

## **General Disclaimer**

### **One or more of the Following Statements may affect this Document**

- This document has been reproduced from the best copy furnished by the organizational source. It is being released in the interest of making available as much information as possible.
- This document may contain data, which exceeds the sheet parameters. It was furnished in this condition by the organizational source and is the best copy available.
- This document may contain tone-on-tone or color graphs, charts and/or pictures, which have been reproduced in black and white.
- This document is paginated as submitted by the original source.
- Portions of this document are not fully legible due to the historical nature of some of the material. However, it is the best reproduction available from the original submission.

Second Quarterly Report

MHD BOUNDARY LAYERS WITH  
NON-EQUILIBRIUM IONIZATION AND  
FINITE RATES

(June 1, 1969 - September 1, 1969)

by

Arthur Sherman  
Hsuan Yeh  
Eli Reshotko (Consultant)

N70-16932

FACILITY FORM 602

|                               |            |
|-------------------------------|------------|
| (ACCESSION NUMBER)            | (THRU)     |
| 38                            | /          |
| (PAGES)                       | (CODE)     |
| CR-107804                     | 25         |
| (NASA CR OR TMX OR AD NUMBER) | (CATEGORY) |

Prepared under Contract No. NASw - 1866 by

Computer and Applied Sciences, Inc.  
Philadelphia, Pennsylvania

for

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

## TABLE OF CONTENTS

| <u>Section</u>                                                 | <u>Page</u> |
|----------------------------------------------------------------|-------------|
| I. Introduction                                                | 2           |
| II. Reduction of Governing Equations to Finite Difference Form | 3           |
| III. Programming                                               | 14          |
| IV. Review of Results                                          | 15          |
| V. Future Efforts                                              | 16          |

## **I. INTRODUCTION**

The present report is the second quarterly report under Contract NASw-1866 and covers the period June 1, 1969 to September 1, 1969. The problem being considered is the boundary layer that will develop over the segmented electrode wall in an MHD Channel when a non-equilibrium plasma is flowing.

As a result of work completed under NASw-1586 we have developed a theoretical analysis for a non-equilibrium boundary layer explicitly taking into account the electron temperature differential equation and the plasma sheath. This analysis was developed using a finite difference format so that finite electrodes along a wall could be treated in a non-similar manner.

The present program seeks to extend this earlier study to include finite ionization and recombination rates in order to make the analysis apply in a more realistic way to physically important situations.

## I. INTRODUCTION

The present report is the second quarterly report under Contract NASw-1866 and covers the period June 1, 1969 to September 1, 1969. The problem being considered is the boundary layer that will develop over the segmented electrode wall in an MHD Channel when a non-equilibrium plasma is flowing.

As a result of work completed under NASw-1586 we have developed a theoretical analysis for a non-equilibrium boundary layer explicitly taking into account the electron temperature differential equation and the plasma sheath. This analysis was developed using a finite difference format so that finite electrodes along a wall could be treated in a non-similar manner.

The present program seeks to extend this earlier study to include finite ionization and recombination rates in order to make the analysis apply in a more realistic way to physically important situations.

## II. REDUCTION OF GOVERNING EQUATIONS TO FINITE DIFFERENCE FORM

In our first Quarterly we reported on the proper development of the fundamental equations governing an MHD boundary layer with non-equilibrium and finite ionization rates. It was then shown how these equations could be transformed to new variables  $\xi$  and  $\eta$  to simplify the solution of the problem.

During the second quarter we have reduced these transformed equations to finite difference format. As before we write all four difference equations in the following compact notation.

$$A_n W_{n+1} + B_n W_n + C_n W_{n-1} = D_n$$

where

$$W_n = \begin{bmatrix} f'_{m+1,n} \\ g_{m+1,n} \\ \Theta_{m+1,n} \\ \alpha_{m+1,n} \end{bmatrix}$$

and

$$A_n = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix}$$

$$B_n = \begin{bmatrix} B_{11} & B_{12} & B_{13} & B_{14} \\ B_{21} & B_{22} & B_{23} & B_{24} \\ B_{31} & B_{32} & B_{33} & B_{34} \\ B_{41} & B_{42} & B_{43} & B_{44} \end{bmatrix}$$

$$C_n = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} \\ C_{21} & C_{22} & C_{23} & C_{24} \\ C_{31} & C_{32} & C_{33} & C_{34} \\ C_{41} & C_{42} & C_{43} & C_{44} \end{bmatrix}$$

$$D_n = \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{bmatrix}$$

The individual matrix elements are for the most part new and revised and are tabulated below:

### Momentum Equation

$$A_{11} f'_{m+1,n+1} + B_{11} f'_{m+1,n} + B_{12} g_{m+1,n} + C_{11} f'_{m+1,n-1} = D_1$$

where

$$A_{11} = \frac{V \Delta \xi}{8 \xi f' \Delta \eta} - \frac{l \Delta \xi}{4 \xi f' (\Delta \eta)^2} - \frac{l' \Delta \xi}{8 \xi f' \Delta \eta}$$

$$B_{11} = 1 + \frac{\Delta \xi}{2 u_{\infty}} \frac{du_{\infty}}{d\xi} + \frac{l \Delta \xi}{2 \xi f' (\Delta \eta)^2}$$

$$B_{12} = - \frac{\Delta \xi}{2 u_{\infty} f'} \frac{du_{\infty}}{d\xi}$$

$$C_{11} = - \frac{V \Delta \xi}{8 \xi f' \Delta \eta} - \frac{l \Delta \xi}{4 \xi f' (\Delta \eta)^2} + \frac{l \Delta \xi}{8 \xi f' (\Delta \eta)}$$

$$D_1 = f'_{m,n} \left( 1 - \frac{\Delta \xi}{2 u_{\infty}} \frac{du_{\infty}}{d\xi} \right) - \frac{V \Delta \xi}{4 \xi f'} f'_{\eta} + \frac{\Delta \xi}{2 u_{\infty} f'} \frac{du_{\infty}}{d\xi} g_{m,n} \\ + \frac{l \Delta \xi}{4 \xi f'} f'_{\eta \eta} + \frac{l' (\Delta \xi)}{4 \xi f'} f'_{\eta}$$



### Energy Equation

$$A_{21} f'_{m+1,n+1} + A_{22} g_{m+1,n+1} + A_{23} \Theta_{m+1,n+1} +$$

$$A_{24} \alpha_{m+1,n+1} + B_{21} f'_{m+1,n} + B_{22} g_{m+1,n}$$

$$+ B_{23} \Theta_{m+1,n} + B_{24} \alpha_{m+1,n} + C_{21} f'_{m+1,n-1}$$

$$+ C_{22} g_{m+1,n-1} + C_{23} \Theta_{m+1,n-1} + C_{24} \alpha_{m+1,n-1}$$

$$= D_2$$

$$A_{21} = - \frac{\Delta \xi}{2 \xi f'} \frac{u_{\infty}^2}{C_p T_{\infty}} \frac{l f_k}{2 \Delta \eta}$$

$$A_{22} = L_1 \left[ v - \frac{l'}{P_R} - \frac{2}{\Delta \eta} \left( \frac{l}{P_R} \right) \right]$$

$$A_{23} = -L_1 \left[ \frac{2\lambda T_{\infty}}{\Delta \eta T_{\infty}} + \frac{\lambda' T_{\infty}}{T_{\infty}} + \frac{5}{2} \frac{\sqrt{2\xi} k T_{\infty} i y_{\infty}}{(\rho\mu)_r u_{\infty} C_p T_{\infty} e} \right]$$

$$A_{24} = 0$$

$$B_{21} = L_2 g_{m,n} \left[ \frac{2\xi}{C_p T_{\infty}} \left( C_p \frac{dT_{\infty}}{d\xi} + u_{\infty} \frac{du_{\infty}}{d\xi} - \frac{i y_{\infty} B_z}{(\rho\mu)_r \rho_{\infty} u_{\infty}} \right) \right]$$

$$B_{22} = 1 + 4 \left( \frac{l}{P_R} \right) \frac{L_1}{\Delta \eta} + \frac{2\xi}{C_p T_{\infty}} L_2 \left[ \left( C_p \frac{dT_{\infty}}{d\xi} + u_{\infty} \frac{du_{\infty}}{d\xi} - \frac{i y_{\infty} B_z}{(\rho\mu)_r \rho_{\infty} u_{\infty}} \right) f'_{m,n} \right]$$

$$\begin{aligned}
& - \frac{1}{(\rho\mu)_r \rho_\infty u_\infty^2} \frac{(\sigma Ex_\infty^2 + \frac{jy_\infty^2}{2}(1+\beta e^2))}{2} \\
& + \frac{I \rho_\infty^2 (\tilde{\alpha}_{rec})_\infty}{(\rho\mu)_r u_\infty^2 M_c^3} \frac{\alpha_{m,n}}{g_{m,n}^3 \Theta_{m,n}^{9/2}} \left( \alpha_{m,n}^2 - \frac{\alpha_{eq. m,n}^3}{2P - \alpha_{eq. m,n}} \right)
\end{aligned}$$

$$\begin{aligned}
B_{23} = & \frac{4 L_1 \lambda T_\infty}{\Delta \eta T_\infty} + \frac{2 \xi}{C_p T_\infty} L_2 \left[ \frac{2 I \rho_\infty^2 (\tilde{\alpha}_{rec})_\infty}{(\rho\mu)_r u_\infty^2 m_c^3} \cdot \frac{\alpha_{m,n}}{g_{m,n}^2 \Theta_{m,n}^{11/2}} \right. \\
& \left. (\alpha_{eq. m,n}^2 \left\{ \frac{2(P - \alpha_{eq. m,n})}{(2P - \alpha_{eq. m,n})} \left( \frac{3}{2} - \frac{I}{k T_{e\infty} \Theta_{m,n}} \right) - \frac{9}{4} \right\} + \frac{9}{4} \alpha_{m,n}^2) \right]
\end{aligned}$$

$$B_{24} = \frac{2 \xi}{C_p T_\infty} L_2 \frac{I \rho_\infty^2 (\alpha_{rec})_\infty}{(\rho\mu)_r u_\infty^2 m_c^3} \left( \frac{\alpha_{eq. m,n}^2}{g_{m,n}^2 \Theta_{m,n}^{9/2}} - 3 \alpha_{m,n}^2 \right)$$

$$C_{22} = -L_1 \left[ v - \frac{l'}{P_R} + \frac{2}{\Delta \eta} \frac{l}{P_R} \right]$$

$$C_{23} = L_1 \left[ \lambda' \frac{T_{e\infty}}{T_\infty} + \frac{5}{2} \frac{\sqrt{2} \xi k T_{e\infty} j y_\infty}{(\rho\mu)_r u_\infty C_p T_\infty e} - \frac{2 \lambda T_{e\infty}}{(\Delta \eta) T_\infty} \right]$$

$$C_{24} = 0$$

$$\begin{aligned}
D_2 = g_{m,n} + L_2 \left[ \left\{ \left( \frac{\lambda'}{P_R} \right) - V \right\} \theta_{\eta} + \frac{\lambda}{P_R} \theta_{\eta\eta} + \frac{\lambda T_{e\infty}}{T_{\infty}} \theta_{\eta\eta} \right. \\
+ \left( \frac{\lambda' T_{e\infty}}{T_{\infty}} + \frac{5}{2} \frac{\sqrt{2\xi} k T_{e\infty} i y_{\infty}}{(\rho\mu)_r u_{\infty} C_p T_{\infty} e} \right) \theta_{\eta} \\
+ \frac{2\xi}{C_p T_{\infty}} \left( \frac{\sigma E_{x\infty}^2 + \frac{d^2 y_{\infty}^2}{dx_{\infty}^2}}{(\rho\mu)_r \rho_{\infty} u_{\infty}^2} (1 + \beta e) \right) g_{m,n} \\
- \frac{2\xi}{C_p T_{\infty}} \frac{2|\rho_{\infty}^2}{(\rho\mu)_r} \frac{(\alpha_{rec.})_{\infty}}{u_{\infty}^2 m_c^3} \frac{\alpha_{m,n}}{g_{m,n}^2 \theta_{m,n}^{9/2}} \left\{ \left[ \frac{15}{4} - \frac{2(P - \alpha_{eq.m,n})}{(2P - \alpha_{eq.m,n})} \right. \right. \\
\left. \left. \left( \frac{5}{2} + \frac{1}{k T_{e\infty} \theta_{m,n}} \right) \right] \alpha_{eq.m,n}^2 - \frac{11}{4} \alpha_{m,n}^2 \right\} \left. \right]
\end{aligned}$$

where

$$L_1 = \frac{\Delta \xi}{8 \xi f \Delta \eta}$$

$$L_2 = \frac{\Delta \xi}{4 \xi f}$$

### Electron Energy Equation

$$\begin{aligned}
 & A_{31} f'_{m+1,n+1} + A_{32} g_{m+1,n+1} + A_{33} \Theta_{m+1,n+1} + A_{34} \alpha_{m+1,n+1} \\
 & + B_{31} f'_{m+1,n} + B_{32} g_{m+1,n} + B_{33} \Theta_{m+1,n} + B_{34} \alpha_{m+1,n} \\
 & + C_{31} f'_{m+1,n-1} + C_{32} g_{m+1,n-1} + C_{33} \Theta_{m+1,n-1} + C_{34} \alpha_{m+1,n-1} \\
 & = D_3
 \end{aligned}$$

$$A_{31} = 0$$

$$A_{32} = L_1 \frac{\theta_{m,n}}{g_{m,n}} \left\{ \frac{k T_{e\infty} (\rho\mu) r}{2 \xi m_c} \rho_{\infty} u_{\infty}^2 \alpha_{m,n} \nu - \frac{k i \gamma_{\infty} \rho_{\infty} u_{\infty} T_{e\infty}}{e \sqrt{2 \xi}} \right\}$$

$$\begin{aligned}
 A_{33} = L_1 \left\{ \frac{\frac{3}{2} k T_{e\infty} \rho_{\infty} u_{\infty}^2 (\rho\mu) r}{2 \xi m_c} \alpha_{m,n} \nu - \frac{\rho_{\infty} u_{\infty}^2 C_p (\rho\mu) r T_{e\infty}}{2 \xi} \right. \\
 \left. \left( \frac{2 \lambda}{\Delta \eta} + \lambda' \right) - \frac{\rho_{\infty} u_{\infty}}{\sqrt{2 \xi}} \frac{3}{2} \frac{k T_{e\infty}}{e} i \gamma_{\infty} \right\}
 \end{aligned}$$

$$A_{34} = L_1 \frac{k i \gamma_{\infty} \rho_{\infty} u_{\infty} T_{e\infty}}{\sqrt{2 \xi}} \frac{\theta_{m,n}}{\alpha_{m,n}}$$

$$B_{31} = L_2 \frac{k T_{e\infty} \rho_{\infty} u_{\infty}^2 (\rho\mu) r}{2 \xi m_c} \alpha_{m,n} \theta_{m,n} \left\{ \frac{6 \xi}{T_{e\infty}} \frac{dT_{e\infty}}{d\xi} - \frac{4 \xi}{\rho_{\infty}} \frac{d\rho_{\infty}}{d\xi} \right\}$$

$$B_{32} = \frac{k T_{e\infty} (\rho\mu)r \rho_{\infty} u_{\infty}^2}{2 \frac{4}{3} m_c} \frac{\alpha_{m,n} \theta_{m,n}}{g_{m,n}} + L_2 \left[ \left( \frac{3}{2} k T_{e\infty} \theta_{m,n} + I \right) \times \right. \\ \left. \frac{\rho_{\infty} (\tilde{\alpha}^{rec})_{\infty}}{m_c} \times \frac{2 \alpha_{m,n}}{g_{m,n}^3} \frac{\theta_{m,n}^{3/2}}{\theta_{m,n}^{3/2}} (\alpha_{m,n}^2 - \left\{ \frac{\alpha_{eq,m,n}}{2P - \alpha_{eq,m,n}} \right\} \alpha_{eq,m,n}^2) \right. \\ \left. - \left\{ \sigma E_{x\infty}^2 + \frac{i \gamma_{\infty}}{\sigma} (1 + \beta_e^2) \right\} - \frac{3 me}{m_c} k \rho_{\infty} T_{e\infty} \alpha_{m,n} \sum_s \frac{V_{es}}{m_s} \right]$$

$$B_{33} = \frac{\frac{3}{2} k T_{e\infty} \rho_{\infty} u_{\infty}^2 (\rho\mu)r}{2 \frac{4}{3} m_c} \alpha_{m,n} + L_2 \frac{k T_{e\infty} \rho_{\infty} u_{\infty}^2 (\rho\mu)r}{2 \frac{4}{3} m_c} \alpha_{m,n} f'_{m,n} \times \\ \left\{ \frac{6 \xi}{T_{e\infty}} \frac{dT_{e\infty}}{d\xi} - \frac{4 \xi}{\rho_{\infty}} \frac{d\rho_{\infty}}{d\xi} \right\} + L_2 \left( \frac{3}{2} k T_{e\infty} \theta_{m,n} + I \right) \times \\ \frac{2 \rho_{\infty} (\tilde{\alpha}^{rec})_{\infty}}{m_c} \times \frac{\alpha_{m,n}}{g_{m,n}^2} \frac{\theta_{m,n}^{1/2}}{\theta_{m,n}^{1/2}} \left[ \left\{ \frac{2(P - \alpha_{eq,m,n})}{(2P - \alpha_{eq,m,n})} \left( \frac{3}{2} - \frac{I}{k T_{e\infty} \theta_{m,n}} \right) - \frac{9}{4} \right\} \times \right. \\ \left. \alpha_{eq,m,n}^2 + \frac{9}{4} \alpha_{m,n}^2 \right] + \frac{L_1}{4 \Delta \eta} \frac{\rho_{\infty} u_{\infty}^2 C_p (\rho\mu)r T_{e\infty} \lambda}{2 \frac{4}{3}} \\ + L_2 \frac{3 me}{m_c} k \rho_{\infty} T_{e\infty} \alpha_{m,n} \sum_s \frac{V_{es}}{m_s}$$

$$B_{34} = L_2 \left[ \frac{k T_{e\infty} \rho_{\infty} u_{\infty}^2 (\rho\mu)r}{2 \frac{4}{3} m_c} f'_{m,n} \theta_{m,n} \left\{ \frac{6 \xi}{T_{e\infty}} \frac{dT_{e\infty}}{d\xi} - \frac{4 \xi}{\rho_{\infty}} \frac{d\rho_{\infty}}{d\xi} \right\} \right. \\ \left. + \left( \frac{3}{2} k T_{e\infty} \theta_{m,n} + I \right) \frac{\rho_{\infty} (\tilde{\alpha}^{rec})_{\infty}}{m_c} \frac{1}{g_{m,n} \theta_{m,n}} (\alpha_{eq,m,n}^2 - 3 \alpha_{m,n}^2) \right. \\ \left. + 3 \frac{me}{m_c} k \rho_{\infty} T_{e\infty} \sum_s \frac{V_{es}}{m_s} \left( \theta_{m,n} - g_{m,n} \frac{T_{e\infty}}{T_{e\infty}} \right) \right]$$

$$C_{31} = 0$$

$$C_{32} = -A_{32}$$

$$C_{33} = -L_1 \left[ \frac{\frac{3}{2} k T_{e\infty} \rho_{\infty} u_{\infty}^2 (\rho \mu) r}{2 \xi m_c} \alpha_{m,n} V + \frac{\rho_{\infty} u_{\infty}^2 C_p (\rho \mu) r T_{e\infty}}{2 \xi} \left( \frac{2\lambda}{4\eta} - \lambda' \right) - \frac{\rho_{\infty} u_{\infty}}{\sqrt{2} \xi} \frac{3}{2} \frac{k T_{e\infty}}{e} i y_{\infty} \right]$$

$$C_{34} = -A_{34}$$

$$D_3 = \frac{\frac{3}{2} k T_{e\infty} \rho_{\infty} u_{\infty}^2 (\rho \mu) r}{2 \xi m_c} \alpha_{m,n} (\theta_{m,n} + L_2 V \theta_{\eta})$$

$$+ L_2 \alpha_{m,n} f'_{m,n} \theta_{m,n} \frac{k T_{e\infty} \rho_{\infty} u_{\infty}^2 (\rho \mu) r}{2 \xi m_c} \left\{ \frac{6\xi}{T_{e\infty}} \frac{dT_{e\infty}}{d\xi} - \frac{4\xi}{\rho_{\infty}} \frac{d\rho_{\infty}}{d\xi} \right\}$$

$$+ L_2 \left( \frac{3}{2} k T_{e\infty} \theta_{m,n} + I \right) \frac{2 \rho_{\infty} (\tilde{\alpha} \text{rec})_{\infty}}{m_c} \frac{\alpha_{m,n}}{g_{m,n}^2 \theta_{m,n}^{9/2}} \left\{ \frac{11}{4} \alpha_{m,n}^2 - \right.$$

$$\left[ \frac{15}{4} - \frac{2(P - \alpha_{eq,m,n})}{(2P - \alpha_{eq,m,n})} \times \left( \frac{5}{2} + \frac{1}{k T_{e\infty} \theta_{m,n}} \right) \right] \alpha_{eq,m,n}^2 \left. \right\}$$

$$+ \frac{k T_{e\infty} (\rho \mu) r \rho_{\infty} u_{\infty}^2}{2 \xi m_c} \alpha_{m,n} \theta_{m,n} \left[ 1 - \frac{L_2 V g_{\eta}}{g_{m,n}} \right]$$

$$+ \frac{\rho_{\infty} u_{\infty}^2 C_p (\rho \mu) r T_{e\infty}}{2 \xi} L_2 \left[ \lambda \theta_{\eta}^2 + \lambda' \theta_{\eta} \right]$$

$$+ \frac{\rho_{\infty} u_{\infty}}{\sqrt{2} \xi} \frac{3}{2} \frac{k T_{e\infty}}{e} i y_{\infty} L_2 \theta_{\eta} + \left[ \sigma E_{x_{\infty}}^2 + \frac{i y_{\infty}^2}{\sigma} (1 - \beta_e^2) \right] L_2 g_{m,n}$$

$$+ \frac{k i y_{\infty}}{e} \frac{\rho_{\infty} u_{\infty} T_{e\infty}}{\sqrt{2} \xi} \theta_{m,n} L_2 \left[ \frac{g_{\eta}}{g_{m,n}} - \frac{\alpha_{\eta}}{\alpha_{m,n}} \right]$$

### Electron Density Equation

$$\begin{aligned}
 & A_{41} f'_{m+1,n+1} + A_{42} g_{m+1,n+1} + A_{43} \Theta_{m+1,n+1} + A_{44} \alpha_{m+1,n+1} \\
 & + B_{41} f'_{m+1,n} + B_{42} g_{m+1,n} + B_{43} \Theta_{m+1,n} + B_{44} \alpha_{m+1,n} \\
 & + C_{41} f'_{m+1,n-1} + C_{42} g_{m+1,n-1} + C_{43} \Theta_{m+1,n-1} + C_{44} \alpha_{m+1,n-1} \\
 & = D_4
 \end{aligned}$$

$$A_{41} = A_{42} = A_{43} = 0$$

$$B_{41} = 0$$

$$C_{41} = C_{42} = C_{43} = 0$$

$$A_{44} = L_1 V$$

$$C_{44} = -A_{44}$$

$$B_{42} = L_2 \frac{4\pi \rho_\infty^2 (\tilde{\alpha} \text{ rec})_\infty}{(\rho/\mu)_r u_\infty^2 m_c^2} \left\{ \frac{\alpha_{m,n}}{g_{m,n}^3 \theta_{m,n}^{7/2}} \left( \frac{\alpha_{eq,m,n}^3}{2P - \alpha_{eq,m,n}} - \alpha_{m,n}^2 \right) \right\}$$

$$\begin{aligned}
 B_{43} = L_2 \frac{4\pi \rho_\infty^2 (\tilde{\alpha} \text{ rec})_\infty}{(\rho/\mu)_r u_\infty^2 m_c^2} & \left\{ \frac{\alpha_{m,n}}{g_{m,n}^2 \theta_{m,n}^{11/2}} \left[ \left\{ \frac{9}{4} - \frac{2(P - \alpha_{eq,m,n})}{(2P - \alpha_{eq,m,n})} \right. \right. \right. \\
 & \left. \left. \left( \frac{3}{2} + \frac{1}{k T_{e\infty} \theta_{m,n}} \right) \right\} \alpha_{eq,m,n}^2 - \frac{9}{4} \alpha_{m,n}^2 \right] \right\}
 \end{aligned}$$

$$B_{44} = 1 - L_2 \frac{2 \xi \rho_\infty^2 (\tilde{\alpha}^{\text{rec.}})_\infty}{(\rho \mu)_r u_\infty^2 m_c^2} \left\{ \frac{1}{g_{m,n}^2 \theta_{m,n}^{9/2}} (\alpha^{\text{eq. m, n}} - 3 \alpha_{m,n}^2) \right\}$$

$$D_4 = L_2 \frac{4 \xi \rho_\infty^2 (\tilde{\alpha}^{\text{rec.}})_\infty}{(\rho \mu)_r u_\infty^2 m_c^2} \left\{ \frac{\alpha^{\text{eq. m, n}} \alpha_{m,n}}{g_{m,n}^2 \theta_{m,n}^{9/2}} \left\{ -\frac{15}{4} - \frac{2(P - \alpha^{\text{eq. m, n}})}{(2P - \alpha^{\text{eq. m, n}})} \times \right. \right. \\ \left. \left. \times \left( \frac{5}{2} + \frac{I}{k T_{e\infty} \theta_{m,n}} \right) \right\} - \frac{11}{4} \frac{\alpha_{m,n}^3}{g_{m,n}^2 \theta_{m,n}^{9/2}} \right\}$$

$$+ \alpha_{m,n} - L_2 V(\alpha_2)$$



### III. PROGRAMMING

Reprogramming of our earlier computer program for the IBM 360/75 has been completed. Some additional checks of accuracy are being made as the 360/75 used a 32 bit word whereas the GE 635 used a 36 bit word. This results in a loss of one digit of accuracy, and in some cases may be significant.

The new computer program for the finite rate problem has been written and compiled, and is being checked out. A listing of this program is enclosed with this quarterly report.

#### IV. REVIEW OF SHEATH ANALYSIS

The boundary conditions that we have used so far are based on the matching of electron particle flux and electron energy flux at the interface between the continuum region and the free-fall region. This model is valid as long as the Debye length is much smaller than the smallest of the mean free paths, i.e. the electron-neutral mean free path. However, even in this case there still exists a transition region between the continuum and the free-fall regions. It is expected that, in this transition region, both the potential and the electron number density undergo a re-distribution; these quantities naturally match their values in the continuum and the free-fall regions in addition to the matching of particle and energy fluxes. Unfortunately, in order to investigate the transition region we have to use a kinetic theory approach and also take into account any charge separation. The formulation is therefore extremely complicated. Bienkowski (Physics of Fluids, February, 1967) has concluded, on the basis of the asymptotic behavior of the equations, the existence of two transitional layers; but he gave no calculated values which can be only obtained by the numerical integration of a set of non-linear differential equations. We are exploring the possibility of solving several typical situations using the actual parameters of our problem in order to evaluate the accuracy of our assumed boundary conditions.

## V. FUTURE EFFORTS

During the second quarter our principal effort was to formulate the new finite difference equations for the finite rate problem and to reprogram these new equations. This has now been completed.

A second effort was to review the sheath model that we have been using to establish our boundary condition on  $T_e$ .

In the immediate future we plan to concentrate on numerical calculations using our two computer programs. We will also formulate the problem for the insulator wall as contrasted to the electrode wall which we are studying now.

```

COMMON/COM1/EK,EM,EC,C1,PI,AKO,P,Y(1000),TE,DUM(10),TW,EKSI,EKSI
1,DUM(7),MEDGE,EKSI
COMMON/COM3/ SMLPH(4),V(1000),WLST(1000,4),W(1000,4),TEEKSI(10)
COMMON/COM5/ CONST,EP1,EP2,EP3,EP4,ERR,N,NPL1,NWRIT,KK,NK,NRTST,
? NTIMES,NPRINT,NTAB,NVMN
C RETURN HERE FOR START OF NEXT CASE
10 MEDGE=1
NRTST = 1
NTIMES = 0
NK = 1
BK = 1.36E-23
EM = 9.107E-31
EC = 1.612E-19
C1 = 2.42E21
PI = 3.1416
AKO = 8.854E-12
Y(1) = 0.
R = 8.317E3
CALL READIN
40 CALL EDGE1
SMLPH(2) = TW/TEEKSI(MEDGE)
50 N = 0
CALL NXTLST
60 N = 1
CALL NXTLST
70 N = N+1
CALL ABCD
CALL EKPK
IF (NRTST-N) 80,80,90
80 CALL TEST
NRTST = N
IF (CONST) 90,200,200
90 IF(N-999) 70,110,110
C CONVERGENCE NOT ATTAINED, PRINT AND GO TO NEXT CASE
110 WRITE (6,903)
NWRIT = NPL1
CALL WOSUB
GO TO 10
C CONVERGENCE ATTAINED, GO TO NEXT PROFILE
200 CALL WNSUB
202 IF (KK-NK) 220,220,210
C ITERATE UP TO (KK-1) TIMES
210 NVMN = 2
NK = NK+1
CALL VMNSUB(850)
C INITIALIZE FOR NEXT PROFILE
220 NK = 1
NVMN = 1
NWRIT = NPL1
CALL VMNSUB(850)
IF (EKSI-EKSI) 260,260,40
260 WRITE (6,902)
N=NWRIT-1
PUNCH 900,((W(I,J),I=1,N),J=1,4)
PUNCH 900,(V(I),I=1,N)
WRITE(7,901) N
GO TO 10
900 FORMAT (4E18.8)

```

901 FORMAT (7H NWRT=,14)

902 FORMAT (16H THATS ALL FOLKS)

903 FORMAT (36H CONVERGENCE NOT ATTAINED, TRY AGAIN)

END

CREADIN

SUBROUTINE READIN

DIMENSION ARAKSI(1)

COMMON/COM1/COM1(1,18),TW,EKSI,DKSI,COM2(2),DETA,RCMR,DELTA,EDGE,EKSI

1A,MEGE,EKSI

COMMON/COM2/ EDGE(1,2),TEMPRA(5),QEAT(5),QECST(5)

COMMON/COM3/ SMLHH(4),V(100),WLST(100,4),W(100,4),TEEKSI(1)

COMMON/COM5/ CONST,EP1,EP2,EP3,EP4,EP5,N,NPL1,NWRIT,KK,NK,NRTST,  
? NTIMES,NPRINT,NTAB,NVNV

COMMON/COM6/ ELLS,EL,ELNX,YAMLS,YAM,YAMNX,PRNLS,PRN,PRNXX,S,GNX,

? THENX,ENX,CONX,BETAX,QEI,QEA,QFCS,C2,PER,CP,RCMR,CM,TIW,TIWC,PIZ

? ,SM,BZ,ALFANX

DIMENSION FF(12)

NAMLIST/NAME1/DELTA,EKSI,DKSI,DETA,CM,SM,TIWC,CP,PER,P

1IZ,C2,BZ,EKR,EP1,EP2,EP3,EP4

NAMLIST/NAME2/TEMPRA,QEAT,QECST,EDGE,SMLHH,TEEKSI,NWRIT

NAMLIST/NAME3/KK,NTAB

READ (5,NAME1)

READ (5,NAME2)

7 READ (5,NAME3)

WRITE (6,666)

666 FORMAT (' \*\*\*\*\* THE FOLLOWING IS A LIST OF THE INPUT \*\*\*\*\*')

WRITE (6,NAME1)

WRITE (6,NAME2)

WRITE (6,NAME3)

IF (NWRIT .EQ. 0) GO TO 8

READ(5,899) ((W(I,J),I=1,NWRIT),J=1,4)

READ(5,899) (V(I),I=1,NWRIT)

899 FORMAT (4E18.8)

GO TO 9

8 NWRIT =N

9 CONTINUE

WRITE (6,667)

667 FORMAT (' \*\*\*\*\* END OF INPUT LISTING \*\*\*\*\*')

RCMR=3.1E-7\*EDGE(1,7)\*EDGE(1,2)\*\*.75

WRITE (6,900)EKSI,DKSI,DETA

900 FORMAT (7HDEKSI=E16.8,3X,6HEKSI=E16.8,3X,5HDKSI=E16.8,3X,5HDETA=  
1E16.8,3X//)

WRITE (6,901)DELTA,RCMR,CM,SM,CP

901 FORMAT (7HDELTA=E16.8,3X,6HRCMR =E16.8,3X,5HCM = E16.8,3X,  
15HSM = ,E16.8,3X,4HCP = ,E16.8//)

WRITE (6,902)TIW,PER,PIZ,C2,BZ

902 FORMAT(7HCTIW =E16.8,3X,6HPER = E16.8,3X,5HPIZ = E16.8,3X,  
15HC2 = ,E16.8,3X,4HBZ = ,E16.8//)

CALL INITIL

CALL WCSUB

WRITE (6,905)

905 FORMAT (1H1/14X6HTEMPRA20X4HQEAT21X5HQECST/)

WRITE (6,906)(TEMPRA(I),QEAT(I),QECST(I),I = 1,NTAB)

906 FORMAT (1H 3E25.8)

NN = (EKSI-DKSI)/DKSI+1.5

WRITE (6,907)

907 FORMAT (1H1/1X4HEKSI15X2HUE16X2HTE16X3HETE15X3HDUE15X3HDTE,  
?13X5HALFAE)

ARAKSI(1) = EKSI

DO 10 I = 2,NN

10 ARAKSI(I) = ARAKSI(I-1)+DKSI

WRITE(6,908)(ARAKSI(I),(EDGE(I,J),J = 1,5),I = 1,NN)

909 FORMAT (1H 7E18.8)

WRITE (6,909)

909 FORMAT (1H1/7X4HEKS113X4HDETE13X4HEHDE13X5HDKHDE12X4HAJYE14X3HEXE1  
13X4HPPES/)

WRITE (5,910)(APAKSI(I),(EDGE(I,J),J = 6,12),I = 1,NN)

910 FORMAT (1H 7E17.8)

911 FORMAT (12A6)

9999 RETURN

END

```

SUBROUTINE INITIL
COMMON/COM1/COM1(1007),TE,COM2(10),TW,COM3(3),TDA,TDDA,DETA2,DETA,
1DUM4(4)
COMMON/COM5/ SMLHH(4),V(1000),WLST(1000,4),W(1000,4),TEEKSI(10)
COMMON/COM5/ CONST,EP1,EP2,EP3,EP4,ERR,H,NPL1,NWRIT,KK,NK,NRTST,
? NTINES,NPRINT,NTAB,NVMN
DO 10 I = 1,NWRIT
DO 11 J = 1,4
11 WLST(I,J) = W(I,J)
10 CONTINUE
IF (NWRIT-1000) 9,14,14
C EXTEND PROFILES
9 NPL1 = NWRIT+1
DO 12 I = NPL1,1000
V(I) = V(I-1)-DETA
DO 13 J = 1,4
WLST(I,J) = 1.
13 W(I,J) = 1.
12 CONTINUE
14 TW = SMLHH(2)*TEEKSI(1)
TDA = 2.*DETA
TDDA = 2./DETA
DETA2 = DETA**2
9999 RETURN
END

```



CEDGE1

SUBROUTINE EDGE1

COMMON/COM1/DUM1(10,7),TE,ETE,FEES,AJYE,UE,DUE,DTE,DETE,RHOE,DRHOE,  
1,EXE,TW,EKSIS,EKSI,DKSI,DUM2(4),TSQSI,DELTA,MEDGE,EKSI  
COMMON/COM2/ EDGE(10,12),TEMPRA(50),QEAT(50),QECST(50)  
COMMON/COM3/ ELLS,EL,ELNX,YANLS,YAM,YAMNX,PNLS,PRN,PRNNX,S,GNX,  
? THENX,ENX,CUNX,BETAX,QEI,QEA,QECS,C2,PER,CP,FCMK,CM,TIW,TIWC,PIZ  
?,SM,BZ,ALFANX

UE = EDGE(MEDGE,1)

TE = EDGE(MEDGE,2)

ETE = EDGE(MEDGE,3)

DUE = EDGE(MEDGE,4)

DTE = EDGE(MEDGE,5)

DETE = EDGE(MEDGE,6)

RHOE = EDGE(MEDGE,7)

DRHOE = EDGE(MEDGE,8)

AJYE = EDGE(MEDGE,9)

EXE = EDGE(MEDGE,10)

PREX = EDGE(MEDGE,11)

ALFAE = EDGE(MEDGE,12)

RCMR=FHOE\*3.1E-7\*TE\*\*0.75

IF (AJYE) 1,2,2

1 TIW=TIWC

GO TO 3

2 TIW=0.

3 AA=MEDGE-1

EKSI=EKSIS+AA\*DKSI

MEDGE=MEDGE+1

TSQSI=SQRT(2.\*EKSI)

9999 RETURN

END

CNXTLST

SUBROUTINE NXTLST

```
COMMON/COM1/BK,EP,EC,C1,P1,AKO,R,Y(1000),TE,ETE,PRES,AJYE,UE,DUM(5  
1),EXE,DUM1(7),DETA,TSQSI,DUM2(3)  
COMMON/COM2/EDGE(10,12),TEMPRA(50),QEAT(50),QECST(50)  
COMMON/COM3/SMLHH(4),V(1000),WLST(1000,4),W(1000,4),TEEKSI(10)  
COMMON/COM4/E(3000,4),PHI(1000,4),H(4,4),EF(4,4),TEMP(4,4),  
?AA(4,4),B(4,4),C(4,4),D(4,4),A(4,4),CKMAT(4,4),TEMPP(4)  
COMMON/COM5/CONST,EP1,EP2,EP3,EP4,ERR,N,NPL1,NWRIT,KK,NK,NRTST,  
?NTINES,NPRINT,NTAB,NVMN  
COMMON/COM6/ELLS,EL,ELNX,YAMLS,YAM,YAMNX,PRNLS,PRN,PRNNX,S,GNX,  
?THENX,ENX,CUNX,BETAX,QEI,QEA,QECS,C2,PER,CP,ROMR,CM,TIW,TIWC,PIZ  
?,SM,BZ,ALFANX  
QEAQAA = .1  
NPL1 = N+1  
ELLS = EL  
EL = ELNX  
YAMLS = YAM  
YAM = YAMNX  
PRNLS = PRN  
PRN = PRNNX  
GNX = .54*(W(NPL1,2)+WLST(NPL1,2))  
THENX = .5*(W(NPL1,3)+WLST(NPL1,3))  
ALFANX = .5*(W(NPL1,4)+WLST(NPL1,4))  
ET = ETE*THENX  
PRNNX = 2./3.  
ELNX = GNX**(-.25)  
3 ENX = (ALFANX*PRES)/(BK*GNX*TE)  
CALL TABLE(TEMPRA,QEAT,QECST,ET,NTAB,N,QEA,QECS)  
QEI = PI*((EC/AKO)*(EC/(16.*BK*ETE*THENX)))*2  
QEI = QEI*ALOG(32.*(BK*ETE)**1.5/EC/EC*AKO**1.5/(EC*ENX**0.5)  
1*(THENX**1.5))  
ENUE = PRES*QEA/(BK*GNX*TE)+(PER*PRES/BK/TE/ENX-ENX)*QECS+ENX*QEI  
SQT = SQRT(8.*BK/EM*ETE*THENX/PI)  
ENUE = ENUE*SQT  
CONX = EC/EM*EC*ENX/ENUE  
BETAX = BZ*EC/ENUE/EM  
A1 = 1.24E7*ET**1.5/ENX**0.5  
A2 = 1.8E5*ET/ENX**0.5  
GAM = ENX*(BK*TE/PRES)*GNX  
BKSKA = 3.E-6*ET**2.5/(TE*GNX)**0.75/ALOG(55.+A1**4+A2**4)  
BKEKA = BKSKA/(1.+BETAX**2)/(1.+1.414*(1.-GAM)/GAM*BKSKA*  
1(QEAQAA)*SQRT(EM/CM*GNX*TE/ET))  
YAMNX = ELNX/PRNNX*BKEKA  
IF (NPL1-1) 5,5,9999  
5 GRP = YAMNX*CP*ROMR*UE*(EC/BK)/TSQSI/DETA  
AJEYW = ALF1NX*AJYE+ALF2NX*EXE  
SCRB = (AJYE+TIW)/EC/ENX*SQRT(2.*PI*EM/(BK*ETE*THENX))+SQRT((2.*PI  
1*EM)/SM)  
B2 = 2.*GRP/((2.-ALOG(SCRB))*(AJYE+EC*ENX*SQRT(BK*ETE*THENX/SM  
1))+1.5*GRP-2.5*AJEYW+2.*TIW)  
B3 = -.25*B2  
DO 10 J = 1,4  
DO 11 I = 1,4  
SMLHH(I) = 0.  
H(I,J) = 0.  
11 EF(I,J) = 0.  
10 CONTINUE
```

```

      H(3,3) = B2
      EF(3,3) = B2
      SMLHH(3) = B2 + WLST(2,3) + B3 * WLST(3,3) - WLST(1,3) + 4. * TE * GNX * TIW / ETE
      1 / (2. * GRP / P2)
      S = C1 * (ET) ** 1.5 * EXP(-C2 / ET)
      GRP = 4. * PER / (BK * S) * PRES / TE / GNX
      IF (.C1 - GRP) 2,1,1
1     ENEQX = GRP / 4. * (1. - GRP / 4.) * S
      GO TO 119
2     ENEQX = S / 2. * (SQRT(1. + GRP) - 1.)
119  SMLHH(4) = ENEQX
9999 RETURN
      END

```

CTABLE

```
SUBROUTINE TABLE(TEMP,QEA,QECS,ARG5,KTAB,N,XQEA,XQECS)
  DIMENSION TEMP(50),QEA(50),QECS(50)
  XTEMP = ARG5
  9 IF (N) 10,101,10
  10 IF (XTEMP-XTEMP1) 101,11,11
  101 J = 1
  NTAB = KTAB
  11 NTAB1 = NTAB+1
  K = J-1
  CALL TLU1(XTEMP,NTAB,TEMP(J),J,IERR)
  IF (IERR) 13,14,13
  13 WRITE (6,901)XTEMP
  GO TO 9999
  14 NTAB = NTAB1-J
  J = J+K
  IF (NTAB) 9999,9999,15
  15 XQEA=QEA(J)+(XTEMP-TEMP(J))/(TEMP(J+1)-TEMP(J))*(QEA(J+1)-QEA(J))
  XQECS=QECS(J)+(XTEMP-TEMP(J))/(TEMP(J+1)-TEMP(J))*(QECS(J+1)-
: QECS(J))
  16 XTEMP1 = XTEMP
  901 FORMAT (34H THIS TEMPERATURE IS NOT IN TABLE-,E16.8)
  9999 RETURN
  END
```

```

SUBROUTINE TLU1(X,N,TABLE,J,IEPR)
REAL TABLE(N)
IF(TABLE(1)-X) 10,20,30
20 J=1
   IEPR=0
   RETURN
30 J=1
   IEPR=-1
   RETURN
10 NN=N-1
   DO 40 J=2,NN
   IF(TABLE(J)-X) 40,60,70
60 IEPR=0
   RETURN
70 J=J-1
   IEPR=C
   RETURN
40 CONTINUE
IF(TABLE(N)-X) 80,90,100
80 J=N
   IEPR=1
   RETURN
90 J=N
   IEPR=C
   RETURN
100 J=N-1
   IEPR=0
   RETURN
END

```

(A8C0)

SUBROUTINE ABCD

```
COMMON/COM1/BK,EM,EC,C1,PI,AKO,I,Y(1000),TE,ETE,PRES,AJYE,UE,DUE,D  
ITE,DETE,RHCE,DRHCE,EXE,TW,EKSI,EKSI,DKSI,TDA,TDDA,DETA2,DETA,TSQ  
ISI,DELTA,MEDGE,EKSI  
COMMON/COM2/ SMLHH(4),V(1000),WLST(1000,4),W(1000,4),TEEKSI(10)  
COMMON/COM3/ E(3000,4),PHI(1000,4),H(4,4),EF(4,4),TEMP(4,4),  
?AA(4,4),B(4,4),C(4,4),D(4),A(4,4),CKMAT(4,4),TEMPP(4)  
COMMON/COM4/ CONST,EP1,EP2,EP3,EP4,ERR,N,NPL1,NWRIT,KK,NK,NRTST,  
? NTIMES,NPRINT,NTAB,NVMN  
COMMON/COM5/ ELLS,EL,ELNX,YAMLS,YAM,YAMNX,PRNLS,PRN,PRNNX,S,GNX,  
? THENX,ENX,CONX,BETAX,QEI,QEA,QECS,C2,PER,CP,ROMR,CM,TIW,TINC,PIZ  
?,SM,BZ,ALFANX  
FP = .5*(W(N,1)+WLST(N,1))  
G = GNX  
THETA = THENX  
BETA = BETAX  
CON = CONX  
ENE = ENX  
ALFA = ALFANX  
DO 10 I=1,4  
10 D(I) = 0.  
DO 11 I=1,4  
DO 11 J=1,4  
AA(I,J)=0.  
B(I,J)=0.  
11 C(I,J)=0.  
SNUS = (PRES/BK*QEA)/TE/G/CM+(QECS/SM)*(PER*PRES/TE/BK/G-ENE)+(EN  
1E*QEI)/CM  
SNUS = SNUS*DELTA*SQRT(8.*BK*ETE*THETA/PI/EM)  
FPA = (WLST(N+1,1)-WLST(N-1,1))/TDA  
FPAA = (WLST(N+1,1)-2.*WLST(N,1)+WLST(N-1,1))/DETA2  
GA = (WLST(N+1,2)-WLST(N-1,2))/TDA  
GAA = (WLST(N+1,2)-2.*WLST(N,2)+WLST(N-1,2))/DETA2  
THA = (WLST(N+1,3)-WLST(N-1,3))/TDA  
THAA = (WLST(N+1,3)-2.*WLST(N,3)+WLST(N-1,3))/DETA2  
ALFAA = (WLST(N+1,4) - WLST(N-1,4))/TDA  
RHOS = R/PRES/40.*TE*(WLST(N,2)+WLST(N-1,2))  
Y(N) = Y(N-1)+TSQSI*RHOS*.5*DETA/UE  
CALL NXTLST  
ELA = (ELNX-ELLS)/TDA  
YAMA = (YAMNX-YAMLS)/TDA  
PRA = (PRNNX-PRNLS)/TDA  
V1 = V(N)  
Q1 = DKSI/(4.*EKSI*FP*TDA)  
Q2 = 2.5*BK*ETE/EC*TSQSI/(ROMR*UE*CP*TE)  
Q3 = DKSI*DETE  
Q4 = PIZ/(RHCE*CP*TE)  
Q5 = 1.5*BK*ETE*THETA+PIZ  
Q6 = 2.5*BK*ETE*THETA+PIZ  
Q7 = 5.*BK/EC*RHCE*ETE*UE/TSQSI/G  
Q9 = ROMR*UE**2  
Q8 = RHCE*CP*ETE*Q9  
Q6 = RHCE*CP*TE*Q9  
Q10 = 1.5*BK*ENE  
F1 = FP  
Q = G  
FP = WLST(N,1)
```

```

G = WLST(N,2)
THETA = WLST(N,3)
ALFA = WLST(N,4)
ET = ETE*THETA
S=C1*(ET)**1.5*EXP(-C2/ET)
GRP = 4.*PER/(BK*S)*PRES/TE/GNX
IF (.C1-GRP) 2,1,1
1 ENQX = GRP/4.*(1.-GRP/4.)*S
GO TO 3
2 ENQX = S/2.*(SQRT(1.+GRP)-1.)
3 ALFAEQ = ENQX*BK*TE*G/PRES
ALFAP = (5.6E-27)*(BK*ETE/EC)**(-4.5)
L2 = DKSI/(4*EKSI*FP)
L1 = L2/(2*DETA)
AA(1,1) = Q1*(V1-2.*EL/DETA-ELA)
AA(1,2) = 0.
AA(1,3) = 0.
AA(1,4) = 0.
B(1,1) = 1.+DKSI*DUE/2./UE+4.*Q1*EL/DETA
B(1,2) = -DKSI*DUE/(2.*UE*F1)
B(1,3) = 0.
B(1,4) = 0.
C(1,1) = Q1*(-V1-2.*EL/DETA+ELA)
C(1,2) = 0.
C(1,3) = 0.
C(1,4) = 0.
D(1) = FP*(1.-DKSI*DUE/2./UE)+(DKSI/4./EKSI/F1)*(-FPA*V1+2.*EKSI*G
1 *DUE/UE+ELA*FPA+EL*FPAA)
AA(2,1) = -2.*Q1*UE**2/CP/TE*EL*FPA
XTEMP = (2.5*SQRT(2*EKSI)*BK*ETE*AJYE)/(ROMR*UE*CP*ETE*EC)
AA(2,2) = L1*(V1 - ELA/PRN - TODA*EL/PRN)
AA(2,3) = -L1*((TODA*YAM*ETE/ET) + YAMA*ETE/ET + XTEMP)
AA(2,4) = 0.
B(2,1) = L2*G*(2*EKSI/(CP*ET)*(CP*DTE+UE*DUE - AJYE*BZ/
: (ROMR*RHOE*UE)))
XTEMP = (CP*DTE + UE*DUE - AJYE*BZ/(ROMR*RHOE*UE))*FP
YTEMP = -1.0/(ROMR*RHOE*UE**2)*.5*(COND*EXE**2 +
% AJYE**2/COND*(1.0 + BETAE**2))
ZTEMP = (PIZ*RHOE**2*ALFAR)/(ROMR*UE**2 *CM**3)*(ALFA/(G**3
% *THETA**4.5))*(ALFA**2 - ALFAEQ**3/(2.*PER - ALFAEQ))
B(2,2) = 1. + 4.*EL/PRN*L1/DETA + 2.*EKSI/(CP*TE)*L2*XTEMP+YTEMP+
% ZTEMP
UTEMP = 4.*L1*YAM*ETE/(DETA*TE)
VTEMP = 2.*PIZ*RHOE**2 *ALFAR/(ROMR*UE**2 *CM**3)*ALFA/(G**2 *
: THETA**5.5)
WTEMP = ALFAEQ**2*(2.*(PER - ALFAEQ)/(2.*PER - ALFAEQ)*(1.5 - EC*
: PIZ/(BK*ETE*THETA)) - 2.25) + 2.25*ALFA**2
B(2,3) = UTEMP + 2.*EKSI/(CP*TE)*L2*VTEMP*WTEMP
B(2,4) = 2.*EKSI/(CP*TE)*L2*PIZ*RHOE**2 *ALFAR/(ROMR*UE**2 *CM**3)
: *(ALFAEQ**2 - 3.*ALFA**2)/(G**2 *THETA**4.5)
C(2,1) = -AA(2,1)
C(2,2) = -L1*(V1 - ELA/PRN + TODA*EL/PRN)
UTEMP = 2.5*SQRT(2.*EKSI)*BK*ETE*AJYE/(ROMR*UE*CP*TE*EC)
C(2,3) = L1*(YAMA*ETE/TE + UTEMP - 2.*YAM*ETE/(DETA*TE))
C(2,4) = 0.
AA(3,1) = 0.
VTEMP = (ELA/PRN - V1)*GA + EL/PRN*GAA + YAM*ETE/TE*THAA
UTEMP = (YAMA*ETE/TE + UTEMP)*THA

```

```

WTEMP = 2.*EKS1/(CP*TE)*((COND*EXE**2 + AJYE**2/COND*
: (1. + BETAE**2))/(ROMR*RHOE*UE**2))*G
XTEMP = -2.*EKS1/(CP*TE)*((3.75-2.*(PER - ALFAEQ)/(2.*PER -
: ALFAEQ)*(2.5 + EC*PIZ/(BK*ETE*THETA)))*ALFAEQ**2 - 2.75*ALFA**2)
YTEMP = 2.*PIZ*RHOE**2 *ALFAR*ALFA/(PCMR*UE**2 + CM**3 * G**2
: *THETA**4.5)
D(2) = G + L2*(VTEMP + UTEMP + WTEMP + XTEMP + YTEMP)
UTEMP = (BK*ETE*ROMR*RHOE*UE**2)/(2.*EKS1*CM)*ALFA*V1
VTEMP = (BK*AJYE*RHOE*UE*ETE)/(EC*SQR(2.*EKS1))
AA(3,2) = L1*THETA/G*(UTEMP - VTEMP)
UTEMP = (1.5*BK*ETE*RHOE*UE**2 *ROMR*ALFA*V1)/(2.*EKS1*CM)
VTEMP = ((RHOE*UE**2 *CP*ROMR*ETE)/(2.*EKS1))*(YAM*TDDA + YAMA)
WTEMP = (RHOE*UE)/SQR(2.*EKS1)*1.5*(BK*ETE)/EC*AJYE
AA(3,3) = L1*(UTEMP - VTEMP - WTEMP)
AA(3,4) = L1*(BK*AJYE*RHOE*UE*ETE)/SQR(2.*EKS1)*THETA/ALFA
UTEMP = (BK*ETE*ROMR*RHOE*UE**2 *ROMR)/(2.*EKS1*CM)
VTEMP = (6.*EKS1)/ETE*DETE - 4.*EKS1*DRHOE/RHOE
B(3,1) = L2*UTEMP*ALFA*THETA*VTEMP
UTEMP = (BK*ETE*ROMR*RHOE*UE**2)/(2.*EKS1*CM)*ALFA*THETA/G
VTEMP = (1.5*BK*ETE*THETA + PIZ)*RHOE*ALFAR/CM*(2.*ALFA/(G**3 *
% THETA**4.5))
WTEMP = ALFA**2 - ALFAEQ/(2*PER - ALFAEQ)*ALFAEQ**2
XTEMP = COND*EXE**2 + AJYE**2 /COND*(1 + BETAE**2)
YTEMP = 3*EM/CM*BK*RHOE*TE*ALFA*SNUS
B(3,2) = UTEMP + L2*(VTEMP*WTEMP - XTEMP - YTEMP)
UTEMP = (1.5*BK*ETE*RHOE*UE**2 *ROMR)/(2.*EKS1*CM)*ALFA
VTEMP = L2*(BK*ETE*RHOE*UE**2 *ROMR)/(2.*EKS1*CM)*ALFA*FP*
% (6.*EKS1/ETE*DETE - 4.*EKS1/RHOE*DRHOE)
WTEMP = L2*(1.5*BK*ETE*THETA + PIZ)*(2.*RHOE*ALFAR)/CM*ALFA/(G**2
% *THETA**5.5)
XTEMP = (2.*(PER - ALFAEQ)/(2.*PER - ALFAEQ)*(1.5 - EC*PIZ/
% (BK*ETE*THETA)) - 2.25)*ALFAEQ**2 + 2.25*ALFA**2
YTEMP = L1/(4.*DETA)*RHOE*UE**2 *(CP*ROMR*ETE*YAM)/(2.*EKS1)
ZTEMP = L2*3.*EM/CM*BK*RHOE*ETE*ALFA*SNUS
B(3,3) = UTEMP + VTEMP + WTEMP*XTEMP + YTEMP + ZTEMP
UTEMP = (BK*ETE*RHOE*UE**2 *ROMR)/(2.*EKS1*CM)*FP*THETA
VTEMP = 6.*EKS1/ETE*DETE - 4.*EKS1/RHOE*DRHOE
WTEMP = (1.5*BK*ETE*THETA + PIZ)*RHOE*ALFAR/CM
XTEMP = 1./(G**2 *THETA**4.5)*(ALFAEQ**2 - 3.*ALFA**2)
YTEMP = 3.*EM/CM*RHOE*ETE*SNUS*(THETA - G*TE/ETE)*BK
B(3,4) = L2*(UTEMP*VTEMP + WTEMP*XTEMP + YTEMP)
C(3,1) = 0.
C(3,2) = -A(3,2)
UTEMP = 1.5*BK*ETE*RHOE*UE**2 *ROMR/(2.*EKS1*CM)*ALFA*V1
VTEMP = RHOE*UE**2 *CP*ROMR*ETE/(2.*EKS1)*(2.*YAM/DETA - YAMA)
WTEMP = RHOE*UE/SQR(2.*EKS1)*1.5*BK*ETE/EC*AJYE
C(3,3) = -L1*(UTEMP + VTEMP - WTEMP)
C(3,4) = -AA(3,4)
UTEMP = 1.5*BK*ETE*RHOE*UE**2 *ROMR/(2.*EKS1/CM)*ALFA*(THETA +
: L2*V1*THA)
VTEMP = L2*ALFA*FP*THETA*((BK*ETE*UE**2 *RHOE*ROMR)/(2.*EKS1*CM))*
: (6.*EKS1/ETE*DETE - 4.*EKS1/RHOE*DRHOE)
WTEMP = L2*(1.5*BK*ETE*THETA + PIZ)*2.*RHOE*ALFAR/CM*ALFA/
: (G**2 *THETA**4.5)*(2.75*ALFA**2 - (3.75 - 2.*(PER - ALFAEQ)/
: (2.*PER - ALFAEQ)*(2.5 + EC*PIZ/(BK*ETE*THETA)))*ALFAEQ**2)
XTEMP = (BK*ETE*ROMR*RHOE*UE**2)/(2.*EKS1*CM)*ALFA*THETA*
: (1. - (L2*V1*GA)/G)
YTEMP = RHOE*UE**2 *CP*ROMR*ETE/(2.*EKS1)*L2*(YAM*THAA + YAMA*THA)

```



```

: ZTEMP = RHCE*UE/SQRT(2.*EKS1)*1.5*BK*ETE/EC*AJYE*L2*THA +
: (CONDUCTE**2 + AJYE**2/COND*(1. + BETAE**2))*L2*G
ATEMP = PK*AJYE/EC*RHCE*UE*ETE/SQRT(2.*EKS1)*THETA*L2*
: (GA/G - ALFAA/ALFA)
D(3) = UTEMP + VTEMP + WTEMP + XTEMP + YTEMP + ZTEMP + ATEMP
B(4,1) = 0.
AA(4,1) = 0.
AA(4,2) = 0.
AA(4,3) = 0.
C(4,1) = 0.
C(4,2) = 0.
C(4,3) = 0.
AA(4,4) = L1*V1
C(4,4) = -AA(4,4)
UTEMP = L2*(4.*EKS1*RHOE**2 *ALFAR)/(ROMR*UE**2 *CM**2)
VTEMP = ALFA/(G**3 *THETA**4.5)
WTEMP = ALFAEQ**3/(2.*PER - ALFAEQ) - ALFA**2
B(4,2) = UTEMP*VTEMP*WTEMP
VTEMP = ALFA/(G**2 *THETA**5.5)
WTEMP = (2.25 - 2.*(PER - ALFAEQ)/(2.*PER - ALFAEQ))*(1.5 + EC*PIZ/
: (BK*ETE*THETA)))*ALFAEQ**2 - 2.25*ALFA**2
B(4,3) = UTEMP*VTEMP*WTEMP
UTEMP = L2*(2.*EKS1*RHOE**2 *ALFAR)/(ROMR*UE**2 *CM**2)
VTEMP = 1./(G**2*THETA**4.5)*(ALFAEQ**2 - 3.*ALFA**2)
B(4,4) = 1. - UTEMP*VTEMP
UTEMP = L2*(4.*EKS1*RHOE**2 *ALFAR)/(ROMR*UE**2 *CM**2)
VTEMP = ALFAEQ**2 *ALFA/(G**2 *THETA**4.5)
WTEMP = 3.75 - 2.*(PER - ALFAEQ)/(2.*PER - ALFAEQ)*
: (2.5 + EC*PIZ/(BK*ETE*THETA))
XTEMP = 2.75*ALFA**3/(G**2 *THETA**4.5)
D(4) = UTEMP*(VTEMP*WTEMP - XTEMP) + ALFA - L2*V1*ALFAA
9999 RETURN
END

```

CEKPK

SUBROUTINE EKPK

COMMON/COM3/ SMLHH(4),V(1000),WLST(1000,4),W(1000,4),TEFKSI(10)

COMMON/COM4/ E(3000,4),PHI(1000,4),H(4,4),EF(4,4),TEMP(4,4),

?AA(4,4),B(4,4),C(4,4),D(4),A(4,4),CKMAT(4,4),TEMPP(4)

COMMON/COM5/ CONST,EP1,EP2,EP3,EP4,ERR,N,NPL1,NWKIT,KK,NK,NPTST,

? NTIMES,NPRINT,NTAB,NVMN

DIMENSION SCRACH(3,3)

IF (N-2) 10,10,140

10 DO 40 I = 1,4

DO 30 J = 1,4

A(I,J) = B(I,J)

TEMP(I,J) = AA(I,J)

DO 20 K = 1,4

A(I,J) = C(I,K)\*H(K,J)+A(I,J)

20 TEMP(I,J) = C(I,K)\*EF(K,J)+TEMP(I,J)

30 CKMAT(I,J) = A(I,J)

40 CONTINUE

50 CALL MXINV

DO 80 I = 1,4

DO 70 J = 1,4

AA(I,J) = C.

DO 60 K = 1,4

60 AA(I,J) = A(I,K)\*TEMP(K,J)+AA(I,J)

70 CONTINUE

80 CONTINUE

DO 110 I = 1,4

E(5,I) = -AA(1,I)

E(6,I) = -AA(2,I)

E(7,I) = -AA(3,I)

E(8,I) = -AA(4,I)

TEMPP(I) = 0.

DO 100 J = 1,4

100 TEMPP(I) = C(I,J)\*SMLHH(J)+TEMPP(I)

110 D(I) = D(I)-TEMPP(I)

DO 130 I = 1,4

PHI(2,I) = C.

DO 120 J = 1,4

120 PHI(2,I) = A(1,J)\*D(J)+PHI(2,I)

130 CONTINUE

GO TO 9999

140 MN = (N-1)\*4-3

DO 160 J = 1,4

TEMP(4,J) = E(MN + 3,J)

A(4,J) = B(4,J)

TEMP(3,J) = E(MN+2,J)

A(3,J) = B(3,J)

TEMP(2,J) = E(MN+1,J)

A(2,J) = B(2,J)

TEMP(1,J) = E(MN,J)

160 A(1,J) = B(1,J)

DO 190 I = 1,4

DO 180 J = 1,4

DO 170 K = 1,4

170 A(I,J) = C(I,K)\*TEMP(K,J)+A(I,J)

180 CONTINUE

190 CONTINUE

200 CALL MXINV

```

      DO 230 I = 1,4
      DO 220 J = 1,4
      TEMP(I,J) = 0.
      DO 210 K = 1,4
210  TEMP(I,J) = A(I,K)*AA(K,J)+TEMP(I,J)
220  CONTINUE
230  CONTINUE
      M = 4*N-3
      DO 250 I = 1,4
      E(M+3,I) = -TEMP(4,I)
      E(M+2,I) = -TEMP(3,I)
      E(M+1,I) = -TEMP(2,I)
250  E(M,I) = -TEMP(1,I)
      DO 270 I = 1,4
      TEMPP(I) = 0.
      PHI(N,I) = 0.
      DO 260 J = 1,4
260  TEMPP(I) = C(I,J)*PHI(N-1,J)+TEMPP(I)
270  D(I) = D(I)-TEMPP(I)
      DO 290 I = 1,4
      DO 280 J = 1,4
280  PHI(N,I) = A(I,J)*D(J)+PHI(N,I)
290  CONTINUE
      P1=PHI(N,1)
      P2=PHI(N,2)
      P3=PHI(N,3)
      P4 = PHI(N,4)
9999 RETURN
      END

```

# CTEST

## SUBROUTINE TEST

```

COMMON/CC3/ SMLHH(4),V(1000),LIST(1000,4),W(1000,4),TEPKSI(1000)
COMMON/CC4/ E(3000,4),PHI(1000,4),H(4,4),EF(4,4),TEPP(4,4),
?AA(4,4),B(4,4),C(4,4),D(4,4),A(4,4),CKVAT(4,4),TEMPP(4,4)
COMMON/CC5/ CONST,EP1,EP2,EP3,EP4,ERR,N,NPL1,NWPIT,KK,NK,NRTST,
? NTIMES,APRINT,NTAB,NVMA
M = 4*N - 3
TERM1 = ABS(1.-E(M,1)-E(M,2)-E(M,3)-E(M,4)-PHI(N,1))
TERM2 = ABS(1.-E(M+1,1)-E(M+1,2)-E(M+1,3)-E(M+1,4)-PHI(N,2))
TERM3 = ABS(1.-E(M+2,1)-E(M+2,2)-E(M+2,3)-E(M+2,4)-PHI(N,3))
TERM4 = ABS(1.-E(M+3,1)-E(M+3,2)-E(M+3,3)-E(M+3,4)-PHI(N,4))
IF (TERM1-EP1) 1,100,100
1 IF (TERM2-EP2) 2,100,100
2 IF (TERM3-EP3) 3,100,100
3 IF (TERM4-EP4) 4,100,100
100 CONST = -1.
GO TO 9999
4 CONST = 0.
9999 RETURN
END

```

CWNSUB

SUBROUTINE WNSUB

DIMENSION AAA(3),BBB(3)

COMMON/COM2/ SMLHH(4),V(1000),WST(1000,4),W(1000,4),TEPKSI(10)

COMMON/COM4/ E(3000,4),PHI(1000,4),H(4,4),EF(4,4),TEMP(4,4),

?AA(4,4),B(4,4),C(4,4),D(4),A(4,4),CKMAT(4,4),TEMPP(4)

COMMON/COM5/ CONST,EP1,EP2,EP3,EP4,EKR,N,NPL1,NVRIT,KK,NK,NRTST,

?NTIMES,NPRINT,NTAB,NVMN

NPL1=N

DO 10 I = 1,4

10 W(N,I) = 1.

35 N = N-1

MN = 4\*N-3

DO 50 I = 1,4

TEMP(4,I) = E(MN+3,I)

TEMP(3,I) = E(MN+2,I)

TEMP(2,I) = E(MN+1,I)

50 TEMP(1,I) = E(MN,I)

DO 70 I = 1,4

W(N,I) = 1.

DO 60 J = 1,4

60 W(N,I) = TEMP(I,J)\*W(N+1,J)+W(N,I)

70 W(N,I) = W(N,I)+PHI(N,I)

IF (N-2) 75,75,35

75 DO 90 I = 1,4

AAA(I) = 0.

BBB(I) = 0.

DO 80 J = 1,4

AAA(I) = H(I,J)\*W(2,J)+AAA(I)

80 BBB(I) = EF(I,J)\*W(3,J)+BBB(I)

90 W(1,I) = BBB(I)+AAA(I)+SMLHH(I)

9999 RETURN

END

CVMSUB

SUBROUTINE VMASUB(A)

COMMON/COM1/DUM1(1019),EKSIS,EKSIM,DKSI,IDA,IDCA,DETA2,DETA,DUM2(3  
1),EKS1

COMMON/COM3/ SMLFH(4),V(1000),WLST(1000,4),W(1000,4),TEEKSI(10)

COMMON/COM5/ CONST,EP1,EP2,EP3,EP4,ERR,N,NPL1,NWRIT,KK,NK,NRTST,

? NTIMES,NPRINT,NTAB,NVMN

TERM1 = EKS1\*DETA/DKSI

TERM2 = DETA/4.

DO 10 I = 2,1000

10 V(I) = V(I-1)-(TERM1+TERM2)\*(W(I,1)+W(I-1,1))+(TERM1-TERM2)\*(WLST(I  
11,1)+WLST(I-1,1))

GO TO (29,99),NVMN

29 DO 11 I = 1,1000

DO 12 J = 1,4

12 WLST(I,J) = W(I,J)

11 CONTINUE

CALL WCSUB

GO TO 9999

99 RETURN 1

9999 RETURN

END

CWFSUB

SUBROUTINE WCSUB

COMMON/COM1/DUM(7),Y(1000),DUM1(12),EKSIS,EKSIM,DKSI,DUM2(6),MEDGE

1,EKSI

COMMON/COM3/ SMLHH(4),V(1000),WLST(1000,4),W(1000,4),TEEKSI(10)

COMMON/COM5/ CONST,EP1,EP2,EP3,ERR,N,NPL1,NWRIT,KK,NK,NRTST,

? NTIMES,NPRINT,NTAB,NVMN

IF (1-MEDGE) 1,2,1

2 EKSI = EKSIS-.5\*DKSI

DO 10 I = 2,NWRIT

10 Y(I) = Y(I-1)+1.

GO TO 3

1 EKSI = EKSI+.5\*DKSI

3 WRITE (6,902) EKSI

902 FORMAT (1H1/5HEKSI=E16.8/)

WRITE (6,903)

903 FORMAT (1H11X2HFP19X1HG16X5HTHETA18X1HV19X1HY/)

WRITE (6,904)((W(I,J),J = 1,3),V(I),Y(I),I = 1,NWRIT)

904 FORMAT (1H 5E20.8)

9999 RETURN

END

## SUBROUTINE MXINV

COMMON/CLM4/ F(3000,4), PHI(1000,4), H(4,4), EF(4,4), TEMP(4,4),  
 ?AA(4,4), B(4,4), C(4,4), D(4,4), A(4,4), CKMAT(4,4), TEMPP(4)

DIMENSION S(4,4)

S(1,1) = (A(2,2)\*A(3,3)\*A(4,4) + A(2,3)\*A(3,4)\*A(4,2) +  
 : A(2,4)\*A(4,3)\*A(3,2) - A(2,2)\*A(4,3)\*A(3,4) -  
 : A(2,3)\*A(3,2)\*A(4,4) - A(2,4)\*A(3,3)\*A(4,2))

S(2,1) = -(A(2,1)\*A(3,3)\*A(4,4) + A(2,3)\*A(3,4)\*A(4,1) +  
 : A(2,4)\*A(4,3)\*A(3,1) - A(2,1)\*A(4,3)\*A(3,4) -  
 : A(2,3)\*A(3,1)\*A(4,4) - A(2,4)\*A(3,3)\*A(4,1))

S(3,1) = (A(2,1)\*A(3,2)\*A(4,4) + A(2,2)\*A(3,4)\*A(4,1) +  
 : A(2,4)\*A(4,2)\*A(3,1) - A(2,1)\*A(4,2)\*A(3,4) -  
 : A(2,2)\*A(3,1)\*A(4,4) - A(2,4)\*A(3,2)\*A(4,1))

S(4,1) = -(A(2,1)\*A(3,2)\*A(4,3) + A(2,2)\*A(3,3)\*A(4,1) +  
 : A(2,3)\*A(4,2)\*A(3,1) - A(2,1)\*A(4,2)\*A(3,3) -  
 : A(2,2)\*A(3,1)\*A(4,3) - A(2,3)\*A(3,2)\*A(4,1))

S(1,2) = -(A(1,2)\*A(3,3)\*A(4,4) + A(1,3)\*A(3,4)\*A(4,2) +  
 : A(1,4)\*A(4,3)\*A(3,2) - A(1,2)\*A(4,3)\*A(3,4) -  
 : A(1,3)\*A(3,2)\*A(4,4) - A(1,4)\*A(3,3)\*A(4,2))

S(2,2) = (A(1,1)\*A(3,3)\*A(4,4) + A(1,3)\*A(3,4)\*A(4,1) +  
 : A(1,4)\*A(4,3)\*A(3,1) - A(1,1)\*A(4,3)\*A(3,4) -  
 : A(1,3)\*A(3,1)\*A(4,4) - A(1,4)\*A(3,3)\*A(4,1))

S(3,2) = -(A(1,1)\*A(3,2)\*A(4,4) + A(1,2)\*A(3,4)\*A(4,1) +  
 : A(1,4)\*A(4,2)\*A(3,1) - A(1,1)\*A(4,2)\*A(3,4) -  
 : A(1,2)\*A(3,1)\*A(4,4) - A(1,4)\*A(3,2)\*A(4,1))

S(4,2) = (A(1,1)\*A(3,2)\*A(4,3) + A(1,2)\*A(3,3)\*A(4,1) +  
 : A(1,3)\*A(4,2)\*A(3,1) - A(1,1)\*A(4,2)\*A(3,3) -  
 : A(1,2)\*A(3,1)\*A(4,3) - A(1,3)\*A(3,2)\*A(4,1))

S(1,3) = (A(1,2)\*A(2,3)\*A(4,4) + A(1,3)\*A(2,4)\*A(4,2) +  
 : A(1,4)\*A(4,3)\*A(2,2) - A(1,2)\*A(4,3)\*A(2,4) -  
 : A(1,3)\*A(2,2)\*A(4,4) - A(1,4)\*A(2,3)\*A(4,2))

S(2,3) = -(A(1,1)\*A(2,3)\*A(4,4) + A(1,3)\*A(2,4)\*A(4,1) +  
 : A(1,4)\*A(4,3)\*A(2,1) - A(1,1)\*A(4,3)\*A(2,4) -  
 : A(1,3)\*A(2,1)\*A(4,4) - A(1,4)\*A(2,3)\*A(4,1))

S(3,3) = (A(1,1)\*A(2,2)\*A(4,4) + A(1,2)\*A(2,4)\*A(4,1) +  
 : A(1,4)\*A(4,2)\*A(2,1) - A(1,1)\*A(4,2)\*A(2,4) -  
 : A(1,2)\*A(2,1)\*A(4,4) - A(1,4)\*A(2,2)\*A(4,1))

S(4,3) = -(A(1,1)\*A(2,2)\*A(4,3) + A(1,2)\*A(2,3)\*A(4,1) +  
 : A(1,3)\*A(4,2)\*A(2,1) - A(1,1)\*A(4,2)\*A(2,3) -  
 : A(1,2)\*A(2,1)\*A(4,3) - A(1,3)\*A(2,2)\*A(4,1))

S(1,4) = -(A(1,2)\*A(2,3)\*A(3,4) + A(1,3)\*A(2,4)\*A(3,2) +  
 : A(1,4)\*A(3,3)\*A(2,2) - A(1,2)\*A(3,3)\*A(2,4) -  
 : A(1,3)\*A(2,2)\*A(3,4) - A(1,4)\*A(2,3)\*A(3,2))

S(2,4) = (A(1,1)\*A(2,3)\*A(3,4) + A(1,3)\*A(2,4)\*A(3,1) +  
 : A(1,4)\*A(3,3)\*A(2,1) - A(1,1)\*A(3,3)\*A(2,4) -  
 : A(1,3)\*A(2,1)\*A(3,4) - A(1,4)\*A(2,3)\*A(3,1))

S(3,4) = -(A(1,1)\*A(2,2)\*A(3,4) + A(1,2)\*A(2,4)\*A(3,1) +  
 : A(1,4)\*A(3,2)\*A(2,1) - A(1,1)\*A(3,2)\*A(2,4) -  
 : A(1,2)\*A(2,1)\*A(3,4) - A(1,4)\*A(2,2)\*A(3,1))

S(4,4) = (A(1,1)\*A(2,2)\*A(3,3) + A(1,2)\*A(2,3)\*A(3,1) +  
 : A(1,3)\*A(3,2)\*A(2,1) - A(1,1)\*A(3,2)\*A(2,3) -  
 : A(1,2)\*A(2,1)\*A(3,3) - A(1,3)\*A(2,2)\*A(3,1))

DETER = A(1,1)\*S(1,1) + A(1,2)\*S(2,1) + A(1,3)\*S(3,1) +  
 : A(1,4)\*S(4,1)

DO 11 I=1,4

DO 10 J=1,4

10 A(I,J)=S(I,J)/DETER



11. CONTINUE  
9999 RETURN  
END