NASA TECHNICAL REPORT

NASA TR R-328

NASA TR R-328

AUTOMATED FAULT DIAGNOSTICS FOR FREQUENCY-DEPENDENT CIRCUITS BY CORRELATION AND VARIANCE TECHNIQUES

by Henry B. Metcalf

Electronics Research Center
Cambridge, Mass.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • FEBRUARY 1970

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TR R-328		

4. Title and Subtitle	5. Report Date
Automated Fault Diagnostics for Frequency-Dependent Circuits by Correlation and Variance Techniques	February 1970
	6. Performing Organization Code
7. Author(s)	8. Performing Organization Report No.
Henry B. Metcalf	C-95
9. Performing Organization Name and Address	10. Work Unit No.
Electronics Research Center Cambridge, Mass.	125-25-04-63-25
	11. Contract or Grant No.
12. Sponsoring Agency Name and Address	13. Type of Report and Period Covered
National Aeronautics and Space Administration	Technical Report
	14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

A computer program, NASAP (Network Analysis for Systems Applications Program), has been adapted to perform fault diagnosis in frequency-dependent systems.

The program simulates a short circuit and an open circuit for each network parameter and prints out a modified transfer function of the network corresponding to each simulated failure. Experimental and simulated data are subjected to correlation and variance analysis in order to rank all parameters according to probability of failure in the network model.

The program lists, at present, single failures in order to decreasing probability. Possible extensions of the program to multiple failures are examined.

17. Key Words	18. Distribution Statement
·Circuits ·Fault Diagnosis ·Probability of Failure ·Multiple Failure	Unclassified-Unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	22	$3.00

*For sale by the Clearinghouse for Federal Scientific and Technical Information
Springfield, Virginia 22151

AUTOMATED FAULT DIAGNOSTICS FOR FREQUENCY-DEPENDENT CIRCUITS BY CORRELATION AND VARIANCE TECHNIQUES

By Henry B. Metcalf
Electronics Research Center
Cambridge, Massachusetts

SUMMARY

A computer program, NASAP (Network Analysis for Systems Applications Program), has been adapted to perform fault diagnosis in frequency dependent systems.

The program simulates a short circuit and an open circuit for each network parameter and prints out a modified transfer function of the network corresponding to each simulated failure. Experimental and simulated data are subjected to correlation and variance analysis in order to rank all parameters according to probability of failure in the network model.

The program lists, at present, single failures in order to decreasing probability. Possible extensions of the program to multiple failures are examined.

I.- INTRODUCTION

Review of Work

The computer program, NASAP, has been adapted to perform fault diagnosis in frequency-dependent circuits. The present version of NASAP can handle linear circuits, which consist of constant-value passive elements, and independent or dependent current and voltage sources. The program has been written in CDC FORTRAN IV to make it easy for the intended user to understand the algorithms and to make changes within the program. The version of NASAP described within this report has been run on the IBM 7094.

The technique employed in the failure diagnostic method has evolved from three sources. The first of these is linear-graph theory and flow-graph theory. Each circuit is given a linear graph description which is then formulated into flow-graph terms by a computer subroutine thus forming the basis of the computer algorithms for later output requests.

The second area is that of correllation and variance analysis. The correllation coefficient is the statistical index used to measure the similarity observed between like sets of data. It is used here to provide a standardized measure of the degree of deviations observed in the simulated failures. The variance measure is employed to provide an index of the percentage deviation caused by a simulated failure.

The last source and also the foundation of the technique is NASAP. The fault-isolation technique developed in this report has been formulated for use with NASAP.

Strategy for Identification of Faulty Elements

For a given network, a computer program simulates the failure of a component in a multi-element network. This simulation is accomplished by modifying NASAP as follows:

1) Solve for the transfer function of the network of interest, designated by F(0).

2) Compute F(k) for 2N distinct cases; namely, when each of the N elements is short-circuited and open-circuited. The functions F(k) with $k = 1,2,...,2N$ but $k \neq 0$ define a set of modified transfer functions where F(2J-1) are short circuits and F(2J) are open circuits, where $J = 1,2,...,N$.

3) Evaluate F(0) and F(k) numerically at specified complex frequencies by taking the absolute value of the transfer functions and the modified transfer functions; this yields 2N sets of data for the modified transfer functions, F(k), and one set of data for the original transfer function, F(0).

4) Solve for the correlation coefficients and the variance coefficients between the original transfer function and the modified transfer functions. This process yields a table of identification coefficients for all possible failures.

5) Measure the correlation and variance coefficients between the observed response and the desired response, when a variant response is observed in a circuit. These coefficients are then compared to the table of identification coefficients and the possible failures are printed out in their order of most probable occurrence.

Computational Procedures for Locating Fault Elements

To pinpoint the failure from a knowledge of:

F(0) Transfer Function
F(k) Modified Transfer Functions

two coefficients are derived.

a) Correlate F(0) with each of the F(k) modified transfer functions to yield the correlation coefficient.

b) Measure the variance caused by each short circuit or open circuit at the specified frequencies and the variance coefficient results.

c) Both coefficients can be calculated for every F(k).

Experimentally the identification is performed as follows:

1) Construct the network experimentally and measure the transfer function.

2) Determine how the modified transfer functions deviate from the calculated transfer function.

3) Use criteria (a) or (b) or both to identify the faulty element.

4) List a small number of elements that are expected failures in their order of most probable occurrence.

Application of Results

The computer program output provides the following:

1) The transfer function of the network of interest.

2) The modified transfer functions corresponding to solutions for each short circuit and open circuit.

3) The numerical evaluation of each transfer function at specified frequencies.

4) The correlation coefficient between the calculated transfer function and each modified transfer function.

5) The variance coefficient computed for each modified transfer function.

6) The most probable failures in terms of the experimental results.

7) The program is written in CDC FORTRAN IV and is presently being run on the IBM 7094. The program is available from project COSMIC.

II.- METHODS

The Topology Equation

The basic algorithm employed in the computer program is the topology equation which provides a method for finding the desired cause-effect relationship between an input and output variable. All applications of the topology equation in this report deal with closed-signal flowgraphs.

The closing element P is the performance parameter. Q refers to the element where failure is being simulated and the topology equation, H = 0, can be expanded as follows:

$$H(P,Q) = H(\overline{P},\overline{Q}) + PH(P',\overline{Q}) + QH(\overline{P},Q') + PQH(P',Q')$$

It states that P can be expressed in terms of Q, since H(P,Q) = 0 is linear in P and Q. Although the topology equation applies to all types of linear systems, it can be illustrated by a flowgraph (such as the one) in Figure 1.

In the case of flowgraphs the topology equation yields:

$H(P')$ - loops with the P tag present

$H(\overline{P})$ - loops that are deprived of the P tag and similarly for $H(Q')$ and $H(\overline{Q})$.

The combination of loops in the topology equation of a closed network is given by:

$$H = \Sigma_m (-1)^m H(m) \text{ where}$$

$$H(m) = \Sigma_n L(m,n) \text{ and}$$

m is the order of the loop and n is the number of loops belonging to each order. L(m,n) is the n^{th} loop of order m. A loop is defined as a sequence of transmittances such that every node is common to two and only two transmittances of the loop, one terminating at the node and one emanating from the node. In a first-order loop every node can be reached from every other node. A loop of order m is a set of N disjoint first-order loops.

For Figure 1 the loops are listed in Table I. The topology equation then is:

$$H(\overline{P},\overline{Q}) = 1 - MN$$

$H(P',\overline{Q}) = -PKL$

$H(\overline{P},Q') = JQR$

$H(P',Q') = JKL\ QPR$

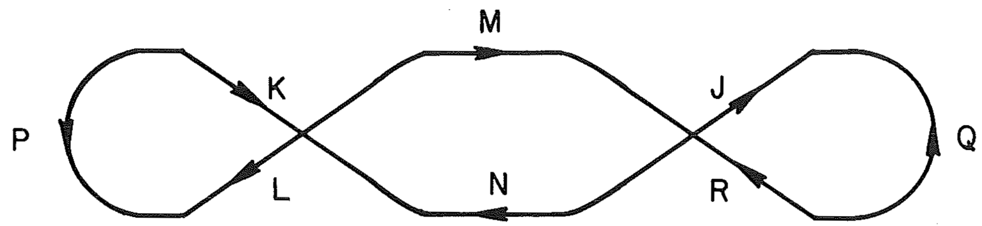

Figure 1.- Loop example

TABLE I.- LISTING OF LOOPS FROM FIGURE 1

n	m	Loop
1	0	1
1	1	MN
2	1	PKL
3	1	QJR
1	2	PKL QJR

The Modified Transfer Function

Solving H = Q for 1/P gives

$\frac{1}{P} = \frac{H(P')}{H(\overline{P})} \triangleq F(0)$, the transfer function

$H(P') = H(P',\overline{Q}) + QH(P',Q')$

$H(\overline{P}) = H(\overline{P},\overline{Q}) + QH(\overline{P},Q')$

implies

$$\frac{1}{P} = \frac{H(P',\overline{Q}) + QH(P',Q')}{H(\overline{P},\overline{Q}) + QH(\overline{P},Q')} = F(0)$$

Table II defines the variance in impedance short circuit and open circuit of parameter Q, as derived from the above formula for F(0).

TABLE II.- MODIFIED TRANSFER FUNCTIONS

Cause of Variance	$Q = Y$	$Q = Z$
Short Circuit (SC)	$1/Q = 0$	$Q = 0$
Open Circuit (OC)	$Q = 0$	$1/Q = 0$
$Y^{OC} = Z^{SC}$	$-\dfrac{H(P',\overline{Q})}{H(\overline{P},\overline{Q})}$	$-\dfrac{H(P',\overline{Q})}{H(\overline{P},\overline{Q})}$
$Y^{SC} = Z^{OC}$	$-\dfrac{H(P',Q')}{H(\overline{P},Q')}$	$-\dfrac{H(P',Q')}{H(\overline{P},Q')}$

Illustrative Example for Numerical Data

Experimental data of the transfer function F(0,i) and the modified transfer function F(k,i) at specified frequencies $\omega(i)$ are printed out by the computer program. This will be illustrated for the circuit in Figure 2.

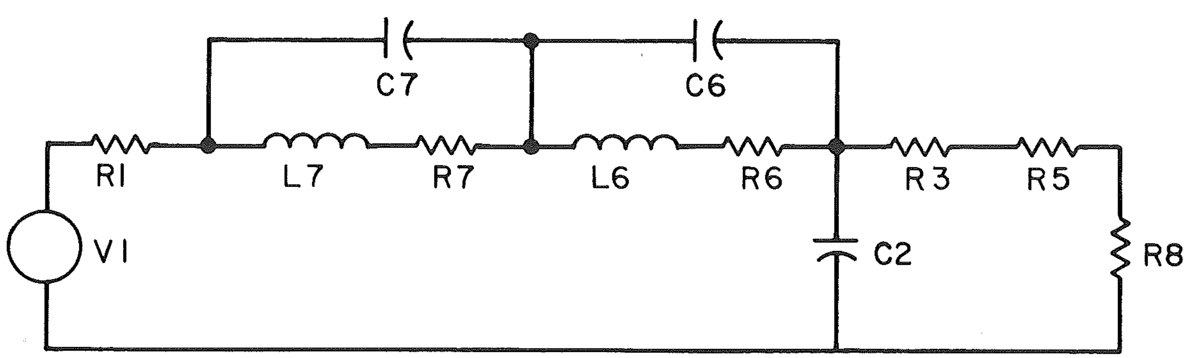

Figure 2.- Illustrative circuit

The circuit was constructed with components listed in Table III and I_{R8}/V_{V1} was measured experimentally.

TABLE III.- COMPONENT VALUES FOR ILLUSTRATIVE NETWORK

ELEMENT	VALUE UNIT
V1	1.000 volt
R1	1.000 K ohm
L7	5.412 MH
R7	1.262 k ohm
C7	200.000 PF
L6	5.645 MH
R6	1.307 K ohm
C6	212.800 PF
R3	1.228 K ohm
C2	281.800 PF
R5	1.254 K ohm
R8	119.000 ohm

Table IV gives the data sets for F(0), the original transfer function, and F(1), F(3), F(4), three modified transfer functions. F(1) corresponds to the short circuit of R1, F(2) corresponds to the open circuit of R1, and F(3) is the short circuit of L7.

Since F(2,i) = 0, the data set of R1 open circuit, it is not listed in the table.

It is important to note that in many of the following tables the data have been carried to five or six places. This is for illustrative purposes only since the input data imply only four-figure accuracy at the most.

TABLE IV.- DATA SETS

ω(i)	F(0,i)	F(1,i)	F(3,i)	F(4,i)
$10^{1.5}$	.362874 E-03	.193424 E-03	.156740 E-03	.632455 E-08
10^{2}	.362874 E-03	.193424 E-03	.156740 E-03	.200000 E-07
$10^{2.5}$	.362873 E-03	.193424 E-03	.156740 E-03	.632455 E-07
10^{3}	.362879 E-03	.193423 E-03	.156740 E-03	.200000 E-06
$10^{3.5}$	.362824 E-03	.193419 E-03	.156739 E-03	.632455 E-06
10^{4}	.362378 E-03	.193376 E-03	.156630 E-03	.199998 E-05
$10^{4.5}$	.357991 E-03	.192952 E-03	.156641 E-03	.632382 E-05
10^{5}	.320148 E-03	.188740 E-03	.155739 E-03	.199782 E-04
$10^{5.5}$	.160346 E-03	.150203 E-03	.145178 E-03	.629477 E-04
10^{6}	.124238 E-04	.204408 E-04	.387241 E-04	.342242 E-04

The Correlation Coefficient

The correlation coefficient (R) for F(0,i) and some set F(k,i) is the mean of the paired products of the deviations of each score from their respective means when these deviations are measured in units of their respective standard deviations, D(0,i) and D(k,i).

$$R = \frac{1}{M} \Sigma \left(\frac{F(0,i) - G(0,i)}{D(0,i)}\right)\left(\frac{F(k,i) - G(k,i)}{D(k,i)}\right)$$

$$= \frac{1}{M} \Sigma \frac{F(0,i)\ F(k,i) - G(0,i)\ G(k,i)}{D(0,i)\ D(k,i)}$$

where the summation extends over the frequency interval specified by M measurements; hence, the means are:

$$G(0,i) = \Sigma\ F(0,i)/M, \text{ and similarly } G(k,i) = \Sigma\ F(k,i)/M$$

and the standard deviation is:

$$D(0,i) = \left(\frac{\Sigma\ F(0,i)^2}{M} - G(0,i)^2\right)^{\frac{1}{2}}$$

Similarly,

$$D(k,i) = \left(\frac{\Sigma\ F(k,i)^2}{M} - G(k,i)^2\right)^{\frac{1}{2}} \text{ for } K \neq 0$$

Each point evaluated corresponds to a point on a curve and each data set describes a curve; hence, a value of R is assigned for each short circuit and open circuit. That is, the deviations of each modified transfer function, F(k), from the original transfer function, F(0), result in a separate value of R as shown in Table V. The absolute value of R will ensure that it is positive.

TABLE V.- CORRELATION COEFFICIENTS

k	FAILURE	SC	OC	R
1	R1	X		.952979
2	R1		X	.000
3	L7	X		.897051
4	L7		X	.794069
5	R7	X		.940948
6	R7		X	.794069
7	C7	X		.905976
8	C7		X	.938431
9	L6	X		.889213
10	L6		X	.741361
11	R6	X		.943269
12	R6		X	.741361
13	C6	X		.897955
14	C6		X	.929121
15	R3	X		.961076
16	R3		X	.000
17	C2	X		.000
18	C2		X	.948035
19	R5	X		.961537
20	R5		X	.000

Variance Due to the Failure of a Single Component

The correlation technique is not always sensitive enough to detect a failure. The variance coefficient, T, can be an alternative or is used in conjunction with the correlation coefficient, R. Both methods are measures of the deviation observed between the original transfer function and the modified transfer functions. When used together they make the fault isolation more effective.

Table VI defines the variance, E, observed in P = 1/F(0) at the occurrence of a short circuit and open circuit of each element; that is, each F(K).

The variance defined in Table VI is computed for the example in Figure 2. The results for $E(\overline{P},X)$ are listed in Table VII and the observed error is plotted in Figure 3 over the frequency range.

TABLE VI.- DEFINITION OF VARIANCE

$P(\overline{Q})$	$-\dfrac{H(\overline{P},\overline{Q})}{H(P',\overline{Q})}$	$\dfrac{1}{F(k)} = \dfrac{1}{Z_{SC}} = \dfrac{1}{Y_{OC}}$
$P(Q')$	$-\dfrac{H(\overline{P},Q')}{H(P',Q')}$	$\dfrac{1}{F(k)} = \dfrac{1}{Z_{OC}} = \dfrac{1}{Y_{SC}}$
$E(p',Q')$	$\dfrac{P - P(Q')}{P}$	$\dfrac{\frac{1}{F(0)} - \frac{1}{F(k)}}{\frac{1}{F(0)}}$
$E(P',\overline{Q})$	$\dfrac{P - P(\overline{Q})}{P}$	$\dfrac{\frac{1}{F(0)} - \frac{1}{F(k)}}{\frac{1}{F(0)}}$
$E(\overline{P},Q')$	$\dfrac{\frac{1}{P} - \frac{1}{P(Q')}}{\frac{1}{P}}$	$\dfrac{F(0) - F(k)}{F(0)}$
$E(\overline{P},\overline{Q})$	$\dfrac{\frac{1}{P} - \frac{1}{P(\overline{Q})}}{\frac{1}{P}}$	$\dfrac{F(0) - F(k)}{F(0)}$

From Table VI:

$$E = \frac{F(0,i) - F(k,i)}{F(0,i)} = E(\overline{P},X)_k$$

where X is either $\overline{Q}$ or Q' depending on whether we are dealing with a short circuit or an open circuit and k designates a particular modified transfer function. That is, if K = 1;

$$E(\overline{P},X)_{k=1} = \frac{F(0,i) - F(1,i)}{F(0,i)}$$

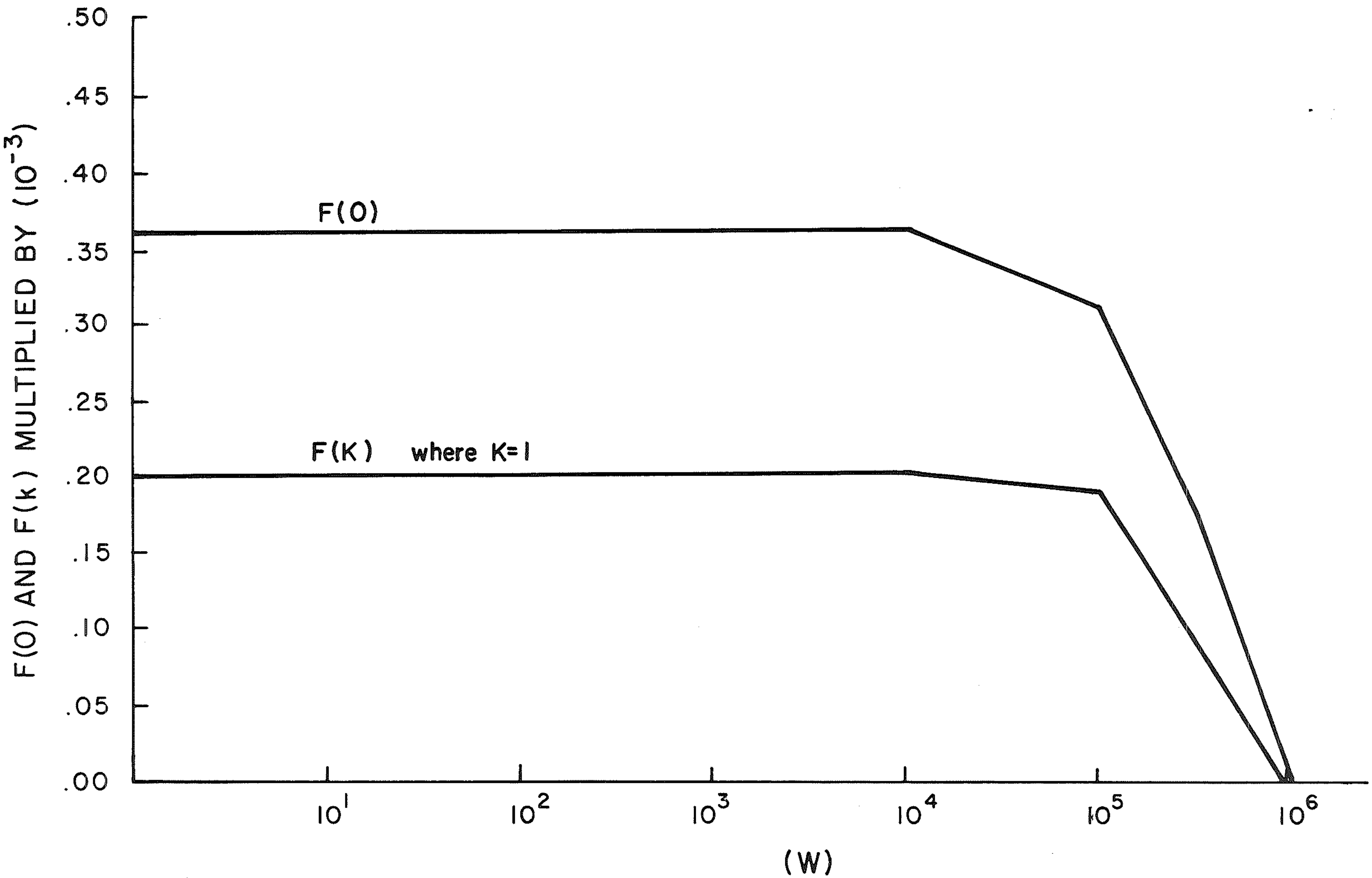

Figure 3.- Observed variance for F(0,i) and F(1,i)

As before, F(0) is the original transfer function, F(1) is R1 short circuit, F(2) is R1 open circuit, F(3) is L7 short circuit, and F(4) is L7 open circuit.

$E(\overline{P},X)_{k=2} = 1$ over the frequency range, since $F(2,i) = 0$ for all i and these data are left out of the table.

TABLE VII.- FRACTIONAL VARIANCE OBSERVED AND VARIANCE COEFFICIENT

$\omega(i)$	$E(\overline{P},X)$ k = 1	$E(\overline{P},X)$ k = 3	$E(\overline{P},X)$ k = 4
$10^{1.5}$	-.466968	-.568060	-.999983
10^{2}	-.466968	-.568060	-.999945
$10^{2.5}$	-.466976	-.568059	-.999826
10^{3}	-.466976	-.568066	-.999449
$10^{3.5}$	-.466907	-.568003	-.998257
10^{4}	-.466368	-.567496	-.994481
$10^{4.5}$	-.461015	-.562444	-.982335
10^{5}	-.410460	-.513541	-.937597
$10^{5.5}$	-.063257	-.094596	-.607426
10^{6}	-.645297	2.11693	1.754730
T(K)	T(1)	T(3)	T(4)
	$\Sigma \mathrm{Log}_{10} \lvert E(\overline{P},X)_{K=1} \rvert$	$\Sigma \mathrm{Log}_{10} \lvert E(\overline{P},X)_{K=3} \rvert$	$\Sigma \mathrm{Log}_{10} \lvert E(\overline{P},X)_{K=4} \rvert$
	17.90326	17.28895	17.98819

Definition of the Variance Coefficient

The variance coefficient, T, for the deviation between the original transfer function, F(0), and the modified transfer function F(k) is defined by B.

$$D(\overline{P},X)_k = \frac{F(0,i) - F(k,i)}{F(0,i)} \qquad (A)$$

Where X is either Q or Q', and k designates the modified transfer function being considered, and then:

$$T = \Sigma \mathrm{Log}_{10} |E(\overline{P},X)_k| \qquad (B)$$

where the summation is taken over all M in the frequency range.

In Table VII for example,

$$T(1) = \Sigma \mathrm{Log}_{10} |E(\overline{P},X)_{k=1}| = \Sigma \mathrm{Log}_{10} \left| \frac{F(0,i) - F(1,i)}{F(0,i)} \right|$$

and similarly for T(3) and T(4). Since:

$E(\overline{P},X)_{k=2} = 1$, clearly T(2) = 0.

Table VIII gives a complete listing of all variance coefficients, (T), and correlation coefficients, (R), for the example in Figure 2.

Fault Isolation Procedure

In review:

1) The network was constructed experimentally.

2) The desired transfer function, F(0), was solved for.

3) The modified transfer functions, F(k), were established for all possible single failures.

4) The correlation coefficients (R) between each modified transfer function and the desired original transfer function were formulated.

5) Similarly, the variance coefficients (T) were established.

TABLE VIII.- CORRELATION COEFFICIENTS - R
VARIANCE COEFFICIENTS - T

ELEMENT	SC	OC	T	R
R3	X		18.16555	.961076
R5	X		18.13207	.961537
R1	X		17.90326	.952979
R6	X		17.03180	.943269
R7	X		16.96023	.940948
L7		X	17.98819	.794069
L6		X	17.87923	.741361
R7		X	17.98819	.794069
R6		X	17.87923	.741361
C2		X	17.32274	.948035
C7		X	15.67600	.938431
C6		X	15.44000	.929121
L7	X		17.28895	.897051
L6	X		17.11118	.889213
C7	X		14.62646	.905976
C6	X		14.53590	.897955
R1		X	0	0
R3		X	0	0
C2	X		0	0
R5		X	0	0

Now to move away from the experimental situation, consider an actual network and assume that this network has undergone all the processes in items 1 through 5; that is, it has been simulated experimentally.

Therefore, the desired response is known and any deviations from this response can be considered as failures. If a deviation is observed, it is desirable to establish the cause of these variances or in other words to isolate the failure.

The isolation procedure is as follows:

1) F(0), the desired network response, is known.

2) $F(X)$ is the observed variant response.

3) $F(X)$ is compared to $F(0)$ and, in the same manner as previously defined, a correlation coefficient, R^*, and a variance coefficient, T^*, are evaluated.

4) Now consider the variance coefficients, $T(k)$, found experimentally from the modified transfer functions $F(k)$. There exists a T for all possible single failures.

5) Next, all variance coefficients, $T(k)$, which are a certain specified percentage less than, and a certain specified percentage greater than, T^* are designated. For example, the set $\{T(1), T(2), T(3), T(4), T(5), T(6)\}$ when

$$T_3 \leq T^* \leq T_4 .$$

6) The set of variance coefficients, $\{T(1), T(2), T(3), T(4), T(5), T(6)\}$ and also the variance coefficient T^* have a corresponding correlation coefficient R^*.

7) Solve for the absolute value of the differences observed in the correlation coefficients $\{R(1), R(2), R(3), R(4), R(5), R(6)\}$ of the modified transfer functions $F(k)$: $k = 1,2,...6$ and the correlation coefficient R^* evaluated for the variant response $F(X)$.

$$|R^* - R(1)| = Z_1$$
$$|R^* - R(2)| = Z_2$$
$$|R^* - R(3)| = Z_3$$
$$|R^* - R(4)| = Z_4$$
$$|R^* - R(5)| = Z_5$$
$$|R^* - R(6)| = Z_6$$

8) The order of the numbers $\{Z_1, Z_2, Z_3, Z_4, Z_5, Z_6\}$ serves as the indicator of the likelihood of failure. The smallest Z has the highest probability of being the failure and so on.

9) The entire process described above is applicable to the multi-failure situation and the algorithms for multi-failures will be considered later in this section.

Examples of the Isolation Process

To illustrate the detection techniques we compile the expected response from the component data for the network of Figure 2 and examine the circuit after failure occurs. Two cases will be considered (Tables X and XI).

TABLE IX.- FAILURE CASES

$\omega(i)$	BEFORE FAILURE F(0,i)	AFTER FAILURE CASE I F(X,i)	AFTER FAILURE CASE II F(X,i)
$10^{1.5}$	.362874 E-03	.632455 E-08	.156740 E-03
10^{2}	.362874 E-03	.200000 E-07	.156740 E-03
$10^{2.5}$	.362873 E-03	.632455 E-07	.156740 E-03
10^{3}	.362879 E-03	.200000 E-06	.156739 E-03
$10^{3.5}$	.362824 E-03	.632455 E-07	.156736 E-03
10^{4}	.362372 E-03	.199998 E-05	.156705 E-03
$10^{4.5}$	.357991 E-03	.632382 E-05	.156393 E-03
10^{5}	.320148 E-03	.199782 E-04	.153314 E-03
$10^{5.5}$	.160346 E-03	.629477 E-04	.125939 E-03
10^{6}	.124238 E-04	.342242 E-04	.247579 E-04

CASE I:

$$T^* = 17.98726, \quad R^* = .794069$$

TABLE X.- DATA FOR ISOLATION PROCESS – Case I

SET A T* Interval	SET B Corresponding Component	SET C Associated R-Score
16.96023	R7-SC	.940948
17.0313	R6-SC	.943269
17.11118	L6-SC	.889213
17.28895	L7-SC	.897051
17.32274	C2-OC	.948035
17.87923	R6-OC	.741361
17.87923	L6-OC	.741361
17.90236	R1-SC	.952979
17.98726	R7-OC	.794069
17.98819	L7-OC	.794069
18.13207	R5-SC	.961537
18.16555	R3-SC	.961076

TABLE XI.- DATA FOR ISOLATION PROCESS – CASE II

T = 17.32274   R = .948035

2% Interval Specification for T gives

LOWER LIMIT = 16.97629

UPPER LIMIT = 17.66919

SET A T* Interval	SET B Corresponding Component	SET C Associated R-Scores
17.0317	R6-SC	.943269
17.1118	L6-SC	.889213
17.28895	L7-SC	.897051
17.32274	C2-OC	.948035

To obtain a set of T scores all experimental variance coefficients within 6% (above or below) T* will be taken. It should be noted that T* will not always equal some T(k) score and R* will not always equal some R(k) score.

The 6% specification gives a variance coefficient interval around T* with a lower limit of 16.06650 and an upper limit of 19.0665. Now, from the experimental scores in Table VIII, the necessary data for fault isolation are illustrated in Table X. The following ordered interval around T* is obtained.

Next, taking R* and Set C, the computation of $|R^* - R| = Z$ is performed

$$|.794069 - .940948| = .146879 = Z_1$$
$$|.794069 - .943269| = .1492 = Z_2$$
$$|.794069 - .889213| = .09514 = Z_3$$
$$|.794069 - .897051| = .10298 = Z_4$$
$$|.794069 - .948035| = .153966 = Z_5$$
$$|.794069 - .741361| = .052708 = Z_6$$
$$|.794069 - .741361| = .052708 = Z_7$$
$$|.794069 - .952979| = .15891 = Z_8$$
$$|.794069 - .794069| = 0 = Z_9$$
$$|.794069 - .794069| = 0 = Z_{10}$$
$$|.794069 - .961537| = .167468 = Z_{11}$$
$$|.794069 - .961076| = .167007 = Z_{12}$$

The order of parameter failure going from high probability to low is: $\{Z_9 = Z_{10}, Z_6 = Z_7, Z_3, Z_4, Z_1, Z_2, Z_5, Z_8, Z_{12}, Z_{11}\}$ which corresponds to {L7-OC = R7-OC, L6-OC = R6-OC, L6-SC, L7-SC, R7-SC, R6-SC, C2-OC, R1-SC, R3-SC, R5-SC} = the probability order of failure. The 6% interval for T* was used for illustrative purposes. Usually a smaller specification would be employed.

$$|R^* - R| = Z$$

$$|.948035 - .940948| = .007087$$
$$|.940835 - .889213| = .058822$$
$$|.948035 - .897051| = .050984$$
$$|.948035 - .948035| = .0000$$

Order of Parameter Failure by Probability

$= \{Z_4, Z_1, Z_3, Z_2\}$

$= \{$C2-OC, R6-SC, L7-SC, L6-SC$\}$

Simultaneous Failure of Several Elements

For a single failure, the topology equation was expanded in two variables and H(P,Q) was solved for P in terms of element Q.

For two simultaneous failures the expansion of H(P,Q,R) in three variables is employed where Q and R are network elements and P is the performance parameter.

Solving H(P,Q,R) = 0 for 1/P defined to be F(0) yields

$$\frac{1}{P} = \frac{H(\overline{P},\overline{Q},\overline{R}) + QH(\overline{P},Q',\overline{R}) + RH(\overline{P},\overline{Q},R') + QRH(\overline{P},Q',R')}{H(P',\overline{Q},\overline{R}) + QH(\overline{P},Q',\overline{R}) + RH(P',Q,R') + QRH(P',Q',R')}$$

When B parameters can be short-circuited or open-circuited simultaneously, 2^B distinct modes of failure exist. Each mode depends on the combination of short-circuit and open-circuit.

For B = 2 four formulas are listed.

(1) $-\dfrac{H(\overline{P},\overline{Q},\overline{R})}{H(P',\overline{Q},\overline{R})}$ (3) $-\dfrac{H(\overline{P},Q',\overline{R})}{H(P',Q',\overline{R})}$

(2) $-\dfrac{H(\overline{P},\overline{Q},R')}{H(P',\overline{Q},R')}$ (4) $-\dfrac{H(\overline{P},Q',R')}{H(P',Q',R')}$

Since B = 2 elements, each Q parameter and each R parameter can function as either an impedance or as an admittance.

There will be (2^{2B}) combinations to be considered in Table XII. In general, the modified transfer functions can now be defined as $F(k): k = 1,2,3,\ldots,2\binom{N}{B}$ where N is the number of elements that can fail and B is the number that can fail simultaneously.

The computations of the correlation coefficient and the variance coefficient do not change in the multiple failure situation. These coefficients are complied and utilized in the same manner as the single failure case.

TABLE XII.- POSSIBLE DOUBLE FAILURE CASES

FAILURE TYPES	Q = Z R = Z	Q = Z R = Y	Q = Y R = Z	Q = Y R = Y
SC-Q; SC-R	1	2	3	4
SC-Q; OC-R	2	1	4	3
OC-Q; SC-R	3	4	1	2
OC-Q; OC-R	4	3	2	1

III.- FUTURE TASKS

The technique presented may be extended as follows:

a) Expand the computer program for multiple failures. Three simultaneous failures are a feasible goal.

b) Test the technique for a wider range of circuits to establish its limitations and potential.

c) Establish whether the coefficients can be utilized more effectively, particularly for multiple failures.

The increasing number of components in microcircuits has established the necessity of failure analysis. The results of this analysis have provided one method for the isolation of failure in the frequency-dependent circuit.

BIBLIOGRAPHY

Carpenter, R.M.; and Happ, W.W.: Computer-Aided Design: Part 3 – Analyzing Circuits with Symbols. Electronics, vol. 39, no. 25, December 1966, pp. 92-98.

Devaney, M.J., Zobrist, G.W.; and Happ, W.W.: Sensitivity Analysis. Tenth Midwest Circuit Symposium, Purdue University, Lafayette, Indiana, May 18-19, 1967.

Happ, W.W.: Flowgraph Techniques for Closed Systems. Trans. IEEE AES-2, no. 3, May, 1966, pp. 252-264.

Happ, W.W.: Present Capabilities of a Maintained Program. IEEE International Convention Record, part 5, 1967, pp. 64-88.

Happ, W.W.: Tagging: Systematics of a Computer-Oriented Symbol Manipulation Technique. Proc. Twelfth Midwest Circuit Conference, June 1969.

Happ, W.W.; and Moody, D.E.: Sensitivity Analysis. Computer-Aided Solid-State Circuit Design Institute, University of Wisconsin, May 1966.

Herskowitz, G.: Computer-Aided Integrated Circuit Design. McGraw-Hill, 1968.

McNamee, L.P.; and Potash, H.: A Users' Guide and Programmers' Manual for NASAP. University of California at Los Angeles, report no. 68-38, August 1968.

Pritsker, A.A.B.: GERT: Graphical Evaluation and Review Technique. J. of Ind. Eng., vol. XVII, no. 5, pp. 267-274.

Weinstein, M.C.; and Happ, W.W.: Symbol-Oriented Failure Analysis in Multicomponent Systems. Proc. First Southeast Symposium on Systems Theory, Blackburg, Virginia, May 1969.

NASA-Langley, 1970 —— 10 C-95