NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Technical Memorandum 33-422

A Digital Technique for Determining 1/3-Octave Sound-Pressure Levels With a More Uniform Confidence Level

John W. Shipley
Ronald A. Slusser

JET PROPULSION LABORATORY
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Preface

The work described in this report was performed by the Environmental Sciences Division of the Jet Propulsion Laboratory. The report was originally presented as a paper at a meeting of the Acoustical Society of America held in Cleveland, Ohio, on Nov. 22, 1968.

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Abstract

A computer program written to analyze acoustical test data is described. The program employs a fast Fourier subroutine to calculate the discrete Fourier coefficients that transform the time-domain data to frequency-domain data.

To achieve a more equal confidence level between the upper and lower 1/3 octaves, the digital data record is sampled at a high rate, and the fast Fourier coefficients are calculated. The same digital data record is then sampled at a much lower rate to provide greater low-frequency resolution for the lower 1/3 octaves.

Digital windowing techniques are considered, and their effect on spectral representation is compared with that of analogous analysis methods.

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I. Introduction

It is often desirable to obtain sound-pressure level values that have almost equal confidence levels and similar spectral shapes for each 1/3-octave window. Unfortunately, both digital sampling theory and algorithms for computing Fourier coefficients, such as the fast Fourier transform (FFT), depend upon equal spacing of data in time and present data equally spaced in frequency.

II. Description of the Problem

If the acoustical information is confined to the 1/3 octaves centered between 20 and 5000 Hz, and if a 512-point FFT is used, the resulting resolution is approximately 20 Hz. This result is unsatisfactory because there are no data points in the 25-, 31.5-, and 50-Hz 1/3 octaves (see Table 1). Even if more than one data set is used, no data will result in these 1/3 octaves. Also, computer time and storage are wasted in calculating the sound-pressure levels in the higher 1/3 octaves, since there are so many degrees of freedom.

If a higher resolution (more than 512 points) were used to get data points in the lower 1/3 octaves, the waste of computer time in the higher 1/3 octaves would be even greater.

III. Proposed Solution

The following alternative is proposed: Sample the digitized data twice. Sample at 10,000 Hz for the higher 1/3 octaves and then resample the data at 500 Hz for the lower 1/3 octaves. A second operation is required, however: the data sampled at the lower rate must be digitally low-pass filtered to avoid folding of information from higher frequencies that are present. This step is not necessary for data sampled at the higher frequency, since an analog filter can be used when the analog data are digitized. The second column in Table 1 shows the result in degrees of freedom for one data set when a 256-point low-sampling-rate transform is matched at the 200-Hz 1/3 octave with a 512-point high-sampling-rate transform. Table 2 shows the confidence levels resulting if 16 data sets are averaged in each case. The confidence level

Table 1. Comparison of degrees of freedom (one data set)

1/3-octave center frequency, Hz	Standard 20-Hz resolution	Improved 2- to 20-Hz resolution	
		Low bands	High bands
20	2	6	
25	0	6	
31.5	0	6	
40	2	10	
50	0	12	
63	2	14	
80	2	18	
100	2	24	2
125	4	28	4
160	2	36	2
200	6	48 ⟷	6
250	6	60	6
315	6		6
400	10		10
500	12		12
630	14		14
800	18		18
1000	24		24
1250	28		28
1600	36		36
2000	48		48
2500	60		60
3150	72		72
4000	92		92
5000	140		140

Arrow shows point at which transfer is made between high- and low-resolution calculations.

Table 2. Comparison of confidence levels (16 data sets)

1/3-octave center frequency, Hz	Standard 20-Hz resolution, %	Improved 2- to 20-Hz resolution, %	
		Low bands	High bands
20	64	88	
25	0	88	
31.5	0	88	
40	64	96	
50	0	97	
63	64	98	
80	64	98	
100	64	99	64
125	80	99	80
160	64	99	64
200	88	99 ⟷	88
250	88	99	88
315	88		88
400	96		96
500	97		97
630	98		97
800	98		98
1000	99		99
1250	99		99
1600	99		99
2000	99		99
2500	99		99
3150	99		99
4000	99		99
5000	99		

Arrow shows point at which transfer is made between high- and low-resolution calculations.

never drops lower than 88% for a ± 1-dB error in the determination of the sound-pressure level.

A computer program written to do this type of analysis performs as follows:

(1) Read in the data cards.

(2) Adjust the data for zero mean.

(3) Low-pass filter the data sampled at the 500-Hz rate.

(4) Evaluate the FFT coefficients for each data set.

(5) Han the FFT coefficients.[1]

(6) Sum and average each data set for both hanned and unhanned coefficients (16 sets each).

(7) Calculate the sound-pressure level in each 1/3 octave from 20 to 200 Hz.

(8) Match the dB sound-pressure levels in the 200-Hz 1/3 octaves calculated from the low- and high-resolution data.

[1]Hanning is a technique to keep one coefficient from influencing the adjacent coefficients, by smoothing with weights of 0.25, 0.50, and 0.25.

(9) Calculate the sound-pressure level in each 1/3 octave from 250 to 5000 Hz.

(10) Print and plot the resulting sound-pressure levels vs frequency.

Each division of the program is discussed below.

The data are read into both the high-resolution and low-resolution calculation arrays simultaneously. The first point of each data set is common to both arrays in all cases. Then, since the data for the high-resolution calculation are sampled at 1/20 of the frequency that the low-resolution calculation uses (i.e., 500 Hz instead of 10,000 Hz), the 20th point of the low-resolution calculation corresponds to the second point used in the high-resolution calculation.

The dc mean must be subtracted from each point of each data set to ensure that the mean of each data set is zero. This is necessary to avoid the calculation of a dc component in the FFT, since such a component would also be shown in adjacent points.

The details of the low-pass filter are shown in Fig. 1. Such a filter forms 21 element sums of the points in the original sample. The sum is weighted by the center lobe of the sin x/x function. The effect of the filter gain is accounted for when the final sound-pressure levels are calculated.

The filter transfer function $H(f)$ is expressed as follows:

$$H(f) = \frac{1}{\pi}\left(\int_0^a \frac{\sin u}{u}\,du + \int_0^b \frac{\sin u}{u}\,du\right)$$

where

$$a = \frac{\pi (f_0 + f)}{f_0}$$

$$b = \frac{\pi (f_0 - f)}{f_0}$$

u = variable of integration (dummy variable)

f_0 = filter center frequency

f = frequency

The FFT coefficients are evaluated by the method outlined by Singleton (Ref. 1) with the errors in the reference corrected. The entire program is written in FORTRAN. The coefficients were hanned by the method

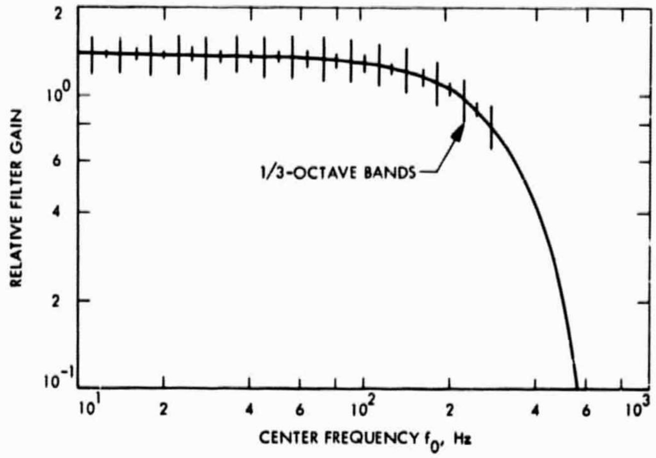

Fig. 1. Filter transfer function for filter center frequency of 500 Hz

outlined by Maling, Morrey, and Lang (Ref. 2) in their paper on digital analysis of acoustical data. The method itself is very simple and involves implementing one equation to get each new coefficient from three old ones:

$$a_n = -\frac{1}{4}x_{n-1} + \frac{1}{2}x_n - \frac{1}{4}x_{n+1}$$

Since the end coefficients ($n = 1$ and $n = 512$) are never used for calculation, hanning those coefficients need not be considered. Figure 2 shows a 1000-Hz sine wave signal analyzed with both hanned and unhanned

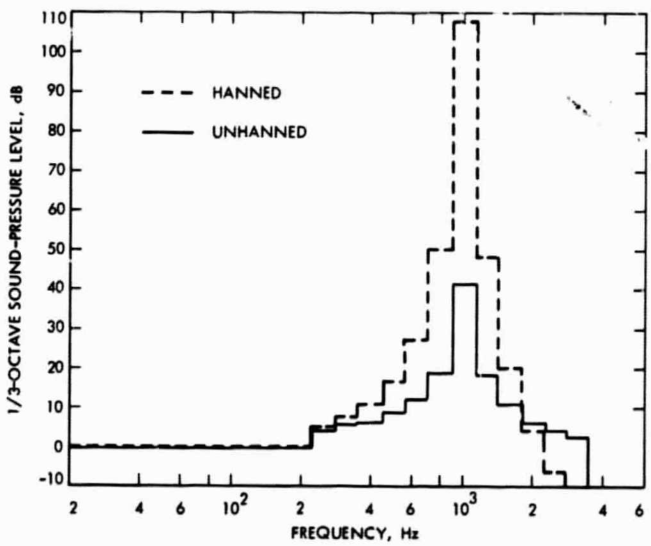

Fig. 2. Digital analysis of a synthetic 1000-Hz pure sine signal, comparing normal and hanning analyses

coefficients. In the unhanned analysis, the 1/3 octaves adjacent to the 1000-Hz 1/3-octave level are 22 dB below the 1000-Hz value, while, in the hanned version, the adjacent 1/3-octave levels are more than 57 dB below the 1000-Hz value.

After the high-resolution coefficients are used to calculate the sound-pressure levels in the 1/3-octave levels from 50 to 200 Hz, a single calculation is made for the 200-Hz, 1/3-octave level with the high-resolution coefficients. The difference in sound-pressure level in dB needed to match the calculations of high- and low-resolution coefficients is then found. Then, the low-resolution coefficients are used to calculate the sound-pressure levels in the 1/3 octaves from 250 to 5000 Hz, with the difference being added to each 1/3 octave level. The spectrum before this difference is added is shown in Fig. 3.

The final results are then printed and plotted. A sample of the plotted output is shown in Fig. 4.

IV. Summary

A computer program has been written to provide a more equal confidence level between the upper and lower 1/3 octaves when analyzing acoustical data. This result is achieved in a two-step process that combines both high-rate and low-rate sampling and uses a fast Fourier subroutine to transform the time-domain data to frequency-domain data.

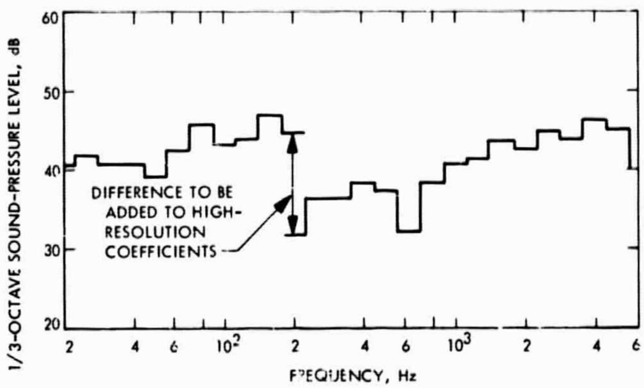

Fig. 3. Digital determination of white noise sound-pressure level before matching levels. The overall sound-pressure level was 70 dB

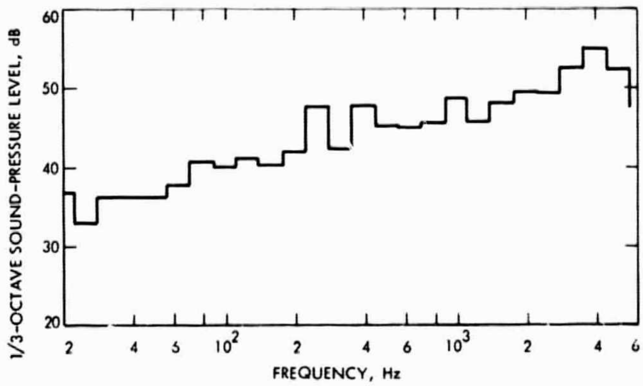

Fig. 4. Digital determination of white noise sound-pressure level after matching levels. The overall sound-pressure level was 73.5 dB

References

1. Singleton, R. C., "A Method for Computing the Fast Fourier Transform With Auxiliary Memory and Limited High-Speed Storage," *IEEE Trans Audio Electroacoustics*, Vol. AU-15, No. 2, p. 91, June 1967.

2. Maling, G. C., Morrey, W. T., and Lang, W. W., "Digital Determination of Third-Octave and Full-Octave Spectra of Acoustical Noise," *IEEE Trans Audio Electroacoustics*, Vol. AU-15, No. 2, p. 98, June 1967.