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GEOPOTENTIAL COEFFICIENT RECOVERY FROM VERY LONG ARCS OF RESONANT ORBITS

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ABSTRACT

Many very long (up to 1100 days) maneuver-free orbital arcs of 12- and 24-hour satellites exist. These orbits are extremely sensitive to low degree and order terms of the geopotential. To utilize these long orbital arcs for geodesy we integrate numerically orbits and variational equations in a mean element coordinate system. Step sizes of 1-10 days are obtained. All gravitational perturbations and drag and radiation pressure effects are included. Tests with published gravity models on these long arcs indicate that considerable improvement in low-order coefficients beyond the second degree is possible and desirable. As an example, a year's arc (in 1968 - 1969) of Minitrack elements on Intelsat 2-F1 (I = 18°, period = 12 hours) has been tested with Smithsonian Astrophysical Observatory (SAO) gravity fields reported in 1966 and 1969. Recovery of the mean anomaly data by a 1969 field is twice as poor as with the 1966 field (standard deviation of fit). On the other hand, a pair of reasonable solved-for coefficients of second and fourth order can recover this data to 0.02° which is ten times better than the recovery with the 1966 Smithsonian field. New results from combined 12- and 24-hour satellite data have also been obtained. A resonance corrected SAO 1969 field is presented which should have the most accurate set of low order coefficients calculated to date.

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GEOPOTENTIAL COEFFICIENT RECOVERY FROM VERY LONG ARCS OF RESONANT ORBITS

INTRODUCTION

It is well known that geopotential effects are greatly amplified on resonant satellite orbits.[1] We recall the reason for this is that a resonant orbit is one whose mean motion is commensurate with the earth's rotation. Over the commensurate (or synodic) period the geographic trace of the satellite repeats and any non-zero earth perturbation in that period, however small, has an opportunity to build up greatly as long as the commensurability is maintained. Such orbits have been recognized since 1960 as the natural ones to observe for the determination of the longitude dependent geopotential.[2] Yet, in spite of the fact that literally scores of these gravitationally tuned orbits exist, the strongest of these are not being incorporated into most of the recent geodetic solutions. The most nearly commensurate orbits are of the communications satellites of 12- and 24-hour period whose accelerations and constraints on the low order geopotential were first presented by the author in 1968.[3]

I think it is time geodesists look upon this high altitude resonant data as providing strength to the low order solution in much the same way as the surface gravity data is now recognized as giving strength to the field beyond about 8th order and degree.[4] At Goddard Space Flight Center, we hope to produce such geodetic solutions from combined data in the coming year. In this report I give further examples of low order geopotential recoveries from long arcs of 12- and

24-hour satellites to emphasize the power of this data to tell us something new about the earth's field. A companion report analyzes the record of well determined 24-hour satellite accelerations through the fall of 1969.[5] The analysis in this report is based not on accelerations but on the direct recovery of mean element data in very long satellite arcs with a rapid numerically integrating orbit determination program. This program, the Resonant Orbit Analysis and Determination Program (ROAD) was described at the April 1969 American Geophysical Union (A. G. U.) meeting[6] and later at the May 1969 Committee for Space Research (COSPAR) meeting[7] in Prague. A similar program for long arc orbit (but not gravity) determination was discussed at the 1969 COSPAR meeting by Foster Morrison.[8]

ANALYSIS OF LONG 12-HOUR SATELLITE ARCS FOR RESONANT GRAVITY EFFECTS

Since the May 1969 AGU meeting, analysis of 12-hour data has concentrated on two satellites with very close commensurabilities, the Cosmos 41 rocket (1964 49E) and the Intelsat 2 F1 spacecraft (1966 96A). Both of these objects appear to be in the deep resonance regime known as libration with respect to the mean longitudes of their equator crossings.[9] The period of libration of the crossings for 1964 49E is about 900 days with an amplitude of about 85 degrees; while the libration period of 1966 96A is at present about 1000 days with an amplitude of about 20 degrees. The most sensitive resonant gravity tests on 12-hour satellites to date have been made with these two objects which have quite distinct orbital elements and dynamic regimes.

Intelsat 2 F1 (1966 96A)

The more discriminating orbit has proved to be Intelsat 2 F1 whose low inclination (18 degrees), high eccentricity (0.63) and rotating perigee (period: 2-1/2 years) have combined to give a very complex resonant evolution (see Gedeon[9]) sensitive to gravity harmonics of as high as 14th order and 6th degree. This orbit has been the poorest to be recovered by the most recent comprehensive gravity fields not using data from these commensurate satellites.

Figure 1 illustrates the great divergence between two numerically integrated trajectories for Intelsat 2 F1 using two similar Smithsonian Astrophysical Observatory gravity fields[10,11] determined from low altitude satellite data. Out of a total crossing excursion of only 40 degrees (with the SAO '66 field) the divergence reaches 6-1/2° at maximum. Considering that the crossing observations I have used for this satellite (derived from the Goddard Space Flight Center's Minitrack Network) are good to about 0.01 degrees, it is clear that this orbit (looked at for at least a year) provides a powerful test object to discriminate between recent geodetic solutions.

In fact I have made many such determinations of this orbit for the first year since Goddard began tracking it intensively in the summar of 1968. The observations I use, as reported previously[6,7], are short arc (typically one week) mean Kepler elements. The reference orbits and variant trajectories for partials (including the geopotential) are computed numerically from the Lagrange planetary equations retaining only forces giving secular or long period effects.

This enables the trajectories to be computed at more than 100 times the speed of conventional Cowell orbit integration. Included in the integration are forces due to atmospheric drag, sun and moon gravity and the sun's radiation pressure (with the earth's shadow accounted for) as well as all relevant geopotential forces. Observations are compared to computed values and the effects on these residuals of variations in the initial Kepler elements and the geopotential are computed (at present) by differencing variant trajectories.

A new development of ROAD in progress is to calculate the partials from numerically integrated variation equations. This promises a calculating speed gain of about ten when a large number of geopotential coefficients are to be estimated.

The partials and residuals are accumulated into normal equations in the usual way for a least squares process.[1] Corrections to the reference trajectory parameters (including gravity coefficients) are then found by inverting the normal equations. Table 1 gives the results of a number of such orbit determinations for Intelsat 2 F1 for a one year data span in 1968-1969 with fixed fields and a solution for a simple low order resonant field. The only data actually fit to in these examples are the mean anomalies of 33 short arc orbit updates. The other elements are generally found to be well recovered when the resonance effects are removed from the mean anomaly. The first four orbit determinations in Table 1 were with recent Smithsonian Astrophysical Observatory fields with only the initial semimajor axis varied to fit the mean anomaly data. They show a clear superiority of the 1966 M1 and 1969 B13.1 fields[13] over the 1969 COSPAR and B6.1 models[12] in recovering this very sensitive resonant data. Because of the complex regime it is difficult to say precisely to what level in the geopotential this resonant data is sensitive. However, if there is a reasonable correspondence of geopotential acceleration to the amplitude of longitude oscillation, then since the amplitude of the geopotential caused oscillation (in mean anomaly) is about 40° (over a 2-1/2 year period), terms with accelerations down to 0.001 of the maximum should be sensitive to this data. A calculation has shown that terms of coefficients of degree (m) 2, 4 and 6 and order (ℓ) 2 through 14 might cause perturbations which the data could discriminate (over a full libration period). Over this one year period, it is felt that most of the discrimination between the gravity fields occurs in the harmonics of order and degree less than or equal to about 6, 6.

The final solution in Table 1 illustrates what appears to be the ultimate recovery possible for the mean anomaly data in this arc. Except for the harmonic 4,4 (which is evidently absorbing in a biased way the effects of neglected resonant harmonics of higher order and degree), the actual solution is quite reasonable based on the recent non-resonant fields. The great improvement in the residuals in this solution, with a small change in the field, illustrates dramatically how sensitive this deeply resonant data is in distinguishing gravity effects. From the trajectories in Figure 1 I would expect much greater differences to show up from the tracking record through 1970.

Cosmos 41 Rocket (1964 49E)

The longest single arc record handled by ROAD is that of 1964 49E between MJD 39157 and 40245, a few days short of 3 years. In fact, the orbit and resonant field determined through the scattered record during this period is a convincing demonstration that this is indeed one free drift object (Figure 2). I had earlier assumed that 1964 49E consisted of a number of arcs of a maneuvered object or a number of objects in distinct orbits.

Figure 2 gives the history of the mean motion (n) of this object (as reported by the North American Air Defense Command — NORAD) during this period and for about 200 days afterward. The solid curve is from the ROAD determined trajectory for the object. After MJD 40245 this is a prediction curve, and as can be seen the ROAD predictions for 1964 49E are turning out to be quite satisfactory even 7 months after the last (ROAD) orbit update.

The ROAD orbit for 1964 49E (in Figure 2) was determined from the mean anomaly data principally. The rms (root mean square) residual for mean anomaly for the determined arc, with its associated gravity constants, is given in line 3 of Table 2. This table gives the rms results of similar orbits determined through this data for recent Smithsonian gravity fields. For the case of these fixed fields, only the initial orbit elements were adjusted to the NORAD data in the arc (as in the fixed field orbits for Intelsat 2 F1).

It is evident from Table 2 that the SAO '69 B13.1 field is a significant improvement over the older M1 field for this very sensitive resonant orbit.

However, other "data fitted" and "realistic" resonant solutions for this arc, including higher order gravity and a better estimation of the radiation pressure effects, have reduced the rms residual in M (mean anomaly) to less than 0.2 degrees. This is a level of accuracy compatible with the NORAD data on other 12-hour satellites.[6]

The general conclusion to be drawn from these tests is the same as from the Intelsat 2 F1 tests: a small, "realistic," change of the gravity field has great leverage in recovering longitude information from deeply resonant satellites.

ANALYSIS OF A LONG 24-HOUR SATELLITE ARC FOR RESONANT GRAVITY EFFECTS

The most dramatic evidence of the leverage of deeply resonant orbits to discriminate between the recent SAO fields is provided by a 675 day arc of the 24-hour, 31-degree inclined orbit of Syncom 2 (1963 31A) in 1965-1968. This arc covers almost a full libration period for the equator crossing of Syncom 2 between 66° East and 86° East. The tracking data for this period consists of 30 sets of short arc Kepler elements determined by the Air Force Systems Command (Sunnyvale. California) from range and range rate observations. Since Syncom 2 has a nearly circular orbit, the mean anomaly alone is not well determined but the combination ω + M, (analogous to the argument of the node) <u>is</u>. Actually I have used the longitude combination ω (argument of perigee) + M + Ω (ascension of the ascending node) - θ_g (hour angle of Greenwich) to fit to. This is the geographic mean longitude of the equator crossing (λ) which is the most convenient single physical reference for the "stationary" orbit as well as being the most significant dynamic variable in deep resonance.[9]

Table 3 shows the results of four orbits determined by ROAD through this long arc of Syncom 2 data, three with fixed SAO fields and one with a 10 parameter resonance field determined with 12 additional long arcs of 24-hour satellites.

Considering the SAO fields, these tests clearly show the SAO '69 COSPAR model to be superior to the others for this critical arc. No other 24-hour satellite arc is nearly as sensitive to the differences in these fields as this one. The interesting feature of this result is that only very small changes in the SAO models 2,2 and 3,3 coefficients give large changes in the quality of fit to this data. Furthermore, the SAO '69 models use substantially the same satellite information so that, as a set, there is probably little to choose between these models as far as their used satellite information is concerned. The fact that the particular 'COSPAR' set has the "correct" set of 2,2 and 3,3 coefficients to fit this critical arc may be coincidental. Nevertheless there is a clear implication in these tests (as well as the previous ones given here and others presented elsewhere[3]) that the SAO data is quite compatible with this data and that a combined data solution would give a significant improvement to the low order field coefficients.

COMBINED DATA SOLUTION

In fact I have just completed such a preliminary combined data (SAO + 24-hour satellites + 12-hour satellites) solutions which I think results in the strongest set of low order and degree coefficients yet available. The SAO data on 24 low altitude satellites and surface gravity anomalies is incorporated as the starting field (SAO '69 B13.1) of which all coefficients are held fixed in subsequent iterations except four pairs of low order coefficients 2,2 3,2 3,3 and 4.4 which have the strongest observable effects on the 12- and 24-hour satellites. The thirteen 12- and 24-hour satellite arcs used in this solution are (I = inclination, e = eccentricity):

I°	e	24-Hour Satellite Arcs	
0	0	E.B. 1 [Early Bird (1965 28A):	Apr.-Dec. 1965]
0	0	SYN. 3,6 [Syncom 3 (1964 47A):	Oct.-Jan. 1964-1965]
32	0	SYN. 2,5 [Syncom 2 (1963 31A):	July-Feb. 1964-1965]
33	0	SYN. 2,1 [Syncom 2:	Aug.-Dec. 1963]
33	0	SYN. 2,4 [Syncom 2:	Apr.-July 1964]
30	0	SYN. 2 DoD [Syncom 2:	Mar.-Jan. 1965-1968]
0	0	INT. 2 F3 [Intelsat 2 F3 (1967 26A):	May-Feb. 1967-1968]
0	0	SYN. 3,11 [Syncom 3:	Mar.-Oct. 1965]
		12-Hour Satellite Arcs	
68	.70	COSMOS 41R [Cosmos 41 Rocket (1964 49E):	1966-1969]
65	.72	MOLNIYA 1 [Molniya 1 (1965 30A):	1965-1966]
66	.73	COSMOS 41 [Cosmos 41 (1964 49D):	1965]
18	.64	INT 2 F1 [Intelsat 2 F1 (1966 96A):	July-Mar. 1968-1969]

The ROAD results of individual and combined arc reductions of the data for these satellites are given in Table 4. The individual arc rms's (in λ for the 24-hour arcs and M for the 12-hour arcs) are from best fits, principally to λ and M, adjusting for only a few low degree resonant gravity constants. The Table 4 values are the actual best fit values rounded up by 0.005°. For the 24-hour arcs, the fit is to λ. For the 12-hour arcs, the fit is to M.

These individual arc results were used as weights in the combined arc solutions whose residuals are given in the last two columns, one for a solution with the B13.1 field fixed and the last with the low order gravity corrections to the harmonics 2,2 3,2 3,3 and 4.4.

It is worth noting that except for the Intelsat 2 F3, Syncom 2, DoD and Cosmos 41R arcs, SAO '69 B13.1 appears to recover the longitude position of these satellites with reasonable accuracy considering data from these very sensitive resonant orbits were not used in the B13.1 solution.

In Table 5 is listed the weighted rms residuals and a field comparison between these two combined data solutions. Again the main points of the B13.1 corrected solution in Table 5 are that:

1. It is a small correction to B13.1.
2. It is a small correction which significantly reduces the weighted residuals, and
3. It is a correction which appears to be compatible with other recent SAO models.

To emphasize the closeness, and apparent compatibility of these recent SAO models with the resonance corrected model reported in Table 5, I have plotted the C, S values (of 2,2 3,3 3,2 and 4,4) for them in Figure 3.

DISCUSSION

A few words should be said about the nature of the Smithsonian gravity models tested in this study. The SAO '66 M1 field was determined from optical tracking data on 16 satellites, is complete to 8,8 and with selected high order resonance coefficients contains a total of 108 gravity parameters. The 1969 Smithsonian fields all are complete to at least 14,14 and contain, in addition to the 16 satellites in the 1966 field, 8 new satellites some of which were tracked with laser and range and range rate equipment. These gravity fields all have more than 200 parameters. The COSPAR and B6.1 are pure satellite data fields, while the B13.1 contains surface gravity anomaly data as well.

Therefore, it should be expected that the B13.1 field will have stronger information than COSPAR and B6.1 in the medium and high order coefficients poorly determined by satellite data alone. According to Strange et al.,[14] these are (roughly) the coefficients ℓ, m where $\ell > 6$, and $2 < m < 11$. Many of these terms (i.e., 7,4 8,4 9,4 . . . 7,6 8,6 9,6 . . . etc.) have significant effects on the 12-hour satellites in deep resonance. On the other hand, the inclusion of ground data may be expected to degrade the low order coefficients ($\ell < 4$) which are probably best determined from the satellite data alone (in COSPAR and B6.1). These coefficients carry almost all the information for the 24-hour satellites. In fact this is exactly what the critical satellite tests (Intelsat 2 F1, Cosmos 41R and Syncom 2, DoD) have revealed. On the 12-hour satellites (i.e., Intelsat and Cosmos) B13.1 is the superior field even though the low order information in it (with strong 12-hour effects) is somewhat degraded over, say, the COSPAR field. It is the satellite only fields (COSPAR and B6.1) which are clearly superior to the others on the most critical 24-hour arc (Syncom 2, DoD). The implication is that these fields have superior low order coefficients (particularly 2,2 and 3,3). Examination of Figure 3 tends to confirm this judgement as far as the resonance coefficients are concerned. COSPAR and B6.1 are closest to the resonance adjusted 2,2 and 3,3 coefficients (conditioning the 24-hour data strongest) whereas B13.1 is closest to the resonance solution for 3,2 and 4,4 (conditioning the 12-hour data strongest).

The SAO B13.1 field was chosen as the best reference field for the combined data solution because of its implied superiority in medium order coefficients important on the critical 12-hour arcs. I felt that any weaknesses in low order coefficients for this field would be largely overcome by the corrections for the principal resonance parameters, all of low order: 2,2 3,2 3,3 and 4,4.

It should be emphasized that the resonance corrected solution given in Table 5 (displayed in Figure 3) is a preliminary solution for which at least four amendments are now being made:

1. The fixed field part of the solution (B13.1) for the 12-hour satellites in deep resonance (Cosmos 41R and Intelsat 2 F1) is being extended to include all geopotential effects above a level of 0.0005 of the maximum.
2. Best fitting area to mass ratios (with regard to radiation pressure) will be solved for in the deep resonance 12-hour arcs.
3. All orbit elements will be differentially corrected for in each arc (instead of semimajor axis and mean anomaly, principally, in the present solution).
4. A minor amount of additional data editing must be made as well as the inclusion of data from nine more long arcs of Early Bird, Syncom 3, ATS 1, 3 and 5, and Molniya 4, 7, 9 and 10.

This additional data is needed to reduce a number of high correlations in the preliminary solution such as between S_{22} and S_{32} (0.71), $S_{2,2}$ and C_{33} (0.71), and S_{22} and C_{22} (-0.61).

CONCLUSIONS

Critical tests of recent Smithsonian Astrophysical Observatory gravity fields on long arcs of 12- and 24-hour satellites reveal:

1. Most of the SAO fields recover the very sensitive resonant tracking information in most of these arcs reasonably well with a few notable exceptions which provide a very discriminating test of the low and medium order portions of these fields.

2. The best available SAO field for the 12-hour data appears to be the 1969 B13.1 set which includes ground gravity information.
3. The best available SAO field for the 24-hour data appears to be the 1969 COSPAR set which is strictly a low altitude satellite field.
4. A small correction of 8 low order resonance coefficients of the SAO 1969 B13.1 field appears to be compatible with the SAO data and produces a significant reduction of the overall residuals in the 12- and 24-hour arcs.
5. The resonance corrected SAO field presented here should have the most accurate set of low order coefficients calculated to date.

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Table 1
Recovery of Mean Anomaly (M) Observations for One Year (1968–1969) on Intelsat 2 F1 with Various Recent Gravity Fields

Gravity Field Used	rms M Residual (degrees)	Strongest Resonant Gravity Coefficients Used (unnormalized): 10^{-6}						Other Resonant Coefficients Used
		2,2		3,2		4,4		
		C	S	C	S	C	S	
SAO '69 - COSPAR	0.39	1.57	-.90	.26	-.24	-.0028	.0078	All through 11,2 13,4 15,6 16,8 and 16,10 giving .0001 of strongest effect
SAO '69 - B6.1[12]	0.33	1.57	-.91	.29	-.18	-.0035	.0086	Same as above
SAO '66 - M1	0.20	1.54	-.87	.25	-.18	-.0011	.0049	All through 11,2 10,4 8,6 and 8,8 giving .0001 of strongest effect
SAO '69 - B13.1[13]	0.18	1.55	-.91	.29	-.22	-.0021	.0075	All through 11,2 13,4 15,6 16,8 and 16,10 giving .0001 of strongest effect
Resonant Field: 3,2 & 4,4 determined from data	0.02	1.56	-.93	.40	-.22	.035	.051	None

Table 2
Recovery of Mean Anomaly Observations for Three Years (1966-1969) on Cosmos 41 Rocket with Various Recent Gravity Fields

Gravity Field Used	rms M Residual (degrees)	Strongest Resonant Gravity Coefficients Used (unnormalized): 10^{-6}						Other Resonant Coefficients Used
		2,2		3,2		4,4		
		C	S	C	S	C	S	
SAO '66 - M1	3.1	1.54	-.87	.25	-.18	-.0011	.0049	All through 7,6; selected to 10,6
SAO '69 - B13.1	1.1	1.55	-.91	.29	-.22	-.0021	.0075	All through 12,2 14,4 16,6 16,8 and 16,10 giving .001 of strongest effect
Resonant Field: 3,2 & 4,4 determined from data	0.5	1.56	-.93	.32	-.19	.0101	.0122	None

Table 3
Recovery of Equator Crossing Observations for Three Years (1965-1968) on Syncom 2 with Various Recent Gravity Fields

Gravity Field Used	rms λ Residual (degrees)	Strongest Resonant Gravity Coefficients Used (unnormalized): 10^{-6}				Other Resonant Coefficients Used
		2,2		3,3		
		C	S	C	S	
SAO '69 - B13.1	0.720	1.553	-.911	.111	.180	3,1 4,2 4,4
SAO '66 - M1	0.680	1.536	-.872	.078	.226	3,1 4,2 4,4
SAO '69 - B6.1	0.293	1.572	-.906	.111	.190	3,1 4,2 4,4
SAO '69 - COSPAR	0.064	1.565	-.897	.096	.198	3,1 4,2 4,4
Resonant Field: 2,2 3,1 3,3 determined from 13 arcs of 24-hour satellites	0.034	1.578	-.905	.100	.198	3,1 4,2 4,4 from SAO '66 M1

Table 4
Residuals in ROAD Solutions with
12- and 24-Hour Satellite Data (13 Arcs)

Arc	Individual Arc rms Residuals σ (λ , M)°	Combined Arc Solutions (See Table 5)	
		With SAO '69 B13.1 σ (λ , M)°	With SAO '69 B13.1 Corrected σ (λ , M)°
24-Hour			
E.B. 1	.025	.035	.024
Syn. 3,6	.015	.013	.014
Syn. 2,5	.035	.071	.040
Syn. 2,1	.025	.021	.021
Syn. 2,4	.015	.017	.014
Syn. 2 DoD	.040	.719	.038
Int. 2 F3	.030	.081	.027
Syn. 3,11	.010	.014	.009
12-Hour			
Cosmos 41R	.500	1.70	.320
Molniya 1	.080	.10	.110
Cosmos 41	.040	.11	.080
Int. 2 F1	.020	.056	.033

Table 5
Weighted rms Residuals in Two Combined Arc Solutions with 12- and 24-Hour Satellites (13 Arcs)

Gravity Field Used	Weighted rms Residual	Strongest Resonant Gravity Coefficients Used (unnormalized): 10^{-6}								Other Resonant Coefficients Used
		2,2		3,2		3,3		4,4		
		C	S	C	S	C	S	C	S	
SAO '69 B13.1	5.90	1.55	-.91	.29	-.22	.111	.180	-.0021	.0075	3,1 4,2 for 24-hour satellites; all through 6,2 8,4 and 10,6 giving .01 of strongest effect for 12-hour satellites
SAO '69 B13.1 corrected	0.99	1.58	-.91	.30	-.21	.097	.198	-.0016	.0075	Same as above

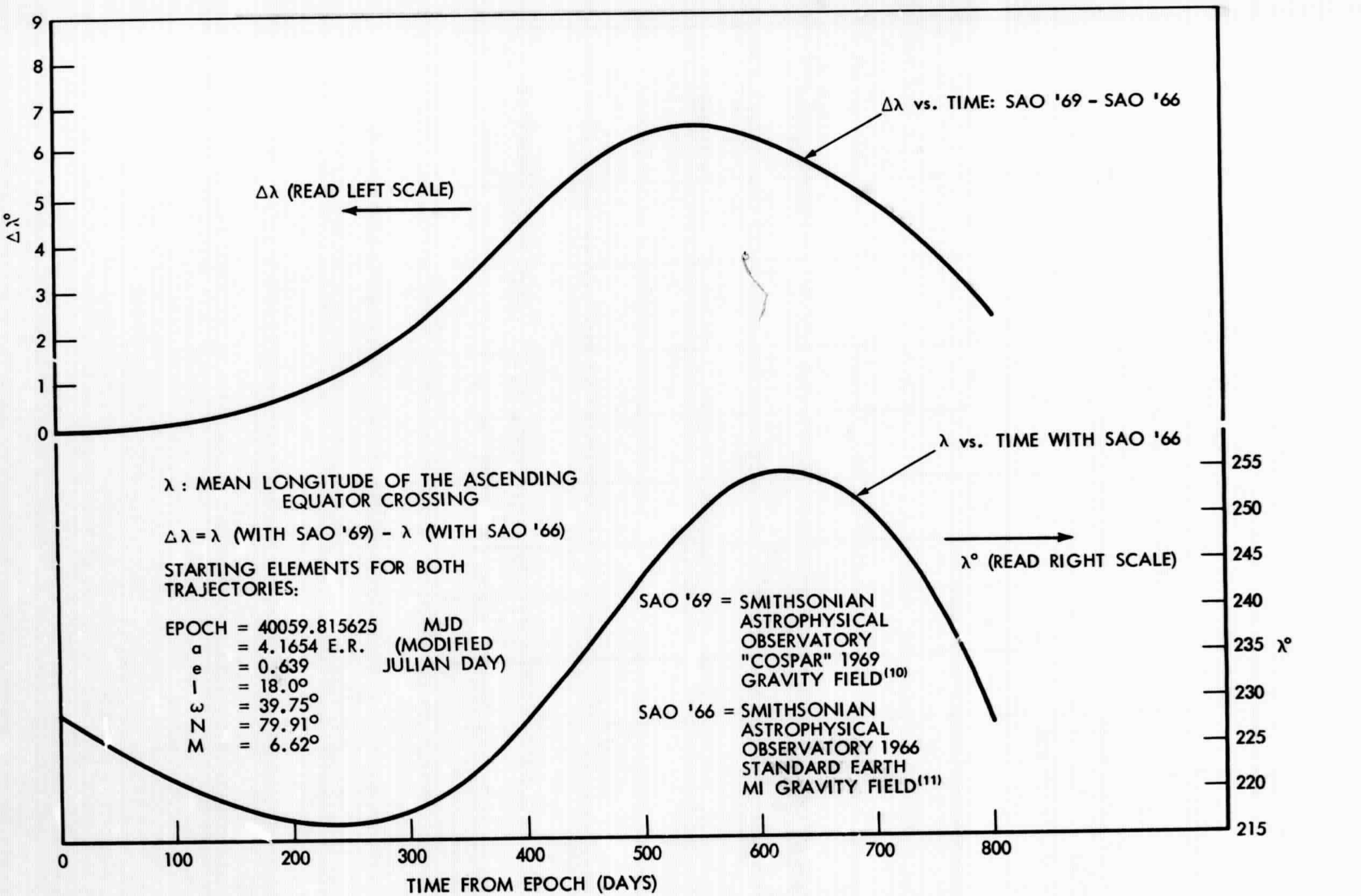

Figure 1. Longitude Changes and Differences in Two Intelsat 2 F1 Trajectories

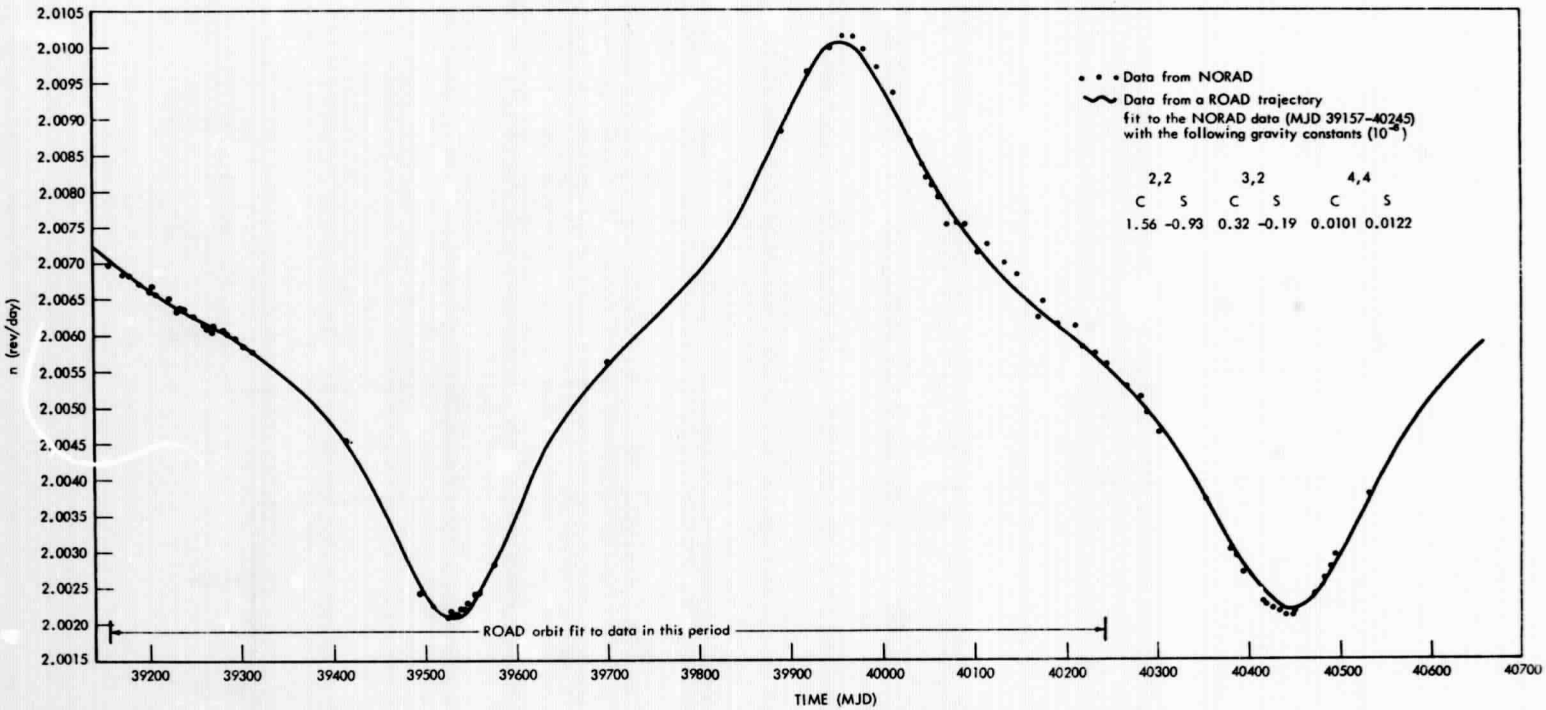

Figure 2. Cosmos 41 Rocket – 1964 49E – Mean Motion Data (n) from NORAD

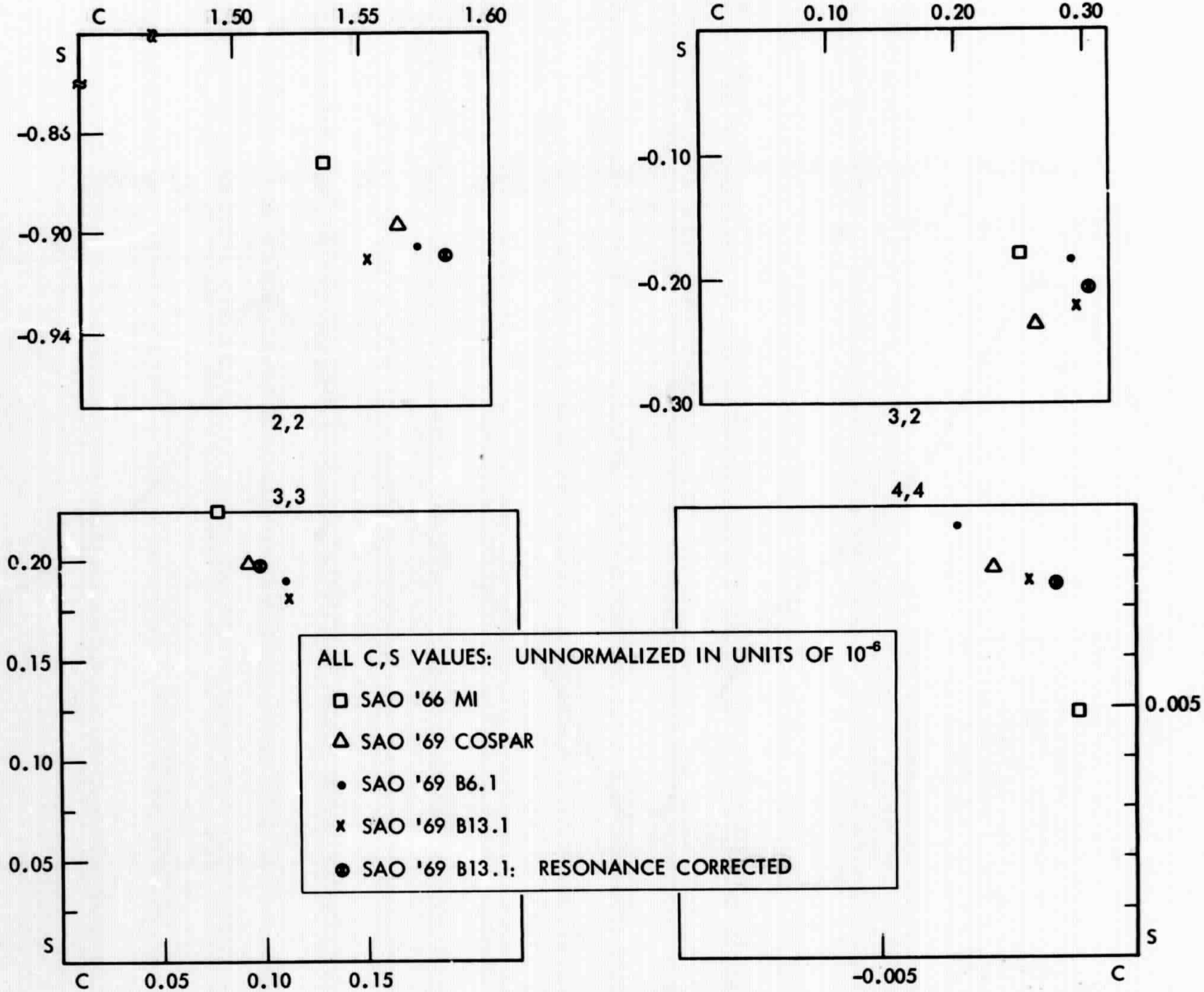

Figure 3. Low Order Gravity Harmonics from Recent Models