A SHORT, CYLINDRICAL ANTENNA AS A DIAGNOSTIC PROBE FOR MEASURING COLLISION FREQUENCIES IN A COLLISION-DOMINATED, NON-MAXWELLIAN PLASMA

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ABSTRACT

Investigations have been made to determine the effects of inter-particle collisions on the antiresonant impedance characteristics of an electrically short, cylindrical antenna in the vicinity of the plasma frequency of an isotropic non-Maxwellian plasma. The dependence of the electron-neutral collision frequency on the electron energy has been taken into account. The experimental results have been compared with the theories proposed by King et al.[1] and Balmain.[2] The use of this antenna as a diagnostic probe for measuring electron-neutral collision frequency and electron density has been investigated. Electron-neutral collision frequencies for Helium measured by this technique are in good agreement with theoretical results calculated from the collision cross-section data of Golden and Bandel.[3]

1. Introduction

Although electrostatic Langmuir probes[4] and r.f. resonance probes[5] have been used for a number of years for making reliable measurements of the electron density and electron temperature of plasmas, no satisfactory r.f. probe technique appears to have been developed as yet, for measuring electron-neutral collision frequencies of either laboratory or ionospheric plasmas. Typical radio frequency diagnostic methods used for measuring collision frequencies of plasmas include a wide variety of r.f. attenuation measurements[6,7] or some form of a cross modulation technique[8].

The primary purpose of this paper is to investigate the impedance behavior of a short, cylindrical antenna in the vicinity of the plasma frequency of a weakly-ionized, isotropic, collision-dominated, non-Maxwellian plasma. Secondly, it is of considerable interest to explore the possibility of using such an antenna as a diagnostic probe for measuring electron-neutral collision frequencies. Since the antenna is electrically very short, the values of the collision frequency and electron density are averaged over the 'near-field' region of the antenna, the dimensions of which are quite small, so that it becomes possible to get information on the local characteristics of the plasma. Missile antennas of this type have been used by Haycock, Baker, and Ulwick[9] and by Jackson and Kane[10] for electron density measurements, by measuring the change in the antenna reactance at the plasma frequency. They have, however, not extended their technique for measuring collision frequencies by considering the change in the input resistance of the antenna. The quasi-static theories used in their calculations also leave much to be desired since the effect of the antenna dimensions and plasma parameters have not been properly accounted for. Mlodnosky and Garriott[11] have used a dipole antenna for measuring electron density and electron temperature in a loss-less plasma; their method consists of measuring the effect of the ion-sheath on the antenna admittance at frequencies much below the plasma frequency. In contrast to these earlier investigations, this paper is devoted mainly to making accurate measurements of collision frequency and electron density in a collision-dominated plasma because, under these conditions, the contribution to the antenna impedance from the electron temperature and ion-sheath effects can be neglected. The probe is, therefore, likely to be useful for diagnostic measurements in high pressure, rare-gas laboratory-discharge plasmas, shock-tube plasmas, and some types of re-entry plasmas, and also in the D-region of the ionosphere[7], provided that the effects of the earth's magnetic field are properly accounted for.[12]

The two principal difficulties encountered in applying such a technique for collision frequency measurements are 1) the lack of an accurate theory which takes into account the effect of the various plasma parameters such as the electron density, collision frequency, electron temperature, and non-collisional damping on the input impedance of the antenna, and 2) the discrepancy in the expression for the plasma conductivity obtained from the Lorentzian model (which is commonly used for determining antenna impedance) and the kinetic theory model derived from the Boltzmann equation.[13] The following two sections of this paper will elaborate on these topics.

2. Theory for the Input Impedance of a Short, Cylindrical Antenna in Cold and Warm Plasmas.

Numerous theoretical papers[14] have been published in recent years describing the impedance and radiation characteristics of antennas immersed in plasmas. As the literature on this problem is very vast only those papers which are most relevant to this investigation are mentioned here. The experimental results, for the most part, have been analyzed using the theory proposed by King, Harrison and Denton[1] for an antenna in a cold, lossy, isotropic plasma. In order to estimate the contribution from the electroacoustic mode to the antenna impedance in the vicinity of the plasma frequency, the results have also been compared with the theory proposed by Balmain[2] for a warm plasma based on a linearized, hydrodynamic model. More recently, Galejs[15] has treated a similar problem using a variational technique; unfortunately, this paper came to our attention too late to permit numerical calculations for comparison. Numerical solutions of the integral equations for cylindrical antennas immersed in warm plasmas have been made by Wunsch,[16] Lin and Mei,[17] Cook and Edgar,[18] and by Kuehl.[19] These authors, however, have ignored collisional effects and, hence, their theory is not applicable to our experiments.

In the theory outlined by King, Harrison and Denton,[1] the plasma is treated as a Lorentzian gas with a collisional damping term that is assumed to be independent of the electron velocity. The current distribution on the antenna is solved by an integral equation technique and the input admittance Y(k) of the antenna is given by the following equations:

$$Y_{in}(k) = G_{in}(k) + jB_{in}(k) \tag{1a}$$

where

$$G_{in}(k) = \frac{2\pi}{\zeta_e \psi_{dl}} \left\{ \frac{2\alpha}{\beta} \left[\beta h + \frac{2}{3}\beta^3 h^3 F\left(1 - \frac{\alpha^2}{\beta^2}\right)\right] + \frac{\beta^4 h^4}{3(\Omega - 3)}\left(1 - \frac{10\alpha^2}{\beta^2} + \frac{5\alpha^4}{\beta^4}\right)\right\} \tag{1b}$$

and

$$B_{in}(k) = \frac{2\pi}{\zeta_e \psi_{dl}} \left\{ \beta h\left(1 - \frac{\alpha^2}{\beta^2}\right) + \frac{1}{3}\beta^3 h^3 F\left(1 - \frac{6\alpha^2}{\beta^2} + \frac{\alpha^4}{\beta^4}\right) - \frac{\beta^4 h^4}{3(\Omega - 3)} \frac{\alpha}{\beta}\left(5 - \frac{10\alpha^2}{\beta^2} + \frac{\alpha^4}{\beta^4}\right) \tag{1c}$$

In the above equation $G_{in}(k)$ and $B_{in}(k)$ are, respectively, the input conductance and input susceptance of the antenna; 2h and a are the length and radius of the cylindrical antenna; $F = 1 + [(3 \ln(2) - 1)/(\Omega - 3)]$, and $\Omega = 2 \ln(2h/a)$.

$$\alpha = \omega\sqrt{\mu|\epsilon_e|}\, f(p) \quad \text{and} \quad \beta = \omega\sqrt{\mu|\epsilon_e|}\, g(p) \quad \text{when } p < 0 \text{ and } \epsilon < 0. \tag{2a}$$

and

$$\alpha = \omega\sqrt{\mu\epsilon_e}\, g(p) \quad \text{and} \quad \beta = \omega\sqrt{\mu\epsilon_e}\, f(p) \quad \text{when } p > 0 \text{ and } \epsilon > 0. \tag{2b}$$

where

$$f(p) = \sqrt{\frac{1}{2}\sqrt{1 + p^2} + 1} \tag{3a}$$

and

$$g(p) = \sqrt{\frac{1}{2}\sqrt{1 + p^2} - 1} \tag{3b}$$

In equations (1-3), $\zeta_e = \omega\mu/\beta$, $\psi_{dl} = 2 \ln(h/a) - 2$, $p = \sigma_e/\omega\epsilon_e$ where

$$\sigma_e = \frac{\epsilon_0 \omega_p^2 \nu_c}{\nu_c^2 + \omega^2} \quad \text{and} \quad \epsilon_e = \epsilon_0 [1 - \frac{\omega_p^2}{\omega^2 + \nu_c^2}]$$

ω_p, ν_c and ω are, respectively, the angular plasma frequency, collision frequency, and signal frequency of the antenna.

The input admittance of the antenna given by equations (1-3) has been computed for a wide range of plasma parameters corresponding to the experimental conditions and has been compared with the measured results.

The theory by King et al. does not take into account the electron temperature which causes a coupling of energy between the electromagnetic and electroacoustic modes, particularly near the plasma frequency. In order to estimate the relative contribution by electroacoustic and collisional effects, computations of the antenna impedance were also made from the theory proposed by Balmain[2] for an isotropic, compressible plasma; the formula for the antenna impedance Z obtained from Poynting's theorem with an assumed triangular current distribution on the antenna is given by (Ref. 2, Eq. 30)

$$Z_{in} = Z_{EM} + Z_P \tag{4a}$$

where

$$Z_{EM} = \frac{1}{j\omega\pi\epsilon_0 k_0 h} [\ln(h/a) - 1] \tag{4b}$$

and

$$Z_P = \frac{1}{j\omega\pi\epsilon_0 k_0 h} \{(1 - k_0) \frac{\pi}{2} [J_0(\tau a) N_0(\tau a) + jJ_0^2(\tau a)]\} \quad \text{when } \omega > \omega_p$$

(4c)

$$Z_P = \frac{-1}{j\omega\pi\epsilon_0 k_0 h} [(1 - k_0) I_0(\rho a) K_0(\rho a)] \quad \text{when } \omega < \omega_p$$

In equation (4c), $\rho = j\tau$ and

$$\rho^2 = \frac{\omega^2(X - U)}{v^2}, \quad X = \frac{\omega_p^2}{\omega^2}, \quad \omega_p^2 = \frac{ne^2}{m\epsilon_0}, \quad U = 1 - jZ, \quad Z = \frac{\nu_c}{\omega}, \quad k_0 = 1 - \frac{X}{U}$$

$v = \sqrt{\frac{3kT_e}{m}}$ is the acoustical velocity in the electron gas. In equations (4a) and (4b) Z_{EM} and Z_P are the contributions from the electromagnetic and electroacoustic modes to the input impedance of the antenna. Equation (4c) shows that even for a lossless plasma, the antenna has a resistive component owing to the contribution from the plasma mode when $\omega > \omega_p$. It is also seen from (4b) that the resistive part R_{em} of the electromagnetic mode impedance is zero when the plasma has no losses; this result is a consequence of the quasi-static approximation made in the analysis. When small collisional losses are introduced, R_{em} is a maximum near the plasma frequency. The values of R_{em} and R_p for typical experimental electron-temperatures and collision frequencies have been calculated and their relative magnitudes have been compared. These results will be discussed later in this paper.

3. Electrical Conductivity of a Non-Maxwellian Plasma.

In deriving an expression for the antenna impedance, King et al.[1] have assumed the plasma to be a 'Lorentzian' gas, where the collision term is independent of the electron energy; the a.c. electrical conductivity of the plasma obtained from Langevin's equation is given by

$$\sigma = \frac{ne^2}{m} \frac{(\nu_e - j\omega)}{m(\nu_e^2 + \omega^2)} \tag{5}$$

where ν_e is the effective collision frequency. Although Balmain has considered the electroacoustic radiation due to the finite electron temperature of the plasma, his results are valid only for a Maxwellian plasma since he has assumed that the collision frequency is independent of electron temperature; hence, his expression for plasma conductivity for the electromagnetic mode is identical to the one used by King.

Although this simple 'conventional' expression for the conductivity has been used quite extensively in the literature, especially by ionospheric physicists, it is still subject to criticism since it is not strictly applicable to a 'statistical' ensemble of electrons having a distribution of energy, but, rather, to a 'microcanonical' assembly with a known, average electron energy. To obtain a more rigorous expression for the plasma conductivity based on statistical considerations one needs to start with the Boltzmann equation; a solution has been obtained by Allis[20] and Margenau[21] by expanding the distribution function in terms of spherical harmonics in velocity space and by Fourier series in time. The high frequency electronic conductivity of the plasma is given by[22]

$$\sigma = \frac{ne^2}{m} [B - jD] \tag{6a}$$

where

$$B = \frac{4\pi}{3} \int_0^\infty f_1 v^4 \, dv \tag{6b}$$

and

$$D = -\frac{4\pi}{3} \int_0^\infty \frac{\omega}{\nu_m} f_1 v^4 \, dv \tag{6c}$$

and

$$f_1 = -\frac{\nu_m}{\nu_m^2 + \omega^2} \frac{1}{v} \frac{dF_0^0}{dv} \tag{6d}$$

where ν_m is the electron-neutral velocity-dependent collision frequency for momentum transfer. F_0^0 is the symmetrical part of the distribution function and depends only on the velocity $\bar{v}$. It is the distribution for electrons prevailing in the plasma when the r.f. field on the antenna is absent. In most cases it is reasonable to assume that this distribution is Maxwellian in nature, with an arbitrary electron temperature T_e.

$$F_0^0 = (\beta/\pi)^{3/2} e^{-\beta v^2} \tag{7a}$$

where

$$\beta = m/2kT_e \tag{7b}$$

The collision frequency of momentum transfer ν_m within the integral sign in (6b-d) can be expressed as

$$\nu_m = \rho Q v \tag{8a}$$

where ρ is the number of gas atoms per cubic centimeter.

$$\rho = 2.687 \times 10^{19} (P/760)(273/T_g) \tag{8b}$$

where P is the neutral gas pressure in Torr, T_g is the neutral gas temperature in degrees Kelvin. Q is the electron-neutral collision cross-section for momentum transfer. Substitution of (7) and (8) into (6b) yields

$$f_1 = \frac{\rho Q v \; 2\beta \; (\beta/\pi)^{3/2} \exp(-\beta v^2)}{\rho^2 Q^2 v^2 + \omega^2} \tag{9}$$

For a Maxwellian gas, the cross-section for momentum transfer Q varies inversely as the electron velocity. Hence, ν_m is a constant with respect to electron velocity. Thus, from equation (6b)

$$B = \frac{4\pi}{3} \frac{\nu_m}{\nu_m^2 + \omega^2} 2\beta (\beta/\pi)^{3/2} \int_0^\infty \exp(-\beta v^2) v^4 dv \quad (10)$$

The integral on the right hand side may be evaluated by using the identity

$$\int_0^\infty e^{-\beta v^2} v^4 dv = \frac{3}{8} (\sqrt{\pi}/\beta^{5/2}) \quad (11)$$

From equations (6), (10) and (11) it follows that for a Maxwellian gas

$$\sigma = \frac{ne^2}{m} \left[\frac{\nu_m - j\omega}{\nu_m^2 + \omega^2} \right] \quad (12)$$

A comparison of equations (5) and (12) indicates that for a Maxwellian gas the effective collision frequency ν_e obtained from the Lorentzian model is equal to the average collision frequency $\nu_m(\bar{v})$ where $\bar{v} = (3kT_e/m)^{1/2}$ is the root mean square velocity.

However, most gaseous plasmas are non-Maxwellian in nature; the collision frequency of these gases has a strong functional dependency on the electron velocity. For example, for air and nitrogen Q is proportional to velocity. For water Q varies as $1/v^2$. In the experiments described in this paper, the gas used for collision frequency measurements was Helium. The collision cross-section Q of Helium as a function of electron energy is shown in Figure 5 of Reference 3. It can be seen from this figure that ionized Helium is a non-Maxwellian plasma since the Q is reasonably constant with respect to electron velocity. Hence, the effective collision frequency ν_e cannot be simply equated with the collision frequency obtained from kinetic theory as defined in equation (6). Numerical and experimental comparisons of the two conductivity expressions for non-Maxwellian plasmas such as air, etc., have been made by Margenau and Stillinger[23] and by Kane[24]; large discrepancies between the two models have been noticed. It follows that in any experiment involving a precise measurement of the collision frequency using electromagnetic methods, it is necessary to discriminate between the approximate and accurate formula for the conductivity.

Several authors have suggested convenient mathematical expedients in order to relate the two models and to define the collision frequency of plasmas in an unambiguous way. Molmud[22] has suggested the use of complex effective collision frequencies in the Lorentz model, while Whitmer and Hermann[25] have defined both an effective collision frequency and an effective plasma frequency; other possibilities are also available. For example, Shkarofsky[26] has used g and h functions. For the purpose of analyzing the experimental data presented in this paper, the prescription suggested by Molmud[23] appears to be the most convenient, even though the method becomes less accurate as the neutral gas pressure is increased. Q for Helium is reasonably independent of electron velocity, which means that the collision frequency ν_m varies directly as the velocity v; the mean free path of the electrons $L = v/\nu_m$ is, hence, a constant.

Molmud[23] has shown that for a constant Q the expressions for B and D as given by equations (6b) and (6c) reduce to the following form:

$$B = \frac{1}{\omega} \frac{4x^{1/2}}{3\sqrt{\pi}} \quad 1 - x - x^2 e^x E_i(-x) \tag{13a}$$

$$D = \frac{1}{\omega}\frac{4x}{3\sqrt{\pi}}\left[\left(\frac{1}{2} - x\right)\pi^{1/2} + \pi x^{3/2} e^{x}\left(1 - \phi(\sqrt{x})\right)\right] \tag{13b}$$

where

$$x = \frac{4\omega^2}{\pi\nu_m^2} \; ; \quad \nu_m = 2\rho Q/(\pi\beta)^{1/2} \tag{13c}$$

$$- E_i(-x) = \int_x^\infty \frac{e^{-t}}{t}\, dt$$

and

$$\phi(z) = \frac{2}{\sqrt{\pi}} \int_0^z \exp(-t^2)\, dt$$

where $E_i(-x)$ and $\phi(z)$ are the Exponential Integral and Error Integral, respectively. On equating the conductivity expressions for the two models given by equations (5) and (6), the complex effective collision frequency for the Lorentzian case necessary for equalizing the two conductivities is given by

$$\nu_e = \nu_{eR} + j\nu_{eI} \tag{14a}$$

When $\nu_m/\omega < 1$,

$$\nu_{eR} = \frac{4}{3}\nu_m\left(1 - 0.22\, \nu_m^2/\omega^2\right) \tag{14b}$$

and

$$\nu_{eI} = 0.18\left(\nu_m^2/\omega^2\right)\left[1 + 2.14\, \nu_m^2/\omega^2\right] \tag{14c}$$

When $\nu_m/\omega > 1$,

$$\nu_{eR} = (3\pi\nu_m/8)\left(1 + 0.28\, \omega^2/\nu_m^2\right) \tag{14d}$$

and

$$\nu_{eI} = 0.18\omega\left(1 - 7.5\, \omega^2/\nu_m^2\right) \tag{14e}$$

when ν_m in the above two equations is strictly defined as $\nu_m = \int \rho Q(v)\ Fv\ dv$. For Helium with a constant Q, this reduces to the form

$$\nu_m = 2\rho Q\ (2kT_e/\pi m)^{1/2} \tag{15}$$

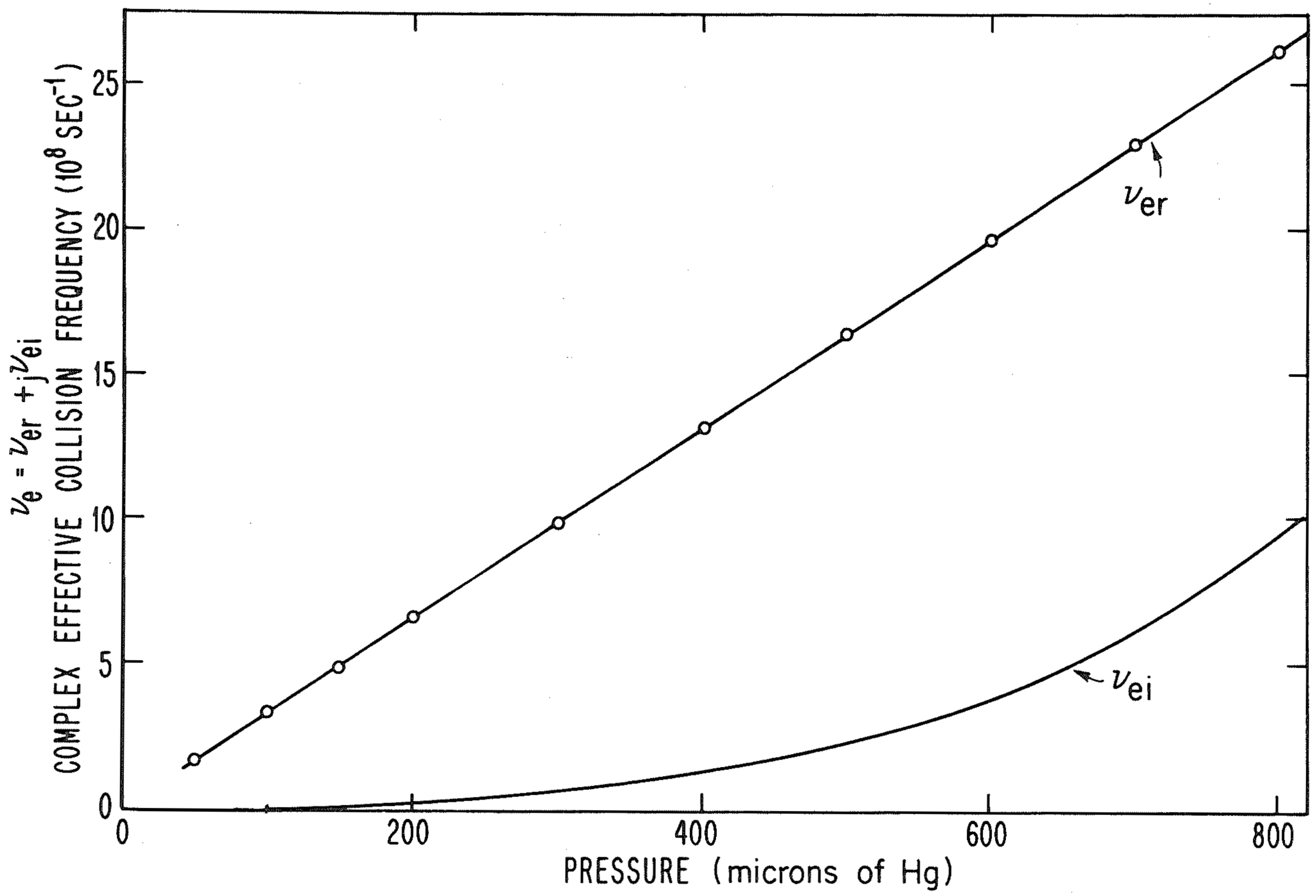
Equations (14a) and (14b) indicate that when the imaginary part of the complex effective frequency is small ($\nu_{eI} << \nu_{eR}$), the collision frequency for momentum transfer ν_m can be obtained from the effective collision frequency ν_e of the Lorentzian model using the relation

$$\nu_e = \frac{4}{3}\nu_m\ (1 - 0.22\ \nu_m^2/\omega^2) \qquad \text{when } \nu_m/\omega < 1 \tag{16a}$$

and

$$\nu_e = (3\pi\nu_m/8)(1 + 0.28\ \omega^2/\nu_m^2) \qquad \text{when } \nu_m/\omega > 1 \tag{16b}$$

In order to obtain the relative magnitudes of ν_{eI} and ν_{eR}, the complex effective collision frequency for Helium was calculated for various neutral gas pressures used in the experimental investigation; in these calculations the electron temperature T_e was 6.5 electron volts and the signal frequency $\omega/2\pi$ was 450 MegaHertz. The results are shown in Figure 1. It is seen from this figure that the assumption $\nu_{eI} << \nu_{eR}$ is valid only at pressures below 500μ Hg. At higher pressures ν_{eI} increases rapidly in value and, thus, there is no simple method of obtaining ν_m from the measured effective collision frequency ν_e. In the high pressure limit it may become necessary to use the method of Whitmer and Hermann[25] and prescribe both an equivalent electron density and collision frequency to relate the Lorentzian and kinetic theory conductivity expressions.



COMPLEX EFFECTIVE COLLISION FREQUENCY REQUIRED FOR EQUATING LORENTZIAN AND KINETIC THEORY MODELS FOR PLASMA CONDUCTIVITY. (ELECTRON TEMPERATURE T_e = 6.5eV COLLISION CROSS SECTION = 4.5A^2 FREQUENCY = 450 MHz)

FIG. 1

4. Experimental Procedure and Apparatus.

In the experimental investigations to be described in the following sections, the collision frequency ν_m is measured by using the following procedure: a) precision impedance measurements are made on the antenna at 11 frequencies over a 2 to 1 range, typically 300 to 600 MHz, which includes the electron plasma frequency. It is preferable to have the plasma frequency fall nearly in the middle of this range (the approximate plasma frequency may be found by using some independent method such as the Langmuir probe). b) Using a 4th degree interpolation polynomial the frequencies at which the maximum values of the resistance, R^{meas}, and magnitude of the impedance, $|Z|^{meas}$, occur are determined as well as the values themselves. c) Using the frequency of $|Z|^{meas}_{max}$ as the plasma frequency ω_p, R^{meas}_{max} is matched to the R^{theo}_{max} value from a family of resistance curves derived from King's theory (eq. 1) with the same plasma frequency, but with the collision frequency as the parameter.* d) The value of the collision frequency for momentum transfer ν_m is then obtained from either equation (16a) or (16b) provided $\nu_{eI} << \nu_{eR}$. e) Lastly, to obtain an estimate of the accuracy of the results, the value of ν_m obtained in this manner is compared with the theoretical results for ν_m calculated using the collision cross-section data of Golden and Bandel.[3]

*This scheme has been formalized and a computer program written to process the measured data and return the plasma frequency and effective collision frequency. At high collision frequencies the assumption that $|Z|_{max}$ occurs at the electron plasma frequency is not accurate. But, for the laboratory plasma measured using the above technique, the error involved in the determination of ν_e and ω_p is only a few percent ($\approx 5\%$). Thus, it is questionable whether one should use a more sophisticated curve fitting procedure.

A block diagram of the experimental apparatus is shown in Figure 2. The experimental investigations were made in a hot-cathode, Helium d. c. discharge tube, 14 cms in diameter and 38 cms in length. The antenna was a copper rod, 4 mms in diameter and 3.5 cms in length. Since the length of the antenna is very short compared to the wavelength of the r. f. signal frequency used in the experiment, the radiation field is negligible and the plasma discharge around the antenna has a significant effect only on the reactive near field of the antenna. Hence, the finite size of the plasma container does not seriously compromise the 'infinite' plasma assumption made in the theoretical analysis. This was investigated experimentally by using plasma columns of larger and larger diameter, until the size of the plasma column had a negligible effect on the antenna impedance. The electron density profile in the plasma column is ambipolar diffusion controlled with a radial variation of the type $n(r) = n(0)\, J_0(2.404r/a)$ where a is the radius of the discharge tube and n(0) is the electron density at the axis. Since the antenna was placed along the axis of the positive column, it is evident that when the radius of the discharge column is sufficiently large, the fields of the antenna see essentially a homogeneous plasma.

The antenna was connected to the inner conductor of a vacuum-tight precision coaxial connector; this eliminates large junction effects near the driving point of the antenna. The coaxial connector was mounted at the center of a copper disc which also served as the anode of the discharge tube. The electron density and electron temperature of the plasma were determined by a planar Langmuir probe. The Langmuir probe V-I characteristics were scanned electronically at the rate of once every two seconds with a semi-log plot displayed on a cathode ray tube. This rate is slow enough to

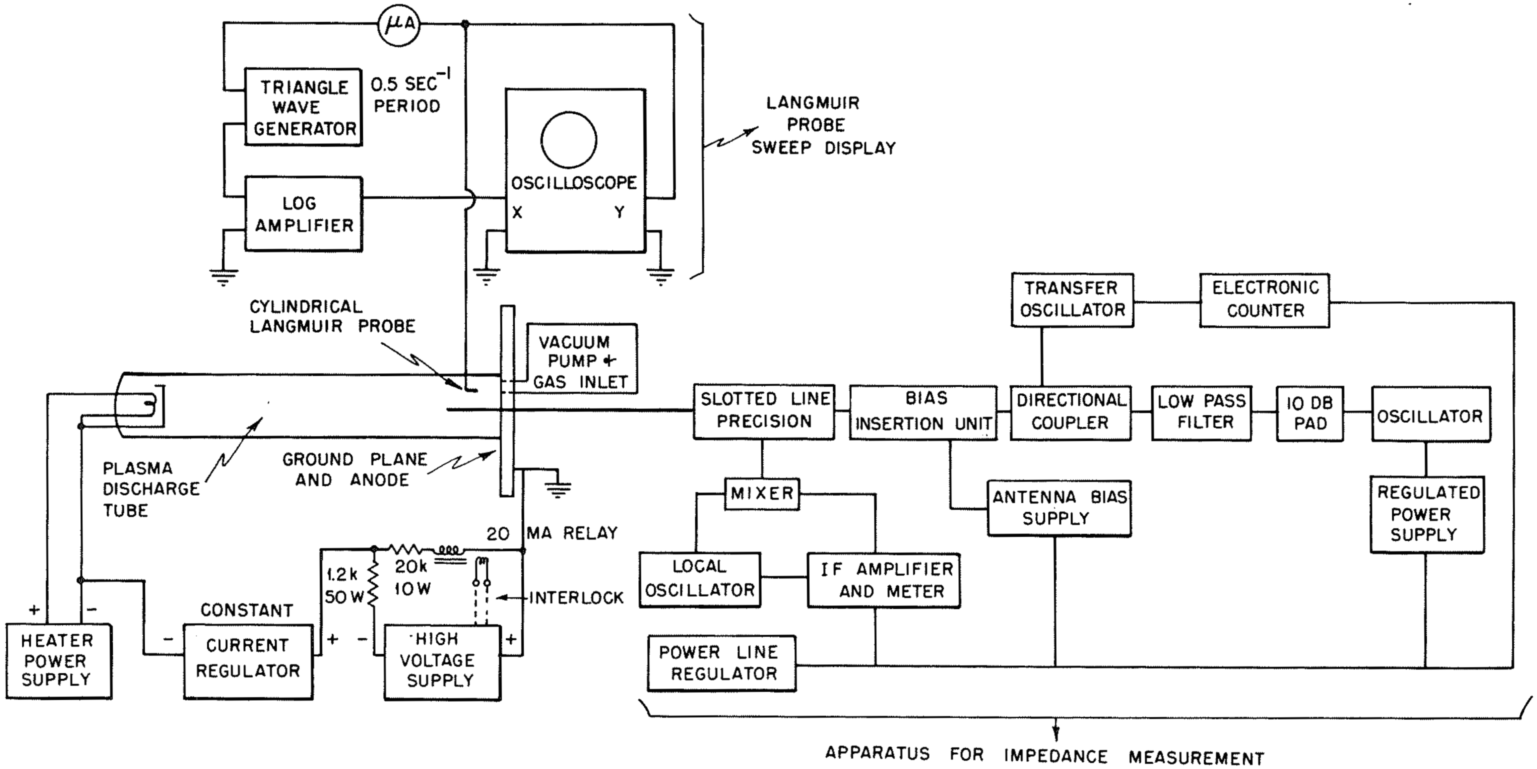


FIG. 2 BLOCK DIAGRAM OF EXPERIMENTAL APPARATUS

make any hysterisis in the system negligible but fast enough to exclude drift effects in the plasma discharge. The semi-log plot allows a direct reading of the electron temperature. This was facilitated by a calibrated overlay placed on the CRT face. Since the electron temperature is important both for calculating collision frequencies as well as estimating electroacoustic effects, great care was exercised in making these measurements. The value of the electron temperature measured by this technique was compared with the theoretical values calculated from the cPR vs T_e/V_i characteristics for Helium (see Ref. 26, Fig. 14-11). The ionization potential for Helium $V_i = 24.46$ volts and $c = 3.9 \times 10^{-3}$. P is the neutral gas pressure in mms of Hg and R is the radius of the discharge column in cms. The results are shown in Figure 3; they indicate good agreement between theory and experiment.

The electron-neutral collision frequency for momentum transfer $\nu_m = 2.66 \times 10^{22} \times \sqrt{T_e}\, Q_m P$ was determined from the collision cross-sections for Helium measured by Golden and Bandel.[3] In the above equation T_e is the electron temperature of the plasma in degrees Kelvin, P is the neutral gas pressure in mms of Hg, and Q_m is the electron-neutral collision cross-section for momentum transfer in cms^2. Since the plasma is weakly ionized it is assumed that the electron-ion collisions are negligible.

5. Comparison of Experimental Results with Theoretical Results of King and Applications to Plasma Diagnostics.

The antenna impedance was measured as a function of ω_p/ω by varying the signal frequency in the vicinity of the plasma frequency. The plasma frequency during the experiment was stabilized by using a constant

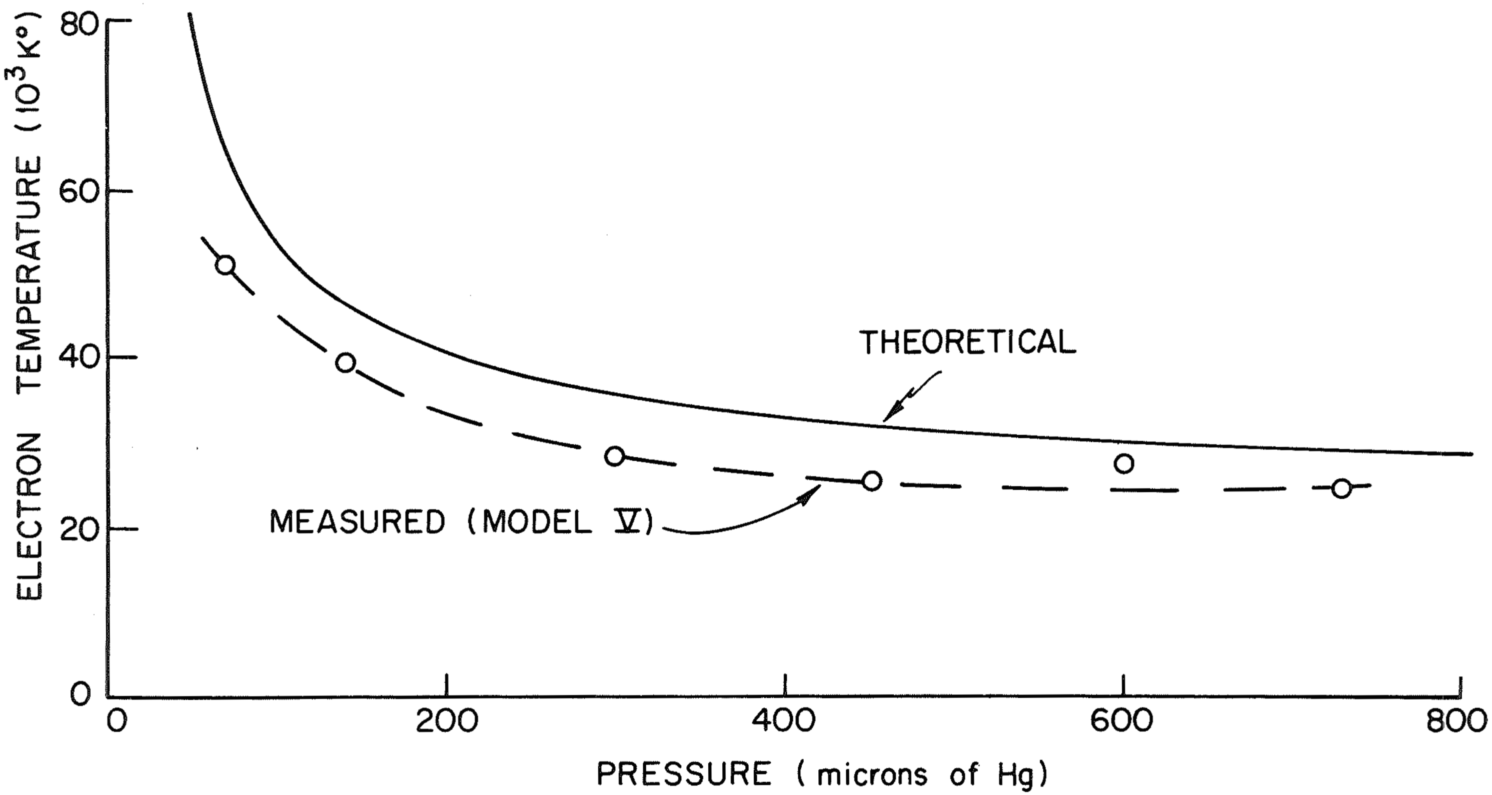
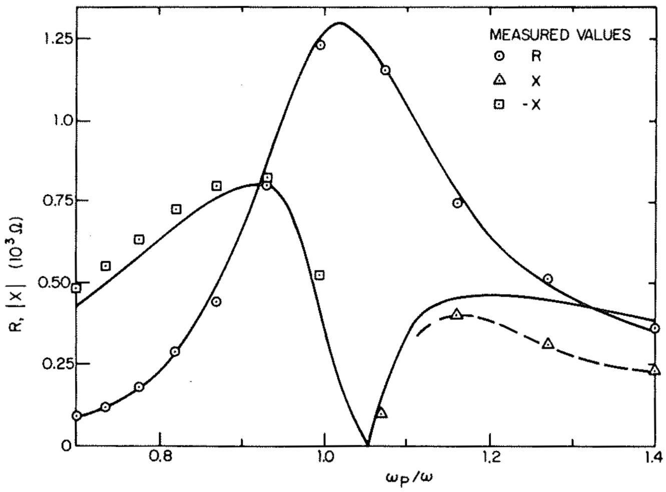


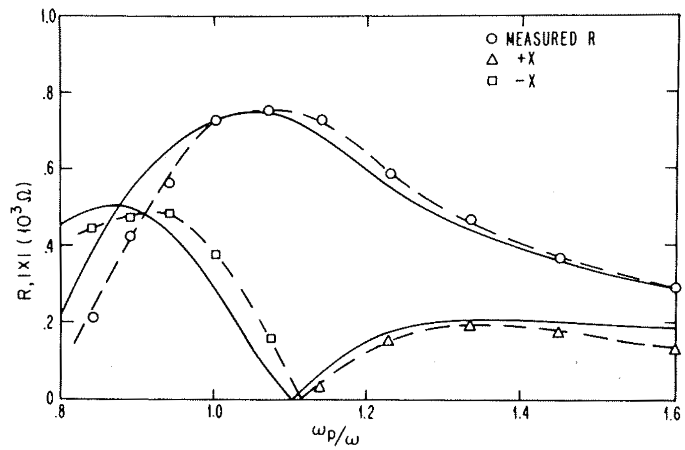
FIG. 3 COMPARISON OF MEASURED ELECTRON TEMPERATURE WITH THEORETICAL VALUES CALCULATED FROM THE CpR vs. Te/Vi CURVE FOR A HELIUM POSITIVE COLUMN (*FIG. 14-11 OF S.C. BROWN'S BASIC DATA OF PLASMA PHYSICS, M.I.T. PRESS 1959*)

current regulator in series with the d. c. power supply to the discharge tube. The impedance measurements were made at various neutral gas pressures and the results were compared with the theoretical values of the impedance obtained from King's theory (Equation 1). The values of ν_e and ω_p were obtained by a curve-fitting technique described earlier.

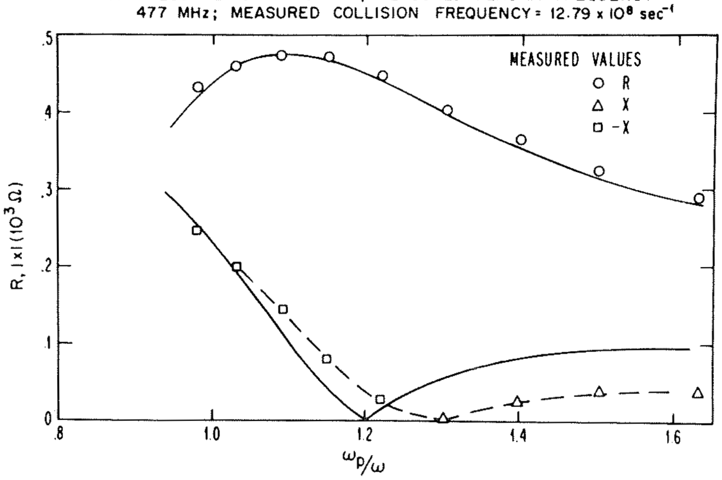
The comparison between theoretical and experimental results for measurements made at neutral gas pressures of 140, 300, and 750 microns of Hg are shown in Figure 4. The theoretical curves were obtained by setting $\omega_p/\omega = 1$ at $Z_{max}^{measured} = \sqrt{R^2 + X^2}$ and $R_{max}^{theoretical} = R_{max}^{measured}$. It can be seen from this figure that there is good agreement between theory and experiment for values of ω_p/ω between 0.8 and 1.6. <u>This demonstrates that the shape of the theoretical curve is correct and that King's theory gives a good description of the impedance behavior of the antenna in the vicinity of the plasma frequency of a collision-dominated plasma</u>. Having obtained the experimental value for ν_e in this manner, ν_m was then obtained from equation 16. In Figure 5 the experimental values for ν_e are compared with the theoretical values obtained from the collision cross-section data of Golden and Bandel.[3] It is seen that there is fairly good agreement, although the experimental values appear to be somewhat larger than that predicted by theory. This increase is probably due to a) errors in measuring neutral gas pressures due to calibration errors in the thermocouple gauge, b) impurities in the gas system which can increase ν_e, and c) non-collisional damping phenomena which have not been taken into account in either of the theories. Non-collisional damping effects have also been noticed by Waletzko and Bekefi[27] and by Crawford and Harp[28] in impedance measurements made



4a PRESSURE = 140 MICRONS; MEASURED PLASMA FREQUENCY = 418 MHz; MEASURED COLLISION FREQUENCY = 7.288 x 10^8 sec^{-1}

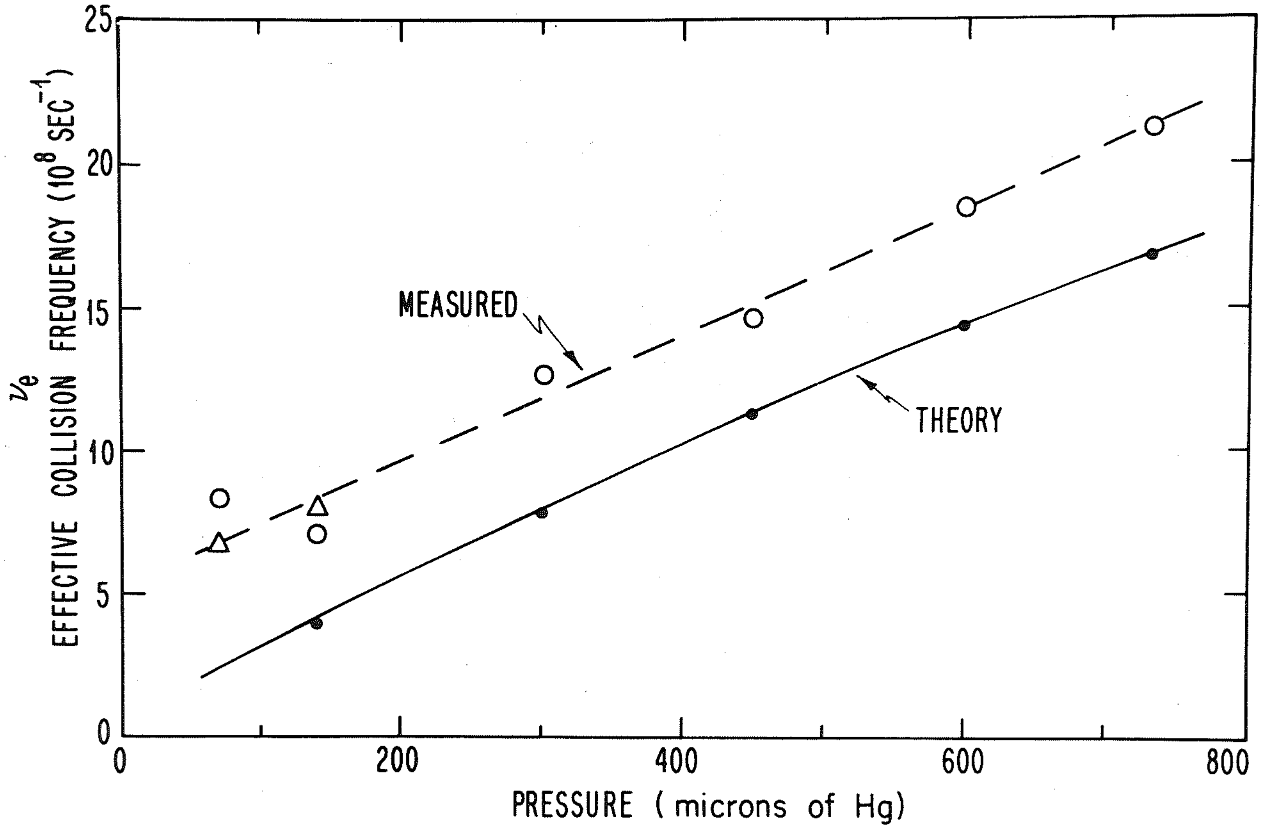


4b PRESSURE = 300 MICRONS; MEASURED PLASMA FREQUENCY = 477 MHz; MEASURED COLLISION FREQUENCY = 12.79 x 10^8 sec^{-1}



4c PRESSURE = 730μ; MEASURED PLASA FREQUENCY = 587 MHz; MEASURED COLLISION FREQUENCY = 21.19 x 10^8 sec^{-1}

FIG 4 COMPARISON OF THEORETICAL AND EXPERIMENTAL RESULTS FOR THE IMPEDANCE OF A SHORT DIPOLE ANTENNA IN A PLASMA (GAS-HELIUM; h = 3.49 x 10^{-2} METERS; a = 2.13 x 10^{-3} METERS)



COMPARISON BETWEEN MEASURED AND THEORETICAL VALUES FOR THE EFFECTIVE ELECTRON-NEUTRAL COLLISION FREQUENCY FOR MOMENTUM TRANSFER.

FIG. 5

with a spherical probe immersed in an isotropic plasma. The results shown in Figure 5, however, indicate that, in general, reasonable values for the collision frequency can be obtained by using the antenna as a diagnostic probe in a collision dominated plasma. The value of the electron density obtained from the 'curve-fitting' procedure shown in Figure 4 was compared with independent density measurements made with a planar Langmuir probe. The results are compared in Table 1 and show once again that the antenna can be used for making reliable measurements of the electron density.

6. Effect of Electron Temperature and Ion-Sheath Effects on Antenna Impedance.

It is obvious from the previous discussion that the experimental results are in good agreement with King's theory[1] based on the cold plasma model. For warm plasmas, having significant electron temperature, Balmain's theory (Equation 4) indicates that an additional resistance term is necessary to account for the electroacoustic radiation by the antenna. If resistance term R_p due to this plasma mode is small compared to the resistance R_{em} due to the electromagnetic mode, the cold plasma assumption is still valid insofar as the diagnostic measurements are concerned. If, however, R_p and R_{em} are comparable, significant errors are likely to occur if the cold plasma theory is used for making collision frequency measurements. In order to estimate the relative magnitudes of R_{em} and R_p calculations were made using equation (4) for typical plasma parameters occurring in the experiments just described. The results are shown in Figure 6; the values of ν_e, T_e, ω and

TABLE 1

Comparison of Plasma Frequency Determined by a Planar Langmuir Probe and from R.F. Impedance Measurements using a Short Cylindrical Antenna

Neutral Gas Pressure of Helium (microns)	Discharge Current (milliamps)	Plasma Frequency Short Antenna (MHz)	Plasma Frequency Langmuir Probe (MHz)
70	350	450	404
140	300	418	420
300	250	477	420
450	250	510	458
600	325	630	543
730	250	587	458

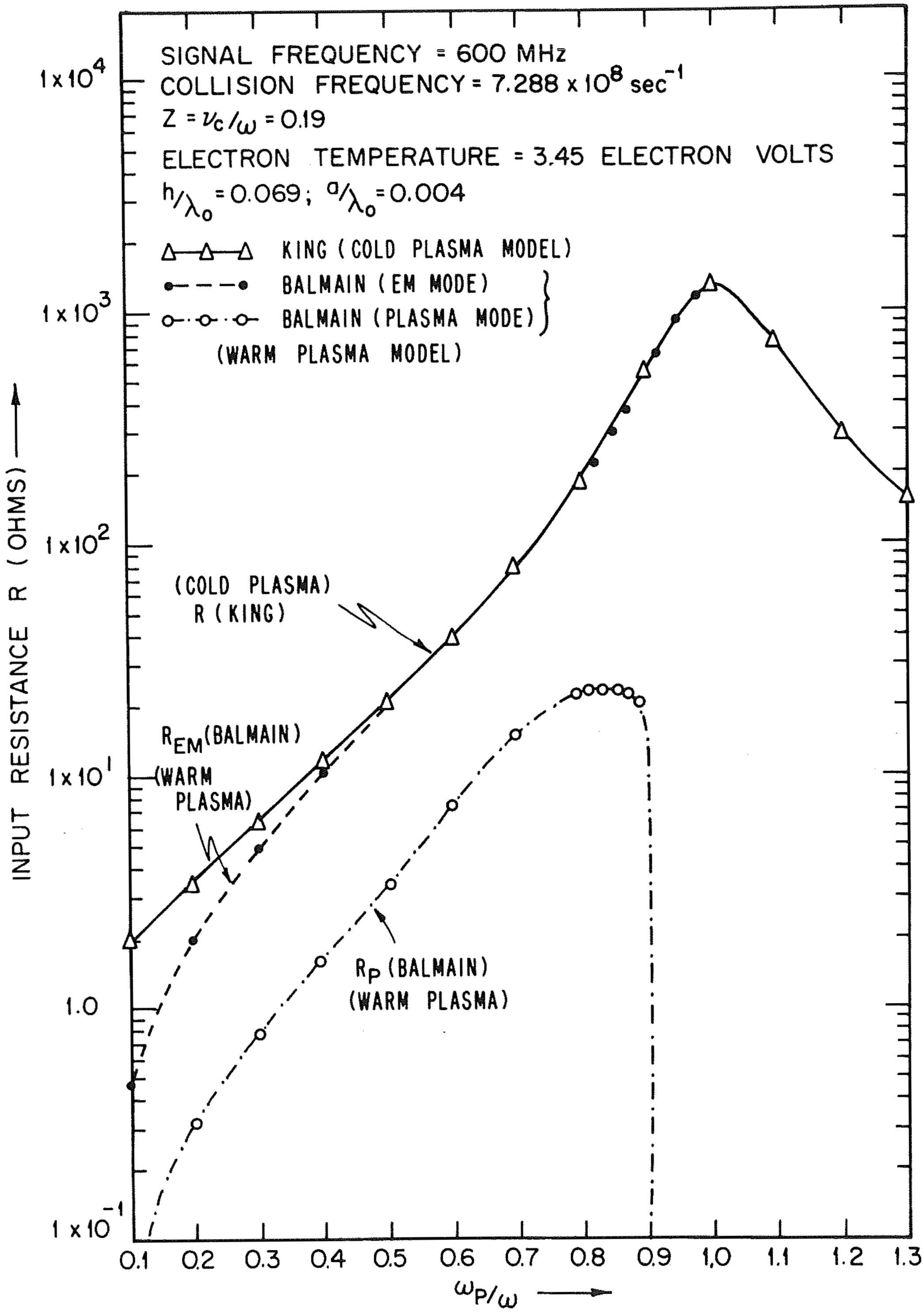


FIG. 6 EFFECT OF ELECTRON TEMPERATURE ON INPUT RESISTANCE OF THE ANTENNA. – COMPARISON OF COLD AND WARM PLASMA MODELS.

the antenna dimensions used in these calculations are similar to those occurring in the impedance measurements shown in Figure 4a. It is seen from this figure that R_p reaches its maximum value of 22.78 ohms at $\omega_p/\omega = 0.84$ where the corresponding $R_{em} = 270$ ohms. Hence, since the maximum value of $R_p/R_{em} = 0.084$, the electroacoustic mode makes a negligible contribution to the input resistance of the antenna when the collision frequency is the dominant loss mechanism. It is also interesting to note that R_p falls off sharply to zero as the plasma frequency is approached. The resistance R calculated from King's theory (Equation 1) is also plotted in Figure 6. It is seen that R_{em} obtained from Balmain's theory agrees closely with R of King's for ω_p/ω between 0.5 and 1.0. For $\omega_p/\omega < 0.4$, R_{em} drops off sharply to zero and differs significantly from King's theory; this discrepancy is due to the quasi-static approximations made by Balmain in his analysis, as a consequence of which the radiation resistance is neglected.

In Figure 7 the theoretical values for R_p, computed for various electron temperatures ranging from 3 to 50 electron volts, have been plotted as a function of ω_p/ω. R_{em} has also been plotted in the same figure for comparison. The signal frequency $\omega/2\pi$ was 600 MHz and the effective collision frequency ν_e was 0.16ω in these calculations. The plasma is assumed to be an idealized Maxwellian gas, so that the effective collision frequency is independent of the electron temperature. This figure indicates that for collision frequencies of this magnitude, the maximum value of R_p/R_{em} = 0.185 even for electron temperatures T_e as high as 10 electron volts. For R_p to become comparable to R_{em}, the electron temperature must correspond to 50 electron volts; these are abnormally high electron temperatures which are rarely encountered in laboratory plasmas where a typical T_e is of the

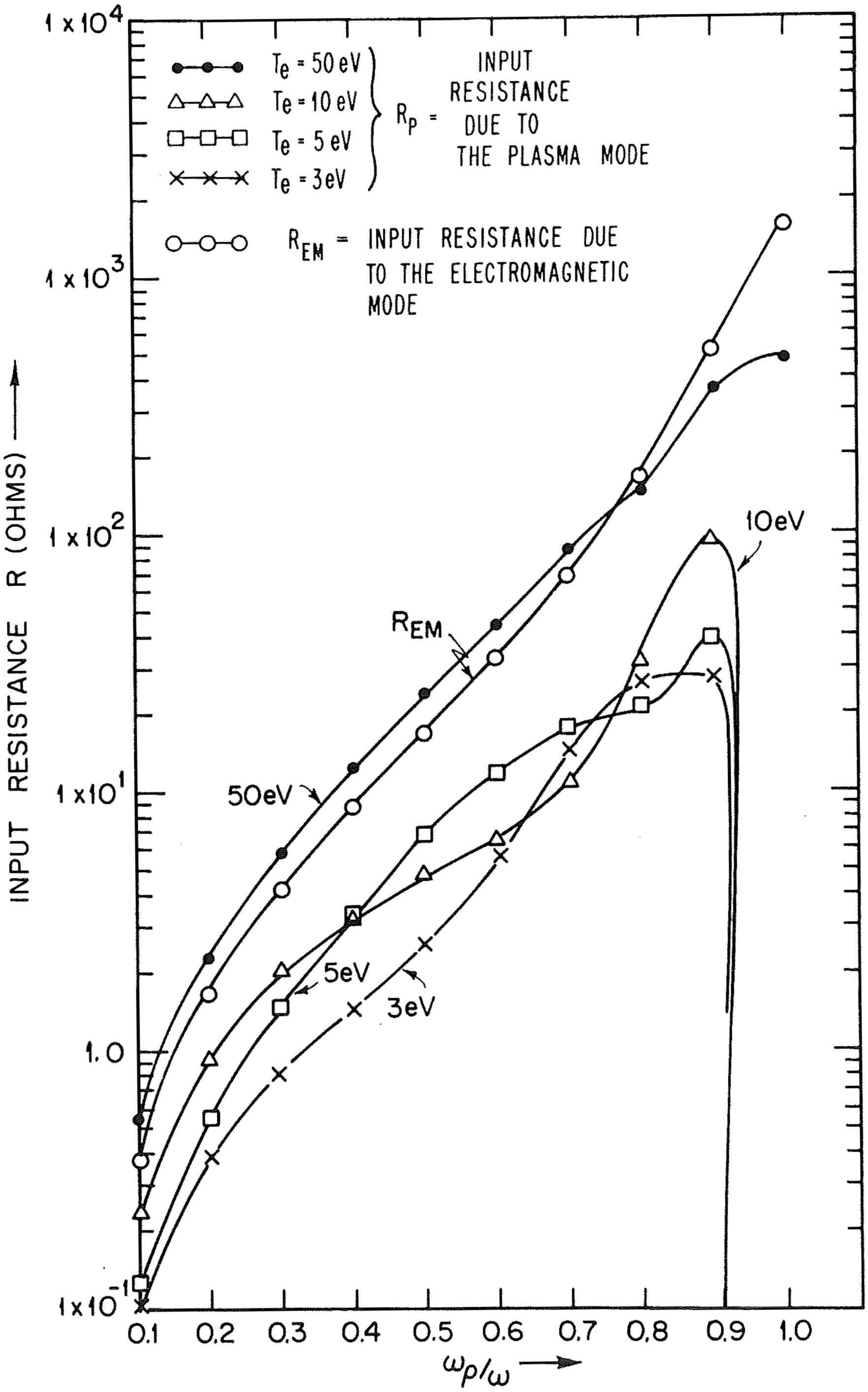
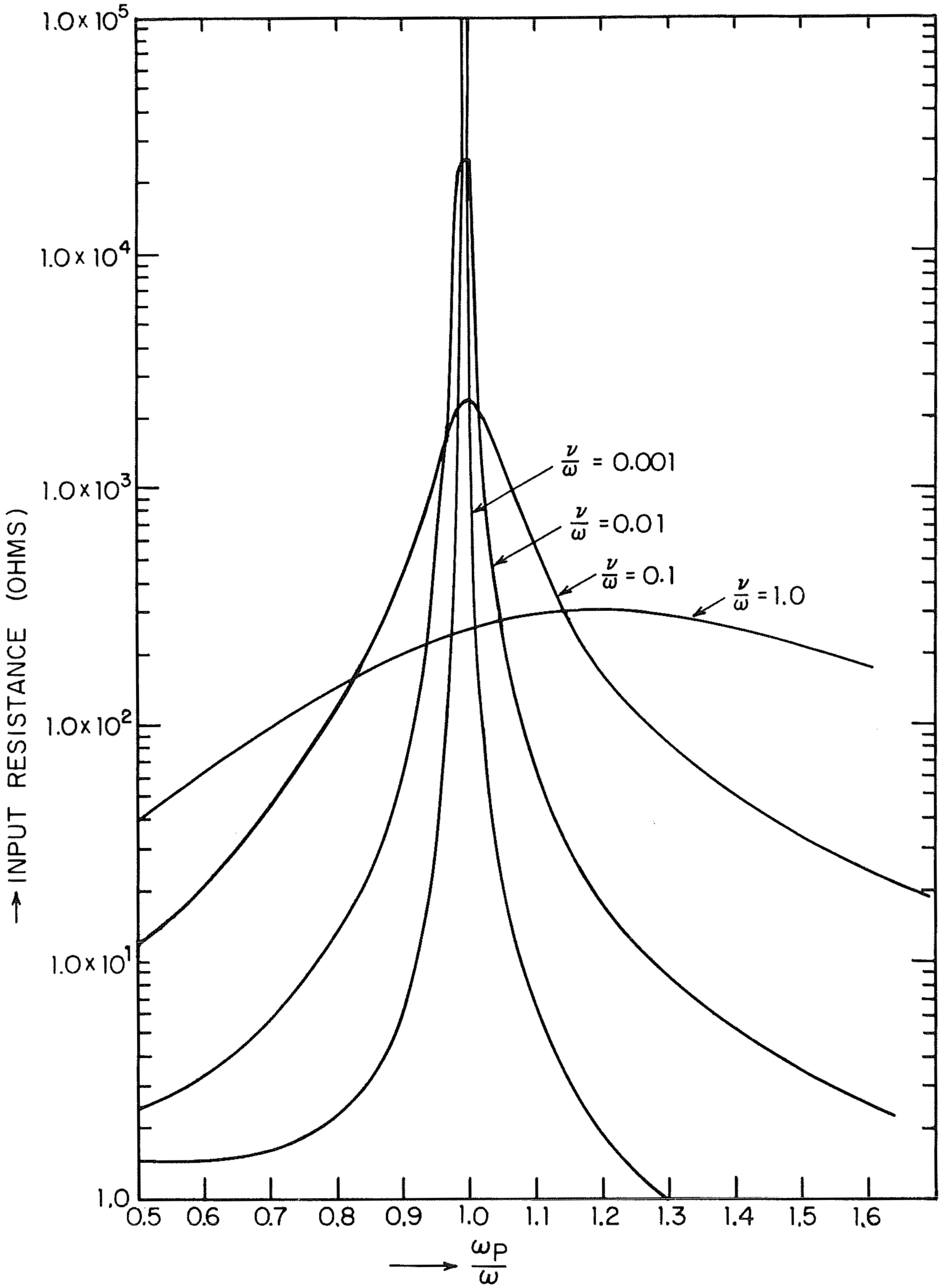


FIG. 7 INPUT RESISTANCE OF ANTENNA FOR VARIOUS ELECTRON TEMPERATURES.

order of a few electron volts. The electron temperatures in the ionosphere vary from 300 to 1000^o K (0.025 to 0.09 electron volts) with the highest collision frequencies occurring in the D-region (60 - 80 kilometers) where ν_e varies from 13 to 0.2 megacycles.[7] The results shown in Figure 7 indicate that for most laboratory plasmas, the collisions rather than the electron temperature play a dominant role in determining the input resistance in the vicinity of the plasma frequency. In Figure 8 the input resistance has been calculated for various ν_e/ω from King's cold plasma theory. These results show that collisional losses as small as $\nu_e/\omega = 0.001$ can result in a significant peaking of the input resistance in the neighborhood of the plasma frequency. For very high collision frequencies ($\nu_e/\omega = 1.0$) the resistance varies in a smooth manner near ω_p. In order to apply this impedance probe technique for plasma diagnostics, an error analysis was performed by considering various values of ν_e and T_e. The range where the probe can be used for determining ν_e within 10% was determined and is shown in Figure 9. Carlin and Mittra[29] have shown that the power in the electroacoustic mode drops off sharply when the dimensions of the antenna are large compared to the electroacoustic wavelength. For an electron temperature of 5.2 electron volts (6×10^4 Kelvin), the electroacoustic wavelength is typically of the order of 10^{-2} cms for a signal frequency of 1 GigaHertz. Hence, by using long antennas it is possible to reduce the maximum value of R_p even further, so that the probe can be used for diagnosing even plasmas having low collisional losses.

The effect of the ion-sheath on the impedance of the bare antenna was investigated by applying various d.c. bias voltages to the antenna to vary the sheath thickness. This was generally found to have a negligible



VARIATION IN INPUT RESISTANCE OF A SHORT ANTENNA AS A FUNCTION OF COLLISION FREQUENCY. $h/\lambda = 0.0698$; $a/\lambda = 0.0042$

FIGURE 8

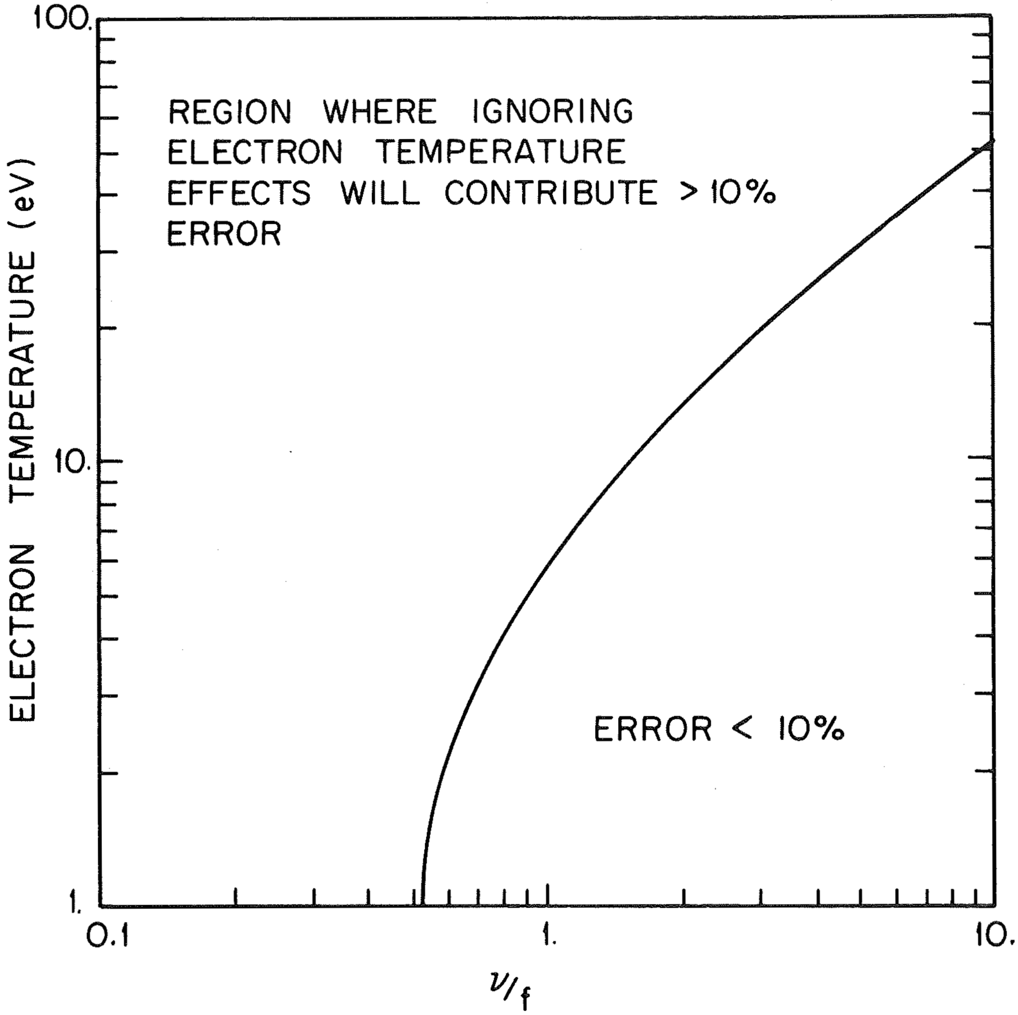


FIG. 9 DOMAIN OF VALIDITY FOR COLLISION FREQUENCY MEASUREMENT USING THE IMPEDANCE PROBE TECHNIQUE. (*DETERMINED FOR $f = 600$ MHz AND $\omega_p/\omega = 0.9$ USING BALMAIN'S FORMULA AS A REFERENCE.*)

effect on the antiresonant impedance behavior of the antenna near ω_p. This is probably due to the antenna radius being much larger than the Debye wavelength of the plasma used in these investigations. For an electron density $n = 2 \times 10^9$ electrons/cm^3 and an electron temperature $T_e = 5$ electron volts, the Debye wavelength is approximately 10^{-2} cms, as compared to the antenna radius of 0.2 cms. Experiments by Waletzko and Bekefi[27] and Harp and Crawford[28] on a spherical probe indicate that ion-sheath effects become noticeable only when the antenna dimensions are of the same order of magnitude as the Debye length. Investigations by Mlodnosky and Garriott[11] indicate that the effects of an ion-sheath on the antenna admittance are noticeable only at frequencies well below the plasma frequency of a low density lossless plasma. Another probable reason is that when the collision frequencies become significant, the 'resonance' conductance peak gets damped rapidly. Buckley[30] has shown that for a spherical resonance probe, the width of the conductance peak is proportional to $0.1\omega_p + 1.6\nu_e$. Since the ν_e in our experiments were quite large, it is quite likely that the resonance effects were too heavily damped to be noticeable.

7. Conclusions

Our investigations show that 1) an electrically short cylindrical antenna can be used for diagnosing the electron density and collision frequency of the plasma, 2) near the plasma frequency the influence of collisional losses on the antenna resistance is similar to that obtained from including electroacoustic wave effects, 3) the resistance contribution due to the electroacoustic mode is negligible when the collision frequency is high, so that the 'cold' plasma theory is applicable for interpreting experimental data, 4) the velocity dependence of the collision frequency can be satisfactorily accounted for by introducing a fictitious complex effective collision frequency, and 5) the effect of the ion-sheath on the antenna impedance near ω_p is small when the electron density and collision frequency are large.

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