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HOLOGRAPHIC OPTICAL DATA PROCESSING

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16. ABSTRACT

To process optical data is to perform mathematical operations on optical inputs. This paper describes the holographic technique and its application to optical subtraction and pattern recognition (matched filtering). Also, experimental results that are presented verify the theory and the appendixes show the Fourier transformation operator property of a lens and input translation invariance of the matched filtered output.

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HOLOGRAPHIC OPTICAL DATA PROCESSING

SUMMARY

To process optical data is to perform mathematical operations, such as addition and multiplication on optical inputs. Using holography, these operations may be performed on optical inputs having complex amplitudes. This report presents the theory and techniques necessary for the application of holography to optical subtraction and pattern recognition.

The parameters affecting the successful recording of a hologram are discussed in detail. These include the photographic process, the coherence of the light source, the vibrational stability of the optical system, and the resolving power of the recording medium.

In optical subtraction it is shown how, by double exposure and the introduction of a 180-degree phase delay, a wave front may be subtracted from another.

In pattern recognition, it is shown how a matched filter may be synthesized by holographic methods. Graphs of the sensitivity of the filtered output power to rotation of the input transparency are given. The appendixes show mathematically that a lens displays the Fourier spectrum of an input in its front focal plane on its back focal plane and how the filtered output is input translation invariant.

INTRODUCTION TO HOLOGRAPHY

Definition

Holography is a method by which a highly complex wave front can be recorded and later reconstructed with identical phase and amplitude from the recording medium. The recording is accomplished by photographing the interference pattern created by a simple wave front, normally plane or spherical, interfering with the wave front to be recorded (Fig. 1). The recorded interference pattern is unique in that only a wave front of identical phase and amplitude could interfere with the same simple wave front to recreate this pattern. This simple wave front is usually called the "reference wave front"; the wave front to be recorded is spoken of as the "signal wave front."

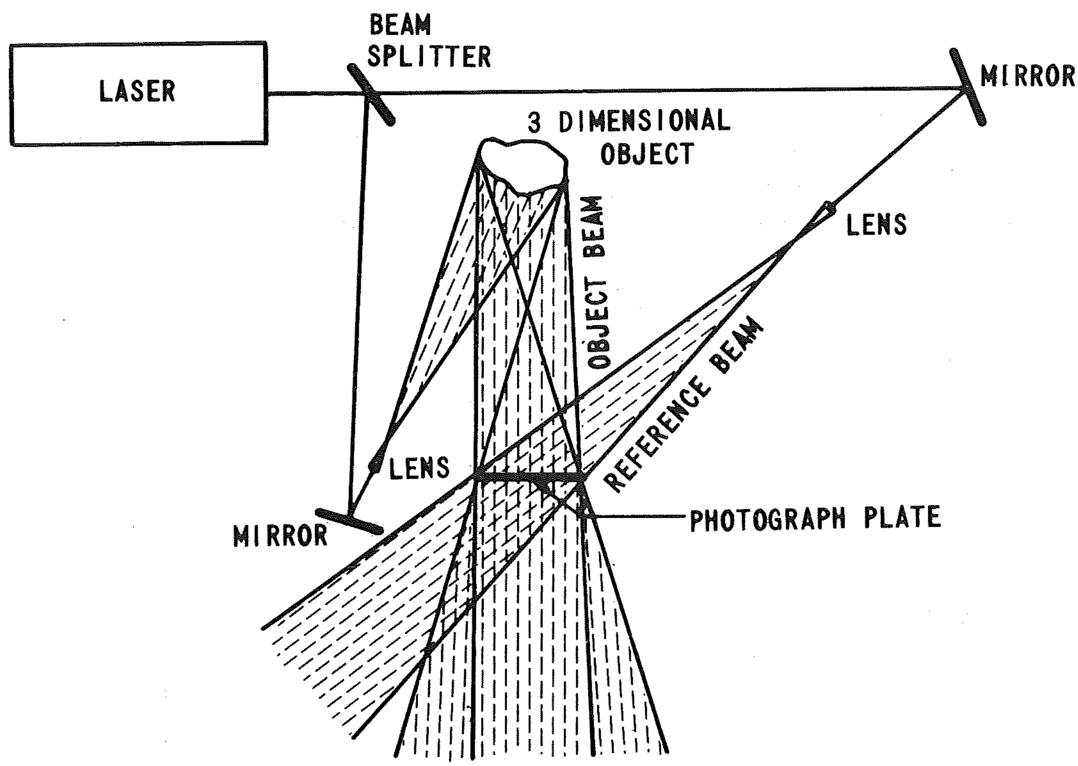

Figure 1. Configuration for recording the wave front reflected from a three-dimensional object.

Reconstruction of the recorded wave front is accomplished by reillumination of the photographic plate with the same reference wave front used in the recording process. The photographic plate behaves as a diffraction grating, diffracting part of the reference wave front into a reconstruction of the stored wave front (Fig. 2).

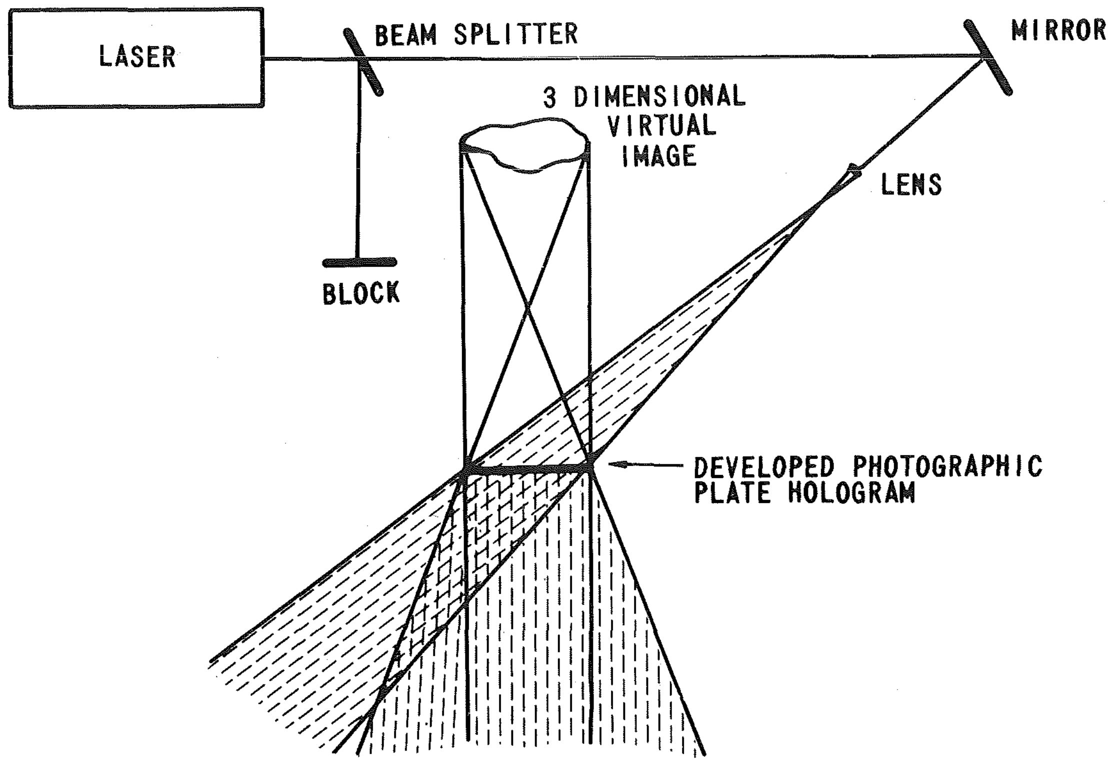

Figure 2. Configuration for reconstructing the wave front reflected from the three-dimensional object.

The Photographic Process

Exposure, development, and object beam to reference beam intensity ratio must be such that the response of the emulsion, measured in amplitude transmittance, is linearly proportional to the intensity being recorded. Unfortunately, emulsions do not have an entirely linear response to energy. Shown in Figure 3 is a typical transmittance versus exposure curve where amplitude transmittance is defined as the ratio of the amplitude of the light incident upon the plate to the amplitude of the light transmitted by the plate; the exposure is in units of energy so that it is proportional to intensity. If the emulsion is to have a linear response, the intensity modulation must be confined to the linear section of this curve.

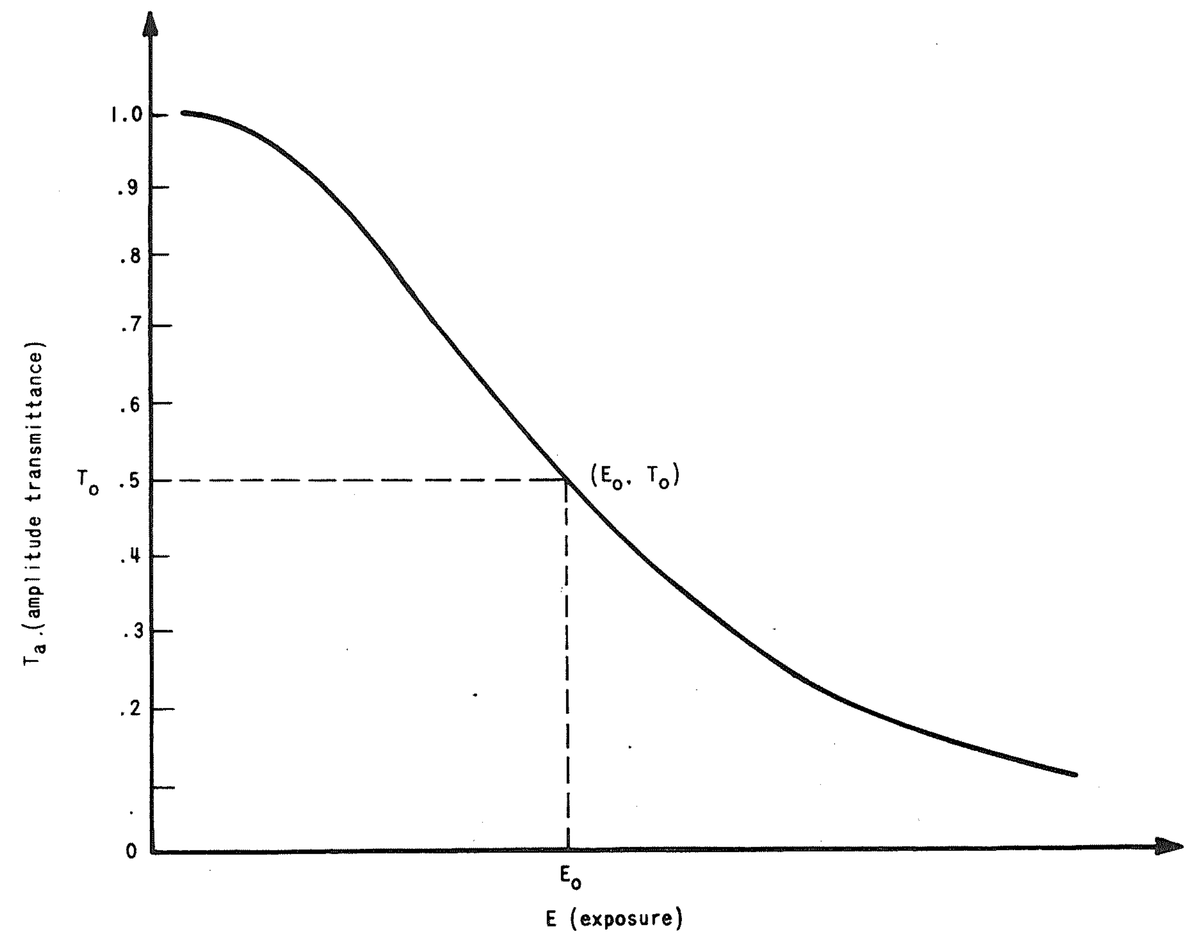

Figure 3. Graph of the typical emulsion response (measured in amplitude transmittance) to exposure.

Let the complex amplitude distribution (at the plate) of the reference wave front and the signal wave front be represented by $R(x,y)$ and $S(x,y)$ respectively. If these wave fronts interfere at the surface of a square-law detector, such as a photographic plate, an intensity distribution, $I(x,y)$, will be recorded, where

$$I(x,y) = |R(x,y) + S(x,y)|^2 = |R(x,y)|^2 + |S(x,y)|^2$$

$$+ R^*(x,y)S(x,y) + R(x,y)S^*(x,y).$$

If the real amplitudes of the reference and signal wave fronts are approximately constant, we may regard the first two terms of the expression as constant. Then, the last two terms are the only modulation terms.

Accordingly, the amplitude transmittance, T_a, of the plate will also have a constant part, T_o, and a fluctuating part, $f(x,y)$. Thus,

$$T_a(x,y) = T_o + f(x,y).$$

To stay on the linear section of the curve, $f(x,y)$ must be much less than T_o. This is satisfied if we pick $|S(x,y)|$ much less than $|R(x,y)|$ so that the intensity modulation terms are much less than the constant terms:

$$R^*(x,y)S(x,y) + R(x,y)S^*(x,y) << |R(x,y)|^2 + |S(x,y)|^2$$

In addition, if we pick $R(x,y)$ such that

$$\left[|R(x,y)|^2 + |S(x,y)|^2\right]^{1/2} \simeq |R(x,y)| = \left(\frac{E_o}{t}\right)^{1/2},$$

where (E_o, T_o) is a point that falls in the middle of the linear section of the curve (E_o being the desired exposure), then transmittance modulation function $f(x,y)$ will be linearly proportional to the intensity modulation. Therefore,

$$T_a(x,y) = T_o + \beta t\left[R^*(x,y)S(x,y) + R(x,y)S^*(x,y)\right],$$

where β is the slope of the curve at (E_o, T_o) and t is the exposure time.

If the plate is reilluminated with the same reference wave front used in the recording, the light transmitted will be

$$R(x,y)T_a(x,y) = R(x,y)T_o + \beta t|R(x,y)|^2 S(x,y) + \beta t R^2(x,y) S^*(x,y).$$

The second term, $\beta t|R(x,y)|^2 S(x,y)$, is a reconstruction of the signal wavefront except for a simple constant (assuming the reference wave front has a constant amplitude distribution in the x,y plate) multiplied times its amplitude. This term creates the virtual image shown in Figure 2. The first and third terms do not contribute to the reconstruction of the signal wave front since they travel in a different direction. (Note that if we use $R^*(x,y)$ instead of $R(x,y)$ to reconstruct the hologram, the third term becomes $\beta t|R(x,y)|^2 S^*(x,y)$, which creates a real image.)

Coherence

Since no source of light emits an infinitely long wave train, even highly monochromatic laser light has associated with it a temporal coherence length, ΔL, which is proportional to the duration, Δt, of a single wave train [1].

$$\Delta L = c\Delta t \simeq c/\Delta\nu = \frac{\left(\bar{\lambda}_o\right)^2}{\Delta\lambda_o},$$

where $\Delta\nu$ is the effective frequency range of the Fourier spectrum and λ_o is the mean wavelength. This distance determines the maximum path difference between two wave fronts from a common source that will give satisfactory fringe contrast. If the path length difference between the reference and the signal wave fronts exceeds the coherence length of the laser, a hologram will not be recorded. In practice, the coherence length must be experimentally determined for each light source since laser cavity expansion because of heating will ultimately limit the theoretical value.

It is also important to have a spatially coherent reference wave front. This can be accomplished by focusing the reference beam through a pinhole before it strikes the plate. The diameter of the pinhole should be on the order of the size of detail desired to be resolved in the image [2]. It is important to note that the spot size of the beam at its focal point should be less than or equal to the diameter of the pinhole if the system is to be diffraction limited.

$$d \geq SS = 1.22\ \lambda f/D,$$

where d is the diameter of the pinhole, SS is the spot size, λ is the wavelength, f is the focal length of the focusing lens, and D is the effective diameter of the lens.

Stability

The relative phase of the two interfering beams must be held constant during the exposure if the interference pattern is to be recorded. Hence the path length difference between the reference wave front and the signal wave front must be held constant to within a fraction of a wavelength of the light used to record the hologram. This is usually accomplished by either rigidly mounting all components of the optical system to a common heavy support which is isolated from floor vibrations by air mounts (or some cushioning material) or by using a high-powered pulse ruby laser to shorten the necessary exposure time to nanoseconds.

Emulsion Resolving Power

If the spacing between the interference fringes becomes smaller than the grain size of the emulsion, the fringes will not be recorded. Since the angle between the signal and reference wave fronts determines the spacing, this angle must not exceed a certain critical angle, Θ, determined by

$$\Theta = 2 \arcsin (\lambda/2p),$$

where λ is the wavelength of the recording light and p is the resolving power of the emulsion.

APPLICATIONS

Optical Subtraction

Holography can be used to optically subtract one complex wave front from another. If the wave fronts are identical except for an additional term in one, the additional term can be emphasized by subtracting the two wave fronts.

The technique is to superimpose two holograms on the same photographic plate by double exposure. If the same reference wave front is used in each exposure, the reconstruction of the signal wave front will be the sum of the signal wave fronts used in each exposure. To make the wave fronts subtract rather than add, a 180-degree phase delay is introduced into the signal wave front to be subtracted.

If S and $S + \Delta S$ are the complex amplitude distributions of the signal wave fronts for the first and second exposures, respectively, and R is the complex amplitude distribution of the reference wave fronts for both exposures (omitting the coordinate notation for abbreviation), the light transmitted by the doubly exposed hologram will be

$$RT_a = RT_o + \beta t\,[R^2 S + R^2 S^*] + \beta' t'\,[R^2 (S + \Delta S) + R^2 (S + \Delta S)^*] .$$

(For multiple exposure holography, it is desirable to prebias the emulsion to a transmittance T_o at the beginning of the linear section of the T-E curve. Then the reference-to-signal amplitude ratio can be unity and the successive exposures can be chosen such that $\beta_1 t_1 \doteq \beta_2 t_2 = \beta_3 t_3 \ldots\ldots$)

If a 180-degree phase delay is introduced into the wave front to be subtracted and if $\beta t = \beta' t'$, we can write

$$RT_a = RT_o + \beta t \Big[|R|^2 S \exp(i\pi) + R^2 S^* \exp(i\pi) + R^2 (S+\Delta S)$$

$$+ R^2 (S+\Delta S)^* \Big] = RT_o + \beta t |R|^2 \Delta S + \beta t R^2 \Delta S^* .$$

Effectively, S has been erased, leaving only a reconstruction of ΔS.

This was first done with Fourier transform holography [3] (Fig. 4). Because the Fourier spectrum of a signal input created by backlighting of a transparency is relatively insensitive to translation of the transparency (see Appendix A), such a configuration is much easier to use than a system whose signal wave front is the backscatter off an object. However, in principle, it should be possible to subtract backscattered signals also. Figure 5 is a photograph of the holographic image resulting from the subtraction of the signal diffusely reflected by the right half of a small card from the signal diffusely reflected from the entire card (the right half of the card is much dimmer than the left half). The holographic configuration that was used for this subtraction is shown in Figure 6.

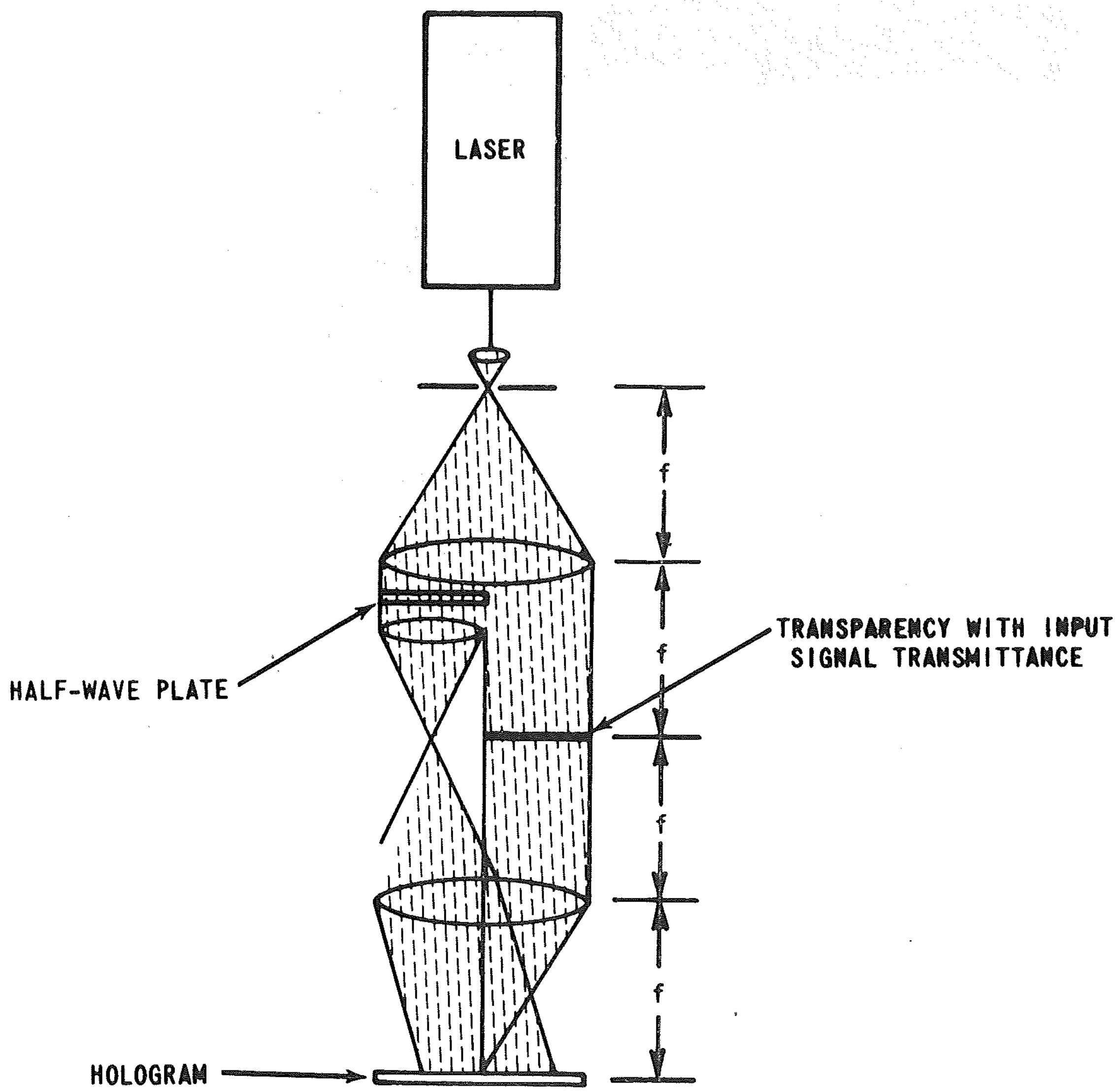

Figure 4. Configuration for optical subtraction by Fourier transform holography.

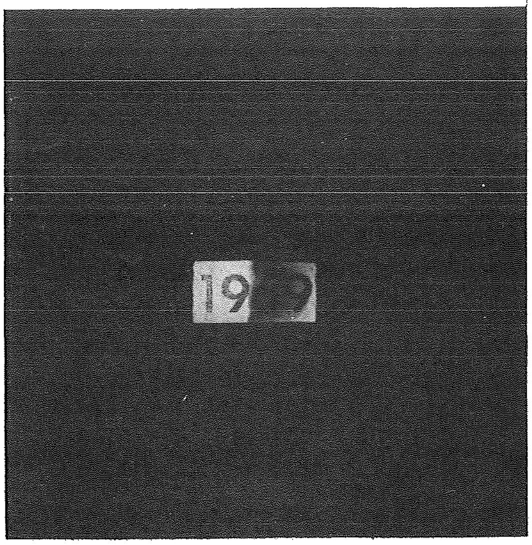

Figure 5. Photograph of the image reconstructed from a subtraction hologram made by the configuration shown in Figure 6 (The background for the number 69 was erased by optical subtraction.)

Pattern Recognition

Pattern recognition can be accomplished by optical spatial filtering using holograms as matched filters. The purpose of such a system is to detect the presence of a given pattern by enhancing its optical signal and suppressing any extraneous signals caused by other patterns. This is accomplished by causing the desired signal to have a higher energy density at the surface of a detector than any of the extraneous signals.

The basic system for accomplishing this task is shown in Figure 7. The pattern to be recognized is placed at P_1 in the form of a transparency (such as a photographic negative). A plane parallel beam of coherent light is incident upon the transparency at P_1 and is transmitted with complex amplitude $s(x_1, y_1)$. The lens L_2 displays a complex amplitude distribution $S(x_2/\lambda f, y_2/\lambda f)$, at P_2 which is the Fourier transform of $s(x_1, y_1)$. (See Appendix B: Fourier Transformation by A Lens.) Placing a transparency at P_2 with complex amplitude transmittance $T_a(x_2/\lambda f, y_2/\lambda f)$ causes $S(x_2/\lambda f, y_2/\lambda f)$ to be multiplied by $T_a(x_2/\lambda f, y_2/\lambda f)$. Lens L_3 then displays the Fourier transform of this product at plane P_3.

If the signal $s(x,y)$ is to be recognized, the transparency at P_2 should have

$$T_a(x_2\lambda f, y_2 \lambda f) = kS^*(x_2/\lambda f, y_2/\lambda f)$$

so that

$$T_a(x_2/\lambda f, y_2/\lambda f)S(x_2/\lambda f, y_2/\lambda f) = k|S(x_2/\lambda f, y_2/\lambda f)|^2.$$

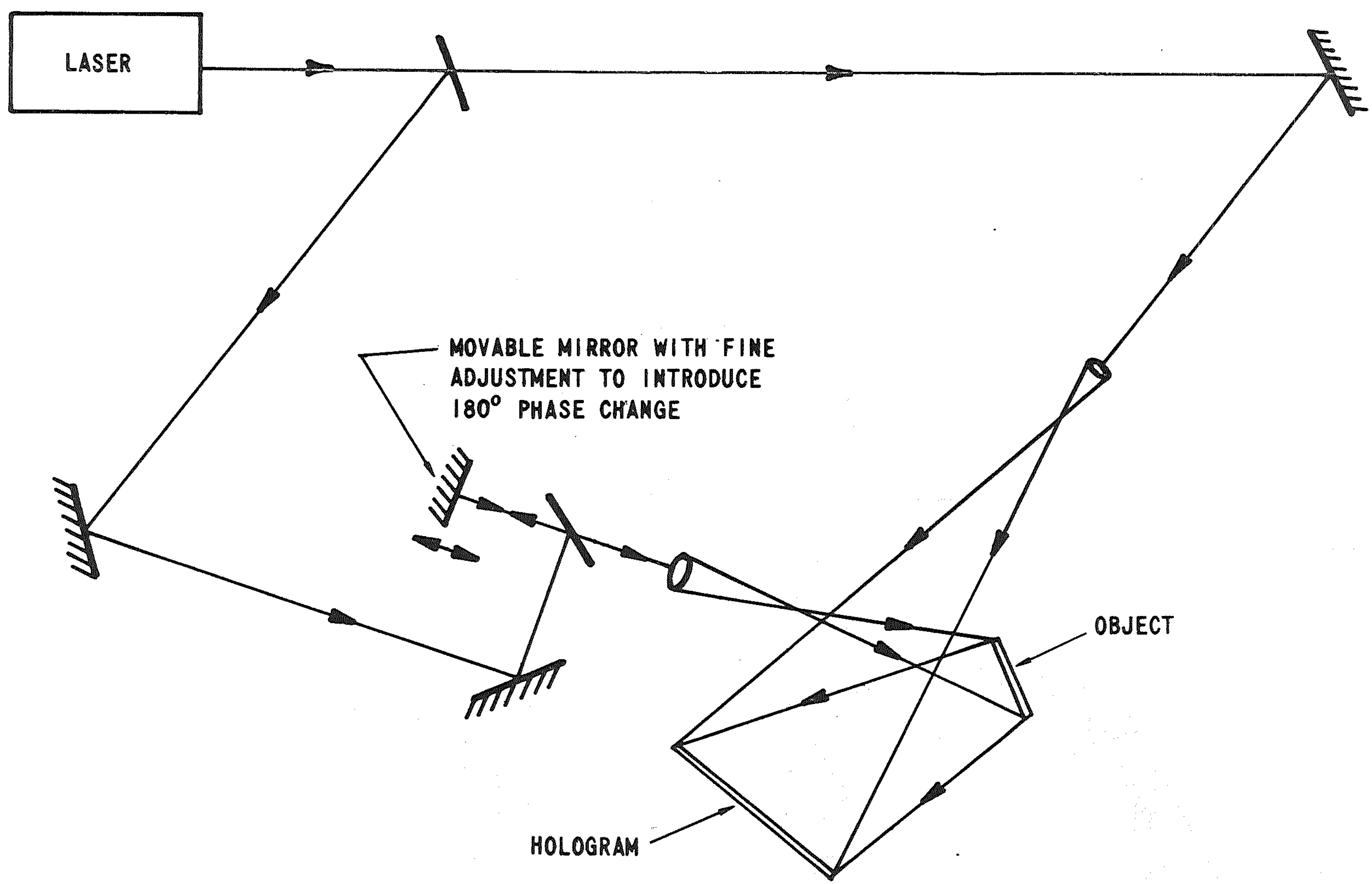

Figure 6. Configuration for optical subtraction of backscattered signals.

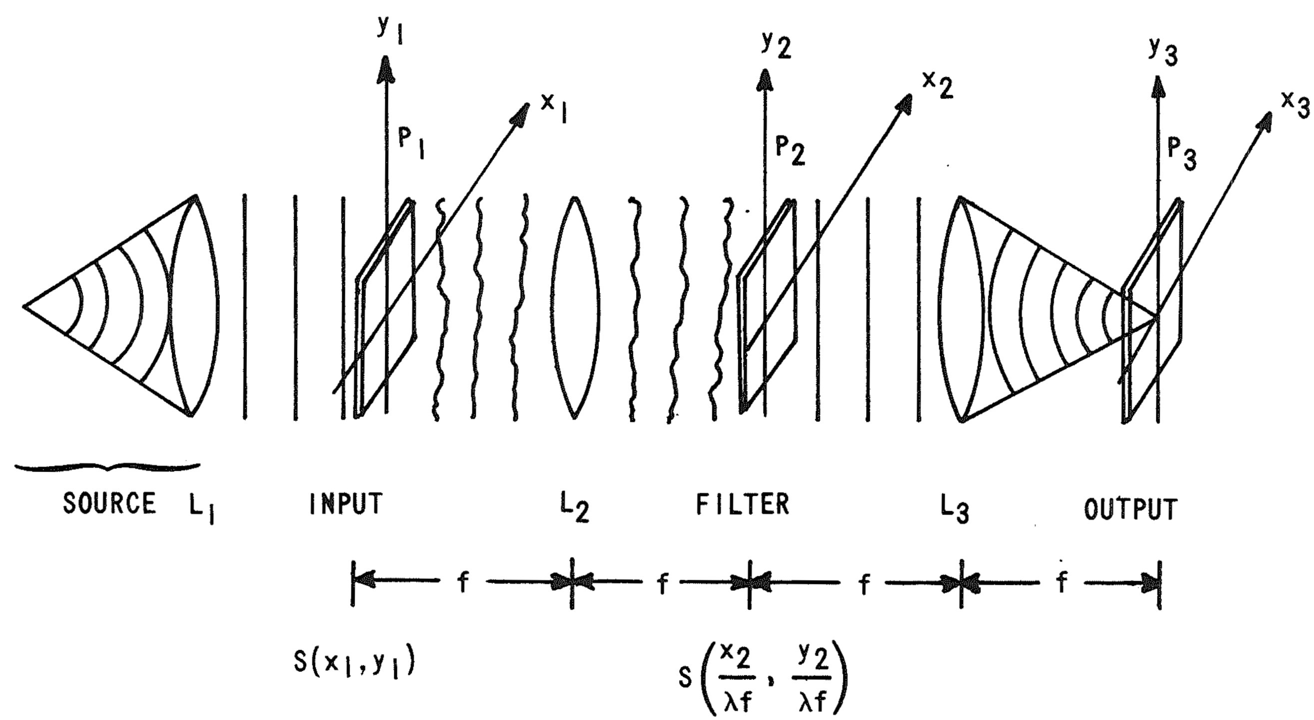

Figure 7. Basic optical system for pattern recognition.

The curvature of the wave front incident at P_2 is thereby canceled, leaving a plane wave which can be focused to an approximate point on P_3 by lens L_3 as in Figure 7. Such a transparency is called a matched filter for input signal $s(x, y)$.

However, until the invention of holography, it was not possible to make a transparency with complex amplitude transmittance $kS^*(x_2/\lambda f,\ y_2/\lambda f)$. With holography, a matched filter for any given input signal can be easily made. Shown in Figure 8 is the system for making the matched filter. To make the matched filter for an input signal, $s(x,y)$, $s(x,y)$ is Fourier transformed onto plane P_2. Also incident on plane P_2 is a plane parallel beam that has complex amplitude distribution $R \exp(i2\pi\alpha y_2)$ on P_2, where $\alpha = \sin\theta/\lambda$ and R = constant. If a photographic plate is placed at P_2, a hologram can be recorded with complex amplitude transmittance

$$T_a(x_2/\lambda f,\ y_2/\lambda f) = R^2 + S^2 + RS\exp(-i2\pi\alpha y_2) + RS^*\exp(i2\pi\alpha y_2).$$

The fourth term is the desired transmittance except for the linear phase term $\exp(i2\pi\alpha y_2)$. However, this term is desirable since it separates the desired transmittance from the other terms by inclining the desired transmittance at an angle, θ, off axis (Figure 9). Multiplying $T_a(x_2/\lambda f,\ y_2/\lambda f)$ by $S(x_2/\lambda f,\ y_2/\lambda f)$ gives a reconstruction of the plane parallel wave front incident upon P_2 except for the $|S(x_2/\lambda f,\ y_2/\lambda f)|^2$ multiplier. Then, lens L_3 will take the Fourier transform of this term which approximates a Dirac delta function centered at coordinates $(0,\ \alpha\lambda f)$ on P_3 at which a light-sensitive detector can be located (Appendix B).

To experimentally verify the theory, a matched filter was made for a pattern of vertical slits by the configuration shown in Figure 10 on a 4-by 5-inch Kodak 649-F photographic plate. To make the filter fairly selective, a stop one-half inch in diameter was located at the center of signals Fourier spectrum to act as a high-pass filter. This eliminated the dc term and the lower frequency (spatial) diffraction. It was found that, as was expected mathematically, the effect of translation of the input transparency in the $(x_1,\ y_1)$ plane was only an equal shift of the output point in the $(-x_3,\ -y_3)$ output plane. However, the output was sensitive to rotation of the input transparency. Graphs of the sensitivity of the matched filter to rotation of the input transparency are plotted in Figures 11 and 12.

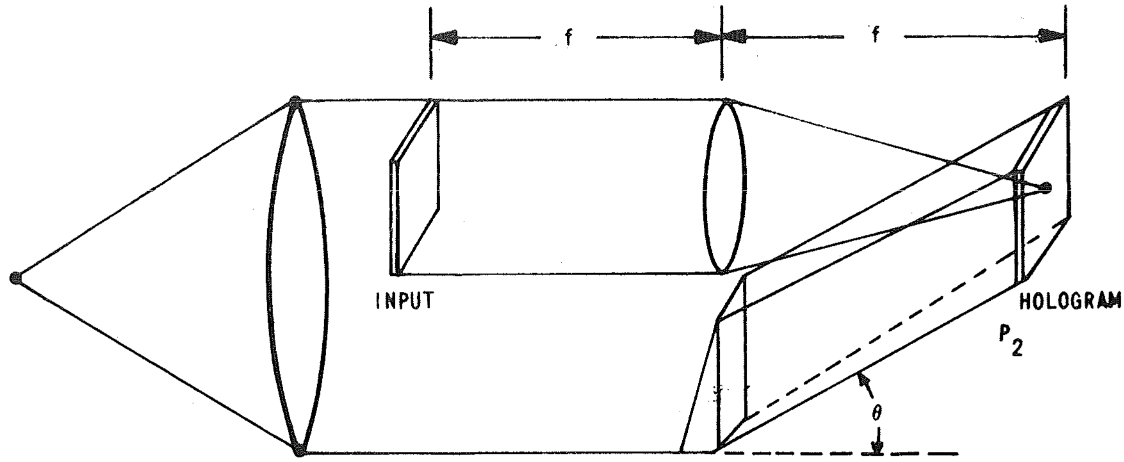

Figure 8. Configuration for making a matched filter by holography.

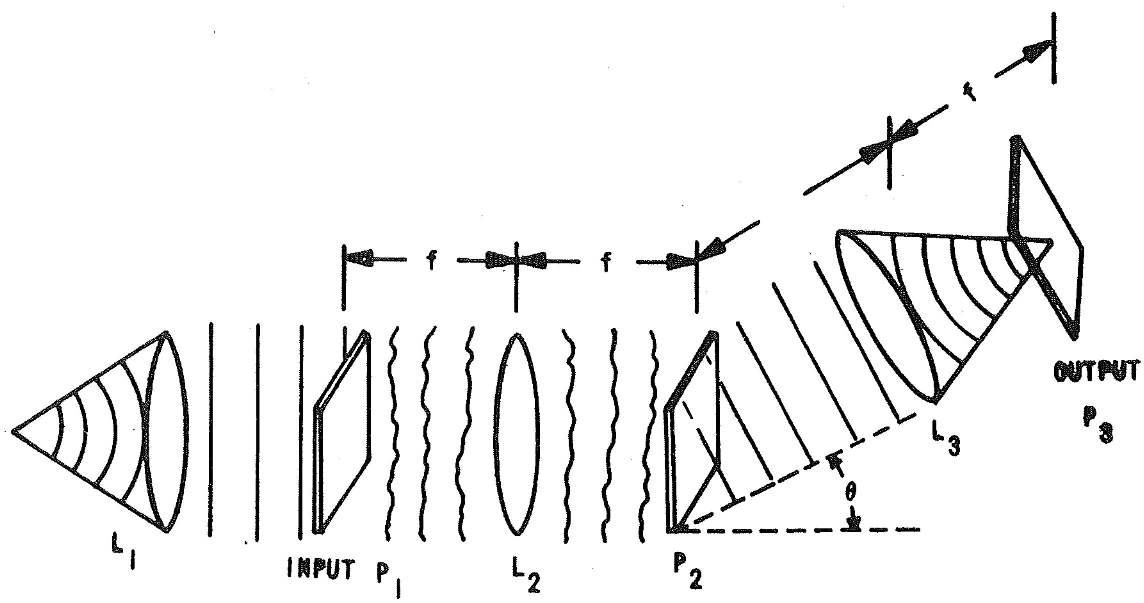

Figure 9. Schematic showing off-axis displacement of a holographically constructed matched filters output.

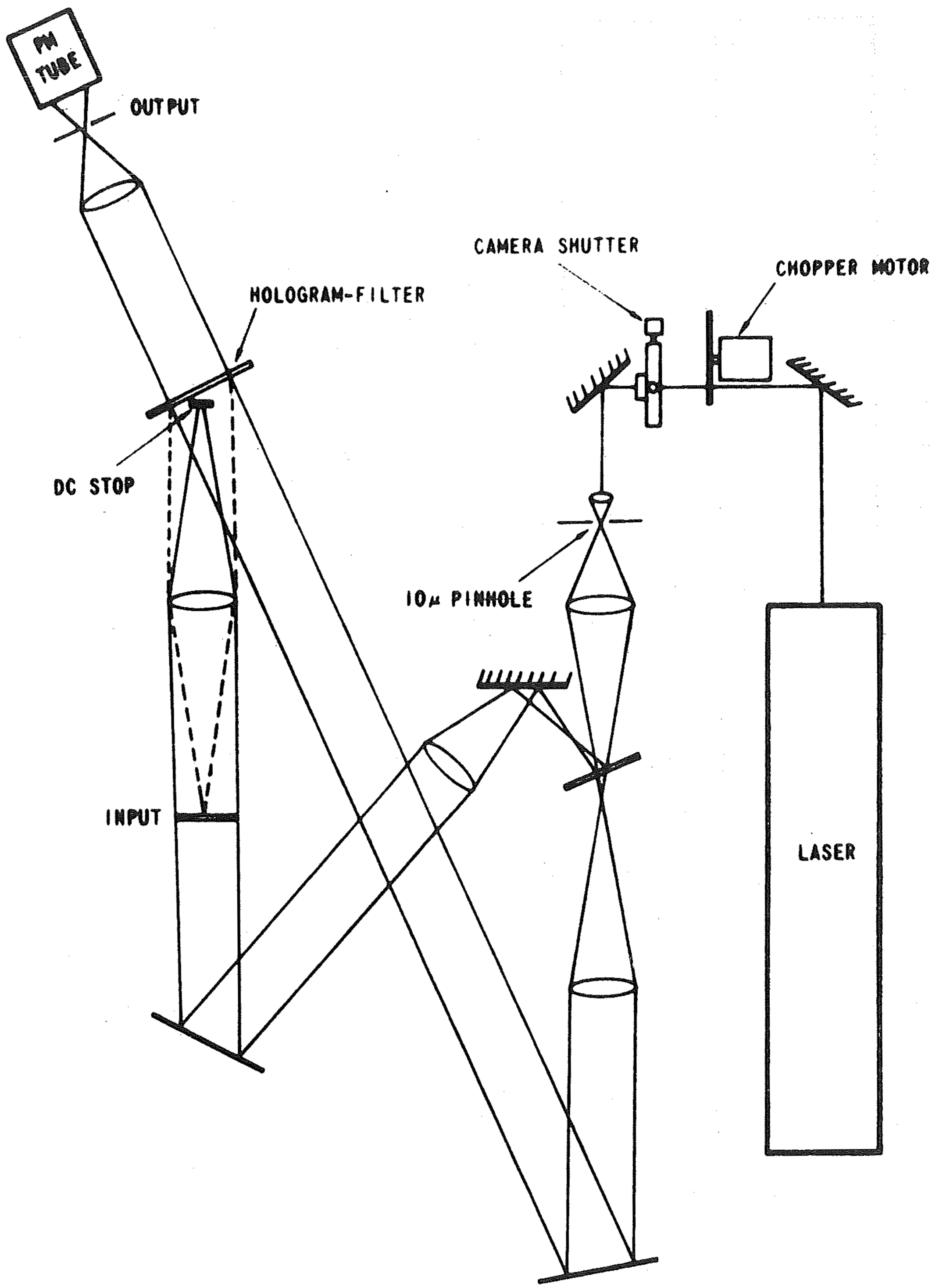

Figure 10. Experimental pattern recognition configuration used to obtain results plotted in Figures 11 and 12.

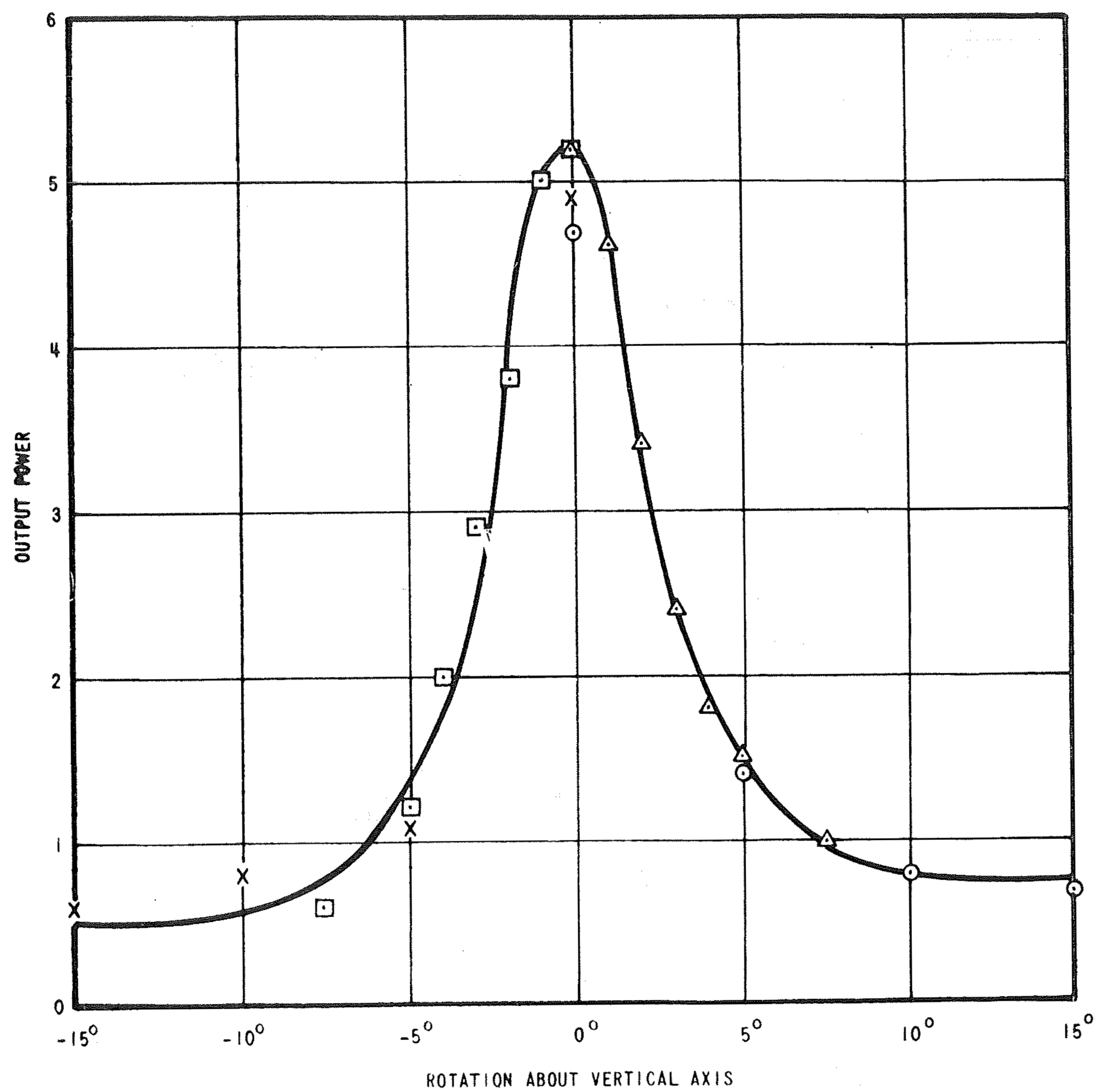

Figure 11. Graph of the response of the matched filters output (measured in units of power) to a rotation of the input transparency about its vertical axis.

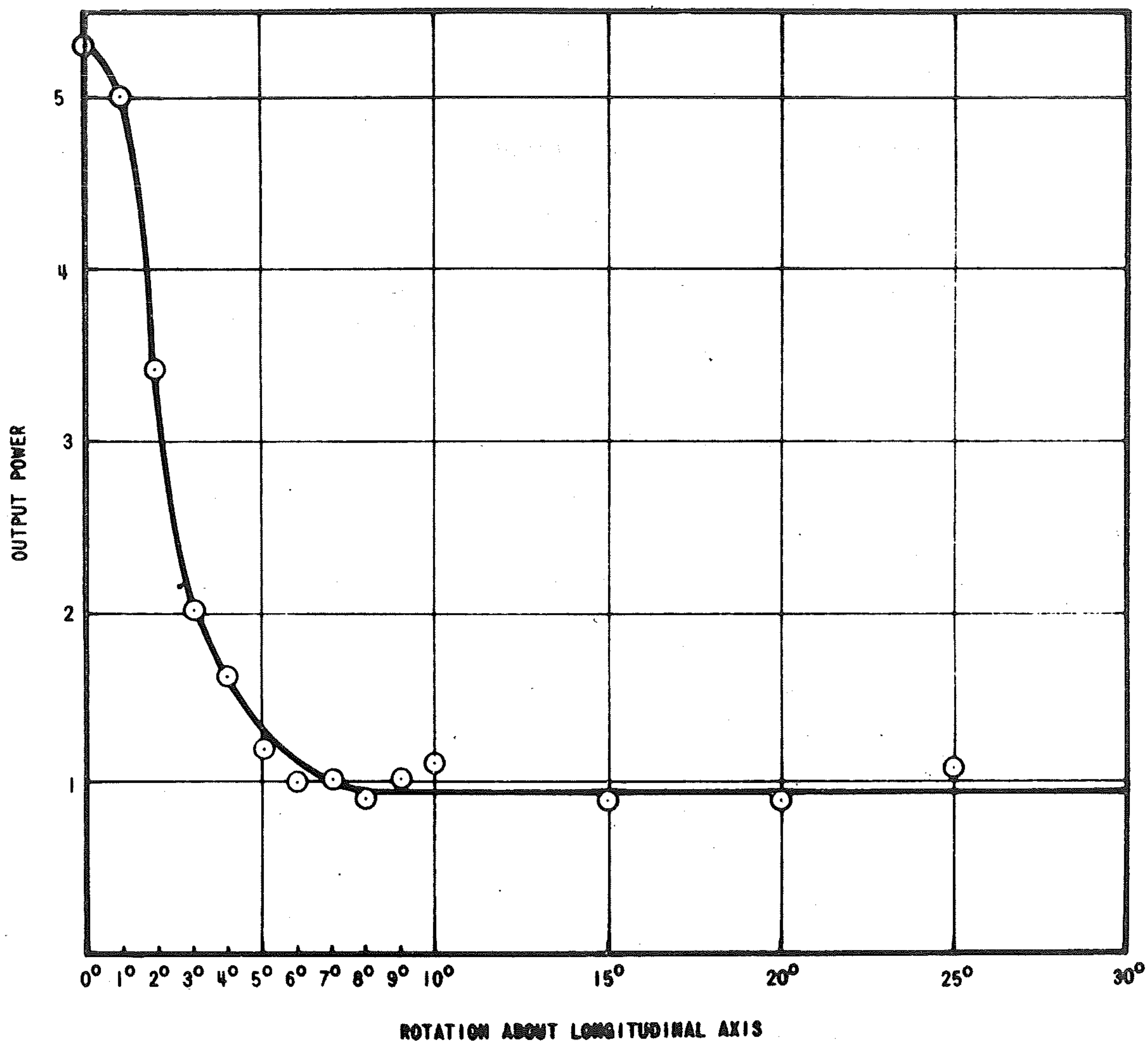

Figure 12. Graph of the response of the matched filters output (measured in units of power) to a rotation of the input transparency about its longitudinal axis.

APPENDIX A

SHIFT INVARIANCE OF MATCHED FILTER OUTPUT

To find the effect on the filtered output of a shift of the input, let the input signal be shifted by an amount, a, such that $s(x_1)$ becomes $s(x_1-a)$. Then, the Fourier transform of $s(x_1-a)$ will be

$$F\left[s(x_1-a)\right] = \int_{-\infty}^{\infty} s(x_1-a) \exp(-2\pi i f_x x_1) dx_1.$$

Let $x_1-a = x'$, then $x_1 = x' + a$ and $dx_1 = dx'$ so that,

$$F\left[s(x_1-a)\right] = \int_{-\infty}^{\infty} s(x') \exp\left[(-2\pi i f_x)(x' + a)\right] dx'$$

$$= \exp(-2\pi i f_x a) \int_{-\infty}^{\infty} s(x') \exp(-2\pi i f_x x') dx' = \exp(-2\pi i f_x a) F\left[s(x_1)\right]$$

$$= \exp(-2\pi i f_x a) S(f_x).$$

Thus, the only effect in the frequency plane is a linear phase shift.

Multiplying by the amplitude transmittance of the matched filter,

$$T_a(f_x)\left[\exp(-2\pi i f_x a) S(f_x)\right] = R|S(f_x)|^2 \exp(-2\pi i f_x a) \exp(2\pi i \alpha x_2). \quad p$$

To find the distribution on plane P_3, we must take the Fourier transform of this product:

$$F\ [T_a(f_x)\ \exp\ (-2\pi i f_x a) S(f_x)]$$

$$= \int_{-\infty}^{\infty} R\,|S(f_x)|^2 \exp\ [(-2\pi i f_x)(a-\alpha\lambda f)]\ \exp\ (-2\pi i f_x x_3)\,df_x$$

where $y_2 = f_x \lambda f$. If $R\,|S(f_x)|^2 \simeq K$, which is a constant, then

$$F\ [T_a(f_x)\ \exp\ (-2\pi i f_x a) S(f_x)] = K \int_{-\infty}^{\infty} \exp\ [(-2\pi i f_x)(x_3+a-\alpha\lambda f)]\,df_x$$

$$= K\delta(x_3 + a - \alpha\lambda f),$$

where $\delta(x_3+a-\alpha\lambda f)$ is a Dirac delta function centered at coordinate $(-a+\alpha\lambda f)$ on the x_3 axis of plane P_3.

Thus the only effect of a shift in the (x_1, y_1) plane is an equal shift in the $(-x_3, -y_3)$ plane.

APPENDIX B

FOURIER TRANSFORMATION BY A LENS[1]

A thin lens acts as a phase transformation on an incident wave front. Referring to Figure B-1, the total phase delay may be written

$$\Phi(x,y) = k\, n\, \Delta(x,y) + k[\Delta_0 - \Delta(x,y)],$$

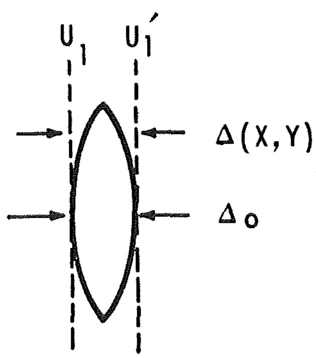

Figure B-1. Schematic of a wave front with complex amplitude U_1 incident upon a lens and the complex amplitude U_1' transmitted by the lens.

where n is the index of refraction of the lens material and the index of the surrounding medium is unity. Hence, the transmitted wave front is related to the incident wave front by

$$U_1'(x,y) = \exp\left(j\Phi(x,y)\right)U_1(x,y).$$

The thickness function, $\Delta(x,y)$, may be written

$$\Delta(x,y) = \Delta_1(x,y) + \Delta_2(x,y),$$

and, by reference to Figure B-2, it is seen that

$$\Delta_1(x,y) = \Delta_{0_1} - \left(R_1 - \sqrt{R_1^2 - x^2 - y^2}\right) = \Delta_{0_1} - R_1\left(1 - \sqrt{1 - \frac{x^2 + y^2}{R_1^2}}\right)$$

$$\Delta_2(x,y) = \Delta_{0_2} - \left(-R_2 - \sqrt{R_2^2 - x^2 - y^2}\right) = \Delta_{0_2} + R_2\left(1 - \sqrt{1 - \frac{x^2 + y^2}{R_2^2}}\right)$$

---

1. This development follows the approach made by J. W. Goodman in his book, Introduction to Fourier Optics.

$$\Delta(x,y) = \Delta_o - R_1 \left(1 - \sqrt{1 - \frac{x^2+y^2}{R_1^2}}\right) + R_2 \left(1 - \sqrt{1 - \frac{x^2+y^2}{R_2^2}}\ .\right)$$

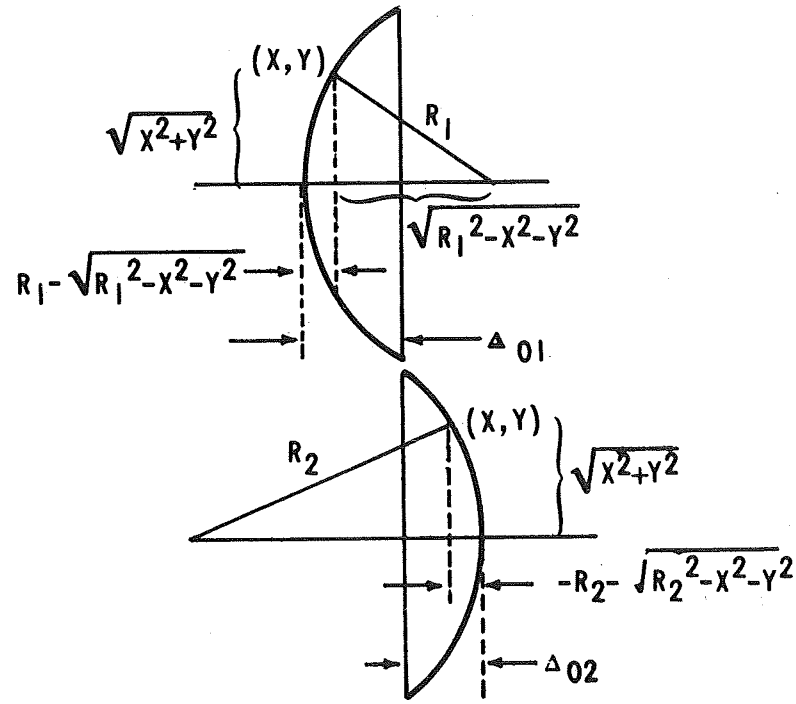

Figure B-2. Schematic for calculating the thickness function of a lens.

If only paraxial rays are considered, then the following approximations can be made:

$$\sqrt{1 - \frac{x^2+y^2}{R_1^2}} \simeq 1 - \frac{x^2+y^2}{2R_1^2}$$

$$\sqrt{1 - \frac{x^2+y^2}{R_2^2}} \simeq 1 - \frac{x^2+y^2}{2R_2^2}\ .$$

Substitution of these approximations gives:

$$\Delta(x,y) = \Delta_0 - \frac{x^2+y^2}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right).$$

With the substitution of this expression, the lens phase transformation term can be written

$$\exp\left(j\Phi(x,y)\right) = \exp\left(jkn\Delta_0\right) \exp\left[-j\frac{k}{2f}\left(x^2+y^2\right)\right],$$

where f, the focal length, is defined as

$$\frac{1}{f} \equiv (n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right).$$

To find the distribution $U_f(x_f, y_f)$ in the back focal plane of the lens, the Fresnel diffraction formula can be applied:

$$U_f(x_f,y_f) = \frac{\exp(jkf)}{j\lambda f} \iint_{-\infty}^{\infty} U_1'(x_1,y_1) \exp\left\{ j\frac{k}{2f}\left[(x_f-x_1)^2 \right.\right.$$

$$\left.\left. + (y_f-y_1)^2 \right] \right\} dx_1\, dy_1.$$

Expanding the quadratic terms is the exponent

$$U_f(x_f,y_f) = \frac{\exp(jkf)}{j\lambda f} \exp\left[j\frac{k}{2z}\left(x_f^2+y_f^2\right)\right] \iint_{-\infty}^{\infty} U_1'(x_1,y_1)$$

$$\exp\left[j\frac{k}{2f}\left(x_1^2+y_1^2\right)\right] \exp\left[-j\frac{k}{2f}\left(x_f x_1 + y_f y_1\right)\right] dx_1\, dy_1.$$

The expression for U_1' in terms of U_1 is

$$U_1' = \exp\left(j\Phi(x_1,y_1)\right) U_1(x_1,y_1) = \exp\left(jkn\Delta_0\right) \exp\left[-j\frac{k}{2f}\left(x_1^2+y_1^2\right)\right].$$

Substitution causes the quadratic phase terms within the integral to vanish leaving

$$U_f(x_f,y_f) = \frac{\exp(jkf)}{jkf} \exp\left[j\frac{k}{2z}\left(x_f^2 + y_f^2\right)\right]$$

$$\iint_{-\infty}^{\infty} U_1(x_1,y_1)\exp\left[-j\frac{2\pi}{\lambda f}(x_1x_f + y_1y_f)\right] dx_1dy_1$$

$$= \frac{\exp(jkf)}{jkf} \exp\left[j\frac{k}{2z}\left(x_f^2 + y_f^2\right)\right] F\left[U_1(x_1,y_1)\right],$$

where the symbol F [ ] denotes the Fourier transform operation and the transform is evaluated at frequencies

$$\left(f_x = \frac{x_f}{\lambda f},\ f_y = \frac{y_f}{\lambda f}\right)$$

Consider a plane object with amplitude transmittance $t_o(x_o,y_o)$ as in Figure B-3. If a monochromatic plane wave of amplitude A is incident on the object, the disturbance transmitted will be

$$U_o(x_o,y_o) = A\ t_o(x_o,y_o).$$

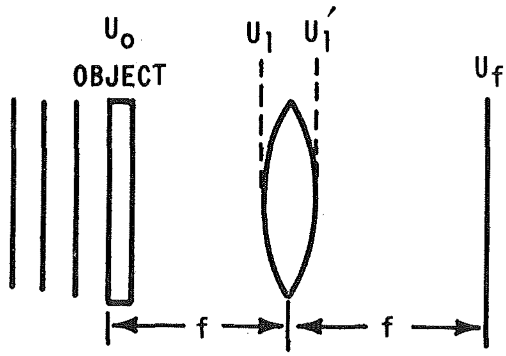

Figure B-3. Configuration for Fourier transformation of a plane object amplitude transmittance by a lens.

Again, the Fresnel diffraction formula can be used, this time to relate U_1 to U_o:

$$U_1(x_1,y_1) = \frac{\exp(jkf)}{j\lambda f} \iint U_o(x_o,y_o) \exp\left\{ j\frac{k}{2f}\left[(x_1-x_o)^2 + (y_1 - y_o)^2\right]\right\} dx_o dy_o.$$

The form of the formula is that of a convolution of $U_o(x_o, y_o)$ with $h(x_1-x_o, y_1-x_o)$, where

$$h(x_1-x_o, y_1-y_o) = \frac{\exp(jkf)}{j\lambda f} \exp\left\{ j\frac{k}{2f} \left[(x_1-x_o)^2 + (y_1-y_o)^2 \right] \right\}.$$

Using the convolution theorem, we can write:

$$F\left[U_1(x_1, y_1) \right] = F\left[U_o(x_o, y_o) \right] \; F\left[h(x_1, y_1) \right].$$

Recalling our expression for $U_f(x_f, y_f)$ and substituting for $F\left[U_1(x_1, y_1) \right]$:

$$U_f(x_f, y_f) = \frac{\exp(jkf)}{jkf} \exp\left[j\frac{k}{2f} (x_f^2 + y_f^2) \right] F\left[U_o(x_o, y_o) \right] \; F\left[h(x_1, y_1) \right],$$

where

$$F\left[h(x_1, y_1) \right] = \exp(jkf) \exp\left[-\frac{jk}{2f} (x_f^2 + y_f^2) \right].$$

Substituting for $F\left[h(x_1, y_1) \right]$ and dropping the constant phase delay terms, it is found that

$$U_f(x_f, y_f) = F\left[U_o(x_o, y_o) \right].$$

It is therefore seen that a lens Fourier transforms an optical input in its front focal plane onto its back focal plane.

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APPROVAL

HOLOGRAPHIC OPTICAL DATA PROCESSING

By Rodney W. Jenkins

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This document has also been reviewed and approved for technical accuracy.

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