MODIFIED THEORY FOR CAVITY PERTURBATION MEASUREMENT OF PLASMA PARAMETERS

Scientific Report No. 5

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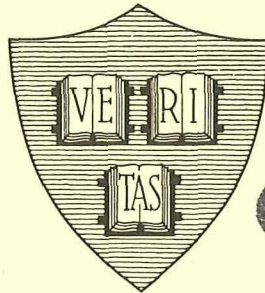
Y. S. Yeh and W. A. Saxton

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Y. S. Yeh and W. A. Saxton[*]

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ABSTRACT

This paper proposes a cavity method, based on a first order perturbation theory, for the measurement of plasma density and electron collision frequency. Effects of fringing fields around a finite plasma column are considered, making it possible to remove the restriction that cavity dimension be much larger than the diameter of a circular cylindrical plasma tube. This greatly reduces the size of the cavity used normally for plasma diagnostics, and provides longitudinal resoulution along the plasma column. Theory was experimentally verified with measurements on standard dielectric materials and on discharge plasmas whose electron density data were checked with Lagmuir probes.

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Modified Theory for Cavity Perturbation
Measurement of Plasma Parameters

by

Y. S. Yeh and W. A. Saxton

INTRODUCTION

One of the common ways to determine the electron
density and effective electron-neutral collision frequency
of a gaseous plasma is to measure the perturbations of a
microwave cavity due to the presence of the plasma within
the cavity.[1] In this technique the plasma, usually bounded
by a glass discharge tube, is inserted into a cavity whose
resonance frequency and bandwidth are known beforehand.
When the plasma appears, the resonance frequency and band-
width are perturbed and take on new values which depend on
the plasma's geometrical configuration, its physical orienta-
tion within the cavity, the shape and dimensions of the
cavity, and the distribution of the RF fields within it.[2]
Based upon the assumption that the plasma is a well-described
dielectric with a relative dielectric constant and loss tan-
gent dependent on the plasma and collision frequencies, the
differences between the resonant frequencies and bandwidths,
with and without the plasma present, are used to calculate
the electron density and collision frequencies.

Since many laboratory discharge tubes are circular cylindrical in shape, it is convenient to choose a circular cylindrical cavity geometry. If the diameter of the discharge tube is much smaller than the diameter of the cavity, and field fringing at the holes on the faces of the cavity through which the tube is inserted is negligible, it is usually possible to get a relatively good measure of the plasma's equivalent dielectric constant and loss tangent. At best, though, one gets an average measure of the discharge parameters, since electron distributions in both the radial and axial directions are usually a matter for speculation.

There are, however, situations in which the discharge has a large diameter compared to practical cavity dimensions, protrudes from holes in the cavity where sizable fringing fields are produced, and is not axially uniform as in the usual discharge.[3] In this case, if one were to use a circular cylindrical cavity it would have to be unusually large to apply the usual cavity perturbation theory, with resonance frequencies which would probably be too low for the range of plasma frequencies encountered; and, still, axial resolution of the plasma properties would be almost impossible since so much of the discharge would be inside the cavity.

One solution to the problem of diagnosing a discharge with these characteristics is to construct a very long rectangular cavity whose cross section has a relatively small dimension parallel to the axis of the discharge tube. As long

as the cavity is narrow compared to the length of the discharge it will serve to resolve axial variations. The other dimensions can be adjusted according to the desired range of resonance frequencies and bandwidths; but, more often than not, it will be impossible to avoid a situation in which the discharge tube diameter is comparable to, and not much less than, at least one of these dimensions and/or the cavity width. In this case, fringing at the holes becomes more complex, as if the fields there were not already greatly disturbed from their distributions in the totally closed unperturbed cavity on which the conventional perturbation theory is based.

This paper analyzes the cavity perturbation method for a thin rectangular cavity with a large plasma tube protruding from both of its sides. Modified theory is developed for a general plasma column in which the gross disturbances of the fields at the holes are taken into account as well as the fact that certain cavity dimensions are comparable to and even smaller than the plasma column. A rectangular cavity which is smaller overall than a circular cylindrical cavity with the same range of resonance frequency and bandwidth was constructed. Experimental data were taken on dielectric constant and loss tangent for several sample liquid and solid dielectrics. This data corroborated the theoretical predictions based on the modified theory and indicated the applicability of this general approach to actual plasmas. Plasma

measurements were then performed which yielded data on electron densities that agreed with separate measurements using Langmuir probes.

EXPERIMENTAL SETUP

The rectangular cavity was constructed from 1/8 inch thick copper sheet as shown in Figure 1. Holes at the center of the top and bottom mate to circular brass "ears," 5.08 cm in length, that function as circular waveguides for which the lowest cut-off frequency is 3600 MHz for the TM_{01} mode. Since the planned operating frequency was in the 500-1000 MHz range, the waves in the circular guides are attenuated more than 30 db before they reach the open end of the guides. As a result, the ear may be considered as an infinitely long closed structure which prevents any radiation from the rectnagular cavity itself into free space. This allows one to insert a plasma discharge tube, or a conventional dielectric rod, of any length into the ear-cavity combination. Even though the plasma or dielectric cylinder would protrude from the cavity, the characteristics of that part of the medium within the cavity may be determined, as will be shown subsequently.

An RF coupling loop, 3 inches in diameter, was connected to the center part of an N-type connector and soldered to each end plate (Figure 2). The loop was oriented to excite or detect TE modes. Cavity ends were identical and interchangeable so that both could be used for excitation or detection.
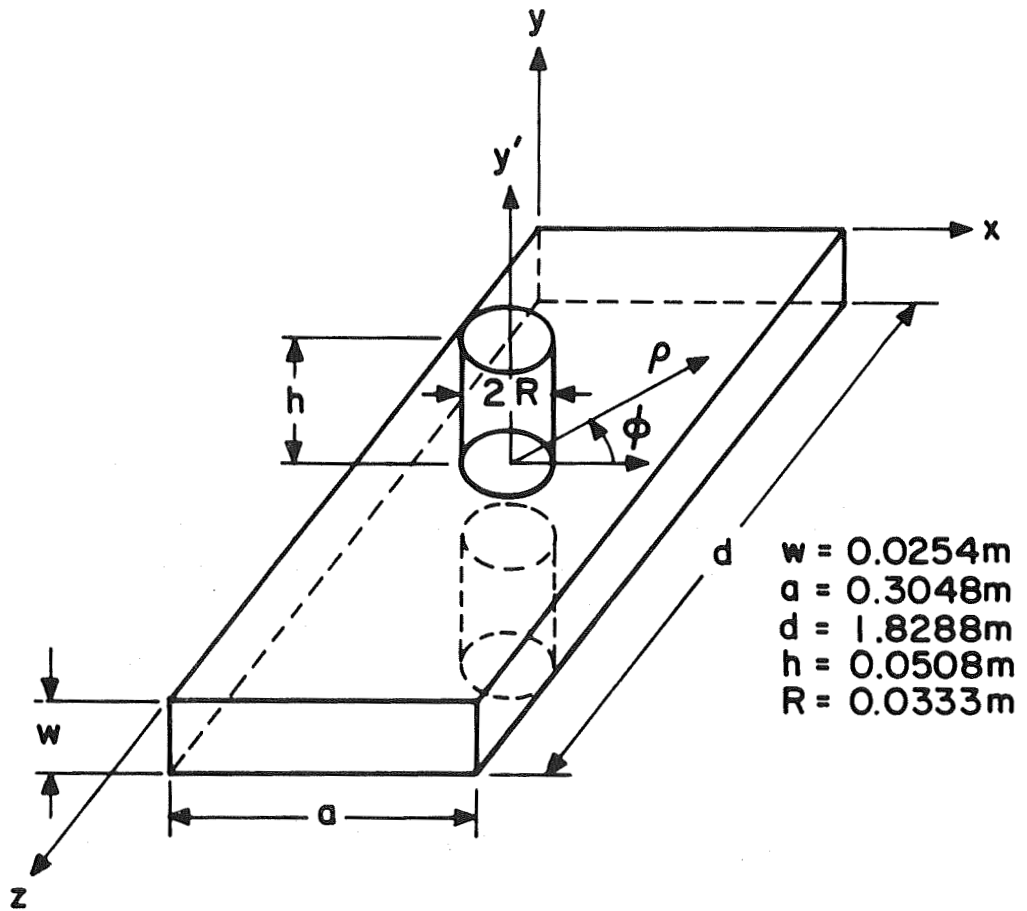
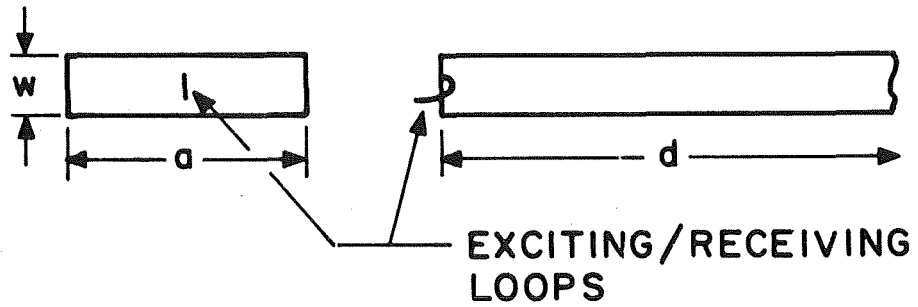FIG. I RECTANGULAR CAVITY WITH PROTRUDING
METALLIC EARS

FIG. 2 ORIENTATION OF EXCITING/RECEIVING LOOPS

The plasma tube was made of pyrex glass whose inside and outside diameters were 5.9 and 6.35 cm, respectively. It contained a hot cathode discharge in argon at a pressure of 1 torr.

Input RF to the cavity was provided by a stable, continuously tunable CW signal generator which was constantly monitored on a Hewlett-Packard 524C frequency counter. Cavity output was fed through a 9 db pad, a calibrated variable attenuator, a 6 db pad, a crystal diode modulator, an audio amplifier and a tuned voltmeter, in that order.

DETERMINATION OF ELECTRON DENSITY

Resonance frequency shifts in the modified cavity due to the presence and the absence of the sample consist of two parts. Estin and Bussey[4] pointed out that in such a case there would be a frequency pulling effect caused by the ear, and a perturbation resulting from the effect of the dielectric material itself.

The rectangular cavity was operated in TE_{10p} modes, with measurements made for p = 3, 5, 7 and 9. Referring to the coordinates of Figure 1, the field distribution, the natural frequency f, and the stored energy W_o for a normal rectangular cavity are given by [5]

$$E_y = E_o \sin \frac{\pi x}{a} \sin \frac{p \pi z}{d} \tag{1}$$

$$H_x = - j \frac{E_o}{\eta} \frac{p}{2d} \sin \frac{\pi x}{a} \cos \frac{p \pi z}{d} \tag{2}$$

$$H_z = j \frac{E_o}{\eta} \frac{\lambda}{2a} \cos \frac{\pi x}{a} \sin \frac{p \pi z}{d} \qquad (3)$$

$$f = 1.5 \times 10^8 \sqrt{(1/a)^2 + (p/d)^2} \qquad (4)$$

$$W_o = \frac{\epsilon_o}{8} awd \, E_o^2 \qquad (5)$$

where E_o is a constant, η is the free space wave impedance, and λ is the wavelength.

For a closed cavity, the presence of the plasma tube results in a cavity resonance frequency perturbation δf_1 given by

$$\frac{\delta f_1}{f} = - \iiint_{v'} (\epsilon_r - \epsilon_o) |E_y|^2 dv' / 4W_o \qquad (6)$$

where v' is the volume occupied by the tube, ϵ_r is the real part of the dielectric constant of the plasma. Here, one usually assumes that E_y is that given by Equation (1).

Consider now this closed cavity is deformed such that the ear is produced by pulling a cylindrical disc section of the cavity wall, with the diameter of the hole, to infinity against the time average electromagnetic stress $(\epsilon_r |E|^2 - \mu |H|^2)/4$ on the disc, as indicated in Figure 3. This action causes change in stored energy and hence shift
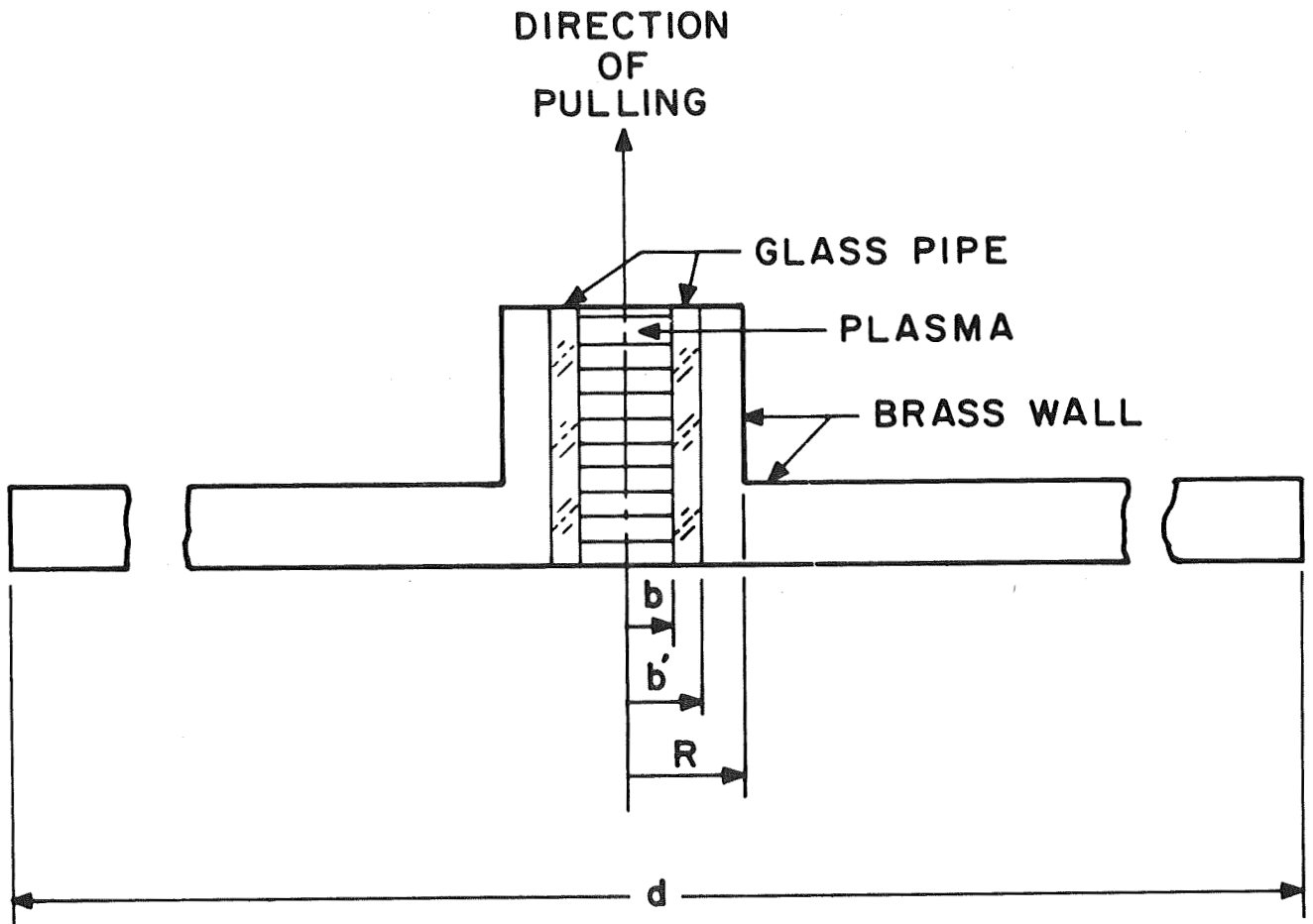
FIG. 3 POSITION OF PLASMA TUBE WITHIN ONE OF THE CAVITY'S EAR-HOLE ASSEMBLIES

in resonance frequency. It is the combination of this frequency change and that of Equation (6) which gives the measured frequency shift in the actual measurement. According to the resonator reactor theorem,[6] the frequency pulling effect of the ear is,

$$\frac{\delta f_2}{f} = \delta w / W_o \qquad (7)$$

where f_2 and w are the change in resonance frequency and stored energy, respectively.

The change in stored energy in the process of producing the ear is obtained from the work done on the system or extracted from it. This is obtained if we approximate the field distribution in the ear by one that would be excited by the original electric field at the ear cavity interface. At the interface, E_y is fairly constant;

$$E_y \cong E_o \left[1 - \frac{\pi^2}{2a^2} (x - a/2)^2 \right] \left[1 - \frac{p^2\pi^2}{2d^2} (z - d/2)^2 \right] . \qquad (8)$$

Using cylindrical coordinates at the center of the hole (Figure 1),

$$E_y = E_o - \frac{1}{4} \left[\left(\frac{\pi}{a} \right)^2 + \left(\frac{p\pi}{d} \right)^2 \right] E_o \rho^2 - \frac{1}{4} \left[\left(\frac{\pi}{a} \right)^2 - \left(\frac{p\pi}{d} \right)^2 \right] E_o \rho^2 \cos 2\phi .$$

$$(9)$$

This is a field distribution which will excite TM_{0m} and TM_{2m} modes in the ear if it is free space.

The electron density distribution is radially nonuniform, and is approximated by

$$N = n_o n = n_o J_o(k\rho) \tag{10}$$

where

J_o is the Bessel function of zeroth order

n_o is the electron density at the center of the tube

k is given by 2.405/b

The complex dielectric constant is

$$\varepsilon = \varepsilon_o - \varepsilon_o \frac{\omega_p^2}{\omega(\omega-j\nu)} \tag{11}$$

where

ν is the collision frequency

ω is the microwave angular frequency

$\omega_p = 56500 \ N^{1/2}$

Equation (11) can be rewritten as

$$\varepsilon = \varepsilon_o - \varepsilon_o \alpha n \tag{12}$$

where

$$\alpha = \alpha_r + j\alpha_i = n_o \frac{56500^2}{\omega^2 + \nu^2} + jn_o \frac{56500^2}{\omega^2 + \nu^2} \frac{\nu}{\omega} \ . \qquad (13)$$

Since the excitation field E_y has an extremely small component with ϕ variation, the field excited in the circular cylindrical wave guide could be predominately of the TM_{0m} type. For a solution of the form $E_y' = R(\rho)e^{-\gamma y'}$, we have

$$\frac{1}{\rho} \frac{\partial}{\partial \rho} R(\rho) + (k^2 + \gamma^2) R(\rho) = 0 \qquad (14)$$

where

$$k^2 = \omega^2 \mu \varepsilon = k_o^2 - k_o^2 \alpha n \ .$$

Equation (14) can be rewritten as

$$\frac{1}{\rho} \frac{\partial}{\partial \rho} R(\rho) + \lambda^2 R(\rho) = \alpha k_o^2 n R(\rho) \qquad (15)$$

where

$$\lambda^2 = k_o^2 + \gamma^2 \ .$$

For $\alpha = 0$, i.e., without plasma, the general solutions are the well-known mode functions, $J_o(\lambda_m \rho)$ with λ_m's given by $J_o(\lambda_m R) = 0$, and R is the radius of the ear.

In the presence of the plasma, the eigenvalues and the eigenfunctions are modified. In order to apply the perturbation theory, we must show that the right-hand side of Equation (15) is much smaller than λ^2. The smallest λ^2 is $(2.405/R)^2 = 2.1/cm^2$. For an electron density of 10^{11} electrons/cc and an operating frequency of 886 mHz, i.e., TE_{109} mode of the cavity, the right-hand side is $0.36 J_o(k\rho)/(1-j\nu/\omega)$ and is indeed much smaller. The first order perturbation theory,[7] taking only three terms for the approximation of ψ, yields the following modified eigenvalues and eigenfunctions,

$$\lambda_i'^2 = \lambda_i^2 + \alpha k_o^2 u_{ii} \qquad (16)$$

$$\gamma_1'^2 = \gamma_i^2 + \alpha k_o^2 u_{ii} \qquad (17)$$

$$\psi_1 = J_o(\lambda_1 \rho) + \alpha\delta_{12} J_o(\lambda_2 \rho) + \alpha\delta_{13} J_o(\lambda_3 \rho) \qquad (18)$$

$$\psi_2 = + \alpha\delta_{21} J_o(\lambda_1 \rho) + J_o(\lambda_2 \rho) + \alpha\delta_{23} J_o(\lambda_3 \rho) \qquad (19)$$

$$\psi_3 = + \alpha\delta_{31} J_o(\lambda_1 \rho) + \alpha\delta_{32} J_o(\lambda_2 \rho) + J_o(\lambda_3 \rho) \qquad (20)$$

$$u_{ij} = \int_0^b n(\rho) j_o(\lambda_i \rho) j_o(\lambda_j \rho) \rho \, d\rho \tag{21}$$

$$\delta_{ij} = k_o^2 \frac{u_{ij}}{\lambda_i^2 - \lambda_j^2} \tag{22}$$

where $j_o(\lambda_i \rho)$ is the normalized Bessel function such that

$$j_o(\lambda_i \rho) = J_o(\lambda_i \rho) \left/ \left[\frac{R^2}{2} J_o'^2(\lambda_i R) \right]^{1/2} \right. .$$

Without the presence of the plasma, the axial electric field E_y' in the cutoff waveguide can be approximated by

$$E_y' = A_1 E_o j_o(\lambda_1 \rho) e^{-\gamma_1 y'} + A_2 E_o j_o(\lambda_2 \rho) e^{-\gamma_2 y'} + A_3 E_o j_o(\lambda_3 \rho) e^{-\gamma_3 y'} .$$

$$\tag{24}$$

The expansion coefficients are given by matching E_y' to that of Equation (9) at the interface, i.e., $y = 0$.

$$A_i = \int_0^R f(\rho) j_o(\lambda_1 \rho) \rho \, d\rho \tag{25}$$

where

$$f(\rho) = 1 - \frac{1}{4}\left[\left(\frac{\pi}{a}\right)^2 + \left(\frac{p\pi}{d}\right)^2\right]\rho^2 . \qquad (26)$$

The energy added to the system in producing both ears, referring to Figure 3, is

$$\delta w_{air} = \frac{1}{2}\int_0^\infty\int_0^{2\pi}\left[\int_0^b \epsilon_o|E_y'|^2 + \int_b^{b'} \epsilon_g|E_y'|^2 + \int_{b'}^{R} \epsilon_o|E_y'|^2\right]\rho d\rho d\phi dy' \qquad (27)$$

The axial field E_y'' in the presence of the plasma is

$$E_y'' = A_1'E_o\psi_1 e^{-\gamma_1'y'} + A_2'E_o\psi_2 e^{-\gamma_2'y'} + A_3'E_o\psi_3 e^{-\gamma_3'y'} \qquad (28)$$

where

$$A_i' = \int_0^R f(\rho)\psi_i \rho d\rho \qquad (29)$$

The energy added to the system is:

δw_{plasma}

$$= \frac{1}{2} \int_0^\infty \int_0^{2\pi} \left[\int_0^b (\epsilon_o - \epsilon_o \alpha_r n) |E_y''|^2 + \int_b^{b'} \epsilon_g |E_y''|^2 + \int_{b'}^R \epsilon_o |E_y''|^2 \right] \rho d\rho d\phi dy' \quad . \tag{30}$$

The difference is

$$\delta w \simeq \frac{1}{2} \int_0^\infty \int_0^{2\pi} \int_0^b \left[(\epsilon_o - \epsilon_o \alpha_r n) |E_y''|^2 - \epsilon_o |E_y'|^2 \right] \rho d\rho d\phi dy'. \tag{31}$$

Here we have neglected terms from b to b' and b' to R because the ranges are much less than 0 to b. Furthermore, the difference between Equations (29) and (30) tends to cancel each other. The integration of $|E_y'|^2$, $n(\rho)|E_y''|^2$, and $|E_y'^2|$ can be carried out. To be consistent with the first order perturbation, all the higher order terms, i.e., terms involving α^2 and up, are neglected in these three integrals; thus we obtain

$$\delta w = - \alpha_r^2 \sum_{i=1}^3 \sum_{j=i}^3 T_{ij} U_{ij} \pi \epsilon_o E_o^2$$

$$+ \alpha_r \left[\sum_{i=1}^3 \sum_{j=i}^3 T_{ij} V_{ij} - \sum_{i=1}^3 \frac{A_i^2}{2\gamma_i} U_{ii} - \sum_{i=1}^3 \sum_{j=i+1}^3 \frac{2A_i A_j}{\gamma_i + \gamma_j} U_{ij} \right] \pi \epsilon_o E_o^2 \tag{32}$$

where

$$V_{ij} = \int_0^b j_0(\lambda_i \rho) j_0(\lambda_j \rho) \rho d\rho$$

$$T_{ii} = B_{ii}/2\gamma_i + A_i^2 R_{ii} + \sum_{\substack{i=1 \\ j \neq i}}^{3} \frac{2A_i A_j}{\gamma_i + \gamma_j} \delta_{ji}$$

$$T_{ij} = B_{ij}/\gamma_i + \gamma_j + A_i A_j R_{ij} + \delta_{ij}\left(\frac{A_i^2}{\gamma_i} - \frac{A_j^2}{\gamma_j}\right) + \frac{2A_k A_i}{\gamma_k + \gamma_i} \delta_{ki} + \frac{2A_k A_j}{\gamma_k + \gamma_j} \delta_{kj}$$

where $k \neq i$ or j.

$$R_{ij} = - 0.5 \, k_0^2 (U_{ii}/\gamma_i + U_{jj}/\gamma_j)/(\gamma_i + \gamma_j)^2$$

$$B_{ii} = 2A_i \sum_{\substack{i=1 \\ j \neq i}}^{3} A_j \delta_{ij}$$

$$B_{ij} = \delta_{ij}\left(A_j^2 - A_i^2\right) + A_i A_k \delta_{jk} + A_j A_k \delta_{ik}$$

where $k \neq i$ or j.

The difference in resonant frequency between plasma and free space due to this ear pulling effect is

$$\frac{\delta f_2}{f} = \delta w / W_o .$$ (33)

Equation (6), while expressed in terms of α, is,

$$\frac{\delta f_1}{f} = \alpha_r w' / W_o$$ (34)

where

$$w' = \varepsilon_o \iiint n |E_y|^2 dv' / 4 .$$

The measured frequency shift is the combination of Equation (33) and (34),

$$\frac{\delta f_m}{f} = \delta w / W_o + \alpha_r w' / W_o .$$ (35)

Substituting Equation (32) into (35) and replacing W_o by Equation (5), we obtain the following quadratic equation for α_r

$$- \alpha_r^2 \sum_{i=1}^{3} \sum_{j=i}^{3} T_{ij} U_{ij} \frac{8\pi}{awd}$$

$$+ \alpha_r \left(\sum_{i=1}^{3} \sum_{j=i}^{3} T_{ij} V_{ij} - \sum_{i=1}^{3} \frac{A_i^2}{2\gamma_i} U_{ii} - \sum_{i=1}^{3} \sum_{j=i+1}^{3} \frac{2A_i A_j}{\gamma_i + \gamma_j} U_{ij} \right) \frac{8\pi}{awd}$$

$$+ \alpha_r \left(8w'/awdE_o^2 \varepsilon_o \right) = \delta f_m/f . \tag{36}$$

We can now determine α_r from the measured frequency shift. As a typical example, for TE_{109} mode, numerical calculation shows that Equation (36) becomes

$$6.335 \times 10^{-5} \alpha_r^2 + 1.316 \times 10^{-3} \alpha_r = \delta f_m/f . \tag{37}$$

To check the convergence of the three term approximation of ψ as in Equations (18) through (20), we employ only two terms to evaluate Equation (36). The resultant equation is

$$0.651 \times 10^{-5} \alpha_r^2 + 1.263 \times 10^{-3} \alpha_r = \delta f_m/f . \tag{38}$$

It is therefore clear that three term approximation already yields convergent results.

α_r is related to electron density n_o by

$$\alpha_r = n_o \frac{56500^2}{\omega^2 + \nu^2} . \qquad (39)$$

In case of small collision loss, the electron density can be directly evaluated by neglecting ν^2 in Equation (39). If ν is appreciable in comparison to ω, we must obtain ν first. This is described in the next section.

DETERMINATION OF COLLISION FREQUENCY

Referring to Equation (13), the complex dielectric constant of the plasma can be expressed by

$$\varepsilon = \varepsilon_r - j \, \sigma/\omega \qquad (40)$$

where

$$\varepsilon_r = \varepsilon_o - \varepsilon_o \alpha_r n$$

$$\sigma/\omega = \varepsilon_o \alpha_1 n$$

$$\alpha_1/\alpha_r = \nu/\omega$$

Consider now that the cavity is composed of the cavity itself and the two ears. The field distribution in this arrangement

is E and H and is different from that of the closed cavity.
Ordinary cavity perturbation theory leads to the following
relations,

$$\frac{\delta f}{f} = - \frac{\iiint_{v'} (\varepsilon_r - \varepsilon_o)|E|^2 dv'}{\iiint_v (\varepsilon_o |E|^2 + \mu_o |H|^2) dv} \tag{41}$$

$$\frac{1}{Q} - \frac{1}{Q_o} = 2 \frac{\iiint_{v'} \frac{\sigma}{\omega} |E|^2 dv'}{\iiint_v (\varepsilon_o |E|^2 + \mu_o |H|^2) dv} \tag{42}$$

where

v' is the volume occupied by the plasma tube

v is the total volume of the cavity-ear configuration

δf is the resonance frequency shift

Q and Q_o is the Q factor for plasma and free space,
respectively

f is the resonance frequency of the cavity

It is noticed that $\varepsilon_r - \varepsilon_o = - \varepsilon_o \alpha_r n$ and $\sigma/\omega = \varepsilon_o \alpha_i n$, therefore
the integrals in Equations (41) and (42) can be eliminated to
obtain

$$\nu/\omega = \frac{1}{2} \left[\frac{1}{Q} - \frac{1}{Q_O} \right] f/\delta f \qquad (43)$$

Equation (43) now yields ν and can be used in Equation (39) to obtain n_O.

COMPARISON OF EXPERIMENTAL RESULTS

We first checked the validity of this modified theory by performing measurements on dielectric materials with known dielectric constants and loss tangents. It is quite obvious that the only modification required is to change the density function $n(\rho)$ to $n(\rho) = 1$. The measurement results and the standard values are compared in Table I.

Electron density and the collision frequency for various discharge currents were measured, using the TE_{109} mode with f = 886 MHz. The collision frequency varied slightly around 3800 MHz, which yielded a $(\nu/\omega)^2$ of about 0.45. Measured electron density compared to some of the Langmuir probe measurements appear in Figure 4.

ACKNOWLEDGMENT

TABLE I

Measurements of Dielectric Constants and Loss Tangents Using TE_{109}

Mode f = 886 mHz

Dielectric Material	ϵ/ϵ_o Measured	ϵ/ϵ_o Standard	$10^4 \times \tan \delta$ Measured	$10^4 \times \tan \delta$ Standard
Balsa	1.19	1.173**	183	199*
Fir	1.97	1.86**	730	660**
Heptane	1.95	1.97*	not measurable	3.5*
Polysterene	2.51	2.55*	not measurable	< 2.5*
Micarta[1]	3.70	3.64**	610	660**
Aroclor 1232[2]	3.12	3.20*	2460	2500*
Aroclor 1242[2]	2.84	2.86*	1245	1200*
Aroclor 1248[2]	2.86	2.78*	550	570**
Aroclor 1252[2]	2.78	2.72*	70	65*

* Standard value given by A. R. von Hipple.[7]

** Standard values are checked and corrected using the ordinary cavity perturbation method by inserting a small cylindrical sample of each material, 2.54 cm in length and diameter, into the closed cavity.

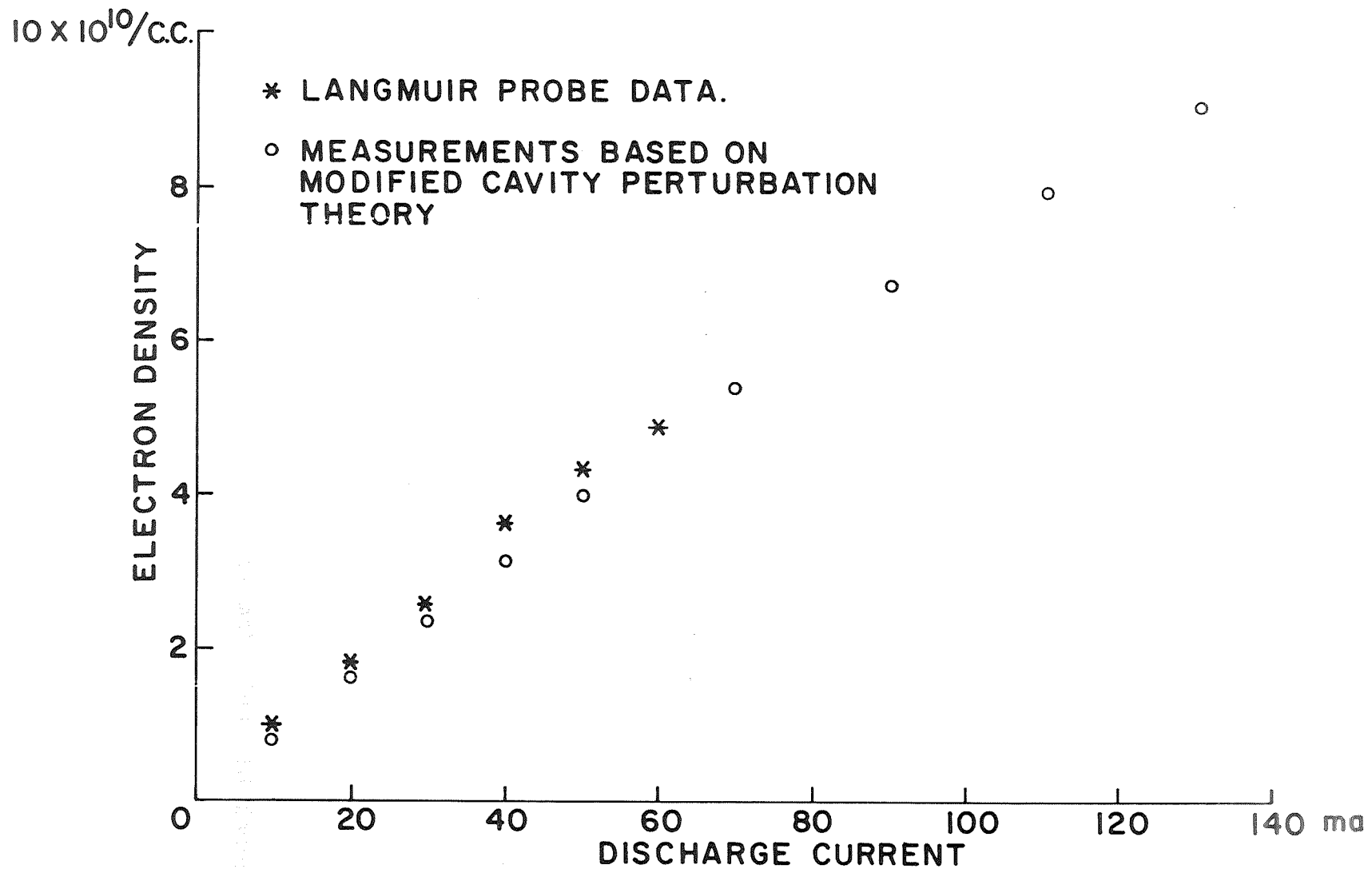[1] Monsanto Chemical Company.

[2] General Electric Company.

FIG. 4 ELECTRON DENSITIES MEASURED IN HOT−CATHODE ARGON
DISCHARGES

of the U. S. Army Electronics Command at Fort Monmouth,

New Jersey, and Mr. Kenton Garoff, Chief of the Laboratory's

Electron Tubes Division.

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