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TECHNICAL REPORT

MICROWAVE MICROELECTRONIC TECHNIQUES
APPLIED TO PHASE SHIFTERS AND OTHER CONTROL DEVICES

FINAL REPORT

29 January 1968 through 29 August 1969

Contract No. NAS8-21268

Prepared for:

GEORGE C. MARSHALL SPACE FLIGHT CENTER
Huntsville, Alabama 35812

Prepared by:

G. H. Robinson
Microwave Integrated Modules Department

November 1969

SPERRY

MICROWAVE ELECTRONICS DIVISION • CLEARWATER, FLORIDA

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1. SUMMARY

This report describes the results of work performed by Sperry Microwave Electronics Division, Sperry Rand Corporation, Clearwater, Florida, for the George C. Marshall Space Flight Center under Contract No. NAS8-21268 from 29 January 1968 through 29 August 1969. Work accomplished prior to 29 November 1968 has also been reported in interim reports of July and November of 1968.

1.1 PROGRAM OBJECTIVES

The prime objective of this program has been to further the development of microwave phase shifters and other microwave control devices in microelectronic form. To this end, the effort has included analytical studies and experimental investigations of materials and devices for application to microwave signal control and processing functions. Particular emphasis has been placed on utilization of the unique properties of ferrimagnetic materials.

1.2 PRINCIPAL RESULTS

The effort has been directed toward:

- 1. Generating design data for microstrip transmission lines on ferrimagnetic and dielectric substrates with emphasis on minimizing losses.**
- 2. Generating design data for fixed bias and latched microstrip junction circulators.**
- 3. Investigating techniques for increasing the peak power capability of microstrip circulators.**
- 4. Evaluating operating characteristics of both fixed bias and latched microstrip junction circulators including peak and average power performance and temperature effects.**
- 5. Temperature stabilization of fixed bias microstrip circulators.**
- 6. Determining repeatability of manufacturing and performance of fixed bias microstrip circulators.**
- 7. Development of a latching microstrip circulator and measurement of performance characteristics such as temperature effects, switching speed, and energy.**

8. Evaluation of slot line as an alternative for miniature components, and advancing the theoretical description of slot line.
9. Development of slot line transitions to microstrip and coax.
10. Development of slot line ferrimagnetic digital phase shifters.

Very briefly, the results obtained indicate:

1. To minimize ohmic losses (which typically represent 90% of the total losses) in microstrip lines at X band, the substrate surface finish should be better than 10 microinches, the rf conductor thickness about 0.2 mil and the substrate thickness as great as is consistent with single-mode propagation without radiation.
2. Microstrip circulators have bandwidth and loss characteristics comparable to their stripline counterparts. In a slightly modified form, the analytical procedures utilized in the design of stripline circulators can be used for microstrip circulators.
3. Peak power capability can be improved by rare earth doping of the ferrimagnetic substrate (e.g., adding 2% Dy to YIG increases peak power capability of a typical microstrip circulator from about 60 watts to 1.5 kw). Magnetic losses increase as Dy content increases (e.g., the circulator using the 2% Dy-YIG has insertion losses of about 0.4 db while the circulator of the same design using undoped YIG has insertion loss of 0.3 db).

4. Circulator Performance

Component	Substrate	Insertion Loss (db)	Isolation (db)	Minimum Insertion Loss (db)	Peak Power Threshold
Typical Fixed Bias Microstrip Circulator 6.3 - 9.3 GHz	YIG	≤ 0.5	> 20	≈ 0.3	≈ 60 w
	2% Dy YIG	≤ 0.6	> 20	≈ 0.4	≈ 1.5 kw
	5% Dy YIG	≤ 1.0	> 20	≈ 0.7	> 8.5 kw
Typical Latched Microstrip Circulator 9.0 - 9.7 GHz	YIG	≤ 1.0	> 16 (> 20 db, 9.2 - 9.6 GHz)	0.5	Not Measured

5. Good temperature stability in X band microstrip circulators can be achieved with gadolinium-substituted YIG substrates.
6. Good manufacturing and performance repeatability is achieved in microstrip circulators without any adjustment of individual units, including magnets.
7. Latching microstrip circulators can be built with switching circuitry external to the substrate. A 20 db isolation bandwidth in X band is on the order of 500 MHz with losses under 1 db. A switching time of 3 μ sec has been demonstrated with 175 μ joules switching energy (including driver). Shorter switching times and lower energy appear feasible.
8. Slot transmission line losses are no greater than microstrip in X band.
9. Slot line on 0.055 thick D16 substrate shows little interaction with conductors within 1/4 inch of the substrate.
10. A slot line to microstrip transition has exhibited VSWR under 1.1:1 from 8 to 10 GHz.
11. Theoretical prediction of the H-field of slot line has been developed* and detailed experimental investigation of the fields performed.
12. Latched slot line phase shifters have exhibited figures of merit of 250°/db and phase shift of 110 degrees/inch.

1.3 SCOPE OF REPORT

The following paragraphs indicate the contents of this report.

Section 2 . . . Discusses the general features of microstrip and slot line devices, pointing out their relative advantages by comparison with more conventional microwave devices.

Section 3 . . . Gives theoretical characteristics of microstrip line and discusses choice of materials.

Section 4 . . . Gives theoretical characteristics of slot transmission line and discusses possible applications.

* The analysis was developed by Dr. S. B. Cohn under funds from this program with additional support from the U. S. Army Electronics Command and Stanford Research Institute under Contract DAAB07-68-C 0088. (See Appendix A.)

Section 5 . . . Describes various configurations and the design considerations for fixed bias and latched microstrip circulators. Basic circulator theory is referenced and utilized as required. Differences in design considerations between microstrip and strip transmission line circulators are pointed out.

Section 6 . . . Describes various configurations and design considerations for latched ferrite phase shifters in slot line.

Section 7 . . . Describes the results of experimental work accomplished, including measurement techniques and a summary of data acquired.

Section 8 . . . Details conclusions which may be drawn from the results of Section 7.

Section 9 . . . Recommends a course of investigation based on contents of Section 8.

Appendix A . . . Contains an analytical approach to slot line yielding the magnitude of magnetic field components.

2. BACKGROUND

2.1 THE NEED FOR NEW MICROWAVE COMPONENTS

The rapidly increasing complexity of microwave systems and the diversification of their applications had led in recent years to increased emphasis on the need for small, highly reliable, low cost microwave circuitry. The relatively low volume production of present day microwave components is in part responsible for and, in part due to, the essentially hand-tailored aspects of many microwave devices. Methods of assembly and manufacture of most discrete microwave components are not compatible with either high volume or low cost production.

The recent success of integrated circuits in dramatically reducing the cost of low frequency electronic circuits has stimulated the microwave industry to take a second look at their designs and methods of manufacturing microwave components.

A result of this introspective look has been the development of new microwave components that result in substantial savings in the cost, size and weight of microwave systems. These savings can in many cases be realized with little or no sacrifice in performance. While development costs for these systems are in many cases substantially more than would be incurred in the simple assembly of conventional microwave components, the long range cost savings in production far outweigh this initial investment.

2.2 BASIC MIC TECHNIQUES

Microwave integrated circuits (MICs) can be approached from two basic techniques:

- Monolithic-active devices and rf circuits on semiconductor substrates, and
- Hybrid circuits - rf circuits and semiconductor chips on insulating substrates.

The monolithic approach permits multifunction circuits to be formed on a single semiconductor chip. This approach inherently lends itself to extremely high volume production of the same circuit function at a relatively low cost but offers little or no flexibility, and the start costs are extremely high. Also, testing of individual elements in the circuit is complicated, and replacement of faulty parts impossible. This technique does, however, represent the ultimate in miniaturization.

The hybrid technique offers circuit flexibility while maintaining a reasonably low production cost. The development phase is simplified since each device in a module can be developed and evaluated individually prior to circuit integration. This technique allows utilization of both ferrite and semiconductor devices in the circuit using presently available techniques. Hybrid circuitry, then, appears the most logical choice for development at this time for distributed element circuitry.

2.3 TRANSMISSION LINE TECHNOLOGY

Advantages of slot and microstrip transmission lines for MIC applications stem from the fact that both microstrip and slot lines are planar structures and that complicated circuitry can be easily realized with the use of photoetching techniques. For example:

- Assembly tolerances normally associated with interconnecting conventional devices are eliminated.
- Strip-to-ground-plane spacing for microstrip circuits fabricated on ceramic substrates can be very accurately controlled, thereby eliminating the VSWR and moding problems that often arise from this source in conventional stripline circuits.
- Absence of a dielectric-to-dielectric interface with an accompanying need for many screws to pull the dielectric together to prevent air spaces is another advantage of the ceramic substrate microstrip or slot line circuit over the conventional stripline circuit.
- Achieving the "identicalness" for hundreds or thousands of circuits so necessary for systems using electronic beam steering techniques appears to be relatively easy with photo-etching techniques.

Microstrip transmission line -- the basis of most microwave integrated circuit design -- uses a high dielectric constant material as a substrate with a conductor applied to one side and a ground plane applied to the other (see Figure 1). In recent years, tremendous strides have been made in the development of microstrip microwave components utilizing both semiconductor and ferrite technology.

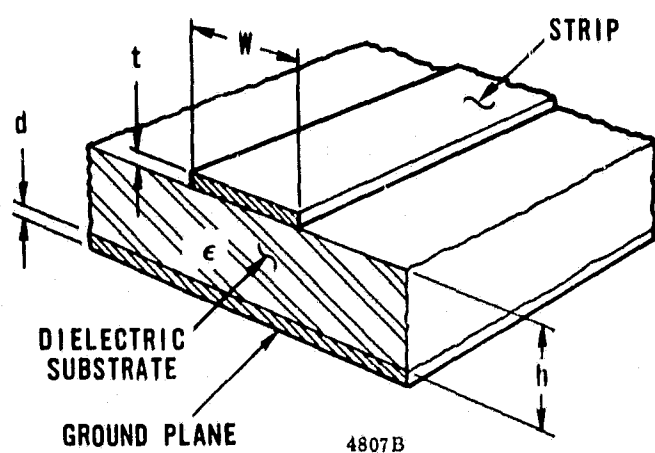


Figure 1. Dimensions in a Microstrip Transmission Line

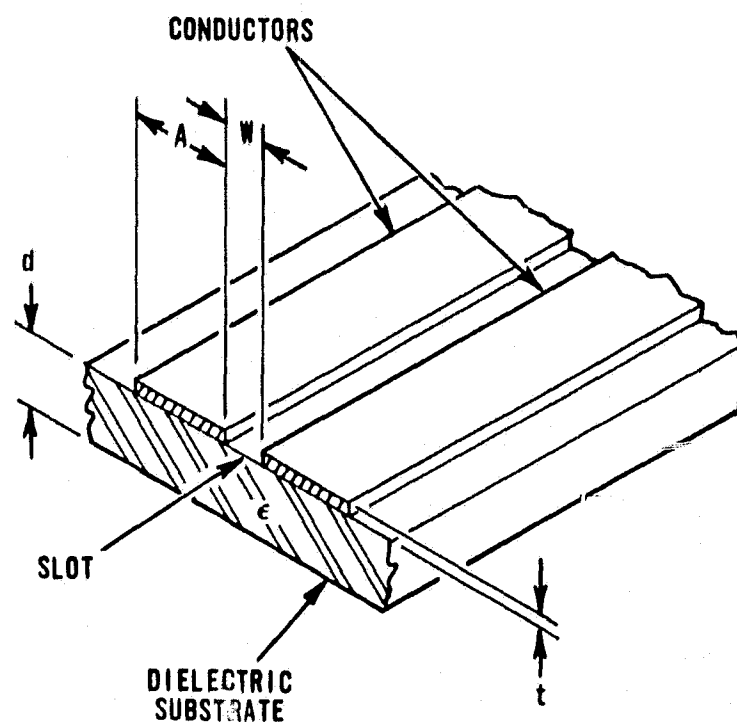


Figure 2. Dimensions in Slot Transmission Line

The microstrip circuit technology permits photoetching a complete rf circuit on a dielectric, ferrimagnetic, and/or semiconductor substrate. These circuits can combine many active and passive components to provide overall signal processing functions. A complete transmitter circuit (or receiver circuit) can be photoetched in one or two operations. This new technology is expected to have a great impact on advanced system designs. In particular, the low power microwave components market should be radically affected by this new technology.

Slot transmission line is a new type of transmission line described by Cohn.¹ This line appears to offer certain advantages in the design of ferrimagnetic components. The transmission line consists of a slot in the conducting plane on one side of a dielectric sheet (Figure 2). Fabrication techniques are basically the same as for microstrip so that it has the same advantages of photoetching, etc.

The prime reason for considering slot line as well as microstrip is the existence of a region of elliptically polarized rf magnetic field in the dominant mode of propagation. This has distinct advantages over microstrip for ferrimagnetic device

application where special structures such as meander lines must be used to provide a region of elliptical polarization. In the microstrip structure, only a small part of the total transmission line length exhibits the circular polarization which interacts with the ferrite. The entire length of slot line would be active. The line is also well suited to shunt-mounted elements since both conductors are on one side of the substrate. Thin film resistors shunting the line could be deposited with ease, for example. Need for holes through the substrate for shunt-mounted elements is eliminated. Straightforward methods of coupling between slot and microstrip lines exist so that they would be compatible in a system, with each type of line used to best advantage.

3. CHARACTERISTICS OF MICROSTRIP LINE

The properties of microstrip line have been investigated at Sperry Microwave's Laboratory and by a number of other workers.^{2,3,4,5,6,7} The following sections describe briefly some of the criteria for microstrip line and the significant properties of the substrate used.

3.1 DESIGN CRITERIA FOR MICROSTRIP LINE

3.1.1 Microstrip Impedance and Effective Wavelength

Dielectric Substrates. Although microstrip cannot support propagation in a pure TEM mode, it appears that the TEM assumption is a very good first approximation. Wheeler² has utilized a TEM solution in a homogeneous medium having an effective dielectric constant, ϵ_{eff} , to determine the characteristic impedance of the inhomogeneous dielectric line (refer to Figure 1). The effective dielectric constant has a value intermediate between that of the substrate and that of the surrounding medium. Wheeler utilized an "effective filling factor" determined by conformal mapping to evaluate ϵ_{eff} . It is important to recognize that the TEM approximation is accurate only if the substrate thickness is sufficiently small compared to a wavelength, i. e., if

$$\frac{\sqrt{\epsilon} h}{\lambda_0} \ll 1,$$

where h is as defined in Figure 1.

Figure 3 presents the characteristic impedance as a function of w/h for several values of substrate relative dielectric constant, ϵ_r . These data have proven to be quite accurate for use in the design of microstrip lines.

In contrast to TEM lines, the velocity of propagation and hence the "guide" wavelength of the microstrip line varies with strip width w . This occurs because the part of the field that extends in the air above the conductor becomes more significant for small strip width. The ratio of wavelength in the microstrip line to the wavelength of a TEM wave in a homogeneous dielectric with a dielectric constant equal to that of the substrate is given as a function of w/h in Figure 4. For values of strip width-to-substrate thickness ratios and dielectric constants commonly used, the actual wavelength in microstrip is typically 20 and 25 percent longer than the TEM value.

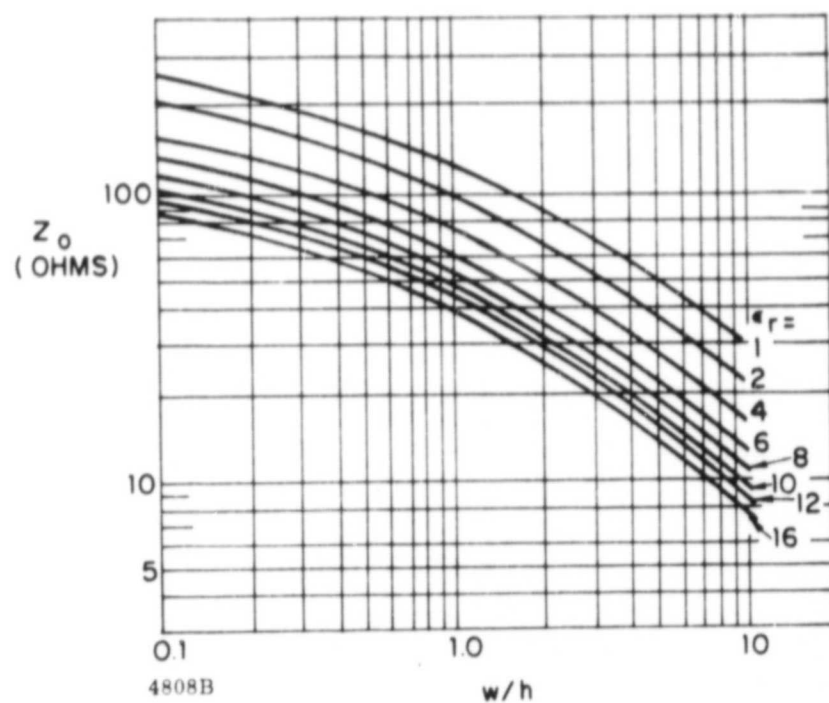


Figure 3. Calculated Characteristic Impedance Z_0 of Microstrip Lines as a Function of w/h and ϵ_r (after Caulton, et al)⁵

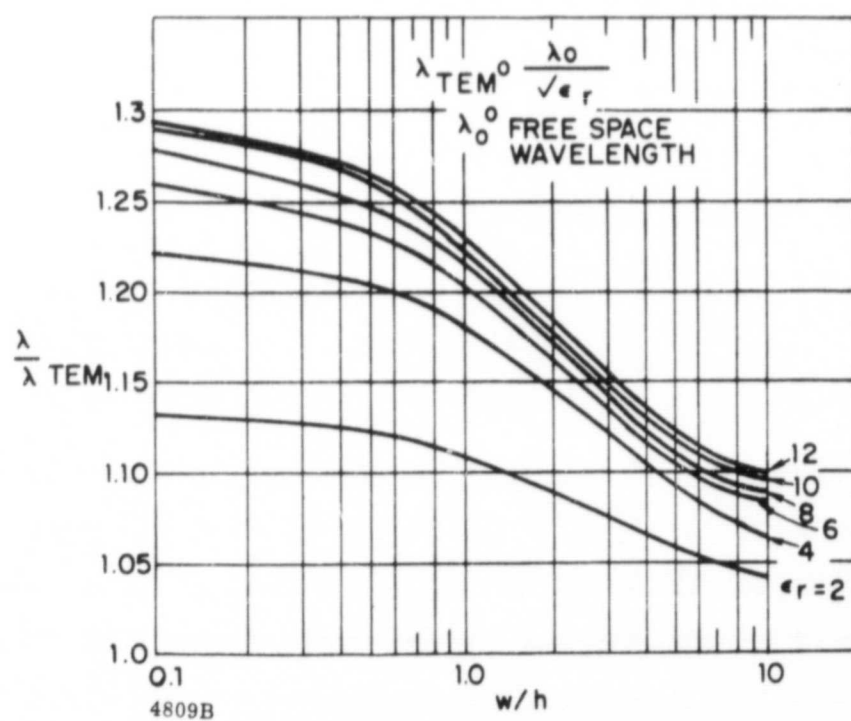


Figure 4. Calculated Normalized Wavelength as a Function of w/h and ϵ_r (after Caulton, et al)⁵

Ferrite Substrates. The basic design considerations relative to use of ferrite substrates in microstrip line are very similar to those already discussed for dielectric substrates except for an additional perturbation of propagation characteristics due to the microwave permeability of the ferrite. The departure of permeability (μ) from 1.0 and the rf frequency dependence of the permeability cause a resulting change in both wavelength and microstrip line impedance. The permeability is, of course, influenced by the applied field and this dependence forms the basis for device operation.

3.1.2 Losses in Microstrip Lines

Minimum insertion loss is a requirement for most microwave circuits. The total insertion loss for a microstrip circuit is made up of dielectric, resistive and reflection losses; if ferrite materials are used, magnetic losses must also be considered. The reflection losses can be minimized by careful attention to impedance levels and avoidance of discontinuities. For ferrite devices, such as phase shifters, that operate at bias field values far from resonance, magnetic losses can be minimized by selecting a material with an appropriate value of $\frac{\gamma^4 \pi M_s}{\omega}$ and a small intrinsic linewidth. For a first order approximation, the magnetic losses far from resonance can be accounted for by an equivalent magnetic loss tangent. The following discussion pertains specifically to resistive and dielectric losses in microstrip; by appropriate substitution the discussion can, through the artifice of the equivalent magnetic loss tangent, apply to magnetic losses as well.

Resistive Loss. From well-known TEM transmission line theory, the resistive or conductor loss in db/unit length for a microstrip line is given by

$$L_c = \frac{(\pi \mu f)^{1/2}}{Z_0 W} \left(\frac{1}{\sqrt{\sigma_{c1}}} + \frac{1}{\sqrt{\sigma_{c2}}} \right) \times 4.343 \text{ db/unit length}$$

where

- μ = permeability = $\mu_r \mu_0$ (of conductors)
- σ_{c1} = conductivity of strip
- σ_{c2} = conductivity of ground plane
- f = frequency in Hz
- Z_0 = characteristic impedance
- W = effective strip width

Dielectric losses can be calculated from

$$L_d = 4.343 \cdot \omega (\mu \epsilon')^{1/2} \tan \delta \text{ db/unit length}$$

$$\omega = 2\pi f$$

$$\mu = \text{permeability (of substrate)}$$

$$\epsilon' = \text{real part of the permittivity}$$

$$\tan \delta = \text{dielectric loss factor}$$

An interesting result is obtained if the losses per wavelength are considered. In the case of conductor loss:

$$L_c \cdot \lambda = \frac{8.686 (\pi \mu)^{1/2} v_c}{Z_0 W \sqrt{\sigma_{c_0} \cdot \epsilon' \cdot f}} \text{ db}/\lambda$$

where

$$\sigma_{c_0} = \text{conductivity of both ground plane and strip}$$

$$v_c = \text{velocity of light}$$

As indicated by the above equation, the conductor loss per wavelength actually decreases as the inverse of the square root of frequency. This means that where line lengths are fractional parts of a wavelength, the total conductor loss will be less at the higher frequencies. In particular, this would apply to microwave filter networks where quarter wave stubs and lines are used. Resistive losses for a typical case are shown in Figure 5.

Dielectric Loss. For the dielectric loss case:

$$L_d \cdot \lambda = 8.686 \pi (\mu)^{1/2} \tan \delta \cdot v_c$$

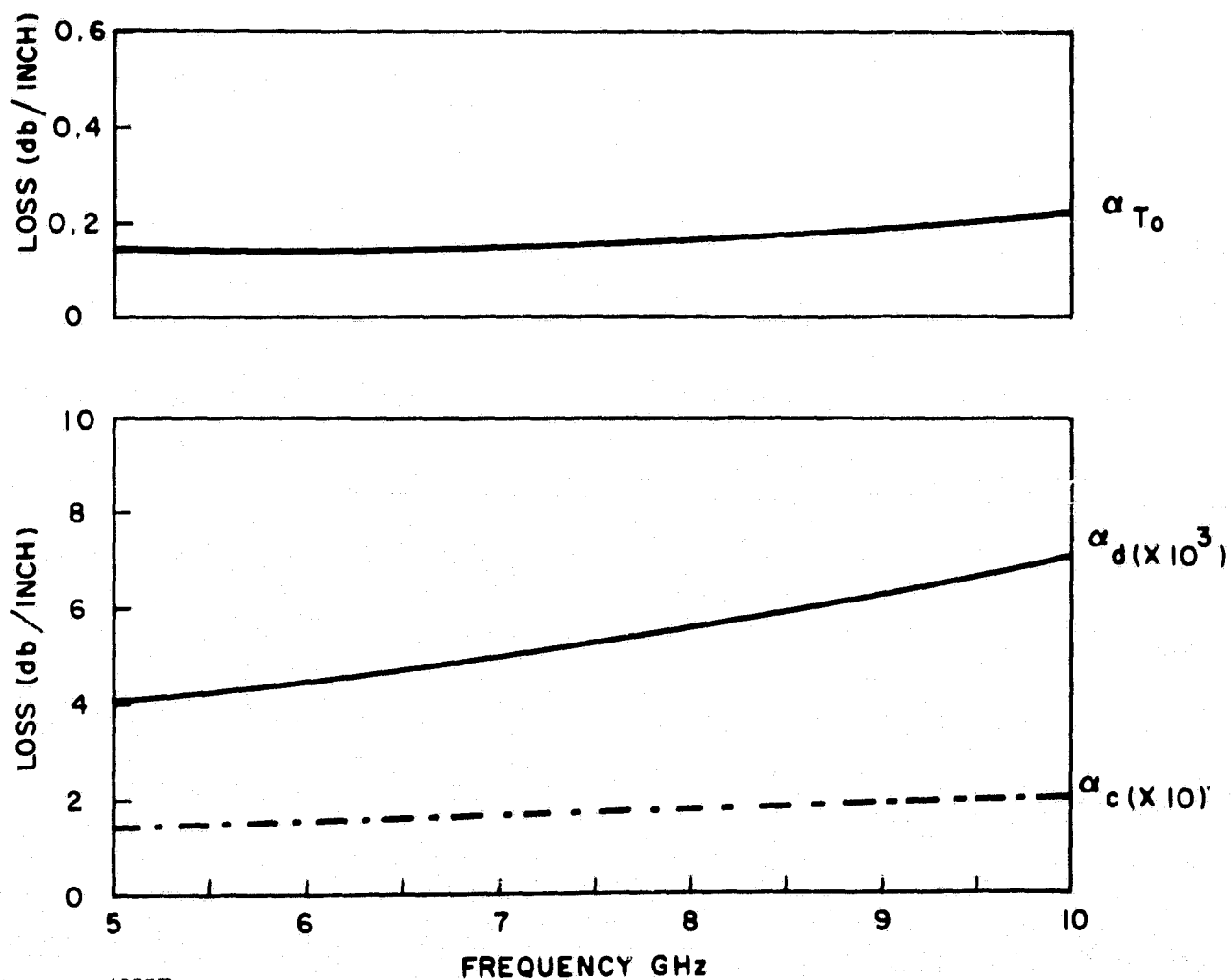
This equation shows that dielectric losses per wavelength are independent of frequency and dielectric constant, but depend only on the dielectric loss factor. Dielectric losses for a typical case are shown in Figure 5.

Total Loss. The total loss in a microstrip circuit on a dielectric substrate is the sum of the conductor and dielectric losses. Total loss in db/inch as a function of frequency is shown for a typical substrate thickness in Figure 5. Figure 6 presents computed total loss as a function of frequency for 50 ohm microstrip lines on aluminum oxide substrates of several thicknesses. It can be seen that losses are lower for the thicker substrates.

The net effect of the last two equations is to show that where wavelength is a parameter in the microwave circuit, scaling these circuits to higher frequencies does not necessarily increase the loss. This, of course, depends on how the transmission

line is being used; i. e., if, as the frequency is increased, the line lengths remains a fraction of a wavelength, the loss is reduced, but if the fractions of a wavelength are multiplied with frequency, the losses will increase.

$$\begin{aligned} Z_0 &= 50 \text{ ohms} \\ W &= 0.055", h = 0.055" \\ \mu &= 1 \\ \tan \delta &= 0.0001 \\ \epsilon' &= 9.5 \\ \sigma &= 10^7 \text{ mhos/meter} \\ \alpha_{\text{TOTAL}} &= \alpha_{\text{DIELECTRIC}} + \alpha_{\text{CONDUCTOR}} \end{aligned}$$



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Figure 5. Dielectric and Copper Loss in Microstrip

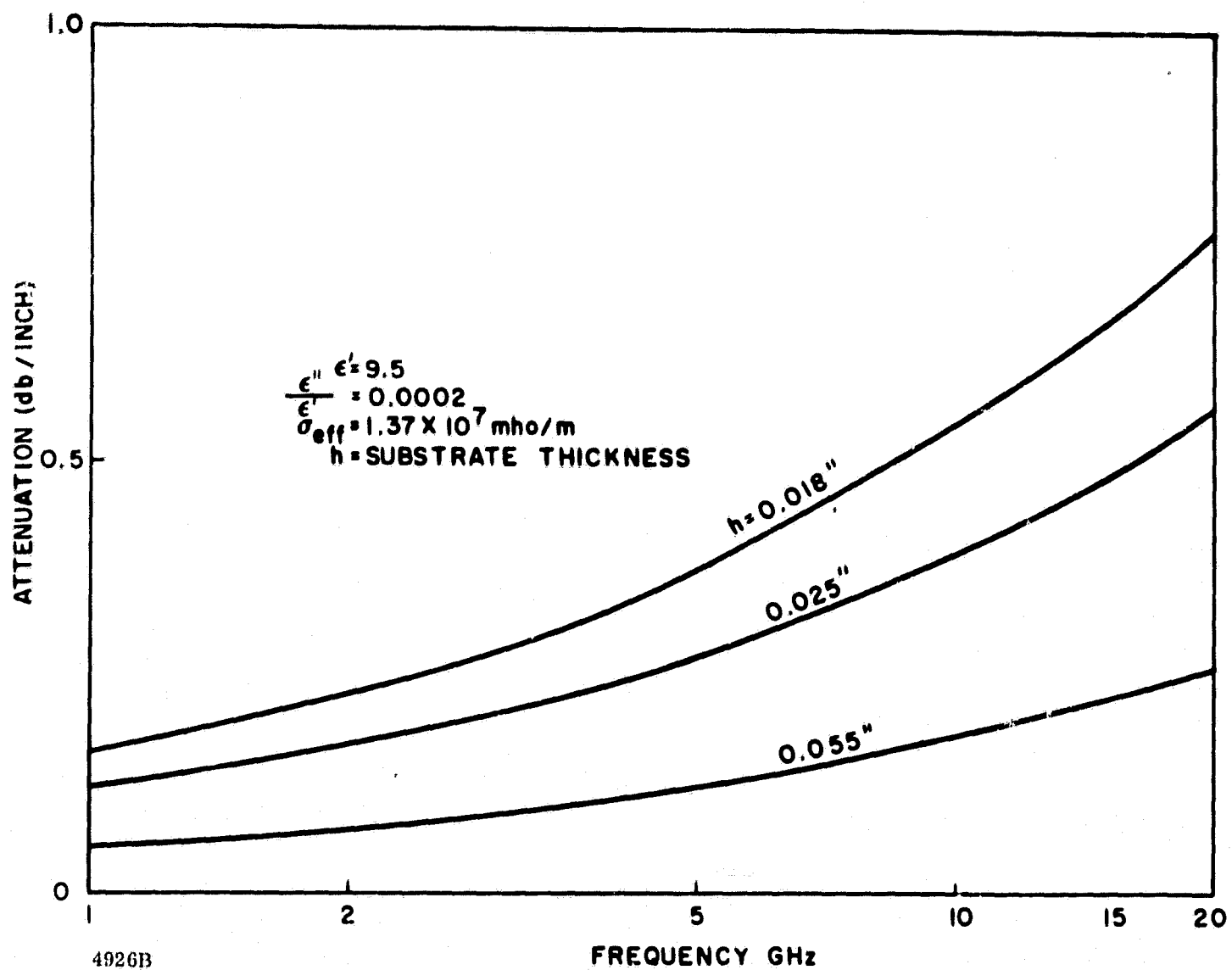


Figure 6. Attenuation Versus Frequency for 50 Ohm Microstrip Line

3.2 SUBSTRATES

3.2.1 Dielectric Substrates

Two properties of the substrate must be considered:

- Electrical characteristics (i.e., dielectric constant and dielectric losses at microwave frequencies), and
- Mechanical stability.

Electrical Characteristics. Losses should always be minimized and it is desirable to have available a choice of materials which provide a wide range of dielectric constants. This selection of materials provides a means which will allow the electrical wavelength (λ_{eff}) in the line to be compatible with the size of a

standardized module. Therefore, at the low frequency end of the microwave spectrum, a substrate with a high dielectric constant is selected in order to reduce λ_{eff} . At higher frequencies, a lower dielectric constant may be more desirable to avoid "over-miniaturization."

Mechanical Stability. The second important property of the substrate is the mechanical stability. At short wavelengths, the requirements imposed upon the mechanical tolerances become quite stringent. Variations in mechanical lengths will be reflected as inconsistencies in the line impedance. Obviously, organic materials with a high thermal coefficient of expansion and a relatively soft texture do not meet these requirements. On the other hand, ceramic materials offer an ideal combination of stability, low loss, and wide range of dielectric constants. Table 1 shows some of the typical characteristics of these materials and the corresponding wavelength λ_{TEM} in the material.

TABLE 1. TYPICAL CHARACTERISTICS OF MATERIALS

Material	ϵ_r	$\tan \delta$	λ (1 GHz) TEM	λ (8 GHz) TEM	λ (16 GHz) TEM
Fused Silica at 100 MHz 1 GHz 10 GHz	3.78 3.78 3.78	1×10^{-4} 0.6×10^{-4} 1×10^{-4}	150 mm	18.8 mm	9.2 mm
Alsimag 614* at 100 MHz 1 GHz 10 GHz	9.3 9.2 9.0	3×10^{-4} 5×10^{-4} 7×10^{-4}	100 mm	12.5 mm	6.25 mm
Rutile (TiO_2) at 100 MHz 1 GHz 10 GHz	100 - 90	2.5×10^{-4} 20×10^{-4}	30 mm	3.75 mm	1.88 mm
Lucalox ⁺ at 9.72 GHz	9.9	2.5×10^{-5}	96 mm	12 mm	6 mm

Table 1. Typical Characteristics of Materials (Continued)

Material	ϵ_r	$\tan \delta$	$\lambda(1 \text{ GHz})$ TEM	$\lambda(8 \text{ GHz})$ TEM	$\lambda(16 \text{ GHz})$ TEM
"Tellite" or "Polyguide" at 10 GHz	2.32	1.5×10^{-4}	197 mm	24.7 mm	12.3 mm
Microwave Ferrite at 9 GHz	9 to 12	5×10^{-4}	~100 mm	~12 mm	~6 mm
Microwave Garnet at 9 GHz	16	$<0.5 \times 10^{-5}$	75 mm	9.4 mm	4.7 mm
Beryllium Oxide	6	$<5 \times 10^{-4}$	~140 mm	~15 mm	~8 mm
Silicon [□] (1000 ohm-cm) at 1 GHz	11.7	~0.15	87.8 mm	10.9 mm	5.45 mm
GaAs [□] (Semi-Insulating) at 1 GHz	11.1		90 mm	11.2	5.6 mm

* American Lava Corporation

+General Electric

□ R. R. Webster, "Integrated Microwave Oscillators, Amplifiers, Switches and Converts," Digest of Technical Papers, Solid State Circuits Conference, University of Pennsylvania, February 15-17, 1967 (page 38).

4. CHARACTERISTICS OF SLOT TRANSMISSION LINE

4.1 GENERAL DESCRIPTION

Slot transmission line consists of a gap in a conducting plane on one side of a dielectric or ferrite substrate. There is no conductor on the opposite surface. (See Figures 2 and 7.) This type of transmission line was originally introduced by S. B. Cohn¹ in 1968. A more detailed theoretical analysis has since been presented by Cohn⁸ and experimental work reported by Cohn and others.^{9, 10}

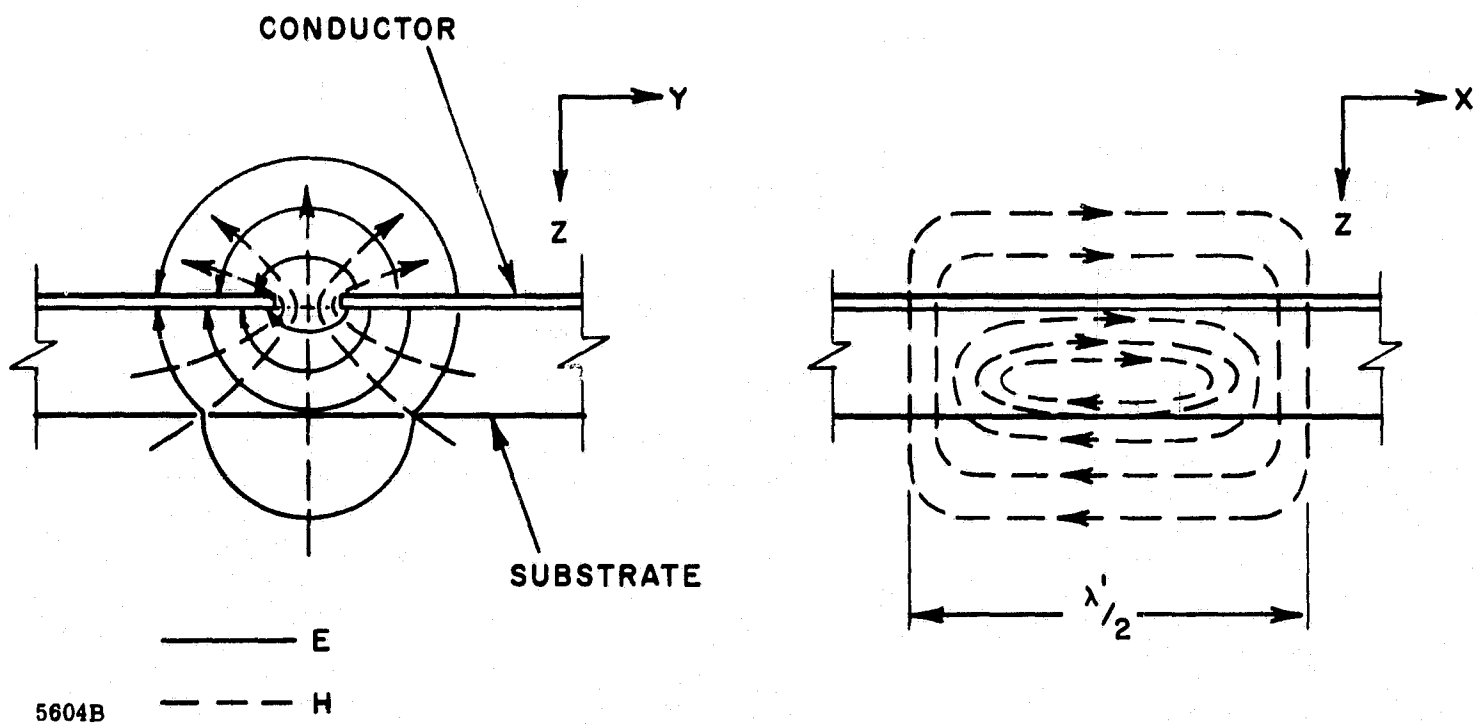


Figure 7. Field Distribution in Slot Line

The field distribution of the dominant mode of slot line is shown in Figure 7. The mode is similar to the TE_{10} mode of rectangular waveguide. Qualitatively, the line may be visualized as derived from doubly ridged waveguide with dielectric loading along one side of the ridge. The qualitative effect of various dimensions on the propagation constant is then relatively clear.

The existence of a region where the rf H-field is elliptically polarized makes the line attractive for ferrite applications. To achieve similar elliptically polarized regions in microstrip requires rather complicated configurations such as a coupled meander line circuit. Slot line has the further advantage of having both conductors on one side of the substrate, thus simplifying shunt-mounting of diodes or other elements.

4.2 PRACTICAL DESIGN INFORMATION

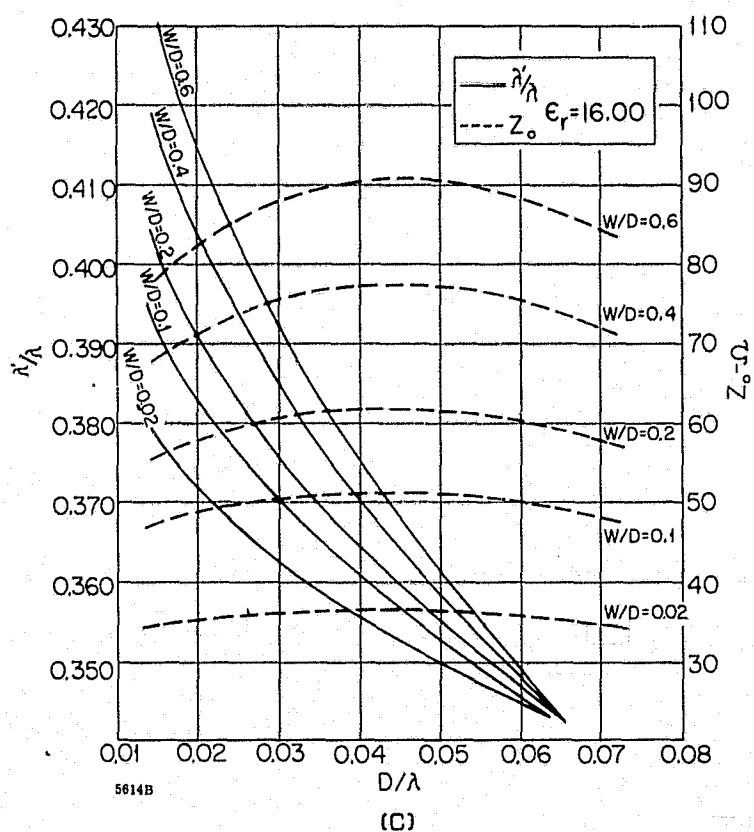
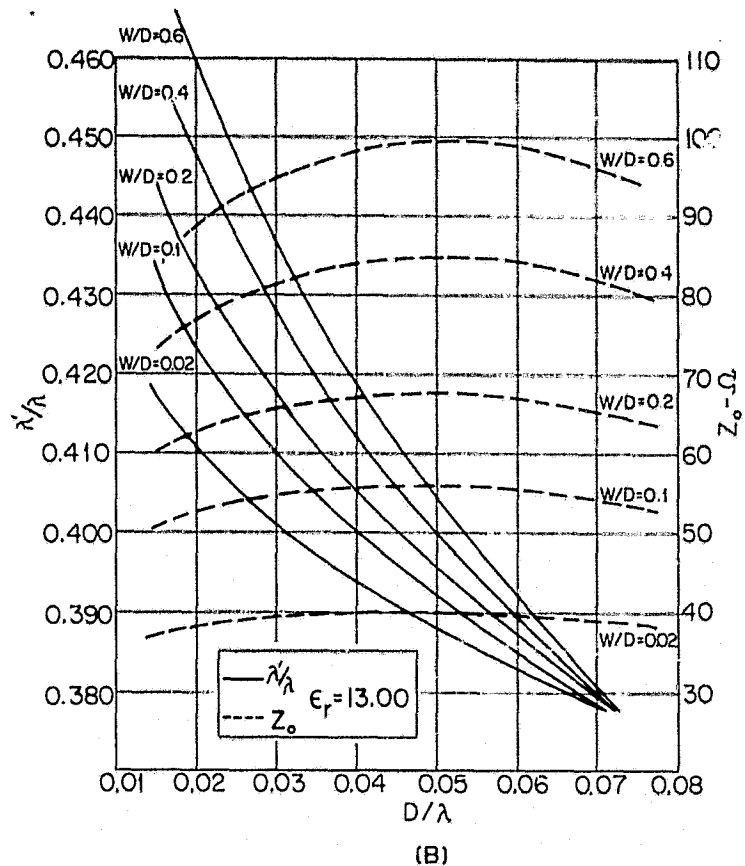
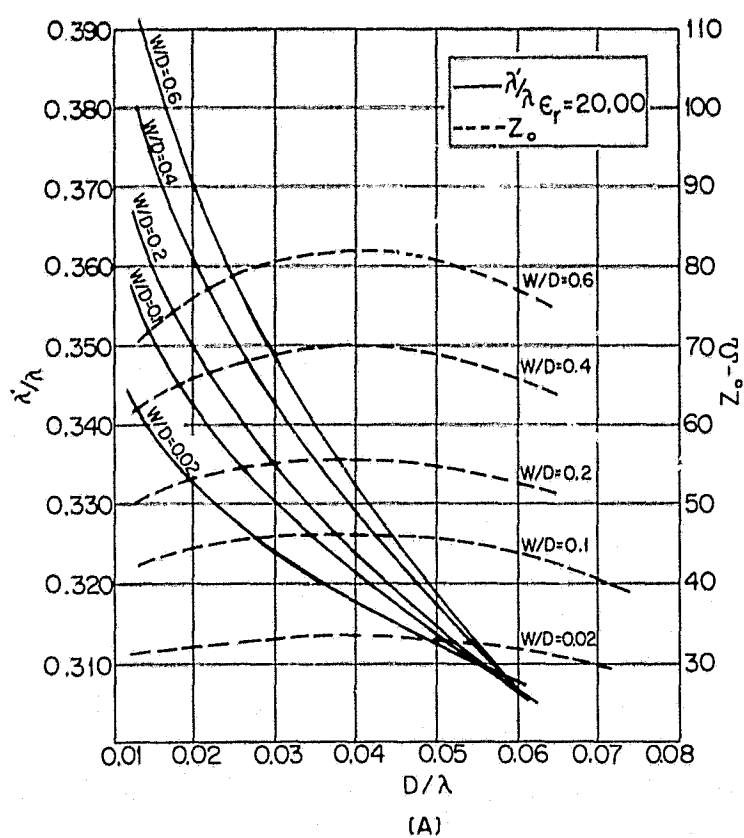
The practical application of slot line requires that radiation be minimized and that the fields be closely confined to the slot. These requirements are met by choosing a high dielectric constant material for the substrate and a relatively narrow slot. In general, adequate containment will be provided by choosing dimensions and dielectric constant yielding a ratio of guided-to-free space wavelength of 0.3 to 0.4 or smaller. For example, at X band frequencies, a dielectric constant of 16 with slot widths less than 0.035 inch and a substrate thickness of 0.055 inch would be sufficient. Figure 8 shows theoretical wavelength ratios for three dielectric constants for various substrate thicknesses and slot widths normalized to the free space wavelength.

An additional requirement in normal uses of slot line is that only propagation in the dominant mode be permitted; i. e., the configuration must be such that other possible modes are cut off or not excited. In typical experimental setups, the substrates are not enclosed. Under these circumstances, surface modes could conceivably cause difficulties. The dominant surface mode¹¹ is the TM_0 which may be propagated regardless of dimensions. Higher order TM and TE modes are cut off at frequencies below

$$F_c = \frac{n}{Yt \sqrt{\epsilon_d \mu_d - \epsilon_0 \mu_0}}$$

At frequencies below cutoff, these modes propagate with radiation losses. In our experiments, surface modes have not presented an appreciable problem. Where the slot line is placed in an enclosure, the modes of concern would be those supported by a dielectrically loaded, ridged waveguide. In addition, the enclosure would introduce a cutoff frequency for the dominant slot line mode. Because of the loading, the effect of the enclosure on propagation constant should be minimal.

The width of the conductor on either side of the slot must be considered. The determining factor is the width required to support the slot line mode. Based on the theory presented by Cohn⁸ (see Appendix A), the width of the conductor and substrate should be on the order of $\lambda_0/2$ where the slot is centered on the substrate.



LEGEND

λ = FREE SPACE WAVELENGTH
 λ' = SLOT WAVELENGTH
 D = SUBSTRATE THICKNESS
 W = SLOT WIDTH
 Z_0 = POWER VOLTAGE IMPEDANCE

Figure 8. Wavelength and Impedance in Slot Line (after Marriani, et al')

4.3 TRANSITIONS TO OTHER TRANSMISSION LINES

Slot line is easily coupled to microstrip in configurations where the lines are on opposite sides of a substrate. The slot line is then simply a slot in the microstrip ground plane and the lines would be coupled at any point of intersection. The coupling is maximum when the intersection is at right angles and minimum when the lines are parallel.

A transition between slot line and microstrip can be formed at a right-angle junction by terminating one arm of the microstrip in an open circuit one-quarter wavelength from the junction. (See Figure 9.) The bandwidth of the transition can be improved by lowering the microstrip impedance in the one-quarter wave section and raising the slot impedance in the one-quarter wave section.

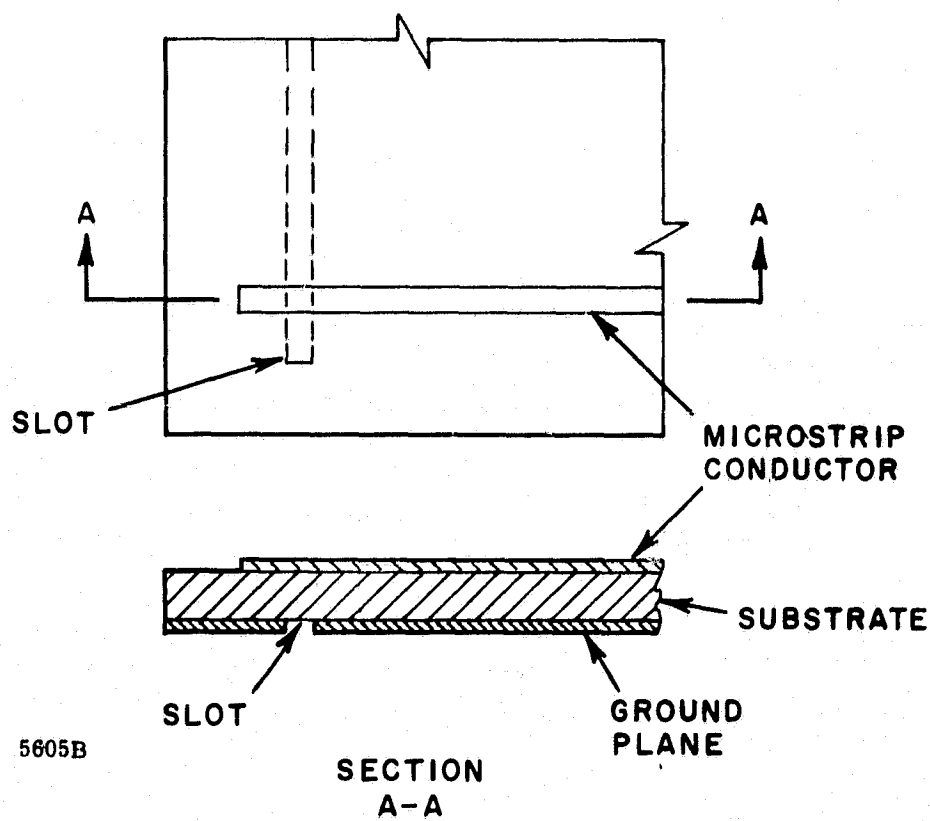


Figure 9. A Simple Microstrip-slot Transition

A transition from coax to slot can be accomplished in a manner similar to the feed for a slot antenna.¹² A balun is not required. Figure 10 shows an assembly of such a transition. This type of transition has a reasonably low VSWR over a broad frequency range. A shunt slot line stub would improve the performance of the transition over a narrower band and the transition could be made tunable by an adjustable slot line short.

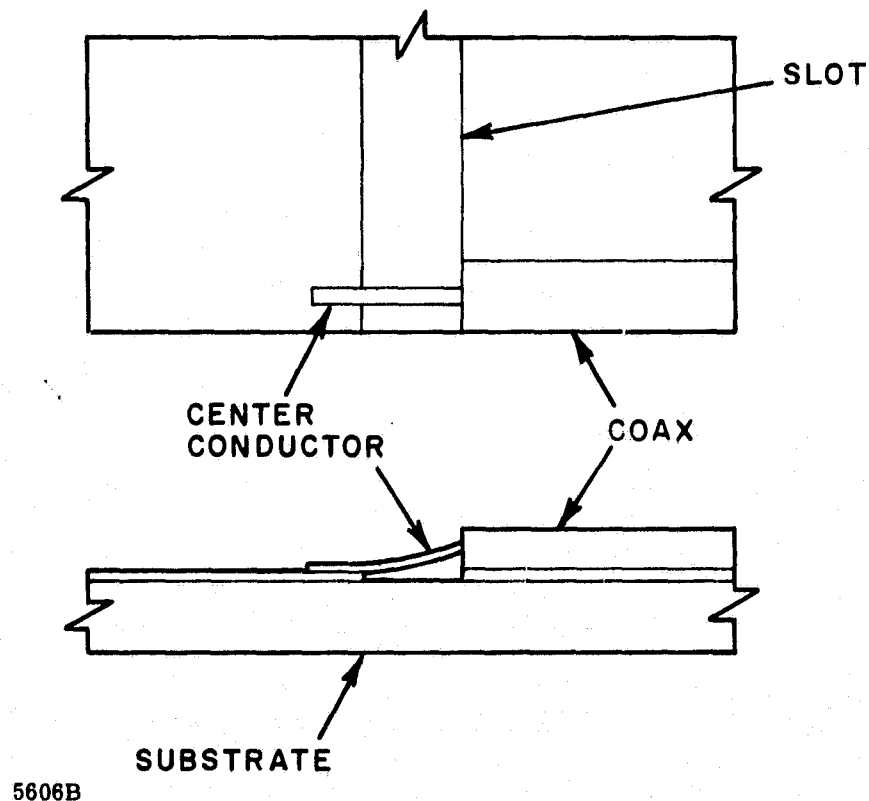


Figure 10. A Simple Coax-slot Transition

4.4 THEORETICAL CALCULATIONS

In both of the above transitions, the impedance of the slot line must be considered. The impedance calculated on the power-voltage basis for slot is shown in Figure 8. There are, of course, other definitions of impedance which may be chosen. A mode transformation is also involved which makes analysis of the transition difficult. Transitions have been developed experimentally from 50-ohm coax microstrip to 0.021-inch wide slot on 0.055-inch thick substrates with a relative dielectric constant of 16. The calculated impedance level for such a line is about 72 ohms rather than 50. These calculated impedances may prove more useful in the design of transformers between slot sections and in the application of shunt elements.

Thus far, losses in slot line have not been treated theoretically. Experimental results indicate that the loss in slot line is about the same as microstrip in db/in. of line length for useful impedances. The loss is not affected appreciably by substrate thickness, but increases when the slot becomes very narrow.

Additional theory has been developed under this program by Cohn* which allows calculation of the longitudinal H-field and one of the transverse components for dielectric substrates. These calculated values give a much better understanding of slot line.

Figure 7 shows the conventions used for reference. The fields calculated are h_x (along the direction of propagation) and h_z (perpendicular to the substrate). A digital computer was used to calculate these field values at discrete points for particular dimensions, materials, and frequencies. The results were presented both numerically and as digital plots. Some of the characteristics noted were:

- h_x reverses direction at $Z \geq 0.5 d$ for $d \geq 0.055$, $Y = 0$,
- Decreasing W/d increases h_x and h_z near the slot and causes more rapid decay of fields with distance from the slot,
- Increasing d/λ increases h_x and h_z near the slot,
- Increasing d/λ or increasing W/d increases h_x/h_z (causes more nearly circular polarization).

In general, it appears very nearly maximum nonreciprocal interaction in a ferrimagnetic substrate can be obtained with magnetic bias flux everywhere parallel to the substrate surface rather than the curved paths suggested by the field cross section. Plots of h_x are shown in Figure 11. These plots are computer-generated and show the quadrant bounded by $Y = 0$, $Z = 0$, $Y = 0.160$ in. and $Z = 0.160$ in. The contours are for constant magnitude of h_x . The plots are normalized to the field at $Y = Z = 0$ so that a scale factor must be used in comparing plots. Note that the largest magnitude of negative longitudinal field is concentrated in the upper half of the substrate (away from the conductor). Also, $h_z > h_x$ throughout the area of larger longitudinal fields. These points are discussed further in Section 6 regarding slot line phase shifter design.

* Reference cit. page 1-3.

4.5 GENERAL CONCLUSION

Slot transmission line has proven to be a practical planar transmission line. Field containment is adequate with practical substrate materials and losses are no greater than other planar transmission line presently used. Slot line is compatible with microstrip in that both types of line can be used on a common substrate and good, broadband transitions of simple design can be developed and have been demonstrated. This allows exploitation of each line to full advantage even in the design of single substrate integrated circuitry.

SCALE: The contours are for constant H_x

PLOT SYMBOL	$H_x/H_x \quad z=y=0$	PLOT SYMBOL	$H_x/H_x \quad z=y=0$
\$	1.000	7	0.466
=	0.933	6	0.400
+	0.866	5	0.333
x	0.800	4	0.266
-	0.734	3	0.200
*	0.666	2	0.133
9	0.599	1	0.066
8	0.533	0	< 0.0066

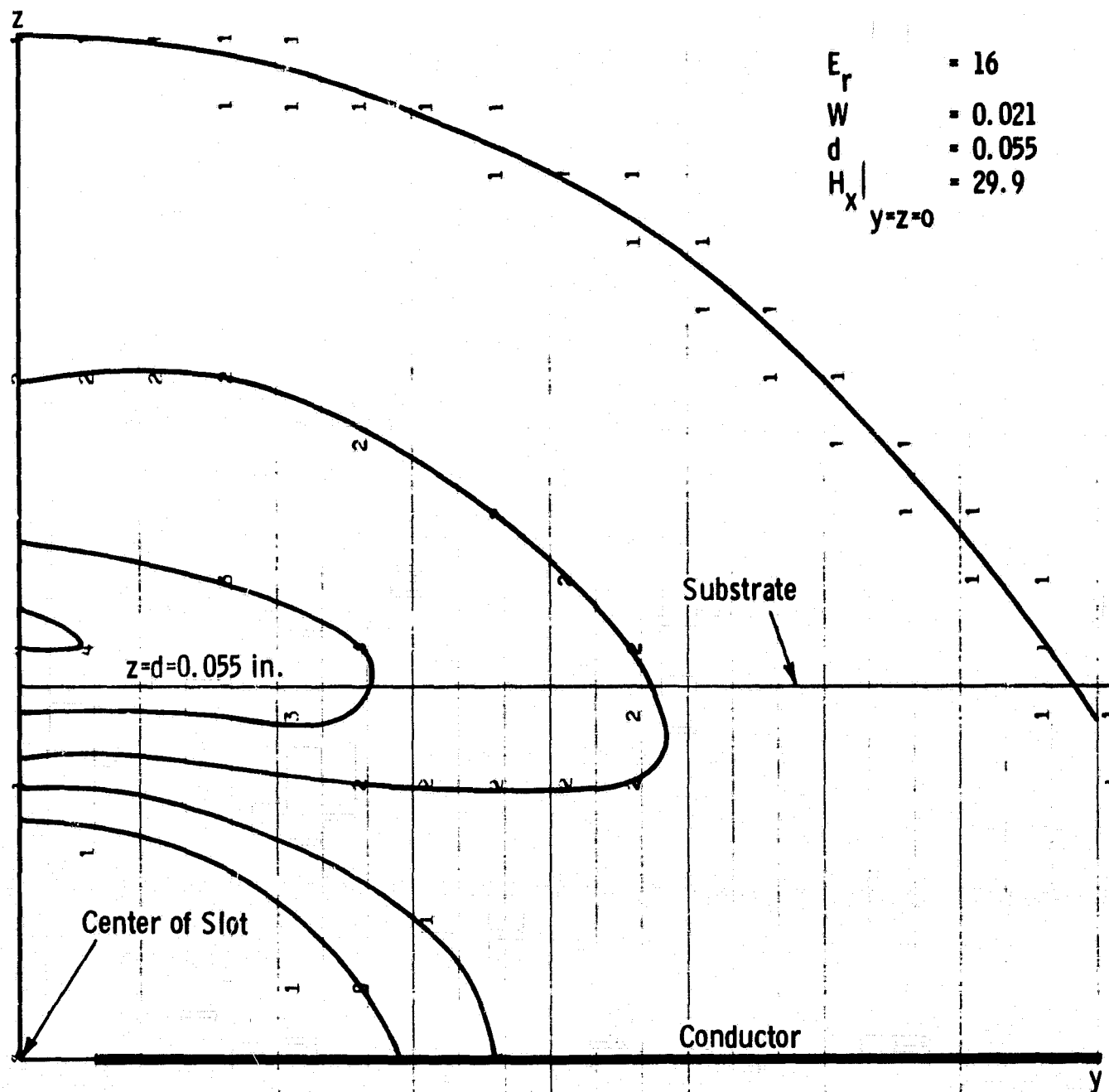


Figure 11. Calculated Magnetic Field in Slot Line (sheet 1 of 3)

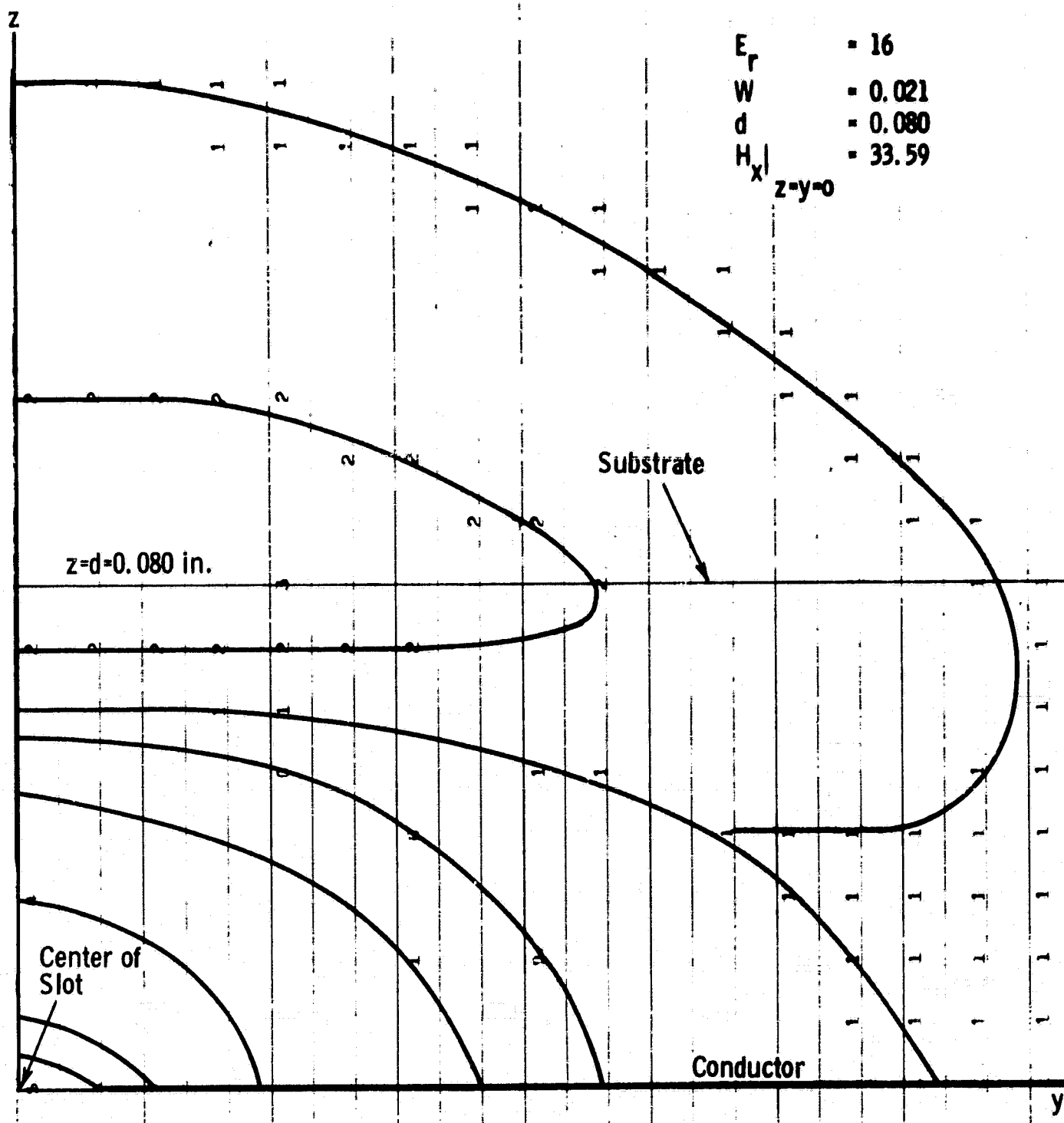


Figure 11. Calculated Magnetic Field in Slot Line (sheet 2 of 3)

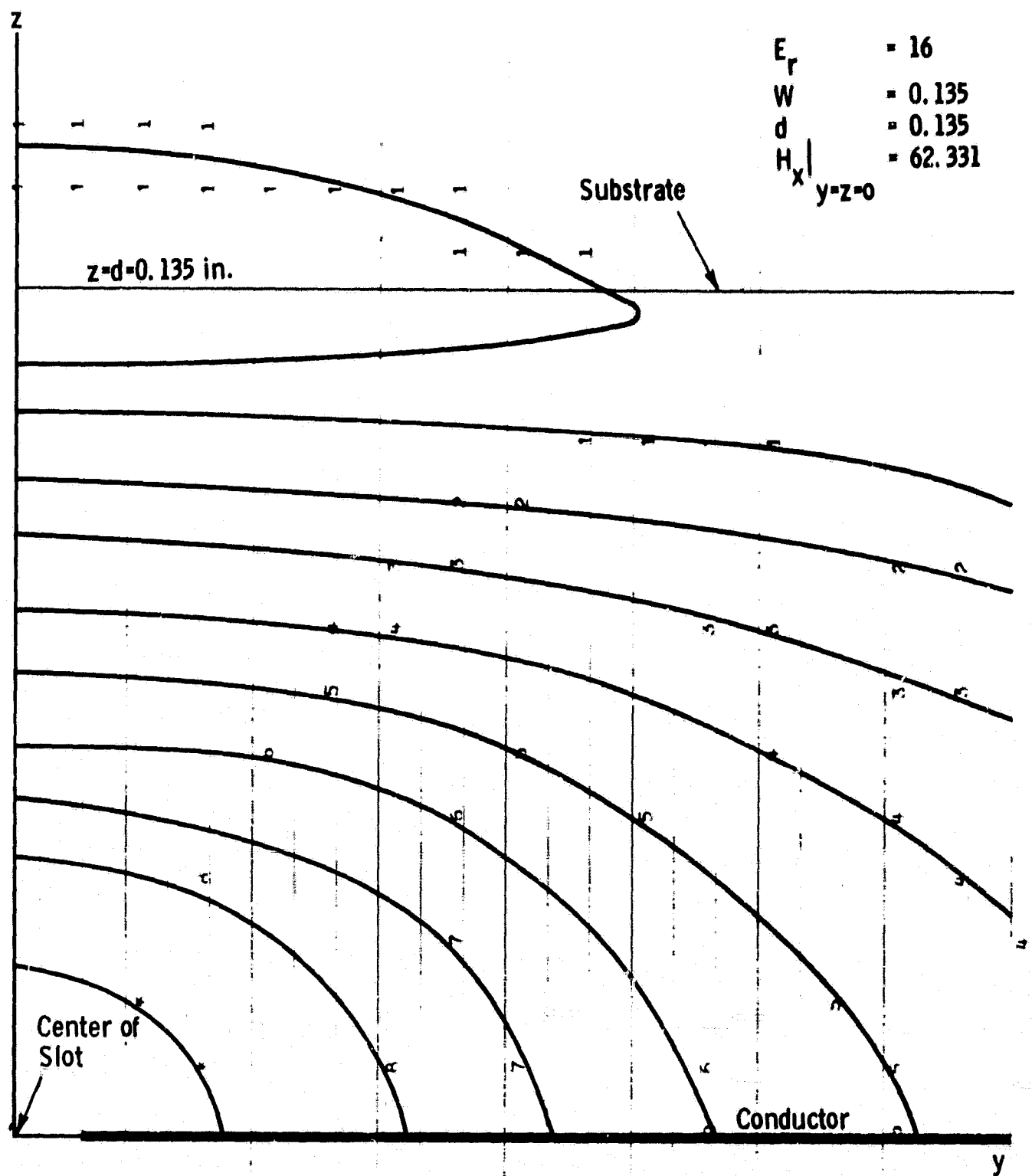


Figure 11. Calculated Magnetic Field in Slot Line (sheet 3 of 3)

5. MICROSTRIP CIRCULATORS

This section describes fixed bias and latching microstrip circulators and gives design considerations.

5.1 MICROSTRIP CIRCULATOR CONFIGURATIONS

Circulators have been one of the more versatile and useful components in microwave systems. Circulators can be built easily in microstrip and are similar in many respects to those in strip transmission line. They may take one of two forms, each of which employs a symmetrical Y-junction with a conducting disc at the junction:

- Ferrimagnetic disc cemented into a nonmagnetic ceramic substrate at the junction. This is sometimes referred to as the "plug-in" type (Figure 12).
- All-ferrimagnetic substrates magnetized only in the circular area of the junction (Figure 13).

The operation of both configurations can be explained by the results of Fay and Comstock¹³ and Bosma.¹⁴ The equations developed for the stripline models can be used to good advantage in determining the dimensions to be used and predicting performance of microstrip circulators. Particularly, equations 15 and 2 of Fay and Comstock¹³ can be used to calculate, from basic material parameters, the diameter of the disc for the plug-in type or the diameter of the magnetized area for the all-ferrimagnetic type. The shield diameter is then made approximately 80 percent of the disc or magnet diameter to gain good matching characteristics. The results obtained using these equations have been very good. Selection of materials for use in a microstrip circulator is based on the same well known guidelines used in design of their stripline counterparts.

In a microstrip system application, the substrate thickness is usually fixed so that this parameter cannot be used to control performance. This does not present a serious problem in the design of fixed bias circulators. It is significant in the design of latching circulators as is discussed in following paragraphs. (Where microstrip circulators are used as components with connectors in conventional systems, a limitation on substrate thickness would not exist except that surface waves and moding must be avoided.)

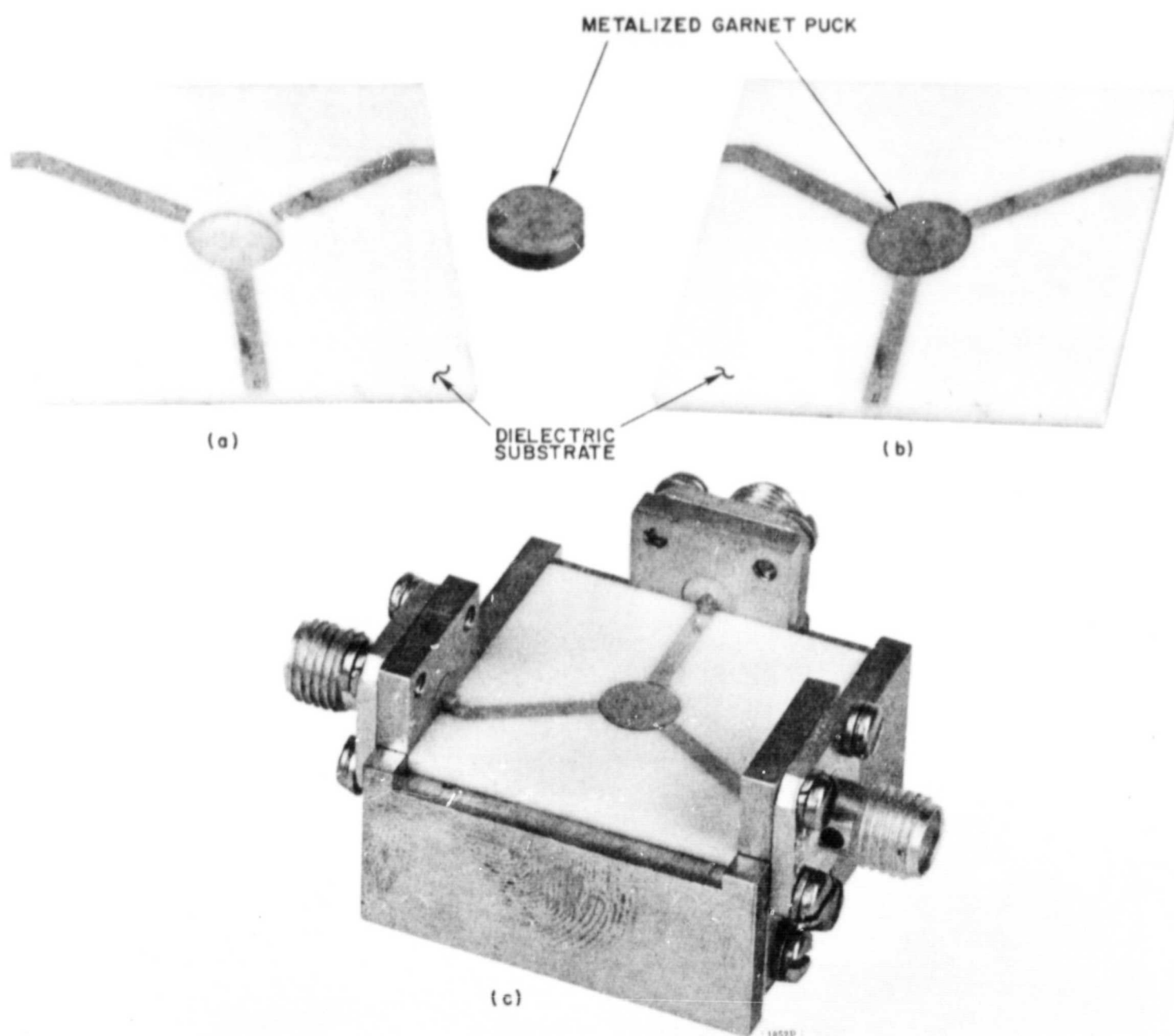


Figure 12. X Band Plug-in Microstrip Circulator

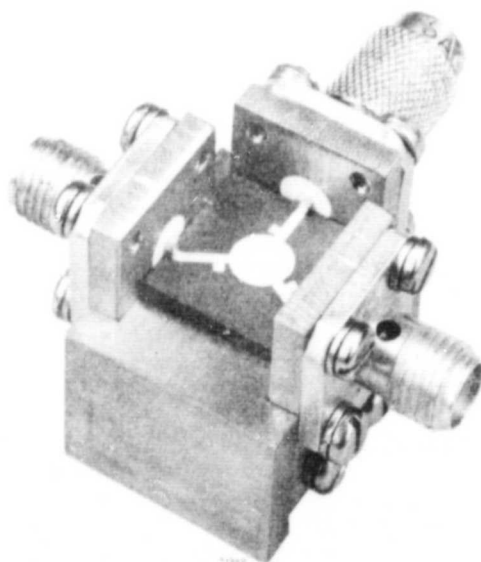


Figure 13. X Band All-ferrimagnetic-substrate Circulator

Stripline circulators often employ variable tuning elements in their designs to compensate for small changes from unit to unit. Such compensation does not appear necessary in microstrip units.

Peak power capability of microstrip circulators can be enhanced by using rare-earth doped substrates. Losses increase with increased doping but substantial improvement of peak power is attained with moderate increase in loss.

5.2 LATCHING CIRCULATORS IN MICROSTRIP

Design of latching circulators in microstrip presents a different set of conditions from either stripline or waveguide. There are two basic approaches:

- **External Loop:** Place the switching wire outside the substrate and form part of the closed magnetic loop outside the substrate (Figure 14).
- **Internal Loop:** Place a switching wire internal to the substrate itself and form a closed magnetic loop within the substrate (Figure 15).

The external loop latching configuration has proven superior to the internal loop in ease of fabrication and in performance. The external loop configuration can be formed in three basic geometries:

1. A flat plate of ferrimagnetic material with a annular groove for the switching wire may be placed against the ground plane of the substrate, or
2. A plate similar to (1) above may be placed against the top of the circuit, or
3. A "C" structure contacting the upper and lower surfaces of the junction and returning around the edge of the substrate may be used to complete the magnetic loop.

All of the configurations, except the latter, can be used only with the all-ferrimagnetic substrate and in these cases the substrate thickness is very important. All of the magnetic flux passing through the circular area of the junction perpendicular to the ground plane must return parallel to the ground plane through the circular cylindrical surface surrounding the junction (see Figure 14). To avoid a reduction in remanent magnetization (flux limiting during switching), this surface should have an area at least equal to the circular area of the junction. This is true when the substrate height is equal to one half the junction radius.

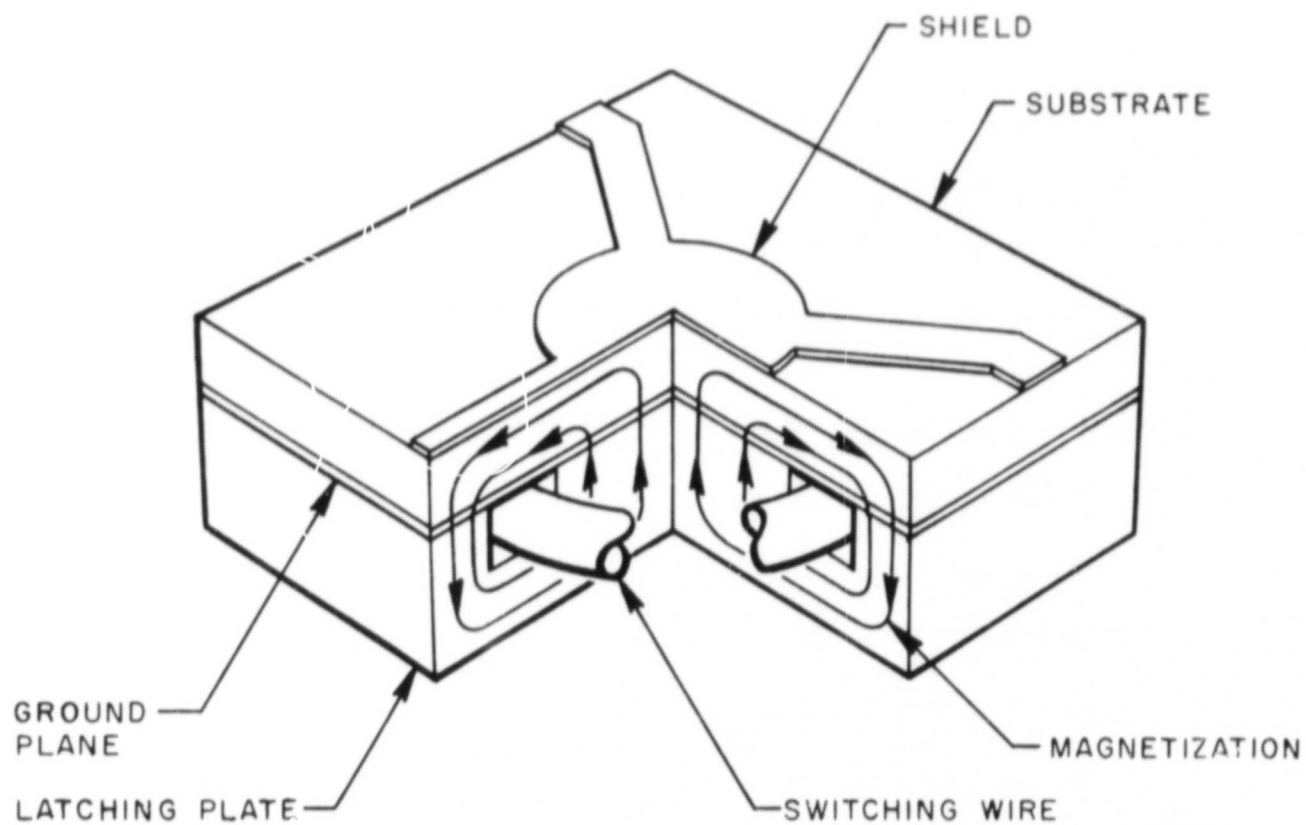


Figure 14. Cross Section of Latching Microstrip Circulator with Switching Wire External to Substrate

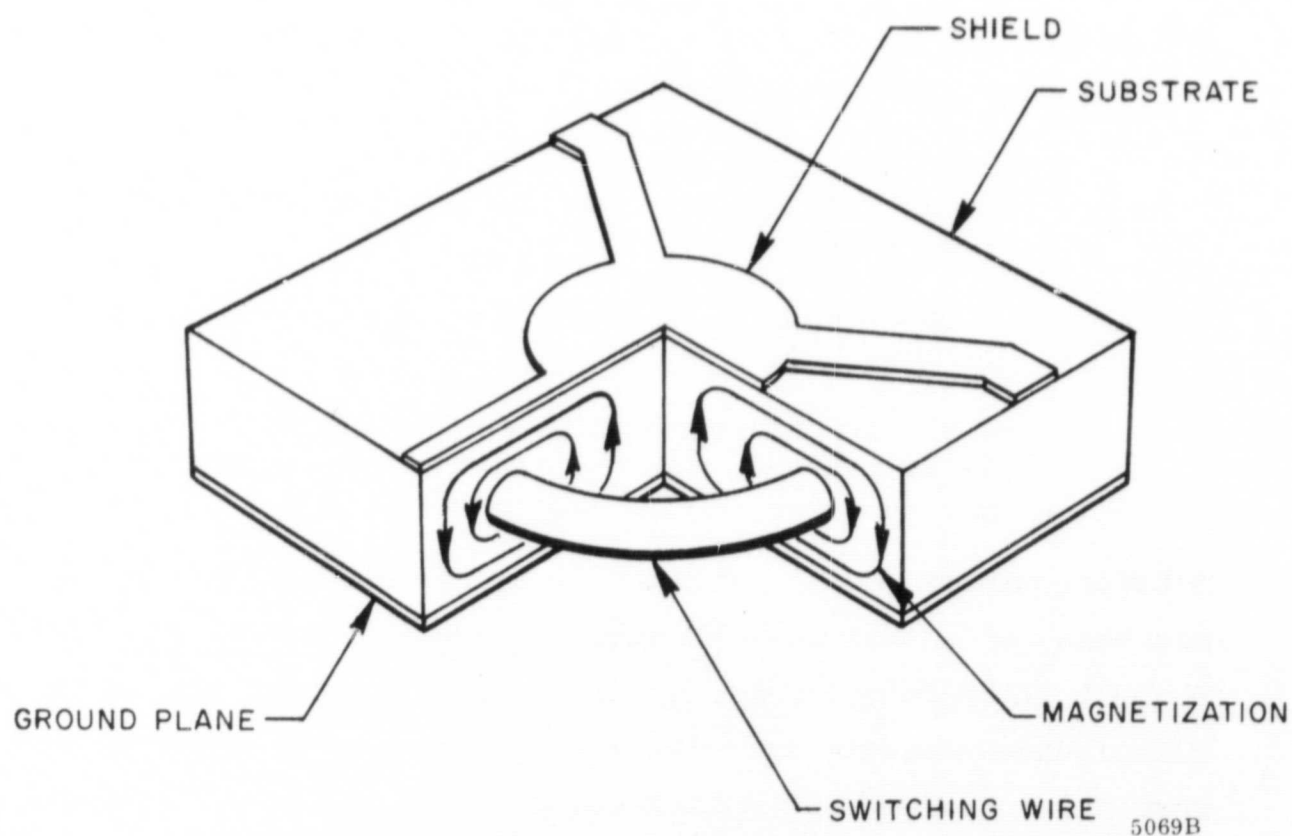


Figure 15. Cross Section of Latching Microstrip Circulator with Switching Wire Internal to Substrate

As has been stated, the substrate thickness must be restricted to avoid surface modes and to be compatible with other components. At the lower frequencies with resulting larger junction diameters this restriction may dictate the use of a "C" type structure where return path isn't dependent on substrate thickness.

The prime limitation in the external loop latching configuration is attaining a high remanence flux level in the junction. A limiting factor in this configuration is the necessary gap (ground plane) between the latching plate and the substrate. While it is a very short gap (~ 0.0002 inch), it still has a large effect on remanent field. To optimize the results, the material used for the latching plate should have a somewhat high coercive field, a saturation magnetization higher than the microwave material, and a high squareness ratio (ratio of driven flux to remanent flux). To minimize switching energy, the coercive field of the latching plate should be no larger than required for good latching action. The switching energy and switching speed are also limited by the ground plane through which the magnetic flux passes.

6. SLOT LINE PHASE SHIFTERS

This section presents design consideration for slot line phase shifters and several basic design approaches.

6.1 DESIGN CONSIDERATIONS

Achieving nonreciprocal phase shift with ferrite in a particular guiding structure is dependent upon having significant energy in a circularly polarized H-field in a region where the dc field in the ferrite can be perpendicular to the polarization plane. The existence of elliptically polarized fields in slot line was shown in Section 4; hence, nonreciprocal phase shift can be achieved. Because the H-field loops are curved in cross section (Figure 7), it would appear that the dc field on the ferrite would have to follow a curved path so that the applied field and the transverse H-field would be perpendicular. This would certainly be the case if the longitudinal and transverse fields were equal. The transverse component, which is perpendicular to the substrate, is in fact larger than the longitudinal field over most of the region where the fields are strong. Thus a magnetization parallel to the substrate surface and perpendicular to the slot will give very nearly maximum effectiveness.

An examination of Figure 7 shows that the longitudinal field reverses direction within the substrate; therefore, the magnetization must reverse direction within the substrate or the ferrite must be limited to one region of the substrate and an external return path provided.

6.2 SLOT LINE PHASE SHIFTER CONFIGURATIONS

There are several design approaches which may be taken with slot line phasers. Three latched geometries are shown in Figure 16. In Figure 16(a) and (c), the entire latched ferrite loop is within the microwave propagating region while in (b), the return is external. In the first case, the toroid can be formed by one flat plate and a separate U-shaped piece. Both pieces would have to have the proper microwave characteristics since they are in the propagating region. The single piece construction would give better latching but would require specially formed substrates or a separate phase shifter substrate. In the second case, the external material of the return path would be chosen to have good "square loop" properties and higher coercive field to overcome gap reluctance. It need not have good microwave properties. This construction would be somewhat larger than the former and would require higher switching energy.

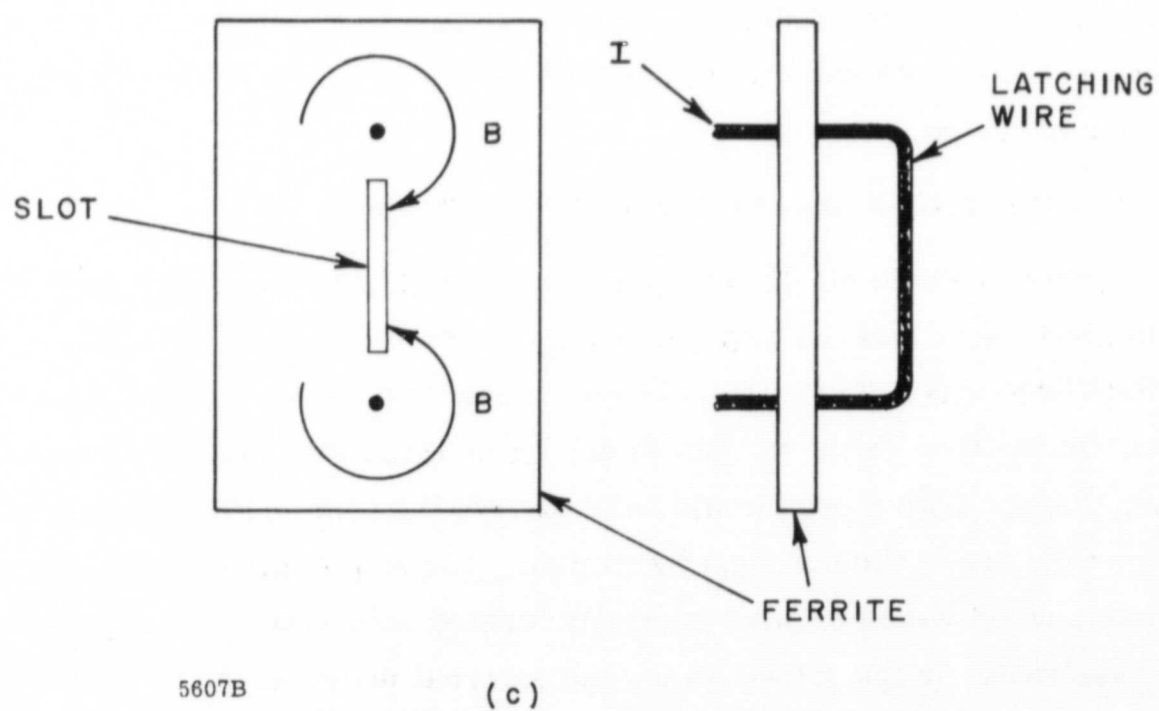
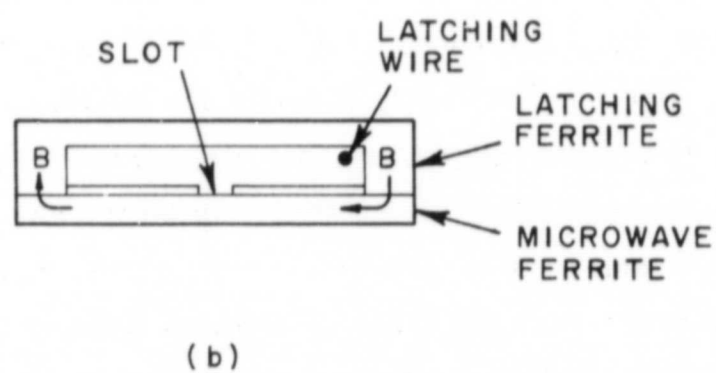
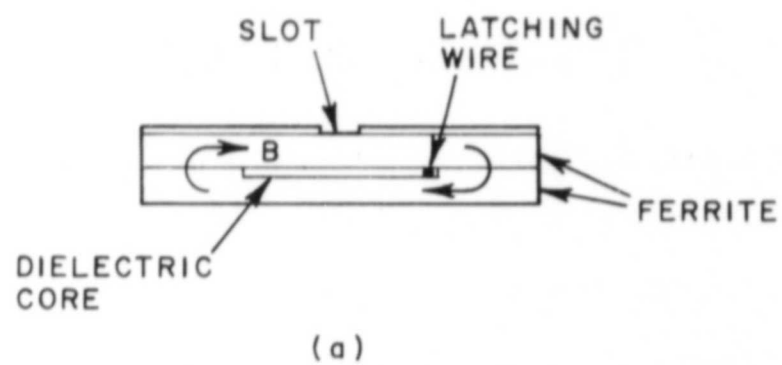


Figure 16. Latched Ferrimagnetic Slot Line Phasers

The construction in (c) has the advantage of good latching characteristics. This approach does not require special substrates and could be easily incorporated on substrates with other components. Since the latched fields return through the substrate, components affected by change in magnetization would have to be located at least one-half the slot length from the slot line.

In all of the above cases, the magnetization is unidirectional through areas having an opposite sense of polarization. This should not result in serious degradation of performance since the energy content in one of these areas is low. The only strong negative longitudinal fields in Figure 11 are near the nonconductor surface of the substrate above the slot. In this region, the magnetization is reversed for the configuration of Figure 16(a) and dielectric replaces the ferrite in the configurations of Figure 16(b) and (c).

The choice of dimensions for each design must be made carefully to achieve the optimum fields for the particular approach. The design of Figure 16(a) would require a thinner substrate so that the longitudinal field reverses nearer $Z = d/2$ at $Y = 0$. The remaining designs would require a thicker substrate to achieve the maximum magnitude of longitudinal field and to cause the reversal of direction to occur near the dielectric-air interface. A lower impedance slot (narrower) would raise field levels and should improve results for all three cases with little effect on the location of longitudinal field reversal.

Another approach which can be taken is shown in Figure 17. This is not truly slot line since there is ferrite on both sides of the conductor. However, the fields are very similar to slot line and the transition from slot line would be minor. The advantages lie in the field pattern being more nearly symmetrical and concentrated near the slot, simplifying design and latching. By dc-isolating this section of ground plane, switching current can pass through the ground conductor or a separate conductor could be provided. This design requires no special substrate but the latching loop is necessarily formed from two pieces and requires good microwave material throughout.

Another approach (Figure 18) which has not yet been explored involves a modification of the line itself rather than the ferrite geometry. A slot line periodically loaded by either variations of slot width or addition of small dielectric pieces across the slot exhibits a larger phase constant. This would improve the performance in terms of deg./in., allowing smaller phase shifters requiring less switching energy to be built. Based on some early results reported on a periodic waveguide structure,¹⁵ improvement might well be very significant.

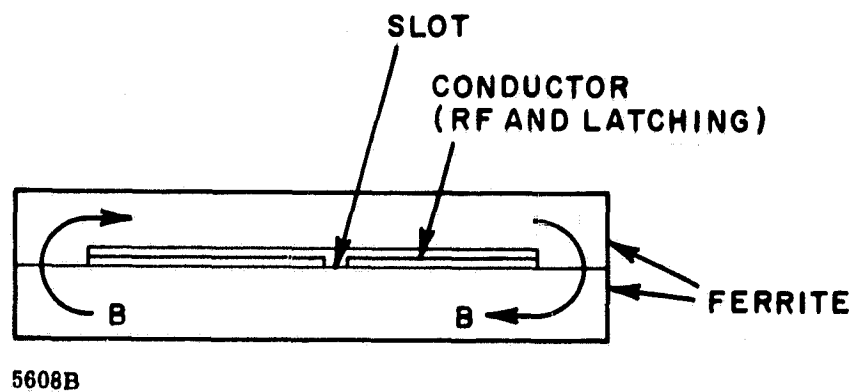


Figure 17. Symmetrically Loaded Slot Phaser

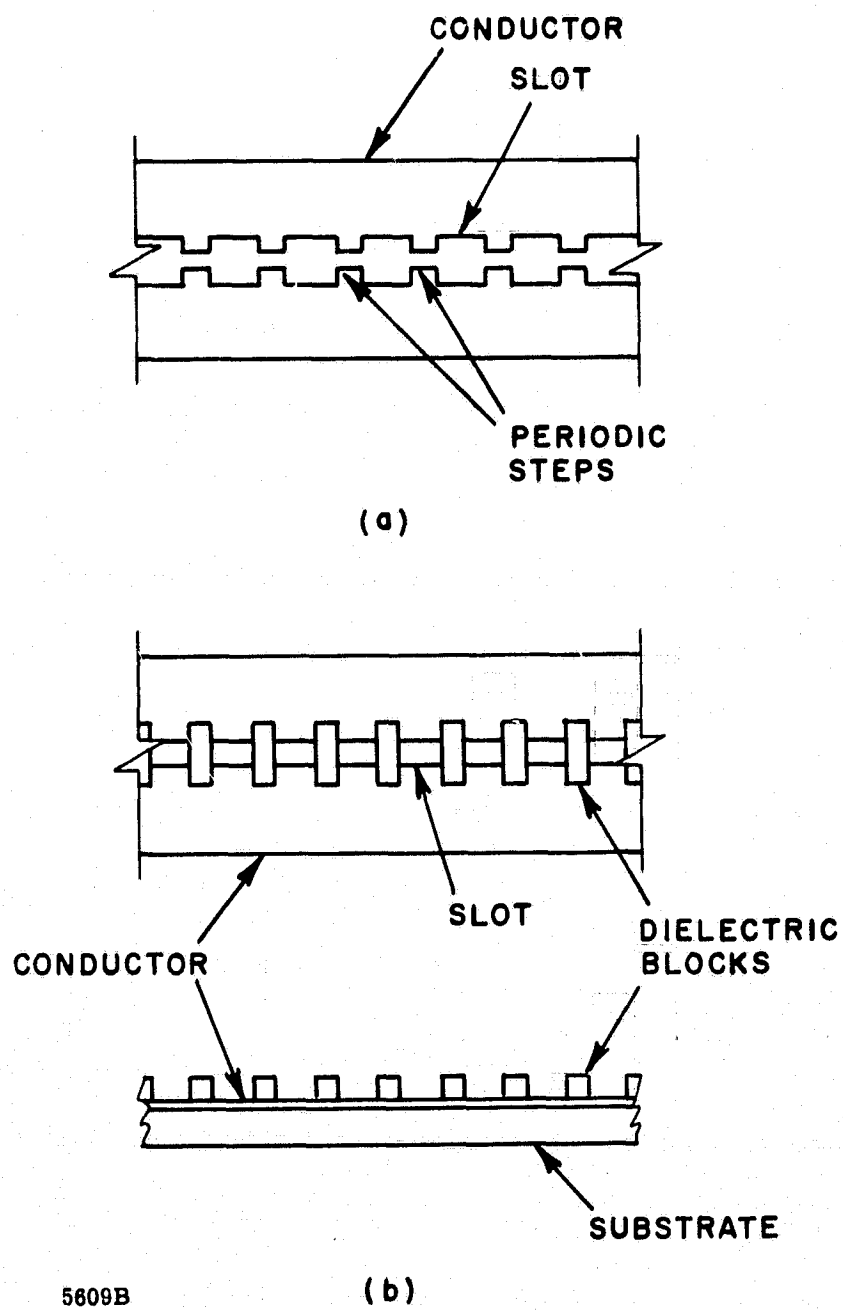


Figure 18. Periodically Loaded Slot Line

7. SUMMARY OF EXPERIMENTAL WORK

This section summarizes the experimental work conducted under this program.

7.1 MICROSTRIP PARAMETERS

One of the first investigations conducted was an experimental investigation of microstrip parameters with various dielectric and ferrimagnetic substrates. The object was to establish:

- The magnitude of losses and parameters controlling loss
- Accuracy of calculated wavelength and impedance
- Effect of the permeability of unmagnetized ferrimagnetic substrates on impedance and wavelength

7.1.1 Losses

Losses were measured with the special equipment shown in Figure 19. A straight test line which is short circuited at both ends is very lightly coupled to a signal source and a detector. The equipment used allows the measurement of very small signals so that coupling to the test line is very small and may be ignored in the measurement. The signal frequency is set to a resonance of the line section and the Q of the resonance measured. This allows a calculation of the losses of the test line in db per unit length.

In all of the measurements made, conductor losses were much larger than either dielectric or magnetic losses. Factors affecting conductor loss have been investigated in detail. The results are shown in Figures 20, 21, 22 and 23.

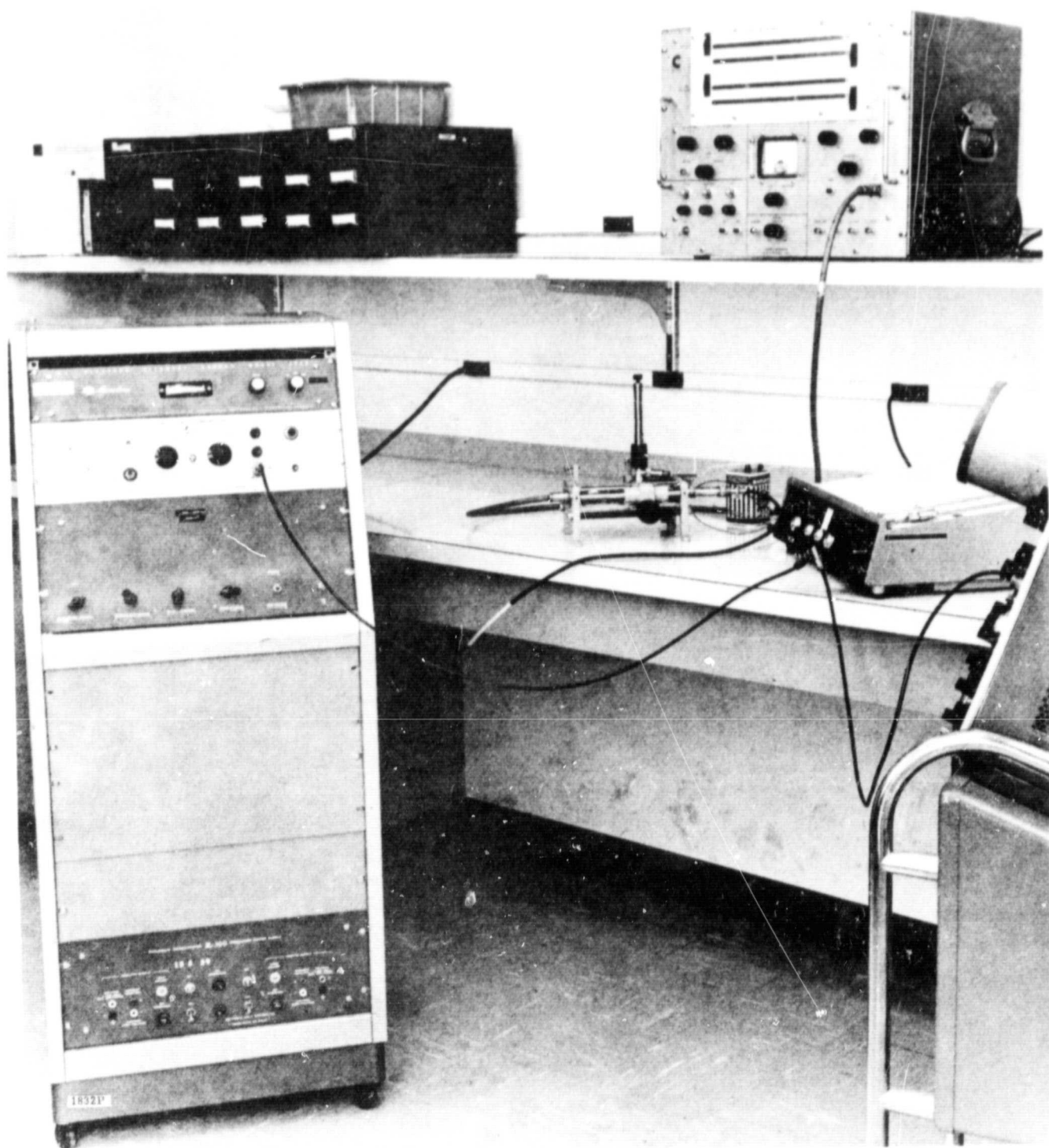
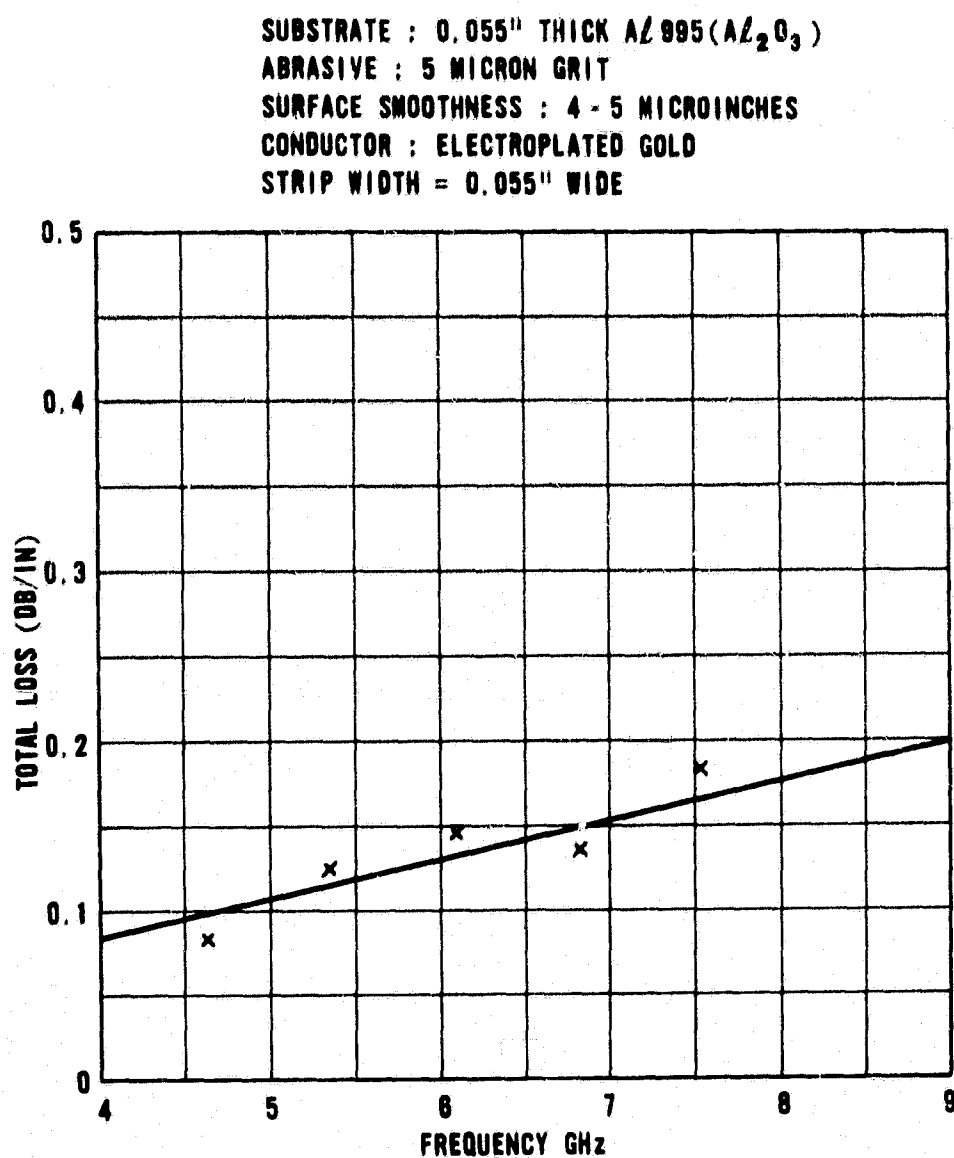


Figure 19. Measurement Apparatus for Loss and Wavelength

Figure 20 shows the total loss (dielectric and conductor) as a function of frequency for a typical 50 ohm line and 0.055 inch thick Al_2O_3 . It is significant to note that the conductivity of the gold conductors was measured to be approximately 10^7 mhos/meter or about 40 percent of the theoretical value for gold. If films of higher conductivity can be formed, the loss will be improved.

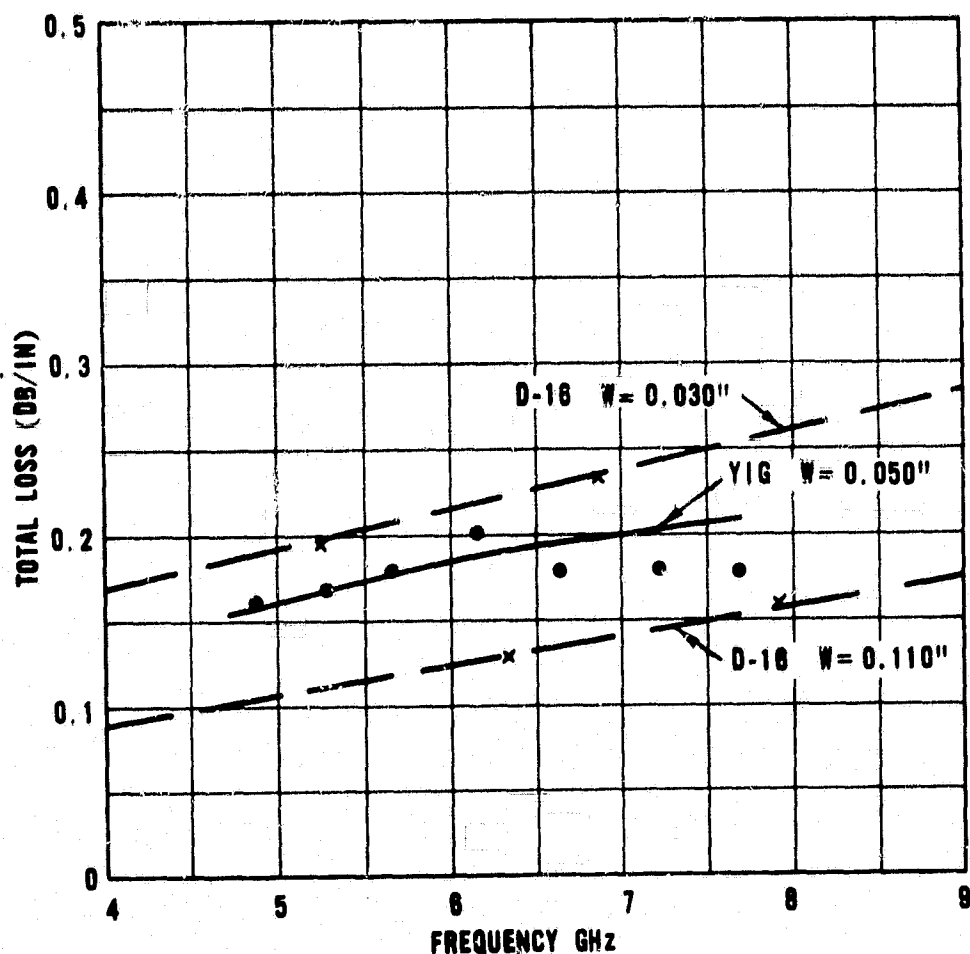


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Figure 20. Total Loss Measured as a Function of Frequency on a Three Inch Long, 50 Ohm Microstrip Line ($\text{Al}-995$ Substrate)

In Figure 21, the effect of strip width on loss is demonstrated. As would be expected, the loss is lower for the wider conductors. A 50 ohm strip on this substrate would be 0.033 inch wide or very near the upper curve. The impedance for a microstrip system might be chosen less than 50 ohms for loss reduction. The 50 ohm lines are presently employed for compatibility with existing test equipment and connectors during component development.

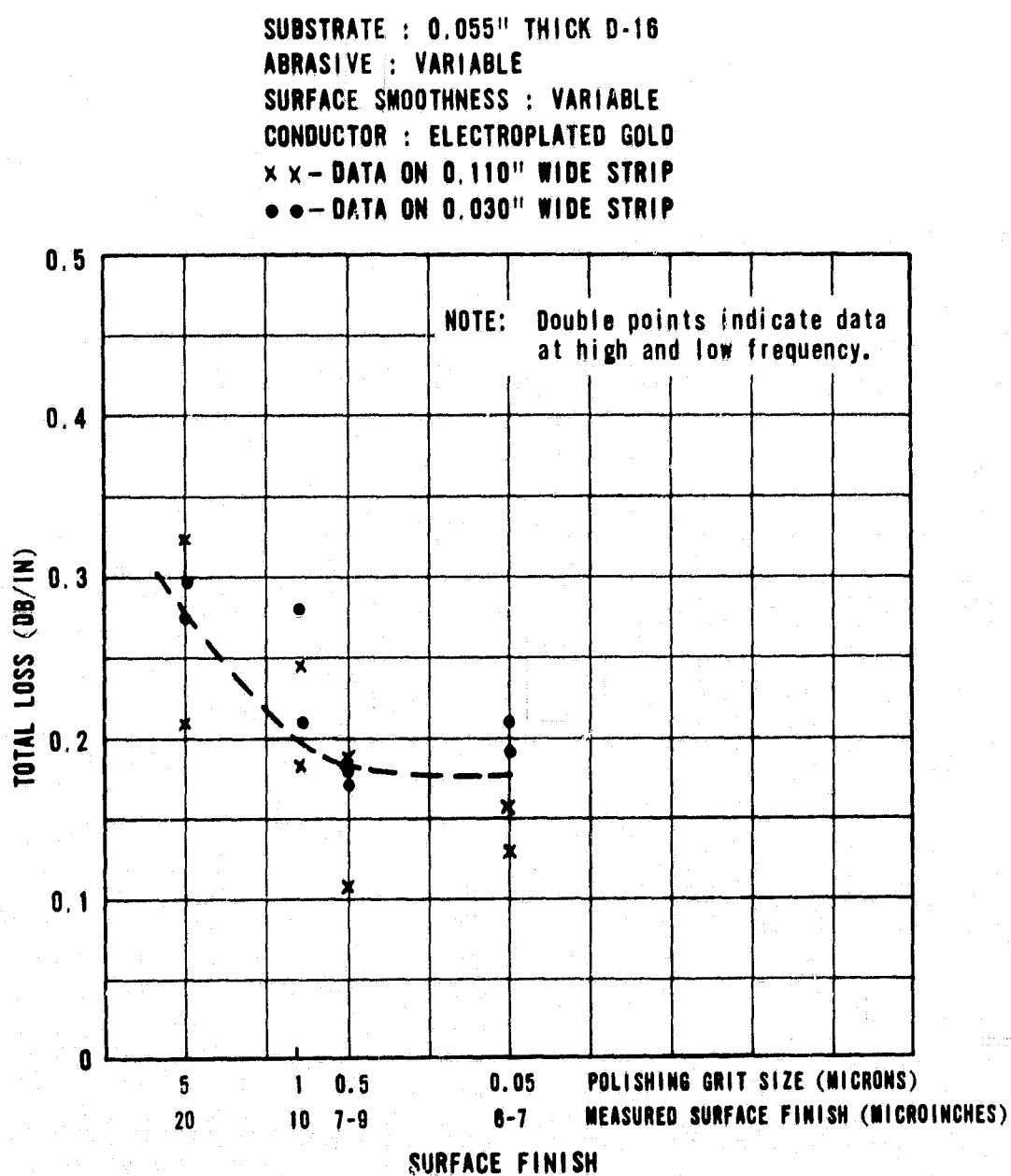
SUBSTRATE : 0.055" THICK, D-16 AND YIG
 ABRASIVE : 0.05 MICRON
 SURFACE SMOOTHNESS : 4-6 MICROINCHES
 CONDUCTOR : ELECTROPLATED GOLD
 STRIP WIDTH : PARAMETER



4965B

Figure 21. Total Loss Measured as a Function of Frequency with Stripwidth as a Parameter (D-16 and YIG Substrates)

The effect of surface finish on loss is demonstrated in Figure 22. It is certainly an important factor in determining loss. The effect is expected to be more pronounced at higher frequencies but the data are too scattered to determine trends of this type with certainty. It can be concluded, however, that the surface finish should be at least as good as 10 microinches. To achieve this quality of surface finish the substrates must be lapped.

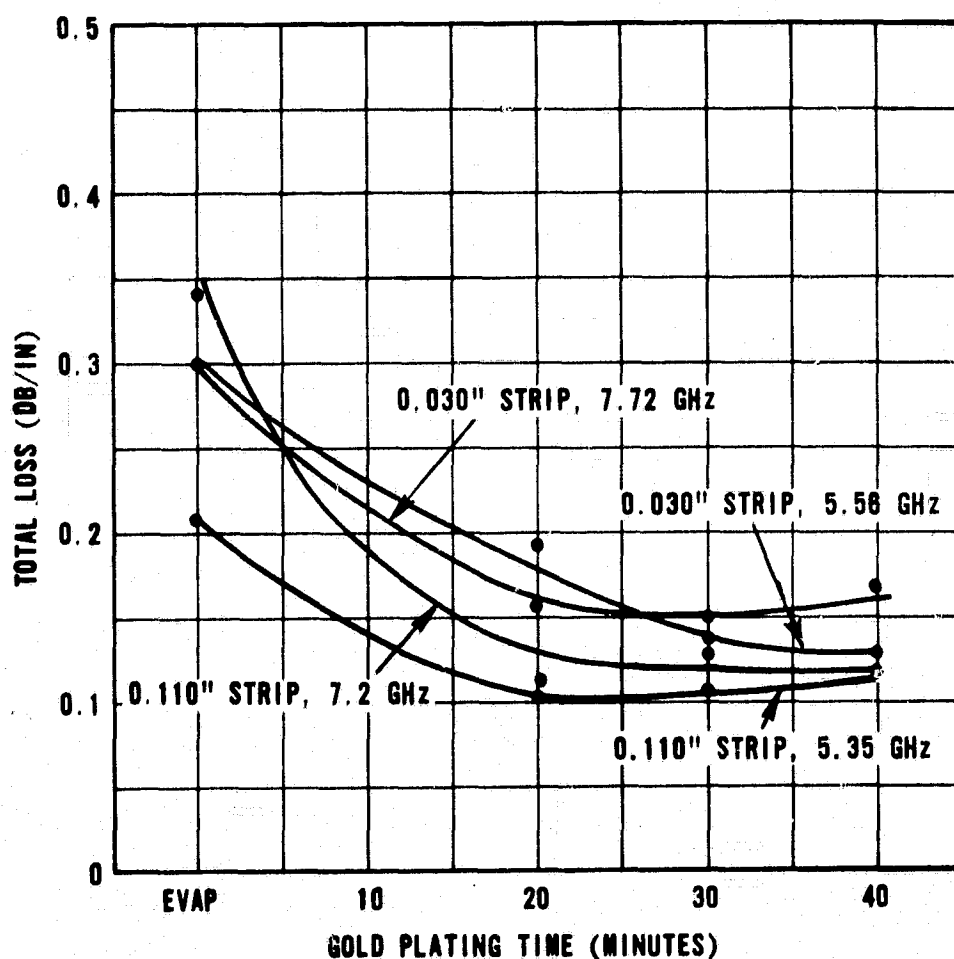


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Figure 22. Total Loss as a Function of Surface Finish (D-16 Substrate)

Figure 23 shows loss as a function of plating time which is equivalent to conductor thickness (10 minutes plating time corresponds to about 40 microinches or 1 micron). The loss is seen to decrease until the thickness becomes several skin depths. Then loss decreases little as conductor thickness increases. The higher the frequency (and thus the thinner the skin depth), the thinner the conductor required for minimizing loss. The required thickness at X band frequencies is about 0.0002 inch.

SUBSTRATE : 0.055" THICK D-13
 ABRASIVE : 0.05 MICRON GRIT
 SURFACE SMOOTHNESS : 4-5 MICROINCHES
 CONDUCTOR : ELECTROPLATED GOLD
 STRIP WIDTH : PARAMETER



4962B

Figure 23. Total Loss as a Function of Plating Time (D-13 Substrate)

7.1.2 Wavelength

Measurement of wavelength is not a simple matter as has been pointed out in previous sections. Most of the energy in microstrip is concentrated in the substrate. If a coupling probe is brought near enough to obtain significant coupling, the line parameters are affected. Special equipment shown in Figures 19 and 24 have been developed for this purpose. The test line is mounted in a holder and a fixed frequency signal is coupled into it as was described in measurement of loss. The holder is mounted on a slotted line carriage as is shown in Figure 24. The probe shown in the figure provides a small perturbation on the line. This perturbation of the probe is a maximum when positioned at an E field maximum on the line. If the probe is moved along the line, and the transmitted signal monitored, maxima and minima of signal will be noted. The distance the probe moves between two adjacent maxima or minima is then one half wavelength on the line. A potentiometer was attached to the probe carriage to provide a voltage referenced to probe position. This voltage was applied to the X input of an X-Y recorder and a voltage corresponding to the transmitted signal to the Y input. Figure 25 shows X-Y plot as the probe was moved along a particular line.

This technique was employed for measuring wavelength on different dielectric substrates. The data in all cases compared well with the theoretical values. In addition, this technique can be used to determine the effective permeability of ferrimagnetic substrates and its frequency dependence.

The ferrimagnetic materials used in this program have been primarily YIG and Dy doped YIG. At X band, the effective microwave permeability measured by the above technique is very nearly one when the substrate is unmagnetized. This means that the microstrip parameters are essentially the same as for a dielectric substrate with the same permittivity. The initial permeability would normally be expected to be somewhat lower than this value. The reason for the difference probably lies in the fact that most of the magnetic vectors lie in the plane of the substrate with little magnetization perpendicular to the substrate due to its thin plate geometry. This would reduce the microwave interaction and cause the increase in effective permeability.

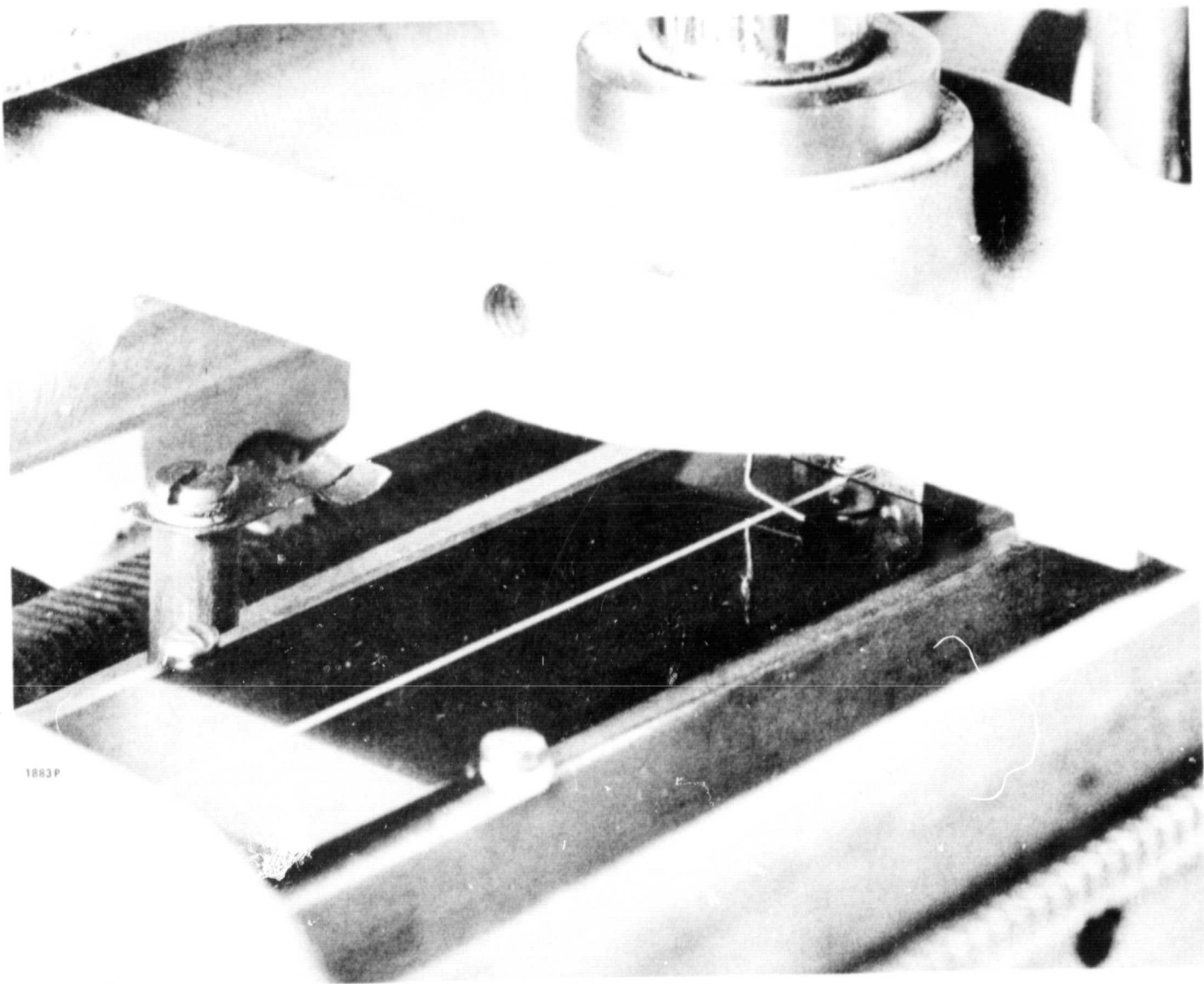


Figure 24. Test Line in Holder Showing Coupling Loop
(Reflected in Ferrite) and Perturbing Probe

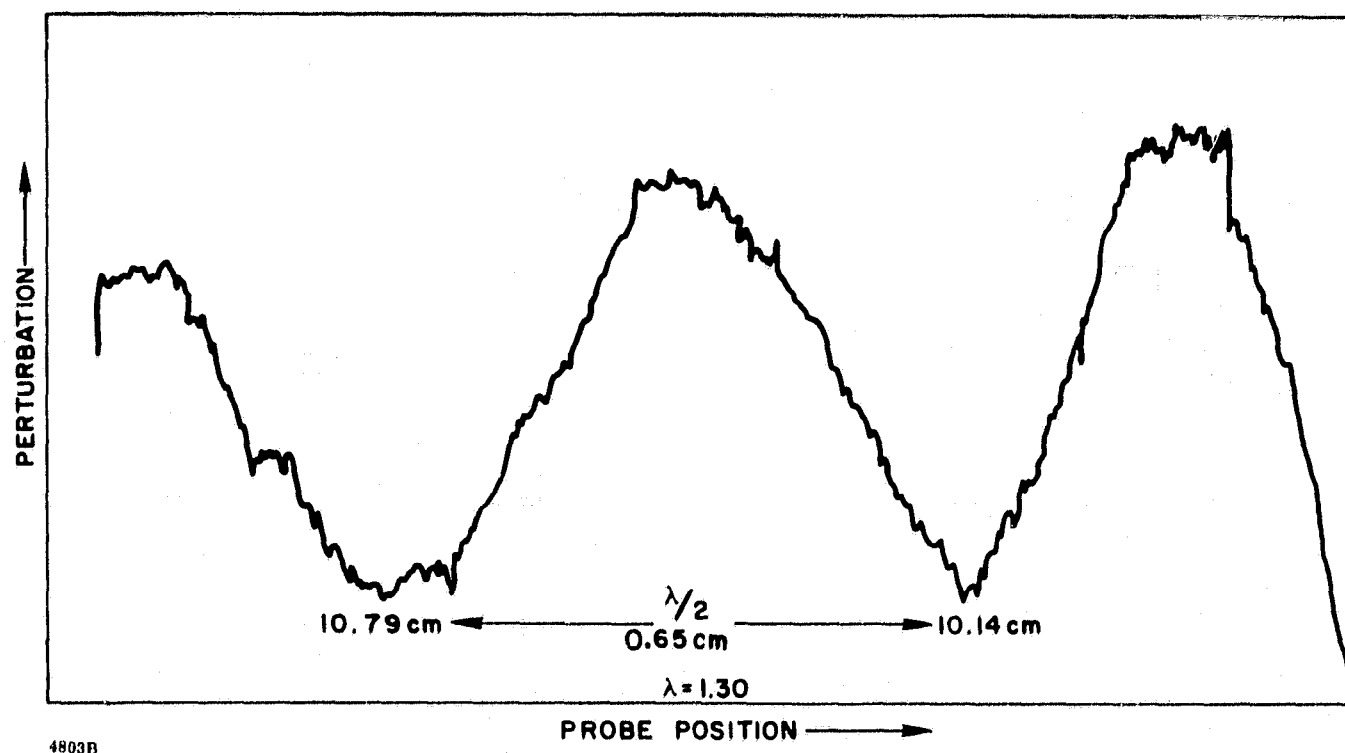


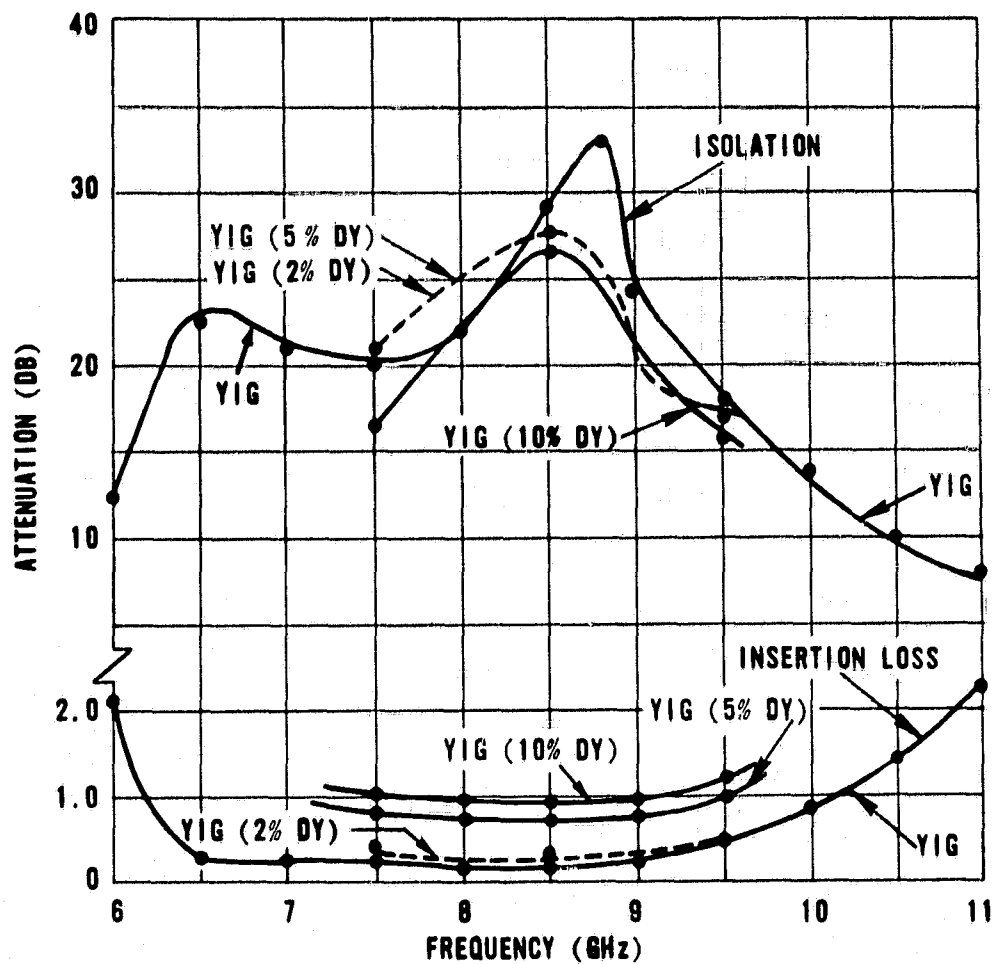
Figure 25. Sample Plot of Standing Wave on Microstrip Line on Ferrite Substrate, No Field Applied (7420 MHz)

7.2 FIXED BIAS CIRCULATORS

7.2.1 Basic Design and Performance

Microstrip circulators have been studied in considerable detail at Sperry. On this contract, the major effort has been directed towards improving peak power performance and temperature characteristics of fixed bias microstrip circulators and towards a thorough investigation of latched microstrip circulators. X band was selected as the frequency band for these studies. Microstrip circulators have also been studied in L, S, C, and Ku bands on a company-funded effort.

The broadband circulator developed for use on this program in the study of peak and average power handling and temperature stability is shown in Figure 13. The substrate is all ferrimagnetic, 0.482 inch square x 0.055 inch thick. The biasing magnet is 0.230 inch diameter x 0.450 inch long. Data indicate that the magnet may be reduced to 0.300 inch long. Both gold and silver conductors have been used with similar results. The low power performance of this circulator with YIG and dysprosium doped YIG substrates of varying doping levels is shown in Figure 26. The dysprosium primarily affects the resonance linewidth of the material which in turn affects peak power capability and insertion loss. Note that adding 2% Dy barely increases the insertion loss but, as is shown in later sections, produces very dramatic increase in peak power capability.



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Figure 26. Performance Data of the Microstrip Circulator Shown in Figure 13

Figure 27 shows a 5-port circulator formed from three 3-port junctions similar to the one described above. In this case, the substrate is 1 inch x 1 inch x 0.055 inch. The data are shown in Figure 28. The design of the junctions is very nearly identical to similar isolated junctions, demonstrating that junctions may be brought into close proximity on the substrate without serious interaction problems.

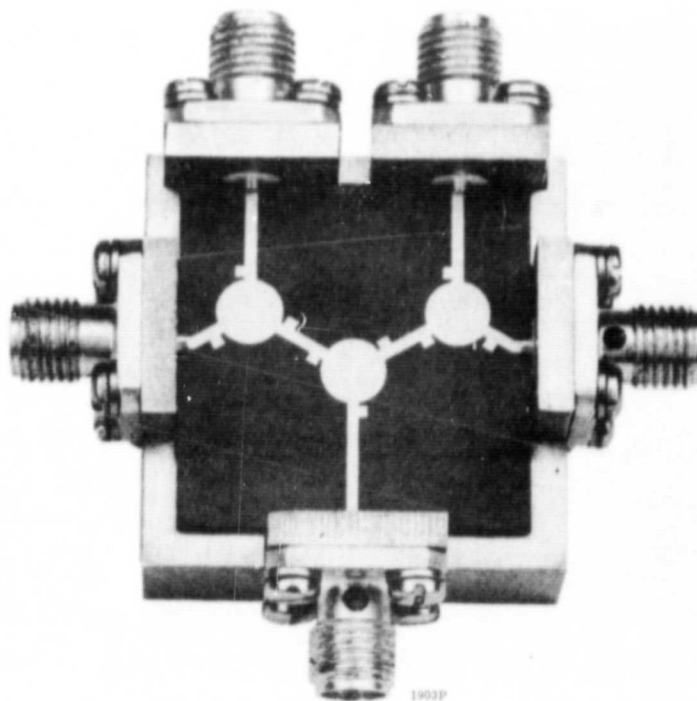


Figure 27. X Band, Five Port, Microstrip Circulator

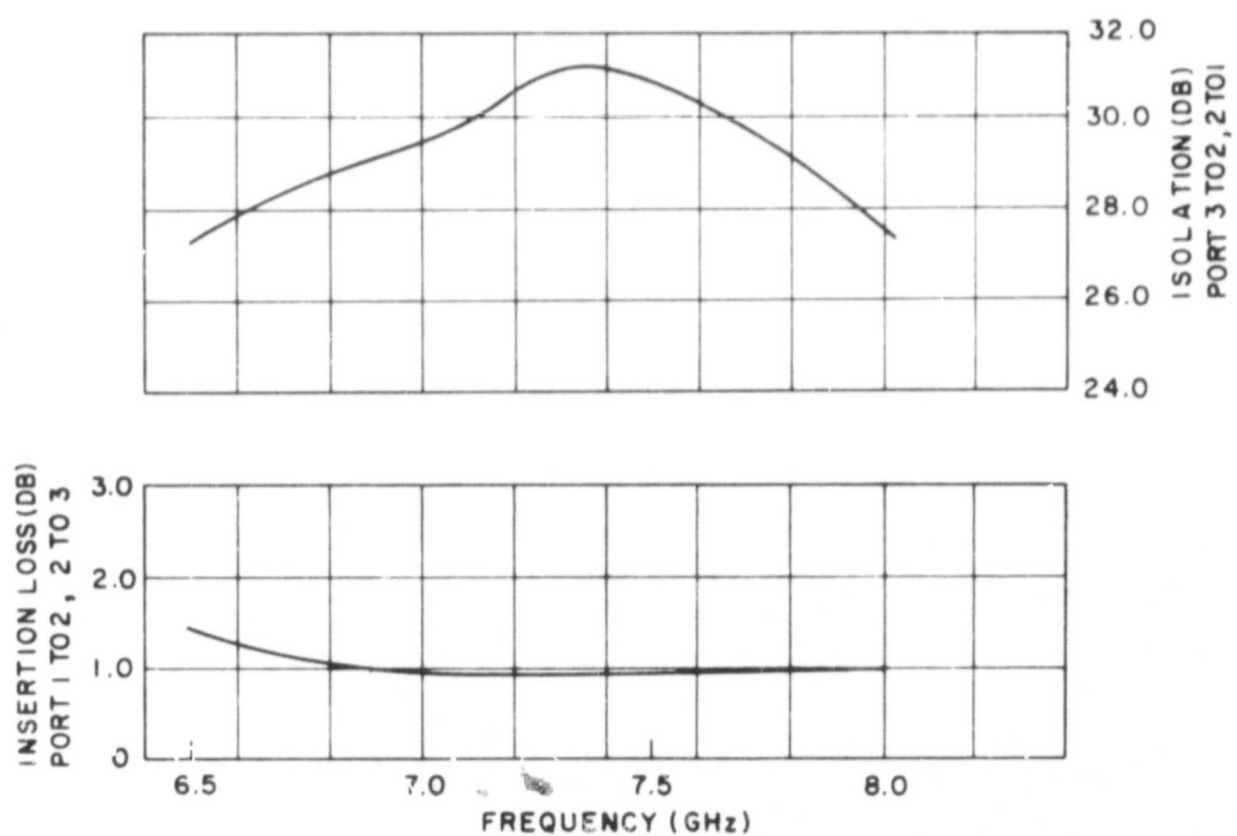


Figure 28. Performance of X Band, Five Port, Microstrip Circulator with Two Ports Terminated

7.2.2 Peak Power Performance

An experimental investigation was conducted regarding the use of dysprosium doping in YIG to increase the peak power handling ability. The rf magnetic field, and thus the peak power, above which nonlinearities occur is directly related to the intrinsic linewidth (ΔH_{int} or ΔH_k) of the material.

$$\left[P_{\text{crit}} \sim \left(\frac{\gamma 4\pi M_s}{\omega} \Delta H_k \right)^2 \right]$$

Dysprosium doping broadens the intrinsic linewidth of YIG and thus increases its peak power handling capability. Magnetic losses are also related to H_{int} and would be expected to increase. The loss with pure YIG substrates is found to be mainly ohmic as was stated in previous sections so that the increase in overall loss resulting from the increased magnetic loss should not be too severe for moderate doping levels. For higher doping levels, magnetic losses will eventually be the dominant contributor to losses.

Substrates with 0%, 2%, 5% and 10% dysprosium substitution were prepared. Each was etched with an identical circulator circuit of evaporated-electroplated gold. They were each tested in the configuration shown in Figure 13. The low power performance is shown in Figure 26. Note that an appreciable increase in overall loss occurs for the 5% Dy material.

The peak power tests were run in the test setup diagrammed in Figure 29.

The average power into the device provides the "X" drive for an X-Y recorder while the average power out of the device provides "Y" drive. The input arm is well calibrated to provide an accurate measurement of average power which is converted to peak power based on duty factor and pulse shape. The output arm is balanced with input with the level set attenuator to give equal readings. If the power is increased from zero to maximum, the X-Y recorder will generate a diagonal line representing equal input and output power. The output calibrated attenuator can then be set to give additional lines on the recorder as the power is increased, which represent various values of insertion loss. With the device in place and the output calibrated attenuator set to its zero value, the power is increased from zero to maximum. If there is no limiting, the recorder trace will follow the

curvature of those generated in calibration. With limiting, the trace follows the calibration at low power, but deviates as power is increased. An example of three such plots is given in Figure 30.

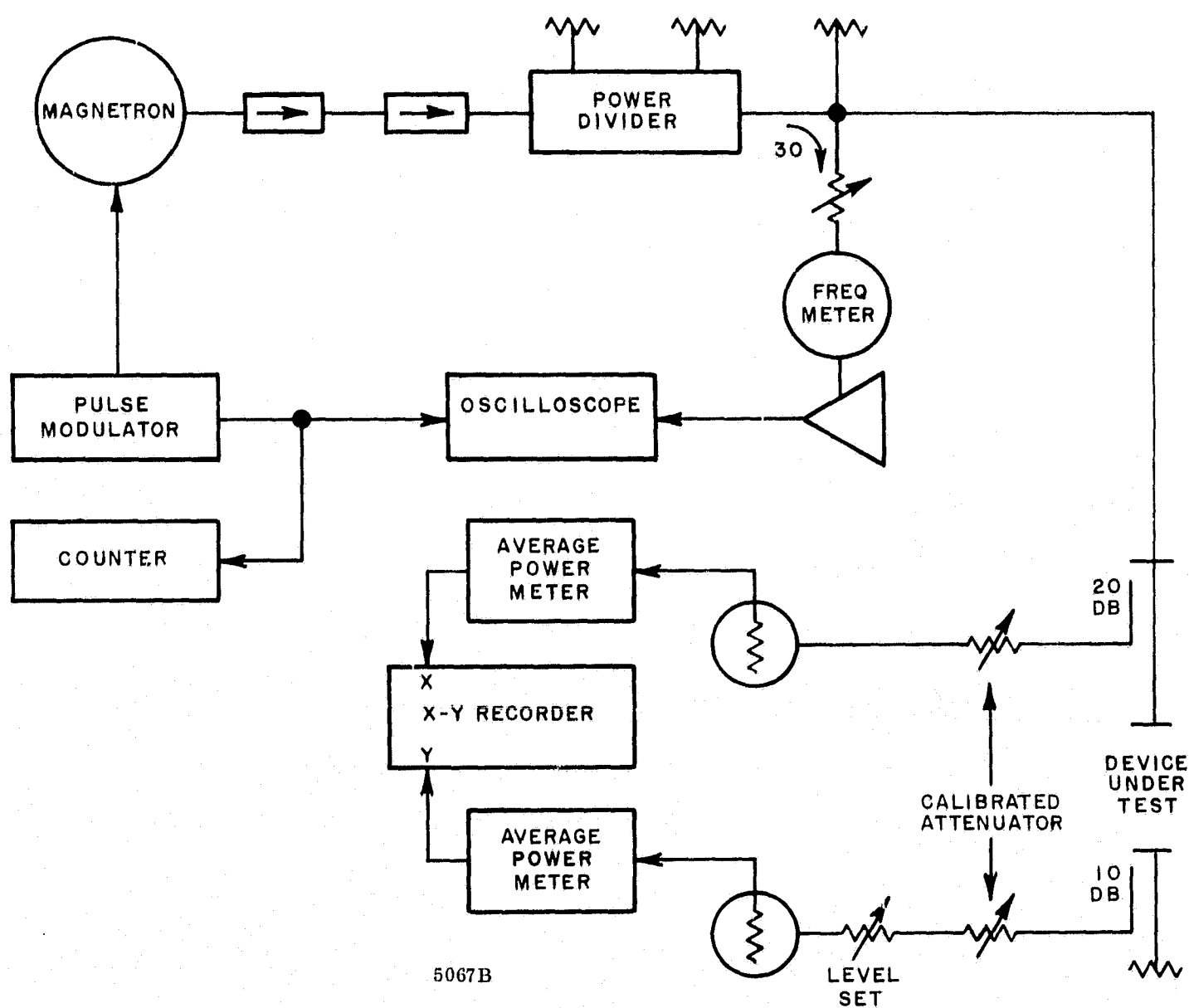


Figure 29. Peak Power Test Setup

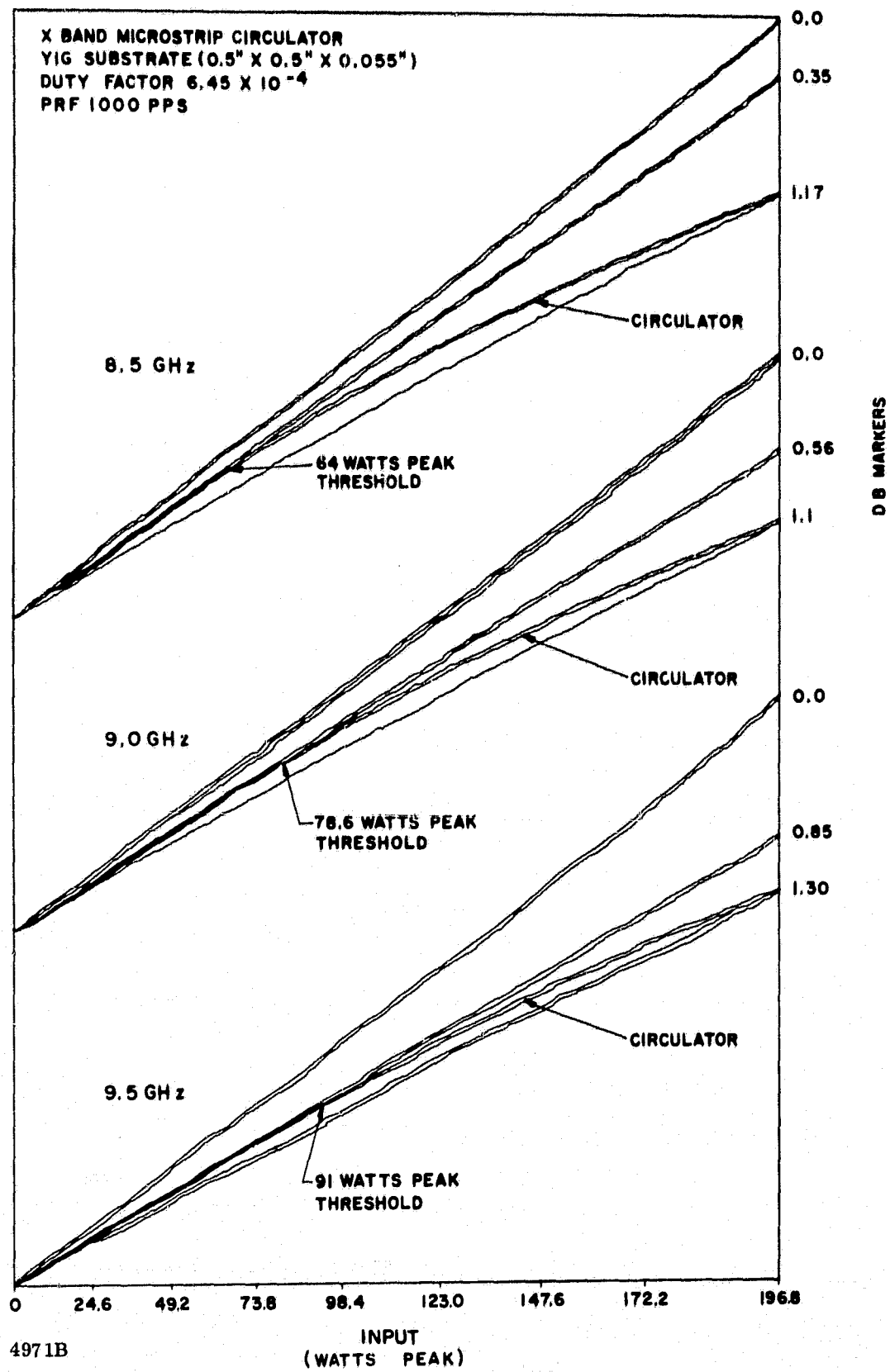


Figure 30. Peak Power Threshold for X Band Microstrip Circulator at Three Different Frequencies

Data obtained by this method are presented in Figure 31 showing the peak power variation of loss for 0%, 2% and 5% dysprosium doped substrates. No limiting was observed to a level of 8.5 kw peak with 5% doping. At this level, an arc occurred at the connector substrate junction. No damage resulted and there was no evidence that the substrate itself had failed in any way during testing. A summary of the loss and limiting threshold data for the circulator is tabulated below.

Dy %	Insertion Loss at Band Center (db)	Limiting Threshold
0	0.3	60 watts
2	0.4	1.5 kw
5	0.7	>8.5 kw
10	0.9	Not measurable

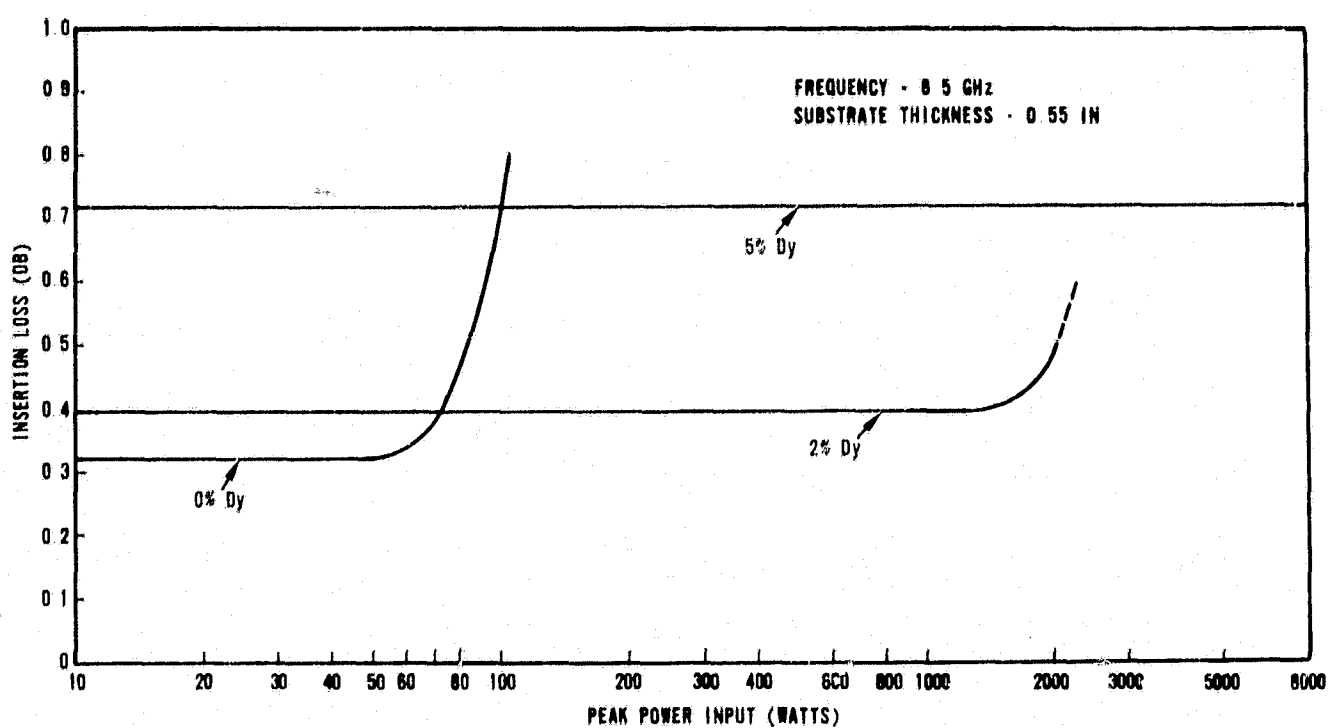


Figure 31. Peak Power Performance X Band Microstrip Circulator (YIG with Various Dy Doping)

Based on these data and the dependence of limiting threshold on ΔH_k , the 10% dysprosium doping should give a threshold of about 50 kw. These power levels are higher than one might expect to encounter in microstrip systems, indicating there will be no difficulty in obtaining ferrite devices to meet the needs. There is very little increase in insertion loss with 2% doping as was expected. This provided a limiting threshold of 1.5 kw, however, an improvement of 25 times the value for the undoped substrate.

7.2.3 Temperature

Temperature tests were run on the circulator shown in Figure 13 with 2% and 5% dysprosium doped YIG substrates. The data are presented in Figures 32 and 33. The performance as a function of temperature is similar to that experienced with stripline circulators. The variation of isolation and loss is due to changes in saturation magnetization as in conventional circulators. The 2% dysprosium doped substrate maintained greater than 20 db isolation from 7.6 to 8.7 GHz over the range of -20°C to 50°C . The 5% dysprosium doped substrate maintained 20 db isolation from 7.7 to 8.7 GHz over the same temperature range.

The maximum losses over the above bandwidths were 0.8 db and 1.0 db for 2% and 5% dysprosium doping, respectively. These losses can be reduced with no sacrifice of isolation by using a pure YIG substrate. The peak power threshold would, of course, be lowered as indicated in the previous section. The variation in loss with temperature is most probably due to low field losses. As the temperature is decreased the saturation magnetization increases, reaching approximately 2000 gauss at -40°C . Low field losses are then very evident just below 7.5 GHz with losses reaching 3 db at 6.5 GHz.

The saturation magnetization can be stabilized to remain reasonably constant over a temperature range by gadolinium substitution. The saturation magnetization is also reduced which lowers the attainable bandwidth at this frequency. However, low field losses are moved to a lower frequency which makes the substitution very desirable for a device operating at frequencies below 7.5 GHz.

A substrate of 15% gadolinium substituted YIG [YIG (15% Gd)] was used for a circulator as shown in Figure 13. Temperature tests on this circulator showed a great improvement in stability over the dysprosium substituted YIG substrates which exhibited a significant loss increase at -20°C . Performance as a function of temperature of the YIG (15% Gd) substrate is shown in Figure 34. The gadolinium substituted substrate shows only a slight loss change with the temperature reduced to -40°C . Similarly, the isolation is more nearly stable over the wider temperature range. This improved temperature performance is due to the more stable saturation magnetization of the gadolinium substituted material.

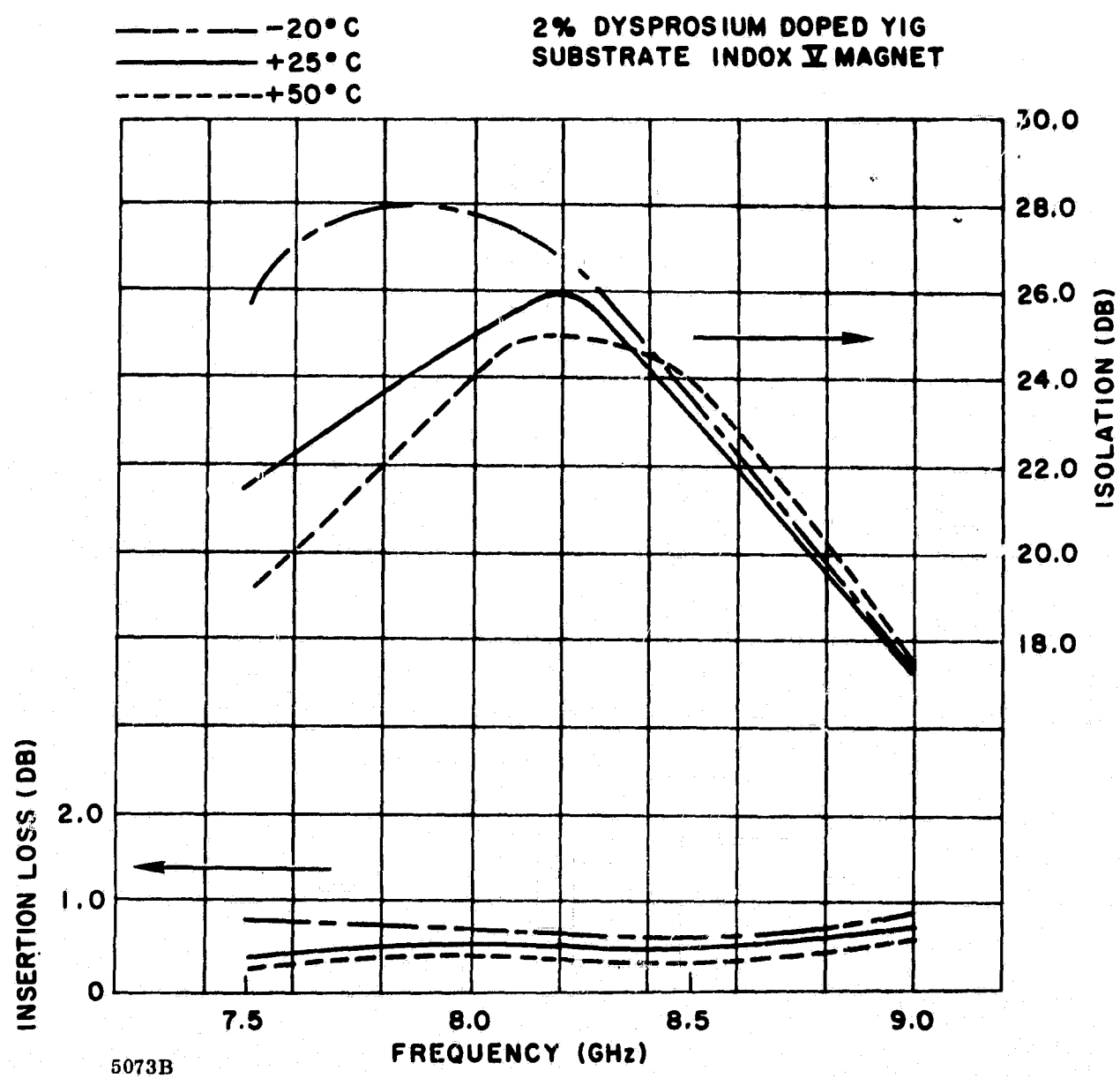


Figure 32. Temperature Effects, Fixed Bias Circulator [YIG (2% Dy)]

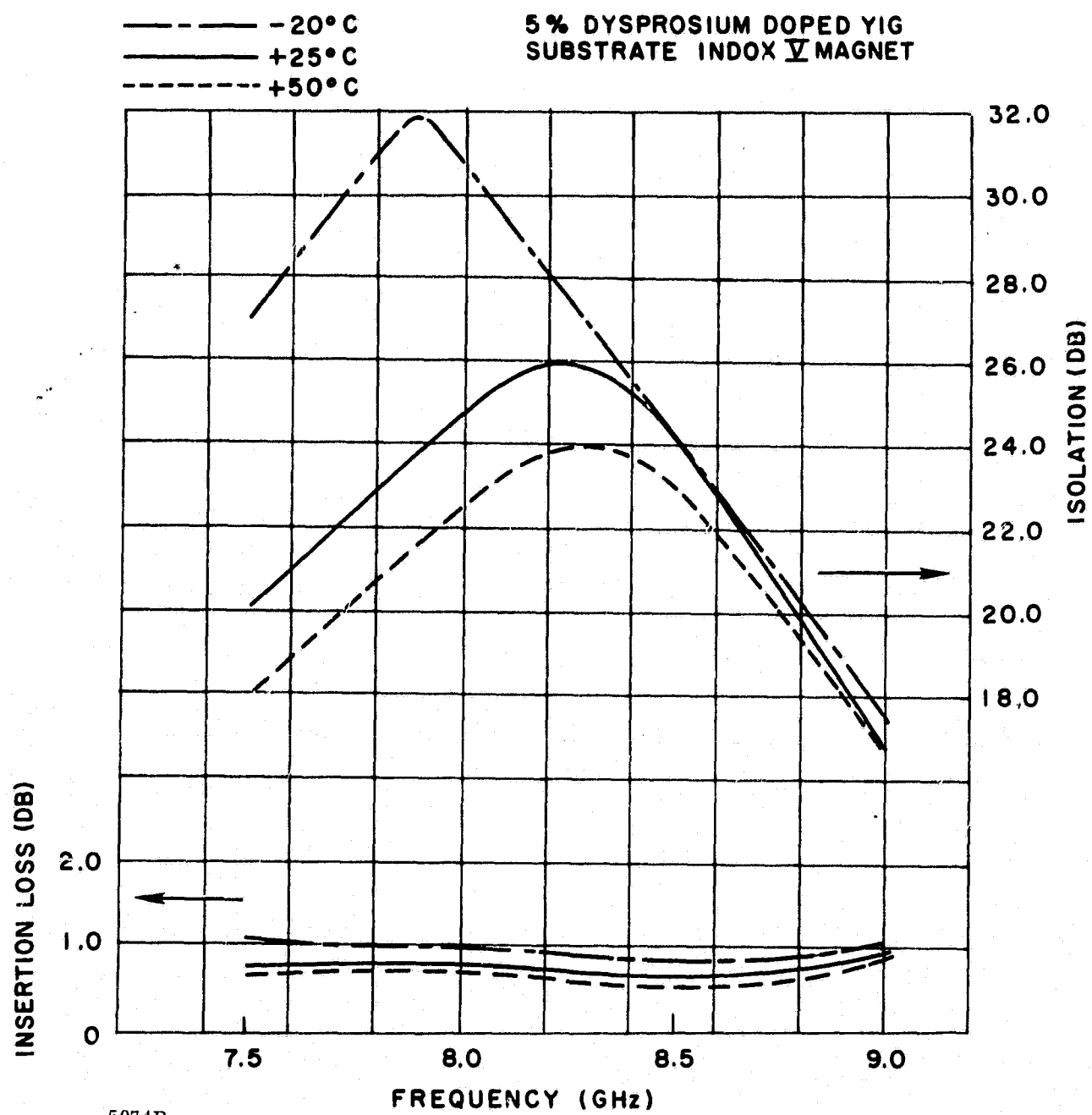


Figure 33. Temperature Effects, Fixed Bias Circulator [YIG (5% Dy)]

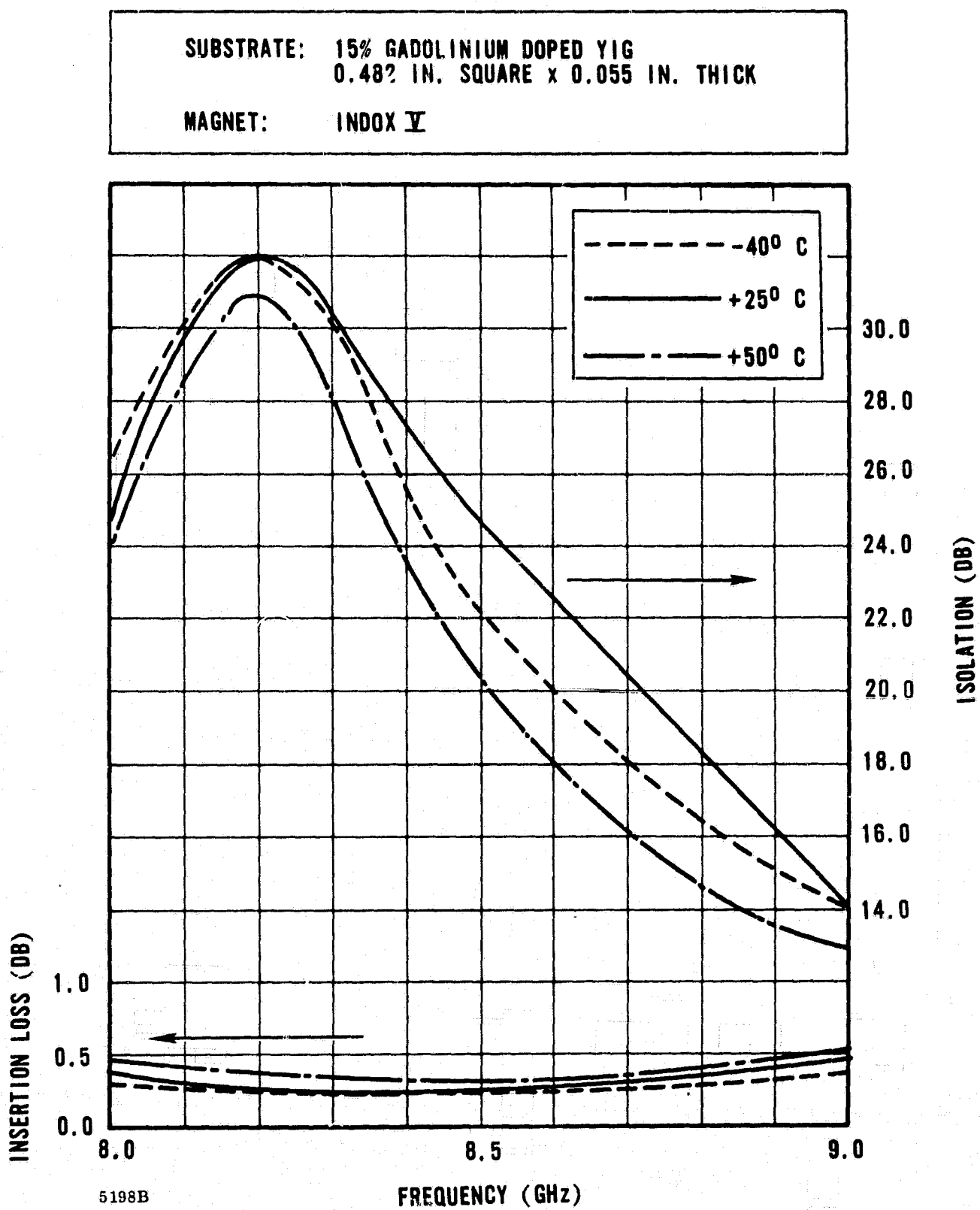


Figure 34. Performance as a Function of Temperature - Fixed Bias Circulator

7.2.4 Average Power

Average power tests were run with a 2% dysprosium substituted substrate in the circulator pictured in Figure 13. The input was CW at 9.0 GHz. At this frequency, the loss was 0.5 db, somewhat higher than at the band center so that the average power effects should be more pronounced. Both isolation and housing temperature were monitored after stabilization at each temperature. No special materials, parts or cooling air were used to aid in heat dissipation. Significant heating began to occur at an average power input of 8 to 10 watts with the housing temperature reaching 10°C over ambient. The dissipated power at this input level was about 1 watt. When the input power was increased to 16 watts average (approximately 1.8 watts dissipated), the housing temperature reached about 17°C above ambient. It would appear that the average power input to an X band junction would be limited to less than 10 watts unless cooling is provided. At lower frequencies with larger junction area the allowable power may be higher.

7.2.5 Biasing Techniques

Techniques for providing the magnetic bias have been examined to give more flexibility in module design and incorporation of modules into arrays, and to reduce costs. The magnet used initially to bias the X band circulator discussed above was .600 in. long (Indox V) and was located below the ground plane. It has been found that this length can be reduced to 0.300 to 0.450 in. in the same configuration without sacrificing performance. Several other configurations have been investigated including:

- Shielded magnet below the ground plane
- Different magnet material to reduce size
- Magnet above the substrate

The design of a shielded magnet should eliminate problems of interaction which might occur when modules are stacked or interaction with other components on the same substrate. An Indox V magnet with cold rolled steel shield is shown with circulator in Figure 35. It is placed against the ground plane to bias the circulator. The dimensions are 0.212 in. x 0.750 in. diameter. Isolation of 20 db from 6.5 to 9.5 GHz has been attained with this configuration. This is equivalent to results previously attained with the longer magnet. It should be possible to reduce the diameter

of the magnet shield to 0.500 inch without loss of performance. The other magnet shown in Figure 35 is an Indox magnet 0.230 inch diameter by 0.300 inch long which is used unshielded.

A platinum cobalt magnet used successfully was 0.075 inch long. However, the cost of raw materials for this magnet is very high and, although it does reduce the size considerably, it would not be generally desirable. If the size can be reduced further (lowering cost) it might find use in some applications.

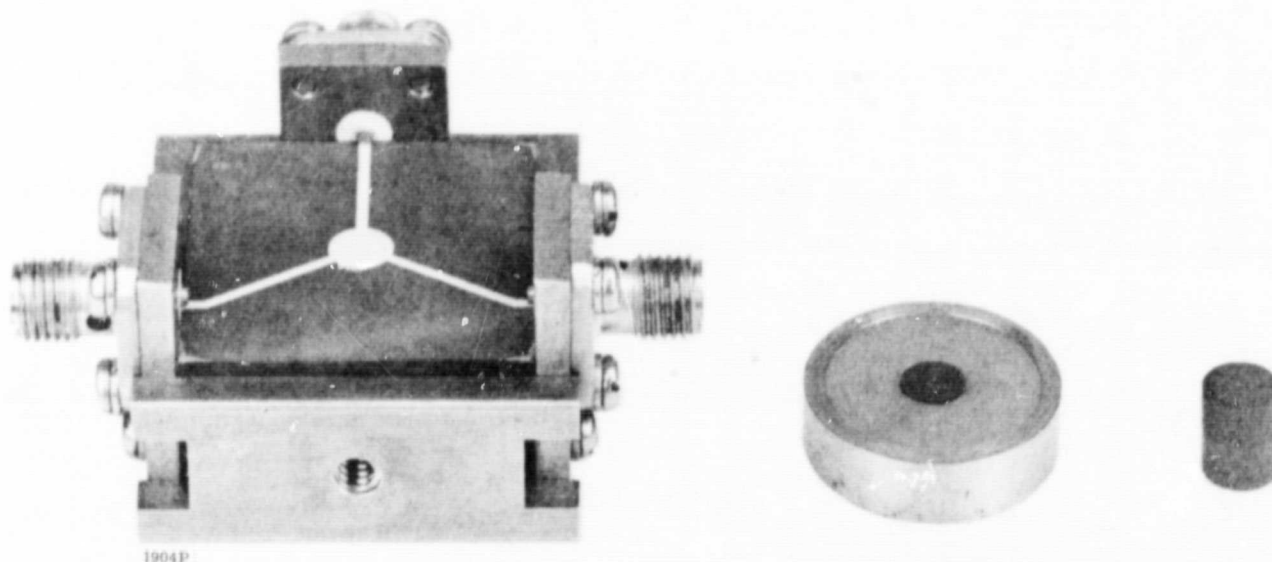


Figure 35. X Band Microstrip Circulator with Two Different Biasing Magnets

Placing the biasing magnet on the circuit side of the substrate has been considered for two reasons. First, with all the magnet above the substrate, there would be no need for locating holes in the mounting base of the substrate. Second, with some magnet above and some below it might be possible to reduce the amount of magnet required. Investigation of both of these techniques indicate that they are feasible.

7.2.6 Repeatability of Production and Performance

A series of six 0.482 inch square by 0.055 inch thick YIG substrates was prepared and coated with a fired silver conductor. The same circuit was etched on all substrates. Three magnets, 0.230 inch diameter by 0.300 inch long, charged to 1300 gauss, were used with each substrate.

All of the substrates tested exhibited a 20 db isolation bandwidth of 800 to 1000 MHz. The losses were in all cases 0.3 db to 0.4 db within the band. Because of the slight variations in the band center, the effective band covered by all of the units tested was 700 MHz. Some of the variations which were noted were due to magnet irregularities (which should be correctable) and some variations were due to the laboratory housing (which would not be involved in actual applications). The circulators were in all cases assembled and tested in a simulated production environment with no "tweaking" or adjustments to optimize parameters.

This should certainly be an acceptable variation for the circulator. It is very good considering the fact that even the field level was not adjusted from unit to unit as would normally be done in conventional circulators.

7.3 LATCHED MICROSTRIP CIRCULATORS

7.3.1 Basic Design Approaches and Performance

Two designs have been investigated. These are the "internal loop" and the "external loop." (See Figures 36 and 37.)

7.3.1.1 Internal Loop (Figure 37)

A bar of YIG was fired with platinum loops of 0.005 inch and 0.010 inch diameter wire in place. The bar was machined to form a substrate 0.482 inch square which showed little evidence of cracks. The loop in this substrate was of 0.005 inch diameter wire. The substrate was initially 0.100 inch thick, subsequently reduced to 0.075 inch and 0.055 inch. At each thickness, an untuned circulator junction with 50 ohm input microstrip lines was etched on the substrate. The circulator was tested latched and with permanent magnet bias at each thickness to determine the latching action. No attempt was made to optimize performance. In general, the fixed bias performance, insertion loss and isolation improved as thickness decreased while latching performance was degraded. The more pronounced change in both cases

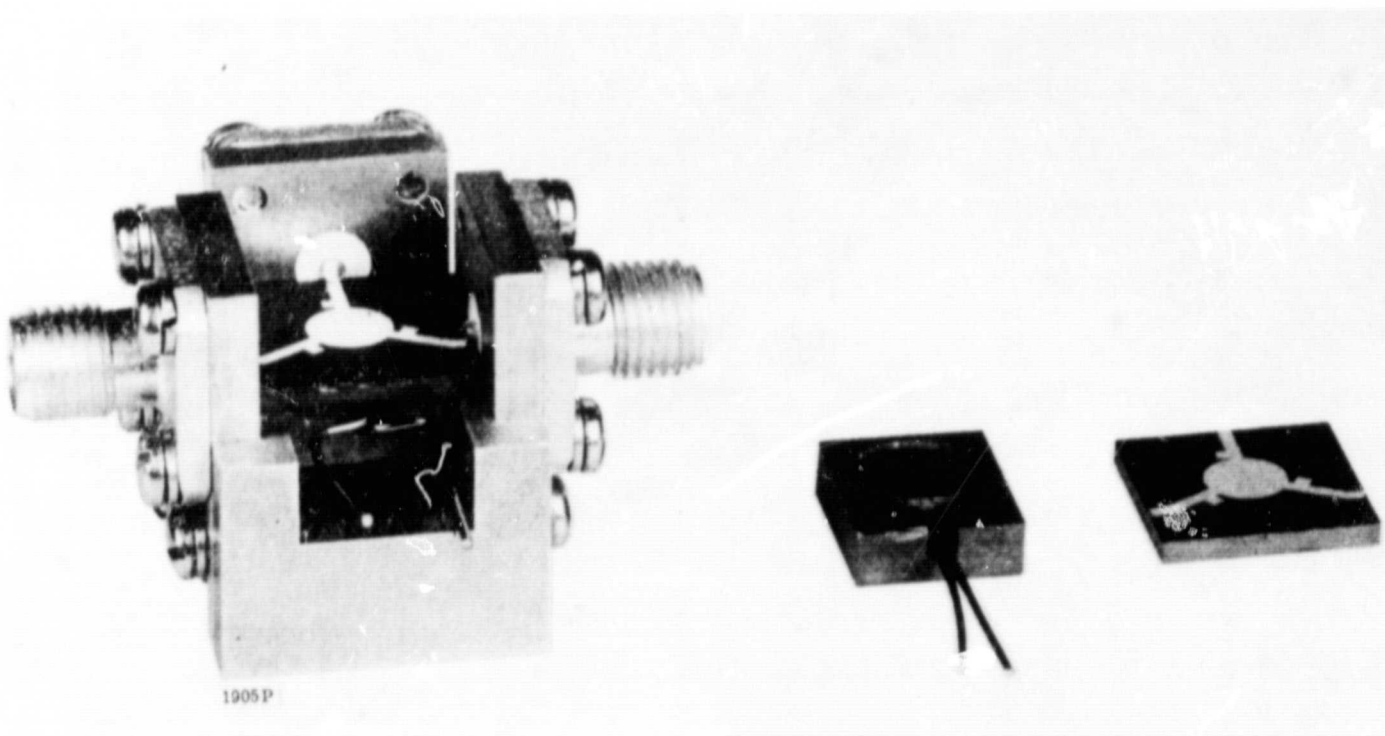


Figure 36. X Band Latching Microstrip Circulator (External Switching Loop)

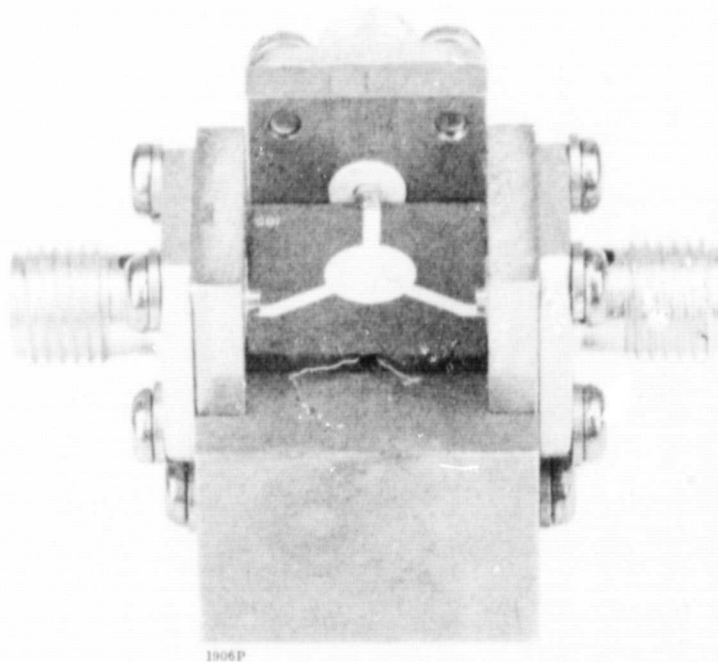


Figure 37. X Band Latching Microstrip Circulator (Internal Switching Loop)

occured between 0.075 inch and 0.055 inch thickness. At 0.055 inch thickness, the fixed bias loss was 0.4 db from 8.0 to 9.0 GHz while the latched loss was 3 to 5 db over the same range. The latched isolation was about 8 db over the range and the fixed bias isolation was greater than 20 db from 8.0 to 8.6 and greater than 14 db from 8.6 to 9.0 GHz. The best losses achieved in the latched configuration was at 0.100 thickness and was 1.4 db over about 500 MHz. The performance of the internal loop circulator is definitely inferior to the external loop at X band and its performance should be even poorer at lower frequencies. Because of its performance, more complicated fabrication techniques, and lack of flexibility, further development was not pursued.

7.3.1.2 External Loop (Figure 36)

Several "external loop" designs similar to that in Figure 36 have been fabricated and tested. The latched performance achieved is shown in Figure 20 along with the dimensions of the device. The latching loop was five turns of No. 28 wire. This was used to match the particular driver being used in testing. Performance of a latched microstrip circulator is shown in Figure 38.

Temperature Effects

The circulator shown in Figure 38 was subjected to temperatures from -40 to +50°C without large changes in characteristics. The performance over this temperature range is shown in Figure 39. The circulator was latched at each temperature. The loss remained less than 1 db and the isolation greater than 18 db from 9.1 to 9.7 GHz over the temperature range. This is somewhat better than might have been expected.

Switching Characteristics

An existing electronic driver was attached to the latched circulator. This driver was not ideally matched to this particular configuration, but it was sufficient to obtain some information regarding switching. The required switching energy was measured as 175 microjoules and switching time was 3 to 4 microseconds. This switching energy is based on average power required by the driver and includes driver inefficiency. The driver which was used was relatively simple, employing three transistors. It could be easily modified to match the circulator and to improve the switching time. In practice, an integrated driver would be used.

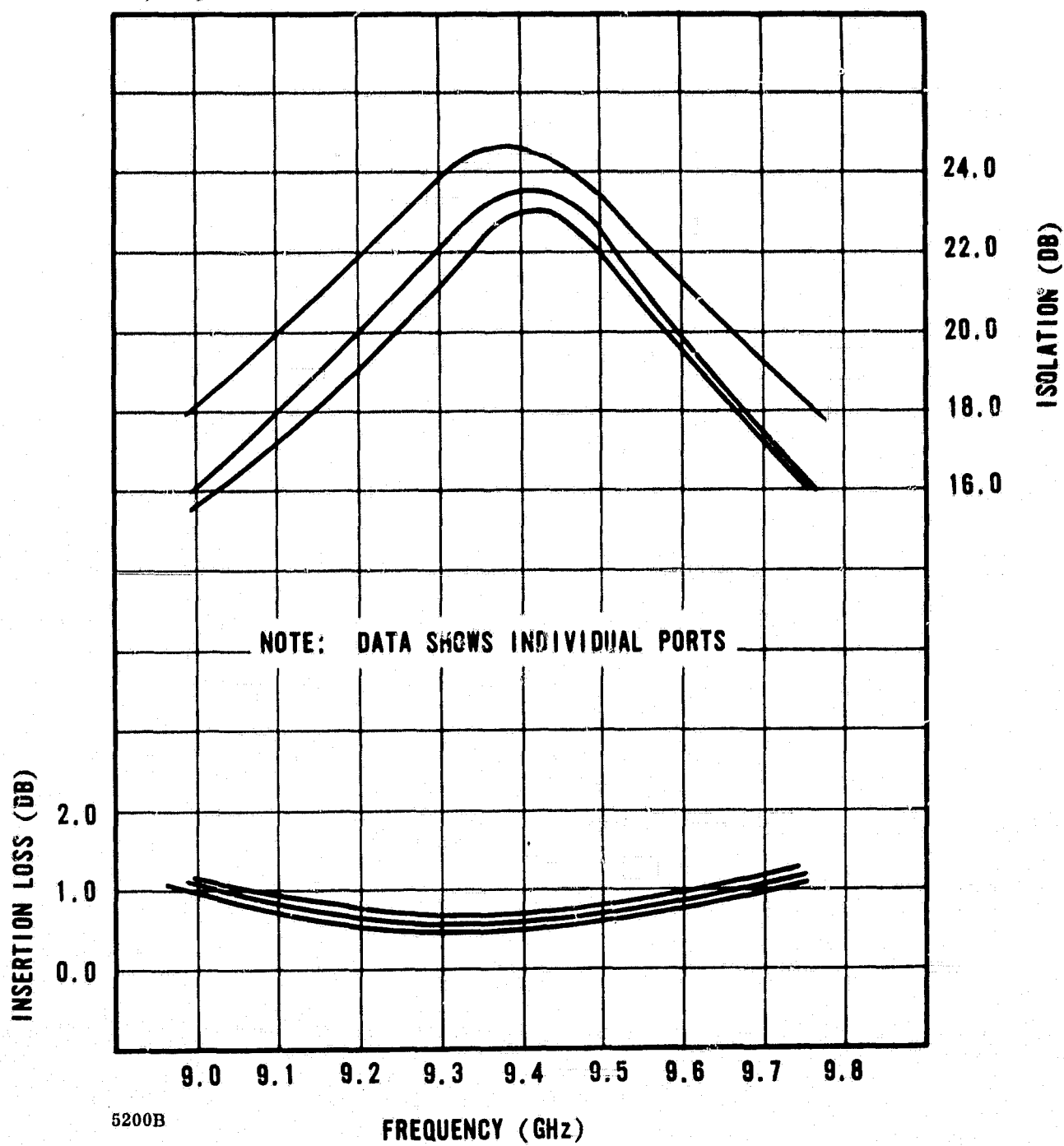
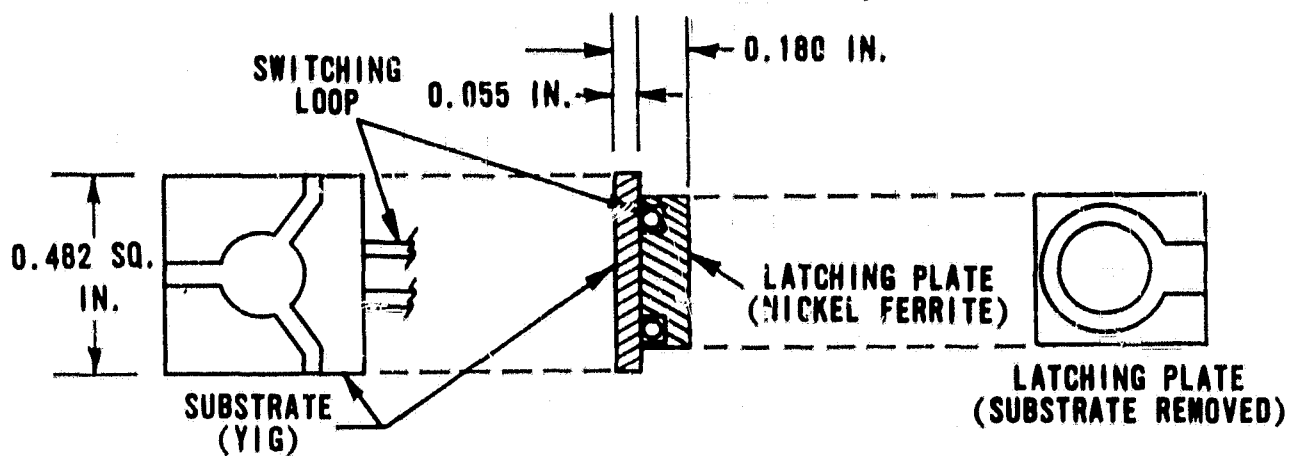


Figure 38. Performance of Latched Microstrip Circulator

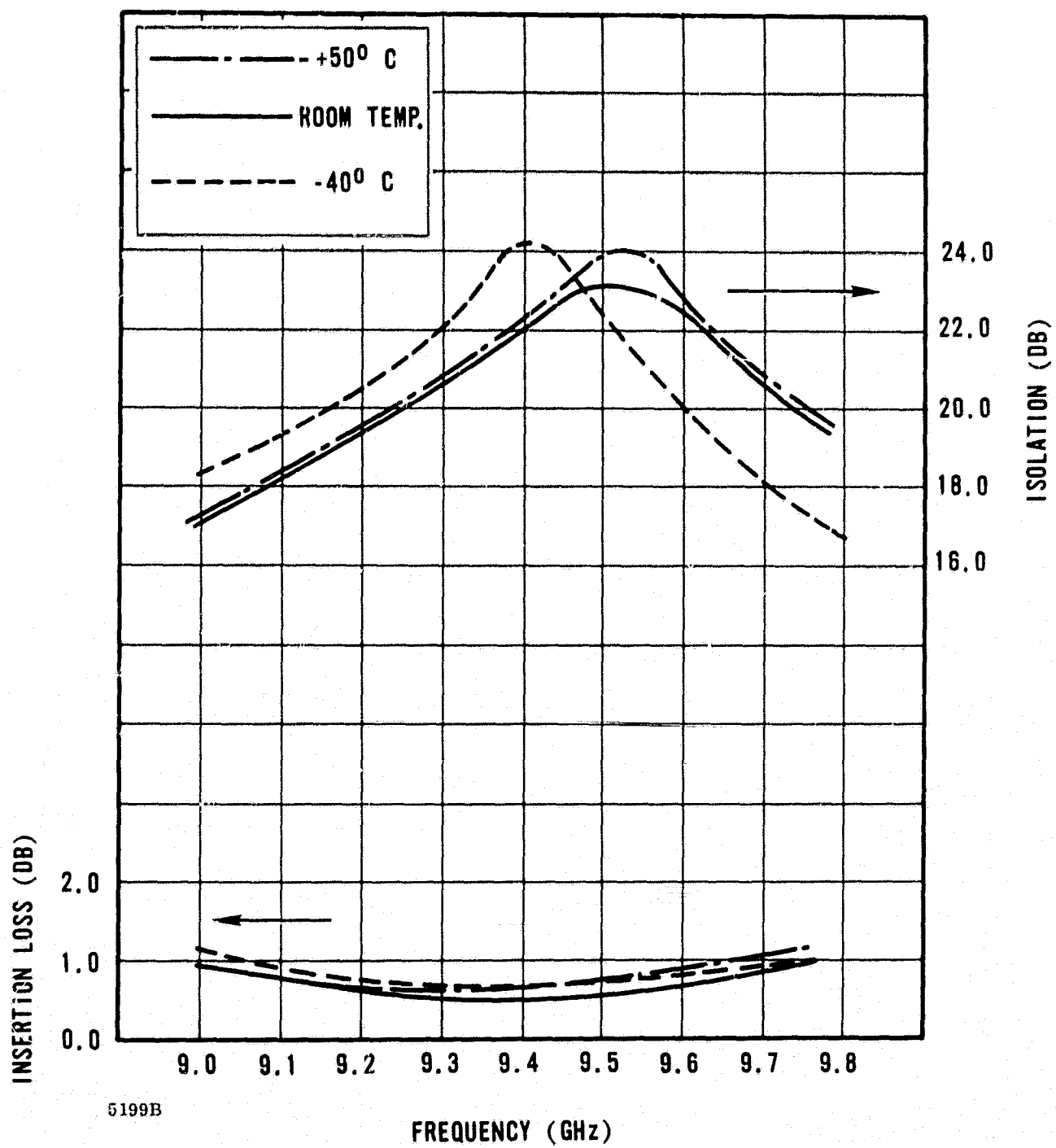


Figure 39. Performance as a Function of Temperature for Latched Microstrip Circulator

7.4 SLOT LINE PARAMETERS

An experimental investigation has been pursued to determine the loss, wavelength, and group velocity for slot line of various geometries and materials. The measurements have included effect of substrate thickness, conductor thickness, slot width, and conductor width for dielectric substrates with $\epsilon_r = 16$.

The measurements were made using lightly coupled resonant line sections. The amplifier and other equipment used is shown in Figure 19. The coupling structure is shown in Figure 40 with and without a substrate mounted.

The energy stored in a resonant transmission line section of length l is:

$$W_s = \frac{2Pl}{V_g}$$

where

P = power in each of two oppositely traveling waves comprising the standing wave

$V_g = \frac{d\omega}{d\beta}$ = group velocity of the traveling waves

$\beta = \frac{2\pi}{\lambda}$ = phase constant of line

λ = line wavelength

If there is assumed no localized loss at the ends of the resonator, the energy dissipated per cycle is

$$W_d = \frac{4\alpha Pl}{\omega}$$

where α = propagation loss in nepers per unit length.

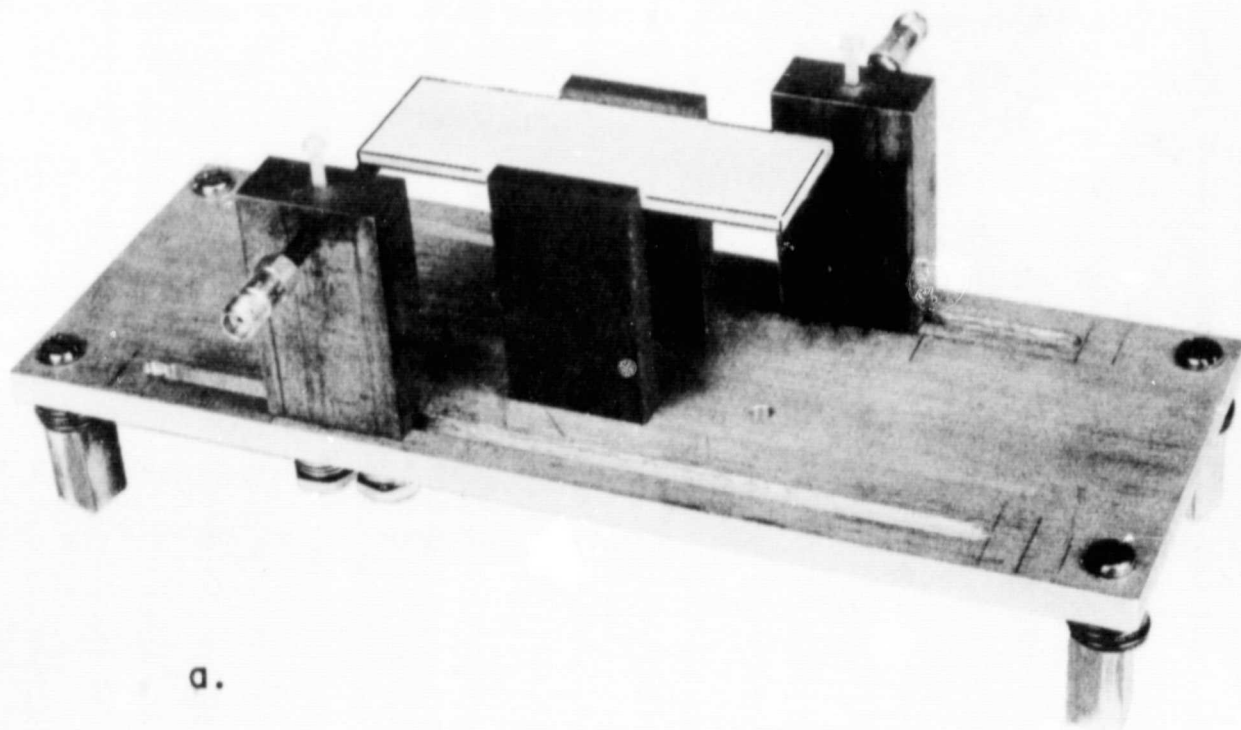


Figure 40. Slot Line Coupling Structure for Loss and Wavelength

where

$$Q = \frac{F}{\Delta F} = \frac{W_s}{W_d}$$

$$F = \text{resonant frequency} = \frac{\omega}{2\pi}$$

$$\Delta F = 3 \text{ db bandwidth}$$

$$Q = \frac{\omega}{2\alpha V_g}$$

$$\alpha = \frac{\omega}{2QV_g} \text{ (nepers per unit length)}$$

$$\alpha \text{ db} = \frac{4.34\omega}{QV_g} = \text{db per unit length}$$

If a resonator is coupled very loosely, the unloaded Q can be determined by measuring the frequency and bandwidth of the resonance. The number of half waves in the resonant section, and thus the wave length, can be determined by moving an E-field perturbing probe along the line. At each E-field maximum, the resonance is

shifted slightly due to the perturbation. By measuring the wavelength and frequency at several resonances, the group velocity can be determined. The above equation may then be used to determine propagation loss.

These calculations are simple but time-consuming and repetitious. To allow rapid analysis of a large quantity of data, a computer program was written to calculate wavelength, phase constant, group velocity, Q , and propagation loss. The input data required are resonant frequency, 3 db frequencies, and the number of half wavelengths of line. These data are punched on cards for processing. While the program was written specifically for slot line, it is readily usable for other transmission lines.

The effect of substrate thickness on wavelength can be seen in Figures 41 and 42. Figure 41 shows β for TEM and 50-ohm microstrip propagation. The line is dispersive and has a somewhat longer wavelength than microstrip. A more direct presentation of β versus substrate thickness is given in Figure 42. There is a reasonably close correlation between β/β_{TEM} and d/λ (d is substrate thickness), as would be expected.

The losses measured are shown in Figure 43. These measurements are for several different substrates. The thickness of the substrate does not appear to have a measurable effect on the loss. These measurements agree closely with losses experienced in measurements discussed in paragraph 7. 5.

7. 5 TRANSITIONS (SLOT LINE)

Both coaxial and microstrip transitions to slot line have been made. A coaxial transition is shown in Figure 44. The transition is made by soldering the outer conductor of 0.085 in semirigid cable at right angles to the slot and ending the outer conductor at one edge of the slot. The center conductor of the cable bends down at roughly a 45° angle and is soldered at the opposite edge of the slot. The junction is made at the open-circuited end of the slot. The connectors used may be miniature or subminiature. The performance of this transition attached to a 3-inch section of slot line is shown in Figure 45.

SLOT LINE: SLOT WIDTH - 0.030 IN.
 SUBSTRATE THICKNESS - d VARIABLE
 CONDUCTOR THICKNESS - 10 MICRONS
 CONDUCTOR WIDTH - 0.485 IN.

DIELECTRIC CONSTANT $\epsilon_r = 16$ FOR ALL CASES

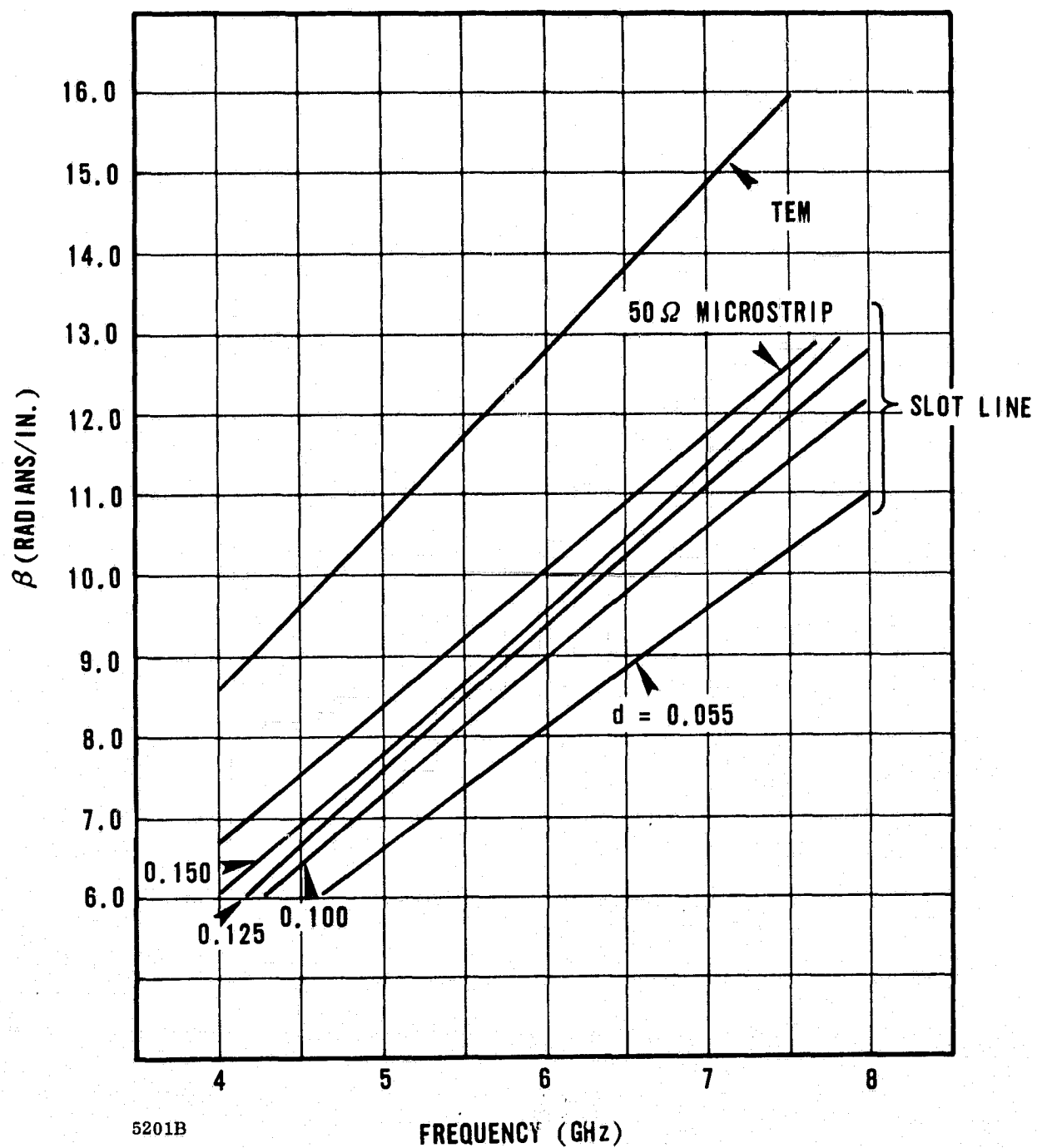


Figure 41. β versus Frequency, Slot Transmission Line

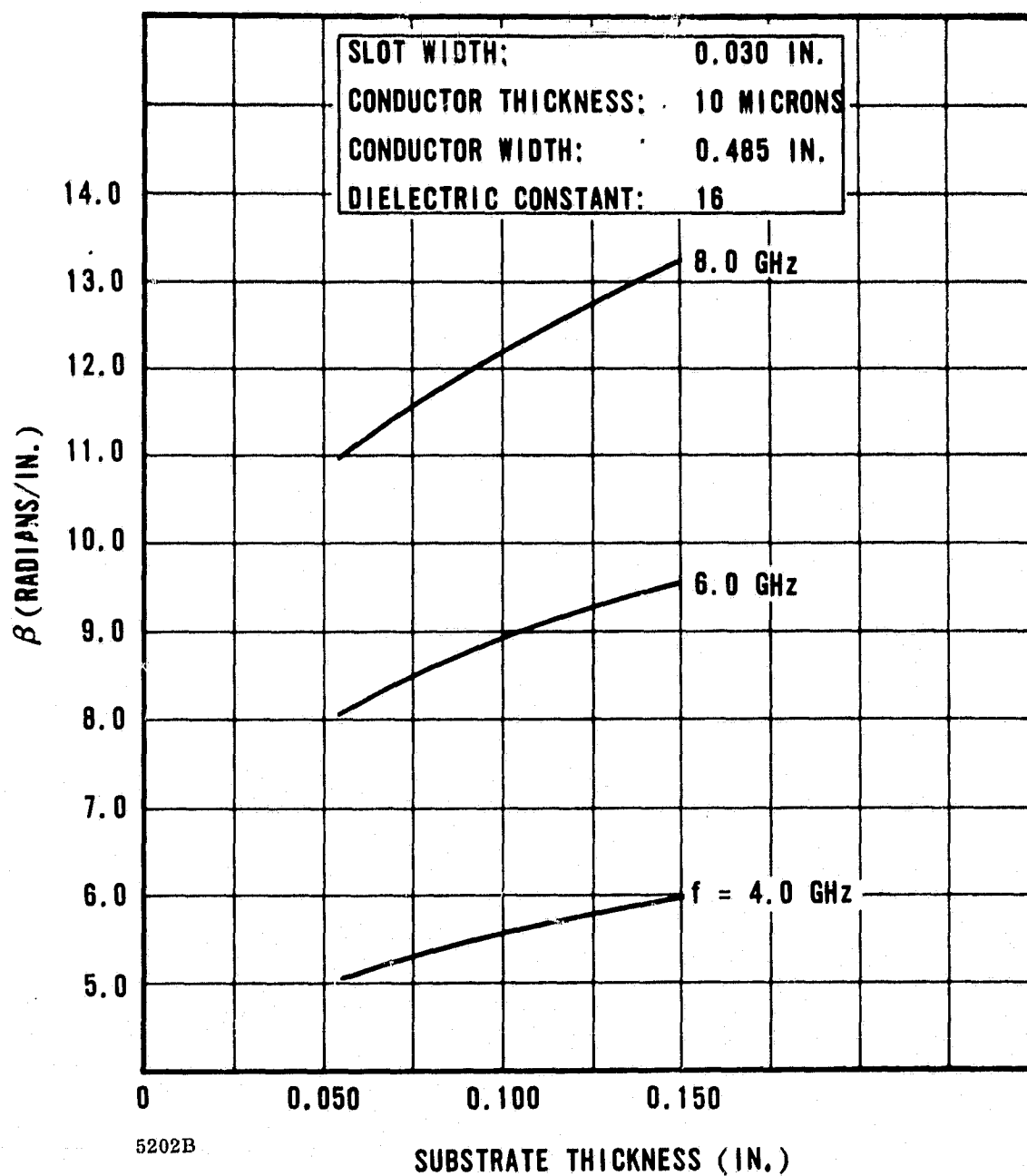


Figure 42. β versus Substrate Thickness, Slot Transmission Line

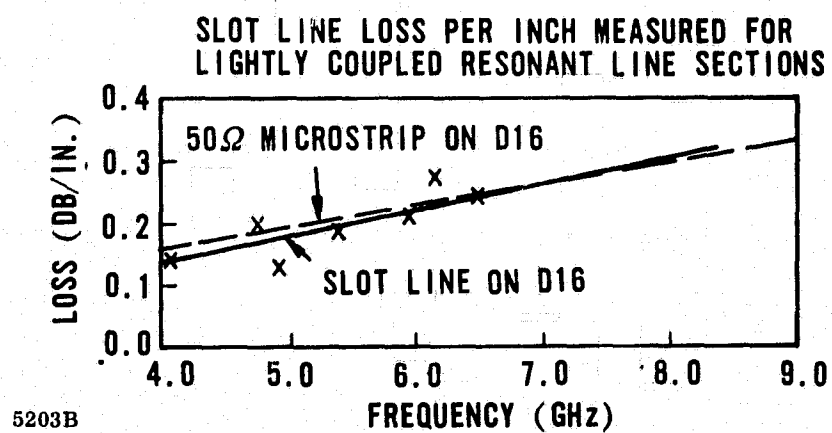


Figure 43. Loss Per Inch, Slot Line Compared with Microstrip

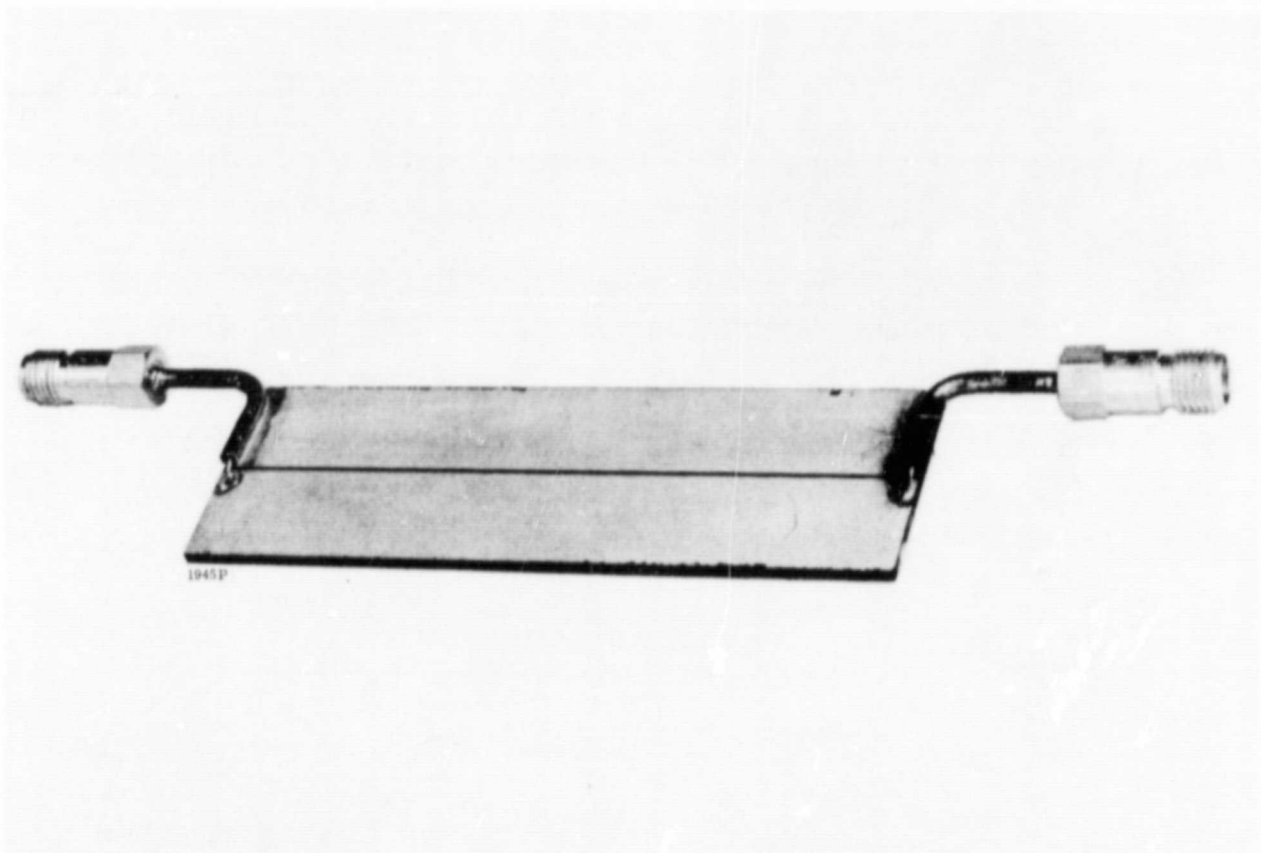


Figure 44. Coax to Slot Line Transition

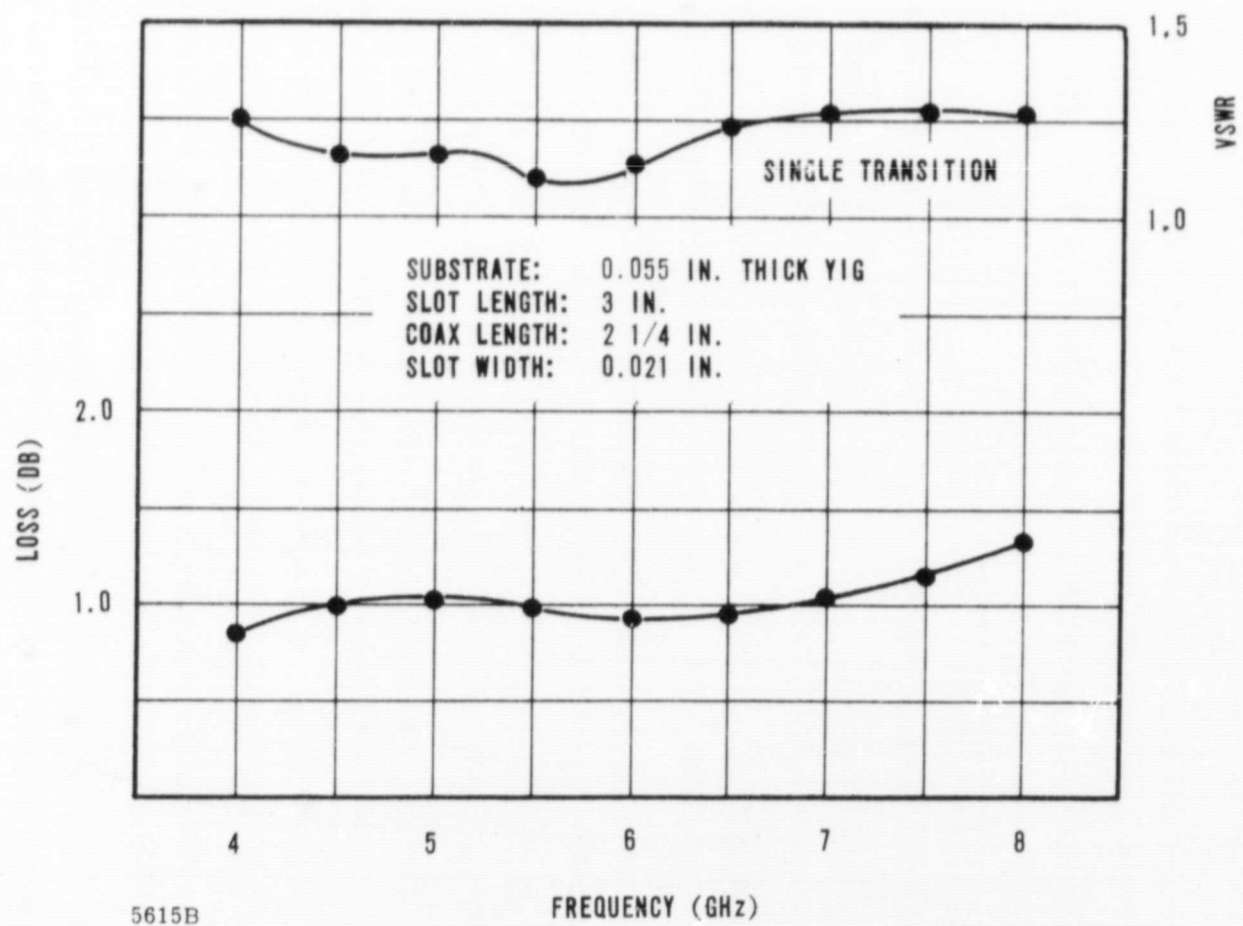


Figure 45. Performance of Coax to Slot Line Transitions

A microstrip-to-slot line transition is easily formed by crossing a slot at right angles with a microstrip line on the opposite side of a substrate (Figure 46). One end of the microstrip junction is open-circuited one-quarter wavelength from the junction and the slot is short-circuited one-quarter wavelength from the junction. The bandwidth and performance is improved by using a "radial line" stub for the microstrip open section. Performance of this transition is shown in Figure 46.

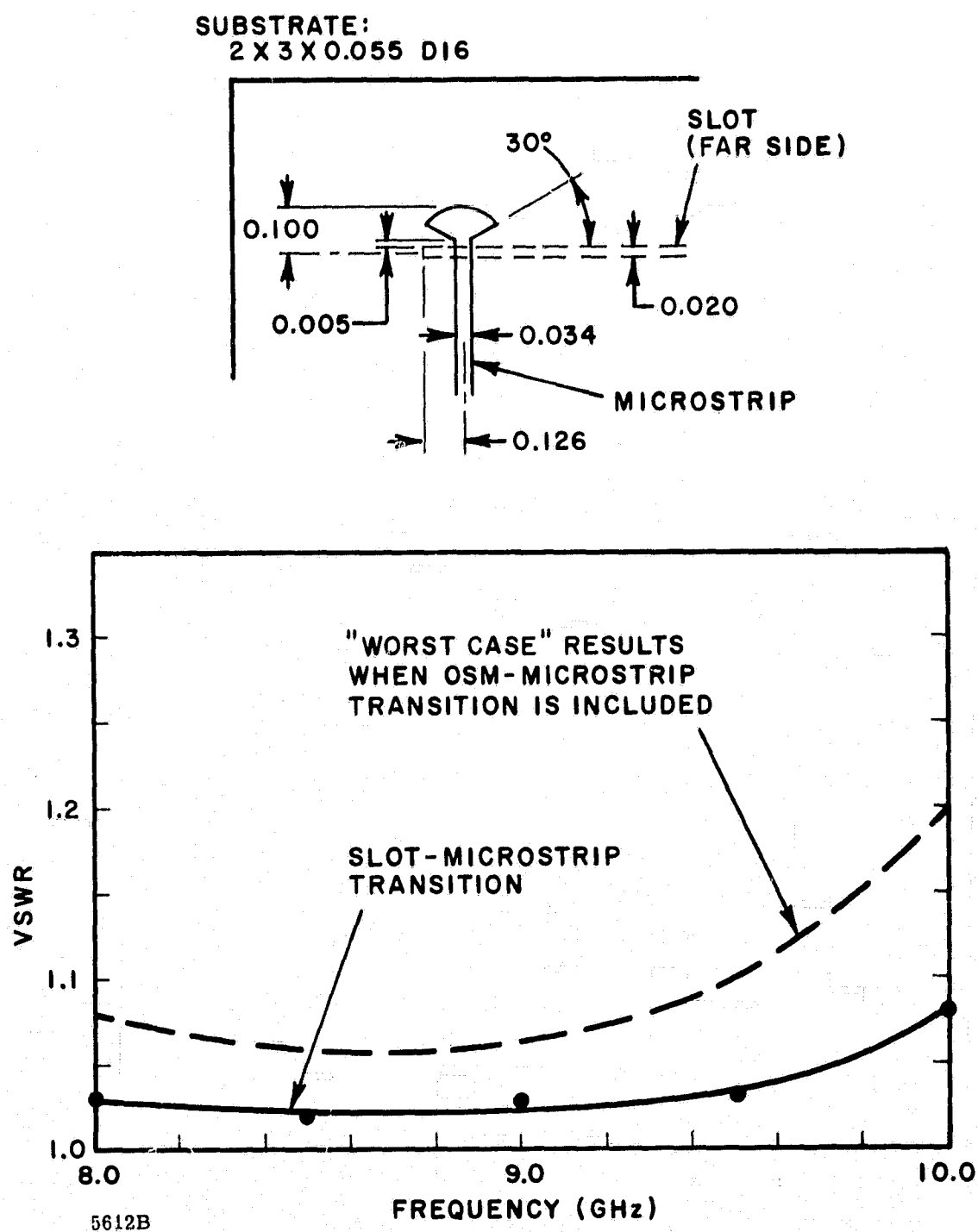


Figure 46. Performance of Slot Line to Microstrip Transition

Two of these transitions were used to form a test section of slot line as shown in Figure 47. The loss and VSWR for the structure are shown in Figure 48. The VSWR shown includes both transitions (including miniature connectors and load).

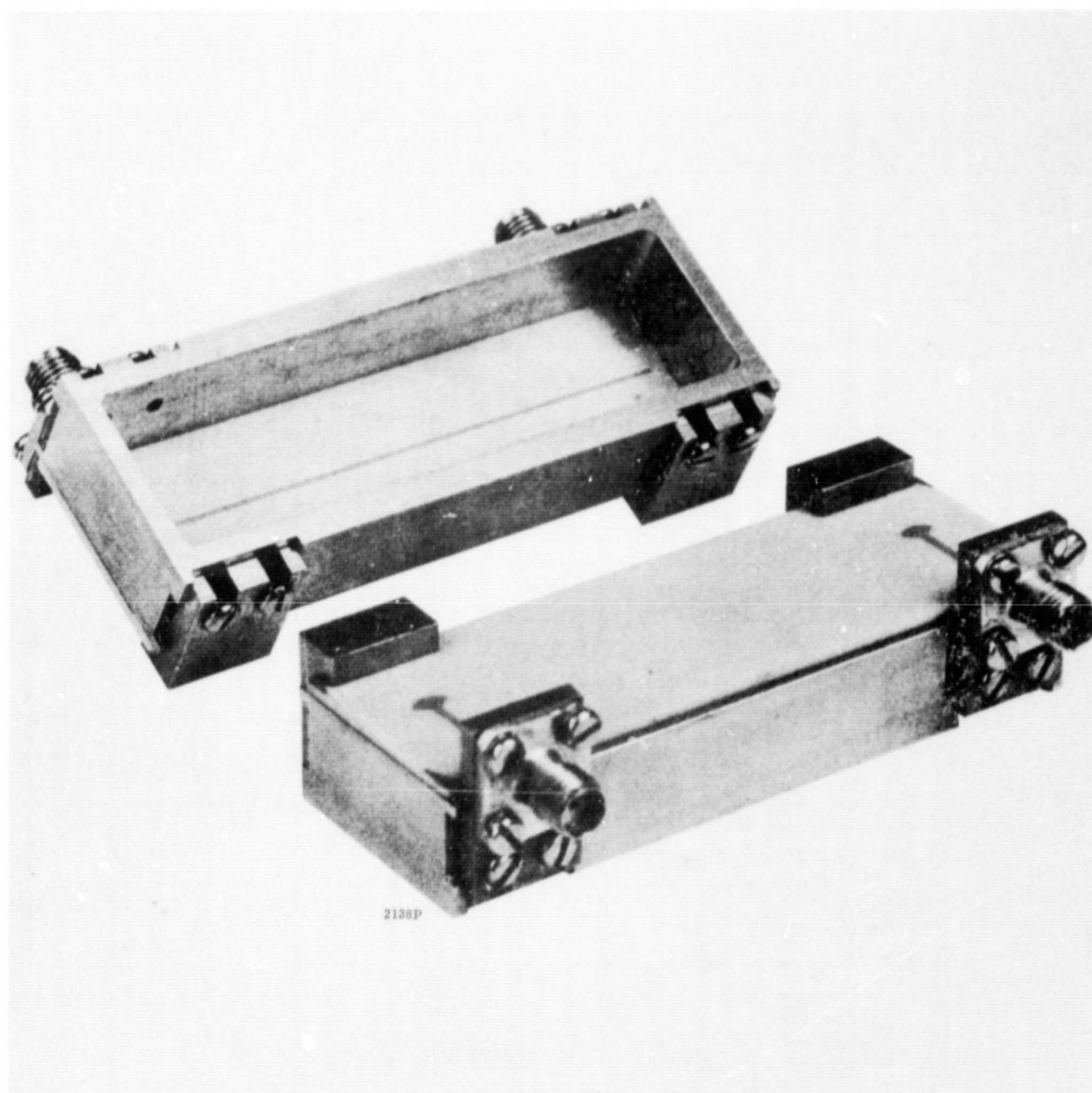


Figure 47. Microstrip to Slot Transitions

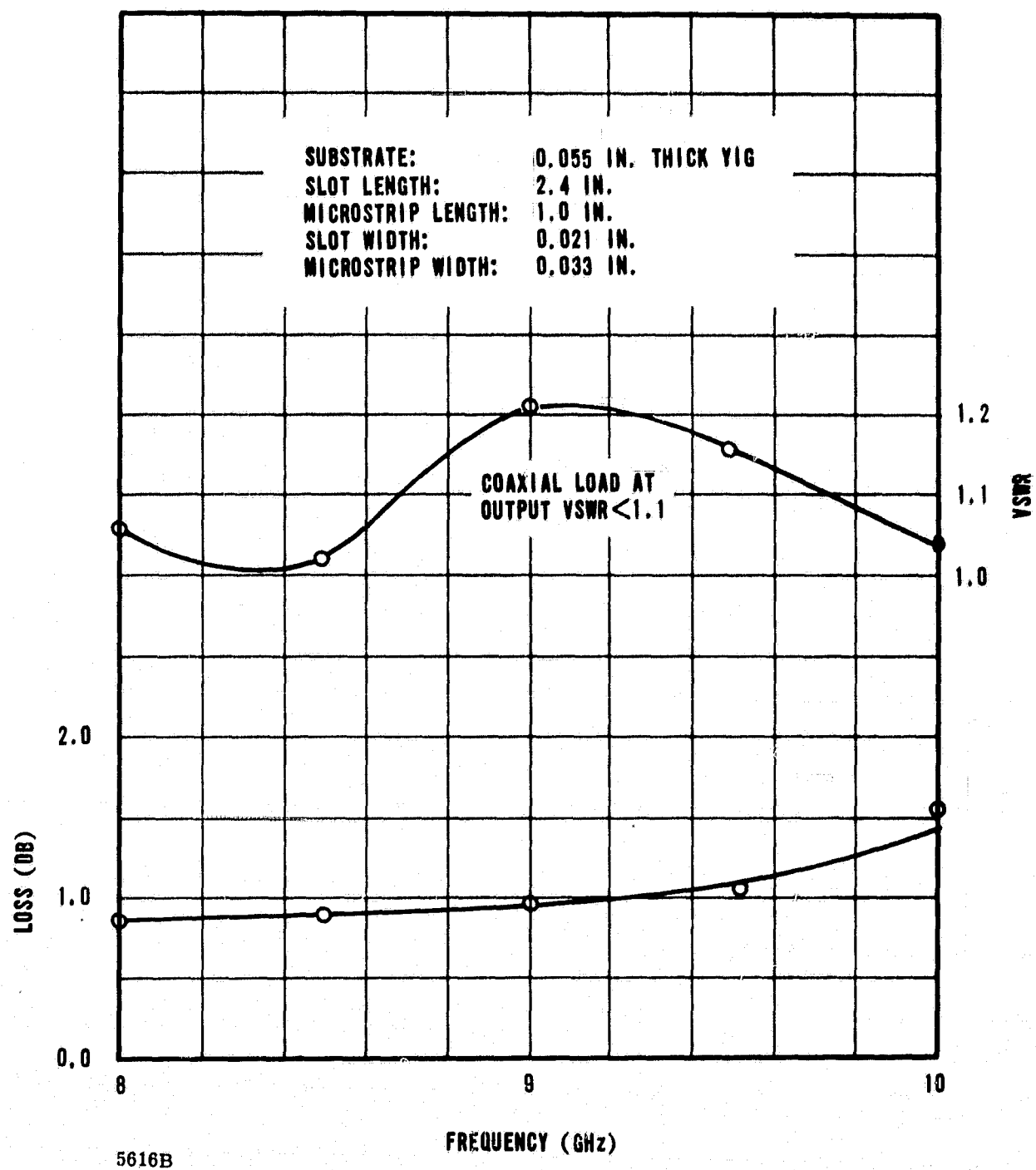


Figure 48. Performance of Microstrip to Slot Transitions

7.6 SLOT LINE PHASE SHIFTERS

7.6.1 Initial Investigation

The achievable phase shift obtainable using several different substrate geometries has been measured in X band. These measurements were made using a Rantec phase measuring system along with a gaussmeter having a recorder output. The field was supplied by an electromagnet in a direction parallel to the substrate. Recorder plots were made of phase versus field from zero gauss through saturation for both directions of magnetization. From these plots, differential phase shift at saturation was determined.

Three geometries tested are shown in Figure 49. At 8 GHz, using aluminum-doped YIG in the configuration shown in Figure 49(a), a differential phase shift of 27 degrees per inch per 0.5 db was achieved. Configuration (b) with 0.055-inch thick YIG substrate gave 25 degrees per inch, and with 0.110-inch thick YIG substrate, 55 degrees per inch. Configuration (c) with a composite substrate of 0.055-inch thick YIG backed by 0.055-inch thick D16 achieved 60 degrees per inch. These results became more meaningful after the development of analytical expressions for the H-field. The fact that the longitudinal field did not reverse sign in the air or at the slot was determined experimentally. The location of the reversal and relative magnitudes of various field components, however, could be determined rapidly only by the analysis developed during the program (Section 4 and Appendix A).

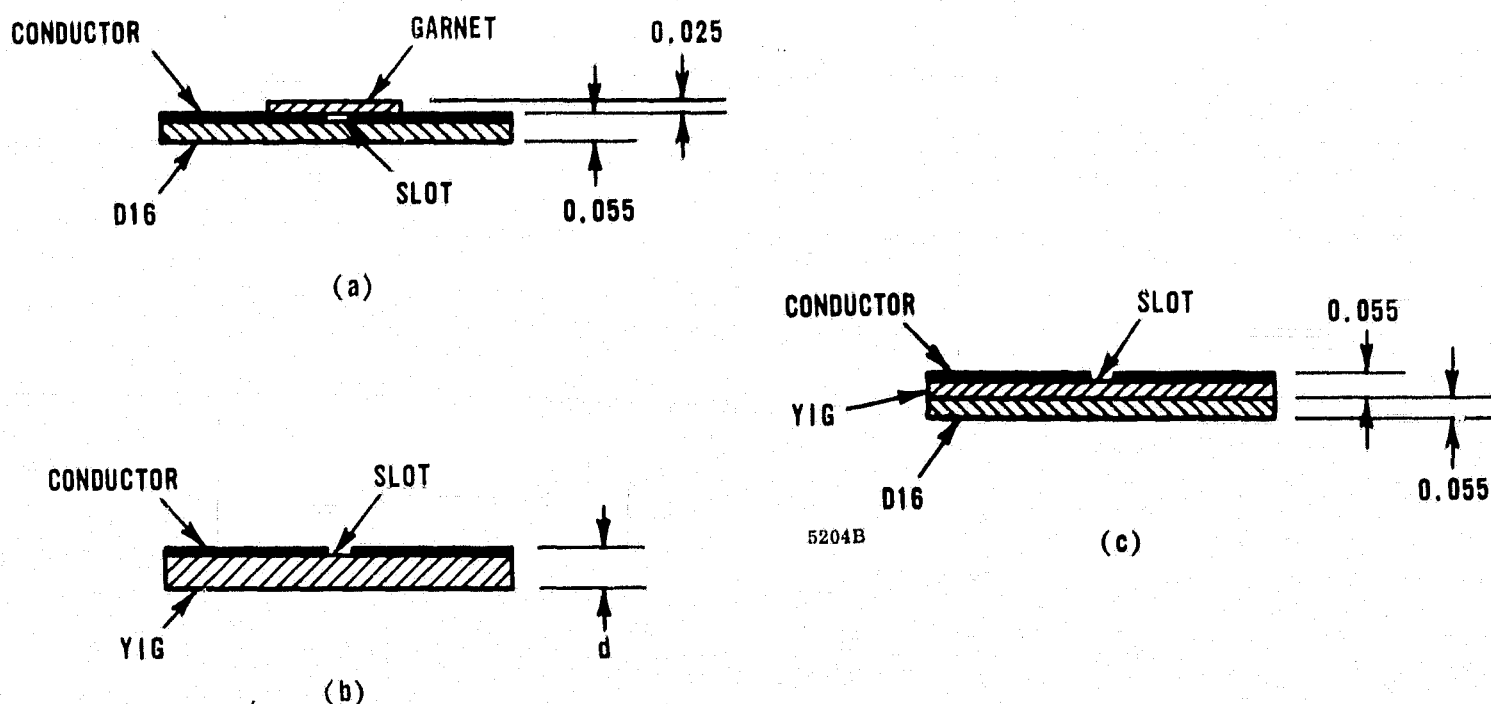


Figure 49. Slot Line Phaser Configurations

The results of this analysis were useful in gaining an understanding of experimental results and in directing experimental work and design efforts. For example, a comparison of the results with configurations (b) and (c) of Figure 49 could lead to the conclusion that there is little nonreciprocal magnetic interaction at depths greater than 0.055 inch from the slot into the substrate for this geometry. The analysis shows that the reversal in the direction of the longitudinal field occurs between 0.055 and 0.110 mils, resulting in a general cancellation of the effects of the interaction in that region. This then explains the small difference in phase shift between the all-YIG and YIG-D16 substrates. Other experiments have been performed which have agreed well with this analysis.

7.6.2 Latched Phase Shifters

The design of latched phasers is discussed in Section 6. Several approaches have been examined experimentally. A photograph of a latched slot line phaser with microstrip input and output is shown in Figure 50. This phaser is shown in cross section in Figure 51. The dimensions and performance of this design are shown in Figure 52.

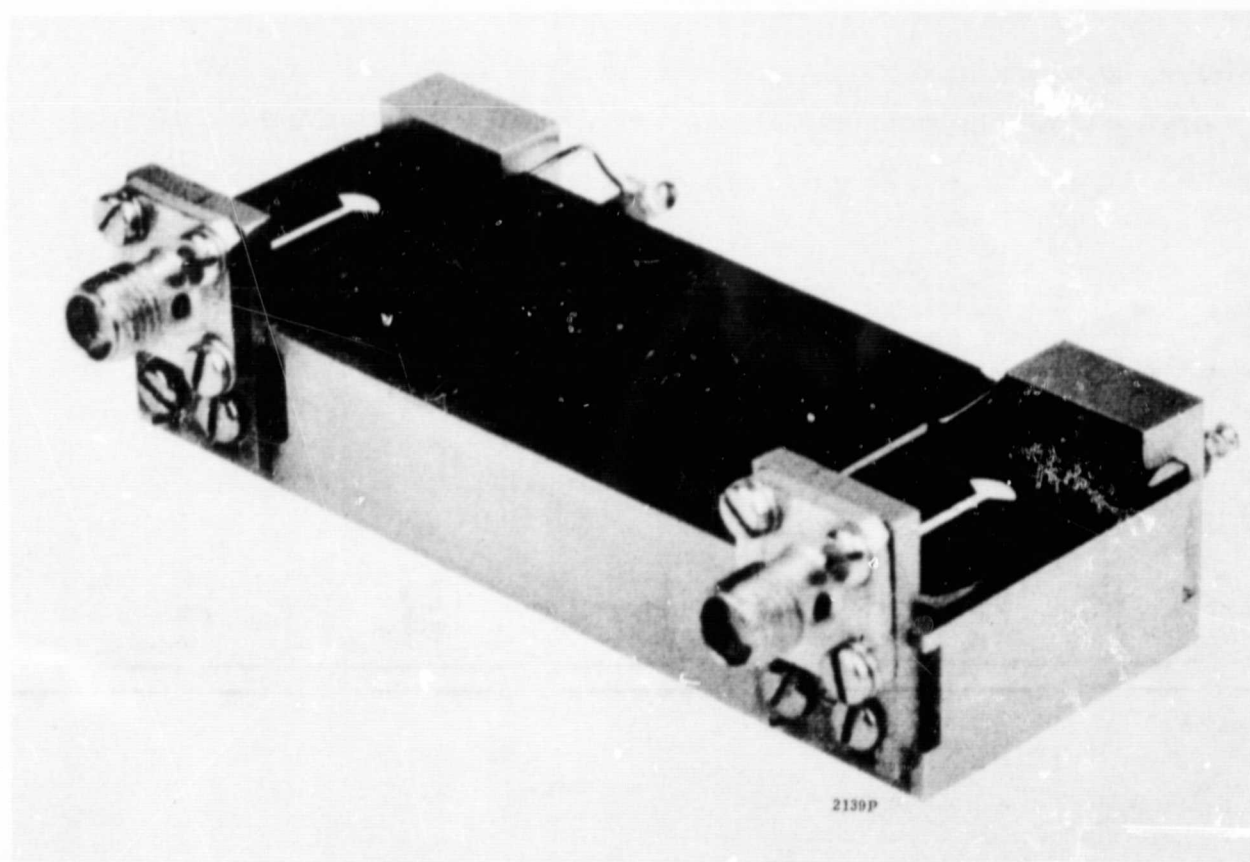


Figure 50. Latched Slot Line Phase Shifter

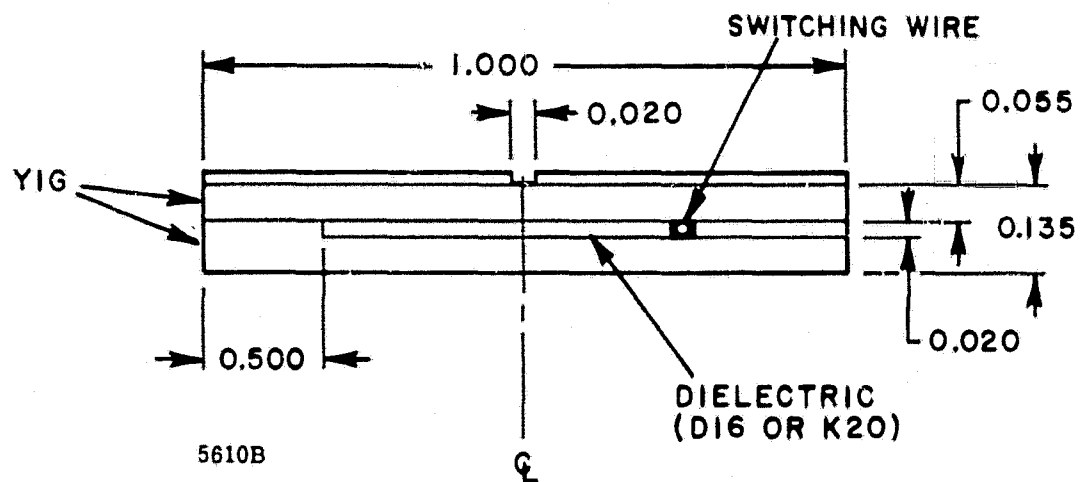


Figure 51. Latched Slot Line Phaser in Cross Section

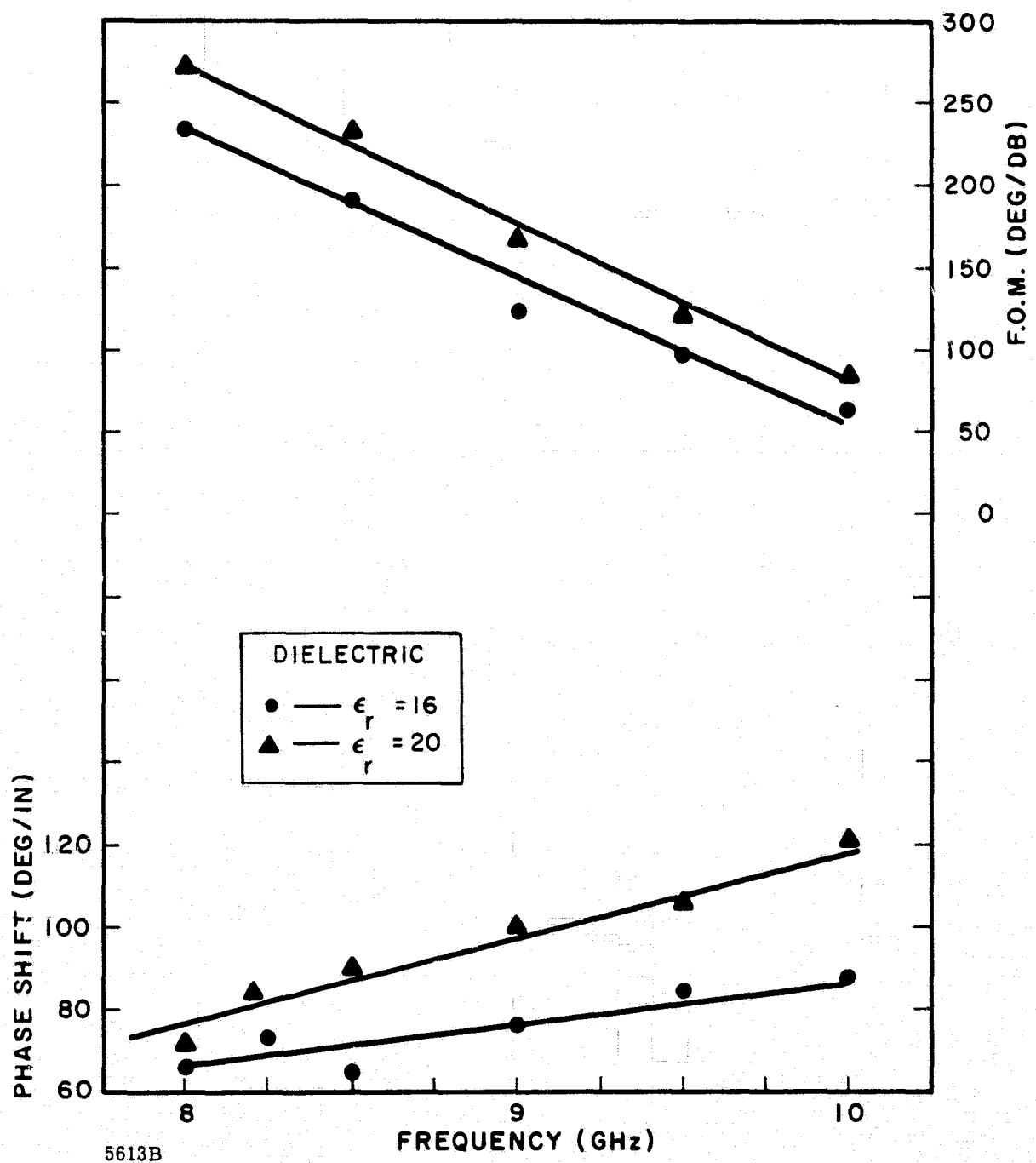


Figure 52. Performance of Phaser Shown in Figure 50

Several things are apparent from the data. The differential phase shift varies strongly with frequency and is improved by increasing the dielectric constant of the core. While the phase shift per inch increases with frequency, the figure of merit (deg/db) decreases. Note that this design could be fabricated from a solid toroid rather than the two-piece structure used. This would eliminate the gaps in the magnetic loop and improve latching performance. This design was tested with thinner substrates which gave inferior performance.

Another design approach was tested but has not been investigated in detail. This approach uses a symmetrical loading of the slot as shown in Figure 53. The slot conductors were dc-isolated in the test piece and latching current passed through the plated conductor. The phase shift was almost constant at 28 to 31 degrees from 8 to 10 GHz. While this phase shift is relatively low, the approach is well worth investigating further.

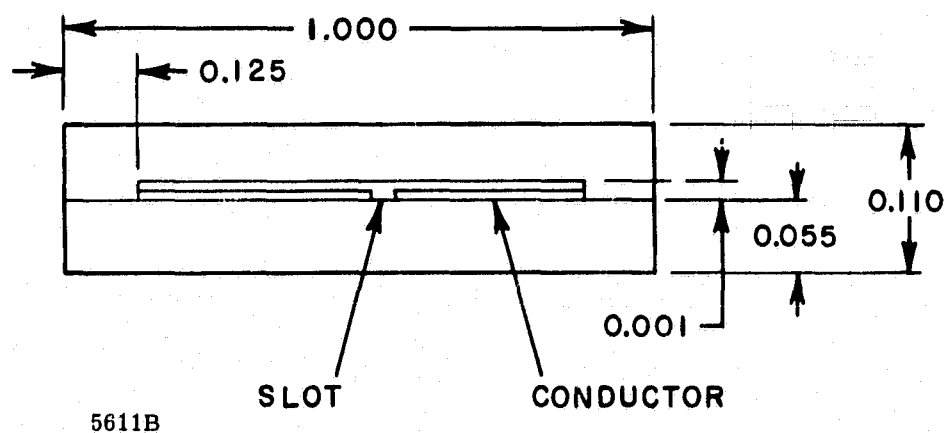


Figure 53. Symmetrically Loaded Slot Line Phaser

8. CONCLUSIONS

The work under this contract has been directed toward:

- Obtaining practical design and fabrication information for microstrip transmission line.
- Obtaining practical design information for fixed bias and latched microstrip circulators.
- Determining performance characteristics for latched and fixed bias microstrip circulators.
- Obtaining practical design information on slot transmission line.
- Development of coaxial and microstrip transitions to slot transmission line.
- Determining the feasibility of using slot transmission line for miniature phase shifters.

The following comments summarize the results of these investigations:

- Characteristics of microstrip lines such as impedance and effective wavelength can be calculated from dimensions and material parameters using techniques like the conformal mapping described by Wheeler^{2,3} or by other schemes such as numerical solution based on difference equations. Calculations of microstrip line characteristics have been programmed by Sperry Microwave for digital computation on the UNIVAC 1108. Coupling between microstrip lines can be evaluated by difference equation techniques or by using a Green's function technique recently reported by Bryant and Weiss.¹⁶
- At X band (above about 8 GHz), the initial permeability may be ignored in wavelength and impedance calculations when using substrates of YIG or materials of similar $4\pi M_s$. At lower frequencies, the initial permeability is significant and must be considered.
- In general, ohmic losses represent about 90 percent of the total microstrip loss so that conductor material, deposition methods, thickness and surface finish are the most important parameters controlling loss. A conductor (gold, silver, copper, or aluminum) 4 to 5 microns thick on a surface finish of 10 microinches is required for minimizing loss.

- Other factors remaining constant (substrate thickness, dielectric constant, etc.), the impedance level employed in module design might be chosen smaller than 50 ohms, leading to smaller losses due to the decreased ohmic losses in the wider center conductor.
- To achieve minimum losses the substrate thickness should be as large as possible consistent with avoiding surface modes.
- A number of substrate materials have mechanical and electrical parameters suitable for use in microstrip. (See Table I, Section 3). These materials provide a good range of dielectric constant.
- Fixed bias microstrip circulators may be designed with performance comparable to strip transmission line types.
- Analytical means are available to aid in the design of fixed and latched bias circulators.
- Fast relaxing ions may be used to substantially increase peak power performance with moderate increase in insertion loss. For example, by dysprosium doping, the peak power threshold of a YIG circulator was increased from 60 watts peak to 1.4 kw with an increase in loss of about 0.1 db. Thresholds as high as 50 kw are attainable with losses of about 1 db.
- Temperature effects are similar to those in strip transmission line circulators. X band microstrip circulators using YIG substrates have been operated over the -20 to -50°C temperature range with moderate changes in characteristics. Choice of a material with a more temperature-stable $4\pi M_s$ should improve the circulator temperature performance.
- Average power input to an X band microstrip circulator must presently be limited to less than 10 watts unless special provisions for cooling are made.
- A variety of magnetic biasing configurations may be used; designs with magnets below or above the substrate appear feasible. Design using shielded magnets less than 1/4-inch long have been successful in X band. The shielding would eliminate possible interactions. The variety of techniques available allows flexibility in module design.

- Latching microstrip circulators with the switching loop external to the substrate give better performance and are less difficult and expensive to fabricate than those with the switching wire in the substrate.
- Substrate thickness must be at least equal to half the junction radius to maintain good latching action with an external loop. This may cause difficulty in lower frequency circulators having larger junctions.
- The attainable bandwidth in a latched circulator will be smaller than a similar fixed bias design.
- The material used in the external portion of the magnetic loop should have a $4\pi M_s$ slightly greater than the substrate, a somewhat high coercive field and high squareness ratio. Lithium ferrite is a good external material choice for use with substrates such as YIG.
- Fixed bias, X band microstrip circulators can be operated over the -40 to $+50^\circ\text{C}$ temperature range with little change in performance characteristics by using gadolinium substituted in YIG.
- The photoetching and substrate fabricating techniques used in producing microstrip yield fixed bias circulators with repeatable performance. Adjustments to individual circuits to optimize performance are not necessary.
- Switching times for latched microstrip circulators of 3 to 4 microseconds have been demonstrated with 175 microjoules switching energy (including driver). Shorter switching times appear to be attainable.
- Latched X band, microstrip circulators have good temperature stability over the -40 to $+50^\circ\text{C}$ temperature range using YIG substrate with nickel or lithium ferrite latching plates.
- Slot transmission line has losses no greater than microstrip in X band.
- Transitions from both coax and microstrip into slot line are easily designed and fabricated. A microstrip-slot transition with a VSWR of less than 1.1 from 8 to 10 GHz has been demonstrated.
- Several design options are available for miniature ferrite phasers using slot line.

- Figures of merit similar to meander line have been achieved in X band although the volume efficiency has been somewhat poorer.
- Slot line phase shifters having the slot on one side of a dielectric-cored ferrite toroid exhibit a strong frequency variation. For example, the data of Figure 52 show a 2:1 variation in differential phase shift between 8 and 10 GHz. However, initial investigation of a symmetrically loaded slot (that is, ferrite on both sides of the slot) has given very flat frequency performance.
- A periodic loading of slot line has been proposed which should improve volume efficiency of the phase shifter.
- A theoretical analysis of slot line rf magnetic fields has been provided and is well supported by experimental evidence. This analysis allows computer calculation of the rf fields in slot line so that effects of design changes can be more readily predicted.

An examination of the above conclusions shows that the microstrip investigation has provided extensive practical information and is relatively complete in scope. The conclusions also show that the slot line investigation has similarly provided detailed analytical and experimental information regarding the design and performance of slot line and slot line phase shifters.

The phase shift performance already attained with slot line is comparable to other types of miniature phase shifters such as meander line. However, several design approaches which show promise for significant improvement were not investigated because of the limitation of time within this program. Slot line phase shifters and slot line circuitry in general have been shown to be compatible with microstrip. Based on these results, a superior miniature phase shifter which could be immediately applied to present module design is certainly a possibility with the slot line approach. Slot line also offers advantages in the design of other components such as filters, diode switches, couplers and detectors. The conclusion is that slot line should be considered as a transmission medium for microwave integrated circuits.

9. RECOMMENDATIONS

The objectives of this program have been:

- To provide practical design information for miniature microwave devices currently entering into microwave integrated circuit design.
- To investigate the performance of some of these devices.
- To investigate new and advanced techniques for the realization of components for future needs in microwave integrated circuitry.

The ultimate goal was to incorporate devices resulting from this program into a functional module to meet specific NASA requirements. The type of circuitry which has been investigated will be suitable for a broad range of space applications. One important application is that of spacecraft communications. Accordingly, a module suitable for a spacecraft-mounted phased array has been considered as a goal for component utilization. It is recommended that functional application of the developed techniques continue to be the goal of any future effort.

One of the key devices in reaching this goal is the miniature phase shifter. It is considered appropriate, therefore, that the development of a miniature phase shifter continue to be emphasized. An examination of Sections 7 and 8 of this report shows the status of slot line phase shifters to be similar to that of other types of phase shifters (e. g., meander line) some years ago. The results are promising and techniques for improving the performance appear feasible. There are also other approaches to miniature phase shifter design, such as H-guide, which offer possible advantages but have not been thoroughly studied. Therefore, it is recommended that further effort include further development of slot line phase shifters, particularly the techniques suggested in Section 6-2, and investigations of other types of phasers. A critical comparison and evaluation of the relative merits of ferrite and diode phase shifters would then be made based on the module requirements.

Slot line also has possible advantages for use in other components such as diode switches and filters as suggested in Section 4. These applications should be considered during investigation of module design.

The proposed course of investigation is considered to be of key importance in the development of microwave systems for spacecraft applications. Accordingly, Sperry Microwave Electronics Division urges the support of these recommendations.

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APPENDIX A*

This appendix presents an analytical approach to slot line yielding the magnitude of magnetic field components. Reference should be made to S. B. Cohn's earlier analytical presentation,⁸ some of which is summarized here.

Boundaries are introduced in slot line such that the problem is reduced to a rectangular waveguide problem. Thus the infinite, orthogonal sets of relatively simple waveguide modes apply rather than sets of cylindrical modes embracing all orders of Hankel junctions.

The following definitions and conventions will be used:

$$\eta = 376.7,$$

$$\delta = w/b,$$

$$w = \text{slot width},$$

$$b = \text{distance between electric walls perpendicular to Y.}$$

Subscripts x, y, and z will denote the X-, Y-, or Z-directed field component. The subscripts TE and TM denote components associated with TE and TM waveguide modes. The subscript n will refer to the particular TE or TM mode as defined in the text. The subscript 1 will denote the dielectric region of the model. For example:

$$H_x = \text{complex magnitude of the X-directed rf magnetic field,}$$

$$H_{xTE} = \text{complex magnitude of the X-directed rf magnetic field of the sum of the TE waveguide modes,}$$

$$H_{xTE_n} = \text{complex magnitude of the X-directed rf magnetic field of the nth TE mode where n is defined in the text.}$$

* This analysis was developed by Dr. S. B. Cohn under funds from this program with additional support from the U.S. Army Electronics Command and Stanford Research Institute under Contract DAAB07-68-C-0088.

Also,

$$F_n = \sqrt{1 + \left(\frac{bv}{n\lambda}\right)^2}$$

$$F_{n1} = \sqrt{1 - \left(\frac{bu}{n\lambda}\right)^2}$$

$$v = \sqrt{\left(\frac{\lambda}{\lambda'}\right)^2 - 1}$$

$$u = \sqrt{\epsilon_r - \left(\frac{\lambda}{\lambda'}\right)^2}$$

λ = free space wavelength

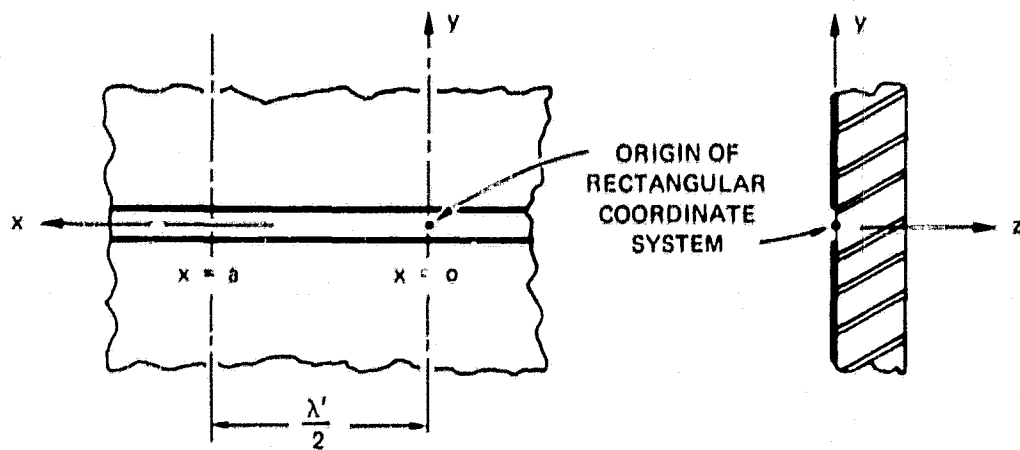
λ' = slot line wavelength

b is associated with the boundaries in the model. For the electric wall case, boundaries are located at $y = \pm b/2$ and for the magnetic wall case, boundaries are located at $\hat{y} = \pm b$.

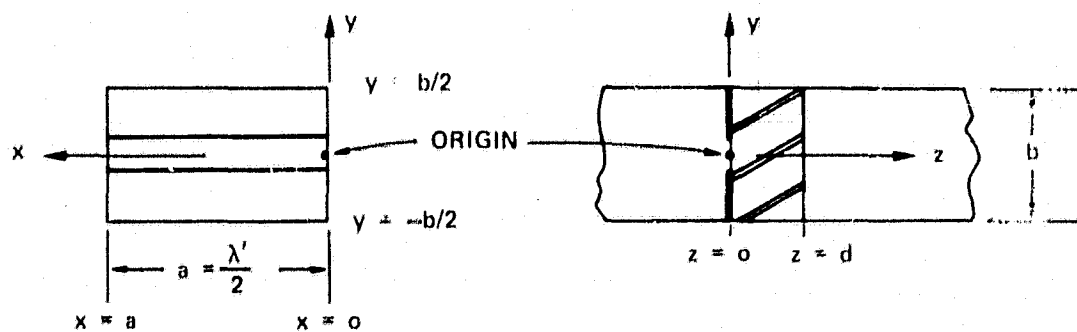
Conversion of slot line into a waveguide problem is illustrated in Figure 1. First, assume that waves of the dominant slot line mode and of equal amplitude are traveling in the $+x$ and $-x$ directions. Planes transverse to x then exist at $\lambda'/2$ intervals (λ' = slot wavelength) where the transverse E and normal H cancel to zero. Infinite conducting planes can be introduced at these points without disturbing the fields between the planes. Let such planes occur at $x = 0$ and $x = \lambda'/2$. The section of slot line between the planes supports a resonant slot wave with no loss if the dielectric and conducting walls are assumed dissipationless.

Next, conducting walls are inserted in planes parallel to the slot and perpendicular to the substrate at $y = \pm b/2$. The region separated from the original infinite space has the rectangular waveguide boundary shown in Figure 1(b). Then walls at $y = \pm b/2$ will have negligible effect for b sufficiently large (typically 1 inch at 3 GHz), yet they allow the use of the rectangular waveguide mode sets. Magnetic wall boundaries may also be used.

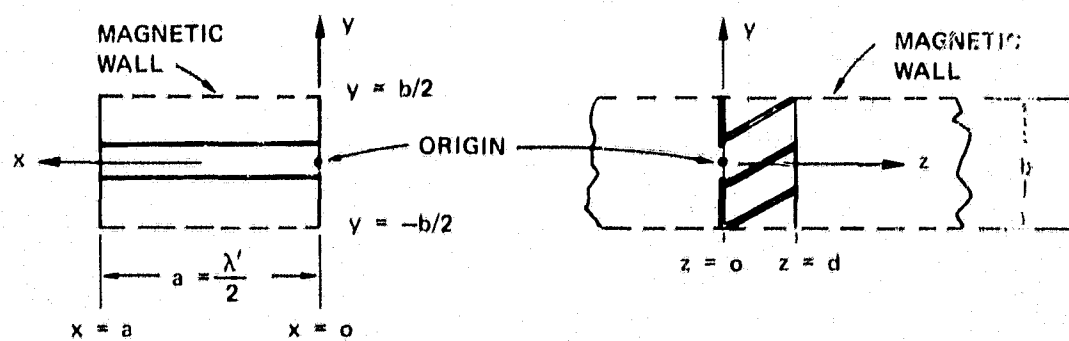
The configuration of Figure 1(b) is a capacitive iris in a rectangular waveguide, with air and dielectric regions as indicated. All waveguide modes must have the $\lambda'/2$ variation of the slot wave in the x direction. Also, because of symmetry, all modes must have an E -field maximum at the center of the slot. Therefore, the full set of modes satisfying the boundary conditions are $TE_{1,2N}$, $n \geq 0$, and $TM_{1,2N}$, $N \geq 1$ for N an integer.



(a) INSERTION OF TRANSVERSE CONDUCTING PLANES AT $x = 0$ AND a



(b) INSERTION OF CONDUCTING PLANES AT $y = \pm b/2$



(c) INSERTION OF MAGNETIC WALLS AT $y = \pm b/2$

Figure 1. Development of Waveguide Models for Slot Line Solution

The T_{10} and all higher modes are cut off in the air regions since $\lambda' < \lambda$ and thus $a < \lambda/2$. In the dielectric region, the TE_{10} wave is propagating, and the first few higher modes will propagate or all higher modes will be cut off, depending on the size of b . The energy is trapped near the slot since the air regions are cut off. The amplitude of each mode in each region must be such that the boundary conditions in the iris plane at $z = 0$ will be met and all field components on either side of the dielectric-air interface will be matched when the full set of modes are superimposed.

Using the above approach, the following equations may be derived² at $z = 0$, $y = a/2$.

$$E_y|_{z=0} = R_0 + \sum_{n>0} R_N \cos \frac{2\pi ny}{b} \quad (1)$$

$$H_x|_{z=0} = -Y_{i0} R_0 - \sum_{n>0} Y_{in} R_n \cos \frac{2\pi ny}{b} \quad (2)$$

where

$$Y_{i0} = -\left(\frac{H_x}{E_y}\right)_{TE_{10}} \quad (3)$$

$$Y_{in} = -\left[\frac{(H_x)_{TE_{1,2n}} + (H_x)_{TM_{1,2n}}}{(E_y)_{TE_{1,2n}} + (E_y)_{TM_{1,2n}}}\right]$$

Assuming that only the TE_{10} mode exists in the slot,

$$E_y = \begin{cases} C_0, & |y| \leq w/2 \\ 0, & w/2 < |y| \leq b/2 \end{cases} \quad (4)$$

E_y in Eq. (1) has the form of a Fourier series. Let this series equal E_y of Eq. (4).

Then,

$$R_o = C_o \delta, \quad R_n = 2C_o \delta \frac{\sin \pi n \delta}{\pi n \delta}, \quad \delta = \frac{w}{b} \quad (5)$$

Note that this is an approximation since E_y is constant in the slot and $E_z \neq 0$ although $E_y \gg E_z$.

Then,

$$E_y|_{z=0} = C_o \delta + 2C_o \delta \sum_{n>0} \frac{\sin \pi n \delta}{\pi n \delta} \cdot \cos \frac{2\pi n y}{b} \quad (6)$$

In the slot,

$$E_y = C_o = V_o/w, \quad V_o = \text{voltage across the slot} \quad (7)$$

$$\therefore E_y|_{z=0} = \frac{V_o}{b} \left(1 + 2 \sum_{n>0} \frac{\sin \pi n \delta}{\pi n \delta} \cdot \cos \frac{2\pi n y}{b} \right) \quad (8)$$

and for the TE_{10} mode

$$E_{y0}|_{z=0} = \frac{V_o}{b} \quad (9)$$

For $n > 0$, we have superimposed $TE_{1,2n}$ and $TM_{1,2n}$ modes with

$$D_n = \frac{(E_y)_{TE_{1,2n}}}{(E_y)_{TM_{1,2n}}} \Big|_{z=0}$$

when

$$\left[(E_x)_{TE_{1,2n}} + (E_x)_{TM_{1,2n}} \right]_{z=0} = 0 \quad (10)$$

The last restriction is required to meet the boundary condition $E_x=0$ at $z=0$. From field component values for the TE and TM modes,¹⁷

$$D_n = \left(\frac{b}{2an} \right)^2 \quad (11)$$

Now,

$$E_{yn}|_{z=0} = (E_{yTE_{1,2n}} + E_{yTM_{1,2n}})|_{z=0} = \frac{2V_o}{b} \left(\frac{\sin \pi n \delta}{\pi n \delta} \right) \left(\cos \frac{2\pi n y}{b} \right) \quad (12)$$

where $n > 0$ and

$$E_{yn}|_{z=0} = \left[E_{yTE_n} + \frac{E_{yTE_n}}{D_n} \right]_{z=0} = E_{yTE_n}|_{z=0} \left(\frac{D_n + 1}{D_n} \right) = \left[D_n E_{yTM_n} + E_{yTM_n} \right]_{z=0} = E_{yTM_n}|_{z=0} (D_n + 1) \quad (13)$$

$$\therefore E_{yTE_n}|_{z=0} = \frac{D_n}{1 + D_n} E_{yn}|_{z=0}, \quad E_{yTM_n}|_{z=0} = \frac{E_{yn}|_{z=0}}{1 + D_n} \quad (14)$$

so that

$$E_{yTE_n}|_{z=0} = \frac{V_o}{b} \cdot \frac{2D_n}{1 + D_n} \cdot \frac{\sin \pi n \delta}{\pi n \delta} \cdot \cos \frac{2\pi n y}{b} \quad (15)$$

$$E_{yTM_n}|_{z=0} = \frac{V_o}{b} \cdot \frac{2}{1 + D_n} \cdot \frac{\sin \pi n \delta}{\pi n \delta} \cdot \cos \frac{2\pi n y}{b}$$

For $n > 0$

and

$$E_{y0}|_{z=0} = E_{yTE_0}|_{z=0} = \frac{V_o}{b} \quad (16)$$

For $n = 0$

The value of E_y at $z \neq 0$ will be established in terms of $E_y|_{z=0}$ by means of a transmission line analogy.

On the transmission line shown in Figure 2, the following equations hold:

$$\begin{bmatrix} V_0 \\ I_0 \end{bmatrix} = \begin{bmatrix} \cosh \gamma z & \frac{1}{Y_0} \sinh \gamma z \\ Y_0 \sinh \gamma z & \cosh \gamma z \end{bmatrix} \begin{bmatrix} V \\ I \end{bmatrix} \quad (17)$$

and

$$\begin{bmatrix} V \\ I \end{bmatrix} = \begin{bmatrix} \cosh \gamma z - \frac{1}{Y_0} \sinh \gamma z \\ -Y_0 \sinh \gamma z & \cosh \gamma z \end{bmatrix} \begin{bmatrix} V_0 \\ I_0 \end{bmatrix} \quad (18)$$

so that if γ , Y_0 , V_0 and I_0 are known, V and I at an arbitrary z can be calculated.

Let $I_0/V_0 = Y_i$ and

$$\begin{bmatrix} V \\ I \end{bmatrix} = \begin{bmatrix} \cosh \gamma z - \frac{Y_i}{Y_0} \sinh \gamma z \\ -Y_0 \sinh \gamma z & Y_i \cosh \gamma z \end{bmatrix} \begin{bmatrix} V_0 \\ V_0 \end{bmatrix} \quad (19)$$

so that V and I are expressed in terms of V_0 .

For a wave in the $+z$ direction, the following analogy may be used:

<u>Field Parameters</u>		<u>Transmission Line Parameters</u>
E_y	analogous to	V
$-H_x$	analogous to	I
$y = -\frac{H_x}{E_y}$	analogous to	Y
γ_n	analogous to	γ

where

$$y = \text{wave admittance} = \frac{2b}{a} Y'$$

$Y' = \text{voltage-power definition of admittance in rectangular guide.}$

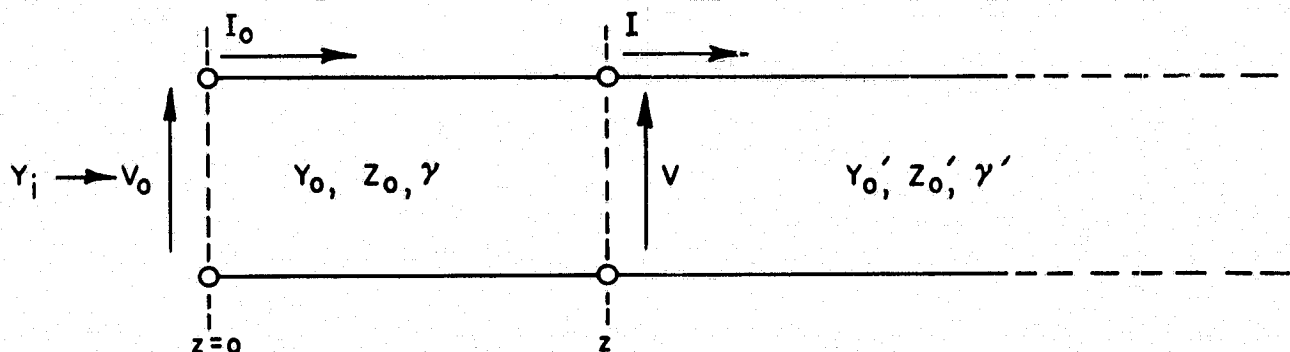


Figure 2. Transmission Line Model

∴ for the dielectric region $0 \leq z \leq d$

$$\begin{aligned} E_{yTE_n} &= \left[\cosh \gamma_{n1} z - \frac{Y_{in}}{Y_{o1n}} \sinh \gamma_{n1} z \right] \left(E_{yTE_n}|_{z=0} \right) \\ H_{xTE_n} &= \left[-Y_{o1n} \sinh \gamma_{n1} z + Y_{in} \cosh \gamma_{n1} z \right] \left(-\frac{2b}{a} E_{yTE_n}|_{z=0} \right) \end{aligned} \quad (20)$$

where the subscript n refers to the mode and the subscript 1 refers to parameters in the dielectric region. The same matrix relates

$$E_{yTM_n} \text{ and } H_{xTM_n} \text{ for TM modes to } E_{yTM_n}|_{z=0}$$

where the admittances are defined for each mode number and type as below.

For TE modes

$$\begin{aligned} Y_{o1n} &= \frac{a \gamma_{n1}}{j 2 b \eta_1 k_1} \\ Y_{in} &= Y_{o1n} \coth \left(\gamma_{n1} d + \coth^{-1} \frac{Y_{on}}{Y_{o1n}} \right) \\ Y_{on} &= \frac{a \gamma_n}{j 2 b \eta k} \end{aligned} \quad (21)$$

For TM modes

$$\begin{aligned} Y_{o1n} &= \frac{j a k_1}{2 b \eta_1 k_1} \\ Y_{in} &= Y_{o1n} \tanh \left(\gamma_{n1} d + \tanh^{-1} \frac{Y_{on}}{Y_{o1n}} \right) \\ Y_{on} &= \frac{j a k}{2 b \eta \gamma_n} \end{aligned} \quad (22)$$

where

$$\eta = \sqrt{\mu_0/\epsilon_0} = 376.7 \text{ ohms}, \quad \eta_1 = \eta/\sqrt{\epsilon_r} \quad (23)$$

$$k = 2\pi/\lambda, \quad k_1 = \frac{2\pi\sqrt{\epsilon_r}}{\lambda} \quad (24)$$

$$\gamma_{n1} = \sqrt{\left(\frac{2\pi n}{b}\right)^2 + \left(\frac{\pi}{a}\right)^2 - \left(\frac{2\pi}{\lambda}\right)^2 \epsilon_r} = \frac{2\pi n}{b} \sqrt{1 - \left(\frac{b}{n\lambda_{g1}}\right)^2} \quad (25)$$

$$\gamma_n = \frac{2\pi n}{b} \sqrt{1 + \left(\frac{b\gamma_{n1}}{2\pi n}\right)^2} \quad (26)$$

In the air region $z \geq d$ the input admittance Y_{in} equals the characteristic impedance Y_{on} so that the terms in brackets in Eq. (20) become

$$e^{-\gamma(z-d)}$$

and equations with $z = d$ as the input point can be written:

$$\begin{aligned} E_{yTE_n} &= E_{yTE_n|z=d} \cdot e^{-\gamma(z-d)} \\ H_{xTE_n} &= Y_{oTE_n} \frac{2a}{b} \cdot E_{yTE_n|z=d} \cdot e^{-\gamma(z-d)} \end{aligned} \quad (27)$$

For $z \geq d$

Again, similar equations apply to the TM modes.

$$\text{Let} \quad E_y(x, y, z) = E_y \cdot e^{j(\omega t - \beta' x)}$$

and

$$H_x(x, y, z) = H_x \cdot e^{j(\omega t - \beta' x)} \quad (28)$$

where

$$\beta' = 2\pi/\lambda'$$

λ' = slot line wavelength

$$E_y = E_{yTE} + E_{yTM}$$

$$H_x = H_{xTE} + H_{xTM}$$

and E_y and H_x are functions of y and z .

The electric walls at $y = \pm b/2$ may be replaced by magnetic walls at $x = \pm b$, eliminating the TE_{10} mode and giving

$$E_y = \sum_n (E_{yTE_n} + E_{yTM_n}) \quad (29)$$

where $n = 1/2, 3/2, 5/2, 7/2, \dots$ because of the change in boundaries.

Using Eq. (15) and (20) (for $0 \leq z \leq d$)

$$E_{yTE_n} = \frac{2V_o D_n}{b(1+D_n)} \cdot \frac{\sin \pi n \delta}{\pi n \delta} \cdot \cos \frac{2\pi n y}{b} \cdot \left(\cosh \gamma_{n1} z - \frac{Y_{iTE_n}}{Y_{o1TE_n}} \sinh \gamma_{n1} z \right) \quad (30)$$

and

$$E_{yTM_n} = \frac{2V_o}{b(1+D_n)} \cdot \frac{\sin \pi n \delta}{\pi n \delta} \cdot \cos \frac{2\pi n y}{b} \cdot \left(\cosh \gamma_{n1} z - \frac{Y_{iTM_n}}{Y_{o1TM_n}} \sinh \gamma_{n1} z \right) \quad (31)$$

From Eq. (29), (30), and (31)

$$E_y = \frac{2V_o}{b} \sum_n \frac{\sin \pi n \delta}{\pi n \delta} \cdot \cos \frac{2\pi n y}{b} \left[\cosh \gamma_{n1} z - \left(D_n \frac{Y_{iTE_n}}{Y_{o1TE_n}} + \frac{Y_{iTM_n}}{Y_{o1TM_n}} \right) \frac{\sinh \gamma_{n1} z}{1+D_n} \right] \quad (32)$$

$n = 1/2, 3/2, 5/2, \dots$

Similarly,

$$H_x = -\frac{2V_o}{b} \cdot \frac{2b}{a} \sum_n \frac{\sin \pi n \delta}{\pi n \delta} \cdot \cos \frac{2\pi n y}{b} \cdot \frac{1}{(1+D_n)} \cdot \left[\left(D_n Y_{iTE_n} + Y_{iTM_n} \right) \cosh \gamma_{n1} z - \left(D_n Y_{o1TE_n} + Y_{o1TM_n} \right) \sinh \gamma_{n1} z \right] \quad (33)$$

Eq. (32) and (33) define E_y and H_x as functions of y and z throughout the dielectric region.

$$H_z = H_z^+ + H_z^- = \sum_n H_{zTE_n} \quad (34)$$

where the superscripts denote propagation in the positive and negative z directions, respectively. Since the TM modes have no H_z component, only the TE_n modes must be included. Now:

$$H_{xTE_n}^\pm = \mp \frac{\gamma_{n1}}{k_{cn1}^2} \cdot \frac{\partial H_{zTE_n}^\pm}{\partial x} = \pm j \frac{\beta' \gamma_{n1} H_{zTE_n}^\pm}{k_{cn1}^2} \quad (35)^{17}$$

where

$$\gamma_{n1}^+ = \gamma_{n1} = -\gamma_{n1}^-$$

$$\therefore H_z = \sum \frac{-jk_{cn1}^2}{\gamma_{n1}\beta'} H_{xTE_n}^+ - H_{xTE_n}^- \quad (36)$$

where

$$\beta' = 2\pi/\lambda' \quad (37)$$

$$k_{cn1}^2 = \left(\frac{2\pi n}{b}\right)^2 \left(1 + \left(\frac{b}{2an}\right)^2\right) \quad (38)$$

$$= \left(\frac{2\pi n}{b}\right)^2 (1 + D_n)$$

$$a = \lambda'/2 \text{ (defined in setting up model)} \quad (39)$$

$$\gamma_{n1} = \frac{2\pi n}{b} \sqrt{1 - \left(\frac{b}{\eta\lambda_{g1}}\right)^2} \quad (40)$$

Then

$$\begin{aligned} H_z &= \sum_n -j \left(\frac{2\pi n}{b}\right)^2 (1 + D_n) \cdot \frac{1}{\left(\frac{2\pi n}{b}\right) F_{n1}} \cdot \frac{\lambda'}{2} \cdot \left(H_{xTE_n}^+ - H_{xTE_n}^-\right) \\ &= -j \frac{\lambda'}{b} \sum_n \frac{n(1 + D_n)}{F_{n1}} \left(H_{xTE_n}^+ - H_{xTE_n}^-\right) \end{aligned} \quad (41)$$

The difference $H_{xTE_n}^+ - H_{xTE_n}^-$ can be obtained from Eq. (33). First, the TM_n modes are eliminated by setting $Y_{iTM_n} = Y_{01TM_n} = 0$. This yields $H_{xTE_n} = H_{xTE_n}^+ + H_{xTE_n}^-$. Noting that $H_{xTE_n}^+ \propto e^{\gamma_{n1}z}$ and $H_{xTE_n}^- \propto e^{-\gamma_{n1}z}$ and expressing the hyperbolic functions in exponential form, the following expression is derived:

$$H_{xTE_n}^+ - H_{xTE_n}^- = -\frac{2V_o}{b} \cdot \frac{2b}{a} \cdot \frac{\sin(\pi n \delta)}{\pi n \delta} \cdot \frac{D_n}{(1 + D_n)} \cdot \cos\left(\frac{2\pi n y}{b}\right) \cdot \left[-Y_{iTE_n} \sinh \gamma_{n1} z + Y_{01TE_n} \cosh \gamma_{n1} z \right] \quad (42)$$

Substituting Eq. (42) into the parentheses of Eq. (41),

$$H_z = +j \frac{\lambda'}{b} \cdot \frac{2V_o}{b} \cdot \frac{2b}{a} \sum_n \frac{n D_n}{F_{n1}} \frac{\sin \pi n \delta}{\pi n \delta} \cdot \cos \frac{2\pi n y}{b} \cdot \left[-Y_{iTE_n} \sinh \gamma_{n1} z + Y_{01TE_n} \cosh \gamma_{n1} z \right] \quad (43)$$

From Eq. (21)

$$Y_{01TE_n} = \frac{a \gamma_{n1}}{j 2 b \eta_1 k_1} = \frac{a}{j 2 b} \cdot \frac{2 \pi n F_{n1}}{b} \cdot \frac{\sqrt{\epsilon_r}}{\eta} \cdot \frac{\lambda}{2 \pi \sqrt{\epsilon_r}} = \frac{a}{j 2 b} \cdot \frac{n \lambda}{\eta b} \cdot F_{n1} \quad (44)$$

$$Y_{iTE_n} = Y_{01TE_n} \coth q_n$$

where $q_n = \alpha_{n1} d + \coth^{-1} \frac{\gamma_n}{\gamma_{n1}}$

$$= \frac{2 \pi n d F_{n1}}{b} + \coth^{-1} \left(\frac{F_n}{F_{n1}} \right)$$

and $F_n = \sqrt{1 + \left(\frac{b \gamma}{2 \pi n} \right)^2}$

So that, for $0 \leq z \leq d$

$$H_z = \frac{2V_o}{\eta b} \cdot \frac{\lambda}{\lambda'} \sum_n \frac{\sin \pi n \delta}{\pi n \delta} \cdot \cos \left(\frac{2\pi n y}{b} \right) \cdot \left[-\coth q_n \cdot \sinh \gamma_{n1} z + \cosh \gamma_{n1} z \right] \quad (45)$$

$n = 1/2, 3/2, 5/2, \dots$

Eq. (33) may be simplified by factoring out Y_{01TE_n} as expressed in Eq. (44).

Then,

$$H_x = \sum_n -\frac{2V_o}{b} \cdot \frac{2b}{a} \cdot \frac{a}{j2b} \cdot \frac{n\lambda}{\eta b} \cdot F_{n1} \cdot \frac{\sin \pi n \delta}{\pi n \delta} \cdot \frac{1}{1 + D_n} \cdot \cos \frac{2\pi n y}{b} \left[\left(D_n \frac{Y_{iTE_n}}{Y_{01TE_n}} + \frac{Y_{iTM_n}}{Y_{01TM_n}} \cdot \frac{Y_{01TM_n}}{Y_{01TE_n}} \right) \cdot \cosh \gamma_{n1} z - \left(D_n + \frac{Y_{01TM_n}}{Y_{01TE_n}} \right) \sinh \gamma_{n1} z \right] \quad (46)$$

But

$$\begin{aligned} \frac{Y_{01TM_n}}{Y_{01TE_n}} &= j \frac{ak_1}{2b\eta_1\gamma_{n1}} \cdot \frac{j2b\eta_1k_1}{a\gamma_{n1}} = -\frac{k_1^2}{\gamma_{n1}^2} \\ &= -\left(\frac{2\pi}{\lambda}\right)^2 \epsilon_r \cdot \frac{b^2}{(2\pi n)^2 F_{n1}^2} = \frac{\epsilon_r D_n}{F_{n1}^2} \cdot \left(\frac{\lambda'}{\lambda}\right)^2 \end{aligned} \quad (47)$$

(since $2a = \lambda'$ from the model). Also,

$$\frac{Y_{iTE_n}}{Y_{01TE_n}} = \coth q_n, \quad \frac{Y_{iTM_n}}{Y_{01TM_n}} = \tanh r_n \quad (48)$$

where

$$r_n = \gamma_1 d + \tanh^{-1} \frac{\gamma_{n1}}{\epsilon_r \gamma_n} = \frac{2\pi n d F_{n1}}{b} + \tanh^{-1} \left(\frac{F_{n1}}{\epsilon_r F_n} \right)$$

and

$$H_x = j \frac{2V_o}{\eta b} \cdot \left(\frac{\lambda'}{\lambda}\right)^2 \cdot \frac{b}{\lambda} \sum_n \frac{F_{n1}}{n} \cdot \frac{\sin \pi n \delta}{\pi n \delta} \cdot \frac{\cos \frac{2\pi n y}{b}}{1 + D_n} \cdot \left\{ \left(\coth q_n - \frac{\epsilon_r}{F_{n1}^2} \left(\frac{\lambda'}{\lambda}\right)^2 \tanh r_n \right) \cosh \gamma_{n1} z - \left(1 - \frac{\epsilon_r}{F_{n1}^2} \left(\frac{\lambda'}{\lambda}\right)^2 \right) \sinh \gamma_{n1} z \right\} \quad (49)$$

$$n = 1/2, 3/2, 5/2, \dots$$

Eq. (49) can be further simplified by factoring $1/F_n^2$ from the braces. By noting the definition of D_n in Eq. (11) and that $2a = \lambda'$ in the model and the definition of F_{n1} on page A-2, we have for $0 \leq z \leq d$

$$H_x = j \frac{2V_o}{\eta b} \left(\frac{\lambda}{\lambda'}\right)^2 \frac{b}{\lambda} \sum_n \frac{1}{nF_{n1}} \cdot \frac{\sin \pi n \delta}{\pi n \delta} \cdot \cos \frac{2\pi n y}{b} \cdot \left\{ \left[\frac{F_{n1}^2 \coth q_n - \epsilon_r (\lambda'/\lambda)^2 \tanh r_n}{1 + (b/n\lambda')^2} \right] \cosh \gamma_{n1} z - \left[1 - \epsilon_r \left(\frac{\lambda'}{\lambda}\right)^2 \right] \sinh \gamma_{n1} z \right\} \quad (50)$$

$$n = 1/2, 3/2, 5/2, \dots$$

While further cancellation of terms is possible, the forms of these equations are more easily adaptable to computerized solution.

For solutions on the air side of the slot ($Z \leq 0$), Eq. (20) may be used by using the absolute value of Z and reversing the sign of the X-directed components. This is simply a transformation of the coordinate system to fit this case.

Also in this region

$$d = 0, \epsilon_r = 1 \quad (51)$$

and

F_{n1} becomes F_n ,

γ_{n1} becomes γ_n , and

$\coth q_n = \tanh r_n = 1$.

Eq. (50) then becomes

$$H_x = -j \frac{2V_o}{\eta b} \cdot \left(\frac{\lambda}{\lambda'}\right)^2 \frac{b}{\lambda} \sum_n \frac{1}{nF_n} \cdot \frac{\sin \pi n \delta}{\pi n \delta} \cdot \frac{\cos \frac{2\pi n y}{b}}{1 + D_n} \cdot \left\{ \left[1 - \left(\frac{\lambda'}{\lambda}\right)^2 \right] \cosh \gamma_n |Z| - \left[1 - \left(\frac{\lambda'}{\lambda}\right)^2 \right] \sinh \gamma_n |Z| \right\} \quad (52)$$

Then,

$$H_x = j \frac{2V_o}{\eta b} \cdot \left(\frac{\lambda}{\lambda'}\right)^2 \frac{b}{\lambda} \sum_n \frac{1}{nF_n} \cdot \frac{\sin \pi n \delta}{\pi n \delta} \cdot \frac{\cos \frac{2\pi n y}{b}}{1 + D_n} \cdot \left(1 - \left(\frac{\lambda'}{\lambda}\right)^2\right) \left(\cosh \gamma_n |Z| - \sinh \gamma_n |Z|\right) \quad (53)$$

Now, $\cosh \gamma_n |Z| - \sinh \gamma_n |Z| = e^{-\gamma_n |Z|}$

So that, for air side of slot $Z \geq 0$

$$H_x = -j \frac{2V_o}{\eta b} \left(\frac{\lambda}{\lambda'}\right)^2 \frac{b}{\lambda} \sum_n \frac{\sin \pi n \delta}{\pi n \delta} \cdot \frac{\cos \frac{2\pi n y}{b}}{1 + D_n} \cdot \left(\frac{1 - \left(\frac{\lambda'}{\lambda}\right)^2}{nF_n}\right) \left(e^{-\gamma_n |Z|}\right) \quad (54)$$

Similarly, Eq. (45) becomes

$$H_z = \frac{2V_o}{\eta b} \cdot \frac{\lambda}{\lambda'} \sum_n \frac{\sin \pi n \delta}{\pi n \delta} \cdot \cos \frac{2\pi n y}{b} \cdot e^{-\gamma_n Z} \quad (55)$$

$n = 1/2, 3/2, 5/2, \dots$

For solutions on air side of the dielectric

$$z \geq d, \epsilon_r = 0 \quad (56)$$

Then

$$H_x = \sum_n H_{xn}|_{z=d} e^{-\gamma_n(z-d)}$$

and

$$H_z = \sum_n H_{zn}|_{z=d} e^{-\gamma_n(z-d)}$$

and

$$H_{xn}|_{z=d}$$

and

$$H_{zn}|_{z=d}$$

are obtained by evaluating Eq. (45) and (49) with $z = d$.

∴ For air side of dielectric $z \geq d$

$$H_x = \frac{2V_o}{\eta b} \left(\frac{\lambda}{\lambda'}\right)^2 \frac{b}{\lambda} \sum_n \frac{\sin \pi n \delta}{\pi n \delta} \cdot \cos \frac{2\pi n y}{b} \cdot \frac{1}{n F_n} \left[\left(\frac{F_{n1}^2 \coth q_n - \epsilon_r (\lambda'/\lambda)^2 \tanh r_n}{1 + (b/n\lambda')^2} \right) \cosh \gamma_{n1} d - \left(1 - \epsilon_r \left(\frac{\lambda'}{\lambda}\right)^2 \right) \sinh \gamma_{n1} d \right] e^{-\gamma_n(z-d)} \quad (57)$$

Similarly, for $z \geq d$

$$H_z = \frac{2V_o}{\eta b} \cdot \frac{\lambda}{\lambda'} \sum_n \frac{\sin \pi n \delta}{\pi n \delta} \cdot \cos \frac{2\pi n y}{b} \cdot \left[\cosh \gamma_{n1} d - \coth q_n \sinh \gamma_{n1} d \right] e^{-\gamma_n(z-d)}$$

$n = 1/2, 3/2, 5/2, \dots$