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RESPONSE OF BOX-TYPE STRUCTURES  
TO SONIC BOOMS

Neil Popplewell

Ph.D. Thesis, 1969.

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By Neil Popplewell

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## ABSTRACT

A Finite Element Displacement Method of analysis is developed, in conjunction with a modelling technique, to determine the response of a box-type structure to sonic booms. Both techniques should be regarded as being complementary. Both are developed systematically in order to assess their accuracy at each intermediate stage.

The basic formulation of the Finite Element Method is checked by comparing the natural frequencies and mode shapes of a freely vibrating box with known solutions. A theoretical description of the flow of an arbitrarily shaped wave over a box-type structure is then formulated. The resulting theoretical wave-forms are compared with the forms of sonic booms observed on different faces of a structure in order to test the validity of the simplifying assumptions. Then the response of the box's mathematical model to this theoretical flow is checked with the experimental model results. Subsequently, the effects of varying the model and wave properties are discussed in detail.

The usefulness of both models is assessed by predicting the dynamical behaviour of a full scale structure and comparing the predicted response with experimentally observed results. This procedure requires a knowledge of the important structural and loading parameters in order to develop scaling laws for the experimental model. Conversely, the mathematical model only requires different computer data describing the material properties and physical dimensions of the full scale structure's components.

The results are used to predict a safe damage criterion for the walls of a single storey house which are subjected to a sonic boom. Although this criterion is based upon damage from blasting operations, it gives results which are consistent with known damage from sonic booms.

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## NOTATION

### Acoustic properties:

|               |                                                                           |
|---------------|---------------------------------------------------------------------------|
| $a_o$         | speed of propagation of the shock front                                   |
| $v_T$         | trace speed across the structure                                          |
| $\lambda$     | free field wavelength of the sonic boom                                   |
| $T_1, T_2, T$ | time characteristics of the sonic boom                                    |
| $T_R$         | rise time of the sonic boom                                               |
| $p_{in}$      | incident pressure                                                         |
| $\phi$        | velocity potential                                                        |
| $p_b$         | internal backpressure                                                     |
| $\rho_b$      | equivalent mass/volume                                                    |
| $a_b$         | speed of sound                                                            |
| $\lambda_b$   | wavelength                                                                |
|               | } of acoustic wave inside<br>the enclosed cavity                          |
| $f_N$         | excitation frequency                                                      |
| $p_o$         | peak positive pressure                                                    |
| $p_-$         | peak negative pressure                                                    |
| $\theta$      | angle of incidence of the shock front with respect to<br>the structure    |
| $\psi$        | angle of inclination of the shock front with respect to<br>the horizontal |

### Structural properties:

|             |                                                                                                       |
|-------------|-------------------------------------------------------------------------------------------------------|
| $\rho_{ST}$ | mass/area                                                                                             |
| $\rho_v$    | mass/volume                                                                                           |
| $L$         | trace length (i.e. the maximum structural length in the<br>direction of the sonic boom's propagation) |
| $\xi_r$     | damping factor of $r^{th}$ mode                                                                       |
| $g$         | acceleration due to gravity                                                                           |
| $E$         | Young's modulus                                                                                       |
| $\nu$       | Poisson's ratio                                                                                       |
| $\hat{d}$   | thickness                                                                                             |

|                       |                                |                                                          |
|-----------------------|--------------------------------|----------------------------------------------------------|
| $D$                   | $= \frac{Ed^3}{12(1 - \nu^2)}$ | flexural rigidity                                        |
| $\mu^2$               | $= \rho_{ST}\omega^2/Dg$       | frequency parameter                                      |
| $w$                   |                                | transverse displacement                                  |
| $f_{11}$              |                                | fundamental frequency                                    |
| $\underline{M}$       |                                | overall mass matrix                                      |
| $\underline{M}_c$     |                                | constrained mass matrix                                  |
| $\underline{S}$       |                                | overall stiffness matrix                                 |
| $\underline{S}_c$     |                                | constrained stiffness matrix                             |
| $\{\underline{F}_c\}$ |                                | constrained forcing vector                               |
| $Q_x, Q_y$            |                                | shear forces                                             |
| $M_x, M_y$            |                                | bending moments                                          |
| $M_{xy}$              |                                | twisting moment                                          |
| $\{F\}_e$             |                                | nodal forces and moments of a rectangular finite element |

#### Acousto-structural property:

|        |                                                                          |
|--------|--------------------------------------------------------------------------|
| $\eta$ | ratio of the internal acoustic to mechanical stiffness of the structure. |
|--------|--------------------------------------------------------------------------|

#### General:

|           |                           |
|-----------|---------------------------|
| $t$       | time                      |
| $x, y, z$ | global co-ordinate system |

#### Suffices:

|     |                     |
|-----|---------------------|
| $M$ | model quantity      |
| $P$ | prototype quantity. |

CHAPTER 1  
INTRODUCTION

An aircraft flying supersonically produces a pressure disturbance on the ground which is commonly known as a sonic boom or bang. With the prospect of supersonic passenger-transport in service within the next few years, the annoyance of people and the response of buildings to sonic booms are of immediate concern. This thesis considers the response of single-storey houses to these booms. People inside buildings will only hear that pressure disturbance which has been transmitted through the structure. Consequently, a knowledge of the structure's response is also a prerequisite to assessing people's reactions to sonic booms.

At a great distance from the source of disturbance and under idealised uniform atmospheric conditions Whitham (1) showed that the shape of the pressure-time history could be described by a capital N. This explains why the sonic boom is sometimes called an 'N'-wave. In general, the shape of the wave will be modified by aircraft manoeuvres (2), atmospheric conditions (3) and reflections from the ground and neighbouring buildings, etc.

Methods for determining the structural response to these wave-forms fall into two categories. From a practical point of view, it is important to develop a simplified analysis which is easily and quickly applicable to any structure. Such a method should use a minimum number of parameters and give a gross estimate of the structure's response and possible damage from a sonic boom. More refined methods are needed to assess the validity of the simplifying assumptions and to determine the magnitude of any additional amplification factors. These two categories are essentially complementary although most of the published work to date falls in the first class.

Early work (4) assumed that the significant dynamical response of a structure was entirely due to its fundamental mode. The structure was idealised as a set of independent subelements. Then the response of each subelement was determined by considering the

response of an equivalent single degree of freedom system to an N-shaped wave. The assumption that the interaction between structural subelements is negligible is common to all the simplified analyses. Crocker and Hudson (5) extended essentially the same analysis to include experimentally observed waveforms. Cheng (6,7), using a Fourier transform technique, and Crocker (8), using the Duhamel integral, independently derived the response of beams and plates to normal and travelling N-waves. Craggs (9) subsequently developed a numerical technique based on the Duhamel integral to deal with the general case of a travelling arbitrary shaped wave acting on an inclined, simply supported plate. Both Crocker and Craggs showed that the higher modal contributions can be neglected in the estimation of the plate's displacements. The higher modes, however, provide a significant contribution to the plate's strains and accelerations.

Experimental and computational methods are presented in this thesis which determine the response of a box-type structure to travelling arbitrary shaped sonic booms. The computational method assumes that the pressure inside the structure can be neglected. Thus, there is no internal acoustic coupling between structural subelements. In order to assess the limitations of this assumption, the magnitude of the pressure inside the structure must be known. This internal pressure usually provides an additional contribution to the stiffness of each individual structural subelement. The most flexible subelements will be most affected by the internal pressure. Hence, the present theory is mainly applicable to the response of the stiffest subelements such as walls and ceilings rather than flexible subelements such as windows. The method can be extended to include the effects of internal acoustic coupling. In conjunction with this work, P.G. Vaidya (10) has computed the form of the pressure-time history due to an incident N-wave inside a rigid, rectangular structure containing an opening. Using similar techniques to those employed in this thesis, A. Craggs (11) has computed the effect on the structure of the pressure inside a rigid, rectangular enclosure with one flexible wall. Their conclusions agree with the assumptions outlined above.

Fig. 1.1 describes the subroutines used in the formulation of the computational, finite element method. This figure also illustrates the general layout of the thesis. In order to check the finite element

formulation, the natural frequencies and normal modes of a freely vibrating box are evaluated and compared with known results in Chapter 2. This formulation is extended in Chapter 3 to include the transient response of a box to a travelling sonic boom which has an arbitrary shape. The mathematical model idealises the system's loading and response as a set of second order differential equations. These equations can then be solved on a high-speed digital computer. Experimental results are presented in Chapter 4. These results were obtained from tests conducted on a physically scaled-down version of an actual structure. The results from the experimental and mathematical models are compared and discussed in Chapter 5. This comparison provides a useful cross check between both sets of results.

Both the experimental and mathematical models have their advantages and disadvantages. Within the limitations of manufacture, the experimental model may have any desired form compatible with an actual structure. The mathematically idealised system may not have the same degree of similarity as the experimental model but it has greater flexibility. Changes in material properties, structural dimensions and the introduction of structural discontinuities can easily be accommodated by using different data in the computer programme of the mathematical model. The model must represent an actual structure within reasonable limits in order to be useful. Full scale sonic boom tests on actual buildings are not only expensive but they can cause a great deal of inconvenience to the general public. Simulation techniques, on the other hand, are not only comparatively cheap but their nuisance-value can be localised. They can also provide much useful information about the main boom loading and structural response parameters. Consequently, advantage was taken of the explosive technique developed at E.R.D.E. (12) for simulating full scale sonic booms.

The relative merits of various types of simulated booms are presented in Chapter 6. Practical difficulties encountered in evaluating the structure's response to these booms are also discussed in this chapter. Scaling laws are introduced in Chapter 7 in order

to compare the model and full scale results. The validity of the simplifying assumptions used in the model work is tested. The results are then compared to results of actual sonic boom tests which are obtained from other sources in order to assess the accuracy of the simulation techniques. Subsequently, a simple damage criterion is developed which is applicable to the walls of single-storey dwellings.

Chapter 8 contains a review of the important conclusions of this thesis.

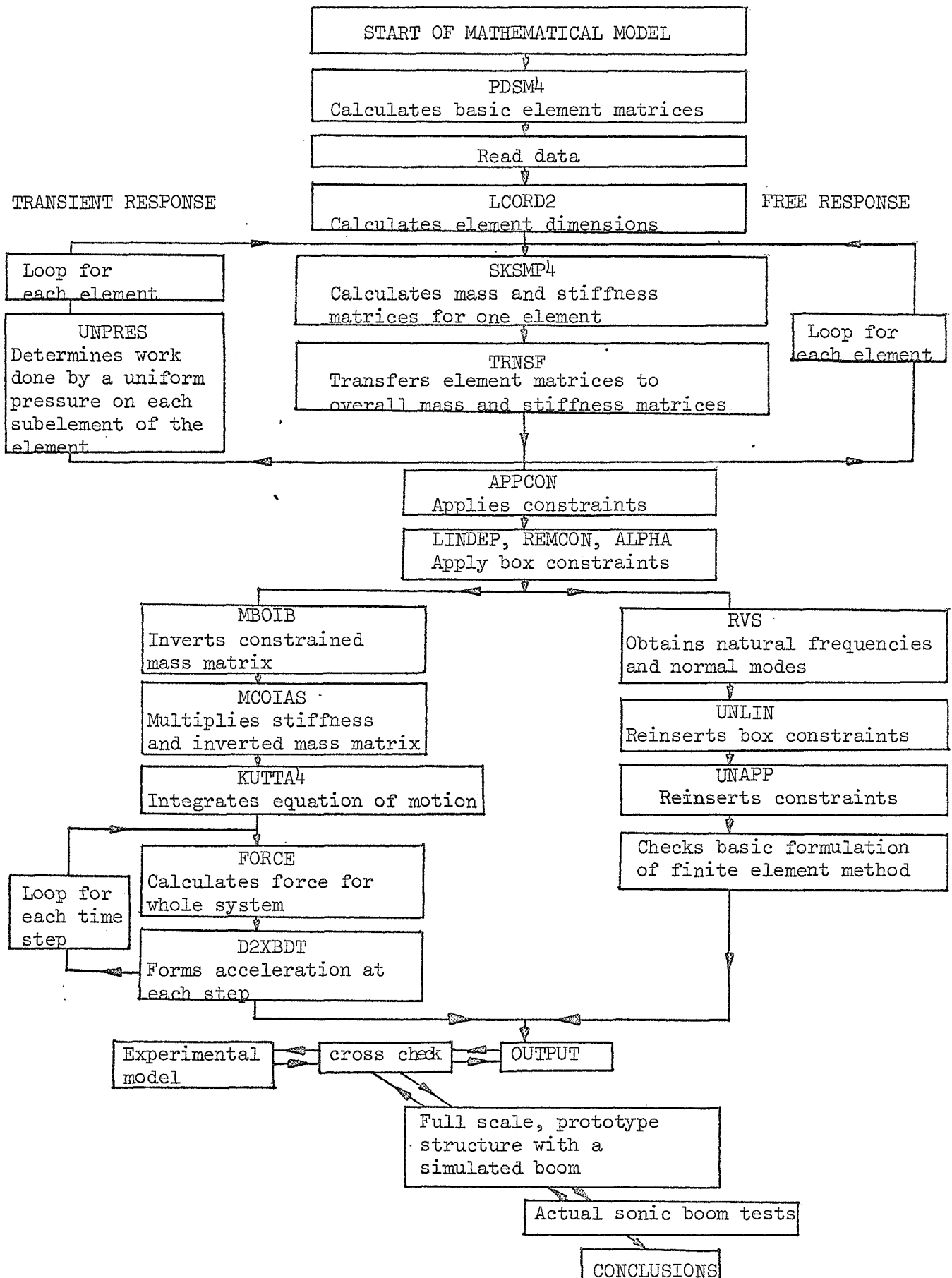


Fig. 1.1 Flow visualisation of thesis layout.

CHAPTER 2  
FREE VIBRATIONS OF RECTANGULAR PLATE SYSTEMS

Matrix methods of structural analysis have been developed in the last fifteen years in the fields of Aeronautical and Civil Engineering. The fundamental assumption of all these methods is that the structure may be represented by a finite number of basic elements. The analysis of the individual elements and the assembled structure is formulated in the notation of matrix algebra in order to utilise the facilities of a digital computer. Using one of these methods, the Finite Element Displacement Method, the procedure for analysing the free vibrations of a box-type structure is developed in this chapter.

The general procedure for analysing such a problem may be summarised as follows:

- (i) The idealisation of the structure into a finite number of individual elements, interconnected at a finite number of nodal points.
- (ii) The evaluation of the properties of each element.
- (iii) The analysis of the whole structure in terms of the unknown displacements.

The idealisation process approximates the box by an assemblage of triangular, rectangular or quadrilateral elements. Only rectangular elements are considered in the following analysis. Careful consideration of the required characteristics of displacements or forces must be taken into account in performing the box's idealisation. The free vibrations of a simply supported plate are analysed using different finite element approximations. Then the approximate results are compared with an exact solution. This comparison provides an assessment of the relative accuracy of the plate's vibration characteristics for a given finite element idealisation.

The basic finite element formulation for a box-type structure is checked by comparing the box's natural frequencies and normal modes to those obtained with a series solution. The evaluation of the box's natural frequencies and normal modes should be regarded as an intermediate stage to computing the box's transient response.

## 2.1 Finite Element Representation of Rectangular Plates

The total energy involved in the transverse motion of a flat, uniform plate is given by

$$E_n = \frac{D}{2} \int_A \{ (\nabla^2 w)^2 - 2(1 - \nu) \left[ \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 \right] \} dA - \frac{\rho_{ST} \omega^2}{2g} \int_A w^2 dA - \int_A f w dA \quad (2.1)$$

For free vibrations, the forcing function  $f(x,y)$  is zero and the two remaining integrals in equation (2.1) correspond to the strain and kinetic energy integrals for the plate. Using Hamilton's variational principle (13), the required solution is given by the displacement function  $w(x,y)$  which makes the energy expression stationary. In applying the finite element technique, an approximate form is chosen for the displacement function in each element, such that the unknown constants make expression (2.1) stationary over the entire plate.

Assume that the deflected shape of an element (with side lengths  $a$  and  $b$ ) can be written in terms of the truncated series

$$w(x,y) = \sum_{i=1}^{2(n+1)} \sum_{j=1} a_{ij} f_i(x/a) f_j(y/b). \quad (2.2)$$

The function  $f_i(r)$  is defined by

$$f_{p+1}(r) = \frac{r^p}{p!} (1-r)^{n+1} \sum_{m=0}^{n-p} \frac{(m+n)!}{m! n!} r^m$$

$$f_{p+n+2}(r) = (-1)^p f_{p+1}(1-r)$$

$$\text{for } p = 0, 1, 2, \dots, n \quad (2.3)$$

$$\text{and } 0 \leq r \leq 1$$

Then the unknown constants,  $a_{ij}$ , can be uniquely determined in terms of the element corner displacements. Note that the maximum number of

degrees of freedom per element is  $4(n + 1)^2$ .

Mason (14) showed that the function which has continuity of displacement ( $W$ ), slopes ( $\frac{\partial W}{\partial x}$  and  $\frac{\partial W}{\partial y}$ ), and twist ( $\frac{\partial^2 W}{\partial x \partial y}$ ) between adjacent elements gives much more accurate natural frequencies and mode shapes for rectangular plates with either simply-supported or clamped edges than the function obtained by excluding the twist. He also found that no significant increase in accuracy was obtained by including the curvatures of the element as additional unknowns. In view of these results, the problem is formulated using sixteen degrees of freedom per element (i.e.  $n = 1$ ).

Consequently, the displacement function (2.2) may be written in matrix notation as

$$w(x, y) = \sum_{i=1}^4 \sum_{j=1}^4 a_{ij} f_i(X) f_j(Y) = \{w_e\}^T \{f_X f_Y\} \quad (2.4)$$

where  $X = x/a$ ,  $Y = y/b$ ,  $\{w_e\}$  is a vector containing the displacements at the corners of the elements and  $\{f_X f_Y\}$  contains the required product of functions (2.3) and element dimensions  $a$  and  $b$ . The kinetic energy integral in equation (2.1) then becomes

$$\begin{aligned} \int_A w^2(x, y) dA &= \sum_{e=1}^N \{w_e\}^T \int_{A_e} \{f_X f_Y\} \{f_X f_Y\}^T dx dy \{w_e\} \\ &= \{w\}^T \underline{M} \{w\} \end{aligned} \quad (2.5)$$

where  $\{w\}$  is a vector containing the nodal displacements for the whole plate. The summation process is over the  $N$  elements into which the plate has been idealised. The overall mass matrix,  $\underline{M}$ , is obtained by applying continuity conditions at the element nodal points.

The strain energy integral can be similarly evaluated as

$$\begin{aligned}
& \int_A \{ (\nabla^2 w)^2 - 2(1 - \nu) \left[ \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 \right] \} dA \\
&= \sum_{e=1}^N \{w_e\}^T \int_{A_e} \left[ \{f''_{XY}\} \{f''_{XY}\}^T + \{f_X f''_Y\} \{f_X f''_Y\}^T + \right. \\
&\quad + \nu \{f''_{XY}\} \{f_X f''_Y\}^T + \nu \{f_X f''_Y\} \{f''_{XY}\}^T + \\
&\quad \left. + 2(1 - \nu) \{f'_X f'_Y\} \{f'_X f'_Y\}^T \right] dA \{w_e\} \\
&= \{w\}^T \underline{S} \{w\} . \tag{2.6}
\end{aligned}$$

$\underline{S}$  denotes the overall stiffness matrix for the plate. The dashes indicate differentiation with respect to  $x$  or  $y$  as appropriate. The only non-symmetric matrices in the above expression are the third and fourth products. These last two products can be integrated by parts and summed to produce a symmetric matrix.

Using equations (2.5) and (2.6), equation (2.1) can be written in matrix form as

$$E_n = \{w\}^T \left[ \frac{D}{2} \underline{S} - \frac{\rho_{ST} \omega^2}{2g} \underline{M} \right] \{w\} . \tag{2.7}$$

The displacement vector  $\{w\}$  now contains all the possible nodal displacements. Some of these nodal displacements may be constrained to be zero in order to include the plate's edge conditions. Then the quadratic energy matrix (2.7) can be partitioned and reduced in size to produce

$$E_n = \{w_c\}^T \left[ \frac{D}{2} \underline{S}_c - \frac{\rho_{ST} \omega^2}{2g} \underline{M}_c \right] \{w_c\} \tag{2.8}$$

where  $\{w_c\}$  is the vector containing only the non-zero nodal displacements.  $\underline{S}_c$  and  $\underline{M}_c$  are the constrained stiffness and mass matrices respectively for the whole plate.

The total energy  $E_n$  is made stationary by differentiating equation (2.8) with respect to each of the nodal displacements in turn and equating it to zero. This procedure produces the eigenvalue problem

$$\left[ \underline{S}_c - \frac{\rho_{ST} \omega^2}{Dg} \underline{M}_c \right] \{w_c\} = \{0\} . \quad (2.9)$$

The solution of equation (2.9) is discussed in section 2.4.1. In practice, there is no formal differentiation since the constrained mass and stiffness matrices are unaffected by the differentiation process.

Solutions of the eigenvalue problem produce the natural frequencies and displacements of the constrained system. The mode shapes of the original system are determined by reinserting all the zero nodal displacement constraints.

### 2.2.1 Evaluation of the shear forces and bending moments

In the Finite Element Displacement Method, the system's force-like quantities can only be approximated in terms of the system's nodal displacements. Timoshenko and Woinowsky-Krieger (15) give the force-like quantities (the shear forces, bending moments and twisting moment) in terms of a plate's transverse displacement as

$$\begin{aligned} Q_x &= -D \frac{\partial}{\partial x} \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) \\ M_x &= -D \left( \frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} \right) \\ M_{xy} &= D(1 - \nu) \frac{\partial^2 w}{\partial x \partial y} . \end{aligned} \quad (2.10)$$

Similar relationships can be obtained for  $Q_y$  and  $M_y$ . Using equation (2.4), equation (2.10) can be written in matrix notation as

$$\begin{aligned} Q_x &= -D \{w_e\}^T \left( \frac{1}{a^3} \{f_X'' f_Y\} + \frac{1}{ab^2} \{f_X' f_Y''\} \right) \\ M_x &= -D \{w_e\}^T \left( \frac{1}{a^2} \{f_X'' f_Y\} + \frac{\nu}{b^2} \{f_X' f_Y''\} \right) \\ M_{xy} &= \frac{D(1 - \nu)}{ab} \{w_e\}^T \{f_X' f_Y'\} . \end{aligned} \quad (2.11)$$

The force-like quantities can now be computed at any point of a finite element in terms of that element's nodal displacements. In practice, the force-like quantities are only evaluated at the elements' nodal points. This gives some idea of the degree of discontinuity of the force-like quantities across element interfaces.

The functions defined in equation (2.3) take the form

$$\begin{aligned} f_1(r) &= (1-r)^2(1+2r) & f_3(r) &= r^2(3-2r) \\ f_2(r) &= (1-r)^2r & f_4(r) &= -r^2(1-r) \end{aligned} \quad (2.12)$$

for a rectangular finite element with sixteen degrees of freedom. These functions have the limiting values shown in the table below.

Table 2.1

| r        | 0 | 1 | r         | 0 | 1 | r          | 0  | 1  | r           | 0,1 |
|----------|---|---|-----------|---|---|------------|----|----|-------------|-----|
| $f_1(r)$ | 1 | 0 | $f'_1(r)$ | 0 | 0 | $f''_1(r)$ | -6 | 6  | $f'''_1(r)$ | 12  |
| $f_2(r)$ | 0 | 0 | $f'_2(r)$ | 1 | 0 | $f''_2(r)$ | -4 | 2  | $f'''_2(r)$ | 6   |
| $f_3(r)$ | 0 | 1 | $f'_3(r)$ | 0 | 0 | $f''_3(r)$ | 6  | -6 | $f'''_3(r)$ | -12 |
| $f_4(r)$ | 0 | 0 | $f'_4(r)$ | 0 | 1 | $f''_4(r)$ | -2 | 4  | $f'''_4(r)$ | 6   |

Substituting these limiting values in equations (2.11) gives the nodal force-like vector,  $\{F\}_e \equiv \{Q_x \ Q_y \ M_x \ M_y \ M_{xy}\}_e$ , in terms of the nodal displacement vector, as

$$\{F\}_e = -D\{w_e\}^T [P] . \quad (2.13)$$

$[P]$  is a 16 x 20 matrix defined in Table 2.2.

### 2.2.2 The Relative Accuracy of the Dynamic Plate Parameters for a given Finite Element Representation

This section assesses the accuracy of the force-like quantities obtained using the Finite Element Displacement Method. Bending moments, shear forces and twisting moments were evaluated for simply supported plates of different aspect ratios  $\beta (=a/b)$ . These approximate results are compared with exact solutions.

For a simply supported plate with sidelengths  $a$  and  $b$ , the exact normalised transverse displacement  $w$  is given by

$$w = \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad (2.14)$$

when the plate is vibrating in its  $(m,n)$ th mode. Using equation (2.10), the exact shear forces and bending moments take the form

$$\begin{aligned} Q_x &= \frac{D\pi^3}{3} (m^3 + mn^2\beta^2) \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \\ M_x &= \frac{D\pi^2}{2} (m^2 + vn^2\beta^2) \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \\ M_{xy} &= \frac{D\pi^2}{2} (1 - \nu)mn\beta \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \end{aligned} \quad (2.15)$$

Similar expressions can be obtained for  $Q_y$  and  $M_y$ . From these relationships it can be seen that the higher modes induce much greater shear forces and bending moments in the plate than the lower modes. Thus, for any arbitrary motion, these force-like quantities will depend critically upon the highest modes excited.

Throughout this thesis, the percentage error is defined as

$$\left( \frac{\text{Exact value} - \text{Approximate value}}{\text{Exact value}} \right) \times 100\% .$$

The percentage errors in the maximum shear forces, bending moments and twisting moments are shown in Table 2.3 for various plate aspect ratios. For each aspect ratio, the elements are of equal size. The shear forces and bending and twisting moments tend to converge to their exact values with an increasing number of elements. It should be noted that, unlike the natural frequencies, the approximate values of these

force-like quantities can be either under or over estimates to their exact values. The percentage error increases for the higher modes. Consequently, more elements are needed in the higher modes to obtain an accuracy for the force-like quantities which is comparable to that in the lowest modes.

The twisting moment can be evaluated much more accurately than either the bending moment or shear force. The twist is included as one of the unknowns in the four unknown displacements at each nodal point. Consequently the twisting moment can be determined directly from the eigenvectors of equation (2.9). On the other hand, the displacement derivatives which determine the bending moments and shear forces have to be obtained indirectly from the eigenvectors by differentiation. This extra process is bound to introduce additional errors to the already approximate eigenvectors.

The percentage errors in the maximum values of the bending moments are often greater than those of the shear forces. This is a surprising result because the bending moments involve approximations to lesser order derivatives of the eigenvector than the shear forces. It should be remembered that the maximum shear forces are evaluated at the edge of the plate whereas the maximum bending moments occur at the plate's centre. Some of the components of the eigenvector are determined absolutely at the plate's edges due to the imposition of the boundary conditions. Thus the basic information from which the maximum shear forces are obtained is more accurate than that for the maximum bending moments. This greater accuracy seems to have more than compensated the error involved in an additional differential approximation which is necessary to compute the maximum shear forces.

Table 2.3 shows that the bending moments are less sensitive to the geometry of the finite elements than the shear forces. For greater overall accuracy, it seems advantageous to take square elements wherever practicable. The (1,3) and (3,1) modes have equal eigenvalues for a square plate. Wilkinson (16) shows that any vector in the subspace

spanned by the two correct eigenvectors of equation (2.9) will also be an eigenvector. The errors in the bending moments and shear forces are greatest in these two modes because the mathematical solution of equation (2.9) cannot differentiate between the two modes.

Using Mason's results (14), it can be seen that the eigenvalues and vectors can be more accurately determined, for a given finite element representation, than the shear forces and bending moments. Careful consideration must be given at the outset to decide upon the required vibration characteristics of the plate. There are two alternatives using a displacement approach, if the higher order modes are significant and the bending moments and shear forces are required. Either the number of finite elements must be increased or the higher displacement derivatives in the unknown nodal displacements must be included. Both alternatives require more computer storage and it is a matter of conjecture to decide upon the better of the two alternatives.

### 2.3 Application of rectangular finite elements to box-type structures

The box structure is assumed to be built up from rectangular plates. Each plate is considered to be perfectly elastic, homogeneous and isotropic. The displacement of each plate is assumed small compared with the wavelength of flexural vibrations. Stretching of the middle surface of the constituent plates is taken to be negligible. Base edges of the box are either fixed or simply supported. The remaining common edges are free to rotate so that adjacent plate edges remain at right angles but do not move in translation. Thus, corner deflections are disallowed.

Using these assumptions, Dickinson and Warburton (17) not only give a comprehensive survey of the possible analytical approaches to the problem but they also develop a series solution. The series approach becomes impracticable for the analysis of complex structures whereas the introduction of structural discontinuities and non-uniformities presents no inherent difficulty with the Finite Element Method. Even so, the series approach provides useful results with which it is possible to check the accuracy and convergence of the approximate finite element results.

The box may be treated as a flat, cross-shaped plate if, in addition to the previous assumptions, the corners of the box are not allowed to twist. Then the only difference between a three-dimensional box and a two-dimensional flat plate analysis is that additional box constraints have to be applied to the flat plate. These constraints stipulate that corresponding points on each side of the joints have the same displacements. Not only is the theory of section 2.1 applicable but, with minor modifications, most of the subroutines developed by V. Mason for the analysis of flat plates can still be used. The flow diagram and an accompanying description of the subroutines used in formulating the box problem are shown in Fig. 1.1. The flat plate approach to the box problem has the added advantage that the transformation of the element matrices from a relative to a global co-ordinate system is simplified.

It is implicitly assumed in this theory that the effect of the air enclosed within the box is negligible. Dickinson and Warburton showed that this was a reasonable assumption for the particular box they considered.

#### 2.4 The eigenvalue problem and storage difficulties

There is a great deal of literature which deals with the problem of determining the eigenvalues of equation (2.9). The two practical methods of solving equation (2.9) are the iterative and the transformation method. It is more convenient to use an iterative method if only a few eigenvalues are required. Such a method evaluates  $\det[\underline{S}_c - \mu^2 \underline{M}_c]$ ,  $\mu^2 = \rho_{ST} \omega^2 / Dg$ , for some choice of  $\mu$ . Using this method, full advantage can be made of the planar storage scheme (18) for the sparsely populated matrices  $\underline{S}_c$  and  $\underline{M}_c$ . This method however has the disadvantage of being inefficient. If all or a large number of eigenvalues are required then equation (2.9) is usually reduced to the standard form

$$[\underline{L}^{-1} \underline{S}_c (\underline{L}^{-1})^T - \mu \underline{I}] \{z\} = \{0\}$$

$$\text{where } \{z\} = \underline{L}\{w_c\} \quad \text{and} \quad \underline{M}_c = \underline{L}\underline{L}^T. \quad (2.16)$$

This reduction is accomplished by a Choleski symmetric decomposition of the positive definite, constrained mass maxtrix  $\underline{M}_c$  (19). The eigenvalues are then evaluated with a transformation technique such as Householder's method (20). Although the transformation method is a relatively fast computer process it has the disadvantage that the matrix  $\underline{L}^{-1} \underline{S}_c (\underline{L}^{-1})^T$  is fully populated. Hence more storage space is required than with the iterative method.

Experience has shown that both methods are sensitive to numerical overflow problems when the order of the constrained mass and stiffness matrices are greater than fifty. The computer simply cannot cope with numbers produced by multiplying together a large number of matrix elements. This problem still exists even if the modulus of the largest element of matrix  $(\underline{L}^{-1}) \underline{S}_c (\underline{L}^{-1})^T$  is reduced to unity. Overflow then occurs due to the division of the matrix elements by the difference of two nearly equal numbers. The problem can be alleviated by allocating separate storage locations to the moduli and exponents of each matrix element. This method is comparable to using double precision arithmetic which requires greater running time and computer storage space. Thus, there seems to be a delicate balance between numerical stability and computer storage requirements. Due to the overriding requirement of numerical stability, it has been found essential to use the smallest number of degrees of freedom of the mechanical system which is compatible with an acceptable inaccuracy in the results.

It is possible to reduce computer storage requirements at the expense of greater running time by using the symmetrical properties of a freely vibrating uniform box. The assumption of symmetrical or antisymmetrical displacements about the box's planes of symmetry reduces the closed and open boxes to eight and four equivalent but smaller plate systems respectively. This simplification reduces the eigenvalue problem to about an eighth or quarter of the size of the original problem with a corresponding reduction in the computer storage requirements.

## 2.5 Comparison of the eigenvalues and vectors of the series and finite element solutions

Tables 2.4 and 2.5 show direct comparisons between Dickinson and Warburton's series solution (17) and the finite element results for the natural frequencies of a 9.6" x 12" x 14.4" x 0.125" six-plate closed, and five plate open, steel box. The open box is essentially the same as the closed box but with the 14.4" x 9.6" face removed. All base edges are considered to be simply supported. The series solution gives errors of less than 0.1% and 0.5% for the closed and open box respectively. Hence the results from the series solution will be assumed to be 'exact' for the present purpose of comparison.

A direct comparison of the finite element results for the open and closed boxes using three (21) and four degrees of freedom per nodal point shows the overall improvement in accuracy obtained by including the twist as an unknown displacement. For example, consider the six lowest natural frequencies of the closed box. If the twist is excluded at least twice as many elements are required to give comparably accurate results. Thus there are obvious advantages, in the light of the discussion in the preceding section, of including the twist as an unknown displacement. This observation is in agreement with Mason's conclusions (14).

Both sets of finite element results give an upper bound and converge to the exact natural frequencies with an increasing number of elements. In general, the percentage errors increase with increasing mode number. The greatest inaccuracies occur when the mode shapes have nodal lines whose separation is smaller than the dimensions of the finite elements. In these cases, the estimated frequencies can easily be improved by increasing the number of elements.

Using four degrees of freedom per nodal point and the same number of finite elements per individual plate sub-system, the closed box's natural frequencies always converge more quickly and are generally more accurate than the corresponding natural frequencies of the open box. Exceptions occur when the separation of the nodal lines for the closed box mode become much smaller than that of the corresponding open box mode. The deterioration in the accuracy of the open box results occurs because the boundary conditions for a free edge cannot be strictly applied. The boundary conditions require that the shear force and bending moment be zero at the free edge. These force-like quantities must be translated

to displacement edge conditions before they can be applied to equation (2.7) as constraints (14). From equation (2.10) it can be seen that the displacement conditions are  $(\partial^3 w / \partial x^3 + \partial^3 w / \partial x \partial y^2) = 0$  and  $(\partial^2 w / \partial x^2 + \nu \partial^2 w / \partial y^2) = 0$  for a free edge which is parallel to the  $y$  axis. Similar expressions can be obtained for a free edge which is parallel to the  $x$  axis. None of these higher derivatives of displacement have been included in the three or four unknowns at the corner of each finite element. Hence these displacement constraints cannot be applied. When four unknowns per corner are used, it is possible to revert to Poisson's condition for a free edge. In this case the twist,  $\partial^2 w / \partial x \partial y$ , is constrained to be zero at the free edge. It is impossible to apply even this constraint when three unknowns per corner are used. Similarly it was found that the closed box's natural frequencies were generally more accurate than the corresponding natural frequencies of the open box when three unknowns per corner were used. Exceptions occurred when the rate of change of twist over the closed box became much greater than that over the open box.

Fig. 2.1 shows the good overall agreement between the mode shapes of the open and closed boxes obtained using the series and finite element solutions. A closer inspection of these mode shapes, however, reveals that they differ in detail even for the lowest mode numbers. Mode shapes obtained for the open and closed boxes with the smallest possible number of finite elements indicate that there is only a slight deterioration from the agreement shown in Fig. 2.1. It should also be noted that a marked lack of detail in the mode shapes obtained with the Finite Element Method occurs when any mode has nodal lines whose separation is smaller than the dimensions of the finite elements.

Dickinson and Warburton obtained close agreement between their theoretical and experimental results even though the mathematical model did not fully represent the experimental edge restraints. They also showed that the frequencies of the air modes within the closed box could be accurately predicted assuming the box to have rigid faces. It can be concluded that, in this particular case, the magnitude of the internal air pressure was so small that it had little effect on the motion of the closed box. Bending modes were experimentally observed for the closed box which could not be predicted by the mathematical model. In these modes the box was behaving like a short cantilever with its root at the

base. This behaviour is similar to the racking motion observed by Cawthorn (22) in houses with certain wood constructions. Although the present theory cannot account for these modes, there is no inherent difficulty preventing the inclusion of this type of motion in the finite element formulation of the problem - if it is found to be significant. Difficulties would occur however in trying to extend the series solution to cover this contingency. The main advantage of the series solution is that the bending moments and shear forces are analytically related to the assumed mode shapes. Even if the higher modes are significant, these force-like quantities can be accurately evaluated once the modes have been determined. It has been shown in section 2.2.2 that this is not the case with the Finite Element Method.

The purpose of this thesis is to extend the theoretical model in order to predict the transient response of complex structures. Thus, the Finite Element Method is used with the limitation that the shear forces and bending moments are not evaluated due to the prohibitive number of elements necessary to accurately compute these variables. It should be emphasised that this limitation is due to practical and not theoretical difficulties. The practical difficulties are caused by the excessive demands on the time and storage requirements for the computer when a large number of finite elements are used.

Dickinson and Warburton showed that, for certain base supports, the open box's fundamental mode corresponds to a torsional mode. This mode cannot be predicted by the theoretical model because the base supports are effectively 'free'. However, this mode will be relatively unimportant in most single storey, residential-type structures. The base edges of this type of structure can generally be approximated by fully fixed edge conditions. Hence the natural frequency of the torsional mode will be usually much higher than the structure's fundamental frequency. Consequently the contribution of the torsional mode to the structure's overall response will be relatively small if the structure's lowest modes are predominantly excited.

## 2.6 Comparison of individual plate modes and the flexural modes of the box

A practical structure should be reduced to the simplest idealised form which is consistent with an acceptable accuracy in the computation of the structure's motion. The effort required to compute the theoretical motion will then be minimised. The comparative merits of computing the motion of

a structure with a simplified plate theory or a more complex box theory are considered in this section. Thus the natural frequencies and normal modes predicted by the plate and box theories are compared as the box's edge restraints are gradually relaxed. This comparison gives an indication of the disparities between the results obtained using one or other of the two methods.

Table 2.6 shows the effect of gradually relaxing the non-base edge restraints on the first six modes of a uniform closed box. This box is representative of a large windowless room which has relative dimensions 40 : 28 : 11.5. The floor of this room is assumed to be rigid and all the base edges are considered to be fully fixed.

When all the joints are fully fixed the five constituent faces of the box act independently. Hence the simple plate theory can be applied to each of the individual walls. The six lowest modes involve a predominant roof motion for this particular case because the roof is easily the largest single plate. Consider next the case where the roof edge restraints have been relaxed so that the side walls are only coupled through the roof. All the corners of the box are still considered to be motionless. In this case the frequency parameter lies approximately midway between the values obtained for the fully fixed and simply supported edge conditions of the independent roof. The shapes of the lowest frequency modes correspond to those of the previous model even though the side walls are now forced to move by the roof's inertia. Hence it would still be practicable to use a simplified plate approach for the lowest modes.

Finally, consider the effect of applying the edge restraints of section 2.3 so that there is 'complete' coupling between all five faces of the box. The frequency parameter now decreases below the values obtained for the independent, simply supported roof. This further decrease in the frequency parameter corresponds to a decrease in the box's overall stiffness due to the further relaxation of the edge restraints. Also, the modal density significantly increases due to the increased subsidiary motion of the side faces. This subsidiary motion, however, only affects the roof's plate mode shapes in the higher modes. Thus, the main discrepancy between the two theories for this set of edge conditions is the prediction of all the box's natural frequencies.

Depending upon the relative sizes of the box's faces, not all side

faces participate equally in a box mode type of vibration (see, for example, modes 13 and 16 in Fig. 2.1). Also, a practical structure usually has a high modal density in which the coupling between faces cannot easily be determined. Hence it could be extremely difficult to decide whether the plate or box theory is more appropriate for even the lowest modes. Unless the walls have been specifically constructed to act independently, a box theory should always be used if the higher modes are considered to be significant. It is also advisable to use the box theory if the structure's modal spacing and density are considered to be important parameters.

## 2.7 Effects of structural discontinuities

In order to illustrate the flexibility of the Finite Element Method, structural discontinuities were introduced into a uniform, five-plate closed box. The base and vertical edges of this box were always constrained to be fully fixed. Table 2.7 shows the resulting changes in the factored frequency parameter,  $\mu \times 10^{-2}$ , for the first three modes in which both  $m$  and  $n$  are odd. The material properties  $\rho_{ST}$  and  $D$  used in the calculation of  $\mu$  were always the same as the corresponding values for the uniform closed box.

The motion in all three modes is dictated by the motion of the roof in the case of the uniform closed box. Thus, the introduction of openings in the side faces of the closed box has relatively little effect on the frequency parameter of the two lowest modes. In these cases the loss of mass inertia compensates the loss of stiffness. The motion of the faces with openings then becomes progressively more significant due to their greater flexibilities.

The main effect, in the fundamental mode, of introducing a central opening in the roof of the closed box is due to greater loss of mass inertia than stiffness. Hence the frequency parameter increases for this mode. The frequency parameter is relatively unaffected in the other two modes because the loss of stiffness compensates the loss of mass inertia. On the other hand, halving the thickness of the uniform closed box's roof reduces the frequency parameter by a factor of two or three. In this case the loss of stiffness more than compensates for the loss of mass inertia.

In all cases, the changes in the frequency parameter are in quantitative agreement with the changes predicted by intuitive reasoning. The corresponding changes in the mode shapes, however, could not be so easily predicted.

## 2.8 Conclusions

Providing the internal acoustic backpressure can be neglected, Dickinson and Warburton have shown that a simplified mathematical model of a box-type structure gives good agreement between theoretical and experimental natural frequencies and mode shapes. In order to extend the same mathematical representation to more complex structures an approximate finite element technique is used in place of the exact series solution. It has been shown that four unknowns per corner give more accurate results for a given number of finite elements than three unknowns per corner. In all cases the finite element results give an upper bound and converge to the exact frequencies with an increasing number of elements. More important, if only the lowest modes are significant, good agreement is obtained for the natural frequencies and mode shapes with only a small number of elements. Many more elements are necessary for the same degree of accuracy, if the shear forces and bending moments are required. In this case it would be advantageous to use the series solution.

Difficulty could be experienced in deciding whether a simplified plate theory or a box theory is more appropriate in a practical situation. The best approach will depend critically upon the degree of coupling between individual faces and the effect of the higher modes. In either case, it is possible to intuitively assess the quantitative effects of introducing discontinuities in the structure on the natural frequencies but not on the corresponding mode shapes.

The evaluation of the free vibration characteristics of a box-type structure was an interim stage to the more practical problem of determining its transient response. In the direct evaluation of the transient vibration problem the stability of the numerical methods depend upon the highest eigenvalue whereas the lowest modes usually dictate the system's response. Thus, the transformation method of solving the eigenvalue problem was found to be more useful than the iterative method.

### CHAPTER 3

#### THEORETICAL FORMULATION OF AN ARBITRARILY SHAPED WAVE TRAVELLING OVER A BOX-TYPE STRUCTURE

The assumptions necessary to reduce a complex structure to an independent set of simple subelements are briefly reviewed in this chapter. The wave's convection is seen to have a significant effect on a particular mode of a subelement when its convection frequency coincides with the natural frequency of the mode. The concept of a convection frequency is subsequently extended to a box-type structure whose subelements do not usually act independently. Then the circumstances for which the wave's convection is significant, for this particular kind of structure, are briefly discussed.

Difficulty is experienced in measuring an actual sonic boom's total duration  $T_2$ . It is comparatively easy, with modern analogue/digital techniques, to evaluate the boom's Fourier transform or frequency spectrum. An excitation frequency can then be defined from this spectrum, which is more easily obtained than  $T_2$ .

The theoretical analysis of the last chapter is extended in order to evaluate the box's response to a travelling wave. The wave can have any arbitrary shape, providing it can be approximated by a set of discontinuous straight lines. Each pressure discontinuity is treated as an impulse so that the nett response can be built up by the process of superposition. At any given instant of time, the wave is assumed to be acting normally to some segment of the structure. Consequently the wave can be approximated by an equivalent uniform pressure on this segment. This assumption however, will only be realistic for those waves whose lengths are much greater than the physical dimensions of the segment. By the introduction of suitable time delays, corresponding to the wave's travel over the box, the pressure-time variation over the various segments is developed for a digital computer. The pressure-time history over the entire box is obtained by assuming pressure doubling on all the front face's segments. The validity of all the approximations is tested by comparing the theoretical pressure-time histories, on individual faces, with experimentally observed waveforms.

The equations of motion are formulated assuming an approximate viscous damping factor. Subsequently, the stability and convergence of the numerical methods used to solve these equations are discussed.

### 3.1.1 A brief review of the transient response of simple structural subelements to an idealised sonic boom

The maximum response of a simple oscillator to an ideal, symmetrical N-wave is presented in this section. The simplifying assumptions necessary to reduce a complex structure to a set of discrete oscillators are then briefly reviewed. The results are extended to the case where the same N-wave travels over the structure. The wave's convection is seen to be significant when its speed across the structure satisfies a relationship which depends upon the mode of vibration. A convection frequency is deduced from this relationship. Then the basic concepts are extended in the following sections to a box-type structure which does not satisfy all the simplifying assumptions.

Consider the response of an undamped oscillator to a symmetrical N-wave which has the properties

$$T_R = 0, T_1 = T_2 = T \text{ and } p_0 = p_-. \quad (3.1)$$

These variables are defined in Fig. 3.1(a). Cheng and Benevise (48) show that the oscillator's maximum displacements for the forced oscillation ( $0 \leq t \leq T$ ) and the free oscillation ( $t > T$ ) may be written in the form

$$|T_n|_{\max} = \frac{C}{p_n^2} \phi_n \quad (3.2)$$

and

$$|T_n'|_{\max} = \frac{C}{p_n^2} \phi_n' \quad (3.3)$$

respectively for zero initial displacement and velocity conditions.  $p_n$  is the oscillator's natural frequency and  $C$  is a function of its mass and  $p_0$ . They show that  $\phi_n$  and  $\phi_n'$  are simply the ratios of the oscillator's maximum dynamic displacement (in the forced and free oscillations respectively) to its static displacement under load  $p_0$ . Thus  $\phi_n$  and  $\phi_n'$  are called the displacement dynamic amplification factors (daf) of the oscillator. They are plotted in Fig. 3.2 against the non-dimensional period ratio  $R = T/\tau$  where  $\tau$  is the oscillator's natural period.

It can be shown that the maximum acceleration during the forced oscillation is given by

$$\left| \frac{\ddot{T}_n}{C} \right|_{\max} = \phi_n'' = \left| \cos \left\{ \tan^{-1} \left( -\frac{1}{\pi R} \right) \right\} - \frac{1}{\pi R} \sin \left\{ \tan^{-1} \left( -\frac{1}{\pi R} \right) \right\} \right|$$

subject to the restriction  $0 \leq \frac{1}{2\pi R} \tan^{-1} \left( -\frac{1}{\pi R} \right) \leq 1$

otherwise

$$\left| \frac{\ddot{T}_n}{C} \right|_{\max} = \left| \cos \left( 2\pi R \frac{t}{T} \right) - \frac{1}{\pi R} \sin \left( 2\pi R \frac{t}{T} \right) \right|_{\max} \quad \text{for } 0 \leq t \leq T \quad (3.4)$$

and, during the free oscillation, it takes the form

$$\left| \frac{\ddot{T}_n}{C} \right|_{\max} = \phi_n' \quad \text{for } t > T. \quad (3.5)$$

C is simply the oscillator's peak acceleration to an instantaneous step forcing function with magnitude  $p_0$ . Hence the acceleration dafs  $\phi_n''$  and  $\phi_n'$  may be defined in a manner similar to the displacement dafs. They are the ratios of the oscillator's maximum forced and free dynamic accelerations to its maximum acceleration under the action of an instantaneous step forcing function  $p_0$ .  $\phi_n'$  retains the same value.  $\phi_n''$  is numerically evaluated from the first equation of (3.4). The inequality does not hold for this equation when R is less than 0.392. In these cases  $\phi_n''$  is calculated from the modulus of the second equation for constant increments h in t such that  $0 \leq rh \leq T$ . r is the total number of increments. The resulting values are shown in Fig. 3.2

The accelerations are measured in preference to the displacements in the experimental work. Therefore the present thesis is mainly concerned with the acceleration dafs. Observations from Fig. 3.2 can now be outlined for these dafs as:

- (i) the forced motion gives the absolute maximum acceleration daf for R smaller than 0.48. The maximum value in this region is 1.33 and it occurs at about  $R = 0.33$ ;
- (ii) the motion in the free vibration period predominates for the accelerations at most large values of R. But, for large R, the maximum accelerations which occur in the forced motion for this N-wave are similar to those observed for an instantaneous step pressure  $p_0$ ;
- (iii) the overall maximum acceleration daf occurs during the free vibrations at R around 0.86. It then has the value of 2.20.

These observations apply to an undamped oscillator. Cheng (48) notes that the presence of moderate damping tends to reduce sharply the peak values of the overall displacement daf ( $\max [\phi_n'' \text{ or } \phi_n']$ ) which occur for values of R greater than 2.0. The humps at which these peak values occur disappear. The overall acceleration daf ( $\max [\phi_n'' \text{ or } \phi_n']$ ) will be similarly affected by damping. Consequently, for values of R greater than 2.0, the effect of damping is to bring the overall maximum values of acceleration due to an N-wave closer to the values observed for an instantaneous step pressure  $p_0$ .

A structure is composed of many components or subelements. If the structural and acoustic coupling between the subelements is negligible then each subelement will act independently. The response of a subelement may then be evaluated separately in terms of a series of its own normal modes. The subelement is equivalent to the simple oscillator if it is also assumed that it vibrates in a single mode and that the mode shape has no influence on the time history of the forcing function. The last condition is fulfilled when the sonic boom's direction of propagation is normal to the subelement. If the first condition is also satisfied then the dafs of the subelement will simply have the same values as the dafs of the oscillator for the appropriate value of R. The mode shape does have some influence on the forcing function's time history, however, when the boom travels over the subelement. Cheng (7) showed that, as a result of the boom's convection over plates and beams, there is a critical velocity to every normal mode which causes resonance to occur in that particular mode. For example, the critical velocity of a plate is shown to conform to the relationship

$$p_{mn} = \frac{n\pi v_T}{L} \quad (\text{rads./sec}) \quad (3.6)$$

for the (m, n) th. mode.  $v_T$  is the boom's speed of propagation in the y direction and L is the plate's length in the direction of propagation. Craggs (11) defined the same relationship in the frequency domain by introducing a convection frequency

$$(f_c)_{mn} = \frac{p_{mn}}{2\pi} = \frac{nv_T}{2L} \quad (\text{c.p.s.}) \quad (3.7)$$

in order to emphasise that resonance occurs in a particular mode when its frequency corresponds to the convection frequency. Cheng (7) then assumed that the subelement's displacement is dictated by its fundamental mode. A

resonant condition will occur in this mode when its frequency coincides with the fundamental convection frequency ( $n = 1$  in (3.7)). The worst possible case under these conditions was found to occur when a sonic boom, having the properties defined in (3.1), travels in a direction which is at right angles to the subelement. Consequently the oscillator's displacement dafs will give an upper bound to the displacement dafs produced by the travelling boom. The corresponding acceleration dafs however were not evaluated.

The range of structures is extended in the following chapters to a box-type structure whose subelements do not usually act independently. The subelements of such a structure cannot be reduced to a set of discrete oscillators because they are both structurally and acoustically coupled. Thus the conclusions presented for the maximum responses are enlarged to include this particular set of structures.

### 3.1.2. Definition of a sonic boom's excitation frequency

Actual sonic booms which are observed on the ground cannot usually be described by a perfect N-wave. Their gross properties however are still characterised by either the boom's total duration  $T_2$  or its wavelength  $\lambda$ . Difficulty was experienced in measuring  $T_2$  for the simulated booms. For example, the full line simulated boom in the freefield waveforms of Fig. 6.2 shows that the easiest measurable duration is the time between a boom's peak positive and negative pressures ( $T_1 - T_R$ ). Cheng (6) suggests an alternative method of defining  $T_2$ . He introduces a frequency  $\omega_N$  of the N-shaped wave which is defined in terms of its total duration  $T_2$  as

$$\omega_N = \frac{2\pi}{T_2} = 2\pi f_N. \quad (3.8)$$

Fig. 3.3 (27) indicates that the frequency spectrum of a theoretical N-wave with a non-zero rise time  $T_R$  has many frequency components. The only difference in the spectra of theoretical N-waves with zero and non-zero rise times is in their high frequency contents above a frequency which corresponds to  $1/\pi T_R$  ( $T_R \neq 0$ ). Thus the theoretical N-wave with  $T_R = 0$  will have a similar low frequency content to that illustrated in Fig. 3.3.  $f_N$ , as defined by Cheng, corresponds to the prominent lowest frequency peak in this spectrum. For convenience,  $f_N$  will be called the 'excitation' (as it refers to the exciting N-wave) frequency. It should be noted however

that the structure is excited by the N-wave, only for a limited duration  $T_2$ . Hence the term excitation frequency should not be taken to infer that the structure's response to the N-wave is analogous with its response to a harmonic disturbing force.  $T_2$  is simply the inverse of the wave's excitation frequency.

With the use of an analogue to digital converter, combined with a digital computer, it is comparatively simple to evaluate the sonic boom's frequency spectrum - and hence  $f_N$  or  $T_2$ .

### 3.2.1 Work done by a uniform pressure on a finite element rectangular subelement

Consider a uniform pressure  $p_0$  acting on a rectangular strip of any one rectangular finite element. Suppose that the diagonal points of this strip are  $(x_0, y_0)$  and  $(x_1, y_1)$  where  $x_0 < x_1$  and  $y_0 < y_1$ . Then the forcing function  $f(x, y)$  over the whole finite element takes the form

$$f(x, y) = \begin{cases} p_0 & \begin{cases} x_0 \leq x \leq x_1 \\ y_0 \leq y \leq y_1 \end{cases} \\ 0 & \text{elsewhere.} \end{cases} \quad (3.9)$$

The forcing integral in the energy equation (2.1) becomes

$$I = p_0 \int_{x_0}^{x_1} \int_{y_0}^{y_1} w \, dx dy \quad (3.10)$$

where the displacement function,  $w(x, y)$ , is given by equation (2.4).

Hence

$$I = p_0 (x_1 - x_0)(y_1 - y_0) \{w_e\}^T \int_0^1 \int_0^1 \{f_{X_1} \quad f_{Y_1}\} dX_1 dY_1 \quad (3.11)$$

where  $X_1 = \frac{x - x_0}{x_1 - x_0}$  and  $Y_1 = \frac{y - y_0}{y_1 - y_0}$ .

The following integrals are needed in the evaluation of the double integral.

$$\int_0^1 f_1 dX_1 = (1 - k_2)^2(1 + k_1 + 2k_2) - k_1(1 - k_2)\left(1 + \frac{4}{3}k_1 + 2k_2\right) + k_1^2\left(\frac{1}{3} + \frac{1}{2}k_1 + \frac{2}{3}k_2\right)$$

$$\int_0^1 f_2 dX_1 = (1 - k_2)^2\left(\frac{1}{2}k_1 + k_2\right) - k_1(1 - k_2)\left(\frac{2}{3}k_1 + k_2\right) + k_1^2\left(\frac{1}{4}k_1 + \frac{1}{3}k_2\right)$$

$$\int_0^1 f_3 dX_1 = k_2^2(3 - k_1 - 2k_2) + k_1k_2\left(3 - \frac{4}{3}k_1 - 2k_2\right) + k_1^2\left(1 - \frac{1}{2}k_1 - \frac{2}{3}k_2\right)$$

$$\int_0^1 f_4 dX_1 = k_2^2(-1 + \frac{1}{2}k_1 + k_2) + k_1k_2(-1 + \frac{2}{3}k_1 + k_2) + k_1^2\left(-\frac{1}{3} + \frac{1}{4}k_1 + \frac{1}{3}k_2\right)$$

$$\text{where } k_1 = \frac{x_1 - x_0}{a} \quad \text{and } k_2 = \frac{x_0}{a}. \quad (3.12)$$

For a uniform pressure over the entire finite element,  $x_0 = 0$  and  $x_1 = a$ , so that  $k_1 = 1$  and  $k_2 = 0$ . The above integrals then reduce to

$$\begin{aligned} \int_0^1 f_1 dX_1 &= \int_0^1 f_3 dX_1 = \frac{1}{2} \\ \text{and} \\ \int_0^1 f_2 dX_1 &= - \int_0^1 f_4 dX_1 = 1/12. \end{aligned} \quad (3.13)$$

Subroutine UNPRES evaluates equation (3.11) using equations (3.12) for each individual finite element.

### 3.2.2 Theoretical description of a wave travelling over a box-type structure

In this section the general forcing function of an arbitrary shaped wave which travels over a box-type structure is formulated, using the results of the previous section. The validity of this formulation is discussed in section 3.2.4 by comparing theoretical pressure - time histories on different faces of the structure with experimentally observed waveforms. A convection frequency is defined for a box-type structure which is analogous to that of a plate. Then the circumstances for which the convection effects could be important for the box-type structure are discussed.

The pressure-time history  $p(t)$  of the sonic boom shown in Fig. 3.1(a) can be written as:

$$\begin{aligned}
 p(t) &= p_o \cdot \frac{t}{T_R} & 0 < t < T_R \\
 p(t) &= \left( \frac{p_o}{T_1 - T_R} \right) \left\{ \left( \frac{p_o}{p_o} T_R + T_1 \right) - \left( 1 + \frac{p_o}{p_o} \right) t \right\} & T_R \leq t \leq T_1 \\
 p(t) &= \left( \frac{-p_o}{T_2 - T_1} \right) \{ T_2 - t \} & T_1 < t < T_2 \\
 p(t) &= 0 & \begin{cases} t \geq T_2 \\ t \leq 0. \end{cases} & (3.14)
 \end{aligned}$$

The equations of the more usual N-wave are obtained from the second and last of these equations when  $T_R = 0$  and  $T_1 = T_2$ .

The experimental free field waveform is approximated by a straight line in a given time interval. Then the corresponding equation of the pressure-time history is given by the second equation in (3.14) for any time interval  $(T_R, T_1)$ . In this way any arbitrary shape can be approximated by a set of discontinuous straight lines. In practice the free field waveform is usually approximated by three straight lines in order to minimise its computational description. This approximation is valid providing the effect on the structure of the wave's high frequency content is negligible.

Consider a sonic boom to be incident on a single plate. It will generally travel across the plate with its shock front at an angle of inclination  $\psi$ . The general forces are then not only functions of  $p(t)$ , that would be measured at a single point, but also of the convection velocity and the plate's dimension in the direction of traverse. It is necessary to account for the convection effects in the formulation of the general forcing function.

For simplicity, consider the boom to be travelling parallel to the plate's  $y$  axis (Fig. 3.1(b)). Suppose that the plate is divided into  $M$  equal rectangular segments each with length  $\ell$  normal to the shock front. If  $\ell$  is small compared with the sonic boom's resolved wavelength on the plate,  $\lambda_T = \lambda \operatorname{cosec} \psi$ , then the boom may be assumed to be as if at zero inclination to each segment. Next, consider the shock front to be just moving onto the first segment. The wave actually moves across this segment but it is assumed to act instantaneously over the whole segment. Hence the magnitude of the initial theoretical pressure wave will be greater than the true pressure magnitude by some unknown margin. An artificial linear rise time correction is applied to the front of the actual sonic boom in order to compensate for this effect. It is calculated in the following manner. If the shock front's speed of propagation is  $a_0$  then the trace speed across the plate  $V_T$  will be

$$V_T = a_0 \operatorname{cosec} \psi. \quad (3.15)$$

There will be a time delay  $T_R + \ell/V_T$  before the actual wave's maximum positive pressure  $p_0$  reaches the far side of the first segment. The theoretical wave is assumed to have the same positive pressure  $p_0$  as the actual wave at this time. This is achieved by replacing the rise time  $T_R$  of the actual wave with the artificial rise time  $T_R + \ell/V_T$ . The theoretical and actual wave then have the same pressure-time variation between their respective maximum positive and negative pressures.

The total time that the actual wave remains on any part of the first segment is the total duration of the sonic boom  $T_2$  plus the time for the rear of the wave to traverse the segment,  $\ell/V_T$ . The theoretical pressure wave should be released instantaneously from acting on the first segment after this time. But, in the Finite Element Displacement Method, the work done by the forcing function over an element is calculated in terms of an equivalent expression at the element's nodal points. For reasons which are

discussed in Chapter 5, a finite element is not subdivided into smaller segments as in section 3.2.1. In this case, work will be done at the nodal points which interconnect the first and second segments as the pressure wave moves over the second segment. Hence another linear time correction  $\ell/V_T$  is applied to the rear of the actual wave which corresponds to the time taken for the rear of the actual wave to reach the far edge of the second segment. This correction is used for all the segments. However, it only strictly holds for the last segment if the boundary edge at the exit of the wave is fully fixed.

The resulting modified pressure-time history is shown in Fig. 3.1(c). It is described by equation (3.14) but with

$$\begin{aligned} T_R &\rightarrow T_R + \ell/V_T \\ T_1 &\rightarrow T_1 + \ell/V_T \\ T_2 &\rightarrow T_2 + 2\ell/V_T \end{aligned} \quad (3.16)$$

for the idealised wave..

The pressure-time histories, for the idealised wave, on the remaining segments of the plate will be identical to that on the first segment but they will act at different times. These time delays correspond to the time taken for the shock front to reach the leading edge of the particular segment under consideration. The general forcing function of the whole plate is then built up by the process of superposition.

Consider now the order of magnitude of the convection correction terms  $\ell/V_T$ . It has been assumed that

$$\ell \ll \lambda_T \quad (3.17)$$

and

$$a_0 = \frac{\lambda}{T_2}.$$

Thus, providing  $\psi \neq 0^\circ$  for the actual boom,

$$\lambda_T = V_T \cdot T_2. \quad (3.18)$$

Substituting this expression in (3.17) gives

$$\frac{\ell}{V_T} \ll T_2. \quad (3.19)$$

As  $V_T$  and  $T_2$  are known from the properties of the sonic boom,  $\ell$  must be

adjusted in the plate's idealisation so that inequality (3.19) is satisfied. No restriction is imposed on the ratios of the correction term  $\lambda/V_T$  to the other time properties of the sonic boom ( $T_R$  for example). The application of such restrictions will depend upon the significance of their contributions to the plate's response. It is implicitly assumed in the present theory that the overriding contribution to the plate's response is created by the actual sonic boom's total duration  $T_2$ .

The extension of this theory to a box is relatively simple if the box can be regarded as being composed of a set of single plates at different angles of incidence and inclination to the shock front. A single angle of incidence  $\theta$  and an angle of inclination  $\psi$  of the wave's shock front relative to the box, are defined.  $\psi$  is always measured from the horizontal as shown in Fig. 4.2.  $\theta$ , however, can have two definitions depending on whether the response of a particular face is being considered for all angles of incidence or whether all the faces' responses are being considered for a particular value of incidence. More details of these definitions are given in section 4.3.2.

The theoretical sonic boom's shock front will always be assumed to be at zero incidence to one of the box's side faces. Then the computational difficulties, which are discussed in section 3.3, will be minimised. Each of the box's faces are divided, as before, into a number of equal rectangular segments. The roof and vertical faces normal to the shock front are divided into segments which are at right angles to the wave's direction of propagation. The modified pressure-time histories on these faces will then be similar to that on the single plate.

The front face of the box is that face, parallel to the shock front, which is the first face to be acted upon by the sonic boom. Consider the shock front to be at  $90^\circ$  inclination to the horizontal. If the front face is initially assumed to be 'rigid', the total pressure magnitude on this surface will be double that of the incident wave. The pressure-time history over the whole front face takes a form similar to the actual sonic boom because the boom acts normally to this face. (The phrase 'acts normally' is often used in connection with a particular face of a box-type structure as a shorthand form of implying that the wave's direction of action is completely normal to this face. Thus, Fig. 4.2 shows that when the wave acts normally to the South face of the model  $\theta = 0^\circ$  with respect to this face and  $\psi = 90^\circ$ . This face would also act as the model's front face.) It is obtained by multiplying equation (3.14) by a factor of two. In practice the front face is not rigid

so that, depending upon this face's flexibility, the true factor is between one and two. However, it is always taken to be two. A pressure disturbance is created within the box when the front face is displaced from its initial position under the action of the incident wave. This disturbance is assumed to have no effect on the box's response.

The pressure-time history over the whole of the back face is taken to be similar to that on the front face with the exception that no pressure doubling occurs on this face. The wave will directly force the back face after a time delay corresponding to the time taken for the shock front to traverse the roof and side faces.

Using the data from subroutine UNPRES, subroutine FORCE evaluates the general forcing function for the whole box. It is assembled manually in order to account for the structure's boundary conditions. Listings of these subroutines are shown in the Appendix.

The concept of a convection frequency is extended from simple subelements such as plates and beams to a box-type structure. Then the circumstances for which the wave's convection can have a significant effect on the box's response are briefly discussed.

It has been shown in section 3.1.1 that the wave's convection is important when its convection frequency satisfies the relationship

$$(f_c)_{mn} = \frac{nV_T}{2L} . \quad (3.20)$$

This relationship can be extended to a box-type structure by defining  $L$  to be its maximum dimension in the direction of the wave's propagation;  $V_T$  to be the trace speed across the box; and  $n$  to be the integer number of equivalent half sine waves in the direction of the boom's propagation. The sine waves are similar to the shape of a particular mode in that direction. It is then possible for a particular box mode to have different values of  $n$  on the roof and side faces. Consider, for example, a wave which is travelling parallel to the 14.4" x 12" sides of the uniform closed box discussed in the previous chapter. Fig. 2.1 shows that the equivalent sine functions for the complete roof and 14.4" x 12" sides have values of four and two, respectively, for  $n$  when the box is vibrating in its seventh mode.

The convection frequencies will depend upon the shock front's angle of inclination  $\psi$  and angle of incidence  $\theta$  to the box. Fig. 4.2 shows that

$$L = a \cos \theta + b \sin \theta \quad (3.21)$$

while

$$V_T = a_0 \operatorname{cosec} \psi \quad (3.22)$$

for the box.

Consider next the circumstances for which the convection effects could provide a significant contribution to the generalised forces of the box. The total time that the travelling wave remains on the box is approximately

$$\frac{1}{V_T} (\lambda + L)$$

where  $\lambda$  is the wave's wavelength. Then the manner in which the box responds depends on whether  $\lambda$  is greater than  $L$ , or vice versa, and how the terms  $(\lambda/V_T)$  and  $(L/V_T)$  are related to the fundamental period of the box  $\tau$ . If  $\lambda$  is much greater than  $L$  and  $(\lambda/V_T)$  is of the same order as  $\tau$ , the box will effectively respond as though the wave were acting normally to each individual face of the box. The convection effects are small under these conditions and the travelling wave is similar to a uniform, instantaneous step pressure acting on each of the faces. If  $L$  is much greater than  $\lambda$ , the box will respond as though the wave were a moving line force on the roof and those side faces which are not at zero incidence to the shock front. The box's mode shapes will then have an important influence on the time history of the generalised forces so that the convection effects can be significant. The effects on a box's accelerations due to changes in  $\lambda$  and  $L$  are investigated experimentally in the next chapter.

### 3.2.3 General observations

The structure's maximum response may be plotted against  $\lambda/L$ . The reasons for using  $\lambda/L$  instead of the more usual parameter  $T/\tau$  are discussed in this section. Then the details involved in the calculation of  $\lambda/L$  for a particular boom/structure combination are given.

A sonic boom which travels over a box-type structure usually excites many of its modes. This behaviour contrasts to that of the oscillator or to the behaviour of the simple subelements in which only a single mode is assumed to be excited. In order to understand the detailed nature of a structure's response to a sonic boom, it becomes necessary to correlate the structure's excited modes with the sonic boom's frequency spectrum and with its convection frequencies. This is a laborious and extremely difficult task. It is much more convenient for a designer to obtain a gross estimate of the

structure's response from the main parameters of the structure and sonic boom. Consequently the structure's maximum response is plotted against the non-dimensional parameter  $\lambda/L$  in preference to the more usual parameter  $f_{11}/f_N (= T/\tau)$  used by Cheng. This is because it is easier to obtain a gross dimension  $L$  of a general structure than its fundamental frequency  $f_{11}$ .

It was not possible to calculate  $f_N$  with analogue/digital techniques until late in the research programme. Consequently  $T_2$  was replaced by  $(T_1 - T_R)$  in equation (3.8) for the model experiments and for the comparison of the model and prototype structure's responses. Hence the calculated excitation frequency of a simulated boom is always

$$f_{N_c} = \frac{1}{T_1 - T_R} . \quad (3.23)$$

But  $(T_1 - T_R)$  is identical to  $T_2$  for an idealised N-wave which has zero rise and fall times. Hence, the shorter the rise and fall times, the more accurate  $f_{N_c}$  approximates  $f_N$ . Now

$$\frac{\lambda}{L} = \left(\frac{a_0}{L}\right) \frac{1}{f_N} \quad (3.24)$$

so that the calculated values of this parameter are given by

$$\frac{\lambda_c}{L} = \left(\frac{a_0}{L}\right) \frac{1}{f_{N_c}} = \left(\frac{a_0}{L}\right) (T_1 - T_R). \quad (3.25)$$

$a_0$  was measured to be 1100 ft./sec. and  $L$  is defined by (3.21). The values of  $\lambda/L$  were always calculated using (3.25) but, for convenience, the subscript  $c$  was omitted. It is then possible to obtain a first approximation of the structure's maximum response for a given value of  $\lambda/L$ . Details of the natural frequencies,  $f_{ij}$ , of the structure and internal cavity must be known before a more accurate estimate of this response can be calculated.

Thus the  $f_{ij}$ s, for the particular model/structure considered, are marked on the abscissae of the various graphs. The values of  $(\lambda/L)_{ij}$  which correspond to  $f_{ij}$  are given by

$$\left(\frac{\lambda}{L}\right)_{ij} = \left(\frac{a_0}{L}\right) \cdot \frac{1}{f_{ij}} . \quad (3.26)$$

Note, however, that  $L$  depends upon the shock front's angle of incidence  $\theta$  relative to the box. But  $a_0/L$  is constant for a particular value of  $\theta$ , so that

$$\lambda/L \propto 1/f_N. \quad (3.27)$$

The 'constant' of proportionality,  $a_o/L$ , varies for different values of  $\theta$  so that the  $(\lambda/L)_{ij}$ , defined by (3.26), will have different values for the various  $\theta$ . The minimum values of  $(\lambda/L)_{ij}$ , calculated from equation (3.26) for the different  $\theta$ , are shown on those graphs which plot the structure's response for various values of  $\theta$ .

#### 3.2.4 Comparison of the theoretical and experimental waveforms over the box

Up to the present time, most experimental work on the flow of a shock wave over a building has been done with blast waves (26). These waves have no negative impulse and produce positive pressures at least an order of magnitude greater than the anticipated peak positive pressure of the SST produced sonic boom. Fig. 3.4 shows how the form of the blast wave is modified as it moves over a closed rectangular structure. This modification is due to the formation of vortices on the edges of the structure and the creation of rarefaction and diffraction waves.

Fig. 3.5 (27) illustrates the combined effects of these phenomena on the roof and front face of a full sized structure for sonic booms with different wavelengths. It is assumed in the following discussion that there is little change in the pressure-time histories observed on the roof and front face for small differences in the angles of incidence and inclination. The effect of a shallow sloping roof on the modification of the free field pressure-time history is also considered to be small.

A comparison of the pressure-time histories measured at stations 2 and 3 shows that there is a variation (up to 10%) of peak positive pressure with height on the front face. This variation seems to depend upon the boom's wavelength. The main modification to the free field pressure signature on the front face is due to the reflected wave. The combined incident and reflected waves produce an amplification of 1.4 to 1.5 on the free field's peak positive pressure. Note that the vortices formed at the edges tend to produce a double peaked shock front. The theoretical forcing function, however, takes the amplification to be two. All points on the theoretical model's front face are subjected to the same forcing function. Consequently there is no spatial variation of the wave's peak positive pressure over the front face of the

theoretical model. In fact, all points on the front face are subjected to a forcing function whose pressure-time history is identical in shape to the free field's pressure-time history. Thus, for the experimental case under consideration, there would only be a single peaked shock front.

The pressure-time history measured on the roof (station 1) shows that the peak positive pressure in this position fluctuates but tends to be similar to the free field's peak positive pressure. Also, the second peak of the pressure-time history on the roof is less pronounced than that on the front face. On the whole, the experimental and theoretical pressure-time histories tend to be similar at a point which is close to the roof's centre.

The theoretical forcing function not only ignores the effects of the vortices but it can be seen from Fig. 3.4 that the theoretically assumed normal pressure distribution on the back face is far removed from the practical situation. A more realistic theoretical model, although possible, would require a significant increase in the number of segments on the back face to describe the complicated movement of the three component waves. However, the theoretical approach should still give some idea of the liberties allowed in the description of the wave's shape.

The assumption that the building's dimensions are much smaller than the sonic boom's wavelength should be valid for most residential-type buildings with respect to an envisaged SST passenger aircraft.

### 3.3 Difficulties associated with the theoretical description of a general forcing function

A wave travelling over a symmetrical box-type structure at any angle of incidence (such that  $\theta \neq 0^\circ$  or  $90^\circ$  in Fig. 4.2) will excite the anti-symmetrical box modes (modes with opposite side walls moving in phase) in both directions. It then becomes necessary to consider the motion of the whole box so that the computational advantages of the box's symmetrical properties are lost. If, however, the wave is at zero incidence to one of the faces then the antisymmetric box modes parallel to the shock front will not be excited. In this case it is only necessary to consider the motion of half of the complete box.

Twice the number of finite elements must be used to describe the whole box as half the box for the same degree of accuracy. Not only could this present computer storage difficulties but at least six times more detail is required in describing the general forcing function. This is because the force corresponding to each displacement must be described at least three

times due to the discontinuities in the boom's pressure-time history. Consequently, it is desirable to use a minimum number of finite elements in the box's idealisation. The actual number of finite elements, however, will depend upon the required accuracy of the analysis.

#### 3.4 Introduction of structural damping

Assuming viscous damping proportional to the system's distributed mass, Hurty and Rubinstein (28) show that the Lagrange equations may be written as

$$\underline{M}_c \{\ddot{w}_c\} + \beta \underline{M}_c \{\dot{w}_c\} + \underline{K}_c \{w_c\} = \{F_c\}. \quad (3.28)$$

The dots denote differentiation with respect to time,  $\beta$  is the viscous damping co-efficient and  $\{F_c\}$  is the constrained forcing vector of the whole system. The overall mass and stiffness matrices are assembled and constrained as in Chapter 2. In practice, a large proportion of the structural damping occurs at the joints whereas, for convenience, the present theory artificially 'averages' this localised effect over the entire structure. As the actual damping in a complex structure is by no means certain the present approximation is regarded as adequate.

$\beta$  is assumed constant for all modes so that the damping factor in the  $r^{\text{th}}$  mode,  $\xi_r$ , is given by standard theory as

$$\xi_r = \beta / 2\omega_r. \quad (3.29)$$

This equation implies that less damping is experienced by the higher modes.

It can be seen from Table 4.1 that  $\xi_r$  generally increases for the higher modes of a Xylonite beam. On the other hand, it was found that the damping of the full scale structure tended to decrease with increasing frequency. The theoretical damping of the Xylonite box was always assumed to be zero ( $\beta = 0$ ) in the computation of the theoretical results. Similarly,  $\xi_1$  was usually assumed to be 0.02 for the full scale structure. The particular values for the damping constant  $\beta$  were selected somewhat arbitrarily because the form and actual values of damping for both the model and full scale structure were not known at the time of computation. In practice, the inclusion of the damping term generally made little difference in the overall forms of both the model's and the full scale structure's theoretical responses. The main difference in either case occurred after the pressure

wave had completely travelled over the whole structure. Then the structure's response decayed more quickly for higher values of the damping constant  $\beta$ .

### 3.5 Numerical solution of the equations of motion

Equations (3.28) may be solved using the normal modes of the freely vibrating system (28) or by step by step integration techniques (29). These methods are all well known so that few details will be given.

The viscous damping of the system has been assumed to be proportional to its distributed mass so that, by expressing the displacements in terms of the normal modes, the set of simultaneous differential equations (3.28) can be reduced to an equal number of independent differential equations. Each individual differential equation can then be integrated to obtain the normal displacement of that mode. The system's total displacement is obtained by summing for all modes the products of the individual mode shape and normal displacement. Usually the lower modes of vibration will dictate the system's response. In this case the higher modes' contributions to the overall response are assumed negligible so that the number of terms in the displacement series can be significantly reduced. When the natural frequencies lie close together, a large number of modes affects the system's response. Then it is advantageous to use a step by step integration technique. This is due to the difficulties associated with the solutions of the eigenvalue problem. A step by step integration technique is always used in this thesis.

Hildebrand (29) discusses the relative merits and gives details of the various step by step integration techniques. The main disadvantage of all these techniques is that the stability of the integration process depends upon the highest eigenvalue of the complete set of equations. The particular method chosen was the standard fourth order Runge-Kutta procedure. This method rewrites equations (3.28) in the form

$$\{\ddot{w}_c\} = -\underline{M}_c^{-1}\underline{K}_c\{w_c\} - \beta\{\dot{w}_c\} + \underline{M}_c^{-1}\{F_c\}$$

$$\text{or } \{\ddot{w}_c\} = G(\{w_c\}, \{\dot{w}_c\}, \{F_c\}) \quad (3.30)$$

for  $\underline{M}_c$  non-singular. Truncation of a Taylor Series expansion gives the approximations:

$$\begin{aligned} \{w_c(t+h)\} &= \{w_c(t)\} + h\{\dot{w}_c(t)\} + \frac{h^2}{2}(\{\ddot{w}_c\}_0 + \{\ddot{w}_c\}_1 + \{\ddot{w}_c\}_2) \\ \text{and} \\ \{\dot{w}_c(t+h)\} &= \{\dot{w}_c(t)\} + \frac{h}{2}(\{\ddot{w}_c\}_0 + 2\{\ddot{w}_c\}_1 + 2\{\ddot{w}_c\}_2 + \{\ddot{w}_c\}_3) \end{aligned} \quad (3.31)$$

where

$$\begin{aligned}
 \{v\}_0 &= h.G(\{w_c(t)\}, \{\dot{w}_c(t)\}, \{F_c(t)\}) \\
 \{v\}_1 &= h.G(\{w_c(t)\} + \frac{h}{2}\{\dot{w}_c(t)\}, \{\dot{w}_c(t)\} + \frac{1}{2}\{v\}_0, \{F_c(t + \frac{h}{2})\}) \\
 \{v\}_2 &= h.G(\{w_c(t)\} + \frac{h}{2}\{\dot{w}_c(t)\} + \frac{h}{4}\{v\}_0, \{\dot{w}_c(t)\} + \frac{1}{2}\{v\}_1, \{F_c(t + \frac{h}{2})\}) \\
 \{v\}_3 &= h.G(\{w_c(t)\} + h\{\dot{w}_c(t)\} + \frac{h}{2}\{v\}_1, \{\dot{w}_c(t)\} + \{v\}_2, \{F_c(t + h)\})
 \end{aligned}
 \tag{3.32}$$

for a step size  $h$  and initial conditions

$$\{w_c(0)\} = \{\dot{w}_c(0)\} = \{\ddot{w}_c(0)\} = \{0\}.
 \tag{3.33}$$

This technique is employed in subroutines KUTTA4 and D2XBTD written by A. Craggs.(11.)

The above method is equivalent to a 4th order Taylor Series having a truncation error of order  $(h^5)$ . If the step size  $h$  is decreased then the truncation error will also decrease. In the limit, as  $h$  tends to zero, this method gives a solution which tends to the true solution. The effort required to compute the results over a given time interval depends upon the step size  $h$ . A smaller value of  $h$  requires a greater computational effort. Thus, it is required to optimise the value of  $h$  so that the conflicting requirements of accuracy and computational effort are satisfied as far as practicable.

### 3.6 Discussion on stability and convergence

Classical fourth order Runge-Kutta theory stipulates that the highest eigenvalue  $\mu_{\max}$  and the step size  $h$  must satisfy the inequality

$$\mu_{\max} h < 2.8
 \tag{3.34}$$

for stability. This criterion only guarantees a solution which does not diverge; it does not necessarily guarantee a good approximation to the true solution. The most favoured method of checking the result's accuracy is to repeat the computation with half the original step size and to compare the two sets of results 'by eye'. The results are usually regarded as being 'accurate' if the difference between the two sets of results is 'small'. The decision

whether some 'small' difference is acceptable can only be based on an assessment of the errors involved in the actual system's mathematical idealisation.

From the consideration of a single degree of freedom system, Craggs (9) recommends an 'accuracy' step size  $h_{acc}$  satisfying the more stringent inequality

$$\mu_{max} h_{acc} < 1.0 \quad (3.35)$$

for a 'reasonable' approximate solution. In order to assess this criterion for a set of dependent equations, maximum and minimum values of acceleration were evaluated for a thirty two degrees of freedom system using various step sizes and different damping ratios (Table 3.1). The maximum and minimum values of acceleration were selected for two reasons. Firstly, by using only the maximum and minimum values, the amount of data to be presented is drastically reduced. Secondly, only the experimental accelerations were measured. Hence the assessment of the theoretical model's accuracy could only be determined on the basis of a comparison of the theoretical and experimental accelerations.

The accelerations which were obtained with the smallest step size ( $h = 0.26/\mu_{max}$ ) will be regarded as the true theoretical accelerations. Table 3.1 shows that the accuracy criterion (3.35) is a useful guide in selecting the initial step size. The results, however, must still be checked with the 'by eye method'. This statement particularly applies if high values of the damping ratio  $\xi_1$  are used in the computation.

The damping factors ( $\xi_1$ ) of the Xylonite box and full scale structure were assumed to be zero and 0.02 respectively. Consequently the theoretical accelerations were always computed with step sizes corresponding to  $h = 0.52/\mu_{max}$  for the box and  $h = 0.26/\mu_{max}$  for the full scale structure. (Since the full scale structure is non-uniform, the factor  $(\rho_{ST}/Dg)$  in equation (2.9) has the corresponding constant value of the side walls. The elements of  $\underline{M}_c$  and  $\underline{S}_c$  in equation (2.9) are modified in order to account for this constant factor.) The errors in the peak theoretical accelerations, compared with the true theoretical accelerations, were generally of the order of  $\pm 0.5$  g. The results were always checked by eye. The peak theoretical accelerations of the model, however, are quoted to two decimal places so that they can serve as a check for any future mathematical model.

## CHAPTER 4

### EXPERIMENTAL MODEL STUDIES

A device for simulating sonic booms under laboratory conditions is described in this chapter. Only small scale models could be tested, due to the limiting size of the working section. The model took the form of a rectangular box which was made from a plastic material in order to simplify the construction.

The dynamical properties of the material and model were evaluated experimentally. Subsequently, the results of the model's response to various booms are presented and discussed.

#### 4.1.1 Description of the sonic boom simulator

The device for simulating a sonic boom is essentially an aluminium, conical shock tube about 47 ft long and with a 4 ft diameter open end. It is convenient to describe the simulator in two parts: the driver and the driven section (see the upper two photographs in Fig. 4.1). In operation, the driver section at the apex of the cone is pressurized with a gas until the diaphragm enclosing the volume of gas is ruptured. The rupture produces a shock wave which propagates and expands down the driven section and a rarefaction wave which moves towards the cone's apex. The rarefaction wave is continuously reflected from the walls of the driver section so that the overall effect is to produce an N-wave characteristic in the driven section of the tube. More details of the simulator are given by Webb and Pallant (30).

#### 4.1.2 Experimental limitations

The peak positive pressure of the simulated boom,  $p_o$ , is a function of the diaphragm's bursting pressure  $p_b$  and the ratio of the diameters of the diaphragm and that of the section of interest. Mathematically,

$$p_o = \frac{p_b}{2} \cdot \frac{d_b}{d_o} \quad (4.1)$$

where  $d_b$  and  $d_o$  are the diameters of the diaphragm and section of interest respectively. Early experiments with diaphragms made from thin paper produced peak positive pressures far greater than those anticipated from an SST produced sonic boom. Equation (4.1) shows that the peak positive pressure  $p_o$  can be reduced by increasing the working section's diameter  $d_o$ . This proved to be too expensive. Consequently weaker materials such as polystyrene and metal foil were developed to produce peak positive pressures as low as  $1.5 \text{ lb/ft}^2$ . These materials also have the added advantage that they rupture more cleanly than paper so that they produce a cleaner N-shape characteristic. The simulated boom's initial rise time  $T_R$  depends upon the manner in which the diaphragm ruptures. At the moment this parameter cannot be controlled although it can be arbitrarily varied by using different diaphragm materials. These remarks also apply to the weaker rear shock front with its associated rise time  $(T_2 - T_1)$ . This time was invariably found to be much greater than  $T_R$ .

The duration of the simulated boom is governed by the length of the driver section and the speed of sound in the enclosed gas. Using air, the wavelength of the simulated boom is about twice the driver section's length. Consequently the wavelength can be changed by simply adjusting the diaphragm's position in the shock tube. For convenience, the driver length is quoted in Figs. 4.5 to 4.7 and Fig. 4.12 because there are slight variations in the pressure-time histories of the simulated booms for any given combination of driver length and diaphragm. The quoted driver length will then give a measure of the simulated boom's wavelength and duration. The simulator was limited to four driver lengths which corresponded to lengths of 3 in, 6 in, 2 ft and 7 ft. Thus the simulated boom's extreme wavelengths are 6 in and 14 ft. Their durations are restricted between 0.4 and 14.0 milliseconds because the boom's speed of propagation in air was found to be about 1100 ft/sec.

The wave interaction between the model and shock tube was minimised in order to simulate free field conditions. This was achieved by restricting the model's physical size to less than 2 ft. It was found impracticable to vary the angle of inclination ( $\psi$  in Fig. 4.2) from  $90^\circ$ .

#### 4.2 Physical properties, natural frequencies and normal modes of the model box

A 12" x 9" x 7" model box was constructed on a plaster foundation (see Fig. 4.2) using Xylonite. Xylonite was used for ease of construction. It suffers, like all plastics, from the disadvantage that its physical properties vary with temperature, humidity, thickness and frequency (31). A strict control of temperature and humidity was impossible so their variations were minimised, as far as possible, by using the same laboratory. Using the apparatus shown in Fig. 4.3, the amplitudes of vibration which were in phase and at quadrature to the excitation force were measured at different frequencies for a free-free Xylonite beam having the same thickness as the model. A typical example of the complex receptance plotted on an Argand plane is shown in Fig. 4.4. The natural frequencies and modal damping were evaluated from these plots using the method described by Kennedy and Pancu (32).

Assuming negligible damping, the principle of conservation of energy (33) gives the natural frequencies of a free, transversely vibrating, uniform beam with cross sectional area  $A$  and length  $\ell$  as

$$\omega^2 = \frac{EI \int_0^\ell \left( \frac{d^2 \phi}{dx^2} \right)^2 dx}{\left[ \rho_v A \int_0^\ell \phi^2 dx + \sum_{r=1}^n m_r \phi_{x_r}^2 \right]} \quad (4.2)$$

when the displacement of the beam is given by  $w(x,t) = \phi(x) \sin(\omega t + \alpha)$ .  $m_1, m_2, \dots, m_r, \dots, m_n$  are added concentrated masses at points  $x_1, x_2, \dots, x_r, \dots, x_n$  and  $\phi_{x_r}$  is the value of  $\phi(x)$  at  $x_r$ . As the mass of the coil

and strain gauge attachment is small and the mass-beam system is excited purely in its  $p^{\text{th}}$  mode, the mode shape of the mass-beam combination,  $\phi(x)$ , and its derivatives can be approximated by the free-free beam's normal mode,  $\phi_p(x)$ , and its corresponding derivatives. Thus, for the  $p^{\text{th}}$  mode of the mass-beam combination, equation (4.2) reduces to

$$\omega_p^2 = \frac{EI (\lambda_p \ell)^4}{\rho_v A \ell^4 + \ell^3 W \phi_p^2 (x = \ell/2)} \quad (4.3)$$

For a particular mode,  $\lambda_p$  is a constant which depends upon the beam's material properties, and

$$E = \left[ \frac{4\pi^2 f_p^2}{I} \right] \left[ \frac{1}{[\lambda_p \ell]^4} \right] \{ \rho_v A \ell^4 + W \ell^3 \phi_p^2 (x = \frac{\ell}{2}) \} \quad (4.4)$$

For free-free edge conditions, Bishop and Johnson (34) tabulate the beam functions

| Mode Number | $(\lambda_p \ell)^4$ | $\phi_p (x = \frac{\ell}{2})$ |
|-------------|----------------------|-------------------------------|
| 1           | 500.564              | -1.21565                      |
| 3           | 14617.6              | 1.42238                       |

(4.5)

Substituting the mass-beam's experimental natural frequencies  $(f_p)_c$  for  $f_p$  in equation (4.4) gives the approximate Young's dynamic modulus  $E_c$  for that frequency. Different values of  $(f_p)_c$  were obtained by using different beam lengths  $\ell$ . Each length was at least seven times greater than the beam's width in order to ensure that a simple beam theory was applicable. The resulting values of  $E_c$  for various frequencies are shown in Table 4.1.

Tables 4.2 and 4.3 show the theoretical natural frequencies of the closed and open Xylonite boxes. The open box is essentially the same as the closed box, but with the 12" x 7" face removed. These natural frequencies were evaluated using the same mathematical model as that discussed in section 2.3. In addition, the base edges of both boxes were assumed to be fully fixed. The theoretical frequencies of the open box are compared with the experimental natural frequencies obtained from resonance tests in which two exciters were driven in, and then out, of phase on opposite side faces (Table 4.3). The two sets of results tend to show reasonable agreement. The differences seem to follow the same trends as

the variations in Table 4.1 of Young's dynamic modulus. An additional stiffness is superimposed on the experimental model's stiffness below the fundamental acoustic cavity frequency due to the effect of the air enclosed within the model. Although resonance tests were not conducted on the closed model it seems reasonable to assume that trends similar to the open model's would be observed.

Both the mathematical and experimental models are more representative of brick walls than stud-plaster board combinations which are typical of American houses. In the latter type of construction the individual walls move independently (36) whereas the models' walls move as single units. Hence, the present investigation is mainly concerned with the response of brick houses, although both models can be extended to include other types of construction. A detailed discussion of the loading and response parameters necessary for the experimental model to simulate a single-storey house is given in Chapter 7. Most of the important parameters can be satisfied using Xylonite for the model and air as the enclosed gas.

#### 4.3.1 Difficulties associated with the interpretation of the results

The peak accelerations and internal pressures were normalised with respect to the sum of the moduli of  $p_o$  and  $p_-$  in order to account for differences in their values. Care should be exercised in the interpretation of the results for the boom with the smallest wavelength (the 3 in driver in Fig. 4.5). This waveform was effectively triangular with little negative impulse, whereas positive and negative impulses were more or less equal for all the other simulated booms.

At the open end of the shock tube a rarefaction occurs which produces a reflected wave of almost equal amplitude but  $180^\circ$  out of phase with the incident wave. The reflected wave may be ignored in determining the model's transient response provided there is a sufficient time interval between the incident wave leaving and the reflected wave reaching the model. This proves to be the case for all but the largest wavelength (7 ft driver) where the general form of the transient response can still be observed. It should also be noted that the wavelength associated with the 7 ft driver

has a much longer rise time ( $T_R/T_L \approx 0.18$ ) than the other drivers.

#### 4.3.2 General observations

The acceleration-time histories at the centre of each face of the model and the internal pressure fluctuations were measured for different angles of incidence, wavelengths and peak pressures of the simulated booms. Typical examples of these responses are shown in Figs. 4.5 to 4.7. The acceleration-time histories of the North, South, West and East faces and the roof of the model are symbolised by N, S, W, E, and R respectively in these figures. Similarly the internal pressure-time history is denoted by M. The North face corresponds to one of the 9" x 7" faces. Once selected, the same symbol is used for a particular face of either the open or closed model.  $\psi$  is not quoted in any of these figures because it was not varied from 90°. The boom's angle of incidence  $\theta$  is referred to a particular face of the model when the responses of all the faces of the model and the internal pressure are considered for a given value of  $\theta$ . This is the case in Figs. 4.6 and 4.7 for example.

For Figs. 4.10 and 4.11, the angles of incidence  $\theta$  are measured anticlockwise from a reference line to the direction of the boom's propagation. The reference line is taken to be that line for which the boom's direction of propagation is completely normal to some given face. The given faces in Figs. 4.10 and 4.11 are the West and North faces respectively. The same convention is also adopted for the full scale structure in section 6.7 but with the window face acting as the reference face.

There was an inadequate number of oscilloscope channels to ascertain the whole closed model's response to a given simulated boom. Thus two nominally identical tests had to be conducted for the same driver/model configuration. The response of at least one face of the model was evaluated for both tests in order to check whether the two tests were actually identical. For this reason there are two curves labelled N for example in Figs. 4.6 and 4.7. It may be noted that the forms of these curves are similar. There is an apparent difference, however, in the

magnitude of the two N curves which are shown in Fig.4.6. This difference is only apparent because the acceleration scale of the left hand curve is twice that of the right hand curve. Thus the minute details in the boom's general waveform have little effect on the model's overall response. These details correspond to the high frequency content of the boom's frequency spectrum. It was also found that, within the experimental limitations, the model's response varies linearly with peak positive pressure of simulated booms produced by the same driver.

The peak acceleration for any side face occurs when the simulated boom is acting normally to that face. The curves shown in Figs. 4.10 and 4.11 give the general trends of the closed model's peak normalised accelerations with  $\lambda/L$ . The added points on these curves relate to the full scale structure and are introduced in section 7.2 for comparison with the experimental model results. They are not discussed in this chapter. The general trends in these figures are obtained from the experimental results in a way similar to that illustrated in Fig. 4.8. The points in Fig. 4.8 are the actual experimental model results and the curves give the general trends. Due to the limited number of results, these trends should not be treated as conclusive. Further investigations are needed to supplement them. They do show however that, irrespective of the angle of incidence, the peak acceleration decreases with increasing  $\lambda/L$ .

The acceleration-time histories show a small amplitude, high frequency burst when the simulated boom initially forces the front and back faces. The load, due to the pressure differential across the model, is transferred through the roof and side faces from the front to the back face by a shearing action. This phenomenon is the same as that observed by Dickinson and Warburton (17) to be a bending mode of the box. The bursts are less prominent for the 7 ft driver. This is the only driver which produces a wave with a finite rise time. Thus the magnitudes of the bursts seem to be a function of the simulated boom's rise time.

Longitudinal waves propagate faster in the model's material than the simulated boom travels in air. Even so, there is still a short time delay before the back face is excited. This delay corresponds to the

time taken for the longitudinal wave to travel from the front to the back face. Subsequently, the simulated boom excites the back face directly but at much lower frequencies (Fig. 4.5).

The critical frequency of an infinite plate is the lowest frequency, for any angle of inclination  $\psi$ , at which an incident harmonic wave passes through the plate as though it were not there. Thus the plate is acoustically 'transparent' at this particular frequency. The same phenomenon can occur in finite plates but at frequencies which are lower than those associated with the infinite plate (47). For the purpose of definition, the boom is assumed to be equivalent to the harmonic wave whose frequency corresponds to the lowest predominant frequency in its frequency spectrum. Then the critical frequency of the model is considered to be the lowest frequency at which any of its component, finite-sized plates can become acoustically transparent to the boom's equivalent harmonic wave. It is assumed that this frequency is an order of magnitude greater than the fundamental acoustic cavity frequency. This situation is typical of most full scale structures.

Following Lyon's example (25), it is convenient to study the closed structure's response and sound transmission in three frequency ranges: at 'low' excitation frequencies below the fundamental structural frequency where the structure and enclosed gas are both stiffness controlled; at 'intermediate' excitation frequencies between the fundamental structural and acoustic cavity frequencies (in this region, the structure is resonant whereas the volume is still stiff); at 'high' excitation frequencies above the fundamental acoustic cavity frequency where the structure and enclosed gas both display resonant behaviour.

The acoustic cavity frequencies are approximately calculated from the formula

$$f_b = \frac{a_0}{2} \left[ \left( \frac{n_x}{l_x} \right)^2 + \left( \frac{n_y}{l_y} \right)^2 + \left( \frac{n_z}{l_z} \right)^2 \right]^{\frac{1}{2}} \quad (4.6)$$

given by Morse (38) for a rigid rectangular room. Then the fundamental

acoustic cavity frequency  $f_{b_i}$  is given by  $n_x = 1, n_y = 0 = n_z$ , where

$$\ell_x = 12 - (2 \times 0.36) = 11.28 \text{ in.}$$

is the closed model's maximum internal dimension. Hence

$$f_{b_i} = a_o / 2\ell_x = .585 \text{ c.p.s.} \quad (4.7)$$

This frequency is shown as i in Fig. 4.8. The 3 in, 6 in, 2 ft and 7 ft drivers produce simulated booms which correspond to high, high or intermediate, intermediate and low excitation frequencies respectively for the particular closed model.

#### 4.3.3 Closed model's response at low excitation frequencies

The main frequency components of the individual faces acceleration-time histories were approximately estimated from oscilloscope photographs. The resulting frequencies, which are shown in Table 4.4, indicate that the model's accelerations are dictated by the six lowest modes at low excitation frequencies. Fig. 4.13 shows that in the closed model's fundamental mode the relative motions of the side faces are small compared with that of the roof. Consequently this mode contributes little to the motion of the side faces. Hence it remains undetected on the side faces in the approximate analysis. Similarly, the contributions of the next five modes to the roof's overall motion are relatively small and they remain undetected on the roof. In general, the relative modal contribution to the response of an individual face depends upon the relative frequency and magnitude of the box modes affecting that face.

Fig. 4.7 is typical of the model's acceleration response at low excitation frequencies. This figure shows that all the faces' accelerations decay before the wave reflected from the shock tube's opening reaches the model. The effect of the simulated boom's rear shock front cannot be absolutely determined because of interference from the reflected wave. Due to the model's symmetry, similar trends exist for both the simulated and reflected waves leaving the model. Consequently, the accelerations will not suddenly increase when the simulated boom leaves the model. Although the accelerations increase

due to the additional forces of the reflected wave, they are still generally smaller than the initial accelerations. Hence, the model's absolute peak accelerations will tend to occur before the reflected wave reaches the model.

The combined effect on the model of both the incident and reflected waves is representative of the situation where a building is situated close to the prototype structure. In practice, there can be any number of reflected waves from nearby buildings affecting the prototype's response. Their effects will mainly depend upon the distance and orientation of the buildings from the prototype structure. The prototype's response will not have decayed to any extent before the reflected wave reaches it if the building is closely situated. Consequently, the response will increase above that due to the incident boom alone. In general, the magnitude of the increase will depend upon the relative direction and position of an individual face's motion when the reflected wave reaches the prototype structure. If the building is not close to the prototype, the overall response characteristics will be similar to those of the model.

Halving the rise time  $T_R$  of nominally identical booms produced by the 7 ft driver nearly doubles the front face's peak normalised accelerations. The peak accelerations of the other faces are relatively unaffected. Thus, apart from the front face, Figs. 4.8, 4.10 and 4.11 show that there is little difference in the closed model's peak accelerations at low excitation frequencies, for different angles of incidence of nominally identical booms.

Fig. 4.7 and Table 4.4 show that, apart from the initial high frequency burst, the higher modes are not excited. When the boom's wavelength is much greater than the model's dimensions, the direct forcing of the pressure wave provides a much greater contribution to the box's generalised forces than the forces due to its convection. The boom is similar to a uniform step pressure which acts instantaneously on each face. Such a pressure will force the box to vibrate in its lowest modes. The main difference between the model's responses will be

due to the uniform pressure not exciting the antisymmetric box modes. Figs. 4.8, 4.10 and 4.11 show that these antisymmetric modes are usually insignificant. The greatest discrepancy occurs for the West face at small angles of incidence relative to this face. In these cases the West face is predominantly affected by the second mode. Fig. 4.13 shows this mode to be antisymmetric.

Figs. 4.10 and 4.11 show that, for the lowest excitation frequencies, the theoretical peak accelerations computed for a unit, instantaneous step pressure on all faces, give upper values for the accelerations of all except the experimental model's front face, at small angles of incidence. The greater response of this face is probably due to the effect of pressure doubling on it. In computing the theoretical values, the material properties of Xylonite were taken to be

$$E = 3.1 \times 10^5 \text{ lb/in}^2 ; \quad \rho_{ST} = 1.818 \times 10^{-2} \text{ lb/in}^2 ;$$

$$\nu = 0.39 \text{ and } \beta = 0$$

for the mathematical model which is described in the introduction of Chapter 5.

Fig. 4.7 shows that the internal pressure-time history is similar to that of the simulated boom when the latter is forcing the model. A frequency which corresponds to the fundamental box mode is superimposed on the internal pressure wave's general form. This mode will tend to dictate the model's mean displacement. Both these observations indicate that the internal pressure is a function of the model's mean displacement during the forced vibration period.

The closed model's fundamental mode is shown in Fig. 4.13 to have a predominant roof motion. Hence the motion of the roof will tend to dictate the model's mean displacement. But the roof's response is insensitive to changes in the simulated boom's angle of incidence  $\theta$  (Fig. 4.11). Consequently it is not surprising that Fig. 4.9 shows that the peak internal pressure is also insensitive to changes in  $\theta$  for very low excitation frequencies.

The closed model's stiffness is much greater than its internal acoustic stiffness ( $\eta < 0.1$ ). Both Pretlove (23,24) and Lyon show that in this case the internal pressure would have little effect on the model's response.

#### 4.3.4 Closed model's response at intermediate excitation frequencies

The important factor, which changes with decreasing wavelength of a simulated boom, is the relative contributions of the convection and direct pressure forces to the model's generalised forces. At intermediate excitation frequencies, the simulated boom's wavelength is usually of the same order as the model's physical dimensions. However, for the 2 ft driver the simulated boom's wavelength is still three or four times the model's greatest dimension. Hence the boom's convection has only a marginal effect on the closed model's response. Thus Table 4.4 shows that the first six box modes are predominantly excited again, although some higher modes are observed on the roof and, for  $\theta \neq 0^\circ$  or  $90^\circ$ , on the side and back faces.

Fig. 4.8 shows that when the excitation frequency is similar to the fundamental box frequency (denoted by I) there is a large upsurge in the model's accelerations. It is anticipated that there will be similar upsurges in the accelerations when the excitation frequency corresponds to the other natural frequencies of the model.

Fig. 4.6 shows that, during the forced vibration period, the form of the internal pressure wave does not follow the simulated boom to the same extent as before. Even so, it still depends upon the closed model's mean displacement. The mean displacement becomes increasingly dependent upon the side faces' motions at the higher intermediate excitation frequencies. Thus the fundamental mode no longer dictates the detailed form of the internal pressure-time history in Fig. 4.6. Large rises in the magnitude of the internal pressure (and the model's mean displacement) occur (Fig. 4.9) when the excitation frequency  $f_N$  has values similar to the first,  $ST_1$ , and the second natural frequencies of the closed model. There is a significant drop in the peak internal

pressure for excitation frequencies between these two values. The overall peak internal pressure in the intermediate frequency region of Fig. 4.9 gives the maximum value of  $\eta$  to be less than 0.1. Thus the internal pressure should not generally affect significantly the response of the closed model at intermediate excitation frequencies.

#### 4.3.5 Closed model's response at high excitation frequencies

Fig. 4.5 shows that the lower modes still dictate the closed model's overall acceleration characteristics at high excitation frequencies, even though the higher modes are excited. The acceleration in each mode is approximately proportional to that mode's generalised force and the inverse of the mode's generalised mass. The generalised mass of a simply supported plate is constant for all modes whereas the generalised force for the  $(m,n)$ th mode is inversely proportional to the product  $mn$  (9). Thus the generalised forces (and hence the accelerations) are greatest for the lowest modes so that these are still predominantly excited. Similar observations will be applicable to the model if each face may be considered to be analogous to a simply supported plate.

The most striking feature of Table 4.4 is that the predominant frequencies bear no resemblance to the high excitation frequencies, which are generally greater than 600 c.p.s when produced by the 3 in and 6 in drivers. Consider, for example, the cases where  $\theta$  equals  $90^\circ$  (West face normal). None of the modes which correspond to even multiples of the fundamental convection frequency will affect the motion of the centres of the individual faces. Fig. 4.13 shows that, in the direction of the boom's propagation, modes 1, 3, 4, 5, 6 and 12 have shapes which are similar on at least one face to the assumed mode shape of the fundamental convection frequency. Only modes 4, 6 and 12, however, have the same shape on the roof, North and South faces. Table 4.4 indicates that, on these faces, the experimental frequencies of the accelerations are similar to the computed natural frequencies of the last three modes. The lower frequency modes are still predominantly excited for the reasons already discussed.

The excited modes have been shown to be dependent upon the simulated boom's convection forces. These forces depend upon the boom's angle of incidence. Hence the model's response depends upon the simulated boom's angle of incidence for high excitation frequencies. Higher modes are excited which tend to cause the model to have less correlation between its individual faces and to increase its peak accelerations.

It is difficult to generalise about the peak internal pressure for high excitation frequencies due to

- (a) the lack of experimental data, and
- (b) the closed model's natural frequencies' being so closely spaced within certain frequency ranges.

#### 4.3.6 Open model's response

The main effect of introducing an opening in one of the closed models' side faces is to increase substantially the magnitude of the internal pressure due to a given simulated boom. The open model acts as a Helmholtz resonator. Given the model and opening dimensions and assuming all its faces are rigid, the Helmholtz frequencies can be determined using standard theory (38). They are given in Table 4.5. This standard theory may be used even for the highest excitation frequencies and the largest openings. (The accuracy of the standard theory usually deteriorates when the model's dimensions are not small compared with the wavelength of the simulated boom).

The internal pressure-time history not only follows the boom at low excitation frequencies but also has a contribution from the Helmholtz effect. The 7 ft driver in Fig. 4.12 illustrates this point for a 6" x 3" opening. Thus, during the forced vibration period, the internal pressure is a function of the open model's mean displacement and the Helmholtz effect. During the subsequent free vibrations, it is mainly created by the Helmholtz effect. The internal pressure is similar to that already discussed for excitation frequencies which are much greater than the Helmholtz frequency. However, it depends upon the open model's mean acceleration rather than its mean displacement.

The overall peak internal pressure occurs for intermediate excitation frequencies. The critical factor at these excitation frequencies

is the similarity of the excitation and Helmholtz frequencies. Consequently, for a given excitation frequency, the peak internal pressure depends upon the size of the opening because the Helmholtz frequency depends directly upon the opening's dimensions.

When the opening is in a face which is parallel to the boom's direction of propagation, the cantilever action for the open model is more noticeable than for the closed model. This action is particularly marked when the East face is removed. Its contribution to the model's overall response remains relatively small because it is quickly attenuated.

Introducing an opening into the closed model changes the ratio of its natural frequencies to the excitation and fundamental convection frequencies. The peak accelerations of the open model, however, still have the same general trends as those of the closed model.

## CHAPTER 5

### COMPARISON OF THE THEORETICAL AND EXPERIMENTAL MODEL RESULTS

The inaccuracies and limitations inherent in an idealised mathematical model can only be assessed by comparing the results from this model with experimental values. Thus the errors associated with the experimental results must be known in order to assess the mathematical inaccuracies. The boom and prototype structure's parameters vary with weather conditions in an undetermined manner. Hence, for the purpose of comparison, experimental results under controlled laboratory conditions are preferred to field results. However, the usefulness of any model can only be assessed by comparing the model results with field results observed on actual buildings.

The 6 in and 2 ft driver results suffer least from the experimental limitations. The simulated booms produced by these drivers are idealised to booms which correspond to intermediate excitation frequencies for the closed theoretical model. The 6 in driver was chosen so that the computer time was minimised. The experimental free field pressure-time history, produced by this driver, was idealised as the broken lines shown in Figs. 5.1 and 5.2. The theoretical pressure-time history around the mathematical model was then formulated as in section 3.2.2. This idealisation was always used in the formulation of the mathematical model's forcing function for convecting booms. A symmetrical N-wave, which has zero rise and fall times, is only introduced in section 5.2 so that the effect of small changes in a sonic boom's pressure-time history on the zeros of its frequency spectrum can be assessed.

It was found that artificial discontinuities in the mathematical model's forced response were created by subdividing the finite elements into smaller segments. These discontinuities correspond to the discontinuous time history of the theoretical forcing function as the boom travels across the segments of a finite element. They were eliminated by assuming a linear pressure rise and fall over the complete finite element. In this case, the theoretical boom's wavelength is only twice the largest dimension of an element. This could invalidate the assumption that the boom effectively acts normally to the finite element. This problem is

discussed in the present chapter.

The mathematical formulation of the closed experimental model is similar to that described in section 2.3. The open mathematical model is essentially the same as the closed model but with one of the 12" x 7" faces completely removed. The base edges of both the open and closed mathematical models are always considered to be fully fixed. The material properties, which are assumed for either model, are shown in the relevant figure. It should be noted that the damping is always assumed to be zero for both models. The idealised simulated boom is fixed at zero incidence to one face so that the storage difficulties, which are described in section 3.3, are minimised. Apart from section 5.5, where the boom's angle of inclination  $\psi$  is varied,  $\psi$  is taken to be at  $90^\circ$  to the horizontal. Details of the model's configuration with respect to the boom are given in Fig. 4.2.

Results are computed for different Young's moduli of the mathematical model in order to indicate the effect on the experimental model of its stiffness variation with frequency. Once this effect has been assessed for the experimental response, the only other extraneous effect - that due to the internal acoustic pressure - is isolated. Hence this effect can be quantitatively estimated. Neither of these effects can be absolutely determined because errors are introduced by the mathematical idealisation of the experimental model. They can only be estimated by showing that the mathematical model is a reasonable representation of the actual model. This is shown to be the case.

Subsequently, the effects of the antisymmetric box modes, and the idealised boom's convection and changes in the angle of inclination are assessed for the closed model. Then the effect of introducing a structural discontinuity (an opening) into the closed model is discussed in detail.

### 5.1 Effect of the variation in the Young's modulus of Xylonite

Figs. 5.1 and 5.2 compare the computed accelerations of the closed model for the same N-wave but two different Young's moduli of Xylonite. The mathematical model assumes that a particular Young's modulus is constant for all modes. Although the ratios of consecutive natural frequencies remain the same for both systems, the difference in any two frequencies of one system is obtained by multiplying the corresponding difference of the other system by a constant factor. This factor is the square root of the

ratio of the different Young's moduli. The Young's dynamic modulus however, varies with frequency so that this property varies for different modes. Hence the experimental box's actual modal spacing differs from that of the mathematical model. The mathematical model should indicate the effects on the experimental box of these variations.

Due to differences in the assumed and measured values of Xylonite's Young's modulus and density, the equivalent theoretical Young's modulus,  $E_{eq}$ , should be

$$E_{eq} = E_{measured} \times \frac{\rho_{assumed}}{\rho_{measured}} \quad (5.1)$$

for the experimental and mathematical models to have equivalent natural frequencies.  $E_{eq}$  will have the same variation with frequency as the measured Young's modulus because both densities are independent of frequency. Thus  $E_{eq}$  varies between  $3.1$  and  $3.5 \times 10^5$  lb/in<sup>2</sup> in the frequency range indicated in Table 4.1. Differences in the theoretical results (computed assuming constant values of  $3.1$  and  $3.9 \times 10^5$  lb/in<sup>2</sup>) due to the different modal spacings should be at least as great as the experimental differences.

Different Young's moduli of the mathematical model change the ratio of the model's natural frequencies relative to the excitation frequency. This changes the relative contribution of the individual modes to the model's transient response. Both Figs. 5.1 and 5.2 show that the acceleration characteristics of the mathematical model do not differ significantly for small changes in this ratio. The theoretical curves for each face have the same form but with a time difference due to the frequency shift associated with the change in Young's modulus. Also, the acceleration's amplitudes remain the same. The magnitude of any mode's acceleration is approximately proportional to the stiffness and displacement of that mode. However, the displacement is roughly proportional to the inverse of the stiffness so that the two factors compensate each other. The displacement magnitude will decrease in any mode for increasing stiffness (Young's modulus) in that mode. Hence the overall displacement magnitude of the mathematical model will decrease because the stiffness increases in each mode. The corresponding effect on the experimental model will not be so marked because the stiffness can either increase or decrease for any mode.

If the natural frequencies of the box modes are widely separated, any small frequency shift of these modes relative to the excitation frequency will make little difference to the individual modal contributions. The variation of the experimental model's stiffness with frequency is approximately 6% whilst its average modal spacing is about 83 c.p.s. Consequently, the stiffness variation makes little difference to the model's overall response. Detailed differences could occur when relatively close modes, having similar mode shapes, are predominantly excited.

## 5.2 Effect of the internal pressure at intermediate excitation frequencies

The mathematical model neglects the internal pressure so that a direct comparison of the theoretical and experimental accelerations should indicate the significance of the internal pressure. If the internal pressure is significant, only the model's volume displacing modes will be affected at intermediate excitation frequencies. The side faces respond equally in volume and non-volume displacing modes. Thus the effect of the internal pressure on these faces is minimal and can be safely neglected ( $\eta \approx 0.07$ ). The roof's centre only responds in volume displacing modes. Although it is more affected than the side faces, low internal pressures can usually be neglected. Under certain circumstances however, even low internal pressures can be significant. These circumstances are outlined next and then a specific example is considered as an illustration.

Craggs (9) has shown that the free response in any mode is small if the sonic boom has a small frequency content at that particular mode's frequency. That is

$$O = \int_{-\infty}^{\infty} p(t) e^{-i\omega_n t} dt$$

where

$$p(t) = \begin{cases} \left(\frac{p_o}{T}\right) \{T - (1 + \frac{p_-}{p_o})t\} & 0 \leq t \leq T \\ 0 & |t| > T \end{cases} \quad (5.2)$$

for a simplified N-wave with zero rise and fall times (i.e.  $T_R = 0$  and  $T_1 = T_2 = T$  in Fig. 3.1(a).)

Integrating by parts gives

$$e^{-if} = \frac{p^2 - (p-1)f^2 - ip^2f}{p^2 + (p-1)^2f^2}$$

$$\text{where } p = 1 + \frac{p_-}{p_0} \geq 1 \quad \text{and} \quad f = \omega_n T = \frac{2\pi\omega_n}{\omega_N}. \quad (5.3)$$

Equating real and imaginary parts gives

$$\cos f = \frac{p^2 - (p-1)f^2}{p^2 + (p-1)^2f^2} \quad \text{and} \quad \sin f = \frac{p^2f}{p^2 + (p-1)^2f^2} \quad p \neq 1. \quad (5.4)$$

Providing  $\cos \frac{f}{2} \neq 0$ , equation (5.4) reduces to

$$\tan(f/2) = \frac{f/2}{1 + \frac{2(p-1)(p-2)}{p^2} \left(\frac{f}{2}\right)^2}, \quad p \neq 1. \quad (5.5)$$

Consider two slightly different sonic booms in order to determine the effect of small changes in a boom's pressure-time history on the zeros of its frequency spectrum. For convenience, these zeros will be called the boom's zero pressure components. The two booms correspond to a symmetrical N-wave ( $p = 2$ ) and to an idealised experimental boom produced by the sonic boom simulator. The latter boom is shown as the broken lines in Figs. 5.1 and 5.2.

When  $p = 2$ , equation (5.5) reduces to

$$\tan(f/2) = (f/2). \quad (5.6)$$

To an engineering approximation, the roots of this equation are given by

$$\frac{\omega_n}{\omega_N} \approx \frac{2n+1}{2} \quad n = 1, 2, 3, \dots \quad (5.7)$$

This implies that the symmetrical N-wave has zero pressure components at those frequencies which correspond to an odd number of half the wave's fundamental frequency.

For the idealised boom,  $p = 4/3$  so that equation (5.5) becomes

$$\tan(f/2) = \frac{f/2}{1 - \frac{1}{4}\left(\frac{f}{2}\right)^2}. \quad (5.8)$$

The roots of this equation are approximately given by

$$\frac{\omega_n}{\omega_N} \approx n \quad n = 2, 3, 4, \dots \quad (5.9)$$

Both (5.7) and (5.9) are better approximations for large  $n$ . A direct comparison between these equations shows that slight deviations in the boom's pressure-time history can give significant variations in the zero pressure components.

Fig. 3.3 indicates the form of the frequency spectrum of a theoretical N-wave with a non-zero rise time. It can be seen that the spectrum has large pressure components on either side of a zero pressure component. Hence, a small frequency shift in this region produces a large variation in the magnitudes of the pressure components. The extent to which any particular structural mode is excited depends upon the magnitude of the sonic boom's pressure component at that mode's frequency. If the frequencies of a box mode and a zero pressure component of the boom nearly coincide, then the model's response in this mode will be extremely sensitive to small changes in the boom, internal pressure and model parameters. The overall response of the model however, will only be affected significantly if any face predominantly vibrates in this mode. To illustrate this point, consider the accelerations shown in Figs. 5.1 and 5.2. There is good agreement between the theoretical and experimental accelerations of all the faces, except those of the roof in Fig. 5.1 during the free response. Even so, the roof's theoretical and experimental accelerations have similar frequency contents so that the discrepancy is not due to the mathematical idealisation of the experimental model. A comparison of the two experimental pressure-time histories indicates that the one in Fig. 5.1 has a lower frequency component (about 330 c.p.s.) than the one in Fig. 5.2. Using equation (5.9), the first zero pressure component of the first boom's spectrum occurs at about a frequency of  $(2 \times 330)$ , or 660 c.p.s. This value is only approximate as the effect of the rear rise time  $(T_2 - T_1)$  has been neglected. It indicates however that the first zero pressure component of the simulated boom in Fig. 5.1 occurs at a lower frequency than that in Fig. 5.2 because the rise times of the two booms are similar. Fig. 4.13 shows that the closed model's fifth mode (at 488.4 c.p.s.) is the closest frequency structural mode to 660 c.p.s. which also affects the

motion of the roof's centre. Thus, of all the box modes which affect the motion of the roof's centre, this mode is the one which is most sensitive to small changes in the boom's frequency spectrum. All the other faces' accelerations have a small frequency content corresponding to this mode during the free vibration period. Fig. 4.13 shows that this mode dictates the motion of the roof's centre but has little effect on the other faces' centres. This is because the box modes affecting the roof's centre are more widely spaced in frequency than the modes affecting the centres of the other faces. Consequently, slight differences in the experimental and theoretical boom's frequency spectra could give marked differences between the roof's experimental and theoretical free responses, with little difference in the other faces' responses.

If the particular mode is volume displacing then the internal pressure increases this mode's natural frequency relative to the excitation frequency. As the fifth mode is volume displacing, the internal pressure could also be a contributory factor to this difference - even though its magnitude is still small.

### 5.3 Comparison of the theoretical and experimental acceleration-time histories

Investigations to supplement the scarcity of information on the precise waveform details as a simulated boom moves over a rectangular model are now being undertaken. Preliminary investigations have shown (37) that the pressure-time history on the front face of a closed rigid model, having the same dimensions as the Xylonite model, differs significantly from the free field pressure signature. It mainly consists of two sharp, positive impulses at the front and rear of the shock wave. Between these impulses is a rounded signature enveloping approximately the same negative impulse as the free field waveform. The pressure-time histories on the remaining faces of the rigid model are similar to the free field waveform.

The free field waveforms produced by the sonic boom simulator were idealised as the broken lines shown in Figs. 5.1 and 5.2. The theoretical pressure-time history around the mathematical model was formulated as in section 3.2.2, using four equal finite elements per individual face. Despite the approximations of this theory and the variation in the experimental model's Young's modulus, a comparison of the theoretical and experimental accelerations shows reasonable overall agreement. The causes of the differ-

ences depend upon the position of each face of the model relative to the incident boom's shock front. They will now be enumerated.

The greatest discrepancy between the theoretical and experimental accelerations occurs on the closed model's back face. It is due to inadequacies in the mathematical model and the theoretical description of the forcing function on this face. All in-plane motion of the individual faces has been neglected in the mathematical model. Thus, the initial high frequency effect of the longitudinal waves is not apparent in the theoretical accelerations. This high frequency component quickly disappears from the experimental accelerations, and is replaced by much lower frequencies, when the simulated boom directly forces the back face. The discrepancies are then generally the greatest. They are mainly due to the inadequate theoretical description of the experimental forces.

The differences in the forced response of the front face are due to the over-simplification in the theoretical idealisation of the experimental pressure-time history on this face. The second peak of the experimental acceleration is greater than the corresponding theoretical value, due to the experimental boom's quicker release of positive pressure. The magnitude of the subsequent experimental peak depends upon the matching of the second positive impulse with the front face's motion. Providing the theoretical and experimental displacement and velocity conditions are similar when the boom completely leaves either model, the subsequent free responses show no further deterioration in agreement. It should be pointed out that the mathematical model had no damping. As the critical damping ratio of Xylonite is about 3%, the experimental free response should, and does, decay much faster than the theoretical response.

The experimental and theoretical accelerations of the roof and side faces, parallel to the boom's direction of propagation, show reasonable agreement; the theoretical forcing function being more realistic on these faces. All these faces are at grazing incidence to the boom. If the theoretical assumption that the forcing function effectively acts normally to each face of the model is invalidated, the responses of these faces would show the greatest inaccuracies. This is not the case. Consequently, the assumption is a reasonable approximation even when the boom's wavelength is only twice the largest dimension of any one finite element. But no

significant reduction in the number of finite elements was found to be possible in the theoretical idealisation of the experimental model. Any reduction resulted in an inadequate theoretical description of the normal modes which dictated the experimental model's response.

It is concluded that the differences between the theoretical and experimental accelerations of all the model's faces, except the back face, are sufficiently small to be acceptable. The discrepancy on the model's back face is mainly due to the inadequate theoretical description of the experimental forces on this face. The theoretical model is sufficiently flexible, however, to allow for any forces when more details of the boom's flow over the model are known. But, for convenience, the simplest finite element idealisation of the experimental model is used so that the back face's theoretical response is usually neglected.

#### 5.4.1 Variation of the theoretical accelerations and displacements on the front face, at intermediate excitation frequencies

A comparison of Figs. 5.2 and 5.3 shows that points in Fig. 5.3 which are away from the centre of the West (front) face, are more affected by the model's higher frequency box modes than the centre point (which is shown in Fig. 5.2). These modes are not necessarily the antisymmetric box modes. Their frequencies roughly correspond to integer values of the fundamental convection frequency, while their mode shapes indicate a large degree of coupling between the individual faces of the model. Thus these effects are essentially due to the boom's convection over the model exciting the isochronous box modes. The theoretical model assumes that the boom is only convected over the roof and side faces (the North and South faces) for zero incidence relative to the front face. Consequently, the front and back faces will only be affected by those isochronous box modes which have a large degree of coupling between these faces and the roof and side faces. The centre of the front face is less affected than the other points of this face because the shapes of the isochronous box modes indicate a relatively small response at this point. The effect of these higher frequencies is to substantially increase the acceleration magnitudes. Fig. 5.3 shows that the higher frequencies do not affect the displacements of the front face. They are still dictated by the lower modes.

#### 5.4.2 Significance of the antisymmetric modes for intermediate excitation frequencies

It is sometimes convenient to simulate a wave moving over a model by similar waves acting simultaneously and normally to each individual face of the model. Each of the simultaneous waves is assumed to have the same free field pressure-time history as the travelling wave, but to act directly on only a single face of the mathematical model. As each of the simultaneous waves is acting normally to a particular face, pressure doubling is assumed to occur on each face. Other than this doubling factor, the pressure-time history on each face of the mathematical model due to the simultaneous waves, is identical to its free field pressure-time history. The theoretical convecting wave, however, only has a pressure doubling effect on the front face. (See section 3.2.2.). Also, the simultaneous waves are not convected over the model. This situation is a theoretical idealisation of an experimental model enclosed within a booth which has loudspeakers producing waves from each face.

A convecting wave will excite all the box modes whereas the non-convecting waves, which act out of phase on the opposite side faces, will only excite the doubly-symmetric box modes. Hence the degree of correlation between the mathematical model's response for the two simulations will depend critically upon the significance of the antisymmetric box modes. It has been found by experiment that the antisymmetric box are more likely to be excited by high and intermediate excitation frequencies than by a low excitation frequency. Thus the more severe case of a theoretical convecting wave which corresponds to an intermediate excitation frequency (for the particular mathematical model under consideration) is assessed in this section. This wave is identical to the idealised experimental boom, (broken lines in Figs. 5.1 and 5.2) which is discussed in the previous two sections. The significance of the antisymmetric box modes for low excitation frequencies is subsequently discussed in the next section. Experimental convecting waves are then compared with a theoretical uniform step pressure which acts instantaneously on all the mathematical model's faces.

Fig. 5.4 shows that more modes are excited by the convecting wave than the five simultaneous waves. Their frequencies are similar to the natural frequencies of those box modes which have a shape corresponding to the assumed shape of the appropriate convection frequency. The effect of these modes is much greater on the accelerations of the roof and side faces

parallel to the travelling boom's direction of propagation. Figs. 5.5 and 5.6 show, however, that the displacement and velocity of each face is unaffected by these higher frequencies. Using the results of the last section, it is concluded that the antisymmetric modes, excited by the boom's convection over the model, significantly affect all the faces' accelerations. They do not affect the faces' displacements and velocities.

It is important to simulate the model's peak response because this parameter is used in the simple damage criterion of section 7.4. Unfortunately it is difficult to generalise about the magnitude of the model's response to either form of excitation, because it is determined, at any instant of time, by the relative amplitudes, phases and frequencies of the excited modes. As the West face predominantly vibrates in the same modes when acted normally upon by both forms of excitation, there is relatively little difference in the magnitude of the two responses. Conversely, the two corresponding responses of the North face show initial agreement and then diverge. This is because the effects of the antisymmetric box modes are more prominent on this face. Figs. 5.4 to 5.6 show that the magnitude of response of the side faces to the convecting wave is greatest when they form the model's front face which is acted upon normally by the wave. Similarly the roof's response is greater for the five simultaneous waves (as one of these waves is acting normally to the roof) than for the grazing incidence of the convected wave.

A closed experimental model usually contains a gas so that either form of excitation would create a pressure disturbance within the model. But the effect of this disturbance has been neglected in the theoretical model. Thus, providing the internal pressure is insignificant, it is possible to simulate the experimental model's displacement and velocity characteristics due to a convecting wave with non-convecting waves. These characteristics are unaffected by the high frequency, isochronous box modes excited by the wave's convection. A detailed knowledge of the experimental model's natural frequencies and mode shapes must be known before attempting a similar simulation for the accelerations.

#### 5.4.3. Significance of the antisymmetric modes for low excitation frequencies

Fig. 5.7 shows that an experimentally simulated boom, which corresponds to a low excitation frequency, only excites the model's lowest modes - even though it travels over the closed model. Even the accelerations are unaffected

by the higher modes associated with the wave's convection. Experimental acceleration-time histories of a given face have the same form irrespective of the wave's angle of incidence. The only difference is a time lag associated with the difference in times for the shock front to reach the point under consideration. This implies that those modes with a large degree of coupling between adjacent faces are predominantly excited. Fig. 4.13 shows that these modes are the doubly-symmetric modes. Hence the antisymmetric modes tend to become unimportant for low excitation frequencies. Consequently, the experimental and theoretical accelerations (due to the instantaneous, uniform step pressure) generally agree because the uniform pressure will only excite the doubly-symmetric modes.

The uniform step pressure acts normally to each of the mathematical model's faces. Hence the model's mathematical peak accelerations are greater for the uniform step pressure than for the idealised convected wave. They are still only half the value of the peak accelerations due to the five simultaneous waves because of the push-pull effect of these waves.

#### 5.5 Effect of changing the boom's angle of inclination

The effects on the closed mathematical model's displacement and accelerations of independently varying the idealised boom's angle of inclination are shown in Figs. 5.8 and 5.9. The North face is assumed to be acting as the model's front face. The boom's shock front always remains at zero incidence relative to this face. The theoretical free field pressure-time history is taken to be the same as the idealised experimental boom which is shown as the broken lines in Figs. 5.1 and 5.2. As the exact pressure distribution around the model is not known, it is assumed to lie between the two cases of pressure doubling on the front face alone and pressure doubling on both the front face and roof. All the remaining faces are forced by the free field's pressure-time history. It can be seen from equation (3.22) that decreasing the angle of inclination  $\psi$  increases the wave's trace speed  $V_T$ . Thus, for the same model configuration, the boom's fundamental convection frequency is increased.

The  $30^\circ$  and  $60^\circ$  booms excite the same low frequency box modes as the normally inclined boom. Different high frequency modes are superimposed on the model's overall accelerations. The frequencies are similar to integer values of the fundamental convection frequency. Figs. 5.6 and 5.9 show that the faces' displacements are unaffected by these high frequency modes.

The greatest response of any face has been experimentally shown to occur when the simulated boom is acting normally to that face. Consequently, the roof's peak response should tend to increase whilst the side faces' peak responses should tend to decrease with decreasing angle of inclination. The theoretical results do not generally vindicate these trends for a given pressure distribution. The pressure distribution around the model will change for different angles of inclination so that the true model response will depend upon the actual pressure distribution. Hence the theoretical results only approximate the true response but the general observations still apply.

#### 5.6 Effect of introducing a structural discontinuity

A comparison of Figs. 4.13 and 4.14 shows that the theoretical, in-vacuo mode shapes and natural frequencies change significantly when the East face is removed from the closed model. It can be seen from Figs. 5.2 and 5.10, however, that the same theoretical boom still excites similar frequency modes. These modes, which dictate the overall response at intermediate excitation frequencies, have natural frequencies which are similar to the excitation frequency. Thus, the degree of matching between the model's natural frequencies and the excitation frequency is the most important single parameter for intermediate excitation frequencies.

The open model's fundamental natural frequency is about half that of the closed model. But, due to the free edges of the opening, the assumed wavelength of the equivalent sine wave (which is at right angles to a free edge) of the fundamental open box mode is twice that of the closed model. Or, the open model's fundamental convection frequency in this direction is half that of the closed model. Thus the ratios of the theoretical boom's fundamental convection frequency to the open and closed model's fundamental natural frequencies are approximately the same when the West face is normal to the boom's direction of propagation. In this case, Figs. 5.2 and 5.10 show that the theoretical boom's convection excites the high frequency modes only slightly more for the open model than for the comparable closed model. A contributory factor towards this difference could be created by the open model having slightly greater structural coupling between its faces than the closed model.

The open model's theoretical and experimental accelerations (Fig. 5.10) show good agreement. The major differences are an increase in the experimental peak normalised accelerations and a frequency shift between the two

acceleration-time histories of the West face. The corresponding closed model case (Fig. 5.2) shows that there are comparable differences between its theoretical and experimental accelerations. But the internal pressure has little effect on the closed experimental model's response. Hence the differences in the open model's experimental and theoretical accelerations are inherent in the structural idealisation of the experimental model.

The closed model's peak normalised accelerations are generally greater than corresponding open model values because of additional forces on the closed model's back face.

### 5.7 Conclusions

Reasonable theoretical estimates of a model's response to a travelling wave can usually be obtained with a minimum description of the wave and model characteristics. The experimental model's stiffness variation with frequency does not affect the model's overall response because its natural frequencies are widely separated. Thus the main difference between the theoretical and experimental results is due to the inadequate theoretical description of the boom's flow over the experimental model. This flow is most complicated over the model's back face so that this face has the greatest discrepancy. It should be emphasised, however, that the mathematical model is sufficiently flexible to allow for any flow when more details are known.

The internal pressure does not usually affect the closed model's response significantly at low and intermediate excitation frequencies. The model's response is extremely sensitive to small changes in either the travelling wave, internal pressure or model characteristics when the travelling wave has a zero pressure component at a frequency which corresponds to the natural frequency of a significant box mode. Then even small changes in the details of the travelling wave and internal pressure will be significant.

It has been shown that normally acting, non-convecting waves will generally give a greater model response than a similar single wave travelling over the model. The model's accelerations are much more sensitive to the higher frequencies associated with the wave's convection than either the displacements or velocities. Consequently, it is possible to simulate the model's displacements and velocities, due to a travelling wave, with non-convecting waves. A detailed knowledge of the model's natural frequencies and mode shapes must be known before attempting a similar simulation for the accelerations.

The most important parameter, at intermediate excitation frequencies, is the degree of matching between the frequencies of the box modes and the excitation frequency.

If the internal pressure remains insignificant, the open model's peak response, to the same boom, will be less than that of the closed model. This effect is due to the additional forces on the closed model's back face. It will only remain true so long as there is reasonable structural coupling between individual faces of the model.

## CHAPTER 6

### FULL SCALE STUDIES

Resonance tests were conducted on a single brick wall in order to assess the validity of the assumption that it behaves like a plate. Subsequently, the natural frequencies and mode shapes of a full scale structure were investigated as a preliminary stage to evaluating its transient response to a sonic boom. Difficulties encountered in the investigation are enumerated in this chapter. Data acquisition techniques which are required to evaluate the structure's transient response are then briefly described together with their associated problems.

Sonic booms were simulated with explosive charges. The repeatability of these simulations was checked. Then the structure's response to various forms of simulations was evaluated in order to determine the importance of exactly simulating the pressure-time history of a sonic boom. Differences in the responses of the opposite side walls were appraised in order to test the validity of symmetry assumed for the mathematical model. The forced and free response of a structural subelement to a given simulated boom were compared in order to determine any important differences between them. Results were then presented for the response of various structural subelements to different wavelengths and angles of incidence of the simulated booms.

#### 6.1 Natural frequencies and normal modes of the prototype structure

Webb showed that the natural frequencies and normal modes of an individual brick wall could be accurately predicted using a plate theory. Subsequently, the single storey building (shown in Fig. 6.1) was harmonically excited in order to determine its natural frequencies and normal modes. The building had single thickness brick walls with supporting columns, a plaster board ceiling suspended from equally-spaced wooden beams and a large window in one of the walls. A sloping roof was constructed over the ceiling in order to protect it from the weather. Fig. 6.1 also shows that the test building was situated well

away from neighbouring buildings in order to simulate free field conditions as far as possible.

The structure's natural frequencies were determined from vector diagrams by a method which was the same as that described in Chapter 4. The resulting natural frequencies are shown in Table 6.1. It was found that the window's natural frequencies were in close agreement with the natural frequencies which were theoretically predicted assuming the window's edge restraints were simply supported. Table 6.1 shows that the structure's modal density, a mode every 2.4 c.p.s., was extremely high. It should also be added that there was a variation of up to 10% in the structure's natural frequencies. This variation seemed to depend upon the water content of the brick walls. The profusion of modes together with the variation in the structure's natural frequencies created enormous practical difficulties. Consequently only qualitative estimates of the structure's mode shapes were made because their absolute determination proved to be too time consuming.

Experimental and theoretical natural frequencies were compared as a check on the practical limitations of the idealised mathematical model. The structure was reduced to its simplest form for the theoretical analysis in order that the computational effort was minimised. It was idealised by neglecting the effects of columns, sills and doors and assuming symmetry about the vertical central plane which was at right angles to the window. The ceiling's beam-plaster board combination was idealised as a homogeneous, isotropic plate with the beams contributing the total stiffness of the plate. The plaster board was assumed to contribute only an additional mass to the total mass of the beams. The whole building was assumed to be rectangular. The results were evaluated using 22 rectangular finite elements with 4 unknowns per corner for two different heights of this 'equivalent' rectangular structure. These heights corresponded to the mean height of the sloping roof (13.625 ft) and the height of the back wall (11 ft). With these assumptions the box model described in section 2.3 (with the base edges fully fixed) can be applied. It is interesting to note that very little information is readily available on the dynamic properties of building materials. The various material properties of the main

building components were taken to be :

| Material           | $\nu$ | Material Property           |                     |                |
|--------------------|-------|-----------------------------|---------------------|----------------|
|                    |       | $\rho_{ST}(\text{lb/in}^2)$ | $E(\text{lb/in}^2)$ | $d(\text{in})$ |
| Glass              | 0.23  | 0.0212                      | $10 \times 10^6$    | 0.25           |
| Brick              | 0.15  | 0.2473                      | $1.5 \times 10^6$   | 4.5            |
| Wood/Plaster board | 0.2   | 0.0955                      | $1.3 \times 10^6$   | 5.0            |

The main application of this box theory is the prediction of the response of those structural subelements for which the internal acoustic pressure is usually neglected. This internal pressure mainly affects the least stiff structural subelements such as the window. It usually has little effect on the stiffest structural subelements such as the brick walls and ceiling. Hence the complete window was idealised with only two finite elements. This idealisation is inadequate to determine those modes which have a predominant window motion. Table 6.1 shows that the mathematical model with a mean height of 13.625 ft gives the best correlation with the experimental results. Even so, there are twice as many experimental modes as theoretical modes. Discrepancies also occur between the experimental and theoretical natural frequencies and normal modes in even the lowest modes. A comparison of the two sets of theoretical results indicates that small changes in the structure's theoretical description significantly changes its natural frequencies and normal modes.

The theoretical box model was further simplified with the assumption that each face of the full scale structure acts as an independent plate. Only the natural frequencies of the rear, right and left hand side walls for those modes which predominantly affect the motion of the centre points of these faces were computed with the independent plate theory. The ground edge of each plate was assumed to be fully fixed and all the remaining edges were considered to be simply supported. Each plate had material properties which were identical to the properties of the box model's corresponding face. Then the formulae contained in reference (46) can be applied. Note however that the  $(m,n)$ th mode defined in (46) is the same as the  $(m+1, n+1)$ th mode defined in section 2.2.2. Although the dimensions of the two side plates were identical to the corresponding

faces of the 13.625 ft high box model; the dimensions of the rear plate were taken to be the true dimensions of the full scale structure (11 ft x 15 ft). Table 6.2 compares the natural frequencies predicted by the plate and 13.625 ft high box theories with the experimental frequencies observed near the centre of the relevant face during the resonance tests. It should be noted that the box theory gives additional frequencies to those shown in Table 6.2. These frequencies correspond to those box modes in which the centre of the particular face has a relatively small motion compared with the motions of the other faces. The plate theory however gives no intermediate frequencies because each plate acts independently. Thus the box theory gives a much higher modal density for the structure than the plate theory. Neither of these theories however give modal densities which are as high as that of the actual structure.

Most structural coupling occurred between the opposite side walls through the interconnecting ceiling beams. Table 6.1 shows that little coupling was observed during the resonance tests at frequencies below 59 c.p.s. Thus it can be seen from Table 6.2 that the plate theory was nearly as accurate as the box theory in the prediction of the structure's lowest natural frequencies. The main discrepancy between the plate and experimental frequencies occurred for the higher natural frequencies of the rear wall. The box theory gave much better estimates of these higher frequencies than the plate theory.

The natural frequencies of the actual full scale structure changed with humidity and temperature in an undetermined manner. This phenomenon, combined with the structure's high modal density, made an accurate prediction of the structure's natural frequencies and normal modes even more difficult. Thus gross structural parameters, which only need a minimum knowledge of the structure's modal behaviour, are required to predict the possibility of structural damage from a sonic boom. Such parameters are developed and discussed in the next Chapter.

## 6.2 Accuracy of data analysis

Due to the full scale structure's high modal density, better modal detection techniques are required than the "by eye" method applied to the experimental model's results. Digital techniques were employed which represent the time history of a given record in terms of its Fourier

series. Signals were recorded on-site and analysed at the Data Analysis Centre, Southampton University. The recordings were digitised by an analogue to digital converter and analysed on a digital computer. Data acquisition was initiated by a trigger pulse on one of the tape tracks. The digital programme was a complete package which displayed automatically the modulus and phase of the Fourier series in graphical form. To obtain a convenient display format, the modulus of the Fourier series was always normalised with respect to its own peak value. In other words, the relative amplitude scale shown in each frequency spectrum has no absolute value.

The analogue to digital conversion can introduce two sources of error. Firstly, a quantization error is introduced by the necessity of representing the analogue signal with a finite number of binary digits. The maximum quantization error was less than 0.1% for the present investigation (39). Secondly, there is an aliasing error. This error is due to an insufficient number of samples describing the significant high frequency information. Gertel and Holland (40) suggest that

$$E = 100 \left( 1 - \cos \frac{\pi}{n} \right) \% \quad (6.1)$$

(where  $n$  is the number of digital points per cycle).

This gives reasonable estimates of the maximum aliasing error for a transient signal. An initial investigation of the transient records showed that the maximum frequency of interest was 300 c.p.s. Thus the aliasing error was less than 10% for the least number of digital points selected. The same authors also suggest that low frequency variations can be expected when the dimensionless frequency parameter of the shock satisfies the inequality  $f\tau \leq 1$ .  $f$  is the frequency in c.p.s. and  $\tau$  is the shock record's duration in seconds.  $\tau$  was fixed at half a second for the simulation of free field conditions because a significant wave was reflected from a neighbouring building. Thus low frequency variations should occur below 2 c.p.s. This value is sufficiently low that the variations cause little concern.

The resolution (or frequency spacing of the displayed data) of the Fourier series is determined by the inverse of the record's total

duration  $1/\tau$  . Hence the resolution is 2 c.p.s. and it is independent of the sampling rate. Gertel and Holland indicate that there are large variations in the local maximum and minimum values of the Fourier series' amplitude which depend upon the selected sampling rate. Only qualitative estimates were required for the present investigation because it was only needed to determine those modes which dictated the structure's response. Hence a sampling rate of 2,500 samples per second was found to be adequate.

### 6.3 Repeatability

Sonic booms were simulated using the explosive technique described in reference 12. Fig. 6.2 compares nominally the same simulated boom observed on different days. Although measurements were only taken once a day, they should give some indication of any grave discrepancies in the weather conditions occurring on different days. These conditions were observed to be :

|                                    | DAY 1           | DAY 2           |
|------------------------------------|-----------------|-----------------|
| Wind speed and direction           | 8 kts. easterly | 5 kts. easterly |
| Humidity (%)                       | 66              | 57              |
| Temperature ( $^{\circ}\text{C}$ ) | 7.5             | 20              |
| Pressure (mm of mercury)           | 754.8           | 750.5           |

Thus the temperature difference has the greatest variation.

Fig. 6.2 shows that there is little difference between the frequency spectra of the two booms. The differences could be due to either small differences in the atmospheric conditions or differences in the material and packing of the explosives. Fig. 6.3 shows the corresponding variability in the frequency spectra of the accelerations of various structural subelements. It can be immediately seen that the side wall's spectrum is insensitive to small changes in either the simulated booms or the weather conditions. On the other hand, the accelerations of the centre of the glass window have markedly different frequency spectra. The same differences were also reflected in the maximum normalised accelerations and displacements of these subelements (shown in Table 6.3).

Although the window was seen to deflect under the action of wind gusts, the gusts occurred on most days due to the site's exposed nature. Table 6.3 indicates that the marked differences in the window's response occurred consistently throughout the two days. This factor can thus be eliminated. Loosening of the window's supports can also be eliminated because the greatest responses occurred on the first day. The maximum normalised internal pressures created by the longest duration booms (approximately 200 msec) are similar. Thus the differences in the window's responses are not due to the internal acoustic backpressure. Changes in the dynamical properties of glass with temperature, age and humidity could be contributory factors affecting the window's response. Further detailed investigations are necessary however to pinpoint the root cause of these differences. In particular, the effects of the non-linear behaviour of glass and its supports should be considered.

Present investigations are confined to the prediction and analysis of the response of brick walls and ceilings. These subelements are relatively insensitive to small changes in the weather conditions.

#### 6.4 Difference in the responses of opposite side walls

Fig. 6.5 illustrates the difference in the frequency response of the right (RH) and left hand (LH) opposite side walls to a given simulated boom. It should be noted that the relative amplitude of the right hand wall (dotted curve) is 45% greater than that of the left hand wall. This difference was apparent for all the simulated booms whose shock fronts were at zero incidence to the window.

The main frequency response of both side walls occurs at about 22 c.p.s. with an additional frequency of 38 c.p.s. being observed only on the right wall for the nominal 200 msec boom. The greater overall amplitude of the right hand wall's frequency spectrum for all zero incident booms is also reflected in its greater maximum normalised peak accelerations shown in Table 6.3. This difference in the opposite side walls' responses was also observed during the resonance tests at the mode corresponding to 23 c.p.s.

The main differences in the two responses occur at low frequencies in the region of 22 to 38 c.p.s. They are the result of stiffness differences between the two walls which are due to the different positions

of the supporting columns and lintels and the introduction of a door in the left hand wall. It can be seen from Fig. 6.5 however, that the theoretical assumption of the opposite walls being identical is a reasonable approximation.

#### 6.5 Comparison of the Mk.I and Mk.II simulated booms

The Mk.I simulated boom corresponds to an 'N'-shaped waveform whereas the Mk.II boom consists of two distinct positive impulses. Although the Mk. II boom does not simulate the exact shape of a sonic boom's pressure-time history, the explosive charges are easier to prepare and install than for the Mk. I boom. Only two lump charges are required with a time delay circuit for the Mk. II simulation, whereas explosive line charges hung lengthways on supporting stakes are needed for the Mk.I simulation.

Fig. 6.4 shows both types of simulated booms (which correspond to nominal durations of 200 msec), together with their respective frequency spectra. The relative amplitude of the Mk. I frequency spectrum is 3.9 times greater than that of the Mk. II. The main differences between the two spectra are :

- (i) The Mk. I spectrum has a predominant frequency of 2 c.p.s. compared with 16 c.p.s. for the Mk. II simulation.
- (ii) The Mk. II spectrum contains discrete frequencies whose amplitudes are similar to that of the predominant frequency. The Mk. I spectrum, on the other hand, mainly consists of the predominant frequency.

Fig. 6.4 also illustrates the differences in the acceleration-time histories of the structural subelements due to the two types of booms. The accelerations caused by the Mk. II boom generally attenuate more quickly during the second cycle due to the quicker release of pressure. The overall maximum accelerations usually occur shortly after the second impulse. This behaviour contrasts with the structure's accelerations caused by the Mk. I boom. The overall maximum accelerations due to the Mk. I boom usually occur during the initial forced vibration period - there is only a slight increase in the attenuated accelerations when this boom leaves the structure. The overall maximum, normalised

accelerations of the window and front wall however are significantly greater for the Mk. I boom. In this case the increased acceleration of the window creates an increase in the front wall's acceleration due to the structural coupling between these subelements. There is little difference, on the other hand, between the overall maximum normalised accelerations of the remaining structural subelements caused by either boom.

Although the overall maximum normalised accelerations of the side walls are relatively unaffected by the different simulated booms, Fig. 6.4 shows that there is greater differentiation between the excitation of closely spaced modes for the Mk. II boom due to its discrete frequencies. This figure also illustrates the fact that the high frequency content (over 100 c.p.s.) of either boom does not significantly affect the structure's response. The high frequency content is due to the very fine details in the shapes of the simulated waves. This observation agrees with the similar qualitative assessment of the model's response to the comparable shock tube generated booms.

It is concluded that the general shape of the sonic boom is important if it is required to determine the accelerations of the most flexible structural subelements. If the response of these subelements significantly affects the response of the stiffer subelements then the general shape of the sonic boom must be simulated. The general shape of the boom is unimportant however if it is only required to estimate the overall maximum accelerations of the structure's stiffest subelements.

## 6.6 Comparison of the structure's forced and free response

In order to discover any important differences in the modes which dictate the structure's forced and free response it is necessary to compare the relevant frequency spectra directly. A direct comparison is only possible, however, if they have the same resolution. The resolution is dictated by the total duration of the sampled record. Thus, zeros were substituted in place of the free response of the original signal so that the total duration of the forced response, plus zeros, was the same as that of the forced response plus free response. Any differences in the amplitudes of the two frequency

spectra are then entirely due to the action of the excited modes in the free vibration period. It must be emphasised that the insertion of zeros does not increase the accuracy of the frequency spectra. It only gives equal frequency spacing of the estimated amplitudes.

The frequency spectrum of the forced response plus zeros is the same as that of the forced response alone because :

$$F(\omega) = \int_0^{T_F} f(t)e^{-i\omega t} dt = \int_0^{\tau} f(t)e^{-i\omega t} dt$$

as  $f(t) = 0 \quad T_F < t \leq \tau.$  (6.2)

$T_F$  corresponds to the time of the forced response and  $\tau$  is the total time of the analysed record.

Fig. 6.6 illustrates the differences in the structure's forced and free response. It should be noted that the relative amplitude scale of the forced plus free response is 1.58 times greater than that of the forced response alone. The high frequency content in the structure's response mainly occurs during its forced response. Although the structure's damping factor decreases with frequency, the response's high frequency components complete many more cycles in a given time than the low frequency components. Consequently, the high frequency content of the structure's response attenuates much more quickly than the low frequency content. In fact, the structure tends to vibrate in its lowest modes during its free response. These modes even dictate the structure's accelerations during this period.

## 6.7 Discussion of the experimental results

Fig. 6.7 illustrates the relative frequency response of various structural subelements to a Mk.I simulated boom. Observations from several of these records may be briefly summarised as :

- (i) The internal acoustic pressure-time history always has a similar form to the window's displacement-time history. Superimposed upon its general form are small pressure

fluctuations created by the effects of the pressure disturbance within the cavity. Thus the internal pressure will be coupled to the window. It does not significantly affect the response of the ceiling and side walls.

(ii) A comparison of the results for the nominal 200 msec booms which were detonated on the same day (Tables 6.4 and 6.5) indicates that the window's acceleration is sensitive to the simulated boom's rise time  $T_R$ . The window has a greater acceleration for a smaller rise time. Its displacement and the accelerations of the remaining subelements however are relatively insensitive to  $T_R$ . This phenomenon is in qualitative agreement with the experimental model's response to the longest duration (7 ft driver) boom.

(iii) The structure acts as a Helmholtz resonator when the side door is open. The internal pressure disturbance still mainly affects the window's response. As for the experimental model, the Helmholtz frequency of 7 c.p.s. could be accurately predicted by again assuming all the structure's boundaries to be rigid.

It is intended to consider in more detail the response of the window, ceiling and right hand side wall to simulated booms which have different durations ( $T_1 - T_R$ ) and angles of incidence  $\theta$ . ( $\theta$  is measured anticlockwise relative to the window face). The booms were all detonated on the same day in order to minimise any possible variation in the subelement's responses due to different weather conditions. They were designed to have nominal durations of 40, 100 and 200 msec. There were variations in these nominal durations caused by small differences in the explosive material and its packaging. For convenience, the shortest duration boom will be referred to as the 40 msec boom; the longest duration boom as the 200 msec boom; and the intermediate duration boom as the 100 msec boom.

Fig. 6.8 shows that, irrespective of the boom's angle of incidence, the accelerations of the side walls tend to be dominated by one or two modes for the 40 and 100 msec booms. These modes are the lowest frequency modes in which the side walls have a relatively large motion compared with

the other boundaries. The response of the side walls to the 200 msec boom differs markedly from that already described. As well as the two lowest frequency modes this boom forces the side walls to vibrate in several other modes. This is particularly true at non-zero angles of incidence when the structure's antisymmetric modes are excited. The relative amplitudes of these additional modes depend upon the relative amplitudes of the simulated boom's frequency spectrum at the frequencies which correspond to those of the modes. The important difference between the 200 msec and 40/100 msec booms is due to the 200 msec boom having a greater time to impart the wave's energy to the structure. Thus the subsidiary peaks in the range 0 to 50 c.p.s. of its frequency spectrum have a significant effect on the structure's accelerations. Conversely the structure's displacements and velocities are still mainly affected by the two lowest frequency modes. Hence, if the structure's accelerations are of primary interest, it is important to simulate the pressure-time history details of the 200 msec and longer duration sonic booms. The same degree of simulation is unnecessary if only the structure's displacements and velocities are required.

Similar trends to those outlined for the side wall's accelerations are also observed in Fig. 6.10 for the effects on the window's acceleration of changes in the duration and angle of incidence of the boom. More modes are excited in the case of the window due to its greater flexibility. Assuming that the window is simply supported, the modes which are excited correspond to the (1,1), (3,1) and (5,1) plate modes. It is interesting to note that the 200 msec boom also excites consistently the (2,1) plate mode - even when the shock front is at zero incidence! This frequency (13.5 c.p.s.) corresponds to a subsidiary peak in the boom's frequency spectrum. The excitation of this antisymmetric mode could be due to the shock front not being completely at zero incidence to the window. This phenomenon is associated with the spherical and not plane shock front of the simulated boom. Doubt is also cast on the validity of the assumption of simply supported edge conditions for the window because this frequency is observed at the window centre. Further investigations are required into the effects of the possible non-linear behaviour of the window and its supports.

The modes which affect the ceiling's response are shown in Fig. 6.9 to be relatively insensitive to changes in the boom's angle of incidence. This phenomenon was also observed for the experimental model. The additional low frequency modes excited by the nominal 40 msec boom are more likely to be associated with the effects of the boom's convection than the internal acoustic pressure. The internal pressures for the 40 and 100 msec booms were similar in magnitude and frequency but the additional ceiling modes were not excited by the 100 msec boom.

It is interesting to note that the lowest structural modes were predominantly excited by all the simulated booms. Thus it seems possible to use the independent plate rather than the box theory in determining the response of the side walls and ceiling. The plate theory should give reasonable results for the 40 and 100 msec booms but not for the 200 msec boom. The latter boom excites the structure's antisymmetric modes and these modes involve opposite side wall coupling. Further investigations are needed to assess the true accuracy of the plate theory.

## CHAPTER 7

### COMPARISON OF MODEL AND FULL SCALE RESULTS

Scaling laws are developed in this chapter in order to make a direct comparison between the results of a prototype structure and its experimental model. Two approaches are adopted. Initially an approximate method is formulated which requires a knowledge of the important loading and structural parameters. This method assumes that the model and prototype are geometrically similar. This assumption however is not strictly correct. Hence a more refined, computer-orientated method is subsequently developed.

The responses of the mathematical model and prototype structure are compared. The mathematical model has the advantage that, once developed, it only requires different computer data describing the material properties and physical dimensions of the structural subelements.

Finally, all the results are compared with published results in order to determine their accuracy. A simple 'rule of thumb' guide to possible structural damage from a sonic boom is then developed for the walls of single-storey buildings.

#### 7.1 Model scaling laws

A dynamic structural model should be a physical system which duplicates the dynamic characteristics of the prototype structure such that measurements on the model can be used to predict the prototype's behaviour. The model is usually constructed of a material which is different from the prototype, so that it is essential to determine the relevant structural and loading parameters. These parameters should be the same for both the model and prototype. The model must also be constructed so that the model/boom parameters are within the simulator's working range.

The simplified case of the response of a simply supported, flexible panel to a non-symmetrical N-wave, whose direction of propagation is normal to the plate, is considered. This plate is assumed to form one face of an otherwise rigid box. The theory accounts for the effect of the enclosed gas and is a first approximation to a room with very stiff walls containing a large window. The parameters obtained will be the most significant ones

for the more general case of a room with flexible walls, providing they can be represented as plates.

Neglecting the structural and acoustic damping, the equations of motion for an N-wave with a finite rise time  $T_R$  (as shown in Fig.3.1(a) when  $T_1 = T_2 = T$ ) whose direction of propagation is normal to a plate, may be written as :

(i) For the plate

$$\frac{\partial^4 w}{\partial x^4} + \frac{\partial^4 w}{\partial y^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\rho_{ST}}{D} \frac{\partial^2 w}{\partial t^2} = \begin{cases} p_R/D & 0 < t < T_R \\ \frac{p_{in} + p_b}{D} & T_R \leq t < T \\ \frac{p_b}{D} & t \geq T \end{cases}$$

where

$$p_R = p_o \frac{t}{T_R}, \quad p_b = -\rho_b \frac{\partial \phi}{\partial t}$$

$$p_{in} = p_o \left[ \left( \frac{1 + \frac{T_R}{T}}{1 - \frac{T_R}{T}} \right) - \frac{2}{(1 - \frac{T_R}{T})} \left( \frac{t}{T} \right) \right] \quad (7.1)$$

providing the internal acoustic pressure is not significant during the rise time.

(ii) For the acoustic backing of the enclosed gas

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = \frac{1}{a_b^2} \frac{\partial^2 \phi}{\partial t^2} \quad (7.2)$$

subject to the boundary conditions

(a) simply supported edges parallel to the x-axis at  $y = b$  and parallel to the y-axis at  $x = a$  respectively:

$$w|_{y=b} = 0 = \frac{\partial^2 w}{\partial y^2} \Big|_{y=b} \quad \text{and} \quad w|_{x=a} = 0 = \frac{\partial^2 w}{\partial x^2} \Big|_{x=a} \quad (7.3)$$

(b) plate initially at rest:

$$w = 0 = \frac{\partial w}{\partial t} \quad \text{at } t = 0 \quad (7.4)$$

(c) equal velocities at the plate and acoustic medium interface,

$z = g$  say:

$$\left. \frac{\partial w}{\partial t} \right|_{z=g} = \left. \frac{\partial \phi}{\partial z} \right|_{z=g} \quad (7.5)$$

Introducing the non-dimensional parameters

$$\begin{aligned} \xi &= \frac{x}{\ell} & \eta &= \frac{y}{\ell} & \delta &= \frac{z}{\ell} & \zeta &= \frac{w}{\ell} \\ \tau &= f_{11} t & \phi &= \frac{\phi}{a_b \lambda_b} \end{aligned} \quad (7.6)$$

then equations (7.1) to (7.5) respectively become

$$\frac{\partial^4 \zeta}{\partial \xi^4} + \frac{\partial^4 \zeta}{\partial \eta^4} + 2 \frac{\partial^4 \zeta}{\partial \xi^2 \partial \eta^2} + \left( \frac{\rho_{ST}}{D} \cdot \ell^4 f_{11}^2 \right) \frac{\partial^2 \zeta}{\partial \tau^2} = \begin{cases} \left( \frac{p_o \ell^3}{D} \right) \left( \frac{f_R}{f_{11}} \right) \tau, & 0 < \tau < \frac{f_{11}}{f_R} \\ \left( \frac{p_o \ell^3}{D} \right) \left| \left( \frac{1 + f_N/f_R}{1 - f_N/f_R} \right) - \frac{2}{(1 - f_N/f_R)} \frac{f_N}{f_{11}} \right| \\ - \left( \frac{\ell \rho_b}{\rho_{ST}} \right) \left( \frac{f_c}{f_{11}} \right) \left( \frac{\rho_{ST} \ell^4}{D} f_{11}^2 \right) \left( \frac{\lambda_b}{\ell} \right) \frac{\partial \phi}{\partial \tau}, & \frac{f_{11}}{f_R} \leq \tau < \frac{f_{11}}{f_N} \\ - \left( \frac{\ell \rho_b}{\rho_{ST}} \right) \left( \frac{f_c}{f_{11}} \right) \left( \frac{\rho_{ST} \ell^4}{D} f_{11}^2 \right) \left( \frac{\lambda_b}{\ell} \right) \frac{\partial \phi}{\partial \tau}, & \tau \geq \frac{f_{11}}{f_N} \end{cases} \quad (7.7)$$

where  $f_R = 1/T_R$ ,  $f_N = 1/T$

$$\frac{\lambda_b}{\ell} = \left( \frac{a_b}{\ell} \right) \left( \frac{1}{a_b/\lambda_b} \right) = \frac{f_c}{f_b}$$

$$\frac{\partial^2 \phi}{\partial \xi^2} + \frac{\partial^2 \phi}{\partial \eta^2} + \frac{\partial^2 \phi}{\partial \delta^2} = \left( \frac{f_{11}}{f_c} \right)^2 \frac{\partial^2 \phi}{\partial \tau^2} \quad (7.8)$$

with the boundary conditions taking the form

$$\zeta|_{\eta = b/\ell} = 0 = \frac{\partial^2 \zeta}{\partial \eta^2}|_{\eta=b/\ell}, \quad \zeta|_{\xi=a/\ell} = 0 = \frac{\partial^2 \zeta}{\partial \xi^2}|_{\xi=a/\ell} \quad (7.9)$$

$$\zeta = 0 = \frac{\partial \zeta}{\partial \tau} \quad \text{at } \tau = 0 \quad (7.10)$$

and

$$\frac{\partial \Phi}{\partial \delta}|_{\delta=g/\ell} = \left( \frac{f_{11}}{f_c} \right) \left( \frac{f_b}{f_c} \right) \frac{\partial \zeta}{\partial \tau}|_{\delta=g/\ell}. \quad (7.11)$$

Note that the purely structural boundary conditions, (7.9) and (7.10), are parameter free because the most critical case of a free edge, which involves Poisson's ratio, has been excluded by the initial assumptions. The arbitrary structural length  $\ell$  is now defined such that  $f_c$  corresponds to the fundamental internal acoustic convection frequency.

The equations of motion are now independent of size and must apply directly to both the model and prototype structures. Taking the equations in order, the required parameters are :

1. The modal and aspect ratio parameter,  $(\frac{\rho_{ST}}{D} \ell^4 f_{11}^2)$ .

This ratio reduces to the plate's modal ratio parameter  $f_{11}/f_{mn}$  for complete geometric similarity because

$$\frac{\rho_{ST}}{D} \ell^4 = \frac{R_{mn}}{f_{mn}^2} \quad (\text{where } R_{mn} \text{ is a constant depending upon the plate's aspect ratio})$$

for a simply supported plate vibrating in its (m,n)th mode. This parameter will be the same for model and prototype if the model is geometrically similar to the prototype and the inertia and stiffness distributions are the same.

2. The static stiffness ratio parameter,  $(\frac{p_o \ell^3}{D})$ . It reduces to  $(\frac{p_o}{E} \{1 - \nu^2\})$  for complete geometric similarity; dictates the magnitude of the response; but is relatively unimportant if the model and prototype responses are both linear and the effect of Poisson's ratio is small.

3. The frequency ratio parameter,  $f_{11}/f_N$ . This parameter is the ratio of the fundamental structural frequency to the excitation frequency of the N-wave.
4. The ratio of the rise time to the fundamental N-wave duration,  $T_R/T$ . Intuitively, this parameter will be most important when the maximum structural response occurs while the N-wave is still forcing the structure. Craggs (9) showed that it is also important when the N-wave excites a plate under isochronous conditions, ( $f_{11}/f_N = 1$ ).
5. The internal fundamental convection frequency ratio,  $f_{11}/f_c$ . Apart from the isochronous condition,  $f_{11}/f_c = 1$ , the internal pressure disturbance will be relatively unimportant for panels much stiffer than the internal acoustic stiffness. In any case, the fundamental external convection frequency ratio, associated with the N-wave moving over a flexible structure, will be more important due to its greater pressure magnitude.
6. The ratio of the fundamental structural to the fundamental acoustic cavity frequency,  $f_{11}/f_b$ .
7. The parameter  $\rho_b/\rho_{ST}$ . For geometric similarity, this parameter reduces to  $\rho_b/\rho_v$  which is the ratio of the internal gas and structural densities.

Both Pretlove (23,24) and Lyon (25) have shown that the last two parameters are important in the determination of the magnitude of the sound pressure level transmitted into the cavity from outside the structure.

## 7.2 Comparison of the prototype and experimental model results

In order to assess the accuracy of simulating the prototype with a model it is necessary to compare their respective results. There are certain discrepancies in the present comparison because the doors, supporting columns and window of the prototype were not included in the model. Also, the model was not completely geometrically similar to the prototype structure. It is implicitly assumed in this comparison that the side

wall's response is insensitive to small changes in geometry and is unaffected by the inclusion of structural discontinuities in the other faces.

The material properties and dimensions of the model and prototype (using a mean height for the sloping roof) are :

|                           | $E(\text{lb/in}^2)$         | $\rho_{ST}(\text{lb/in}^2)$ |
|---------------------------|-----------------------------|-----------------------------|
| Model (Xylonite)          | $4.2 \times 10^5$           | 0.022                       |
| Prototype (Brick)         | $15 \times 10^5$            | 0.25                        |
| Model dimensions (ins.)   | 12 : 9 : 7 : 0.36           |                             |
| Prototype dimensions (ft) | 17.25 : 15 : 13.625 : 0.375 |                             |

while their fundamental frequencies were found to be

$$(f_{11})_M = 243.5 \text{ c.p.s.} \quad (f_{11})_P = 16 \text{ c.p.s.} \quad (7.12)$$

The mean geometric scaling ratio is 18.3, so that

$$\frac{l_M}{l_P} = \frac{1}{18.3} \quad (7.13)$$

In order to account for incomplete geometric similarity, the constant static stiffness ratio gives

$$\frac{(p_o)_M}{(p_o)_P} = \frac{D_M}{D_P} \left(\frac{l_P}{l_M}\right)^3 = 0.87 \quad (7.14)$$

if it is assumed that the effect of Poisson's ratio can be neglected.

Consider the simplest case when the wave's shock front is at zero incidence to one of the side faces of the model and prototype structure and is also at  $90^\circ$  inclination to the horizontal. As the prototype's response was only evaluated with the window face being the front face, then,

$$\frac{L_P}{L_M} = 17.25$$

The fundamental cavity frequency is given approximately by

$$f_{b_i} = f_b = \frac{a_o}{2L}$$

if air is used as the gas enclosed inside the model. This formula is obtained by assuming that all the structural boundaries are rigid so that the fundamental acoustic mode can be approximated by a half sine wave extending over length  $L$ . Note that this mode is the fundamental mode because  $L$  also happens to be the maximum structural length.

The fundamental external convection frequency satisfies equation (3.20) with  $n=1$ . This relationship, when used in conjunction with equation (3.22), gives

$$\frac{(f_c)_M}{(f_c)_P} = \left( \frac{a_o}{L} \right)_M / \left( \frac{a_o}{L} \right)_P \quad (\psi = 90^\circ)$$

If the wave's speed of propagation  $a_o$  is the same for model and prototype, both the fundamental external convection frequency parameter and the ratio of the fundamental structural and cavity frequencies reduce to

$$\frac{(f_{11})_M}{(f_{11})_P} = \frac{L_P}{L_M}$$

But, using equation (7.12),

$$\frac{(f_{11})_M}{(f_{11})_P} = \frac{243.5}{16} = 15.2$$

which is compared with the value of

$$\frac{L_P}{L_M} = 17.25.$$

This difference will not affect the structures' displacements and velocities as they are dictated by the lower structural modes. It could become increasingly important for the accelerations, particularly at high excitation frequencies.

The ratio of the densities of the model and prototype gases should be given by

$$\frac{(\rho_b)_M}{(\rho_b)_P} = \frac{\ell_P}{\ell_M} \cdot \frac{(\rho_{ST})_M}{(\rho_{ST})_P} = \frac{18.3}{11.4} = 1.6. \quad (7.15)$$

In fact, the experimental ratio is unity. Lyon (25) shows that the magnitude of the internal pressure is approximately proportional to the density of the enclosed gas for excitation frequencies below the panel's critical frequency. Hence the magnitude of the model's internal pressure should be less than that in the prototype structure - all other factors being equal.

The model and prototype are neither completely geometrically similar nor are their inertia and stiffness distributions exactly the same. In fact, the prototype's modal density is much higher than that of the model due to additional structural components which were not present on the model. The modal and aspect ratio parameter reduces, however, to

$$\frac{(\frac{\rho_{ST}}{D} \ell^4 f_{11}^2)_M}{(\frac{\rho_{ST}}{D} \ell^4 f_{11}^2)_P} = \frac{0.022}{0.25} \times \frac{1}{0.87} \times \frac{1}{18.3} \times 15.2^2 = 1.28. \quad (7.16)$$

If this parameter is assumed to be equivalent to the modal ratio, then this relationship implies that the modal density of the model is greater than that of the prototype. It must be emphasised, however, that this relationship is only approximate because neither a model nor a prototype face is acting entirely independently from the remaining faces. Its accuracy will depend upon the similarity of structural and acoustic coupling between individual faces of the model and prototype and the effect of structural discontinuities. Below the structure's fundamental frequency, however, the structure is stiffness controlled so that the modal ratio parameter is relatively unimportant, providing the boom's frequency spectrum has a small high frequency content. It will become important when the structure is resonant. In this case more modes are excited for the prototype than the model. These additional modes will be much more important in determining the acceleration than either the displacement or velocity characteristics. The last two parameters are relatively insensitive to

all but the lowest modes.

For the same rise time and frequency ratio parameters, the non-dimensional response will be the same for prototype and model. The acceleration relationship is given by

$$\begin{aligned} \left( \frac{\partial^2 \zeta}{\partial \tau^2} \right)_M &= \left( \frac{\partial^2 \zeta}{\partial \tau^2} \right)_P \\ \text{or} \quad \frac{\ddot{w}_M}{\ddot{w}_P} &= \frac{l_M}{l_P} \cdot \frac{(f_{11}^2)_M}{(f_{11}^2)_P} \end{aligned} \quad (7.17)$$

so that

$$\begin{aligned} \frac{\left( \frac{\ddot{w}}{p_o} \right)_M}{\left( \frac{\ddot{w}}{p_o} \right)_P} &= \frac{l_M}{l_P} \cdot \frac{(f_{11}^2)_M}{(f_{11}^2)_P} \cdot \frac{(p_o)_P}{(p_o)_M} = \frac{1}{18.3} \times 15.2^2 \times \frac{1}{0.87} \\ &= 14.5. \end{aligned} \quad (7.18)$$

In deriving this relationship it has been implicitly assumed that the model and prototype behave in an identical manner with varying peak pressures of the simulated boom. This relationship is approximately linear for the model. No information, however, is available for the prototype structure. Hence it is assumed to be linear.

Ratio (7.18) is compared in Table 7.1 with the ratios of the model's accelerations (shown in Fig. 4.10) to the prototype wall's accelerations. It can be seen that this approximate analysis gives reasonable agreement at very low excitation frequencies but not for low and intermediate excitation frequencies. In order to extend the model's application to the higher excitation frequencies, a more refined method - which takes into account the different modal densities and geometries - is described below. The method inherently assumes that the internal pressure affects the model and prototype in the same way and to the same extent. Although this method does not require the faces of the model to act independently, it still does not account for the different coupling characteristics exhibited by the prototype structure from those of the experimental model.

Using the finite element representation of the box model in section 6.1, the maximum theoretical accelerations of the prototype structure were computed to be :

|                                                                     | R.H. Wall    | Ceiling  | Rear Wall    |
|---------------------------------------------------------------------|--------------|----------|--------------|
| Position with respect to centre of each individual face in ft.      | (0, +0.6875) | (0, 0)   | (0, -0.6875) |
| Maximum acceleration<br>(Equivalent height of prototype: 13.625 ft) | 0.044 g.     | 0.147 g. | 0.041 g.     |

for a uniform, unit instantaneous pressure all around the structure. A critical damping ratio,  $\xi_1$ , of 0.02 was assumed for the whole structure. Each of the theoretical points is sufficiently close to the centre of a face that differences in their magnitudes due to the effects of the antisymmetric modes will be small. The prototype's accelerations due to a simulated boom are referred to the corresponding theoretical values due to the instantaneous pressure. The factored prototype values are then plotted for a given value of  $(\lambda/L)$  in Figs. 4.10 and 4.11 as a multiple of the experimental model's accelerations due to the uniform unit step pressure. For example, suppose the right hand wall of the prototype structure has an acceleration of 0.44 g. for a particular value of  $(\lambda/L)$  and angle of incidence  $\theta$ . (It should be noted that  $\theta$  is adjusted for the prototype structure so that it is referred to the left hand wall and not to the window face. This notation is then identical to that used for the model). This acceleration is ten times the theoretical acceleration due to the uniform, unit instantaneous pressure shown in the above table. Then the prototype's acceleration is plotted in Fig. 4.10 as ten times the value of the model's acceleration due to the uniform unit step pressure for the same values of  $(\lambda/L)$  and  $\theta$ .

The prototype's peak accelerations have the same overall tendency as the model results to decrease with increasing  $\lambda/L$  (decreasing excitation frequency). This trend is also observed for actual booms. Reference (41) states that :

'B-58 booms, when normalised to peak pressure, cause lower response of wall and ceiling elements than smaller F106 or F104 aircraft in 4 out of 5 tests of sample data taken at Oklahoma City and White Sands'

even though 'no large differences in structural motions were visually observable'.

The prototype's peak accelerations have the same magnitude as the corresponding peak accelerations of the model below the model's fundamental frequency (i.e. in the low excitation frequency region). Large discrepancies in the magnitudes of the peak accelerations occur above the model's fundamental frequency. They are due to differences in the modal behaviour of the two structures which are caused by their different geometries.

In general, the prototype's accelerations follow similar trends to the model's accelerations for the same change in the boom's angle of incidence. The greatest discrepancy occurs for the rear walls. It is probably due to the difference in pressure flows over the two structures. Fig. 4.10 shows that the peak acceleration of the model's front face can be four times greater than the peak value on a rear face. This factor occurs in the intermediate frequency range and decreases to between one and two for low excitation frequencies. The last situation is representative of both F104 and B-58 booms forcing most residential type structures. In this case it has been 'statistically proved at the 95% confidence level that the inbound response (of walls) in both cases is more than twice as large as the trailing response' (41). This effect is mainly due to pressure doubling on the front face.

The same reference also states that the strains of a 10 ft x 5 ft window were 300% greater for an inbound vector than a trailing vector. This effect was due to the participation of higher modes. The window was excited in the region of the intermediate and low excitation frequency boundary where the ratio of the corresponding model accelerations is about four. These results are in general agreement because the accelerations are more affected than the strains by the higher modes. The variation of the window's strains and the model's accelerations with changing angle of incidence also agree.

Blume (42) noted that the larger B-58 caused large, low frequency windows to displace twice as much as the F104, for a given angle of incidence. This phenomenon occurs because the natural frequencies of

these windows are more closely matched to the excitation frequency of the B-58 than that of the F104. The window's peak accelerations will increase for the F104 as they are dictated by the higher modes which correspond more closely to this plane's higher excitation frequency. The magnitude of the displacements, on the other hand, are weighted towards the lower frequencies which are more efficiently excited by the B-58.

Fig. 4.11 shows that the ceiling of the prototype structure is relatively unaffected by changes in the boom's angle of incidence. The maximum ceiling accelerations of the prototype structure, unlike those of the model, are no longer governed by the initial impulse. Instead, the variation with incidence of the ceiling's peak accelerations is similar to that of the side walls. The model ceiling's accelerations are much greater than those of the prototype. These differences are probably due to the simulated boom not forcing the ceiling of the prototype structure directly because of the false roof's presence, whereas the shock tube generated booms flow directly over the model's ceiling.

### 7.3 Comparison of the prototype and finite element results

Figs. 7.1 and 7.2 compare the theoretical finite element accelerations with the experimentally observed accelerations of the prototype structure. The theoretical box model of section 6.1 was used again with a critical damping ratio  $\xi_1$  of 0.02. Then the theory of section 3.2.2 was applied. The Mk.I simulated bang, shown in Fig. 6.4, usually resembles an N-shaped wave more closely than the booms formed by the sonic boom simulator. Thus it was idealised as an N-wave whose properties are given in Figs. 7.1 and 7.2.

The accelerations of the side walls show reasonable overall agreement. The theoretical accelerations, however, have a marked high frequency content, due to the wave's convection, which is hardly noticeable on the experimental accelerations. Using equation (3.29), the theoretical damping factor for this mode is  $5/(2 \times 2\pi \times 190) = 0.002$  compared with a damping factor of about 0.02 observed for the prototype structure. In

general, the damping factor of the prototype structure decreased with increasing frequency. It was found to be in the range of 0.02 to 0.06 for frequencies below 100 c.p.s. These values are higher than those assumed for the mathematical model, particularly at high frequencies. Thus, the high frequency content of the mathematical model's accelerations is much more prevalent in the free vibration period than that of the prototype structure. Differences in the high frequency content of the forced responses are due to differences in the theoretical and experimental mode shapes.

This high frequency content is not so noticeable on the ceiling's theoretical accelerations as the ceiling's motion in this mode is small in relation to the side walls' motions. Although the ceiling frequencies of the theoretical and experimental accelerations do not correspond, the lowest frequency mode with a predominant roof motion is excited in both cases. This difference is mainly due to the poor representation of the beam-plaster board combination by an 'equivalent' isotropic plate. The magnitude of the experimental acceleration is less than the theoretical acceleration. This is because of (i) the higher frequency of the theoretical acceleration and (ii) the simulated boom not forcing directly the prototype ceiling.

The response of the structure is mainly stiffness controlled as the excitation frequency is well below the fundamental structural frequency. The ratio of acoustic to mechanical stiffness of the prototype structure is less than 0.3. Most of the internal pressure is due to the window's motion as the window is much more flexible than any of the other structural sub-elements. In this particular case, the window is independent of the side walls. The internal pressure affects the window's response but does not affect significantly the much stiffer side walls. This, in fact, accounts for the reasonable agreement between these walls' theoretical and experimental accelerations. The internal pressure provides an additional stiffness to the structural components which will increase slightly their natural frequencies. However, the ceiling's theoretical response is at a much higher frequency than the experimental response. This difference is due to the ceiling's poor mathematical idealisation, and not to the effect of the internal pressure.

If the structure is linear, its displacements are given approximately by the external force pattern divided by the structure's stiffness. Fig. 7.3 illustrates this point as the theoretical displacements follow the N-waves during the forced vibration period. Consequently, decreasing the N-wave's peak negative pressure  $p_-$  from 2 to 1.308 lb/ft<sup>2</sup> gradually decreases the magnitude of the displacement. Superimposed on this motion is a frequency corresponding to the lowest frequency mode which has a predominant side wall motion. This happens to be the fundamental structural mode. Decreasing the idealised structure's height not only increases the natural frequencies but changes the relative motion of the individual walls, for a given box mode. Thus, the smaller (11 ft) structure's displacement magnitude is less than that of the 13.625 ft idealised structure due to an increase in the effective stiffness and a decrease in the relative side wall motion of the fundamental box mode.

#### 7.4 Discussion on the possibility of damage

It has been observed (43) that the peak velocity gives the best indication of possible structural damage from blast waves. A recommended safe limit of 2 in/sec has been found to include 94% of the known damage data. Only accelerations were determined in the present investigation. For an accurate assessment of the velocity response, the accelerations should be integrated using either an analogue or digital computer (44). A simple damage criterion is useful however as a quick 'rule of thumb' guide. It is only applicable to those cases where the sonic boom's peak pressures are so great that instantaneous damage occurs as the aircraft flies overhead.

Assuming a sinusoidal response, the corresponding safe acceleration limit is given by

$$\left(\frac{\ddot{w}}{f}\right)_P = \frac{2\pi}{386.4} \dot{w}_P = 3.25 \times 10^{-2} \text{ g/c.p.s.} \quad (7.19)$$

for a prototype structure. This gives an 'energy ratio',  $(\ddot{w}/f)_P^2$ , of 1.6 compared with a recommended safe value of unity (41).

Even though the ceiling of the prototype structure is not isotropic, the maximum normalised accelerations of the prototype and model show

reasonable agreement on all faces for low excitation frequencies. Hence the model results can be used to indicate the possibility of damage to the prototype structure for low excitation frequencies. For the non-dimensioned velocity,  $\partial \zeta / \partial \tau$ , to be the same for model and prototype

$$\frac{\dot{w}_M}{\dot{w}_P} = \frac{l_M}{l_P} \frac{(f_{11})_M}{(f_{11})_P} = \left( \frac{\ddot{w}}{f_{11}} \right)_M / \left( \frac{\ddot{w}}{f_{11}} \right)_P = 0.83 \quad (7.20)$$

where  $\dot{w} = \frac{\partial w}{\partial t}$  ,  $\ddot{w} = \frac{\partial^2 w}{\partial t^2}$  .

Using this relationship in (7.19) gives

$$\begin{aligned} \left( \frac{\ddot{w}}{\dot{w}} \right)_M &= \frac{2\pi}{386.4} \frac{(f_{11})_M}{(f_{11})_P} f_P \\ &= \frac{2\pi}{386.4} f_M . \end{aligned} \quad (7.21)$$

For damage

$$\dot{w}_M = 0.83 \quad \dot{w}_P = 1.66, \quad (7.22)$$

hence

$$\ddot{w}_M = \frac{2\pi}{386.4} \times 1.66 f_M = 2.7 \times 10^{-2} f_M . \quad (7.23)$$

Difficulty is experienced in defining  $f_M$  because the response is not sinusoidal and it is affected by more than one mode. This is particularly true for the peak accelerations which are much more affected by the structure's higher natural frequencies than the peak velocities. A compromise criterion, giving a conservative estimate, is obtained by using the overall peak acceleration and the lowest pre-dominant acceleration frequency. This frequency will tend to dominate the velocity response. For low excitation frequencies,  $f_M$  will correspond to the fundamental structural frequency. Using this approximation the requirement of a detailed knowledge of the structure's modal behaviour is unnecessary.

For low excitation frequencies,  $f_M = 243.5$  c.p.s. so that equation (7.23) becomes

$$\ddot{w}_M = 2.7 \times 10^{-2} \times 243.5 = 6.57 \text{ g.} \quad (7.24)$$

Fig. 4.10 shows that the maximum normalised acceleration of those model's side faces which are at  $90^\circ$  incidence to the wave's shock front, is  $0.6 \text{ g/lb/ft}^2$  in this frequency region. This maximum value occurs when the excitation frequency  $f_N$  coincides with the model's fundamental structural frequency (which is denoted by  $M_I$  in Fig. 4.10). Thus, using equation (7.24), there is a possibility of structural damage to this face if a sonic boom has a peak positive pressure  $p_0$  which is greater than  $\frac{6.57}{2 \times 0.6} \approx 5.5 \text{ lb/ft}^2$ . It has been assumed that the structure's response varies linearly with increasing peak pressure of a sonic boom and that the boom has equal peak positive and negative pressures. The safe limit value obtained from the model results agrees with the value obtained for the prototype structure using equation (7.19) and a fundamental wall frequency of 16 c.p.s. Fig. 4.10 shows however that the greatest acceleration of a face of the model (and hence the greatest possibility of damage) occurs when the shock front is at zero incidence to that face. The maximum normalised acceleration is then  $1.7 \text{ g/lb/ft}^2$ . It occurs when the excitation frequency coincides with the model's fundamental structural frequency. Using the same assumptions as before, the safe limit of damage in this case will be reached by a peak positive pressure of  $6.57 / 2 \times 1.7 = 1.9 \text{ lb/ft}^2$ . It should be noted that these values are evaluated for a simulated boom whose shock front is at  $90^\circ$  inclination to the horizontal. This angle is compatible with an aircraft speed of Mach 1.

The natural frequencies of the walls of American and British houses are about the same (7 to 25 c.p.s.), even though the materials and construction techniques differ (45). The likely excitation frequency of a future SST is about 3 c.p.s. so that most walls will be excited in the low frequency region. For a fundamental wall frequency of 16 c.p.s., the corresponding value of  $\lambda/L$  in Fig. 4.10 will be greater than  $(\frac{16}{3} \times 3 = 16)$  depending upon the angle of incidence. This reduces the previously calculated maximum normalised accelerations by a factor of three or more and consequently increases the safe limit peak pressures to  $5.7(3 \times 1.9)$  and  $16.5 \text{ lb/ft}^2$  ( $3 \times 5.5$ ).

Edwards and Northwood (44) indicate that damage is more likely to occur at a peak velocity of 4 to 5 in. per sec. than the safe limit of 2 in. per sec. This increased velocity level corresponds to a peak positive pressure of between 11.4 (2 x 5.7) and 33 lb/ft (2 x 16.5). At this upper velocity level damage is due to 'opening of old cracks and formation of new plaster cracks, dislodging of loose objects'. These theoretical results are in general agreement with the damage reported from actual booms even though they had different angles of inclination from the simulated booms. Reference (41) states that :

'Other than the 16 day old plaster ceiling, which fell on a 12.1 lb/ft<sup>2</sup> boom, a piece of ceramic tile which fell on 14 lb/ft<sup>2</sup> booms, and glass breakage only very short hairlike extensions and some spalling were observed in old drywall, stucco, and plaster under booms of up to 38 lb/ft<sup>2</sup> in strength'.

## CHAPTER 8

### REVIEW AND SUMMARY OF CONCLUSIONS

This chapter briefly reviews the important conclusions presented more comprehensively in the previous chapters.

The Finite Element Displacement Method has been applied to the problem of determining the dynamical characteristics of a simple box-type structure. The free vibration characteristics were evaluated using rectangular finite elements and the results were compared with an exact solution. Excellent agreement was found for the natural frequencies and mode shapes when only a small number of elements were used in the box's idealisation. If the box's bending moments and shear forces are required, however, many more elements are needed for the same degree of accuracy.

This method was extended to determine the time history of the mathematical model's response to a travelling wave. All internal acoustic coupling was neglected. The theoretical results were compared with the experimental response of a physically scaled-down model to a simulated sonic boom. The comparison indicates that reasonable theoretical estimates of the experimental model's response to a travelling wave can usually be obtained with a minimum description of the wave and model characteristics. Discrepancies occur, however, when a travelling wave has a small pressure component at a frequency corresponding to the natural frequency of a significant mode. Under these circumstances, the experimental model's response is extremely sensitive to small changes in the internal pressure, travelling wave or model characteristics.

The main difference between the theoretical and experimental models' responses was due to an inadequate theoretical description of the wave's flow over the model. In particular, the theoretical description of the flow over the model's back face was found to be too much simplified. Consequently, the greatest discrepancy between the experimental and theoretical results occurred on this face. It should be emphasised that the mathematical model

is sufficiently flexible to include more precise details of the wave's flow when they are known.

The evaluation of the full scale structure's response in terms of its normal modes is very difficult because its natural frequencies are closely spaced. The frequencies also have a variation of about 10% which seems to depend upon the water content of the brickwork. These factors present difficulties to any method which tries to simulate the response of the full scale structure in any detail.

It is convenient to try to simplify the theoretical analysis of a structure's transient response by using an independent plate rather than a box theory. The best approach depends critically upon the degree of coupling between individual faces and the effect of the higher modes. For the particular full scale structure considered, structural coupling occurred mainly between the opposite side walls. A plate theory should give reasonable approximate results for the shorter but not for the longer duration (200 msec) boom. Further investigations are needed to clarify these differences.

Due to the complexity of a normal mode approach, it is advantageous to use gross parameters which rely to a minimum on the structure's modal behaviour. Even though there are detailed differences in the responses of the experimental model and full scale structure, reasonable agreement is shown between their overall normalised peak accelerations for low excitation frequencies. Evidence from blasting operations indicates that there is good correlation between the damage experienced by a structure and its overall peak velocity. Defining a corresponding safe damage criterion in terms of an overall peak acceleration requires some knowledge of the structure's modes. A conservative peak acceleration criterion is suggested which minimises the required knowledge. The predominant structural frequency is taken to be the fundamental structural frequency at low excitation frequencies. This criterion gives results for the walls which are consistent with known damage from actual sonic booms. It is not applicable to glass windows. Large discrepancies in the magnitude of the window's response to nominally identical booms were observed on different days. Further investigations are needed to evaluate and predict the response of glass to sonic booms. In particular, it is necessary to investigate the effect of non-linearities on the behaviour of glass.

It is intended now to make some observations about the extreme limits of a box-type structure's response to a sonic boom and then to review the important results which are only applicable for certain excitation frequencies. A detailed investigation of the variation in the overall peak acceleration shows that it tends to decrease with increasing  $\lambda/L$ . This was found to be the case for both the experimental model (open and closed boxes) and the full scale structure. The experimental peak accelerations of the closed model can be seven times greater than its peak accelerations due to a unit, instantaneous step pressure acting on all faces. The greatest acceleration on any face always occurs when the travelling wave is acting normally to that face. Introducing a large opening into the model's front face can increase the peak accelerations of the back and side faces by a factor of two. This amplification is due to the increased internal pressure.

The peak acceleration tends to be independent of the sonic boom's excitation frequency, convection and angle of incidence at low excitation frequencies. The rise time  $T_R$  is significant. Its effect is mainly restricted to the structure's front face. Decreasing the rise time increases the peak acceleration of the front face.

The most important parameter at intermediate excitation frequencies is the degree of matching between the structure's natural frequencies and the sonic boom's excitation frequency. There may be an upsurge in the structure's peak accelerations when the excitation frequency is similar to any one of the structure's natural frequencies. Not all the faces are equally affected, however, because a given box mode does not equally affect the various faces of the structure. This parameter was also found to be important for the full scale structure at very low excitation frequencies. In this case, the sonic boom has a greater time to impart the wave's energy to the structure. Hence its subsidiary excitation frequencies have a significant effect on the structure's accelerations. The effect on the structure's displacements or velocities still remains relatively unimportant.

The wave's convection is most significant at high excitation frequencies. Changes in the shock front's angle of incidence also affect significantly the structure's response. Higher structural modes are

excited in the case of the model. The accelerations are sensitive to these higher modes whereas the displacements and velocities are mainly affected by the lowest modes.

At low and intermediate excitation frequencies, the internal pressure depends on the closed structure's mean displacement. In the case of the full scale structure, the mean displacement is dictated by the displacement of the most flexible subelement - the window. Introducing an opening into one of the side faces increases substantially the magnitude of the internal pressure. The open structure acts as a Helmholtz resonator at all excitation frequencies. The Helmholtz frequency can be accurately determined using standard theory and assuming all the faces to be rigid. This is true for even the highest excitation frequencies and the largest openings where the accuracy of the standard theory usually deteriorates because the cavity's physical dimensions are not small compared with the wavelength of the simulated boom.

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[P] is a (16 x 20) matrix which is defined by a set of (4 x 5) submatrices as

$$[P] = \begin{bmatrix} \underline{X} & | & \underline{P_{12}} & | & \underline{P_{13}} & | & \underline{P_{14}} \\ \underline{P_{12}} & | & \underline{X} & | & \underline{P_{23}} & | & \underline{P_{24}} \\ \underline{P_{13}} & | & \underline{P_{32}} & | & \underline{X} & | & \underline{P_{34}} \\ \underline{P_{14}} & | & \underline{P_{24}} & | & \underline{P_{34}} & | & \underline{X} \end{bmatrix}.$$

The submatrices are then defined by the following relationships:

$$P_{12} = \begin{bmatrix} 0 & -\frac{2}{a} & 0 & 0 & 0 \\ 0 & \frac{6}{a^2} & 0 & 0 & 0 \\ \frac{6}{a^2} & 0 & -\frac{2}{a} & -\frac{2v}{a} & 0 \\ -\frac{12}{a^3} & 0 & \frac{6}{a^2} & \frac{6v}{a^2} & 0 \end{bmatrix} \quad P_{13} = \begin{bmatrix} -\frac{2}{b} & 0 & 0 & 0 & 0 \\ 0 & \frac{6}{b^2} & -\frac{2v}{b} & -\frac{2}{b} & 0 \\ \frac{6}{b^2} & 0 & 0 & 0 & 0 \\ 0 & -\frac{12}{b^3} & \frac{6v}{b^2} & \frac{6}{b^2} & 0 \end{bmatrix}$$

$$P_{14} = \begin{bmatrix} \frac{4}{b} & \frac{4}{a} & 0 & 0 & (v-1) \\ 0 & (-\frac{6}{a^2} + \frac{6}{b^2}) & \frac{4v}{b} & \frac{4}{b} & 0 \\ (\frac{6}{a^2} - \frac{6}{b^2}) & 0 & \frac{4}{a} & \frac{4v}{a} & 0 \\ -\frac{12}{a^3} & -\frac{12}{b^3} & (-\frac{6}{a^2} - \frac{6v}{b^2}) & (-\frac{6}{b^2} - \frac{6v}{a^2}) & 0 \end{bmatrix}$$

cont/d...

TABLE 2.2

[P] matrix for the determination of a rectangular finite element's force-like quantities.

(Table 2.2 continued)

$$P_{23} = \begin{bmatrix} -\frac{4}{b} & \frac{4}{a} & 0 & 0 & (v-1) \\ 0 & (-\frac{6}{a^2} + \frac{6}{b^2}) & -\frac{4v}{b} & -\frac{4}{b} & 0 \\ (\frac{6}{a^2} - \frac{6}{b^2}) & 0 & \frac{4}{a} & \frac{4v}{a} & 0 \\ -\frac{12}{a^3} & \frac{12}{b^3} & (-\frac{6}{a^2} - \frac{6v}{b^2}) & (-\frac{6}{b^2} - \frac{6v}{a^2}) & 0 \end{bmatrix}$$

$$P_{24} = \begin{bmatrix} \frac{2}{b} & 0 & 0 & 0 & 0 \\ 0 & \frac{6}{b^2} & \frac{2v}{b} & \frac{2}{b} & 0 \\ \frac{6}{b^2} & 0 & 0 & 0 & 0 \\ 0 & \frac{12}{b^3} & \frac{6v}{b^2} & \frac{6}{b^2} & 0 \end{bmatrix}$$

$$P_{32} = \begin{bmatrix} \frac{4}{b} & -\frac{4}{a} & 0 & 0 & (v-1) \\ 0 & (-\frac{6}{a^2} + \frac{6}{b^2}) & \frac{4v}{b} & \frac{4}{b} & 0 \\ (\frac{6}{a^2} - \frac{6}{b^2}) & 0 & -\frac{4}{a} & -\frac{4v}{a} & 0 \\ \frac{12}{a^3} & -\frac{12}{b^3} & (-\frac{6}{a^2} - \frac{6v}{b^2}) & (-\frac{6}{b^2} - \frac{6v}{a^2}) & 0 \end{bmatrix}$$

$$P_{34} = \begin{bmatrix} 0 & \frac{2}{a} & 0 & 0 & 0 \\ 0 & \frac{6}{a^2} & 0 & 0 & 0 \\ \frac{6}{a^2} & 0 & \frac{2}{a} & \frac{2v}{a} & 0 \\ \frac{12}{a^3} & 0 & \frac{6}{a^2} & \frac{6v}{a^2} & 0 \end{bmatrix}$$

cont/d...

(Table 2.2 continued)

$$P_{41} = \begin{bmatrix} -\frac{4}{b} & -\frac{4}{a} & 0 & 0 & (v-1) \\ 0 & (-\frac{6}{a^2} - \frac{6}{b^2}) & -\frac{4v}{b} & -\frac{4}{b} & 0 \\ (\frac{6}{a^2} - \frac{6}{b^2}) & 0 & -\frac{4}{a} & -\frac{4v}{a} & 0 \\ \frac{12}{a^3} & \frac{12}{b^3} & (-\frac{6}{a^2} - \frac{6v}{b^2}) & (-\frac{6}{b^2} - \frac{6v}{a^2}) & 0 \end{bmatrix}$$

X is a (4 x 5) submatrix whose elements are all zero.

| Aspect ratio, $\beta$ | Number of elements | Mode | $Q_x$<br>% error | $Q_y$<br>% error | $M_x$<br>% error | $M_y$<br>% error | $M_{xy}$<br>% error |
|-----------------------|--------------------|------|------------------|------------------|------------------|------------------|---------------------|
| 0.5                   | 16<br>36           | 11   | 6.29             | -1.99            | -4.83            | -4.80            | -0.02               |
|                       |                    |      | 2.82             | -0.93            | -2.22            | -2.22            | 0.00                |
|                       | 16<br>36           | 13   | -13.46           | 21.77            | -6.78            | -23.89           | -0.29               |
|                       |                    |      | -9.59            | 22.70            | -3.58            | -15.42           | -0.11               |
|                       | 16<br>36           | 31   | 19.29            | -24.18           | -25.03           | -17.77           | 0.86                |
|                       |                    |      | 32.56            | -15.30           | -15.85           | -10.99           | 0.00                |
|                       | 16<br>36           | 33   | 18.52            | -10.42           | -24.59           | -22.31           | -4.10               |
|                       |                    |      | 23.97            | -6.24            | -15.77           | -15.00           | -0.80               |
| 0.99                  | 16<br>36           | 11   | 2.17             | 2.03             | -4.92            | -4.87            | -0.06               |
|                       |                    |      | 0.92             | 0.87             | -2.32            | -2.27            | -0.05               |
|                       | 16<br>36           | 13   | -21.46           | 19.98            | -10.97           | -24.83           | 0.58                |
|                       |                    |      | -13.84           | 29.90            | -6.44            | -15.81           | -0.06               |
|                       | 16<br>36           | 31   | 19.95            | -21.59           | -24.84           | -11.15           | 0.60                |
|                       |                    |      | 30.03            | -13.91           | -15.82           | -6.55            | -0.06               |
|                       | 16<br>36           | 33   | 16.14            | 16.02            | -23.66           | -23.62           | -2.99               |
|                       |                    |      | 9.38             | 8.87             | -15.52           | -15.57           | -0.54               |
| 1.0                   | 16<br>36           | 11   | 2.12             | 2.12             | -4.82            | -4.82            | -0.01               |
|                       |                    |      | 0.90             | 0.90             | -2.22            | -2.22            | 0.00                |
|                       | 16<br>36           | 13   | -77.40           | 51.36            | -412.01          | 39.46            | 0.64                |
|                       |                    |      | 28.55            | 49.13            | -240.97          | 21.81            | -0.01               |
|                       | 16<br>36           | 31   | 42.72            | 92.27            | 30.78            | -558.77          | 0.64                |
|                       |                    |      | 28.62            | 84.84            | 7.09             | -276.11          | -0.01               |
|                       | 16<br>36           | 33   | 16.09            | 16.09            | -23.58           | -23.58           | -2.94               |
|                       |                    |      | 9.15             | 9.15             | -15.52           | -15.52           | -0.49               |

$$\% \text{ error} = \frac{(\text{Exact Value} - \text{Approximate Value})}{\text{Exact Value}} \times 100\%$$

TABLE 2.3

Percentage error in the maximum shearing forces, bending moments and twisting moment of a simply supported plate using a finite element approximation.

| Mode<br>no.      | p | m | n | Series<br>solution | Finite element solutions    |                      |                            |       |       |
|------------------|---|---|---|--------------------|-----------------------------|----------------------|----------------------------|-------|-------|
|                  |   |   |   |                    | Three degrees of<br>freedom |                      | Four degrees of<br>freedom |       |       |
|                  |   |   |   |                    | Total number of elements    |                      |                            |       |       |
|                  |   |   |   |                    | Symmetric<br>96             | Antisymmetric<br>128 | 24                         | 40    | 96    |
| 1                | 0 | 0 | 0 | 179                |                             |                      | 183.2                      | 181.9 | 179.7 |
| Percentage error |   |   |   |                    | -3.0                        |                      | -2.3                       | -1.6  | -0.4  |
| 2                | 0 | e | 0 | 203                |                             |                      | 206.8                      | 205.1 | 203.3 |
|                  |   |   |   |                    | -3.0                        |                      | -1.9                       | -1.0  | -0.1  |
| 3                | e | 0 | 0 | 258                |                             |                      | 262.9                      | 262.4 | 258.1 |
|                  |   |   |   |                    | -4.8                        |                      | -1.9                       | -1.7  | -0.0  |
| 4                | 0 | 0 | 0 | 272                |                             |                      | 277.8                      | 276.3 | 272.9 |
|                  |   |   |   |                    | -2.5                        |                      | -2.1                       | -1.6  | -0.3  |
| 5                | 0 | 0 | e | 283                |                             |                      | 290.5                      | 287.5 | 284.2 |
|                  |   |   |   |                    | -4.7                        |                      | -2.7                       | -1.6  | -0.4  |
| 6                | 0 | 0 | 0 | 333                |                             |                      | 342.8                      | 339.7 | 334.0 |
|                  |   |   |   |                    | -2.7                        |                      | -2.9                       | -2.0  | -0.3  |
| 7                | 0 | e | e | 384                |                             |                      | 431.9                      | 428.2 | 386.0 |
|                  |   |   |   |                    | -3.1                        |                      | -12.5                      | -11.5 | -0.5  |
| 8                | 0 | 0 | e | 397                |                             |                      | 460.8                      | 460.1 | 399.5 |
|                  |   |   |   |                    | -12.4                       |                      | -16.1                      | -15.9 | -0.6  |
| 9                | e | 0 | e | 437                |                             |                      | 480.3                      | 476.2 | 439.0 |
|                  |   |   |   |                    | -4.4                        |                      | -9.9                       | -9.0  | -0.5  |
| 10               | e | e | 0 | 455                |                             |                      | 509.4                      | 462.4 | 457.0 |
|                  |   |   |   |                    | -2.1                        |                      | -12.0                      | -1.6  | -0.4  |
| 11               | 0 | 0 | e | 486                |                             |                      | 551.4                      | 546.1 | 489.2 |
|                  |   |   |   |                    | -2.9                        |                      | -13.5                      | -12.4 | -0.7  |
| 12               | e | 0 | 0 | 499                |                             |                      | 581.0                      | 503.7 | 501.8 |
|                  |   |   |   |                    | -10.7                       |                      | -16.4                      | -0.9  | -0.6  |
| 13               | e | 0 | 0 | 570                |                             |                      | 642.4                      | 584.8 | 573.9 |
|                  |   |   |   |                    | -5.2                        |                      | -12.7                      | -2.6  | -0.7  |
| 14               | e | 0 | e | 577                |                             |                      | 693.8                      | 611.0 | 580.8 |
|                  |   |   |   |                    | -4.3                        |                      | -20.2                      | -5.9  | -0.7  |
| 15               | 0 | e | 0 | 624                |                             |                      | 711.1                      | 705.7 | 630.5 |
|                  |   |   |   |                    | -8.3                        |                      | -14.0                      | -13.1 | -1.0  |
| 16               | e | e | e | 648                |                             |                      | 758.8                      | 699.3 | 653.2 |
|                  |   |   |   |                    | -9.2                        |                      | -17.1                      | -7.9  | -0.8  |

The 0 and e indicate whether the integers p, m and n are odd or even for the particular mode using the notation of (17).

TABLE 2.4

Natural frequencies of Dickinson and Warburton's. (17) closed box(c.p.s.).

| Mode<br>no.      | m | n | Series<br>solution | Finite element solutions    |                     |                            |       |
|------------------|---|---|--------------------|-----------------------------|---------------------|----------------------------|-------|
|                  |   |   |                    | Three degrees of<br>freedom |                     | Four degrees of<br>freedom |       |
|                  |   |   |                    | Total number of elements    |                     |                            |       |
|                  |   |   |                    | Symmetric<br>48             | Antisymmetric<br>64 | 20                         | 48    |
| 1                | 0 | 0 | 94                 |                             |                     | 95.7                       | 95.2  |
| Percentage error |   |   |                    | -3.2                        |                     | -1.8                       | -1.3  |
| 2                | e | 0 | 113                |                             |                     | 115.4                      | 114.6 |
|                  |   |   |                    | -2.4                        |                     | -2.1                       | -1.4  |
| 3                | 0 | e | 190                |                             |                     | 195.8                      | 192.1 |
|                  |   |   |                    | -2.3                        |                     | -3.1                       | -1.1  |
| 4                | 0 | 0 | 219                |                             |                     | 225.0                      | 222.3 |
|                  |   |   |                    | -4.7                        |                     | -2.7                       | -1.5  |
| 5                | 0 | 0 | 234                |                             |                     | 240.2                      | 238.7 |
|                  |   |   |                    | -2.5                        |                     | -2.7                       | -2.0  |
| 6                | e | 0 | 246                |                             |                     | 253.5                      | 252.1 |
|                  |   |   |                    | -3.7                        |                     | -3.0                       | -2.5  |
| 7                | 0 | 0 | 287                |                             |                     | 293.9                      | 290.8 |
|                  |   |   |                    | -4.0                        |                     | -2.4                       | -1.3  |
| 8                | e | e | 298                |                             |                     | 350.0                      | 301.8 |
|                  |   |   |                    | -1.8                        |                     | -17.5                      | -1.3  |
| 9                | 0 | e | 341                |                             |                     | 356.3                      | 350.9 |
|                  |   |   |                    | -3.7                        |                     | -4.5                       | -2.9  |
| 10               | 0 | e | 348                |                             |                     | 421.5                      | 355.1 |
|                  |   |   |                    | -3.8                        |                     | -2.1                       | -2.0  |
| 11               | 0 | 0 | 370                |                             |                     | 385.1                      | 381.5 |
|                  |   |   |                    | -4.9                        |                     | -4.1                       | -3.1  |
| 12               | 0 | e | 433                |                             |                     | 496.2                      | 438.4 |
|                  |   |   |                    | -8.7                        |                     | -14.6                      | -1.2  |
| 13               | e | e | 451                |                             |                     | 524.4                      | 463.6 |
|                  |   |   |                    | -4.8                        |                     | -16.3                      | -2.8  |
| 14               | 0 | e | 504                |                             |                     | 587.3                      | 518.6 |
|                  |   |   |                    | -3.6                        |                     | -16.5                      | -2.9  |
| 15               | e | 0 | 522                |                             |                     | 580.9                      | 546.4 |
|                  |   |   |                    | -5.1                        |                     | -11.3                      | -4.7  |
| 16               | e | 0 | 537                |                             |                     | 630.5                      | 569.0 |
|                  |   |   |                    |                             |                     | -17.4                      | -6.0  |

The 0 and e indicate whether the integers m and n are odd or even for the particular mode using the notation of (17).

TABLE 2.5

Natural frequencies of Dickinson and Warburton's (17) open box (c.p.s.)

| Edges<br>fully<br>fixed<br><br>$\times 10^{-2}$ | Coupling<br>between roof<br>and side<br>faces only<br><br>$\times 10^{-2}$ | Roof simply<br>supported,<br>sides fully<br>fixed<br><br>$\times 10^{-2}$ | Roof mode<br>description |   | Complete<br>coupling<br><br>$\times 10^{-2}$ | Mode description<br>Roof mode            |   | Subsidiary<br>motion<br>of face with<br>dimensions<br>(in.): |
|-------------------------------------------------|----------------------------------------------------------------------------|---------------------------------------------------------------------------|--------------------------|---|----------------------------------------------|------------------------------------------|---|--------------------------------------------------------------|
|                                                 |                                                                            |                                                                           | m                        | n |                                              | m                                        | n |                                                              |
| 3.54                                            | 2.83                                                                       | 1.88                                                                      | 1                        | 1 | 1.20                                         | 1                                        | 1 |                                                              |
| 5.66                                            | 4.77                                                                       | 3.73                                                                      | 2                        | 1 | 1.38                                         | 2                                        | 1 | 28 x 11.5                                                    |
| 8.54                                            | 6.91                                                                       | 5.65                                                                      | 1                        | 2 | 1.99                                         | 1                                        | 2 | 40 x 11.5                                                    |
| 9.20                                            | 8.00                                                                       | 6.81                                                                      | 3                        | 1 | 2.56                                         | 2                                        | 2 | 40 x 11.5                                                    |
| 10.51                                           | 8.88                                                                       | 7.50                                                                      | 2                        | 2 | 3.21                                         | 3                                        | 1 |                                                              |
| 13.84                                           | 12.16                                                                      | 10.59                                                                     | 3                        | 2 | 3.50                                         | 3                                        | 2 | { 40 x 11.5<br>28 x 11.5                                     |
| 3.91                                            | 4.30                                                                       | 5.63                                                                      |                          |   | 2.92                                         | Ratio of $\frac{\mu_{\max}}{\mu_{\min}}$ |   |                                                              |

Using the number of half wavelengths for the roof's mode description

TABLE 2.6

Effect of relaxing edge restraints on the frequency parameter,  $\mu = \sqrt{\frac{\rho_{ST} \omega^2}{Dg}}$ ,  
for a 40" x 28" x 11.5" closed box having no base plate and clamped base  
edges.

| Uniform closed box; base and vertical edges fully fixed. | Properties as uniform closed box but roof half as thick. | Central opening in roof. (Open to remaining face area being 1:4). | Central, doorway. Opening in both 40" x 11.5" side faces. (Open to remaining face area being 1:4). |
|----------------------------------------------------------|----------------------------------------------------------|-------------------------------------------------------------------|----------------------------------------------------------------------------------------------------|
| 2.83                                                     | 1.20                                                     | 4.44                                                              | 2.57                                                                                               |
| 8.00                                                     | 3.21                                                     | 8.09                                                              | 7.89                                                                                               |
| 12.30                                                    | 5.54                                                     | 14.75                                                             | 8.73                                                                                               |

Dimensions, edge restraints and material properties are kept the same as the corresponding properties of the uniform, closed box. Only the three lowest modes for which m and n are both odd are considered.

TABLE 2.7

Effect on the frequency parameter,  $\mu \times 10^{-2}$ , of introducing structural discontinuities in a 40" x 28" x 11.5" closed box.

| $\mu_{\max}^h$ | Damping ratio $\xi_1$ | Maximum and Minimum Accelerations (g) |           |       |            |
|----------------|-----------------------|---------------------------------------|-----------|-------|------------|
|                |                       | North face                            | West face | Roof  | South face |
| 0.26           | 0                     | 10.59                                 | 9.01      | 7.02  | 11.17      |
|                |                       | -15.72                                | -7.74     | -6.33 | -11.94     |
| 0.52           | 0                     | 10.87                                 | 8.96      | 7.07  | 10.70      |
|                |                       | -15.52                                | -7.71     | -8.50 | -11.50     |
| 1.04           | 0                     | 9.83                                  | 8.26      | 7.10  | 9.66       |
|                |                       | -15.11                                | -7.50     | -8.14 | -10.31     |
| 2.08           | 0                     | 9.61                                  | 7.91      | 6.87  | 10.05      |
|                |                       | -14.31                                | -8.28     | -7.25 | -14.29     |
| 0.26           | 0.002                 | 10.51                                 | 8.97      | 7.02  | 11.16      |
|                |                       | -15.72                                | -7.67     | -6.26 | -11.91     |
| 0.52           | 0.002                 | 10.72                                 | 8.92      | 7.06  | 10.69      |
|                |                       | -15.52                                | -7.64     | -8.39 | -11.37     |
| 0.26           | 0.02                  | 9.84                                  | 8.62      | 6.95  | 11.09      |
|                |                       | -15.72                                | -7.10     | -6.20 | -11.63     |
| 0.52           | 0.02                  | 13.03                                 | 9.16      | 7.05  | 11.16      |
|                |                       | -15.52                                | -7.96     | -9.15 | -13.71     |

North face normal to N-wave

TABLE 3.1

Effect of step size and damping ratio on the peak accelerations.

| Frequency c.p.s. | $E_c$ lb/in <sup>2</sup> | $\xi_r$ |
|------------------|--------------------------|---------|
| STATIC           | $2.76 \times 10^5$       |         |
| 129.9            | $4.09 \times 10^5$       | 0.028   |
| 435.0            | $3.83 \times 10^5$       | 0.033   |
| 608.7            | $4.12 \times 10^5$       | 0.031   |
| 704.5            | $4.20 \times 10^5$       | 0.043   |

$$\rho_{ST} = 0.0222 \text{ lb/in}^2$$

Thickness of beam,  $d = 0.36 \text{ in.}$

TABLE 4.1

Experimental results for the Young's modulus of Xylonite.

| Mode<br>No. | Theoretical natural<br>frequency c.p.s. | m | n |
|-------------|-----------------------------------------|---|---|
| 1           | 243.5                                   | 0 | 0 |
| 2           | 399.0                                   | 0 | e |
| 3           | 420.7                                   | 0 | 0 |
| 4           | 432.7                                   | e | 0 |
| 5           | 488.4                                   | 0 | 0 |
| 6           | 548.0                                   | e | 0 |
| 7           | 624.7                                   | e | e |
| 8           | 688.2                                   | e | e |
| 9           | 740.2                                   | e | e |
| 10          | 851.3                                   | 0 | e |
| 11          | 914.1                                   | 0 | 0 |
| 12          | 1039                                    | 0 | 0 |
| 13          | 1042                                    | 0 | 0 |
| 14          | 1166                                    | 0 | 0 |
| 15          | 1214                                    | 0 | 0 |
| 16          | 1295                                    | e | e |
| 17          | 1415                                    | e | e |
| 18          | 1470                                    | e | e |
| 19          | 1527                                    | e | e |
| 20          | 1535                                    | e | e |
| 21          | 1648                                    | e | e |
| 22          | 1651                                    | e | e |
| 23          | 1829                                    | 0 | 0 |
| 24          | 1843                                    | 0 | 0 |
| 25          | 1924                                    | 0 | e |
| 26          | 2067                                    | e | e |
| 27          | 2114                                    | 0 | 0 |
| 28          | 2303                                    | e | e |
| 29          | 2354                                    | e | e |
| 30          | 2529                                    | 0 | e |
| 31          | 2633                                    | e | e |
| 32          | 2891                                    | 0 | 0 |

Overall modal density      82.7 c.p.s. per mode

$$E = 4.2 \times 10^5 \text{ lb/in}^2 \quad \rho_{ST} = 0.0222 \text{ lb/in}^2$$

TABLE 4.2

In-vacuo natural frequencies of the closed Xylonite model.

| Mode No. | Theoretical natural frequency c.p.s. | m | Experimental frequency c.p.s. |
|----------|--------------------------------------|---|-------------------------------|
| 1        | 128.4                                | 0 | 132                           |
| 2        | 316.5                                | 0 | 330                           |
| 3        | 329.0                                | e | 290                           |
| 4        | 388.1                                | 0 | 410                           |
| 5        | 448.8                                | 0 | 445                           |
| 6        | 510.3                                | e | 390                           |
| 7        | 552.1                                | e | 470                           |
| 8        | 582.5                                | 0 | 560                           |
| 9        | 654.9                                | e | *                             |
| 10       | 752.5                                | e | 850                           |
| 11       | 856.2                                | 0 | 875                           |
| 12       | 982.8                                | 0 | 950                           |
| 13       | 1012                                 | 0 | *                             |
| 14       | 1087                                 | e | *                             |
| 15       | 1225                                 | 0 | 1240                          |
| 16       | 1261                                 | e | 1300                          |
| 17       | 1285                                 | e |                               |
| 18       | 1392                                 | 0 |                               |
| 19       | 1457                                 | 0 |                               |
| 20       | 1501                                 | e |                               |
| 21       | 1722                                 | e |                               |
| 22       | 1767                                 | e |                               |
| 23       | 1856                                 | 0 |                               |
| 24       | 1903                                 | 0 |                               |
| 25       | 2058                                 | 0 |                               |
| 26       | 2089                                 | e |                               |
| 27       | 2155                                 | e |                               |
| 28       | 2226                                 | 0 |                               |
| 29       | 2302                                 | e |                               |
| 30       | 2629                                 | 0 |                               |
| 31       | 2764                                 | e |                               |
| 32       | 2894                                 | e |                               |

Overall modal density 86.4 c.p.s. per mode

\* Modes not excited due to position of exciters.

$$E = 4.2 \times 10^5 \text{ lb/in}^2$$

$$\rho_{ST} = 0.0222 \text{ lb/in}^2$$

TABLE 4.3

In-vacuo and experimental natural frequencies of the open Xylonite model.

| Driver                                                                                          | Wall | $\theta = 0^\circ$<br>(north face<br>normal) | $22\frac{1}{2}^\circ$ | $45^\circ$       | $67\frac{1}{2}^\circ$ | $90^\circ$<br>(west face<br>normal) |
|-------------------------------------------------------------------------------------------------|------|----------------------------------------------|-----------------------|------------------|-----------------------|-------------------------------------|
| 3 in.<br>high<br>excitation<br>frequencies                                                      | N    | 490                                          | 455                   | 475              | 475                   | <u>475</u> , 1430                   |
|                                                                                                 | W    | 455, <u>820</u>                              | <u>467</u> , 1020     | <u>446</u> , 776 | 416                   | <u>384</u> , 794                    |
|                                                                                                 | S    | 645                                          | 650                   | 665              | 685                   | 475                                 |
|                                                                                                 | R    | 1100                                         | 835                   | 862              | 475, 1300             | <u>475</u> , 835                    |
|                                                                                                 | E    | -                                            | 1000                  | 1000             | <u>416</u> , 1350     | <u>382</u> , 835                    |
| 6 in.<br>high/<br>intermed-<br>iate<br>excitation<br>frequencies                                | N    | 488                                          | 475                   | 483              | 475                   | 483                                 |
|                                                                                                 | W    | <u>445</u> , 820                             | <u>400</u> , 1000     | 400              | 386                   | <u>455</u> , 1075                   |
|                                                                                                 | S    | 654                                          | 645                   | 614              | 475                   | 483                                 |
|                                                                                                 | R    | 455, <u>1100</u>                             | 475, 1450             | 258              | 475                   | <u>400</u> , 1250                   |
|                                                                                                 | E    | -                                            | 400, 1000             | <u>440</u> , 960 | 372                   | <u>380</u> , 1250                   |
| 2 ft.<br>intermed-<br>iate<br>excitation<br>frequencies                                         | N    | 458                                          | 465                   | 475              | 475                   | 475                                 |
|                                                                                                 | W    | 488                                          | <u>453</u> , 910      | 418              | 470                   | 475                                 |
|                                                                                                 | S    | 587                                          | 555                   | 500              | 478                   | 475                                 |
|                                                                                                 | R    | <u>450</u> , 1250                            | <u>240</u> , 665      | <u>260</u> , 500 | 233, 475              | <u>238</u> , 830                    |
|                                                                                                 | E    | -                                            | 757                   | <u>461</u> , 910 | 475                   | 475                                 |
| 7 ft.<br>low<br>excitation<br>frequencies                                                       | N    | 475                                          | 475                   | 455              | 455                   | 475                                 |
|                                                                                                 | W    | ↑                                            | 435                   | 364              | 376                   | 370                                 |
|                                                                                                 | S    | ↓                                            | 490                   | 455              | 500                   | 370                                 |
|                                                                                                 | R    | 225                                          | 240                   | 246              | 238                   | 238                                 |
|                                                                                                 | E    | -                                            | 364                   | 364              | 376                   | 345                                 |
| Fundamental con-<br>vection frequen-<br>cy (n=1) in<br>direction of<br>boom's propa-<br>gation. |      | 550                                          | 453                   | 445              | 511                   | 734                                 |

Note: An underlined value indicates the predominant frequency of the two frequencies shown.

TABLE 4.4: MAIN WALL FREQUENCIES AT A GIVEN ANGLE OF INCIDENCE ( $\theta$ ) OBSERVED AT THE CENTRE OF EACH FACE OF THE CLOSED MODEL.

| Size of opening                                                   | Normally incident face | Driver                                       | Internal acoustic frequencies (c.p.s.) |                |
|-------------------------------------------------------------------|------------------------|----------------------------------------------|----------------------------------------|----------------|
| 3 x 1½ in. Helmholtz resonator frequency [38] equals 123.8 c.p.s. | E                      | High and intermediate excitation frequencies | 3 in. }<br>6 in. }                     | 124.5      862 |
|                                                                   |                        | Intermediate excitation frequency            | 2 ft.                                  | 126.5      870 |
|                                                                   |                        | Low excitation frequency                     | 7 ft.                                  | 126.5          |
|                                                                   | N                      |                                              | 3 in. }                                | 125.0      885 |
|                                                                   |                        |                                              | 6 in. }                                | 140.2      840 |
|                                                                   |                        |                                              | 2 ft. }                                | 125.0          |
|                                                                   |                        |                                              | 7 ft. }                                | 126.5          |
|                                                                   | W                      |                                              | 3 in. }<br>6 in. }                     | 142.7      894 |
|                                                                   |                        |                                              | 2 ft. }<br>7 ft. }                     | 142.9          |
|                                                                   |                        |                                              |                                        |                |
| 6 x 3 in. Helmholtz resonator frequency equals 183.5 c.p.s.       | E                      |                                              | 3 in. }<br>6 in. }                     | 188.9      894 |
|                                                                   |                        |                                              | 2 ft. }                                | 200            |
|                                                                   |                        |                                              | 7 ft. }                                | 192.1          |
|                                                                   | N                      |                                              | 3 in. }<br>6 in. }                     | 188      900   |
|                                                                   |                        |                                              | 2 ft. }<br>7 ft. }                     | 187            |
|                                                                   |                        |                                              |                                        |                |
|                                                                   | W                      |                                              | 3 in. }                                | 186      934   |
|                                                                   |                        |                                              | 6 in. }                                | 195      934   |
|                                                                   |                        |                                              | 2 ft. }                                | 196            |
|                                                                   |                        |                                              | 7 ft. }                                | -              |

TABLE 4.5      Effect of a variable sized opening in the East face

cont/d...

| Size of opening                                                   | Normally incident face | Driver  | Internal acoustic frequencies (c.p.s.) |      |  |
|-------------------------------------------------------------------|------------------------|---------|----------------------------------------|------|--|
| No east face<br>Helmholtz resonator frequency equals 276.9 c.p.s. | E                      | 3 in.   | 279.6                                  | 2500 |  |
|                                                                   |                        | 6 in.   | 279.6                                  |      |  |
|                                                                   |                        | 2 ft. } | 294                                    |      |  |
|                                                                   |                        | 7 ft. } |                                        |      |  |
|                                                                   | N                      | 3 in. } |                                        | 700  |  |
|                                                                   |                        | 6 in. } |                                        |      |  |
|                                                                   |                        | 2 ft.   | 294                                    | 770  |  |
|                                                                   |                        | 7 ft.   | -                                      |      |  |
|                                                                   | W                      | 3 in. } | 286                                    | 3333 |  |
|                                                                   |                        | 6 in. } |                                        |      |  |
|                                                                   |                        | 2 ft. } | 300                                    |      |  |
|                                                                   |                        | 7 ft. } |                                        |      |  |

TABLE 4.5 (continued) Effect of a variable sized opening in the East face.

| Mode No.                                | Theoretical Frequencies (c.p.s.) |    |   |        |     | Experimental frequencies (c.p.s.) |                 |
|-----------------------------------------|----------------------------------|----|---|--------|-----|-----------------------------------|-----------------|
|                                         | Equivalent height (ft.)          | ll | m | 13.625 | m   | Structure                         | Window          |
| 1                                       | 24.8                             |    | o | 19.9   | o   | 16                                | 7.5 }<br>10.3 } |
| 2                                       | 28.4                             |    | e | 20.8   | e   | 17.5<br>20<br>23                  | 13.7<br>21/22   |
| 3                                       | 29.5                             |    | o | 22.2   | o   | 24.8<br>26.5                      | 28.5            |
| 4                                       | 34.7                             |    | o | 30.2   | o   | 27<br>29<br>30<br>31              | 33/34           |
| 5                                       | 47.1                             |    | e | 38.5   | e   |                                   |                 |
| 6                                       | 49.1                             |    | o | 41.6   | o   | 35<br>36                          | 42              |
| 7                                       | 54.5                             |    | e | 47.6   | e   | 37<br>38                          | 48.3            |
| 8                                       | 63.8                             |    | o | 51.0   | e   | 39                                |                 |
| 9                                       | 64.2                             |    | e | 52.6   | o   | 49<br>50                          |                 |
| 10                                      | 74.7                             |    | o | 54.7   | o   | 55<br>56                          | 54              |
| 11                                      | 79.5                             |    | o | 63.1   | o   | 59*                               | 62              |
| 12                                      | 82.4                             |    | e | 69.8   | e   | 71                                | 66              |
| 13                                      | 83.0                             |    | o | 74.1   | e   | 73                                | 69              |
| 14                                      | 91.0                             |    | e | 74.5   | o   | 75*                               | 75              |
| 15                                      | 94.2                             |    | o | 75.4   | o   |                                   |                 |
| 16                                      | 95.0                             |    | e | 77.7   | e   | 81*                               |                 |
| 17                                      | 109.7                            |    | e | 87.1   | o   | 87                                | 85              |
| 18                                      | 111.6                            |    | e | 94.2   | e   | 94                                |                 |
| 19                                      | 122.4                            |    | o | 99.1   | e   | 96<br>98                          |                 |
| 20                                      | 126.3                            |    | o | 103.9  | o   | 101                               |                 |
| 21                                      | 130.3                            |    | o | 105.9  | o   | 103*<br>105                       | 102             |
| 22                                      | 132.6                            |    | o | 110.9  | o   |                                   |                 |
| 23                                      | 146.4                            |    | e | 114.3  | o   |                                   |                 |
| 24                                      | 150.3                            |    | e | 115.9  | e   |                                   |                 |
| 25                                      | 156.7                            |    | o | 125.1  | e   |                                   |                 |
| Overall modal density (c.p.s. per mode) |                                  |    |   | 5.3    | 4.2 | 2.4                               |                 |

The o and e indicate odd or even m using the notation of (17).

\*Large correlated box mode detected in resonance tests.

TABLE 6.1 Comparison of the experimental and theoretical natural frequencies of the full scale structure.

| Rear wall       |             |                           | Right hand wall | Left hand wall  |             |                                  |
|-----------------|-------------|---------------------------|-----------------|-----------------|-------------|----------------------------------|
| Resonance tests | Theoretical |                           | Resonance tests | Resonance tests | Theoretical |                                  |
|                 | Box theory  | Plate theory<br>11' x 15' |                 |                 | Box theory  | Plate theory<br>13.625' x 17.25' |
| 16              | 22.2        | 22.3 (1,1)                | 23              | 20              | 19.9        | 15.2 (1,1)                       |
| 37              | 41.6        |                           | 59              | 59              | 54.7        | 50.4 (1,3)                       |
| 49              | 52.6        |                           | 75              | 75              | 74.5        |                                  |
| 59              | 63.1        | 68.2 (1,3)                | 81              | 81              | 75.5        | 81.8 (3,1)                       |
| 75              | 74.5        |                           | 98              | 98              | 103.9       |                                  |
| 81              | 75.5        | 123.8 (3,1)               |                 | 105             | 105.9       | 123.4 (1,5)                      |

(The integers in the brackets define the (m,n) mode of the theoretical plate)

TABLE 6.2 Comparison of the theoretical and experimental natural frequencies (c.p.s.) which predominantly affect the centres of the rear and opposite side walls of the full scale structure.

Absolute normalised maximum response and RMS for angle of incidence  $0^\circ$  on Day 1.

| Physical variable                        | Location                  | $(T_L - T_R) = 194.7$ ms.<br>$T_R = 6.8$ ms. |                    |              | $(T_L - T_R) = 193.9$ ms.<br>$T_R = 7.1$ ms. |                    |              | $(T_L - T_R) = 80.2$ ms.<br>$T_R = 2.0$ ms. |                    |              |
|------------------------------------------|---------------------------|----------------------------------------------|--------------------|--------------|----------------------------------------------|--------------------|--------------|---------------------------------------------|--------------------|--------------|
|                                          |                           | RMS                                          | Abs. max. response | Time (msecs) | RMS                                          | Abs. max. response | Time (msecs) | RMS                                         | Abs. max. response | Time (msecs) |
| Displacement<br>(in/lb/ft <sup>2</sup> ) | Window centre             | 0.108                                        | 0.263              | 203.5*       | 0.091                                        | 0.245              | 10.0         | 0.041                                       | 0.387              | 143.6*       |
|                                          | Window centre             | 0.772                                        | 2.342              | 104.5        | 0.603                                        | 1.809              | 102.9        | 1.381                                       | 3.512              | 172.0*       |
|                                          | Ceiling                   | 0.035                                        | 0.082              | 81.8         | 0.029                                        | 0.070              | 300.4        | 0.034                                       | 0.083              | 152.8*       |
|                                          | Front wall                | 0.005                                        | 0.028              | 43.9         | 0.006                                        | 0.032              | 6.0          | 0.008                                       | 0.032              | 6.0          |
| Acceleration<br>(g/lb/ft <sup>2</sup> )  | Front lintel              | 0.009                                        | 0.041              | 4.8          | 0.009                                        | 0.036              | 0.4          | 0.010                                       | 0.043              | 12.0         |
|                                          | RH wall                   | 0.020                                        | 0.068              | 69.8         | 0.013                                        | 0.045              | 41.9         | 0.027                                       | 0.063              | 80.2         |
|                                          | LH wall                   | 0.014                                        | 0.054              | 39.1         | 0.010                                        | 0.040              | 8.6          | 0.015                                       | 0.051              | 3.6          |
|                                          | Rear wall                 | 0.012                                        | 0.042              | 17.6         | 0.044                                        | 0.105              | 12.0         | 0.010                                       | 0.048              | 20.7         |
| Pressures                                | Inside window microphone  | 0.048                                        | 0.136              | 60.2         | 0.040                                        | 0.116              | 198.7        | 0.077                                       | 0.201              | 54.3         |
|                                          | Outside window microphone | 0.316                                        | 2.243              | 6.4          | 0.379                                        | 2.160              | 7.2          | 0.265                                       | 2.241              | 2.0          |

\*during subsequent free response period. Angle of incidence  $\theta$  measured with respect to window face.  
TABLE 6.3

Normalised with respect to peak positive, free field pressure.

cont/d....

Absolute normalised maximum response and RMS for angle of incidence  $0^\circ$  on Day 2.

| Physical variable                     | Location                  | $(T_L - T_R) = 28.3$ ms.<br>$T_R = 9.7$ ms. |                    |              | $(T_L - T_R) = 55.0$ ms.<br>$T_R = 6.8$ ms. |                    |              | $(T_L - T_R) = 183.1$ ms.<br>$T_R = 7.5$ ms. |                    |              |
|---------------------------------------|---------------------------|---------------------------------------------|--------------------|--------------|---------------------------------------------|--------------------|--------------|----------------------------------------------|--------------------|--------------|
|                                       |                           | RMS                                         | Abs. Max. response | Time (msecs) | RMS                                         | Abs. max. response | Time (msecs) | RMS                                          | Abs. max. response | Time (msecs) |
| Displacement (in/lb/ft <sup>2</sup> ) | Window centre             | 0.029                                       | 0.084              | 183.5*       | 0.016                                       | 0.041              | 88.2*        | 0.001                                        | 0.028              | 189.5*       |
| Acceleration (g/lb/ft <sup>2</sup> )  | Window centre             | 0.967                                       | 2.326              | 41.9*        | 0.195                                       | 0.601              | 38.7         | 0.090                                        | 0.405              | 14.4         |
|                                       | Ceiling                   | 0.045                                       | 0.132              | 55.5*        | 0.018                                       | 0.048              | 201.1*       | 0.022                                        | 0.071              | 8.8          |
|                                       | Front wall                | 0.007                                       | 0.034              | 29.9         | 0.006                                       | 0.023              | 29.5         | 0.008                                        | 0.046              | 17.9         |
|                                       | Front lintel              | 0.033                                       | 0.110              | 57.5*        | 0.006                                       | 0.027              | 17.2         | 0.011                                        | 0.034              | 15.6         |
|                                       | RH wall                   | 0.067                                       | 0.224              | 87.8*        | 0.015                                       | 0.040              | 110.5*       | 0.019                                        | 0.068              | 71.0         |
| Pressures                             | LH wall                   | 0.026                                       | 0.104              | 47.1*        | 0.012                                       | 0.036              | 95.8*        | 0.017                                        | 0.052              | 52.0         |
|                                       | Rear wall                 | 0.026                                       | 0.084              | 49.9*        | 0.006                                       | 0.018              | 179.9*       | 0.011                                        | 0.034              | 34.7         |
|                                       | Inside window microphone  | 0.050                                       | 0.105              | 90.2*        | 0.023                                       | 0.081              | 155.2*       | 0.049                                        | 0.117              | 86.6         |
|                                       | Outside window microphone | 0.402                                       | 2.605              | 13.2         | 0.212                                       | 1.389              | 9.6          | 0.516                                        | 2.145              | 8.0          |

\* during subsequent free response period.

TABLE 6.3 (contd.)

Normalised with respect to peak positive, free field pressure.

Absolute normalised maximum response and RMS for angle of incidence  $22\frac{1}{2}^\circ$  (with respect to window face)

| Physical Variable                     | Location                  | $(T_1 - T_R) = 26.4$ ms.<br>$T_R = 9.8$ ms. |                   |             | $(T_1 - T_R) = 77.8$ ms.<br>$T_R = 2.3$ ms. |                    |             | $(T_1 - T_R) = 187.9$ ms.<br>$T_R = 8.4$ ms. |                    |             | $(T_1 - T_R) = 192.3$ ms.<br>$T_R = 1.5$ ms. |                    |             |
|---------------------------------------|---------------------------|---------------------------------------------|-------------------|-------------|---------------------------------------------|--------------------|-------------|----------------------------------------------|--------------------|-------------|----------------------------------------------|--------------------|-------------|
|                                       |                           | RMS                                         | Abs. max response | Time (msec) | RMS                                         | Abs. max. response | Time (msec) | RMS                                          | Abs. max. response | Time (msec) | RMS                                          | Abs. max. response | Time (msec) |
| Displacement (in/lb/ft <sup>2</sup> ) | Window centre             | 0.037                                       | 0.077             | 180.3*      | 0.021                                       | 0.058              | 81.8        | 0.002                                        | 0.033              | 183.5       | 0.001                                        | 0.036              | 35.5        |
|                                       | Window centre             | 0.722                                       | 2.488             | 39.9*       | 0.265                                       | 0.832              | 32.7        | 0.166                                        | 0.458              | 5.6         | 0.204                                        | 0.672              | 27.7        |
| Acceleration (g/lb/ft <sup>2</sup> )  | Ceiling                   | 0.047                                       | 0.156             | 57.1*       | 0.026                                       | 0.081              | 36.7        | 0.046                                        | 0.092              | 33.1        | 0.019                                        | 0.108              | 36.7        |
|                                       | Front wall                | 0.007                                       | 0.029             | 47.9*       | 0.008                                       | 0.043              | 3.2         | 0.007                                        | 0.042              | 7.6         | 0.008                                        | 0.049              | 2.4         |
|                                       | Front lintel              | 0.032                                       | 0.104             | 30.3*       | 0.007                                       | 0.043              | 16.4        | 0.012                                        | 0.045              | 15.6        | 0.010                                        | 0.039              | 16.7        |
|                                       | RH wall                   | 0.057                                       | 0.202             | 91.4*       | 0.017                                       | 0.064              | 41.5        | 0.017                                        | 0.057              | 48.3        | 0.016                                        | 0.049              | 46.3        |
| Pressures                             | LH wall                   | 0.030                                       | 0.112             | 69.8*       | 0.018                                       | 0.074              | 6.8         | 0.025                                        | 0.102              | 25.5        | 0.021                                        | 0.070              | 25.9        |
|                                       | Rear wall                 | 0.017                                       | 0.076             | 55.1*       | 0.009                                       | 0.042              | 34.7        | 0.013                                        | 0.045              | 21.1        | 0.010                                        | 0.039              | 40.4        |
|                                       | Inside window microphone  | 0.042                                       | 0.115             | 92.2*       | 0.032                                       | 0.104              | 75.8        | 0.057                                        | 0.144              | 109.3       | 0.044                                        | 0.115              | 114.5       |
|                                       | Outside window microphone | 0.375                                       | 2.341             | 11.6        | 0.307                                       | 1.262              | 78.6        | 0.562                                        | 2.288              | 2.4         | 0.434                                        | 2.091              | 2.4         |

\*during subsequent free response period.

TABLE 6.4

Normalised with respect to peak positive, free field pressure.

Absolute normalised maximum response and RMS for angle of incidence  $45^\circ$  (with respect to window face)

| Physical Variable                     | Location                  | $(T_1 - T_R) = 27.6$ ms.<br>$T_R = 9.8$ ms. |                    |              | $(T_1 - T_R) = 73.1$ ms.<br>$T_R = 6.7$ ms. |                    |              | $(T_1 - T_R) = 185.5$ ms.<br>$T_R = 8.2$ ms. |                    |              | $(T_1 - T_R) = 189.5$ ms.<br>$T_R = 8.3$ ms. |                    |              |
|---------------------------------------|---------------------------|---------------------------------------------|--------------------|--------------|---------------------------------------------|--------------------|--------------|----------------------------------------------|--------------------|--------------|----------------------------------------------|--------------------|--------------|
|                                       |                           | RMS                                         | Abs. max. response | Time (msecs) | RMS                                         | Abs. max. response | Time (msecs) | RMS                                          | Abs. max. response | Time (msecs) | RMS                                          | Abs. max. response | Time (msecs) |
| Displacement (in/lb/ft <sup>2</sup> ) | Window centre             | 0.033                                       | 0.067              | 120.5*       | 0.026                                       | 0.073              | 90.6*        | 0.003                                        | 0.031              | 18.6         | 0.002                                        | 0.036              | 33.5         |
|                                       | Window centre             | 0.672                                       | 1.775              | 39.1*        | 0.329                                       | 1.164              | 10.0         | 0.146                                        | 0.657              | 83.0         | 0.187                                        | 0.576              | 41.9         |
| Acceleration (g/lb/ft <sup>2</sup> )  | Ceiling                   | 0.044                                       | 0.197              | 61.8*        | 0.034                                       | 0.143              | 32.7         | 0.039                                        | 0.130              | 14.4         | 0.032                                        | 0.115              | 61.0         |
|                                       | Front wall                | 0.005                                       | 0.026              | 21.9         | 0.008                                       | 0.052              | 5.2          | 0.006                                        | 0.037              | 0.4          | 0.006                                        | 0.040              | 0.79         |
|                                       | Front lintel              | 0.023                                       | 0.094              | 51.9*        | 0.010                                       | 0.043              | 21.5         | 0.010                                        | 0.046              | 17.2         | 0.009                                        | 0.047              | 41.5         |
|                                       | RH wall                   | 0.044                                       | 0.155              | 102.9*       | 0.022                                       | 0.071              | 23.5         | 0.017                                        | 0.063              | 102.9        | 0.019                                        | 0.067              | 86.7         |
|                                       | LH wall                   | 0.030                                       | 0.101              | 47.0*        | 0.027                                       | 0.098              | 7.2          | 0.026                                        | 0.104              | 25.9         | 0.025                                        | 0.099              | 21.5         |
| Pressures                             | Rear wall                 | 0.018                                       | 0.058              | 103.0*       | 0.010                                       | 0.046              | 37.5         | 0.012                                        | 0.035              | 74.7         | 0.015                                        | 0.034              | 87.5         |
|                                       | Inside window microphone  | 0.030                                       | 0.083              | 91.0*        | 0.026                                       | 0.149              | 84.6         | 0.047                                        | 0.127              | 241.4*       | 0.049                                        | 0.126              | 238.6        |
|                                       | Outside window microphone | 0.640                                       | 2.220              | 10.4         | 0.376                                       | 3.294              | 6.0          | 0.478                                        | 2.225              | 5.6          | 0.493                                        | 1.972              | 2.0          |

\*during subsequent free response period.

TABLE 6.5

Normalised with respect to peak positive, free field pressure.

| Zero incidence<br>$\lambda/L$<br>(w.r.t.window face)<br>(for prototype) | FACE       | Normalised<br>Model<br>Acceleration | Normalised<br>Prototype<br>Acceleration                                                  | RATIO                                                                              |
|-------------------------------------------------------------------------|------------|-------------------------------------|------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------|
| 1.8<br>Intermediate<br>excitation<br>frequency                          | SIDE FACES | 1.1                                 | $\begin{cases} 0.112 \\ 0.052 \end{cases}$                                               | $\begin{cases} 9.8 \\ 21.2 \end{cases}$                                            |
|                                                                         | BACK FACE  | 1.3                                 | 0.042                                                                                    | 30.9                                                                               |
| 3.5<br>Low excitation<br>frequency                                      | SIDE FACES | 0.75                                | $\begin{cases} 0.020 \\ 0.018 \end{cases}$                                               | $\begin{cases} 37.5 \\ 41.6 \end{cases}$                                           |
|                                                                         | BACK FACE  | 0.8                                 | 0.009                                                                                    | 88.9                                                                               |
| 12.4<br>Very low<br>excitation<br>frequency                             | SIDE FACES | 0.25                                | $\begin{cases} 0.034 \\ 0.022 \end{cases}$                                               | $\begin{cases} 7.4 \\ 11.7 \end{cases}$                                            |
|                                                                         | BACK FACE  | 0.4                                 | $\begin{cases} 0.027 \\ 0.020 \end{cases}$<br>$\begin{cases} 0.021 \\ 0.053 \end{cases}$ | $\begin{cases} 9.3 \\ 12.5 \end{cases}$<br>$\begin{cases} 19.0 \\ 7.6 \end{cases}$ |

TABLE 7.1

Acceleration comparison of experimental model and prototype structure.

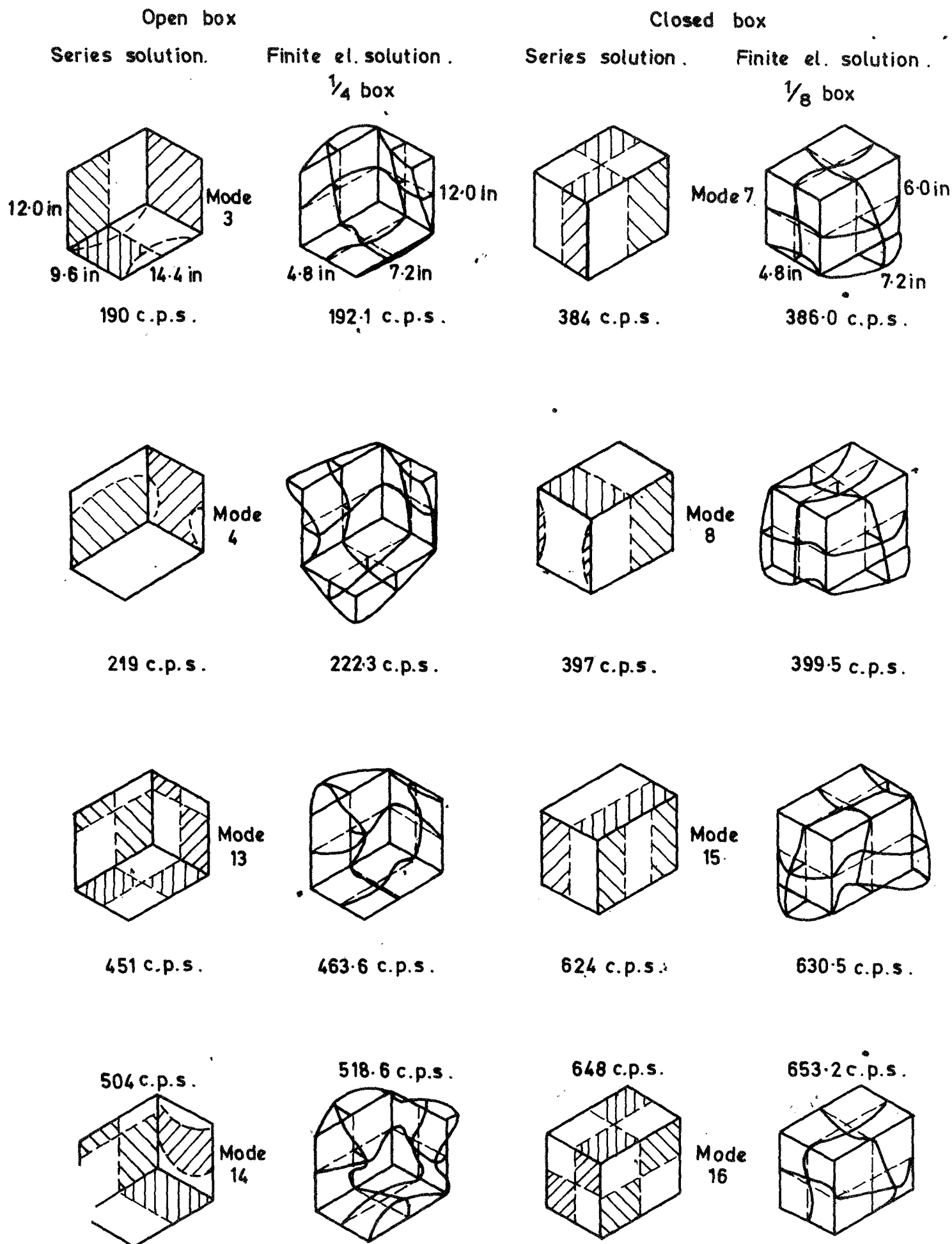
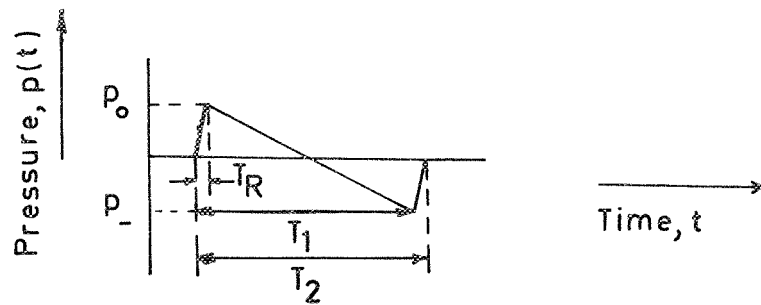
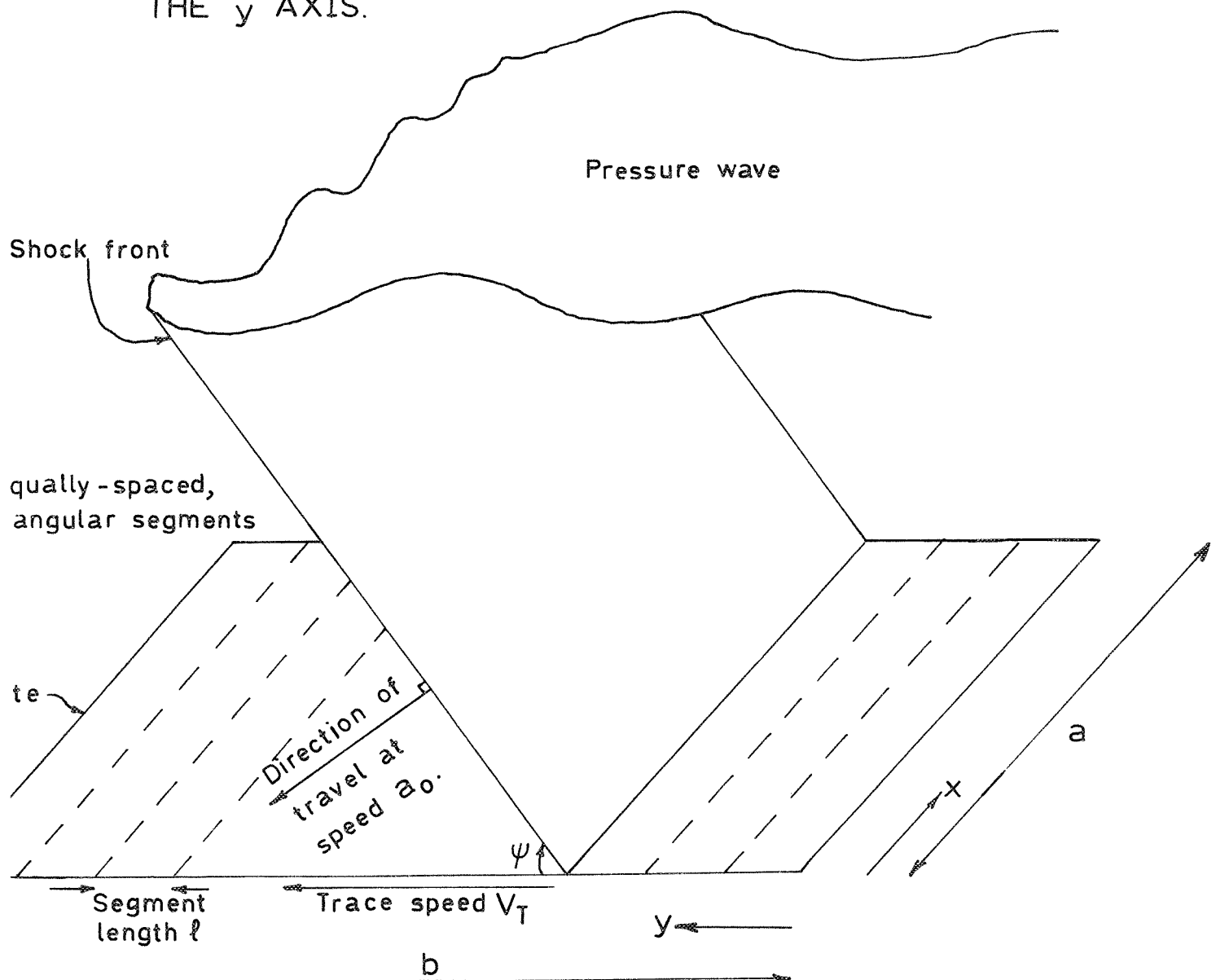


Fig. 2.1 Mode shape comparison of the series and finite element solutions.

(a) N-WAVE WITH RISE TIME  $T_R$ .



(b) PLATE LOADED BY A WAVE TRAVELLING PARALLEL TO THE  $y$  AXIS.



(c) MODIFIED PRESSURE-TIME HISTORY.

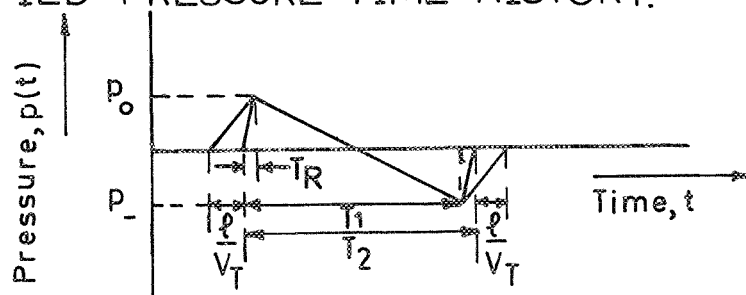


Fig. 3.1

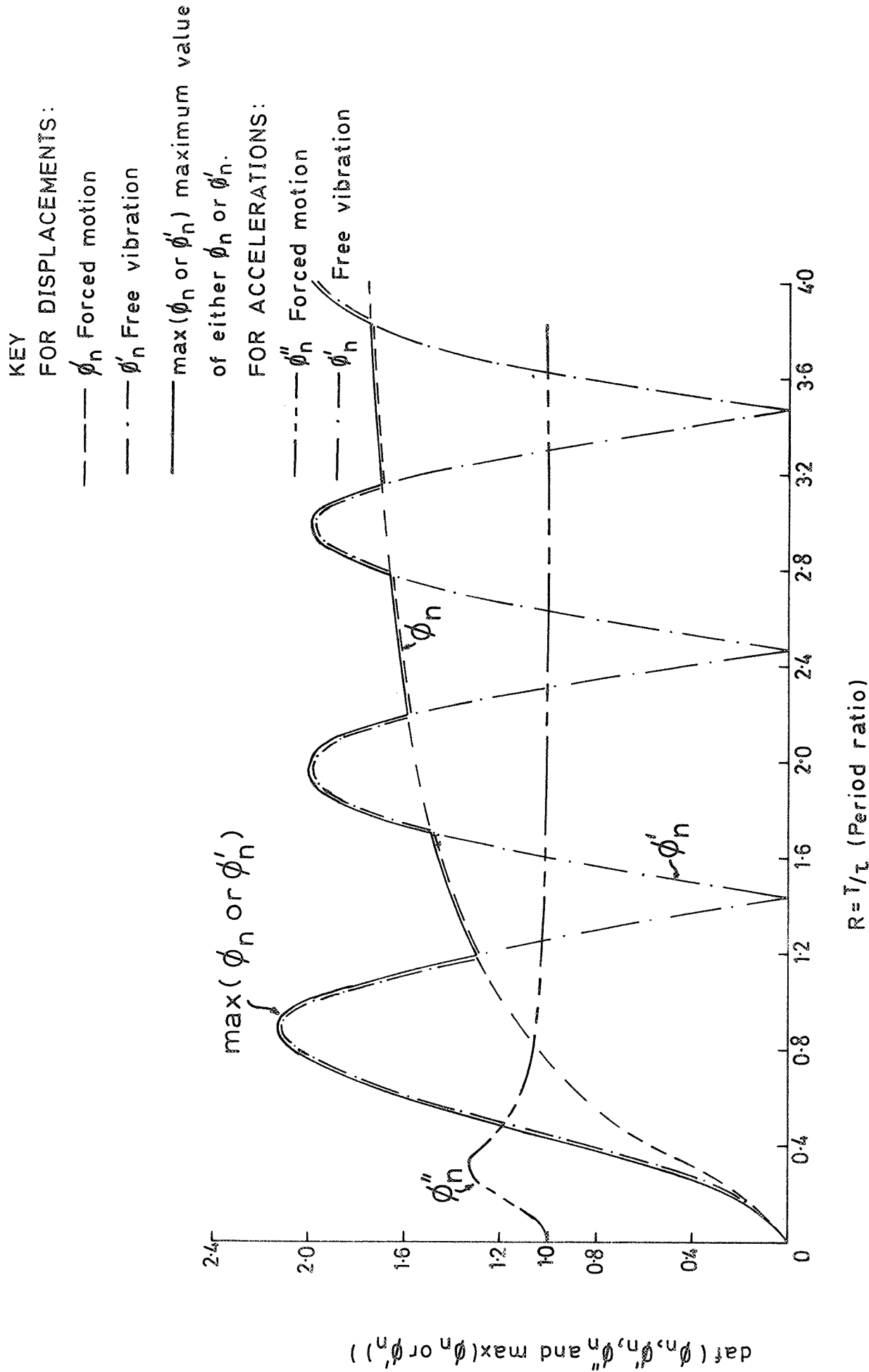
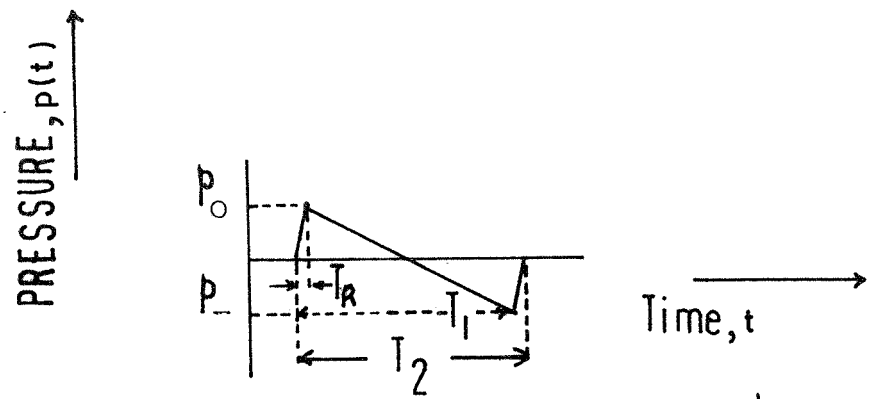
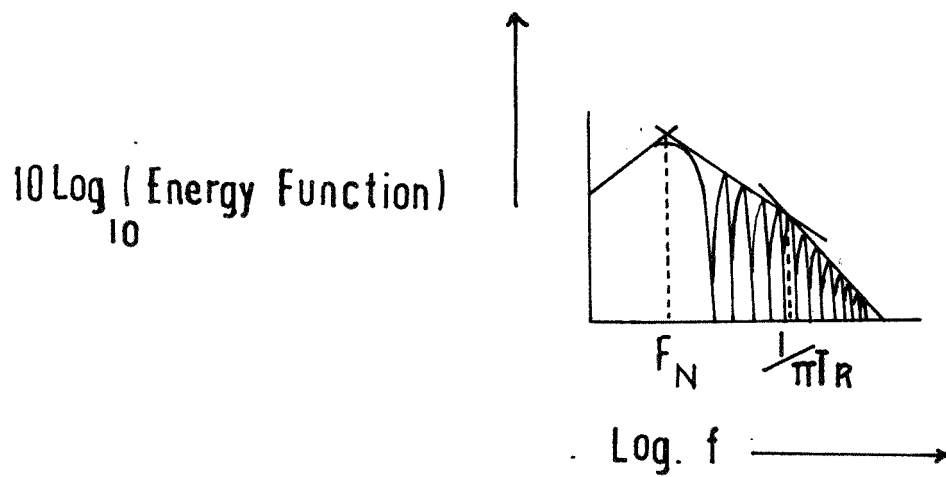


Fig.3.2 Dynamic amplification factor (daf) for a simple,undamped oscillator subjected to a symmetrical N - shaped wave having zero rise and fall times.



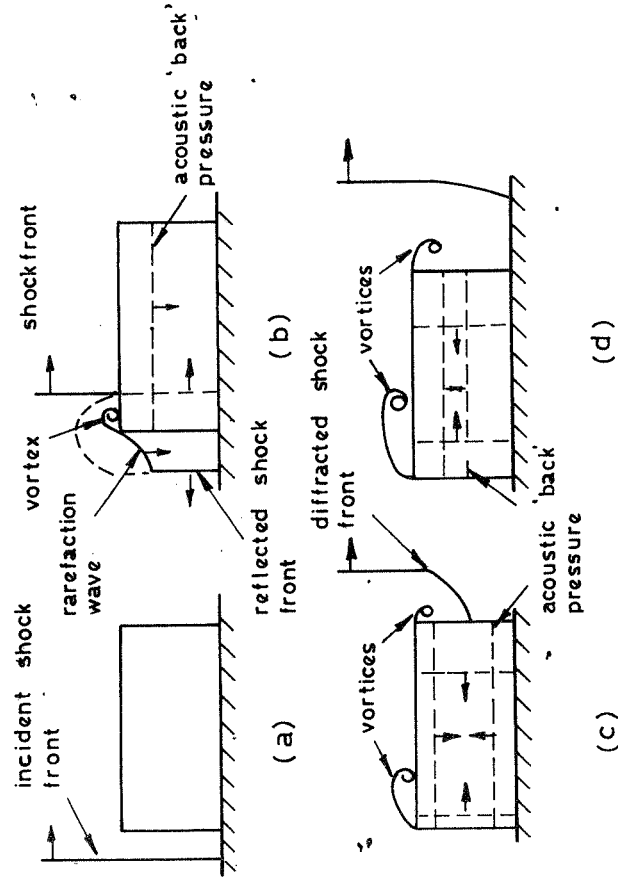
N-WAVE WITH RISE TIME  $T_R$



SPECTRUM OF N-WAVE WITH RISE TIME  $T_R$

Fig 3.3

Side elevation



Plan

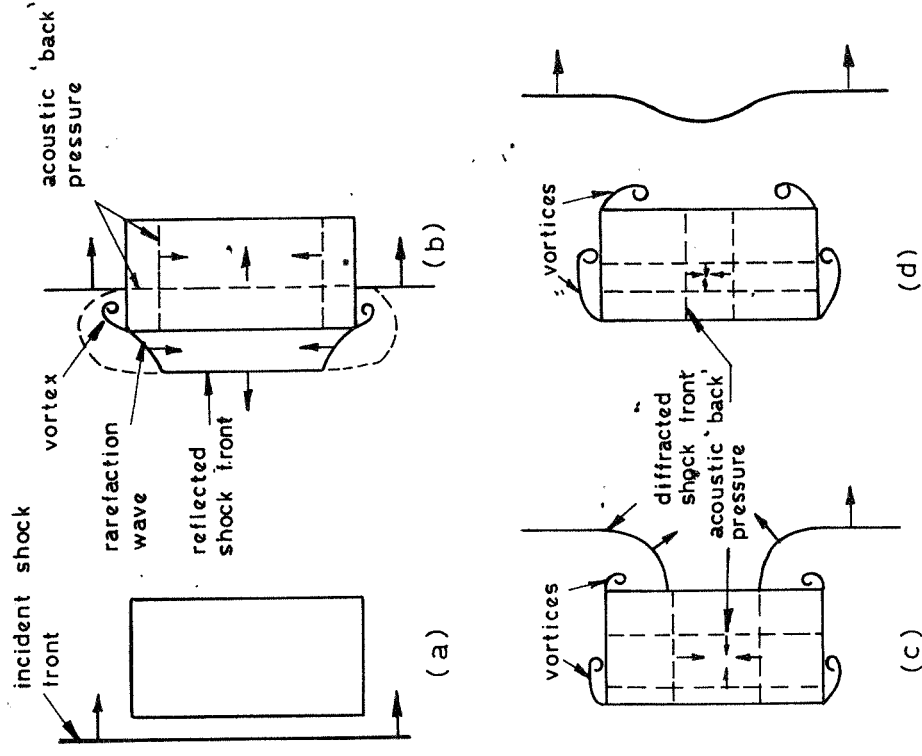


Fig.3.4 Behavior of a blast wave over a closed rectangular structure

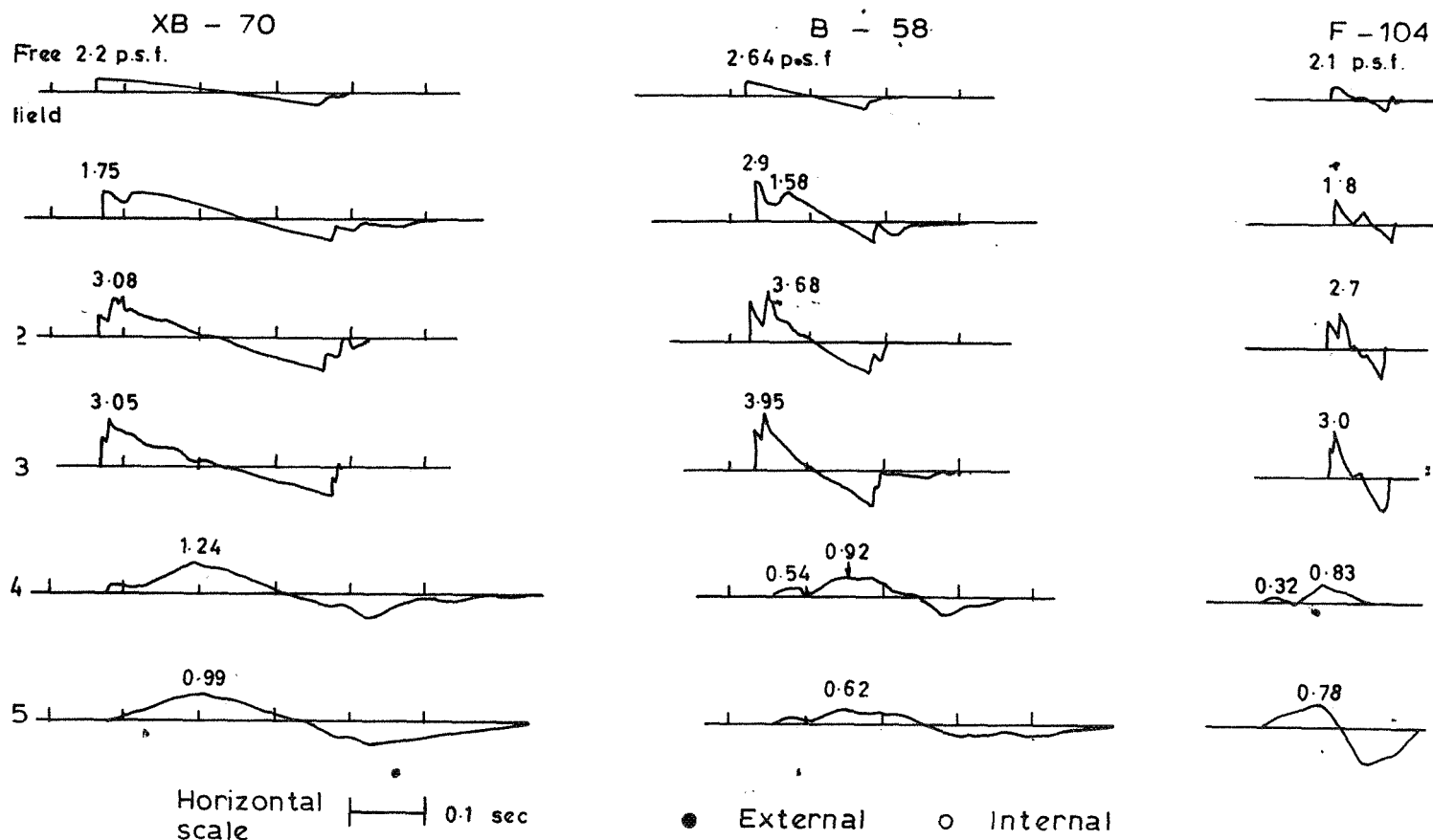
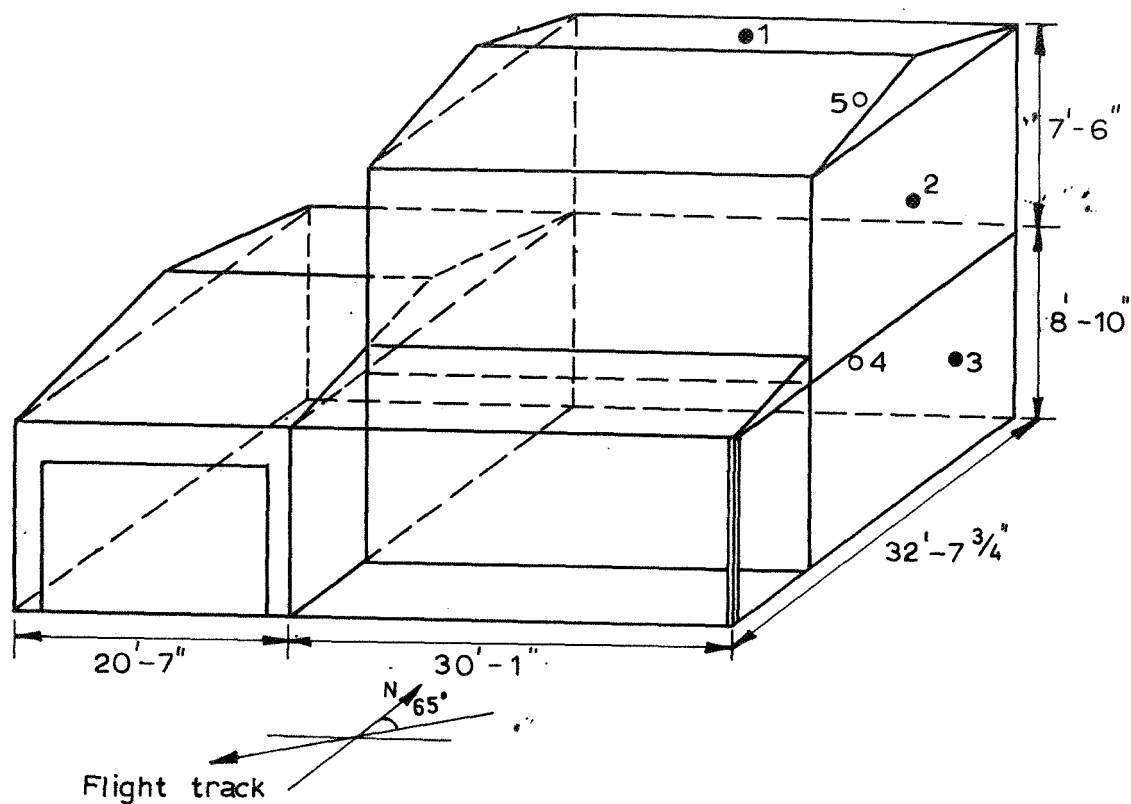
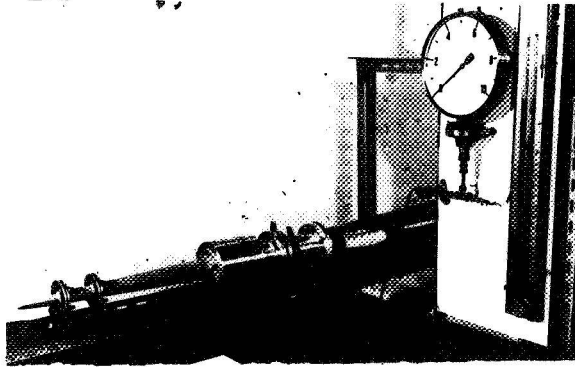
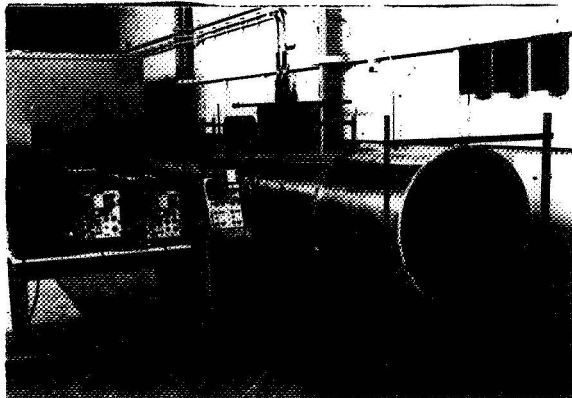


Fig.3.5 Form of the pressure signature over a full sized structure .

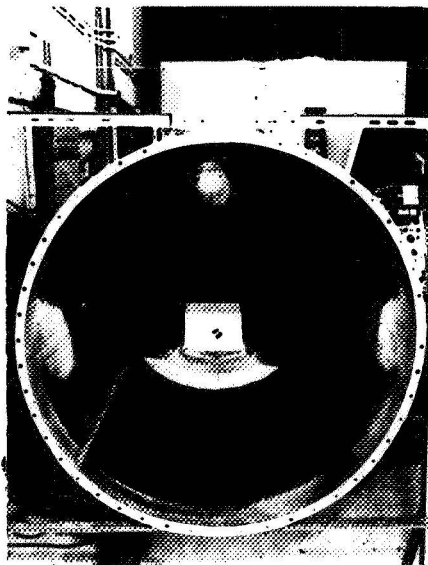
## EXPERIMENTAL SETUP



Driver



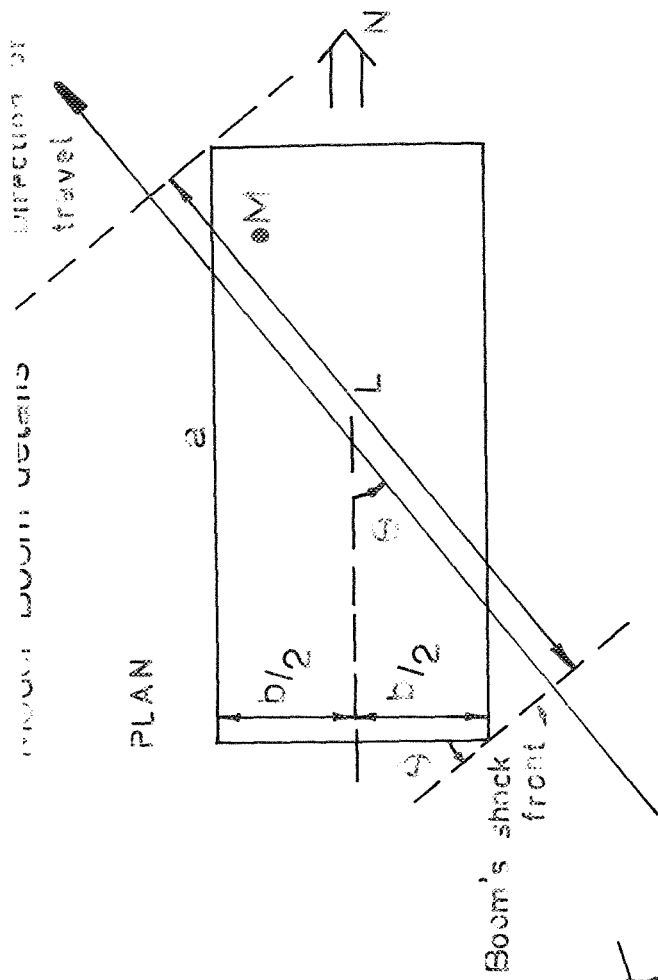
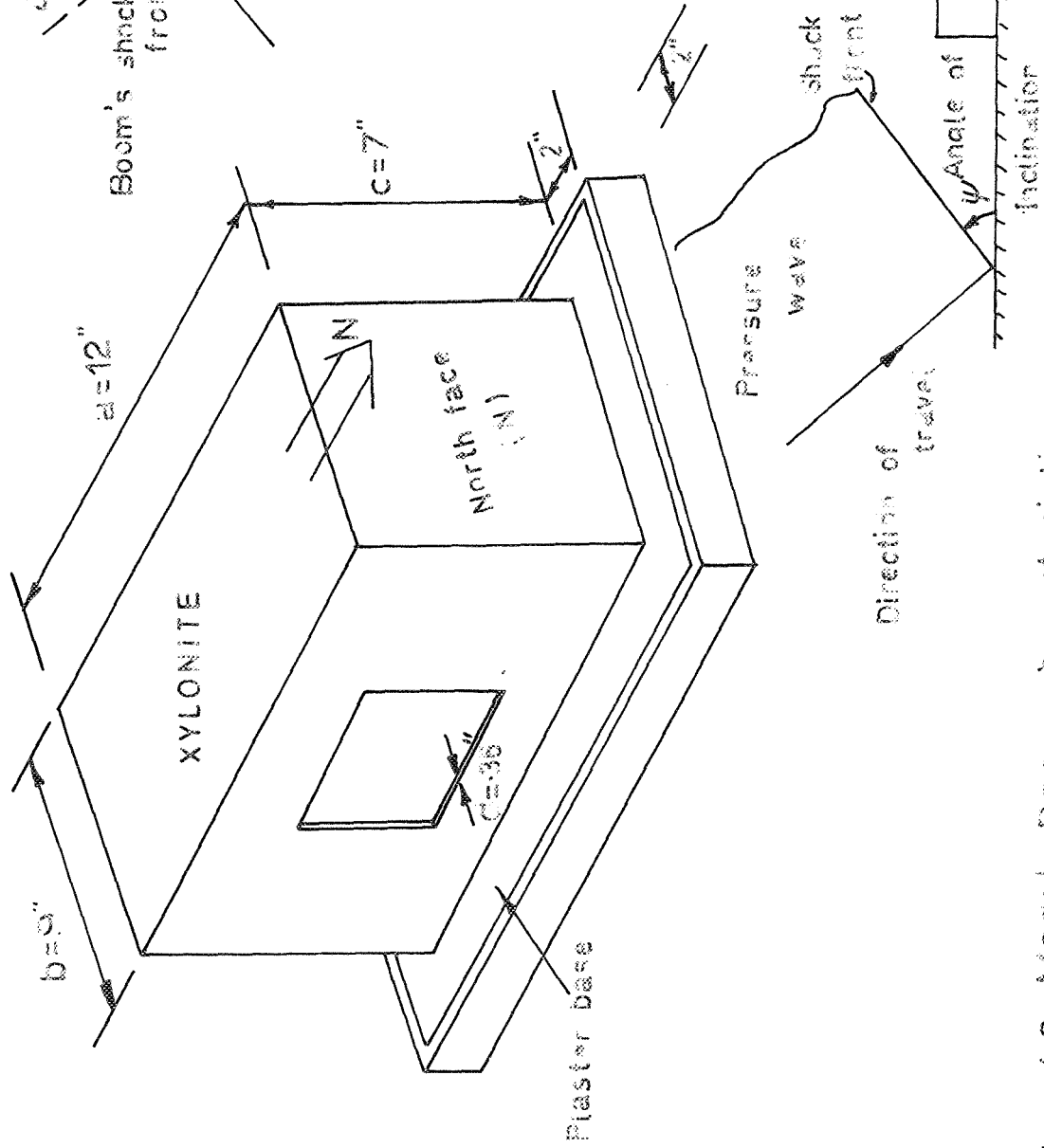
Conical shock tube



Position of xylonite model

Fig. 4.1

# Basic Model



$$L = a \cos \theta + b \sin \theta$$

• M Internal microphone location

$\theta$  Angle of Incidence

## SIDE ELEVATION

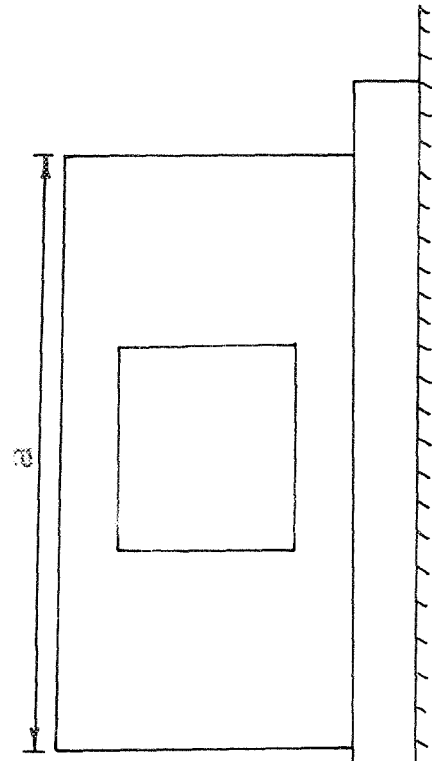


Fig. 4.2 Model - Boom characteristics

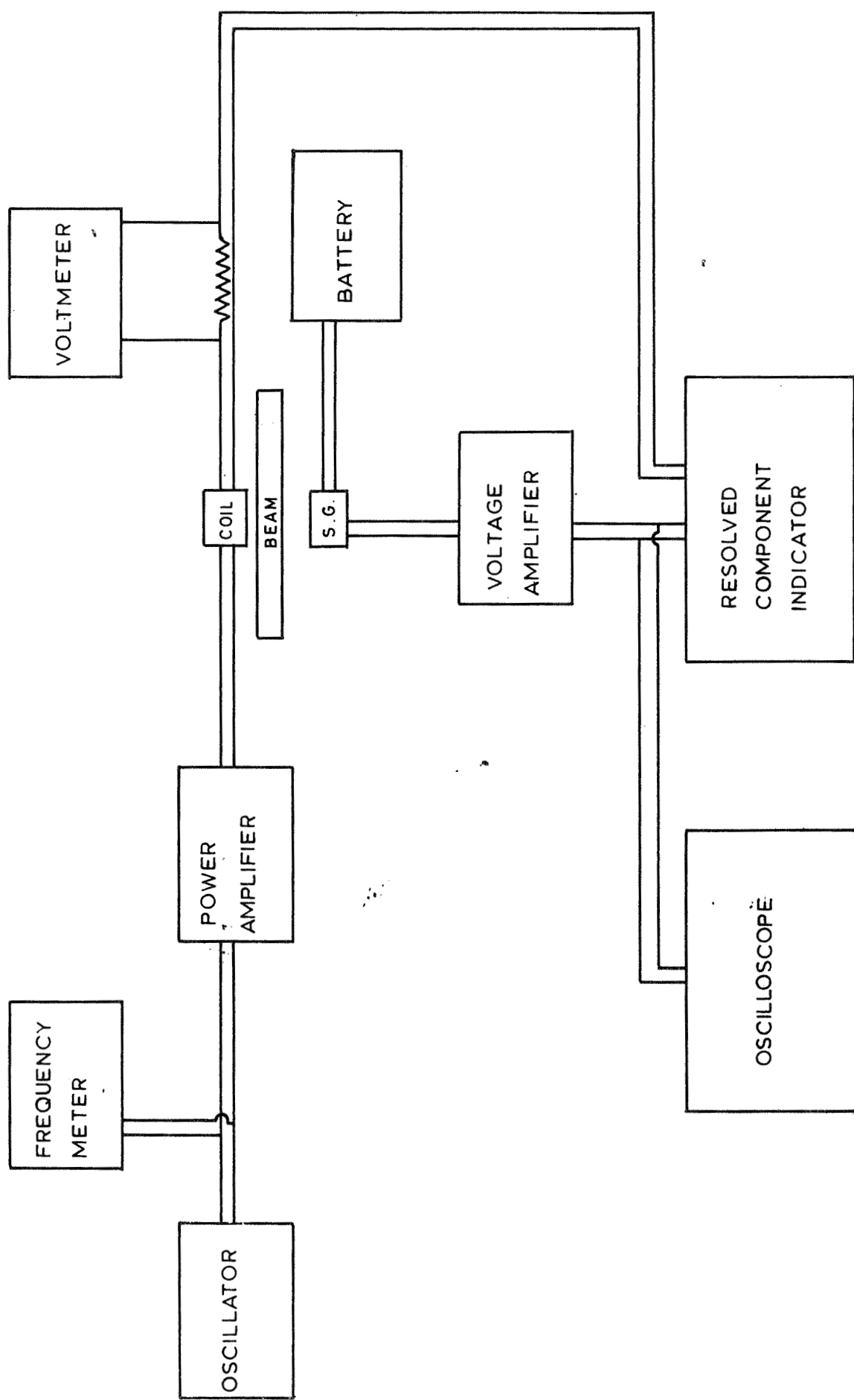


Fig.4.3 Circuit diagram to determine the bending stiffness of Xylonite

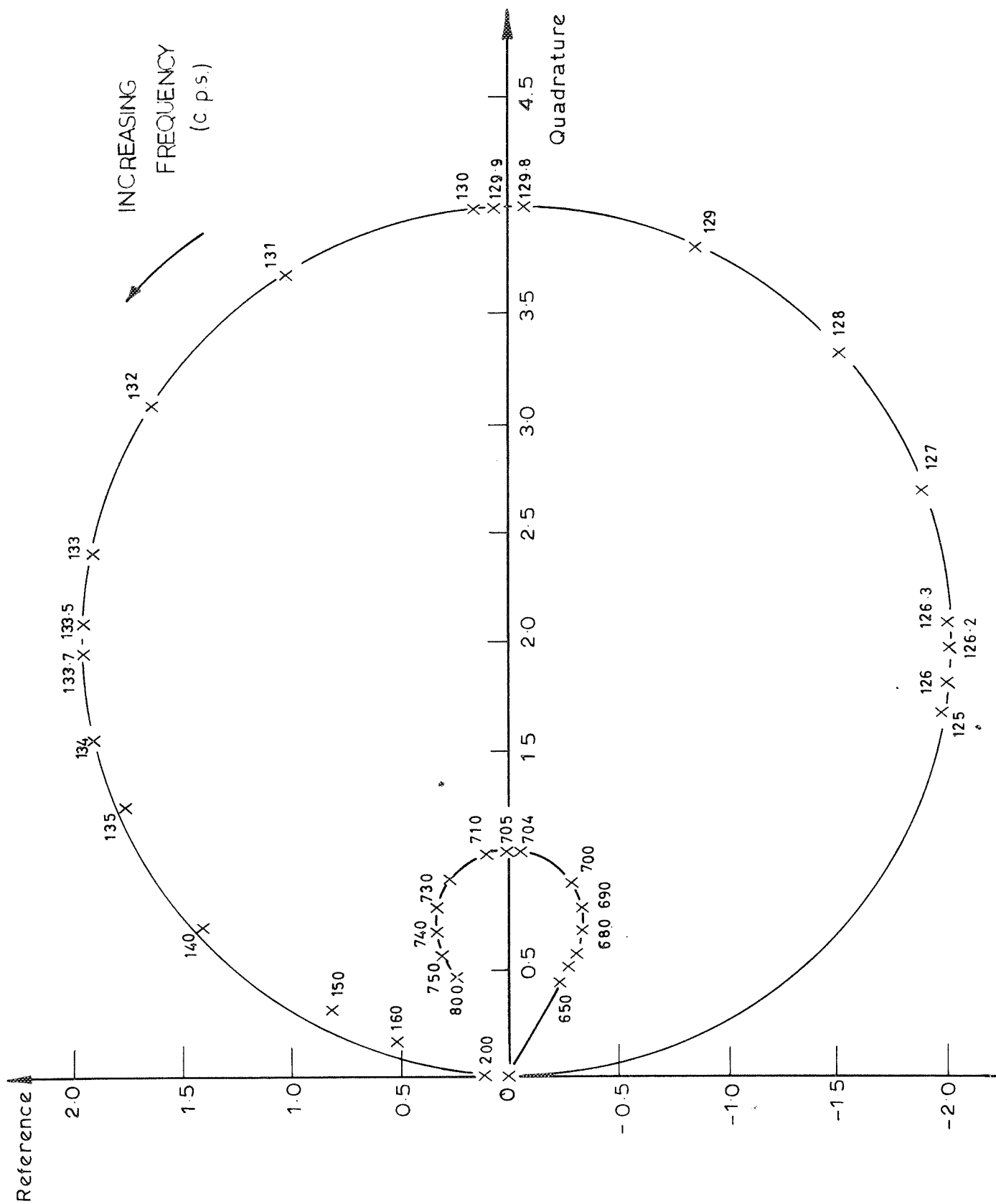
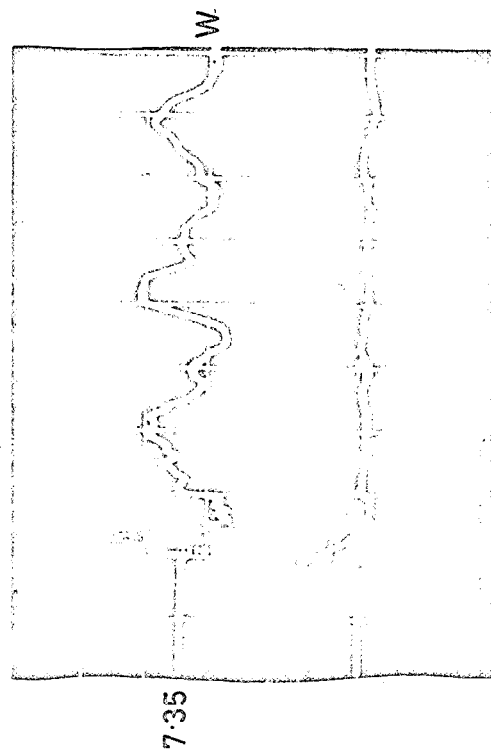
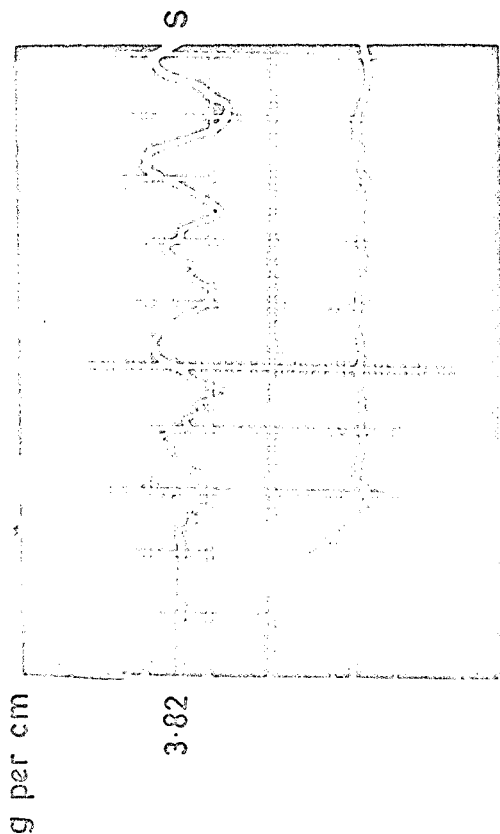


Fig 4.4

3 in. DRIVER ; NORTH FACE NORMAL ; ACCELERATIONS AND PRESSURES OF THE LENGTH

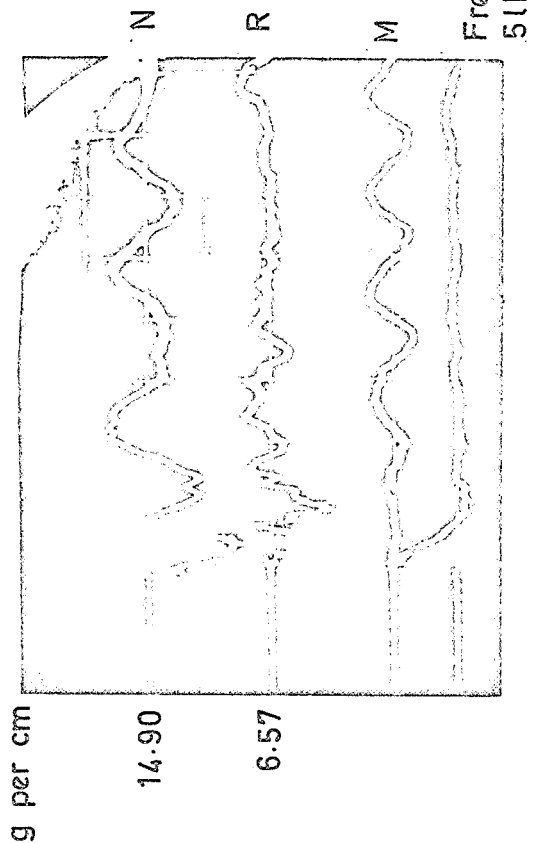
CLOSED MODEL.



NOMENCLATURE:

N,S,W,R accelerations of the midpoints of the closed model's North, South, West faces & Roof respectively.

M Internal microphone's pressure-time history.

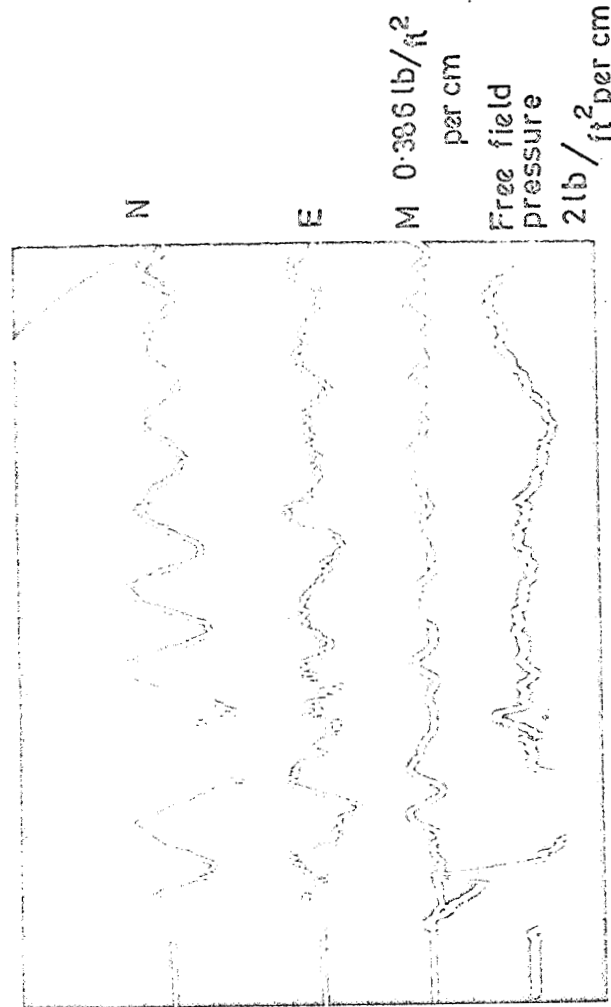
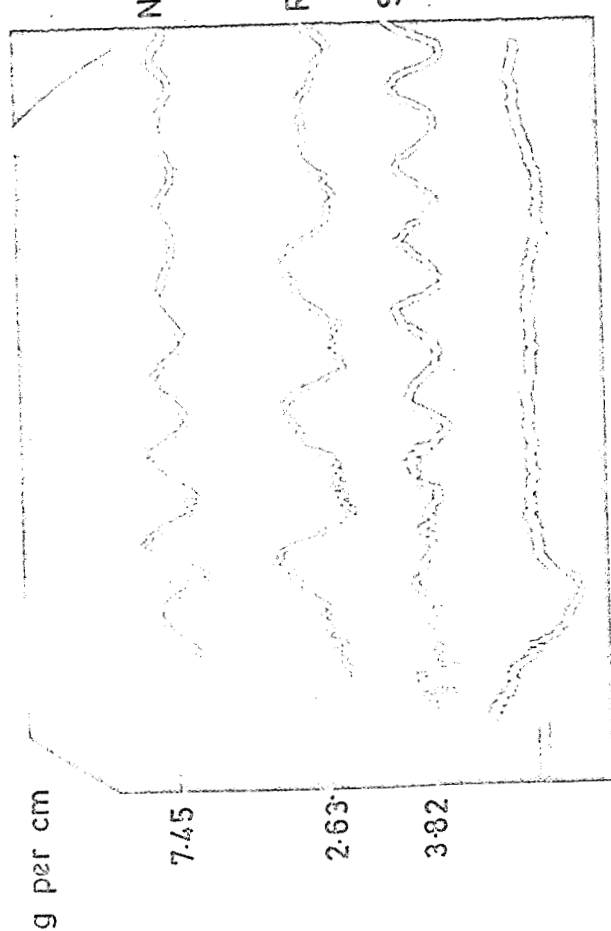


TIME SCALE 1 msec per cm .

Fig.4.5

LENGTH

THE CLOSED MODEL.



NOMENCLATURE:

N,S,W,E,R accelerations of the midpoints of the

closed model's North, South, West, East faces &

Roof respectively.

M Internal microphone's pressure-time history.

TIME SCALE : 2 msec per cm .

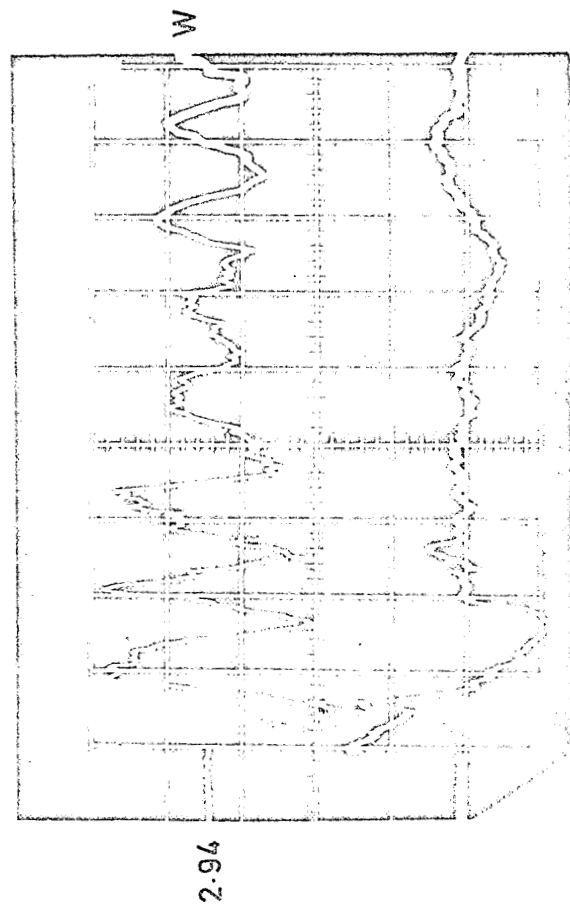
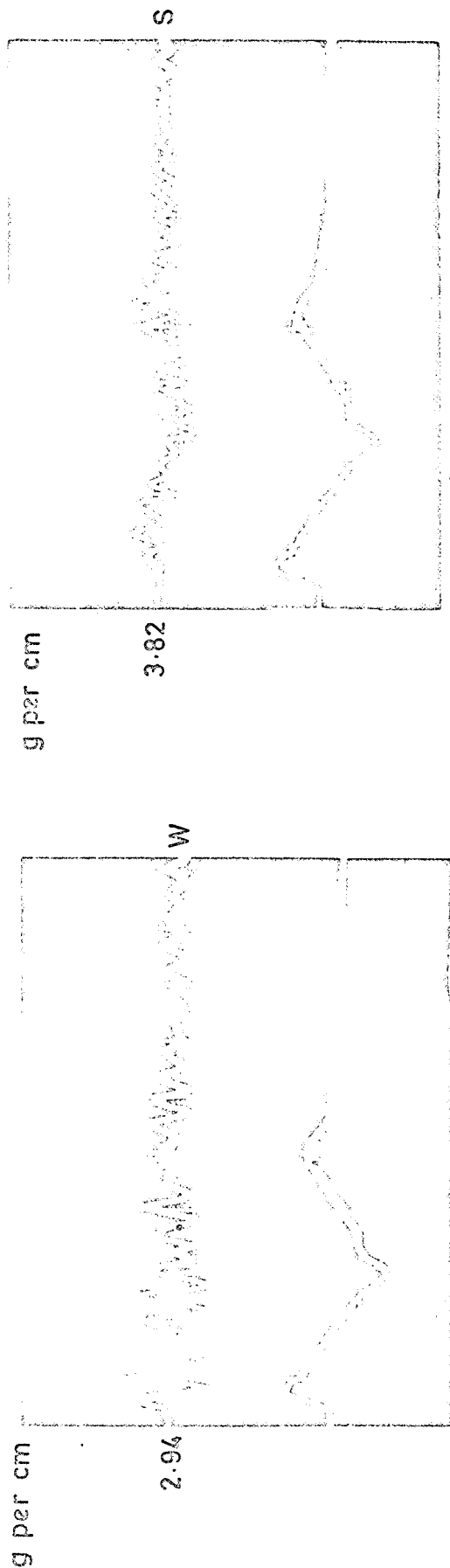


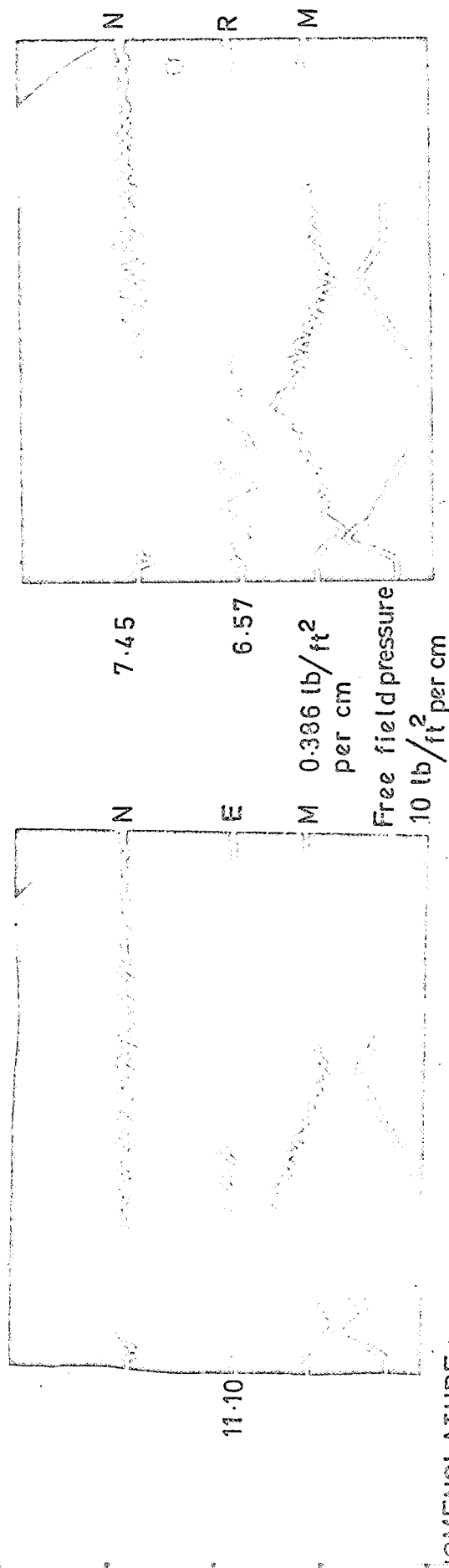
Fig .4.6

LENGTH

# THE CLOSED MODEL.



TIME SCALE : 5 msec per cm .



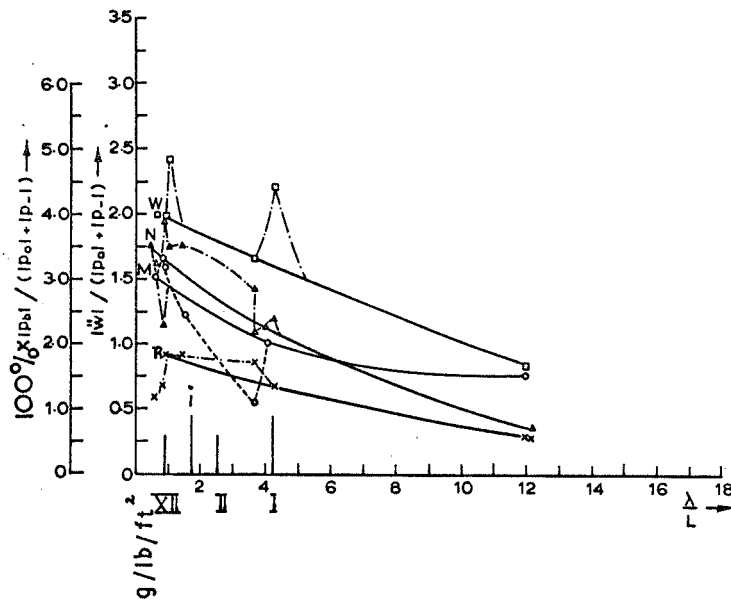
10 MENCLATURE :

W, S, W, E, R accelerations of the midpoints of the closed model's North, South, West, East faces & Roof respectively.

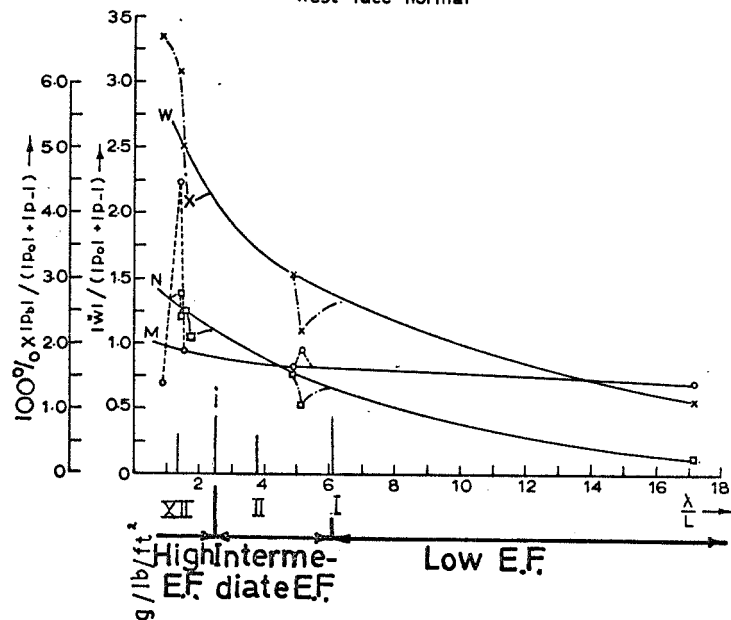
Fig. 4.7

M Internal microphone's pressure-time history.

67 1/2° Incidence on north face.



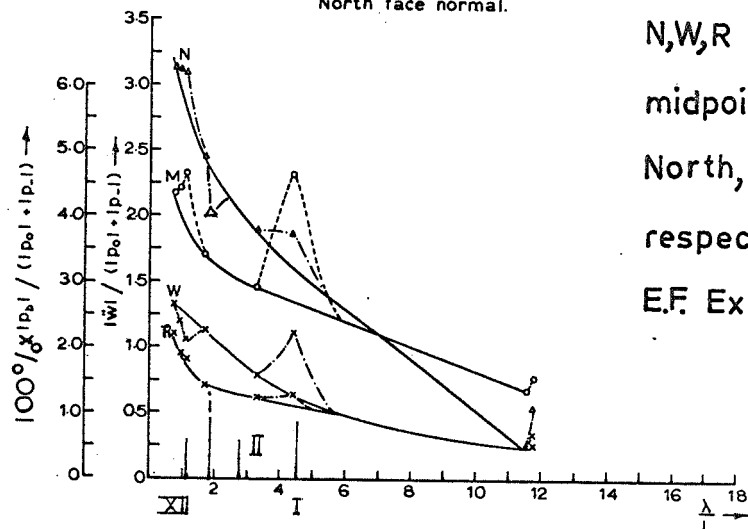
West face normal



KEY

Capital and small Roman numerals indicate appropriate structural and cavity acoustic frequencies respectively  
 $|p_b|$  Modulus of peak internal pressure at microphone location M.  
 $|p_0|, |p_-|$  Peak positive and negative pressures of simulated boom.

North face normal.



N, W, R Peak accelerations at midpoints of closed model's North, West faces and Roof respectively.

E.F. Excitation frequency  $f_N$ .

Fig. 4.8 Maximum response of the closed model.

$$\propto \frac{1}{f_N}$$

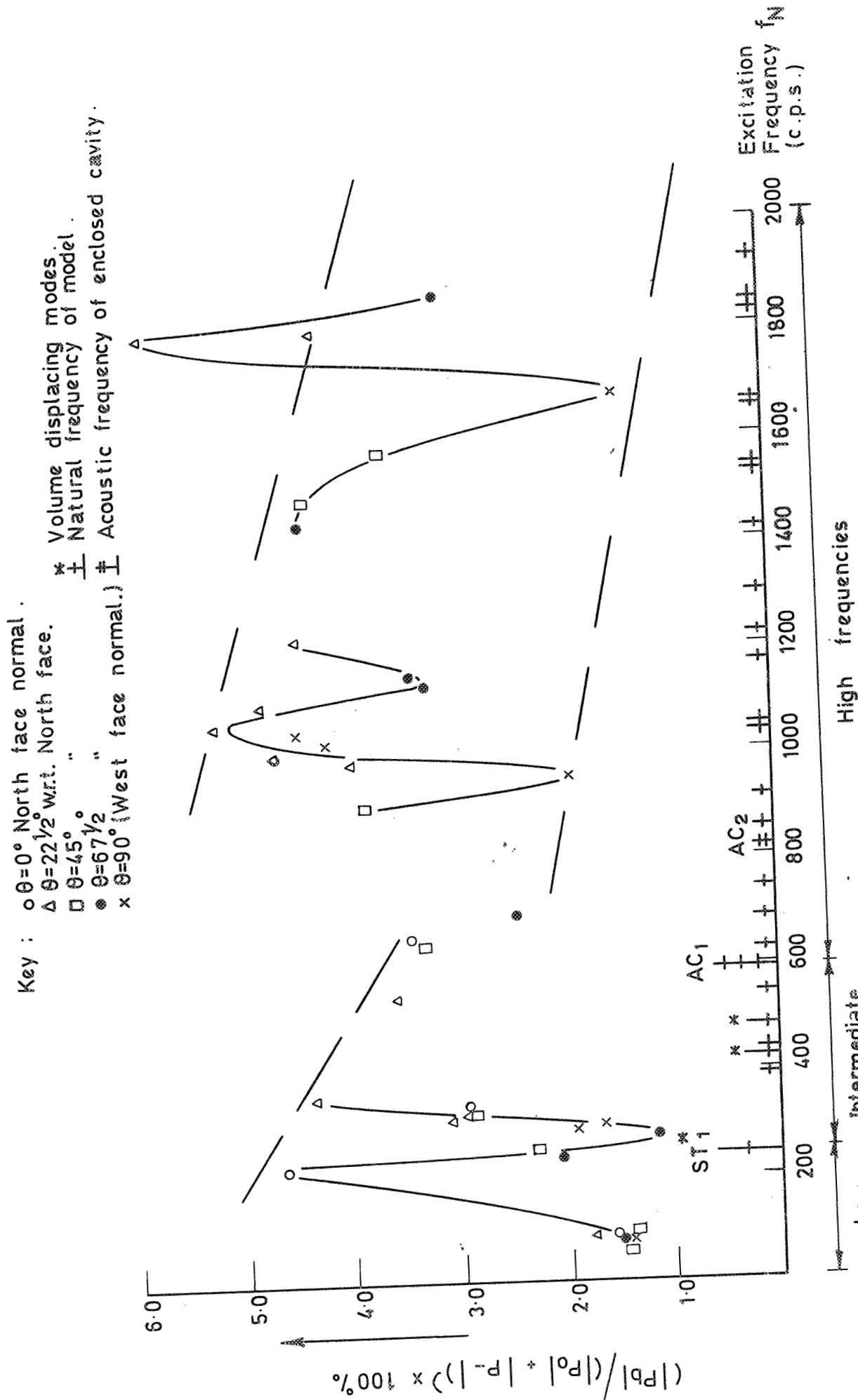


Fig. 4.9 Peak internal pressure for various angles of incidence on the closed model.

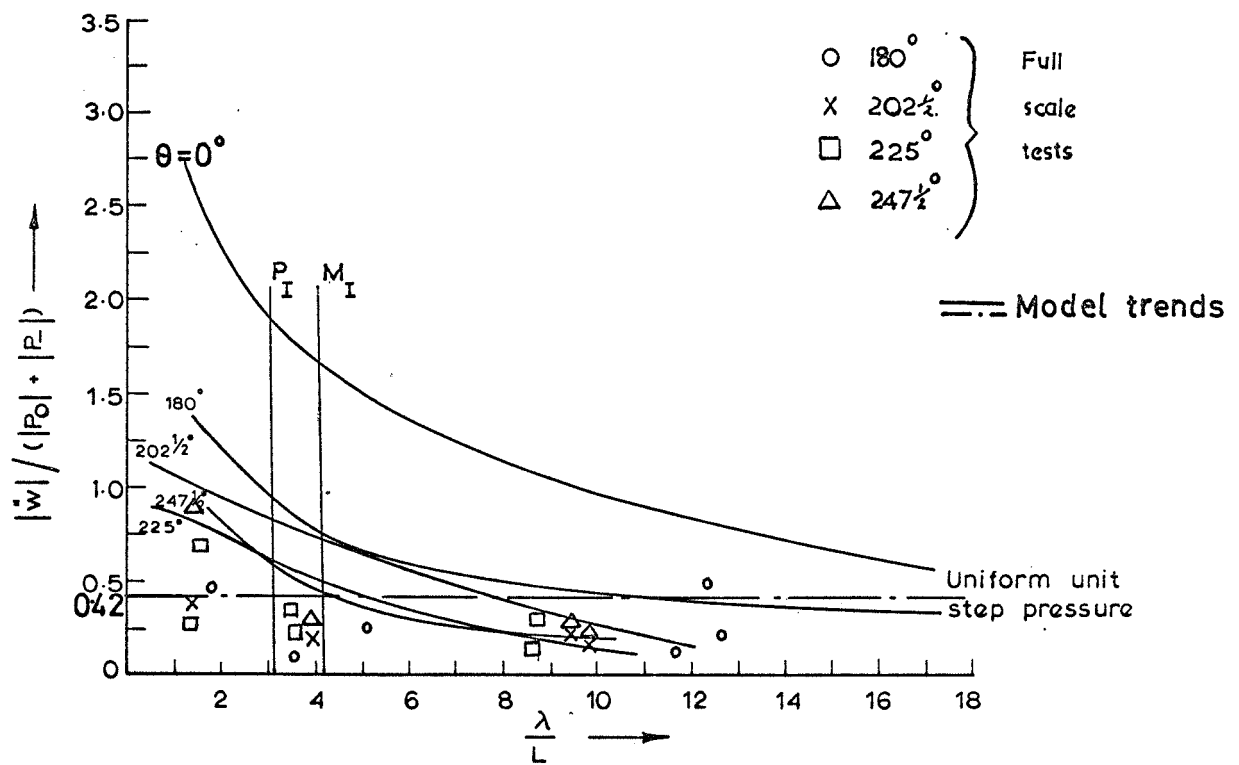
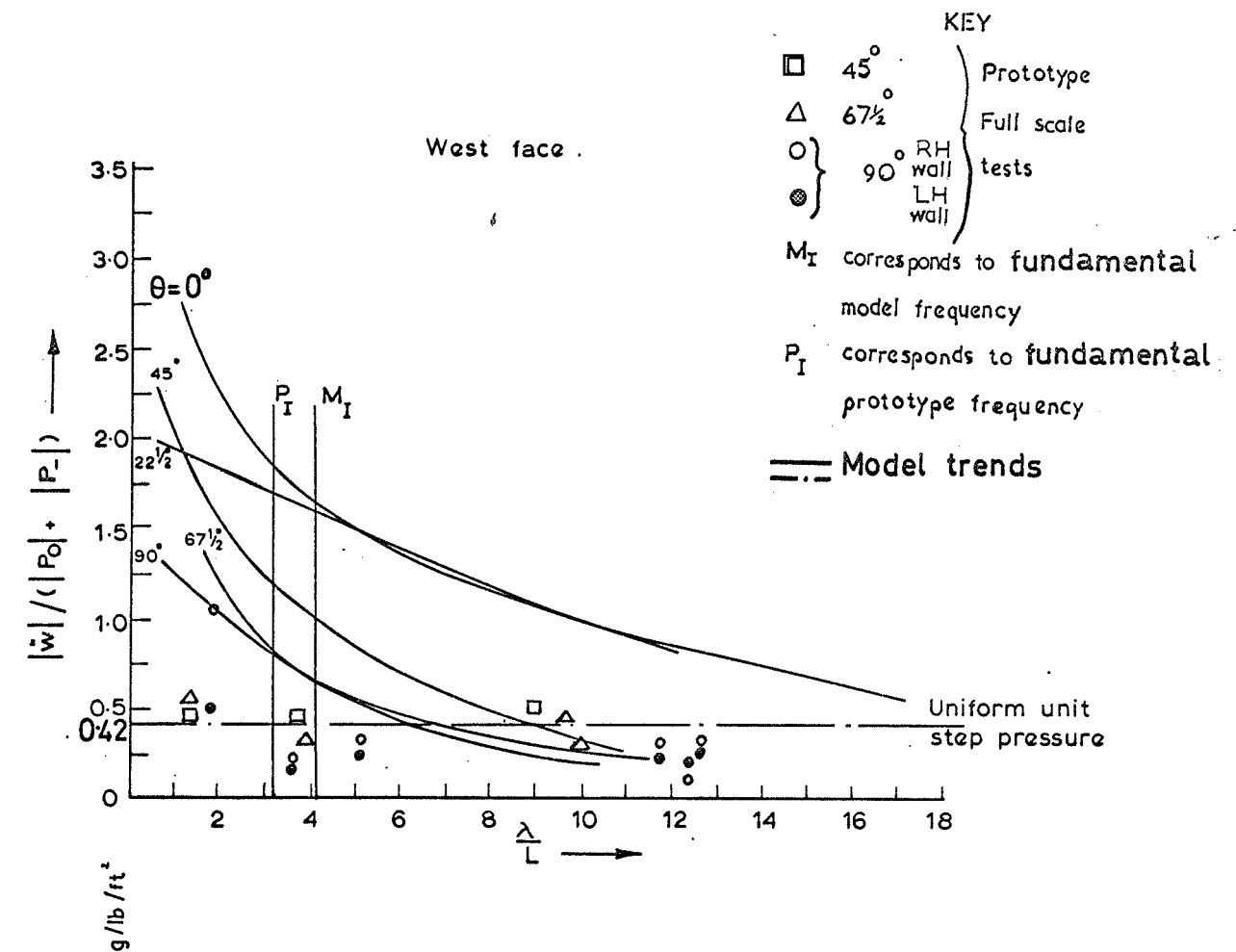


Fig. 4.10 Maximum closed model response with varying angles of incidence. ( $\theta$  given w.r.t. West face.)

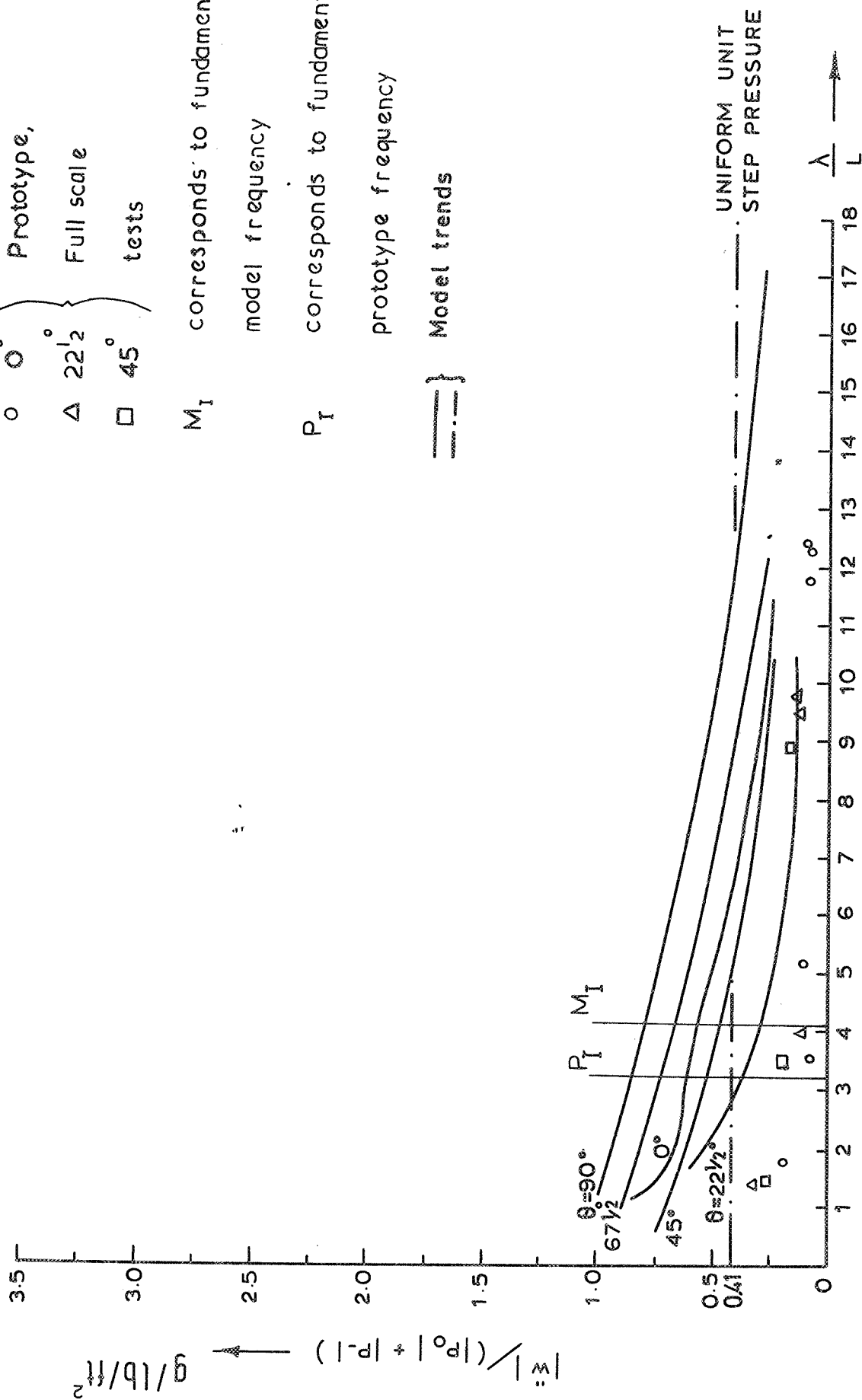


Fig. 4.11

Maximum closed model response with varying angles of incidence,  $\theta$ .  
Roof ( $\theta$  given with respect to North face)

6 x 3 opening in east face; east face normal .

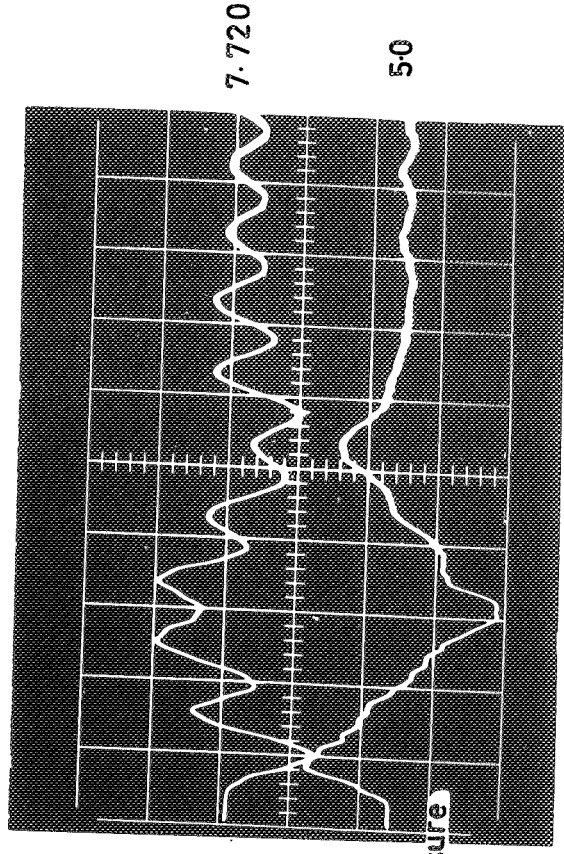
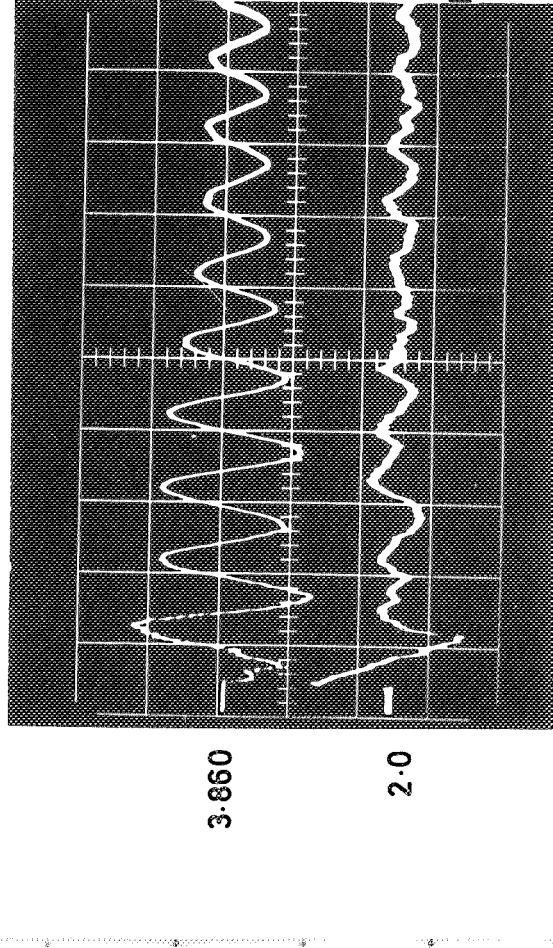
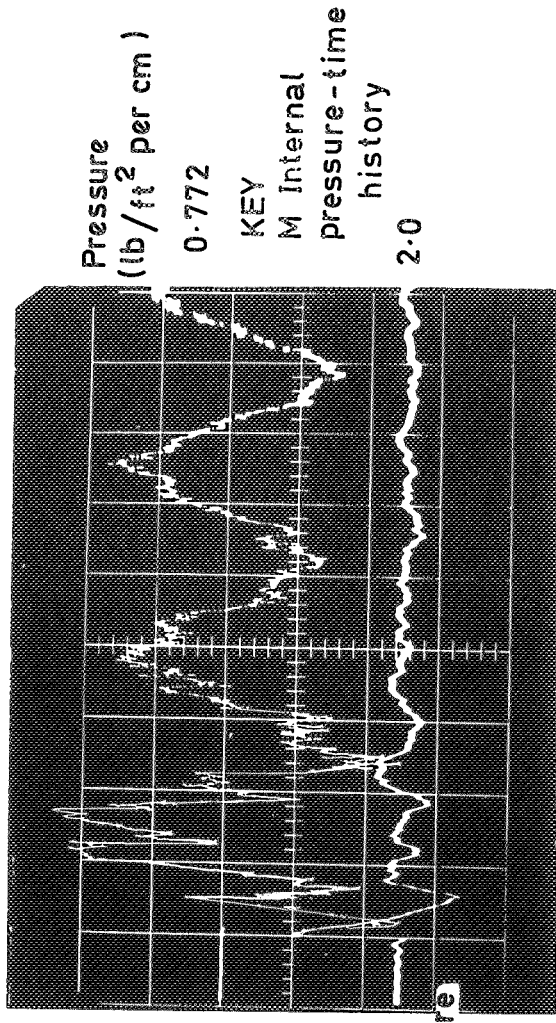
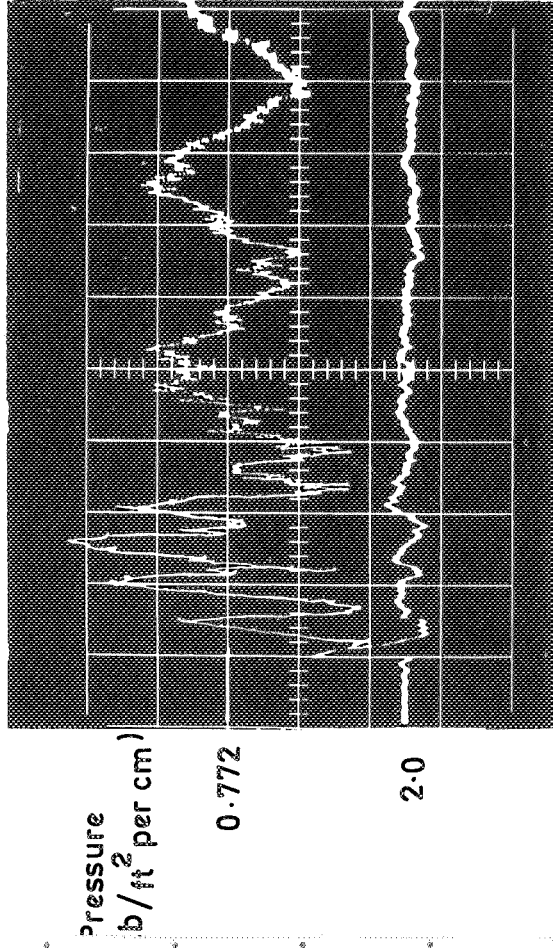
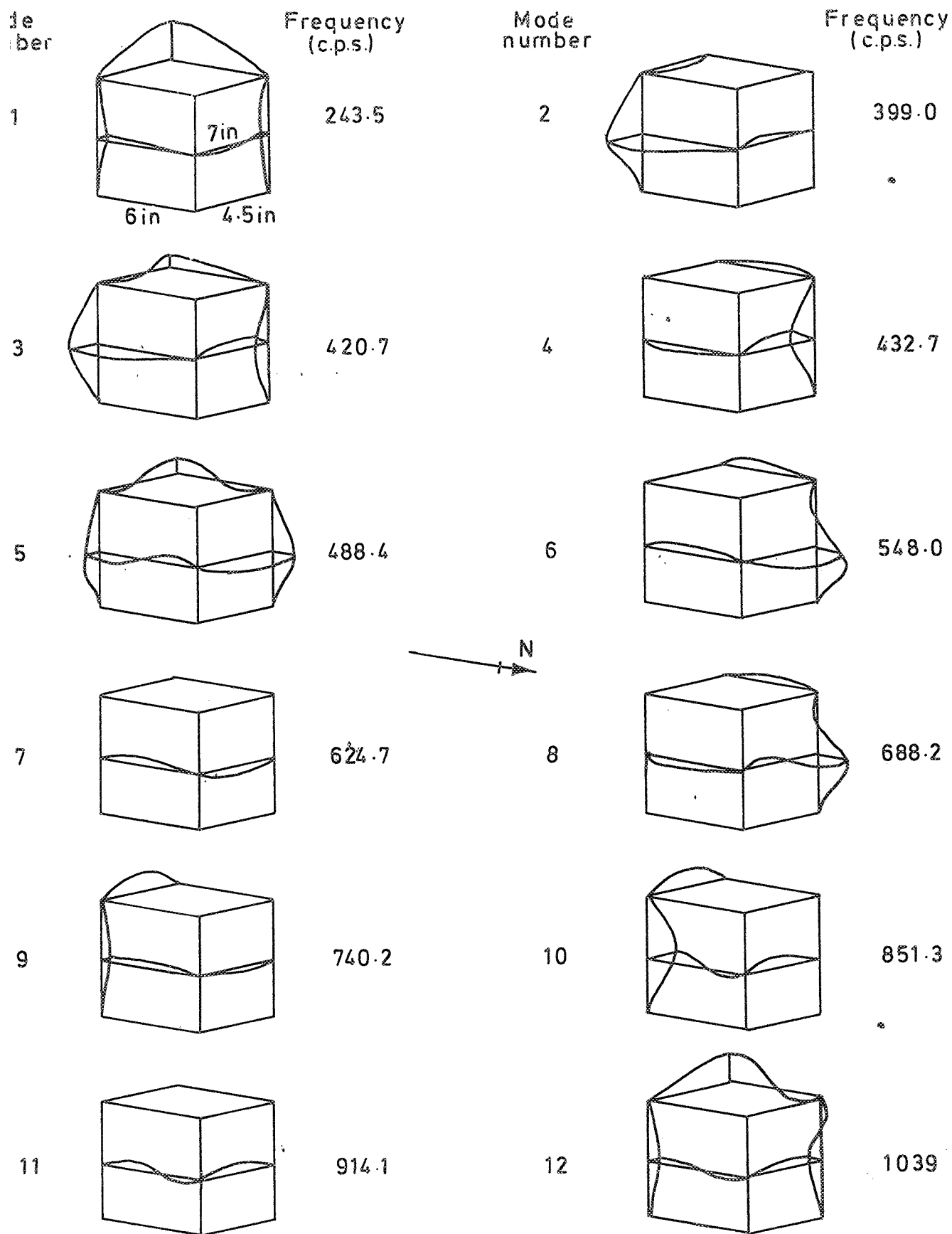
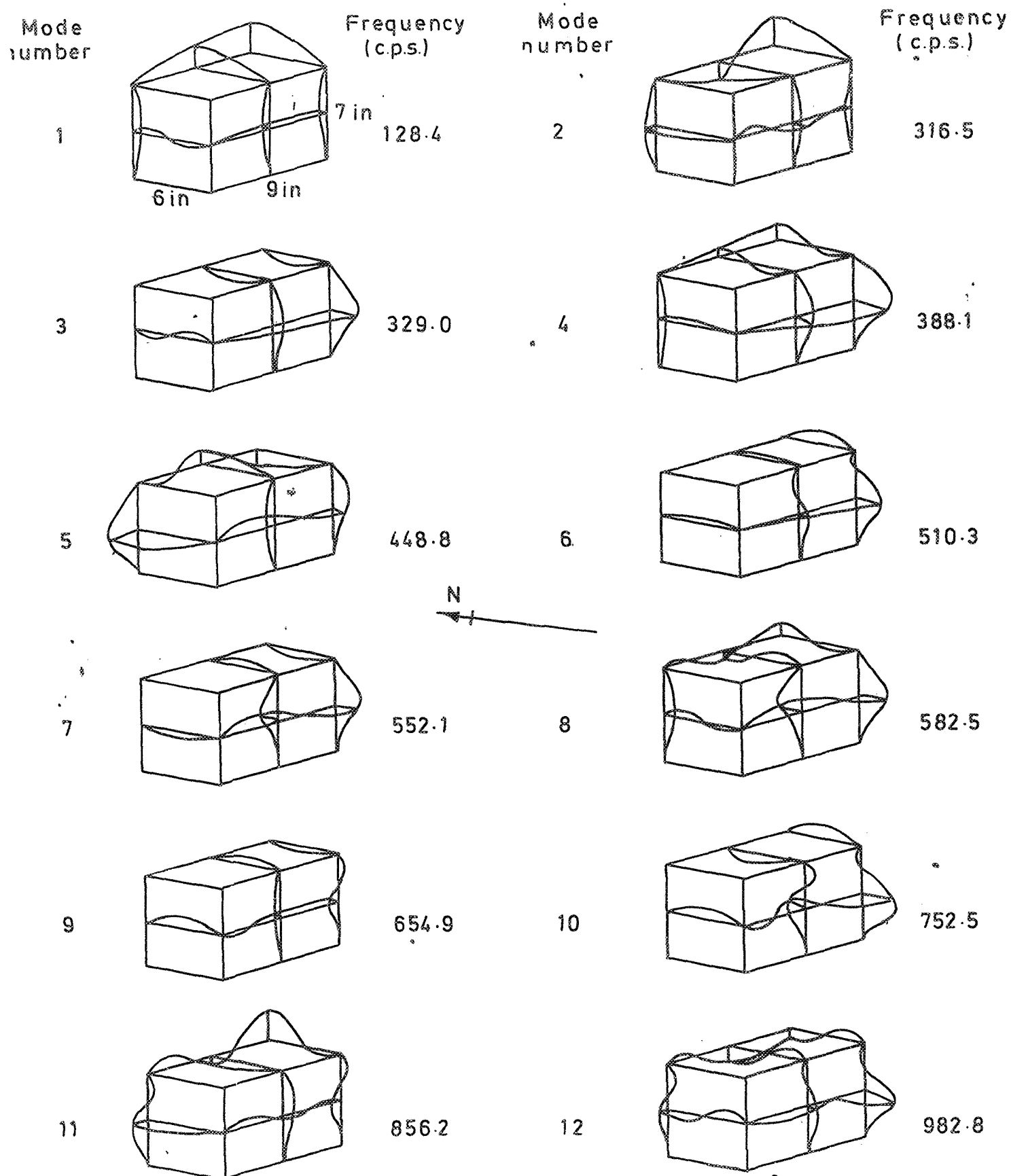


Fig. 4.12



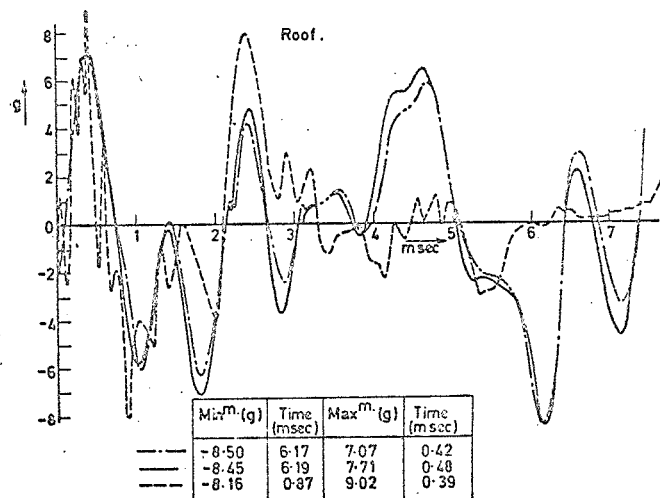
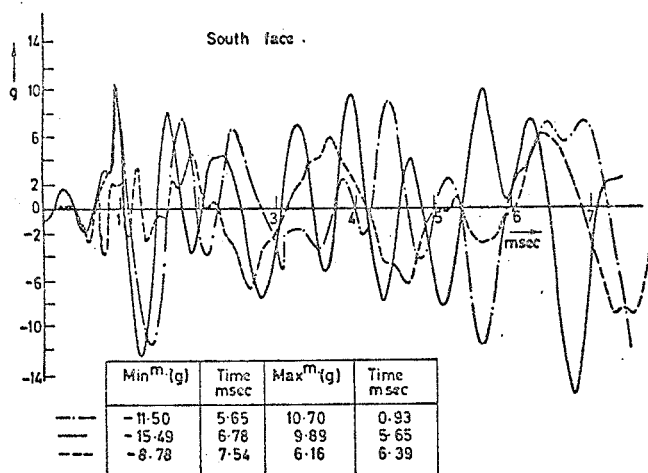
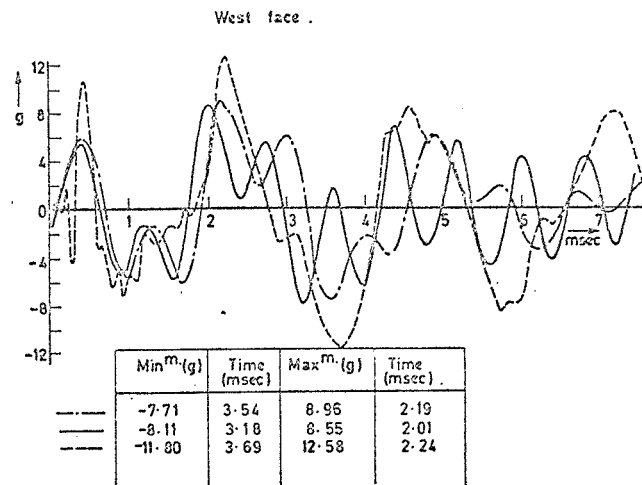
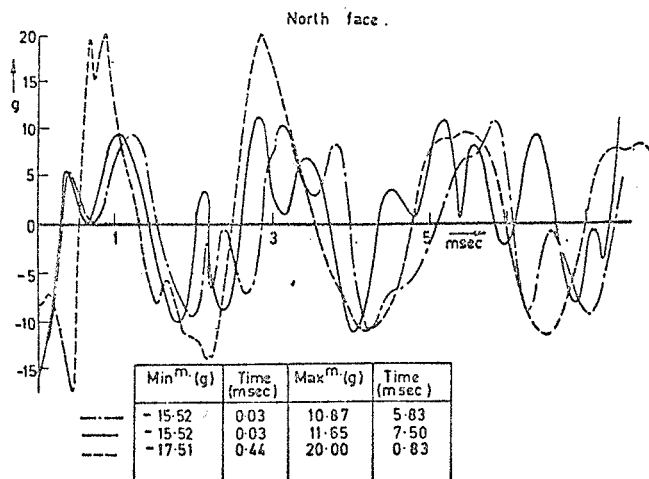
$$E = 4.2 \times 10^5 \text{ lb/in}^2 \quad \rho_{ST} = 0.0222 \text{ lb/in}^3$$

Fig. 4.13 Computed mode shapes for quarter of the closed Xylonite box

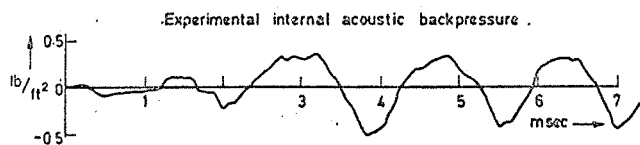
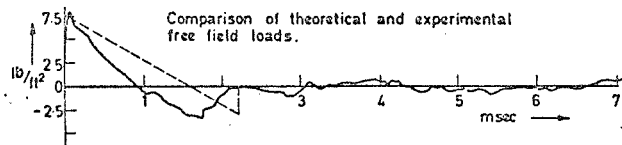


$$E = 4.2 \times 10^5 \text{ lb/in}^2 \quad P_{ST} = 0.0222 \text{ lb/in}^2$$

Fig. 4.14 Computed mode shapes for half the Xylonite model with no east face.

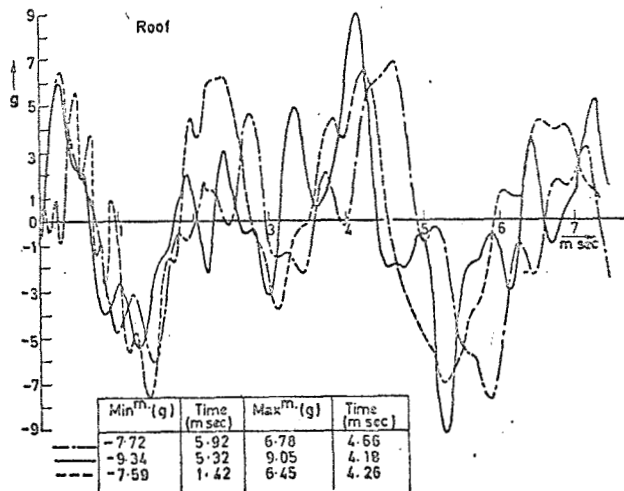
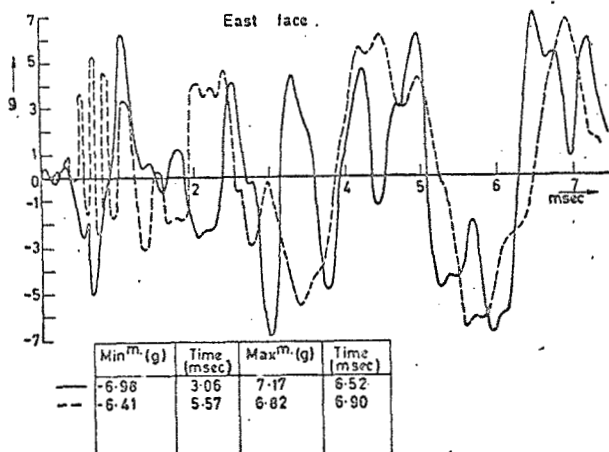
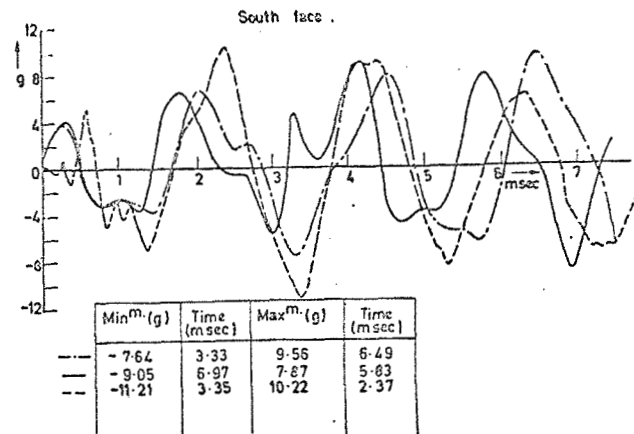
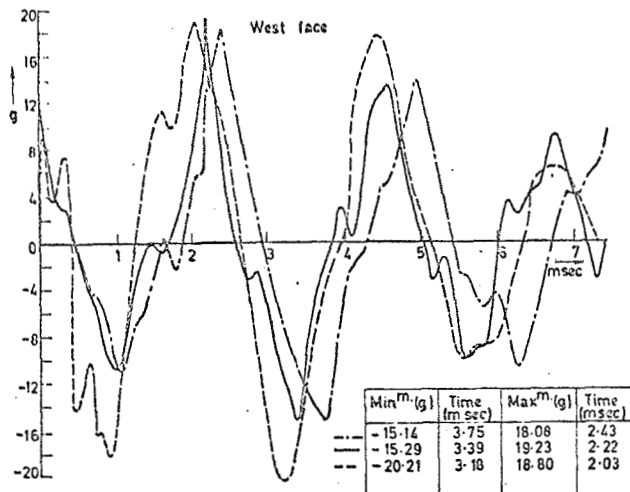


—  $E = 3.1 \times 10^9 \text{ lb/in}^2$ ,  $E = 3.9 \times 10^9 \text{ lb/in}^2$  —, Experimental —

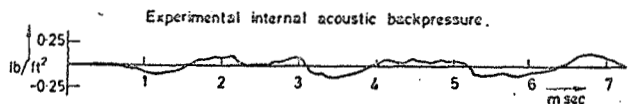
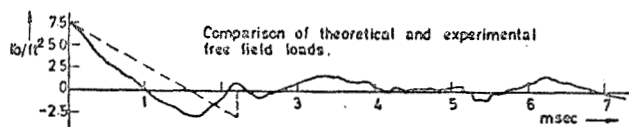


North face normal,  $\nu = 0.39$ ,  $\rho c = 1.016 \times 10^2 \text{ lb/in}^2$ ,  $\beta = 0$

FIG5. COMPARISON OF THE EXPERIMENTAL AND THEORETICAL ACCELERATIONS OF THE CLOSED MODEL BOX ASSUMING DIFFERENT DYNAMIC YOUNG'S MODULI.



—  $E = 3.1 \times 10^5 \text{ lb/in}^2$  —  $E = 3.9 \times 10^5 \text{ lb/in}^2$  - - - Experimental



West face normal,  $\nu = 0.39$ ,  $P_{st} = 1.018 \times 10^{-2} \text{ lb/in}^2$ ,  $\beta = 0$

FIG 52 COMPARISON OF THE EXPERIMENTAL AND THEORETICAL ACCELERATIONS OF THE CLOSED MODEL BOX ASSUMING DIFFERENT DYNAMIC YOUNG'S MODULI.

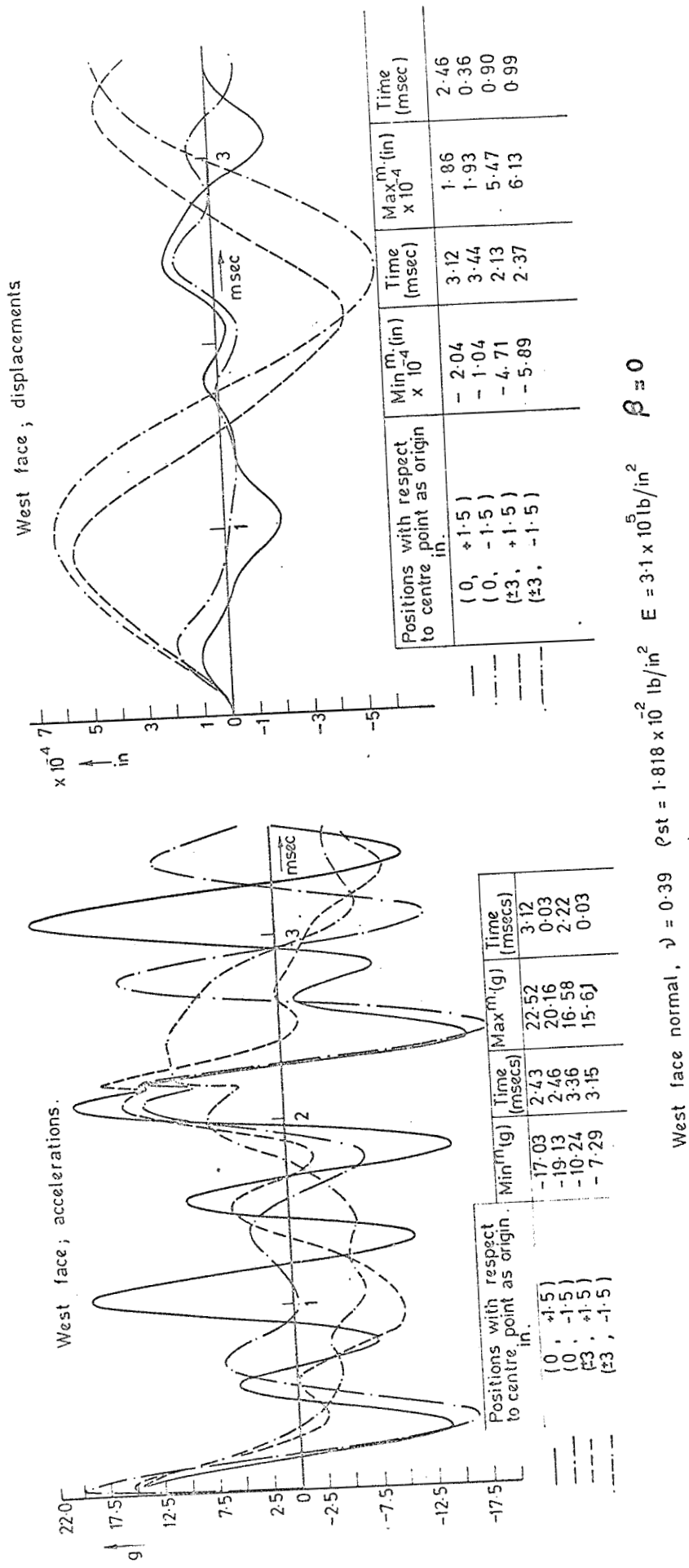


FIG. 5.3 EFFECT OF THE ANTISYMMETRIC MODES ON THE ACCELERATIONS AND DISPLACEMENTS OF THE FRONT FACE.

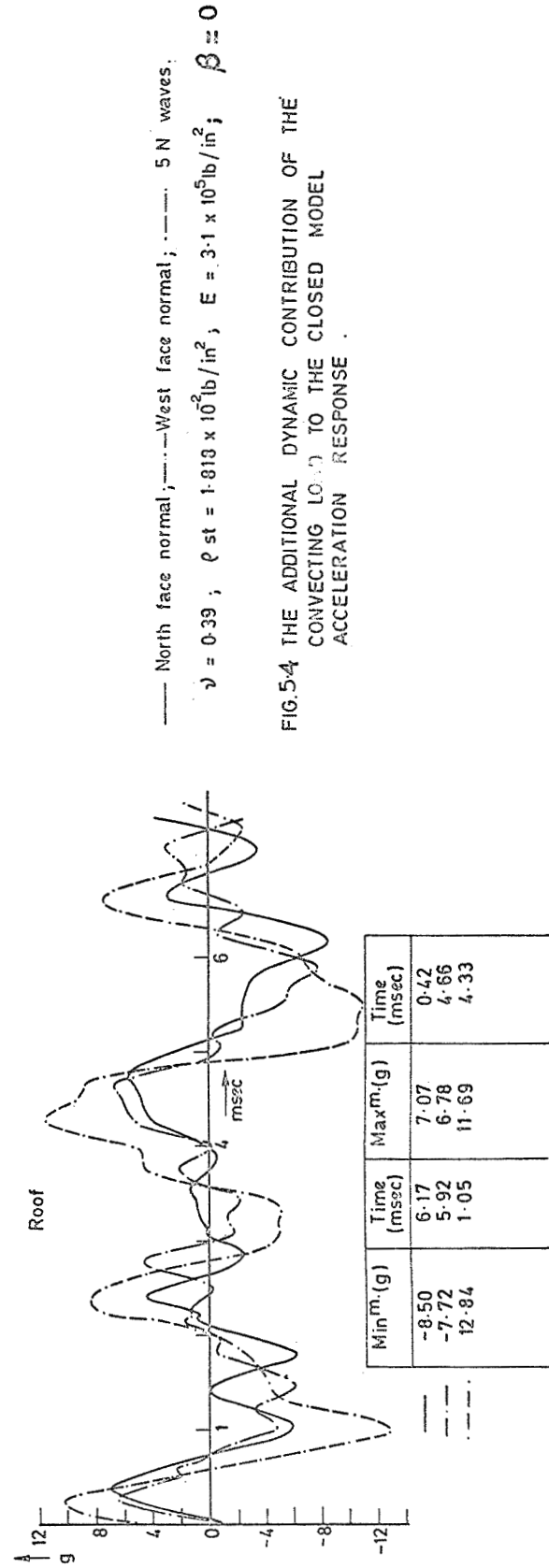
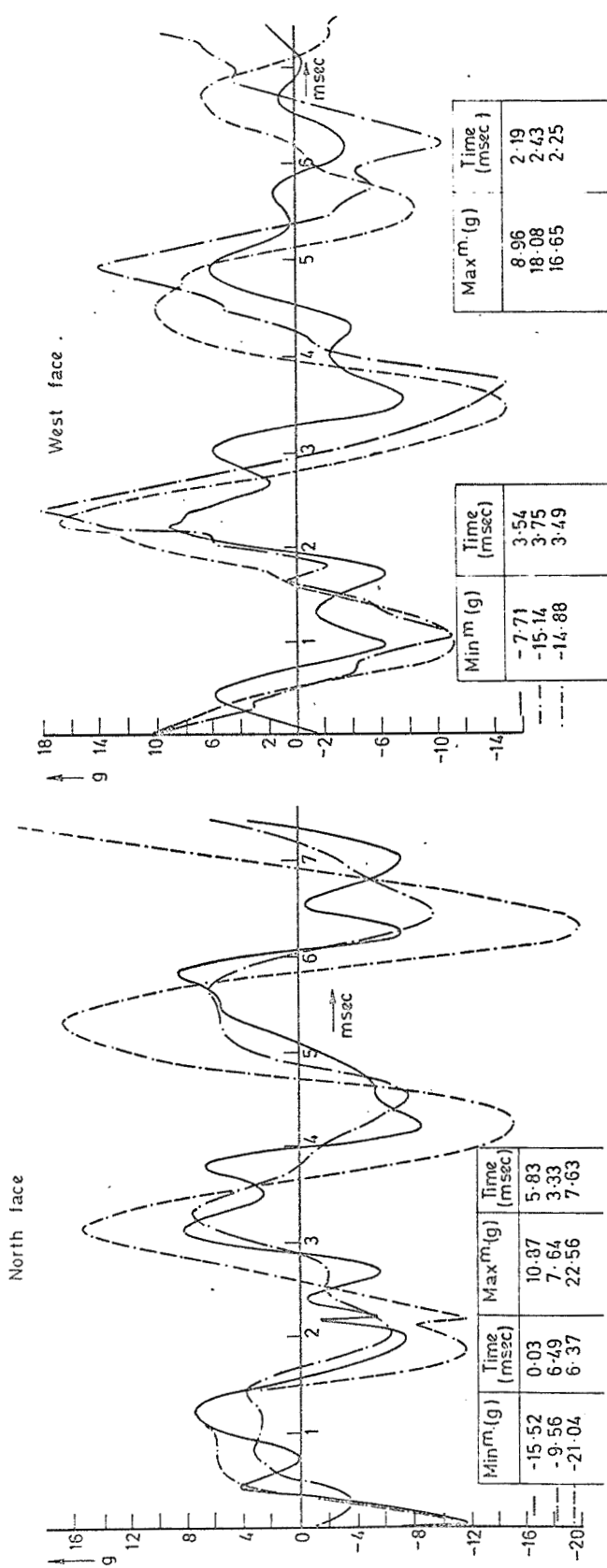
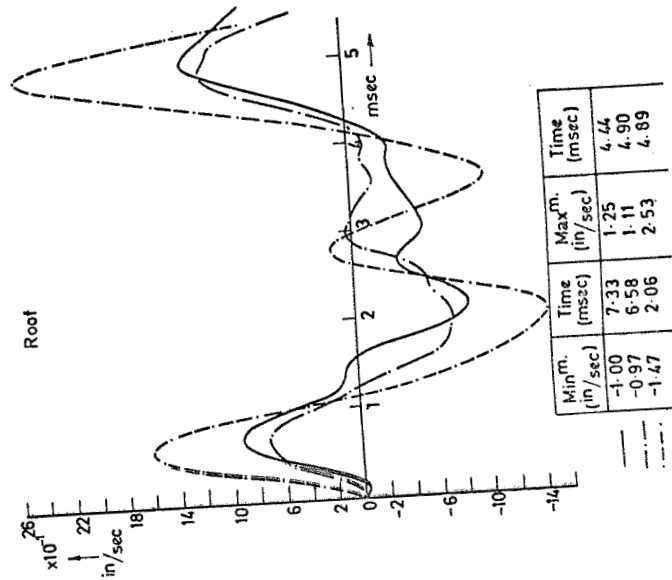
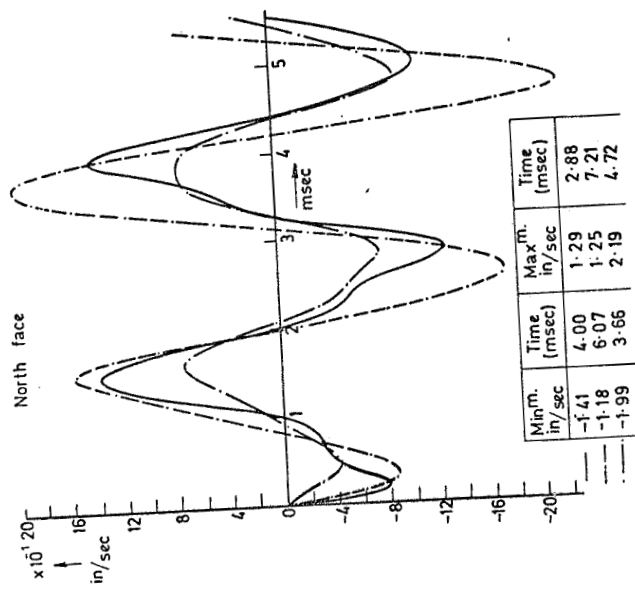
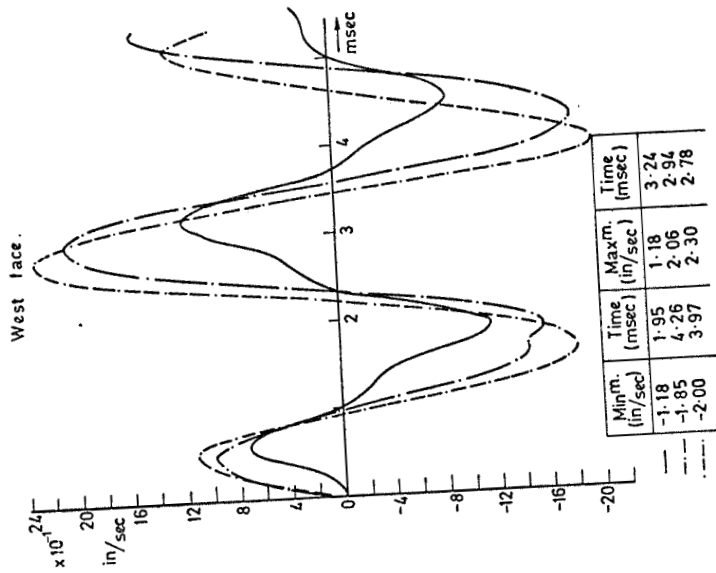
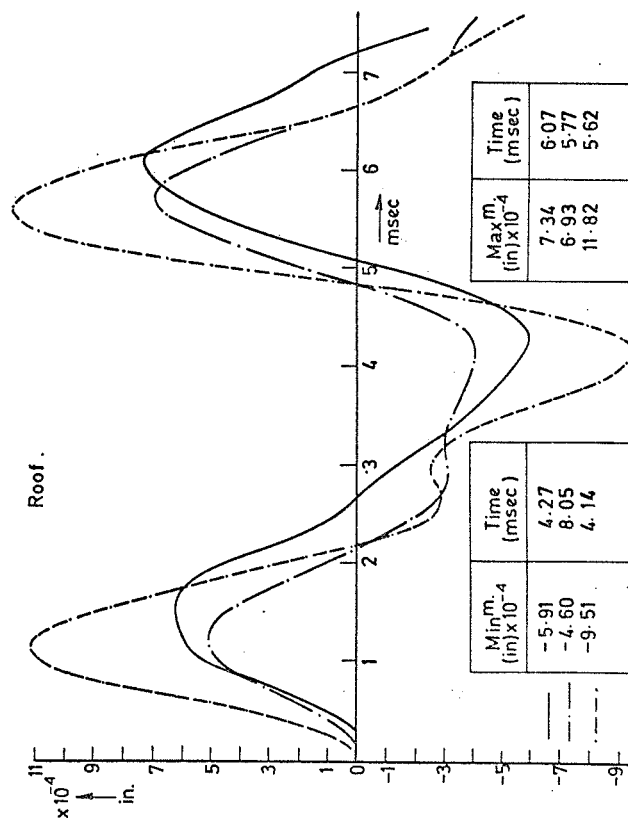
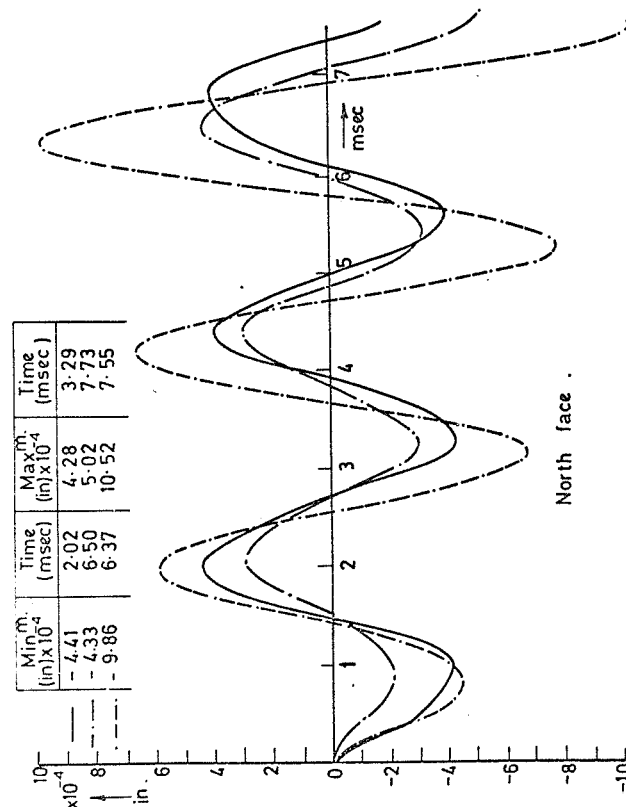
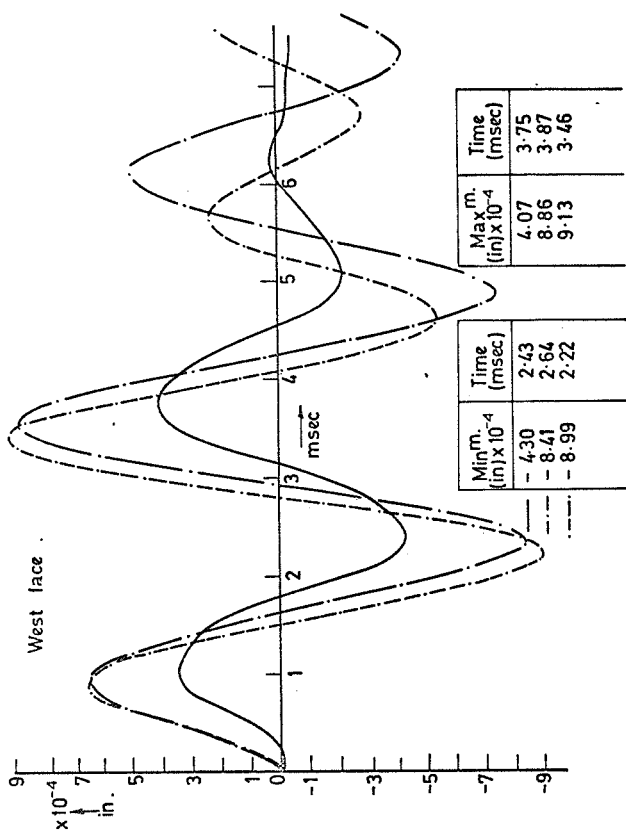


FIG. 5.4 THE ADDITIONAL DYNAMIC CONTRIBUTION OF THE CONNECTING LOAD TO THE CLOSED MODEL ACCELERATION RESPONSE



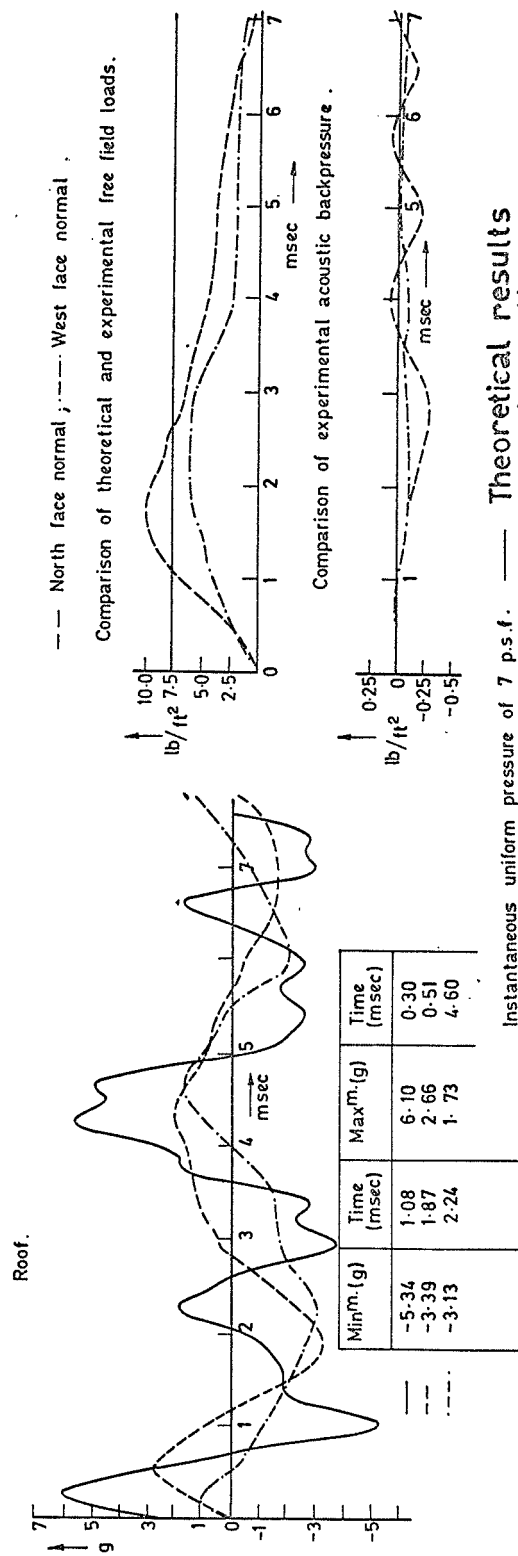
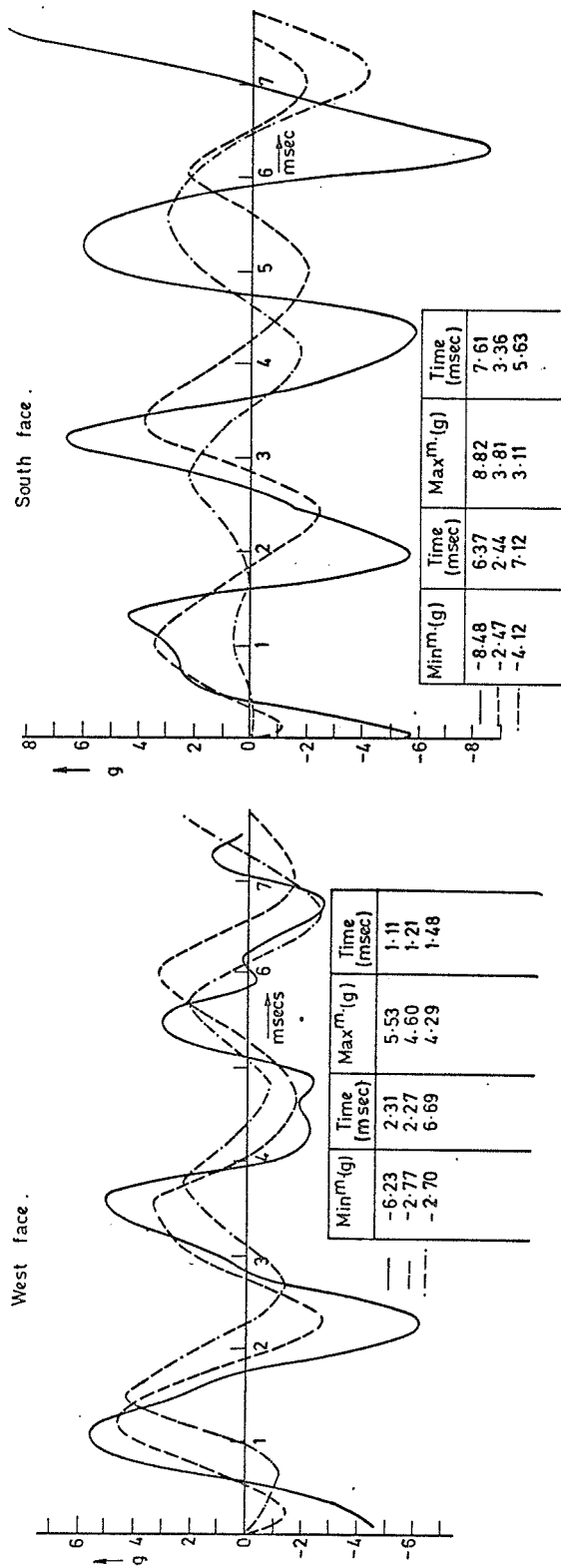
— North face normal; --- West face normal; - - - 5 N-waves.  
 $\nu = 0.39$ ;  $\rho_{st} = 1.818 \times 10^{-2} \text{ lb/in}^2$ ;  $E = 3.1 \times 10^5 \text{ lb/in}^2$ ;  $\beta = 0$

FIG. 5-5 THE ADDITIONAL DYNAMIC CONTRIBUTION OF THE CONVECTING LOAD TO THE CLOSED MODEL VELOCITY RESPONSE



— North face normal; --- West face normal; -.-.- 5 N-waves  
 $\nu = 0.39$ ;  $\rho_{st} = 1.818 \times 10^{-2} \text{ lb/in}^2$ ;  $E = 3.1 \times 10^5 \text{ lb/in}^2$ ;  $\beta = 0$

FIG 5-6 THE ADDITIONAL DYNAMIC CONTRIBUTION OF THE CONVECTING LOAD TO THE CLOSED MODEL DISPLACEMENT RESPONSE.

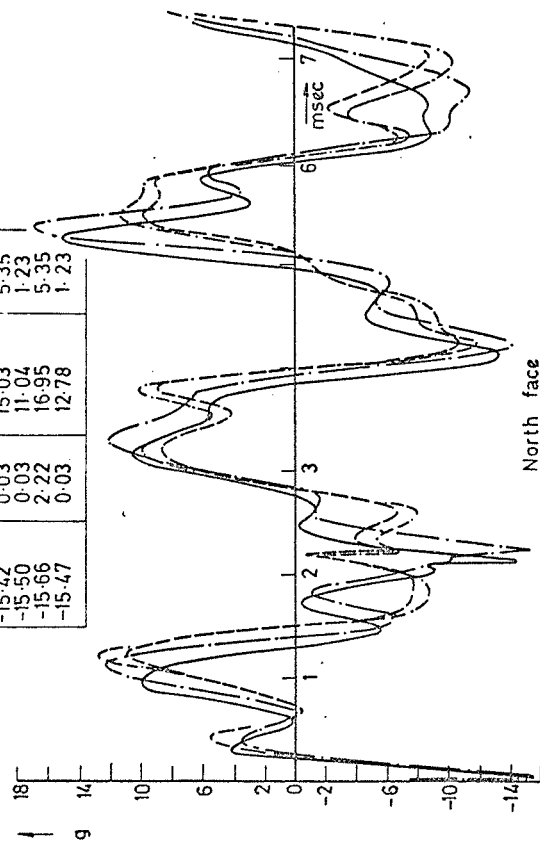


Instantaneous uniform pressure of 7 p.s.f. — Theoretical results  
 North face normal, 7' driver. --- Experimental results  
 West face normal, 7' driver. - - -

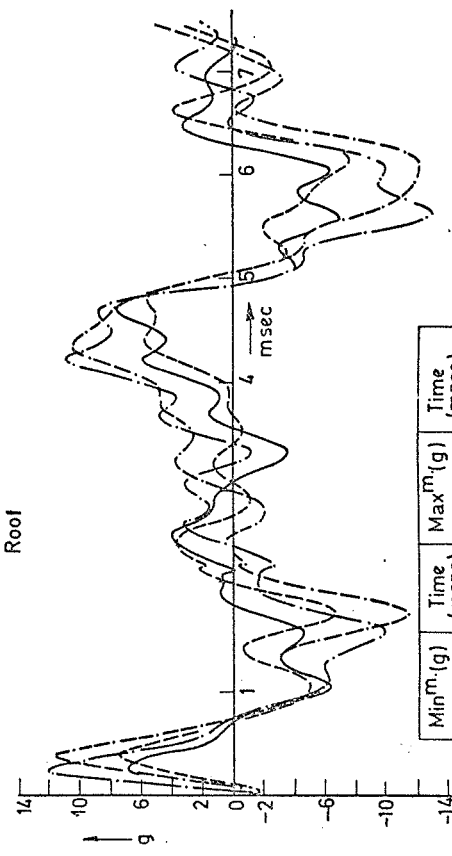
$\gamma = 0.39$ ,  $\rho_{st} = 1.818 \times 10^2 \text{ lb/in}^2$ ,  $E = 3.1 \times 10^5 \text{ lb/in}^2$ ;  $\beta = 0$

FIG.5.7 THE EFFECT ON THE CLOSED MODEL'S ACCELERATIONS OF INCREASING THE N-WAVE'S WAVELENGTH.

| Min <sup>m</sup> (g) | Time (msec) | Max <sup>m</sup> (g) | Time (msec) |
|----------------------|-------------|----------------------|-------------|
| -15.42               | 0.03        | 15.03                | 5.35        |
| -15.50               | 0.03        | 11.04                | 1.23        |
| -15.66               | 2.22        | 16.95                | 5.35        |
| -15.47               | 0.03        | 12.78                | 1.23        |

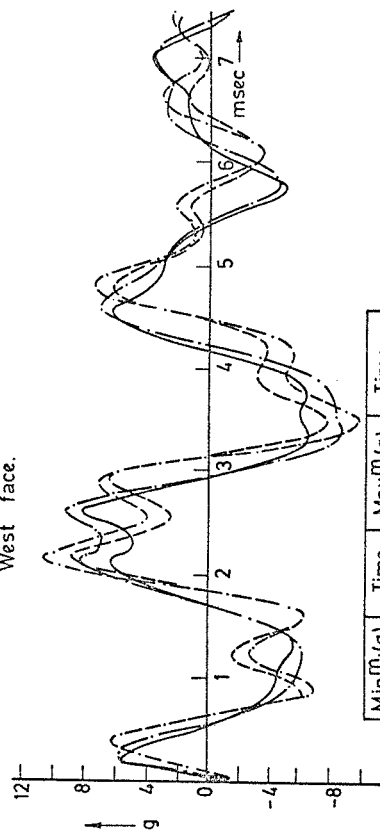


Roof



| Min <sup>m</sup> (g) | Time (msec) | Max <sup>m</sup> (g) | Time (msec) |
|----------------------|-------------|----------------------|-------------|
| -7.05                | 5.62        | 7.51                 | 4.75        |
| -7.65                | 6.16        | 7.63                 | 0.39        |
| -13.14               | 5.65        | 12.09                | 2.40        |
| -12.21               | 6.10        | 11.89                | 0.39        |

West face.



| Min <sup>m</sup> (g) | Time (msec) | Max <sup>m</sup> (g) | Time (msec) |
|----------------------|-------------|----------------------|-------------|
| -6.50                | 3.36        | 8.07                 | 2.64        |
| -7.80                | 3.51        | 8.86                 | 2.19        |
| -8.72                | 3.42        | 9.26                 | 2.64        |
| -9.87                | 3.54        | 10.68                | 2.19        |

Angle of inclination.

- Pressure doubling on front face.
- - - Pressure doubling on front face.
- · - · - Pressure doubling on front face and top.
- · · · · Pressure doubling on front face and top.

30°  
60°  
30°  
60°

$$\gamma = 0.39; \quad \rho_{st} = 1.818 \times 10^2 \text{ lb/in}^2; \quad E = 3.1 \times 10^5 \text{ lb/in}^2; \quad \beta = 0$$

FIG.5.8 THE EFFECT ON THE MODEL'S ACCELERATIONS OF CHANGING THE N-WAVE'S ANGLE OF INCLINATION . NORTH FACE NORMAL .

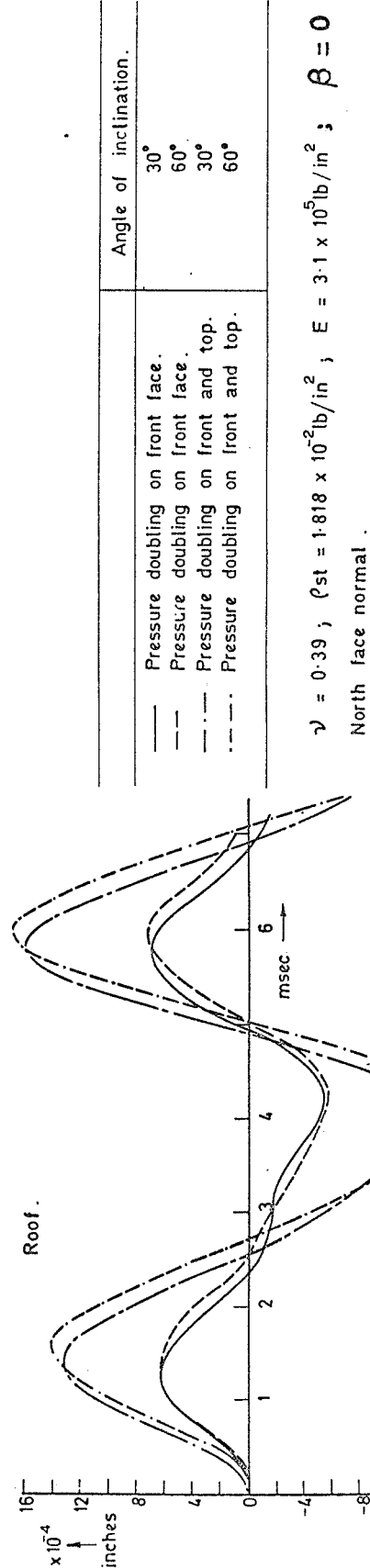
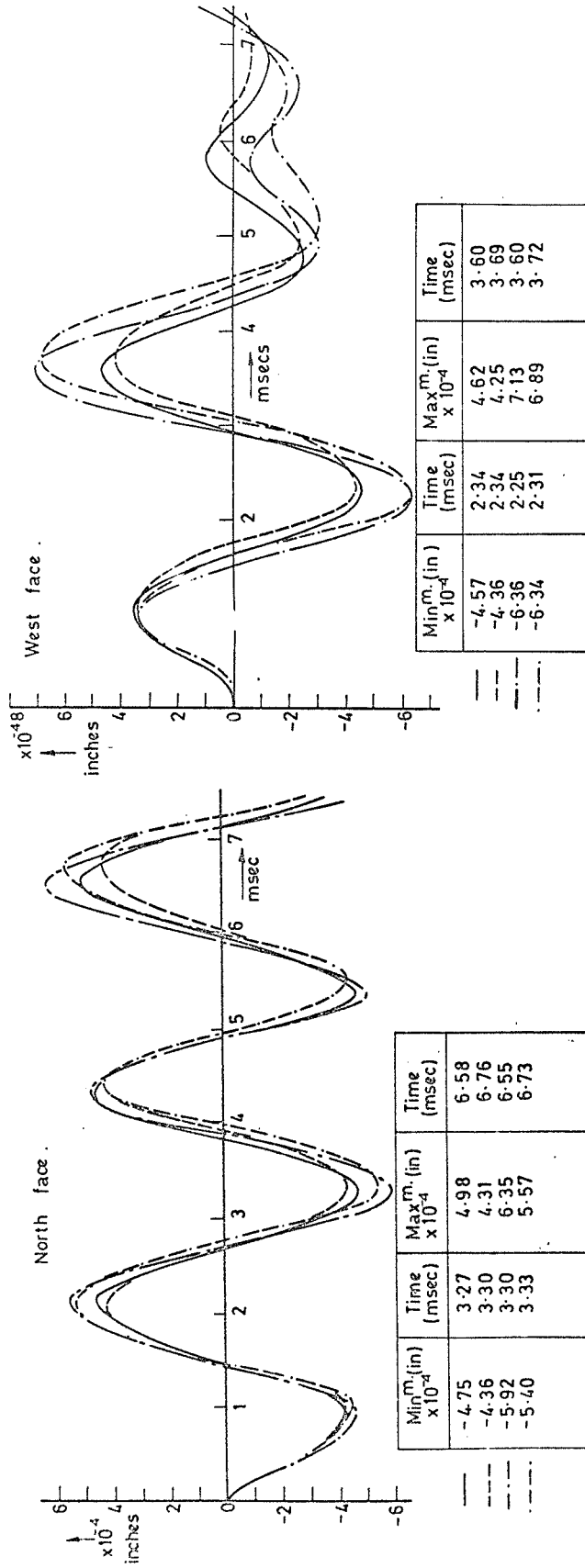
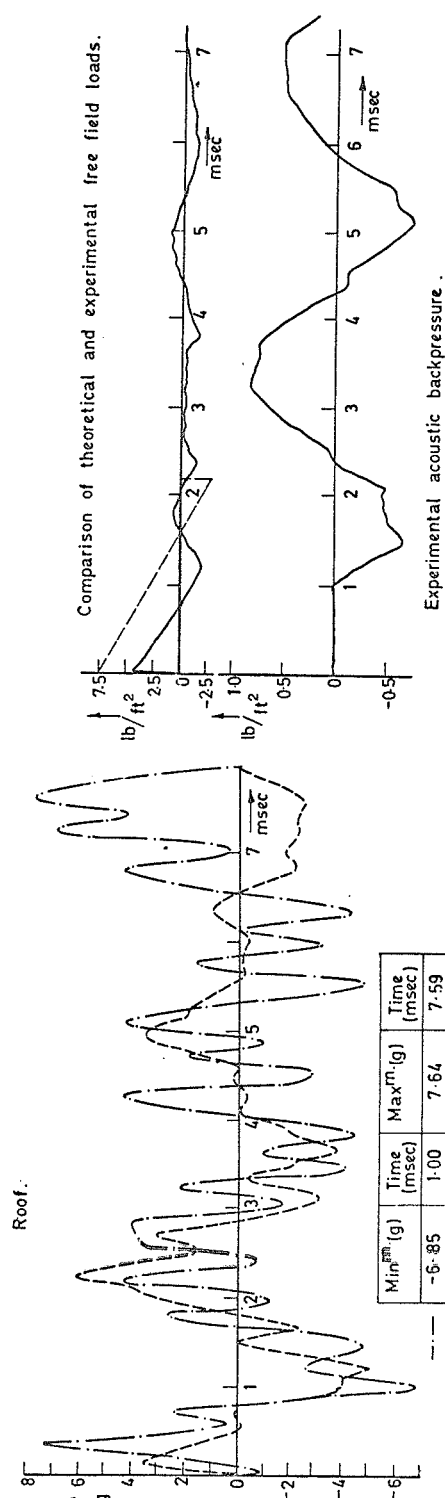
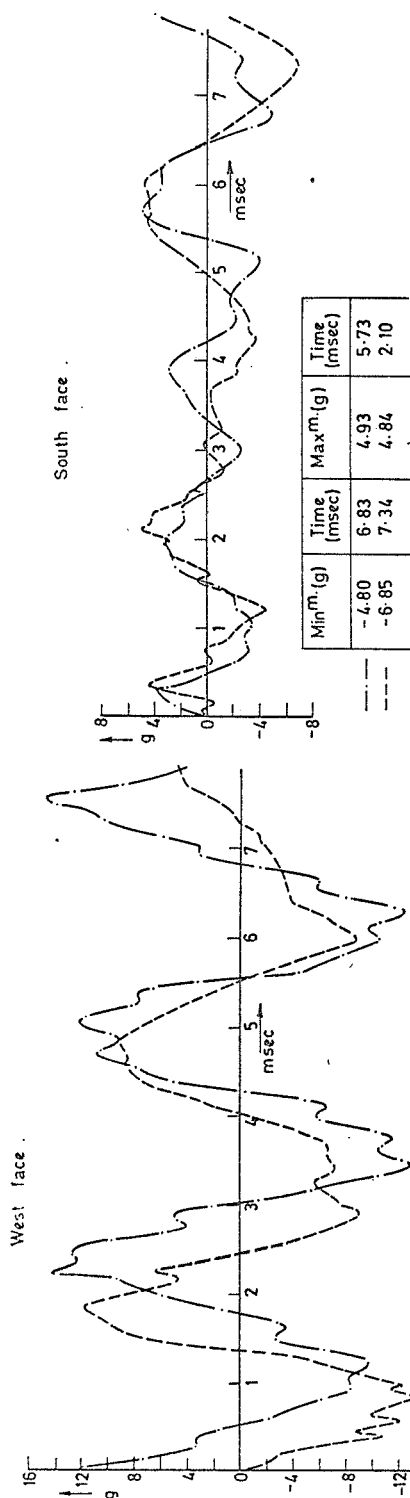


FIG. 5.9 THE EFFECT ON THE MODEL'S DISPLACEMENTS OF CHANGING THE N-WAVE'S ANGLE OF INCLINATION.

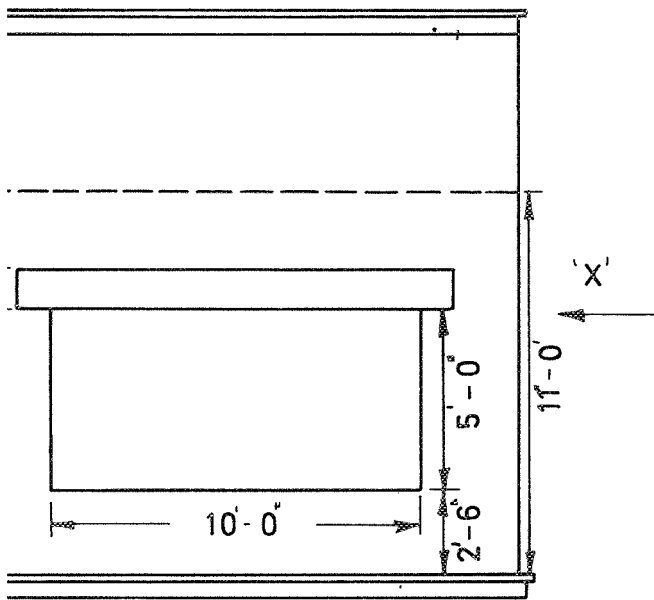


West face normal .

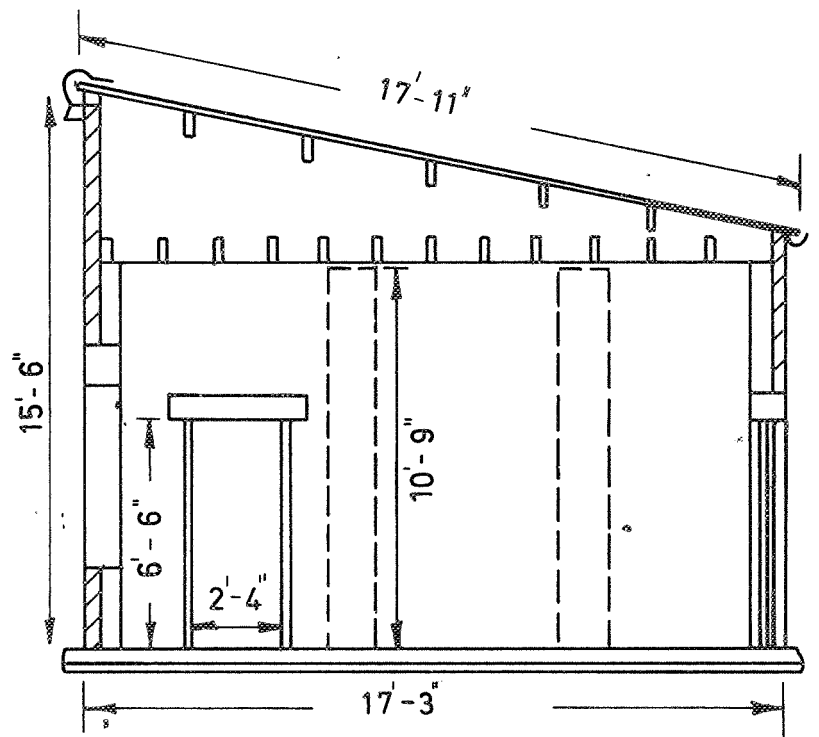
--- No east wall , theoretical . --- No east wall , experimental .

$\gamma = 0.39$  ;  $\rho_{st} = 1.818 \times 10^{-2} \text{ lb/in}^2$  ;  $E = 3.1 \times 10^5 \text{ lb/in}^2$  ;  $\beta = 0$

FIG. 10 THE EFFECT OF CHANGING THE STRUCTURAL PARAMETERS - NO EAST FACE .

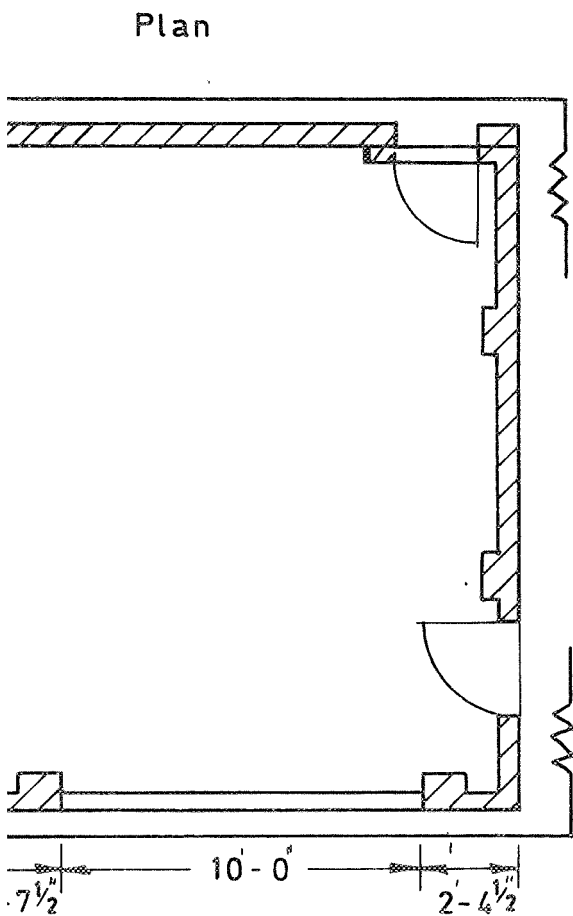


Front elevation.

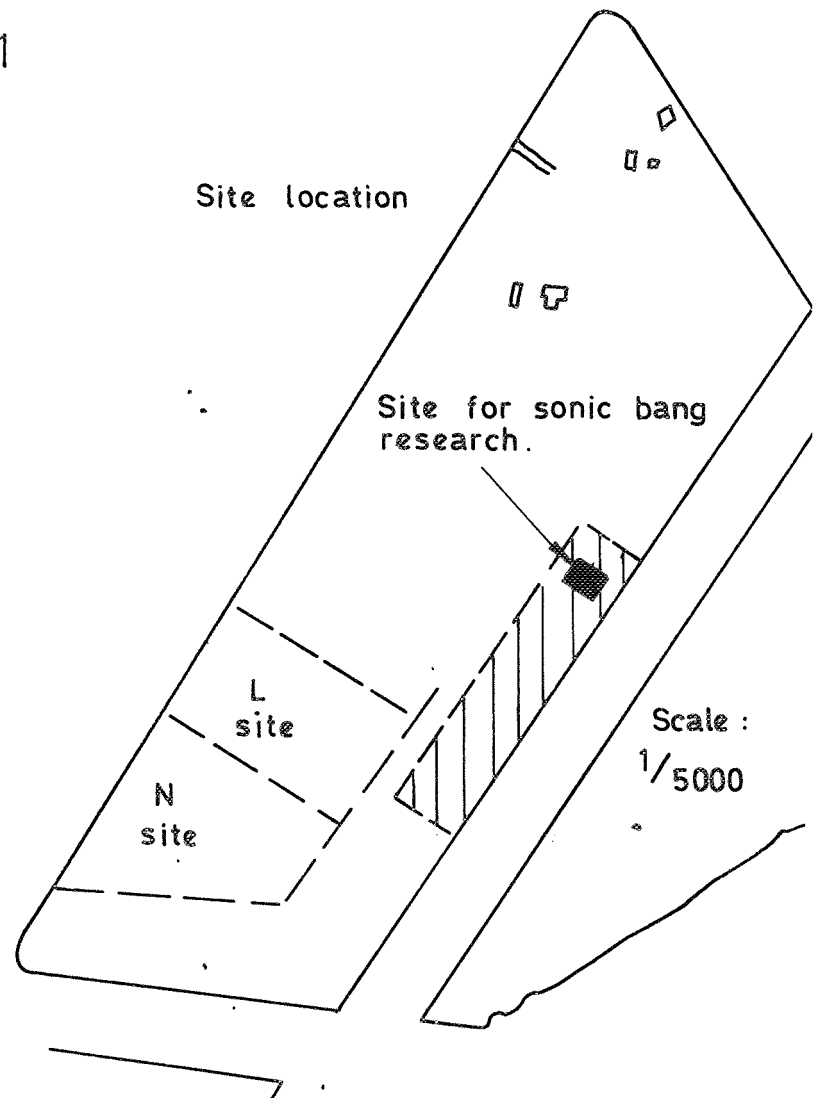


Side elevation 'X'

Fig. 6.1



Plan



Site location

Site for sonic bang research.

L site

N site

Scale : 1/5000

# Variability of simulated waves

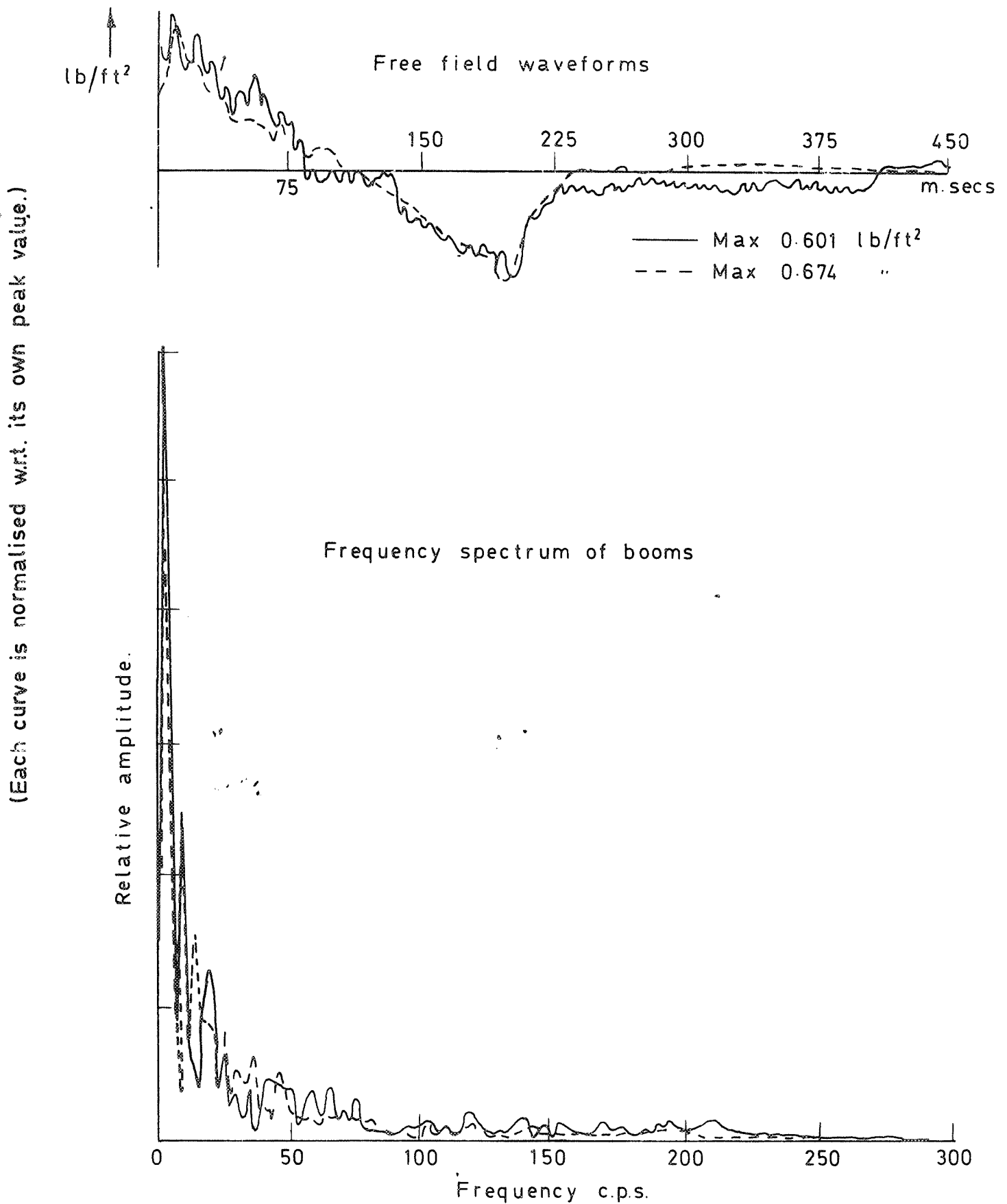


Fig. 6.2

# Variability of corresponding structural components.

(Each curve is normalised w.r.t. its own peak value.)

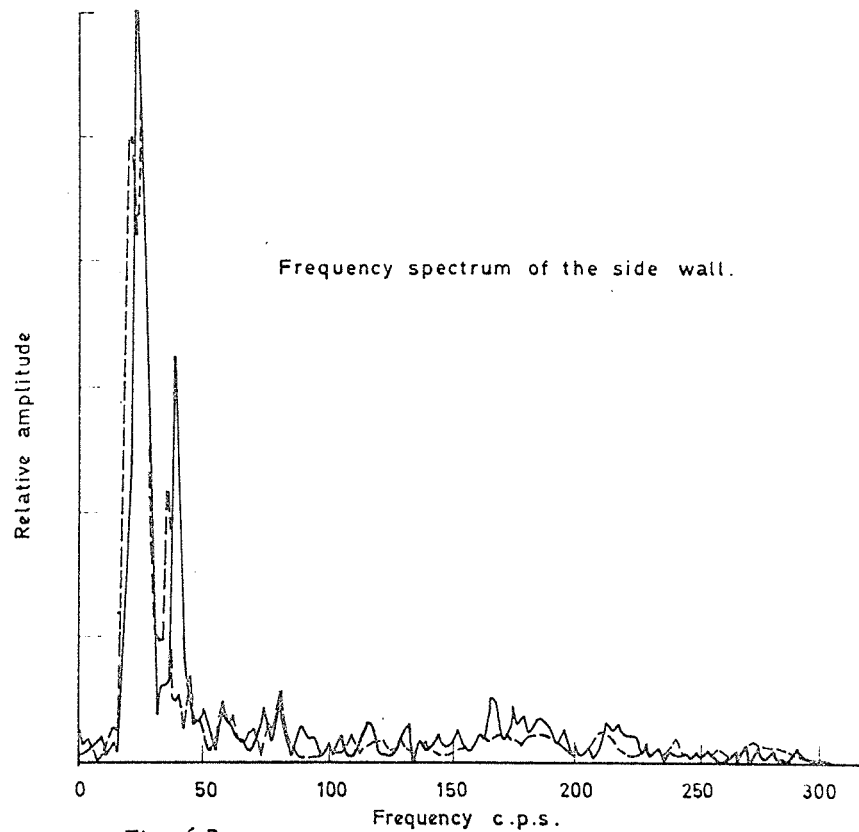
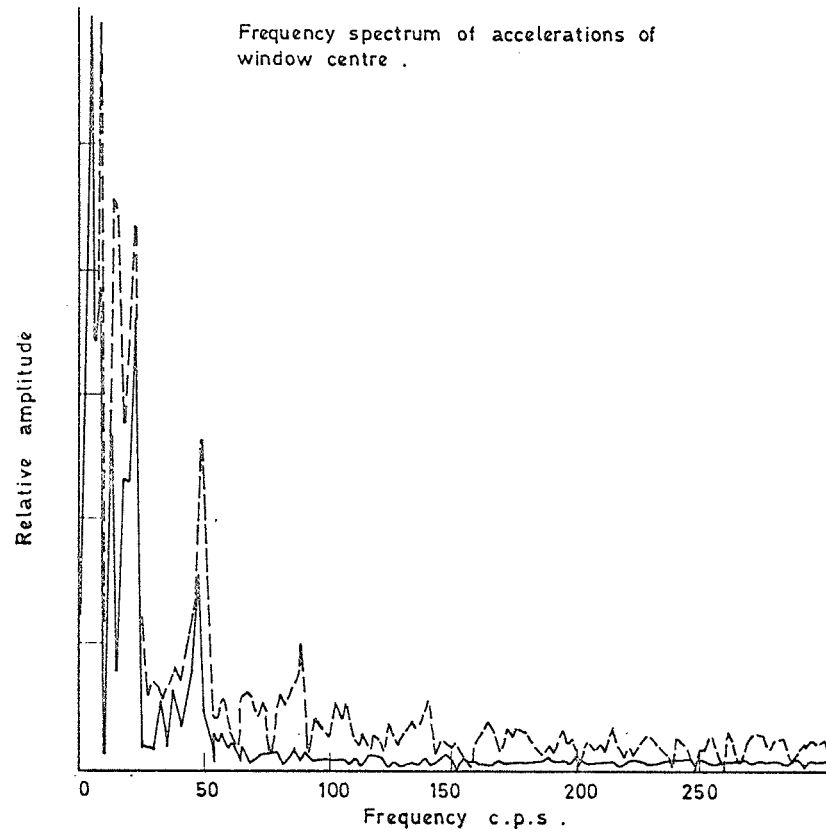
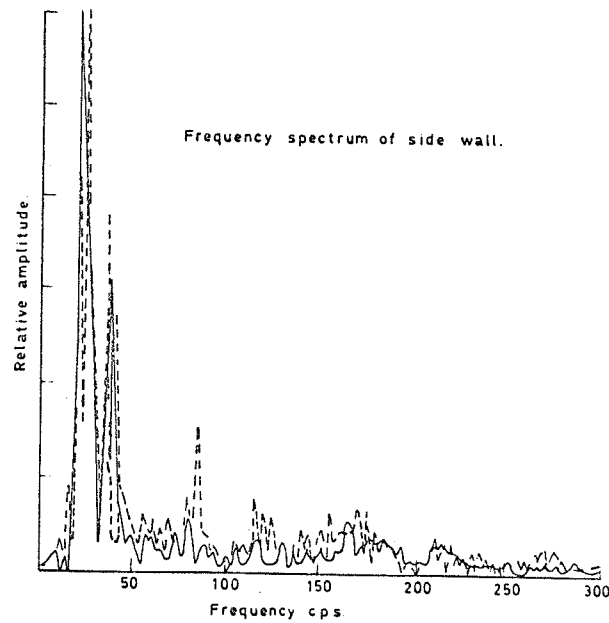
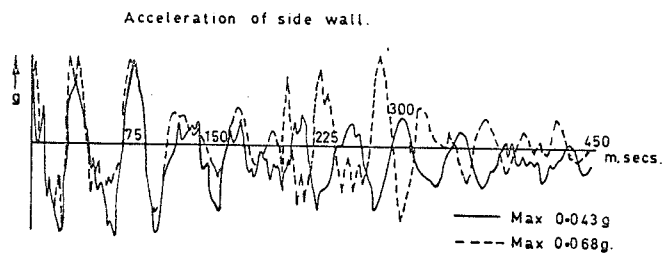
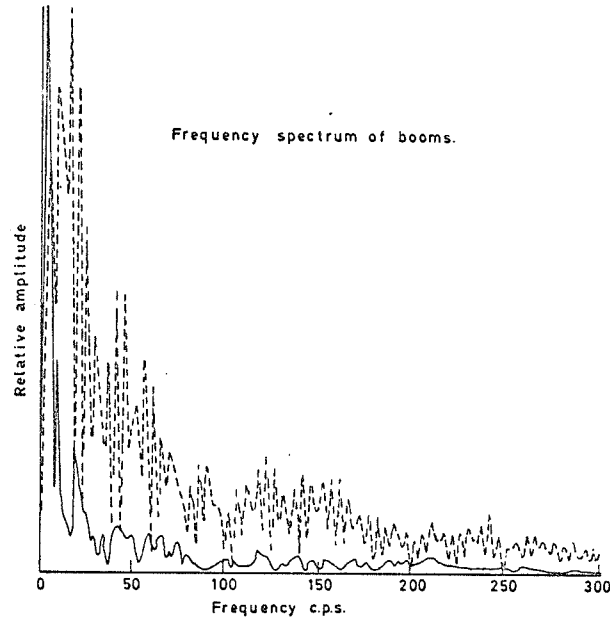
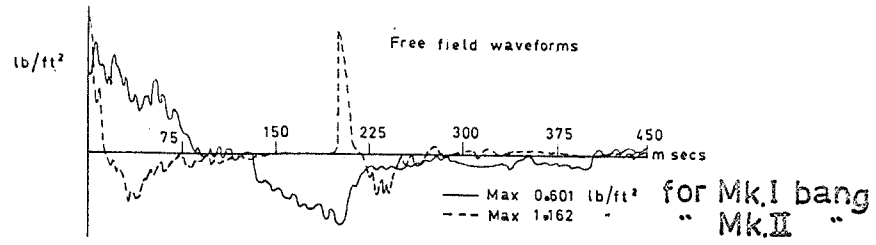


Fig. 6.3

# Comparison of Mk I and Mk II bangs.



(Each curve is normalised w.r.t. its own peak value.)

Fig 6.4

(Each curve is normalised w.r.t. its own peak value.)

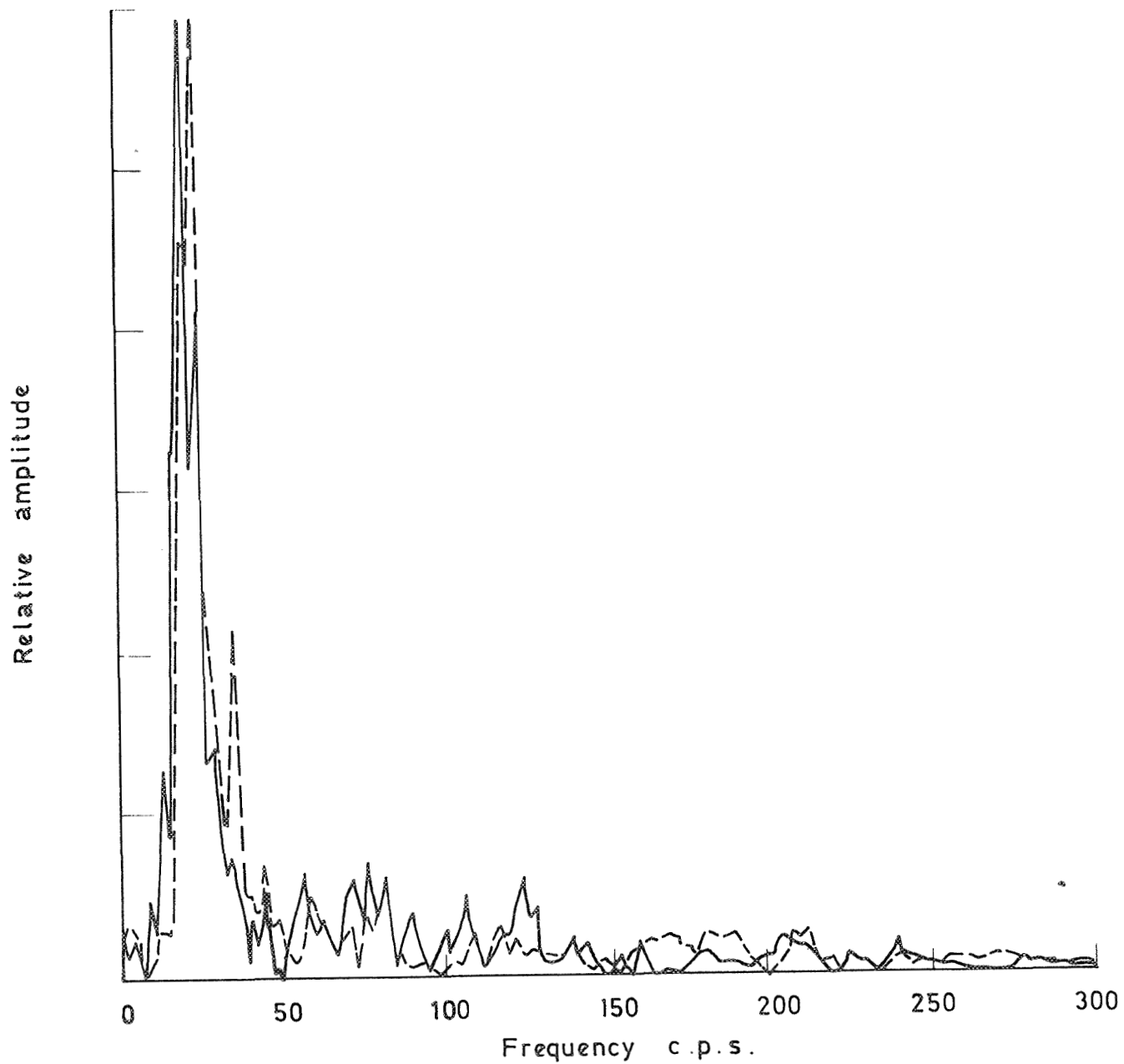


Fig.6.5 Comparison of the frequency spectra of opposite side walls for the 200 ms mk.1. bang.

(Each curve is normalised w.r.t. its own peak value.)

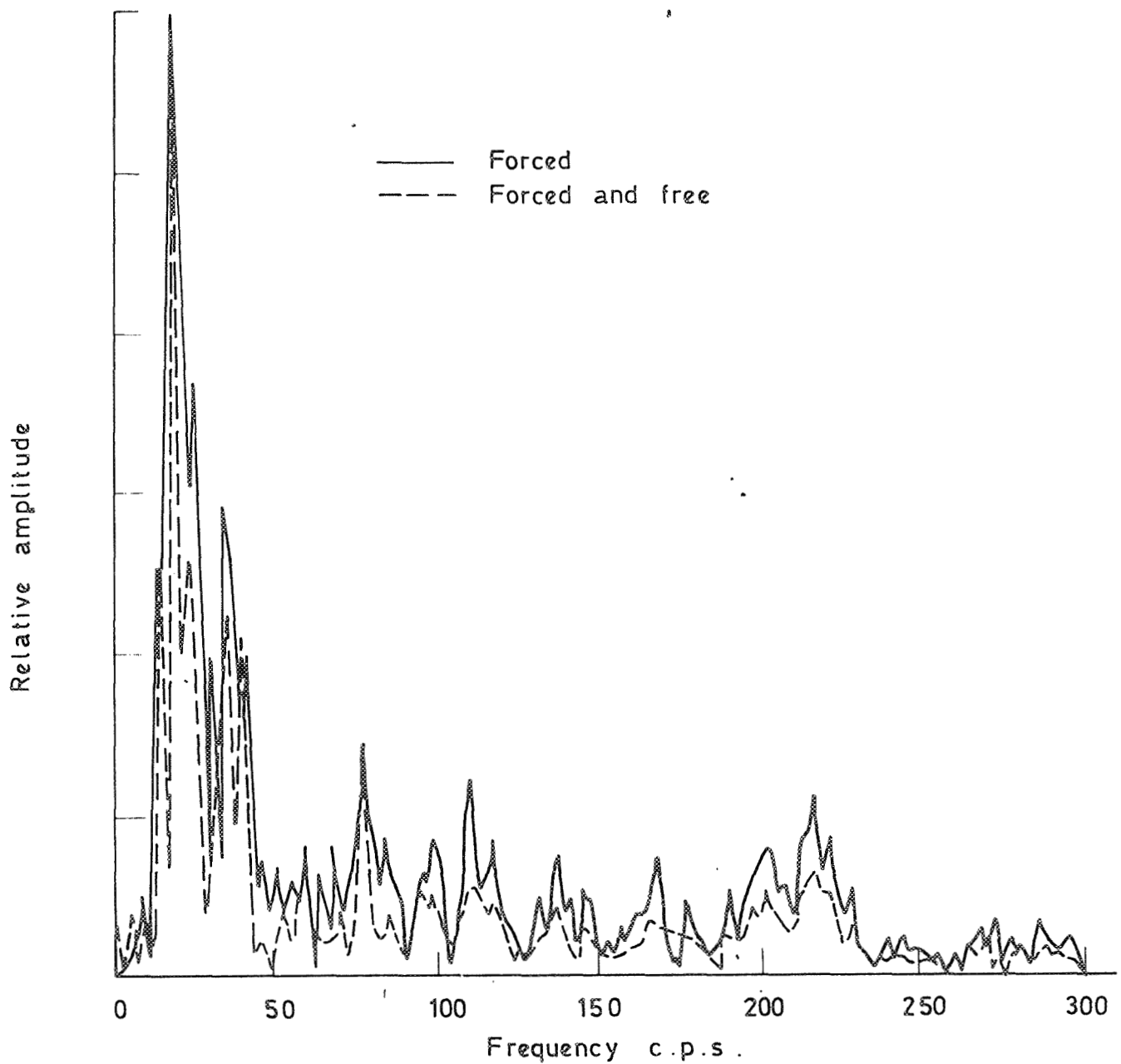


Fig. 6.6 Comparison of the forced and free frequency response of the rear wall

Comparison of acceleration frequency response  
of various faces for a 100 m.sec Mk.I bang .

(Each curve is normalised w.r.t. its own peak value.)

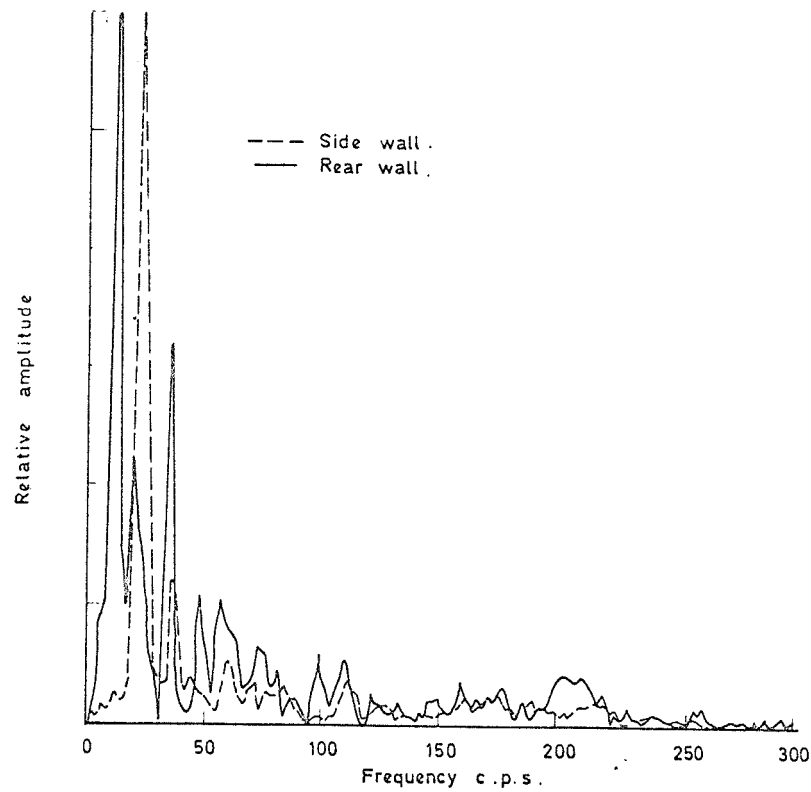
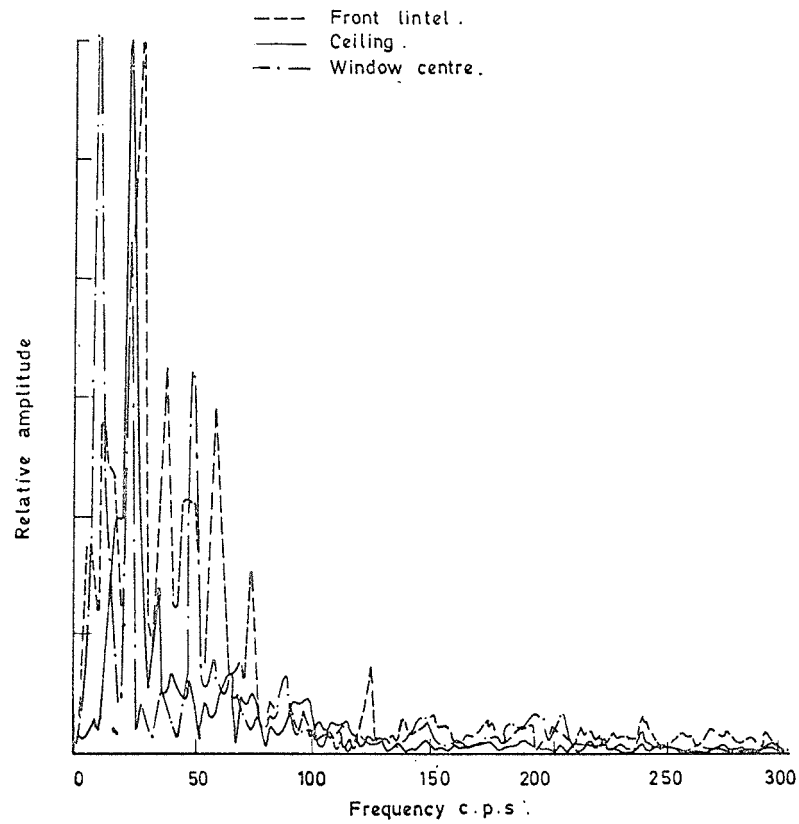


Fig. 6.7.

INCREASING INCIDENCE  $\theta$  (W.R.T. WINDOW FACE)

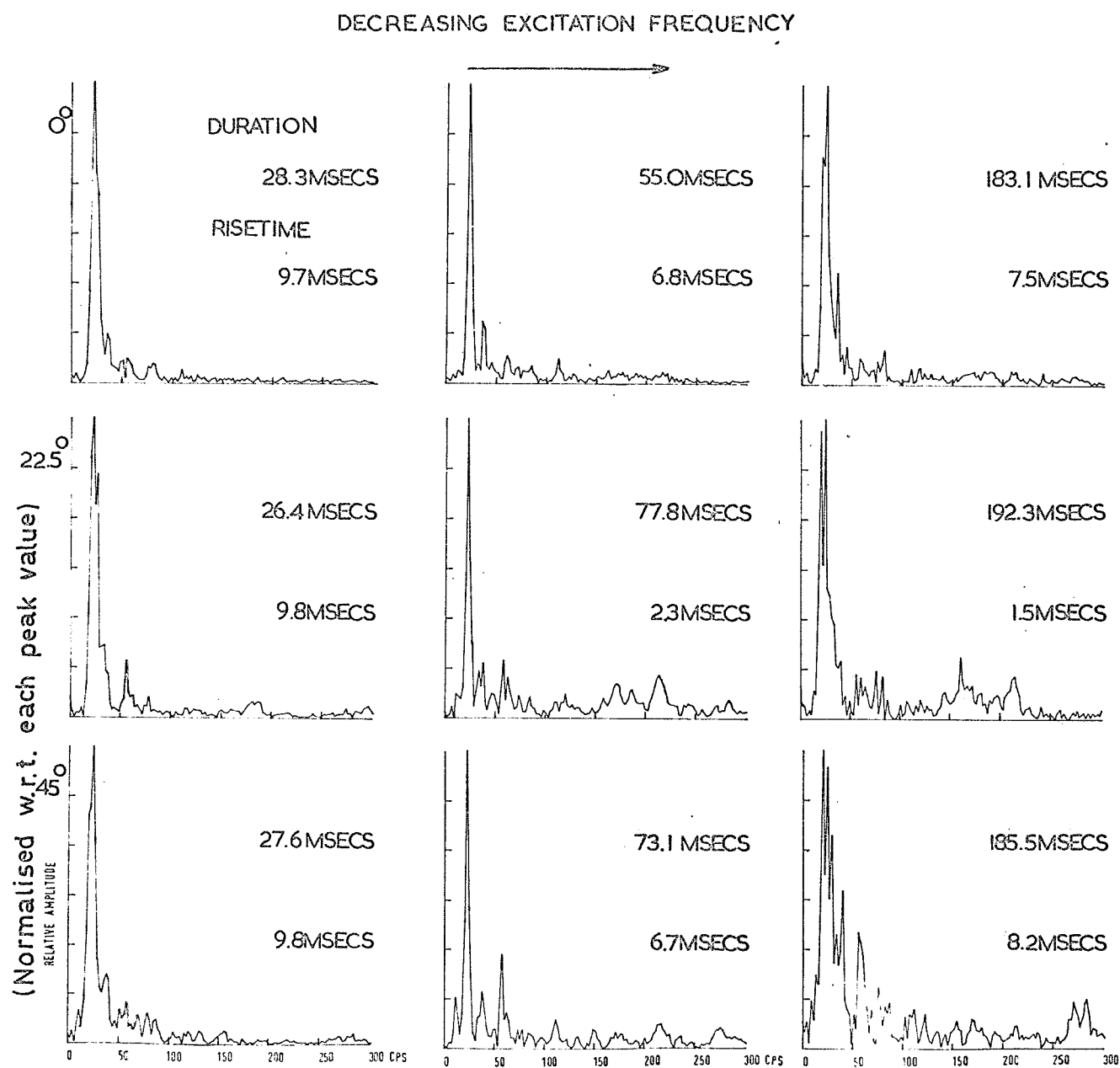


FIG.6.8 FOURIER ANALYSIS OF RIGHT HAND SIDE WALL'S ACCELERATION

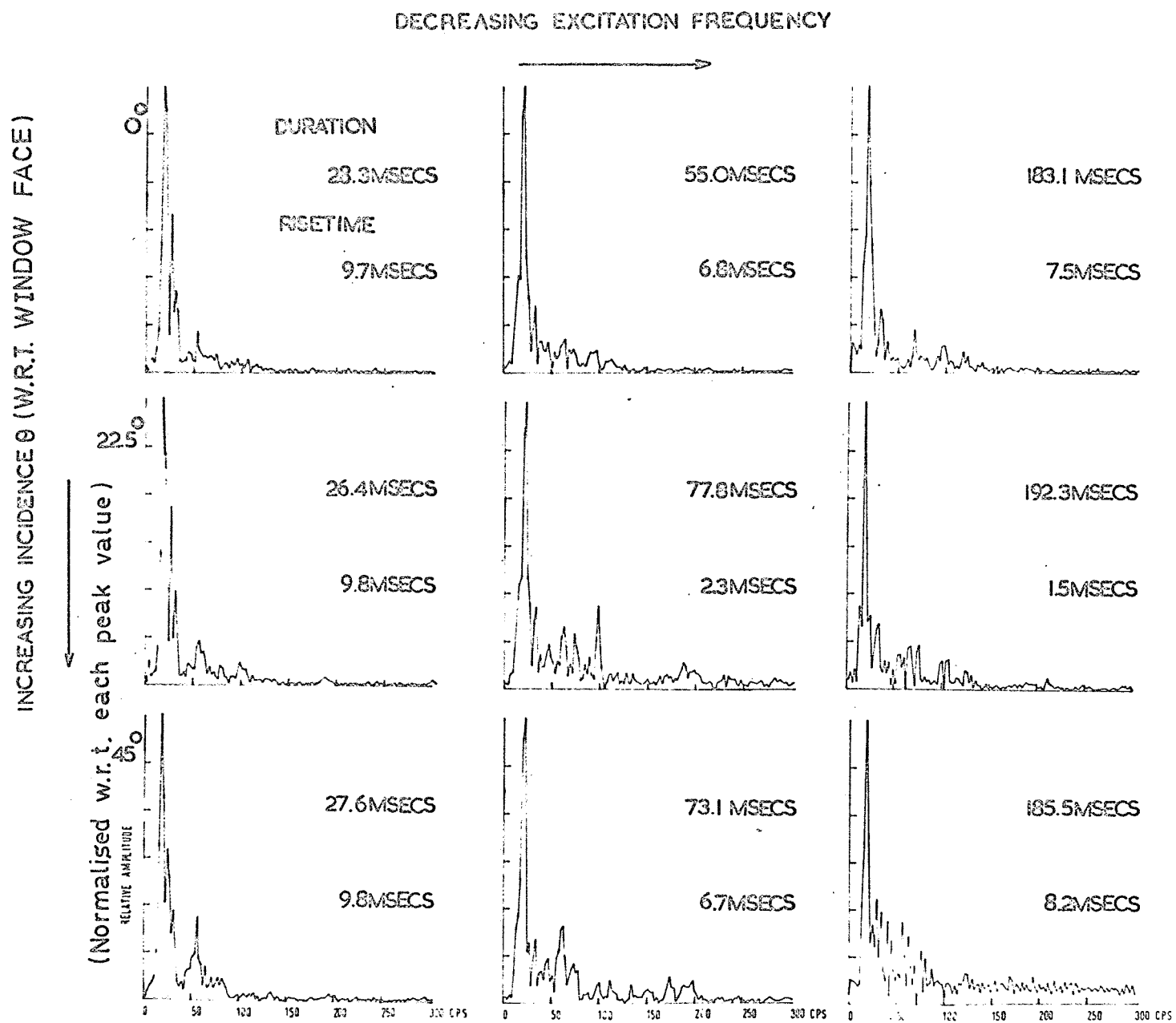


FIG.6.9 FOURIER ANALYSIS OF CEILING'S ACCELERATION

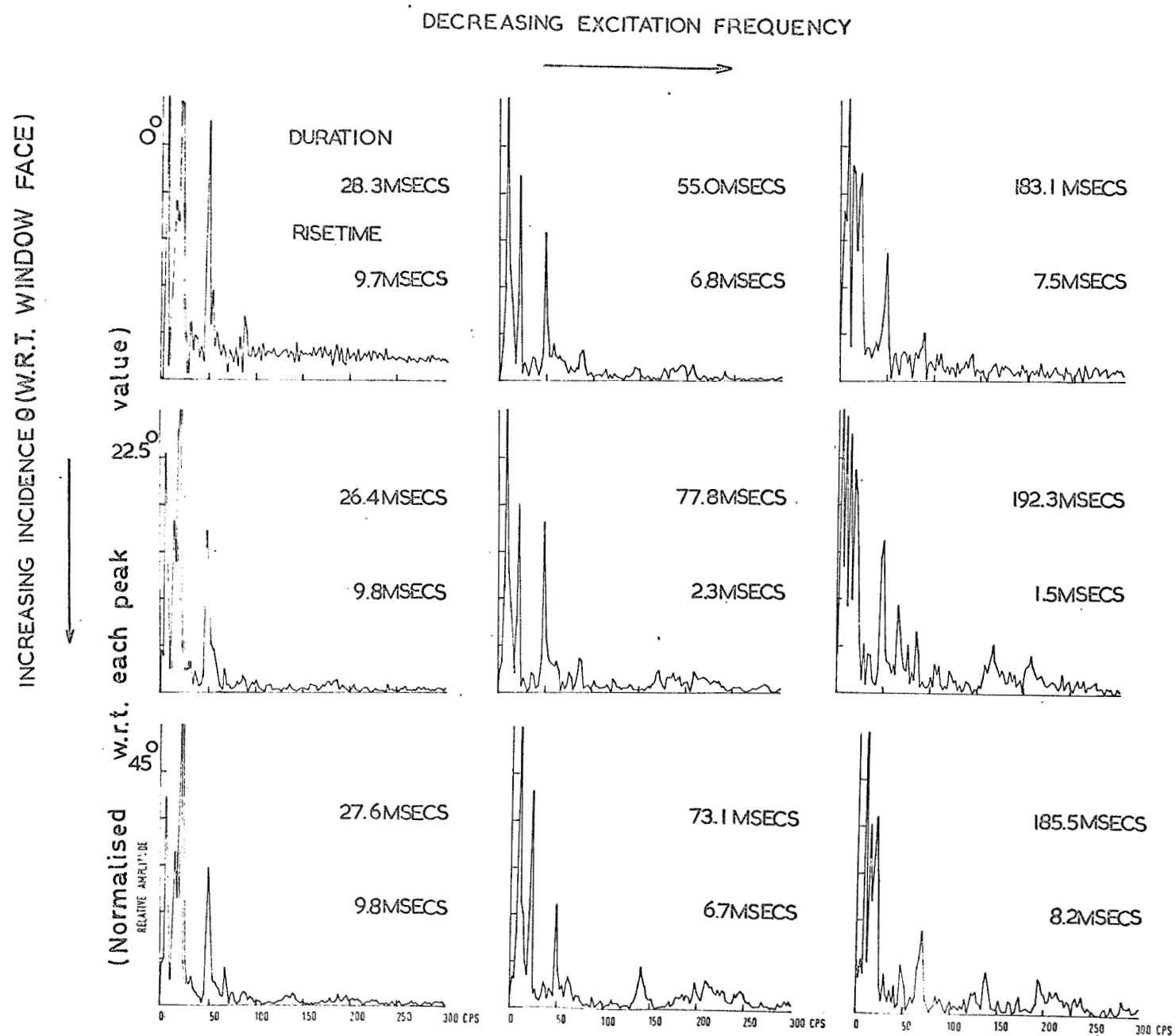


FIG.6.10 FOURIER ANALYSIS OF WINDOW CENTRE'S ACCELERATION

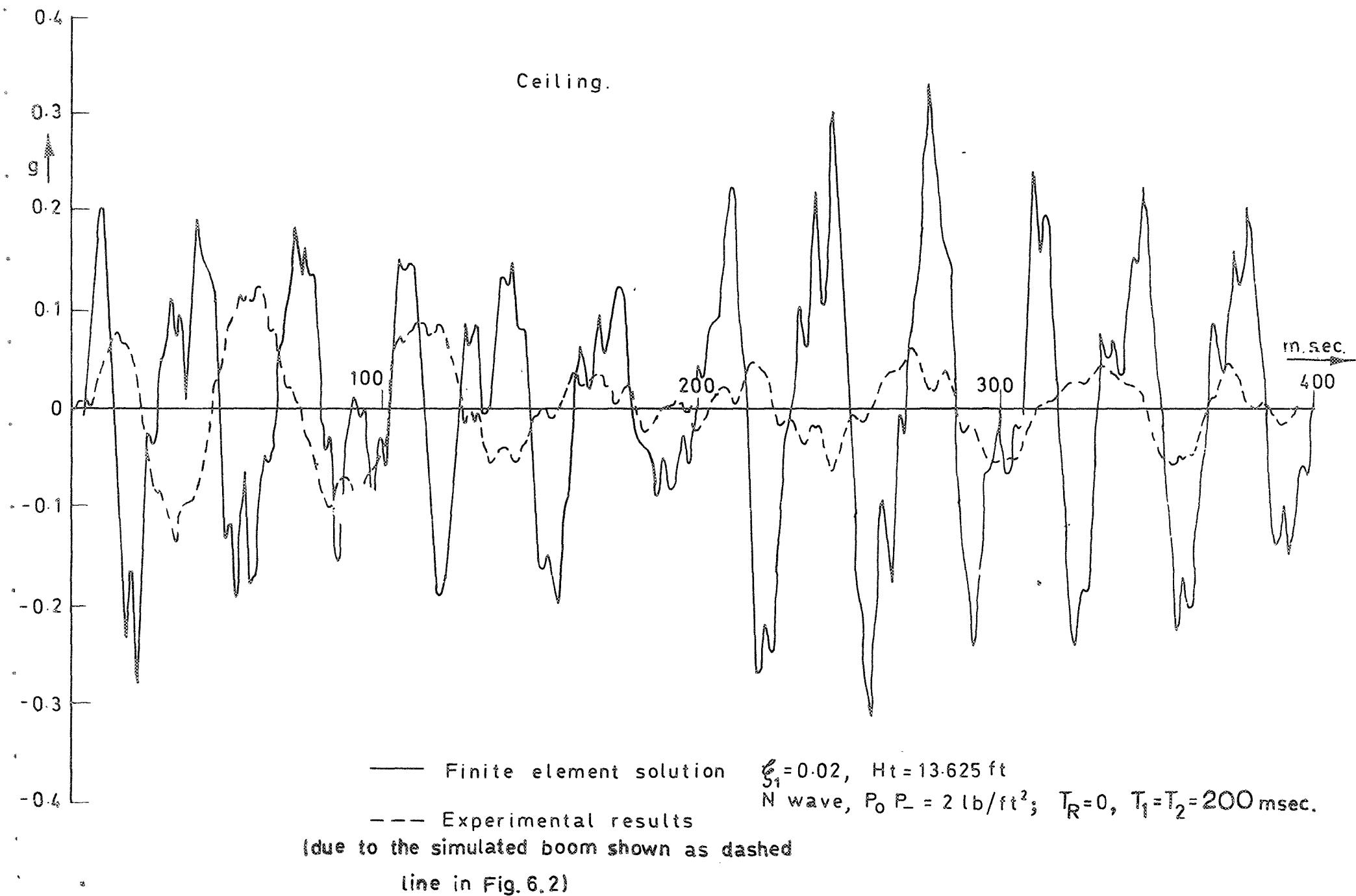


Fig. 7.1

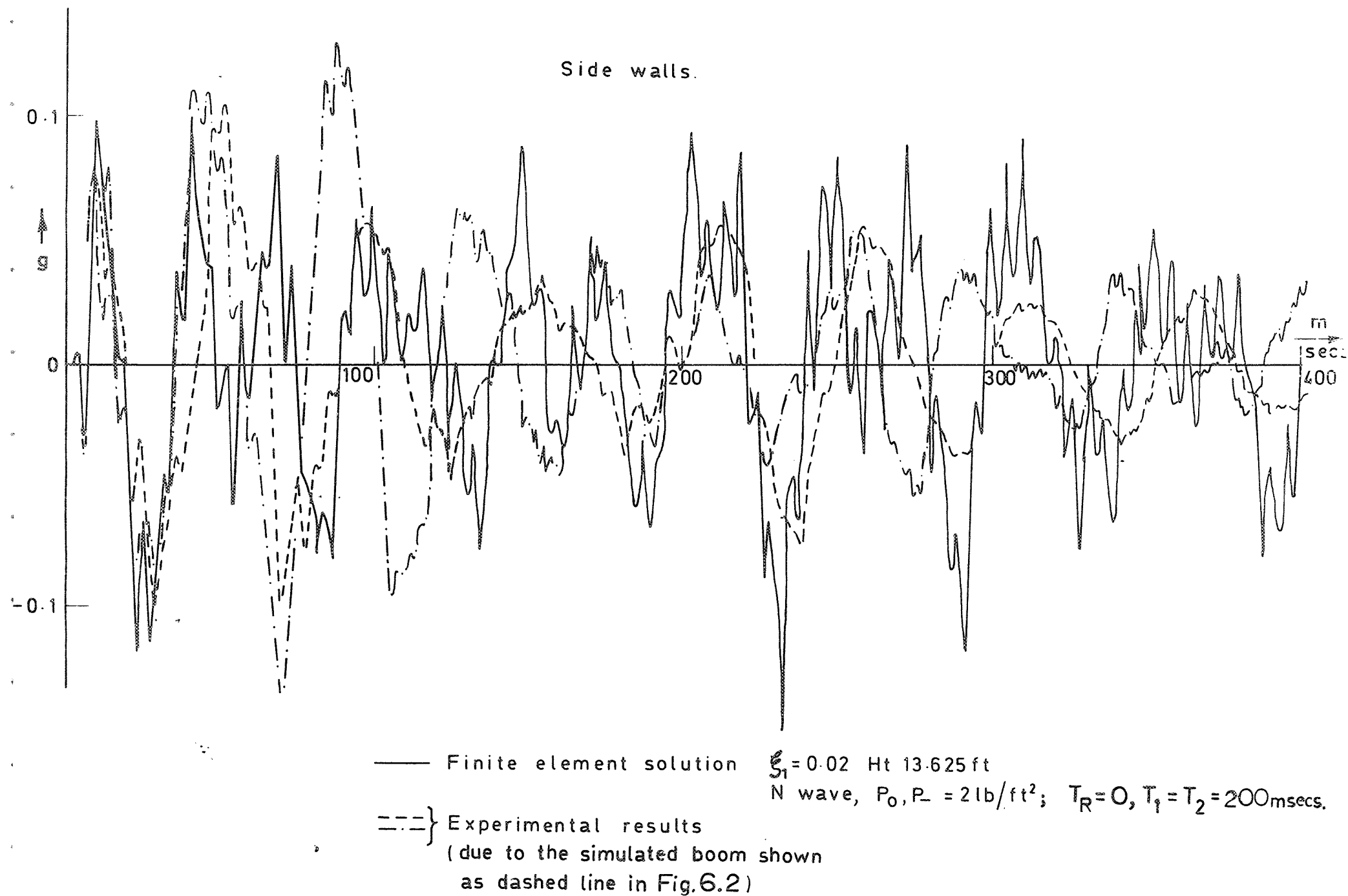


Fig. 7.2 Comparison of finite element and full scale experimental accelerations.

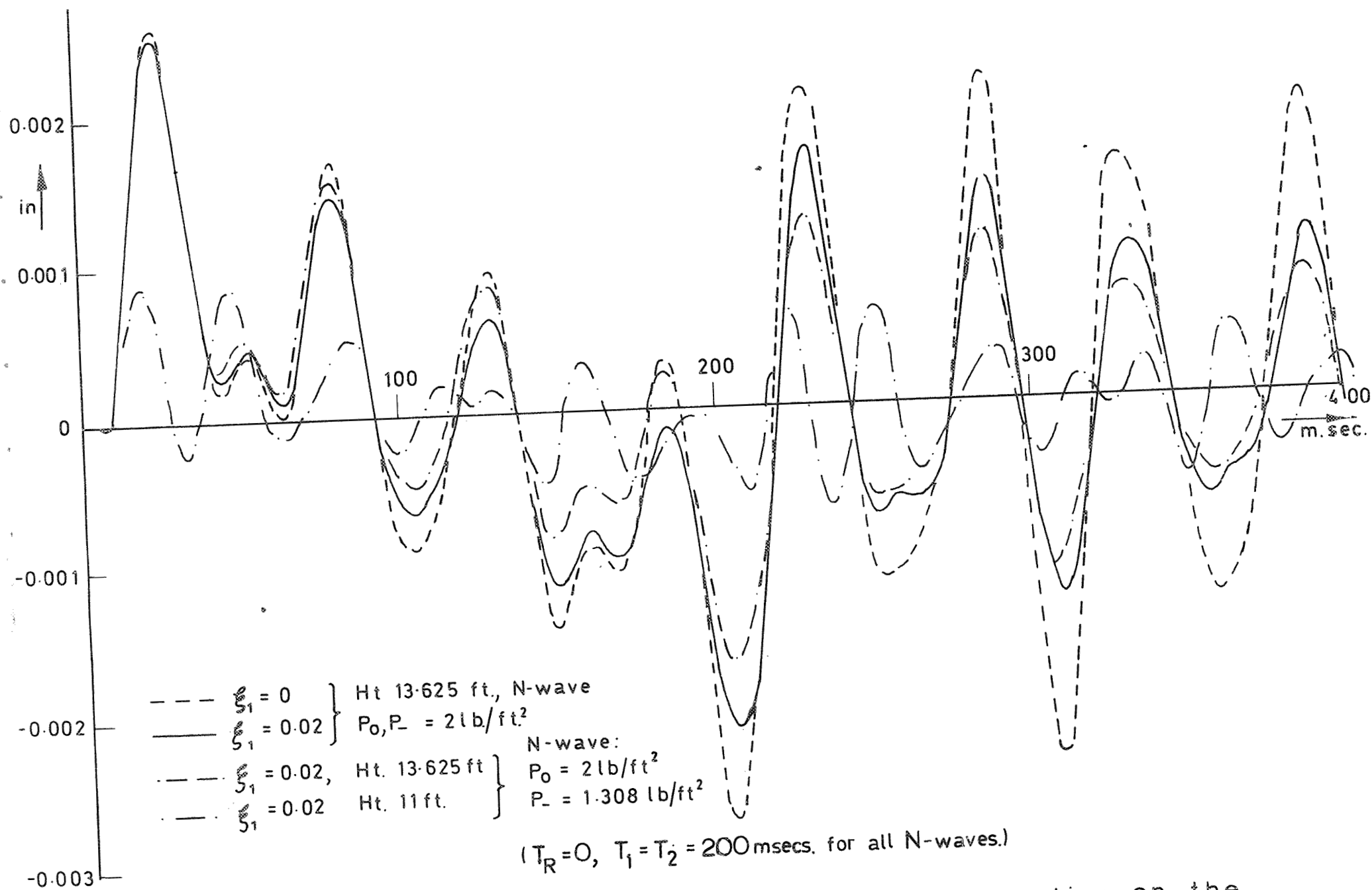


Fig.7.3 Effect of changing structural and boom properties on the displacement of the side wall.