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SPECTRAL ANALYSIS OF DATA FROM THE "SIRIO" FLUXGATE MAGNETOMETER

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1.0 INTRODUCTION

The magnetic field experiment on the SIRIO spacecraft uses a triaxial set of sensors to obtain precise measurements of the vector magnetic field (Ness et al., 1969). An early assignment of telemetry allocated a total of 96 words of 8 bits each in the telemetry format, as shown in Figure 1.

The sampling rate of the instrument was 4 times per frame, uniformly spaced, so that the Nyquist frequency was 2 times the frame rate (of 1 per second) or f_N = 2 Hz. This frequency is twice the nominal spin frequency of the satellite but only slightly higher than the proton cyclotron frequency, f_p = 1.7 Hz, assuming a field of 110 gamma. Time variations of the field of up to 50% are expected so that fp will range between 0.85 and 3.4 Hz.

In mid-October, discussions of the possible use of more telemetry for the magnetic field experiment were held. The net result was that the sampling rate could be doubled in frames 0-6, but not in frame 7, and of equal importance an increase in the precision for each A/D conversion from 8 to 9 bits was also permitted. This substantially improves the experiment since the digitization uncertainty is now $\pm$85 gamma/512 = $\pm$0.16 gamma while the Nyquist frequency is 4 Hz, well above the cyclotron frequency.

However, since in frame 7 the sampling rate is only one half that of the other frames, a question arises about the effects in spectral analysis using data which periodically contain missing values. It is the purpose of this note to investigate this problem in the specific situation of the SIRIO experiment.

This note will also outline the general principles for analyzing the effects on spectral analysis of uniformly sampled data streams which contain missing values.

2.0 MATHEMATICAL BASIS (Bracewell, 1965)

It is well-known that a continuous time function, $f(t)$ whose spectrum $F(f)$ is band limited from 0 to f_N Hz, is uniquely representable for all values of t by uniformly spaced discrete samples at a rate $f_s = f_N * 2$.

That is, given $f(t)$ then multiplication by

$$\sum_{K=-\infty}^{+\infty} \delta(t-k\Delta t)$$

will produce a discrete time series at the sample points $t_k = k\Delta t$ such that $F(f)$ can be uniquely determined ($\Delta t = 1/f_s = 1/2\, f_N$). $\delta(t-t_o)$ is an impulse at $t=t_o$ such that $\delta\,(t-t_o) = 0$ everywhere except at t_o when it $= \infty$ and normalized so that.

$$\int_{-\infty}^{\infty} \delta(t-t_o)dt = 1.$$

The spectrum $F(f)$ is then replicated in the frequency domain so that the discrete time series spectrum is given by

$$\sum_{k=-\infty}^{+\infty} F\,(f-k/\Delta t) \tag{2.1}$$

For the analysis to be performed, use shall be made of the following three Fourier Transform theorems:

Addition: the spectrum of time series A plus time series B is simply the sum of the corresponding spectra, i.e.

(2.2) $$F_{A+B}(f) = F_A(f) + F_B(f)$$

Shifting: The spectrum of a time series A offset in time by an amount Δt is given by the produce of the spectrum and exp $(-i2\pi f\Delta t)$ i.e.

(2.3) $$F_{A(t+\Delta t)}(f) = F_{A(t)}(f) * e^{-i2\pi f\Delta t}$$

Similarity: the spectrum of a time series A', distorted in time linearly by a factor a, is given by the original spectrum distorted in frequency linearly by a^{-1} and in amplitude by $1/|a|$

(2.4) $$F_{A'(f)} = \frac{1 \quad F(f/a)}{|a|}$$

where $f_{A'}(t) = f(at)$

3.0 APPLICATIONS

In our study of the SIRIO uniformly sampled time series with periodic data gaps, we will assume the original time series, $f(t)$, to be appropriately band limited so that $F(f)=0$ for $f \geq f_N$ where f_N is computed on the basis of 8 samples per frame, i.e. $F_N = 4$ Hz.

We assume that the repetitive time base for the time series is 8 frames or one format. If data points were available at the uniform rate of f_s, then the length of this time base would be $8 \times 8 \times \Delta t = 64\Delta t$. The real window can be considered to be of equal length but with some of the data points missing.

The SIRIO sampling window is then given by the following function:

$$(3.1) \qquad w(t) = \sum_{k=-\infty}^{+\infty} W_k \, \delta(t-k\Delta t)$$

where

$$(3.2) \qquad \begin{array}{llrl} W_K = 1 & \text{for} & -28 \leq K \text{ (modulo 64)} \leq & +28 \\ W_K = 1 & \text{for} & K \text{ (modulo 64)} = & \pm 30 \\ W_K = 0 & \text{for} & K \text{ (modulo 64)} = & \pm 29;\ \pm 31 \\ W_K = 0.5 & \text{for} & K \text{ (modulo 64)} = & \pm 32 \end{array}$$

The origin has been selected so that the sampling window is an even function of time and the repetition rate is the format rate. (See Figure 2).
The weights for the end values are one half because the window overlaps at these terminal points.

Spectral analysis of the SIRIO data will yield a result which is the convolution of the true spectrum F(f) with the spectrum of the window given in (3.1). The spectrum of the window in 3.1 is readily obtained by the use of the three theorems given in (2.2) to (2.4).

The principal effect to be noticed from application of the theorems is that the spectrum of the window (3.1) will contain lines (impulses of variable amplitude) at frequencies equal to 1/64 of the sampling frequency. We shall refer to these as "parasitic lines". That is, the spectrum of (3.1) is

$$(3.4) \qquad W_{SIRIO}(f) = \sum_{k=-\infty}^{+\infty} b_k \, \delta\left(f - \frac{k}{64} f_s\right)$$

The values of the b_K are determined by the values of the W_K as follows:

$$W_{SIRIO}(f) = 1 + \sum_{k=0}^{32} W_K \exp (i\ 2\pi\ K\ f/f_N) \tag{3.5}$$

Note that if all the $W_K = 1.0$ except for K (modulo 64)=32 when W_K-0 5, then b_K=0 for all K except K(modulo 64 = 0 when $b_K = 1.0$. This of course is what would be expected since the completely filled data window for SIRIO should be invariant to a selection of time base for repetition. The actual time window for SIRIO is as given in (3.2) and the corresponding spectral window is shown in Figure 3a for $0 \leq f \leq f_N$.

The effect of the missing sample points is to introduce relatively small amplitude lines (< 7% of the main line at f=0). These lines will lead to a contamination of all spectral estimates since the estimated spectrum from the SIRIO data will be related to the true spectrum through the convolution of (3.3) with F(f). That is

$$F_{estimated}\ (f) = \int_{-\infty}^{+\infty} F_{true}(\xi) * W_{SIRIO}(f-\xi)\ \ d\xi \tag{3.6}$$

The contaminating lines are seen to be both small and of alternating sign. Thus, depending upon the exact properties of F(f), i.e. the position of lines and the general shape, F_{est} (f) may or may not be a valid approximation to F(f). Especially in the interpretation of the spectral results, should one be aware of these contaminating effects.

As an example, if the true spectrum of $F(f)$ contained only one line of amplitude A at $f = \alpha f_N$, then the estimated spectrum would be given by

$$F_{est}(f) = A * W_{SIRIO}(f - \alpha f_N) \tag{3.7}$$

and a whole set of parasitic lines would be computed and appear to be present in the estimated spectrum.

One other note here is that in this treatment we have assumed an infinitely long time series so that the spectra are all composed of lines. In any real computation we shall be faced with use of a finite length record (time interval = T), the effect of which is to broaden each line by convolution by the transform $(\sin(\pi fT)/\pi fT)$ of the data window corresponding to the available time interval. In order to avoid overlapping the individual parasitic lines, at a spacing of $\frac{1}{32} f_N$, it would be necessary to consider using a time series on the order or greater than 32 times the sequence length.

Another important note is that the present discussion has dealt exclusively with the amplitude spectra. It is most probable that in practice the power spectra will be computed. The effect of this is to multiply the true power spectrum by the square of the data window's transform.

Thus the maximum contamination for the raw SIRIO window will be $(0.065)^2 = 0.42\%$ of the main peak. At the same time, we should be aware that any overall amplitude spectral slope such as $f^{-\alpha}$ will now become $f^{-2\alpha}$ and the problem of contamination is improved only to the extent that the alternating signs of the parasitic lines leads to cancellation of their individual contribution. This cancellation occurs only for the amplitude spectra.

4.0 IMPROVEMENTS

It is possible to consider improving the accuracy of SIRIO spectral estimates, at least to some degree, by using techniques similar to those employed for treating spurious data points which occur at random in any real experimental data set. These are due to a variety of causes, such as poor transmission, reception, encoding, decoding, etc. In these cases the conceptually simple and quantitatively defined method of numerical interpolation can be employed.

In this method the value for a missing data point is obtained from a weighted sum of neighboring data points. Classical formulae used in the well known techniques of data smoothing (Hildebrand, 1956) are especially convenient for solution to this problem.

As a first approximation consider using the zeroth order, 3 point formula:

$$y_i = \frac{1}{3} \, (f_{i-1} + f_i + f_{i+1}) \tag{4.1}$$

where y_i represents the smoothed output for three data points, centered about the ith point. If in fact f_i is not available (as would be the case for K (modulo 64) = ± 29, ± 31, then solving for f_i (assumed = y_i) we obtain the estimate

$$y_i = \frac{1}{2} \, (f_{i-1} + f_{i+1}) \tag{4.2}$$

Applying this to the case of SIRIO we have the following weights

$$W_K = 1 \text{ for } K \text{ (modulo 64)} \leq -27$$

$$W_K = 1+\frac{1}{2} \text{ for } K \text{ (modulo 64)} = \pm 28$$

$$W_K = 2 \quad \text{ for } K \text{ (modulo 64)} = \pm 30$$

$$W_K = 1 \quad \text{ for } K \text{ (modulo 64)} = \pm 32$$

Substituting these values in (3.4) yields the corresponding data window shown in Figure 3b. Here is is seen that there is a substantial reduction in the amplitude of the parasitic lines for $f < 0.5f_N$, a slight increase for $0.5f_N < f < 0.75f_N$ and an increase by almost a factor of two for $f > 0.8f_N$.

This window is more attractive, from an analysis viewpoint, since contamination greater than 1% does not occur until $f/f_N > 0.6$. This means that if the true spectrum falls off rapidly with frequency, such as $f^{-\alpha}$ where $\alpha > 1$, only the values near the folding frequency will possibly be contaminated. If the bandpass of the instrument is chosen to be less than f_N, say $f_{LP} < 0.8\ f_N$, then almost undistorted spectral estimates can be obtained.

This technique of using smoothing formulae to estimate the values associated with t_k where K (modulo 64) = ± 29, ± 31 can be extended to higher order, which naturally requires additional data points. In those instances where the estimated value for t_K, K (modulo 64) = ± 31, depends upon the value of t_K K(modulo 64) = ± 29 or viceversa, then repeated or chained use of the formulae is required. Given here without further derivation are the modified weights (which are non-unity) for the following cases:

1st degree, 5 points
See Figure 3c

$$\begin{aligned} W_{\pm 25} &= 1/4 \\ W_{\pm 27} &= 1 + 1/4 \\ W_{\pm 28} &= 2 + 1/2 \\ W_{\pm 30} &= 2 \\ W_{\pm 32} &= 1/2 \end{aligned} \tag{4.4}$$

Also 1st degree, 5 points
See Figure 3d

$$\begin{aligned} W_{\pm 22} &= 1 + {}^{8}/49 \\ W_{\pm 28} &= 1 + {}^{16}/49 \\ W_{\pm 30} &= 1 + {}^{40}/49 \\ W_{\pm 32} &= 1 + {}^{1}/14 \end{aligned} \tag{4.5}$$

3rd degree, 5 points
See Figure 3e

$$\begin{aligned} W_{\pm 28} &= 1 + {}^{1}/4 \\ W_{\pm 30} &= 2^{1}/2 \\ W_{\pm 32} &= 3/4 \end{aligned} \tag{4.6}$$

3rd degree, 7 points
See Figure 3f

$$\begin{aligned} W_{\pm 24} &= 1 + \frac{116}{529} \\ W_{\pm 25} &= 1 - \frac{203}{529} \\ W_{\pm 26} &= 1 - \frac{24}{529} \\ W_{\pm 27} &= 1 + \frac{13}{529} \\ W_{\pm 28} &= 1 + \frac{372}{529} \\ W_{\pm 30} &= 1 + \frac{600}{529} \\ W_{\pm 32} &= \frac{1}{2} + \frac{8}{23} \end{aligned} \tag{4.6}$$

The improvements, or lack thereof, can be seen in Figures 3c, 3d, 3e and 3f respectively.

5.0 COMMENTS

As a part of this study, the effect of changing the location of the missing data points within the data set was investigated. The purpose here was to see whether or not an improvement was realized simply by redistributing the sampling times. The intuitively least influential distribution might seem to be, at first consideration, that of a uniform distribution of missing values. In the case at hand, SIRIO, this would be 7 consecutive values followed by one missing data value. We would expect this to introduce a partioning of the spectral window by eighths and it is precisely what happens, as seen in Figure 3g. Here the parasitic lines, of alternating sign, have equal magnitude = 6.5%.

We conclude that the spacing of gaps periodically is not an improvement since the cancellation enjoyed by alternating signs occurs now over a much wider frequency interval than for the present SIRIO distribution. In addition, the amplitudes are all at the same relatively large value. This result might have been anticipated if we had considered the effects of constructive interference of the missing values, with respect to this time distribution.

The effect of estimating the values in the data gaps by using the simple formula in (4.2) yields the data window shown in Figure 3h. Again the amplitudes of the parasitic lines at frequencies near the main line f=0 are reduced while those near f_N are increased. An improvement is made which is similar in spirit to that of cases 3a, 3b.

If the missing points are immediately adjacent to each other then a slight alteration of these parasitic lines is effected.

6.0 FUTURE STUDIES

It is clear that the use of higher order formula for interpolation (and smoothing) of the data may lead to significant improvements beyond those already achieved. Possibly these shall be worth the computational effort and they definitely should be investigated.

In addition it might be of interest to construct some artificial time series, representative of the anticipated data, and examine the effects of these various windows.

7.0 REFERENCES

Bracewall, R., The Fourier Transform and Its Application, McGraw Hill, N. Y., 1965.

Hildebrand, F. B., Introduction to Numerical Analysis, McGraw Hill, N. Y., 1956.

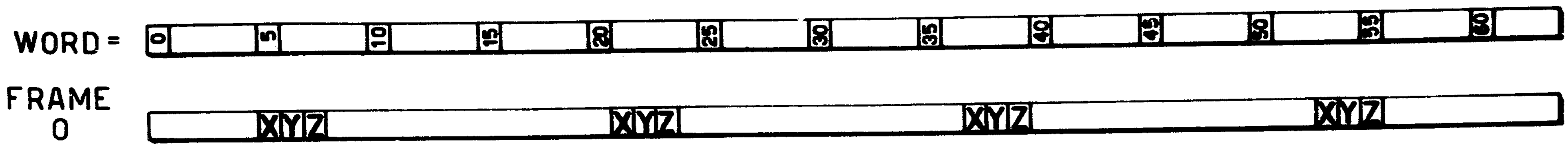
Ness, N. F., D. H. Fairfield, F. Mariani and S. Cantarano, Magnetic Field Experiment for SIRIO Spacecraft, NASA-GSFC X-616-69-293.

8.0 LIST OF FIGURES

9.0 APPENDIX A

The following computer pointouts represent the results of the numerical evaluation of formula (3.5). The weights used are listed in the format:

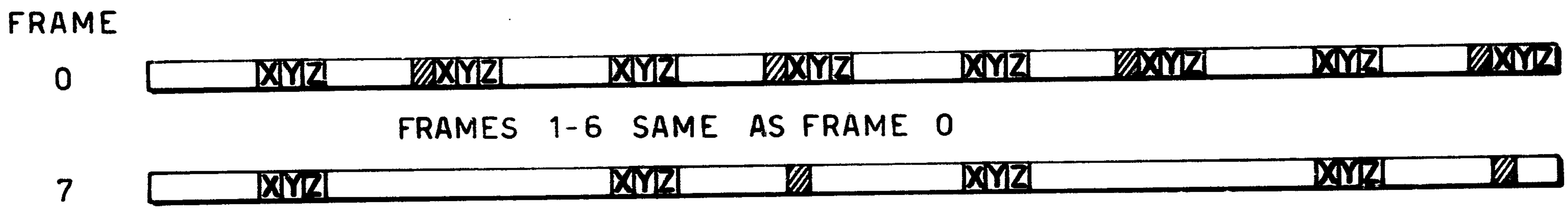
$$
\begin{array}{l}
W_1 \cdots\cdots\cdots\cdots\cdots\cdots W_8 \\
W_9 \cdots\cdots\cdots\cdots\cdots\cdots W_{16} \\
W_{17} \cdots\cdots\cdots\cdots\cdots\cdots W_{24} \\
W_{25} \cdots\cdots\cdots\cdots\cdots\cdots W_{32}
\end{array}
$$



FRAMES 1-7 SAME AS FRAME 0

X Y Z ARE 24 BIT BYTES REPRESENTING THREE COMPONENTS

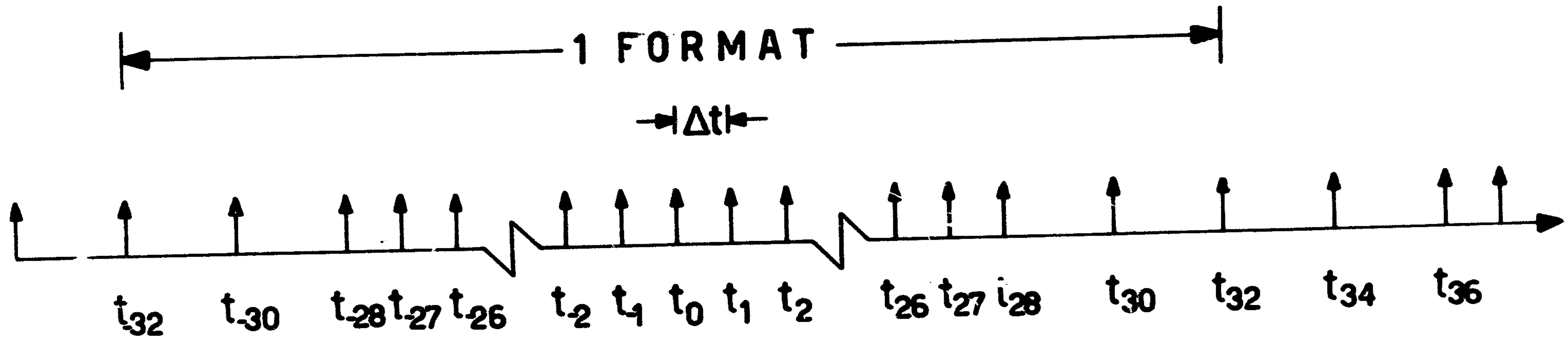
OLD SIRIO FORMAT - Sampling at Words 0,16,32,48



▨ CONTAINS THE 9TH BITS OF 27 BIT BYTES FOR TWO SETS OF XYZ READOUTS

NEW SIRIO FORMAT - Sampling at Words 0,8,16,24,32,40,48,56
(FOR FRAME 7 DELETE 8,24,40,56)

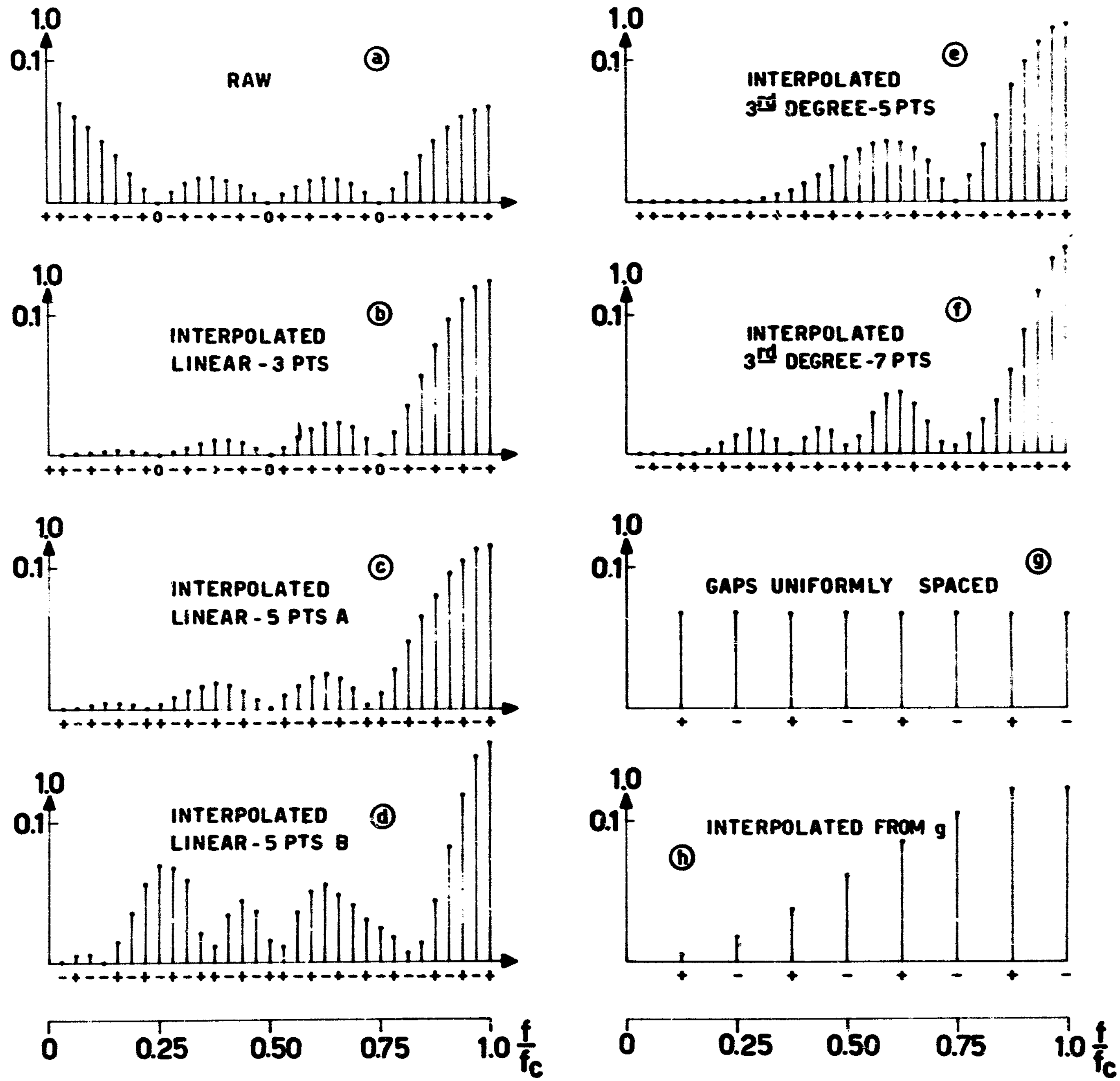
FIGURE 1



Missing Samples @ $t_{\pm 29}, t_{\pm 31}, t_{\pm 33}, t_{\pm 34}, \ldots$

SIRIO Magnetic Field Sampling Window

Fig. 2



1.0
0.1
RAW
a
INTERPOLATED
LINEAR - 3 PTS
b
INTERPOLATED
LINEAR - 5 PTS A
c
INTERPOLATED
LINEAR - 5 PTS B
d
0
0.25
0.50
0.75
1.0 f/fc
INTERPOLATED
3rd DEGREE-5 PTS
e
INTERPOLATED
3rd DEGREE-7 PTS
f
GAPS UNIFORMLY SPACED
g
INTERPOLATED FROM g
h

Fig. 3

SIRIO WINDOW USING SIMILARITY,+,SHIFT THEOREMS

WEIGHTS FROM 1 TO 32 ARE 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000
1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000
1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000
1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 .500000

F/FC	AMP	F/FC	AMP
.00000	1.00000	1.03125	-.00000
.03125	.00000	1.06250	.00000
.06250	-.00000	1.09375	-.00000
.09375	.00000	1.12500	.00000
.12500	-.00000	1.15625	-.00000
.15625	.00000	1.18750	.00000
.18750	-.00000	1.21875	-.00000
.21875	.00000	1.25000	.00000
.25000	-.00000	1.28125	-.00000
.28125	.00000	1.31250	.00000
.31250	-.00000	1.34375	-.00000
.34375	.00000	1.37500	.00000
.37500	-.00000	1.40625	-.00000
.40625	.00000	1.43750	.00000
.43750	-.00000	1.46875	-.00000
.46875	.00000	1.50000	.00000
.50000	-.00000	1.53125	-.00000
.53125	.00000	1.56250	.00000
.56250	-.00000	1.59375	-.00000
.59375	.00000	1.62500	.00000
.62500	-.00000	1.65625	-.00000
.65625	.00000	1.68750	.00000
.68750	-.00000	1.71875	-.00000
.71875	.00000	1.75000	.00000
.75000	-.00000	1.78125	-.00000
.78125	.00000	1.81250	.00000
.81250	-.00000	1.84375	-.00000
.84375	.00000	1.87500	.00001
.87500	-.00000	1.90625	-.00002
.90625	.00000	1.93750	.00003
.93750	-.00000	1.96875	-.00005
.96875	.00000	2.00000	1.00000
1.00000	.00000	2.03125	.00006

REFERENCE AND CHECK

SIRIO WINDOW USING SIMILARITY,+,SHIFT THEOREMS

WEIGHTS FROM 1 TO 32 ARE 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000
1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000
1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000
1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 .500000

F/FC	AMP	F/FC	AMP
.00000	1.00000	1.03125	-.00000
.03125	.00000	1.06250	.00000
.06250	-.00000	1.09375	-.00000
.09375	.00000	1.12500	.00000
.12500	-.00000	1.15625	-.00000
.15625	.00000	1.18750	.00000
.18750	-.00000	1.21875	-.00000
.21875	.00000	1.25000	.00000
.25000	-.00000	1.28125	-.00000
.28125	.00000	1.31250	.00000
.31250	-.00000	1.34375	-.00000
.34375	.00000	1.37500	.00000
.37500	-.00000	1.40625	-.00000
.40625	.00000	1.43750	.00000
.43750	-.00000	1.46875	-.00000
.46875	.00000	1.50000	.00000
.50000	-.00000	1.53125	-.00000
.53125	.00000	1.56250	.00000
.56250	-.00000	1.59375	-.00000
.59375	.00000	1.62500	.00000
.62500	-.00000	1.65625	-.00000
.65625	.00000	1.68750	.00000
.68750	-.00000	1.71875	-.00000
.71875	.00000	1.75000	.00000
.75000	-.00000	1.78125	-.00000
.78125	.00000	1.81250	.00000
.81250	-.00000	1.84375	-.00000
.84375	.00000	1.87500	.00001
.87500	-.00000	1.90625	-.00002
.90625	.00000	1.93750	.00003
.93750	-.00000	1.96875	-.00005
.96875	.00000	2.00000	1.00000
1.00000	.00000	2.03125	.00006

SIRIO WINDOW USING SIMILARITY,+,SHIFT THEOREMS

WEIGHTS FROM 1 TO 32 ARE 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000
1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000
1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000
1.000000 1.000000 1.000000 1.000000 .000000 1.000000 .000000 .500000

F/FC	AMP	F/FC	AMP
.00000	1.00000	1.03125	-.06507
.03125	.06507	1.06250	.06041
.06250	-.06041	1.09375	-.05304
.09375	.05305	1.12500	.04355
.12500	-.04355	1.15625	-.03266
.15625	.03266	1.18750	.02121
.18750	-.02121	1.21875	-.01005
.21875	.01005	1.25000	-.00000
.25000	.00000	1.28125	.00825
.28125	-.00825	1.31250	-.01418
.31250	.01417	1.34375	.01746
.34375	-.01746	1.37500	-.01804
.37500	.01804	1.40625	.01609
.40625	-.01609	1.43750	-.01202
.43750	.01202	1.46875	.00641
.46875	-.00641	1.50000	.00000
.50000	-.00000	1.53125	-.00641
.53125	.00641	1.56250	.01202
.56250	-.01202	1.59375	-.01609
.59375	.01609	1.62500	.01804
.62500	-.01804	1.65625	-.01746
.65625	.01746	1.68750	.01417
.68750	-.01417	1.71875	-.00825
.71875	.00825	1.75000	-.00000
.75000	.00000	1.78125	.01005
.78125	-.01006	1.81250	-.02121
.81250	.02122	1.84375	.03266
.84375	-.03267	1.87500	-.04355
.87500	.04355	1.90625	.05303
.90625	-.05305	1.93750	-.06038
.93750	.06041	1.96875	.06502
.96875	-.06507	2.00000	1.00000
1.00000	.06667	2.03125	.06513

SIRIO WINDOW USING SIMILARITY,+,SHIFT THEOREMS

WEIGHTS FROM 1 TO 32 ARE 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000
1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000
1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000
1.000000 1.000000 1.000000 1.500000 .000000 2.000000 .000000 1.000000

F/FC	AMP	F/FC	AMP
.00000	1.00000	1.03125	-.12171
.03125	.00029	1.06250	.11217
.06250	-.00109	1.09375	-.09731
.09375	.00214	1.12500	.07855
.12500	-.00311	1.15625	-.05762
.15625	.00362	1.18750	.03641
.18750	-.00335	1.21875	-.01670
.21875	.00214	1.25000	-.00000
.25000	-.00000	1.28125	.01265
.28125	-.00283	1.31250	-.02067
.31250	.00591	1.34375	.02409
.34375	-.00865	1.37500	-.02339
.37500	.01044	1.40625	.01946
.40625	-.01071	1.43750	-.01346
.43750	.00907	1.46875	.00660
.46875	-.00542	1.50000	.00000
.50000	-.00000	1.53125	-.00542
.53125	.00660	1.56250	.00907
.56250	-.01346	1.59375	-.01071
.59375	.01946	1.62500	.01044
.62500	-.02338	1.65625	-.00865
.65625	.02408	1.68750	.00591
.68750	-.02067	1.71875	-.00283
.71875	.01264	1.75000	.00000
.75000	.00000	1.78125	.00213
.78125	-.01672	1.81250	-.00334
.81250	.03643	1.84375	.00361
.84375	-.05764	1.87500	-.00310
.87500	.07856	1.90625	.00212
.90625	-.09732	1.93750	-.00106
.93750	.11218	1.96875	.00024
.96875	-.12172	2.00000	1.00000
1.00000	.12500	2.03125	.00035

SIRIO WINDOW USING SIMILARITY,+,SHIFT THEOREMS

WEIGHTS FROM 1 TO 32 ARE 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000
1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000
1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000
.250000 1.000000 1.250000 2.500000 .000000 2.000000 .000000 .500000

F/FC	AMP	F/FC	AMP
.00000	1.00000	1.03125	-.14618
.03125	-.00172	1.06250	.11888
.06250	.00515	1.09375	-.08183
.09375	-.00601	1.12500	.04425
.12500	-.00007	1.15625	-.01365
.15625	.01481	1.18750	-.00667
.18750	-.03572	1.21875	.01802
.21875	.05641	1.25000	-.02478
.25000	-.06897	1.28125	.03155
.28125	.06726	1.31250	-.04043
.31250	-.04978	1.34375	.04982
.34375	.02078	1.37500	-.05520
.37500	.01100	1.40625	.05157
.40625	-.03549	1.43750	-.03647
.43750	.04502	1.46875	.01190
.46875	-.03722	1.50000	.01563
.50000	.01562	1.53125	-.03722
.53125	.01190	1.56250	.04502
.56250	-.03648	1.59375	-.03548
.59375	.05158	1.62500	.01100
.62500	-.05520	1.65625	.02078
.65625	.04982	1.68750	-.04978
.68750	-.04043	1.71875	.06726
.71875	.03154	1.75000	-.06897
.75000	-.02477	1.78125	.05640
.78125	.01801	1.81250	-.03570
.81250	-.00665	1.84375	.01480
.84375	-.01366	1.87500	-.00005
.87500	.04427	1.90625	-.00603
.90625	-.08184	1.93750	.00518
.93750	.11889	1.96875	-.00178
.96875	-.14619	2.00000	1.00000
1.00000	.15625	2.03125	-.00167

SIRIO WINDOW USING SIMILARITY,,,SHIFT THEOREMS

WEIGHTS FROM 1 TO 32 ARE 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000
1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000
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1.000000 1.000000 1.163265 1.326531 .000000 1.938776 .000000 1.071429

F/FC	AMP	F/FC	AMP
.00000	1.00000	1.03125	-.11256
.03125	.00045	1.06250	.10597
.06250	-.00162	1.09375	-.09538
.09375	.00308	1.12500	.08138
.12500	-.00418	1.15625	-.06481
.15625	.00432	1.18750	.04675
.18750	-.00302	1.21875	-.02845
.21875	.00016	1.25000	.01125
.25000	.00404	1.28125	.00355
.28125	-.00896	1.31250	-.01487
.31250	.01370	1.34375	.02196
.34375	-.01726	1.37500	-.02452
.37500	.01874	1.40625	.02279
.40625	-.01753	1.43750	-.01754
.43750	.01348	1.46875	.00990
.46875	-.00692	1.50000	-.00127
.50000	-.00128	1.53125	-.00693
.53125	.00990	1.56250	.01348
.56250	-.01754	1.59375	-.01753
.59375	.02279	1.62500	.01874
.62500	-.02451	1.65625	-.01726
.65625	.02195	1.68750	.01370
.68750	-.01487	1.71875	-.00896
.71875	.00354	1.75000	.00405
.75000	.01126	1.78125	.00015
.78125	-.02847	1.81250	-.00302
.81250	.04676	1.84375	.00431
.84375	-.06482	1.87500	-.00417
.87500	.08139	1.90625	.00306
.90625	-.09539	1.93750	-.00160
.93750	.10598	1.96875	.00039
.96875	-.11256	2.00000	1.00000
1.00000	.11480	2.03125	.00050

SIRIO WINDOW USING SIMILARITY,+,SHIFT THEOREMS

WEIGHTS FROM 1 TO 32 ARE 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000
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1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000
1.000000 1.000000 1.000000 1.250000 .000000 2.500000 .000000 .750000

F/FC	AMP	F/FC	AMP
.00000	1.00000	1.03125	-.12201
.03125	.00000	1.06250	.11327
.06250	.00000	1.09375	-.09950
.09375	-.00005	1.12500	.08178
.12500	.00013	1.15625	-.06148
.15625	-.00024	1.18750	.04010
.18750	.00034	1.21875	-.01916
.21875	-.00031	1.25000	-.00000
.25000	-.00000	1.28125	.01629
.28125	.00082	1.31250	-.02894
.31250	-.00236	1.34375	.03759
.34375	.00485	1.37500	-.04225
.37500	-.00842	1.40625	.04326
.40625	.01309	1.43750	-.04124
.43750	-.01871	1.46875	.03695
.46875	.02494	1.50000	-.03125
.50000	-.03125	1.53125	.02493
.53125	.03695	1.56250	-.01870
.56250	-.04123	1.59375	.01308
.59375	.04326	1.62500	-.00842
.62500	-.04224	1.65625	.00485
.65625	.03759	1.68750	-.00236
.68750	-.02893	1.71875	.00081
.71875	.01628	1.75000	.00000
.75000	.00000	1.78125	-.00032
.78125	-.01917	1.81250	.00035
.81250	.04012	1.84375	-.00025
.84375	-.06149	1.87500	.00014
.87500	.08179	1.90625	-.00007
.90625	-.09951	1.93750	.00004
.93750	.11328	1.96875	-.00005
.96875	-.12201	2.00000	1.00000
1.00000	.12500	2.03125	.00006

SIRIO WINDOW USING SIMILARITY,+,SHIFT THEOREMS

WEIGHTS FROM 1 TO 32 ARE 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000

1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000

1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.219282

.616257 .954631 1.024575 1.703214 .000000 2.134216 .000000 .847826

F/FC	AMP	F/FC	AMP
.00000	1.00000	1.03125	-.13920
.03125	-.00000	1.06250	.11715
.06250	.00007	1.09375	-.08818
.09375	-.00018	1.12500	.06012
.12500	.00004	1.15625	-.03843
.15625	.00105	1.18750	.02417
.18750	-.00377	1.21875	-.01444
.21875	.00822	1.25000	.00476
.25000	-.01328	1.28125	.00787
.28125	.01671	1.31250	-.02302
.31250	-.01608	1.34375	.03685
.34375	.01009	1.37500	-.04426
.37500	.00016	1.40625	.04174
.40625	-.01111	1.43750	-.02946
.43750	.01787	1.46875	.01154
.46875	-.01641	1.50000	.00567
.50000	.00567	1.53125	-.01641
.53125	.01155	1.56250	.01787
.56250	-.02946	1.59375	-.01111
.59375	.04174	1.62500	.00016
.62500	-.04426	1.65625	.01009
.65625	.03684	1.68750	-.01607
.68750	-.02301	1.71875	.01671
.71875	.00786	1.75000	-.01327
.75000	.00477	1.78125	.00821
.78125	-.01446	1.81250	-.00376
.81250	.02419	1.84375	.00103
.84375	-.03844	1.87500	.00005
.87500	.06013	1.90625	-.00020
.90625	-.08819	1.93750	.00009
.93750	.11716	1.96875	-.00006
.96875	-.13920	2.00000	1.00000
1.00000	.14745	2.03125	.00005

SIRIO WINDOW USING SIMILARITY,+,SHIFT THEOREMS

WEIGHTS FROM 1 TO 32 ARE 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 .000000
1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000
1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 .000000
1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 .500000

F/FC	AMP	F/FC	AMP
.00000	1.00000	1.03125	-.00000
.03125	.00000	1.06250	.00000
.06250	-.00000	1.09375	-.00000
.09375	.00000	1.12500	.06667
.12500	.06667	1.15625	.00000
.15625	.00000	1.18750	-.00000
.18750	-.00000	1.21875	.00000
.21875	.00000	1.25000	-.06667
.25000	-.06667	1.28125	-.00000
.28125	.00000	1.31250	.00000
.31250	-.00000	1.34375	-.00000
.34375	.00000	1.37500	.06667
.37500	.06667	1.40625	.00000
.40625	.00000	1.43750	-.00000
.43750	-.00000	1.46875	.00000
.46875	.00000	1.50000	-.06666
.50000	-.06667	1.53125	-.00000
.53125	-.00000	1.56250	.00000
.56250	.00000	1.59375	-.00000
.59375	-.00000	1.62500	.06667
.62500	.06667	1.65625	-.00000
.65625	.00000	1.68750	.00000
.68750	-.00000	1.71875	-.00000
.71875	.00000	1.75000	-.06666
.75000	-.06667	1.78125	-.00001
.78125	-.00000	1.81250	.00001
.81250	.00000	1.84375	-.00001
.84375	-.00000	1.87500	.06668
.87500	.06667	1.90625	-.00001
.90625	.00000	1.93750	.00003
.93750	-.00000	1.96875	-.00005
.96875	.00000	2.00000	1.00000
1.00000	-.06667	2.03125	.00006

SIRIO WINDOW USING SIMILARITY,+,SHIFT THEOREMS

WEIGHTS FROM 1 TO 32 ARE 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.500000 .000000
1.500000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000
1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.500000 .000000
1.500000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 .500000

F/FC	AMP	F/FC	AMP
.00000	1.00000	1.03125	-.00000
.03125	.00000	1.06250	.00000
.06250	-.00000	1.09375	-.00000
.09375	.00000	1.12500	.12024
.12500	.00476	1.15625	.00000
.15625	.00000	1.18750	-.00000
.18750	-.00000	1.21875	.00000
.21875	.00000	1.25000	-.10669
.25000	-.01831	1.28125	-.00000
.28125	.00000	1.31250	.00000
.31250	-.00000	1.34375	-.00000
.34375	.00000	1.37500	.08642
.37500	.03858	1.40625	.00000
.40625	.00000	1.43750	-.00000
.43750	-.00000	1.46875	.00000
.46875	.00000	1.50000	-.06250
.50000	-.06250	1.53125	-.00000
.53125	-.00000	1.56250	.00000
.56250	.00000	1.59375	-.00000
.59375	-.00000	1.62500	.03859
.62500	.08642	1.65625	-.00000
.65625	.00000	1.68750	.00000
.68750	-.00000	1.71875	-.00000
.71875	.00000	1.75000	-.01830
.75000	-.10669	1.78125	-.00000
.78125	-.00000	1.81250	.00000
.81250	.00000	1.84375	-.00001
.84375	-.00000	1.87500	.00477
.87500	.12024	1.90625	-.00002
.90625	.00000	1.93750	.00003
.93750	-.00000	1.96875	-.00005
.96875	.00000	2.00000	1.00000
1.00000	-.12500	2.03125	.00006