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PLANNING R & D PROJECTS USING GERT

by

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ABSTRACT

This study investigates the use of GERT in describing and analyzing the planning of research and development projects. Research and development programs are characterized by logical building blocks which are used to describe the underlying structure of the planning process. Five milestones are defined: problem definition, completion of the research activity, completion of the evaluation of proposed solutions, completion of a prototype, and implementation of the solution. The activities leading to the achievement of these milestones are described in terms of the fundamental processes of creative thought, time estimation, cost estimation and evaluation.

Utilizing these descriptive tools, the R & D effort is analyzed with respect to the logical relationships between activities and the alternate paths by which the milestones may be realized. This information characterizes the effort which is then drawn in network form. A computer program is available to simulate the network and examples are presented.

The ability of GERT to provide information on which the choice of alternate R & D structures can be based is indicated and the problem of projecting completion times, probability of success and man-hour requirements is considered. Concepts for using GERT for management review of R & D projects are then presented.

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INTRODUCTION

During the past decade, a great deal of money and effort has been expended on Research and Development programs. Until the past few years, however, little attention was given to studying the process itself. In 1965 Dr. Raoul J. Freeman in his paper R&D Management Research [11]* described some of the past quantitative work done in this area. In his discussion of network planning systems, he states:

> Underlying these methods are the assumptions that an appropriate network representation of an R&D project can be set down, and that realistic estimates of "activity" characteristics can be made.

More importantly, Freeman suggested some "new directions" for further research on the R&D process, a natural area of interest being the "decision pattern" of the research project where such a pattern is interpreted to mean the sequence of decisions which the researcher could be expected to make. It is an underlying assumption of the work to follow that the research project does possess an ordering of decisions peculiar to the type of project. It is to this particular ordering and the ability of GERT networks to model it that this paper is addressed.

A DEFINITION OF THE R&D PROCESS

Assume the existence of a well-defined need. The goal of the research project is to establish a means of fulfilling this need. Assume also the existence of explicit criteria or measures of value by which successive proposals can be evaluated and assume that intangible factors

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*Numbers in brackets refer to the bibliography at the end of this report.

do not play a role in the evaluations. The purpose of these assumptions is to abstract the structure of the research process from the structure of the decision-making process. That these assumptions are not unreasonable is substantiated in a recent paper. [1]

> Data were collected on about 300 ideas created in a divisional laboratory of a major U. S. corporation. . . . Further, data analysis suggests that two pieces of information are required before an idea is generated: (1) knowledge of a need, problem, or opportunity relevant to the company, and (2) a knowledge of a means or technique for satisfying the need, solving the problem, or capitalizing on the opportunity.

In this report, attention is concentrated on the process which exists between knowledge of the need (which is assumed given) and the solution (the object of the process). It is Freeman's hypothesis, that types of research ranging from "pure" research to "applied" development can only be defined, [12] ". . . in terms of the differing decision structures that are inherent in them." The following structure is proposed and will in itself be considered as the defining structure of the R&D process to be considered. <u>The R&D process consists of the serial realization of five milestones, or five objectively recognizable achievements, and is accomplished through four processes</u>.

The first milestone is the agreement of the researchers to an explicit definition of the problem. Certainly the expression of a need does not explicitly assert the definition of a problem. The first task of an R&D group is thus defined to be the successive breaking down of the need into component parts. These component parts are thought of as being a set of objectives, the attainment of which satisfies the need. From the recognition of these objectives the researchers define a set of procedures, hardware, or perhaps the mathematics necessary to model the situation, which constitute a means of obtaining the objectives, and are therefore problem definitions.

For example, consider Figure 1, which within the context of integrated circuit manufacturing, considers this process leading to the definition of the problem. The need which initiates the R&D process is for extremely pure silicon crystals. These crystals are to form the substrate upon which the microcircuit is to be deposited. This need is now broken down into component parts which constitute the set of objectives. As shown in Figure 1, the result of this breakdown is three objectives. Objective one is the solution of the problem by elimination, i.e. finding an acceptable substitute for silicon which is already available at the necessary purity. Objective two is the development of a means of refining the crystals to a sufficient purity after they have been grown. Objective three is the improvement of the crystal growing process to the point where the crystals are acceptably pure without further work.

After the researchers have defined the various objectives, they consider means of realizing each objective. In Figure 1, the second objective is shown broken down into possible means of solution. At the time this need occurred, a process called zone leveling had already been used to provide germanium with a uniform distribution of impurities for the manufacture of transistors and the application of it to silicon crystals is considered. Zone leveling (or zone refining as it sometimes is called) [26] utilizes a change in phase to accomplish a redistribution of impurities. It was decided that this process should be investigated with respect to liquid to solid, solid to vapor and liquid to vapor phase changes. These decisions constitute the last step in the problem definition phase of the R&D activity.

Steps Leading to Problem Definition	Example
Definition of a Need	A silicon crystal of a very high and thus far unobtainable purity is needed for semiconductor manufacturing.
The need is expressed in terms of objectives	Find a substitute for the silicon which is already available at the needed purity. / Refine the crystals until the needed purity is obtained / Arrange the growing process so that the crystals are grown initially at the needed purity.
Problem definition	Investigate the zone leveling with respect to the following phase changes. Liquid to Solid / Solid to Vapor / Liquid to Vapor

Figure 1. The Problem Definition Phase

In order to make clear the process involved, three additional examples are now given from the areas of aerospace manufacturing, military hardware and communications industry.

A possible need of an aerospace manufacturer may be a new means of forming metals. One of the consequent objectives might be to form metals in a fashion other than chipping. The problem is then defined to be the exploration of electrochemical milling, electromagnetic forming or explosive bonding. A solution is considered feasible if such a procedure can be developed within a mass production environment.

Recently a need for the development of an anti-missile missile has been expressed again. In this case a possible objective would be described in terms of the type and performance envelope of a particular variety of missile: hypersonic, solid fuel, X pounds of warhead capacity, performance envelope envisioned, etc. The solution to the problem could be a paper feasibility study or it may actually consist of test shots with prototype hardware.

The telephone companies have a long history of successfully applying mathematical modeling. If service at an exchange is to be expanded, a typical need of management is to know how many lines must be added to maintain a certain level of service. The problem definition phase consists of determining that this is a queueing problem and that a good approximation to the service and arrival rates are needed so that models can be used. Hopefully, these examples clarify the concept of the problem definition stage.

The second milestone is considered to be the completion of the research activity _per se_, at which time the proposed solutions have been generated. At this point we take a significantly sharp turn from the body of material on R&D. We do not propose to examine the techniques of problem resolution from the standpoint of decision theory. We assume that problem solving, like problem definition, has an underlying logical structure, apart from the essentially creative process. An examination of this structure and how it can affect the time and money expended for a solution is the subject under investigation. An example (in the structure involved in problem solving) may help to illustrate the difference. In certain areas of management, the technique of "brainstorming" has been shown to be beneficial in the solution of certain types of problems. In solving mathematical problems, where the approach or the context in which the problem is embedded is often responsible for the solution, a technique such as "brainstorming" does not apply. If a group of mathematicians are employed, the efforts are independent and not necessarily parallel. That is to say that the structure underlying these solution procedures is different.

The third milestone is the completion of the evaluation of the proposed solutions with respect to cost and time. Here again the original assumptions enter the picture. Whether to use the principle of choosing the solution which offers the minimum expected cost, or minimum variance in the costs or perhaps some cost aspiration level is not at question. The assumption is that such a rule has been picked and that it will be applied consistently. In the world of working engineers, the factors of cost and time do enter the picture and no solution will be implemented if the resulting expenditures of money and time are prohibitive. Therefore if the network analysis of R&D efforts is to model the requirements of reality, such evaluations cannot be ignored. Solutions will be unacceptable on this basis and hence evaluation enters the logical structure of the process.

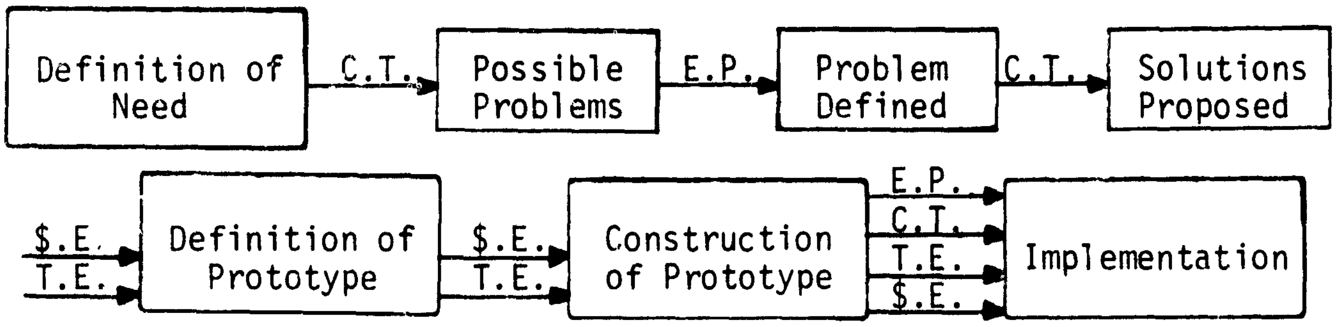
The fourth milestone is that of the completion of a prototype. For the purposes of this paper, a prototype may be understood to be either a physical model, an analytic (in the sense of mathematical) or logic (such as a simulation program) representation of the proposed solution.

The fifth milestone is the implementation of the solution and represents the natural conclusion of an R&D project. At this end of the network, the possible manifestations of a milestone are the most diverse. Implementation could range from a presentation to management, which for the case of the telephone company example would result in an order for X trunk lines, to the first production unit of a Sprint anti-missile missile or perhaps a pilot zone refining facility, the products of which would be subject to intensive analysis as to the purity and distribution of contaminates.

It is considered that four processes are involved in achieving the milestones. These four processes are: creative thought; evaluation of proposals, cost estimation and time estimation.

Without becoming ensnarled in the metaphysics of the creative process, assume that it is possible to delineate a mental process which will simply be called creative thought. This process is the essential mortar which binds the milestones into a definitive structure. Again, it is not the purpose of the network analysis to inquire into the nature or genius nor to investigate why engineer X consistently solves problems that engineer Y cannot solve. Further, it is reasonable to assume that the process of evaluating proposed definitions and the process of evaluating proposed solutions have different characteristics with respect to duration and probability of success or failure than the process of problem solution. Therefore, this process of evaluation is considered separately, as are the processes of evaluating the cost of a solution and evaluating the time required to implement the solution.

The process of evaluation is considered to be a group process and is to be distinguished from whatever evaluation the individual researchers undertake. A block diagram of the milestones and processes described is given in Figure 2.



C.T. = Creative Thought
E.P. = Evaluation of Proposals
$.E. = Cost Estimation
T.E. = Time Estimation

Figure 2. Defining Structure of the R&D Project

REVIEW OF GERT

The structure of an R&D project as it has been defined, is by the very nature of the work, probabilistic. That is to say, there is no _a priori_ knowledge which enables management to predict the exact combination of activities which will result in a solution. In fact, there is serious doubt that one such combination even if it were to be established would characterize the solution procedure uniquely. It is this probabilistic nature of events which originally necessitated the development of management tools such as PERT and CPM and has resulted in the extension to more general networks.

GERT is a general procedure for the formulation and evaluation of systems using a graphical approach. The GERT approach to problem solving utilizes the following steps:

1. Convert a qualitative description of a system or problem to a model in stochastic network form.
2. Collect the necessary data to describe the branches of the network.
3. Determine the equivalent function or functions of the network.
4. Convert the equivalent function into performance measures associated with the network.
5. Make inferences concerning the system under study from the information obtained in 4 above.

The components of GERT-type networks [27] are directed branches (arcs, edges, transmittances) and logical nodes (vertices). Two parameters are associated with the branch:

1. The probability that a branch is taken, p, given that the node from which it emanated is realized.
2. A time, t, required, if the branch is taken, to accomplish the activity which the branch represents.

the time t can be a random variable. If the branch is not part of the realization of the network then the time for the activity represented by the branch is zero.

A node in a stochastic network consists of an input (receiving, contributive) function and an output (emitting, distributive) function. Three logical relations on the input side and two types of relations on the output side will be considered. This yields six types of nodes which are described in Table 1 below.

TABLE I. NODE CHARACTERISTICS AND SYMBOLS

Output \ Input	Exclusive-or ⧏	Inclusive-or ◁	and ◖
Deterministic D	⧏D	◁D	◖D
Probabilistic ▷	⧏▷	◁▷	◖▷

EXCLUSIVE-OR The realization of any branch leading into the node causes the node to be realized; by definition one and only one of the branches leading into this node can be realized at a given time. (However, feedback branches can cause the node to be realized again.)

INCLUSIVE-OR The realization of any branch leading into the node causes the node to be realized. The time of realization is the smallest of the completion times of the activities leading into the INCLUSIVE-OR node.

AND	The node will be realized only if all the branches leading into the node are realized. The time of realization is the largest of the completion times of the activities leading into the AND node.
DETERMINISTIC	All branches emanating from the node are taken if the node is realized, that is, all branches emanating from this node have a p-parameter equal to one.
PROBABILISTIC	At most one branch emanating from the node is taken if the node is realized.

It is easy to see that a PERT/CPM network is equivalent in node logic to a GERT network with all AND-DETERMINISTIC nodes and thus the set of possible PERT networks is a subset of the set of all GERT networks.

The question which naturally arises is why are the traditional methods of PERT/CPM inapplicable to the study of R&D projects? The basic reason is that these techniques require that every path of the network be taken. In other words, the structure of problems amenable to such network techniques must be completely deterministic. In no PERT/CPM exposition is any accommodation made for a probabilistic choice of a path between two nodes. Further, no successive review or re-evaluation is permitted. Thus, PERT/CPM techniques do not allow for an unrealized alternative nor cycling, i.e. feedback branches. It is this basic capability of GERT to allow for such behavior that characterizes the basic difference and qualifies GERT as an aid in the planning of R&D projects.

THE APPLICATION OF GERT TO R&D PROJECTS

With this background it is possible to study the R&D process using GERT. The intent here is not to present general model of the R&D process but to illustrate that a graphical model can be developed. Other researchers can build their versions of an R&D model and a communications as well as an analysis tool will be established.

Activities Leading to Milestone 1: Definition of the Problem

First an analysis of the steps leading to the first milestone is performed, assuming that a need has been defined. Based on the need, personnel begin to define the problem. Each individual meets with success or failure. If all men meet with failure, the problem is not successfully defined, an evaluation of the proposals occurs, and perhaps the statement of the need is revised before the process begins "anew". If one of the men is successful, then the problem is considered defined and the steps to obtain a solution procedure may begin. To represent this process in terms of a PERT/CPM network would be difficult in that one would have to represent each alternative outcome as a separate network. A possible representation of a GERT network of the problem definition process is presented as Figure 3.

At point A it is established that the problem is not sufficiently well defined to begin the research process, and the problem definition process must be repeated. The question is whether or not the next cycle can be thought of as starting under identical circumstances as the first. If this is thought to be the case then a loop is introduced. Should one feel that the research does not begin again from scratch, a loop cannot be formed but rather another process, identical in structure, but probably differing in probabilities and durations is formed in series with the original. Figure 4 illustrates such a construction.

Care must be exercised in introducing loops just as in a PERT network care is taken to assure the absence of loops. Logically, in order

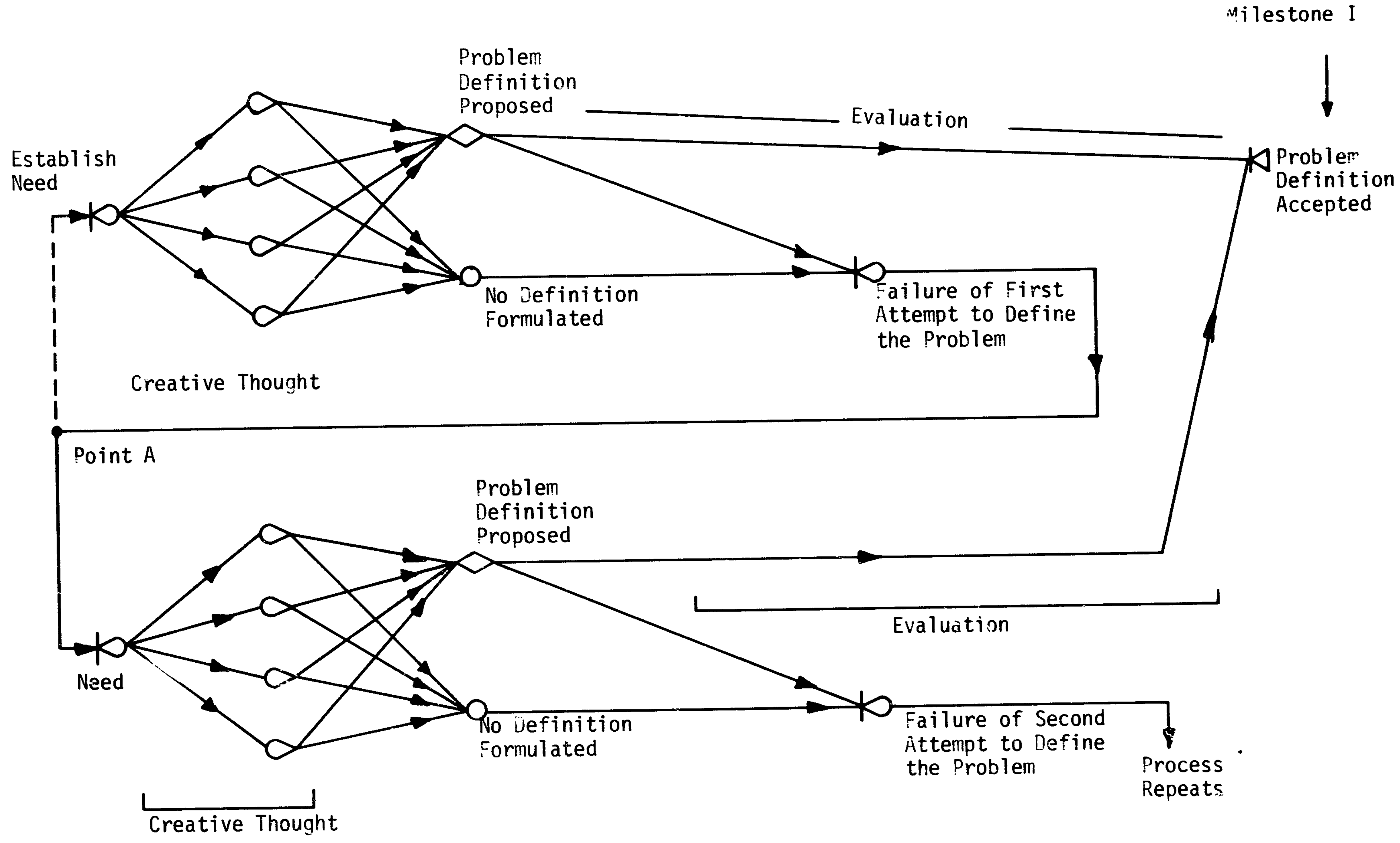


Figure 3. GERT Problem Definition

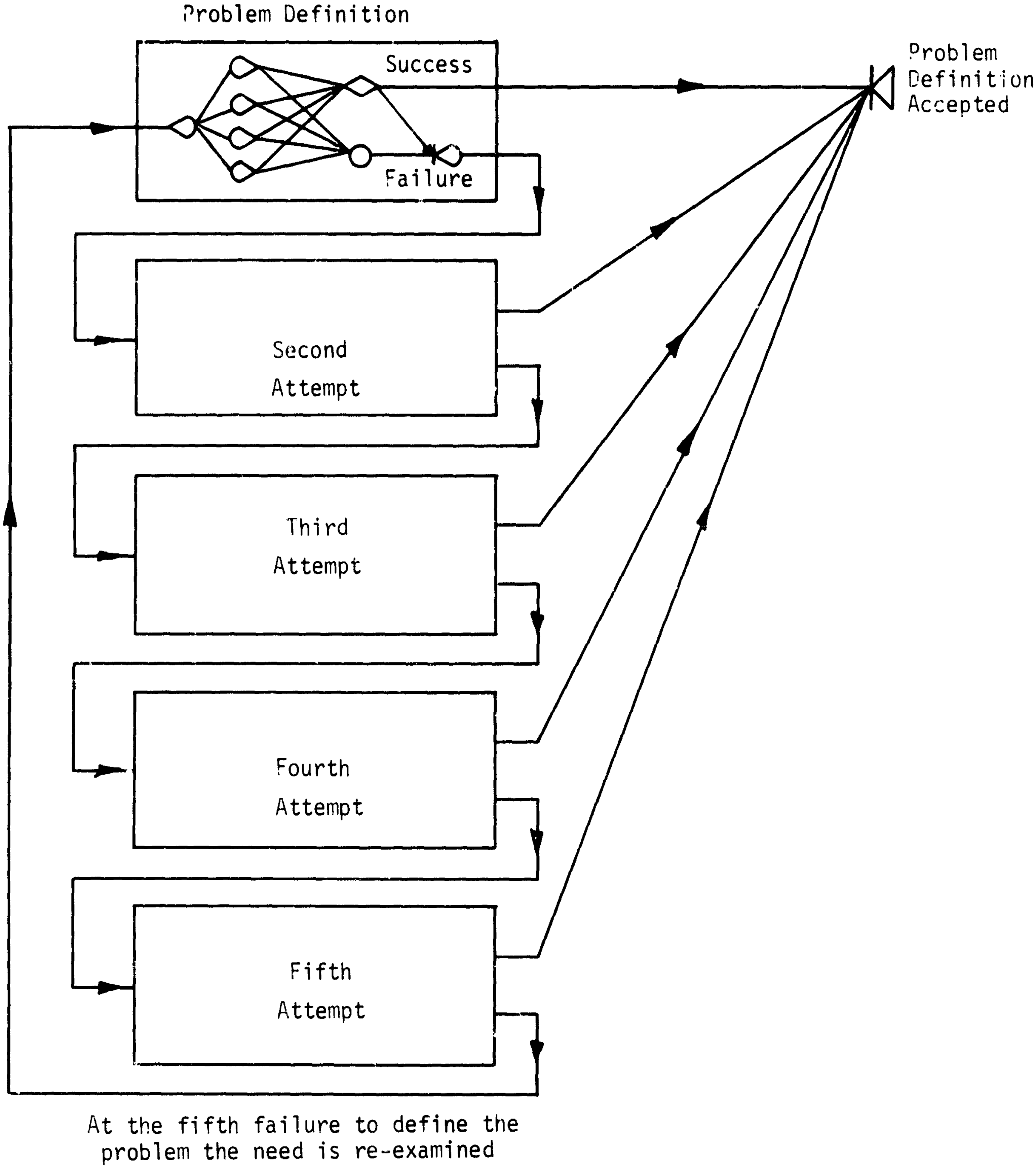


Figure 4. Series Problem Definition

for a loop to properly be included, the process being described must contain a point of regeneration such as occurs in the repeated independence throws of a die. A serious doubt exists as to whether a true point of regeneration could ever exist in such a real life process. It would be necessary to suppose that the researchers do not have memory of past work and consequently would not be able to learn from their mistakes, hardly an assumption which would lead to a realistic modeling of an R&D project. It may be argued that this is an extreme position, but it serves to illustrate the subtleties involved by the incorporation of logical nodes.

At what point then would the introduction of a loop be justified? It would be necessary to suppose that the entire problem is changed. For example, the initial need may have been to formulate a means to start an automobile in the middle of the Alaskan winter. Subsequently, it is found that the problem really is how to enable men to travel from a housing area to a large factory. The attention of the group is then changed from the problems of warming motor oil to the economics of potential Alaskan public transportation systems. It is possible to defend this type of change in the statement of the need as a legitimate point of regeneration.

Successive reviews and the improvement of a solution clearly do not constitute points of regeneration and should not be represented by loops. These processes are part of a series of sub-networks. In a certain sense, this restrictive use of loops constitutes a major strength of GERT analysis. Projects do not run forever--time, money, or the initial need change and the project, if it is not completed, must be dropped. Certainly this has been evident in the history of such military R&D projects as Dyna-Soar and Skybolt. This restriction then can be used to force management to look closely at the long term commitment of time, money and physical resources much as PERT forced management to formalize the definition of large scale construction and repetitive manufacturing processes. Restating various bidders research proposals in terms of GERT networks should provide the manager with sufficient input to apply current game or decision theory to the choice of a contractor. This is an area which should be investigated further.

Returning to the proposed representation of the problem definition phase, it is now clear that the sequence of networks may continue to be added in series as long as the cost of effort or the expected time to success does not exceed the limitations imposed on parameters of the project. It is likely that as the problem definition phase is repeated, successive attempts at problem definition should have consecutively higher overall probabilities of success and shorter durations.

Activities Leading to Milestone 2: Proposed Solutions

After the problem has been defined, effort will be directed towards the completion of the research activity and the subsequent advancement of solutions for evaluation. Suppose that three research groups are involved in the effort, their activities are independent, and there is no reason to suppose that the underlying structure of their individual activity varies to a significant extent.

Consider the need of the previous example, i.e. a method of preparing extremely pure silicon crystals and suppose that investigation of solid to liquid transformations constitutes the problem definition. An engineer might begin his problem solving by searching the literature. In so doing, many sources will be consulted. This search is represented by Figure 5.

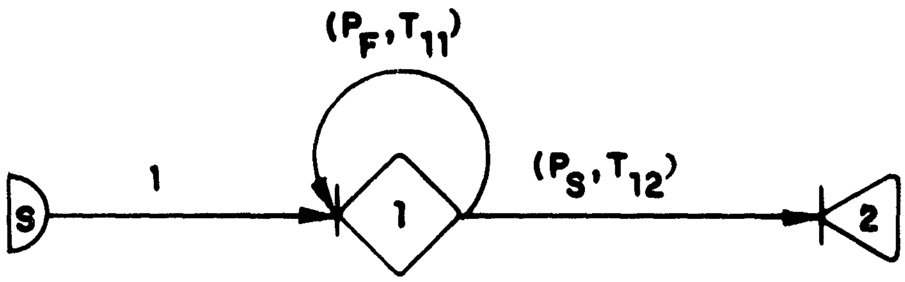


Figure 5. Literature Search

The probability of realizing the self loop is defined to be the probability of consulting anotner source. Success is achieved when the researcher "escapes" from the self loop and node 2 is realized, signifying that this solution process has advanced beyond the literature search phase. The probability of success is p_s. p_f is the probability of consulting another source; presumably p_f is rather large in comparison to p_s.

Suppose the engineer has learned that the purification obtainable is highly sensitive to the number of zone lengths in a charge and to the distribution coefficient (the concentration of the solute in the solid divided by its concentration in the liquid). He may decide to test various configurations of the zone refining apparatus. This is represented by a structure similar to that at node 1 of Figure 6.

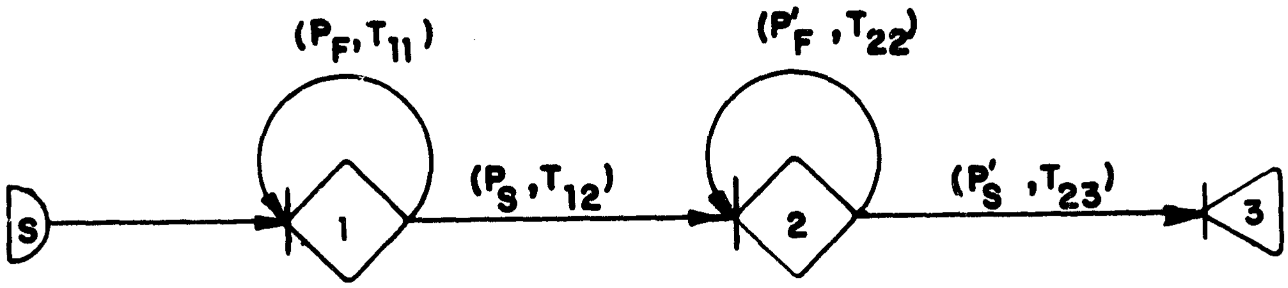


Figure 6. Literature Search and Testing of Configurations

The probability of testing still another configuration is defined to be p_f' while the probability of finding some particularly advantageous configuration is p_s'. Since consecutive attempts are distinctly different, a self loop is allowed. Once again p_s' is small in relation to p_f'.

If the resources of the laboratory allow, node 3 might well represent consecutive attempts to establish the most advantageous speeds at which the molten zones are to be moved through the charge for a given configuration. This process can be represented by a self loop at node 3. The three consecutive phases are shown in Figure 7, where each realization of a loop around node 3 represents another test.

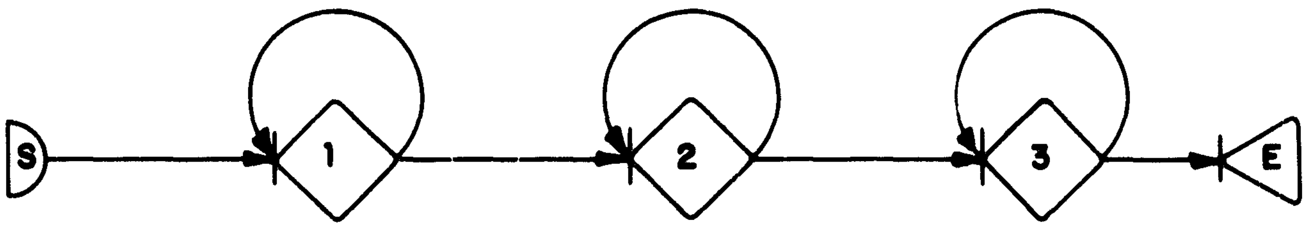


Figure 7. Consecutive Phases

Suppose that tests of the various configurations reveal that the silicon was being contaminated by the container. A completely different configuration would have to be derived, and a return from node 3 to node 2 or node 1 would be necessary. Similarily the determination of the distribution coefficient may be so imprecise as to necessitate additional research and a return to node 1 from node 2 would occur. Here again the problem of whether or not to include a loop occurs. For the sake of the example, they will be included. The final structure is illustrated in

Figure 8. The solution, i.e. desired final result, is defined to be the specification of the diffusion coefficient, the apparatus for (in this case) floating zone refining, and the parameters specifying a production facility.

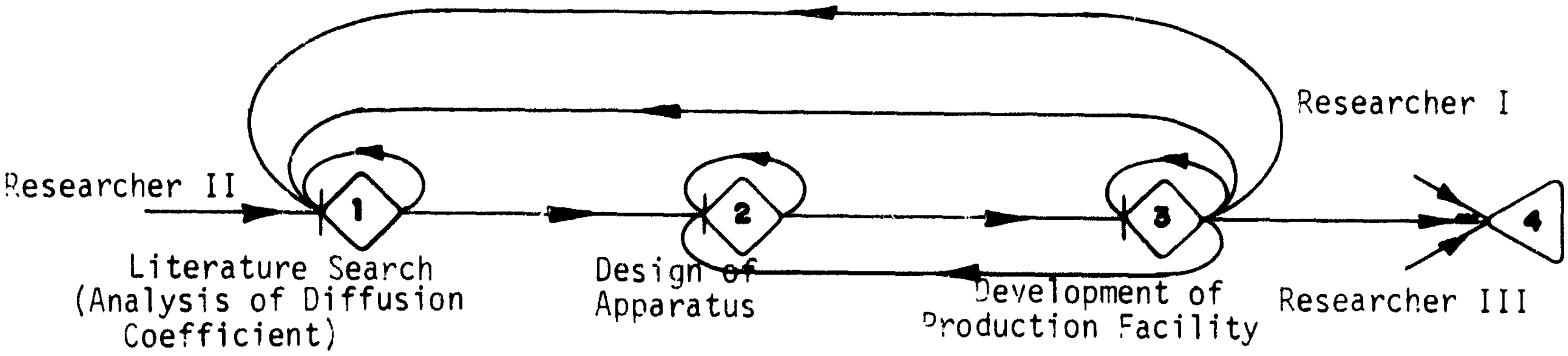


Figure 8. Network for Generation of Proposed Solutions

Activities Leading to Milestone 3: Acceptable Solutions

Next, the proposed solutions are evaluated with respect to time and cost. Structurally speaking the evaluation process is comparatively simple. The solution must meet both the cost restraints and the time restraints to be acceptable; hence, an AND node represents the acceptance of the solution. Node 7 is an INCLUSIVE-OR node since if either time or cost is unacceptable the proposed solution is unacceptable.

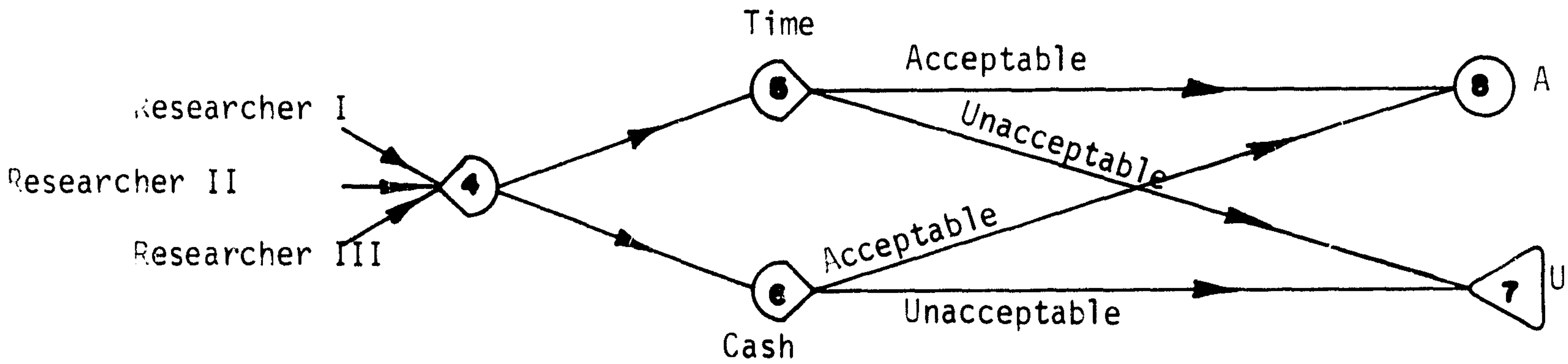


Figure 9. First Evaluation of Solution

Suppose that the floating zone process proposed was judged to be too expensive. Failing the dollar requirements, the proposed solution is discarded, this results in the node labeled 7 being realized. Here again the problem of whether or not to include a loop occurs. The second time that a solution is proposed the researcher has knowledge of the failure of the first solution. The researcher presumably benefits from this knowledge and therefore a loop is not justified. The second attempt at a solution is represented by an identical structure to that which represented the first attempt and is connected in series as is illustrated in Figure 10.

It is the desire of the researchers that once three successive proposals are rejected that further research be abandoned and the need re-examined. A completely new need must be generated if the third failure node is to be connected to the node which initiated the process. This structure is shown in Figure 11. Note that the time/cost evaluation is combined into a single activity in Figure 11 which permits the representation of the network in terms of EXCUSIVE-OR, PROBABILISTIC node types. Networks of this type are linear and easier analyzed. [27]

Of course, if a loop cannot be justified, then the entire network from problem definition to evaluation must be repeated.

Activities Leading to Milestone 4: The Construction of a Prototype

Three possibilities are considered as prototypes: a physical model; an analytical model; or both. Figure 12 allows for all three possibilities. In the example of purifying silicon, a mathematical study of the possible size and shape of the molten zone might be undertaken as well as simultaneous experimental modeling. For this particular situation, the probability of the top and bottom branches would be zero.

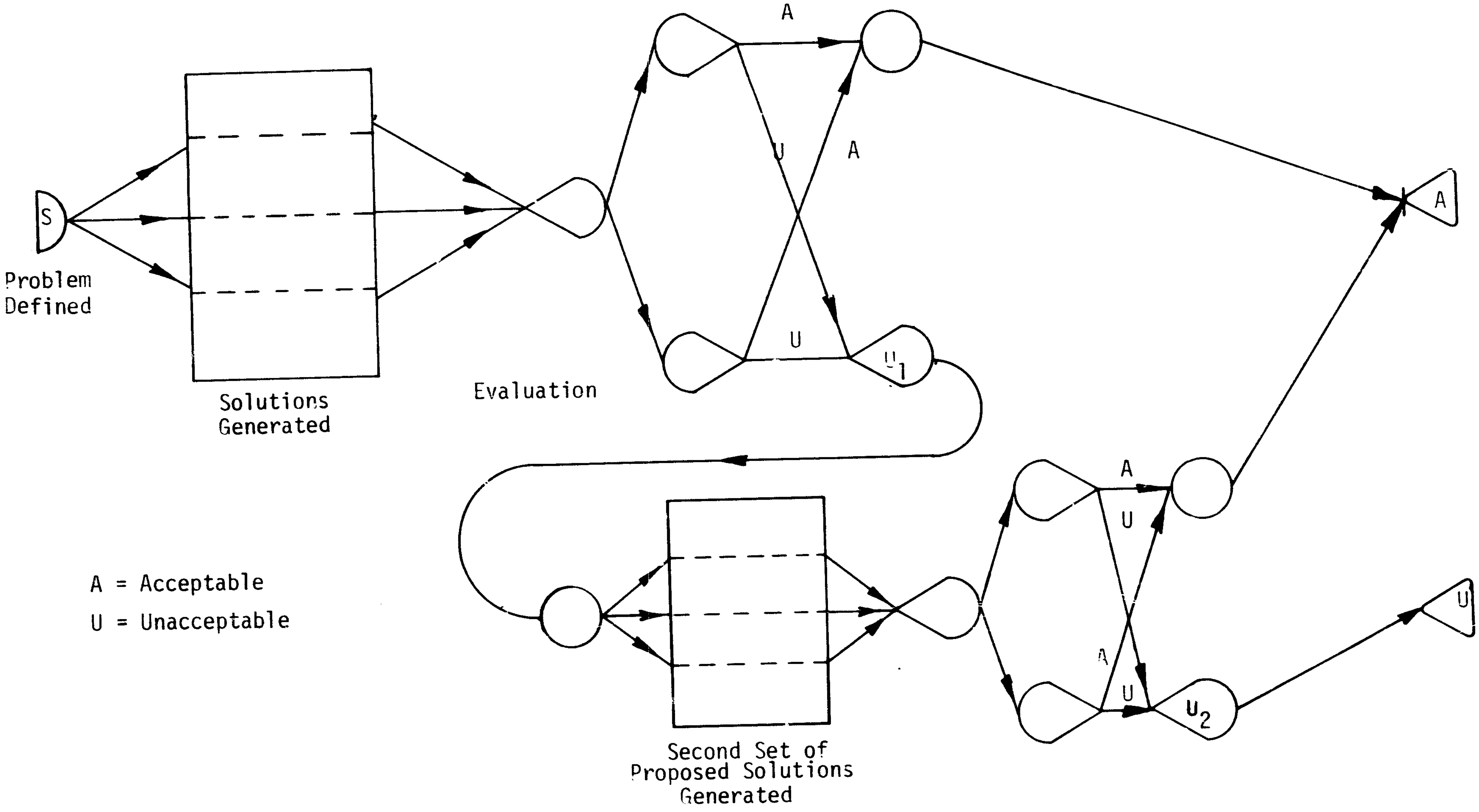


Figure 10. Series Proposal

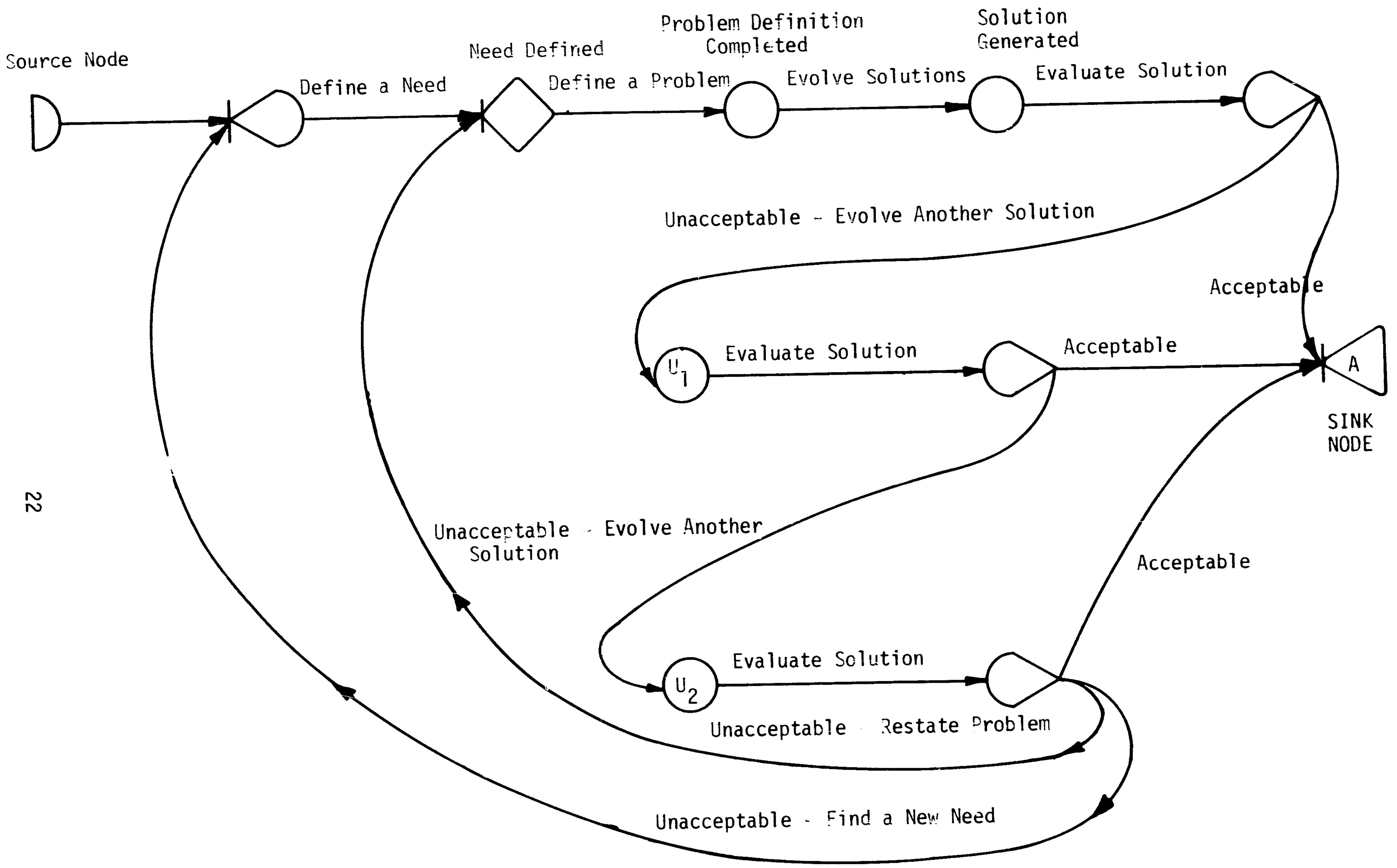


Figure 11. Series Proposals with Restatements of the Problems and the Need

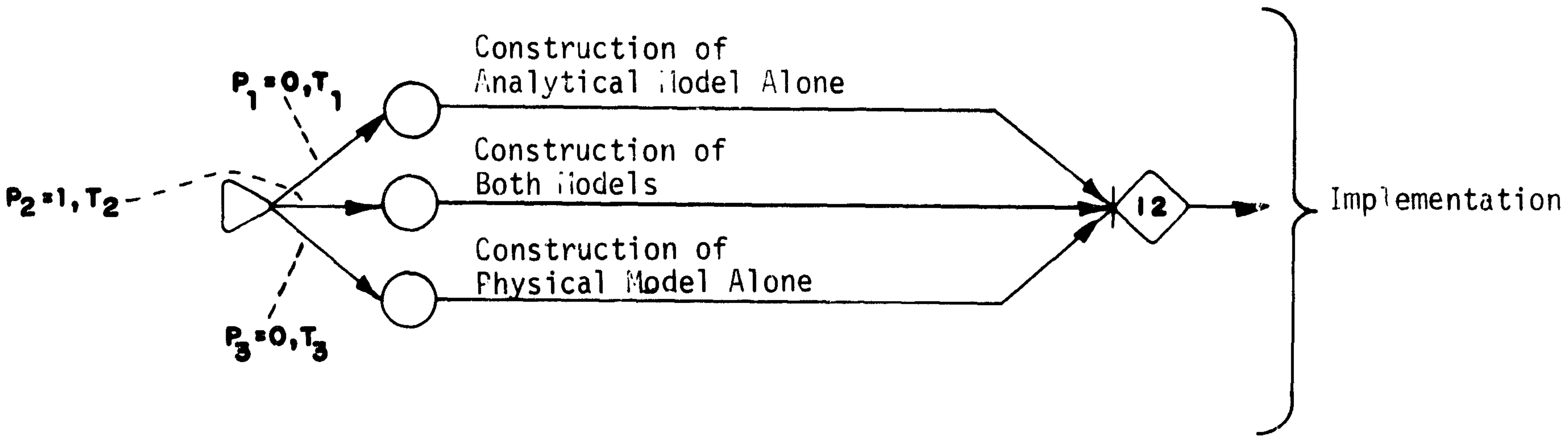


Figure 12. Prototype Phase

Activities Leading to Milestone 5: Implementation

In the example problem implementation might be a series of production runs. This is simply represented by a single branch.

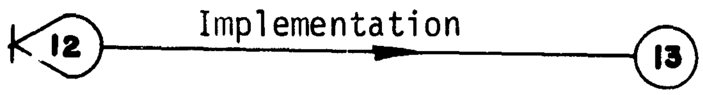


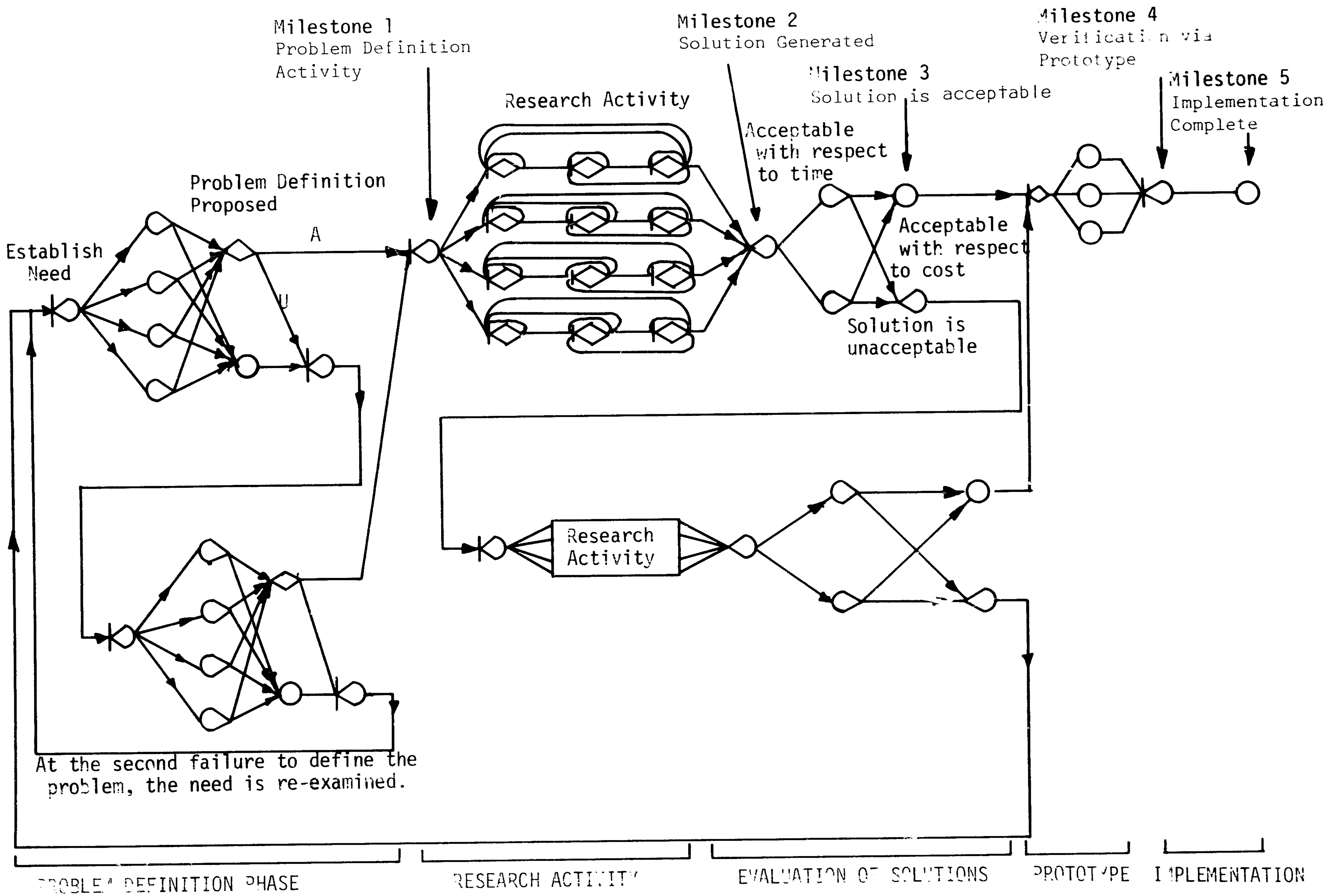
Figure 13. Implementation Phase

As the R&D project progresses, the network becomes progressively simple. The reason for this is that the structure is increasingly problem oriented. For specific situations where the implementation stage consists of a known sequence; plant site selection, purchase of site, plant construction, plant start up, personnel training, etc., the branch between

12 and 13 can easily be replaced by the specific sequence. For the purpose of this paper it is sufficient to point out the flexibility of the representation.

The complete network involving the 5 milestones is given in Figure 14. One final characteristic of this representation needs to be mentioned. There is no failure node per se. This is not necessary since a time parameter has been incorporated in the overall analysis. It is assumed that an R&D project has failed if the expected time to realize the last node, given that it is realized, exceeds the time limit set for the duration of the project. This ability to project the time to solve a problem is one of the reasons for applying GERT to R&D projects.

Figure 14.



Milestone 1
Problem Definition
Activity
Milestone 2
Solution Generated
Milestone 3
Solution is acceptable
Milestone 4
Verification via
Prototype
Milestone 5
Implementation
Complete
Research Activity
Problem Definition
Proposed
Establish
Need
A
U
Acceptable
with respect
to time
Acceptable
with respect
to cost
Solution is
unacceptable
Research
Activity
At the second failure to define the
problem, the need is re-examined.
PROBLEM DEFINITION PHASE
RESEARCH ACTIVITY
EVALUATION OF SOLUTIONS
PROTOTYPE
IMPLEMENTATION

THE GERT REPRESENTATION AS A MANAGEMENT PLANNING TOOL

A major feature of GERT network analysis is the ability it gives management to model processes which are intrinsically non-deterministic. Since it is exactly this kind of non-deterministic situation which introduces risk in management decision making, it is natural that the GERT representation of an R&D project be examined as a possible aid to formal decision making. The underlying structure of the management problem we are considering will be taken as follows: management has decided that a new area demands immediate attention and that an R&D effort is indicated. The question to be answered is whether or not a reasonable risk can be assumed for a given dollar expenditure, and indirectly then, whether or not the R&D project is to be initiated.

Two types of information are needed to analyze this problem. The first type concerns management's attitudes toward risk. What dollar amounts is management willing to commit over what period of time and what is an acceptable probability of success level associated with this commitment? The second type of information concerns the R&D effort itself. Since a major feature of GERT analysis is the incorporation of a probability distribution for the time required to complete an activity, a given network configuration of the R&D process can be parameterized in terms of the expected time to termination and the expected costs incurred. This output concerning the expected behavior of the R&D configuration can then be used in conjunction with management's attitudes toward risk to improve decision making. The major portion of this section will be devoted to a description of the procedure used within GERT to obtain this information and how the results based on this information can be incorporated into the formal decision-making process.

A simulation technique employing the GASP simulation language [32] was the principal tool used to determine the expected characteristics of the R&D networks. To illustrate the analysis procedure to be used, the problem definition phase of the R&D process has been chosen. This particular sub-network is sufficiently general to illustrate the analysis procedure and in addition serves to illustrate the modeling process at a more detailed level than previous examples. Three researchers are involved in the problem definition process which is thought of as consisting of two phases. The initial phase of the process might be thought of as being general discussion while the final phase could be the advancement of specific proposals. Interaction is allowed between the three men at each phase of the definition process and an allowance is made for this interaction to return the process from phase 2 to phase 1. One idea, generated at the start node of the network passes from researcher to researcher, from phase to phase, until in phase 2, one of the men feels that the definition of the problem is sufficiently well advanced as to warrant a vote by all three researchers. The voting process is a simple majority rule. The GERT network representation of the complete process appears as Figure 15. Since one idea is generated to start the process, the start node, node 2 consists of a probabilistic output side. The problem definition portion of the network, nodes 3 to 20 contain feedback loops. For presentation purposes, ancillary nodes 3, 5, 7, 9, 10, 12, 13, 15 and 16 appear to indicate that a decision regarding an idea is first made and then a transition to any of the three researchers is possible. This

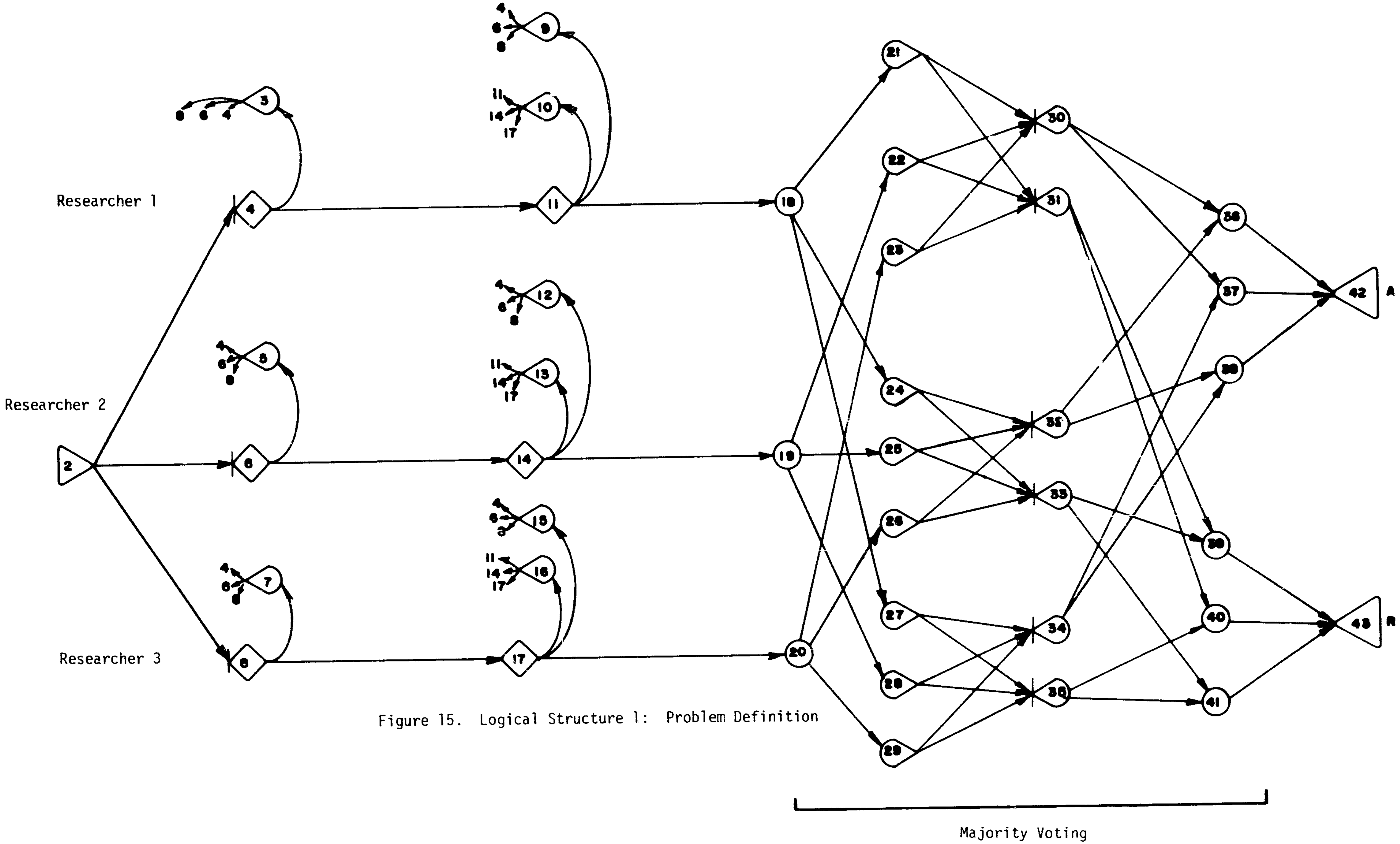


Figure 15. Logical Structure 1: Problem Definition

process could also be modeled without these ancillary nodes in a composite form as illustrated in Figure 16.

Ancillary Node Representation Composite Representation

Figure 16. Two Feasible Network Representations

The two phases of the problem definition process are made up of nodes 3 to 8 and 9 to 17 respectively. The possibility that an idea currently being considered by researcher 1 is not advanced to the second phase is represented by the branch from node 4 to node 3 (4-3). The possibility that the idea remaining in phase one is picked up by researcher 3 is represented by branch 3-8, the possibility that the idea is picked up by researcher 2 is represented by branch 3-6, and the possibility that the idea remains with researcher 1 for further consideration is represented by branch 3-4.

An idea may be advanced from phase 1 to 2 by the realization of branches 4-11, 6-14, or 8-17. While in the second phase of the definition process, the idea may be picked up by other researchers and remain in this phase, represented by branches emanating from nodes 10, 13, and 16 or it may be returned by any of the researchers for more general discussion in phase one, by any of the branches emanating from nodes 9, 12 or 15. When the idea is ready to be voted upon, one of the researchers must advance it to the voting process, represented by the realization of branches 11-18, 14-19, or 17-20.

The majority voting process is sufficiently general so as to allow for each researcher to vote differently depending upon who proposed the problem definition for a vote. This can be illustrated by a consideration of researcher 1. Researcher 1 proposes a solution for final evaluation and voting if branch 11-18 is realized. Node 18 is an AND-DETERMINISTIC node hence each branch emanating from it is realized. In particular, branch 18-21 is realized. Node 21 is an AND-PROBABILISTIC node, and the probability that branch 21-30 is realized is the probability that researcher 1 will vote favorably for one of his own proposed solutions, whereas the probability that branch 21-31 is realized is the probability that researcher 1 will unfavorably evaluate and vote for his own solution. In an analogous fashion, node 22 represents the decision to vote favorably or unfavorably for an idea proposed by researcher 2 and node 23 represents the decision to vote favorably or unfavorably for a solution proposed by researcher 3. A majority voting scheme will return a favorable vote if either researchers 1 and 2, 1 and 3, or 2 and 3 vote favorably. This is represented by the realization of branches 30-36 and 32-36, 30-37 and 34-37 or 32-38 and 34-38 respectively. Nodes 36, 37, and 38 are of the AND-DETERMINISTIC type and the branches emanating from these nodes are realized only if every branch leading into the node is realized i.e. a majority affirmative vote. Node 42 is attained if any one of the possible combinations of an affirmative majority is realized and therefore represents the positive termination of the problem definition phase of the R&D process. Node 43 is another sink node and represents the negative termination of the problem definition phase.

For purposes of comparison, another structure has been developed for the problem definition phase of the R&D process. This structure is similar to the one in Figure 15 but has been simplified by restricting interaction in the problem definition process. This network, shown in Figure 17, consists of three researchers working in parallel, with a three phase process leading to a vote.

In the discussion to follow, the structure appearing in Figure 17 will be called Logical Structure II and the previous structure with two phases will be called Logical Structure I.

Each network has been simulated and a representation of the output from the GERT simulation program is shown in Figure 18.

The first two lines of the final results give the probability of reaching the sink nodes 42 and 43, the mean time to realize these nodes, the standard deviation of the times, the minimum time and the maximum time to realize the node in 400 simulations of the network. The first two lines of the histogram section of the output give a histogram of the times to realize the sink nodes. The histograms themselves are described in terms of the lower limit of the histogram and the width of the individual cell. The first cell is a count of all realizations of the network which occurred in a time less than the lower limit and the tenth cell is a count of all realizations of the network which occurred in time greater than or equal to the lower limit plus eight times the cell width.

From this information concerning the distribution of the time to realize the sink nodes, one can construct a chart of the cumulative probability of success given that success occurs within a specific interval of interest, $[0, \dot{T}] = I$. The ratio of the number of simulations successfully terminated in time interval I to the total number of

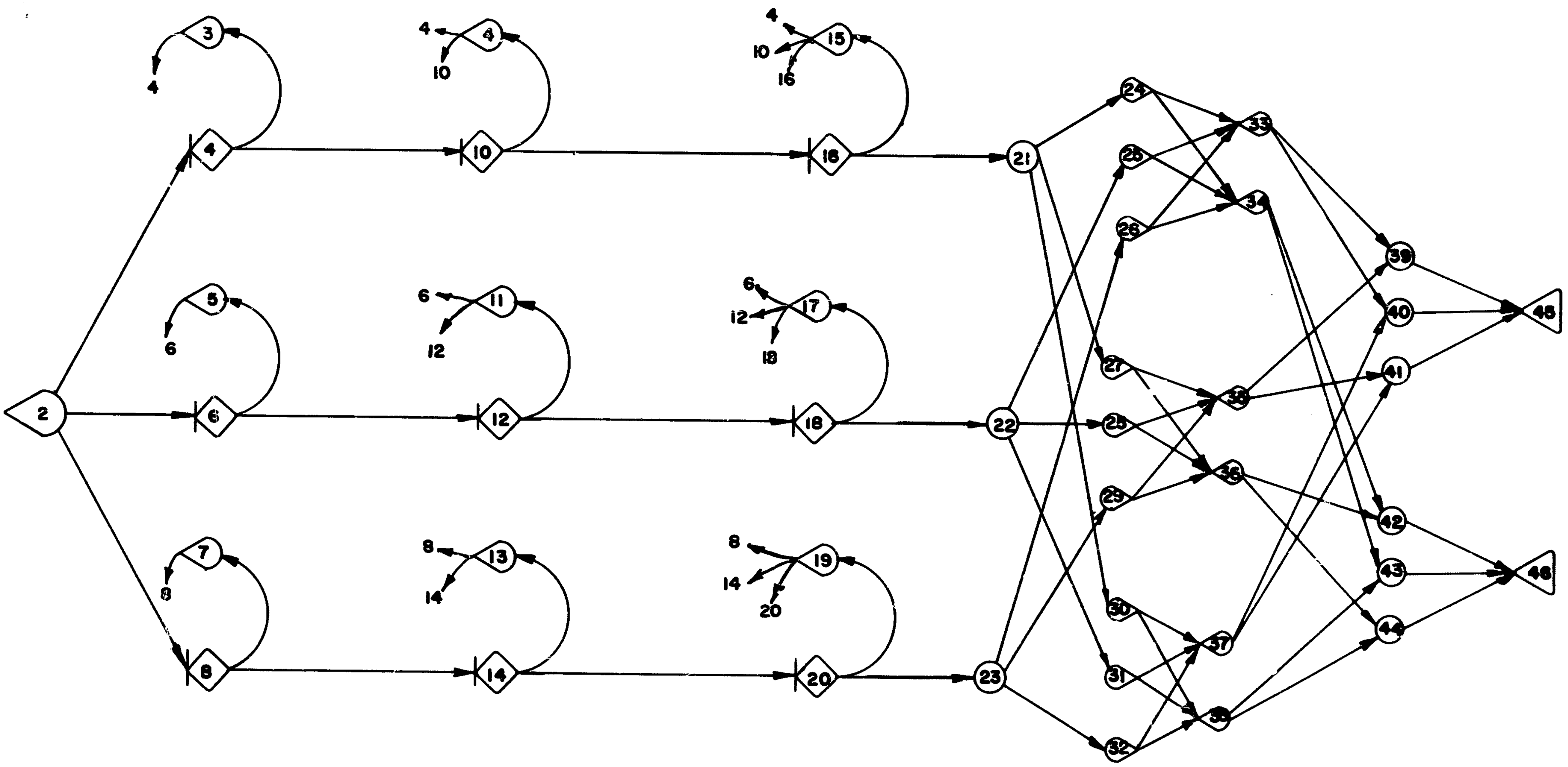


Figure 17. Logical Structure 2: Problem Definition

GERT SIMULATION PROJECT 1 BY RONALD ENLOW

DATE - 5/ 3/ 1969

** Final Results for 400 Simulations **

Node	Prob	Mean	Std. Dev.	Min.	Max.
42	0.5350	587.7046	317.1953	127.6343	1491.5898
43	0.4650	611.0015	318.0273	130.3877	1555.2927

** Histograms **

Node	Lower Limit	Cell Width	Frequencies									
42	360.0	200.0	63	50	39	27	27	5	3	0	0	0
42	360.0	200.0	49	42	38	28	18	6	5	0	0	0

GERT SIMULATION PROJECT 2 BY RONALD ENLOW

DATE - 5/ 3/ 1969

** Final Results for 400 Simulation **

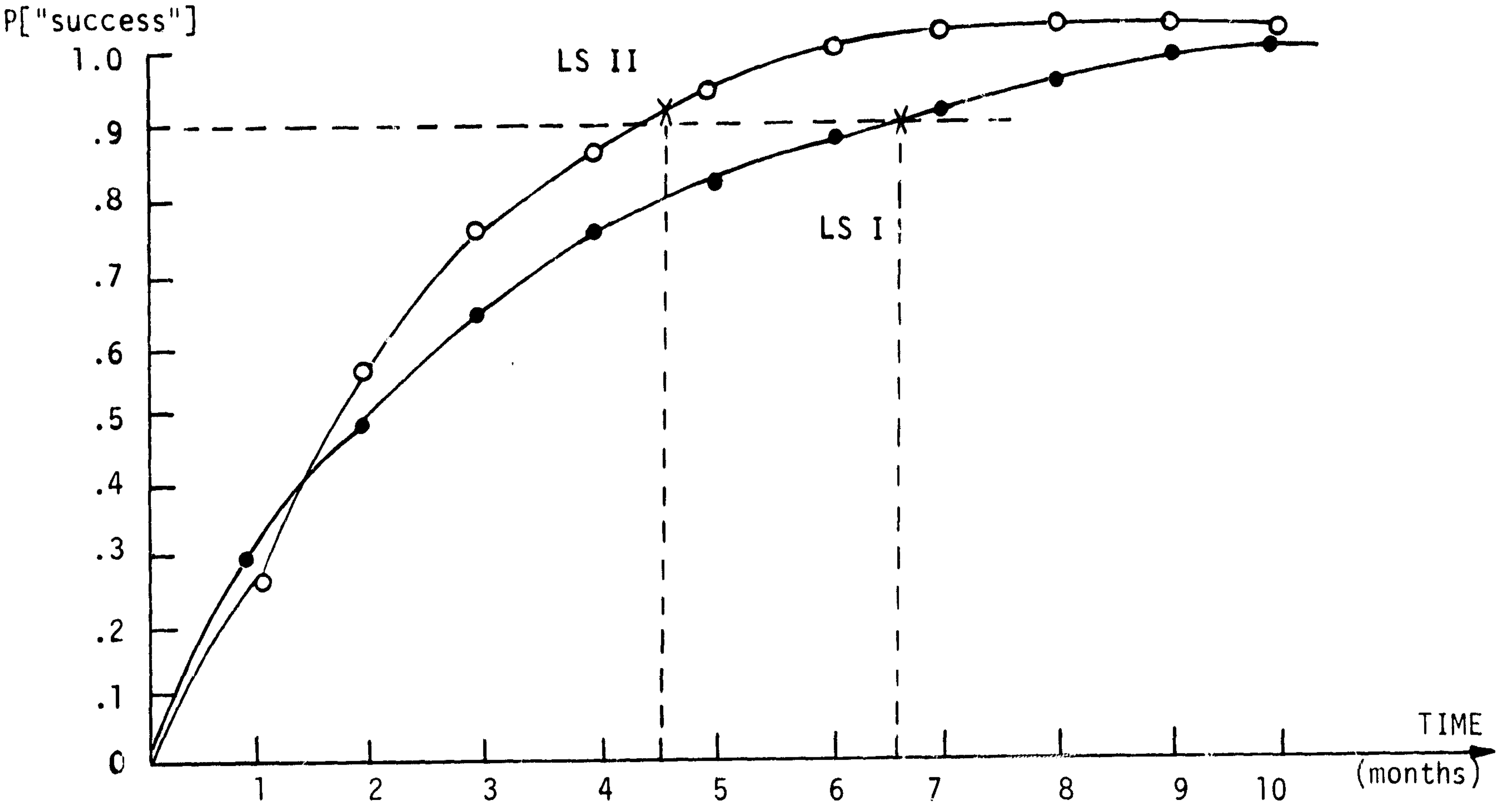
Node	Prob	Mean	Std. Dev.	Min.	Max.
45	0.4925	1470.3874	1411.4508	16.4199	10097.5268
46	0.5075	1351.6122	1405.9566	51.0907	8126.5783

** Histograms **

Node	Lower Limit	Cell Width	Frequencies									
45	200.0	200.0	21	20	24	14	17	11	14	6	8	42
46	200.0	200.0	26	26	27	24	12	10	8	10	3	57

Figure 18. GERT Simulation Output for Logical Structures I and II

simulations successfully terminated approximates the desired probability. The cumulative probability curves for the two logical structures simulated appear in Figure 19.



ex: P[success|successful termination within ore week]

$$= \frac{65}{234} = \frac{\text{\# of runs successfully completed within one week}}{\text{\# of runs successfully completed}}$$

T	P	CUM
1	.277	.277
2	.205	.482
3	.145	.627
4	.111	.738
5	.101	.839
6	.051	.890
7	1047	.937
8	.021	.958
9	.012	.970
10	.017	.987

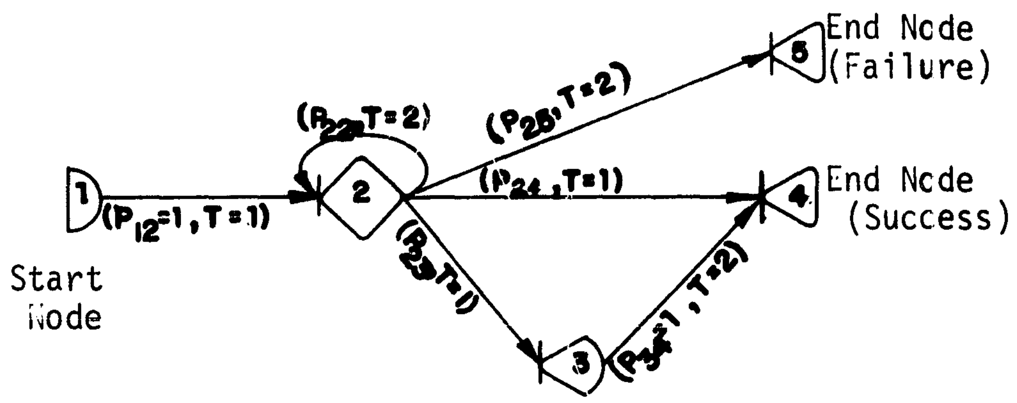
LS I

T	P	CUM
1	.275	.275
2	.290	.565
3	.165	.730
4	.115	.845
5	.100	.945
6	.040	.985
7	.015	1.000
8	0	1.000
9	0	1.000
10	0	1.000

LS II

Figure 19. Cumulative Probability Chart

The cumulative probability curves in Figure 19 give an indication of how the conditional probability of success is enhanced as a function of time. Unfortunately, this information does not describe the R&D effort sufficiently for our purposes. In addition, a knowledge of the cost of the effort must be obtained. The branches of an R&D network will represent different manning levels at the various stages of the project. This implies that the cost of completing a project in a given time interval is not a linear function of time and it is in fact a function of the path taken through the network. The following example illustrates the possibility that two paths might result in an equal termination time, yet incur distinctly different costs.



Branch	Cost/Unit Time
1-2	$10
2-2	$20
2-3	$30
2-4	$10
2-5	$35
3-4	$40

Figure 20. GERT Network with a Cost Counter Added

Consider Figure 20--if branches 1-2, 2-3, and 3-4 were successively realized, the time to success would be four time units and the cost would be \$120. If branches 1-2, 2-2 (self loop), and 2-4 were successively realized, the time to success would also be four time units, but the cost would be \$60. The GERT Simulation Program has been modified to record this variance in paths by accumulating the branch costs as the branches are realized and to print the cumulative costs in histogram form as part of the standard output. The modifications required of the GERT simulation program are listed in the Appendix. Histograms of the following three variables are obtained.

1. The cost given that the effort terminated successfully and given that the time to success was less than or equal to a pre-assigned constant, T;
2. The cost given that the effort terminated successfully and given that the time to success was greater than a pre-assigned constant T; and
3. The cost given that the effort terminated unsuccessfully irrespective of the time to failure.

Simulation of structures I and II yielded the results shown in Table 2.

From the information in these Tables, it is possible to estimate the expected cost of success given that the time to success T_s is less than or equal to T. The expected cost is computed by the program. Clearly, the expected cost associated with a particular network configuration is a function of time and this dependence is illustrated in Figure 21.

Table 2. Network Costs for T = 700

Logical Structure I

Variable	Cell Lower Limit									
	0	400	600	800	1000	1200	1400	1600	1800	2000
Cost | $T_s \le T$ & Success	1	28	29	42	24	12	5	3	0	0
Cost | $T_s > T$ & Success	0	0	0	0	11	9	16	9	8	7
Cost | Failure	6	22	17	31	27	31	26	13	8	4

Logical Structure II

Variable	Cell Lower Limit									
	0	400	600	800	1000	1200	1400	1600	1800	2000
Cost | $T_s \le T$ & Success	5	5	10	4	13	4	10	9	8	8
Cost | $T_s > T$ & Success	0	0	0	0	0	0	0	0	0	120
Cost | Failure	3	12	15	6	9	5	7	13	10	123

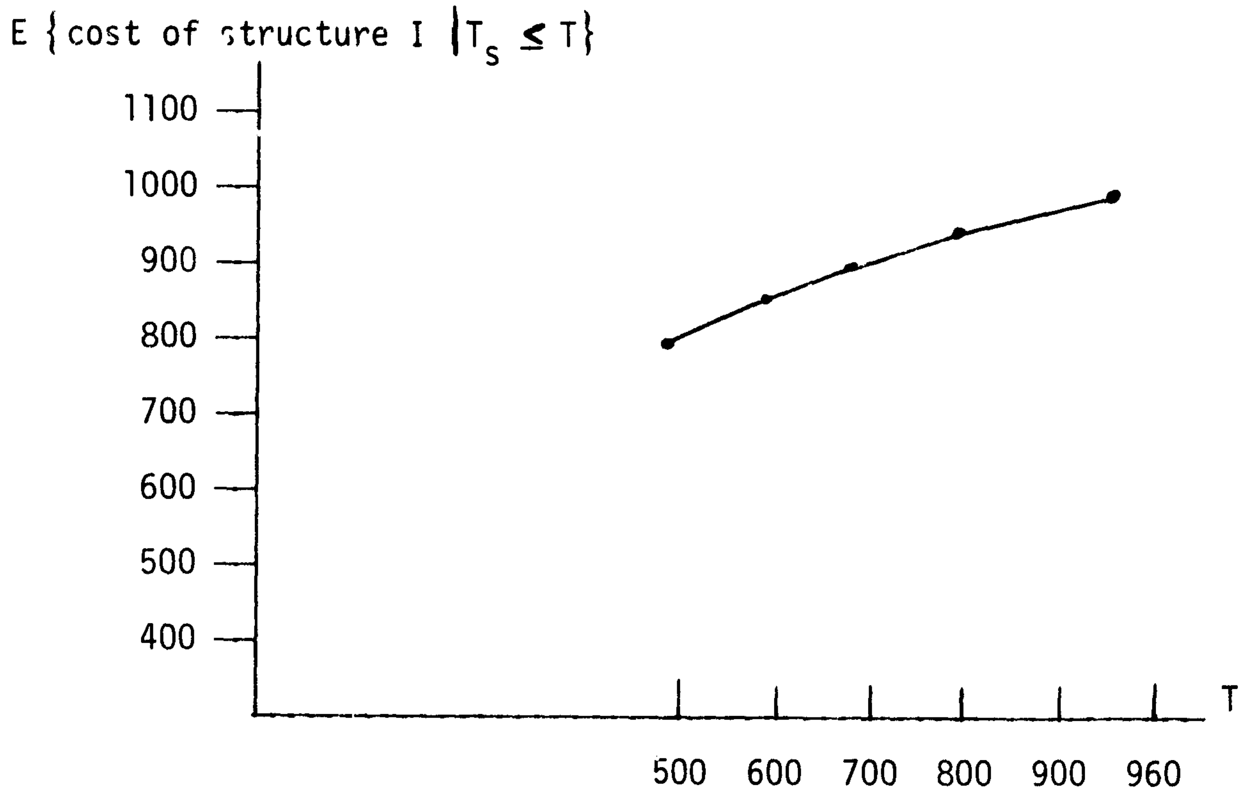


Figure 21. Network Cost as a Function of Time

The cost figures appearing in Figure 21 were obtained from the simulation of logical structure I. Each branch of the network was assigned a unit cost per unit time. Consider the effect of the way we have assigned costs upon the expected cost function. Since unit costs have been assigned, the effect of looking at increasingly long time intervals is to restrict our attention to increasingly expensive realizations of the network. The natural result is that the expected cost function appearing in Figure 21 is a monotone increasing function.

Another set of runs was made in which researcher three was assigned a cost of 10 cost units per unit time, the probability of advancing a proposed problem definition was increased from 0.5 to 0.8 and the mean and standard deviation of the distribution of the time for this activity was reduced. The probabilities associated with affirmative or negative votes remained the same as before. The effect of these changes on the expected cost function is illustrated in Figure 22.

$E \{\text{cost of structure I} | T_s \leq T \text{ and success}\}$

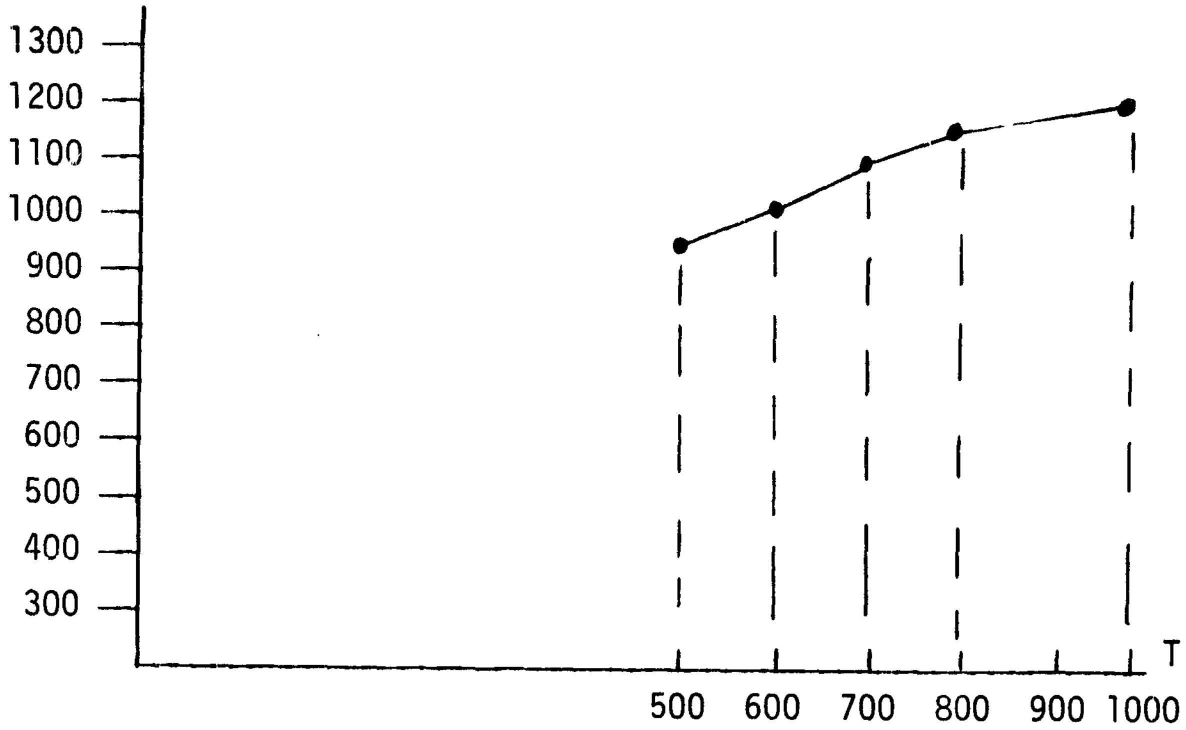


Figure 22. Network Costs as A Function of Time for Increased Probability of Success

A plot of the two cumulative probability curves before and after the modification gives an indication of what the extra dollars have purchased. The curves are shown in Figure 23 and presented in tabular form in Table 3.

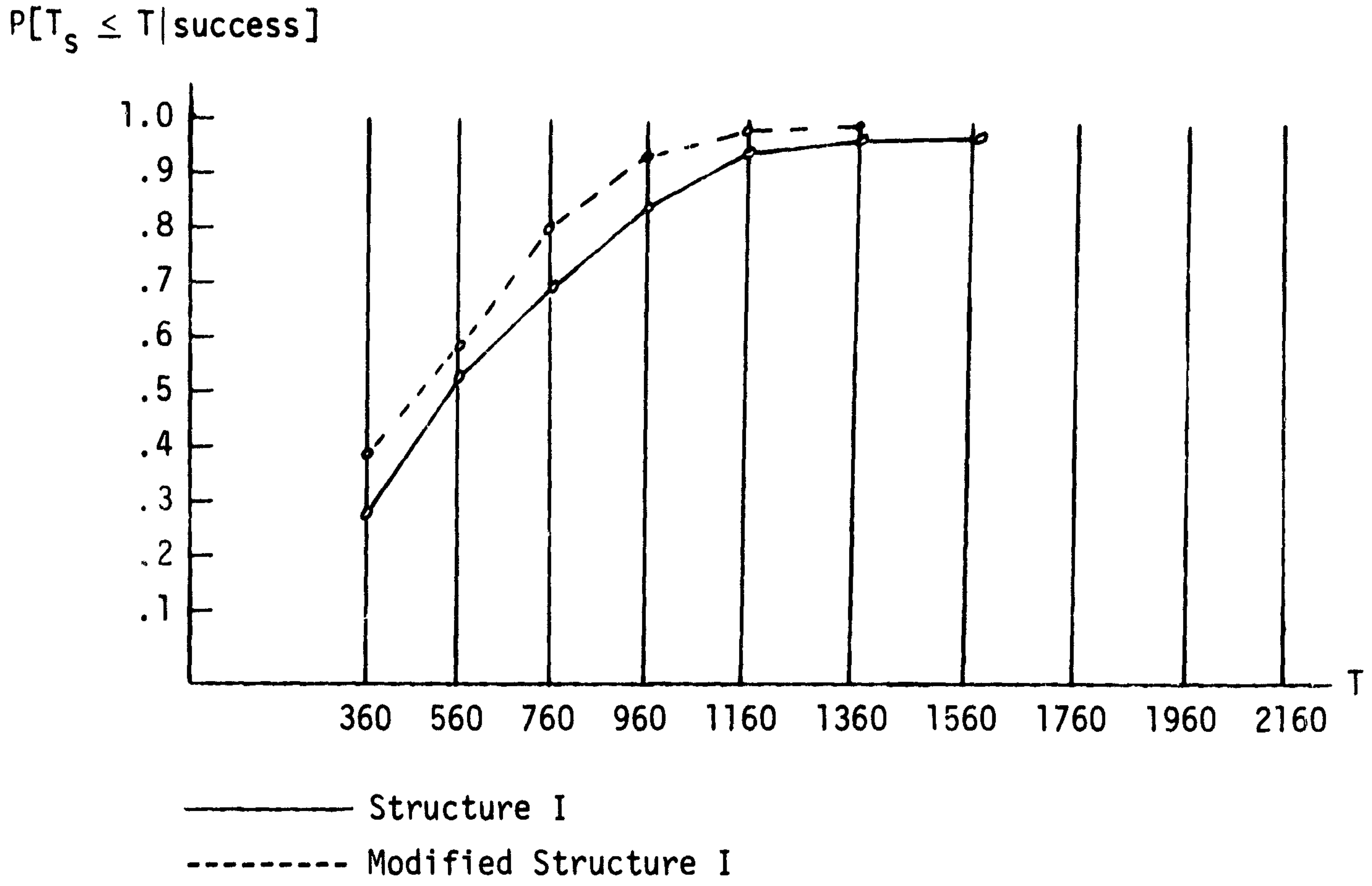


Figure 23. Cumulative Probability Curves for Modified and Un-modified Networks.

Despite knowledge of the expected cost of a project as a function of time, and a knowledge of the conditional probability of success over time, there is still no rule by which one can choose one logical structure over another. Management's attitudes towards risk must be incorporated into the decision making process. In most situations there are budgetary constraints, the violation of which is undesirable. This fact is reflected in the assumption that the manager is aware of a dollar cost amount which

Table 3. Conditional Probability Mass Functions of Successful Project Completion For Structure I and Modified Structure I.

Modified Structure I			Structure I		
Time	Probability	Cumulative Probability	Time	Probability	Cumulative Probability
[0,200)	$P[T_s<200 \vert success] = .118$	.118	[0,360)	$P[T_s \leq 360 \vert success] = .294$	.294
[200,400)	$P[200 \leq T_s <400 \vert success] = .112$	.230	[360,560)	$P[360 \leq T_s<560 \vert success] = .233$	.527
[400,600)	$P[400 \leq T_s <600 \vert success] = .135$	.365	[560,760)	$P[560 \leq T_s<760 \vert success] = .182$	.709
[600,800)	$P[600 \leq T_s <800 \vert success] = .079$	.444	[760,960)	$P[760 \leq T_s<960 \vert success] = .127$	.836
[800,1000)	$P[800 \leq T_s <1000 \vert success] = .096$	.540	[960,1160)	$P[960 \leq T_s<1160 \vert success] = .127$	.963
[1000,1200)	$P[1000 \leq T_s<1200 \vert success] = .062$	.602	[1160,1360)	$P[1160 \leq T_s<1360 \vert success] = .023$	.986
[1200,1400)	$P[1200 \leq T_s<1400 \vert success] = .079$	.681	[1360,1560)	$P[1360 \leq T_s<1560 \vert success] = .014$	1.000
[1600,1800)	$P[1400 \leq T_s<1600 \vert success] = .033$	.714	[1560,1760)	$P[1560 \leq T_s<1760 \vert success] = 0.0$	1.000
[1600,1800)	$P[1600 \leq T_s<1800 \vert success] = .049$	.763	[1760,1960)	$P[1760 \leq T_s<1960 \vert success] = 0.0$	1.000
[1800,+∞)	$P[T_s \geq 1800 \vert success] = .237$	1.000	[1960,+∞)	$P[T_s \geq 1960 \vert success] = 0.0$	1.000

represents the maximum allowable expenditure for a given time interval. Define this maximum cost as CMAX. No project whose cost for the interval exceeds CMAX will be implemented.

Concomitant with the assumption of a known budget restraint is the assumption that the manager requires a certain confidence in the successful termination of the R&D project before he will implement the project. Let PMIN be this level of aspiration with regard to the probability of success. No project whose probability of successful termination for the interval is less than PMIN will be implemented.

Define T_s to be the time to successful completion and apply our information concerning the network to the interval ($T_s \leq T$). Output from the GERT simulation of a network configuration can be displayed graphically. The expected value of the cost of the configuration given that the time to success was less than or equal to T is plotted on the abscissa of a graph. The probability that the time to success, T_s, was less than or equal to T given successful completion is plotted on the ordinate of a graph. The points CMAX and PMIN divide the area of the first quadrant into a region of acceptability and a region of unacceptability as illustrated in Figure 24.

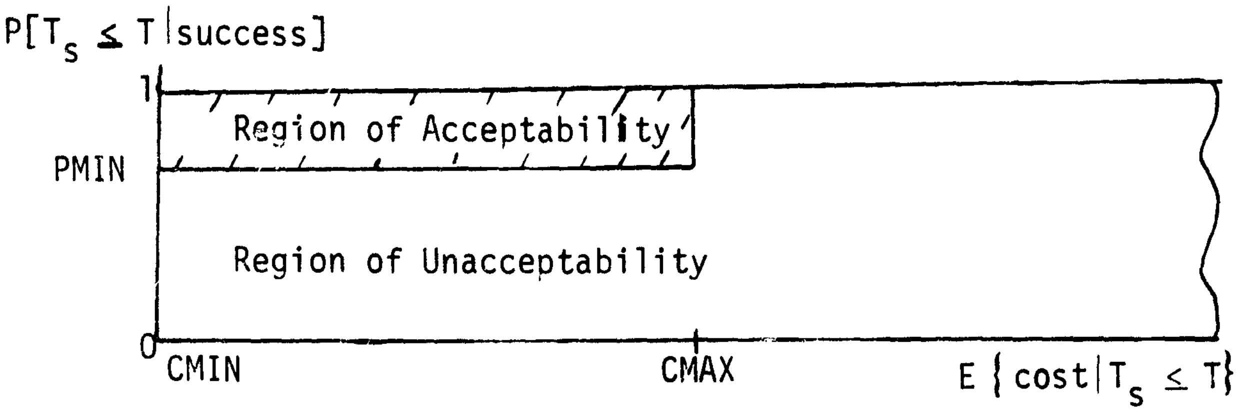


Figure 24. Decision Graph for Selection of a Network Configuration.

The region of acceptability (as it is now defined) represents a gross delineation of the manager's attitudes toward risk as it is defined only as a function of the two limits or aspiration levels CMAX and PMIN.

Suppose the manager is faced with the problem of choosing between structure I: $P[T_S \leq T | \text{success}] = .90 > \text{PMIN}$ and $E\{\text{cost} | T_S \leq T\} = \$100,000 < \text{CMAX}$ and structure II: $P[T_S \leq T | \text{success}] = .90 > \text{PMIN}$ and $E\{\text{cost} | T_S \leq T\} = \$200,000 <$ CMAX. Since the probabilities of successful termination are equal it would not be surprising if the manager choose structure I.

This simple example illustrates the possibility of further refining the manager's decision criteria with respect to choosing from various alternatives each of which lies within the region of acceptability.

The process of R&D Project selection proceeds as follows. For a key point in time, perhaps a quarterly review point, or in the case of response to an emergency, a deadline imposed from without, a value of T is chosen and the various configurations of the R&D effort are simulated. One result of the simulation of network structure I (referred to as S_i) is a point (C_i, P_i) where $C_i = E\{\text{cost of structure I} | T_{S_i} \leq T\}$ and success and $P_i = P[T_{S_i} \leq T | \text{success}]$.

Those configurations whose points fall within the region of unacceptability are discarded immediately. Suppose the points (C_i, P_i) for configurations I and II fall within the region of acceptability.

If it happens that $P_{S_i} \geq P_{S_j}$ and $C_{S_i} < C_{S_j}$ then clearly $S_i >> S_j$, where >> is read "is preferred to." Similarly, if $C_{S_i} \leq C_{S_j}$ and $P_{S_i} > P_{S_j}$, then $S_i >> S_j$. These situations are rare however, and the more common case would occur in which if $P_{S_i} \leq P_{S_j}$ then $C_{S_i} > C_{S_j}$. In this case the manager's views towards the trade-off between cost and probability must be obtained. The information can be expressed in terms of a family of indifference curves as shown below in Figure 25.

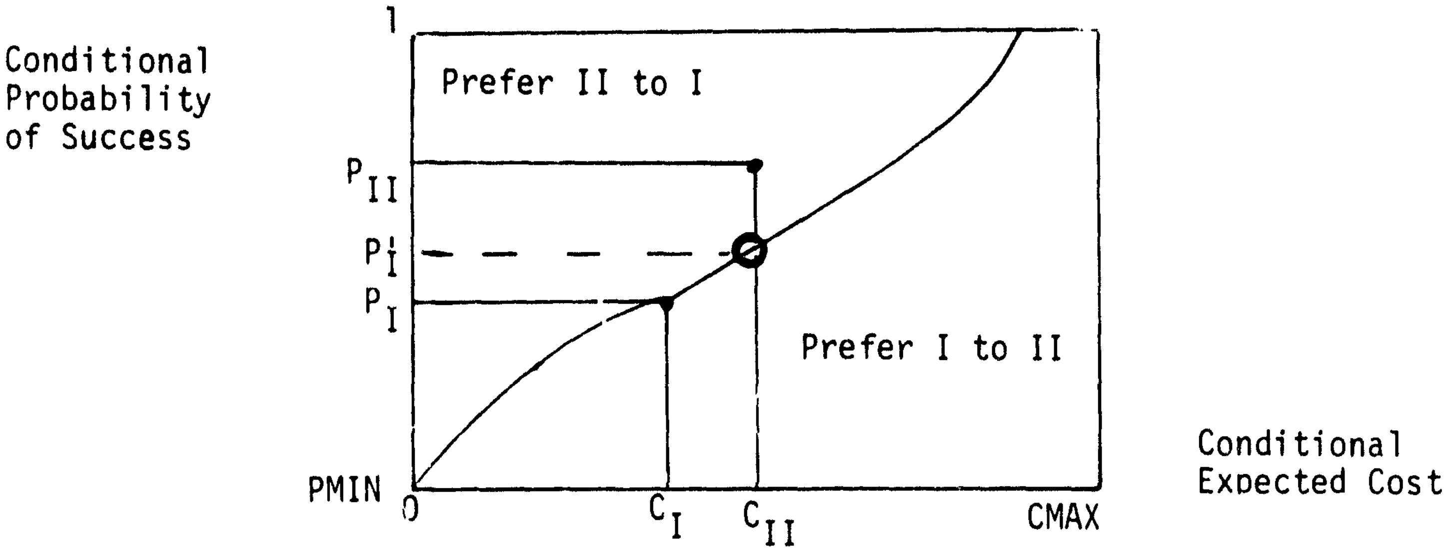


Figure 25. Cost - Probability of Success Trade-Off Decision

The curve represents the manager's cost-probability of success trade-off selected on the basis of the point (C_I, P_I). The manager faced with a choice between (C_I, P_I) and (C_{II}, P_{II}) would prefer to implement structure II as opposed to structure I since he is indifferent between (C_I, P_I) and and (C_{II}, P'_I) and (C_{II}, P_{II}) is lexicographically higher than (C_{II}, P'_I). By utilizing these various procedures it is possible for the manager to utilize the output of the GERT simulation as an aid in this decision-making process. In the next section, decisions facing management after projects have been initiated are considered.

THE GERT REPRESENTATION AS A DECISION TOOL

Recent R & D projects have illustrated that neither the cost of a project nor the probability of success are completely under the control of management. It is a natural requirement of project management that a periodic review be made of existing projects in order to assess their progress and the probable demands they will make upon corporate resources. This section is devoted to the definition of a proposed review procedure which shows promise and should be developed further to make it a useful applied technique.

The problem of review represents a departure from the material of the previous section for now instead of considering what is basically a problem of initiation, management must concern itself with the problem of continuation. Just as the problem has changed, so has the type of information available to management. Once the project is actually underway, historical information becomes available through which past estimates of probabilities and time distributions may be judged. In addition to historical information, the continuing project makes available information concerning the time rate of change of important parameters: the conditional probability of success and the expected cost. Such information is valuable to the decision maker and will be incorporated into a measure of performance.

Before defining a measure of performance, the parameters which determine system performance will be specified. These will be categorized as either subject to or not subject to the direct control of management. Management can directly affect the structure of the project, the probability associated with a branch, and the probability distribution of the time to realize the activity associated with a branch once the branch is taken. Given that the branch is taken, the a posteriori probability associated with it is 1 and once the activity is completed, the time required to complete that activity is known. Since the cost of the effort depends entirely upon the path taken through the network, and this is not directly controlled by management, the resources expended can only be bounded from above, i.e., the value of CMAX of the previous section. The context is which the review procedure is to be embedded is continually changing; moment by moment new historical information becomes available and the option to incorporate this information into a review procedure and to incorporate changes in the controllable variables is therefore continually available to management.

Assume that an absolute time scale is given which is divided into equal segments of fixed length Δt. This time scale will be used to describe the time of initiation of a given project and to schedule reviews (events will occur at the end of intervals). Assume that reviews will be conducted every $k\Delta t$ time units, $k = 1, 2, \cdots$, once the project has been initiated and that for the following development all networks are initiated at the origin of the time scale (the developement may easily be generalized to exclude the assumption). From this point on, we will concentrate on project P_i as it is represented by its corresponding GERT network N_i. In what follows, j will be used as the index to represent the current period and n some future period. Thus T_{ij} is defined as the current duration of project i, i.e., T_{ij} means that project i has been running for j segments of time.

Consider network N_i at time T_{ij} and assume that the project has not been naturally terminated (a sink node realized) in the interval $[0, j\Delta t]$. As indicated in the previous section, a standard output of the GERT Simulation Program is the E{cost of $N_i | T_s \leq T$ and success}. Let $E_n^{(j)}\{C_i\}$ = E{cost of $N_i | T_s \leq T_{in}$ and success} computed at time $j\Delta t$, where $j < n$, and $\Delta E_n^{(j)}\{C_i\} \equiv [E_{n+1}^{(j)}\{C_i\} - E_n^{(j)}\{C_i\}]$ = E{cost of $N_i | T_{in} < T_s \leq T_{i,\, n+1}$ and success}, at time $j\Delta t$. The random variable C_i is the cost of project P_i. The rate at which the expected cost for successful completion is changing is the derivative of the expected cost function evaluated at time T_{in} which is approximated by

$$\frac{\Delta E_n^{(j)}\{C_i\}}{\Delta t} .$$

Another output from the simulation of network N_i is a histogram of the time of successful terminations. This histogram was used to compute the conditional probability curves in Figure 19. Let

$P_{in}^{(j)} \equiv P\{T_s \leq T_{in} | \text{success for } N_i\}$, based on information available at time $j\Delta t$, $j < n$; then

$P_{i,\, n+1}^{(j)} \equiv P\{T_s \leq T_{i,\, n+1} | \text{success for } N_i\}$,

and define

$\Delta P_{i,\, n}^{(j)} \equiv [P_{i,\, n+1}^{(j)} - P_{i,\, n}^{(j)}]$.

The rate at which the conditional probability is changing at time $n\Delta t$ based on the projection made at time $j\Delta t$ is then approximated by

$$\frac{\Delta P_{i,\,n}^{(j)}}{\Delta t}.$$

These measures of system performance can be used by management to perform the program review function. A method for accomplishing this review function will now be explored.

Management is periodically faced with the need to make a decision as to whether or not a project should be cancelled or continued. In making this decision, two comparisons are available to management: 1) comparison to a standard and 2) comparison to other projects. When a number of R & D projects are being supported concurrently, the presence of an overall budget restraint will in general force management to review the individual projects with a view towards reducing the cost of the overall effort by cancelling the least promising of the R & D projects or reallocating the budget among the projects. Reallocation problems will be considered later.

The performance of the R & D effort can be assessed in terms of the value of cost to date, the anticipated probability of success based on progress to date, and the rate at which these are changing. It is assumed in what follows that these are the prime measures of project performance and that other measures are superimposed by management after a preliminary decision is made on these quantitative factors. The evaluation of an R & D effort then must inevitably involve management's attitude toward the cost of the effort, the probability of success, and the trade-off between these variables. In order to make this trade-off precise, consider the information available about the expected behavior of the R & D effort represented by N_i at a future time $n \cdot \Delta t$. Historical information is available concerning the actual cost to date in the C_{ik}

values, $k = 1, 2, \cdots, j$. C_{i1} was the cost of N_i at time Δt, C_{i2} was the cost of N_i at $2\Delta t$, and C_{ij} is the present cumulative cost (i.e., at time $j\Delta t$). Since the inception of the network, estimates of the conditional probability of success have been possible at time $k\Delta t$, one at $k = 0$, the second at $k = 1$, and the j^{th} at $k = j - 1$. Assume all have been made. In addition to historical information, the computed performance measures

$$\frac{\Delta E_n^{(k)} \{C_n\}}{\Delta t} \quad \text{and} \quad \frac{\Delta P_{in}^{(k)}(n)}{\Delta t} \quad \text{for } k = 1, \cdots, j \text{ and } n > k \text{ are}$$

available. A measure of progress of the R & D effort would be a combination of these above four components. The measure should be designed to reflect management's cost-probability of success trade-off and the relation of this project to other concurrent projects. A linear combination will be assumed by assigning weights to each of the performance measures.

Let W_i be a set of real numbers each of which is a member of the interval [0, 1] with the additional property that the sum of the W_i's is unity, i.e.,

$$\sum_{i=1}^{4} W_i = 1.0.$$

Define a vector of weights as $W = (W_1, W_2, W_3, W_4)$ where the components have the following definition:

W_1 = the weight management gives the conditional probability of success for period n based on calculations made in period j, $j < n$.

W_2 = the weight management gives the expected costs incurred up to period n including those spent by j.

W_3 = the weight management gives the rate at which the conditional probability of success is changing in the n^{th} interval based on calculations made in period j, j < n.

W_4 = the weight management gives the rate at which expected costs are changing in the n^{th} interval.

Define $CMAX_i$ as the maximum amount available for project i based on potential profits expected from project i. Assume there are R on-going projects. Define the overall measure by which the progress of network N_i can be judged for the future period nΔt at the present time jΔt as:

$$M_{in}^{(j)} = W_1 \frac{P_{in}^{(j)}}{\sum_{i=1}^{R} P_{in}^{(j)}} + W_2 \frac{CMAX_i - E_n^{(j)}\{C_i\}}{\sum_{i=1}^{R} (CMAX_i - E_n^{(j)}\{C_i\})}$$

$$+ W_3 \frac{\Delta P_{in}^{(j)}}{\sum_{i=1}^{R} \Delta P_{in}^{(j)}} + W_4 \frac{CMAX_i - (C_{ij} + \Delta E_n^{(j)}\{C_i\})}{\sum_{i=1}^{R} (CMAX_i - (C_{ij} + \Delta E_n^{(j)}\{C_i\}))}.$$

The particular choice of terms in this measure was reached after testing several alternatives and represents an attempt to display the performance of the R & D effort based on the four measures mentioned previously. Term 1 measures the anticipated conditional probability of success based on progress to date and is normalized by the probability of success of the competitive projects. Since the project by assumption has not terminated at time jΔt and the review is being made for future time nΔt, term 2 was developed to measure the worth of dollars expended to date. This is accomplished by computing the expected value of the additional funds required for success at time nΔt. Terms 3 and 4 measure the rate at which the cost and probability of success are changing in the interval [nΔt, (n + 1) Δt]. Each term has been normalized and favorable performance is indicated by increasing values of $M_{in}^{(j)}$. Management can control the cost-conditional probability of success trade-off by adjusting the weight W_i accordingly.

The key to understanding the importance of $M_{in}^{(j)}$ as a management tool comes with the realization that this is an objective means of obtaining data concerning the progress of an R & D project. This objectivity is a result of the ability to model the R & D project structure in GERT network form and the ability to simulate the behavior of the project through its network structure. An immediate result of this objectivity is increased confidence in management's inputs to the decision-making structure associated with capital allocation and profitability review techniques.

An example of the application of the measure $M_{in}^{(j)}$ will now be given. Consider the situation in which R = 2, i.e., there are two on-going R & D projects. The example networks for these projects are given in Figures 26 and 27. For the purposes of comparison, the same structures were used as given in Figures 15 and 17. Assume that both projects were initiated at time 0 and that for the purposes of the example, the review horizon is $10\Delta t$. Three points have been chosed for reviews, $0 \cdot \Delta t$, $3 \cdot \Delta t$, and $6 \cdot \Delta t$. At each of these points, $M_{in}^{(j)}$ will be computed for $n = j + 1, \cdots, 10$.

Since the review at $0\Delta t$ was made before the problem definition phase began, no historical information was available; at subsequent times, however, historical information would be available. In the analysis, assumptions have been made concerning the path taken through the networks. At time $3 \cdot \Delta t$, historical information from project 1 indicated that branches 2-6 and 6-14 had been realized. The probabilities associated with these

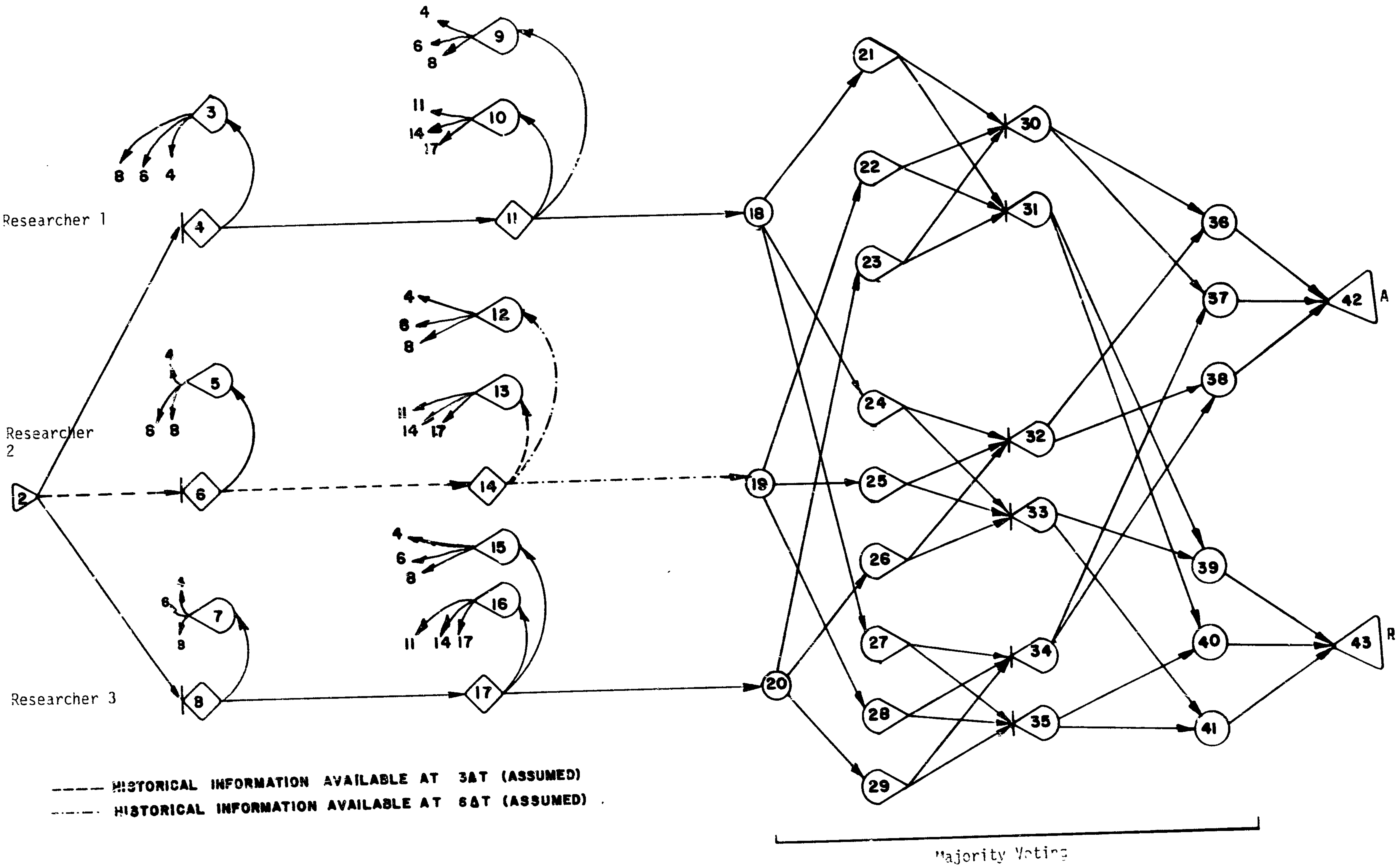
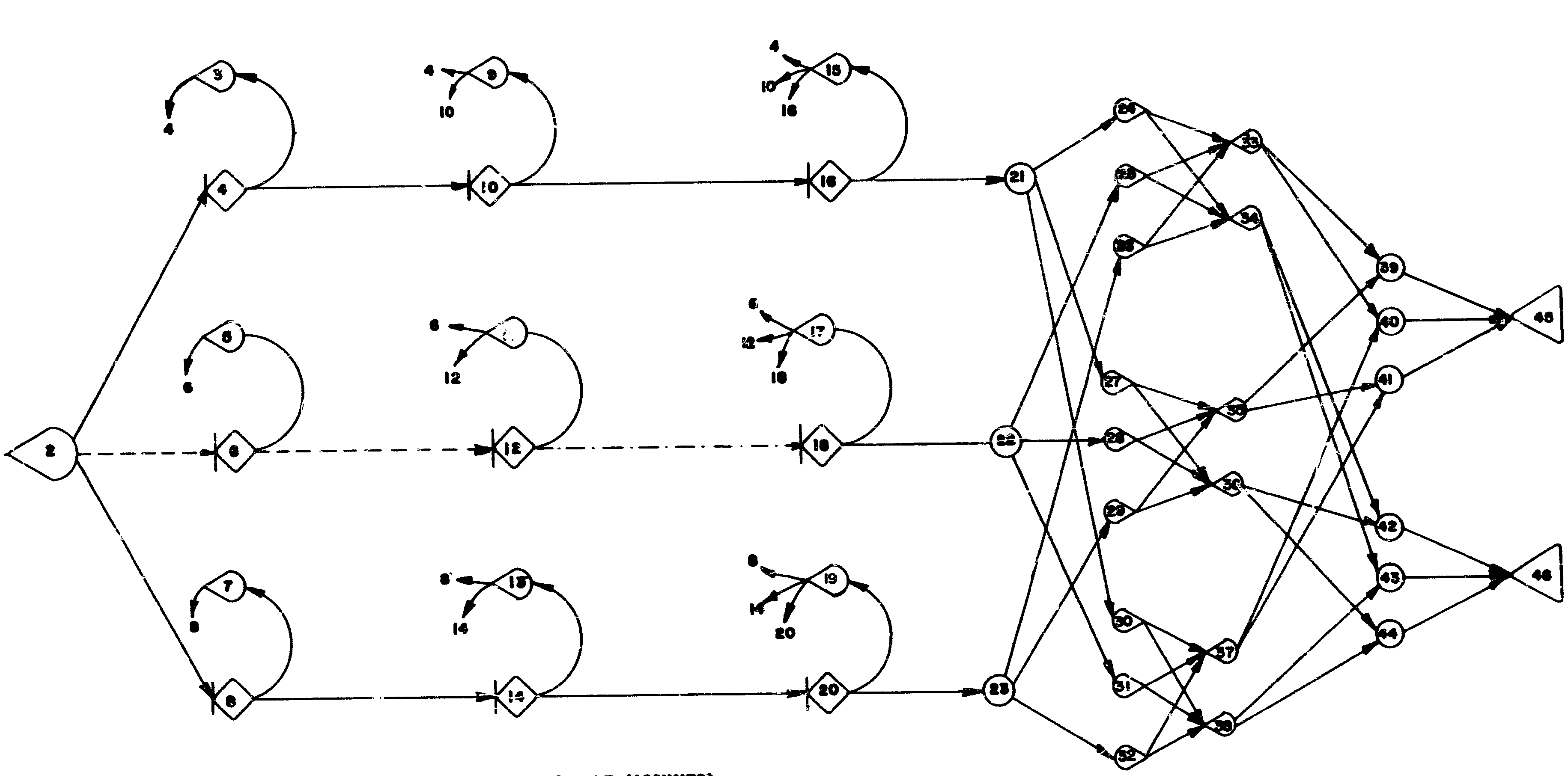


Figure 26. Example: Project I



– – – – HISTORICAL INFORMATION AVAILABLE AT 3ΔT (ASSUMED)

–·–·— HISTORICAL INFORMATION AVAILABLE AT 6ΔT (ASSUMED)

Figure 27. Example: Project II

branches were updated to 1.0. Historical information from project 2 at this time indicated that branches 2-6 (a certain event) and 6-12 had been realized. The probability associated with branch 6-12 was updated to 1.0. At this point the updated networks were simulated to provide information on which the calculation of the review measures

$$\{M_{i,\,n}^{(3)};\ i = 1, 2,\ \ n = 4, \cdots, 10\}$$

are based. At time $6 \cdot \Delta t$, historical information from project 1 indicated that the problem definition process was not advancing past the second phase and had returned to the first phase infrequently. The probability associated with branch 14-13 was set at 0.6. The probability associated with branches 14-19 and 14-12 was set at 0.2. Historical information from project 2 indicated that branch 12-18 had been realized and the probability associated with it was set at 1.0. The updated networks were simulated again in order to compute

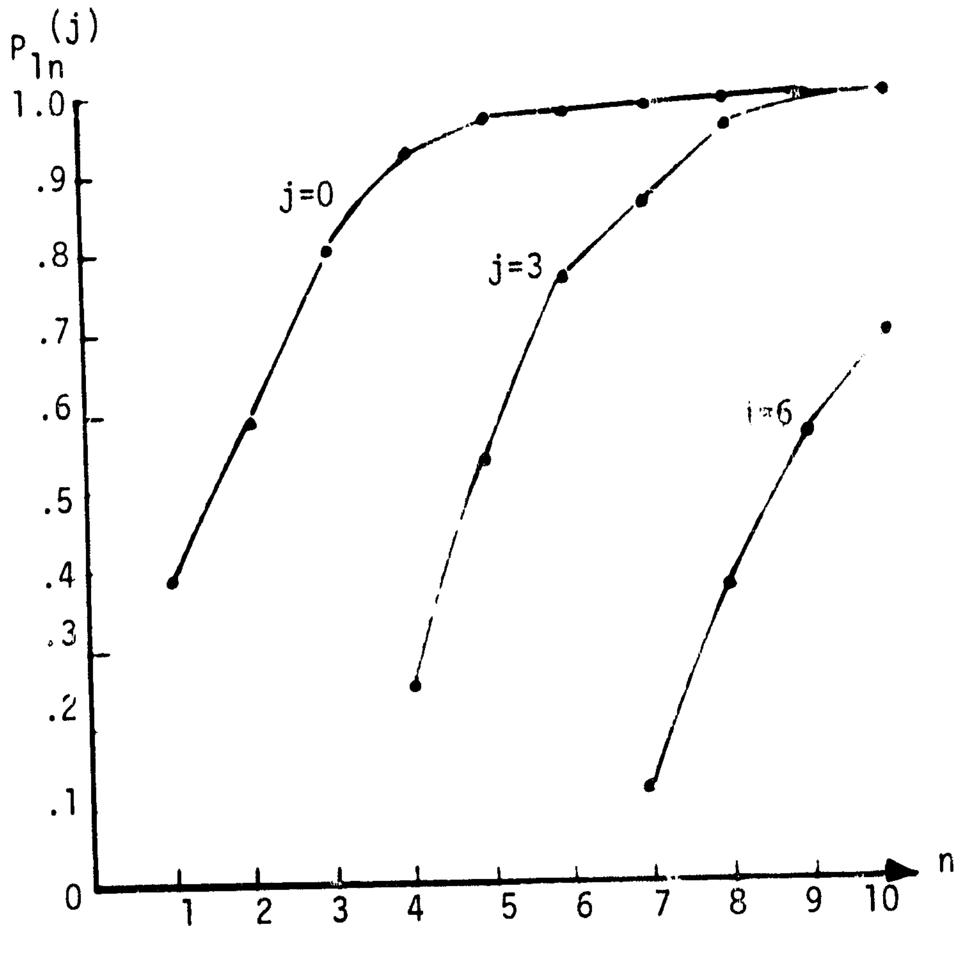
$$\{M_{in}^{(6)};\ i = 1, 2,\ \ n = 7, \cdots, 10\}.$$

Figure 28 represents the results of the simulation of networks N_1 and N_2. Notice that logical structure I is characterized by high early estimates of the probability of success which diminish as time passes; a similar situation exists for expected costs; low initial values, increasing as time passes. Logical structure II presents the opposite picture.

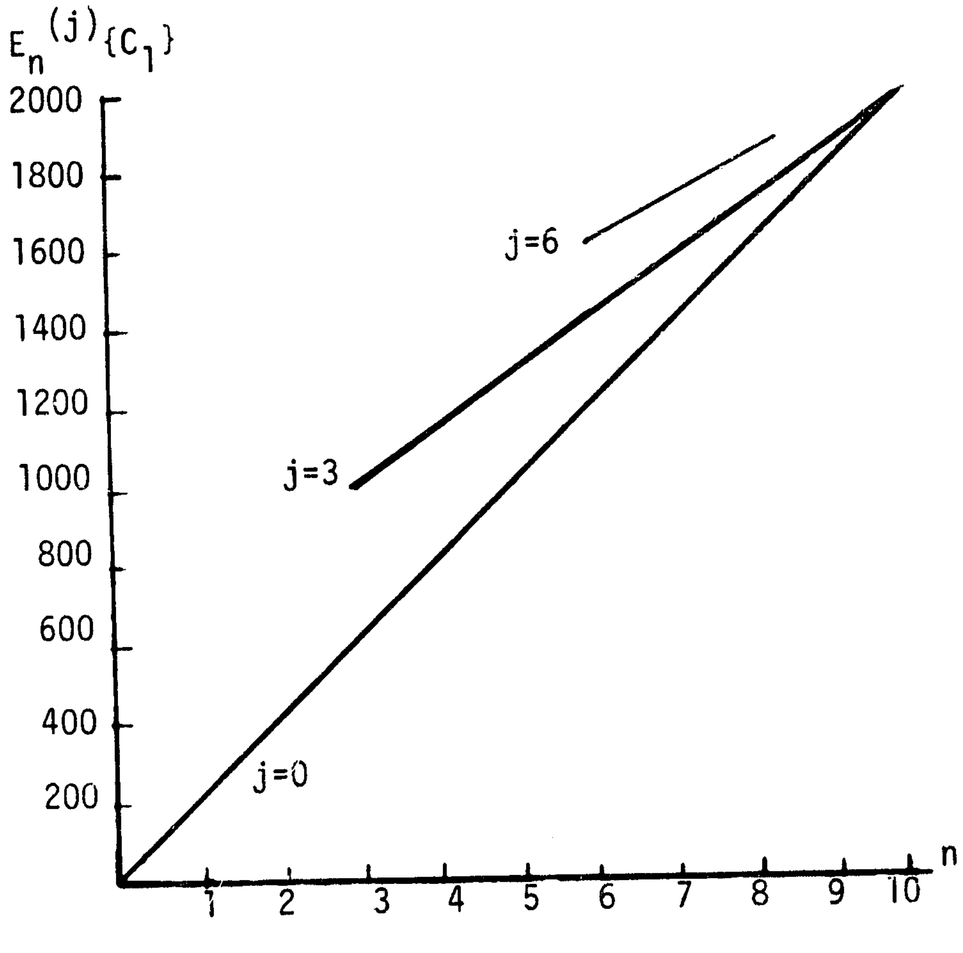
An immediate consequence of the definitions is the fact that, given the W vector, the behavior of the two on-goin R & D projects can now be reduced to a set of sequences, i.e., for j = 0, the set is

$$\{\left\langle M_{11}^{(0)}, M_{12}^{(0)}, \cdots, M_{1,10}^{(0)}\right\rangle, \left\langle M_{21}^{(0)}, M_{22}^{(0)}, \cdots, M_{2,\,10}^{(0)}\right\rangle\}$$

while for j = 3, the set is

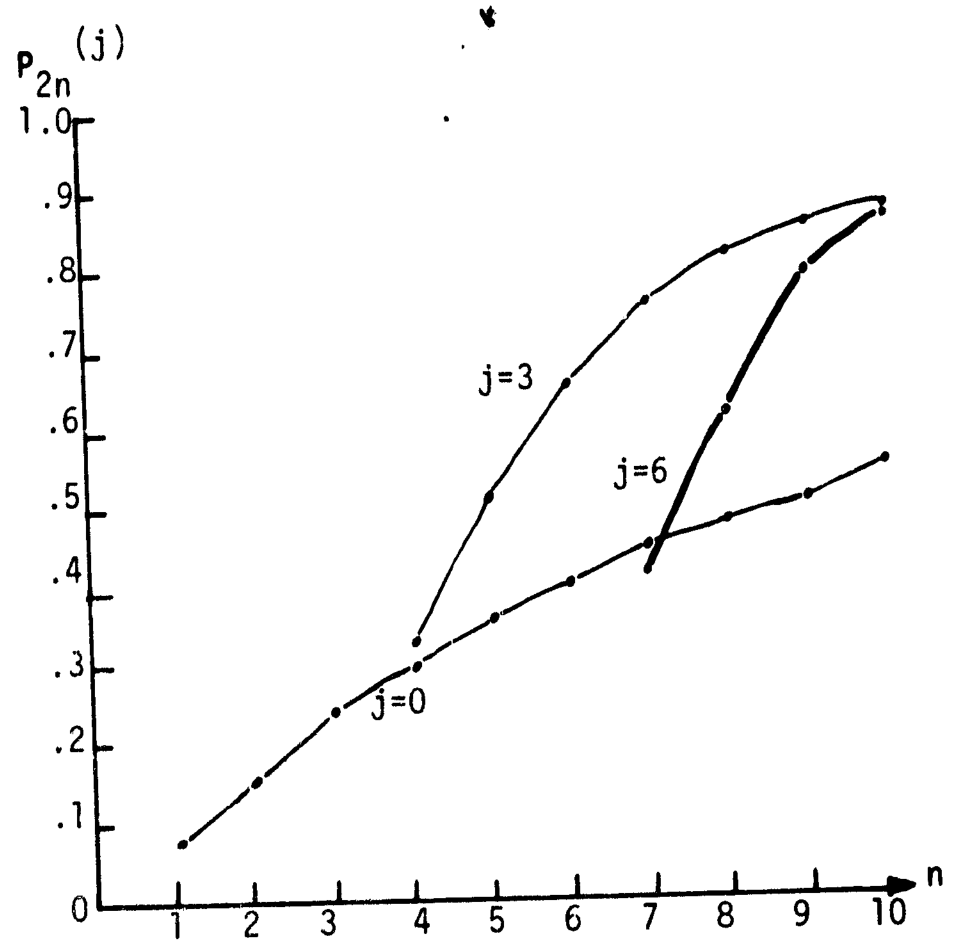


Project 1

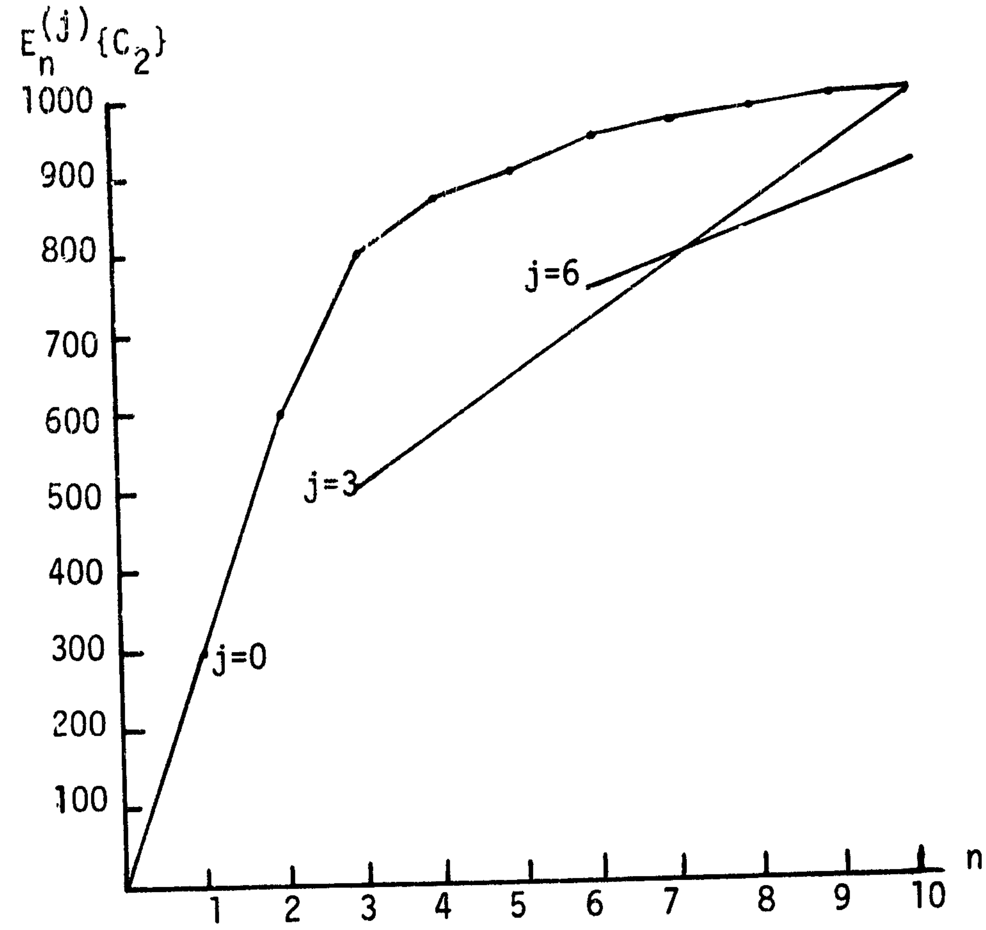


$CMAX_1 = 2100$

Figure 28. Defining Relations for Project 1 and Project 2



Project 2



$CMAX_2 = 1000$

Figure 28. Defining Relations for Project 1 and Project 2

$$\{\langle M_{14}^{(3)}, M_{15}^{(3)}, \cdots, M_{1,10}^{(3)}\rangle, \langle M_{24}^{(3)}, M_{25}^{(3)}, \cdots, M_{2,10}^{(3)}\rangle\}$$

and for j = 6, the set is

$$\{\langle M_{17}^{(6)}, M_{18}^{(6)}, M_{19}^{(6)}, M_{1,10}^{(6)}\rangle, \langle M_{27}^{(6)}, M_{28}^{(6)}, M_{29}^{(6)}, M_{2,10}^{(6)}\rangle\}.$$

Using the basic data of Figure 28, these sequences have been plotted in Figures 29 through 34 for 5 W-vectors and two values of $CMAX_2$.

Figure 29 illustrates the review measure for the case in which all four terms are equally weighted, i.e., $W_1 = W_2 = W_3 = W_4 = .25$. A superiority of performance for P_1 is indicated at review times $0\Delta t$ and $3\Delta t$ since $M_{1n}^{(0)} > M_{2n}^{(0)}$ and $M_{1n}^{(3)} > M_{2n}^{(3)}$ for all n. However, a reversal occurs when j = 6 and P_2 is preferred for part of the last interval when the review is made at $6\Delta t$.

The effect of increasing $CMAX_2$ from $1,000.00 to $1,500.00 is illustrated in Figure 30. For this case, the review at time 0 indicates that project P_2 is expected to be superior to P_1 after time $6\Delta t$. This may mean that if early success is important P_1 is superior. The interpretation and ramifications associated with this performance measure will be studied extensively during the coming year. All terms were equally weighted in the above computations. Figures 31, 32, 33, and 34 illustrate the behavior of the measure for W = (.5, .5, 0, 0), W = (0, 0, .5, .5), W = (.5, 0, .5, 0) and W = (0, .5, 0, .5) respectively. A W-vector of (.5, .5, 0, 0) considers only the expected cost and estimated probability of success, disregarding information concerning their rates of change. For W = (0, 0, .5, .5), only rates of change are considered. In choosing W = (.5, 0, .5, 0) management is purely concerned with the probability of success and the rate at which it is changing. Cost is the concern of management when W = (0, .5, 0, .5) and probability of success and the rate at which it is changing is

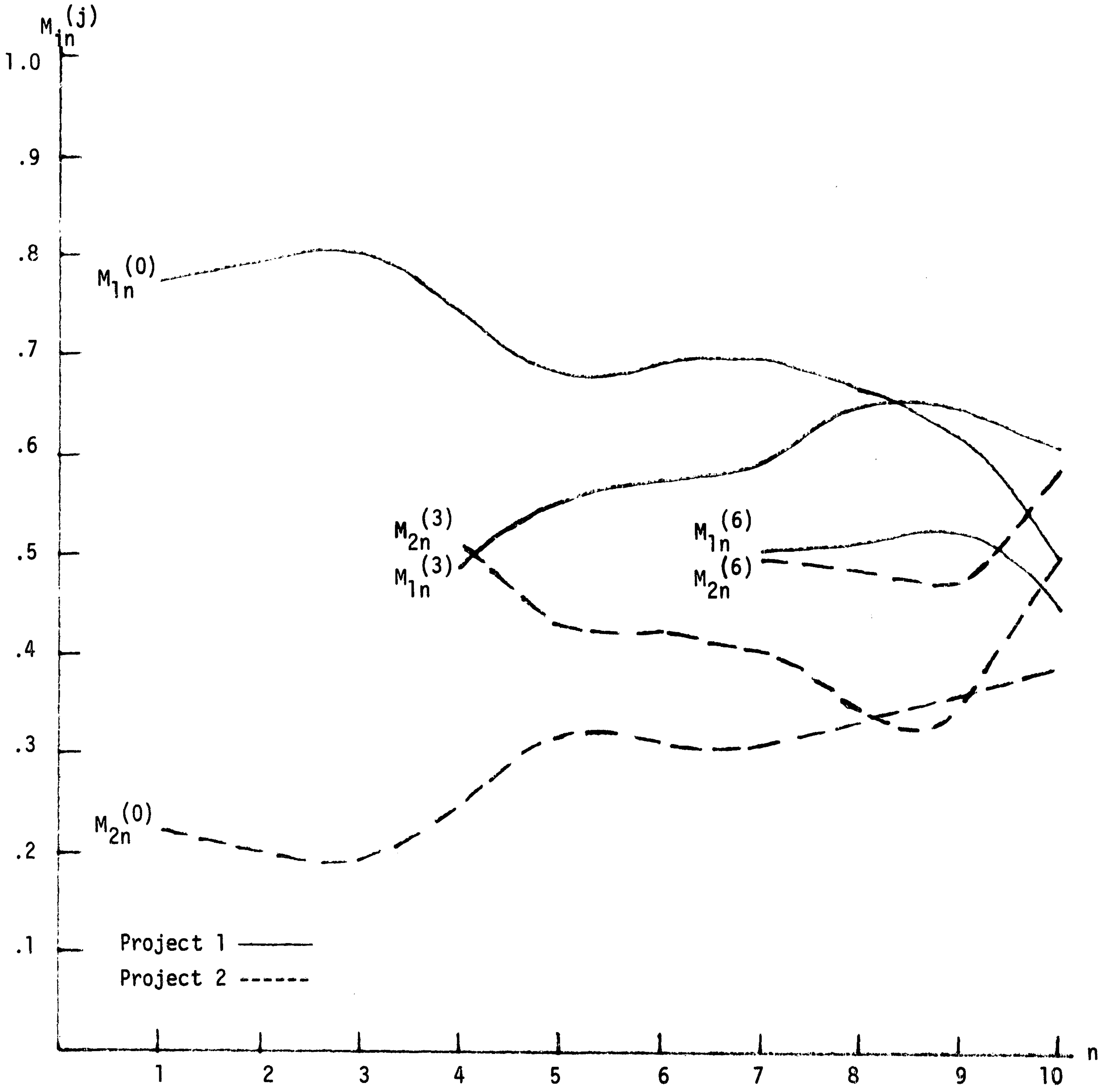


Figure 29. Review Measure for Equal Weights
w_i = .25, i - 1, •••, 4

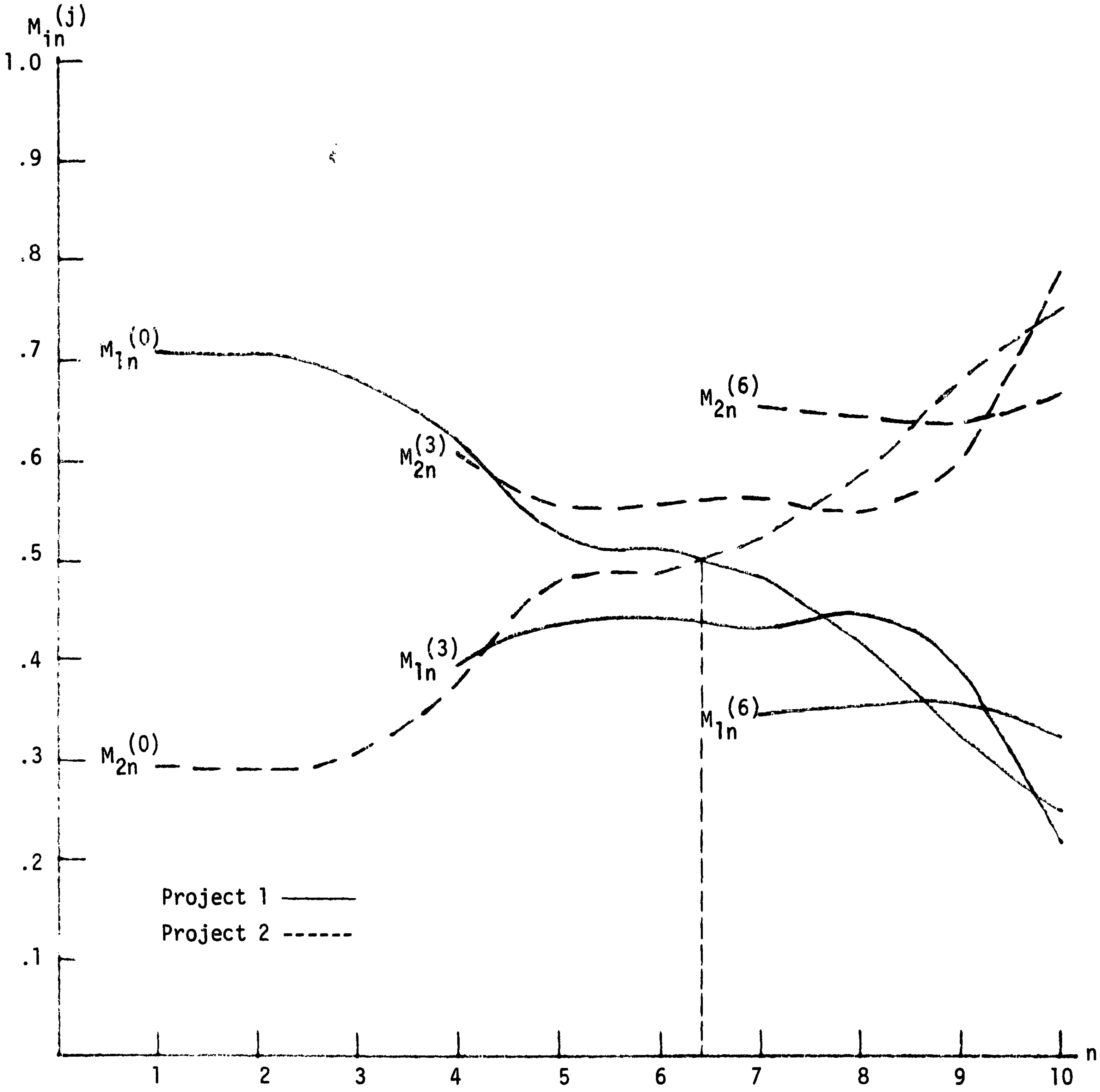


Figure 30. The Effect of Increasing $CMAX_2$ to $1500 on the Measure

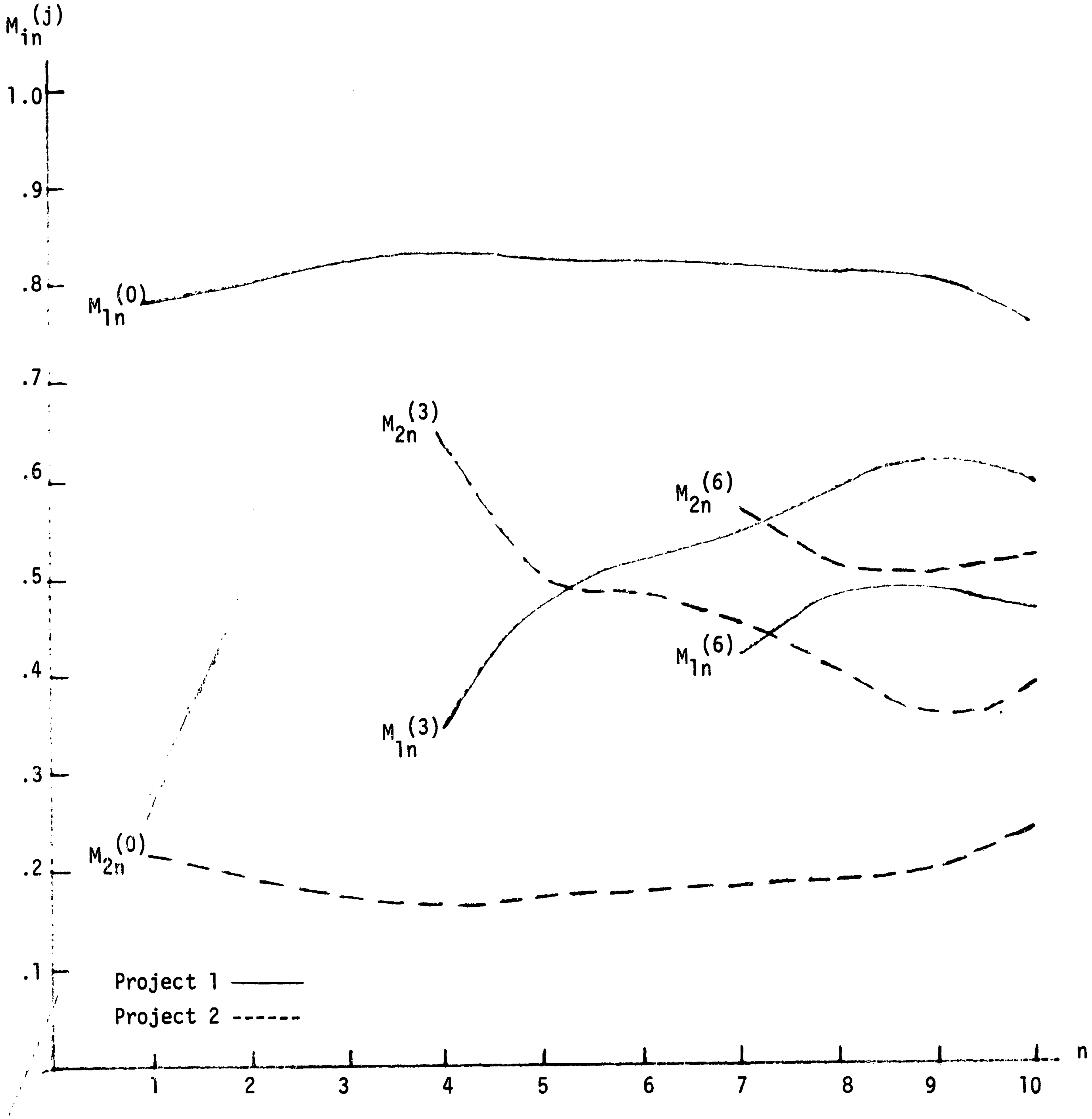


Figure 31. Review Measure for W = (.5, .5, 0, 0)

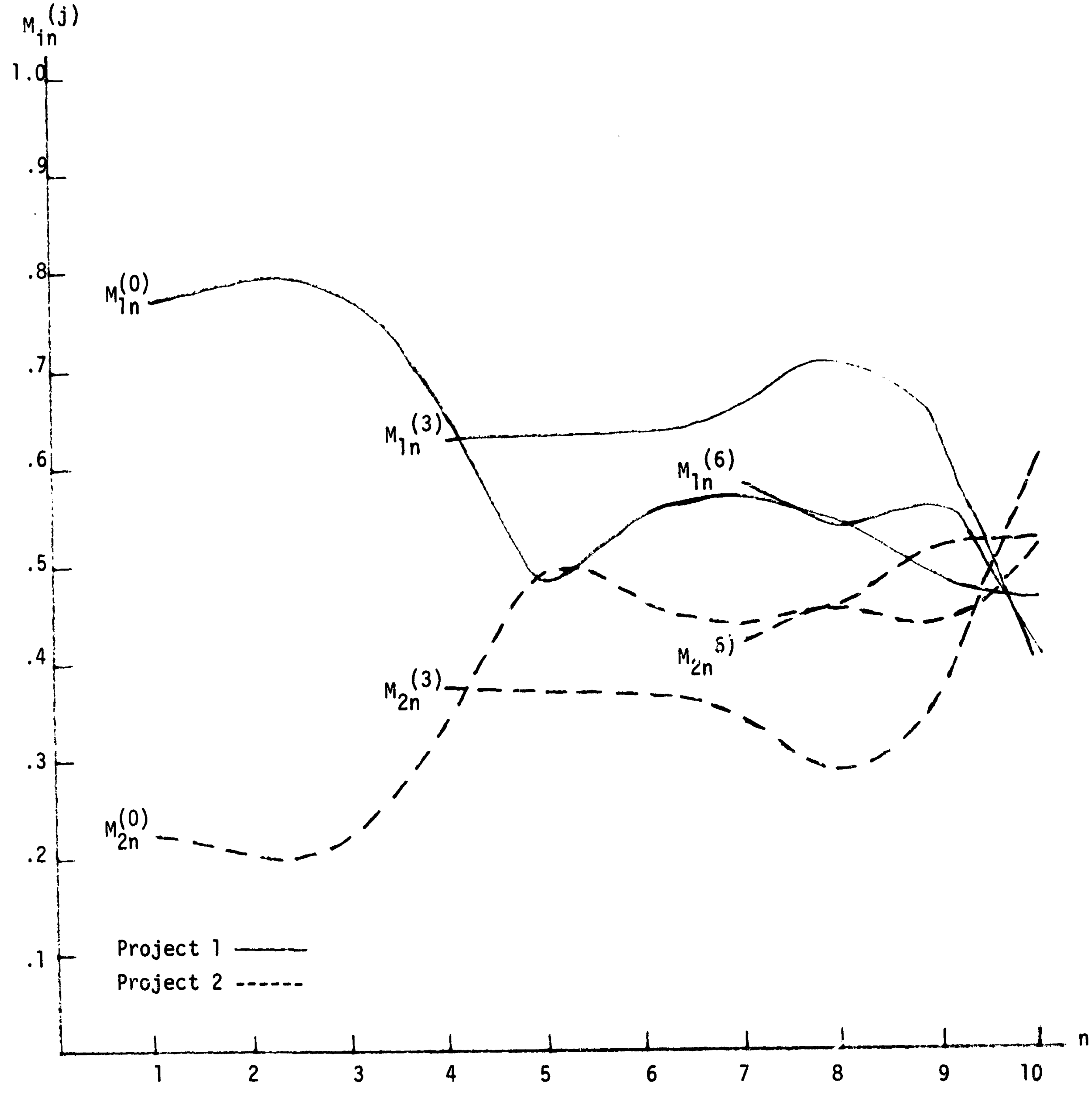


Figure 32. Review Measure for W = (0, 0, .5, .5)

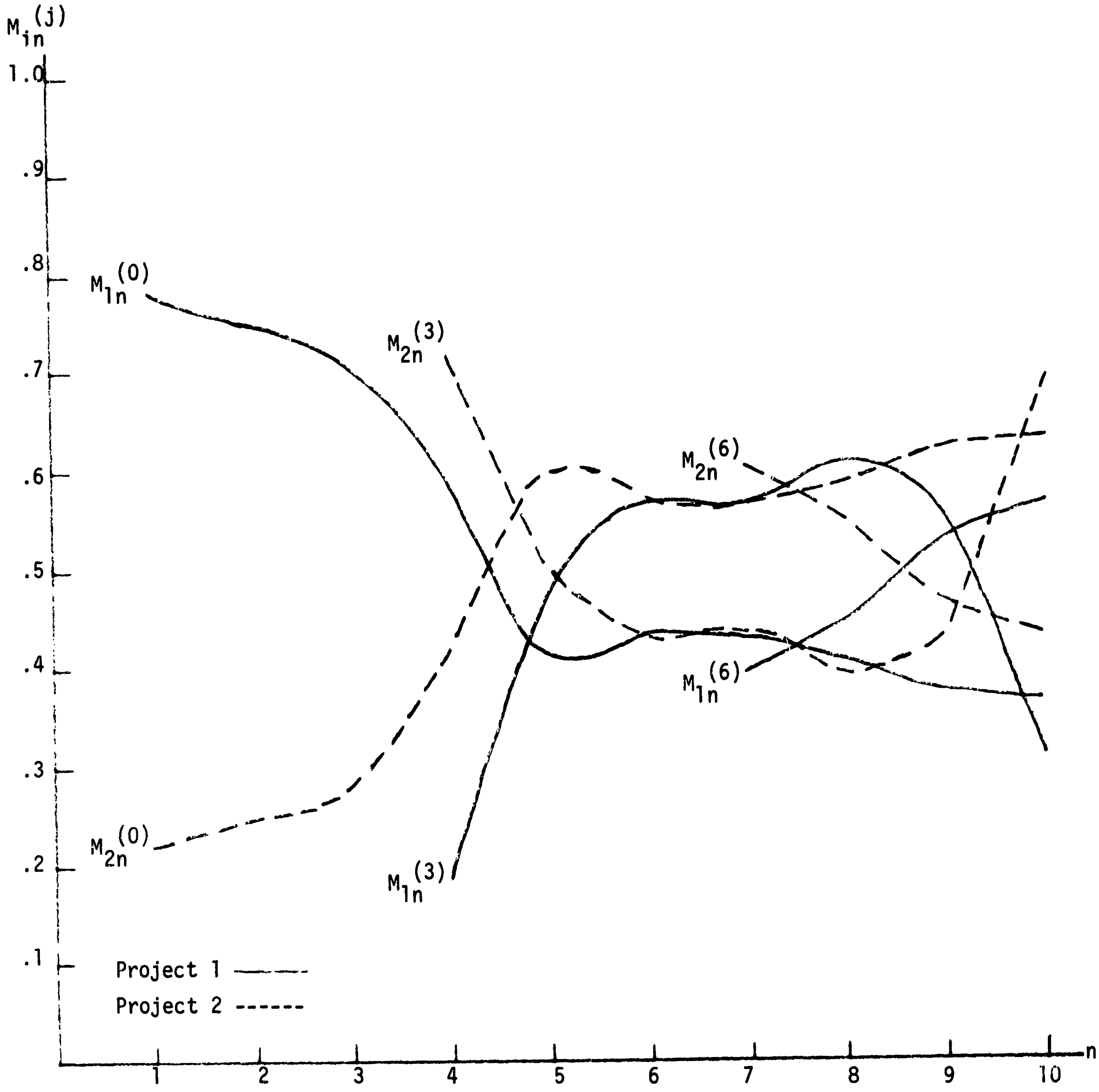


Figure 33. Review Measure for W = (.5, 0, .5, 0)

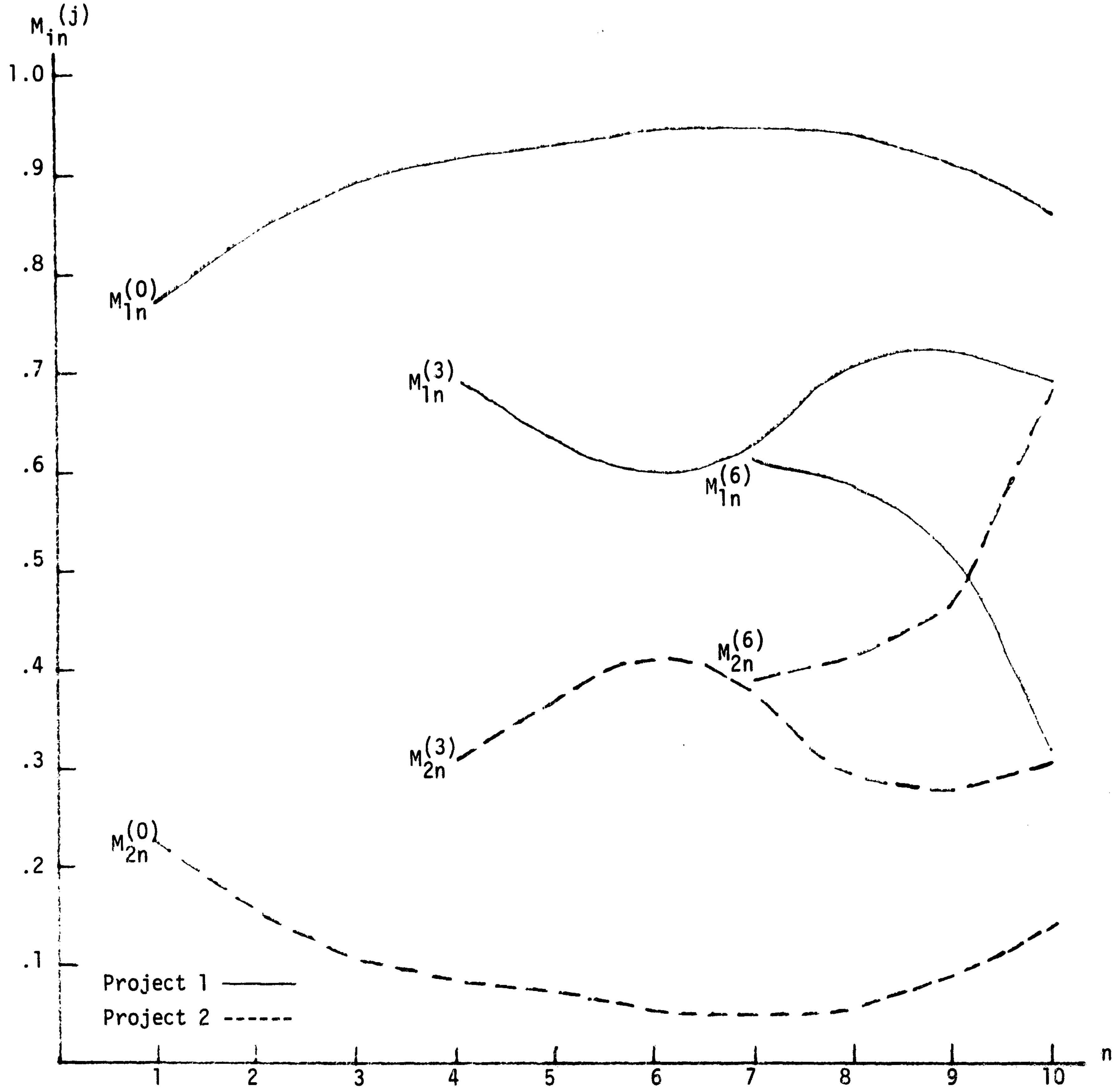


Figure 34. Review Measure for W = (0, .5, 0, .5)

disregarded. No attempt has been made to explain these figures since our current knowlegde of the performance measure is minimal. The material is presented as a basic approach for which future research should bring refinements.

Now that the response of the review measure has been illustrated, it can be applied to two problems: that of selecting projects for continuation in the presence of a budget restraint and that of reviewing the profitability of the effort. These are the methods alluded to earlier as comparison to other projects and comparison to a standard respectively.

Assume the existence of an overall budget restraint B, for the R projects, which must be met at a future time $n\Delta t$ (perhaps the end of the fiscal year) with the property that $B < \sum_{i=1}^{R} CMAX_i$. (This is a capital budgeting problem which has been treated extensively in the literature. At this point, our concern is more with the inputs than the decision-making structure.) Assume that the vector of weights is specified. The problem facing management is that of selecting from the R on-going projects at the present time $j\Delta t$ those which satisfy two requirements: they are favorable in terms of the cost-probability of success trade-off and they will satisfy the budget restraint. Initiate the selection process by computing all elements of the set

$$\{M_{in}^{(j)}\} \quad i = 1, 2, \cdots, R.$$

Rank the measures in descending order $\left\langle M_{\zeta_1 n}^{(j)}, \cdots, M_{\zeta_R n}^{(j)} \right\rangle$ where the ζ_i are members of the set $\{1, 2, \cdots, R\}$ and $\{\zeta_1, \zeta_2, \cdots, \zeta_R\}$ is a permutation of the set $\{1, 2, \cdots, R\}$. The $M_{\zeta_i n}^{(j)}$ have the property that $M_{\zeta_i n}^{(j)} \geq M_{\zeta_1 n}^{(j)}$ if $i < 1$. This ordering summarizes management's views concerning the weighted evaluation of the progress of the respective R & D projects at time $n\Delta t$ considering all information available at time $j\Delta t$. All that remains now is to estimate the cost of the respective projects for the interval $[0, n\Delta t]$ and reserve the noncomitant funds for the sequence of ranked projects until the budget restraint is reached. Those projects for which funds could not be reserved are candidates for cancellation. The expected cost of project P_i for the interval $[0, n\Delta t]$ computed at time $j\Delta t$ is of course $E_n^{(j)}\{C_i\}$. Let d be the greatest integer such that $E_n^{(j)}\{C_{\zeta_1}\} + \cdots + E_n^{(j)}\{C_{\zeta_d}\} \leq B$. In this case, d projects will be chosen for continuation, namely projects $\zeta_1, \zeta_2, \cdots, \zeta_d$ and these projects will have satisfied management's cost-probability of success trade-off as well as having high probability of satisfying the overall budget restraint. Attention can now be directed to the second problem area mentioned, that of comparing an individual project to a standard of profitability.

Initially, when a project was examined for implementation, a figure CMAX was set, relative to an interval $[0, T]$, with the property that project costs were not to exceed this figure. Implicit in this requirement is the assumption that the discounted profits to be realized from the successful termination of the project would at the time of successful termination exceed the costs of the project plus the value which could have been obtained by alternative investments. But, as has been pointed out previously, cost is a function of the path taken through the network and as time passes and costs add up, the forecast of profitability must be updated in order to avoid the possibility that the present worth of future profits can no longer exceed the discounted value of dollars already expended. Note that this eventuality does not require costs to exceed CMAX but rather allows for the potential profits to have diminished more rapidly than expected due to competition taking place in the market, inflation, overoptimistic market estimates or Congressional investigations.

The procedure for evaluating the profitability of project P_i at time $j\Delta t$ is as follows. Assume that when the justification for the initiation of project P_i was given at time $0 \cdot \Delta t$, a profitability of P(i) was hypothesized. Assume also that when the project is successfully terminated the sum P(i) is realized immediately. The network N_i is updated with all available historical information and is simulated over the interval $[j \cdot \Delta t, T(\alpha)]$ where $T(\alpha)$ is a time chosen so that $P[T_s \geq T(\alpha)] < \alpha$. A typical value for α might be .05. Define $Q = [\frac{T(\alpha)}{\Delta t}] + 1$ where [ ] is the greatest integer operator. Compute the sequence $\left\langle \Delta E_j^{(j)}\{C_i\}, \cdots, \Delta E_Q^{(j)}\{C_i\} \right\rangle$. The profitability calculation will proceed by assuming a minimum rate of return I for the research organization and by computing the future worth of expenditures relative to time Q and comparing this sum to P(i). Define the future worth of historical expenditures as $FW[\langle C'_{ik} \rangle, k = 1, \cdots, j]$ where $C'_{ik} = C_{ik} - C_{i,k-1}$ and $FW[\langle C'_{ik} \rangle, k = 1, \cdots, j] = \sum_{k=1}^{j} (C'_{ik})(1 + I)^{Q-K}$

Expenditures made after period j must also enter into the calculations but since the value of these expenditures is essentially a random variable, the expected value of their future worth must be used. Define this sum as

$$E\{FW[\Delta E_k^{(j)}\{C_i\}, k = j, j + 1, \cdots, Q]\} = \sum_{k=j}^{Q} [\Delta E_k^{(j)}\{C_i\}(1+I)^{Q-K}] \cdot \Delta P_{ik}^{(j)}.$$

The difference $P = P(i) - \{FW[C'_{ik}] + E\{FW[\Delta E_k^{(j)}\{C_i\}]\}\}$ is then computed.

If $P \leq 0$ or if $0 < P \leq \Delta E_{j+1}^{(j)}\{C_i\}$ then project P_i should be discontinued. If $P > \Delta E_{j+1}^{(j)}\{C_i\}$ then project P_i is continued for at least another time period of length Δt.

In this section, several new concepts have been presented. These have not been fully explored and represent more of a road map for continuing research than a detailed procedure for application.

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APPENDIX

APPENDIX

CHANGES MADE IN GERT SIMULATION PROGRAM

Main Program

Changes were made in the GERT Simulation Program [28,29] to obtain a histogram of the conditional time and cost information. Below only the changes made are listed. The statement numbers refer to the programs listed in Appendix B of reference 29.

Specify the node whose realization time is compared with T_S. The node number has been given the variable name NSUCS.

Specify the time T_s. This time has the variable name TSUCS.

Between GRTS 300 and GRTS 310

```
      XMM = 0.0
      NSKP = NSKS + 1
      NSKP3 = NSKS + 3
      DO 30 K = NSKP, NSKP3
   30 NSINK(K) = 0
```

GRTS 310; READ (NCRDR,15)(XLOW(K), K = 1, NSKP3)

GRTS 320; READ (NCRDR,15)(WIDTH(K),K = 1, NSKP3)

Subroutine EVNTS

Between EVNT 120 and EVNT 130

```
      JTB = JTRIB(2)
```

EVNT 240; 9 IF(NTYPE(NEND) - 2) 199,199,50

Between EVNT 250 and EVNT 260

```
  199 Y = TNR(NEND) - TNR(JTRIB(2))
      IF(Y) 198,198,197
```

```
    198 TEMP = TNR(JTRIB(2))
        GO TO 196
    197 TEMP = TNR(NEND)
    196 XMM = XMM - ATRIB(2)*(TNOW - TEMP)
        RETURN
Between EVNT 480 and EVNT 490
NNRR = NSKP
IF(NEND - NSUCS) 201,200,201
201 NNRR = NSKP + 2
200 IF(TNOW - TSUCS) 202,202,203
203 NNRR = NSKP + 1
202 CALL COLCT (XMM, NNRR, NSET,QSET)
    CALL HISTO (XMM, XLOW(NNRR), WIDTH(NNRR),NNRR)
    XMM = 0.0
```

Subroutine SCHAT

```
Between SCHAT 120 and SCAT 130
TNR(NODE) = TNOW
Between SCAT 230 and SCAT 240
INDXQ = (NEXT - 1) * IMM + 2
ATRIB(2) = QSET(INDXQ)
Between SCAT 260 and SCAT 270
XMM = XMM + DEV * ATTRIB(2)
JTRIB(2) = NODE
```

Input Changes

Data Card 2	Field 2	A node number 0 is assigned to the cost accumulation. Since three additional histograms are generated three additional dummy node #'s must appear (they need not be distinct #'s) Ex: Logical Structure II Without cost b24546 with cost b2454660b0b0 i.e. the dummy node is #0.
Data Card 5	Field 1	The lower limits for the three cost histograms must be given.
Data Card 6	Field 1	The cell width for the three cost histograms must be given.
Data Card 8	Field 2	The number of nodes on which statistics are collected must be increased by 3.
	Field 9	This field now requires a 2.
Data Card 9	Field 1	The number of cells in each histogram for the number of nodes on which statistics are collected must now include the additional three entries.