TN 310-N-10-69

13 October 1969

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NASA CR 102068

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ABSTRACT

An elementary physical model for the combustion of flammable materials is presented with primary emphasis upon downward burning. The theoretically predicted results are reasonably consistent with available data. The effects of gravity are discussed. A set of system equations required for a more refined model is also presented.

A PHYSICAL MODEL FOR DOWNWARD BURNING

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I. INTRODUCTION

The following is an extension of an earlier document by Mayer[1]; the major improvements being an improved treatment of the simple physical model. From references such as Friedman[2] it appears that a simple, yet universal model for burning rates apparently will not be found. Many factors appear to influence burning rates and these include: geometry, orientation with respect to the "g" field, fuel chemistry, ambient pressure and composition, forced external flow (wind), surface roughness, melt layer integrity and motion, and probably other factors.

To achieve the ultimate goal of theoretically predicting burning rates under varying situations, consideration must be first given to elementary conditions for which experimental data are available. For these conditions, a theoretical or semi-empirical model could then be developed and, eventually, brought into agreement with comparable data. Ultimately, a sophisticated computer program may be developed capable of predicting burning rates at desired accuracy and for (complex) conditions of interest. The present note concerns itself with the first steps of such an effort.

II. DOWNWARD BURNING: PHYSICAL MODEL

In this section, an estimate for downward burning rate will be made. Initially an assumption for the radiant heat flux $\dot{Q}_r$ will be utilized. In a later note, expressions for $\dot{Q}_r$, in terms of more fundamental parameters, such as mixture ratios, will be developed.

Temperature changes are propagated by conduction in a solid and the propagation rate is determined essentially by the materials' thermal diffusivity, α, and geometry. For a given material and geometry, the propagation rate can be easily determined by numerical or analog techniques, in general, and, for simpler geometrics, and initial conditions, analytically.

The time constant τ of a material may be defined as

$$\tau \equiv \frac{\ell_1^2}{\alpha} \tag{1}$$

A simple model for the burning rate of a solid would be one whereby the radiant flame heats the adjacent unburned portion, defined here as the "pre-heat zone", directly by thermal radiation and, indirectly by conduction from the hot, "burning" pyrolysis zone (see figures 1, and 2). The burning rate, V_b, is defined as the rate of advance of the surface interface between the pyrolysis zone and the pre-heat zone. To produce pyrolysis, the material must have its temperature elevated to the critical (pyrolysis) value. In a later, more refined model, Pyrolysis rates as a function of temperature will be given appropriate consideration. When a surface portion of the material reaches the burning pyrolysis temperature, the pyrolysis zone advances if the heat which is released is sufficient to elevate the adjacent section to this temperature also. Other conditions must be met to sustain this burning such as the availability of sufficient oxygen, and mixing of the oxygen with the fuel vapors which are generated by pyrolysis.

The flame temperature is usually so high that much thermal energy is transferred radiantly to that portion of the pyrolysis zone which is closest to the flame. Some of this energy is conducted away from the surface and into the pre-heat zone. In addition, some radiant energy goes directly from the flame to

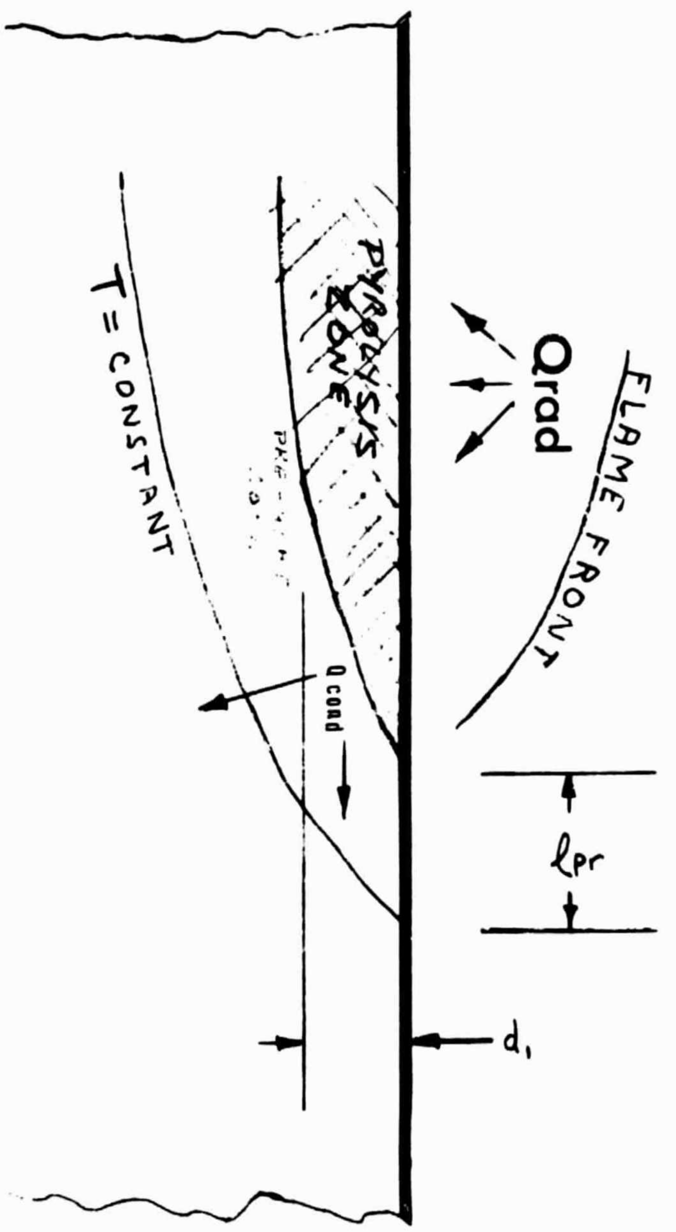

FIGURE 1: SCHEMATIC OF BURNING <u>THICK</u> SPECIMEN

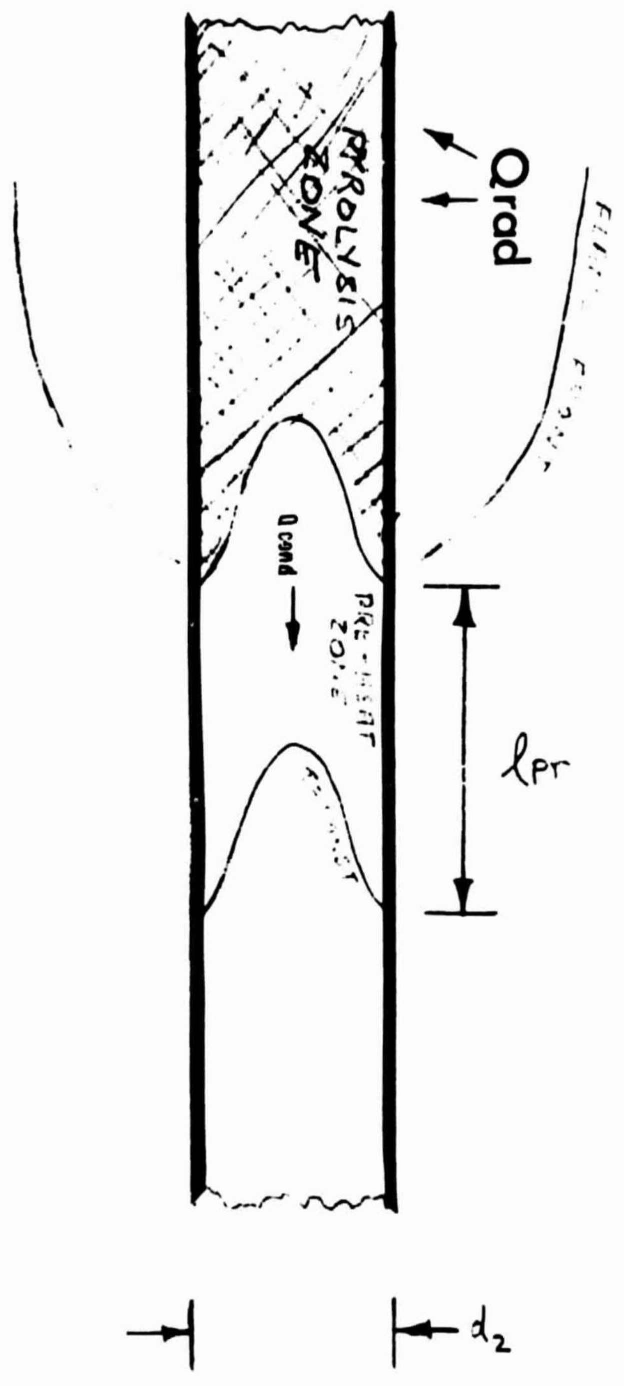

FIGURE 2: SCHEMATIC OF BURNING <u>THIN</u> SPECIMEN

the nearest portion of the pre-heat zone surface. The radiant geometric shape factor is generally significantly lower for the pre-heat zone than for the pyrolysis zone, and so direct radiant heating to the pre-heat zone may be, in general, less than that to the pyrolysis zone (temperature levels and surface characteristics also affect the heat flux). For both zones this energy is conducted away from the surface into the adjacent solid material.

The hot flame also generates a buoyancy force because the hot, combustion product gases have a lower density. This induces a flow of ambient cooler gases past the pre-heat zone. This gas generally provides no cooling to the pyrolysis zone because of the blocking effect of the generated pyrolysis gases. This is covered in detail in Section III.

Consider first the case of conduction away from the pyrolysis zone. This may be approximated by the case of a solid being initially at a uniform temperature T_o and its surface is suddenly changed to a new temperature T_v. Physically this is justified by the fact that the radiant heating is intense and the temperature rapidly increases and then stabilizes at the pyrolysis temperature (or a relatively narrow temperature band), when additional energy is supplied. For this case the normalized temperature profile, $\frac{T-T_v}{T_o-T_v}$, may be represented by the Gaussian error integral G

$$\frac{T-T_v}{T_o-T_v} = G\left(\frac{y}{2(\alpha\theta)^{\frac{1}{2}}}\right) = \frac{2}{(\pi)^{\frac{1}{2}}} \int_0^{\frac{y}{2(\alpha\theta)^{\frac{1}{2}}}} e^{-z^2}\, dz \qquad (2)$$

Figure 3 is a plot of Equation 2. Note that a point at a distance l has an elevated temperature such that its normalized temperature increase is approximately 10% of the rise at the hot surface when

$$\frac{l}{2(\alpha\theta)^{\frac{1}{2}}} = 1.1 \qquad (3)$$

Stated another way, we may define the system time constant $\tilde{\tau}$, and the system characteristic dimension l_1, as

$$\tilde{\tau} \equiv \frac{l_1^{\,2}}{(2.2)^2\,\alpha} \qquad (4)$$

This may be expressed in the form of a velocity, V_1, (by taking their ratio)

$$V_1 \quad \frac{l_1}{\tilde{\tau}} = \frac{4.85\,\alpha}{l_1} \qquad (5)$$

Now, the physical system differs from the simple model in that the pre-heat zone is also convectively cooled, and heat is conducted in <u>two</u> dimensions (for a two dimensional body). If the

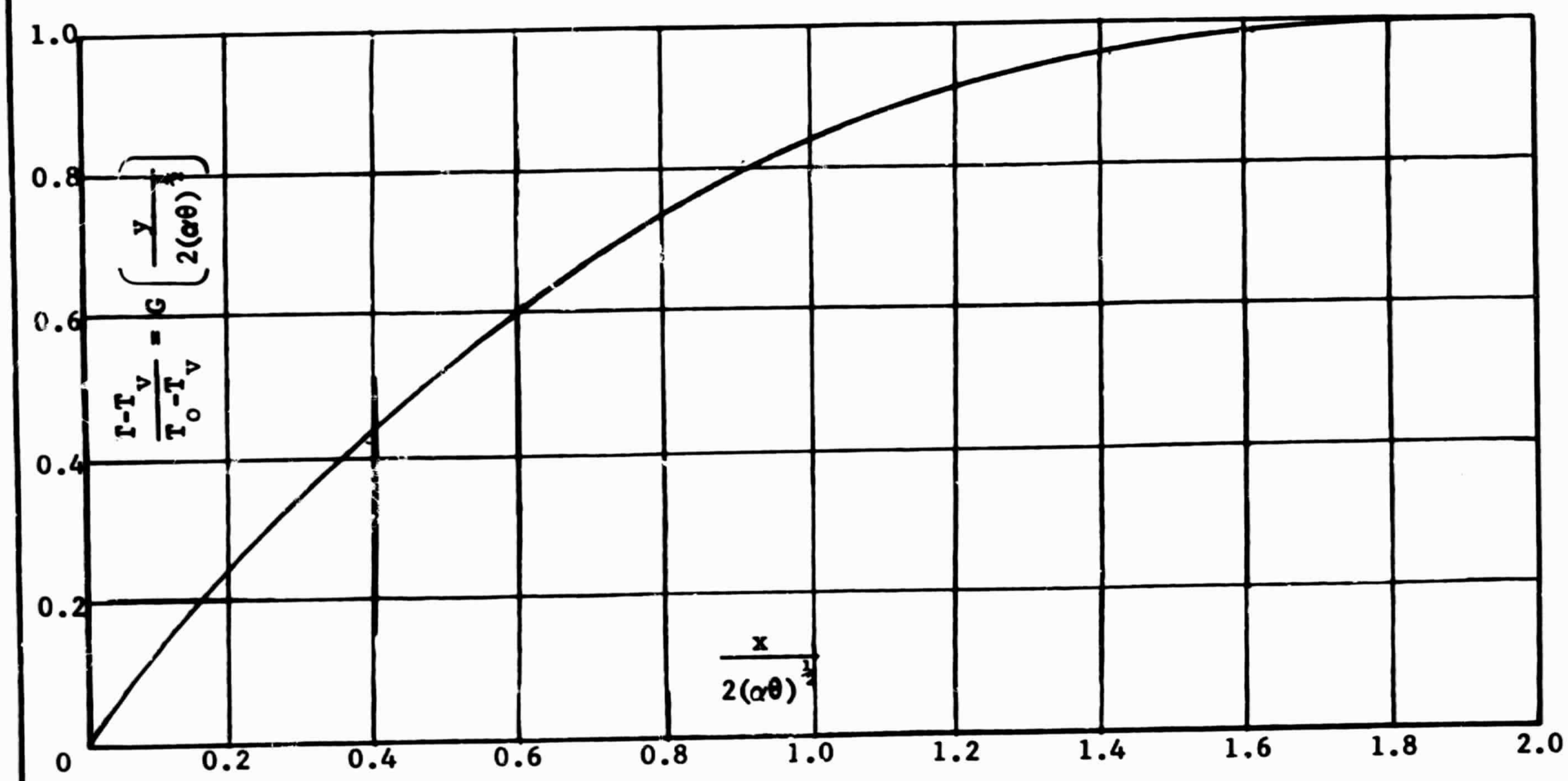

FIGURE 3: GAUSSIAN ERROR INTEGRAL

conduction cooling is assumed to dominate, however, then some approximations regarding the behavior of the real system may be obtained. Further, assume the pre-heat zone, ℓ_1, is defined as that region where at least 10% of the temperature rise is realized

$$\ell_1 \simeq \ell_{pr} \tag{6}$$

then

$$V_b \simeq V_1 \tag{7}$$

For the case when the solid is very thick then Equations 5, 6, and 7 may be reasonable approximations. For a thin geometry being heated (burning) from both sides, the burning rate V_b, would tend to be greater, and ℓ_{pr} would also be greater. On the other hand, buoyancy induced convection cooling will change also, so as to tend to decrease V_b and ℓ_{pr} for this case.

Consider a heat balance and refer to figures 1 and 2. The energy Q_{pr} entering the pre-heat zone over the characteristic time interval $\tilde{\tau}_1$ may be written as:

$$\frac{Q_{pr}}{\tilde{\tau}_1} = c_p \; \rho \; d_1 \; \Delta T \; \Delta z \tag{8}$$

for the thick specimen case where

$$\Delta T \equiv (T_v - T_o)$$

$$d_1 \equiv \text{mean pre-heat depth.}$$

For the thin specimen case, replace d_1 with d_2

$$\frac{Q_{pr}}{\tilde{\iota}_1} = c_p \; \rho \; d_2 \; \Delta T \; \Delta z \qquad (9)$$

where, in this case,

$$d_2 = \text{thickness of specimen}$$

Consider first the thick specimen case. The heat received by radiation, q_{rad}, is

$$q_{rad} = \dot{Q}_{rad} \; \Delta z \; \iota_{pr} \qquad (10)$$

The conducted heat into the pre-heat zone, q_{cond}, is

$$q_{cond} = \frac{k}{\iota_{pr}} \; \Delta T \; \Delta z \; d_1 \qquad (11)$$

The heat convected away by the ambient gases, q_{conv}, is

$$q_{conv} = h \; \Delta z \; \iota_{pr} \; \Delta T \qquad (12)$$

The appropriate heat balance is

$$\frac{Q_{pr}}{\tilde{\iota}_1} = q_{rad} + q_{cond} - q_{conv} \qquad (13)$$

Combining Equations 8, 10, 11, 12, 13 and 7, result in

$$\frac{\iota_{pr}}{\tau_{pr}} = V_b = \frac{\dot{Q}_{rad}\ \iota_{pr}}{c_p\ \rho\ d\ \Delta T} + \frac{\alpha}{\iota_{pr}} - \frac{h\ \iota_{pr}}{c_p\ \rho\ d} \qquad (14)$$

Combining with Equation 5 and rearranging

$$V_b = 4.85 \left[\frac{\dot{Q}_{rad}\ \alpha^2}{k\ d_1\ \Delta T\ (3.85+N_u)} \right]^{\frac{1}{2}} \text{(THICK SPECIMEN)} \qquad (15)$$

where

$$N_u = \text{Nusselt Number}$$

For the thin specimen case (see Figure 2) we have

$$q_{rad} = 2\ \dot{Q}_{rad}\ \Delta\ z\ \iota_{pr} \qquad (10\text{-a})$$

$$q_{cond} = \frac{k}{\iota_{pr}}\ \Delta T\ \Delta\ z\ d_2 \qquad (11\text{-a})$$

$$q_{conv} = 2h\ \Delta\ z\,\iota_{pr}\ \Delta T \qquad (12\text{-a})$$

Combining equations as before results in

$$V_b = 4.85 \left[\frac{\dot{Q}_{rad}\ \alpha^2}{2(1.85+N_u)k\ d_2\ \Delta T} \right]^{\frac{1}{2}} \text{(THIN SPECIMEN)} \qquad (15\text{-a})$$

A numerical estimate will now be made for the magnitude of V_b

Taking the following representative values:

$$Q_{rad} = 1 \ ^B/F.T^2 \text{-sec} \quad \text{(Reference 3)}$$

$$k = 0.1 \ ^B/\text{hr-ft-}^oF$$

$$d_2 = 0.0005 \text{ ft}$$

$$\Delta T = 500^oF$$

$$N_u = 10$$

$$\alpha = 0.01 \ FT^2/\text{hr}$$

results in

$$V_b = 0.05 \text{ IN/sec}$$

which is reasonably consistent with experimental data generated at White Sands Test Facility (See Reference 4. References 5 and 12 summarizes some of this information).

Some feel for the sensitivity of the predicted downward burning rate to acceleration levels may be obtained by noting that

$$N_u \propto g^{\frac{1}{4}} \text{ (laminar)} \tag{16}$$

$$N_u \propto g^{0.4} \text{ (turbulent)} \tag{17}$$

Usually the convective currents are in the laminar regime. In either case, we see that

$$V \propto g^{\frac{-1}{8}} \quad \text{(LAMINAR)} \tag{18}$$

$$V \propto g^{\frac{-1}{8}} \quad \text{(TURBULENT)} \tag{19}$$

A comparison between equations (15) and (15-a) indicates that if $d_2 \simeq d_1$ then the thin speciment, which is heated on both sides, will burn at a rate faster by a factor of approximately $(3.85/1.85)^{\frac{1}{2}}$ or 44%. If d_2 is less than d_1, the burning rate will be still more rapid; by a factor of approximately

$$(3.85/1.85)^{\frac{1}{2}} \ (d_1/d_2)^{\frac{1}{2}}.$$

These last comments are only first order approximations which appear reasonable but may not be valid if there are large variations in Q_{rad}, ΔT or N_u with specimen thickness, or if the assumptions made in the derivation are not applicable.

Figures 4 and 5 illustrate the nature of the burning rate equations.

According to equations 15 and 15-a the burning rate is also sensitive to heat flux. It is interesting to note that the White Sands Reports[4] indicate that g level had no significant effect upon measured heat flux or flame temperature.

In figures 4 and 5 a line representing a typical or nominal case is presented. In each figure three other lines are also included (making a total of four per figure). These other lines.

correspond to a change of one order in magnitude of the parameter

$$\frac{\dot{Q}_{rad}}{k}\frac{\alpha^2}{\Delta T}$$

Thus, as an example, if a new material were being considered and only its thermo-physical properties (α^2/k) changed by one order of magnitude, then the line adjacent to the nominal curve would apply. To be specific, the example considered has a value of $(\alpha^2/k) = 10^{-3}$ in appropriate units. If the new material has a value of 10^{-2}, then the line above the "nominal" curve would apply. If the new value were 10^{-4}, then the line below the "nominal" line would apply. Alternately, if the thermo-physical properties are unchanged, but $\dot{Q}_{rad}$ increased by one order of magnitude, then the line above the "nominal" would apply. If it decreased by two orders of magnitude then the second line below the nominal would apply.

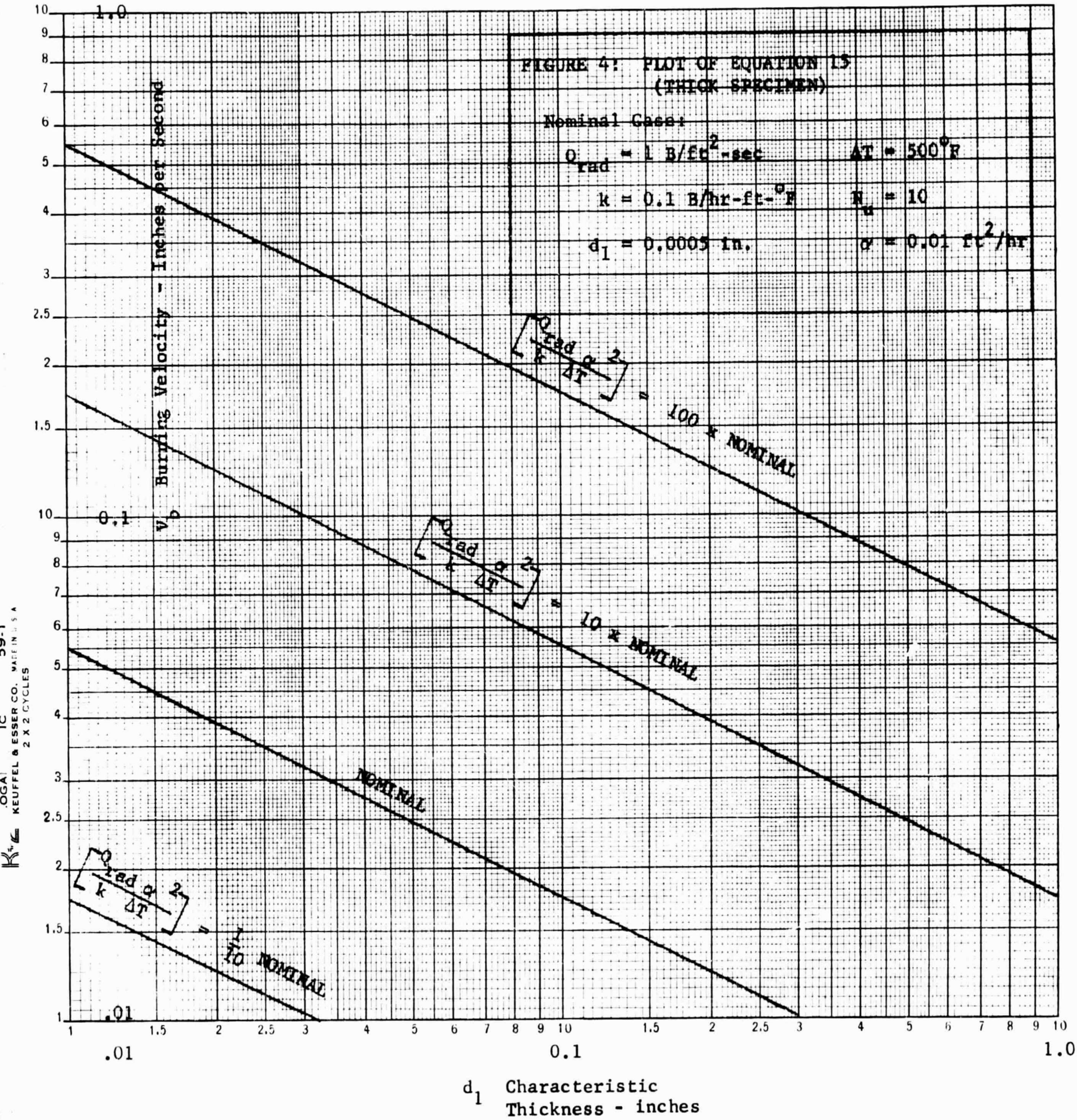

FIGURE 4: PLOT OF EQUATION 15
(THICK SPECIMEN)
Nominal Case:
Q_rad = 1 B/ft²-sec
ΔT = 500°F
k = 0.1 B/hr-ft-°F
N_u = 10
d₁ = 0.0005 in.
α = 0.01 ft²/hr
[Q_rad/k · α/ΔT]² = 100 x NOMINAL
[Q_rad/k · α/ΔT]² = 10 x NOMINAL
NOMINAL
[Q_rad/k · α/ΔT]² = 1/10 NOMINAL
V_b Burning Velocity - Inches per Second
1.0
0.1
.01
0.1
1.0
d₁ Characteristic Thickness - inches
KEUFFEL & ESSER CO.
2 X 2 CYCLES

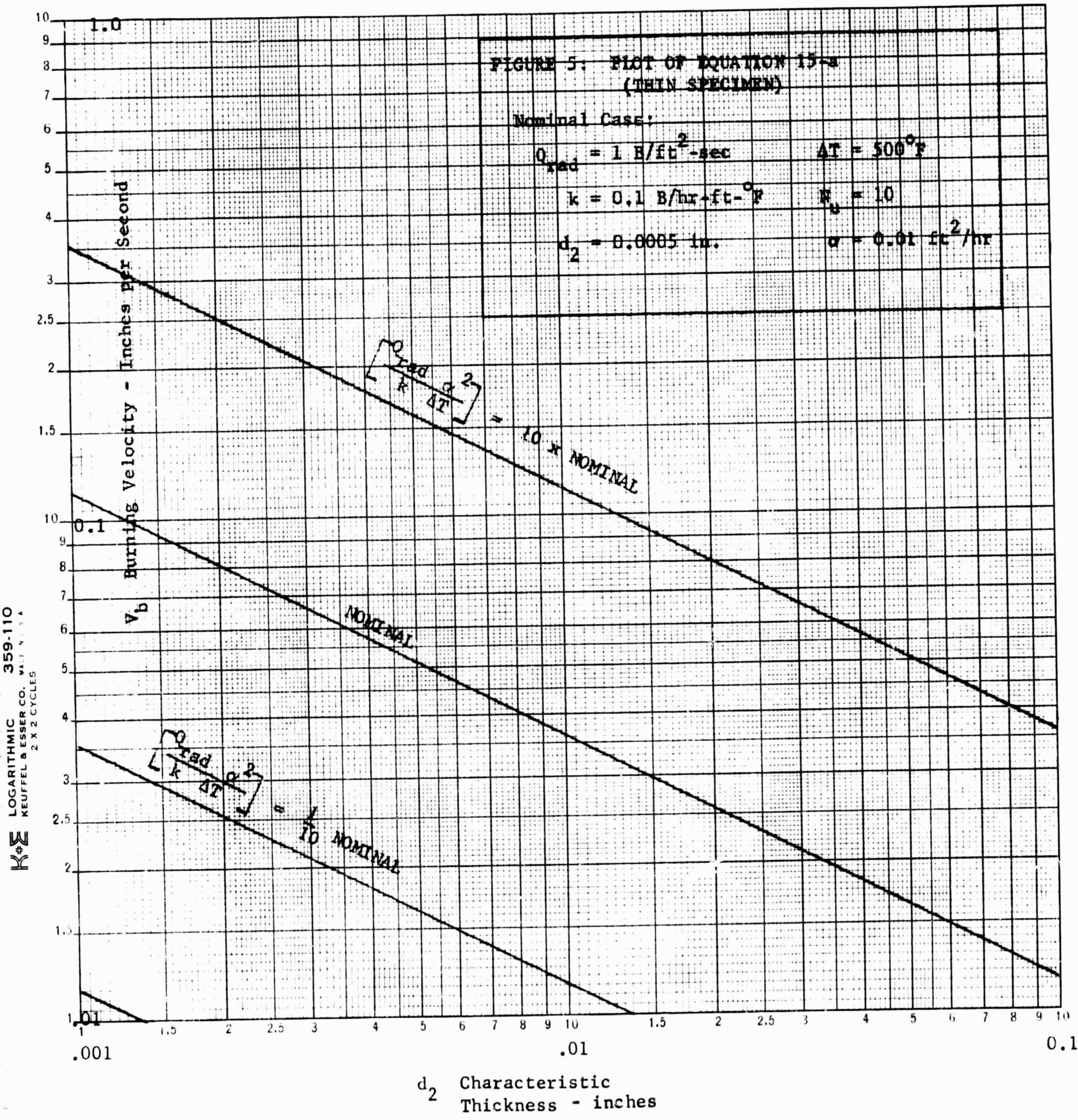

FIGURE 5: PLOT OF EQUATION 15-a (THIN SPECIMEN)

III. NO CONVECTIVE HEAT TRANSFER IN THE PYROLYSIS ZONE

In this section it will be shown that there is to be expected no convective heat transfer in the pyrolysis zone due to the blocking effects of fuel vapor injection. It is well established that there is a critical mass injection rate which, if exceeded, will eliminate convective heat transfer for laminar flows[6], and reasonably well established even for turbulent flows over flat surfaces[7]. This is best expressed in terms of a non dimensional parameter

$$\left[\frac{(\dot{m}/A)_w}{(\dot{m}/A)_e} \quad (Re_x)^{\frac{1}{2}} \quad \left(\frac{M_x}{M_a}\right)^{\frac{1}{3}} \right]_{cr} \simeq 0.6 \quad \text{(LAMINAR)} \tag{20}$$

If the value 0.6 is exceeded, and if the flow is laminar, there will be no convective heat transfer. For purposes of estimating the order of magnitude, assume

$$A_w \simeq A_e \tag{21}$$

$$\left(\frac{M_x}{M_a}\right)^{\frac{1}{3}} \simeq 1 \tag{22}$$

From this we see that no convective heat will be transferred if

$$\frac{\dot{m}_w}{\dot{m}_e} > 0.6 \quad (R_e)^{-\frac{1}{2}} \tag{23}$$

Now $\dot{m}_w/\dot{m}_e$ is approximately equal to the fuel/air ratio and this value is typically on the order of .06. Thus, there will be no convective heat transfer if

$$R_e > 100 \tag{24}$$

In natural convection, the equivalent value for the Grashoff Number is, to a first approximation;

$$G_r \simeq (R_e)^2 \tag{25}$$

and so the corresponding critical Grashoff Number is

$$(G_r)_{cr} > 10{,}000$$

To estimate the magnitude of the Grashoff Number assume

$$L = 0.02 \text{ ft}$$

$$T_{mean} = 1000^\circ F$$

$$\Delta T = 1500^\circ F$$

This results in $G_r \simeq 500{,}000$ which is well above the critical value and so it appears there will be no convective heat transfer in the pyrolysis zone.

IV. FLAME QUENCHING AT ELEVATED "g" LEVELS

There is some evidence to indicate that materials which burn at normal 1-g will not burn, at elevated "g" levels[4]. (There is also evidence of flame extinguishment at reduced "g" levels[8], one possible factor causing this is discussed in reference (9)). In the simplified model currently developed, buoyancy induced cooling of the pre-heat zone is the only factor which could account for this. To demonstrate that this is not unreasonable, consider the following: Under what conditions will the cooling rate compare with the direct radiant heat flux? For the examples considered earlier, typical radiant heat fluxes were measured to be about $1 \ B/ft^2 - sec$[3]. For convective cooling with a temperature difference of 500^oF, a heat transfer coefficient, h, of $7 \ B/hr \ ft^2 - {}^oF$ is required. This corresponds to a value of Nusselt Number of:

$$N_u = \frac{hL}{k} = \frac{(7 \ B/hr \ ft^2 - {}^oF) \ (.02 \ ft)}{.02 \ B/hr \ ft - {}^oF} = 7$$

and for natural convection from a vertical surface, the corresponding Grashoff Number would be approximately 5×10^4. It has been shown in the previous numerical example that

Grashoff Numbers much larger than this are likely and so it appears that acceleration induced convective cooling can quench a flame.

It was also shown earlier that the burning rate is not very sensitive to "g" levels ($V_b \propto g^{-\frac{1}{8}}$) and so buoyancy induced quenching may be expected only for materials whose minimum equilibrium burning rate at 1-g is marginal in the first place.

Other factors also can cause flame quenching. These include a change in the oxidizer/fuel ratio, the oxidizer concentration, the flame temperature, the flame position, and possibly other factors. (See also reference 5 for a discussion of the effect of gravity upon burning rate.)

V. SYSTEM EQUATIONS FOR THE CALCULATION OF V_b

We now illustrate the type of system equations which define V_b uniquely for the model of conduction controlled flame spreading in fuel strips. The essential requirements of steady-state burning velocity V_b are mixing and burning with the ambient fluid accomplished close to the pyrolysis zone. Oxidant supply of the appropriate amount (and exceeding the minimum required) must be induced by buoyancy effects consistent with the products of combustion. Continuous ignition near the fuel injection zone is presumably provided by intermixing with the recirculating combustion products in the wake (downstream) of the injection. The illustrative system equations defining V_b are stated in a sufficiently simple form to enable approximative numerical estimates of their plausibility. Refined formulations for the evaluation of V_b would require numerical or machine calculations.

V. A. Definitions of Principal Parameters

We introduce a number of parameters pertinent to the description of "entrained oxidant" and resultant combustion products responsible for the production of $\dot{Q}_{rad}$. For simplicity we assume that both the entrained ambient fluid and the combustion products in the neighborhood of the pyrolysis zone are contained in a flow layer of thickness ΔY_a. Let $\dot{M}\ \frac{gm}{sec.cm}$ represent mass flow per unit width along the horizontal (see Figure 1) so that

$$\dot{M}_v = \dot{m}_v \, \ell_v \tag{26}$$

May be determined in terms of the fuel burning velocity and thickness. The induced ambient flow rate $\dot{M}_a$, then, defines the mixture ratio

$$R = \dot{M}_a / \dot{M}_v \tag{27}$$

Where $\dot{M}_a$ involves the parameters ΔY_a and the buoyancy-induced air velocity V_a:

$$\dot{M}_a = \rho_a \, V_a \, \Delta \, Y_a \tag{28}$$

From these definitions we have a relation

$$\Delta \, Y_a = \frac{R \, \dot{M}_v}{\rho_a \, V_a} \tag{29}$$

connecting the three parameters ΔY_a, R, and V_a introduced here. Three additional parameters ℓ_f, T_f, V_f represent the properties of the combustion products at the mixture ratio R contained in the flow zone of width ΔY_a. Thus five additional constraints among ΔY_a, R, V_a, ρ_f, T_f, V_f are required to relate these quantities to $\dot{M}_v$. These additional relations describe the mechanisms of (1) mixing, (2) buoyancy acceleration, conservation laws for, (3) mass, (4) thermal energy or heat release and (5) equation of state.

V. B. Mixing Near the Pyrolysis Zone

The simplest mixing law which may be postulated here is (10) equality of the magnitudes of injected momentum flux $\dot{M}_v V_v$ and entrained mass flux $\dot{M}_a V_a$:

$$V_a = \frac{\dot{M}_v}{\dot{M}_a} V_v = V_v / R \tag{30}$$

Note that flame temperature is usually a simple function of mixture ratio so that R can be experimentally determined indirectly (through flame temperature measurements) or directly.

V. C. Buoyancy Acceleration

If the fuel-air mixture in the mixing zone of dimensions l_v, ΔY_a did not burn, its mass content would be obtainable from the low temperature data ρ_v, T_v, ρ_a, $T_a = T_o$. Burning and expansion, within the assumed width ΔY_a, produces the buoyant acceleration resulting in V_f: of the burned products. The buoyant force affecting the velocity change, including the induction of air at the speed V_a, is equated to the net upward momentum flow rate (per unit horizontal width, $\Delta z = 1$)

$$F_{buoyant} \equiv \ell_v \ \Delta Y_a \ (\rho_a - \rho_f) \ g$$

$$F_{buoyant} = \Delta Y_a \ V_f \ (\rho_f \ V_f)$$

$$V_f^{\,2} = g\ell_v \ (\rho_a / \rho_f - 1) \tag{31}$$

In using the density difference $\rho_a - \rho_f$ for the buoyant force, the "cold" mixture density was represented by ρ_a without inclusion of $\rho_v \neq \rho_a$ in the mixture, this is valid if $R \gg 1$, which is the typical case for hydrocarbon/air burning.

V. D. Conservation Laws and the Equation of State

From the mass conservation law applied to the upward flow

$$\rho_f \ V_f \ \Delta Y_a = \dot{M}_a + \dot{M}_v = \dot{M}_v \ (1 + R) \tag{32}$$

In stating the energy release, a distinction is made between the oxidizer-rich flow $R > R_{st}$, wherein the full heat value H of the fuel is released, and the oxidizer-lean flow, wherein only a part of the oxidizer-fuel mixture can react. Thus if $R < R_{st}$ the fuel that can react is $\dot{M}_a / R_{st}$. Accordingly we have from energy conservation

$$T_f - T_a \simeq \frac{H/c}{(1 + R)} \qquad R \geq R_{st}$$

$$T_f - T_a \simeq \frac{HR/c}{R_{st}\ (1 + R)} \qquad R \leq R_{st} \tag{33}$$

Finally we employ a simplified equation of state at constant pressure

$$\rho_f\ T_f \simeq \rho_a\ T_a \tag{34}$$

wherein initial effects $c_v\ T_v \neq c_a\ T_a$ and specific heat variations are considered of second order.

We note in connection with Equation (33) that the magnitude of R for which combustion is in fact achievable is restricted to the neighborhood of the stoichiometric ratio R_{st}; i.e. the temperature R_f is restricted to values above the ignition temperature T_{ig}. For example in hydrocarbon-air combustion ($H \simeq 10^4$ cal/gm, c = 0.3 cal/(gm deg K) $R_{st} \simeq 16$ and the extraneous restriction on R is*

$$\frac{1}{32} < \frac{R}{R_{st}} < 4 \tag{35}$$

The corresponding ignition temperature restriction is

---

*The data quoted here yields stoichiometric temperatures $T_{st} - T_o \simeq 2000^oK$. For $R<R_{st}$ heat release is incomplete due to oxidant deficiency.

$$T_f > T_{ig} \simeq (600 + T_a) \simeq 900^{o}K \tag{35-a}$$

V. E. Summary of the System Equations

Of the six variables ΔY_a, R, V_a, ρ_f, T_f, V_f connected by Equations (29, 30, 31, 32, 33, 34) we may immediately eliminate ΔY_a, V_a, ρ_f using Equations (39, 30, 34) and obtain

$$V_f = \left(\frac{1 + R}{R^2}\right) \frac{T_f}{T_a} V_v \tag{36}$$

$$V_f^{\,2} = g\ell_v \left(\frac{T_f}{T_a} - 1\right) \tag{37}$$

$$T_f - T_a \simeq \frac{H/c}{1 + R} \qquad R \geq R_{st}$$

$$T_f - T_a = \frac{HR/c}{R_{st} \ (1 + R)} \qquad R \leq R_{st} \tag{38}$$

These system equations relating V_f, T_f and R to properties of the fuel and its environment contain V_b as an implicit variable in V_v. On the other hand the properties T_f, ρ_f, ΔY_a obtainable from the system equations illustrated here define the radiant heat flux $\dot{q}_{rad}$ (T_f, ρ_f, ΔY_a) which, therefore, also becomes an implicit function of V_b. Accordingly, the $\dot{q}_{rad} = \dot{q}_{rad}(V_b)$ obtained in this manner can be introduced into the preheat zone equation (15) to obtain the equation which implicitly defines the steady-state value of V_b in terms of given fuel and ambient properties in the model illustrated here.

V. F. Discussion

The system equations developed in the preceding section are conveniently illustrated with the following representative numberical data (see also Reference (1))

$$\dot{m}_v = 5 \times 10^{-3} \frac{gm/sec}{cm^2} \quad V_v = 10 \text{ cm/sec} \quad \ell_v = 0.2 \text{ cm}$$

$$\rho_v \simeq \rho_a / 2 \simeq 5 \times 10^{-4} \text{ gm/cm}^3$$

We first consider Equations (36)(37)(38) to ascertain the mixture ratio R which is consistent with these equations for $H \simeq 10^4$ cal/gm, $c \simeq 1/3$ cal/gm deg, $R_{st} = 16$. The mixture is fuel rich, $R \leq R_{st}$, as shown by a subsequent check, with initially assumed $T_f - T_a \simeq 1500^\circ K$ or $T_f / T_a \simeq 6$. With these data, then, Equation (37) yields

$$V_f \simeq (10^3 \times 0.2 \times 5) \simeq 32 \text{ cm/sec}$$

Next solving the R from Equation (36) one obtains the oxidant-lean mixture

$$R = 2.5 < R_{st}$$

The temperature estimated from Equation (38)

$$T_f - T_a \simeq 1500^\circ K$$

checks with the initially assumed value.

With R = 2.5, the air induction speed V_a, obtained from Equation (30) is

$$V_a \simeq 4 \text{ cm/sec}$$

The flow layer thickness Δy_a obtainable from Equation (29) is

$$\Delta Y_a = \frac{R \rho_v V_v}{\rho_a V_a} l_v = \frac{\rho_v}{\rho_a} l_v = 0.1 \text{ cm}$$

It is of interest to note that this length is sufficient for the diffusive mixing of the "injected" fuel and the induced air stream. Taking the "cold" diffusion coefficient for gases, $\alpha_g \simeq 0.2 \text{ cm}^2/\text{sec}$, the time required for interdiffusion with the fuel "jet" of width l_v is

$$\tilde{\tau}_{diff} \simeq \frac{l_v^2}{4\alpha_g} \simeq 0.05 \text{ sec}$$

During this time the "cold"mixture could travel a distance not exceeding

$$\Delta x = V_a \tilde{\tau}_{diff} \simeq 0.2 \text{ cm} \sim l_v$$

Actually, as noted previously, the burning which accompanies interdiffusion expands the flow region locally and tends to maintain the flame over the pyrolysis zone l_v.

Finally we note the order of magnitude radiation from the burning zone of thickness $\Delta y_a \simeq 0.1$ cm at temperature $T_f \simeq 1800^\circ K$. Unlike in black body radiation, in hydrocarbon air-flames, the radiant heat flux is expressible in the form

$$\dot{Q}_{rad} = \Delta Y_a \; C^a \, T_f^{\,2}$$

where C^a is a constant dependent on radiating combustion products CO, CO_2, H_2O (11). To a rough approximation, constant C^a is given by

$$C^a \simeq 4 \times 10^{-7} \; \frac{\text{cal/sec}}{\text{cm}^3 \; (\text{deg K})^2}$$

Accordingly with the thickness $\Delta Y_a \sim 0.1$ cm and $T_f \simeq 1800^\circ K$ we obtain

$$\dot{Q}_{rad} \simeq 0.12 \; \frac{\text{cal/sec}}{\text{cm}^2}$$

This rough estimate is lower by a factor of two, compared with the test data of Reference 3.

It is evident from the system equations summarized in Section V.E. that in addition to thermal properties of the fuel-atmosphere system, the model considered here contains the effects of g and system chemistry implicit in H and R_{st}. Although comparison of the present model with test data has been made here only for illustrative purpose, the order-of-magnitude agreement obtained is plausible. Further development and refinement of the model for downward burning with more detailed treatment of the test data appears desirable before extension to more general orientation are pursued.

NOMENCLATURE

A	area
B	B.T.U.
C_p	specific heat at constant pressure
d	thickness or depth
e	natural logarithm base
G	Gaussian Error Integral
g	acceleration (normal or induced)
G_r	Grashoff Number
h	convection coefficient
H	heat of combustion
k	thermal conductivity
l	length
l_{pr}	pre-heat zone length
$\dot{m}$	mass flux rate
M	Molecular Weight
$\dot{M}$	Mass flux per unit length
N_u	Nusselt Number
Q	energy
q	heating rate
R	mixture ratio; mass of air to mass of fuel

NOMENCLATURE (continued)

R_e	Reynolds Number
R_{st}	Stoichiometric Mixture Ratio
T	temperature
T_o	original ambient temperature
T_v	temperature of pyrolysis zone of material
V	velocity
V_b	burning rate
y	distance in y direction
z	arbitrary integration variable
Z	distance in z direction

GREEK SYMBOLS

α	thermal diffusivity
o	density
θ	time
$\tilde{\tau}$	time constant

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