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THE DIURNAL PHASE ANOMALY IN THE UPPER ATMOSPHERIC DENSITY AND TEMPERATURE

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ABSTRACT

The heat conduction equation for the neutral atmosphere in the region between 120-1500 km is solved using the concept of dynamic diffusion. It is shown that the present discrepancy in the phase and the diurnal amplitude of the thermospheric temperatures as inferred from incoherent back-scatter and satellite drag measurements can be fully reconciled by such an approach. The numerical solutions show that the phase and the diurnal amplitude of the atmospheric density in the region between 200-500 km are in accord with the satellite drag results; the corresponding neutral temperature, however, follows the diurnal pattern inferred from the back-scatter technique. The characteristics of the density variations above 500 km strongly depend upon the choice of the hydrogen and helium models and their relative concentration. In the region where hydrogen becomes a dominant constituent, the density reaches its maximum value in the nighttime and its minimum in the day-time.

The physical basis for the dynamic diffusion is discussed, and it is suggested this should replace the conventional static

diffusion models in computing temperatures from the atmospheric densities obtained from satellite drag measurements.

INTRODUCTION

The diurnal characteristics of the thermospheric density and temperature and their physical interpretations have been the subject of considerable discussion during the past few years. The satellite drag results which have been the major source of information about the neutral atmosphere over the past decade seem to indicate that the atmospheric density and temperature above 200 km reach their maximum values around 1400 hours local time almost universally. (Jacchia and Slowey 1968, See also Priester el. al (1967) for other references). The theoretical considerations which are based on the solution of the heat conduction equation with solar ultra volet radiation as the main energy input, suggest that at equinox the diurnal temperature should have its peak value around 1700 hours. In order to explain this phase discrepancy of 3 hours, Harris and Priester (1962) introduced the concept of a secondary heat source to which they ascribed a corpuscular origin. In spite of the success of Harris and Priester in bringing their theoretical calculations in accord with the satellite drag measurements, the concept of this new heat source has been seriously questioned in the literature. A number of alternatives have been suggested to this heat source which include the introduction of dynamical effects, coupling with the mesosphere and extending the one dimensional calculation of the heat conduction equation to two or three dimensions. (Freeman 1967, Dickinson et al. 1968, Volland et al. 1969). All these papers succeed to varying degrees in bringing the theoretical calculations in accord with the satellite drag results without the necessity of an additional heat source. The controversy now has taken a new turn. The recent observations on the thermospheric temperatures inferred from the incoherent back-scatter radars suggest the temperature to have its peak value around 1700 hours in agreement with the calculations of Harris and Priester without an additional heat source and in sharp disagreement with the satellite drag results. (Nisbet 1968, Mahajan 1968, McClure 1968, Carru et al. 1968, Waldteufel and McClure 1970). The disagreement between the satellite drag and the incoherent back-scatter extends beyond the time of the maximum. The minimum temperature inferred from the back-scatter measurements occurs about one hour later than the time of the minimum inferred from the drag measurements. The diurnal amplitude of the temperature inferred from the two sets of measurements are also different. The ratio of maximum to minimum temperature derived from the back-scatter data is about 1.5. The corresponding value from the drag measurement is about 1.3.

The purpose of this paper is to show that the phase and the amplitude of the neutral atmosphere derived from the back-scatter and drag measurements do not contradict each other if is recognized that the basic parameters measured from the two experiments are the temperature and the density, respectively. The basic problem lies in computing the temperature from the density and vice versa. It is proposed in this paper that the concept of the dynamic diffusion as opposed to the static diffusion (Jacchia 1964) usually employed in computing the temperature from the density leads to a reasonable reconciliation between the two sets of the observational results.

THE PHYSICAL CONCEPT OF DYNAMIC DIFFUSION

The concept of the dynamic diffusion is based on the assumption that the atmosphere can be divided into two regions according to the relative importance of diffusion or mixing as the controlling process. The turbopause which is usually taken to be the height above which diffusive equilibrium prevails in the atmosphere is assumed here to be also the level separating the mixing and diffusive regions. In reality there is no sharp boundary between the two regions and this reference level is chosen here for convenience only, without implying a new definition for the turbopause. Both the density and temperature in the two regions undergo diurnal variations as determined by the dynamical and heat transfer processes of that region. However, the changes in the two regions are not independent of each other because of the coupling through a common boundary. The solutions of the continuity, momentum and energy balance equations must match at this level. Accordingly, the density and temperature at the turbopause are not invariant but undergo temporal and diurnal variations. Based on this idealized concept, the particle density n_j of the jth constituent of the neutral atmosphere above the turbopause may be expressed in the following form:

$$n_j(z,t) = \frac{n_j(z_o,t)\ T(z_o,t)}{T(z,t)} \exp - \int_{z_o}^{z} \frac{m_j\ g}{kT(z,t)} \tag{1}$$

where k is the Boltzmann constant g the acceleration due to gravity; and n_j (z_o,t) and T (z_o,t) are the particle density and temperature at the turbopause level z_o.

Equation (1) essentially summarizes the basic concept of dynamic diffusion which may be stated as follows: At any given instant, the neutral constituents in the upper atmosphere above the turbopause are in diffusive equilibrium even though the density and temperature at the lower boundary are not invariant. The variations at the lower boundary which are caused by the physical processes lower down tend to create transients in the medium. However, because of an extremely short time constant of the atmosphere, these transients die very quickly and the transition from one diffusive equilibrium state to another is almost instantaneous. Thus, if the variations at the lower boundary are known, the diurnal variations in neutral density and temperature in the diffusive region can very simply be calculated by solving the heat conduction equation in conjunction with equation (1). Because of the variability at the lower boundary, it is evident that the temperature and density at any given height will not be in the same phase and will depend strongly on the phase and amplitude of the diurnal variations at the lower boundary. We shall return to this point in the later part of this paper while discussing the numerical results obtained from the solutions of the heat conduction equation. Since the concept of dynamic diffusion and hence the main conclusion of this paper is based on the assumed variability at the lower boundary, it is important to investigate the nature and extent of these variations. This is considered in the next section.

THE DIURNAL VARIATIONS BELOW THE TURBOPAUSE

The discussions of this section will have a rather qualitative character and are meant mainly to make plausible the possibility of appreciable density variations at the turbopause level. The result, expressed in Eqn. (14), will be a formula giving an approximate relationship between the density and the temperature at the turbopause. Since we are mainly interested in the density, we will have to specify the temperature with respect to its diurnal amplitude and phase, and we will obtain in an approximate way the corresponding density variation. We will simplify our problem by assuming that the atmosphere below the turbopause is completely mixed and the main dynamical effects of this region are controlled by the temperature variations which are specified. This seems to be a reasonable assumption for the region between 70 - 120 km where solar X-rays, Lyman α and the ultraviolet radiation in the Schumann continuum range provide the main heat source in the atmosphere. The equation of continuity and momentum conservation for this region, neglecting the viscosity and the tidal effects, may be written in the following form

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0 \tag{2}$$

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} - 2\vec{v} \times \vec{\omega} = -\frac{1}{\rho} \nabla p + \vec{g} \tag{3}$$

where $\vec{\omega}$ is the angular velocity ($\omega = 7.27 \times 10^{-5}$ rad/sec) of the earth, ρ and $\vec{v}$ are respectively the density and the velocity of the gas, and p is the scalar pressure

$$p = n\,k\,T \tag{4}$$

For a given temperature distribution, (2) and (3) along with (4) can be uniquely solved subject to the boundary conditions. We assume here that the changes in temperature, density and pressure are small compared to their mean values. This enables us to adopt the usual perturbation technique for solving these equations. Thus, if we assume that all the time varying quantities can be expanded in a Fourier series, we may write

$$\begin{aligned} \rho &= \rho_o + \sum_r \rho_{1r}\, e^{ir\sigma t} \\ p &= p_o + \sum_r p_{1r}\, e^{ir\sigma t} \\ T &= T_o + \sum_r T_{1r}\, e^{ir\sigma t} \\ v &= \sum_r v_{1r}\, e^{ir\sigma t} \end{aligned} \tag{5}$$

where σ is the fundamental frequency. The quantities ρ_{1r}, p_{1r},

T_{1r}, and v_{1r} are complex, thus containing a phase term.

Substituting (5) in (2), (3) and (4), the resulting differential equations can be solved for ρ_{1r} and $\vec{v}_{1r}$. For a horizontally stratified atmosphere in which the importance of horizontal pressure gradients can be neglected as compared to the vertical, ρ_{1r} and v_{1rz} are simply given by the following expressions (See Appendix).

$$\rho_{1r} = - \frac{\rho_{oo}\, T_{1r}}{T_o} \left(\frac{g}{K_r H}\right) e^{\frac{-z}{H}} + Ae^{\alpha_1 z} + Be^{\alpha_2 z} \tag{6}$$

$$v_{1rz} = \frac{i\, r\, \sigma\, g\, H}{K_r \rho_{oo}} \left[A \left(\alpha_1 + \frac{1}{H}\right) e^{\left(\alpha_1 + \frac{1}{H}\right) z} + B\left(\alpha_2 + \frac{1}{H}\right) e^{\left(\alpha_2 + \frac{1}{H}\right) z}\right]$$

$$- \frac{i\, r\, \sigma}{K_r}\, g \left(\frac{T_{1r}}{T_o}\right) \tag{7}$$

where A and B are constants of integration and are in general complex. α_1, α_2, K_r, ρ_{oo} and H are defined in the appendix.

The constants A and B can be evaluated by assuming that at the lower boundary (z = o), v_{1rz} = o. At the upper boundary ($z = z_o$) from where the diffusive equilibrium begins we note that

$$\frac{dp}{dt} = o \tag{8}$$

or

$$v_z = - \frac{\partial p}{\partial t} \Big/ \frac{\partial p}{\partial z} \tag{9}$$

corresponding to the definition of the vertical velocity as the velocity of isobaric surfaces. Thus, at $z = z_o$

$$v_{1rz} = i\, r\, H(z_o) \left[\frac{T_{1r}}{T_o} + \frac{\rho_{1r}}{\rho_o}\right]_{z=z_o} \tag{10}$$

From these constraints on the vertical velocity, A and B can be shown to have the following expressions

$$A = \frac{\rho_o(z_o) \left(\frac{T_{1r}}{T_o}\right) e^{\frac{K_r z_o}{g}}}{(2 - g/K_r H)} \tag{11}$$

$$B = \left(\frac{g}{K_r H}\right) \rho_{oo} \frac{T_{1r}}{T_o} + \frac{\rho_o(z_o)}{\left(2 - \frac{g}{K_r H}\right)} e^{\frac{K_r z_o}{g}} \left(\frac{g}{K_r H} - 1\right) \frac{T_{1r}}{T_o} \tag{12}$$

The expression for density variations at the turbopause in which we are primarily interested in this problem is obtained by substituting $z = z_o$ in equation (6). From (6), (11) and (12)

$$\frac{\rho_{1r}(z_o)}{\rho_o(z_o)} = - \frac{T_{1r}}{T_o} \left\{ \left(\frac{g}{K_r H}\right) \left(1 - e^{\frac{K_r z_o}{g}}\right) + \frac{\left(1 - \frac{g}{K_r H}\right)}{\left(2 - \frac{g}{K_r H}\right)} \left[e^{\left(2\frac{K_r}{g} - \frac{1}{H}\right) z_o} + \frac{1}{\left(1 - \frac{g}{K_r H}\right)} \right] \right\} \tag{13}$$

Since both $\frac{K_r H}{g}$ and $\frac{K_r z_o}{g}$ are $\ll 1$, equation (13) can be further simplified and the relation between density and temperature can be expressed in an approximate form

$$\frac{\rho_{1r}(zo)}{\rho_o(zo)} \simeq \left(\frac{z_o}{H}\right) \left(\frac{T_{1r}}{T_o}\right) \tag{14}$$

The physical significance of this appro formula is that it gives us the order of magnitude of lensity amplitude at the turbopause level once the temperature amplitude is specified. However, it does not contain any information on the temperature amplitude itself. The temperature amplitude, therefore, has to be treated here as the principal parameter which, along with the temperature phase, has to be assumed in a plausible way. Eqn. (14) furthermore tells us that temperature and density at the turbopause should be expected to be in phase.

In the region between 70 - 120 km, the main source of heating is the absorption of the Lyman α and ultra violet radiation in the Schumann continuum range and thus the temperature has a strong diurnal component. Considering only the first harmonic terms, the diurnal variations in temperature and density may be written in the following form

$$T = T_o + T_1 \cos (\omega t + \emptyset) \tag{15}$$

$$\rho(z_o) = \rho_o(z_o) + \rho_1(z_o) \cos (\omega t + \emptyset) \tag{16}$$

The variations between density and temperature at the turbopause according to equation (14) may be related by the following expression

$$\rho(z_o) = \rho_o(z_o)\,[1 + R\cos(\omega t + \phi)] \qquad (17)$$

where

$$R = \left(\frac{T_1}{T_o}\right)\left(\frac{Z_o}{H}\right)$$

Since $\left(\frac{T_1}{T_o}\right)$ and $\frac{\rho_1(z_o)}{\rho_o(z_o)}$ are the relative amplitudes for the temperature and density, it is evident from equations (14) and (17) that for a given temperature variation the density variation is amplified by a factor $\frac{zo}{H}$. A simple estimate for this amplification may be made as following. If the lower and upper boundaries in this problem are assumed to be 70 and 120 km, respectively, then, assuming a mean scale height of about 10 km, $\frac{zo}{H} \simeq 5$.
Thus, if $\frac{T_1}{T_o} \simeq 0.1$, $\frac{\rho_1(zo)}{\rho_o(zo)} \simeq 0.5$.

The example given above is a rough estimate and is meant to be only illustrative. The actual amplification may vary over a wide range if the effects of the viscosity and eddy diffusion are taken into account. The discussion of this section is meant to emphasize the point that a large diurnal variation in density at the turbopause level is possible even though the temperature variations may not be very large. In the next section we shall discuss the effect of this variability on the solutions of the heat conduction equation in the diffusive region.

HEAT CONDUCTION EQUATION

The numerical results on density and temperature presented in this section are based on the solution of the heat conduction equation as discussed by Stubbe (1970). (To be referred to as ST 1.) For expediency, we have neglected the energy input from the electron-ion gas and the convection term arising from the horizontal transport. An exclusion of these terms from the heat conduction equation may cause the temperature to be underestimated by 10 - 20%. However, this can easily be compensated by suitably adjusting the heating efficiency of the incoming radiation which is an undetermined parameter in this problem. Both the phase and the day to night ratio of the exospheric temperature are relatively independent of the choice of the heating efficiency, the horizontal convection and the energy input from the ionized gas. The results of this paper can, therefore, be discussed without any loss of generality.

With the assumptions outlined above, the heat conduction equation for the diffusive region may be expressed in the following form:

$$\frac{\partial T}{\partial t} + v_z \frac{\partial T}{\partial z} = \frac{1}{k \sum_{\ell} n_{\ell} c_{p\ell}} \left(\frac{\partial}{\partial_z} \left(\lambda \frac{\partial T}{\partial z}\right) + P_G - P_L \right) \tag{18}$$

where n_{ℓ} is the density of the ℓth constituent of the neutral atmosphere and $c_{p\ell}$ the corresponding specific heat at constant pressure. The neutral constituents of the atmosphere are assumed to be O_2, N_2, O, H_e and H. The other terms in equation

(18) are defined as follows:

λ density weighed thermal conductivity

$$\frac{\sum_\ell A(\ell) n_\ell}{\sum n_\ell} T^{1/2} \text{ with } A(O_2) = A(N_2) = 180;\ A(O) = 360$$

$A(H_e) = 900$, $A(H) = 2100$; all expressed

in units of ergs cm^{-1} sec^{-1} $deg^{-3/2}$

P_G Heat production rate due to the absorption of the solar UV

$$\epsilon_n \sum n_\ell(z) \int_0^\infty \sigma_\ell(\lambda)\, I_\infty(\lambda) \exp \left\{ - \sec \chi \sum_\ell \sigma_\ell(\lambda) \int_z^\infty n_\ell dz \right\} d\lambda \tag{20}$$

where $I_\infty(\lambda)$ is the exospheric photon flux intensity and $\sigma_\ell(\lambda)$ the absorption cross section. The factor of proportionality, ϵ_n, is the efficiency of conversion of solar energy into heat energy. As in ST-1, $\epsilon_n = 0.4$ except for the Schumann Runge continuum range ($1325 < \lambda < 1775$ Å) for which we assumed $\epsilon_n = 0.1$. In computing P_G, sec χ which accounts for the solar zenith angle variation is replaced by an appropriate Chapman function near sunrise and sun-set as discussed in ST-1.

P_L = Heat loss arising from the infrared radiation emitted by thermally excited atomic oxygen (Bates 1951).

$$= \frac{1.67 \times 10^{-18} \exp(-228/T)\, n(O)}{1 + 0.6 \exp(-228/T) + 0.2 \exp(-325/T)} \text{ (ergs } cm^{-3} \text{ } sec^{-1}) \tag{21}$$

v_z "Breathing" velocity of the atmosphere

$$\frac{H(z)}{p(zo)} \frac{\partial p(zo)}{\partial t} + T \int_{zo}^{z} \frac{1}{T^2} \left(\frac{\partial T}{\partial t}\right) dz \qquad (22)$$

where H(z) is the density weighed mean scale height and p(zo) the total pressure at z = zo.

The first term on the right hand side of equation (22) is the additional term arising from the variation in density and temperature at the turbopause.

DIURNAL VARIATIONS IN DENSITY AND TEMPERATURE

The numerical solutions of the heat conduction equation as outlined in the previous section in conjunction with equations (1), (15), and (17) give the diurnal variations in density and temperature above the turbopause. The numerical procedures adopted here are exactly the same as discussed in ST-1. The neutral constituents of the atmosphere are assumed to be O_2, N_2, O, H_e and H, all in diffusive equilibrium. Their values are, however, specified at different levels. The values of H and He are specified at 500 km. As in ST-1

$$n(H,500) = 1.5 \times 10^7 \exp\left\{-\frac{(T-700)}{178}\right\} \quad cm^{-3} \qquad (23)$$

The expression for n(He) at 500 km is adopted from Keating et al. (1970) and is given in the following form

$$\log_{10} n\,(He,500) = -14.552 + 13.104 \log_{10} T(500) - 2.019(\log_{10} T(500))^2 - 0.4\,\delta\theta$$

where and are respectively the solar declination angle and the geographic latitude both measured in radians. The last term in equation (24) is introduced to account for the winter helium bulge as discussed by Keating and Prior.

In the case of O_2, N_2 and O, these values are specified at 120 Km in accordance with (17)

$$
\begin{aligned}
O_2 &= 7.5 \times 10^{10} [1 + R \cos (\omega t + \phi)] \quad cm^{-3} \\
N_2 &= 4.00 \times 10^{11} [1 + R \cos (\omega t + \phi)] \quad cm^{-3} \\
O &= 7.60 \times 10^{10} [1 + R \cos (\omega t + \phi)] \quad cm^{-3}
\end{aligned}
\tag{25}
$$

The diurnal variations in temperature at 120 km is assumed to be of the form

$$
T \text{ (120 Km)} = 500 [1 + 0.1 \cos (\omega t + \phi)] \tag{26}
$$

The numerical solutions presented here correspond to the equator (= 0), equinox conditions (= 0) and for medium solar activity. The solar U.V. flux values are assumed to be 30 percent higher than those given by Hinterreger et al. (1965).

Fig. 1 shows the diurnal variations in density and temperature at 300 Km for R = 0.5 and ϕ = 0.

The assumption of ϕ = 0 implies that the density and temperature at the lower boundary are in phase with their maximum values at midnoon and minimum values at midnight. R = 0.5 implies a factor of 3 variation in density at 120 Km, related to the minimum value. The main features of Fig. 1 may be summarized in the following:

1. The density and temperature at 300 Km do not reach their maximum values at the same time. Whereas the

density maximum occurs at about 1500 hours local time in good agreement with the satellite drag results, the temperature maximum occurs about two hours later as inferred from incoherent back-scatter data.

2. The temperature minimum occurs about two hours later than the density minimum in accordance with both the satellite drag and back-scatter results.

3. The minimum to maximum change in temperature is from about 800 to 1200, an increase of 50% from day to night. This is in very good agreement with the diurnal amplitude in temperature inferred from back-scatter measurements.

Since both R and Ø are assumed parameters in this problem, it is important to study the effects of their variations.

In Fig. 2, the diurnal variations in density and temperature at 300 km are shown for varying values of Ø. The value of R is assumed to be 0.3 in all the cases. We note that the diurnal characteristics of the neutral temperature remains practically unaffected with the variations in Ø. The changes in the neutral density, on the other hand, are very significant both with regard to their phase and magnitude. Both the density and temperature are in the same phase with their maximum values at 1700 hours when $\phi = -90^{\circ}$, i.e. when the density and temperature at the lower boundary have their peak values at 1800 hours. As $\phi \rightarrow 0$, the density peak moves to an earlier hour. The shift in the time of maximum is not in the same proportion as the changes in Ø. For example, when Ø changes from 90° to 0° (a phase change of six hours at the lower boundary), the density peak at 300 km moves only from 1400 hour to 1500 hours. Finally, when $\emptyset = 180^{\circ}$ (the maximum density and temperature at the lower boundary at midnight), the density maximum at 300 km occurs late in the evening.

The change in the diurnal density amplitude with the change in $\emptyset$ is quite noticeable in Fig. 2. The day to night variation in density is maximum for $\emptyset = -90^{\circ}$ and minimum for $\emptyset = +90^{\circ}$.

In Fig. 3 are shown the diurnal variations in density at 200 and 300 km for varying values of R. The phase angle $\emptyset$ is assumed to be zero and R is assumed to vary from 0.0 to 0.5. The shift in the time of maximum with the increasing value of R is quite noticeable, particularly at 300 km. Larger day to night variations in density are seen with increasing values of R. This effect is more noticeable at 200 km where the day to night ratio are respectively 1.2, 2, and 3 for R = 0, 0.3 and 0.5.

There are practically no change in the times of the maximum and minimum and very little change in the magnitude of the neutral temperature with the variations in R [not shown in Fig. 3.]

The diurnal variations in density in the altitude range from 200 to 1400 km are shown in Fig. 4. The values of R and $\emptyset$ for this case are taken to 0.5 and 0.0, respectively. The important points to note in this figure are the gradual shift of the time of maximum to later hours with increasing altitude. In the altitude range of 200-500 km, the peak density occurs between 1400-1500 hours. The time of the minimum also advances gradually to later hours beginning with 1:30 hours at

200 km. The diurnal amplitude is also height dependent, increasing with altitude up to 600 km. Above 600 km, the day to night variation becomes less and less pronounced. At about 1000 km there is practically no variation from day to night. Above 1200 km, the situation is reversed and the day-time density becomes lower than the night-time. This change in the diurnal characteristics of the neutral density above 600 km can be attributed to the presence of hydrogen as an important constituent in this height range. Above 1000 km, H becomes a dominant constituent. According to Eqn. (23), n(H) varies inversely with temperature because of its escape from the gravitational field of the earth.

At present there is no reliable data for the study of the diurnal variations above 1000 km. Most of the satellites in this altitude range are of very low eccentricity and hence not suitable for this purpose. An observational data on diurnal variations in the height-range above 1000 km will prove very useful in determining the role of hydrogen in this region and the processes controlling its escape from the earth's atmosphere.

CONCLUDING REMARKS

We have shown how the present discrepancy in the phase and the diurnal amplitude of the thermospheric temperature inferred from satellite drag and incoherent back-scatter can be reconciled by introducing the concept of dynamic diffusion in the solution of the heat conduction equation. We have been able to explain most of the observed features of the diurnal characteristics in density and temperature without introducing a second heat source or involving the concept of horizontal transport. Rishbeth (1967) has suggested the latter as a mechanism for causing the phase lag between the density and temperature at thermospheric heights. At present, however, this idea is purely qualitative and it is difficult to ascertain its merit without carrying out some detailed numerical analysis.

The concept of dynamic diffusion may appear somewhat artificial, since the atmosphere cannot be arbitrarily divided into two regions with a common boundary at 120 km. The choice of 120 km, however, is not very critical and the conclusions of this paper are equally valid if the boundary is chosen at lower heights. Ideally, one should treat the entire region between 70 and 1500 km as one entity without introducing a boundary in between. The problem of getting numerical solutions in this case, however, is formidable. Because of the increased air mass below 100 km, there is an additional problem in getting converged solutions in an acceptable time. The approach adopted in this paper is a reasonable compromise. It is simple and easy to adopt for the purpose of generating atmospheric models. Thus, in computing the temperature from the density obtained from satellite drag measurements, the static diffusion model can be easily replaced by a dynamic diffusion model as outlined in this pap .

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APPENDIX

Equation (6) and (7) may be derived as follows: From (2), (3), (4) and (5), the first and zero order equations are obtained in the following form:

$$\frac{\partial \vec{v}_1}{\partial t} - 2\vec{v}_1 \times \vec{\Omega} = - \frac{1}{\rho_o} (\nabla p_1 - \rho_1 \vec{g}) \qquad \text{A-1}$$

$$\frac{\partial \rho_1}{\partial t} + \nabla . (\rho_o \vec{v}_1) = o \qquad \text{A-2}$$

$$p_1 = p_o \left(\frac{\rho_1}{\rho_o} + \frac{T_1}{T_o}\right) \qquad \text{A-3}$$

$$p_o = n_o k T_o \qquad \text{A-4}$$

$$\frac{1}{\rho_o} \nabla p_o + \vec{g} = o \qquad \text{A-5}$$

From A-1 and (5), the expression for the rth harmonic in velocity is given by

$$\rho_o \vec{v}_{1r} = \frac{1}{\left(1 - \frac{4\omega^2}{r^2 \sigma^2}\right) ir\sigma} \left\{ - \frac{4}{r^2 \sigma^2} (\vec{\omega} . \vec{A}_{1r}) \vec{\omega} + \vec{A}_{1r} - \frac{2\vec{\omega}}{ir\sigma} \times \vec{A}_{1r} \right\} \qquad \text{A-6}$$

where

$$\vec{A}_{1r} = - \nabla p_{1r} + \rho_{1r} \vec{g}$$

Substituting (A-6) in (A-2) and assuming the atmosphere to be horizontally stratified so that all the horizontal gradients may be neglected compared to the vertical, the following differential equation is obtained

$$\frac{\partial^2 \rho_{1r}}{\partial z^2} + \frac{1}{H}\frac{\partial \rho_{1r}}{\partial z} + \frac{K_r}{gH}\rho_{1r} = -\frac{\rho_o}{H^2}\frac{T_{1r}}{T_o} \qquad \text{A-7}$$

where $K_r = \dfrac{r^2\sigma^2\left(1 - \frac{4\omega^2}{r^2\sigma^2}\right)}{\left(1 - \frac{4\omega^2}{r^2\sigma^2}\sin^2\theta\right)}$

and $H = \dfrac{kT_o}{mg}$ with θ the geographic latitude

In deriving A-7, the altitude variations of T_o and T_{1r} have been neglected for simplicity. The solution of A-7 is simply given by

$$\rho_{1r} = -\frac{\rho_{oo}T_{1r}}{T_o}\left(\frac{g}{K_r H}\right)e^{-z/H} + Ae^{\alpha_1 z} + Be^{\alpha_2 z} \qquad \text{A-8}$$

with ρ_{oo} the density of the lower boundary and

$$\alpha_1 = -\frac{1}{2H} + \frac{1}{2}\left(\frac{1}{H^2} - \frac{4K_r}{gH}\right)^{1/2},$$

$$\alpha_2 = -\frac{1}{2H} - \frac{1}{2}\left(\frac{1}{H^2} - \frac{4K_r}{gH}\right)^{1/2}$$

If the sun creates the main driving force, σ can be taken to be equal to ω; in this case the expression for α_1 and α_2 can be further simplified. Since K_r is of the order of ω^2 [except when $(1 - \frac{4\omega^2}{r^2\sigma^2}\sin^2\theta \approx 0]$ and $H \sim 10$ km, it is easy to verify that $\frac{1}{H} \gg \left|\frac{4K_r}{g}\right|$. Thus, $\alpha_1 \simeq -\frac{K_r}{g}$ and $\alpha_2 \simeq -\frac{1}{H} + \frac{K_r}{g}$. The expression for the vertical velocity as given by (7) can easily be obtained from (A-8) and (A-6).

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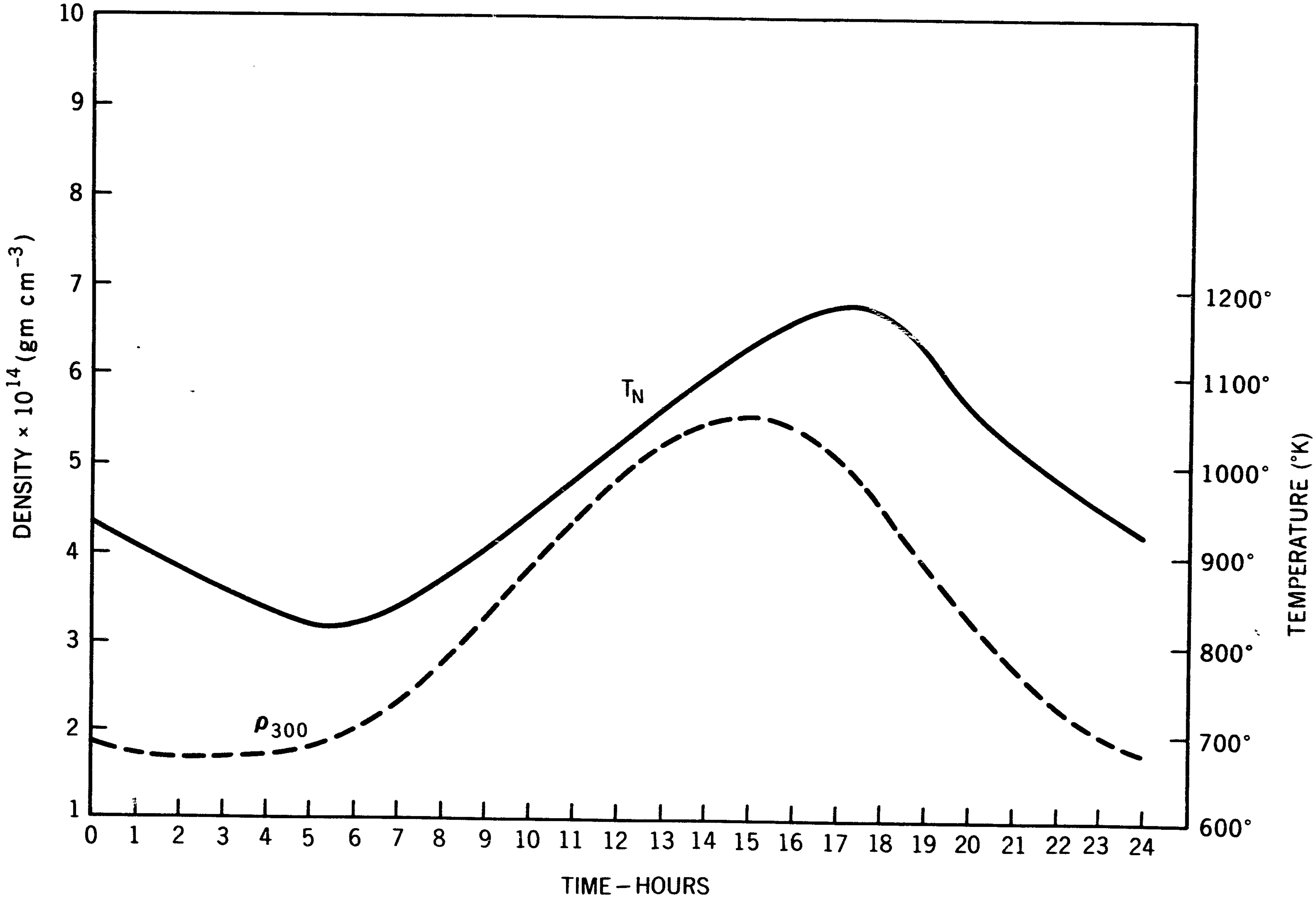

Figure 1. The diurnal variations in density and temperature at 300 km.

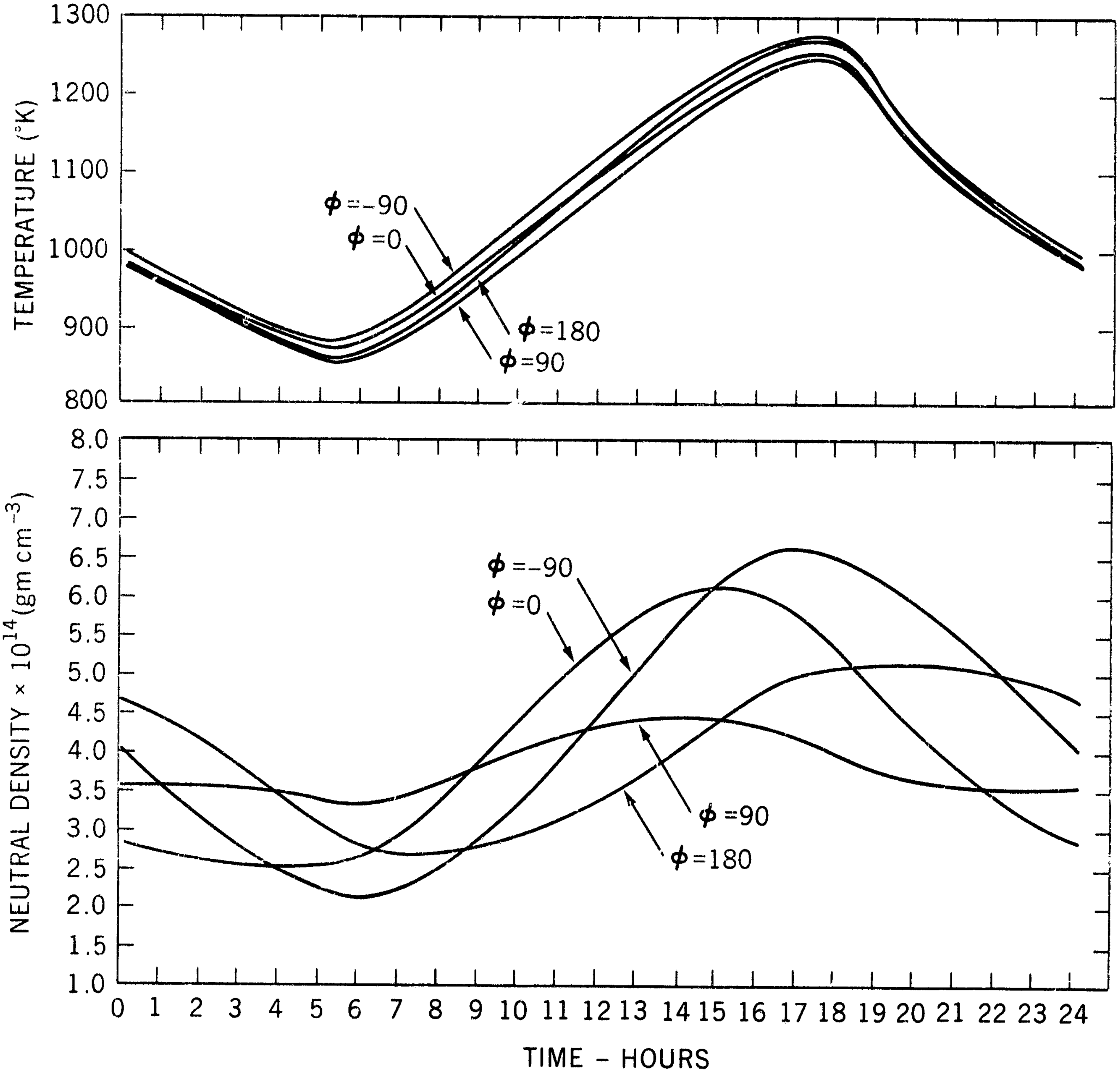

Figure 2. The diurnal variations in density and temperature at 300 km. The effect of phase change at the lower boundary.

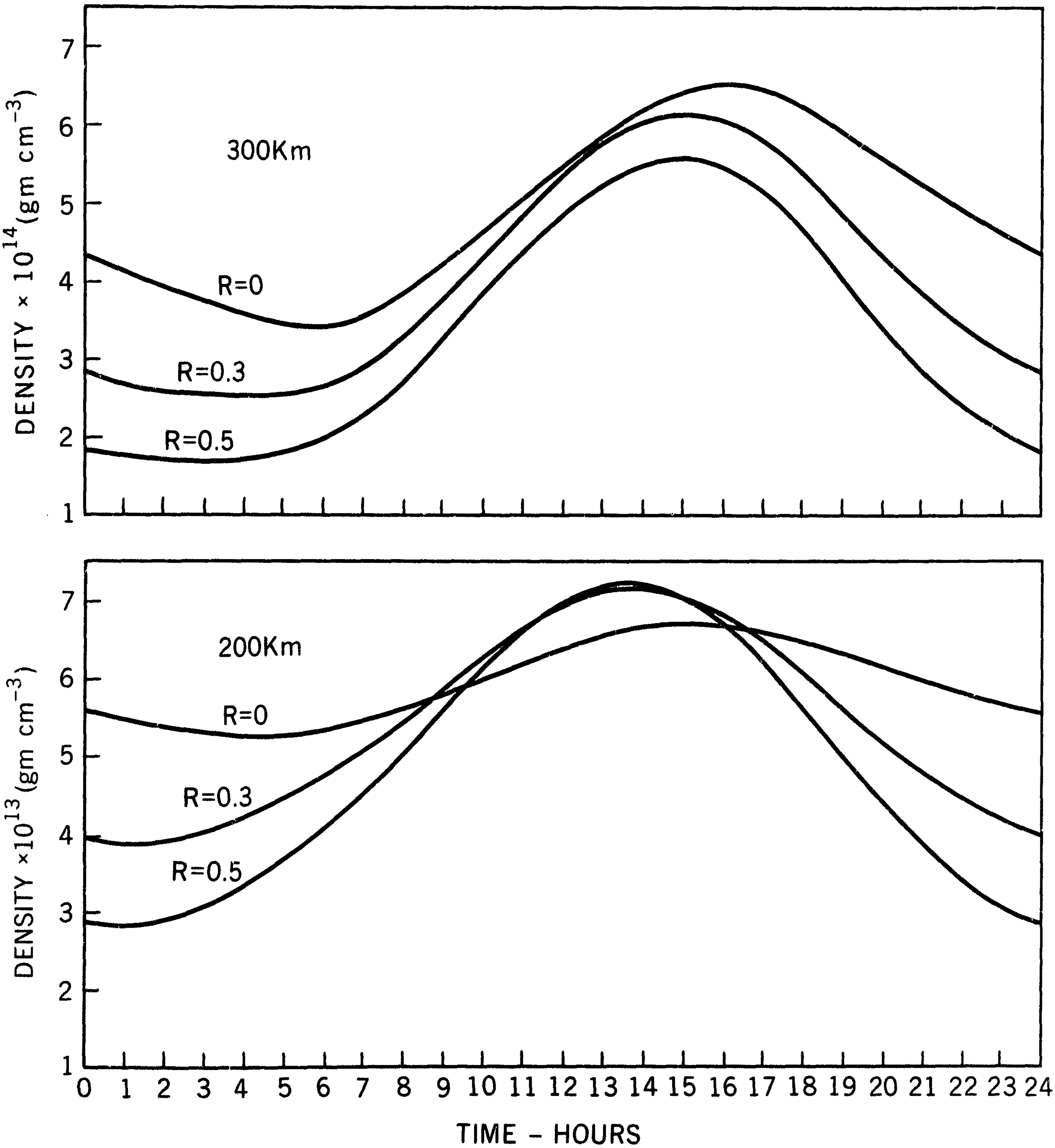

Figure 3. The diurnal variations in density at 200 and 300 km.
The effect of the diurnal amplitude change
at the lower boundary.

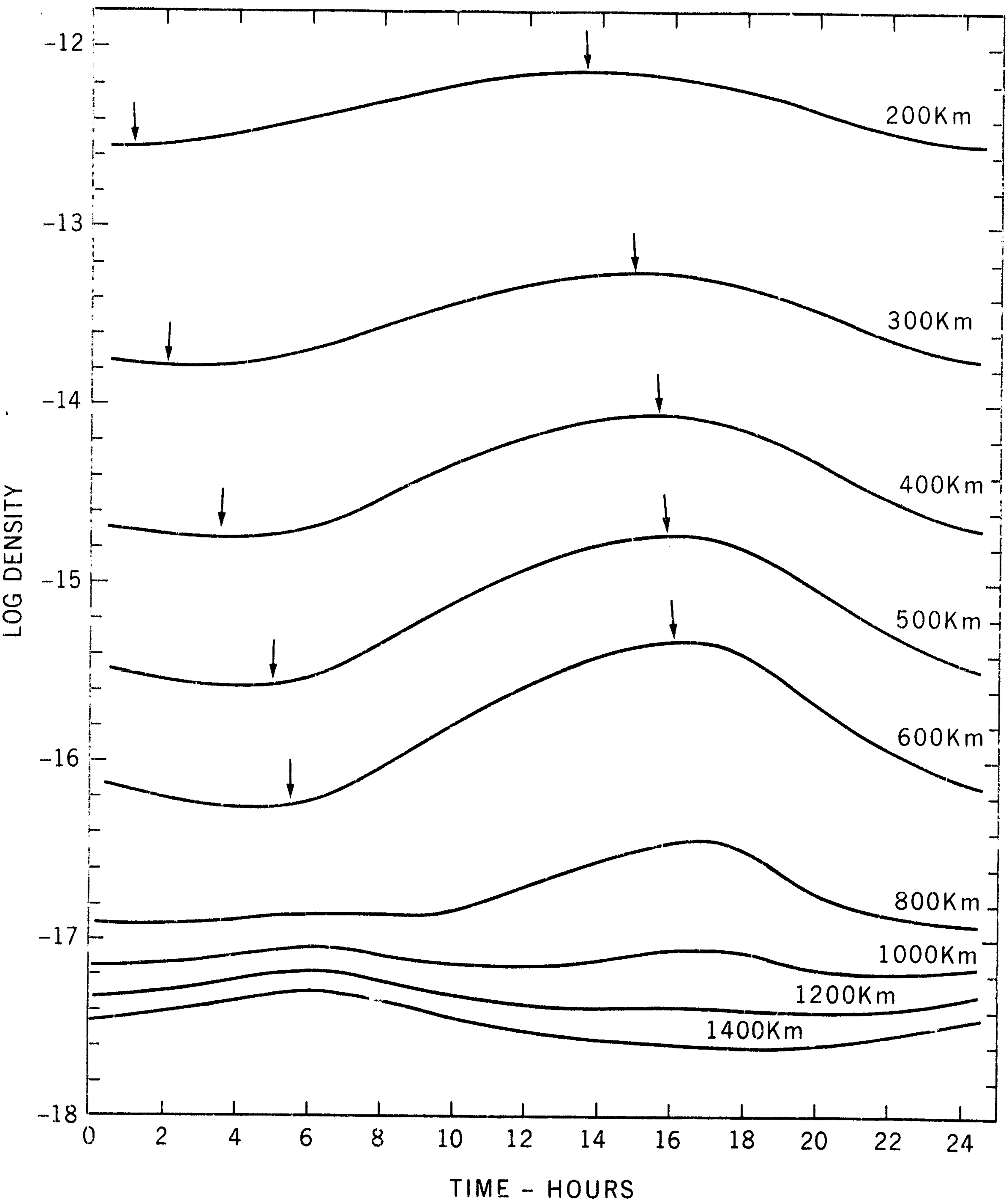

Figure 4. The diurnal variations in density in altitude range of 200 - 1400 km.