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ORDINARY-MODE ELECTROMAGNETIC INSTABILITY IN COUNTERSTREAMING PLASMAS WITH ANISOTROPIC TEMPERATURES

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ABSTRACT

The instability of the electromagnetic linearly polarized mode propagating perpendicular to a uniform magnetic field is studied by using the Vlasov equation for a counterstreaming electron plasma with anisotropic temperatures. An instability occurs if the streaming velocity exceeds a certain threshold value which can be below that required to excite the electrostatic two-stream instability. It is found that temperature perpendicular to the field has a stabilizing effect, while parallel temperature enhances the electromagnetic instability. Typical growth rates are of the order of the electron cyclotron frequency.

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ORDINARY-MODE ELECTROMAGNETIC INSTABILITY IN COUNTERSTREAMING PLASMAS WITH ANISOTROPIC TEMPERATURES

I. INTRODUCTION

A number of papers have recently appeared in the literature on the linear stability of waves propagating perpendicular to the direction of stream motion in plasmas with relative streaming.[1-5] In particular, for a system of two identical electron plasmas, counterstreaming along an external uniform magnetic field, it has been found that in addition to the well-known electrostatic (longitudinal) two-stream instability with propagation vector in the direction of streaming, the electromagnetic (transverse) linearly polarized mode propagating perpendicular to the magnetic field can become unstable.[3,4] The cold plasma theory predicts that the threshold streaming velocity required for the excitation of this "modified ordinary mode" is $c\Omega/\omega_p$, where c is the velocity of light, Ω and ω_p are the electron cyclotron and plasma frequencies respectively.[3] The effect of temperature perpendicular to the field has also been investigated by means of the fluid equations, with the result that the electromagnetic instability is stabilized by perpendicular temperature.[4]

The purpose of this paper is to study the modified ordinary mode instability by taking into account temperatures both perpendicular and parallel to the magnetic field ($T_\perp$ and $T_\parallel$ respectively). Our analysis is based on the Vlasov equation for counterstreaming plasmas with anisotropic bi-Maxwellian velocity distributions. It is found that an electromagnetic instability can be excited at streaming velocities which can be below the threshold value required for the electrostatic two-stream instability. It is shown that while $T_\perp$ is stabilizing, in qualitative agreement with the result of the macroscopic theory, $T_\parallel$ enhances the instability. This is in marked contrast to the electrostatic two-stream instability, which is stabilized by $T_\parallel$ but independent of $T_\perp$.[6]

Typical growth rates for the electromagnetic instability are of the order of the electron cyclotron frequency.

The dispersion relation for the linearly polarized mode for a counterstreaming bi-Maxwellian plasma is given in Section II. Section III presents the instability analysis and some graphical illustrations of the results. Finally, in Section IV the results are summarized.

II. DISPERSION RELATION

Let us consider a homogeneous, infinite plasma in a static, uniform magnetic field $\underline{B}_0$. When the plasma is sufficiently hot, the interparticle collisions may be neglected and our analysis is based on the Vlasov equation for each charged-particle species combined with Maxwell's equations. The system is linearized and Fourier transformed in space and time by assuming perturbations propagating perpendicular to the magnetic field. We shall assume that the unperturbed velocity distribution functions F_{oj} depend only upon the parallel and perpendicular components of the particle velocity ($v_\parallel$ and $v_\perp$, where subscripts $\parallel$ and $\perp$ refer to directions with respect to $\underline{B}_0$) and are even functions of $v_\parallel$. In such a case, the general dispersion relation for waves propagating perpendicular to $\underline{B}_0$ factors into two equations: the dispersion relation for the ordinary mode, which is a purely transverse linearly polarized mode, whose electric field is along $\underline{B}_0$, and the dispersion relation for the extraordinary mode, which is a mixed transverse-longitudinal mode, elliptically polarized in a plane perpendicular to $\underline{B}_0$ and containing the wave vector. We shall restrict ourselves to the investigation of the stability of the linearly polarized mode. The corresponding dispersion equation, relating the frequency ω and the wave-vector $\underline{k}$, can be written as[7]

$$c^2 k^2 = \omega^2 - \sum_j \omega_{pj}^2 + \sum_j \omega_{pj}^2 \Omega_j^2$$

$$\times\ 2\pi \int_{-\infty}^{\infty} dv_{\parallel} \int_0^{\infty} dv_{\perp}\, v_{\parallel}^2 \frac{1}{N_j} \frac{\partial F_{oj}}{\partial v_{\perp}} \sum_{n=-\infty}^{\infty} \frac{n^2 J_n^2\left(\frac{kv_{\perp}}{\Omega_j}\right)}{\omega^2 - (n\Omega_j)^2} \tag{1}$$

where

$$\omega_{pj} = \left(4\pi N_j e_j^2/m_j\right)^{1/2}$$

and

$$\Omega_j = |e_j| B_0/m_j c \ ;$$

N_j is the equilibrium density for particles of type j, J_n is a Bessel function of order n.

We study the above dispersion relation for the case in which particles of the same species are counterstreaming along the magnetic field with random (Maxwellian) energies in directions both perpendicular and parallel to $\underline{B}_0$. The equilibrium velocity distribution F_{oj} is, then, of the form

$$F_{oj} = \frac{N_j}{2} \frac{1}{\pi^{3/2} V_{\perp j}^2 V_{\parallel j}} e^{-v_\perp^2/V_{\perp j}^2} \left[e^{-(v_\parallel - u_j)^2/V_{\parallel j}^2} + e^{-(v_\parallel + u_j)^2/V_{\parallel j}^2} \right] \tag{2}$$

where u_j is the mean directional velocity of particles of species j and

$$V_{\perp j} = \left(\frac{2T_{\perp j}}{m_j}\right)^{1/2}, \qquad V_{\parallel j} = \left(\frac{2T_{\parallel j}}{m_j}\right)^{1/2},$$

are their thermal velocities perpendicular and parallel to $\underline{B}_0$ respectively ($T_{\perp j}$ and $T_{\parallel j}$ are non-isotropic temperatures). The normalization

$$2\pi \int_{-\infty}^{\infty} dv_\parallel \int_0^{\infty} dv_\perp \, v_\perp \, F_{oj} \left(v_\perp^2, v_\parallel^2\right) = N_j$$

holds and no current along the lines of force is associated with a distribution of the form (2). From the specific equilibrium velocity distribution (2) and by using Weber's second exponential integral,[8] the dispersion relation (1) becomes

$$c^2 k^2 = \omega^2 - \sum_j \omega_{pj}^2 - \sum_j \omega_{pj}^2 \Omega_j^2 \left(\frac{T_{\parallel j}}{T_{\perp j}} + 2 \frac{u_j^2}{V_{\perp j}^2}\right) e^{-\mu_j} 2 \sum_{n=+1}^{\infty} \frac{n^2 I_n(\mu_j)}{\omega^2 - (n\Omega_j)^2} \tag{3}$$

where I_n is a modified Bessel function and

$$\mu_j \equiv \frac{1}{2}\left(\frac{kV_{\perp j}}{\Omega_j}\right)^2 .$$

By using the relation

$$2\sum_{n=+1}^{\infty} \frac{n^2 I_n(\mu_j)}{\omega^2 - (n\Omega_j)^2} = -\frac{1}{\Omega_j^2}\left[e^{\mu_j} - I_0(\mu_j) + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{\omega^2 I_n(\mu_j)}{(n\Omega_j)^2 - \omega^2}\right]$$

Equation (3) can be written as

$$\omega^2\left\{1 + \sum_j\left(\frac{T_{\parallel j}}{T_{\perp j}} + 2\frac{u_j^2}{V_{\perp j}^2}\right)\sum_{n\neq 0}\frac{\omega_{pj}^2}{(n\Omega_j)^2 - \omega^2}\, e^{-\mu_j} I_n(\mu_j)\right\}$$

$$= -\left\{\sum_j \omega_{pj}^2\left(\frac{T_{\parallel j}}{T_{\perp j}} + 2\frac{u_j^2}{V_{\perp j}^2} - 1\right) - c^2k^2 - \sum_j \omega_{pj}^2\left(\frac{T_{\parallel j}}{T_{\perp j}} + 2\frac{u_j^2}{V_{\perp j}^2}\right) e^{-\mu_j} I_0(\mu_j)\right\} . \quad (4)$$

In the case of no relative motions, $u_j = 0$, Equation (4) yields the dispersion relation which has been discussed by Hamasaki,[9] and by Davidson and Wu.[10] These authors conclude that a purely growing mode exists in an electron plasma with $\beta_{\parallel e} > 2$ and $T_{\perp e}/T_{\parallel e} < 1$, where $\beta_{\parallel e}$ is the ratio between the electron kinetic energy density in the direction of $\underline{B}_0$ and the magnetic energy density. In the next section, it will appear that in the presence of relative streaming, an instability occurs for $\beta_{\parallel e}$ and $T_{\perp e}/T_{\parallel e}$ which do not necessarily satisfy the above conditions. A broadening of the unstable region is as expected since in our system the relative streaming motion is a cause of instability as well as the temperature anisotropy.

III. INSTABILITY ANALYSIS FOR ELECTRON PLASMAS

A. Instability Criterion

Henceforth we consider the ion mass to be infinite so that only electron dynamics needs to be considered. In such a case an equilibrium distribution function of the form (2) describes a symmetrical double beam system, i.e., two opposing drifting electron beams which interpenetrate in a static, uniform magnetic field.[11] In order to establish the instability criterion it is convenient to use Equation (3), rather than Equation (4), written in the form

$$L_k\left(\omega^2\right) = R_k\left(\omega^2\right) \tag{5}$$

with

$$\begin{cases} L_k\left(\omega^2\right) \equiv \omega^2 - c^2 k^2 - \omega_p^{\,2} \\ R_k\left(\omega^2\right) \equiv \omega_p^{\,2}\,\Omega^2\left(\dfrac{T_\parallel}{T_\perp} + 2\,\dfrac{u^2}{V_\perp^{\,2}}\right) e^{-\mu}\, 2 \displaystyle\sum_{n=+1}^{\infty} \dfrac{n^2\, I_n(\mu)}{\omega^2 - (n\Omega)^2} \end{cases} \tag{6}$$

where the subscript $j = e$ has been dropped and all quantities refer to electrons.

Following Hamasaki,[9] the dispersion relation (5) can be analyzed graphically by plotting $L_k\left(\omega^2\right)$ and $R_k\left(\omega^2\right)$ as a function of ω^2 for given values of k and looking for the intersections of the two curves. If $R_k(0) < L_k(0)$, the smallest root for $\omega_k^{\,2}$ is negative, corresponding to an absolute instability. On making use of

Equations (6) and the relation

$$\sum_{n=1}^{\infty} I_n(\mu) = \frac{1}{2}\left[e^{\mu} - I_0(\mu)\right] ,$$

the instability criterion can then be expressed as

$$\omega_p^2\left(\frac{T_\parallel}{T_\perp} + 2\frac{u^2}{V_\perp^2} - 1\right) - \left[c^2k^2 + \omega_p^2\left(\frac{T_\parallel}{T_\perp} + 2\frac{u^2}{V_\perp^2}\right)e^{-\mu} I_0(\mu)\right] > 0 ,$$

or

$$1 - \frac{1}{U}\frac{T_\perp}{T_\parallel} - F\left(U\beta_\parallel, \mu\right) > 0 , \tag{7}$$

where

$$\begin{cases} \beta_\parallel = \dfrac{V_\parallel^2\,\omega_p^2}{c^2\,\Omega^2} = \dfrac{NT_\parallel}{B_0^2/8\pi} , \\[2ex] U \equiv 1 + 2\dfrac{u^2}{V_\parallel^2} , \\[2ex] F\left(U\beta_\parallel, \mu\right) \equiv \dfrac{2\mu}{\beta_\parallel U} + e^{-\mu} I_0(\mu) . \end{cases} \tag{8}$$

Since F > 0 and

$$\left(1 - \frac{1}{U}\frac{T_\perp}{T_\parallel}\right) < 1 \ ,$$

two necessary conditions for instability follow from inequality (7):

$$\frac{T_\perp}{T_\parallel} < 1 + 2\,\frac{u^2}{V_\parallel{}^2} \tag{9}$$

and

$$\beta_\parallel > \frac{2}{1 + 2\,\dfrac{u^2}{V_\parallel{}^2}} \ . \tag{10}$$

Therefore, due to the presence of relative streaming, an instability can occur also when $T_\perp/T_\parallel > 1$ and $\beta_\parallel < 2$.

B. Stability-Instability Boundaries

The equation corresponding to inequality (7),

$$1 - \frac{1}{U}\frac{T_\perp}{T_\parallel} - F\left(U\beta_\parallel \,,\, \mu\right) = 0 \ , \tag{11}$$

yields the instability-stability boundaries. Solutions of this equation are obtained by looking for values of μ for which the curves $F\left(U\beta_\parallel \,,\, \mu\right)$ and the straight lines $\left(1 - 1/U \ T_\perp/T_\parallel\right)$, corresponding to different values of the parameters $U\beta_\parallel$ and

$(1/U)\ T_\perp/T_\parallel$, are tangent. The dependence of $F(U\beta_\parallel,\ \mu)$ upon μ is such that the curve $F(U\beta_\parallel,\ \mu)$ is tangent to the straight line $\left(1 - (1/U)\ T_\perp\ T_\parallel\right)$ for values of $\mu = \mu_0$ for which the function $F(U\beta_\parallel,\ \mu)$ has a minimum, i.e.

$$\left.\frac{\partial F(U\beta_\parallel,\ \mu)}{\partial \mu}\right|_{\mu=\mu_0} = 0 \ .$$

Therefore, Equation (11) is equivalent to the following two coupled equations:

$$\begin{cases} 1 - \dfrac{1}{U}\dfrac{T_\perp}{T_\parallel} - F(U\beta_\parallel,\ \mu_0) = 0 & \text{(12a)} \\[2ex] \dfrac{2}{\beta_\parallel U} - e^{-\mu_0}\, I_0(\mu_0) + e^{-\mu_0}\, I_1(\mu_0) = 0 \ . & \text{(12b)} \end{cases}$$

The above system of equations determines the range of values of the parameters $\left(U,\ T_\perp/T_\parallel,\ \beta_\parallel\right)$ at the instability boundary. It can be solved numerically to express the boundaries of instability in terms of curves separating the stable from the unstable region in the $T_\perp/T_\parallel$ versus $\beta_\parallel$ plane for various values of U, i.e. $(u/V_\parallel)^2$. The results are shown in Figures 1 and 2. It appears that while for stationary plasmas, $u = 0$, the minimum unstable value of $\beta_\parallel$ is 2, in the presence of relative streaming, an electromagnetic instability can occur in sufficiently low-β plasmas. As an example, when $u = 2V_\parallel$, the minimum value of $\beta_\parallel$ for instability is 0.22.

It is interesting to recall that the threshold streaming velocity required for the electrostatic two-stream (TS) instability is $1.3V_\parallel$ for a Maxwellian equilibrium distribution function.[6] From Figures 1 and 2 it is seen that it is possible to excite the electromagnetic instability alone without simultaneously triggering the TS instability. For example, for $T_\perp/T_\parallel = 1$, i.e., an isotropic plasma, and $\beta_\parallel \geq 3$, the critical streaming velocity for electromagnetic instability is $V_\parallel$, which is below that required for the TS instability. If $T_\perp/T_\parallel < 1$, the threshold is further reduced. It therefore follows that conditions can be arranged in the laboratory such that the TS instability is suppressed while the electromagnetic instability is excited. This conclusion is somewhat different from an earlier one based on the fluid equations, in which only the effect of perpendicular temperature was considered.[4]

Corresponding to a set of parameters in the unstable region, there is a range of wavenumbers for which the waves are unstable. Such a range is determined from the instability criterion (7). Since inequality (7) requires the function $F\left(U\beta_\parallel, \mu\right)$ to be less than a quantity smaller than one, it follows that the unstable range of values of μ is such that $\mu_{min} < \mu < \mu_{max}$. Therefore, an instability occurs for wavenumbers k which are greater than a minimum value, k_{min}, and smaller than a maximum value, k_{max}. In general, μ_{min} and μ_{max} can be determined only numerically. However, by assuming that μ_{min} and μ_{max} are sufficiently smaller and greater than one respectively, the modified Bessel function $I_0(\mu)$, which appears in (7) through $F\left(U\beta_\parallel, \mu\right)$, can be approximated with its power series and asymptotic expansion, respectively, and expressions for μ_{min} and μ_{max} can be obtained. As an example, let us determine μ_{min}. For μ less than one ($\mu < 0.2$), we can approximate $F(U\beta_{\parallel}, \mu)$, at first-order in μ, as

$$F(U\beta_{\parallel}, \mu) \simeq \frac{2\mu}{\beta_{\parallel} U} + 1 - \mu ,$$

and the instability criterion (7) yields $\mu > \mu_{min}$, where

$$\mu_{min} \equiv \frac{\frac{T_{\perp}}{T_{\parallel}}}{1 + 2\frac{u^2}{V_{\parallel}^2} - \frac{2}{\beta_{\parallel}}} . \tag{13}$$

Expressions (13) for μ_{min} is consistent with the assumption $\mu < 0.2$ for values of parameters $\left[T_{\perp}/T_{\parallel}, (u/V_{\parallel})^2, \beta_{\parallel}\right]$ such that

$$5\frac{T_{\perp}}{T_{\parallel}} < 1 + 2\left(\frac{u}{V_{\parallel}}\right)^2 - \frac{2}{\beta_{\parallel}} . \tag{14}$$

Since

$$\mu = \frac{1}{2}\left(\frac{kV_{\perp}}{\Omega}\right)^2 ,$$

by using Equation (13), we obtain

$$k_{min}^2 = \frac{2\frac{T_{\perp}}{T_{\parallel}}}{1 + 2\frac{u^2}{V_{\parallel}^2} - \frac{2}{\beta_{\parallel}}}\left(\frac{\Omega}{V_{\perp}}\right)^2 , \tag{15}$$

which yields the minimum unstable wavenumber under condition (14).

In order to be able to relate the results of the present kinetic theory to those of the cold theory of the modified ordinary mode, it is useful to solve Equations (12) in the limit of low perpendicular temperature, i.e., for μ_0 small. In this case the exponential and Bessel functions in (12a) and (12b) can be expanded in μ_0. At second-order in μ_0 we have

$$\begin{cases} \dfrac{1}{U}\dfrac{T_\perp}{T_\parallel} - \dfrac{3}{4}\mu_0^2 = 0 \, , & \text{(16a)} \\[2ex] \dfrac{2}{\beta_\parallel U} - 1 + \dfrac{3}{2}\mu_0 - \dfrac{5}{4}\mu_0^2 = 0 \, . & \text{(16b)} \end{cases}$$

A quadratic equation in U is obtained by solving one of the above equations with respect to μ_0 and substituting into the other. Only one of the two solutions of the quadratic equation is consistent with the assumption that μ_0 be sufficiently small than one. The acceptable solution can be expressed as

$$u_c^2 = \frac{\Omega^2}{\omega_p^2} c^2 + \alpha^2 \frac{V_\perp^2}{4} - \frac{V_\parallel^2}{2} \, , \tag{17}$$

where

$$\alpha^2 \equiv \left[24\left(\frac{\Omega c}{\omega_p V_\perp}\right)^2 - 11\right]^{1/2} - \frac{1}{3} \, . \tag{18}$$

α^2 is a positive quantity for

$$V_\perp < 1.5 \frac{\Omega}{\omega_p} c \, . \tag{19}$$

Equation (17) yields the threshold value for the streaming velocity when the temperature perpendicular to the magnetic field is sufficiently low and such that condition (19) is satisfied. From Equation (17) it appears explicitly that both the magnetic field and the perpendicular temperature have a stabilizing effect, while the parallel temperature enhances the instability.* In the cold limit the critical value for the streaming velocity becomes $(\Omega/\omega_p)c$, which is the result derived in Reference 3.

C. Growth Rate

If the instability criterion (7) is satisfied, the dispersion equation for the linearly polarized mode has a negative root for ω^2, corresponding to a nonconvective instability. By indicating this solution with $-\omega_k^2 = \gamma_k^2 > 0$ and by taking into account only electrons, Equation (4) yields

$$\gamma_k^2 \left\{1 + U \frac{T_\parallel}{T_\perp} \sum_{n \neq 0} \frac{\omega_p^2}{(n\Omega)^2 + \gamma_k^2} e^{-\mu} I_n(\mu)\right\} = \omega_p^2 U \frac{T_\parallel}{T_\perp} \left\{1 - \frac{1}{U}\frac{T_\perp}{T_\parallel} - F\left(U\beta_\parallel, \mu\right)\right\} . \tag{20}$$

In general, the growth rate can be determined by means of a graphical method. Simple analytical expressions for the growth rate $|\gamma_k|$ can be given only to the extent to which one is able to approximate Equation (20). In the approximation

*In Reference 4, by using the fluid equations it was concluded that the temperature stabilizes the modified ordinary-mode instability. This result applies only to the particular case in which $T_\perp \gg T_\parallel$.

in which

$$1 + U \frac{T_\parallel}{T_\perp} \sum_{n \neq 0} \frac{\omega_p{}^2}{(n\Omega)^2 + \gamma_k{}^2} e^{-\mu} I_n(\mu) \simeq U \frac{T_\parallel}{T_\perp} \frac{\omega_p{}^2}{\Omega^2 + \gamma_k{}^2} 2e^{-\mu} I_1(\mu) \tag{21}$$

Equation (20) yields

$$\frac{|\gamma_k|}{\Omega} \simeq \left\{ \frac{1 - \frac{1}{U}\frac{T_\perp}{T_\parallel} - F(U\beta_\parallel, \mu)}{2e^{-\mu} I_1(\mu) - \left[1 - \frac{1}{U}\frac{T_\perp}{T_\parallel} - F(U\beta_\parallel, \mu)\right]} \right\}^{1/2} \tag{22}$$

For the above to be a valid expression for the growth rate, the right-hand side of (22) has to be real. Hence, in addition to condition (7), which is the instability criterion for the non-approximated Equation (20), the following condition has also to be satisfied:

$$2e^{-\mu} I_1(\mu) - \left[1 - \frac{1}{U}\frac{T_\perp}{T_\parallel} - F(U\beta_\parallel, \mu)\right] > 0 \ . \tag{23}$$

Conditions (7) and (23) are independent conditions. For example, with $T_\perp/T_\parallel = 0.8$; $\beta_\parallel = 1$; $(u/V_\parallel)^2 = 9$ and $2 \leq \mu \leq 3.2$, condition (7), but not (23), is satisfied. In Figures 3 and 4 we show some numerical results obtained by using Equation (22). The quantity $|\gamma_k|/\Omega$ is plotted versus μ. In agreement with the conclusions of Section III B, an instability occurs for $\mu_{min} < \mu < \mu_{max}$. The curves in Figure 3 are obtained for $(u/V_{\parallel})^2 = 2$, $\beta_{\parallel} = 2$ and four different values of the parameter $T_{\perp}/T_{\parallel}$. It appears that the instability is enhanced by increasing parallel temperature and stabilized by increasing perpendicular temperature. In Figure 4 curves are shown for $\beta_{\parallel} = 1$, $T_{\perp}/T_{\parallel} = 0.8$ and two different values of the parameter $(u/V_{\parallel})^2$. By comparing these two curves at the same parallel temperature, it is seen that the instability is enhanced by increasing streaming velocity. The growth rates shown in both Figures 3 and 4 are of the order of the electron cyclotron frequency. Since in these examples $\beta_{\parallel} = \omega_p^2 V_{\parallel}^2/\Omega^2 c^2 \geq 1$ and $V_{\parallel} < c$, it follows that $\Omega < \omega_p$. Therefore, the above growth rates for the electromagnetic instability are smaller than those corresponding to the electrostatic two-stream instability, which are of the order of the electron plasma frequency.

Instead of (21), the following approximation

$$1 + U \frac{T_{\parallel}}{T_{\perp}} \sum_{n \neq 0} \frac{\omega_p^2}{(n\Omega)^2 + \gamma_k^2} e^{-\mu} I_n(\mu) \simeq 1 , \tag{24}$$

has been used by Hamasaki in calculating the growth rate from Equation (20), in the case of no streaming.[9] When condition (24) is satisfied, Equation (20) yields

$$\frac{|\gamma_k|}{\omega_p} \simeq \left\{ U \frac{T_{\parallel}}{T_{\perp}} \left[1 - \frac{1}{U} \frac{T_{\perp}}{T_{\parallel}} - F(U\beta_{\parallel}, \mu) \right] \right\}^{1/2} \tag{25}$$

The growth rate given by Equation (25) is maximum for values of $\mu = \mu_0$ such that the function $F\left(U\beta_{\parallel}, \mu\right)$ is minimum in μ_0, i.e.,

$$\left\{\begin{array}{l} \dfrac{|\gamma_{max}|}{\omega_p} \simeq \left\{U \dfrac{T_{\parallel}}{T_{\perp}} \left[1 - \dfrac{1}{U} \dfrac{T_{\perp}}{T_{\parallel}} - F\left(U\beta_{\parallel}, \mu_0\right)\right]\right\}^{1/2} \\ \\ \dfrac{2}{\beta_{\parallel} U} - e^{-\mu_0} I_0\left(\mu_0\right) + e^{-\mu_0} I_1\left(\mu_0\right) = 0 \end{array}\right. \tag{26}$$

Equations (26) have been solved by Hamasaki in the limit of no streaming, U = 1, and with the assumption that μ_0 be sufficiently smaller than one.[9] It seems, however, that the expression for the growth rate derived in such a way is not consistent with approximation (24).* We may note, moreover, that approximation (24) is unlikely to be satisfied, in general,since it requires one to be smaller than the sum of a series whose first terms are of order one.

IV. CONCLUSION

In conclusion, we have investigated an electromagnetic instability in a counterstreaming plasma in a magnetic field and found that conditions exist under which the electromagnetic instability is excited independently of the electrostatic two-stream instability. We have shown that the effect of temperature perpendicular to the field is toward stabilization while parallel temperature enhances the

*We note also that Hamasaki's result is a second-order quantity in μ_0 while in deriving it an expansion at first-order in μ_0 has been used.

instability. Due to the presence of streaming, an instability can occur in sufficiently low-β plasmas and may then play a role in laboratory experiments.

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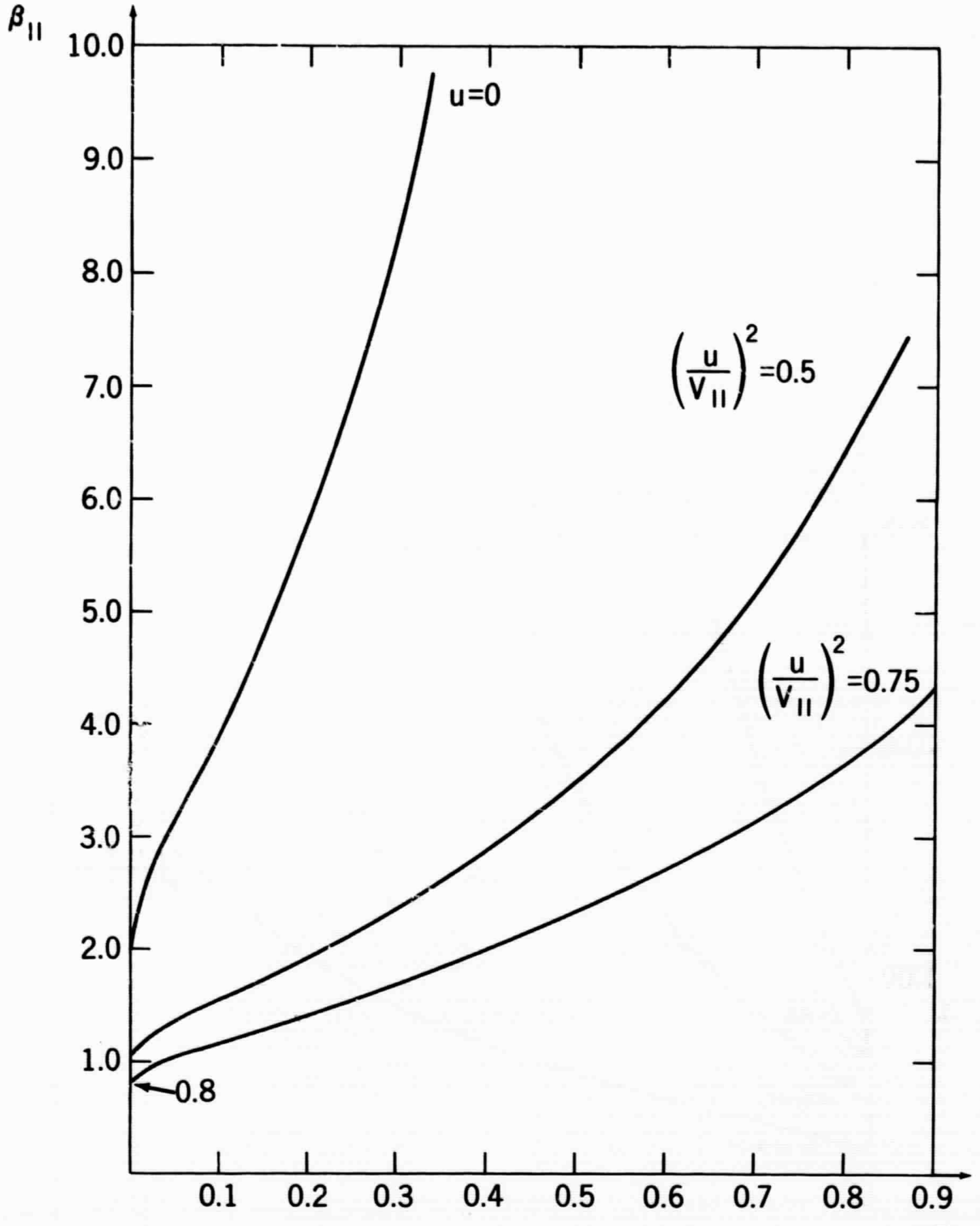

Figure 1. Stability-instability boundaries in the $T_{\perp}/T_{\parallel}$ vs $\beta_{\parallel}$ plane when $(u/V_{\parallel})^2 = 0;\ 0.5;\ 0.75$. Unstable regions lie above the curves.

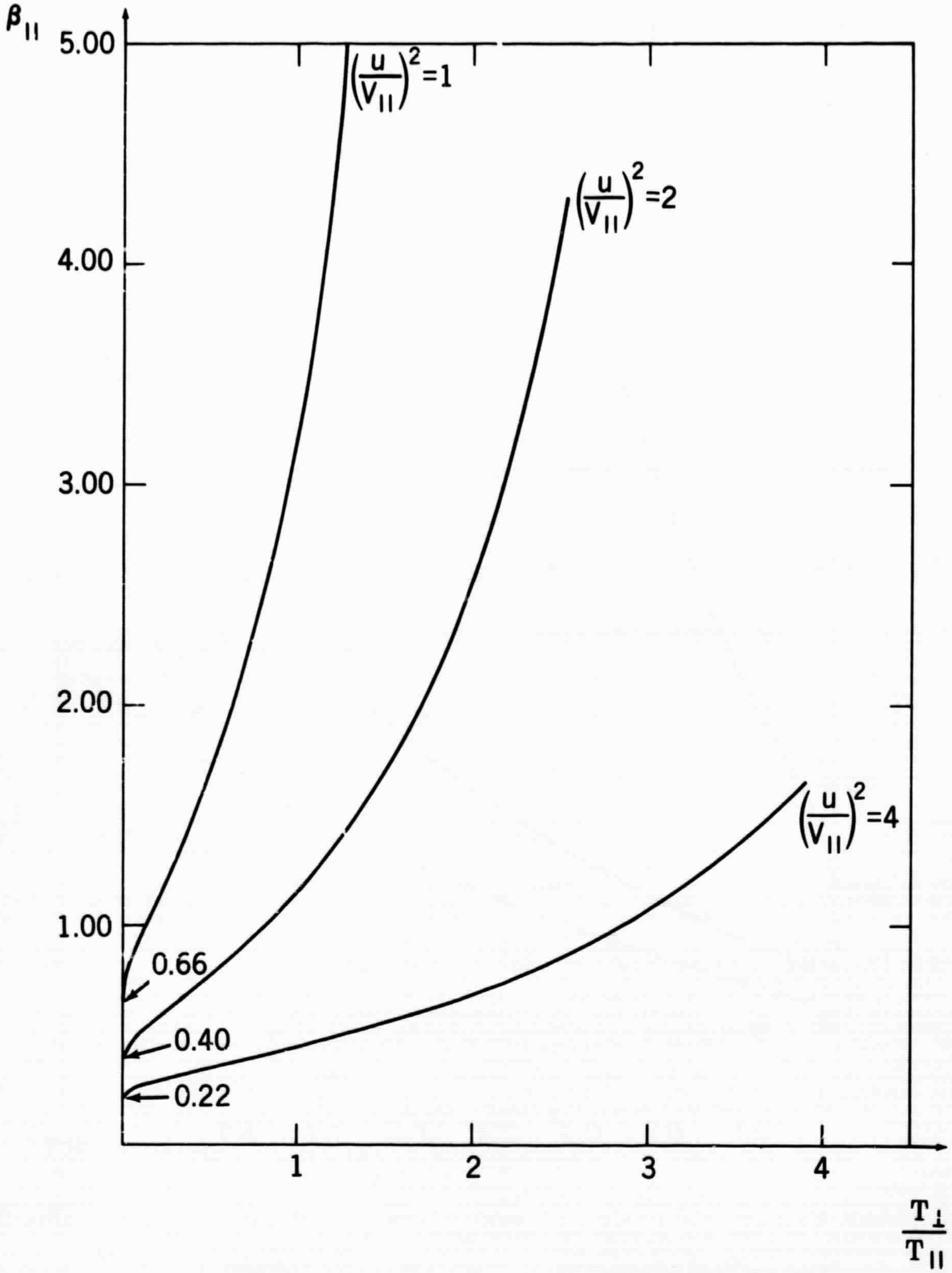

Figure 2. Stability-instability boundaries in the $T_{\perp}/T_{\parallel}$ vs $\beta_{\parallel}$ plane when $(u/V_{\parallel})^2 = 1; 2; 4$.

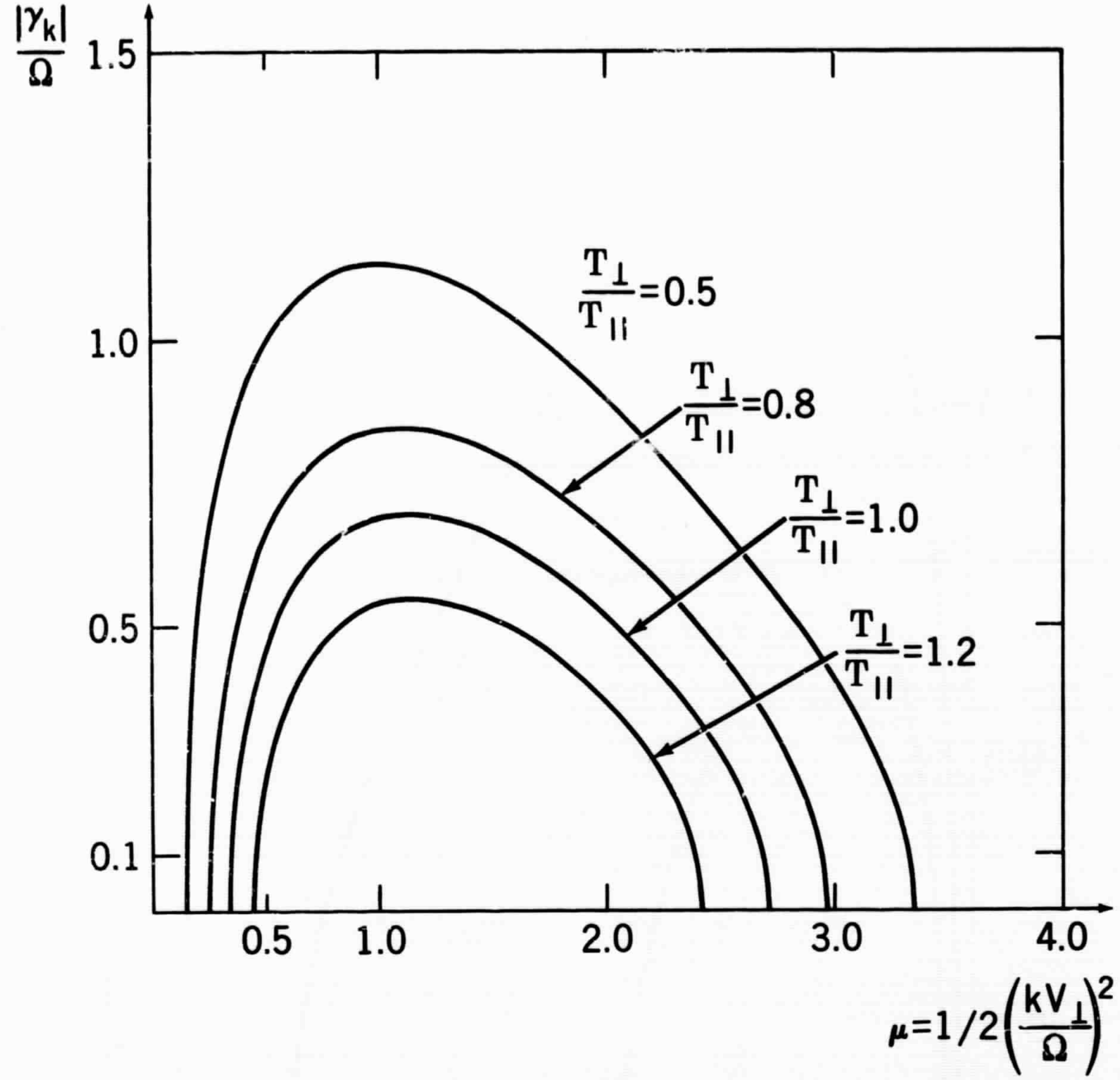

Figure 3. Growth rates as a function of wavenumber for $(u/V_\parallel)^2 = 2$, $\beta_\parallel = 2$ and four different values of $T_\perp/T_\parallel$.

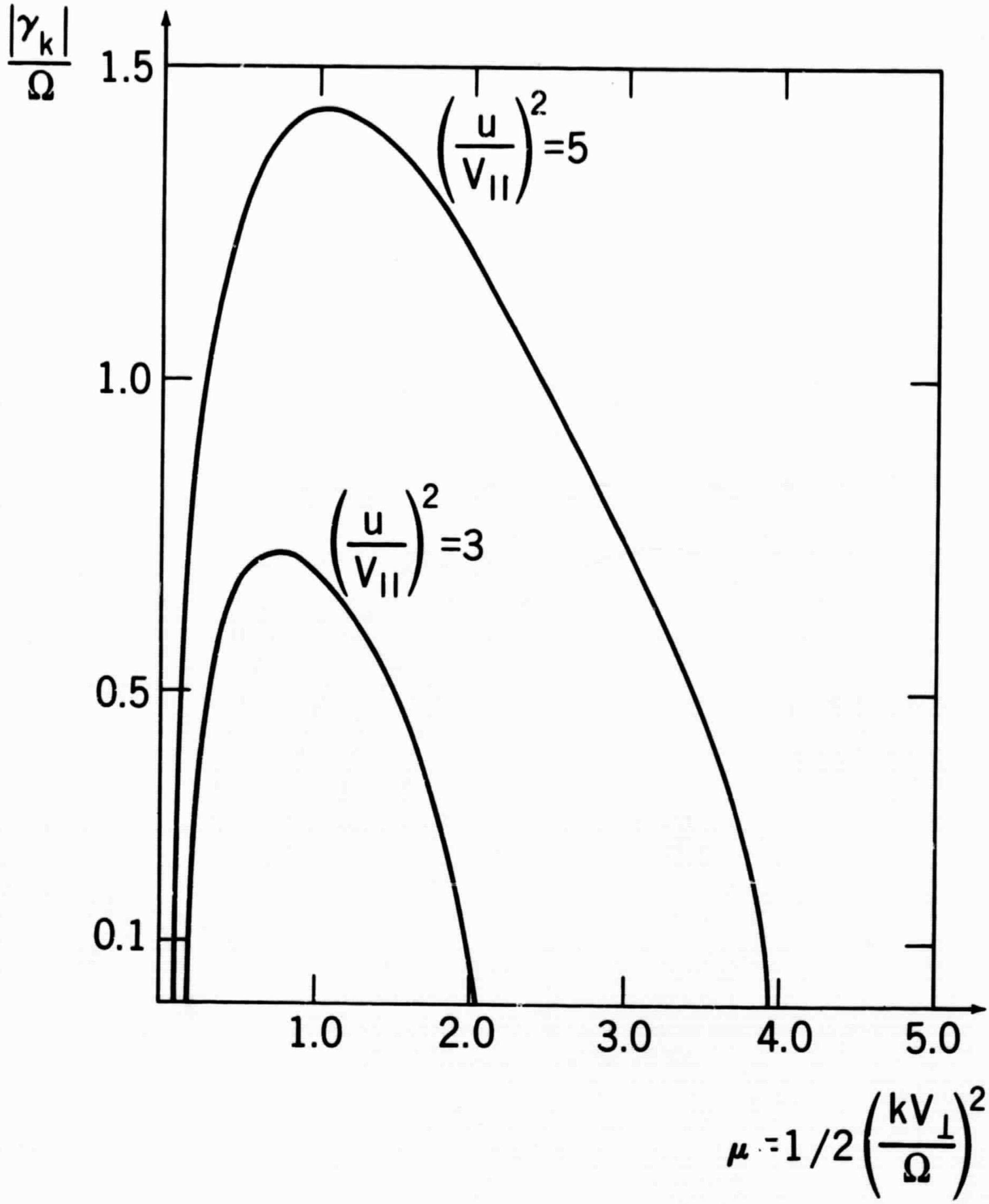

Figure 4. Growth rates as a function of wavenumber for $\beta_{\parallel} = 1$, $T_{\perp}/T_{\parallel} = 0.8$ and two different values of $(u/V_{\parallel})^2$.