CHAPTER 10

Celestial Mechanics

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The use of two-way, phase-coherent range and Doppler tracking data from the NASA/JPL Deep Space Network was required for the accurate navigation of Mariners 6 and 7 to the vicinity of Mars. Because of the importance of these data to the field of celestial mechanics, tracking coverage of Mariners 6 and 7 from launch to Mars encounter and beyond was scheduled in such a way that new information could be obtained on the Earth/Moon mass ratio, the gravity field of Mars (in particular the mass of the planet), and the ephemerides of Mars and Earth. It was recognized that these data complemented earlier Doppler data from Mariner 4 (range measurements were not obtained from that spacecraft), and that a combination of data from the three flyby trajectories and from direct radar range measurements to the planet itself during the 1969 opposition would result in significant improvements in the ephemeris of Mars. Because of these improvements, exploration of the size, shape, and gross topography of Mars would be permitted by using the radar range measurements to its surface. In this regard, it was realized that the times of immersion and emersion as the spacecraft were occulted by Mars would provide important calibration points for the size of the planet. (See ch. 9 of this report.)

Meaningful analysis of the tracking data from Mariners 6 and 7 and the radar range measurements to Mars, which are still being obtained, will require many months to complete. Therefore, this chapter contains a discussion of only the tracking data to determine the Earth/Moon mass ratio and the mass of Mars.

EARTH/MOON MASS RATIO

The Earth revolves about the center of mass of the Earth/Moon system at a distance of about 4671 km and a speed of 12.4 m/sec. It impresses a sinusoidal curve on range and Doppler tracking data with a frequency equal to the sidereal mean motion of the Moon and with an amplitude inversely proportional to $(1+\mu^{-1})$, where μ^{-1} is the ratio of the masses of the Earth and the Moon and is approximately equal to 81.3. The principle involved in the determination of μ^{-1}

is that a value can be found that eliminates the monthly cycle in the tracking data residuals in a least-squares sense. The determination of μ^{-1} is direct and reliable; no other unknown parameter in the representation of the tracking data has a frequency anywhere near that of the Moon's orbit. The lunar ephemeris is good to seven or eight figures and does not introduce any noticeable error into μ^{-1}, which, by comparison, can be determined to the order of 10 to 20 ppm. When searching for possible sources of systematic error in the data, it is very difficult to think of anything significant with a monthly cycle, although a sufficiently great S-band propagation effect correlated with the rotation of the Sun would be close enough to cause difficulty. Periodic variations in the interplanetary medium, the only reasonable possibility, must be very small for Mariners 6 and 7, however, even if present.

For the Mariner Earth-to-Mars trajectories, the total delay for an inverse square distribution of electrons with a density of 6/cc at the Earth's distance is a maximum of only 4 m. Melbourne (ref. 10–1) has suggested that a 28-day sinusoidal solar flux variation of 0.1 percent with the appropriate phase could produce an error of about 0.001 in μ^{-1} because of a similar variation in the solar radiation pressure on the spacecraft. The agreement of the several interplanetary spacecraft, however, seems to indicate that this sort of systematic error is not present unless the phase of the flux variation is the same for each mission, which does not seem likely. Data derived from the spacecraft's temperature control flux monitor, which measures relative variations in solar flux to a better than 0.1 percent accuracy, should determine whether a variation is present that could significantly bias the solutions for μ^{-1}.

Table 10–I shows the results on the Earth/Moon mass ratio, as determined from 12 weeks of Mariner 6 Doppler data[1] and about 11 weeks of Mariner 7 Doppler data.[2] Results from Mariner 2 (ref. 10–2), Mariner 4 (ref. 10–3), Mariner 5 (ref. 10–4), and Pioneers 6 and 7 (ref. 10–5) are also given. The deviations $\mu^{-1}-\bar{\mu}^{-1}$ from the arithmetic mean $\bar{\mu}^{-1}$ of the Mariner and Pioneer values are tabulated for each spacecraft.

A solution obtained by combining data from Rangers 6 through 9 (ref. 10–6) is also shown, although the gravitational constant G_M for the Moon was determined directly by the Ranger impact trajectories; μ^{-1} must be computed with an assumed value for the geometric gravitational constant G_E, which is

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[1] From May 4 to July 28, 1969.

[2] From May 8 to July 22, 1969.

Table 10–I.—Determinations of Earth/Moon mass ratio μ^{-1}, as determined from Mariner, Pioneer, and Ranger tracking data

Spacecraft	μ^{-1}	$\mu^{-1}-\bar{\mu}^{-1}$
Mariner 2 (Venus)	81.3001±0.0013	−0.0007
Mariner 4 (Mars)	81.3015± .0017	.0007
Mariner 5 (Venus)	81.3006± .0008	−.0002
Mariner 6 (Mars)	81.3011± .0015	.0003
Mariner 7 (Mars)	81.2997± .0015	−.0011
Pioneer 6	81.3005± .0007	−.0003
Pioneer 7	81.3021± .0004	.0013
Combined Rangers	81.3035± .0012	.0027

defined by $\mu^{-1}=G_E/G_M$. The error in G_E, however, is about 1 ppm; therefore, the percentage error in the value of μ^{-1} determined from Ranger is almost equal to the percentage error in G_M. The fundamentally different method of determining μ^{-1} from Ranger impact trajectories, as opposed to using interplanetary trajectories such as those of Mariner and Pioneer, is reflected in a significant difference in the values. An arithmetic mean of the seven Mariner and Pioneer values results in $\mu^{-1}=81.3008$, with an rms deviation from the mean of 0.0008. The Ranger value, however, differs from this mean value by 0.0027. There is no reason to suspect that the direct determination of μ^{-1} from Mariner and Pioneer spacecraft is subject to systematic errors of a size that would adjust the value to that provided by Ranger.

The Mariner data have been processed with two computer programs: one in single precision (about 8 figures) and the other in double precision (about 16 figures), with both heliocentric and geocentric formulations of the equations of condition in the method of weighted least squares. The value of μ^{-1} is essentially the same in all cases; therefore, in the authors' opinions, it is necessary to perform new reductions of data from the lunar spacecraft. Since the Ranger solution for μ^{-1} was obtained, about 2½ yr ago, several conditions have changed. First, knowledge of the gravity field and ephemeris of the Moon has increased significantly through analyses of Lunar Orbiter data (ref. 10–7). In addition, it is now possible to perform the necessary computations in double precision; those of 2½ yr ago were performed only in single precision. In any case, the reconciliation of the μ^{-1} values derived from lunar and interplanetary spacecraft is receiving increased attention because of the results of Mariners 6 and 7, which argue for a μ^{-1} value closer to 81.300 or 81.301 than to 81.303 or 81.304, as suggested by the combined Ranger data.

MASS DETERMINATION

For both Mariners 6 and 7, the determination of the mass of Mars is complicated by nongravitational forces acting on the spacecraft. One channel of the infrared spectrometer (IRS), for example, operates in a cryogenic environment produced by expelling hydrogen and nitrogen gases from a pressure-regulated system. This gas expulsion imparts a force of 100 dyn or more to the spacecraft and produces a velocity change in its trajectory on the order of 0.1 m/sec. In normal operation, the system starts jetting about 35 min before encounter (the closest approach to Mars) and continues at a constant pressure through encounter for a period of about 80 min. After this period, the gas is allowed to escape into space without pressure regulation; about 5 hr are required for the gas to decay to an insignificant level at approximately an exponential rate.

On Mariner 6, the system did not operate normally; as a result, the force on the spacecraft acted over a period of 4 or 5 days after encounter instead of over the intended period of a few hours. The system seems to have operated normally on Mariner 7. However, this spacecraft was affected 6 days before encounter by an unknown event, which imparted a velocity increment to the trajectory on the order of 1 m/sec, or a change of 4×10^7 g cm/sec in momentum. It is possible that the spacecraft was hit by a meteoroid; whatever the reason, however, the spacecraft attitude was disrupted. Beyond this point, it is not clear whether additional unknown forces were also acting on the spacecraft; one possibility is that the spacecraft battery, which gave indication of a malfunction following the time of the event, was punctured by the impact and that the leaking electrolyte imparted a low-thrust force over a period of about 1 day. Because of uncertainties regarding the event and the resulting forces on the spacecraft in the few days before encounter, no results from Mariner 7 are available at this time.

Mariner 6 did not experience an event of the same type as Mariner 7. Therefore, data acquired before the initiation of the IRS cryogenic system blowdown can be used to obtain a value for the mass of Mars (fig. 10–1). However, data from about 35 min before encounter and beyond will be analyzed when, similar to Mariner 7, engineering telemetry data can be combined with the tracking data in an analysis of the nongravitational forces acting on the spacecraft.

To obtain the mass of Mars, we fit 670 Doppler points from July 26, 5 days before encounter, to July 31, at 04:39:57 G.m.t., about 7 min before initiation of IRS cooling. The parameters in the least-squares solution were the position

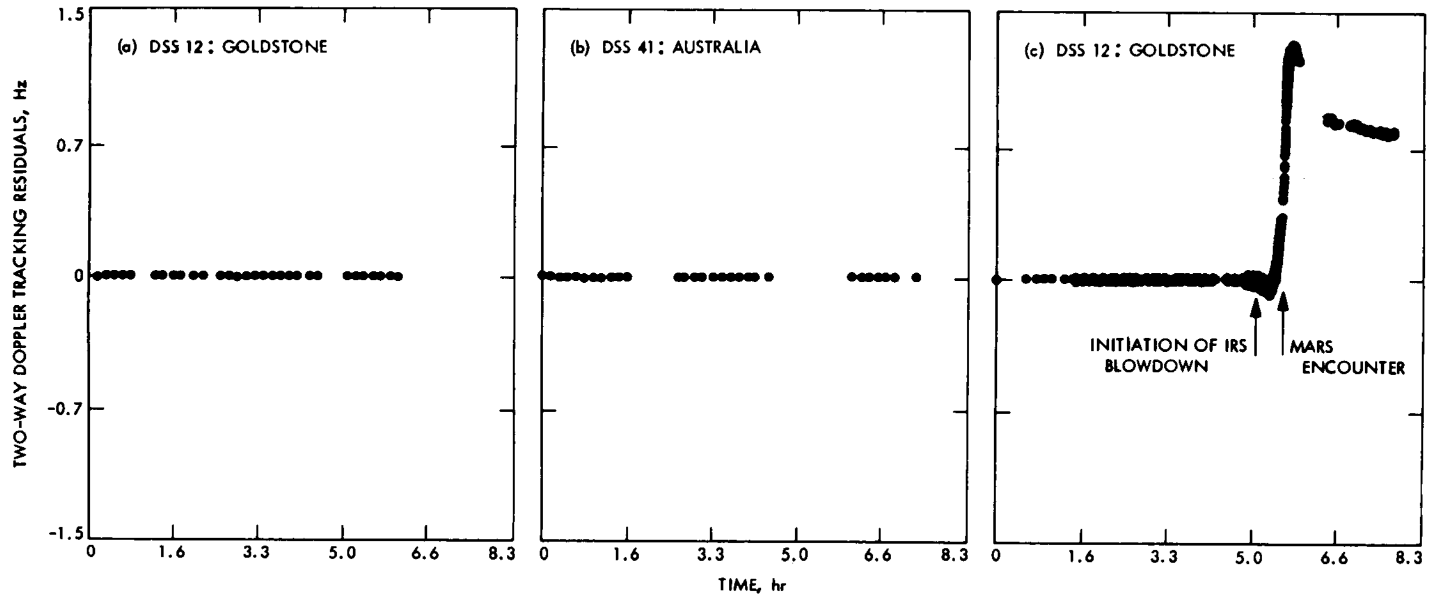

FIGURE 10–1. – Mariner 6 two-way Doppler tracking residuals. Doppler and range data used in the fit cover the period from 5 days before encounter to a few minutes before initiation of IRS gas blowdown. The three plots show the precision with which the orbit was known until the perturbation was introduced by the jetting gas. The Doppler count time C_c was 600 sec until 01:00 G.m.t. on July 31 and then 60 sec until 04:35 G.m.t., at which time T_c became 10 sec. At 06:17 G.m.t., T_c again became 60 sec. The noise on the residuals is proportional to $T_c^{1/2}$. The abscissa is given as time in hours as shown below. (a) From July 29, 1969, 23:48 G.m.t. (b) From July 30, 1969, 08:22 G.m.t. (c) From July 30, 1969, 23:44 G.m.t.

and velocity of the spacecraft at the epoch, the gravitational constant for Mars $G_{M♂}$, the distance off the Earth's axis of rotation, and the longitude of each of the five tracking stations for which data were available. The statistical properties of the fit are summarized in table 10–II. The solution for $G_{M♂}$ is 42828.22 ± 1.83 km^3/sec^2.

A second solution was performed in which the Doppler data were fit along with nine range measurements taken on July 27 from DSS 14 at Goldstone, Calif., with the experimental ranging system installed for Mariner 5. The changes to the statistical properties of the Doppler fit are shown in table 10–III, and the nine range residuals are shown in table 10–IV. As shown in table 10–III, the Doppler fit is not changed significantly by the introduction of range data. The value of $G_{M♂}$ for the range and Doppler fit is 42828.48 ± 1.38 km^3/sec^2, which is not significantly different from the Doppler-only value, either in size or in estimated standard error. This estimate was computed with a standard error of 62×10^{-9} sec on each range point and 0.05 Hz on Doppler data sampled at 1-min intervals.

Although the introduction of range data into the fit does not affect appreciably the solution for the mass of Mars, it does help to determine the orbit of Mariner. This has important implications to other experiments, such as the

Table 10–II.—Statistical properties of fit to preencounter Doppler data

Deep Space Station	Number of points	Data interval, G.m.t.	Residuals, Hz[a]	
			Mean	rms
41 (Australia)	200	July 26, 06:49 to July 30, 15:41	0.000006	0.0028
51 (South Africa)	26	July 27, 16:28 to July 27, 23:02	.000164	.0019
62 (Spain)	80	July 26, 17:29 to July 30, 22:31	–.000086	.0025
12 (California)	311	July 26, 01:33 to July 31, 04:40	–.000105	.0043
14 (California)	53	July 26, 00:35 to July 30, 06:38	–.000098	.0024

[a] Units are Hz at S-band and can be converted approximately to millimeters per second by multiplying by 67.

Table 10–III.—Statistical properties of fit to preencounter range and Doppler data

Deep Space Station	Number of points	Data interval, G.m.t.	Residuals, Hz	
			Mean	rms
41 (Australia)	200	July 26, 06:49 to July 30, 15:41	0.000008	0.0028
51 (South Africa)	26	July 27, 16:28 to July 27, 23:02	.000164	.0019
62 (Spain)	80	July 26, 17:29 to July 30, 22:31	–.000088	.0025
12 (California)	311	July 26, 01:33 to July 31, 04:40	.000127	.0042
14 (California)	53	July 26, 00:35 to July 30, 06:38	.000090	.0024

Table 10–IV.—Range residuals from fit to preencounter range and Doppler data

Reception on July 27, 1969, G.m.t.	Residual, 10^{-9} sec [a]
01:12:02	–42.1
01:59:02	62.7
02:29:02	5.9
02:59:02	6.2
03:29:02	2.9
03:59:02	–16.2
04:29:02	5.5
06:06:02	–8.2
06:36:02	–19.5

[a] Units can be converted to meters in one-way range by multiplying by 0.15. The mean residual is -0.30 and the rms of the 9 residuals is 26.9 or 4.0 m.

S-band occultation and the ultraviolet spectroscopy, both of which will require good orbital data to complete a final analysis. Precise knowledge of the orbit is important in obtaining information on the ephemeris of Mars and will play a major role in later analyses of the tracking data when the nongravitational forces on Mariners 6 and 7 are understood better. We believe, at present, that the Mars-centered orbit for Mariner 6 from the fit to range and Doppler data can predict events along the trajectory to better than 1 sec in time or to better than 8 km along the flightpath. For example, the best estimate of the time of closest approach to Mars is on July 31, 1969, 05:19:06 G.m.t.; it is estimated that this value should be good to ±1 sec.

The only source for an accurate determination of the mass of Mars, other than Mariners 6 and 7, is from Mariner 4. A recent analysis (ref. 10–8) of Doppler data taken over a 10-day interval centered about closest approach has yielded a value for $G_{M♂}$ of 42828.32 ± 0.13 km^3/sec^2. The masses determined from Mariners 4 and 6 are, therefore, in agreement, and there is good reason to accept the spacecraft-determined value when performing analyses with other planetary data.

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APPENDIX A

Preliminary Portraits

A sampling of partially processed pictures obtained in the television experiment on Mariners 6 and 7 was promptly released for publication. The six shown here indicate the details discernible in even raw prints. Further processing is expected to increase the value of these pictures in quantitative studies of Martian phenomena.

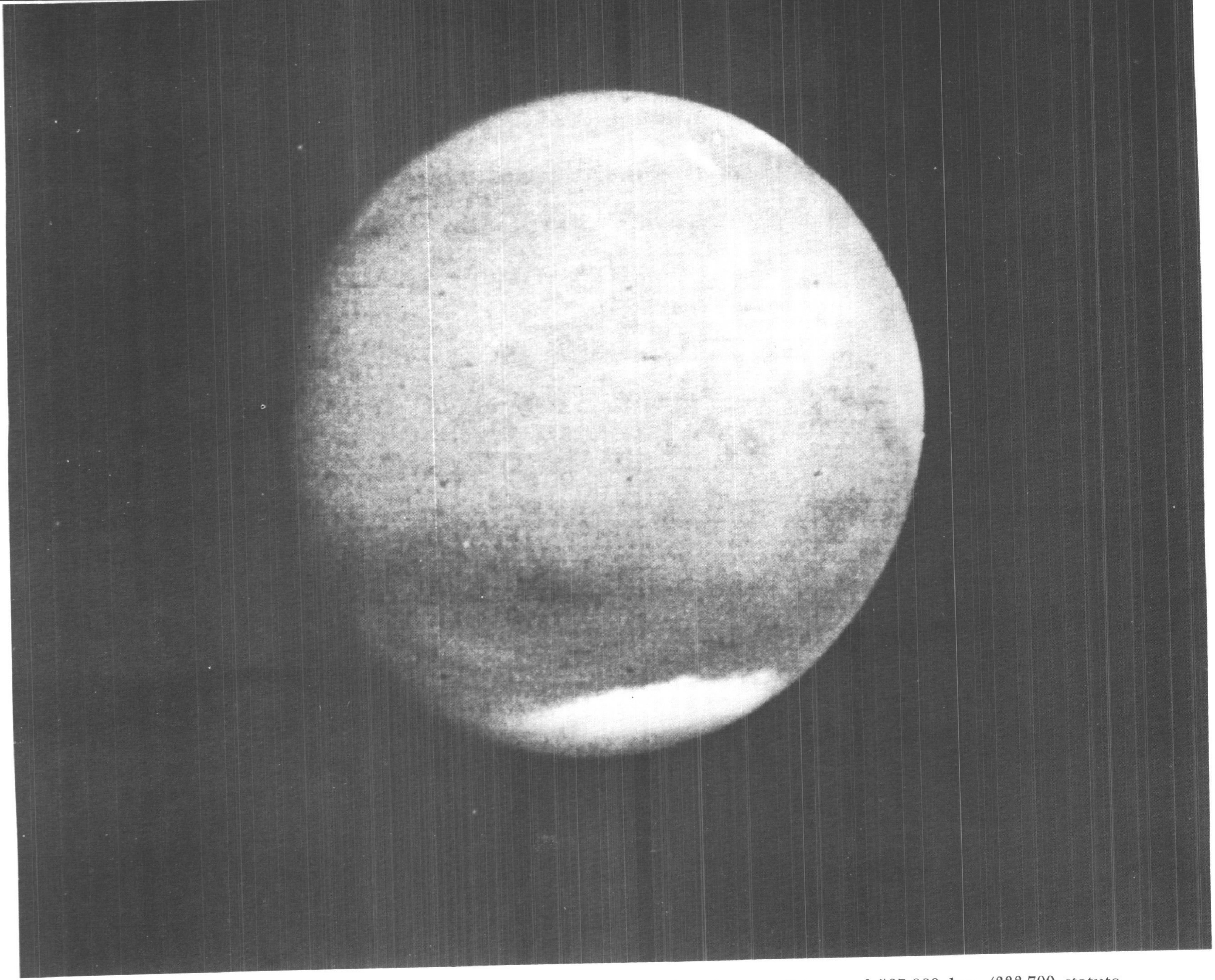
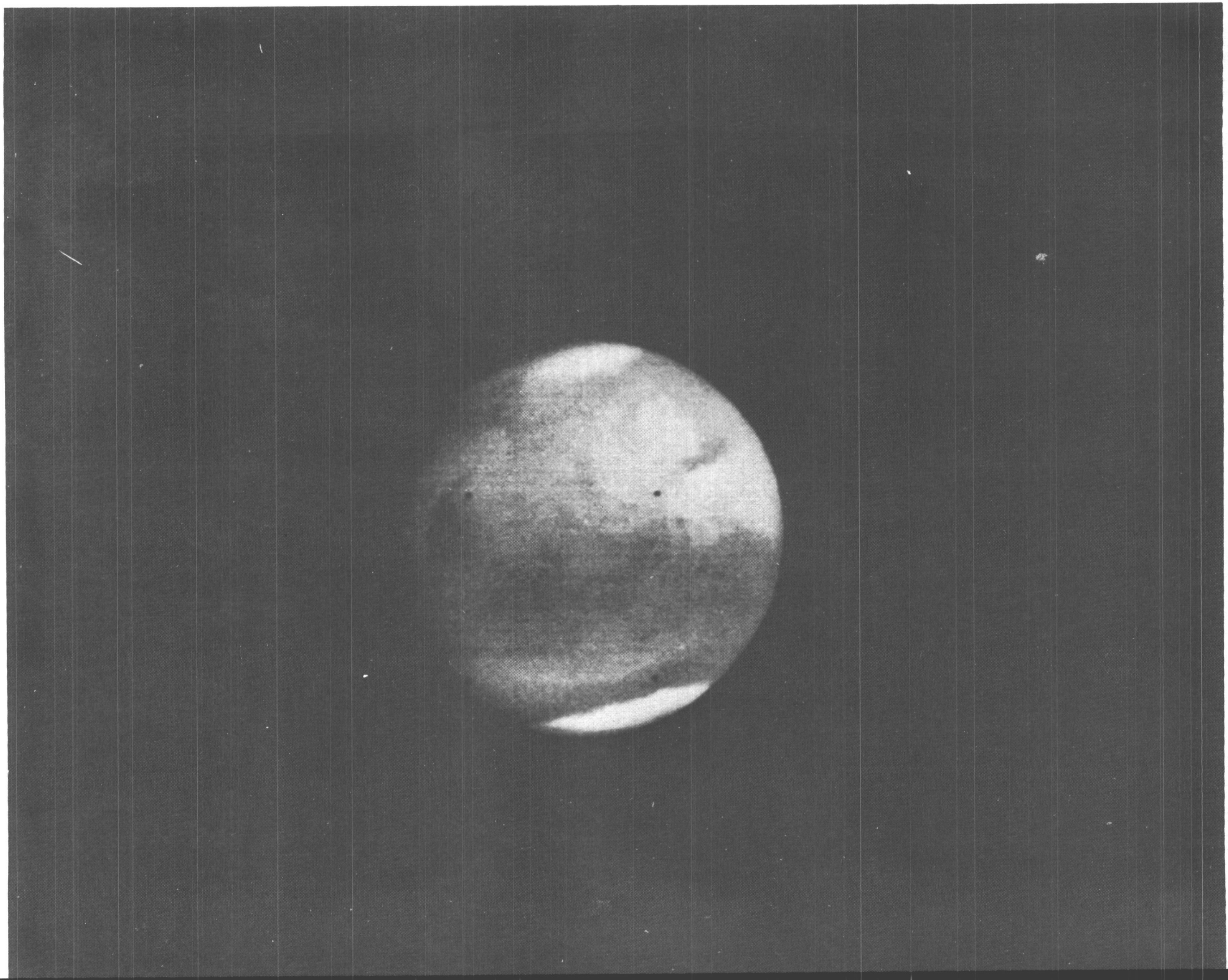


FIGURE A-1. – A far-encounter picture from Mariner 6 shows Mars from a distance of 537 000 km (333 700 statute miles) with the morning terminator at the left. The white rim with a central spot slightly above center has been

FIGURE A-2. – This Mariner 7 far-encounter picture was taken 861 850 km (535 650 statute miles) from Mars. In the upper right is a bright ring; this is the area known as Elysium. Toward the bottom the irregularity of the border around the south polar cap, which was slowly shrinking, can be seen.



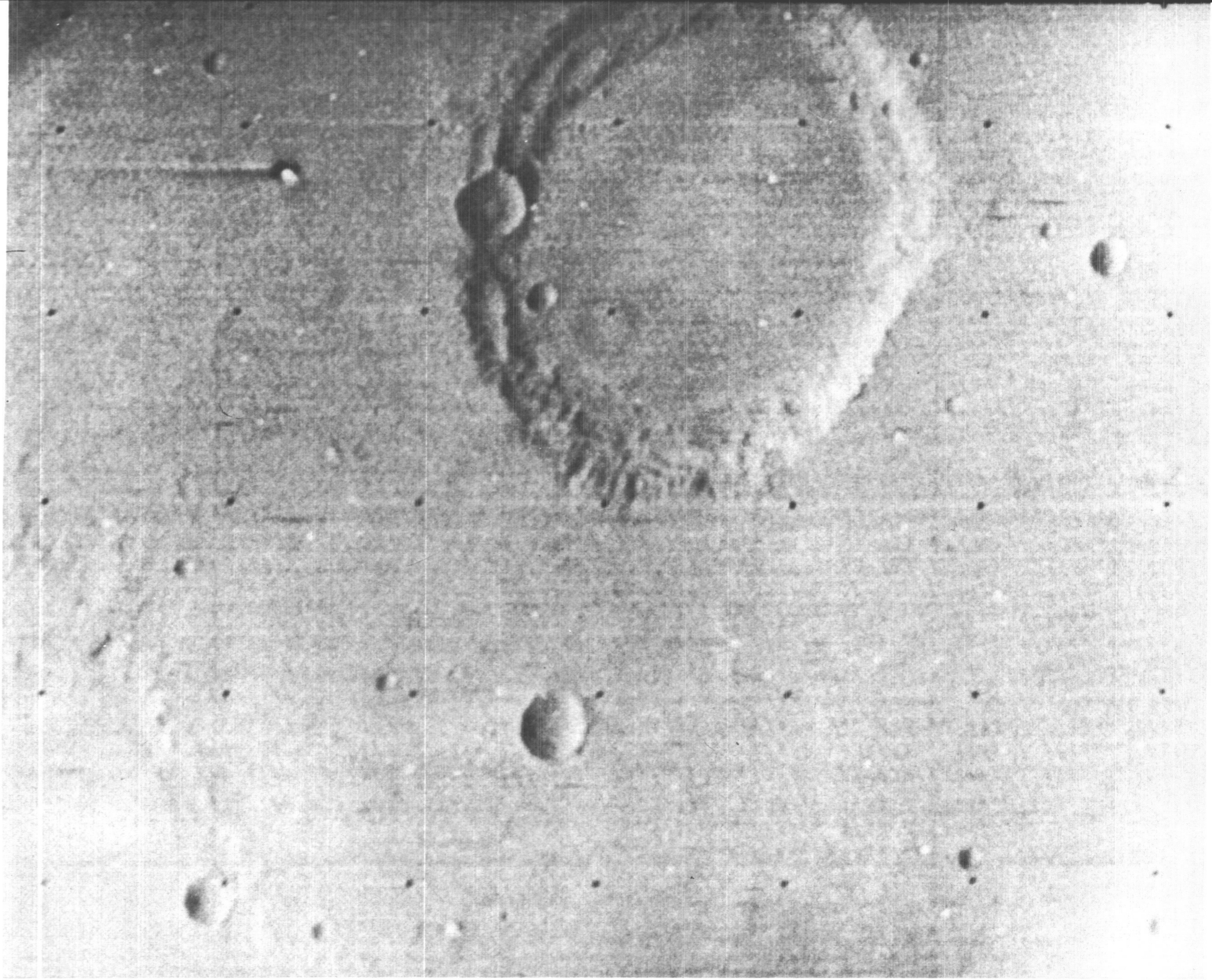
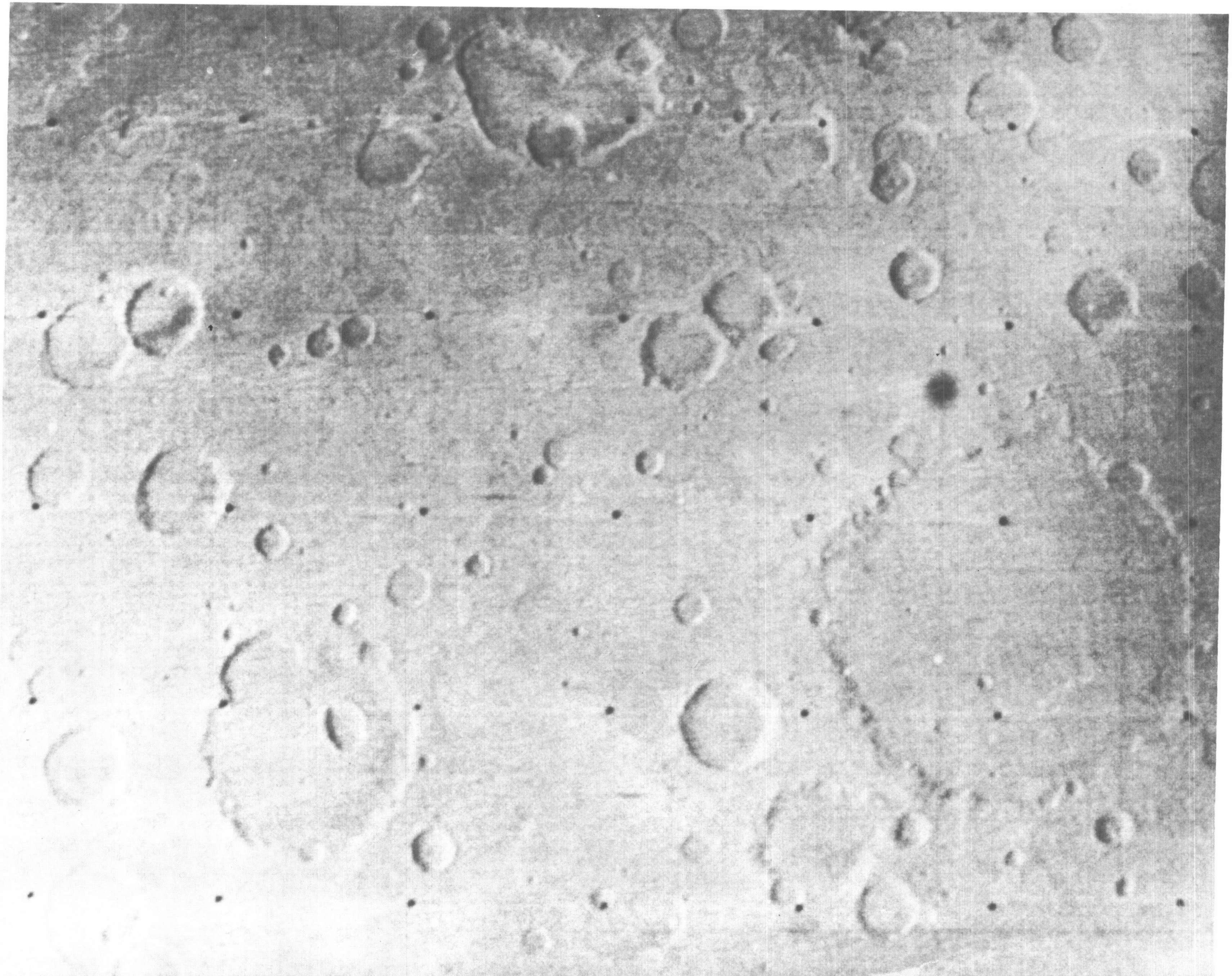


FIGURE A–3. – Mariner 6's narrow-angle camera took this near-encounter picture of a 63- by 48-mile area from a distance of 2300 miles. It shows features similar to findings on the Moon. The large crater is 24 miles wide. *On*

FIGURE A–4. – Mariner 6's wide-angle camera took this picture of a 560- by 430-mile area of Mars from a distance of 2150 miles. The regularly spaced black dots are reference points. At least 100 craters, including some with crudely polygonal outlines, are recorded here.



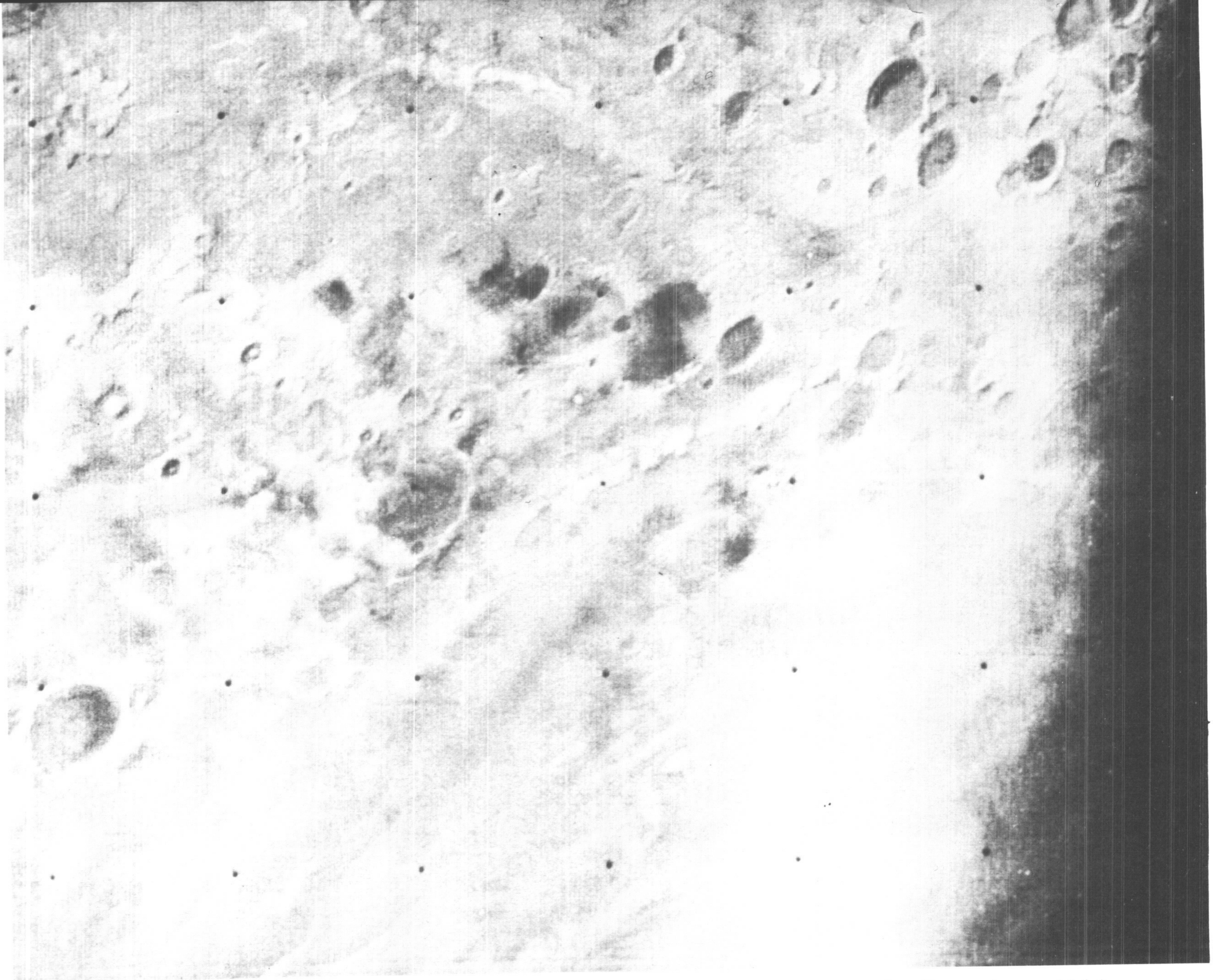


FIGURE A–5. – Mariner 7 sent back this view of the south polar region with the sunset shadow line at the east (right). It includes craters of various sizes and shapes, as well as linear and blotchy features. Two craters in the right

FIGURE A–6. – Mariner 7 also obtained this oblique, high-resolution picture of the craters forming the "giant footprint." The Sun was 8° above the local horizon off to the northwest (upper left). The area shown, about 75° south latitude, is about 85 by 200 miles.