0.9950061E 00	-0.2174263E-02
0.3399998E 01	0.2511868E 04	-0.2819930E-01	0.3192720E 02	0.9967587E 00	-0.1409965E-02
0.3599998E 01	0.3981042E 04	-0.1336743E-01	0.2011768E 02	0.9984622E 00	-0.6683718E-03
0.3799997E 01	0.6309520E 04	-0.5704034E-02	0.1268851E 02	0.9993435E 00	-0.2852017E-03
0.3999997E 01	0.9999914E 04	-0.2338854E-02	0.8004858E 01	0.9997308E 00	-0.1169427E-03
0.4199997E 01	0.1584878E 05	-0.9490312E-03	0.5050494E 01	0.9998907E 00	-0.4745157E-04
0.4399998E 01	0.2511867E 05	-0.3826027E-03	0.3186586E 01	0.9999560E 00	-0.1913014E-04
0.4599998E 01	0.3981041E 05	-0.1708490E-03	0.2010586E 01	0.9999803E 00	-0.8542452E-05
0.4799997E 01	0.6309518E 05	-0.7455189E-04	0.1268592E 01	0.9999914E 00	-0.3727595E-05
0.4999997E 01	0.9999912E 05	-0.2536828E-04	0.8004258E 00	0.9999971E 00	-0.1268414E-05
0.5199997E 01	0.1584877E 06	-0.1708475E-04	0.5050337E 00	0.9999980E 00	-0.8542378E-06
0.5399997E 01	0.2511861E 06	-0.8801237E-05	0.3186552E 00	0.9999990E 00	-0.4400619E-06
0.5599997E 01	0.3981027E 06	-0.8801237E-05	0.2010583E 00	0.9999990E 00	-0.4400619E-06
0.5799996E 01	0.6309503E 06	-0.8801237E-05	0.1268591E 00	0.9999990E 00	-0.4400619E-06
0.5999997E 01	0.9999906E 06	-0.5177194E-06	0.8004248E-01	0.9999999E 00	-0.2588597E-07
0.6199997E 01	0.1584877E 07	0.0	0.5050350E-01	0.1000000E 01	0.0
0.6399997E 01	0.2511860E 07	0.0	0.3186590E-01	0.1000000E 01	0.0
0.6599997E 01	0.3981026E 07	0.0	0.2010581E-01	0.1000000E 01	0.0
0.6799996E 01	0.6309500E 07	0.0	0.1268592E-01	0.1000000E 01	0.0
0.6999996E 01	0.9999769E 07	0.0	0.8004360E-02	0.1000000E 01	0.0
0.7199996E 01	0.1584870E 08	0.0	0.5050369E-02	0.1000000E 01	0.0
0.7399996E 01	0.2511832E 08	0.0	0.3186591E-02	0.1000000E 01	0.0
0.7599997E 01	0.3981021E 08	0.0	0.2010582E-02	0.1000000E 01	0.0
0.7799996E 01	0.6309450E 08	0.0	0.1268602E-02	0.1000000E 01	0.0
0.7999996E 01	0.9999736E 08	0.0	0.8004394E-03	0.1000000E 01	0.0


LOG(FREQ)
20.*LOG(ABS(H))
-2.00E 02 -1.80E 02 -1.60E 02 -1.40E 02 -1.20E 02 -1.00E 02 -8.00E 01 -6.00E 01 -4.00E 01 -2.00E 01 0.0
-2.0000E 00
-1.0000E 00
-9.5367E-07
1.0000E 00
2.0000E 00
3.0000E 00
4.0000E 00
5.0000E 00
6.0000E 00
7.0000E 00
8.0000E 00


LOG(FREQ)
PHI
-2.00E 02 -1.60E 02 -1.20E 02 -8.00E 01 -4.00E 01 0.0 4.00E 01 8.00E 01 1.20E 02 1.60E 02 2.00E 02
-2.0000E 00
-1.0000E 00
-9.5367E-07
1.0000E 00
2.0000E 00
3.0000E 00
4.0000E 00
5.0000E 00
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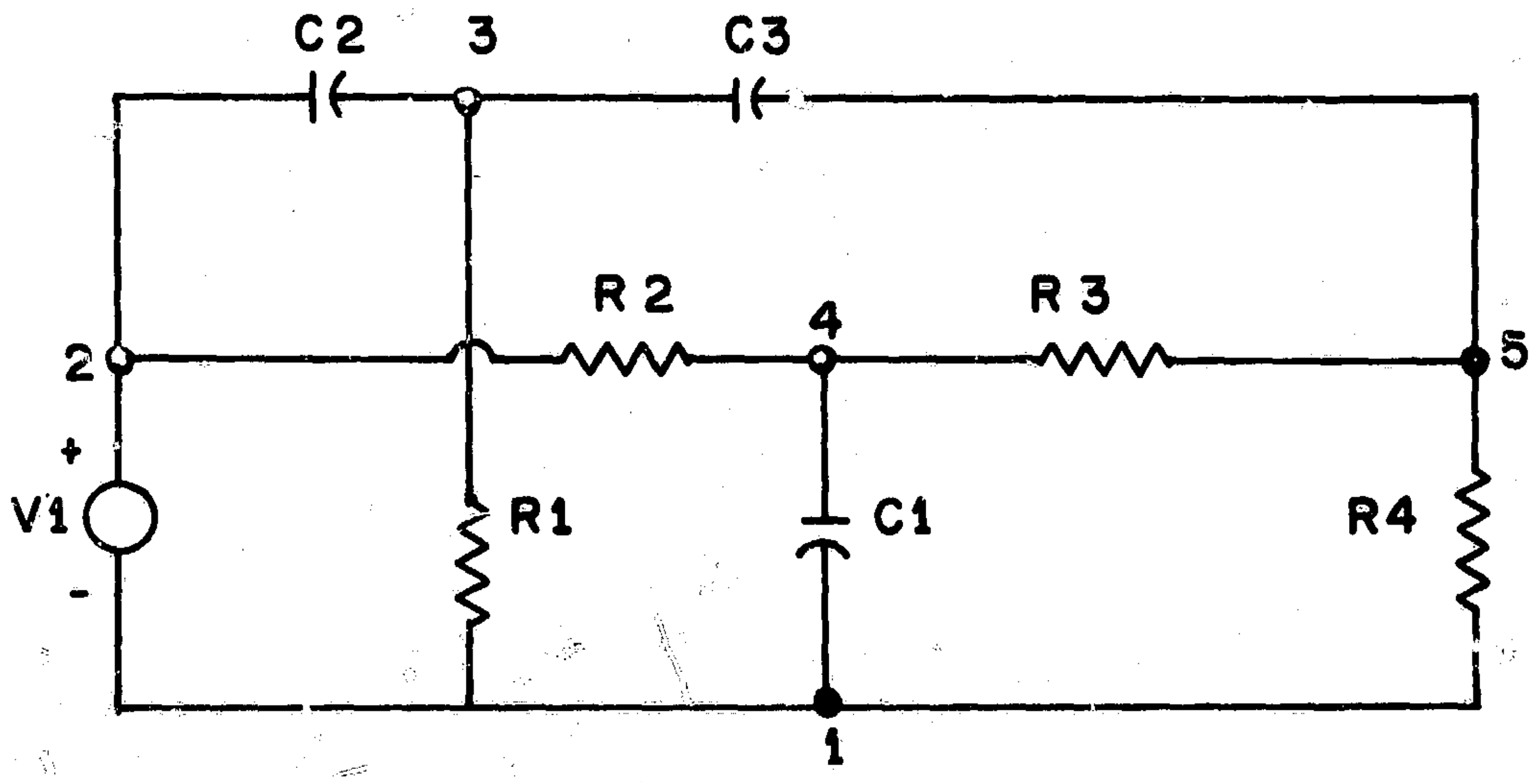
7.0000E 00
8.0000E 00



NASAP PROGRAM TWIN-T, 60 CYCLE ELMINATION

```
C1 4 1 .1UF
C2 2 3  .05UF
C3 3 5  .05UF
R1 3 1 24K
R2 2 4 48K
R3 4 5 48K
R4 5 1 100K
V1 1 2 1
OUTPUT
VR4/VV1
FREQ  -2  8  .2
EXECUTE
```

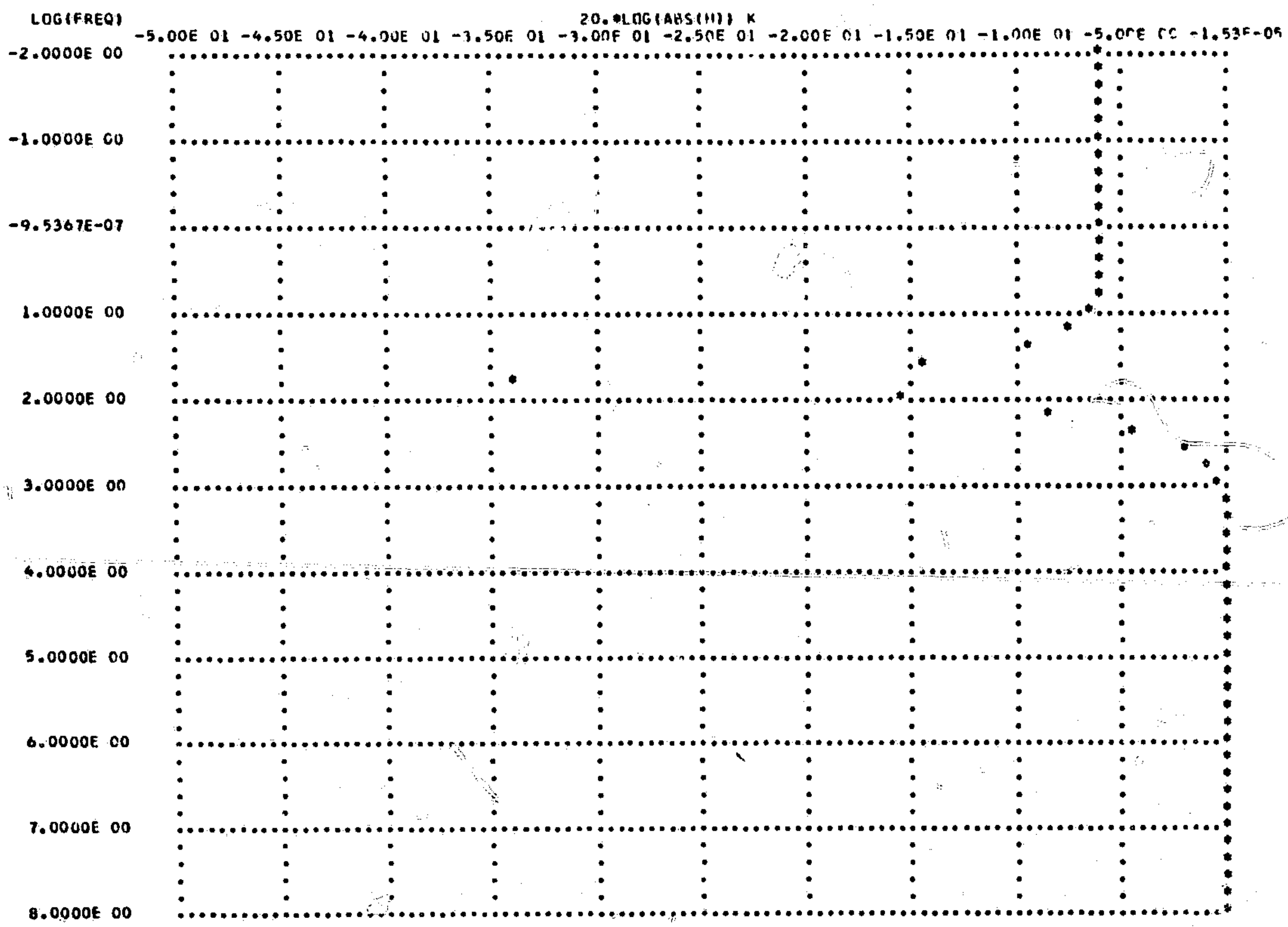
NUMBER OF LOOPS PER ORDER

```
1=   9
2=  17
3=   6
```

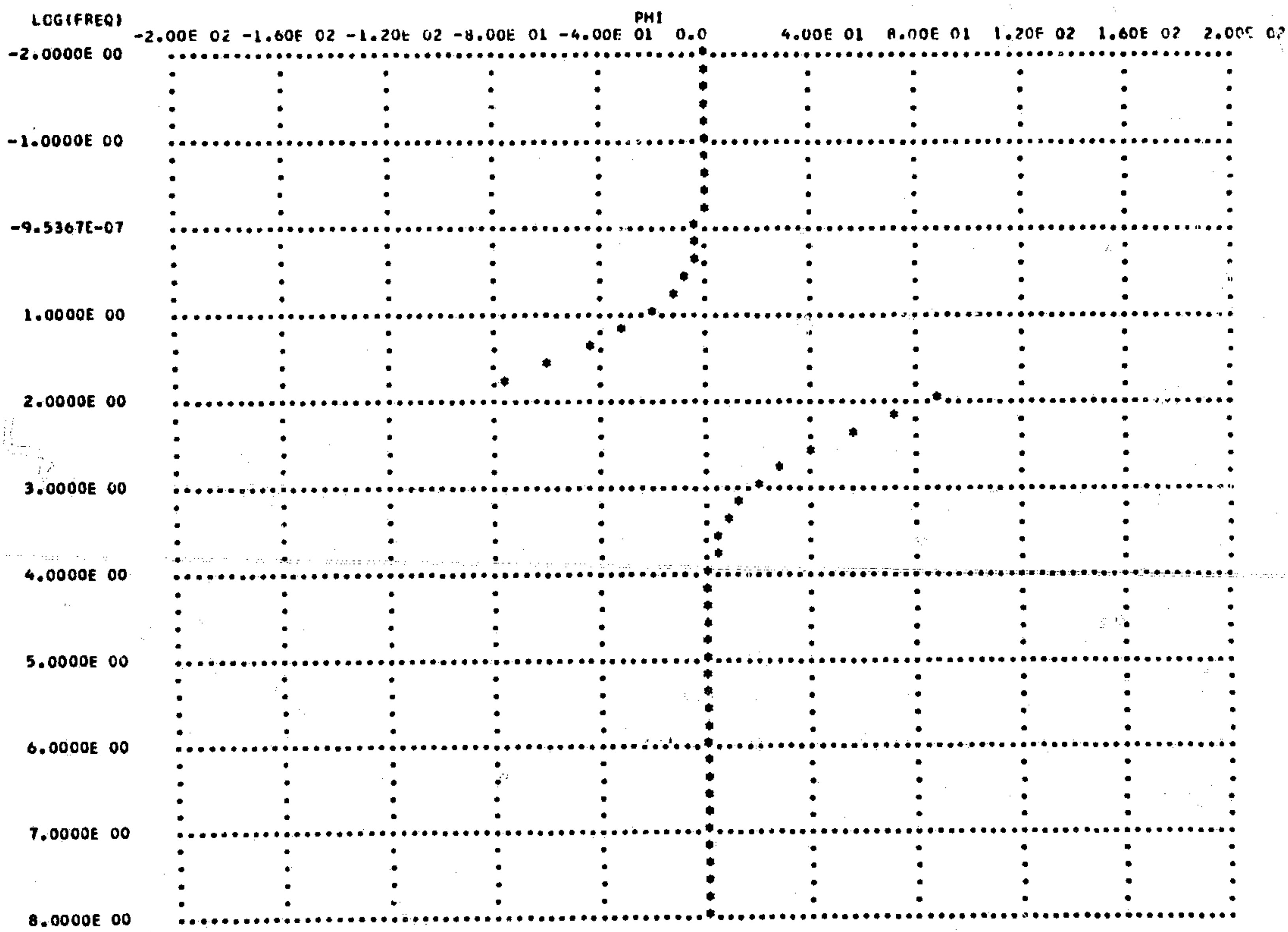
TRANSFER FUNCTION VR4/VV1

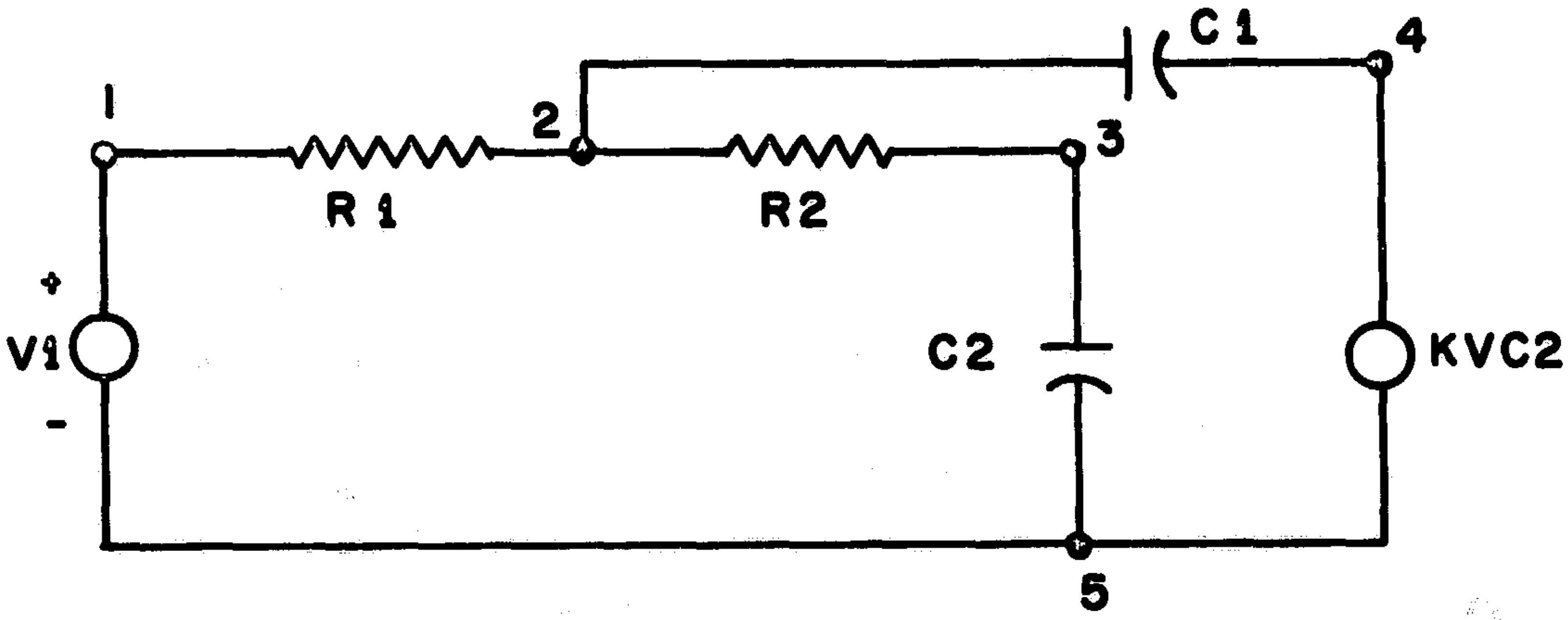
```
                  (                                                 2              3 )
                  (  7.23E 07    +1.74E 05 S    +4.17E 02 S    +1.00E 00 S         )
H(S)= 1.000E 00*--------------------------------------------------------------------
                  (                                                 2              3 )
                  (  1.42E 08    +1.20E 06 S    +2.48E 03 S    +1.00E 00 S         )
```

LOG(FREQ)	FREQ	20.*LOG(ABS(H))	PHI(H)	ABS(H)	LOG(ABS(H))
-0.2000000E 01	0.1000001E-01	-0.5845128E 01	-0.2186439E-01	0.5102037E 00	-0.2922564E 00
-0.1799999E 01	0.1584898E-01	-0.5845127E 01	-0.3465284E-01	0.5102038E 00	-0.2922564E 00
-0.1599999E 01	0.2511893E-01	-0.5845129E 01	-0.5492103E-01	0.5102035E 00	-0.2922565E 00
-0.1400000E 01	0.3981078E-01	-0.5845139E 01	-0.8704376E-01	0.5102032E 00	-0.2922570E 00
-0.1200000E 01	0.6309581E-01	-0.5845154E 01	-0.1379550E 00	0.5102023E 00	-0.2922577E 00
-0.1000000E 01	0.1000000E 00	-0.5845204E 01	-0.2186434E 00	0.5101994E 00	-0.2922602E 00
-0.8000011E 00	0.1584890E 00	-0.5845308E 01	-0.3465234E 00	0.5101931E 00	-0.2922654E 00
-0.6000013E 00	0.2511880E 00	-0.5845586E 01	-0.5491939E 00	0.5101768E 00	-0.2922793E 00
-0.4000006E 00	0.3981067E 00	-0.5846282E 01	-0.8703846E 00	0.5101359E 00	-0.2923141E 00
-0.2000008E 00	0.6309563E 00	-0.5848033E 01	-0.1379346E 01	0.5100331E 00	-0.2924017E 00
-0.9536743E-06	0.9999978E 00	-0.5852423E 01	-0.2185628E 01	0.5097755E 00	-0.2926211E 00
0.1999989E 00	0.1584888E 01	-0.5863430E 01	-0.3462056E 01	0.5091297E 00	-0.2931715E 00
0.3999987E 00	0.2511878E 01	-0.5891029E 01	-0.5479317E 01	0.5075147E 00	-0.2945515E 00
0.5999985E 00	0.3981055E 01	-0.5959967E 01	-0.8653944E 01	0.5035026E 00	-0.2979984E 00
0.7999983E 00	0.6309541E 01	-0.6130875E 01	-0.1359824E 02	0.4936922E 00	-0.3065438E 00
0.9999981E 00	0.9999948E 01	-0.6546991E 01	-0.2111073E 02	0.4705985E 00	-0.3273496E 00
0.1199999E 01	0.1584888E 02	-0.7523237E 01	-0.3192114E 02	0.4205699E 00	-0.3761619E 00
0.1399999E 01	0.2511876E 02	-0.9689080E 01	-0.4596477E 02	0.3277522E 00	-0.4844540E 00
0.1599998E 01	0.3981053E 02	-0.1446031E 02	-0.6175490E 02	0.1892275E 00	-0.7230154E 00
0.1799998E 01	0.6309538E 02	-0.3415910E 02	-0.7740144E 02	0.1959043E-01	-0.1707955E 01
0.1999990E 01	0.9999944E 02	-0.1538213E 02	0.8759645E 02	0.1701741E 00	-0.7691065E 00
0.2199998E 01	0.1584883E 03	-0.8428205E 01	0.7243732E 02	0.3789569E 00	-0.4214103E 00
0.2399998E 01	0.2511869E 03	-0.4537122E 01	0.5660121E 02	0.5931218E 00	-0.2268561E 00
0.2599998E 01	0.3981045E 03	-0.2222244E 01	0.4114478E 02	0.7742615E 00	-0.1111122E 00
0.2799998E 01	0.6309536E 03	-0.9917510E 00	0.2805060E 02	0.8920978E 00	-0.4958755E-01
0.2999998E 01	0.9999939E 03	-0.4166448E 00	0.1835492E 02	0.9531643E 00	-0.2083224E-01
0.3199998E 01	0.1584882E 04	-0.1697488E 00	0.1176374E 02	0.9806467E 00	-0.8487441E-02
0.3399998E 01	0.2511868E 04	-0.6821883E-01	0.7470314E 01	0.9921768E 00	-0.3410942E-02
0.3599998E 01	0.3981042E 04	-0.2727690E-01	0.4725691E 01	0.9968646E 00	-0.1363845E-02
0.3799997E 01	0.6309520E 04	-0.1087528E-01	0.2984816E 01	0.9987487E 00	-0.5437643E-03
0.3999997E 01	0.9999914E 04	-0.4349411E-02	0.1884073E 01	0.9994994E 00	-0.2174707E-03
0.4199997E 01	0.1584878E 05	-0.1738676E-02	0.1188967E 01	0.9997998E 00	-0.8693378E-04
0.4399998E 01	0.2511867E 05	-0.6963606E-03	0.7502347E 00	0.9999198E 00	-0.3481803E-04
0.4599998E 01	0.3981041E 05	-0.2821614E-03	0.4733786E 00	0.9999675E 00	-0.1410807E-04
0.4799997E 01	0.6309518E 05	-0.1164877E-03	0.2986850E 00	0.9999866E 00	-0.5824384E-05
0.4999997E 01	0.9999912E 05	-0.5850250E-04	0.1884584E 00	0.9999933E 00	-0.2925125E-05
0.5199997E 01	0.1584877E 06	-0.1708475E-04	0.1189096E 00	0.9999980E 00	-0.8542378E-06
0.5399997E 01	0.2511861E 06	-0.1656704E-04	0.7502699E-01	0.9999981E 00	-0.8283518E-06
0.5599997E 01	0.3981027E 06	-0.8283510E-05	0.4733885E-01	0.9999990E 00	-0.4141755E-06
0.5799996E 01	0.6309503E 06	0.0	0.2986880E-01	0.1000000E 01	0.0
0.5999997E 01	0.9999906E 06	0.0	0.1884587E-01	0.1000000E 01	0.0
0.6199997E 01	0.1584877E 07	0.0	0.1189097E-01	0.1000000E 01	0.0
0.6399997E 01	0.2511860E 07	0.0	0.7502701E-02	0.1000000E 01	0.0
0.6599997E 01	0.3981026E 07	0.0	0.4733890E-02	0.1000000E 01	0.0
0.6799996E 01	0.6309500E 07	0.0	0.2986881E-02	0.1000000E 01	0.0
0.6999996E 01	0.9999769E 07	0.0	0.1886616E-02	0.1000000E 01	0.0
0.7199996E 01	0.1584870E 08	0.0	0.1189102E-02	0.1000000E 01	0.0
0.7399996E 01	0.2511832E 08	0.0	0.7502781E-03	0.1000000E 01	0.0
0.7599997E 01	0.3981021E 08	0.0	0.4733894E-03	0.1000000E 01	0.0
0.7799996E 01	0.6309450E 08	0.0	0.2986907E-03	0.1000000E 01	0.0
0.7999996E 01	0.9999736E 08	0.0	0.1884623E-03	0.1000000E 01	0.0



LOG(FREQ)
20.*LOG(ABS(H)) K
-5.00E 01 -4.50E 01 -4.00E 01 -3.50E 01 -3.00E 01 -2.50E 01 -2.00E 01 -1.50E 01 -1.00E 01 -5.00E 00 -1.53E-05
-2.0000E 00
-1.0000E 00
-9.5367E-07
1.0000E 00
2.0000E 00
3.0000E 00
4.0000E 00
5.0000E 00
6.0000E 00
7.0000E 00
8.0000E 00


LOG(FREQ)
PHI
-2.00E 02
-1.60E 02
-1.20E 02
-8.00E 01
-4.00E 01
0.0
4.00E 01
8.00E 01
1.20E 02
1.60E 02
-2.0000E 00
-1.0000E 00
-9.5367E-07
1.0000E 00
2.0000E 00
3.0000E 00
4.0000E 00
5.0000E 00
6.0000E 00
7.0000E 00
8.0000E 00



NASAP PROGRAM 2ND ORDER RC ACTIVE LOWPASS FILTER

```
V1   5   1   1.0
R1   1   2   .707
R2   2   3   1.414
C1   2   4   1.0F
C2   3   5   1.0F
V2   4   5   -2.0   VC2
OUTPUT
VV2/VV1
FREQ   -2   8   .2
EXECUTE
```

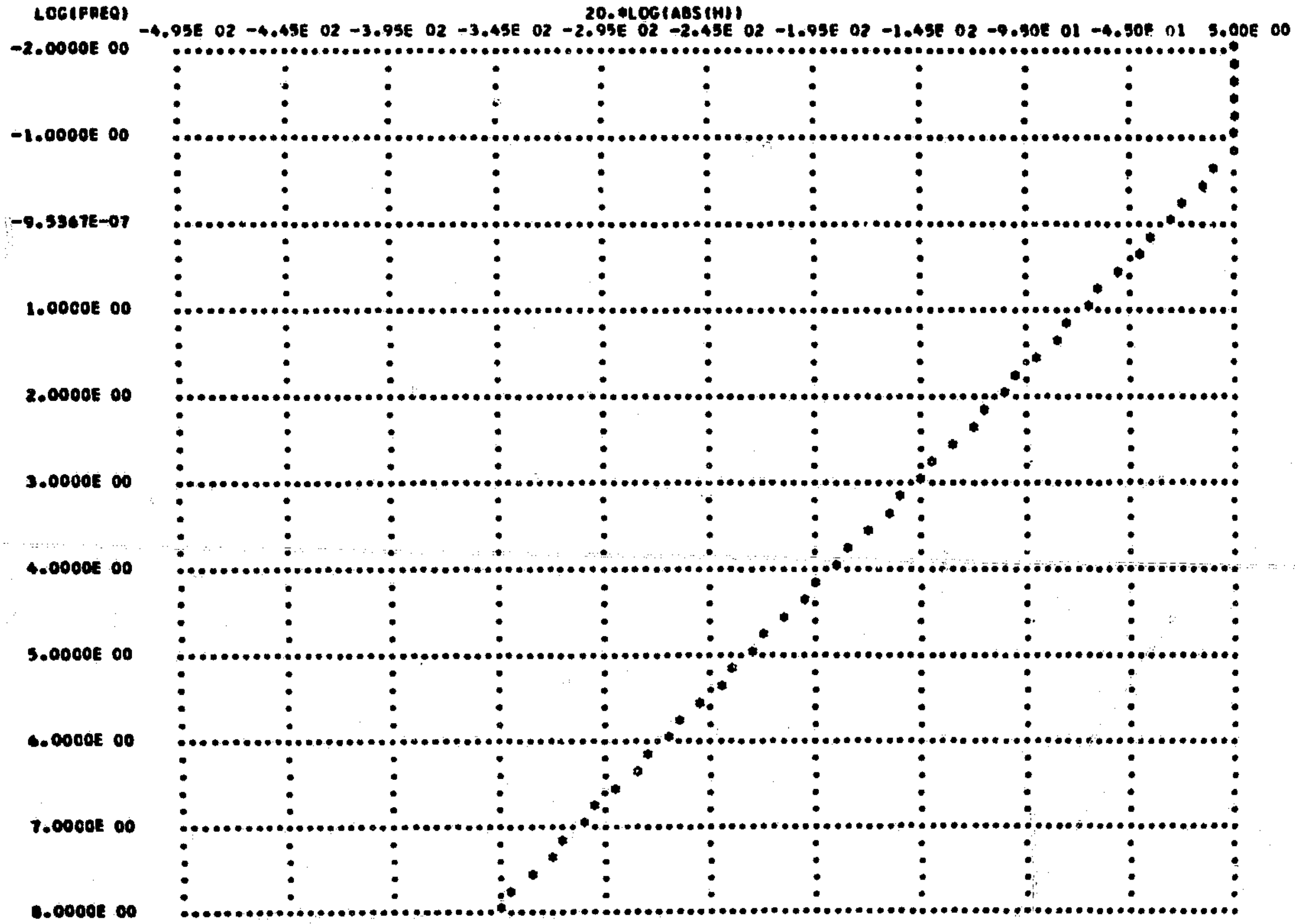
NUMBER OF LOOPS PER ORDER

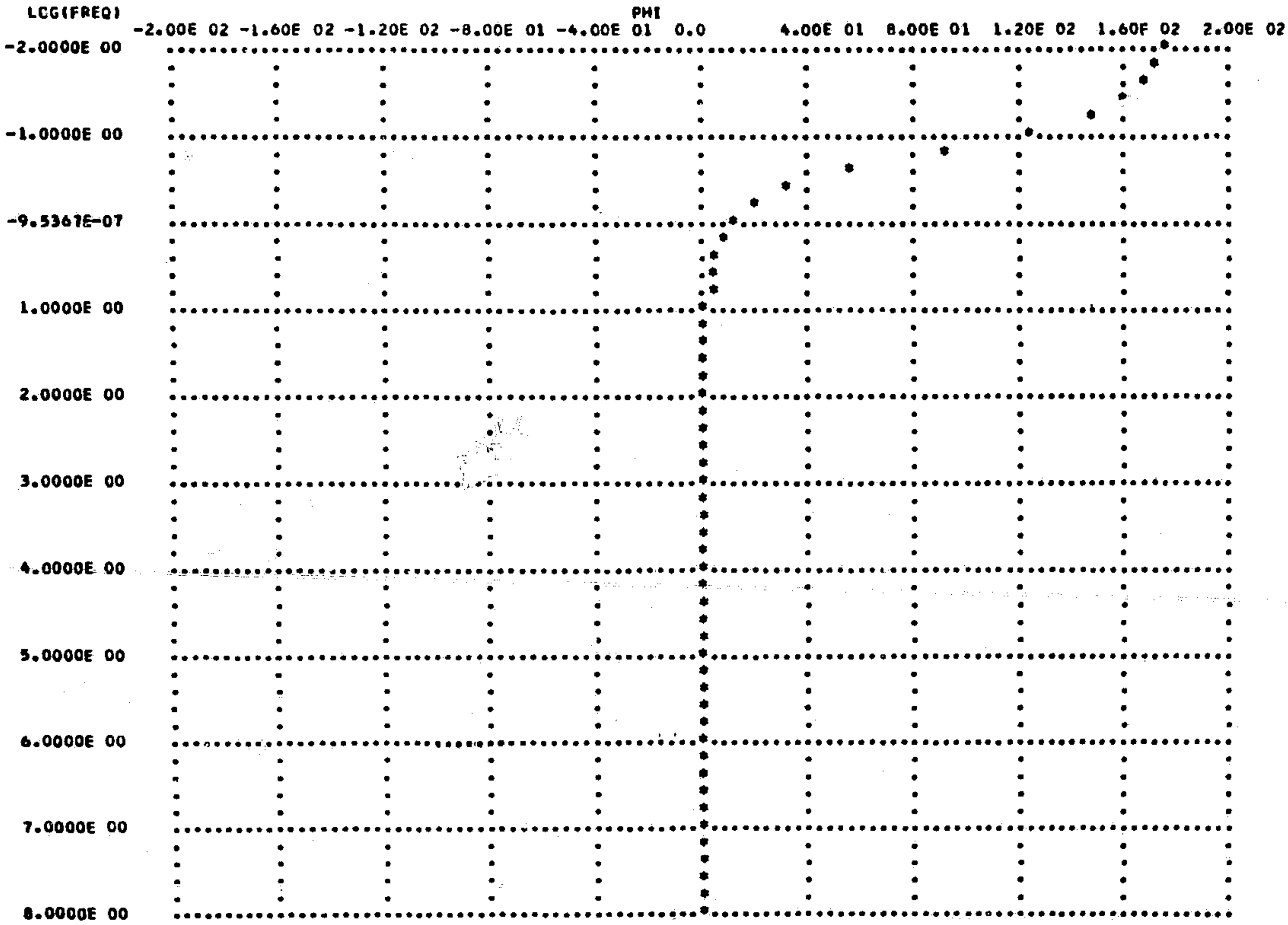
```
    1=     6
    2=     2
```

TRANSFER FUNCTION   VV2/VV1

```
                    (                 )
                    (  1.00E 00       )
H(S)= 2.001E 00*-----------------------------------------------
                    (                                       2 )
                    (  1.00E 00     +1.41E 00 S   +1.00E 00 S  )
```

LOG(FREQ)	FREQ	20.*LOG(ABS(H))	PHI(H)	ABS(H)	LOG(ABS(H))
-0.2000000E 01	0.1000001E-01	0.6020528E 01	0.1749029E 03	0.1999984E 01	0.3010264E 00
-0.1799999E 01	0.1584898E-01	0.6020167E 01	0.1719057E 03	0.1999901E 01	0.3010084E 00
-0.1599999E 01	0.2511893E-01	0.6017906E 01	0.1671089E 03	0.1999380E 01	0.3008953E 00
-0.1400000E 01	0.3981078E-01	0.6003638E 01	0.1593287E 03	0.1996099E 01	0.3001819E 00
-0.1200000E 01	0.6309581E-01	0.5914684E 01	0.1463735E 03	0.1975760E 01	0.2957342E 00
-0.1000000E 01	0.1000000E 00	0.5391904E 01	0.1242682E 03	0.1860353E 01	0.2695952E 00
-0.8000011E 00	0.1584890E 00	0.3047843E 01	0.9035191E 02	0.1420340E 01	0.1523921E 00
-0.6000013E 00	0.2511880E 00	-0.2553255E 01	0.5626759E 02	0.7453105E 00	-0.1276628E 00
-0.4000006E 00	0.3981067E 00	-0.1001357E 02	0.3394301E 02	0.3157339E 00	-0.5006785E 00
-0.2000008E 00	0.6309563E 00	-0.1792151E 02	0.2085843E 02	0.1270353E 00	-0.8960758E 00
-0.9536743E-06	0.9999978E 00	-0.2590671E 02	0.1300530E 02	0.5065981E-01	-0.1295336E 01
0.1999989E 00	0.1584888E 01	-0.3390436E 02	0.8165326E 01	0.2017350E-01	-0.1695218E 01
0.3999987E 00	0.2511878E 01	-0.4190401E 02	0.5141666E 01	0.8031547E-02	-0.2095201E 01
0.5999985E 00	0.3981055E 01	-0.4990392E 02	0.3241576E 01	0.3197452E-02	-0.2495196E 01
0.7999983E 00	0.6309541E 01	-0.5790390E 02	0.2044642E 01	0.1272931E-02	-0.2895195E 01
0.9999981E 00	0.9999948E 01	-0.6590388E 02	0.1289919E 01	0.5067633E-03	-0.3295195E 01
0.1199999E 01	0.1584888E 02	-0.7390392E 02	0.8138409E 00	0.2017456E-03	-0.3695196E 01
0.1399999E 01	0.2511876E 02	-0.8190388E 02	0.5134884E 00	0.8031647E-04	-0.4095195E 01
0.1599998E 01	0.3981053E 02	-0.8990390E 02	0.3239866E 00	0.3197454E-04	-0.4495195E 01
0.1799998E 01	0.6309538E 02	-0.9790387E 02	0.2044216E 00	0.1272935E-04	-0.4895194E 01
0.1999998E 01	0.9999944E 02	-0.1059039E 03	0.1289811E 00	0.5067638E-05	-0.5295195E 01
0.2199998E 01	0.1584883E 03	-0.1139039E 03	0.8138168E-01	0.2017468E-05	-0.5695193E 01
0.2399998E 01	0.2511869E 03	-0.1219039E 03	0.5134832E-01	0.8031696E-06	-0.6095193E 01
0.2599998E 01	0.3981045E 03	-0.1299039E 03	0.3239857E-01	0.3197472E-06	-0.6495193E 01
0.2799998E 01	0.6309536E 03	-0.1379039E 03	0.2044207E-01	0.1272933E-06	-0.6895195E 01
0.2999998E 01	0.9999939E 03	-0.1459037E 03	0.1289811E-01	0.5067649E-07	-0.7295188E 01
0.3199998E 01	0.1584882E 04	-0.1539038E 03	0.8138161E-02	0.2017471E-07	-0.7695189E 01
0.3399998E 01	0.2511868E 04	-0.1619038E 03	0.5134836E-02	0.8031698E-08	-0.8095189E 01
0.3599998E 01	0.3981042E 04	-0.1699038E 03	0.3239861E-02	0.3197479E-08	-0.8495191E 01
0.3799997E 01	0.6309520E 04	-0.1779038E 03	0.2044216E-02	0.1272941E-08	-0.8895191E 01
0.3999997E 01	0.9999914E 04	-0.1859038E 03	0.1289813E-02	0.5067669E-09	-0.9295192E 01
0.4199997E 01	0.1584878E 05	-0.1939037E 03	0.8138185E-03	0.2017482E-09	-0.9695186E 01
0.4399998E 01	0.2511867E 05	-0.2019039E 03	0.5134833E-03	0.8031706E-10	-0.1009519E 02
0.4599998E 01	0.3981041E 05	-0.2099037E 03	0.3239862E-03	0.3197477E-10	-0.1049519E 02
0.4799997E 01	0.6309518E 05	-0.2179038E 03	0.2044217E-03	0.1272943E-10	-0.1089519E 02
0.4999997E 01	0.9999912E 05	-0.2259038E 03	0.1289814E-03	0.5067670E-11	-0.1129519E 02
0.5199997E 01	0.1584877E 06	-0.2339038E 03	0.8138175E-04	0.2017480E-11	-0.1169519E 02
0.5399997E 01	0.2511861E 06	-0.2419038E 03	0.5134851E-04	0.8031747E-12	-0.1209519E 02
0.5599997E 01	0.3981027E 06	-0.2499037E 03	0.3239873E-04	0.3197501E-12	-0.1249518E 02
0.5799996E 01	0.6309503E 06	-0.2579036E 03	0.2044222E-04	0.1272949E-12	-0.1289519E 02
0.5999997E 01	0.9999906E 06	-0.2659036E 03	0.1289814E-04	0.5067680E-13	-0.1329519E 02
0.6199997E 01	0.1584877E 07	-0.2739036E 03	0.8138184E-05	0.2017483E-13	-0.1369519E 02
0.6399997E 01	0.2511860E 07	-0.2819036E 03	0.5134848E-05	0.8031739E-14	-0.1409519E 02
0.6599997E 01	0.3981026E 07	-0.2899036E 03	0.3239876E-05	0.3197506E-14	-0.1449519E 02
0.6799996E 01	0.6309500E 07	-0.2979036E 03	0.2044223E-05	0.1272949E-14	-0.1489519E 02
0.6999996E 01	0.9999769E 07	-0.3059033E 03	0.1289833E-05	0.5067815E-15	-0.1529518E 02
0.7199995E 01	0.1584870E 08	-0.3139036E 03	0.8138219E-06	0.2017501E-15	-0.1569518E 02
0.7399996E 01	0.2511832E 08	-0.3219033E 03	0.5134905E-06	0.8031926E-16	-0.1609517E 02
0.7599997E 01	0.3981021E 08	-0.3299036E 03	0.3239876E-06	0.3197508E-16	-0.1649518E 02
0.7799996E 01	0.6309450E 08	-0.3379033E 03	0.2044241E-06	0.1272971E-16	-0.1689517E 02
0.7999996E 01	0.9999736E 08	-0.3459033E 03	0.1289836E-06	0.5067854E-17	-0.1729517E 02


LOG(FREQ)
20.*LOG(ABS(H))
-4.95E 02 -4.45E 02 -3.95E 02 -3.45E 02 -2.95E 02 -2.45E 02 -1.95E 02 -1.45E 02 -9.50E 01 -4.50E 01 5.00E 00
-2.0000E 00
-1.0000E 00
-9.5367E-07
1.0000E 00
2.0000E 00
3.0000E 00
4.0000E 00
5.0000E 00
6.0000E 00
7.0000E 00
8.0000E 00


LCG(FREQ)
PHI
-2.00E 02 -1.60E 02 -1.20E 02 -8.00E 01 -4.00E 01 0.0 4.00E 01 8.00E 01 1.20E 02 1.60F 02 2.00E 02
-2.0000E 00
-1.0000E 00
-9.5367E-07
1.0000E 00
2.0000E 00
3.0000E 00
4.0000E 00
5.0000E 00
6.0000E 00
7.0000E 00
8.0000E 00

TRANSFER FUNCTION VR2/VV1

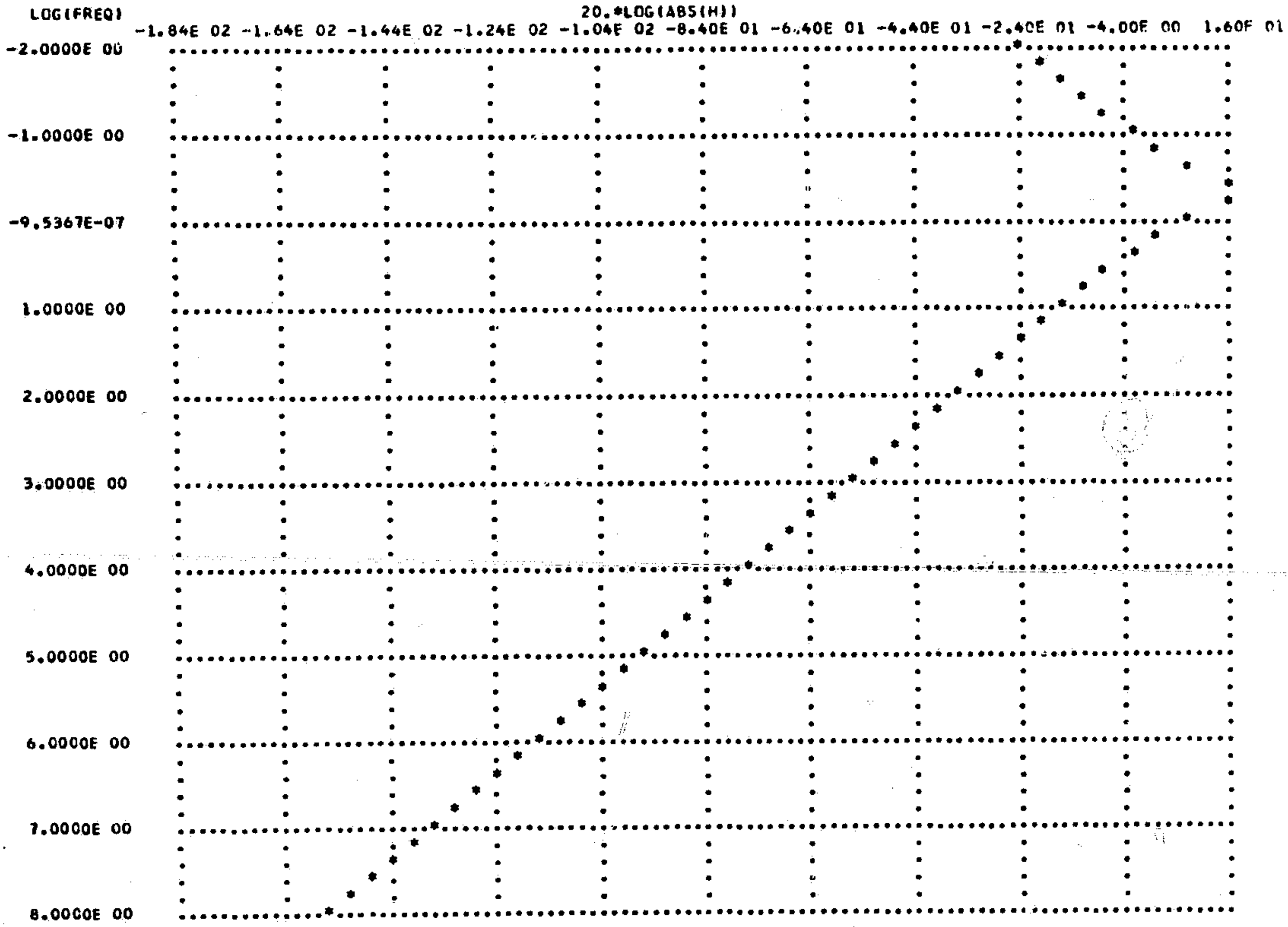
```
              (                    )
              (  1.00E 00          )
H(S)= 2.259E-02*-------------------------------------------------------------------------------------------------------------
              (                                             2           3           4           5           6           7 )
              (  4.52E-02   +3.00E-01 S   +9.94E-01 S   +2.12E 00 S   +3.35E 00 S   +3.72E 00 S   +3.45E 00 S   +1.86E 00 S   )
              (                                                                                                             )
              (              8 )
              ( +1.00E 00 S    )
```

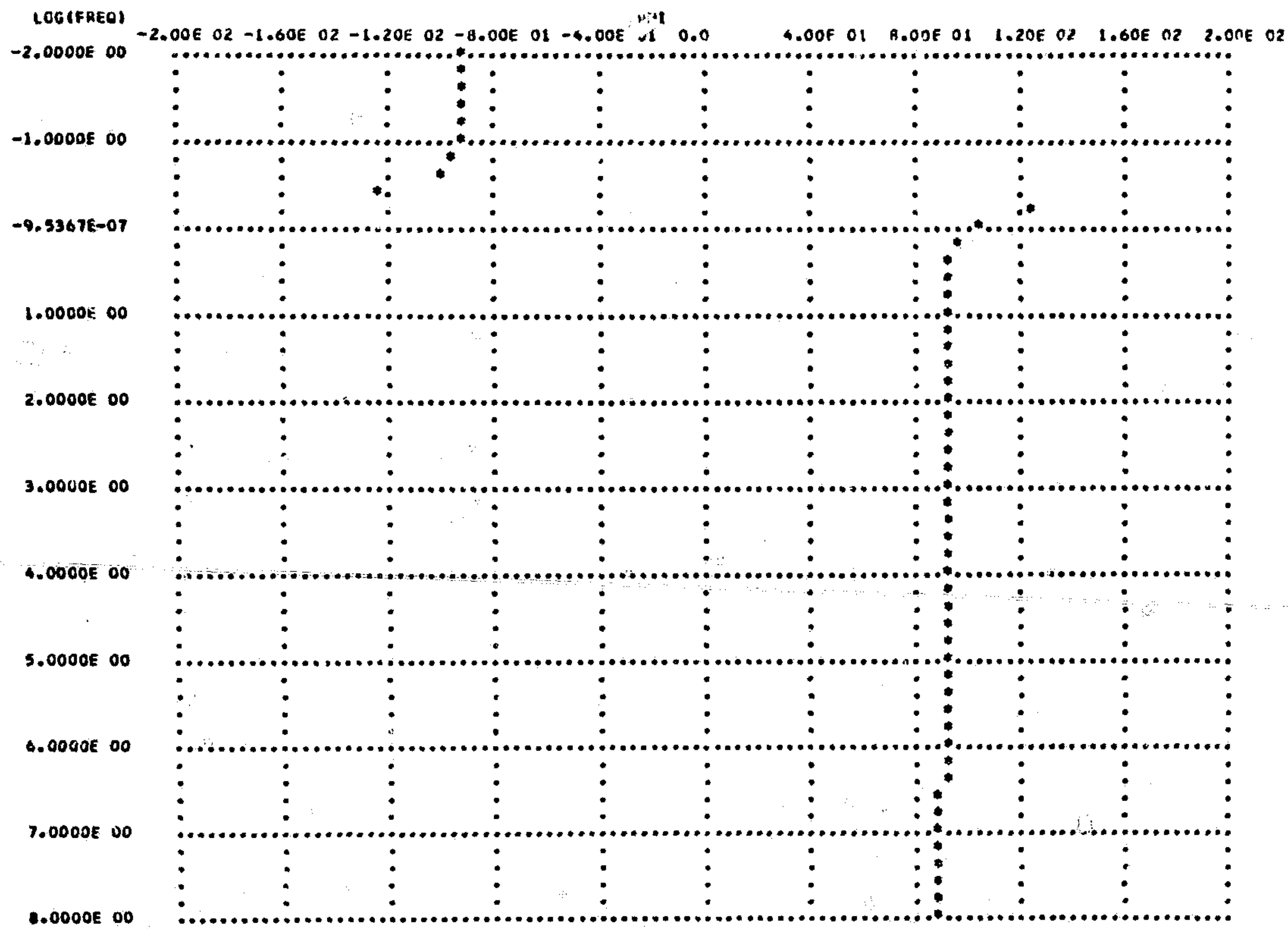
LOG(FREQ)	FREQ	20.*LOG(ABS(H))	PHI(H)	ABS(H)	LOG(ABS(H))
-0.1020000E 01	0.9549946E-01	-0.6262591E 01	0.1193552E 03	0.4862622E 00	-0.3131296E 00
-0.9999995E 00	0.1000003E 00	-0.6289253E 01	0.1064967E 03	0.4847718E 00	-0.3144627E 00
-0.9799995E 00	0.1047130E 00	-0.6307855E 01	0.9273335E 02	0.4837348E 00	-0.3153927E 00
-0.9599996E 00	0.1096480E 00	-0.6315286E 01	0.7788217E 02	0.4833211E 00	-0.3157643E 00
-0.9399995E 00	0.1148156E 00	-0.6315952E 01	0.6170189E 02	0.4832840E 00	-0.3157976E 00
-0.9199995E 00	0.1202266E 00	-0.6326902E 01	0.4392140E 02	0.4826751E 00	-0.3163452E 00
-0.8999996E 00	0.1258528E 00	-0.6374842E 01	0.2430334E 02	0.4800185E 00	-0.3187421E 00
-0.8799996E 00	0.1318259E 00	-0.6475431E 01	0.2676759E 01	0.4744915E 00	-0.3237716E 00
-0.8599995E 00	0.1380388E 00	-0.6591885E 01	-0.2132010E 02	0.4681774E 00	-0.3295943E 00
-0.8399996E 00	0.1445442E 00	-0.6635132E 01	-0.4920020E 02	0.4658471E 00	-0.3317566E 00
-0.8199996E 00	0.1513564E 00	-0.6784216E 01	-0.8547298E 02	0.4579197E 00	-0.3392108E 00
-0.7999996E 00	0.1584895E 00	-0.8683272E 01	-0.1337041E 03	0.3679904E 00	-0.4341636E 00
-0.7799996E 00	0.1659589E 00	-0.1380259E 02	-0.1774649E 03	0.2041129E 00	-0.6901295E 00
-0.7599996E 00	0.1737804E 00	-0.1992108E 02	0.1543309E 03	0.1009125E 00	-0.9960549E 00
-0.7399996E 00	0.1819705E 00	-0.2571396E 02	0.1357748E 03	0.5179666E-01	-0.1285698E 01
-0.7199996E 00	0.1905463E 00	-0.3105486E 02	0.1222454E 03	0.2800639E-01	-0.1552743E 01
-0.6999996E 00	0.1995266E 00	-0.3602693E 02	0.1116135E 03	0.1579986E-01	-0.1801347E 01
-0.6799996E 00	0.2089300E 00	-0.4071323E 02	0.1028473E 03	0.9211656E-02	-0.2035662E 01
-0.6599996E 00	0.2187765E 00	-0.4517616E 02	0.9538510E 02	0.5510505E-02	-0.2258808E 01
-0.6399996E 00	0.2290671E 00	-0.4946109E 02	0.8888998E 02	0.3364688E-02	-0.2473055E 01
-0.6199996E 00	0.2398837E 00	-0.5360126E 02	0.8314491E 02	0.2088991E-02	-0.2680063E 01
-0.5999996E 00	0.2511889E 00	-0.5762181E 02	0.7800009E 02	0.1314948E-02	-0.2881091E 01
-0.5799996E 00	0.2630272E 00	-0.6154224E 02	0.7334846E 02	0.8373121E-03	-0.3077112E 01
-0.5599996E 00	0.2754233E 00	-0.6537784E 02	0.6911020E 02	0.5394036E-03	-0.3268892E 01
-0.5399996E 00	0.2884037E 00	-0.6914085E 02	0.6522441E 02	0.3491060E-03	-0.3457043E 01
-0.5199996E 00	0.3019956E 00	-0.7284122E 02	0.6164320E 02	0.2280019E-03	-0.3642061E 01
-0.4999996E 00	0.3162283E 00	-0.7648743E 02	0.5832817E 02	0.1498402E-03	-0.3824371E 01
-0.4799997E 00	0.3311318E 00	-0.8006624E 02	0.5524817E 02	0.9901149E-04	-0.4004313E 01
-0.4599996E 00	0.3467373E 00	-0.8364339E 02	0.5237762E 02	0.6574007E-04	-0.4182170E 01
-0.4399996E 00	0.3630786E 00	-0.8716403E 02	0.4969485E 02	0.4393270E-04	-0.4358202E 01
-0.4199997E 00	0.3801858E 00	-0.9065224E 02	0.4718172E 02	0.2933513E-04	-0.4532612E 01
-0.3999997E 00	0.3981075E 00	-0.9411176E 02	0.4482297E 02	0.1969753E-04	-0.4705588E 01
-0.3799996E 00	0.4168698E 00	-0.9754573E 02	0.4260396E 02	0.1326519E-04	-0.4877287E 01
-0.3599997E 00	0.4365163E 00	-0.1009569E 03	0.4051430E 02	0.8956820E-05	-0.5047846E 01
-0.3399997E 00	0.4570886E 00	-0.1043477E 03	0.3854321E 02	0.6061978E-05	-0.5217386E 01
-0.3199997E 00	0.4786305E 00	-0.1077202E 03	0.3668178E 02	0.4111386E-05	-0.5386012E 01
-0.2999997E 00	0.5011877E 00	-0.1110764E 03	0.3492186E 02	0.2793719E-05	-0.5553818E 01
-0.2799997E 00	0.5248079E 00	-0.1144179E 03	0.3325632E 02	0.1901547E-05	-0.5720894E 01
-0.2599997E 00	0.5495414E 00	-0.1177460E 03	0.3167876E 02	0.1296284E-05	-0.5887300E 01
-0.2399997E 00	0.5754404E 00	-0.1210623E 03	0.3018314E 02	0.8848800E-06	-0.6053116E 01
-0.2199997E 00	0.6025601E 00	-0.1243678E 03	0.2876447E 02	0.6047969E-06	-0.6218390E 01
-0.1999997E 00	0.6309578E 00	-0.1276635E 03	0.2741780E 02	0.4138309E-06	-0.6383178E 01
-0.1799997E 00	0.6606539E 00	-0.1309505E 03	0.2613884E 02	0.2834494E-06	-0.6547524E 01
-0.1599997E 00	0.6918315E 00	-0.1342295E 03	0.2492345E 02	0.1943234E-06	-0.6711475E 01
-0.1399997E 00	0.7244365E 00	-0.1375013E 03	0.2376797E 02	0.1333316E-06	-0.6875068E 01
-0.1199997E 00	0.7585782E 00	-0.1407665E 03	0.2266901E 02	0.9155224E-07	-0.7038326E 01
-0.9999973E-01	0.7943288E 00	-0.1440260E 03	0.2162337E 02	0.6290708E-07	-0.7201300E 01
-0.7999974E-01	0.8317643E 00	-0.1472800E 03	0.2062822E 02	0.4375123E-07	-0.7364000E 01
-0.5999970E-01	0.8709642E 00	-0.1505291E 03	0.1968074E 02	0.2975375E-07	-0.7526457E 01
-0.3999972E-01	0.9120115E 00	-0.1537739E 03	0.1877861E 02	0.2047884E-07	-0.7688694E 01
-0.1999974E-01	0.9549932E 00	-0.1570145E 03	0.1791891E 02	0.1410173E-07	-0.7850726E 01
0.0	0.1000000E 01	-0.1602516E 03	0.1709995E 02	0.9714462E-08	-0.8012580E 01

TRANSFER FUNCTION VV2/VV1

$$H(S) = 1.100E\ 01 * \frac{(0.0 \quad +1.00E\ 00\ S)}{(1.00E\ 01 \quad +1.00E\ 00\ S \quad +1.00E\ 00\ S^{2})}$$

LOG(FREQ)	FREQ	20.*LOG(ABS(H))	PHI(H)	ABS(H)	LOG(ABS(H))
-0.2000000E 01	0.1000001E-01	-0.2320529E 02	-0.9036017E 02	0.6914085E-01	-0.1160265E 01
-0.1799999E 01	0.1584898E-01	-0.1920035E 02	-0.9057114E 02	0.1096433E 00	-0.9600179E 00
-0.1599999E 01	0.2511893E-01	-0.1518797E 02	-0.9090652E 02	0.1740210E 00	-0.7593983E 00
-0.1400000E 01	0.3981078E-01	-0.1115679E 02	-0.9144196E 02	0.2767963E 00	-0.5578393E 00
-0.1200000E 01	0.6309581E-01	-0.7077999E 01	-0.9230655E 02	0.4426904E 00	-0.3539000E 00
-0.1000000E 01	0.1000000E 00	-0.2877260E 01	-0.9374269E 02	0.7180207E 00	-0.1438630E 00
-0.8000011E 00	0.1584890E 00	0.1645753E 01	-0.9630817E 02	0.1208614E 01	0.8228767E-01
-0.6000013E 00	0.2511880E 00	0.7091908E 01	-0.1018698E 03	0.2262535E 01	0.3545954E 00
-0.4000006E 00	0.3981067E 00	0.1572333E 02	-0.1237535E 03	0.6111763E 01	0.7861664E 00
-0.2000008E 00	0.6309563E 00	0.1594331E 02	0.1247414E 03	0.6268534E 01	0.7971657E 00
-0.9536743E-06	0.9999978E 00	0.7208434E 01	0.1020325E 03	0.2293093E 01	0.3604217E 00
0.1999989E 00	0.1584888E 01	0.1733706E 01	0.9637265E 02	0.1220915E 01	0.8668536E-01
0.3999987E 00	0.2511878E 01	-0.2798712E 01	0.9377673E 02	0.7245433E 00	-0.1399356E 00
0.5999985E 00	0.3981055E 01	-0.7002940E 01	0.9232655E 02	0.4465325E 00	-0.3501470E 00
0.7999983E 00	0.6309541E 01	-0.1108305E 02	0.9145425E 02	0.2791560E 00	-0.5541527E 00
0.9999981E 00	0.9999948E 01	-0.1511478E 02	0.9091417E 02	0.1754935E 00	-0.7557390E 00
0.1199999E 01	0.1584888E 02	-0.1912740E 02	0.9057600E 02	0.1105681E 00	-0.9563702E 00
0.1399999E 01	0.2511876E 02	-0.2313240E 02	0.9036322E 02	0.6972355E-01	-0.1156620E 01
0.1599998E 01	0.3981053E 02	-0.2713440E 02	0.9022914E 02	0.4398251E-01	-0.1356720E 01
0.1799998E 01	0.6309538E 02	-0.3113515E 02	0.9014461E 02	0.2774863E-01	-0.1556758E 01
0.1999998E 01	0.9999944E 02	-0.3513547E 02	0.9009122E 02	0.1750755E-01	-0.1756774E 01
0.2199998E 01	0.1584883E 03	-0.3913560E 02	0.9005756E 02	0.1104637E-01	-0.1956780E 01
0.2399998E 01	0.2511869E 03	-0.4313564E 02	0.9003636E 02	0.6969750E-02	-0.2156782E 01
0.2599998E 01	0.3981045E 03	-0.4713567E 02	0.9002298E 02	0.4397601E-02	-0.2356784E 01
0.2799998E 01	0.6309536E 03	-0.5113568E 02	0.9001451E 02	0.2774694E-02	-0.2556785E 01
0.2999998E 01	0.9999939E 03	-0.5513568E 02	0.9000916E 02	0.1750715E-02	-0.2756784E 01
0.3199998E 01	0.1584882E 04	-0.5913568E 02	0.9000581E 02	0.1104627E-02	-0.2956784E 01
0.3399998E 01	0.2511868E 04	-0.6313568E 02	0.9000369E 02	0.6969727E-03	-0.3156784E 01
0.3599998E 01	0.3981042E 04	-0.6713567E 02	0.9000232E 02	0.4397600E-03	-0.3356784E 01
0.3799997E 01	0.6309520E 04	-0.7113567E 02	0.9000150E 02	0.2774699E-03	-0.3556784E 01
0.3999997E 01	0.9999914E 04	-0.7513568E 02	0.9000095E 02	0.1750718E-03	-0.3756784E 01
0.4199997E 01	0.1584878E 05	-0.7913567E 02	0.9000063E 02	0.1104630E-03	-0.3956783E 01
0.4399998E 01	0.2511867E 05	-0.8313568E 02	0.9000041E 02	0.6969729E-04	-0.4156784E 01
0.4599998E 01	0.3981041E 05	-0.8713570E 02	0.9000029E 02	0.4397600E-04	-0.4356785E 01
0.4799997E 01	0.6309518E 05	-0.9113567E 02	0.9000018E 02	0.2774702E-04	-0.4556784E 01
0.4999997E 01	0.9999912E 05	-0.9513568E 02	0.9000014E 02	0.1750716E-04	-0.4756784E 01
0.5199997E 01	0.1584877E 06	-0.9913567E 02	0.9000008E 02	0.1104629E-04	-0.4956783E 01
0.5399997E 01	0.2511861E 06	-0.1031357E 03	0.9000008E 02	0.6969748E-05	-0.5156783E 01
0.5599997E 01	0.3981027E 06	-0.1071357E 03	0.9000008E 02	0.4397617E-05	-0.5356783E 01
0.5799996E 01	0.6309503E 06	-0.1111357E 03	0.9000008E 02	0.2774709E-05	-0.5556783E 01
0.5999997E 01	0.9999906E 06	-0.1151357E 03	0.9000008E 02	0.1750719E-05	-0.5756783E 01
0.6199997E 01	0.1584877E 07	-0.1191357E 03	0.9000008E 02	0.1104630E-05	-0.5956784E 01
0.6399997E 01	0.2511860E 07	-0.1231357E 03	0.9000008E 02	0.6969743E-06	-0.6156784E 01
0.6599997E 01	0.3981026E 07	-0.1271357E 03	0.8999997E 02	0.4397620E-06	-0.6356783E 01
0.6799996E 01	0.6309500E 07	-0.1311357E 03	0.8999997E 02	0.2774710E-06	-0.6556783E 01
0.6999996E 01	0.9999769E 07	-0.1351355E 03	0.8999997E 02	0.1750744E-06	-0.6756778E 01
0.7199996E 01	0.1584870E 08	-0.1391355E 03	0.8999997E 02	0.1104635E-06	-0.6956777E 01
0.7399996E 01	0.2511832E 08	-0.1431355E 03	0.8999997E 02	0.6969822E-07	-0.7156774E 01
0.7599997E 01	0.3981021E 08	-0.1471355E 03	0.8999997E 02	0.4397620E-07	-0.7356777E 01
0.7799996E 01	0.6309450E 08	-0.1511355E 03	0.8999997E 02	0.2774733E-07	-0.7556774E 01
0.7999996E 01	0.9999736E 08	-0.1551354E 03	0.8999997E 02	0.1750750E-07	-0.7756772E 01


LOG(FREQ)
20.*LOG(ABS(H))
-1.84E 02 -1.64E 02 -1.44E 02 -1.24E 02 -1.04E 02 -8.40E 01 -6.40E 01 -4.40E 01 -2.40E 01 -4.00E 00 1.60E 01
-2.0000E 00
-1.0000E 00
-9.5367E-07
1.0000E 00
2.0000E 00
3.0000E 00
4.0000E 00
5.0000E 00
6.0000E 00
7.0000E 00
8.0000E 00


LOG(FREQ)
PHI
-2.00E 02 -1.60E 02 -1.20E 02 -8.00E 01 -4.00E 01 0.0 4.00E 01 8.00E 01 1.20E 02 1.60E 02 2.00E 02
-2.0000E 00
-1.0000E 00
-9.5367E-07
1.0000E 00
2.0000E 00
3.0000E 00
4.0000E 00
5.0000E 00
6.0000E 00
7.0000E 00
8.0000E 00

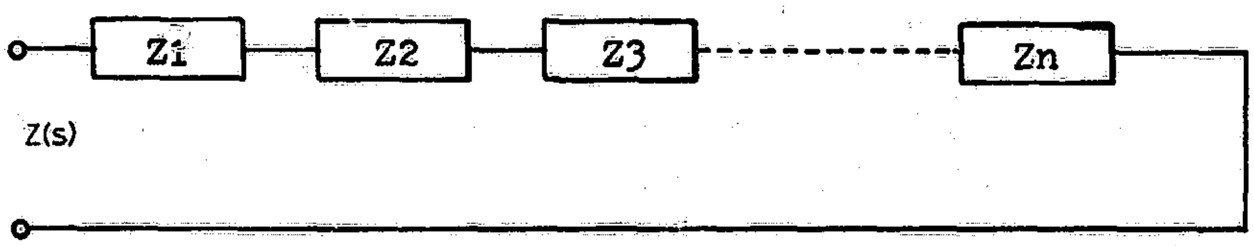
APPENDIX A

Notes from network analysis

Network elements:

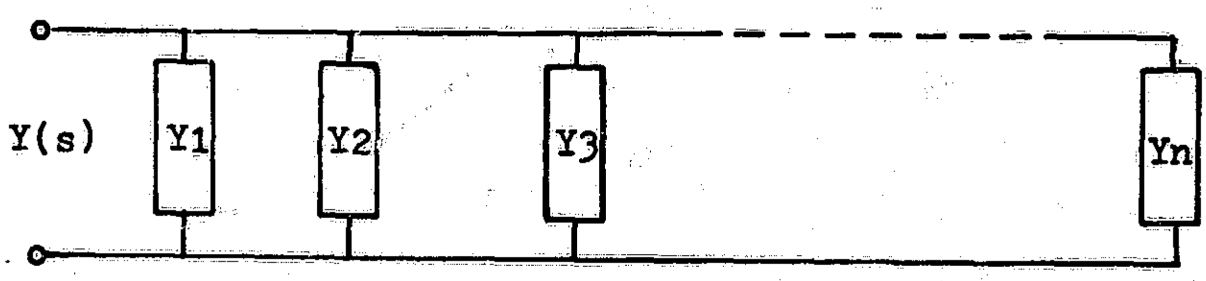
Symbol			
iv relationship	$v = Ri$	$v = L\frac{di}{dt}$	$v = \frac{1}{C}\int i\,dt$
transformed IV relationships, zero initial conditions	$V = RI$	$V = LsI$	$V = \frac{1}{Cs} I$
impedance, Z(s)	R	Ls	$\frac{1}{Cs}$
admittance, Y(s)	$G = \frac{1}{R}$	$\frac{1}{Ls}$	Cs

Network functions for some selected networks



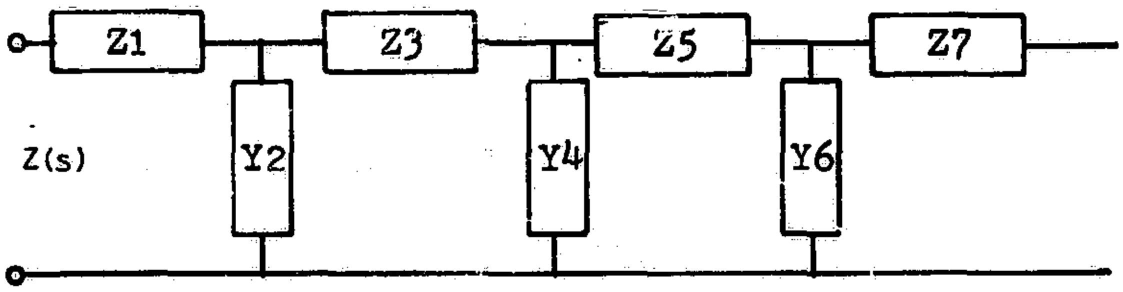
Series connection

$$Z(s) = Z1(s) + Z2(s) + Z3(s) + \ldots + Zn(s)$$



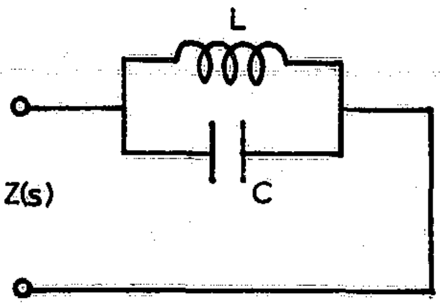
Parallel connection

$$Y(s) = Y1(s) + Y2(s) + Y3(s) + \ldots + Yn(s)$$



Ladder connected

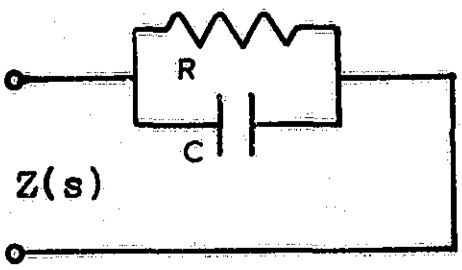
$$Z(s) = Z1(s) + \cfrac{1}{Y2(s) + \cfrac{1}{Z3(s) + \cfrac{1}{Y4(s) + \cfrac{1}{Z5(s) + \cfrac{1}{Y6(s) + \cfrac{1}{Z7(s) + \cfrac{1}{\ddots}}}}}}}$$



Parallel LC circuit

$$Z(s) = \frac{(1/C)\ s}{s^2 + (1/LC)} = \frac{2Ks}{s^2 + \omega^2}$$

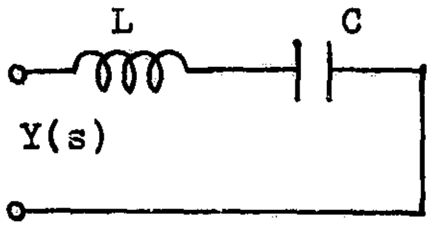
where $L = \frac{2K}{\omega^2}$, $C = \frac{1}{2K}$



$$Z(s) = \frac{(1/C)}{s + (1/RC)} = \frac{K}{s + \sigma}$$

where $C = \frac{1}{K}$ $R = \frac{K}{\sigma}$

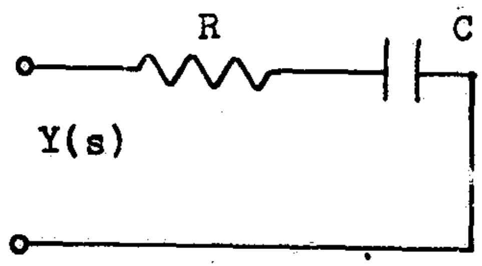
Parallel RC circuit



Series LC circuit

$$Y(s) = \frac{(1/L)\, s}{s^2 + (1/LC)} = \frac{2Ks}{s^2 + \omega^2}$$

where $L = \frac{1}{2K}$, $C = \frac{2K}{\omega^2}$



Series RC circuit

$$Y(s) = \frac{(1/R)\, s}{s + (1/RC)} = \frac{Ks}{s + \sigma}$$

where $R = \frac{1}{K}$, $C = \frac{K}{\sigma}$

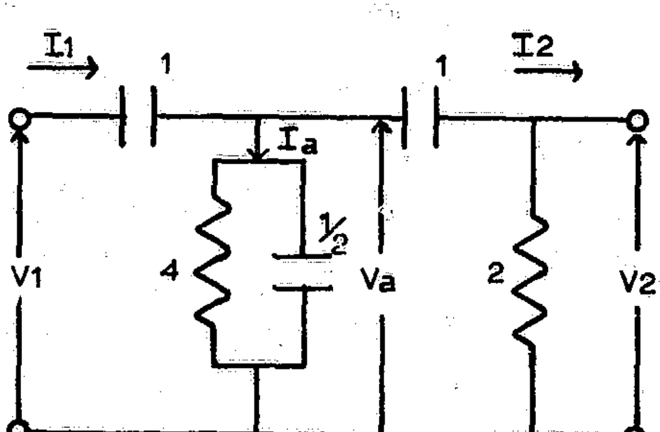
A method of computing the transfer function of a ladder network

Method:

(1) A unit output is assumed.
(2) The input required to produce the unit output is then found.
(3) Then use the relationship

$$\frac{\text{unit output}}{\text{input for unit output}} = \frac{\text{general output}}{\text{input to give general output}}.$$

Example of method: Find the voltage ratio V2/V1 for the ladder network shown.



Step (1) Assume V2 = 1 volt

Step (2) Calculate V1 using V2 = 1 volt

$$I2 = \tfrac{1}{2}V2 = \tfrac{1}{2}$$

$$Va = ((1/s) + 2)I2 = ((1/s + 2)\tfrac{1}{2} = (1/2s) + 1 = \frac{2s + 1}{2s}$$

$$Ia = (\tfrac{1}{4} + \tfrac{1}{2}s)Va = \frac{(\tfrac{1}{4} + \tfrac{1}{2}s)(2s + 1)}{2s} = \frac{4s^2 + 4s + 1}{8s}$$

$$I1 = Ia + I2 = \frac{4s^2 + 4s + 1}{8s} + \frac{1}{2} = \frac{4s^2 + 8s + 1}{8s}$$

$$V1 = \frac{I1}{s} + Va = \frac{1}{s} \cdot \frac{4s^2 + 8s + 1}{8s} + \frac{2s + 1}{2s} = \frac{12s^2 + 12s + 1}{8s^2}$$

Step (3)

$$\frac{V2}{V1} = \frac{1}{\dfrac{12s^2 + 12s + 1}{8s^2}} = \frac{8s^2}{12s^2 + 12s + 1}$$

$$\frac{V2}{V1} = \frac{2}{3}\,\frac{s^2}{s^2 + s + \frac{1}{12}}$$

Loop equations

Form of loop equation

$$\begin{bmatrix} \zeta_{11} & \zeta_{12} & \zeta_{13} & \cdot\ \cdot & \zeta_{1n} \\ \zeta_{21} & \zeta_{22} & \zeta_{23} & \cdot\ \cdot & \zeta_{2n} \\ \zeta_{31} & \zeta_{32} & \zeta_{33} & \cdot\ \cdot & \zeta_{3n} \\ \cdot & \cdot & \cdot & \cdot\ \cdot & \\ \zeta_{n1} & \zeta_{n2} & \zeta_{n3} & \cdot\ \cdot & \zeta_{nn} \end{bmatrix} \begin{bmatrix} I1 \\ I2 \\ I3 \\ \cdot \\ In \end{bmatrix} = \begin{bmatrix} V1 \\ V2 \\ V3 \\ \\ Vn \end{bmatrix}$$

$$\zeta_{ik} = R_{ik} + L_{ik}s + 1/(C_{ik}s)$$

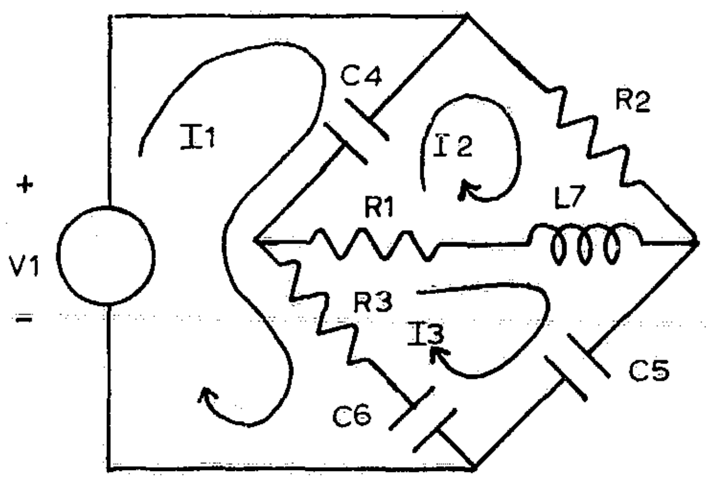
The I_k loop current is found by Crammer's rule to be

$$I_k = \frac{\Delta_{1k}}{\Delta} V1 + \frac{\Delta_{2k}}{\Delta} V2 + \frac{\Delta_{3k}}{\Delta} V3 + \ . \ . \ . + \frac{\Delta_{nk}}{\Delta} Vn$$

where Δ is the determinant, $|\zeta|$, and Δ_{jk} is the $_{jk}$th cofactor of Δ.

Loop equation example

Network



Loop equations for network

$$\begin{bmatrix} R3+\frac{1}{C4s}+\frac{1}{C6s} & \frac{-1}{C4s} & -R3-\frac{1}{C6s} \\ \frac{-1}{C4s} & L7s+R1+R2+\frac{1}{C4s} & -L7s-R1 \\ -R3-\frac{1}{C6s} & -L7s-R1 & L7s+R1+R3+\frac{1}{C5s}+\frac{1}{C6s} \end{bmatrix} \begin{bmatrix} I1 \\ I2 \\ I3 \end{bmatrix} = \begin{bmatrix} V1 \\ 0 \\ 0 \end{bmatrix}$$

Node equations

Form of node equations

$$\begin{bmatrix} \eta_{11} & \eta_{12} & \eta_{13} & \cdots & \eta_{1n} \\ \eta_{21} & \eta_{22} & \eta_{23} & \cdots & \eta_{2n} \\ \eta_{31} & \eta_{32} & \eta_{33} & \cdots & \eta_{3n} \\ \cdots & & & & \\ \eta_{n1} & \eta_{n2} & \eta_{n3} & \cdots & \eta_{nn} \end{bmatrix} \begin{bmatrix} V1 \\ V2 \\ V3 \\ \\ Vn \end{bmatrix} \begin{matrix} \\ . \\ \\ \\ \end{matrix} \begin{bmatrix} I1 \\ I2 \\ I3 \\ \\ In \end{bmatrix}$$

$$\eta_{ik} = G_{ik} + C_{ik}s + 1/(L_{ik}s)$$

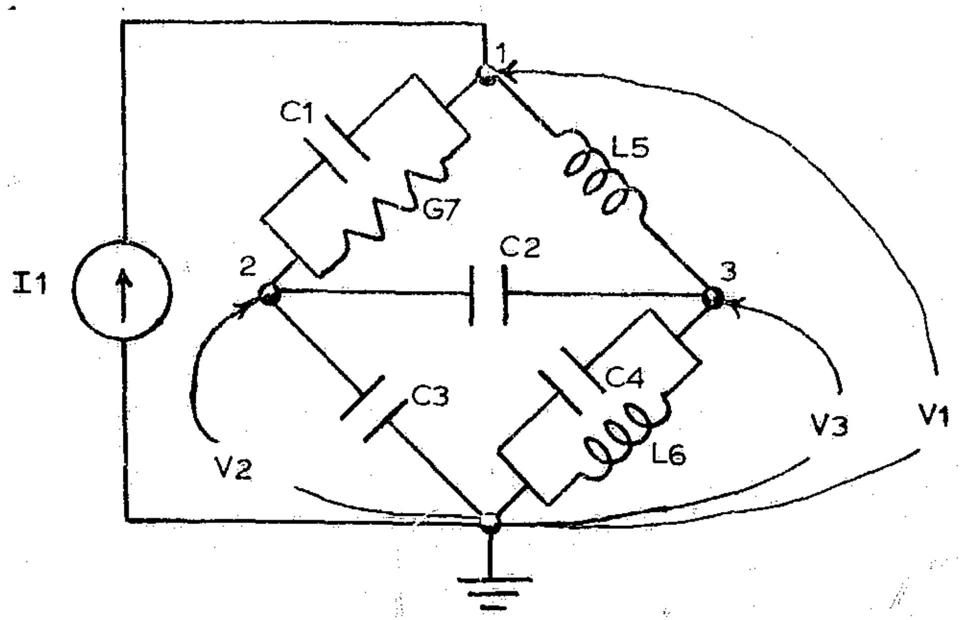
The Vk node voltage is found by Crammer's rule to be

$$V_k = \frac{\Delta'_{1k}}{\Delta'} I1 + \frac{\Delta'_{2k}}{\Delta} I2 + \frac{\Delta'_{3k}}{\Delta'} I3 + \ldots + \frac{\Delta'_{nk}}{\Delta'} In$$

where Δ' is the determinant, $|\eta|$, and Δ_{jk} is the $_{jk}$th cofactor of Δ.

Node equation example

Network

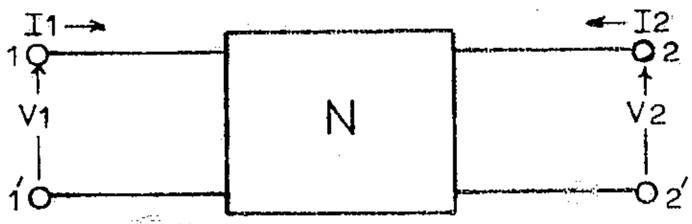


Node equations

$$\begin{bmatrix} C1s+G7+\frac{1}{L5s} & -C1s-G7 & \frac{-1}{L5s} \\ -C1s-G7 & C1s+C2s+C3s+G7 & -C2s \\ \frac{-1}{L5s} & -C2s & C4s+C2s+\frac{1}{L5s}+\frac{1}{L6s} \end{bmatrix} \begin{bmatrix} V1 \\ V2 \\ V3 \end{bmatrix} = \begin{bmatrix} I1 \\ 0 \\ 0 \end{bmatrix}$$

Open-circuit impedance and short-circuit admittance functions

Two port network and the open-circuit impedance parameters.



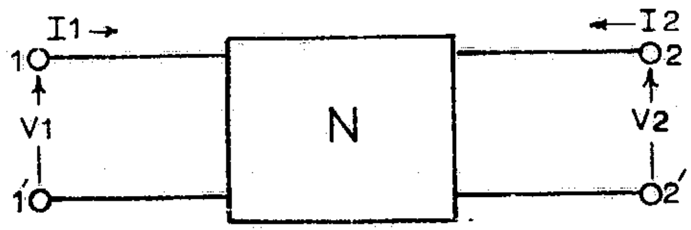
$$V1 = z_{11} I1 + z_{12} I2$$

$$V2 = z_{21} I1 + z_{22} I2$$

$$z_{11} = \left.\frac{V1}{I1}\right|_{I2 = 0} \qquad z_{12} = \left.\frac{V1}{I2}\right|_{I1 = 0}$$

$$z_{21} = \left.\frac{V2}{I1}\right|_{I2 = 0} \qquad z_{22} = \left.\frac{V2}{I2}\right|_{I1 = 0}$$

Two port network and the short-circuit admittance functions.



$$I1 = y_{11}V1 + y_{12}V2$$

$$I2 = +y_{21}V1 + y_{22}V2$$

$$y_{11} = \left.\frac{I1}{V1}\right|_{V2 = 0} \qquad +y_{12} = \left.\frac{I1}{V2}\right|_{V1 = 0}$$

$$+y_{21} = \left.\frac{I2}{V1}\right|_{V2 = 0} \qquad y_{22} = \left.\frac{I2}{V2}\right|_{V1 = 0}$$

Open-circuit impedance parameters in terms of the short-circuit admittance parameters

$$z_{11} = \frac{y_{22}}{\Delta y} \qquad z_{12} = \frac{-y_{12}}{\Delta y}$$

$$z_{21} = \frac{-y_{21}}{\Delta y} \qquad z_{22} = \frac{y_{11}}{\Delta y}$$

where

$$\Delta y = \begin{vmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{vmatrix}$$

Short-circuit admittance parameters in terms of open-circuit impedance parameters

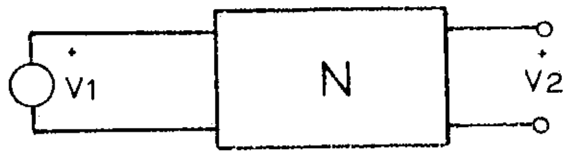
$$y_{11} = \frac{z_{22}}{\Delta z} \qquad y_{12} = -\frac{z_{12}}{\Delta z}$$

$$y_{21} = -\frac{z_{21}}{\Delta z} \qquad y_{22} = \frac{z_{22}}{\Delta z}$$

where

$$\Delta z = \begin{vmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{vmatrix}$$

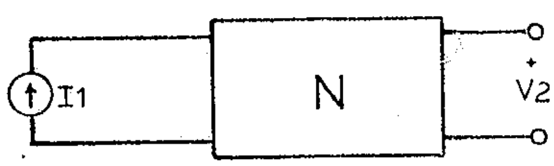
Transfer functions



$$G21 = \frac{V2}{V1} = \frac{-y_{21}}{y_{22}} = \frac{z_{21}}{z_{11}}$$

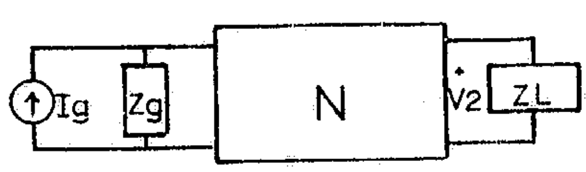
$$G21 = \frac{V2}{V1} = \frac{-y_{21}}{y_{22}+YL} = \frac{z_{21}}{\Delta z+z_{11}ZL}$$

$$G2g = \frac{V2}{Vg} = \frac{z_{21}ZL}{(z_{11}+Zg)(z_{22}+ZL)-z_{21}z_{12}}$$

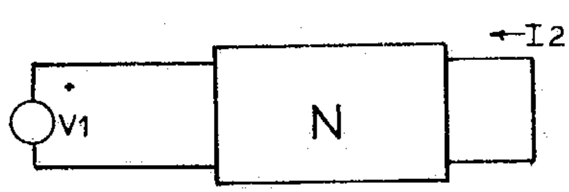


$$Z21 = \frac{V2}{I1} = \frac{-y_{21}}{\Delta y} = z_{21}$$

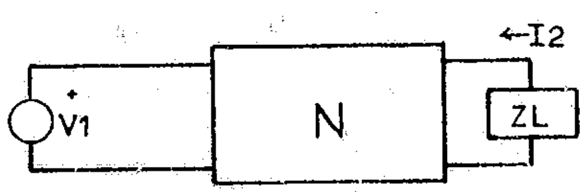
$$Z21 = \frac{V2}{I1} = \frac{-y_{21}}{\Delta y+y_{11}YL} = \frac{z_{21}ZL}{z_{22}+ZL}$$



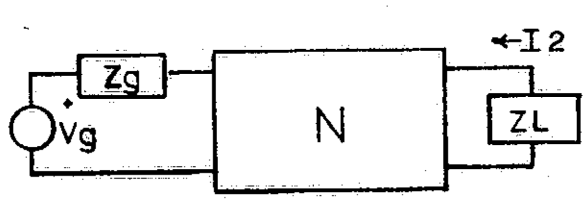
$$Z2g = \frac{V2}{Ig} = \frac{z_{21}ZLZg}{(z_{11}+Zg)(z_{22}+ZL)-z_{12}z_{21}}$$



$$Y21 = -\frac{I2}{V1} = \frac{-z_{21}}{\Delta z} = y_{21}$$



$$Y21 = -\frac{I2}{V1} = \frac{-y_{21}YL}{y_{22}+YL} = \frac{z_{21}}{\Delta z+z_{11}ZL}$$



$$Y2g = \frac{I2}{Vg} = \frac{z_{21}}{(z_{11}+Zg)(z_{22}+ZL)-z_{21}z_{12}}$$

Appendix B

Notes from network synthesis

Hurwitz polynomials

Form of polynomials, $Q(s) = a_n s^n + a_{n-1} s^{n-1} + . . . + a_1 s + a_0$.

A polynomial Q(s) is Hurwitz if

(1) Q(s) is real when s is real.

(2) The roots of Q(s) have real parts which are zero or negative.

Examples:

(1) $Q(s) = (s+2)(s+3+j\sqrt{5})(s+3-j\sqrt{5})$, Hurwitz
(2) $Q(s) = (s-2)(s+3+j\sqrt{5})(s+3-j\sqrt{5})$, not Hurwitz

Properties of Hurwitz polynomials

(1) All the coefficients a_i are nonnegative. None of the coefficients between the highest order term and the lowest order term may be zero unless the polynomial is even or odd.

(2) The odd and the even part of a Hurwitz polynomial have roots on the $j\omega$ axis only. If

$$Q(s) = m(s) + n(s) = \text{even part} + \text{odd part}$$

then all roots of n(s) are on the $j\omega$ axis and all roots of m(s) are on the $j\omega$ axis.

(3) If Q(s) is an even or odd Hurwitz polynomial, all of Q(s) roots are on the $j\omega$ axis.

(4) The continued fraction expansion of the ratio of even to odd or odd to even parts of a Hurwitz polynomial results in all positive quotient terms. This property serves as a test to determine if a polynomial is Hurwitz.

Example: Testing a polynomial to determine if it is Hurwitz

a) Determine if the polynomial

$$Q(s) = s^4 + s^3 + 5s^2 + 3s + 4$$

is Hurwitz by means of a continued fraction expansion.

$m(s) = s^4 + 5s^2 + 4$

$n(s) = s^3 + 3s$

$$
\begin{array}{l}
s^3 + 3s \;\overline{\big)\, s^4 + 5s^2 + 4}\;\; (s \leftarrow \\
\qquad\qquad \underline{s^4 + 3s^2} \\
\qquad\qquad\qquad 2s^2 + 4 \;\overline{\big)\, s^3 + 3s}\;\; (s/2 \leftarrow \\
\qquad\qquad\qquad\qquad\qquad \underline{s^3 + 2s} \\
\qquad\qquad\qquad\qquad\qquad\qquad s \;\overline{\big)\, 2s^2 + 4}\;\; (2s \leftarrow \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad \underline{2s^2} \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad 4 \;\overline{\big)\, s}\;\; (s/4 \leftarrow \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \underline{\underline{s}}
\end{array}
$$

All positive

Since all four quotient terms are positive Q(s) is Hurwitz.

b) Determine if the polynomial

$Q(s) = s^3 + 3s^2 + 6s + 18$

is Hurwitz. Note that there should be three quotient terms in the continued fraction expansion.

$n(s) = s^3 + 6s, \quad m(s) = 3s^2 + 18$

$$
\begin{array}{l}
3s^2 + 18 \;\overline{\big)\, s^3 + 6s}\;\; (s/3 \\
\qquad\qquad\quad \underline{\underline{s^3 + 6s}}
\end{array}
$$

The continued fraction expansion of n(s)/m(s) has terminated after one division, thus yielding only one quotient term. This means that $(s^3 + 6s)$ is a factor of Q(s). To proceed with the test, factor Q(s) as

$Q(s) = (s^2 + 6s)Q'(s).$

Then $(s^2 + 6s)$ and $Q'(s)$ must both be Hurwitz for Q(s) to be Hurwitz.

$$
\begin{array}{l}
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad 1 + 3/s \\
s^3 + 6s \;\overline{\big)\, s^3 + 3s^2 + 6s + 18} \\
\qquad\qquad \underline{s^3 + 0s^2 + 6s} \\
\qquad\qquad\qquad\quad 3s^2 + 0s + 18 \\
\qquad\qquad\qquad\quad \underline{3s^2 + 0s + 18}
\end{array}
$$

$Q(s) = (s^3 + 6s)(1 + 3/s)$

Since $(s^3 + 6s)$ and $(1 + 3/s)$ are both Hurwitz, $Q(s)$ is Hurwitz.

c) Determine if the polynomial

$$Q(s) = (s+3)(s-2) = s^2 + s - 6$$

is Hurwitz by means of a continued fraction expansion.

$m(s) = s^2 - 6$
$n(s) = s$

$$s \,\overline{)\, s^2 - 6 \,}\; (s$$
$$s^2$$
$$-6 \,\overline{)\, s \,}\; (-s/6$$
$$s$$

Since there was a negative quotient the polynomial was not Hurwitz.

Positive real functions

A function $F(s)$ is a positive real function if the following is true:

(1) $F(s)$ is real for real s.

(2) The real part of $F(s)$ is greater than or equal to zero when the real part of s is greater than or equal to zero, $Re[F(s)] \geq 0$ for $Re[s] \geq 0$.

Properties of positive real functions

(1) If $F(s)$ is a positive real function (p.r.f.) then $1/F(s)$ is a p.r.f.

(2) If $F_1(s)$ and $F_2(s)$ are p.r.f. the $F_1(s) + F_2(s)$ is a p.r.f. $F_1(s) - F_2(s)$ need not be a p.r.f.

(3) If $F(s)$ is p.r.f. it has no poles and zeros in the right half of the s plane.

(4) If $F(s)$ is p.r.f. it has only simple poles with real positive residues on the $j\omega$ axis.

(5) If $F(s)$ is p.r.f. its poles and zeros are either real or occur in conjugate pairs.

(6) If F(s) is given as the ratio of two polynomials

$$F(s) = \frac{P(s)}{Q(s)},$$

then the highest powers of the polynomials P(s) and Q(s) may differ at the most by unity. Therefore, there can be no multiple poles or zeros at $s = \infty$.

(7) If F(s) is given as the ratio of the polynomial P(s)/Q(s), then the lowest powers of polynomials Q(s) and P(s) may differ at most by unity. Therefore, there can be no multiple poles or zeros at s = 0.

(8) For a rational function, F(s) = P(s)/Q(s), with real coefficients, the necessary and sufficient conditions to be a p.r.f. are

a. F(s) must have no poles in the right half s plane,

b. F(s) may have only simple poles on the $j\omega$ axis with real and positive residues,

c. $Re[F(j\omega)] \geq 0$ for all ω.

Two special cases

If $F(s) = \frac{s + a}{s^2 + bs + c}$

F(s) is a p.r.f. if

a. $a, b, c, \geq 0$

b. $b \geq a$.

If $F(s) = \frac{s^2 + a_1 s + a_0}{s^2 + b_1 s + b_0}$

F(s) is p.r.f. if

a. $a_1, a_0, b_1, b_0, \geq 0$

b. $a_1 b_1 \geq (\sqrt{a_0} - \sqrt{b_0})^2$

Basic synthesis operations

(1) The removal of a pole at infinity. The function

$$F(s) = \frac{a_{n+1} s^{n+1} + a_n s^n + \ldots + a_1 s + a_0}{b_n s^n + b_{n-1} s^{n-1} + \ldots + b_1 s + b_0}$$

has a pole at $s = \infty$. This pole at infinity may be removed by dividing the numerator by the denominator and then expressing F(s) as the quotient plus the remainder over the denominator.

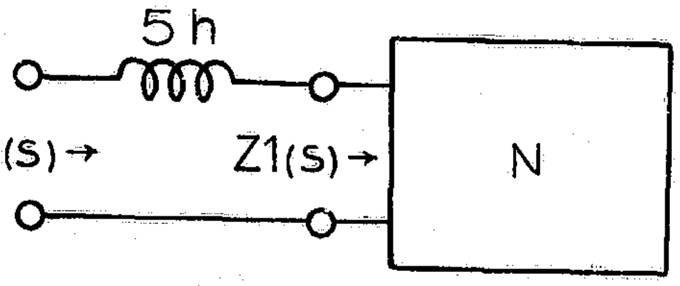
Example:

a) $$Z(s) = \frac{30s^2 + 5s + 2}{6s + 1}$$

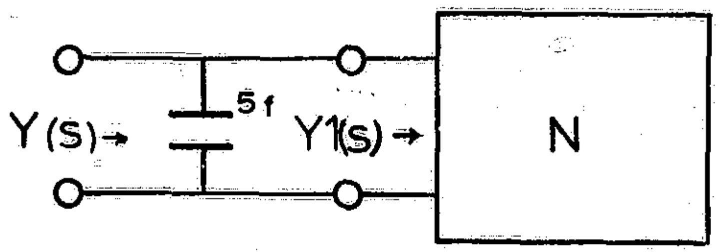
$$6s + 1 \overline{\big)\, 30s^2 + 5s + 2}$$ with quotient $5s$ ← 5 henry inductor

$$\frac{30s^2 + 5s \qquad}{\qquad\qquad 2}$$

$$Z(s) = 5s + \frac{2}{6s + 1} = 5s + Z1(s)$$



b) $$Y(s) = \frac{30s^2 + 5s + 2}{6s + 1} = 5s + \frac{2}{6s + 1} = 5s + Y1(s)$$



(2) The removal of a pole at zero. The function

$$f(s) = \frac{a_0 + a_1 s + \ldots + a_{n-1} s^{n-1} + a_n s^n}{b_1 s + b_2 s^2 + \ldots + b_n s^n}$$

has a pole at $s = 0$. This pole at the origin may be removed by dividing the numerator by the denominator and then expressing F(s) as the quotient plus the remainder over the denominator. Note the order of the coefficients of F(s) here.

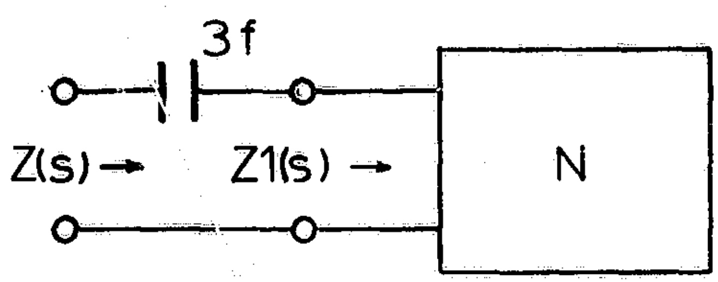
Example:

a) $Z(s) = \dfrac{30s^2 + 5s + 2}{15s^2 + 6s} = \dfrac{2 + 5s + 30s^2}{6s + 15s^2}$

$$6s + 15s^2 \overline{\smash{)}\,2 + 5s + 30s^2} \quad \frac{1}{3s} \longleftarrow \text{3f capacitor}$$
$$\underline{2 + 5s}$$
$$30s^2$$

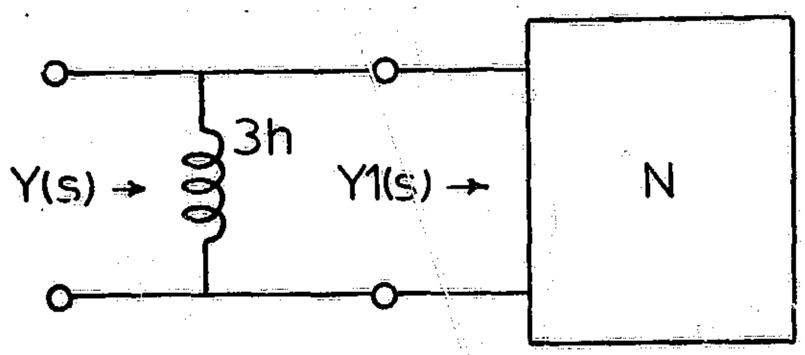
$$Z(s) = \frac{1}{3s} + \frac{30s^2}{6s + 15s^2}$$

$$Z(s) = \frac{1}{3s} + \frac{10s}{5s + 2} = \frac{1}{3s} + Z1(s)$$



b) $Y(s) = \dfrac{30s^2 + 5s + 2}{15s^2 + 6s} = \dfrac{2 + 5s + 30s^2}{6s + 15s^2}$

$$= \frac{1}{3s} + \frac{10}{5s + 2} = \frac{1}{3s} + Y1(s)$$



(3) The removal of conjugate imaginary poles. The function

$$F(s) = \frac{p(s)}{(s^2 + \omega_1^2)q_1(s)}$$

has a pair of conjugate poles on the $j\omega$ axis at $s = \pm j\omega_1$. F(s) may be expanded as

$$F(s) = \frac{2K_1 s}{s^2 + \omega_1^2} + F_1(s)$$

where $F_1(s) = F(s) - \frac{2K_1 s}{s^2 + \omega_1^2}$

Example:

a) $Z(s) = \frac{90s^3 + 15s^2 + 16s}{45s^3 + 18s^2 + 5s + 2} = \frac{P(s)}{Q(s)}$

Now if Q(s) is tested by means of a continued fraction expansion to determine if it is Hurwitz we find

$n(s) = 45s^3 + 5s$

$m(s) = 18s^2 + 2$

$$18s^2 + 2 \,\overline{\big)\, 45s^3 + 5s} \quad (5s/2$$
$$\underline{45s^3 + 5s}$$

← premature termination of continued fraction expansion, thus, $45(s^2 + 1/9)$ must be a factor of Q(s)

Now Q(s) may be written as $(s^2 + 1/9)q_1(s)$, then

$$Z(s) = \frac{90s^3 + 15s^2 + 16s}{(s^2 + 1/9)q_1(s)} = \frac{2K_1 s}{s^2 + 1/9} + Z1(s)$$

where $q_1(s) = 45s + 18$ and was found by dividing $(s^2 + 1/9)$ into Q(s).

$2K_1$ may be found as

$$2K_1 = \frac{(s^2 + 1/9)}{s} \left. \frac{90s^3 + 15s^2 + 16s}{(s^2 + 1/9)(45s + 18)} \right|_{s^2 = -1/9} = 1/3.$$

$$Z1(s) = \frac{90s^3 + 15s^2 + 16s}{45s^3 + 18s^2 + 5s + 2} - \frac{(1/3)s}{(s^2 + 1/9)}$$

$$= \frac{90s^3 + 15s^2 + 16s}{(s^2 + 1/9)(45s + 18)} - \frac{(1/3s)(45s + 18)}{(s^2 + 1/9)(45s + 18)}$$

$$= \frac{90s^3 + 10s}{(s^2 + 1/9)(45s + 18)} .$$

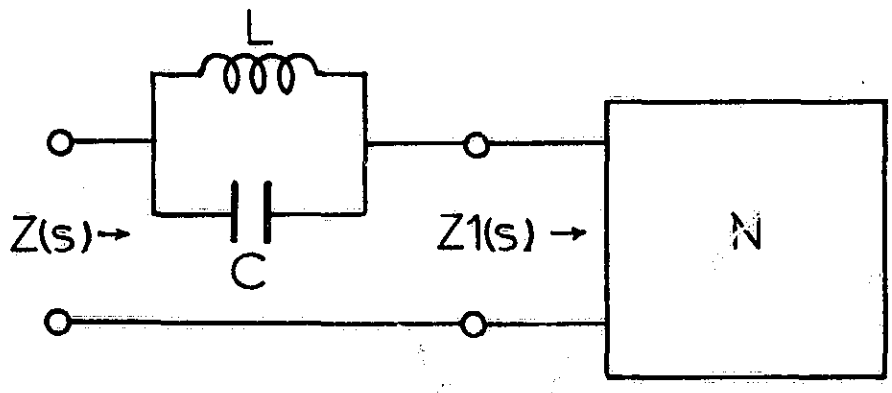
Since the pole at $s = \pm j1/3$ has been removed from Z1(s), the term $(s^2 + 1/9)$ must be a factor of $90s^3 + 10s$ and can be removed by long division.

$$s^2 + 1/9 \,\overline{)\, 90s^3 + 10s}$$
quotient: $90s$
$$\underline{90s^3 + 10s}$$

Z1(s) is then

$$Z1(s) = \frac{90s}{45s + 18} = \frac{10s}{5s + 2}.$$

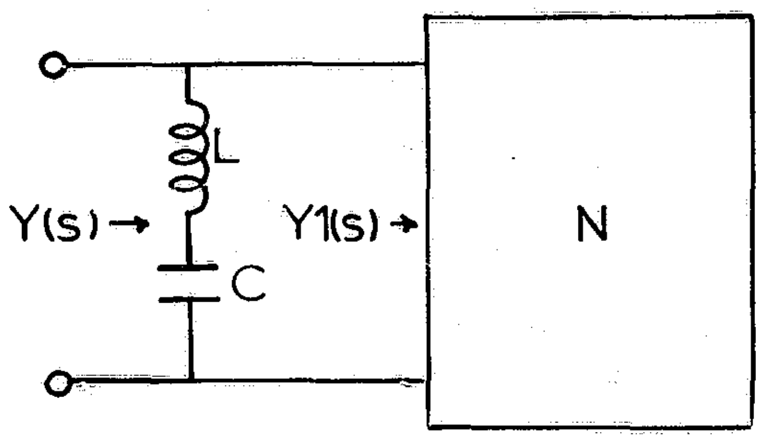
The network interpretation of the removal of this pole is



where $C = \frac{1}{2K} = 3$

$$L = \frac{1}{\omega^2 C} = \frac{9}{1}\,\frac{1}{3} = 3.$$

b) $$Y(s) = \frac{90s^3 + 15s^2 + 16s}{45s^3 + 18s^2 + 5s + 2} = \frac{1/3s}{s^2 + 1/9} + \frac{10s}{5s + 2}$$



where $L = \frac{1}{2K} = 3$

$$Y1(s) = \frac{10s}{5s + 2}$$

$$C = \frac{1}{L}\,\frac{1}{\omega^2} = \frac{9}{1}\,\frac{1}{3} = 3$$

Properties of LC functions

The properties of an LC impedance or admittance function.

(1) The function is a ratio of odd to even or even to odd polynomials.

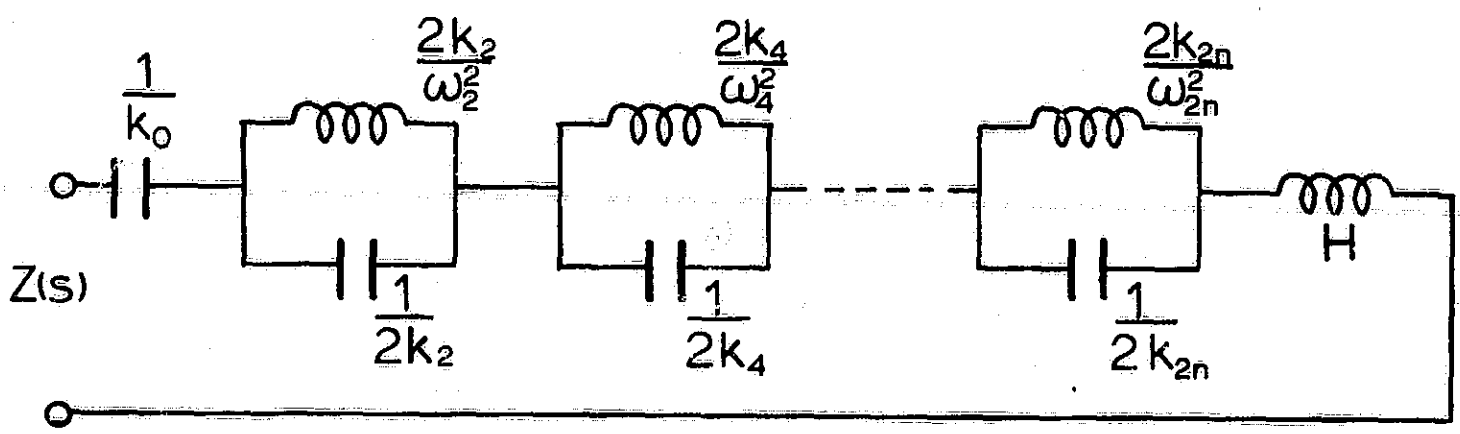
(2) The pole and zeros of the function are simple and lie on the $j\omega$ axis.

(3) The poles and zeros of the function are interlaced on the $j\omega$ axis.

(4) The highest powers of the numerator and denominator must differ by unity; the lowest powers also differ by unity.

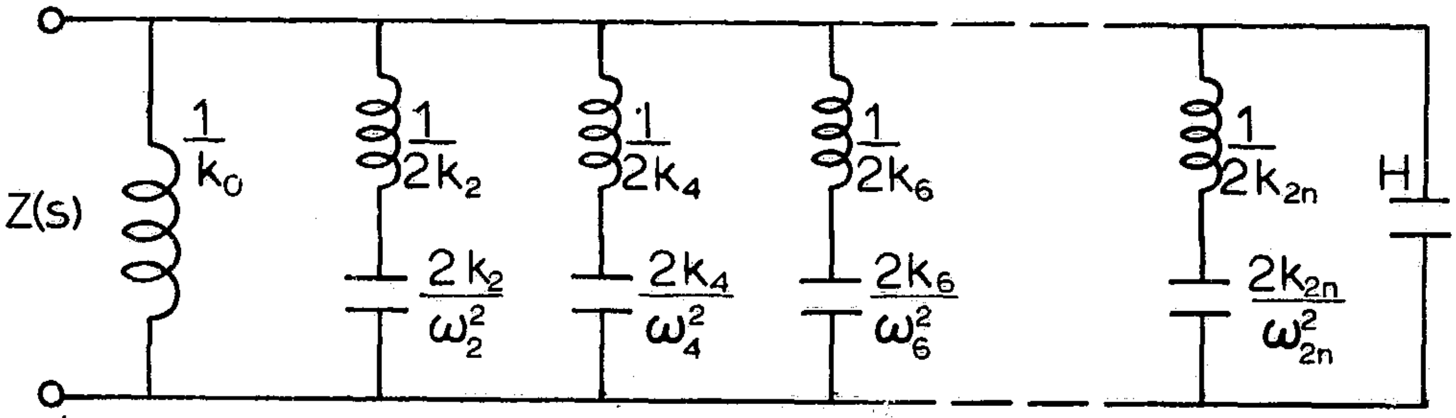
(5) The function must have either a zero or a pole at the origin and at infinity.

First Foster form of a reactive network



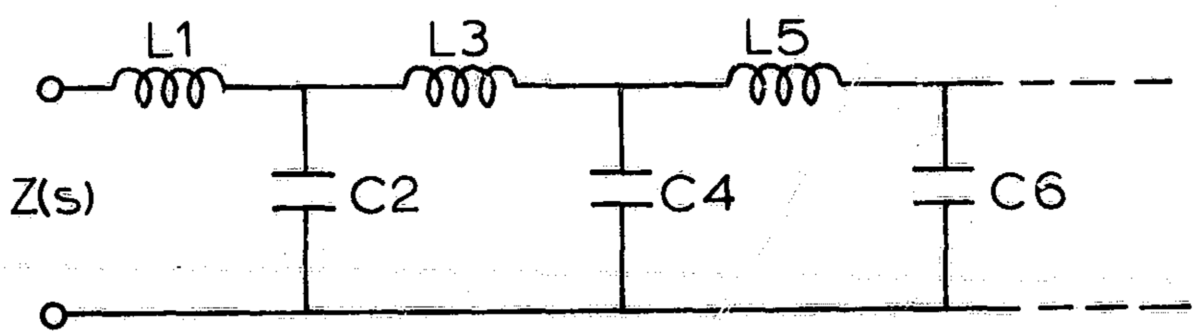
$$Z(s) = \frac{K_0}{s} + \sum_{i=1}^{n} \frac{2K_{2i}\,s}{s^2 + \omega_{2i}^2} + Hs$$

Second Foster form of a reactive network



$$Y(s) = \frac{K_0}{s} + \sum_{i=1}^{n} \frac{2K_{2i}\, s}{s^2 + \omega_{2i}^2} + Hs$$

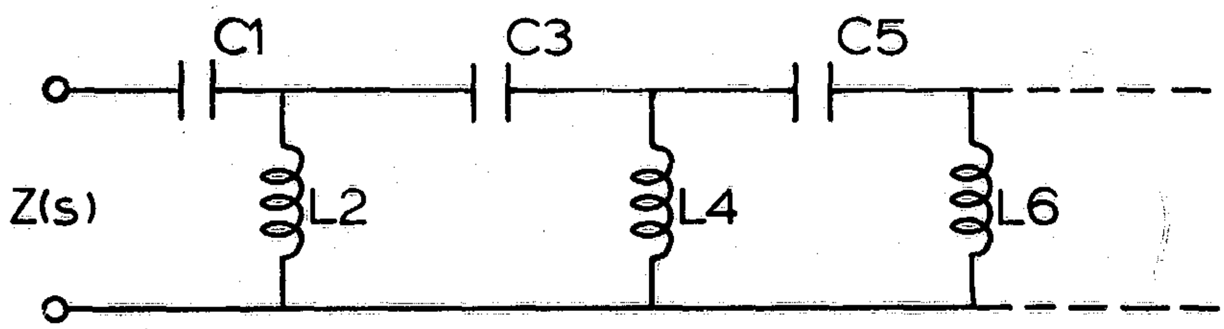
First Cauer form of a reactive network



$$Z(s) = L1s + \cfrac{1}{C2s + \cfrac{1}{L3s + \cfrac{1}{C4s + \cfrac{1}{L5 + \cfrac{1}{C6 + \cdots}}}}}$$

continued fraction expansion

Second Cauer form of a reactive network



$$Z(s) = \cfrac{1}{C1s} + \cfrac{1}{\cfrac{1}{L2s} + \cfrac{1}{\cfrac{1}{C3s} + \cfrac{1}{\cfrac{1}{L4s} + \cfrac{1}{\cfrac{1}{C5s} + \cfrac{1}{\cfrac{1}{L6s} + \cdots}}}}}$$

reversed coefficient continued fraction expansion

Properties of RC impedance function

(1) All poles and zeros are simple and are located on the negative real axis of the s plane.

(2) Poles and zeros interlace.

(3) The lowest critical frequency is a pole which may be at $s = 0$.

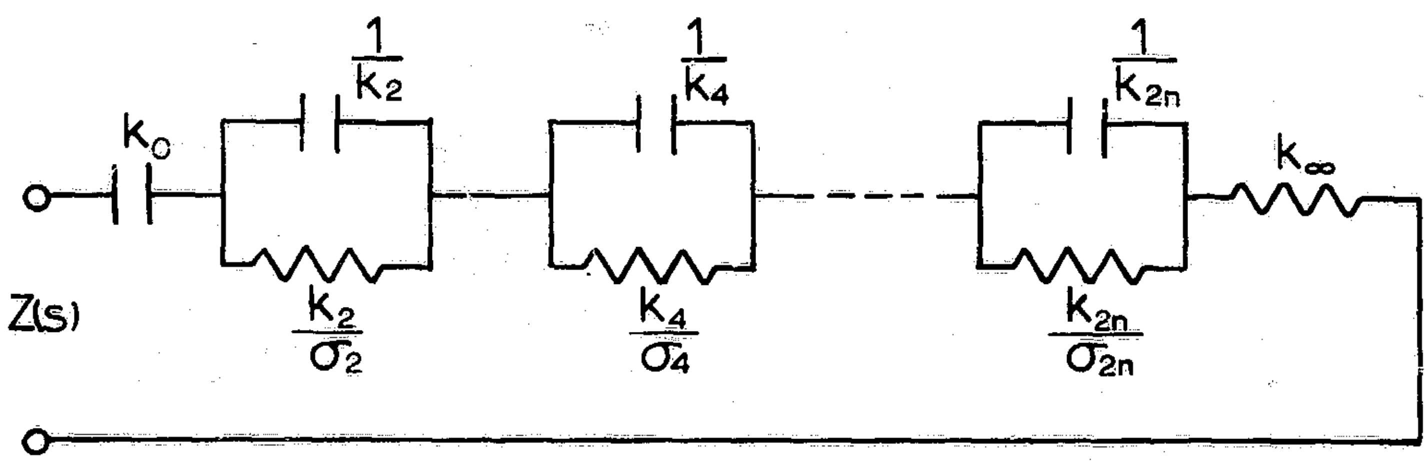
(4) The highest critical frequency is a zero which is at $s = \infty$.

(5) The residues at the poles of Z(s) are real and positive.

(6) The slope $dZ/d\sigma$ is negative.

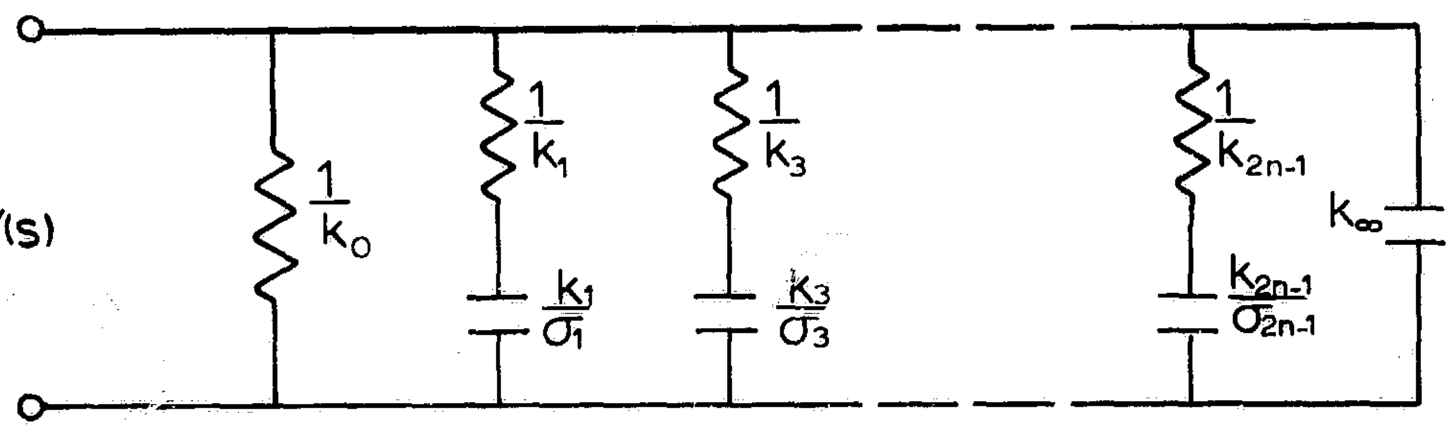
(7) $Z(\infty) < Z(0)$.

The Foster form of an RC impedance function



$$Z(s) = \frac{K_0}{s} + \sum_{i=1}^{n} \frac{K_{2i}}{s + \sigma_{2i}} + K_\infty$$

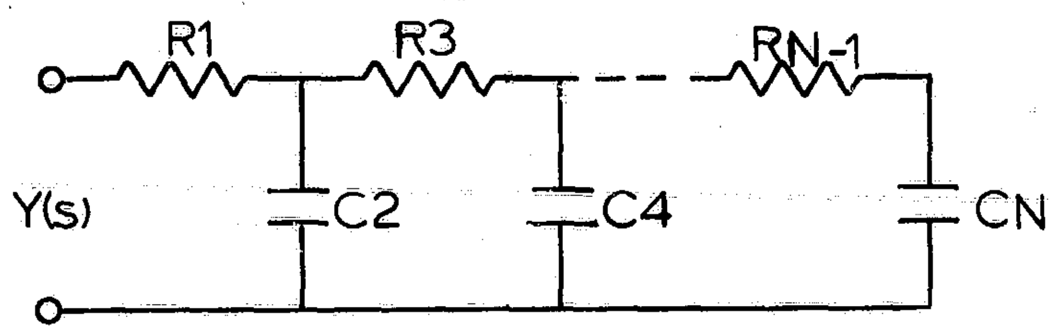
The Foster form of an RC admittance function



$$Y(s) = K_0 + \sum_{i=1}^{n} \frac{K_{2i-1}s}{s + \sigma_{2i-1}} + K_\infty$$

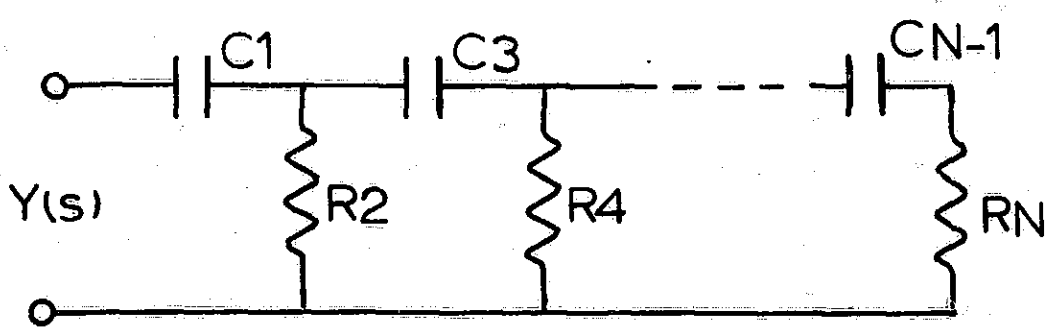
Expand as Y(s)/s.

First Cauer form of an RC admittance function



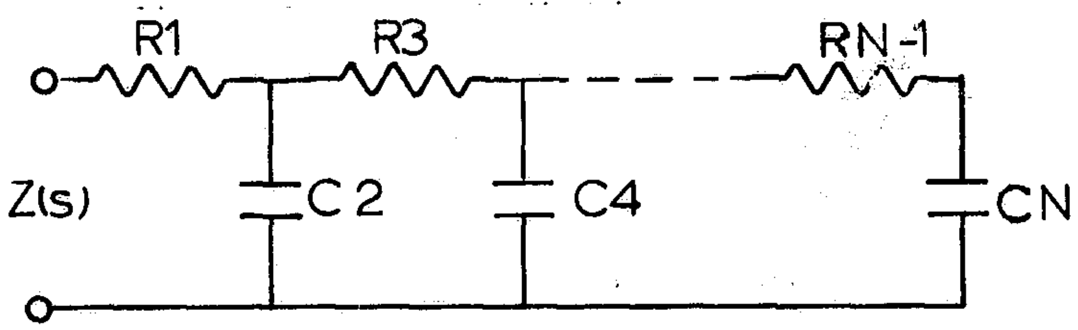
If Y(s) has a pole at s = ∞, the first element is C2.
If Y(s) is a constant at s = ∞, the first element is R1.
If Y(s) has a zero at s = 0, the last element is Cn.
If Y(s) is a constant at s = 0, the last element is Rn-1.
Use the continued fraction expansion to obtain element values.

Second Cauer form of an RC admittance function



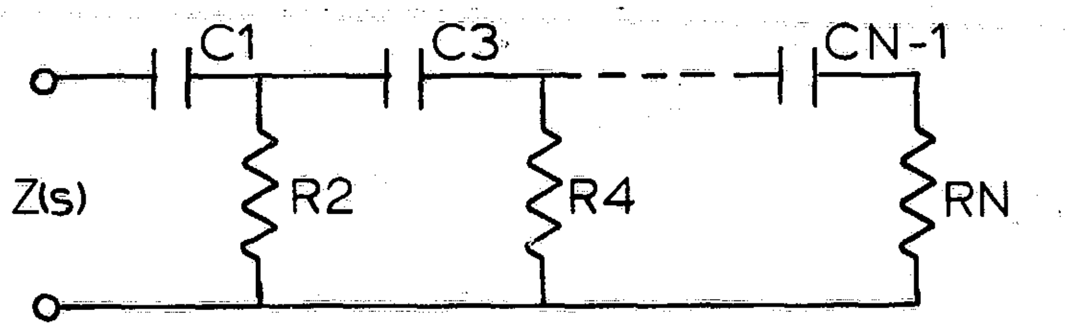
If Y(s) has a zero at s = 0, the first element is C1.
If Y(s) is a constant at s = 0, the first element is R1.
If Y(s) has a pole at s = ∞, the last element is Cn- 1.
If Y(s) is constant at s = ∞, the last element is Rn.
Use the reverse coefficient continued fraction expansion to obtain element values.

First Cauer form of an RC impedance function



If Z(s) has a zero at s = ∞, the first element is C2.
If Z(s) is a constant at s = ∞, the first element is R1.
If Z(s) has a pole at s = 0, the last element is Cn.
If Z(s) is a constant at s = 0, the last element is Rn-1.
Use continued fraction expansion to obtain element values.

Second Cauer form of an RC impedance function



If Z(s) has a pole at s = 0, the first element is C1.
If Z(s) is a constant at s = 0, the first element is R2.
If Z(s) has a zero at s = ∞, the last element is Cn -1.
If Z(s) is a constant at s = ∞, the last element is Rn.
Use a reverse coefficient continued fraction expansion to obtain element values.

Properties of an RC admittance function

(1) All poles and zeros are simple and are located on the negative real axis of the s plane.

(2) Poles and zeros interlace.

(3) The lowest critical frequency is a zero which may be at $s = 0$.

(4) The highest critical frequency is a pole which may be at infinity.

(5) The residues at the poles of $Y(s)$ are real and negative; the residues of $Y(s)/s$ are real and positive.

(6) The slope $dY/d\sigma$ is positive.

(7) $Y(0) < Y(\infty)$.

Properties of a transfer function F(s) for LLFPB network

(1) $F(s)$ is real for real s.

(2) $F(s)$ has no poles in the right-half s plane and no multiple poles on the $j\omega$ axis. If $F(s)$ is given as

$$F(s) = \frac{P(s)}{Q(s)} ,$$

the degree of $P(s)$ cannot exceed the degree of $Q(s)$ by more than unity. In addition, $Q(s)$ must be a Hurwitz polynomial.

(3) Suppose $P(s)$ and $Q(s)$ are given in terms of even and odd parts:

$$F(s) = \frac{P(s)}{Q(s)} = \frac{m_1(s) + n_1(s)}{m_2(s) + n_2(s)}$$

where m_1, m_2 are even and n_1, n_2 are odd.

then

$$F(j\omega) = \frac{m_1(j\omega) + n_1(j\omega)}{m_2(j\omega) + n_2(j\omega)}.$$

The amplitude response of $F(j\omega)$ is

$$\left|F(j\omega)\right| = \left[\frac{m_1(j\omega)^2 + n_1^2(j\omega)}{m_2(j\omega)^2 + n_2(j\omega)}\right]^{\frac{1}{2}}$$

while the phase response of $F(j\omega)$ is

$$\mathrm{Arg}[F(j\omega)] = \arctan\left[\frac{n_1(\omega)}{m_1(\omega)}\right] - \arctan\left[\frac{n_2(\omega)}{m_2(\omega)}\right].$$

<u>Some properties of the open-circuit impedance and short-circuit admittance parameters</u>

(1) The poles of $z_{21}(s)$ are also the poles of $z_{11}(s)$ and $z_{22}(s)$ but a pole of $z_{11}(s)$ or $z_{22}(s)$ need not be a pole of $z_{12}(s)$.

(2) The poles of $y_{12}(s)$ are also the poles of $y_{11}(s)$ and $y_{22}(s)$, but a pole of $y_{11}(s)$ or $y_{22}(s)$ need not be a pole of $y_{12}(s)$.

(3) If $y_{11}(s)$, $y_{22}(s)$, and $y_{12}(s)$ all have poles at $s = s_1$ and K_{11}, K_{22}, and K_{12} represent the residues of these poles, then

$$K_{11}K_{22} - K_{12}^2 \geq 0$$

for an LC, or RL two-port network.

(4) If $z_{11}(s)$, $y_{22}(s)$, and $z_{12}(s)$ all have poles at $s = 1$ and K_{11}, K_{22}, and K_{12} represent the residues of these poles, then

$$K_{11}K_{22} - K_{12}^2 \geq 0$$

for an LC, or RL two-port network.

APPENDIX C

FILTER TERMS AND DEFINITIONS

Arithmetic Symmetry:

Response showing mirror image symmetry about the center frequency when frequency is displayed on an arithmetic scale. Constant envelope delay in band-pass filters is usually accompanied by arithmetic symmetry in phase and amplitude responses.

Attenuation:

Loss of signal in transmission through a filter usually referred to signal amplitude or signal power normally measured in db's but sometimes as a voltage ratio.

Band-Reject Filter:

A filter that rejects one band of frequencies and passes both higher and lower frequencies, sometimes considered a notch filter.

Band-Pass Filter:

A filter that passes one band of frequencies and rejects both higher and lower frequencies.

Bandwidth:

The width of the passband of a band-pass filter. Bandwidth can be expressed in either a percent of center frequency or as the difference between limiting frequencies.

Bessel Function:

A mathematical approximation used to yield maximum constant time delay in a filter with little consideration for amplitude response.

Butterworth Function:

A mathematical approximation used to yield maximally flat dc amplitude response in a filter with little consideration for time delay or phase response.

FILTER TERMS AND DEFINITIONS - Continued

Characteristic Impedance:

Term usually used with the transmission line or delay line and not a filter. The characteristic impedance of a transmission line is the input impedance which the line would have if it were of infinite length. Terminating a finite length transmission line or delay line in its characteristic impedance will cause its input impedance to appear (ideally) to be equal to the characteristic impedance over all of its operating range. In this sense, a filter does not normally have a characteristic impedance. The characteristic impedance of a filter is usually taken as the input impedance to ground and is measured in ohms.

Comeback:

A term used to describe points in the stopband of a filter where spurious response occurs beyond points where the filter is attenuating properly. Comeback usually occurs at frequencies much higher than the passband frequencies due to feedthrough by way of parasitic elements.

Corner Frequency:

Generally used to describe the upper frequency at which 3 db attenuation occurs in a high gain amplifier. A cornering circuit is usually introduced to attenuate the high frequencies before the phase shift of the amplifier exceeds 90°. The well designed cornering circuit prevents high frequency oscillation in feedback amplifiers, sometimes erroneously referred to as the cutoff frequency points.

Cutoff Frequency:

The frequency at which 3 db attenuation occurs in the passband of a monotonic filter. The passband edge closest to the stop band, sometimes called the 3 db point.

Decibels:

A unit of gain or attenuation for expressing the ratio of two powers. It is used in describing power gain, power loss, duty cycle, performance figure or anything which can be considered as a ratio of two powers. A db is defined as ten times the $\log_{10}$ of the ratio of P_1 to P_2, where P_1 and P_2 are two powers such as input and output power or peak power and average power.

FILTER TERMS AND DEFINITIONS - Continued

Discrimination Ratio:

The ratio of passband width to stopband width.

Dissipation:

Energy losses in a filter due to nonideal storage elements. Dissipation is usually due to resistive core losses in the inductors.

Distortion:

Generally considered a modification of signals which produce undesirable end effects. These modifications can relate to phase, amplitude, delay and so on. The distortion is usually defined as a percentage of signal power remaining after fundamental wave component has been removed.

Elliptic Function:

A mathematical approximation which provides the sharpest passband magnitude roll-off, but has the worst phase response and transient response of all the classical filter functions.

Envelope Delay:

Propagation time delay of the envelope of an amplitude modulated signal as it passes through a filter, sometimes called time delay or group delay. Envelope delay is proportioned to the slope of the phase shift response versus frequency curve. Envelope delay distortion occurs when the delay is not constant in all frequencies in the passband.

Gain:

The increase in a signal characteristic, such as power, as it passes through a filter or amplifier. An amplifier or an active filter can produce a power gain, however, a passive filter or transformer, can not produce a power gain. It can produce a voltage gain. Such gains are expressed as a ratio in db for a specific range of frequencies.

FILTER TERMS AND DEFINITIONS - Continued

Gaussian Function:

A mathematical approximation used to design a filter which passes a step function with zero overshoot with a maximum rise time similar to a Bessel function filter.

Geometric Symmetry:

Filter response showing mirror image symmetry about the center frequency when frequency is plotted on a log scale. This is the natural response of many electrical circuits.

Group Delay:

See envelope delay.

High-Pass Filter:

A filter which passes high frequency and rejects low frequencies.

Input Impedance:

The impedance measured at the input terminals of a filter when it is properly terminated at its output. This impedance normally varies considerably over the passband.

Insertion Loss:

The loss of a signal caused by a filter being inserted in a circuit. Generally it is measured in db and is a ratio of power delivered to the load with the filter in the circuit to the power in the load, if a perfect lossless matching transformer replaces the filter. When a filter is inserted between two circuits, whose impedance differs widely, the insertion loss is usually specified in other ways.

Load Impedance:

The impedance that normally must be connected to the output terminals of the filter in order to meet the filter specifications. The filter will drive this load.

FILTER TERMS AND DEFINITIONS - Continued

Low-Pass Filter:

A filter which passes low frequencies and rejects high frequencies.

Matching Loss:

The passband power response variation expressed in db.

Octave:

A term originating with music. One octive represents a frequency ratio of 2 to 1.

Output Impedance:

The impedance looking back into the output terminals of a filter when it is properly terminated at its input. This impedance often varies considerably over the passband.

Overshoot:

The amount in percent by which a signal exceeds the steady state output on its initial rise.

Parasitic Elements:

The undesirable elements in a circuit which are inherent in the circuitry. Examples may be wire resistance, winding capacity, leakage inductance and core losses.

Passband:

The frequency range in which a filter is intended to pass signals.

Phase:

The part of a period through which the independent variable has advanced, measured from an arbitrary origin.

FILTER TERMS AND DEFINITIONS - Continued

Phase Angle:

Phase angle for a periodic wave form is obtained by multiplying the phase by two π if the angle is to be expressed in radians, or by 360 if the angle is to be expressed in degrees.

Phase Shift:

The change of signal phase as it passes through a filter.

Q:

The figure of merit of a capacitor or inductor. The ratio of its reactance to its equivalent series resistance. Also, in bandpass filters, loaded Q is the term used to define the percentage of 3 db bandwidth. For example: loaded Q equals center frequency divided by 3 db bandwidth.

Reflection Coefficient:

A measure of the difference between the driving source impedance and the input impedance of a filter.

Response:

The output signal of a filter, referenced to the input or excitation signal. It is used as a measure of the performance of the filter. Usually, a particular type of response is of interest, such as impulse response, forced (steady state) response, or transient response.

APPENDIX D NASAP - MU

Under grants from NASA/ERC the original NASAP algorithm has been extended into NASAP-MU (Missouri University version). This version computes a true A.C. worst case analysis for both the magnitude and phase of the transfer function. NASAP-MU also outputs the magnitude gradient, phase gradient, magnitude tolerance, and phase tolerance matrices. NASAP-MU, however, does not have the poles, zeros, and transient response capabilities of NASAP-69.

NASAP-MU calculates the sensitivity of any or all elements of a circuit, one at a time, with respect to the transfer function using an algorithm which finds the sensitivity in terms of flowgraph loops. (See reference 7, Chapter 2, for a complete explanation of the sensitivity algorithm). The NASAP-MU user must specify, on the problem input encoding, the elements for which the sensitivity is desired and the element tolerance values for those particular elements.

The specified sensitivities are calculated and stored internally within NASAP-MU for use in the worst case analysis. Since the sensitivities are polynomials of s, it is easy to compute a numerical value of the sensitivity at each particular frequency point of interest. An algorithm has been developed to relate the numerical value of the sensitivity at a particular point to the partial derivatives of the magnitude and phase components of the transfer function at that particular point.

(See reference 7, Chapter 2, for a complete explanation of the worst case algorithm.) The algorithm uses the sign of the partial derivatives to determine a criteria for obtaining a true A.C. worst case, phase and magnitude analysis. NASAP-MU automatically calculates and plots the worst case maximum magnitude, worst case minimum magnitude, and nominal magnitude versus omega (in radians) and the worst case maximum phase, worst case minimum phase, and nominal phase versus omega of the transfer function.

For the benefit of the user, NASAP-MU also prints out the magnitude gradient, phase gradient, magnitude tolerance, and phase tolerance matrices. The gradient matrices are essentially the values of the partial derivatives, (where the columns correspond to the elements and the rows correspond to the frequency range) obtained from the sensitivity calculations. The tolerance matrices are the matrices obtained by multiplying the partial derivatives of an element times the tolerance of the element.

For example, consider the simple RL circuit in Figure D.1.

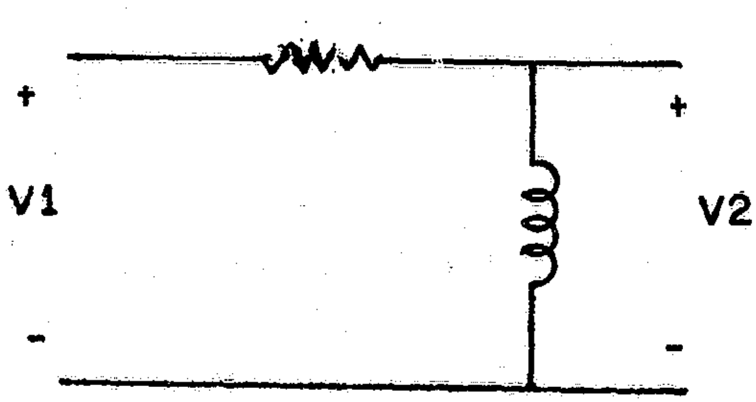


Figure D.1. RL circuit.

The voltage gain of Figure D.1 is $V_g = sL/(R + sL)$ and the sensitivities are:

$$\frac{\partial V_g}{\partial R} = \frac{-sL}{(R+sL)^2} \qquad \text{and} \qquad \frac{\partial V_g}{\partial L} = \frac{sR}{(R+sL)^2}$$

If R = 1 and L = 2 then the magnitude gradient matrix at $\omega = 1$ and $\omega = 2$ is:

$$\begin{bmatrix} \left.\frac{\partial V}{\partial R}\right|_{\omega=1} & \left.\frac{\partial V}{\partial L}\right|_{\omega=2} \\ \left.\frac{\partial V}{\partial R}\right|_{\omega=2} & \left.\frac{\partial V}{\partial L}\right|_{\omega=2} \end{bmatrix} = \begin{bmatrix} \frac{-8}{25} & \frac{-4}{25} \\ -\frac{32}{289} & -\frac{16}{289} \end{bmatrix}$$

With the given tolerances, $\Delta R = .05$ and $\Delta L = .1$, then the magnitude tolerance matrix is:

$$\begin{bmatrix} -\frac{.4}{25} & -\frac{.4}{25} \\ -\frac{1.6}{289} & -\frac{1.6}{289} \end{bmatrix}$$

To illustrate the worst case analysis and the use of the gradient and tolerance matrices consider the following Darlington active filter, figure D.2.

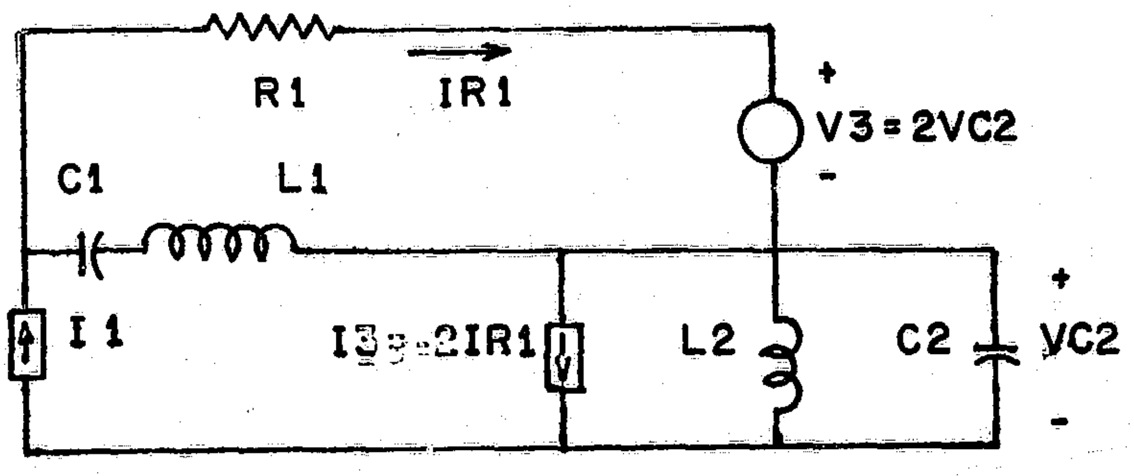


Figure D.2. Darlington Active Filter

The problem was run on NASAP-MU and figure D.3 shows the worst case magnitude analysis of the current gain IR1/I1.

The sensitivity of the current gain with respect to C1, C2, L1, L2, and R1 was encoded for NASAP-MU and figure D.4 contains the magnitude gradient and tolerance matrices.

Suppose for example, the designer of this circuit examined the worst case magnitude plot and found that for the given element tolerances, the negative spike of current gain did not return back to unity gain soon enough in the frequency domain. The designer may wish to try another circuit configuration or he may try to change existing element values to magnitude only, he may examine the gradient matrix observing that above five radians; (1) C1 is the predominant element decreasing the current gain (therefore, 1/sC1 increases the current gain), (2) C2 and L1 are the predominant elements increasing the current gain (i.e. sL1 increases and 1/sC2 decreases the current gain). This information is useful and perhaps the designer will increase the value of C1 and L1 and decrease the value of C2. He may then run another NASAP-MU problem with new element values and examine once again the worst case analysis. Figure D.5 shows the result of the second NASAP-MU run.

If at this point the designer is not satisfied he may repeat the process again. If he wishes he may use the magnitude tolerance matrix which allows him to consider changing element tolerances as well as element values to obtain a better curve fit.

In conclusion, the design by analysis eliminates some guess work on the part of the designer and is perhaps a step in a feedback process in the optimization of a circuit design.

0.192E 01	-0.480E 00	-0.533E-01	-0.484E 00	0.719E 00
-0.904E-02	0.904E-02	0.753E-03	0.273E-01	0.181E-04
-0.755E 00	0.170E 01	0.210E-01	0.171E 01	0.473E 00
-0.479E 00	0.191E 01	0.132E-01	0.192E 01	0.717E 00
-0.266E 00	0.167E 01	0.733E-02	0.166E 01	0.698E 00
-0.152E 00	0.137E 00	0.417E-02	0.136E 01	0.606E 00
-0.907E-01	0.111E 01	0.249E-02	0.111E 01	0.509E 00
-0.568E-01	0.909E 00	0.156E-02	0.905E 00	0.425E 00
-0.371E-01	0.751E 00	0.102E-02	0.749E 00	0.356E 00
-0.252E-01	0.629E 00	0.690E-03	0.626E 00	0.301E 00
-0.176E-01	0.532E 00	0.483E-03	0.530E 00	0.257E 00
-0.127E-01	0.456E 00	0.347E-03	0.454E 00	0.221E 00
-0.933E-02	0.394E 00	0.256E-03	0.392E 00	0.192E 00
-0.701E-02	0.344E 00	0.192E-03	0.342E 00	0.168E 00
-0.537E-02	0.302E 00	0.147E-03	0.301E 00	0.148E 00
-0.418E-02	0.268E 00	0.115E-03	0.266E 00	0.131E 00
-0.330E-02	0.239E 00	0.906E-04	0.237E 00	0.117E 00
-0.264E-02	0.214E 00	0.724E-04	0.213E 00	0.105E 00
-0.214E-02	0.193E 00	0.586E-04	0.192E 00	0.950E-01
-0.175E-02	0.175E 00	0.479E-04	0.174E 00	0.862E-01

(a)

0.128E-01	-0.479E-02	-0.960E-02	-0.161E-02	0.216E-01
-0.602E-04	0.903E-04	0.136E-03	0.907E-03	0.544E-06
-0.503E-02	0.170E-01	0.378E-02	0.569E-02	0.142E-01
-0.319E-02	0.191E-01	0.238E-02	0.636E-02	0.215E-01
-0.177E-02	0.166E-01	0.132E-02	0.532E-02	0.209E-01
-0.101E-02	0.137E-01	0.751E-03	0.452E-02	0.182E-01
-0.604E-03	0.111E-01	0.448E-03	0.368E-02	0.153E-01
-0.378E-03	0.908E-02	0.281E-03	0.301E-02	0.127E-01
-0.247E-03	0.751E-02	0.183E-03	0.249E-02	0.107E-01
-0.168E-03	0.628E-02	0.124E-03	0.208E-02	0.903E-02
-0.117E-03	0.532E-02	0.870E-04	0.176E-02	0.770E-02
-0.843E-04	0.455E-02	0.625E-04	0.151E-02	0.662E-02
-0.621E-04	0.394E-02	0.461E-04	0.130E-02	0.575E-02
-0.467E-04	0.343E-02	0.346E-04	0.114E-02	0.503E-02
-0.358E-04	0.302E-02	0.265E-04	0.999E-03	0.444E-02
-0.278E-04	0.267E-02	0.206E-04	0.885E-03	0.394E-02
-0.220E-04	0.238E-02	0.163E-04	0.788E-03	0.352E-02
-0.176E-04	0.214E-02	0.130E-04	0.707E-03	0.316E-02
-0.142E-04	0.193E-02	0.106E-04	0.637E-03	0.285E-02
-0.116E-04	0.175E-02	0.863E-05	0.577E-03	0.259E-02

(b)

Figure D.4 (a) Gradient Magnitude Matrix (b) Tolerance Magnitude Matrix

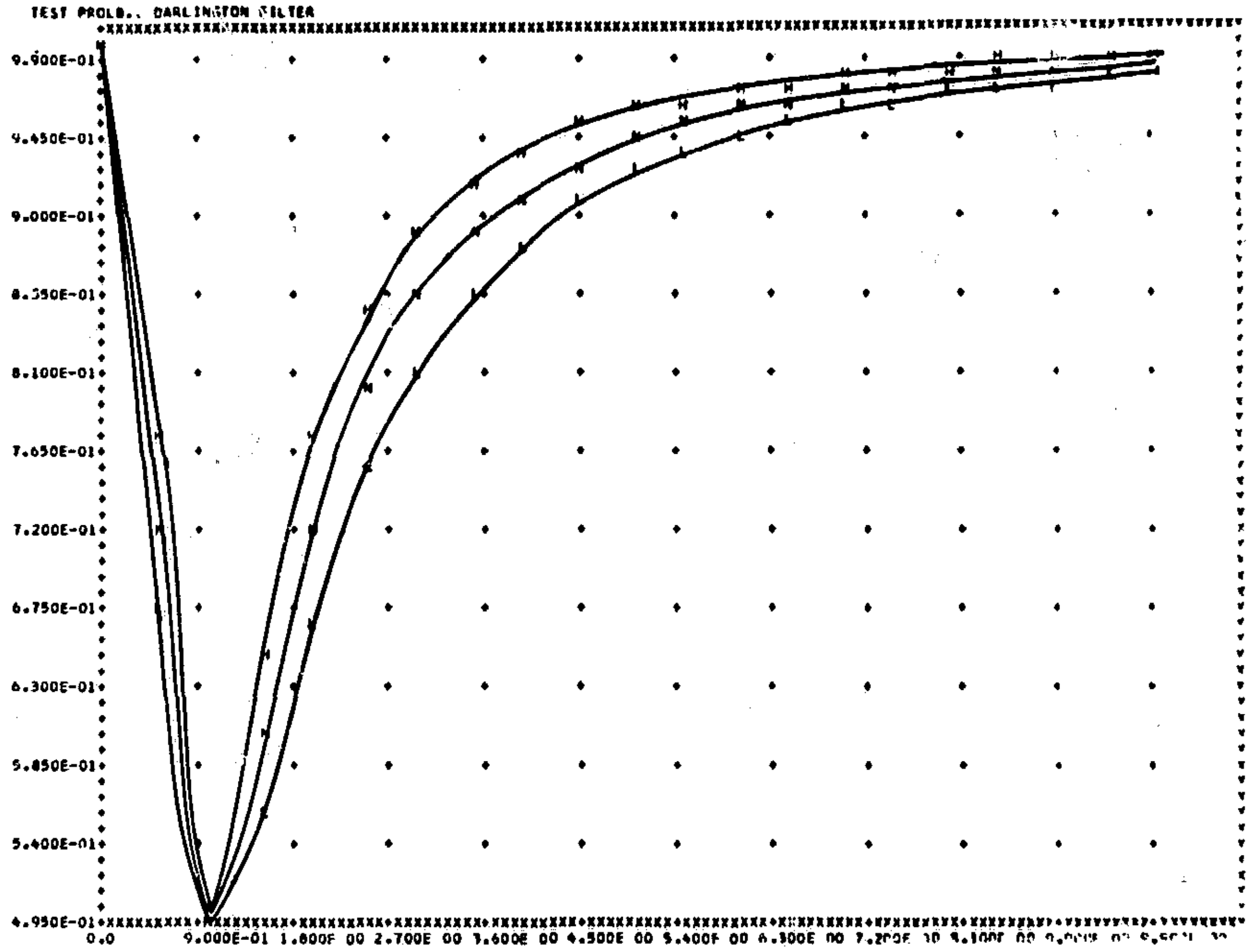


Figure D.3. Worst case magnitude of Darlington filter

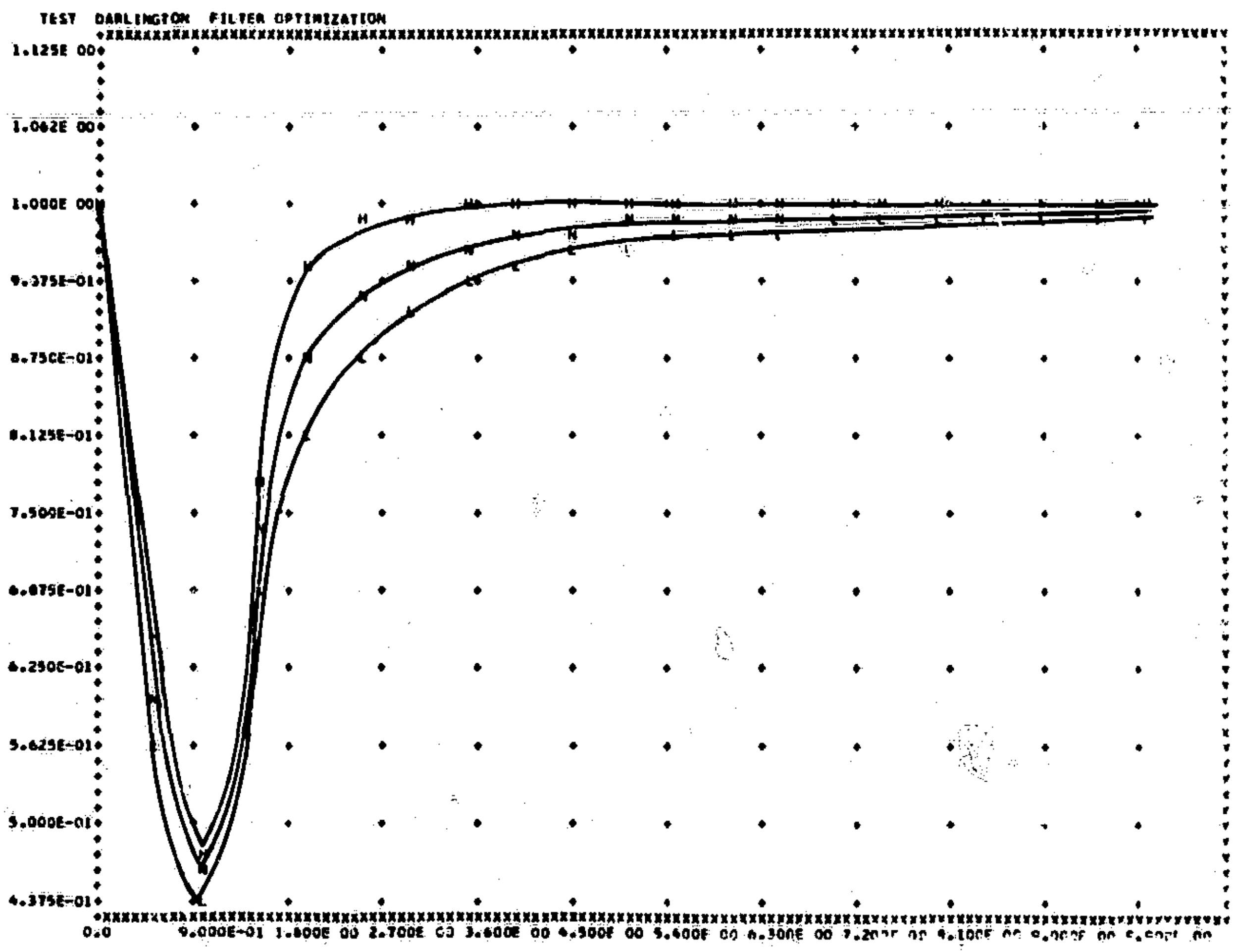


Figure D.5. Worst case magnitude of optimized Darlington filter

THE FOLLOWING DIGITAL COMPUTER PROGRAMS WERE DEVELOPED TO GENERATE THE ELEMENT VALUES FOR NORMALIZED BUTTERWORTH AND CHEBYSHEV LOW-PASS FILTERS. THE EXPLICIT FORMUAS AS DEVELOPED BY DR. LOUIS WEINBERG FOR THE CASE IN WHICH ALL ZEROS LIE IN THE LEFT-HALF S-PLANE WERE UTILIZED IN THE PREPARATION OF THESE PROGRAMS. THE PROGRAMS ARE WRITTEN IN FORTRAN IV LANGUAGE UTILIZING AN I.B.M. 026 KEYPUNCH AND ARE ENCLOSED FOR YOUR CONVENIENCE.

```
C      THIS PROGRAM GENERATES ELEMENT VALUES FOR NORMALIZED CHEBYSHEV
C      LOW-PASS FILTERS WITH ZEROES OF REFLECTION COEFFICIENT LOCATED IN
C      THE LEFT-HALF S-PLANE.
C
       DIMENSION STAR(106), LCOL(2), F84(2), QUOTE(30,2), BLANK(9), A(10)
      $, FORM(37), B(10), S(10,20), C(10,20), R(6), RIN(6), DBRIPL(6)
       REAL LPAREN, LCOL, I2
       LOGICAL EVEN, ODD, NODD, NEVEN
       DOUBLE PRECISION RIN, R1, DBRIPL, RIPLSQ, RIPPLE, THETA, C, AA,
      $NINV, H, K, XI, ETA, B, A, S, R
       DATA RIN/1.D0, 2.D0, 3.D0, 4.D0, 6.D0, 8.D0/,DBRIPL/1.D-1,
      $2.5D-1, 5.D-1, 1.D0, 2.D0, 3.D0/
C      ONLY THE THREE FOLLOWING VARIABLES SHOULD CONCERN THE USER.
C      R1 = LOAD RESISTANCE(NORMALIZED AT 1 OHM)
C      RIN = 1/R2
C      DBRIPL = RIPPLE IN DECIBELS
C      THE REST ARE UTILIZED INTERNALLY BY THE PROGRAM.
       R1 = 1.
       DO 14 MM = 1, 6
       RIPLSQ = 10.**(DBRIPL(MM)/10.) - 1.
       RIPPLE = DSQRT(RIPLSQ)
       DATA LPAREN, LCOL(1), LCOL(2), I2, X1, COL, F84(1), F84(2), X9,
      $RCOL, RPAREN/4H(    ,4HT2,@,4H*@, ,4HI2, ,4H1X, ,4H@*@, ,4HF8.4,
      $4H,1X,,4H9X, ,4H@*@ ,4H)    /
       DATA STAR(2), BLANK(1)/4H****,4H    /
       READ(5,1) QUOTE
 1     FORMAT(20A4)
       DO 2 I = 3,106
 2     STAR(I) = STAR(2)
       STAR(1) = BLANK(1)
       WRITE(6,3) QUOTE
 3     FORMAT(1H1,20A4/2(T2,20A4/))
       WRITE(6,4) STAR
 4     FORMAT(T1,106A1)
       WRITE(6,5)
 5     FORMAT(T2,@*    * C1@,T16,@* L2@,T26,@* C3@,T36,@* L4@,T46,@* L5@,
      *T56,@* L6@,T66,@* C7@,T76,@* L8@,T86,@* C9@,T96,@* L10@,T106,@*@/
      $T2,@* N *   OR    *   OR    *   OR    *   OR    *   OR    *   OR
      $  *   OR    *   OR    *   OR    *   OR    *@/
      $T2@*    *     L1@@ *      C2@@ *      L3@@ *      C4@@ *      L5@@ *
      $  C6@@ *      L7@@ *      C8@@ *      L9@@ *      C10@@*@)
       WRITE(6,6) STAR
 6     FORMAT(T1,106A1/)
       FORM(1) = LPAREN
       FORM(2) = LCOL(1)
       FORM(3) = LCOL(2)
       FORM(4) = I2
       FORM(5) = X1
       FORM(36) = RCOL
       FORM(37) = RPAREN
       DO 12 I = 1, 10
 12    A(I) = 0.
       DO 15 L = 1, 20
       DO 15 LL = 1, 10
       THETA = 3.1415926*FLOAT(L)/(2.*FLOAT(LL))
       C(LL,L) = 2.*DCOS(THETA)
 15    S(LL,L) = 2.*DSIN(THETA)
       DO 13 II = 1, 6
       DO 10 I = 7, 34, 3
       FORM(I) = X9
```

```
10    FORM(I + 1) = BLANK(1)
      DO 9 I = 6, 33, 3
9     FORM(I) = COL
      IF(II.EQ.4) WRITE(6, 16)
16    FORMAT (1H1)
      R(II) = 1./RIN(II)
      RFOUR = SNGL(RIN(II))
      INTR = IFIX(RFOUR)
      IF(INTR.EQ.1) GO TO 123
      WRITE(6,7) INTR
7     FORMAT (T44,@R1 = 1  R2 = 1/@,I1/)
      GO TO 125
123   WRITE(6,124)
124   FORMAT(T44,@R1 = 1  R2 = 1@/)
125   CONTINUE
      WRITE(6,4) STAR
      DO 8 N = 1, 10
      NEVEN = .FALSE.
      NODD = .FALSE.
      NN = N/2
      IF(N.EQ.(NN+NN)) NEVEN = .TRUE.
      IF(N.NE.(NN+NN)) NODD = .TRUE.
      IF(RIN(II).EQ.1..AND.NEVEN) GO TO 120
      IF(MM.GE.4.AND.RIN(II).EQ.2..AND.NEVEN) GO TO 120
      IF(MM.GE.5.AND.RIN(II).NE.8..AND.NEVEN) GO TO 120
      IF(NEVEN) AA = 4.*R1*R(II)*(1.+RIPLSQ)/((R1+R(II))**2)
      IF (NODD) AA = 4.*R1*R(II)/((R1 + R(II))**2)
      GO TO 122
120   WRITE(6, 121) N
121   FORMAT(T2,@ @,I2,1X,@*@,10(9X,@*@))
      GO TO 8
122   CONTINUE
      DO 119 J = 1, N
      ODD = .FALSE.
      EVEN = .FALSE.
      JJ = J/2
      IF(J.EQ.(JJ + JJ)) EVEN = .TRUE.
      IF(J.NE.(JJ + JJ)) ODD = .TRUE.
      NINV = 1./FLOAT(N)
      H =(DSQRT(1. +(1. - AA)/RIPLSQ) +DSQRT((1. - AA)/RIPLSQ))**NINV
      K = (DSQRT(1./RIPLSQ + 1.) + 1./RIPPLE)**NINV
      XI = K - 1./K
      ETA = H - 1./H
      B(J) = XI*XI-C(N,2*J)*XI*ETA+ETA*ETA+S(N,2*J)*S(N,2*J)
      IF(J .EQ. 1)  GO TO 113
      IF(J.EQ.N) GO TO 114
      A(J) = 4.*S(N,2*J-3)*S(N,2*J-1)/(A(J-1)*B(J-1))
      GO TO 119
113   A(1) = 2.*S(N,1)/(R1*(XI - ETA))
      GO TO 119
114   IF (NEVEN) A(N) = 2.*R(II)*S(N,1)/(XI + ETA)
      IF (NODD) A(N) = 2.*S(N,1)/(R(II)*(XI + ETA))
119   CONTINUE
      DO 11 I = 1, N
      IF(A(I).EQ.0.) GO TO 11
      FORM(3*I+4) = F84(1)
      FORM(3*I+5) = F84(2)
11    CONTINUE
       WRITE(6,FORM) N, (A(I), I = 1, N)
8     CONTINUE
```

```
      WRITE(6,6) STAR
13    CONTINUE
      WRITE(6,555) RIPPLE
555   FORMAT(/////T2,@THE VALUE OF RIPPLE IS@,2X,1PD20.14)
14    CONTINUE
      STOP
      END
```

THE FOLLOWING EIGHTEEN DATA CARDS ARE USED.

```
                                     ELEMENT VALUES (IN HENRYS AND FARADS) FOR A
                                           NORMALIZED CHEBYSHEV FILTER
                                               RIPPLE = 1/10 DB
                                     ELEMENT VALUES (IN HENRYS AND FARADS) FOR A
                                           NORMALIZED CHEBYSHEV FILTER
                                               RIPPLE = 1/4 DB
                                     ELEMENT VALUES (IN HENRYS AND FARADS) FOR A
                                           NORMALIZED CHEBYSHEV FILTER
                                               RIPPLE = 1/2 DB
                                     ELEMENT VALUES (IN HENRYS AND FARADS) FOR A
                                           NORMALIZED CHEBYSHEV FILTER
                                               RIPPLE =  1  DB
                                     ELEMENT VALUES (IN HENRYS AND FARADS) FOR A
                                           NORMALIZED CHEBYSHEV FILTER
                                               RIPPLE =  2  DB
                                     ELEMENT VALUES (IN HENRYS AND FARADS) FOR A
                                           NORMALIZED CHEBYSHEV FILTER
                                               RIPPLE =  3  DB
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C       THIS PROGRAM GENERATES ELEMENT VALUES FOR NORMALIZED BUTTERWORTH
C       LOW-PASS FILTERS WITH ZEROES OF REFLECTION COEFFICIENT LOCATED IN
C       THE LEFT-HALF S-PLANE.
C
        DIMENSION STAR(106), LCOL(2), F84(2), QUOTE(30,2), BLANK(9),A(10),
       $FORM(37), B(10), S(10,20), C(10,20), R(6), RIN(6)
        REAL LPAREN, LCOL, I2
        LOGICAL EVEN, ODD, NEVEN, NODD
        DOUBLE PRECISION RIN, R1, THETA, C, NINV, A, S, LAMDA
        DATA RIN(1), RIN(2), RIN(3), RIN(4), RIN(5)/1.D0,2.D0,3.D0,4.D0,
       $6.D0/,RIN(6)/8.D0/
C       ONLY THE TWO FOLLOWING VARIABLES SHOULD CONCERN THE USER.
C       R1 = INPUT RESISTANCE(NORMALIZED AT 1 OHM)
C       RIN = R2
C       THE REST ARE UTILIZED INTERNALLY BY THE PROGRAM.
        R1 = 1.
        DATA LPAREN, LCOL(1), LCOL(2), I2, X1, COL, F84(1), F84(2), X9,
       $RCOL, RPAREN/4H(    ,4HT2,@,4H*@, ,4HI2, ,4H1X, ,4H@*@, ,4HF8.4,
       $4H,1X,,4H9X, ,4H@*@ ,4H)    /
        DATA STAR(2), BLANK(1)/4H****,4H    /
        READ(5,1) QUOTE
1       FORMAT(20A4)
        DO 2 I = 3, 106
2       STAR(I) = STAR(2)
        STAR(1) = BLANK(1)
        WRITE(6,3) QUOTE
3       FORMAT(1H1,20A4/T2,20A4//)
        WRITE(6,4) STAR
4       FORMAT(T1,106A1)
        WRITE(6,5)
5       FORMAT(T2,@*    * L1@,T16,@* C2@,T26,@* L3@,T36,@* C4@,T46,@* L5@,
       $T56,@* C6@,T66,@* L7@,T76,@* C8@,T86,@* L9@,T96,@* C10@,T106,@*@/
       $T2,@* N *    OR     *    OR     *    OR     *    OR     *    OR     *    OR
       $   *    OR     *    OR     *    OR     *    OR     *@/
       $T2@*    *       C1@@ *       L2@@ *       C3@@ *       L4@@ *       C5@@ *
       $  L6@@ *       C7@@ *       L8@@ *       C9@@ *       L10@@*@)
        WRITE(6,6) STAR
6       FORMAT(T1,106A1/)
        FORM(1) = LPAREN
        FORM(2) = LCOL(1)
        FORM(3) = LCOL(2)
        FORM(4) = I2
        FORM(5) = X1
        FORM(36) = RCOL
        FORM(37) = RPAREN
        DO 12 I = 1, 10
12      A(I) = 0.
        DO 15 L = 1, 20
        DO 15 LL = 1, 10
        THETA = 3.1415926*FLOAT(L)/(2.*FLOAT(LL))
        C(LL,L) = 2.*DCOS(THETA)
15      S(LL,L) = 2.*DSIN(THETA)
        DO 13 II = 1,6
        DO 10 I = 7, 34, 3
        FORM(I) = X9
10      FORM(I + 1) = BLANK(1)
        DO 9 I = 6, 33, 3
9       FORM(I) = COL
        IF(II.EQ.4) WRITE(6, 16)
16      FORMAT (1H1)
```

```
      RFOUR = SNGL(RIN(II))
      INTR = IFIX(RFOUR)
      WRITE (6,7) INTR
7     FORMAT (T44,@R1 = 1  R2 = @,I1/)
      WRITE(6,4) STAR
      DO 8 N = 1, 10
      NEVEN = .FALSE.
      NODD = .FALSE.
      NN = N/2
      IF (N.EQ.(NN+NN)) NEVEN = .TRUE.
      IF (N.NE.(NN+NN)) NODD = .TRUE.
      DO 119 J = 1,N
      ODD = .FALSE.
      EVEN = .FALSE.
      JJ = J/2
      IF (J.EQ.(JJ + JJ)) EVEN = .TRUE.
      IF (J.NE.(JJ + JJ)) ODD = .TRUE.
      NINV = 1./FLOAT(N)
      LAMDA = ((RIN(II) - R1)/(R1 + RIN(II)))**NINV
      IF (J .EQ. 1) GO TO 113
      IF(J .EQ. N) GO TO 114
      A(J) = S(N,2*J-3)*S(N,2*J-1)/(A(J-1)*(1.-LAMDA*C(N,2*J-2)+
     $LAMDA**2))
      GO TO 119
113   A(1) = R1*S(N,1)/(1. - LAMDA)
      GO TO 119
114   IF (NEVEN) A(N) = S(N,1)/(RIN(II)*(1. + LAMDA))
      IF (NODD) A(N) = RIN(II)*S(N,1)/(1. + LAMDA)
119   CONTINUE
      DO 11 I = 1, N
      IF(A(I) .EQ. 0.) GO TO 11
      FORM(3*I+4) = F84(1)
      FORM(3*I+5) = F84(2)
11    CONTINUE
      WRITE (6,FORM) N, (A(I), I = 1, N)
8     CONTINUE
      WRITE (6,6) STAR
13    CONTINUE
      STOP
      END
```

THE FOLLOWING TWO DATA CARDS ARE USED.

ELEMENT VALUES (IN HENRYS AND FARADS) FOR A
NORMALIZED BUTTERWORTH FILTER