TRAPPED PROTONS $\geq 100$ keV
AND POSSIBLE SOURCES

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INTRODUCTION

In this brief review we are able only to consider the general characteristics of the trapped \( \geq 100 \text{ keV} \) proton population, the principal time variations observed, and the status of major source and transport mechanisms with primary emphasis on diffusion by violation of the third adiabatic invariant. These items are presented in three separate sections: SURVEY, TIME VARIATIONS, and TRANSPORT and SOURCES.
The initial indication of the existence of a lower energy proton population in the trapping regions was obtained from a rocket-borne emulsion experiment conducted by NAUGLE and KNIFFEN (1961). These early results showed a steep rise in the 8-20 Mev proton intensities in the high latitude portion (L ≥ 1.64) of the region of observation, 1.5 ≤ L ≤ 1.8 and 1400-2000 km. Soon after this observation, the first definitive measurements of trapped protons down to energies of ~ 1 Mev were reported by BAME et al. (1962) using a 1960 rocket observation at L ~ 2.6.

In 1962 the results of the first survey of low energy protons throughout the outer trapping regions and the first observations of protons down to energies of ~ 100 keV were reported by DAVIS and WILLIAMSON (1963) using data from the EXPLORER 12 satellite. Figure 1, from DAVIS and WILLIAMSON (1963), shows three typical spectra obtained at various altitudes in the magnetosphere. They found in general that

a) protons of energies ≥ 100 keV were trapped throughout the region L = 2 to the magnetopause

b) the proton spectrum softened with increasing radial distance (Figure 1) and could be well represented as an exponential

c) during a 1½ month geomagnetically quiet period the proton intensities were stable to ±30%

d) a proton intensity enhancement of a factor of ~ 3 was observed in the L = 3 - 4.5 region during a magnetic storm.
Extensive additional observations from EXPLORERS 14, 15, and 26 (DAVIS, 1965; DAVIS and WILLIAMSON, 1966; SORAAS and DAVIS, 1968), from MARINER 4 (KRIMIGIS and ARMSTRONG, 1966), from satellite 1964 45a (MIHALOV and WHITE, 1966), from EXPLORER 33 (ARMSTRONG and KRIMIGIS, 1968), from INJUN 4 (KRIMIGIS, 1968), and from OGO 4 (FRITZ and KRIMIGIS, 1968) have generally verified the earlier results and have added further significant details to the behavior of low energy (> 100 keV) outer zone protons. Extensions of and additions to the above list of general behavioral characteristics have been

i) the low energy proton population is quite stable during geomagnetically quiet times

ii) the characteristic e-fold energy $E_0$ of the proton spectrum varies, with minor perturbations, as $L^{-3}$ out to $L \sim 5$ during quiet periods

iii) large magnetic storms produce non-adiabatic changes down to $L$ values of 3 while even small magnetic disturbances produce similar changes at $L \geq 5$

iv) the adiabatic redistribution of the low energy proton population due to ring current effects is very important and must be compensated for in order to investigate sources, losses, and invariant-violating transport mechanisms.

The much greater stability of the ≥ 100 keV proton population relative to the low energy (10-100 keV) trapped electron population has been demonstrated by DAVIS (1965). However, long term variations do occur in the proton population and are illustrated in Figure 2.
Note that between the time periods shown, proton intensities have been enhanced at low energies and depleted at high energies. This result has been since verified and shown to be an effect which occurs during geomagnetic storms (SORAAS and DAVIS, 1968).

In addition, the occurrence of proton intensity enhancements in coincidence with magnetic substorms has been reported by DAVIS and WILLIAMSON (1966) and KONRADI (1967). An event discussed by DAVIS and WILLIAMSON (1966) has been studied in detail by BROWN et al., (1968) using all available electron, proton, and magnetic field data from the Explorer 26 satellite. Although the data did not allow an unambiguous interpretation, they did clearly indicate the importance of the magnetic substorm in particle acceleration processes. KONRADI (1967) has further indicated that substorm-associated protons appeared to have drifted to the satellite location following injection in the night side hemisphere.

To end this brief survey of outer zone proton characteristics, we show in Figure 3 an R-\(\lambda\) plot of a model \(> 400\) keV trapped proton population as compiled by the National Space Science Data Center (KING, 1967). Data for Figure 3 were obtained from 7 satellites, 6 research groups, and span the period August 1961 - April 1965. Because of time variations Figure 3 is not necessarily representative of the actual situation at any given time. However, intensity levels at a given point in space are generally stable to factors of 2-3 and it is to within this accuracy that the model of Figure 2 applies.
TIME VARIATIONS

We mentioned earlier that redistributions of the proton population due to ring current effects are significant and must be accounted for to discuss sources, losses, and other non-adiabatic processes. Such changes, resulting from a particle's response to slow variations in the earth's magnetic field while conserving the three adiabatic invariants, have been reported for energetic protons (40-110 Mev) at $L \leq 2.4$ by McILWAIN (1966).

A recent study by SORAAAS and DAVIS (1968) clearly shows similar adiabatic (invariant conserving) variations occurring in the low energy proton population. They have used five months of data (January - June 1965) to study the temporal variations of the trapped 100-1700 keV proton population. Adiabatic effects are removed by transforming the data from the time dependent field to a reference field where the ring current field is effectively zero. The transformation is obtained for protons mirroring at the equator using a disturbance field

$$\Delta B(t,R) = C D_S(t) f(R)$$

where $C = 0.7$ and corrects for the induction field of the earth, $t =$ time of observation and $f(R) =$ radial dependence of a model ring current field. The function $f(R)$ and an example of the effects of the transformation are shown in Figure 4. It should be noted that whether such adiabatic redistributions produce intensity increases or decreases at a given point in space depends strongly on the existing proton energy
spectra and spatial gradients as well as the specific field perturbation in effect at the time. Remaining variations in the time histories of the corrected fluxes are now interpreted as being indicative of the occurrence of non-adiabatic (invariant violating) processes.

Figures 5 and 6 present the uncorrected and corrected time histories of the equatorially mirroring 100-1700 keV trapped proton population at $L = 3$ and 4 respectively. The corrected data are seen to be clearly more ordered than the uncorrected data. In fact SORAAS and DAVIS (1968) report that the corrected data yield a regression coefficient of intensity on $D_{st}$ which is a factor of 5 smaller than that obtained with the uncorrected data. This removal of the intensity-$D_{st}$ correlation is to be expected if the transformation used has properly accounted for the adiabatic variations in the data. Note that the intensity variation at $L = 3$ on day 168 seems to have been purely an adiabatic redistribution during the small magnetic disturbance shown in the $D_{st}$ values.

Figures 5 and 6 indicate and SORAAS and DAVIS (1968) show that three basic large-scale time variations in 100-1700 keV proton intensities were in evidence during the period under study:

1) Adiabatic redistributions of the proton population which vary directly with the magnetic field variations and during which the three particle adiabatic invariants are conserved.

2) Rapid non-adiabatic variations occurring in the main phase of geomagnetic storms during which the low energy proton intensities are enhanced and high energy intensities depleted.
3) A slow, non-adiabatic post-storm variation during which both low and high energy intensities recover toward the pre-storm levels - low energy intensities decay and high energy intensities increase during this period.

The above results directly apply to equatorially mirroring protons. However SORAAS and DAVIS (1968) report that variations in the proton pitch angle distributions which are both correlated and non-correlated with magnetic variations are observed. Compensation must therefore be made for adiabatic variations in proton intensities at all pitch angles before quantitative comparisons can be made with invariant violating processes such as cross-L diffusion.

A next step in the above approach is the extension of the calculations to all pitch angles in order to obtain the pitch angle dependence of the non-adiabatic processes. In addition, it may be necessary to obtain the function $f(R)$ separately for each storm and to extend its functional form to higher altitudes.
TRANSPORT and SOURCES

Steady State

The initial work of KELLOGG (1959), PARKER (1960), and HERLOFSON (1960) showed that the transport of charged particles across L shells under violation of the third adiabatic invariant while conserving the first and second invariants was a potentially effective way of populating the trapping regions with energetic particles. The mechanism envisioned as causing this diffusion was the occurrence of magnetic perturbations on time scales causing a violation of the third invariant. With a source of particles at the magnetopause, the net result of this process would be a group of particles diffusing inward across field lines and becoming energized due to the conservation of the first and second invariants. Following this initial work several excellent theoretical studies have been reported concerning charged particle diffusion in the magnetosphere (DAVIS and CHANG, 1962; TVERSKOY, 1964, 1965; DUNGEY et al., 1965; NAKADA and MEAD, 1965; FÄLTHAMMAR, 1965, 1966, 1968; HAERENDEL, 1968).

An early comparison of proton observations in the outer zone with diffusion theory by DUNGEY et al. (1965) showed reasonable agreement at several pitch angles for the radial dependence of the proton's characteristic e-fold energy, $E_0$. The $L^{-3}$ dependence expected for $E_0$ is simply a consequence of the diffusion of an exponential energy spectrum under conservation of the first and second invariants. As the proton spectrum is reasonably approximated by an exponential, the results of DUNGEY et al. (1965) provide an argument in favor of diffusion. For other initial spectral forms, the evolution of the proton energy spectrum as the protons diffuse across field lines has to be calculated. Thus observation of characteristic energy dependencies other than $L^{-3}$ does not by itself disprove the existence of a diffusive process.
More quantitative studies have been completed using a Fokker-Planck formalism:

\[ \frac{\partial n}{\partial t} = -\frac{1}{\tau} \frac{\partial}{\partial r} \left( r \frac{\partial n}{\partial r} \right) - \frac{1}{2} \frac{\partial^2 n}{\partial r^2} - \frac{\partial}{\partial \mu} \left( \frac{\partial \mu}{\partial t} \right) n - \frac{\partial}{\partial J} \left( \frac{\partial J}{\partial t} \right) n \]

Here

- \( n \) = number of particles in \( dr, d\mu, \) and \( dJ \)
- \( \mu \) = first adiabatic invariant, the magnetic moment
- \( J \) = second adiabatic invariant
- \( D_1 = \left( \frac{\delta r}{\delta t} \right) \) = average radial displacement per unit time
- \( D_2 = \left( \frac{\delta r^2}{\delta t} \right) \) = average radial displacement squared per unit time
- \( \tau \) = e-fold lifetime for charge exchange

Using the relation between \( D_1 \) and \( D_2 \) obtained by Fälthammer (1966) for a dipole field

\[ D_1 = \frac{2(2)}{r^2} \frac{\partial}{\partial r} \left( \frac{2}{r^2} \right) \]

The above may be written in a "diffusion-like" form

\[ \frac{\partial n}{\partial t} = \left( \frac{1}{\tau} \frac{\partial}{\partial r} \left( r \frac{\partial n}{\partial r} \right) - \frac{1}{2} \frac{\partial^2 n}{\partial r^2} - \frac{\partial}{\partial \mu} \left( \frac{\partial \mu}{\partial t} \right) n - \frac{\partial}{\partial J} \left( \frac{\partial J}{\partial t} \right) n \right) \]

The last two terms on the right above represent losses due to coulomb collisions and charge exchange respectively.

This equation has been solved by Davis and Chang (1962), Tverskoy (1964), and Nakada and Mead (1965). The latter authors evaluated the size of the diffusion coefficient, \( D \), by obtaining the rate and size of sudden impulses occurring in the magnetosphere and by using a distorted magnetosphere without a tail configuration (Mead, 1964). They obtained a value of

\[ D = 0.3(10)^{-9} r^{10} \text{ Re}^3 / \text{day} \]
Using this value for D, NAKADA and MEAD (1965) compared the predicted and observed proton steady state \( \frac{\partial n}{\partial t} = 0 \) distribution functions. These are shown in Figure 7.

The basic difference between the observed and calculated curves in Figure 7 is that the peaks of the observed distributions are at a lower altitude than the theoretical curves. NAKADA and MEAD (1965) found that an increase of a factor of eight in D would bring the observed and calculated peaks into coincidence. They also argued that a larger value of D should be expected since

a) inclusion of tail field effects would increase D by about a factor of 4 and

b) perturbations other than sudden impulses probably contribute to the diffusion process. The importance of other variations (including electric fields) may be seen from the fact that D can be expressed in terms of the power spectrum of the disturbance (FÄLTHAMMAR, 1965, 1968)

In addition, TVERSKOY (1965) after an earlier low estimate, has reevaluated D and obtained

\[ D = (4 - 13) \times 10^{-9} r^{10} \text{ Re}^2/\text{day} \]

a value which would produce agreement between the observed and calculated proton intensity distributions.
Therefore, with apparently reasonable values for the diffusion coefficient the steady state equatorially mirroring proton distributions can be described by diffusion theory. Absolute flux comparisons remain to be done, i.e., an observed steady source of low energy protons at high altitudes has still to be used to predict observed fluxes of higher energy protons at low altitudes.

**Non-Steady State**

An important advance in further estimating the importance of diffusion via third invariant violation as a transport process in the outer zone has been reported by SORAAS (1969). He has considered the non-steady state \( \frac{\partial n}{\partial t} \neq 0 \) condition by studying the post-storm long term non-adiabatic recovery of proton intensities toward pre-storm values (the third basic time variation discussed in the previous section). Using the same formalism as NAKADA and MEAD (1965), SORAAS (1969) obtains a value for the diffusion coefficient by best fitting the time evolution of the measured distribution function.

The measured proton distribution function representing the initial conditions for the solution of the Fokker-Planck equation are shown in Figure 8 for the post-storm period following the April 18, 1965 storm. The measured and computed time evolution of this distribution are shown in Figure 9. Calculated curves are shown for the best fit value of

\[
D = 2.4(10)^{-9} r^{10} \text{ Re}^2/\text{day}
\]

as well as for the earlier estimate of NAKADA and MEAD (1965).
The experimental and theoretical results show good agreement at a value required by NAKADA and MEAD (1965) to fit the equilibrium distribution and in good agreement with the estimate of TVERSKOY (1965).

The calculated curves for both values of $D$ in Figure 9 are identical for the three lowest energies shown, indicating that the proton time behavior is dominated by losses at these energies. Also the low energy protons decay faster than the predicted curves. Therefore in the loss dominated region, either the coulomb and charge exchange loss terms are not properly accounted for or additional loss mechanisms are operating. SORAAS (1969) suggests that pitch angle scattering by ion cyclotron noise (KENNEL and PETSCHEK, 1966) may be forcing the enhanced low energy intensities to decay rapidly back to their stable upper limit.

The major role played by loss mechanisms shows the importance of including all appropriate loss terms in the Fokker-Planck equation prior to quantitative comparisons with theory, especially if proton and alpha particle distributions are to be compared as a test of diffusion processes (TVERSKOY, 1965; FRITZ and KRIMIGIS, 1968; KRIMIGIS, 1970).

It thus appears that diffusion theory employing violation of the third adiabatic invariant and conservation of the first and second invariants predicts distribution functions which are in good agreement with a) the observed steady state ($\frac{\partial n}{\partial t} = 0$) proton distribution and b) the observed time evolution ($\frac{\partial n}{\partial t} \neq 0$) of the proton distribution during the slow non-adiabatic recovery toward pre-storm intensities following a magnetic storm.

The value of the diffusion coefficient in both instances is $2.4(10)^{-9} \text{r}^{10} \text{Re}^9$/day.
The cause of the rapid non-adiabatic variations occurring during magnetic storms is as yet unknown. Present evidence indicates they are intimately connected with the occurrence of magnetic substorms. The importance of these substorm-associated variations as a source of outer zone protons as compared to the steady inward diffusion of a high altitude, low energy population remains to be determined.

Note in proof: The conversion from flux to density given by Nakada and Mead (1965) is \( n = n \sigma \rho j \) where \( n \) = number of particles in \( dr, d\mu, dJ \), \( r \) = radial distance and \( j \) = differential directional flux. However, the proportionality term contains a factor of \( \mu^{-1} \) which should be included in the conversion if \( \mu \) is not conserved. The omission of this term by Nakada and Mead (1965) and by Soraas (1969) leads to errors in the Coulomb loss term on page 9 which can be as large as a factor of 2 for the cases considered. However uncertainties in the ambient atmosphere will overshadow a discrepancy of this size and the results reviewed herein are in the main unaffected. I am indebted to T. Birmingham, L. Davis, G. Mead, and M. Walt for discussions on this point.
I wish to thank Mr. L. R. Davis and Drs. T. Birmingham, S. M. Krimigis, and G. D. Mead for many very helpful discussions. I further thank Messers. L. Davis and F. Soraas for allowing me to discuss some of their results prior to final publication.
REFERENCES


FIGURE CAPTIONS

Figure 1. Integral proton spectra measured on three field lines on 26 August 1961. Note exponential representation and spectral softening with increasing radial distance. (from DAVIS and WILLIAMSON, 1963)

Figure 2. Proton intensity profiles from 1962 and 1965 for equatorial pitch angles of 60°. Note low energy enhancement and high energy depletion which occurred between the indicated epochs. (from DAVIS and WILLIAMSON, 1966)

Figure 3. R-λ flux map of model (AP5) proton environment. Compensation not made for time variations. (from KING, 1967)

Figure 4a. Radial dependence of ring current magnetic field used in model calculations of adiabatic effects on trapped protons (from SORAAS and DAVIS, 1968)

Figure 4b. Radial proton integral intensity profile as measured on day 109, 1965 when Dst = -47y, along with transformed profile corresponding to Dst= 0 assuming conservation of the three adiabatic invariants. Radial dependence of ratio of the magnetic field after and before build up of ring current and radial movement ∆R of the particles also shown. Note adiabatic effects can produce both increases and/or decreases at any given position in trapping region (from SORAAS and DAVIS, 1968)
Figure 5a. Uncorrected fluxes. The time behavior of protons mirroring at the equator for the eight integral energies measured at \( L = 3 \). The different curves are marked with letters running from A to H corresponding to the energies 98, 134, 180, 345, 513, 775, 1140 and 1700 keV. The curves are displaced in order to avoid overlap and the values read from the curves A to H must be multiplied by 10 raised to the following exponents -1, -0.75, -0.50, -0.25, 0.0, 0.25, 0.25 and 0.25 in order to get the integral proton intensity above a certain energy in protons/cm\(^2\) sec sr. Below the proton data are plotted the hourly average D\( _{st} \) values. (from SORAAS and DAVIS, 1968)

Figure 5b. Corrected fluxes. The time behavior of protons mirroring at the equator at \( L = 3.0 \) after the adiabatic effects are removed. The different curves are marked with letters running from A to H corresponding to the energies 134, 180, 220, 345, 513, 775, 1140 and 1700 keV. The curves are displaced in order to avoid overlap and the values read from the curves A to H must be multiplied by 10 raised to the following exponents -1, -0.75, -0.50, -0.25, 0.0, 0.25, 0.25 and 0.25 in order to get the integral proton intensities above a certain energy in protons/cm\(^2\) sec sr. Below the proton data are plotted the hourly average D\( _{st} \) values. Note absence of variations at \( \sim \) day 169 indicating perturbations in Figure 5a due entirely to adiabatic redistribution at this time. (from SORAAS and DAVIS, 1968)
Figure 6a. Uncorrected fluxes. The time behavior of protons mirroring at the equator for the eight integral energies measured at L = 4. The different curves are marked with letters running from A to H corresponding to the energies 98, 134, 180, 345, 513, 775, 1140, and 1700 keV. The curves are displaced in order to avoid overlap and the values read from the curves A to H must be multiplied by 10 raised to the following exponents: -1, -0.75, -0.50, -0.25, 0.0, 0.25, 0.25, and 0.25 in order to get the integral proton intensity above a certain energy in protons/cm² sec sr. Below the proton data are plotted the hourly average Dₘ values. (from SORAAS and DAVIS, 1968)

Figure 6b. Corrected fluxes. The time behavior of protons mirroring at the equator at L = 4.0 after the adiabatic effects are removed. The different curves are marked with letters running from A to H corresponding to the energies 134, 180, 220, 345, 513, 775, 1140 keV and 1700 keV. The curves are displaced in order to avoid overlap and the values read from the curves A to H must be multiplied by 10 raised to the following exponents: -1, -0.75, -0.50, -0.25, 0.0, 0.25, 0.25, and 0.25 in order to get the integral proton intensities above a certain energy in protons/cm² sec sr. Below the proton data are plotted the hourly average Dₘ values. (from SORAAS and DAVIS, 1968)

Figure 7. Comparison of integral fluxes as measured by DAVIS et. al. (1964) and calculated integral fluxes. The calculated curves are normalized to the same peak flux for the lowest energy threshold. (from NAKADA and MEAD, 1965)
Figure 8. The initial distribution. The left-hand side of the figure shows the integral proton fluxes above various energies versus radial distance as measured after the April 18 storm on day 111 of 1965. The right-hand side of the figure shows the distribution function $n$ for different values of the magnetic moment plotted vs. radial distance. (from SORAAS, 1969)

Figure 9. The time-behavior of the integral proton intensities of different $L$-values computed from the transport equation with two values of the diffusion coefficient, are compared with the experimentally measured values at $L = 3.0, 3.5, 4.0$ and 4.5. (from SORAAS, 1969)
EXPLORER XII, PASS 8B 26 AUGUST 1961

$J(>E) [\text{cm}^{-2} \text{ sec}^{-1} \text{ sr}^{-1}]$

$e^{-E/E_0}$, $E_0=400$ KEV

$R_o=2.8$ EARTH RADII
$\lambda_m=38^\circ$

$E_0=120$ KEV

$R_o=5.0$
$\lambda_m=10^\circ$

$E_0=64$ KEV

$R_o=6.1$
$\lambda_m=6.5^\circ$

FIGURE 1
PROTON INTENSITIES
AT 60° E.P.A.

FIGURE 2

Plot showing proton intensities as a function of L (Earth radii) for different energies: E = 100 KEV, E = 510 KEV, and E = 1700 KEV.

- October-December 1962
- February 1965
RADIAL PROFILE OF RING CURRENT FIELD

$-f(r)$

EARTH RADII

FIGURE 4a
Figure 4b shows the ratio of magnetic fields $\Delta R$ in Earth radii on the right axis, and the proton flux $J(E)$ in protons/cm$^2$-sec-sr on the left axis. The graph includes curves for different energy levels (1100 KeV, 134 KeV, 180 KeV, 513 KeV, 775 KeV, 345 KeV, and 1700 KeV), with labels for the corrected and uncorrected data. The figure highlights the effect of different energy levels on the proton flux and magnetic field ratio.
TIME DEPENDENCE OF PROTONS

FIGURE 5a
TIME DEPENDENCE OF PROTONS

L = 4, EPA = 90°

DAY OF YEAR, 1965
TIME DEPENDENCE OF PROTONS

L = 4.0
EPA = 90°

J(> E) PROTONS / CM² SEC SR

DST IN Y'S

DAY OF YEAR 1965

FIGURE 6b
Figure 8

For PROTON INTEGRAL INTENSITIES OF DAY III, 1965:

- 8 keV
- 71 keV
- 134 keV
- 220 keV
- 345 keV
- 513 keV
- 775 keV
- 1100 keV
- 1700 keV
- 2200 keV
- 2500 keV
- 3450 keV
- 13400 keV
- 11000 keV
- 17000 keV

Log (F(E)) protons/cm² sr


EARTH RADII:
- 5.5
- 6.0
- 6.5
- 7.0
- 7.5
- 8.0

**Pre-storm Intensities**

**Calculated Curve**

\[ D_2 = 2.40 \times 10^{-9} r^{10} \text{(Earth radii)}^2 \]

\[ L = 3.0 \]

**Calculated Curve**

\[ D_2 = 0.31 \times 10^{-9} r^{10} \text{(Earth radii)}^2 \]

\[ L = 4.0 \]

**Day of Year, 1965**

**Figure 9**