NASA TECHNICAL NOTE

NASA TN D-5560

c. 1

SUMMARY OF AVAILABLE INFORMATION ON REYNOLDS ANALOGY FOR ZERO-PRESSURE-GRADIENT, COMPRESSIBLE, TURBULENT-BOUNDARY-LAYER FLOW

by Aubrey M. Cary, Jr.

Langley Research Center
Langley Station, Hampton, Va.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • JANUARY 1970

1. Report No. NASA TN D-5560	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle SUMMARY OF AVAILABLE INFORMATION ON REYNOLDS ANALOGY FOR ZERO-PRESSURE-GRADIENT, COMPRESSIBLE, TURBULENT-BOUNDARY-LAYER FLOW		5. Report Date January 1970
		6. Performing Organization Code
7. Author(s) Aubrey M. Cary, Jr.		8. Performing Organization Report No. L-6648
		10. Work Unit No. 129-01-20-09-23
9. Performing Organization Name and Address NASA Langley Research Center Hampton, Va. 23365		11. Contract or Grant No.
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546		13. Type of Report and Period Covered Technical Note
		14. Sponsoring Agency Code
15. Supplementary Notes		

16. Abstract

An analysis of available experimental data defining the Reynolds analogy between turbulent heat transfer and skin friction has been conducted. For Mach numbers greater than 5, there are insufficient data and too much scatter in the available data to empirically define within desirable accuracy limits the functional dependence of the Reynolds analogy factor on the boundary variables. For Mach numbers less than 4 or 5 and near-adiabatic wall conditions, an empirical definition of the Reynolds analogy factor as presented by Chi and Spalding appears to be valid as a mean of the data; no simultaneous measurements of skin friction and heat transfer were found for this lower Mach number range. There is an indication from the experimental data that decreasing the ratio of wall-to-local-total enthalpy decreases the Reynolds analogy factor. Of the available prediction methods, the Kármán and Kozlov estimates of the Reynolds analogy factor agree best with the general level of the experimental data. It is concluded that experimental turbulent-heat-transfer data cannot in general be used to validate methods for predicting turbulent skin friction until a comprehensive definition of Reynolds analogy is available.

17. Key Words Suggested by Author(s) Turbulent skin friction; Turbulent heat transfer; Reynolds analogy		18. Distribution Statement Unclassified – Unlimited	
19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 17	22. Price* $3.00

*For sale by the Clearinghouse for Federal Scientific and Technical Information
Springfield, Virginia 22151

SUMMARY OF AVAILABLE INFORMATION ON REYNOLDS ANALOGY FOR ZERO-PRESSURE-GRADIENT, COMPRESSIBLE, TURBULENT-BOUNDARY-LAYER FLOW

By Aubrey M. Cary, Jr.
Langley Research Center

SUMMARY

An analysis of available experimental data defining the Reynolds analogy between turbulent heat transfer and skin friction has been conducted. For Mach numbers greater than 5, there are insufficient data and too much scatter in the available data to empirically define within desirable accuracy limits the functional dependence of the Reynolds analogy factor on the boundary variables. For Mach numbers less than 4 or 5 and near-adiabatic wall conditions, an empirical definition of the Reynolds analogy factor as presented by Chi and Spalding appears to be valid as a mean of the data; no simultaneous measurements of skin friction and heat transfer were found for this lower Mach number range. There is an indication from the experimental data that decreasing the ratio of wall-to-local-total enthalpy decreases the Reynolds analogy factor. Of the available prediction methods, the Kármán and Kozlov estimates of the Reynolds analogy factor agree best with the general level of the experimental data. It is concluded that experimental turbulent-heat-transfer data cannot in general be used to validate methods for predicting turbulent skin friction until a comprehensive definition of Reynolds analogy is available.

INTRODUCTION

There are now available numerous empirical or semiempirical methods which provide reliable estimates of turbulent skin friction for zero-pressure-gradient, compressible, turbulent-boundary-layer flow up to Mach 4 or 5 with zero or moderate heat transfer. (See, for example, refs. 1, 2, and 3.) These same methods can be used to predict heat transfer provided that a Reynolds analogy between skin friction and heat transfer is available. Chi and Spalding (ref. 4) determined a Reynolds analogy factor from available heat-transfer data for Mach numbers less than 5 and ratios of wall-to-total temperature from approximately 0.5 to 1.2 with data restricted to two-dimensional and axisymmetric zero-pressure-gradient flow. By using calculated values of skin friction (from ref. 3) to form the ratio $2N_{St}/C_f$, where N_{St} is the local Stanton number and C_f is the local skin-friction coefficient, Chi and Spalding concluded that for their range of conditions the

Reynolds analogy factor had no apparent dependence on Mach number or heat transfer and that a value of $2N_{St}/C_f = 1.16$ provided the best fit to the experimental data. Since the skin-friction predictions for the data conditions included in Chi and Spalding's investigation were previously validated for near-adiabatic wall conditions, the resulting estimate for the Reynolds analogy factor should be reliable for the specified conditions. A Reynolds analogy factor of 1.16 is significantly lower than predictions from Colburn (ref. 5) or Rubesin (ref. 6) but approximates the Kármán prediction, as modified in reference 7, for Prandtl numbers of 0.7 to 0.75.

For Mach numbers greater than 4 or 5 and for other than near-adiabatic heat-transfer conditions at all Mach numbers, no theory or method for predicting turbulent skin friction and heat transfer has been consistently validated. Though numerous investigators have attempted to show the desirability of various methods (see, for example, refs. 1, 2, 7, and 8), the considerable leeway afforded each investigator in applying the prediction methods (choice of virtual origin for turbulent flow, choice of the Reynolds analogy factor, etc.) has precluded making a definitive choice. Thus, the considerable body of turbulent-heat-transfer data available for higher Mach numbers and for which skin friction was not also measured cannot be used to establish the Reynolds analogy factor as was done by Chi and Spalding in their analysis.

In the present report, for Mach numbers greater than 5, only data from investigations in which both skin friction and heat transfer were measured simultaneously were used to determine values for the Reynolds analogy factor. Included are the published results for high Mach numbers ($M > 4$) and high heat-transfer rates on flat plates or nozzle walls (zero pressure gradient), which were found in an extensive literature search (refs. 4 and 7 to 16). A summary of results presented by Chi and Spalding for near-adiabatic wall conditions is also included for completeness.

SYMBOLS

C_f local skin-friction coefficient

F_c function from the Spalding and Chi method (ref. 3)

H enthalpy

M Mach number

N_{Pr} Prandtl number

N_{St}	local Stanton number, $\dfrac{\dot{q}}{\rho_e u_e \left(H_{aw} - H_w\right)}$
$\dot{q}$	heat-transfer rate
R	local Reynolds number, $\dfrac{\rho_e u_e}{\mu_e}$
$R_{x,w}$	wall Reynolds number (eq. (8))
T	temperature
u	velocity
x	longitudinal distance measured from leading edge
x_{CL}	longitudinal distance along nozzle center line measured from nozzle throat
x_v	longitudinal distance measured from origin of turbulent flow
μ	dynamic viscosity
ρ	density

Subscripts:

aw	adiabatic wall
e	local or edge of boundary-layer conditions
i	incompressible
t	total
w	wall conditions
x	based on longitudinal distance from peak-heating location at transition or beginning of turbulent flow
θ	based on momentum thickness

RESULTS AND DISCUSSION

Available skin-friction and heat-transfer data from which a Reynolds analogy factor may be deduced can be divided into two classes: those data obtained for Mach numbers less than 4 or 5 with near-adiabatic wall conditions and high Mach number data with significant heat transfer. In the first class, heat-transfer data alone will define a Reynolds analogy factor since current prediction methods for the skin friction may be used with confidence for these flow conditions. In the second class, both skin friction and heat transfer must be measured since no generally acceptable method for predicting skin friction for these conditions exists. An attempt was made to obtain all published data from investigations where skin friction and heat transfer were measured simultaneously.

A summary of the Reynolds analogy factors derived from the available data is presented in figure 1 for flat-plate, turbulent-boundary-layer flow and in figure 2 for turbulent-boundary-layer flow on a nozzle wall (approximately zero pressure gradient at the measuring station) with a favorable pressure-gradient history. The conditions for which flat-plate data defining the Reynolds analogy were available were Mach numbers from 0 to 12 and ratios of wall-to-local-total enthalpy from 0.09 to 1.20. The variation of the data for any given flow condition is indicated in the figures by the width of the bar with each symbol. Most of the data shown in figure 1 for Mach numbers greater than 4 were obtained from reference 12 where skin friction and heat transfer were measured simultaneously on a flat plate in a shock tunnel. These data were rereduced for this investigation by using a recovery factor of 0.89 instead of 1.0 as used in reference 12. Additional details of this data reduction are given in reference 8, and the heat-transfer and skin-friction data reduced by using a recovery factor of 0.89 are given in table I. Difficulty in obtaining accurate measurements of skin friction and heating during the short run time associated with shock tunnels accounts for some of the scatter in the data from reference 12.

To delineate functional dependence, an attempt was made to correlate the experimental data with the local Mach number, the ratio of wall-to-local-total enthalpy, and the Spalding and Chi incompressible skin-friction coefficient $C_f F_c$, as presented in figures 1 and 2. Also presented in figures 1 and 2 are predictions of the Reynolds analogy factor from five popular methods; each of these methods is discussed in the appendix. There are two immediate conclusions to be drawn from figure 1. First, there is excessive scatter in the data; part of this scatter may be due to the chosen correlation functions and part due to measurement inaccuracies. Second, the available data are generally either for low Mach numbers $(M_e < 4 \text{ or } 5)$ and near-adiabatic wall conditions (C_f not measured) or for higher Mach numbers and low ratios of wall-to-local-total enthalpy. Because of the excessive scatter in the experimental data and the limited range of conditions, no definitive statements as to the overall variation of the Reynolds analogy factor with the boundary variables can be made. However, for the conditions of Mach number less than 4 or 5 and near-adiabatic wall enthalpy, Chi and Spalding recommend using $2N_{St}/C_f = 1.16$, and for these specified conditions, the Chi and Spalding recommendation appears to be as good a representation of the data as other popular estimates. (See the predictions included in fig. 1.) From the variation of the Reynolds analogy factor with wall-to-local-total enthalpy ratio, a tentative conclusion may be that a decrease in $2N_{St}/C_f$ occurs for the lower range of wall-to-local-total enthalpy ratio $\left(0 < H_w/H_t < 0.4\right)$. Of the predictions shown in figure 1, the estimates of Kozlov (ref. 17) and Kármán (ref. 18) best represent the general level of the data. The Kármán values were determined by using the modification presented in the appendix of reference 7. Kozlov (ref. 17) presented empirical expressions for the skin-friction and heat-transfer coefficients and arrived at a Reynolds analogy expression by forming the ratio of these two empirical equations. (See appendix.) Predictions using this method for representative boundary conditions perhaps best approximate the data in both level and trend. The predictions of Kozlov for turbulent skin friction and heat transfer are similar to the predictions of the T-prime methods as discussed in references 1, 7, 8, and 16; since these studies show the T-prime methods to be questionable at low wall-to-local-total temperature ratios, the Kozlov method is also in doubt.

The nozzle-wall data shown in figure 2 were obtained in a shock tunnel (refs. 12 and 14) or in a boundary-layer channel (ref. 15) where the wall boundary layer has a pressure-gradient history. Heat-transfer and skin-friction data listings were provided by the authors of reference 15 and are presented here in table II. The scatter of the nozzle-wall data is too great and the range of conditions too restricted to experimentally assess the effects of the boundary variables on the Reynolds analogy factor. The data generally fall below Colburn's prediction (ref. 5) and above Reynolds' original analogy (ref. 19). The Kozlov and the modified Kármán predictions agree best with the level of the data, as they did with the flat-plate data.

An assessment of the data in figures 1 and 2 indicates that a relationship between heat transfer and skin friction for turbulent-boundary-layer flow is not well defined. Thus, it is recommended that experimental heat-transfer data not be used in attempting to validate a skin-friction prediction method until accurate measurements of skin friction and heat transfer are made and, thus, a comprehensive definition of the Reynolds analogy is available.

CONCLUDING REMARKS

The main result derived from an analysis of available experimental data on the Reynolds analogy is that for Mach numbers less than 4 or 5 with other than near-adiabatic wall conditions and for Mach numbers greater than 4 or 5, there are insufficient data and too much scatter in the available data to empirically define the Reynolds analogy factor within desirable accuracy limits. Therefore, extreme care must be used in attempting to validate methods for predicting turbulent skin friction by using experimental heat-transfer data until a comprehensive definition of the Reynolds analogy is available. Accurate measurements of skin friction and heat transfer (obtained simultaneously) are needed in the lower range of the wall-to-local-total enthalpy below Mach 5 and through the entire range of the wall-to-local-total enthalpy ratio above Mach 5. The data gathered for this analysis are useful but are not definitive from the point of view of the Reynolds analogy. There is an indication that decreasing the wall-to-local-total enthalpy ratio decreases the Reynolds analogy factor. Of the available prediction methods, the Kârmân and the Kozlov estimates of the Reynolds analogy factor give the best general agreement with the level of the experimental data. It is clear that a definitive formulation of the Reynolds analogy awaits extensive experimental investigation.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., October 8, 1969.

APPENDIX

FORMULATION OF SEVERAL METHODS FOR PREDICTING THE REYNOLDS ANALOGY FACTOR

Reynolds

Reynolds' original expression for the analogy between boundary-layer heat transfer and skin friction for incompressible flow can be obtained in reference 19 as

$$2 \frac{N_{St,i}}{C_{f,i}} = 1.0 \tag{1}$$

When extended to compressible flow, it is merely assumed that

$$2 \frac{N_{St}}{C_f} = 1.0 \tag{2}$$

Colburn

The Colburn form of the Reynolds analogy was empirically derived for incompressible flow in reference 5. This method of predicting the Reynolds analogy factor extended to compressible flow has enjoyed great popularity through the years. The expression for this Reynolds analogy factor is

$$2 \frac{N_{St}}{C_f} = N_{Pr}^{-\frac{2}{3}} \tag{3}$$

where the Prandtl number used for the predictions in the present investigation is taken to be 0.725, and from equation (3) yields a Reynolds analogy factor of 1.24.

Kármán

Kármán's expression for the Reynolds analogy factor, originally derived for incompressible flow in reference 18 and extended to compressible flow in reference 7, is expressed as

$$2 \frac{N_{St,i}}{C_{f,i}} = \left\{1 + 5\left(\frac{C_{f,i}}{2}\right)^{1/2} \left[\left(N_{Pr} - 1\right) + \log_e\left(\frac{5N_{Pr} + 1}{6}\right)\right]\right\}^{-1} \tag{4}$$

where $2N_{St,i}/C_{f,i}$ is assumed to be $2N_{St}/C_f$ when $C_{f,i}$ is taken to be the Spalding and Chi incompressible skin-friction coefficient $F_c C_f$ and the Prandtl number is 0.725. A more detailed discussion of this approach is presented in reference 7. It was also stated in reference 7 that these Kármán values of the Reynolds analogy factor are representative of the predictions from reference 20.

Kozlov

Empirical expressions for the local skin-friction coefficient and Stanton number were used by Kozlov (ref. 17) to define the Reynolds analogy factor. These expressions based on local conditions at the edge of the boundary layer are

$$C_f = 0.085 R_{x,w}^{\left(-0.29+0.01 \log R_{x,w}\right)} \left(\frac{T_w}{T_e}\right)^{-0.61} \left(\frac{T_{aw}}{T_e}\right)^{-0.19} \tag{5}$$

$$N_{St} = 0.0296 R_{x,w}^{-0.2} N_{Pr,w}^{-0.57} \left(\frac{T_w}{T_e}\right)^{-0.61} \left(\frac{T_{aw}}{T_e}\right)^{-0.28} \tag{6}$$

Equations (5) and (6) were derived independently from comparisons with experimental data. According to reference 17, the range of conditions for which equations (5) and (6) are valid within 10 percent are

for equation (5)

$$10^5 \leqq R_{x,w} \leqq 10^9$$

$$0 \leqq M_e \leqq 10$$

$$0.6 \leqq \frac{T_w}{T_{aw}} \leqq 1.0$$

and for equation (6)

$$5 \times 10^5 \leqq R_{x,w} \leqq 10^9$$

$$0 \leqq M_e \leqq 10$$

$$0.1 \leqq \frac{T_w}{T_{aw}} \leqq 5.0$$

The ratio of equations (5) and (6) gives the Reynolds analogy factor as

$$2 \frac{N_{St}}{C_f} = 0.695 R_{x,w}^{\left(0.09-0.01 \log R_{x,w}\right)} N_{Pr,w}^{-0.57} \left(\frac{T_{aw}}{T_e}\right)^{-0.09} \tag{7}$$

APPENDIX

For equations (5), (6), and (7) the wall Reynolds number is defined as

$$R_{x,w} = \frac{\rho_w u_e x_v}{\mu_w} \tag{8}$$

where ρ_w is the density based on wall values of temperature and pressure, μ_w is the dynamic viscosity based on wall values of temperature, and x_v is the distance from the origin of turbulent flow.

Chi and Spalding

From an analysis of experimental data defining the Reynolds analogy, Chi and Spalding (ref. 4) give the Reynolds analogy factor which best fits experimental data as

$$2 \frac{N_{St}}{C_f} = 1.16 \tag{9}$$

REFERENCES

1. Nestler, D. E.; and Goetz, R.: Survey of Theoretical and Experimental Determinations of Skin Friction in Compressible Boundary Layers: Part II. The Turbulent Boundary on a Flat Plate. Tech. Inform. Ser. No. R58SD270 (Contract AF 04(645)-24), Missile Ord. Syst. Dep., Gen. Elec. Co., Jan. 29, 1959.

2. Hornung, H. G.: A Survey of Compressible Flow Boundary Layers – Theory and Experiment. Rep. ACA 67, Dep. Supply, Australian Aeronaut. Res. Comm., Feb. 1966.

3. Spalding, D. B.; and Chi, S. W.: The Drag of a Compressible Turbulent Boundary Layer on a Smooth Flat Plate With and Without Heat Transfer. J. Fluid Mech., vol. 18, pt. 1, Jan. 1964, pp. 117-143.

4. Chi, S. W.; and Spalding, D. B.: Influence of Temperature Ratio on Heat Transfer to a Flat Plate Through a Turbulent Boundary Layer in Air. Proceedings of the Third International Heat Transfer Conference – Vol. II, Amer. Inst. Chem. Eng., c.1966, pp. 41-49.

5. Colburn, Allan P.: A Method of Correlating Forced Convection Heat Transfer Data and a Comparison With Fluid Friction. Trans. Amer. Inst. Chem. Eng., vol. XXIX, 1933, pp. 174-211.

6. Rubesin, Morris, W.: A Modified Reynolds Analogy for the Compressible Turbulent Boundary Layer on a Flat Plate. NACA TN 2917, 1953.

7. Bertram, Mitchel H.; and Neal, Luther, Jr.: Recent Experiments in Hypersonic Turbulent Boundary Layers. Presented at the AGARD Specialists Meeting on Recent Developments in Boundary-Layer Research (Naples, Italy), May 10-14, 1965.

8. Bertram, Mitchel H.; Cary, Aubrey M., Jr.; and Whitehead, Allen H., Jr.: Experiments With Hypersonic Turbulent Boundary Layers on Flat Plates and Delta Wings. Hypersonic Boundary Layers and Flow Fields, AGARD CP No. 30, May 1968.

9. Neal, Luther, Jr.: A Study of the Pressure, Heat Transfer, and Skin Friction on Sharp and Blunt Flat Plates at Mach 6.8. NASA TN D-3312, 1966.

10. Fallis, W. B.: Heat Transfer in the Transitional and Turbulent Boundary Layers of a Flat Plate at Supersonic Speeds. UTIA Rep. No. 19, Inst. Aerophys., Univ. of Toronto, May 1952.

11. Wallace, J. E.: Hypersonic Turbulent Boundary-Layer Studies at Cold Wall Conditions. Proceedings of the 1967 Heat Transfer and Fluid Mechanics Institute, Paul A. Libby, Daniel B. Olfe, and Charles W. Van Atta, eds., Stanford Univ. Press, c.1967, pp. 427-451.

12. Hironimus, G. A.: Hypersonic Shock Tunnel Experiments on the W7 Flat Plate Model – Expansion Side, Turbulent Flow and Leading Edge Transpiration Data. CAL Rep. No. AA-1952-Y-2 (Contract No. AF 33(615)-1847), Cornell Aeronaut. Lab., Inc., Feb. 1966.

13. Young, Frank Levi: Experimental Investigation of the Effects of Surface Roughness on Compressible Turbulent Boundary Layer Skin Friction and Heat Transfer. DRL-532, CR-21 (Contract NOrd-16498), Univ. of Texas, May 1965.

14. Wallace, J. E.: Hypersonic Turbulent Boundary Layer Measurements Using an Electron Beam. CAL Rep. No. AN-2112-Y-1 (Contract No. NSR33-009-029), Cornell Aeronaut. Lab., Inc., Aug. 1968.

15. Lee, Roland E.; Yanta, William J.; and Leonas, Annette C.: Velocity Profile, Skin Friction Balance and Heat Transfer Measurements of the Turbulent Boundary Layer at Mach 5. Proceedings of 1968 Heat Transfer and Fluid Mechanics Institute, Ashley F. Emery and Creighton A. Depew, eds., Stanford Univ. Press, c.1968.

16. Hopkins, Edward J.; Rubesin, Morris W.; Inouye, Mamoru; Keener, Earl R.; Mateer, George C.; and Polek, Thomas E.: Summary and Correlation of Skin-Friction and Heat-Transfer Data for a Hypersonic Turbulent Boundary Layer on Simple Shapes. NASA TN D-5089, 1969.

17. Kozlov, L. V.: Connection of Aerodynamic Heat With Skin Friction. FTD-TT-64-377/1 + 2, U.S. Air Force, Dec. 1964. (Available from DDC as AD-609788.)

18. Von Kármán, Th.: The Analogy Between Fluid Friction and Heat Transfer. A.S.M.E. Trans., vol. 61, no. 8, Nov. 1939, pp. 705-710.

19. Reynolds, Osborne: On the Extent and Action of the Heating Surface for Steam Boilers. Proc. Manchester Lit. Phil. Soc., vol. 14, 1875, pp. 7-12.

20. Deissler, R. G.; and Loeffler, A. L., Jr.: Analysis of Turbulent Flow and Heat Transfer on a Flat Plate at High Mach Numbers With Variable Fluid Properties. NASA TR R-17, 1959. (Supersedes NACA TN 4262.)

TABLE I.- MODIFIED HEAT-TRANSFER AND SKIN-FRICTION DATA

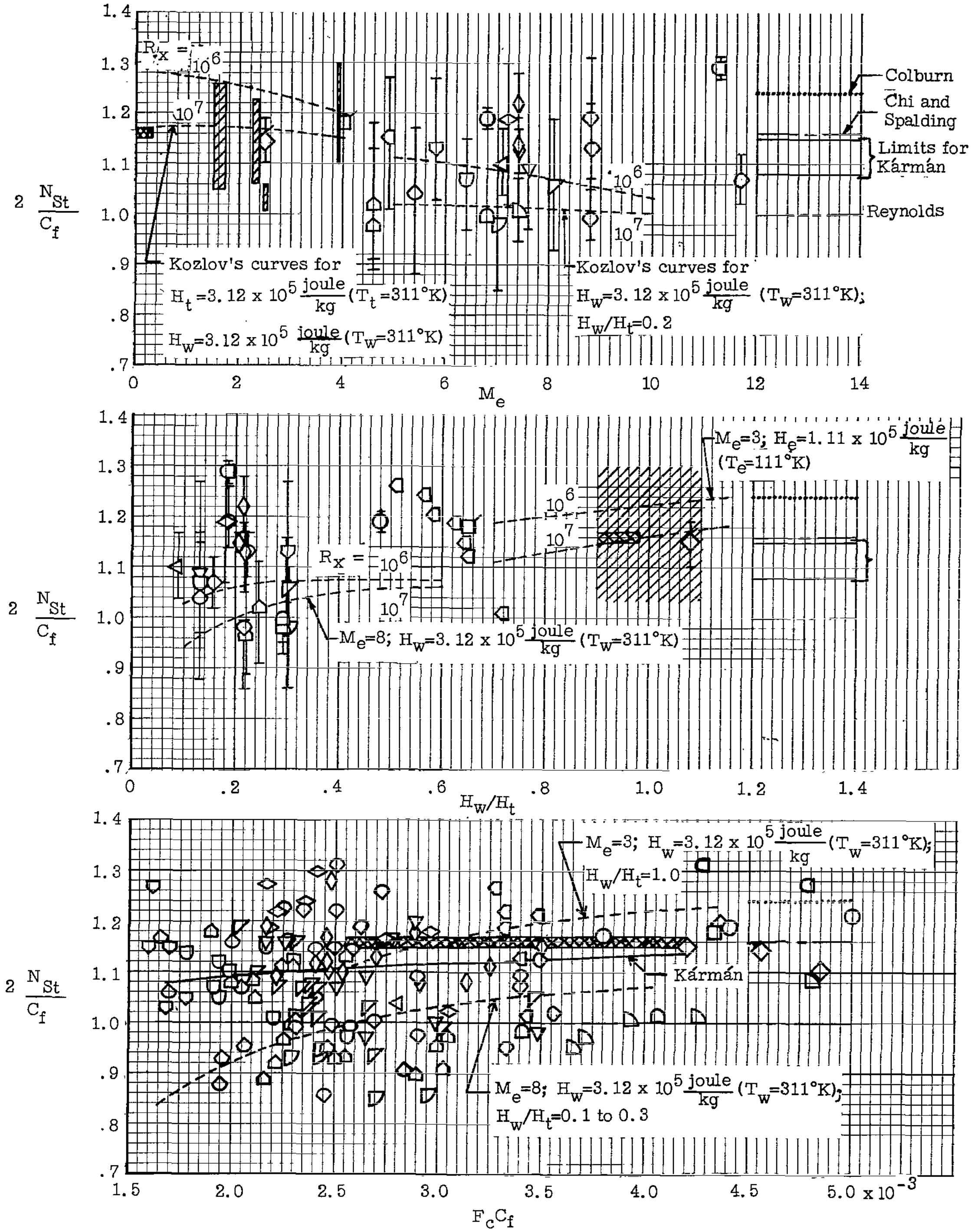
[From ref. 12]

R_x	N_{St}	C_f
Run 5; $M_e = 8.8$; $H_w/H_t = 0.22$		
6.8×10^6	3.82×10^{-4}	8.06×10^{-4}
9.3	3.64	6.05
11.9	3.43	7.02
14.4	3.25	6.50
17.0	2.98	6.00
20.6	3.09	6.22
24.1	2.94	6.92
27.7	2.75	5.54
31.3	2.54	5.93
Run 6; $M_e = 8.8$; $H_w/H_t = 0.22$		
6.9×10^6	4.47×10^{-4}	8.21×10^{-4}
9.5	4.28	6.18
12.1	3.80	6.99
14.7	3.77	6.33
17.3	3.38	5.89
20.9	3.70	6.07
24.5	3.27	6.86
28.1	3.17	5.48
31.7	3.05	5.83
Run 7; $M_e = 7.4$; $H_w/H_t = 0.22$		
1.5×10^6	6.36×10^{-4}	11.10×10^{-4}
4.2	5.73	10.33
6.8	5.39	9.98
14.9	5.02	9.26
17.6	4.71	7.46
20.3	4.45	8.06
23.0	4.38	7.80
25.7	4.07	6.85
29.5	4.30	7.67
33.3	4.08	8.55
37.1	3.85	7.04
Run 8; $M_e = 4.6$; $H_w/H_t = 0.24$		
2.0×10^6	8.48×10^{-4}	17.50×10^{-4}
3.3	8.24	14.60
4.5	7.83	17.30
5.7	7.28	13.20
8.2	7.13	12.80
9.4	6.22	12.90
10.6	6.47	12.00
11.9	6.49	11.40
13.6	6.58	13.10
15.3	6.79	14.60
17.0	6.38	12.10
18.7	5.58	13.00
Run 12; $M_e = 4.6$; $H_w/H_t = 0.22$		
1.3×10^6	8.52×10^{-4}	18.00×10^{-4}
3.4	7.83	17.50
5.4	7.08	14.80
7.5	7.01	15.10
9.6	6.61	13.40
11.7	7.03	13.90
13.7	6.81	11.50
15.8	6.50	12.00
17.9	6.04	12.30
20.0	6.23	9.85
22.9	6.23	12.00
25.8	6.16	14.70
28.7	5.50	11.90
31.6	5.78	13.00
Run 18; $M_e = 7.6$; $H_w/H_t = 0.13$		
2.5×10^6	6.18×10^{-4}	12.60×10^{-4}
4.1	6.28	10.44
5.7	5.42	10.81
7.4	5.86	10.00
9.0	5.31	8.72
10.6	5.73	9.60
12.3	4.90	8.89
13.9	4.73	8.18
16.2	4.86	9.04
18.4	4.60	9.55
20.7	4.54	7.83
23.0	4.46	9.41
Run 19; $M_e = 8.1$; $H_w/H_t = 0.30$		
3.5×10^6	4.34×10^{-4}	8.34×10^{-4}
11.2	3.60	7.27
26.8	3.28	6.38
34.6	3.19	5.51
42.4	2.94	5.84
50.1	3.10	5.78
57.9	2.92	4.63
65.7	2.98	5.58
73.5	2.89	4.86
81.2	2.83	5.20
92.2	2.92	5.68
103.1	2.79	6.49
114.0	2.85	5.34
125.0	3.01	6.47
Run 20; $M_e = 7.0$; $H_w/H_t = 0.30$		
15.6×10^6	3.48×10^{-4}	8.22×10^{-4}
27.8	3.42	6.35
64.4	3.05	6.48
89.0	3.10	6.50
101.0	3.06	5.37
113.0	2.83	6.12
125.0	3.12	5.65
138.0	2.99	5.52
172.0	3.38	7.87
189.0	3.21	6.28
206.0	3.05	7.19
Run 22; $M_e = 6.4$; $H_w/H_t = 0.13$		
4.1×10^6	5.64×10^{-4}	11.60×10^{-4}
6.9	5.81	10.40
9.6	5.12	10.10
12.4	5.64	9.83
15.2	4.99	9.03
17.9	5.33	8.91
20.5	4.65	8.69
23.4	4.59	8.08
27.2	4.96	8.83
31.1	5.03	10.50
35.0	4.59	8.72
38.8	4.42	9.14
Run 23; $M_e = 5.4$; $H_w/H_t = 0.13$		
2.9×10^6	7.34×10^{-4}	16.20×10^{-4}
6.0	6.71	11.50
12.3	6.26	11.70
15.4	5.99	14.10
18.6	5.65	11.81
Run 24; $M_e = 5.8$; $H_w/H_t = 0.30$		
20.5×10^6	4.14×10^{-4}	9.08×10^{-4}
34.7	4.08	6.40
63.1	3.87	6.76
91.0	3.70	7.05
162.0	3.81	6.40
222.0	3.43	6.70
Run 25; $M_e = 7.4$; $H_w/H_t = 0.21$		
2.4×10^6	5.66×10^{-4}	8.75×10^{-4}
3.8	5.37	9.11
6.4	4.88	8.55
7.8	4.98	8.87
Run 26; $M_e = 7.4$; $H_w/H_t = 0.21$		
4.5×10^6	5.20×10^{-4}	8.06×10^{-4}
5.8	5.00	7.80
8.4	4.54	7.75
9.7	4.31	8.37
Run 27; $M_e = 7.1$; $H_w/H_t = 0.09$		
2.3×10^6	6.84×10^{-4}	11.30×10^{-4}
3.0	6.72	11.50
4.5	6.16	11.80
5.2	6.11	11.90
Run 29; $M_e = 11.3$; $H_w/H_t = 0.18$		
2.6×10^6	5.35×10^{-4}	8.71×10^{-4}
4.2	5.14	8.10
5.7	4.74	7.24
7.2	4.67	7.14
Run 30; $M_e = 11.7$; $H_w/H_t = 0.16$		
1.0×10^6	4.36×10^{-4}	7.78×10^{-4}
2.1	4.05	7.92
Run 31; $M_e = 8.8$; $H_w/H_t = 0.18$		
2.7×10^6	4.60×10^{-4}	8.61×10^{-4}
4.6	5.14	6.13
6.5	4.36	6.92
8.4	4.48	8.51
10.3	4.19	6.39
12.2	3.88	5.81
14.9	3.73	6.52
17.5	3.64	5.95
20.2	3.49	5.70
22.8	3.48	6.10
Run 32; $M_e = 7.2$; $H_w/H_t = 0.19$		
5.4×10^6	5.96×10^{-4}	10.10×10^{-4}
7.4	5.00	9.76
9.5	5.73	6.96
11.6	5.34	8.21
13.6	5.31	10.40
15.7	5.00	8.05
17.7	4.78	6.93
20.6	4.72	7.41
23.5	4.62	7.57
26.4	4.45	7.45
29.2	4.38	8.17

TABLE II.- NOZZLE-WALL DATA OBTAINED IN A BOUNDARY-LAYER CHANNEL

[From ref. 15]

M_e	$\frac{H_w}{H_t}$	R_θ	N_{St}	C_f
		x_{CL} = 152.4 cm		
4.6	0.67	5.16×10^3	5.56×10^{-4}	8.9×10^{-4}
4.6	.67	5.16	5.71	8.9
4.8	.70	22.73	4.19	7.5
4.8	.48	15.03	6.13	8.8
4.8	.48	15.03	5.97	8.8
4.8	.48	15.03	5.96	8.8
4.8	.48	15.03	5.95	8.8
		x_{CL} = 182.9 cm		
4.6	0.69	5.96×10^3	7.56×10^{-4}	9.3×10^{-4}
4.6	.69	5.96	6.91	9.3
4.8	.70	24.38	4.00	7.8
4.8	.70	24.38	3.84	7.8
4.7	.73	56.51	3.05	7.0
4.8	.53	18.53	4.48	9.2
4.8	.53	18.53	4.65	9.2
4.8	.46	13.85	5.00	9.9
4.8	.46	13.85	5.09	9.9



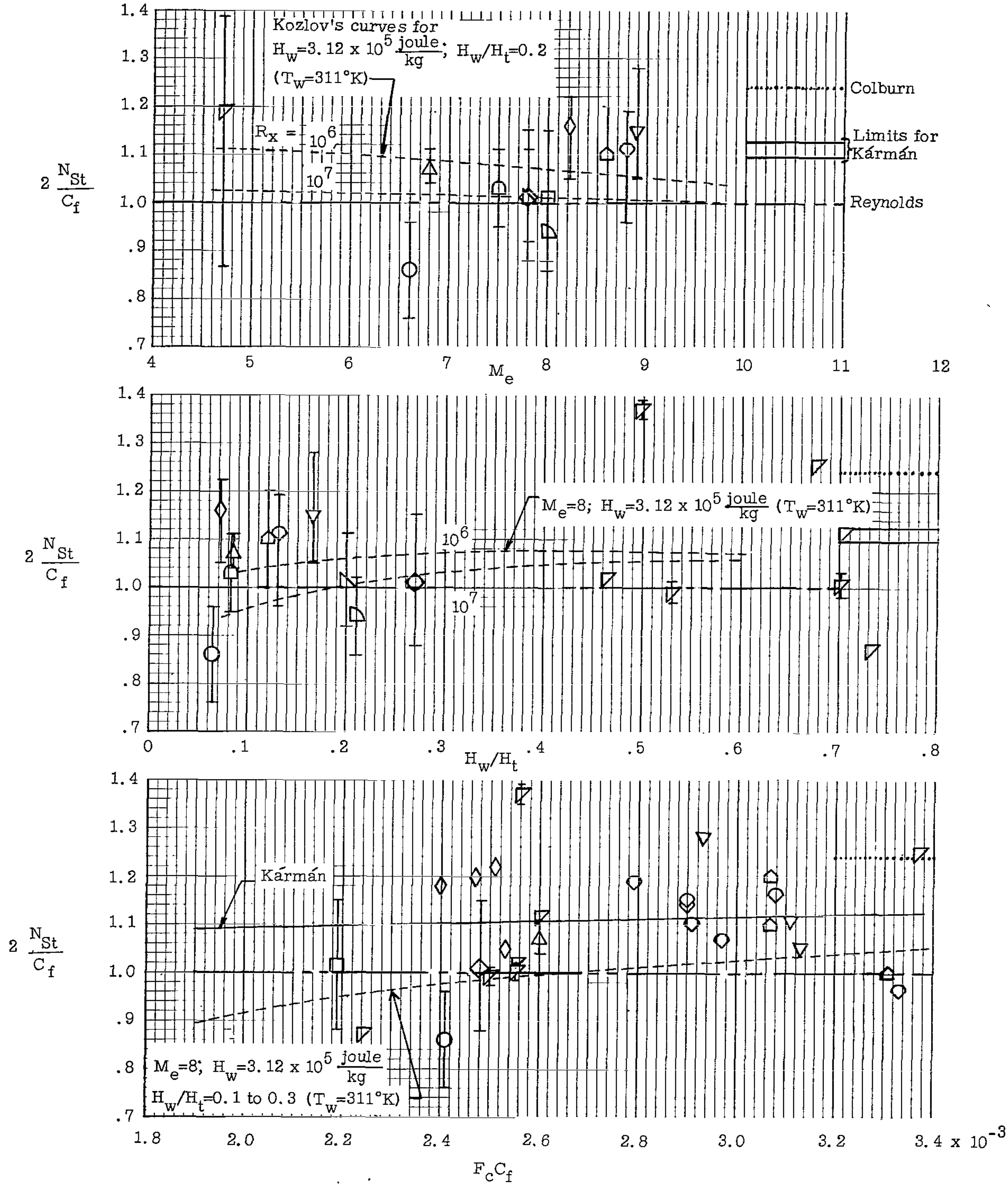
(a) Experimental data and empirical predictions.

Figure 1.- Reynolds analogy for flat-plate flow. All predictions are for N_{Pr} = 0.725.

	M_e	Type of measurement	Reference
	4.0	Local N_{St} (skin friction calculated from ref. 3)	7
	2.5		10
	6.8	Local C_f and N_{St}	9
	7.4		16
	6.8		16
	8.8		12, 11, 8
	7.4		12, 11, 8
	4.6		12, 11, 8
	7.6		12, 11, 8
	8.1		12, 11, 8
	7.0		12, 11, 8
	6.4		12, 11, 8
	5.4		12, 11, 8
	5.8		12, 11, 8
	7.1		12, 11, 8
	11.3		12, 11, 8
	11.7		12, 11, 8
	7.2		12, 11, 8
	4.9		13
	1.5 to 4.0	Local or average N_{St} (skin friction calculated from ref. 3)	4
	0 to 0.3	Local N_{St} (skin friction calculated from ref. 3)	4

(b) Key to data.

Figure 1.- Concluded.



(a) Experimental data and empirical predictions.

Figure 2.- Reynolds analogy for nozzle-wall flow (nominal zero pressure gradient). All predictions for N_{Pr} = 0.725.

	M_e	Type of measurement	Reference
○	6.6	Local C_f and N_{St} (shock tunnel)	11, 14
□	8.0		
◇	7.8		
△	6.8		
◺	7.8		
◗	8.0		
◠	7.6		
⬠	8.8		
◊	8.2		
⌂	8.6		
▽	8.9		
◸	4.7	Local C_f and N_{St} (boundary-layer channel)	15

(b) Key to data.

Figure 2.- Concluded.